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Taxing Market Power

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Abstract

We investigate the effectiveness of tax and transfer policies in correcting market distortions when the economy is imperfectly competitive. We perform this analysis in the context of an exchange model representing a bilateral oligopoly situation, which constitutes a particular example of a Shapley-Shubik strategic market game.

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1 Introduction

The objectives pursued when taxing economic agents are usually to create resources for redistributive purposes or for financing public expenditure (Ramsey (1927), Mirrlees (1971)). Even if less evoked in the public finance literature, taxation can also serve the purpose of correcting distortions generated by the market mechanism, like in the presence of externalities (Pigou (1920), Sandmo (1975)) or imperfect competition (Myles (1989), Guesnerie and Laffont (1978)). Specifically, imperfect competition is a striking example of market failure which, due to strategic interaction, generates market outcomes which are not Pareto optimal.

An approach recently developed by game theorists (Shapley (1976) and Shapley and Shubik (1977)) to investigate market outcomes resulting from strategic interaction, constitutes a natural starting point to identify distortions due to strategic behaviour, and the counter-distortionary measures which are to be used to restore Pareto-optimality. This is the approach followed in the present paper.[1] More precisely, we analyse a *bilateral oligopoly* which constitutes a particular example of a Shapley-Shubik strategic market game introduced in Gabszewicz and Michel (1997). It can be viewed as an extension of the Cournot oligopoly model, in which the market is made of two groups of traders, each group selling a different commodity and buying the commodity sold by the other group. All agents are interested in consuming both goods, but initially own only one of them. In this bilateral oligopoly context, all agents bid strategically in view of manipulating the *relative price*, the exchange rate between the two commodities.

It is well known that the "no-trade" outcome, at which no good is exchanged, is always a Nash equilibrium in strategic market games (the so-called *trivial equilibrium* (see Dubey and Shubik (1978)). Under reasonable conditions, however, the existence of a "nice" equilibrium (besides the trivial one) at which trade occurs among agents, can be established. Conditions for the existence and uniqueness of such an interior equilibrium have been recently studied by Bloch and Ghosal (1997) for the bilateral oligopoly market defined above. Nevertheless, Cordella and Gabszewicz (1998) have constructed a non pathological example of a bilateral oligopoly market at which the *only* Nash equilibrium leads to the no-trade outcome. More precisely, they show in their example that no trade occurs at equilibrium because the perceived advantage obtained from participating to exchange is offset by the perceived advantage of reducing one's supply to manipulate the exchange price. From a welfare point of view, this equilibrium is extremely detrimental to all agents, since it wastes all potential gains from trade: this is the tribute to be paid for the use of market power! In most cases, however, welfare losses due to market power are not as large as in this example, even if strategic manipulation of prices must be expected to lead to market outcomes being far from the competitive one.

[1] This approach relies on the non-cooperative concept of Nash equilibrium. Our analysis should be distinguished from the cooperative framework used in other contexts in which taxation and strategic power have been analysed, like in Aumann and Kurz (1977).

In this paper, we first propose an extension of the Cordella-Gabszewicz's example, covering also such intermediate cases, in which *some* exchange takes place at the non competitive outcome, but without fully exploiting the possibilities offered by the competitive mechanism. Then, using this equilibrium analysis, we raise the question of whether taxing strategic agents could not induce them to restore the competitive outcome while continuing to exert their market power. We find that using taxes and transfers permits to resolve the major distortions introduced by strategic behaviour. To this end we develop a specific tax and transfer scheme which operates according to the following timing. First a tax is levied on the initial endowments of traders. Then agents exchange goods in the strategic market game whose strategies are now restricted to bids which are compatible with the new endowments resulting after taxation. The payoffs of this game include a redistribution of the tax product, to be performed after exchange has taken place. In other words, taxes are levied *before* exchange while transfers are performed *after* it. Accordingly, this tax and transfer scheme affects the initial strategic market game via two channels: taxes alter the strategy set of traders while transfers reshape their payoffs. First we show that, whenever some exchange takes place at the Nash equilibrium *without* taxation, taxes and transfers independent of individual bids can be found to restore the competitive outcome at the Nash equilibrium of the game *with* taxation. Then we show that, when no trade occurs at equilibrium, as in the Cordella-Gabszewicz's example, taxes combined with transfers proportional to the amounts sent for exchange are again sufficient instruments to restore competitive exchange.

