Macroeconomic Implications of Switching the Social Security Trust Fund Towards a Greater Investment in Equities

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ABSTRACT

This paper shows that shifting the portfolio allocation of the social security trust fund towards more equity investment, ceteris paribus, reduces the aggregate capital stock as well as the average consumption level of all individuals except the poor retirees who receive an increase but at the cost of a large increase in uncertainty. If a larger capital stock is desired, reducing the supply of publicly supplied goods is the most effective tool. That change also increases the average private consumption of all the young and the wealthy retirees although it does reduce the average consumption and uncertainty of the old non-savers.

I. Introduction

The social security trust fund can be viewed as a pool of resources invested in financial assets today to provide benefits for retirees in the future. At the present time, the trust fund is fully invested in United States government securities; however, proposals have been made that share a common feature; investing at least part of the social security trust fund in private equities.[1] The intent of these proposals is to raise the rate of return that retirees receive from their social security pension and to increase the size of the private capital stock. While we agree that this type of a change will provide the poorer retirees, those whose retirement income comes completely from social security, with a higher but more variable rate of return, the other effects of the switch to larger equity holdings will depend very much on how the rest of the government reacts. We argue in this paper that too much of the discussion about social security reform has focused on the direct financial implications of the switch and not enough about the macroeconomic effects. If one were to do so, one would find that the reaction of the rest of the government to any changes in investment strategy by social security would be crucially important.

If the rest of the government, for example, continues to supply the same amount of public goods after the switch to more equity investment by social security, the aggregate effect could actually be to reduce the size of the private capital stock. The intuition is that if the total supply of government bonds is not reduced, private savers would be forced to purchase the extra bonds that the government now has to sell to the private sector but previously were purchased by social security. Thus, savers will not only reduce their equity holdings to reconstitute their portfolios to offset the increase in equity income they are receiving through social security, they will also be forced to buy the government bonds that social security was previously buying for the non-savers. Since the interest rates on bonds need to rise to induce the savers to purchase the extra securities, private savers can reduce their equity holdings by more than the increase in purchases by social security and still have enough saved to live comfortably in retirement. Furthermore, this process results in the wealthier retirees facing less risk than they were facing before the adjustment in the social security portfolio. Thus, some of the risk has been shifted from wealthier retirees to poorer ones. In fact, the only participants in the economy that face

increased uncertainty are the old non-savers who rely completely on social security for their retirement income. All the other participants, the young workers who save, the young workers who do not have a large enough income to save, and the retirees who were able to amass a private portfolio to help fund part of their retirement income, face reduced risk. Moreover, the only group that realizes an increase in their average consumption level are the non-saving old, all the other members of the economy find that their average consumption has been reduced by the policy change.

The purchase of U.S. government securities by the social security administration allows the government to use the money raised from selling bonds to the trust fund to provide various types of government supplied goods to the private sector. These can include both public goods and publicly provided private goods. If, as the trust fund reallocates its portfolio towards more equity purchases and fewer resources are provided for the purchase of U.S. government bonds, the rest of the government were to reduce its expenditures on public goods, one would get the more traditional result that the aggregate private capital stock would increase. However, this result is very much dependent on the reduction in spending by the rest of the government on public goods and this outcome would be achieved (and even in greater magnitude) even if the social security trust fund did not reallocate its portfolio. In this case, (one where the trust fund reallocates its portfolio so that its holdings consist of a larger proportion in equity and the rest of the government reduces the supply of publicly provided goods), unlike the previous one, all the members of the economy face an increase in fluctuation of their consumption level across states of the world. Furthermore, the young savers, the young non-savers, and the retired savers all find that their average level of consumption has increased. The only group that sees their average level of consumption decline are the retirees who rely completely on social security for their retirement income. The reason for the decrease is that the decline in return on bonds and equity outweighs the effect of the switch to a larger proportion of equity in the account of the non-savers at social security. The retired savers do not see their income decline since the young savers, who see a large increase in income through the increase in the wage rate on labor, are able to shift some of that income through the purchase of private assets to their old age. Whether this policy of reducing the supply of public goods (either with or without a shift in the portfolio allocation of social security) is

welfare improving is less clear. It will depend in great part on whether or not the existing supply of public goods was suboptimal relative to that of private goods . Moreover, the degree to which the aggregate capital stock increases and the magnitude of the effect on bond interest rates will depend very much on how much the rest of the government reduces its expenditures. Thus, in this paper we argue that any plan that discusses the shift in the portfolio composition of the social security trust fund must also factor in the reaction of the rest of the government in order to determine aggregate effects. To focus on social security without also considering the reaction of the rest of the government can lead to misleading results.

If, in focusing on the rest of the government, one is concerned about the welfare of society, one needs to focus on the proportion of public goods that are being supplied relative to the amount of private goods that are being produced by the economy. However, if one is only concerned about the size of the private capital stock, a reduction in public expenditures on public goods would be the desirable policy, since that type of a change would lead to an increase in the private capital stock. Moreover, if public good expenditures tend to rise more slowly in an expansion than does the overall economy, and if they tend to fall more slowly than does the overall economy in a downturn, this type of expenditure profile by the government on public goods would lead to the capital stock rising faster than the economy as a whole in an upturn and falling more sharply in a downturn.

To study these issues we use an overlapping generations model with uncertainty. The economy contains both savers and non-savers. In our model workers, who labor only when they are young, pay for the social security benefits that they will receive when they are retired through a payroll tax; however, the money needed to repay the government debt (since the government uses the proceeds from its borrowing to provide public goods) should be raised from everyone (since both young and old receive benefit from the public goods). Thus, an income tax (on both labor and capital income) is used to cover the debt repayment. Unfortunately, taxing capital income introduces a great deal of complication. Therefore, we restrict the income tax so that it raises revenue only from labor income in the theoretical part, section III, although we use the tax to raise revenue from both labor and capital in the example, found in section IV. It turns out that the qualitative nature of the results obtained in section

III are not affected by this simplification. However, we retain the more complete model for the example in order to get a better sense of the relative magnitudes of the various changes. Within the above mentioned framework we find the steady state and then examine how the steady state changes when social security alters its investment strategy and the rest of the government reacts to the change in social security behavior.

