3124

REPRINT

Paul Belleflamme and Valeria Forlin

Endogenous vertical segmentation in a Cournot oligopoly

Journal of Economics, online, 2020

CORE CENTER FOR OPERATIONS RESEARCH AND ECONOMETRICS

CORE
Voie du Roman Pays 34, L1.03.01
B-1348 Louvain-la-Neuve
Tel (32 10) 47 43 04
Email:lidam-library@uclouvain.be
https://uclouvain.be/en/research-institutes/
lidam/core/reprints.html

Journal of Economics

Endogenous vertical segmentation in a Cournot oligopoly

Paul Belleflamme · Valeria Forlin

Received: 5 September 2019 / Accepted: 7 June 2020 / The final publication is available at https://link.springer.com/article/10.1007/s00712-020-00706-8

Abstract An arbitrary number of (ex ante symmetric) firms first choose whether to produce a high-quality or a low-quality product and then, the quantity of product to put on the market. We establish the following results: (i) there exists competition within and across quality segments; (ii) firms may be better off producing the low quality if competition within this segment is sufficiently low; (iii) a firm's switch across qualities may benefit all the other firms; (iv) there exists a unique partition of the firms between the two quality segments; (v) if high quality has a larger cost-quality ratio, then the equilibrium exhibits vertical differentiation; (vi) there may be too much differentiation from the consumers' point of view.

Keywords Quality · Differentiation · Oligopolistic competition

JEL classification D43 · L13 · L25

1 Introduction

Many oligopolistic markets share the following features: consumers can choose between two qualities of products or services (high or low), each quality is produced by a separate set of competing firms, and quality levels are mostly exogenous to the firms. These assumptions square well with situations where producing the high quality requires being granted some label or certification,

P. Belleflamme
Université catholique de Louvain, LIDAM/CORE, Voie du Roman Pays 34, 1348 Louvain-la-Neuve, Belgium. Other affiliation: CESifo, Munich, Germany. E-mail: Paul.Belleflamme@uclouvain.be

V. Forlin
Université catholique de Louvain, LIDAM/CORE, and European Commission, DG CLIMA, Avenue de Beaulieu 24, 1160 Brussels, Belgium. E-mail: valeria.forlin@ec.europa.eu.

which is incompatible with continuing to produce the low quality.[1] As for the low quality, it corresponds to some minimum quality standard, above which firms cannot credibly commit (unless they obtain the label or certification).[2]

In these markets, it is important to understand how firms decide which quality to produce. Clearly, firms will base their decision on the relative profitability of producing one or the other quality. Yet, it is not clear how to determine this relative profitability, as it depends on both exogenous and endogenous factors. The exogenous factors are the consumers' willingness to pay for quality upgrades and the respective costs of producing the two qualities; the endogenous factors are the decisions of all firms, as they will jointly determine the level of competition that will prevail on each quality segment. By analysing the interplay between these factors, this note provides new insights on the equilibrium size of the two sets of firms that will emerge when two levels of quality are available; this framework can be useful to explain the segmentation of industries when a new quality certification is introduced.

To address these questions, we analyse a two-stage model of product competition: firms commit first to produce either the high or the low quality; next, they set the quantity of product to put on the market. We establish the following results. In terms of quantity competition (second stage), we highlight the existence of an 'own-competition' and a 'cross-competition' effect: entry in one quality segment reduces equilibrium profits not only in that segment but also in the other segment. A second result is that firms may be better off producing the low quality if competition within this segment is sufficiently low. Regarding quality competition (first stage), we first show that when a firm switches from one quality to the other, this move has ambiguous impacts on the other firms: as expected, it could hurt the firms in the segment that is joined and benefit firms in the segment that is left, but it can also hurt – or benefit – all the other firms. Next, we prove the existence of a unique partition of the firms between the two quality segments. We further show that a sufficient condition for vertical differentiation (i.e., both qualities being chosen at equilibrium) is that the cost-quality ratio be larger for the high than for the low quality. Finally, we illustrate the possibility of disagreement between firms and consumers: from the consumers' viewpoint, the market equilibrium might feature too many or too few firms producing the high quality.

Our analysis contributes to the abundant literature on vertical differentiation. This literature, following the seminal papers of Gabszewicz and Thisse (1979), and Shaked and Sutton (1982), has mostly focused on duopolistic Bertrand competition. Obviously, this setting is ill-suited to represent the case that we are studying here, as each quality can potentially be produced by several firms. We therefore chose to depart from the usual model by considering

[1] The certification for organic food, for example, may not be granted if the non-organic variety is produced on the same farm, to avoid fraud.

[2] For instance, in the context of asymmetric information with credence attributes, a firm investing in a quality level higher than the minimum (but not high enough to deserve the certification) would have no means to communicate this quality increase to its consumers; see Besley and Ghatak (2007).

Cournot competition among an arbitrary number of firms.[3] To the best of our knowledge, this model has not been solved so far; our characterisation of the second-stage equilibrium is thus a novel result. Other papers consider Cournot competition in vertically differentiated markets but adopt different settings and focus on different issues. Gal Or (1983, 1985 and 1987) considers the interplay between quality and quantity oligopolistic competition in similar, yet different, frameworks than ours.[4] Bonanno (1986) examines duopolistic competition with successive quality and quantity decisions; under the assumption that the two qualities are produced at zero cost, he shows that vertical differentiation does not emerge at equilibrium; we generalise this result with more than two firms and potentially different costs for the two qualities. Motta (1993) compares Cournot with Bertrand competition in a duopoly framework, with a continuous choice of quality levels in the first stage. Turrini (2000) considers, like us, a Cournot model of vertical differentiation with two qualities and an arbitrary number of firms, but he adopts the strong assumption that the total output of the high-quality segment is fixed; in our model, we do not impose this restriction. Johnson and Myatt (2006) assume a Cournot industry with N multi-product firms; in a similar setting, with two qualities and a commitment to quality before the choice of quantities, they analyse how profits are affected by the choice of producing only one type of quality or both; however, they do not examine how the number of firms making the same quality choice affects each firm's decision. Miao and Long (2017) adopt a similar setting to the one in Johnson and Myatt (2006), but they look for a free-entry equilibrium, and study the implications of an increase in market size. Finally, our analysis bears some resemblance with the papers on strategic technology adoption in Cournot oligopolies that have followed up on the work of Mills and Smith (1996) and Elberfeld (2003).