Two earlier contributions to the problem of taxing market power in a general equilibrium set-up are those of Myles (1989) and Guesnerie and Laffont (1978). Myles (1989) extends the theory of optimal commodity taxation to a general equilibrium model with imperfect competition. His analysis is focused on the Ramsey problem of choosing commodity tax rates to maximise welfare subject to a given level of public revenue when markets operate in a non competitive framework. Guesnerie and Laffont (1978) study optimal taxation with respect to a social welfare function when the objective of the government is to restore first best Pareto efficiency when this first best is destroyed by the non competitive behaviour of a monopolist. Even if they also tackle the problem of designing tax instruments to offset distortions arising from imperfect competition, their approach differs from ours in several respects. First, their paper considers an economy defined in a much more general set-up than the one considered here. In particular, their model involves a productive sector with a firm behaving non competitively, while consumers are price-takers, as in Gabszewicz and Vial (1972). Furthermore, tax and transfer instruments are linear taxes and lump-sum transfers. Moreover, they do not consider a context of strategic interaction, but only the case of a monopoly. Finally, our approach does not refer to a social welfare function since policy tools are introduced with the sole objective of restoring competitive exchange.

We introduce the model in section 2. The equilibrium analysis without taxation is performed in section 3. Section 4 analyses the effects of taxation introduced in view of overcoming market distortions observed before the tax instruments are introduced.

2 The model

Consider an exchange economy consisting of n traders, falling into two types j, $j = 1, 2$, with $\frac{n}{2}$ of each type; there are two goods, 1 and 2. For traders i, $i = 1, ..., \frac{n}{2}$, preferences and endowments are defined by

$$U_i(x_1, x_2) = \beta_1 x_1 + x_2; \qquad a_i = (1, 0), \tag{1}$$

with $0 < \beta_1 < 1$.
Similarly, for traders i, $i = \frac{n}{2} + 1, ..., n$, we have

$$U_i(x_1, x_2) = x_1 + \beta_2 x_2; \qquad a_i = (0, 1), \tag{2}$$

with $0 < \beta_2 < 1$.
This exchange economy has a unique competitive equilibrium $(\widetilde{p}, \widetilde{x})$, namely the price system $\widetilde{p}$ and the allocation $\widetilde{x}$ defined by

$$\begin{aligned} \widetilde{p} &= (1, 1), \\ \widetilde{x}_i &= (0, 1), \qquad i = 1, ..., \frac{n}{2} \\ \widetilde{x}_i &= (1, 0), \qquad i = \frac{n}{2} + 1, ..., n. \end{aligned}$$

At this competitive allocation, the quantity of good 1 (resp. good 2) sold by agents $i \in \{1, ..., \frac{n}{2}\}$ (resp. $i \in \{\frac{n}{2} + 1, ..., n\}$) is given by $\widetilde{q}_i = \widetilde{q} = 1$ (resp. $\widetilde{b}_i = \widetilde{b} = 1$). To this exchange economy, we may associate a strategic market game Γ defined as follows. For all agents, the *strategy set* consists of the $[0, 1]$ - interval. For an agent $i \in \{1, ..., \frac{n}{2}\}$, we denote by q_i a strategy in $[0, 1]$: q_i is to be interpreted as the quantity of good 1 that agent i sends to the market for trade. Similarly, b_i, $b_i \in [0, 1]$, denotes a strategy of agent i, $i \in \{\frac{n}{2} + 1, ..., n\}$. Given an n-tuple of strategies $(q, b) = (q_1, ..., q_i, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_i, ..., b_n)$, the resulting allocation of goods obtains as

$$(1 - q_i, pq_i)$$

for an agent i in $\{1, ..., \frac{n}{2}\}$, and

$$(\frac{b_i}{p}, 1 - b_i)$$

for an agent i in $\{\frac{n}{2}+1, ..., n\}$, with p denoting the price of good 1 expressed in units of good 2 (good 2 serves as numeraire). This price follows from the equality of demand and supply of good 1, i.e. it satisfies the equation:

$$\sum_{k=\frac{n}{2}+1}^{n} b_k = p \sum_{k=1}^{\frac{n}{2}} q_k;$$

or

$$p = \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k}.$$

Accordingly, given an n-tuple (q, b) of strategies and taking into account (1) and (2), the *payoffs* in the strategic market game obtain as

$$\begin{aligned} U_i(q,b) &= U_i(1-q_i, \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i); \\ &= \beta_1(1-q_i) + \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i, \end{aligned}$$

for $i = 1, ..., \frac{n}{2}$, and

$$\begin{aligned} U_i(q,b) &= U_i(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i, 1-b_i); \\ &= \frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i + \beta_2(1-b_i), \end{aligned}$$

for $i = \frac{n}{2}+1, ..., n$.