Two factors are added to the model in the example section that are not included in the theoretical framework. First, equations are added that permit us to measure the fluctuation in the level of consumption of the various individuals contained in the economy across various states of the world. Second, we allow the government to introduce a guaranteed minimum payment that will become binding if the retired non-savers have a sufficiently low income in the bad state of the world. This introduction is an attempt to allow the non-saving retirees to get some benefit from the superior return on equities but at the same time reduce their downside risk somewhat.

In this paper it is assumed that social security is fully funded. In reality, the social security system at the present time is pay-as-you-go. However, since it has been shown elsewhere that the two methods are equivalent as long as the transition generation is compensated by public borrowing and the benefit rule is unchanged,[2] we feel that the results found in this paper are indicative of what types of changes could be expected if the social security trust fund alters its portfolio allocation and the rest of the government changes the amount of public goods it supplies.

The remainder of the paper is organized as follows. In the next section the model used in the paper is presented. Two versions of the model are discussed. One version, in which the income tax only taxes labor income, is used in the theoretical section, and a second in which the income tax is used to raise revenue from all forms of income, is used for the numerical example. Section III discusses the implications of the theoretical model and Section IV presents a numerical simulation model and discusses what lessons we learn from it. Finally, Section V provides a few conclusions.

II. The Model

We assume a standard type of overlapping generations model with two types of individuals and two states of the world. All individuals work in the first period of their lives and retire in the second. The government provides a fully funded social security scheme that dispenses a pension to the retirees. Type 1 individuals have sufficiently high wages so that in addition to the social security benefits they can expect to get when they retire, they are able to save privately. Type 2 individuals, unfortunately, do not have the wherewithal to save on their own and thus have to rely completely on social security in the second period of their lives.[3] At any point in time, therefore, there are four different groups purchasing goods. There are the young savers, the young non-savers, the retirees who saved when they were young, and the retirees who rely completely on social security for their purchases. The following equations specify the optimization problem and constraints faced by type 1 individuals.

$$(1) \qquad \max .5[u(c^1_{01}) + u(c^1_{02})] + V(G) + .5^*[u(c^1_{11}) + u(c^1_{12})] + ^*V(G)$$

subject to:

$$(2) \qquad c^1_{0j} + B^1_1 + K^1_1 = w_j(1-t_{wj} - t_{Ij})L^1 \quad ; j = 1, 2.$$

$$(3a) \qquad c^1_{1j} = (1+r^k_{1j})K^1_1 + (1+r^B_1) B^1_1 + T^1_{1j} \quad ; j = 1, 2.$$

$$(3b) \qquad c^1_{1j} = (1+r^k_{1j} (1-t_{Ij}))K^1_1 + (1+r^B_1 (1 - t_{Ij})) B^1_1 + T^1_{1j} \quad ; j = 1, 2.$$

In equation (1) u(.) and V(.) are strictly convex utility functions and * is a factor of time preference. Individuals of type 1 maximize utility, u(.), over the two periods and in each period there are two possible states that can occur with probability .5. Individuals are denoted by superscripts and subscripts are used to denote first the period and second the state of nature. The arguments of the utility function are consumption, c, and a publicly provided private good (hereafter referred to as PPPG). The publicly provided private good, G, is considered to be a given constant by individuals. It does not vary either across period or across state. (The analysis would not be altered if the supply of public private goods were to vary across states although the utility level would likely be affected.) For simplicity, we assume that the utility function is separable between privately produced goods and publicly provided ones. Furthermore, in the actual analysis, the utility from consumption is assumed to

take the constant relative risk aversion functional form. (While the functional form for the utility of public goods is less relevant for the analysis, in the example section we assume that the utility of public goods also takes the constant relative risk aversion form.) Equation (2) denotes the budget constraint faced by type 1 individuals when they are young. Type 1 individuals decide how much to consume when they are young, c_{0j}^1 , j = 1,2, and how much to save. Private saving can be done through either bond holding, B_1^1, or the purchase of private capital, K_1^1, or both. The income of a type 1 individual when young consists of after-tax income from labor, $w_j(1-t_{wj} - t_{Ij})L^1$. The labor supply, L^1, can be viewed as efficiency labor and thus the amount provided can differ between type 1 and type 2 individuals. The wage rate, w_j , is state dependent. Young people face two types of taxes, the payroll tax that is used to finance social security, t_{wj} and an income tax, t_{Ij}. Both types of taxes are state dependent. When they are old, type 1 individuals face budget constraints (3). Their income consists of the after tax return on capital income, where the rate of return on capital is, r_{1j}^k, the after tax return on the income from bonds, where the rate of return on bonds is, r_1^B, and the amount received from social security, T_{1j}^1. It should be noted that all sources of income (including social security payments) are taxed at the income tax rate in the example to be found in section IV (equation (3b)), but only labor income is taxed in the model used in section III (equation (3a)). In the discussion of the model, equations that have a number followed by "a" refer to the version of the model where only labor income faces the income tax, and equations with a number followed by "b" refer to the more complete model where both labor and capital income face the income tax. The total amount received by type 1 individuals from social security when they are retired can be denoted as:

(4a) $T_{1j}^1 = (1+r_{1j}^k)K_g^1 + (1+r_1^B)\, B_g^1 \quad ; j = 1,2,$

(4b) $T_{1j}^1 = (1+r_{1j}^k\,(1-t_{Ij}))K_g^1 + (1+r_1^B\,(1 - t_{Ij}))\, B_g^1 \quad ; j = 1,2,$

where B_g^1 and K_g^1 are the amounts of bonds and capital stock, respectively bought by social security for the type 1's. In the more complete model, even the income earned on the bonds and stocks held by the social security trust fund is taxed. It should be noted that social security is fully funded and, therefore,

(5) $B_g^i + K_g^i = w_j t_{wj} L^i \; ; i = 1,2, \; j = 1,2.$

(Since B_g^i and K_g^i are defined benefits and are thus invariant to the state of nature, the payroll tax must

vary across states.)