2 Equilibrium quality and quantity decisions

2.1 The model

A unit mass of consumers are identified by their valuation for quality improvement, θ, which is assumed to be uniformly distributed on the unit interval. A consumer of type θ obtains utility $\theta s_k - p_k$ from one unit of product k that has quality s_k, and is sold at price p_k. Two qualities are available: a high (s_h) and a low (s_l) quality, with $s_h > s_l > 0$. A unit of quality s_k is produced at a constant unit cost c_k, $k = h, l$. The number of firms in the industry, N, is exogenous (i.e., there is no free entry). Finally, all firms are identical: they all have the same marginal cost.

[3] An alternative route would have been to stick to price competition and to introduce horizontal differentiation in each quality segment (in the spirit of Häckner, 2000).

[4] The first paper considers free entry, more than two qualities and the possibility for firms to produce several qualities; in the second paper, quality and quantity decisions are simultaneous, and quality levels are endogenous; the third paper is closer to ours in that quality choices precede quantity choices but the analysis is restricted to a duopoly.

We analyse the following three-stage game. At the first stage, firms simultaneously decide whether to produce the high or the low quality (remember that we assume that firms can only choose to produce a single quality at the first stage of the game, and that both quality levels are exogenously fixed). At the second-stage, firms compete à la Cournot; that is, they choose which quantity to produce given the market-clearing prices. At the third-stage, consumers observe the prices and decide to buy a unit of high quality, a unit of low quality, or nothing. We solve the game for its subgame-perfect equilibrium.

To solve the third-stage of the game, identify as θ_{hl} the consumer who is indifferent between buying either the high or the low quality, and by $\theta_{l\emptyset}$, the consumer who is indifferent between buying the low quality or nothing. We easily find that $\theta_{hl} = (p_h - p_l)/(s_h - s_l)$ and $\theta_{l\emptyset} = p_l/s_l$. As long as $p_h/s_h > p_l/s_l$, consumers with $\theta_{hl} \leq \theta \leq 1$ buy a unit of the high quality, consumers with $\theta_{l\emptyset} \leq \theta \leq \theta_{hl}$ buy a unit of the low quality, and consumers with $0 \leq \theta \leq \theta_{l\emptyset}$ buy nothing. The demand functions for the two goods are therefore $Q_h = 1 - \theta_{hl}$ and $Q_l = \theta_{hl} - \theta_{l\emptyset}$. Inverting this system, we get the following system inverse demand functions:

$$\begin{cases} p_h = s_h(1 - Q_h - \frac{s_l}{s_h}Q_l), \\ p_l = s_l\,(1 - Q_h - Q_l)\,. \end{cases}$$

Moving backwards, we now characterise the equilibrium of the second stage (quantity choices) and first stage (quality choices) of the game.

2.2 Quantity equilibrium

Suppose that at the first-stage, n_h (resp. n_l) firms have decided to produce the high (resp. low) quality, with $n_h + n_l = N$. A typical firm i producing the high quality chooses its quantity $q_{h,i}$ to maximize $s_h(1-(q_{h,i}+q_{h,-i}) - \frac{s_l}{s_h}Q_l)q_{h,i} - c_h q_{h,i}$, where $q_{h,-i}$ denotes the total quantity of the high-quality product produced by the other $(n_h - 1)$ firms. Similarly, a typical firm j producing the low quality chooses its quantity $q_{l,j}$ to maximize $s_l\,(1 - Q_h - (q_{l,j} + q_{l,-j}))\,q_{l,j} - c_l q_{l,j}$. The first-order conditions give, respectively, $2s_h q_{h,i} = s_h - c_h - s_h q_{h,-i} - s_l Q_l$ and $2s_l q_{l,j} = s_l - c_l - s_l Q_h - s_l q_{l,-j}$. As firms are ex ante symmetric, we have that at equilibrium, each of the n_h (resp. n_l) firms produce the same quantity q_h (resp. q_l) of the high- (resp. low-) quality product.[5] The previous equations can thus be rewritten as $2q_h s_h = s_h - c_h - s_h\,(n_h - 1)\,q_h - s_l n_l q_l$ and $2q_l s_l = s_l - c_l - s_l n_h q_h - s_l\,(n_l - 1)\,q_l$. To ease the exposition, we define $v_k \equiv s_k - c_k$ $(k = h, l)$, which can be interpreted as the maximum potential

[5] To see this, suppose that firms 1 and 2 produce the high quality. Their first-order conditions are, respectively, $2s_h q_{h,1} = s_h - c_h - s_h q_{h,2} - s_h q_{h,-1,2} - s_l Q_l$ and $2s_h q_{h,2} = s_h - c_h - s_h q_{h,1} - s_h q_{h,-1,2} - s_l Q_l$, where $q_{h,-1,2}$ is the total quantity produced by the high-quality producers but firms 1 and 2. Suppose now, by contradiction, that the equilibrium quantities are such that $q_2 = q_1 + x$, with $x \neq 0$. The previous two equations become: $3s_h q_{h,1} = s_h - c_h - s_h x - s_h q_{h,-1,2} - s_l Q_l$ and $3s_h q_{h,1} = s_h - c_h - 2s_h x - s_h q_{h,-1,2} - s_l Q_l$, which are clearly incompatible unless $x = 0$.

social value of a unit of quality k: it is the difference between the highest willingness to pay (θs_k evaluated at $\theta = 1$) and the unit cost of production. To make the analysis non-trivial, we assume $v_k > 0$, $k = h, l$. We can then express the equilibrium Cournot quantities as:

$$q_h(n_h, n_l) = \frac{(n_l + 1) v_h - n_l v_l}{s_h (n_h + 1)(n_l + 1) - s_l n_h n_l},$$
$$q_l(n_h, n_l) = \frac{\frac{s_h}{s_l}(n_h + 1) v_l - n_h v_h}{s_h (n_h + 1)(n_l + 1) - s_l n_h n_l}.$$