Notice however that, at the vector of strategies $(\widehat{q}, \widehat{b}) = (0, ..., 0; 0, ..., 0)$, no exchange occurs, and agents' payoffs are then given by

$$U_i(\widehat{q}, \widehat{b}) = \beta_1; \qquad i = 1, ..., \frac{n}{2} \tag{3}$$

and

$$U_i(\widehat{q}, \widehat{b}) = \beta_2; \qquad i = \frac{n}{2}+1, ..., n. \tag{4}$$

A *Nash equilibrium* is a n-tuple of strategies $(q_1^*, ..., q_i^*, ..., q_{\frac{n}{2}}^*; b_{\frac{n}{2}+1}^*, ..., b_i^*, ..., b_n^*)$ chosen by the traders such that no trader has an advantage to deviate unilaterally from his choice.

For further use, we compute

$$\frac{\partial U_i}{\partial q_i} = \frac{(\sum_{k=\frac{n}{2}+1}^{n} b_k)(\sum_{k=1}^{\frac{n}{2}} q_k - q_i)}{(\sum_{k=1}^{\frac{n}{2}} q_k)^2} - \beta_1, \qquad i = 1, ..., \frac{n}{2} \tag{5}$$

and

$$\frac{\partial U_i}{\partial b_i} = \frac{(\sum_{k=1}^{\frac{n}{2}} q_k)(\sum_{k=\frac{n}{2}+1}^{n} b_k - b_i)}{(\sum_{k=\frac{n}{2}+1}^{n} b_k)^2} - \beta_2, \qquad i = \frac{n}{2} + 1, ..., n. \tag{6}$$

Also notice that the second derivatives $\frac{\partial^2 U_i}{\partial q_i^2}$ and $\frac{\partial^2 U_i}{\partial b_i^2}$ are both negative.

The above strategic market game defines a situation of bilateral oligopoly, as the one considered by Cordella and Gabszewicz (1998), in which it was assumed that $\beta_1 = \beta_2 > \frac{n-2}{n}$. They show that, for this particular case, the *only* Nash equilibrium obtains as $(\widehat{q}, \widehat{b})$, leading to the "no-trade" outcome. The extension we consider here allows us to cover intermediate situations at which agents exchange, at equilibrium, *part* of their initial endowment, but not the *total* amount of it (as it is the case at the vector of strategies $(\widetilde{q}, \widetilde{b})$ leading to the competitive allocation $\widetilde{x}$).

In the next section, we analyse the Nash equilibrium as a function of β_1 and β_2, the agents' preference intensities for the good they own initially.

3 Equilibrium analysis without taxation

It is a general property of strategic market games that the vector of strategies where each trader bids and supplies nothing is always a Nash equilibrium (the so-called *trivial equilibrium*). Accordingly, the vector of strategies $(\widehat{q}, \widehat{b})$ is always a Nash equilibrium of our market game Γ. Furthermore, the symmetry of the problem clearly implies that all agents of the same type must play the *same* strategy at a Nash equilibrium $(q_1^*, ..., q_{\frac{n}{2}}^*; b_{\frac{n}{2}+1}^*, ..., b_n^*)$, i.e.

$$\begin{aligned} q_i^* &= q^*, & i \in \{1, ..., \frac{n}{2}\} \\ b_i^* &= b^*, & i \in \{\frac{n}{2} + 1, ..., n\}. \end{aligned}$$

Furthermore, the first derivatives $\frac{\partial U_i}{\partial q_i}$ and $\frac{\partial U_i}{\partial b_i}$ must be equal to zero at an interior equilibrium. Accordingly, at an interior equilibrium, we should have

$$q^* = \frac{b^*}{\beta_1} \frac{n-2}{n} \tag{7}$$

and

$$b^* = \frac{q^*}{\beta_2}\frac{n-2}{n}, \tag{8}$$

with the values q^* and b^* obtained from equating (5) and (6) to zero.

Clearly, except in a very special case, (i.e. when $\beta_1\beta_2 = \left(\frac{n-2}{n}\right)^2$), no value of q^* and b^*, with q^* and b^* both in $]0,1[$, can satisfy this system: *No interior equilibrium exists*. Finally, it is evident that no equilibrium (q^*, b^*) can arise with $q^* > 0$ and $b^* = 0$, or $q^* = 0$ and $b^* > 0$.