The allocation of savings by type 1 individuals between stocks and bonds can be found by substituting the budget constraints, equations (2) and (3), into equation (1). Individuals are left with two choice variables, the amount that will be saved through bonds, B_1^1, and the amount of capital that will be purchased, K_1^1 . Optimizing over the two choice variables yields the first order conditions:

(6a) $-[uN(c_{01}^1) + uN(c_{02}^1)] + * [uN(c_{11}^1) + uN(c_{12}^1)](1+r_1^B) = 0$

(7a) $-[uN(c_{01}^1) + uN(c_{02}^1)] + * [uN(c_{11}^1)(1+r_{11}^k) + uN(c_{12}^1)(1+r_{12}^k)] = 0$

(6b) $-[uN(c_{01}^1) + uN(c_{02}^1)] + * [uN(c_{11}^1)(1+r_1^B (1 - t_{I1})) + uN(c_{12}^1)(1+r_1^B (1 - t_{I2}))] = 0$

(7b) $-[uN(c_{01}^1) + uN(c_{02}^1)] + * [uN(c_{11}^1)(1+r_{11}^k (1 - t_{I1})) + uN(c_{12}^1)(1+r_{12}^k (1 - t_{I2}))] = 0$

Type 2 individuals face a much easier problem since they do not save. Their utility function looks similar to the one for individuals of type 1, but their budget constraints are much simpler. When young, their budget constraint is:

(8) $c_{0j}^2 = w_j(1-t_{wj} - t_{Ij})L^2$; j = 1, 2.

and when old it becomes:

(9) $c_{1j}^2 = T_{1j}^2$; j = 1, 2.

One sees immediately that the type 2 individuals rely solely on social security payments to finance consumption when they are old. The amount of these payments are:

(10a) $T_{1j}^2 = (1+r_{1j}^k)K_g^2 + (1+r_1^B) B_g^2$; j = 1,2.

(10b) $T_{1j}^2 = (1+r_{1j}^k (1-t_{Ij}))K_g^2 + (1+r_1^B (1 - t_{Ij})) B_g^2$; j = 1,2.

Social security purchases assets on behalf of young people through the payroll tax; however, when these people retire, the payment they will receive will come from the gross return earned on the assets held by social security on their behalf. While the return earned on private capital is the marginal product of capital, the return that the government makes on the money it borrows through the sale of bonds is the return received from the supply of public goods. Unfortunately, it is not easy to calculate the return generated by the public goods; however, one can argue that the amount the government pays out in interest payments should be a payment for the provision of the public goods. Since the money borrowed is used for the provision of public goods, we feel that the money required to refund the debt

should be raised through an income tax and not the payroll tax. (While the government in addition to borrowing also raises money for public goods through taxation, since our focus is on the shift of investment strategy by the social security trust fund, we ignore that source of financing.) Therefore, the income tax rate for each state j can be found from the following equation:

(11a) $t_{Ij} \{w_j (L^1 + L^2)\} = (1+r_1^B)(B_g^1 + B_g^2 + B_1^1)$.

(11b) $t_{Ij} \{w_j (L^1 + L^2) + r_{1j}^k (K_g^1 + K_g^2 + K_1^1) + r_1^B (B_g^1 + B_g^2 + B_1^1)\} = (1+r_1^B)(B_g^1 + B_g^2 + B_1^1)$.

As indicated above, all source of income are taxed to pay for the provision of the public goods in the more complete model.

The supply of public goods can now be seen to be equal to

(12) $(B_g^1 + B_g^2 + B_1^1) = 4G$

As mentioned earlier, the government uses the funds raised from selling bonds to both the public and the Social Security Administration to provide individuals with a private good (such as education) that will be identical for all citizens. This model, therefore, introduces a type of symmetry. Bonds issued by the government raise funds for the provision of government supplied goods; equity issued by the private sector raises money for the purchase of private capital. While the funds raised by the government could be used to provide both public goods and government produced private goods, to keep the analysis simple, we restrict government activities to the latter. It should be noted again that we are not trying to imply that the only source of funding for publicly supplied goods is the issuance of government bonds. Most of the funding for public spending comes from taxation such as the personal and corporate income taxes. However, the social security trust fund, as long as at least part of the funds are invested in U.S. government securities, gives the government the potential wherewithal to increase its supply of publicly supplied goods over and beyond what would have been possible if the trust fund were not available. Since the other taxes and the public goods that can be supplied with that revenue do not affect our argument, for simplicity, we omit them here.

To complete the model we need to introduce the production side. We assume a standard constant returns to scale Cobb-Douglas production function that takes the form:

(13) $Y_j = 2_j K^{\$} L^{1-\$}$; j= 1,2,

where $K = K_g^1 + K_g^2 + K_1^1$

and $L = L^1 + L^2$

The θ_j 's refer to the level of productivity in a particular state j, Y_j to the level of output in state j, K to the aggregate size of the capital stock, and L to the size of the total labor force. Finally, the rates of return on the factors of production can be written as:

(14) $\partial Y/\partial L = \theta_j (1-\alpha)K^{\alpha}L^{-\alpha} = w_j \quad ; j= 1,2.$

and

(15) $\partial Y/\partial K = \theta_j \alpha K^{\alpha-1},L^{1-\alpha} = r_{1j}^k \quad ; \quad j = 1,2.$

Since the labor supply in this model is exogenous, the rates of return on capital and labor will only be affected by changes in the size of the aggregate capital stock.

To permit this model to be analyzed one also needs to specify the policy pursued by the government and the Social Security Administration. It is assumed that the payroll tax rate in state one, t_{w1}, is set as a policy parameter. However, since the amount of bonds and capital purchased by the social security trust fund is independent of the state that occurs, t_{w2} is an endogenous variable. In addition to setting the payroll tax rate in state one, social security must make a decision on the composition of the trust fund. It is assumed that the amount of bonds purchased can be represented as:

(16) $B_g^i = \phi A_g^i \quad ; i = 1,2$

and the purchase of stock as

(17) $K_g^i = (1-\phi)A_g^i \quad ; i = 1,2 ,$

where A_g^i is the total amount of assets purchased by social security on behalf of individual i, and ϕ represents the proportion of that total purchase that will be in the form of bonds. The parameter ϕ is the policy parameter used by the Social Security Administration in allocating purchases between bonds and stocks.

To summarize, the model consists of the equations (2)-(17) in the unknowns c_{0j}^1 , c_{1j}^1 , T_{1j}^1 , t_{w2} , A_g^1 , B_1^1 , K_1^1 , c_{0j}^2 , c_{1j}^2 , T_{1j}^2 , t_{lj} , r_1^B , Y_j , w_j , r_{1j}^k , B_g^i , and K_g^i .

III Implications of the Theoretical Model

In this section we use the theoretical model specified in the previous section to analyze the impact of an alteration in the supply of public goods, G, and a change in the proportion of the trust fund held in the form of government bonds, (. To analyze the model, it needs to be differentiated and placed in matrix form. Since that exercise is largely technical in nature and does not provide a great deal of intuition, that analysis is placed in the appendix.

