Foreword

How it all started…

I have always been fascinated by numbers. It is amazing to think that so few elements as the Arabic digits from 0 to 9 can be combined in different, potentially infinite, ways and enable us to express as many different and abstract things as the mass of a proton (1.6726×10^{-27} kg), the speed of light (299.792.458 m/s, generally approximated by 300.000 km/s), the value of π (3.1415918…), etc. Yet, how would we express such knowledge if numbers didn't exist?

Beside my curiosity for numbers, the neuropsychological approach has also captured my attention. In fact, trying to better understand (or even derive) normal brain functioning in healthy individuals from the way the brain dysfunctions after cerebral lesion appears a challenging approach. What has most attracted me along this path is the heuristic power of single case studies. A patient's performance can actually bring you to make hypotheses that cannot be derived from available cognitive models. Single case studies may thus occasionally challenge the evidence gathered so far and shed new light on pre-existing data; they may even drive, sometimes, to question existing models and make new proposals.

It is thus naturally that, combining my fascination for numbers with my interest in single case studies, I started working with brain-damaged acalculic patients during my PhD. My main interest in acalculia was to better understand how numbers are represented and processed at a semantic level. What is/are the meaning(s) conveyed by numbers? Are numbers processed in the same way as other ordered series held in long-term memory? With these questions in mind, I started the investigation of patient CO's number processing abilities. CO was a former accountant who suffered from acalculia after bilateral parietal damage. After testing his number reading and writing abilities, I asked him to perform one of the key tasks used to assess semantic (magnitude) number processing: number comparison. In this task, CO was presented with an Arabic number between 1 and 9 (except for 5) on the centre of a computer screen and asked to judge whether that number was *smaller* or *larger* than 5. He understood the task immediately and showed an almost flawless performance. I then ran the same experiment again and only slightly changed the instructions: I asked CO to judge, for each number on the screen, whether it came *before* or *after* 5 in the counting sequence. My surprise was total: he could not understand the task and repeated "Before what? After what?" This was not due to impaired understanding of the concepts "before" and "after" as he was able to give correct definitions of these terms. The impairment appeared to be profound as it extended to other ordered series, such as letters, days and months. For these latter series, CO was almost at chance in deciding whether an item came before or after another in the sequence (e.g. does September come before or after June during a calendar year?).

In trying to account for CO's pattern of performance, I raised a general hypothesis and asked whether processing the quantity meaning of numbers (e.g. judging 3 as smaller than 5) and processing the relative position of numbers in their conventional sequence (e.g. judging 3 as coming before 5 in the counting sequence) could be dissociated (and thus selectively spared or impaired after cerebral lesion). This hypothesis appeared quite implausible at first sight - how could one imagine that knowing that '3 is smaller than 5' might not help knowing that '3 comes before 5' in the conventional number sequence? - and it could not be derived from existing models of adult number processing. In fact, available models mostly consider the semantic representation of numbers as a representation of quantity (or magnitude), and give little consideration to sequence order as a theoretically important feature of numbers. The study of patient CO suggested, instead, that the relative position of a number within the counting sequence might also be a fundamental feature of the number system, and that it could rely on specific processing mechanisms. Because CO's pattern of dissociation (that is, impaired sequence order processing in the face of preserved quantity processing on numerical material) was so compelling, I thought the proposed hypothesis deserved further consideration. From that time on, my PhD was devoted to testing the initially speculative hypothesis that processing numbers according to the quantity they convey, or to their relative position in the conventional sequence might be potentially dissociated. In parallel to a thorough investigation of the potential dissociation in CO, I also planned to test whether markers/evidence for a similar dissociation could be observed in healthy adult individuals. For this purpose, I devised an event-related potentials (ERP) study and two behavioural experiments in which I contrasted instructions requiring subjects to process either numerical quantity or the order or relative position of numbers in the sequence.

As we will see in the present thesis, I collected some converging evidence in favour of the hypothesis of dissociated mechanisms underlying the processing of quantity and relative position of numbers in the sequence. However, this could not be accounted by any model of adult number processing and a general framework for the interpretation of these data was still lacking. A first hint towards an explanation of my data came from the field of developmental psychology. Research in this area showed, in fact, that children acquire knowledge about the number sequence before (i.e. at a different time) they are able to understand which quantity is associated with each number word (see Fuson, 1988). Hence, because a developmental distinction is observed between sequence knowledge and quantity knowledge, it could be that this distinction remains present up to adulthood. Consequently, although knowledge about the order of numbers in the sequence and the understanding of their associated quantity appear to be tightly linked in normal adults, developmental pathways suggest that they might nonetheless rely on distinct processing mechanisms. Yet, because Fuson (1988) provides the best and most detailed account of the developmental stages guiding to the elaboration of the number word sequence, CO's sequence knowledge will be tested according to Fuson's approach.

With respect to the adult literature, Wiese (2003) recently emphasized that "there is more to numbers than cardinality [(i.e., quantity) and that numbers] can be used to identify a whole range of properties of which cardinality is just one, and crucially not the only one. Apart from cardinality, numbers can be employed to identify, for instance, the ranks of runners (an ordinal number assignment) or the identity of bus lines (a nominal number assignment)." (Wiese, 2003, p.2). In her approach, in fact, Wiese (2003) will go beyond the well known quantity meaning of numbers and stress the importance of numbers as conveying information about the relative position of an item within an ordered sequence. In her perspective, numbers refer to numerical quantity when used in a cardinal assignment (they describe the cardinality of a set); but when used in an ordinal assignment, numbers will refer to numerical rank (and describe the relative position of an element within an ordered sequence). As we will see, sequential order will receive much emphasis in this framework and be even considered as the crucial feature of numbers, thus possibly overruling the importance previously attributed to numerical quantity. For this reason, and because quantity and the sequential order of numbers are given equal consideration, I will start my review of the literature with a description of the position of Wiese (2003). Nonetheless, because this framework was proposed after I collected most of the empirical data presented in this dissertation, it should not be expected to be the starting point of my experimental investigation. The reason why I present this framework is because it acknowledges the importance of numerical order and highlights the grounds upon which it differs from numerical quantity. Yet, once established that numerical quantity and numerical order refer to different number assignments (Wiese, 2003) and have different developmental courses (Fuson, 1988) the hypothesis that they might rely on distinct specific underlying mechanisms appears more reliable. Consequently, the framework proposed by Wiese (2003), together with the data collected by Fuson (1988), enabled me to transform an initially anecdotal observation (CO is unable to tell that 3 comes before 5 in the number sequence, although he knows that 3 is smaller than 5) into a theoretically relevant question (processing the quantity conveyed by a number or its relative position in the sequence might involve distinct and potentially dissociable mechanisms).

In the following section I will first describe the framework proposed by Wiese (2003), to emphasize that both quantity and rank (or relative position in the sequence) are important features of numbers. The work done by Fuson (1988, 1992) on the acquisition of the number sequence and of quantity knowledge by children will be presented later in that section, and more extensively in the paper about patient CO[1].

[1] Let me note that in accordance with scientific writing guidelines, I will refer to my work using the pronoun "we" instead of "I" in the remainder of this dissertation.