Accordingly, at the exclusion of $(\widehat{q}, \widehat{b})$, which is already known to be a Nash equilibrium, we deduce from (7) and (8) and from the non-existence of interior equilibria that *only the vectors of strategies* $(q^*, b^*) = \left(\frac{n-2}{n\beta_1}, 1\right)$ or $(q^*, b^*) = \left(1, \frac{n-2}{n\beta_2}\right)$, or $(q^*, b^*) = (1,1)$ *are possible candidates as Nash equilibria of the game* Γ. At the first candidate, traders in $\{1, ..., \frac{n}{2}\}$ choose the strategy in the interior of the strategy set $[0,1]$, so that the necessary condition $\frac{\partial U_i}{\partial q_i} = 0$ (or (7)) must hold, while traders in $\{\frac{n}{2}+1, ..., n\}$ play at the boundary where they send for trade their whole endowment in good 2. The reverse holds at the second candidate equilibrium. Finally, the third candidate leads to the competitive outcome.

Now we are in a position to find necessary and sufficient conditions under which each of these candidates is, indeed, a Nash equilibrium.

Proposition 1 *When (a)* $\beta_1 > \frac{n-2}{n}$ *and (b)* $\beta_1\beta_2 < \left(\frac{n-2}{n}\right)^2$, *the vector of strategies* (q^*, b^*) *defined by*

$$(q^*, b^*) = \left(\frac{n-2}{n\beta_1}, 1\right) \tag{9}$$

is the only Nash equilibrium of the strategic market game Γ *(apart from the trivial equilibrium* $(\widehat{q}, \widehat{b})$*).*

Proof. To be a Nash equilibrium, the vector of strategies (q^*, b^*) identified by (9) must satisfy the following conditions: *(i)* $q^* \in]0,1[$; *(ii)* $\left.\frac{\partial U_i}{\partial q_i}\right|_{(q^*,b^*)} = 0$; *(iii)* $\frac{\partial^2 U_i}{\partial q_i^2} \leqslant 0$; *(iv)* $\left.\frac{\partial U_i}{\partial b_i}\right|_{(q^*,b^*)} \geqslant 0$ and *(v)* $\frac{\partial^2 U_i}{\partial b_i^2} \leqslant 0$. Condition *(i)* follows from assumption *(a)*; condition *(ii)* is satisfied by the definition of q^* (see (7)); condition *(iv)* follows from

$$\left.\frac{\partial U_i}{\partial b_i}\right|_{(q^*,b^*)} = \left(\frac{n-2}{n}\right)^2 \frac{1}{\beta_1} - \beta_2 \geqslant 0 \qquad \Leftrightarrow \qquad \beta_1\beta_2 \leqslant \left(\frac{n-2}{n}\right)^2,$$

an inequality which holds by assumption *(b)*. Finally, since $\frac{\partial^2 U_i}{\partial q_i^2}$ and $\frac{\partial^2 U_i}{\partial b_i^2}$ are both negative, conditions *(ii)* and *(iv)* are also sufficient conditions for (q^*, b^*) to be a Nash equilibrium.

To show uniqueness, we have to eliminate the other two possible candidates as Nash equilibrium, namely $(q^*, b^*) = \left(1, \frac{n-2}{n\beta_2}\right)$ and $(q^*, b^*) = (1, 1)$. Assume $\left(1, \frac{n-2}{n\beta_2}\right)$ to be a Nash equilibrium under assumptions *(a)* and *(b)*. Then we have $\frac{n-2}{n\beta_2} < 1$, which implies $\beta_2 > \frac{n-2}{n}$. But then it follows from *(a)* that $\beta_1\beta_2 > \left(\frac{n-2}{n}\right)^2$, which violates assumption *(b)*. Assume now $(1, 1)$ to be a Nash equilibrium under the same assumptions. Then, $\left.\frac{\partial U_i}{\partial q_i}\right|_{(q^*,b^*)}$ and $\left.\frac{\partial U_i}{\partial b_i}\right|_{(q^*,b^*)}$ must necessarly be non-negative, i.e.

$$\beta_1 \leq \frac{n-2}{n} \qquad and \qquad \beta_2 \leq \frac{n-2}{n},$$

violating assumption *(a)*. ■

Proposition 2 *When (c)* $\beta_2 > \frac{n-2}{n}$ *and (b)* $\beta_1\beta_2 < \left(\frac{n-2}{n}\right)^2$, *the vector of strategies* (q^*, b^*) *defined by*

$$(q^*, b^*) = \left(1, \frac{n-2}{n\beta_2}\right) \tag{10}$$

is the only Nash equilibrium of the strategic market game Γ *(apart from the trivial equilibrium* $(\widehat{q}, \widehat{b})$*).*

Proof. The proof of this proposition is the same, *mutatis mutandis*, as the proof of proposition 1 ■