High-quality output is positive as long as $\frac{v_h}{v_l} > \frac{n_l}{n_l+1}$, and this condition is the most stringent when $n_l = N - 1$;[6] similarly, low-quality output is positive as long as $\frac{v_h}{v_l} < \frac{s_h}{s_l}\frac{n_h+1}{n_h}$, which is the most stringent when $n_h = N - 1$. Thus, to guarantee positive equilibrium quantities in the subgames in which both types of firm are active, we impose:[7]

$$\text{(A)}\ \frac{N-1}{N} < \frac{v_h}{v_l} < \frac{N}{N-1}\frac{s_h}{s_l}.$$

Using the first-order conditions, we find that at equilibrium, $s_k q_k = p_k - c_k$. It follows that equilibrium profits are computed as $\pi_k = s_k q_k^2$ $(k \in \{h, l\})$. That is,

$$\pi_h(n_h, n_l) = s_h \left(\frac{(n_l + 1) v_h - n_l v_l}{s_h (n_h + 1)(n_l + 1) - s_l n_h n_l}\right)^2, \quad (1)$$
$$\pi_l(n_h, n_l) = s_l \left(\frac{\frac{s_h}{s_l}(n_h + 1) v_l - n_h v_h}{s_h (n_h + 1)(n_l + 1) - s_l n_h n_l}\right)^2. \quad (2)$$

In the corner cases where all firms produce the same quality, we have

$$\pi_h(N, 0) = \frac{v_h^2}{s_h (N+1)^2}, \text{ and } \pi_l(0, N) = \frac{v_l^2}{s_l (N+1)^2}.$$

Before turning to the first stage, we stress two important results. First, the arrival of an additional firm, whichever the quality it produces, reduces the equilibrium profits of any firm (see Appendix **??** for the proof):

$$\pi_h(n_h, n_l) > \pi_h(n_h + 1, n_l) \text{ and } \pi_l(n_h, n_l) > \pi_l(n_h, n_l + 1),$$
$$\pi_h(n_h, n_l) > \pi_h(n_h, n_l + 1) \text{ and } \pi_l(n_h, n_l) > \pi_l(n_h + 1, n_l).$$

That is, there is an 'own-competition' effect (the top line) and also a '*cross*-competition' effect (the bottom line). Second, the relative profitability of producing one or the other quality depends not only on the exogenous parameters (s_k and c_k, $k \in \{h, l\}$) but also on the endogenous split of the firms between

[6] It cannot be $n_l = N$ because at least one firm must be producing the high quality.

[7] The assumption $v_k > 0$ guarantees that equilibrium quantities are positive when all firms produce quality k.

the two qualities. Defining $\sigma \equiv \sqrt{s_h/s_l} > 1$ and comparing equilibrium profits in the two segments, we find

$$\pi_h(n_h, n_l) > \pi_l(n_h, n_l) \iff \frac{v_h}{v_l} > \sigma + \sigma\frac{(\sigma-1)(n_h - n_l)}{\sigma + n_h + \sigma n_l}.$$

We see that if the same number of firms compete in the two segments ($n_h = n_l$), $v_h > v_l$ (or $s_h - c_h > s_l - c_l$) is not a sufficient condition for the high quality to be more profitable than the low quality: it is necessary that $v_h/v_l > \sigma$. We also see that if there is more competition in the high- than in the low-quality segment ($n_h > n_l$), then the ratio v_h/v_l must be even larger for profits to be higher in the high-quality segment. Hence, the overall profitability of the two market segments jointly depends on the comparison between the (exogenous) maximum potential social values (v_h vs. v_l) and the (endogenous) intensities of competition (n_h vs. n_l).

2.3 Quality equilibrium

In the first stage, firms choose which quality to produce, anticipating the equilibrium of the ensuing Cournot competition. This game can be seen as a coalition game with simultaneous decisions and open membership. By choosing to produce either the high or the low quality, firms determine a 'coalition structure' (i.e., a partition of the set of firms into disjoint coalitions), with each firm's profit being a function of the whole coalition structure.[8] Before we characterize the equilibrium partition structure, we want to analyze the externalities among firms.

Externalities. Our analysis of the second-stage equilibrium taught us that a firm's equilibrium profit decreases when an additional firm enters its own coalition or the other coalition, other things being equal. At the first stage of the game, however, other things are not equal, insofar as one additional firm in one coalition necessarily means one less firm in the other coalition (recall that $N = n_h + n_l$ is fixed). Therefore, the impact of entry into a coalition depends on the balance between two conflicting forces: by switching, a firm increases competition in the coalition that it joins (a negative 'own-competition' effect) but also reduces competition in the coalition that it leaves (a positive 'cross-competition' effect). The net effect is a priori ambiguous, as it depends on which of the two externalities is stronger. The same two externalities also condition the evolution of the profits of the firms in the coalition left by the switching firm.

To see this more formally, suppose that one firm switches from the low-quality to the high-quality coalition. The change in profits in the two coalitions

[8] For a survey of approaches to coalition formation, see Bloch (2002).

can be decomposed as follows (with $k \in \{h, l\}$):

$$\pi_k(n_h+1, n_l-1) - \pi_k(n_h, n_l) = \underbrace{\pi_k(n_h+1, n_l-1) - \pi_k(n_h, n_l-1)}_{-} + \underbrace{\pi_k(n_h, n_l-1) - \pi_k(n_h, n_l)}_{+}.$$

We show in Appendix **??** that

$$\pi_h(n_h+1, n_l-1) > \pi_h(n_h, n_l) \Leftrightarrow \frac{v_h}{v_l} < v_1(n_h),$$
$$\pi_l(n_h+1, n_l-1) > \pi_l(n_h, n_l) \Leftrightarrow \frac{v_h}{v_l} < v_2(n_h),$$
$$\text{with } \frac{N-1}{N} < v_1(n_h) < v_2(n_h) < \frac{N}{N-1}\frac{s_h}{s_l}.$$