The macroeconomic effects of a change in the size of G are quite intuitive, but we discuss them here to emphasize that if the goal is to increase the size of the private capital stock, it is this change that is crucial either by itself or at least in conjunction with the change in the composition of the assets held by the social security trust fund. As discussed in the appendix, a decrease in the supply of public goods, G, lowers the financing requirements of the government and thus reduces the amount of government bonds that need to be issued to pay for the public goods supplied. Private savers will in turn not be able to obtain as many government bonds as before the change, and thus, need to shift their portfolio so that it contains more private capital. The increase in the purchase of private capital by the savers will result in an increase in the size of the private capital stock. Moreover, the larger aggregate private capital stock, through the production function, produces an increase in the wage rate of labor and a reduction in the rate of return on capital. Finally, the reduction in the supply of government bonds leads to a relative shortage of bonds and thus reduces the rate of return on bonds. An increase in G, of course, has the exact opposite impact.

There is a great deal of discussion taking place concerning what should be done with the funds that will be made available from the projected surpluses that the federal government is expected to run. The above analysis suggests that returning the extra money to the private sector rather than using it for public expenditures will result in a higher private capital stock. Of course, the size of the capital stock is not the only consideration in deciding on what should be done with the funds that will become available. If one goes back to the utility function, equation (1), one notices that there is a trade-off between private and public consumption. Whether one is better off with more or fewer public goods depends on the weights individuals and the social planner place on the two types of consumption. Therefore, it is quite possible that an increase in public goods expenditures may be utility and welfare enhancing even if

that policy results in a smaller private capital stock. The welfare aspect of these trade-offs is not the focus of this paper; however, these issues are discussed in more detail in Pestieau and Possen (2000).

The second policy change considered is an increase in the proportion of the social security trust fund invested in private equity (a reduction in the parameter (). If all members of society were savers, such a change would have no macroeconomic repercussions. The increase in the purchase of equity by social security would be counterbalanced by a reduction in the purchase of equity by the savers for their private accounts. Since the change in (does not alter the total amount of bonds issued by the government (G remains unchanged), the total amount of bonds held by savers in their private accounts plus the bonds held by social security on their behalf would also remain unchanged. Therefore, the change in policy would have no aggregate effects. However, in our model some members of society are constrained and are unable to save privately. Thus, these individuals are unable to counteract the policy change introduced by the Social Security Administration.

If private savers were to keep their aggregate holdings unchanged (i.e., the stocks and bonds held directly as well as those held for them by social security), there would be an imbalance between supply and demand in the bond market. Since social security has reduced its purchase of bonds, the difference in the amounts of bonds purchased by social security on behalf of the non-savers before as compared to after the policy change would be left unsold. Therefore, the rate of return on bonds needs to increase to induce the private savers to purchase the extra bonds that are no longer being purchased by social security for the accounts of the non-savers. If the savers were to reduce their purchase of equity by an amount equal to the extra bonds they need to purchase, the projected income the savers would obtain after they retire would be higher than before the change in (, (either in absolute terms, or at least in terms of the return to risk trade-off) . Since savers try to equalize expected marginal utility of consumption across periods, savers will try to increase consumption in their youth and reduce consumption in their old age. However, to achieve this goal, savers will reduce their holding of equity by more than their increase in bond purchases. The impact of the decrease in (on the aggregate capital stock is, therefore, to reduce it and <u>not</u> increase it as would be one's initial reaction. Moreover, since the capital stock is reduced, the wage rate on labor falls and the marginal product of

capital rises. Furthermore, as noted above, the increase in bond holdings by the savers, B_1^1 , is exactly equal to the reduction in bond purchases by social security.

The other interesting question that can be asked is by how much does G have to fall when (is decreased in order to insure that the aggregate capital stock rises. Unfortunately, that question is harder to answer although one can get some intuition into that issue by assuming that the decrease in G is equal to the decrease in purchases of government bonds by social security for the accounts of the non-savers. In that situation, the savers can reshuffle their portfolios to keep their aggregate (private plus social security held) holdings unchanged. If there are no further adjustments that combined policy change would result in an increase in the aggregate capital stock. However, the increase in the capital stock results in a decrease in the marginal product of capital and an increase in the wage rate of labor. These changes will result in further adjustments by both the savers and the Social Security Administration. Therefore, as long as these second order effects do not outweigh the initial impact of the combined policy change, the aggregate capital stock will rise. Thus, a reduction of this magnitude would seem to be a reasonable benchmark case to use to show that a combined reduction in G and a portfolio reallocation by the social security trust fund towards more equity investment would yield an increase in the size of the aggregate capital stock.

To get some further insights into these issues, the next section presents some computer simulations on a somewhat expanded version of the theoretical model.

IV. Results from the Numerical Example

This section is intended to provide some insights into the issues raised earlier, but it does not pretend to be an example that uses parameter values that try to mimic the American or any other existing economy. The policy changes introduced in this section are relatively large so that one can easily see the various impacts one would expect from the alterations discussed. The framework used for the simulations was presented in the model section. However, we add two factors here that were not mentioned in that section. To understand how the policy changes discussed in this paper affect the

uncertainty faced by the various members of our economy, four additional equations have been added to the simulation model. They take the form:

(18) $$((c^i_{k1} - (c^i_{k1} + c^i_{k2})/2.)^2 + (c^i_{k2} - (c^i_{k1} + c^i_{k2})/2.)^2)^{.5} = \sigma^i_k \quad ; k = 0,1 \text{ and } i = 1,2.$$

These equations present the standard deviation of consumption (σ^i_k) for the four participants in this economy (young savers, old savers, young non-savers, and old non-savers). The second addition is to allow for the possibility that in the bad state the non-savers may be left with insufficient resources. As a larger proportion of the portfolio held by social security consists of equity, the average return on that portfolio will become more variable across various states of the world. In a bad state, if the trust funds hold a large proportion of stocks, it is quite possible that the income going to the non-savers in their old age will be quite low. To avoid this outcome we allow for the possibility that in the bad state the government may guarantee non-savers a minimum stipend. Thus, in the bad state of the world the consumption of the non-savers will be the maximum of c, the guaranteed minimum, and c^2_{12}, the value of consumption for the old non-savers described in the model section. This change also requires that, when the minimum guarantee is binding, an adjustment needs to be made to the value of the income tax rate in the bad state. If the non-savers are paid their minimum, money needs to be raised to cover the difference between c and c^2_{12} through the income tax.