Proposition 3 *When (d)* $\beta_1 < \frac{n-2}{n}$ *and (e)* $\beta_2 < \frac{n-2}{n}$, *the vector of strategies* $(\widetilde{q}, \widetilde{b}) = (1, 1)$ *is the only Nash equilibrium of the strategic market game* Γ *(apart from the trivial equilibrium* $(\widehat{q}, \widehat{b})$*)*

Proof. First, under assumptions *(d)* and *(e)*, $(1, 1)$ is a Nash equilibrium due to the fact that $\left.\frac{\partial U_i}{\partial q_i}\right|_{(1,1)}$ and $\left.\frac{\partial U_i}{\partial b_i}\right|_{(1,1)}$ are both non-negative and that the second derivatives $\frac{\partial^2 U_i}{\partial q_i^2}$ and $\frac{\partial^2 U_i}{\partial b_i^2}$ are both negative. Furthermore, to prove uniqueness, we notice that the candidate $(q^*, b^*) = \left(\frac{n-2}{n\beta_1}, 1\right)$ is eliminated since $q^* = \frac{n-2}{n\beta_1} < 1 \Rightarrow \beta_1 > \frac{n-2}{n}$, violating assumption *(d)*. In the same manner, the candidate $(q^*, b^*) = \left(1, \frac{n-2}{n\beta_2}\right)$ is eliminated by a similar argument using assumption *(e)*. ■

Proposition 4 *When (f)* $\beta_1\beta_2 > \left(\frac{n-2}{n}\right)^2$, *the vector of strategies* $(\widehat{q}, \widehat{b})$ *is the only Nash equilibrium of the strategic market game* Γ.

Proof. Neither $(q^*, b^*) = \left(\frac{n-2}{n\beta_1}, 1\right)$, nor $(q^*, b^*) = \left(1, \frac{n-2}{n\beta_2}\right)$ can be a Nash equilibrium under assumption *(f)* since, in both cases, these are Nash equilibria only if the reverse of *(f)* holds. Similarly, $(\widetilde{q}, \widetilde{b}) = (1, 1)$ cannot be a Nash equilibrium under assumption *(f)* since the inequalities $\beta_1 \leqslant \frac{n-2}{n}$ and $\beta_2 \leqslant \frac{n-2}{n}$, which are necessary conditions for $(\widetilde{q}, \widetilde{b})$ to be a Nash equilibrium, imply together that the reverse of *(f)* holds. ■

Propositions 1-4, which provide a taxonomy of Nash equilibria as a function of the preference intensities β_1 and β_2, reveal that different levels of distortion with respect to the competitive outcome $\widetilde{x}$, can arise as a consequence of strategic behaviour: no distortion when both β_1 and β_2 are smaller than $\frac{n-2}{n}$ (proposition 3), maximal distortion when $\beta_1\beta_2 > \left(\frac{n-2}{n}\right)^2$ (proposition 4), and "intermediate" distortion when $\beta_1\beta_2 < \left(\frac{n-2}{n}\right)^2$ with at least one β_j larger than $\frac{n-2}{n}$ (propositions 1 and 2). We examine in the next section how these distortions can be resorbed, using appropriate tax and transfer instruments.

4 The effects of taxation

In the framework of a real exchange model as the one considered here, prices express only the exchange rates between commodities. Accordingly, the tax scheme we consider takes the form of a levy on the initial endowments of traders. More precisely, taxation affects the strategy sets of traders and, consequently, the Nash equilibrium of the ensuing strategic market game. This, in turn, affects the equilibrium value of the exchange rate.[2] Thus, the tax which is proposed here consists in a lump-sum, and not in a commodity tax, as in the standard literature dealing with optimal taxation.[3] Our first aim is to find a tax, and a transfer scheme of the product of that tax, in view of resorbing the "intermediate" distortions observed in cases covered by propositions 1 and 2.

First let us consider the case covered by proposition 1, i.e. $\beta_1 > \frac{n-2}{n}$ and $\beta_1\beta_2 < \left(\frac{n-2}{n}\right)^2$. Suppose that a *tax* t, $0 < t < 1$, is levied on good 1, which in our economy affects only agents of type 1 since they are the sole owners of that good. After trade has taken place, suppose that the tax revenue is uniformly *transfered* to all agents i of type 2, i.e. $i \in \{\frac{n}{2}+1, ..., n\}$. This generates a new strategic market game - Γ'_t, say - defined as follows. The strategy set of agents $i \in \{1, ..., \frac{n}{2}\}$ is now the interval $[0, 1-t]$ while the strategy set of agents $i \in \{\frac{n}{2}+1, ..., n\}$ is, as in the game Γ, the interval $[0, 1]$. On the other hand,

[2] This is reminiscent of some effects appearing in exchange economies when there are agents who transfer a part of their initial resources to other agents. In particular, it is well known that, due to the multiplicity of competitive allocations, some agents can gain at equilibrium from free gifts, or loose from accepting such gifts (see Gale (1974)). Similar phenomena can arise with the core of the exchange economy (Aumann (1973)).