For a given value of n_h (and $n_l = N - n_h$), there are thus three configurations of externalities. If $v_1(n_h) < v_h/v_l < v_2(n_h)$, we have that when one firm switches from the low quality to the high quality, high-quality firms are worse off and low-quality firms are better off. This result is in line with what we would expect, as competition increases in the high-quality coalition and decreases in the low-quality coalition. However, the switch could also be detrimental for all firms (i.e., externalities are negative for both coalitions); this is so if $v_h/v_l > v_2(n_h)$. Finally, the switch could benefit all firms (i.e., externalities are *positive* for both coalitions); this is so if $v_h/v_l < v_1(n_h)$. In the latter case, because the low quality has a relatively high maximum potential social value (compared to the high quality), high-quality firms perceive low-quality firms as "tough" competitors, so much so that one fewer competitor from the low-quality coalition is beneficial to their profits, even if this means a stronger level of competition in their own coalition; that is, the positive 'cross-competition' effect outweighs the negative 'own-competition' effect; by the same token, remaining firms in the low-quality competition also benefit from the switch (for them, the positive 'own-competition' effect outweighs the negative 'cross-competition' effect). We show furthermore that both v_1 and v_2 increase in n_h (given $n_l = N - n_h$); this means that, high-quality (resp. low-quality) firms are more likely to welcome entry in their coalition when n_h is large (resp. small). Again, this is explained by the fact that, in these cases, competitors from the other coalition are perceived as tougher competitors, because they face weaker competition within their coalition.

Equilibrium. At equilibrium, an integer number of firms produces the high quality ($n_h \in \mathbb{Z}$) and an integer number of firms produces the low quality ($n_l \in \mathbb{Z}$), with $n_h + n_l = N$. The interior equilibrium with n_h firms producing the high quality and n_l firms producing the low quality (with $n_l = N - n_h$) is such that with $1 \leq n_h \leq N-1$. For this to hold, the following two conditions must be satisfied: $\pi_h(n_h, n_l) \geq \pi_l(n_h-1, n_l+1)$ and $\pi_l(n_h, n_l) \geq \pi_h(n_h+1, n_l-1)$; the former (resp. latter) condition states that no firm producing the high quality (resp. the low quality) wants to switch to the other quality. To facilitate the exposition, we rewrite the two conditions as

$$U(n_h) \leq 0 \leq U(n_h - 1),$$

where $U(n_h) \equiv \pi_h(n_h + 1, n_l - 1) - \pi_l(n_h, n_l)$ measures a firm's '*upgrade incentive*' (i.e., the incentive to move from the low to the high quality) when n_h firms produce the high quality. Using this notation, we express the condition for a corner equilibrium where all firms produce the high quality as $U(N-1) \geq 0$, and the condition for a corner equilibrium where all firms produce the *low* quality as $U(0) \leq 0$.

We are now in a position to state our main results.

Proposition 1 *(1) The equilibrium of the quality game exists and is unique. (2) As the ratio of the maximum potential social values, v_h/v_l, increases, more firms produce the high quality at equilibrium. (3) The equilibrium involves vertical differentiation (i.e., both qualities are produced) unless the maximum potential social values of the two qualities are either very close or very distant from one another.*

We just sketch the proof here (the details can be found in Appendix **??**). Existence and unicity follow from the fact that the conditions $U(n_h) \leq 0 \leq U(n_h - 1)$ can be rewritten as $V(n_h - 1) \leq v_h/v_l \leq V(n_h)$, and that $V(\cdot)$ is an increasing function. So, as n_h increases and assumes all the integer values from 0 to $N-1$, the function $V(\cdot)$ partitions the real line in N intervals, each of which corresponds to an integer number of high-quality firms, meaning that the ratio v_h/v_l necessarily falls in one and only one of these intervals (unicity of the equilibrium).[9] The second statement follows directly: larger values of the ratio v_h/v_l go above higher values of $V(\cdot)$, which correspond to equilibria with more firms choosing to produce the high quality (which is quite intuitive as its relative maximum potential social value increases). Finally, vertical differentiation prevails if the equilibrium is interior, which requires that $U(N-1) < 0 < U(0)$, or equivalently $V(0) < v_h/v_l < V(N-1)$. Simple computations establish that

$$U(N-1) < 0 < U(0) \iff \Phi \frac{s_h}{s_l} > \frac{v_h}{v_l} > \Phi^{-1}$$

$$\text{with } \Phi \equiv \frac{N(N+1)\sigma}{2N\sigma^2 + (N-1)[(N+1)\sigma - 1]} \text{ and } \sigma \equiv \sqrt{s_h/s_l}.$$

It is easily shown that $\Phi < N/(N-1)$, which implies that

$$\frac{N-1}{N} < \Phi^{-1} < \Phi \frac{s_h}{s_l} < \frac{N}{N-1} \frac{s_h}{s_l}.$$

[9] An exception to the unicity property is when n_h is an integer at $U(n_h) = 0$: in this case, a firm would be indifferent between switching and there would be two equilibria. However, this is a special case that does not invalidate the results. To rule it out, we could set a cut-off rule by which either $U(n_h) \leq 0$ or $U(n_h - 1) \geq 0$ needs to be a strict inequality.

As a consequence, corner equilibria remain possible in the space of parameters defined by Assumption (A): all firms adopt the low quality for $(N-1)/N < v_h/v_l < \Phi^{-1}$ and all firms adopt the high quality for $\Phi < v_h s_l / v_l s_h < N/(N-1)$.

We saw in the previous section that if $v_h/v_l > \sigma$, then high quality is 'more profitable' than low quality (in the sense that if competition has the same intensity in the two segments, firms make larger profits when they produce the high quality). Unsurprisingly, if this is so, some firms will choose to produce the high quality at equilibrium. It is indeed easy to show that $\sigma > \Phi^{-1}$, meaning that $v_h/v_l > \sigma$ implies that $U(0) > 0$, or $\pi_h(1, N-1) > \pi_l(0, N)$.