In the simulations, the exponent of the utility of consumption, α, has a value of -2.5 and the exponent of the utility of public goods, γ, is -1.5. The level of productivity of is 3.0 in the good state, θ_1, and 2.0 in the bad state, θ_2. The labor supply of individuals of type 1, L^1, has a value of 20 and of the type 2's, L^2, is 10. In the Cobb-Douglas production function the exponent on capital, β, is .25. In addition, the payroll tax rate in state one, t_{w1}, is assumed to be .08 and the factor of time preference, δ, is given a value of .8. Finally, the initial value of the PPPG supplied by the government is 2.0. The results of the various cases discussed in this section are summarized in Table 1. The one statistic not reported in Table 1 is the value of utility of the participants in the economy. This paper is concerned with the macroeconomic implications of various policy adjustments and not whether the changes are welfare (or Pareto) improving. However, more importantly, the impact of policy changes on utility levels is very dependent on the relative magnitudes of α and γ, parameters for which it is very difficult

to come up with "true" relative values. Thus, for example, a policy that reduces the supply of PPPG and results in an increases in the size of the aggregate capital stock can yield a result that either raises or lowers utility depending on the relative magnitudes of " and <.

The first exercise performed was to shift the proportion of the trust fund devoted to the purchase of government bonds. To insure that our results in the example are not due to starting from a corner solution (the trust fund holding only bonds), we assume that the trust fund portfolio is switched from 60-40 bonds versus equity to 40-60 bonds versus equity. In the first case we assumed that the supply of PPPG remains unchanged as social security changes the composition of its assets held. As in the theoretical section, one finds (from Table 1) that the size of the aggregate capital stock is reduced (from 3.483 to 3.452) and the rates of return on equity and bonds are increased. While the reduction in the size of the capital stock is relatively small, the change, nonetheless, is in a direction opposite of what one would expect. Also, one finds that the policy change results in a shift in the uncertainty faced by the various participants in the economy. The young savers find that the standard deviation of the fluctuation in their consumption is reduced (from 6.074 to 6.058), the old savers likewise face less uncertainty (2.831 to 2.666), and the young non-savers also face less risk (3.037 to 3.0288). However, the retired non-savers face a large increase in the risk that they must face (.441 to .602). Thus, while the expected consumption level of the old non-savers rises from (4.007 + 3.384)/2 = 3.695 to (4.178 + 3.326)/2 = 3.752, they face a larger difference between the amount they can consume in good times as compared to the bad. Moreover, the average consumption level of all the other participants in the economy is reduced by this shift to a higher proportion of capital in the portfolio of the social security trust fund (average consumption of the young savers falls from 14.03 to 13.99, average consumption of the young non-savers falls from 8.18 to 8.15, and average consumption of the old savers falls from 15.87 to 15.80). The young members of the economy see a reduction in the average level of consumption as a consequence of the reduction in the wage rate that is a result of the policy change; whereas, the old savers trade off a lower expected consumption for less risk. Thus, unlike what one normally expects from such a change, the only group that has its average consumption level raised is the old non-saving group and they get the increase in average consumption at the expense

of a large increase in uncertainty. This example shows that the widespread benefit that has been claimed would be received from a switch in the portfolio of the trust fund may not actually occur.

If, on the other hand, social security would like to make the switch but the government wants to limit the downside risk faced by the retired non-savers (for example, not allow their income to fall to 3.326 as in the case presented above) when the social security trust fund increases its holding of equity to 60-40, it could guarantee the non-saving old a guaranteed minimum if the bad state occurs. If the government sets this minimum income at 3.35, one finds that the aggregate capital stock falls to 3.436 instead of to 3.452, the amount obtained when no minimum guarantee is in place. This reduction to 3.436 in the aggregate capital stock results in an even lower wage rate on labor and, therefore, an expected consumption level for both the young savers and non-savers that is even lower than without the guaranteed minimum (average consumption for the young savers falls to 13.963, and average consumption for the young non-savers becomes 8.129). Furthermore, the lower capital stock and higher tax payments result in a lower expected consumption level for the old savers, their average consumption being reduced to 15.772. Thus, the only beneficiaries from the guaranteed minimum are the old non-savers, a result that is not surprising. In fact, the old non-savers have a higher expected consumption level with less uncertainty when there is a guaranteed minimum than when the government provides no such safety net. Moreover, if the government increases the guaranteed minimum (in our model we also allowed the minimum to be 3.375), the expected consumption of the non-saving old rises even more and the uncertainty they face is reduced even further. Their average consumption rises to 3.78 and the standard deviation of consumption falls to .574. Furthermore, the expected consumption of all the other participants in the economy as well as the size of the aggregate capital stock fall even further as the guaranteed minimum gets raised. Even with the guaranteed minimum, the uncertainty faced by the old non-savers is higher than the fluctuations they encountered before social security changed the composition of the trust fund. However, the government (once the trust fund switches to a higher proportion of equity) by reducing the uncertainty and raising the average consumption when it provides the guaranteed minimum, provides the retired non-savers with a benefit (or asset) whose reward-uncertainty trade-off is unavailable to any other individuals. Moreover, their

benefit comes at the expense of all the other participants in the economy. Thus, the old non-savers would clearly be in favor of this switch to a larger holding of equity in their accounts at the social security trust fund and a guaranteed minimum, but whether society as a whole should acquiesce to such a change given that everyone else is made worse off is another question. [4]