[3] In fact, it can be shown that commodity taxation and transfers of the tax product are not sufficient policy tools to restore the competitive outcome.

as a consequence of the transfers, the outcomes of the game Γ_t' at a vector of strategies $(q_1, ..., q_i, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_i, ..., b_n)$ obtain as

$$x_i = \left(1 - t - q_i, \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i\right), \qquad i = 1, ..., \frac{n}{2} \tag{11}$$

$$x_i = \left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i + t, 1 - b_i\right), \qquad i = \frac{n}{2} + 1, ..., n \tag{12}$$

giving rise to utility levels

$$U_i(q, b) = \beta_1(1 - t - q_i) + \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i, \qquad i = 1, ..., \frac{n}{2}$$

and

$$U_i(q, b) = \frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i + t + \beta_2(1 - b_i), \qquad i = \frac{n}{2} + 1, ..., n.$$

Proposition 5 *When (a) $\beta_1 > \frac{n-2}{n}$ and (b) $\beta_1\beta_2 < \left(\frac{n-2}{n}\right)^2$, there exists a tax t on good 1, $1 - \frac{n-2}{n\beta_1} < t < 1 - \frac{n\beta_2}{n-2}$, such that the only Nash equilibrium in the resulting game Γ_t' (apart from the trivial one) has as outcome the competitive allocation $\widetilde{x}$ of the game Γ.*

Proof. First, let us show that, under assumptions *(a)* and *(b)*, the vector of strategies $(q^*, b^*) = (1 - t, 1)$ is a Nash equilibrium of the game Γ_t', with t selected in the interval $\left[1 - \frac{n-2}{n\beta_1}, 1 - \frac{n\beta_2}{n-2}\right]$ which is non-empty by assumption *(b)*. Suppose all agents i, $i = 1, ..., \frac{n}{2}$, play the strategy $q^* = 1 - t$ and all agents i, $i = \frac{n}{2} + 1, ..., n$, play $b^* = 1$. Select an agent i, $i \in \left\{1, ..., \frac{n}{2}\right\}$, and suppose he considers a unilateral deviation from q^*. From (5), we notice that

$$\left.\frac{\partial U_i}{\partial q_i}\right|_{(q^*, b^*)} = \frac{n-2}{n(1-t)} - \beta_1 > 0,$$

where the last inequality follows from $1 - \frac{n-2}{n\beta_1} < t$. Similarly, select an agent i, $i \in \left\{\frac{n}{2} + 1, ..., n\right\}$, and suppose he considers a unilateral deviation from $b^* = 1$. From (6), we notice that

$$\left.\frac{\partial U_i}{\partial b_i}\right|_{(q^*,b^*)} = \frac{(1-t)(n-2)}{n} - \beta_2 > 0,$$

where the last inequality follows from $t < 1 - \frac{n\beta_2}{n-2}$. Then $(q^*, b^*) = (1-t, 1)$ is a Nash equilibrium in Γ'_t. Now we show that it is the unique one. It is easy to see that the only alternative candidate would be

$$\left(\frac{n-2}{n\beta_1}, ..., \frac{n-2}{n\beta_1}; 1, ..., 1\right).$$

However, $\frac{n-2}{n\beta_1}$ must be in $S_i = [0, 1-t]$, $i = 1, ..., \frac{n}{2}$, contradicting $t > 1 - \frac{n-2}{n\beta_1}$. This proves that $(q^*, b^*) = (1-t, 1)$ is the unique Nash equilibrium in Γ'_t, (differing from the trivial equilibrium).

Substituting these equilibrium values q^* and b^* into the outcome functions (11) and (12) shows that the outcomes at equilibrium are the same as at the competitive allocation $\widetilde{x}$ in Γ. ∎

Proposition 5 shows that distortions observed in the case covered by proposition 1 can be resorbed by an adequate tax levied on good 1 and a transfer of the resulting tax revenue to the owners of good 2. From the bounds on t spelled in proposition 5, we can derive the following observations. First, the lower bound imposes that the equilibrium price p^* of good 1 after taxation $\left(\frac{1}{1-t}\right)$ exceeds the equilibrium price before taxation $\left(\frac{n\beta_1}{n-2}\right)$. Accordingly, the tax and transfer scheme operates via two channels. First, it makes poorer the owners of good 1 by confiscating part of their initial endowment. Second, it makes them richer by increasing the relative price of good 1. The combination of the two effects necessarily leads to the competitive outcome. As for the upper bound on t, it guarantees that no owner of good 2 is induced to deviate from the competitive supply, notwithstanding the tax and transfers intervention.