We can also derive a necessary condition on costs and qualities for vertical differentiation to take place at equilibrium. For at least one firm to adopt the low quality at equilibrium, we need $\pi_l(N-1, 1) > \pi_h(N, 0)$, or $U(N-1) < 0$, or $\frac{v_h}{v_l} < \Phi \frac{s_h}{s_l}$. Using the definition of Φ and recalling that $\sigma > 1$, it is readily shown that $\Phi < 1$. So, to have $U(N-1) < 0$, it must be that

$$\frac{v_h}{v_l} < \frac{s_h}{s_l} \Leftrightarrow \frac{c_h}{s_h} > \frac{c_l}{s_l}. \tag{3}$$

We record this result in the following corollary:

Corollary 1 *For both qualities to be produced at equilibrium (i.e., for vertical differentiation to prevail), a necessary condition is that the cost-quality ratio be larger for the high than for the low quality:* $c_h/s_h > c_l/s_l$.

To illustrate the scope of the latter condition, let us examine the two canonical cases that are considered in the literature on vertical differentiation (see, e.g., Motta, 1993). In the first case, it is assumed that there are only fixed costs of quality improvement (while variable costs do not change with quality). That would mean here that $c_h = c_l = c$. Given that $s_h > s_l$, condition (**??**) would then be violated, meaning that vertical differentiation would not endogenously emerge (in fact, all firms would adopt the high quality at equilibrium). The second case takes the opposite assumption: no fixed cost but a variable cost that increases with quality. In particular, it is usually posited that the marginal cost of production of one unit of quality k is proportional to the square of the quality level: $c_k = \alpha s_k^2$. Then, condition (**??**) is equivalent to $s_h > s_l$, which is satisfied by definition; in this case, vertical differentiation may emerge at equilibrium.

3 Consumers' viewpoint

The consumer surplus for a given partition of the N firms between the two segments is computed as:

$$\begin{aligned} CS(n_h, n_l) &= \int_{\theta_{hl}}^{1} [\theta s_h - p_h(n_h, n_l)]\, d\theta + \int_{\theta_{l\emptyset}}^{\theta_{hl}} [\theta s_l - p_l(n_h, n_l)]\, d\theta \\ &= \tfrac{1}{2} s_h [Q_h(n_h, n_l)]^2 + \tfrac{1}{2} s_l [Q_l(n_h, n_l)]^2 + s_l Q_h(n_h, n_l)\, Q_l(n_h, n_l), \end{aligned}$$

where the second line uses the definitions of $p_h(n_h, n_l)$ and $p_l(n_h, n_l)$, as well as the facts that $\theta_{hl}(n_h, n_l) = 1 - Q_h(n_h, n_l)$ and $\theta_{l\emptyset} = 1 - Q_h(n_h, n_l) - Q_l(n_h, n_l)$. This expression shows that the consumer surplus increases with a weighted sum of the total quantity produced of the two qualities. Taking each quality separately, the more firms adopt this quality, the larger the total quantity produced.[10] We therefore expect consumer surplus to be inversely related with the degree of vertical differentiation (i.e., with the dispersion of firms across the two segments). Yet, consumers value more the high quality than the low quality. Hence, we also expect that as v increases, consumers prefer that a larger number of firms produce the high quality, even if this makes the partition of firms more dispersed.

To shed light on this trade-off, we replace $Q_h(n_h, n_l)$ and $Q_l(n_h, n_l)$ by their respective value in the above expression and we find

$$CS(n_h, n_l) = \frac{n_h^2\left(\sigma^2(n_l+1)^2 - n_l(n_l+2)\right)v^2 - 2n_h n_l\left(\sigma^2(n_h n_l - 1) - n_h n_l\right)v + n_l^2\left(\sigma^2(n_h+1)^2 - n_h(n_h+2)\right)\sigma^2}{\left((n_l+1)(n_h+1)\sigma^2 - n_h n_l\right)^2}\frac{v_l^2}{2s_l}, \tag{4}$$

where v stands for v_h/v_l and σ for $\sqrt{s_h/s_l}$. Next, replacing n_l by $N - n_h$, we can express the consumer surplus as a function of n_h only, $CS(n_h)$. The complexity of this function prevents us from fully analysing its first and second derivatives with respect to n_h. We can, however, derive a number of interesting insights by evaluating the derivatives at specific values of n_h:

$$\begin{aligned}
\left.\frac{\partial CS(n_h)}{\partial n_h}\right|_{n_h=0} &= \frac{v_h^2 N}{v^2 s_h (N+1)^3}\left(v(N+1) - \left(N+\sigma^2\right)\right) < 0 \Leftrightarrow v < \frac{\sigma^2+N}{N+1}, \\
\left.\frac{\partial CS(n_h)}{\partial n_h}\right|_{n_h=N} &= \frac{v_h^2 N}{v\sigma^2 s_h (N+1)^3}\left(\left(N+\sigma^2\right)v - \sigma^2(N+1)\right) > 0 \Leftrightarrow v > \frac{\sigma^2(N+1)}{N+\sigma^2}, \\
\left.\frac{\partial CS(n_h)}{\partial n_h}\right|_{n_h=\frac{N}{2}} &= \frac{4\sigma^2 v_h^2 N}{v^2 s_h}\frac{\left(2\sigma^2 + N\sigma^2 - N\right)(v+\sigma)}{(2\sigma + N\sigma - N)^2(2\sigma + N\sigma + N)^2}(v-\sigma) = 0 \Leftrightarrow v = \sigma, \\
\left.\frac{\partial^2 CS(n_h)}{\partial n_h^2}\right|_{n_h=\frac{N}{2}, v=\sigma} &= -32\frac{v_h^2\sigma^2(\sigma-1)\left(\left(N^2+N-2\right)\sigma^2 + N\sigma - N^2\right)}{s_h(2\sigma + N\sigma + N)^3(2\sigma + N\sigma - N)^2} < 0.
\end{aligned}$$

We recall from Assumption (A), which guarantees positive equilibrium quantities in all subgames, that $\frac{N-1}{N} < v < \frac{N\sigma^2}{N-1}$. We also observe the following ranking (assuming $\sigma < N$):

$$\frac{N-1}{N} < \frac{\sigma^2+N}{N+1} < \sigma < \frac{\sigma^2(N+1)}{\sigma^2+N} < \frac{N\sigma^2}{N-1}.$$

The previous results allow us to derive the following conjecture.