The third case explored is one where the government reduces the supply of PPPG in conjunction with the shift in composition of the trust fund from 60-40 bonds versus equity to 40-60. The reduction in PPPG is equal to the reduction in government bonds bought by the trust fund on behalf of the non-savers. While a smaller reduction in PPPG would have provided similar results, the change in PPPG used here is the minimum amount one can lower PPPG to be sure that the reduction in PPPG in conjunction with the shift in the portfolio allocation of the social security trust fund will result in an increase in the aggregate capital stock in a wide class of examples. The most striking outcome of this case is that for even such a relatively small reduction in PPPG the increase in the size of the capital stock as social security switches from 60-40 to 40-60 bonds versus equity is quite substantial (from 3.483 to 4.581, a 31.5% increase). The policy change also increases the uncertainty faced by all the individuals in the economy (the standard deviation of the young savers goes from 6.07 to 6.56, that of the old savers goes from 2.831 to 2.901, the young non-savers see it go from 3.037 to 3.281, and the old non-savers see an increase from .441 to .518) as well as the expected consumption of all the young and the old savers (average consumption of the young savers goes from 14.029 to 15.372, that of the young non-savers from 8.18 to 9.20, and that of the old savers from 15.87 to 16.92). The one surprise is that the expected consumption level of the retired non-savers actually declines (from 3.695 to 3.579). The reason for the decrease appears to be that the decline in return on bonds and equity outweighs the effect of the switch to a larger proportion of equity in the account of the non-savers at social security. The change in policy results in a large increase in income for the young (through the increase in the wage rate on labor). The young savers are able to shift some of that income through the purchase of private assets to their old age and this shift allows the expected consumption of the old savers to rise. However, the young non-savers are not able to shift any consumption directly to their old age. (One can verify that the average consumption of the non-savers over their life-time is higher under the new

policy.) We have not placed much weight in our analysis on the welfare implications of these switches. However, for the parameter values used for our utility functions the switch to a larger proportion of equity in the trust fund and the reduction in the supply of PPPG results in a decline in utility for both the non-savers and savers. However, we could have changed the parameters in the utility function (especially the exponent on the utility from public goods) and turned that result around.

It is interesting as a point of comparison to look at the case where the government reduces the supply of PPPG by the same amount as in the previous example but social security does not change the portfolio allocation of its trust fund. In that situation, the aggregate capital stock rises from 3.483 to 4.592 (slightly more than the 4.581 one obtains when the trust fund reallocates its portfolio to 40-60 bonds versus equity). In this case, the young savers and non-savers and the old savers find that their average consumption rises (and slightly more than when social security reallocates its trust fund), and the uncertainty they face rises (and somewhat more than when the social security trust fund is reallocated.). However, the old non-savers find that their average consumption level falls to an average of 3.532 (somewhat more than to the average of 3.538 obtained when social security reallocates its portfolio), although the uncertainty they face declines (from a standard deviation of .441 to .373) instead of increasing as when there is the portfolio allocation as well (from .441 to .518). Thus, the extra benefit one obtains from social security portfolio reallocation appears to be relatively minor. It is really the reduction in the supply of PPPG that is the driving force contributing to the large changes in the size of the aggregate capital stock and alterations in average consumption and uncertainty of the various agents. Finally, one should point out again, that while the reduction in PPPG raises the capital stock and average consumption of all agents except the old non-savers, the reduction in PPPG can result in a reduction in the utility of the agents (as we obtained for our parameter values). Therefore, the trade-off between PPPG and private goods should be considered in deciding whether one wants to implement the policy.

The last case studied is one where social security switches its trust portfolio towards more equity, the government reduces the supply of PPPG, and the government provides a minimum guaranteed income to the non-saving old in the bad state of the world. When the government provides

such a guarantee (e.g., 3.35), the size of the capital stock is lower than when the guarantee is absent. However, the reduction in the size of the capital stock is quite small (from 4.581 to 4.535) and it is still quite a bit larger than its size (3.483) before the policy change that altered the composition of the social security portfolio and reduced the supply of PPPG. The introduction of the guaranteed minimum also causes a little reduction in average consumption and a small increase in the uncertainty of the young savers, the young non-savers, and the old savers as compared to when there is no minimum guarantee in place. These results can be seen in Table 1.) As expected, the average consumption of the old non-savers is increased (from 3.579 to 3.652) and the uncertainty they face goes down (from .518 to .427). It is interesting, however, that the consumption of the old non-savers rises in the good state as well as the bad when the minimum guarantee is introduced. As discussed earlier, the higher rate of return on bonds and equity results in the increase in the consumption of the old non-savers in the good state. Unlike the savers, they are not able to shift consumption across periods and thus cannot mitigate the reduction in consumption when they are young. Finally, we ran the model for the case where the guaranteed minimum is set at a value of 3.375. However, raising the minimum guarantee of the government resulted in no surprises. The results were similar to the changes that were obtained in going from no guarantee to a guarantee of 3.35 except that in comparing the no guarantee case with the one where the guarantee is 3.375 one found that the magnitudes of the changes are larger.

V. Conclusions

Over the years it has been suggested that social security should shift its portfolio allocation so that at least some of its funds are used to purchase risky equity. The hope from this strategy is that the shift would help to alleviate some of social security's financial problems, give all retirees a 'fairer' rate of return, and help out the economy by increasing the aggregate size of the capital stock. In this paper we have shown that such a shift would have at best a limited effect in achieving these goals. We find that the only group that sees their average consumption rise would be the non-saving retirees and they get this increase at the expense of a large increase in uncertainty. All the other participants (the young

savers, the young non-savers, and the retired savers) find that their average level of consumption falls. Moreover, the policy change results in a decline in the aggregate capital stock and not the increase usually claimed. Thus, while the poorer retirees will see an increase in their expected consumption level, it is not clear that a shift in policy by social security to more equity holding would be beneficial for society as a whole.

One remedy for the limited success of the previous policy would be to combine the shift in portfolio allocation with a reduction in the supply of publicly provided goods by the rest of the government. In that case one does get the expected, and in fact, fairly large increase in the size of the aggregate capital stock. Furthermore, the average consumption level of all participants except the retired non-savers rises although with a comparable increase in uncertainty. Unfortunately, the retired non-savers find that not only does their average level of consumption fall they also face an increase in uncertainty. One remedy could be to provide a minimum guarantee for the retired non-savers to insure that their income does not fall too much when the economy faces bad times. While that policy would raise the average consumption level of the non-saving poor (although still leave it lower than their consumption level in the absence of any policy changes), it would lower the consumption level of all other members of society and leave the size of the capital stock lower than it would be if there were no guarantee (although the amount is still higher than the magnitude of the capital stock in the absence of the reduction in the supply of public goods). Thus, while the capital stock would be increased and most individuals would be made better off by the reduction in the supply of public goods, it is not clear that this policy would be beneficial for the economy.

The policy of reducing the supply of public goods has the additional drawback that one is now trading off publicly provided goods for private goods. Whether such a shift is welfare improving depends on whether before the change there is an undersupply or oversupply of public goods.[5] Of course, if one is only interested in raising the size of the aggregate capital stock then reducing the supply of public goods will yield the desired result. In fact, if one's main focus is on raising the size of the capital stock, the policy that would provide the largest increase in the private capital stock would be one which lowers the supply of public goods but leaves the portfolio of the social security trust fund

unchanged.