The distortion observed in the case covered by proposition 2 can be as well resorbed when a tax is levied on good 2 and the product of that tax is transfered to the owners of good 1. As above, we define, for a given tax t levied on good 2, the resulting game Γ''_t as follows. The strategy set of agents i, $i = 1, ..., \frac{n}{2}$, is, as in Γ, the unit interval $[0, 1]$, while the strategy set of agents i, $i = \frac{n}{2} + 1, ..., n$, now becomes $[0, 1-t]$. The outcomes of Γ''_t, at a vector of strategies $(q_1, ..., q_i, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_i, ..., b_n)$, obtain as

$$x_i = \left(1 - q_i, \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i + t\right), \qquad i = 1, ..., \frac{n}{2}$$

$$x_i = \left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i, 1 - t - b_i\right), \qquad i = \frac{n}{2} + 1, ..., n.$$

Proposition 6 *When (c) $\beta_2 > \frac{n-2}{n}$ and (b) $\beta_1\beta_2 < \left(\frac{n-2}{n}\right)^2$, there exists a tax t on good 2, $1 - \frac{n-2}{n\beta_2} < t < 1 - \frac{n\beta_1}{n-2}$, such that the only Nash equilibrium in the resulting game Γ_t'' (apart from the trivial one) has as outcome the competitive allocation $\widetilde{x}$ of the game Γ.*

Proof. The proof of this proposition is the same, *mutatis mutandis*, as the proof of proposition 5. ■

Propositions 5 and 6 show that the tax and transfer schemes considered above are sufficient to resorb the distortions introduced by strategic behaviour with respect to the competitive allocation, when at least *some* exchange is observed at the Nash equilibrium before taxation. It is easy to check that they are not sufficient when no exchange is observed at equilibrium before taxation, namely, when assumption *(f)* of proposition 4 holds. Does it mean that no *alternative* scheme could do the job in that case? As shown in proposition 7, a tax and transfer scheme, in which the share of the tax revenue received by each agent is positively related to the amount supplied for exchange, succeeds in fully resorbing the maximal distortion observed at the Nash equilibrium in that case.

Suppose that a tax t, $0 < t < 1$, is levied on both goods, 1 and 2, giving rise to a total tax revenue equal to $\frac{nt}{2}$ units of each good, and to resulting strategy sets $[0, 1-t]$ for all traders. Furthermore, suppose that, after trade has occured at an n-tuple of strategies $(q_1, ..., q_i, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_i, ..., b_n)$, a share equal to $\frac{t}{1-t}q_i$ (resp. $\frac{t}{1-t}b_i$) of the total revenue $\frac{nt}{2}$ in good 2 (resp. good 1) is transfered to each agent i in $\left\{1, ..., \frac{n}{2}\right\}$ (resp. $\left\{\frac{n}{2}+1, ..., n\right\}$).[4]

More precisely, the taxes and transfers scheme we have just considered generates a strategic market game Γ_t''' defined as follows. Given a tax t, $0 < t < 1$, the strategy set of all agents is the interval $[0, 1-t]$. As a consequence of transfers the outcomes of the game Γ_t''' at a vector of strategies $(q_1, ..., q_i, ..., q_{\frac{n}{2}}; b_{\frac{n}{2}+1}, ..., b_i, ..., b_n)$ now obtain as

$$x_i = \left(1 - t - q_i, \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i + \frac{t}{1-t} q_i\right), \qquad i = 1, ..., \frac{n}{2} \tag{13}$$

[4] Notice that, for any t, $0 < t < 1$, these transfers are feasible since, $\forall i \in \left\{1, .., \frac{n}{2}\right\}$,

$$\begin{aligned} q_i \leq 1 - t \Rightarrow & \sum_{i=1}^{\frac{n}{2}} q_i \leq \frac{n}{2}(1-t) \\ \Rightarrow & \frac{t}{1-t} \sum_{i=1}^{\frac{n}{2}} q_i \leq \frac{n}{2} t, \end{aligned}$$

and similarly for the transfers to agents of type 2.