Conjecture 1 (1) If v is close to its smallest admissible value, then the consumer surplus is maximised when all firms produce the low quality. (2) If v is equal to σ, then the consumer surplus is maximised when firms split equally

[10] This is a standard result under Cournot competition, which also applies in our setting.

between the two qualities. (3) If v is close to its largest admissible value, then the consumer surplus is maximised when all firms produce the high quality.

Conjecture **??** further suggests that for a given value of σ, the number of high-quality producers that maximises the consumer surplus increases gradually from 0 to N as v increases from its smallest to its largest admissible value. All the numerical simulations that we performed confirm this conjecture. Figure 1 depicts one such simulation, with $N = 4$ and $\sigma = 2$ (meaning that v can go from 3/4 to 16/3).[11] Above the arrow in Figure 1, we indicate the partitions of firms that maximise the consumer surplus; we observe that these partitions involve an increasing number of high-quality producers as v increases. To contrast the consumers' and the firms' preferences, we report below the arrow in Figure 1 the partitions that emerge at the subgame-perfect equilibrium of the game. As stated in Proposition 1, the equilibrium partitions also involve more high-quality producers as v increases. The preferences of the firms are thus globally aligned with those of the consumers, but not exactly, as the thresholds do not coincide. We observe indeed that the high quality may be under- or over-produced at equilibrium with respect to what the consumers would prefer. Intuitively, this divergence is explained by the fact that firms prefer to disperse across the two qualities (as a way to reduce competition among them), whereas consumers prefer the opposite (as more concentration on a given quality induces more production).

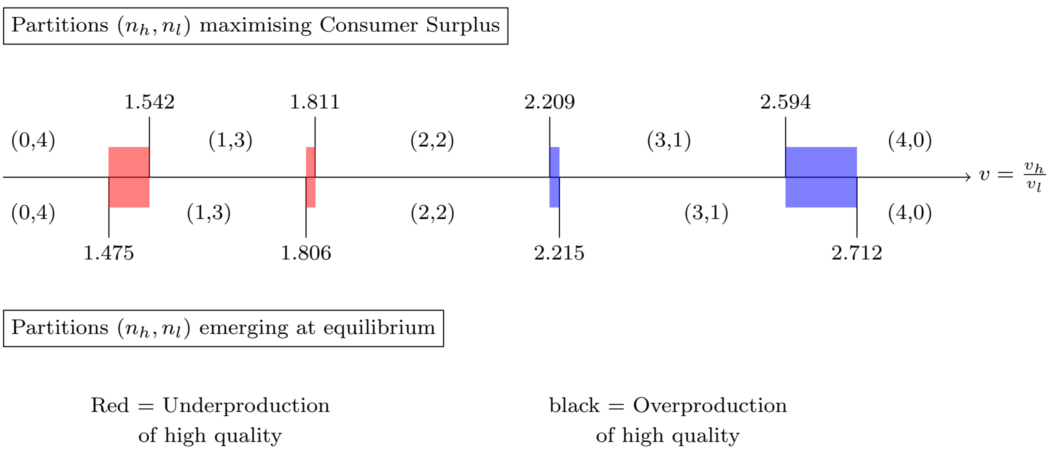

Fig. 1 Values of v at which too many (blue) or too few (red) firms produce high quality ($N = 4, \sigma = 2$).

[11] Given that both σ and v are functions of s_h and s_l, this simulation is compatible with the following scenario: $s_h = 8$, $s_l = 2$, $c_l = 1$ and c_h ranges from 2.67 to 7.25.

4 Conclusion

In this note, we examined the following two-stage game: an arbitrary number of (ex ante symmetric) firms first choose whether to produce a high-quality or a low-quality product and then, the quantity of product to put on the market.

Our main result is that, as long as high quality has a larger cost-quality ratio than low quality, the firms split into two quality segments at the subgame-perfect equilibrium of the game, leading to the endogenous emergence of vertical differentiation. This result can be useful to interpret the way in which an industry splits between certified and non-certified firms when a new labelling scheme becomes available. The model suggests that a label which is mainly associated with fixed costs of quality improvement (e.g., a label participation fee) is likely to be adopted by all firms,[12] while a label which is mainly associated with variable costs that increase with quality is more likely to give rise to a situation where some firms decide to obtain the certification and some do not.

5 Appendix

5.1 Own- and cross-competition effect

We first establish that equilibrium profits decrease in the number of firms in any segment. We have:

$$\pi_h(n_h, n_l) > \pi_h(n_h+1, n_l) \Leftrightarrow \frac{(n_l+1)v_h - n_l v_l}{s_h(n_h+1)(n_l+1) - s_l n_h n_l} > \frac{(n_l+1)v_h - n_l v_l}{s_h(n_h+2)(n_l+1) - s_l(n_h+1)n_l}$$
$$\Leftrightarrow \frac{v_h}{v_l} > \frac{n_l}{n_l+1}, \text{ which follows from Assumption (A);}$$

$$\pi_h(n_h, n_l) > \pi_h(n_h, n_l+1) \Leftrightarrow \frac{(n_l+1)v_h - n_l v_l}{s_h(n_h+1)(n_l+1) - s_l n_h n_l} > \frac{(n_l+2)v_h - (n_l+1)v_l}{s_h(n_h+1)(n_l+2) - s_l n_h(n_l+1)}$$
$$\Leftrightarrow \frac{v_h}{v_l} < \frac{n_h+1}{n_h}\frac{s_h}{s_l}, \text{ which follows from Assumption (A);}$$

$$\pi_l(n_h, n_l) > \pi_l(n_h+1, n_l) \Leftrightarrow \frac{s_h(n_h+1)v_l - s_l n_h v_h}{s_h s_l(n_h+1)(n_l+1) - s_l^2 n_h n_l} > \frac{s_h(n_h+2)v_l - s_l(n_h+1)v_h}{s_h s_l(n_h+2)(n_l+1) - s_l^2(n_h+1)n_l}$$
$$\Leftrightarrow \frac{v_h}{v_l} > \frac{n_l}{n_l+1}, \text{ which follows from Assumption (A);}$$

$$\pi_l(n_h, n_l) > \pi_l(n_h, n_l+1) \Leftrightarrow \frac{s_h(n_h+1)v_l - s_l n_h v_h}{s_h s_l(n_h+1)(n_l+1) - s_l^2 n_h n_l} > \frac{s_h(n_h+1)v_l - s_l n_h v_h}{s_h s_l(n_h+1)(n_l+2) - s_l^2 n_h(n_l+1)}$$
$$\Leftrightarrow \frac{v_h}{v_l} < \frac{n_h+1}{n_h}\frac{s_h}{s_l}, \text{ which follows from Assumption (A).}$$

[12] Assuming firms are symmetric.