Some of the arguments that have been introduced to support a shift in the composition of the trust fund towards a larger equity holding are beyond the scope of this paper. Some advocates of the shift may have the secret agenda of trying to reduce or eliminate redistribution within social security by, in effect, privatizing social security. However, the shift in portfolio allocation will increase the fluctuation in average consumption of the poor retirees. If the government is concerned about the low level of consumption of the poor non-saving retirees in the bad state, it may be forced to introduce a guaranteed minimum stipend. If it were to do so, it would in effect be increasing the amount of redistribution being provided. Moreover, the guarantee would lower the average consumption level of all the other members of society as well as force them to take on the extra risk that has been removed from the shoulders of the poor retirees. Nor do we consider the argument that equity provides a higher return than can be attributed to risk differences as normally measured. This large difference in returns is known as the equity premium puzzle and any discussion thereof is beyond the scope of the present paper.[6] [7] Finally, we do not consider some of the more standard rationales for the establishment of public retirement systems such as myopia of consumers towards retirement savings or the micro uncertainty in life duration.

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Endnotes

1. Aaron and Shoven (1999), Dotsey (1997), Feldstein and Samwick (1998), Geanakopolos and Diamond (1999), Kotlikoff (1998), and Smetters (1997a, 1997b).

2. For a discussion of these issues see Belan and Pestieau (1999) and Pestieau and Possen (2000).

3. The type 2's cannot borrow against future social security benefits in order to invest privately in the stock market.

4. Clearly a Rawlsian objective function would lead to such a measure.

5. For a more complete discussion of these issues see Pestieau and Possen (2000) and Possen and Slutsky (1980).

6. One survey of the equity premium puzzle is Kocherlakota (1996). As explained in that survey, the large equity premium of stock returns compared to the returns of Treasury bills, is still largely a mystery to economists. For us to try to explore that issue would lead us too fare afield from our main focus.

7. One other argument made is that if one could shift the investment of the social security trust fund towards a greater holding of equity one might also be able to give individuals more choice in terms of the types of equity investment social security undertakes on their behalf. However, if social security were to set up these types of "individualized" accounts transactions costs would be increased reducing the benefits of the higher gross returns from stocks as compared to safe bonds. For a discussion of these and related issues see Diamond (1997).

(A1)

Appendix

If one substitutes equations (2)-(5), (8)-(11), (16), and (17) into equations (6), (7), (12), (14), and (15) and differentiates the resultant model, one can express the results as

$$\begin{bmatrix} A_{11}A_{12}A_{13}A_{14}A_{15} \\ A_{21}A_{22}A_{23}A_{24}A_{25} \\ A_{31}0\ A_{33}0\ 0 \\ 0\ 0\ A_{43}0\ 1 \\ 0\ 0\ A_{53}1\ 0 \end{bmatrix} \begin{bmatrix} dk_1^1 \\ dr_1^B \\ dw_1^1 \\ dB_1^1 \\ dr_{11}^k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} dg + \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \\ D_5 \end{bmatrix} dg$$

Where :

$A_{11} = -Q - ½ * "\ [(c_{11}^1)^{-a-1}(1 + r_{11}^k)^2 + ((c_{12}^1)^{-a-1}(1 + r_{12}^k)^2]$

$A_{12} = -QG\ \frac{L^1}{L} - ½\ * "\ T_1[(c_{11}^1)^{-a-1}(1 + r_{11}^k)) + (c_{12}^1)^{-a-1}(1 + r_{12}^k)]$

$A_{13} = ½\ "\ L^1[(c_{01}^1)^{-a-1})(1 - t_{w1}(1 + ((1 + r_1^B))) + (c_{02}^1)^{-a-1}(1 - \frac{q_1}{q_2} t_{w1})(1 + ((1 + r_1^B))\frac{q_1}{q_2}] - ½ * "\ t_{w1} L^1[(c_{11}^1)^{-a-1}(1 + r_{11}^k)[(1 + r_{11}^k)(1 - () + (1 + r_1^B)(] + (c_{12}^1)^{-a-1}$

$(1 + r_{12}^k)[(1 + r_{12}^k)(1 - () + (1 + r_1^B)(]]$

$A_{14} = -Q(1 + (1 + r_1^B)\frac{L^1}{L} - ½ * "\ (1 + r_1^B)[(c_{11}^1)^{-a-1}(1 + r_{11}^k) + ((c_{12}^1)^{-a-1}(1 + r_{12}^k)]$

$A_{15} = ½ * [(c_{11}^1)^{-a-1}\{c_{11}^1 - "\ S_1(1 + r_{11}^k)\} + (c_{12}^1)^{-a-1}\{c_{12}^1 - "\ S_1(1 + r_{12}^k)\}\frac{q_2}{q_1}]$

$A_{21} = -Q - ½ * "\ (1 + r_1^B)\ [(c_{11}^1)^{-a-1}(1 + r_{11}^k) + (c_{12}^1)^{-a-1}(1 + r_{12}^k)]$

$A_{22} = -QG\ \frac{L^1}{L} + ½ * [(c_{11}^1)^{-a-1}\{c_{11}^1 - "T_1(1 + r_1^B)\} + (c_{12}^1)^{-a-1}\{c_{12}^1 - "T_1(1 + r_1^B)\}]$

$A_{23} = ½\ "\ L^1[(c_{01}^1)^{-a-1}(1 - t_{w1}(1 + ((1 + r_1^B))) + (c_{02}^1)^{-a-1})(1 - \frac{q_1}{q_2} t_{w1}(1 + ((1 +$

$r_1^B)) \frac{q_2}{q_1}]$- ½ * " $t_{w1} L^1 (1+ r_1^B) [(c_{11}^1)^{-a-1} [(1+ r_{11}^k) (1-$ ( ) + (1 + r_1^B) ( ] + $(c_{12}^1)^{-a-1}$
$[(1 + r_{12}^k)(1 -$ ( ) + (1 + r_1^B) ( ]]

A_{24} = -Q (1 + (1 + r_1^B)) $\frac{L^1}{L}$ - ½ " * (1 + r_1^B)^2 $[(c_{11}^1)^{-a-1} + (c_{12}^1)^{-a-1}]$

A_{25} = - ½ * " (1 + r_1^B) S_1 $[(c_{11}^1)^{-a-1} +$ $(c_{12}^1)^{-a-1} \frac{q_2}{q_1}]$