$$x_i = \left(\frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i + \frac{t}{1-t} b_i, 1 - t - b_i \right), \qquad i = \frac{n}{2} + 1, ..., n, \tag{14}$$

giving rise to utility levels

$$U_i(q,b) = \beta_1(1 - t - q_i) + \frac{\sum_{k=\frac{n}{2}+1}^{n} b_k}{\sum_{k=1}^{\frac{n}{2}} q_k} q_i + \frac{t}{1-t} q_i, \qquad i = 1, ..., \frac{n}{2}$$

and

$$U_i(q,b) = \frac{\sum_{k=1}^{\frac{n}{2}} q_k}{\sum_{k=\frac{n}{2}+1}^{n} b_k} b_i + \frac{t}{1-t} b_i + \beta_2(1 - t - b_i), \qquad i = \frac{n}{2} + 1, ..., n.$$

Proposition 7 *There exists a tax t, $0 < t < 1$, on goods 1 and 2, such that the only Nash equilibrium of the resulting game Γ_t''' (apart from the trivial one) has as outcome the competitive allocation $\widetilde{x}$ of the game Γ.*

Proof. First let us show that there exists a tax t, $0 < t < 1$, for which the vector of strategies $(q^*, b^*) = (1 - t, 1 - t)$ is a Nash equilibrium of the corresponding game Γ_t'''. This is clearly the case if t satisfies

$$\left. \frac{\partial U_i}{\partial q_i} \right|_{(q^*, b^*)} > 0 \qquad \Leftrightarrow \qquad \frac{t}{1-t} > \beta_1 - \frac{n-2}{n}, \tag{15}$$

and

$$\left. \frac{\partial U_i}{\partial b_i} \right|_{(q^*, b^*)} > 0 \qquad \Leftrightarrow \qquad \frac{t}{1-t} > \beta_2 - \frac{n-2}{n}. \tag{16}$$

If both $\beta_1 - \frac{n-2}{n}$ and $\beta_2 - \frac{n-2}{n}$ are negative, then any t in the unit interval satisfies (15) and (16). Now assume that at least one of these expressions is positive and, without loss of generality, assume that $\beta_1 = \max\{\beta_1, \beta_2\}$. Then $\beta_1 - \frac{n-2}{n} > 0$.

On the other hand, the function $f(t) = \frac{t}{1-t}$ defined on $[0, 1]$ is monotone increasing from 0 to $+\infty$ on this interval. Accordingly, there exists a value of t, say $\bar{t}$, in the same interval such that inequality (15) is satisfied at $t = \bar{t}$. But then, since $\beta_1 \geqslant \beta_2$, inequality (16) is *a fortiori* satisfied with $t = \bar{t}$, which completes the proof that the vector of strategies $(q^*, b^*) = (1 - \bar{t}, 1 - \bar{t})$ is a Nash equilibrium of the corresponding game Γ_t'''. It is easy to check that it is the only one.

Substituting the equilibrium values $q^* = b^* = 1 - \overline{t}$ into the outcome functions (13) and (14) shows that the outcomes at equilibrium are the same as at the competitive allocation $\widetilde{x}$ in Γ. ■

The taxes and transfers scheme underlying proposition 7 appears as a powerful instrument in resorbing distortions with respect to the competitive allocation, since it is effective even when no-trade is the only possible outcome of strategic interaction before taxation. The power of this instrument follows from its inciting content, since the amount of the more prefered good received by each agent in the transfer increases with the amount of the less prefered good he sends to the market.

The main difference between the scheme underlying proposition 7 and those considered in propositions 5 and 6 is that transfers implied by the latter are *anonymous*: they do not distinguish among traders of the same type. On the contrary, the amounts transfered in the former are linked to the individual strategies selected by the agents.

5 Conclusion

In this paper, we have investigated the effectiveness of tax and transfer policies in correcting market distortions when the economy is imperfectly competitive. We have performed this analysis in the context of an exchange model representing a bilateral oligopoly situation, which constitutes a particular example of a Shapley-Shubik strategic market game.

Going back to the equilibrium analysis without taxation developed in section 3, it is easy to see that, besides the trivial equilibrium, the only remaining Nash equilibrium when $n \to \infty$ is $(\widetilde{q}, \widetilde{b}) = (1, 1)$, having as outcome the competitive allocation $\widetilde{x}$. This well known property states that, under strategic interaction in an exchange economy, competition is restored when the number of traders increases without limit. We have shown in this paper that, at least for the particular case of bilateral oligopoly, competition can also be restored, when the number of traders is not large enough, by adequate taxes and transfers. Extending our approach to other non competitive contexts is an open field for further research.

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