Next, we establish the results regarding the externalities among firms in the first stage of the game. As for the equilibrium profits of firms producing the high quality, we have:

$$\pi_h(n_h+1,n_l-1) > \pi_h(n_h,n_l) \Leftrightarrow \frac{n_l v_h-(n_l-1)v_l}{s_h(n_h+2)n_l-s_l(n_h+1)(n_l-1)} > \frac{(n_l+1)v_h-n_l v_l}{s_h(n_h+1)(n_l+1)-s_l n_h n_l}$$

$$\Leftrightarrow \frac{v_h}{v_l} < \frac{s_h(n_h+1)+n_l s_l+n_l^2(s_h-s_l)}{s_l(n_h+1)+n_l s_h+n_l^2(s_h-s_l)} \equiv v_1(n_h).$$

In the low-quality segment, we have:

$$\pi_l(n_h+1,n_l-1) > \pi_l(n_h,n_l) \Leftrightarrow \frac{\frac{s_h}{s_l}(n_h+2)v_l-(n_h+1)v_h}{s_h(n_h+2)n_l-s_l(n_h+1)(n_l-1)} > \frac{\frac{s_h}{s_l}(n_h+1)v_l-n_h v_h}{s_h(n_h+1)(n_l+1)-s_l n_h n_l}$$

$$\Leftrightarrow \frac{v_h}{v_l} < \frac{s_h}{s_l}\frac{s_h(n_h+1)+s_l n_l+(n_h+1)^2(s_h-s_l)}{s_l(n_h+1)+s_h n_l+(n_h+1)^2(s_h-s_l)} \equiv v_2(n_h).$$

We compute

$$v_2(n_h)-v_1(n_h) = \frac{(s_h-s_l)((s_l+n_l(s_h-s_l))n_h+(s_l+n_l(2s_h-s_l)))((s_h+n_l(s_h-s_l))n_h+s_h(n_l+1))}{s_l(s_l(n_h+1)+s_h n_l+(n_h+1)^2(s_h-s_l))\left(s_l(n_h+1)+n_l s_h+n_l^2(s_h-s_l)\right)} > 0.$$

We now show that both $v_1(n_h)$ and $v_2(n_h)$ increase in n_h. Replacing n_l by $N-n_h$, we have:

$$\frac{\partial v_1}{\partial n_h} = (s_h-s_l)\frac{(s_h-s_l)(N+2n_h(N-n_h-1))+s_h(2N+1)+s_l}{[(s_h-s_l)((N-n_h)^2-n_h)+s_h N+s_l]^2},$$

$$\frac{\partial v_2}{\partial n_h} = \frac{s_h(s_h-s_l)}{s_l}\frac{2n_h(s_h-s_l)(N-n_h-1)+(2s_h(N+1)+(N-1)(s_h-s_l))}{[(s_h-s_l)n_h(n_h+1)+s_h(N+1)]^2},$$

both of which are clearly positive.

As a result, $v_1(n_h)$ reaches its smallest value at $n_h=0$ and $n_l=N$, while $v_2(n_h)$ reaches its largest value at $n_h=N-1$ and $n_l=1$; we compute:

$$v_1(0)-\frac{N-1}{N} = \frac{s_l+(2s_h-s_l)N}{N(N+1)(s_l+N(s_h-s_l))} > 0,$$

$$\frac{s_h N}{s_l(N-1)} - v_2(N-1) = \frac{s_h}{s_l}\frac{s_l+(2s_h-s_l)N}{(N-1)(s_h+Ns_l+N^2(s_h-s_l))} > 0,$$

which completes the proof (as we consider any switch from the low-quality to the high-quality coalition, which supposes $1 \le n_l \le N$).

5.2 Proof of Proposition 1

Using the definition of $U(n_h)$, we have that $U(n_h) \le 0$ if and only if $\pi_h(n_h+1,n_l-1) \le \pi_l(n_h,n_l)$, which is equivalent to

$$s_h\left(\frac{n_l v_h-(n_l-1)v_l}{s_h(n_h+2)n_l-s_l(n_h+1)(n_l-1)}\right)^2 \le s_l\left(\frac{\frac{s_h}{s_l}(n_h+1)v_l-n_h v_h}{s_h(n_h+1)(n_l+1)-s_l n_h n_l}\right)^2.$$

Under Assumption (A), all equilibrium quantities are positive; we can thus take the square root on both sides of the inequality. Defining $\sigma \equiv \sqrt{s_h/s_l}$, we can then rewrite the previous inequality as

$$\frac{v_h}{v_l} \leq V(n_h) = \frac{V_{num}(n_h)}{V_{den}(n_h)}, \text{ with}$$

$$\begin{aligned} V_{num}(n_h) &\equiv \sigma(n_l - 1)\left(\sigma^2(n_h+1)(n_l+1) - n_h n_l\right) \\ &\quad + \sigma^2(n_h+1)\left(\sigma^2(n_h+2)n_l - (n_h+1)(n_l-1)\right), \\ V_{den}(n_h) &\equiv \sigma n_l\left(\sigma^2(n_h+1)(n_l+1) - n_h n_l\right) \\ &\quad + n_h\left(\sigma^2(n_h+2)n_l - (n_h+1)(n_l-1)\right). \end{aligned}$$

Hence, $U(n_h) \leq 0$ if and only if $v_h/v_l \leq V(n_h)$. By analogy, $U(n_h - 1) \geq 0$ if and only if $v_h/v_l \geq V(n_h - 1)$. To establish the existence and unicity of equilibrium, we need thus to prove that $V(n_h) > V(n_h - 1)$, meaning that $V(\cdot)$ is an increasing function. It is clear that both $V_{num}(n_h)$ and $V_{den}(n_h)$ are positive. The sign of $V(n_h) - V(n_h - 1)$ is thus given by the sign of

$$\varDelta \equiv V_{num}(n_h)V_{den}(n_h - 1) - V_{num}(n_h - 1)V_{den}(n_h).$$

A few lines of computations establish that (after substituting $N - n_h$ for n_l)

$$\begin{aligned} \varDelta &= \sigma(\sigma - 1)\left[\sigma^2 + N\sigma^2 + n_h\left(\sigma^2 - 1\right)(N - n_h)\right] \\ &\quad \times \left[\left(\sigma^2 - 1\right)(N + 3\sigma + N\sigma + 1)n_h(N - n_h) + \left(N^2\sigma + N^2 - 2N\sigma^2 + \sigma - 1\right)\right]. \end{aligned}$$

The first three terms on the top line are clearly positive. As for the fourth term on the second line, it is easily seen that it reaches a maximum at $n_h = N/2$. The smallest value is thus obtained for either $n_h = 1$ or $n_h = N-1$. Evaluating the fourth term at $n_h = 1$ or at $n_h = N - 1$, we get

$$\sigma\left((N+3)(N-1)\sigma^2 + \left(N^2 - 2N - 1\right)\sigma - 2(N-2)\right).$$

This expression is increasing in σ (its derivative with respect to σ is $2(N\sigma - 1)(N - 2) + \sigma^2(N - 1)(N + 3) + 2\sigma[\sigma(N - 1)(N + 3) - 1]$, all terms of which are positive). As, by definition, $\sigma > 1$, this expression is larger than $(N+3)(N-1) + \left(N^2 - 2N - 1\right) - 2(N-2) = 2N(N-1) > 0$.

We have thus proved that $V(n_h + 1) > V(n_h)$, which implies that the equilibrium exists and is unique: the function $V(\cdot)$ partitions the real line in N intervals, corresponding to increasing sizes of the high-quality segment (expressed as increasing values of $n_h \in \mathbb{Z}$), meaning that the ratio v_h/v_l must fall in one and only one of these intervals. An exception to the unicity property is when n_h is an integer at $U(n_h) = 0$ (in which case two equilibria would arise): to rule it out, we could set a cut-off rule by which either $U(n_h) \leq 0$ or $U(n_h - 1) \geq 0$ is a strict inequality.

To complete the proof, let us show that $V(0) > (N-1)/N$ and $V(N-1) < (s_h/s_l)[N/(N-1)]$. We have

$$V(0) = \frac{2N\sigma^2 + (N-1)[(N+1)\sigma - 1]}{N(N+1)\sigma} \equiv \Phi^{-1} \text{ and}$$
$$V(N-1) = \frac{s_h}{s_l}\frac{N(N+1)\sigma}{2N\sigma^2 + (N-1)[(N+1)\sigma - 1]} \equiv \frac{s_h}{s_l}\Phi.$$

It follows that

$$V(0) - \frac{N-1}{N} = \frac{N\left(2\sigma^2 - 1\right) + 1}{N\sigma(N+1)} > 0,$$
$$\frac{N}{N-1}\sigma^2 - V(N-1) = \frac{N\left(N\left(2\sigma^2 - 1\right) + 1\right)\sigma^2}{(N-1)\left(N\left(2\sigma^2 - 1\right) + \left(N^2 - 1\right)\sigma + 1\right)} > 0.$$

Acknowledgements We thank Olivier Bonroy, Estelle Cantillon and Mathieu Parenti for useful comments on an earlier draft.

References

1. Besley, T. and Ghatak, M. (2007). Retailing public goods: The economics of corporate social responsibility. J Public Econ 91 (9), 1645–1663
2. Bloch, F. (2002). Coalition and Networks in Industrial Organization. Manch School 70, 36–55.
3. Bonanno, G. (1986), Vertical Differentiation with Cournot Competition. Econ Notes 15, 68–91.
4. Elberfeld, W. (2003). A Note on Technology Choice, Firm Heterogeneity and Welfare. Int J Ind Organ 2, 593–605.
5. Gabszewicz, J.J. and J.F. Thisse (1979). Price Competition, Quality and Income Disparities, J Econ Theory 20, 340–359.
6. Gal-Or, E. (1983). Quality and Quantity Competition. Bell J Econ, 14, 590–600.
7. Gal-Or, E. (1985). Differentiated Industries without Entry Barriers. J Econ Theory, 37, 310–339.
8. Gal-Or, E. (1987). Strategic and Non-Strategic Differentiation. Can J Econ 20, 340–356.
9. Häckner, J. (2000) A Note on Price and Quantity Competition in Differentiated Oligopolies. J Econ Theory 93, 233–239.
10. Johnson, J. P. and Myatt, D. P. (2006). Multiproduct Cournot Oligopoly. Rand J Econ 13, 133–153.
11. Miao, Z. and Long, N.V. (2017). Multiple-Quality Cournot Oligopoly and the Role of Market Size. MPRA Paper 82095, University Library of Munich, Germany
12. Mills, D. E. and Smith, W. (1996). It Pays to Be Different: Endogenous Heterogeneity of Firms in an Oligopoly. Int J Ind Organ 14, 317–329.
13. Motta, M. (1993). Endogenous Quality Choice: Price vs. Quantity Competition, J Ind Econ 41, 113–131.
14. Shaked, A. and Sutton, J. (1982). Relaxing Price Competition through Product Differentiation. Rev Econ Stud 49, 3–13.
15. Turrini A. (2000). High-Quality Bias in Vertically Differentiated Oligopolies: A Note on Skills, Trade Welf J Econ 71(2), 133–147.