A_{31} = - $\frac{(1-b)}{L}$ r_{11}^k

A_{33} = [1 - (1-$) (1- ( ) $r_{11}^k t_{w1}$]

A_{43} = $\frac{L}{K}$

A_{53} = (t_{w1} L

D_1 = -Q $(A_g^1 + A_g^2)$ (l+ r_1^B) $\frac{L^1}{L}$ - ½ * " $A_g^1 [(c_{11}^1)^{-a-1}$ (r_1^B - r_{11}^k) (1+ r_{11}^k) +$((c_{12}^1)^{-a-1}$
(r_1^B - r_{12}^k) (l + r_{12}^k)]

D_2 = -Q ($A_g^1 + A_g^2$) $\frac{L^1}{L}$ (l+ r_1^B) - ½ * " A_g^1 (1 + r_1^B)$[(c_{11}^1)^{-a-1}$ (r_1^B – r_{11}^k) +
$((c_{12}^1)^{-a-1}$ (r_1^B - r_{12}^k)]

D_3 = - (1 - $) $r_{11}^k w_1 t_{w1}$

D_5 = - $w_1 t_{w1}$ L

Q = ½ " $[(c_{01}^1)^{-a-1} + (c_{02}^1)^{-a-1}]$

T_1 = B_1^1 + (A_g^1

S_1 = K_1^1 + (l - ( ) A_g^1

Also, the utility of consumption takes the functional form $(c)^{1-"}$ and, therefore, " represents the degree of risk aversion.

One condition that is assumed to hold and that helps insure that the results discussed below go through is:

(A1) $(c_{11}^1)^{-a-1}(1 + r_{11}^k)(r_{11}^k - r_1^B) + (c_{12}^1)^{-a-1}(1+ r_{12}^k)(r_{12}^k - r_1^B) < 0$

When there are large variations in consumption across states this condition is likely to be satisfied.

For parameter values that are within reasonable ranges one can show that the sign of the determinant of the model,), is positive. By reasonable we mean payroll tax rates, for example, less than 50% and average interest rates within empirically reasonable ranges. (In the example in section IV we used average interest rates greater than 100% and still the determinant had a positive sign).

The first exercise performed is to find the impact of an increase in government expenditures, G. The effect of an increase in G on B_1^1 is to increase it as can be seen from an inspection of equation (12). The effect of a change in G on w_1, r_{11}^k, and K_1^1 depends on the sign of $[A_{12} A_{24} - A_{22} A_{14}]$. That term can be shown to be positive. Therefore, an increase in G reduces the size of the capital

since $$\frac{dk_1^1}{dG} = \frac{-4[A_{12}A_{24} - A_{22}A_{14}]A_{33}}{D},$$

reduces the wage rate since

$$\frac{dw_1^1}{dG} = A_{31}\frac{[A_{12}A_{24} - A_{22}A_{14}]}{D}$$

and increases the rate of return on capital since

$$\frac{dr_{11}^k}{dG} = \frac{-A_{43}A_{31}[A_{12}A_{24} - A_{22}A_{14}]}{D}$$

The definition of K can be written as:

(A2) $K = (1-\gamma) w_1 t_{w1} L + K_1^1$ since: $A_g^1 + A_g^2 = t_{w1} w_1 L$

Therefore, $\frac{dk}{dG} = (1-g) t_{w1} L \frac{dw_1}{dG} + \frac{dK_1^1}{dG}$ and hence an increase in G raises the size of aggregate capital stock.. The impact of G on r_1^B is somewhat more complicated, but it can be shown that the increase in G raises r_1^B , the rate of return on government bonds.

The second policy change was to determine the impact of an increase (or decrease) in θ, the proportion of the social security trust fund invested in government bonds. Unfortunately, the solution for a change in θ is more complicated than for a change in G. The effect of a higher θ on the wage rate is:

$$\frac{dw_1}{d\theta} = \frac{D_5 A_{31} [A_{12} A_{24} - A_{22} A_{14}] - A_{31}[A_{12} D_2 - A_{22} D_1] - D_3[A_{11} A_{22} - A_{21} A_{12}]}{\Delta}$$

One can verify that a sufficient condition for an increases in θ to raise the wage rate of labor is (A1). One can also note that the first term of $\frac{dw_1}{d\theta}$ is very similar to the numeration of $\frac{dw_1}{dG}$ except that the term D5 in $\frac{dw_1}{d\theta}$ means that they have opposite signs. While it appears that a change G has a larger impact on w_1 , and the other variables than does a change in θ, that result cannot be proven theoretically for a broad range of parameter values. If one differentiates equation (14), one can also verify that

$$\frac{dK_1}{d\theta} = \frac{K}{\beta W_1} \frac{dw_1}{d\theta}$$

and therefore, the increase in θ raises the size of the aggregate capital stock. If one then differentiates equation (15) one obtains:

$$\frac{dr_{11}^k}{d\theta} = \frac{(\beta - 1) r_{11}^k}{K} \frac{dK}{d\theta}$$

and one therefore has the result that the increase in θ reduces the rate of return on capital.
If one now differentiates equation (A2), the relationship that provides a definition of K, as well as

equation (12), that provides a definition of w_1, with respect to (and eliminates $\frac{dw_1}{d\gamma}$, one obtains:

(A5)

$$\frac{dk_1^1}{d\gamma} = \frac{dK}{d\gamma}(1 - t_{w1}(1-\gamma)(1-\beta)r_{11}^k) + t_{w1}Lw_1$$

Therefore, the amount of capital purchased by the savers for their private accounts is also increased when (is raised. While an increase in (with G unchanged leaves the total amount of bonds issued unchanged, the increase in (means that more bonds will be purchased by the social security trust fund and a smaller amount made available for purchase by the savers. Therefore

$\frac{dB_1^1}{d\gamma} < 0$. This result can also be obtained from the matrix. Finally, one can also show that the rate of return on government bonds falls, i.e. $\frac{dr_1^B}{d\gamma} < 0$.

(	G	Guaranteed minimum	c_{01}^1	c_{02}^1	c_{11}^1	c_{12}^1	c_{01}^2	c_{02}^2	c_{11}^2	c_{12}^2	K_1^1	K	s_0^1	s_1^1	s_0^2	s_1^2
60% bonds 40%equity	2	0.0	18.32	9.73	17.88	13.87	10.327	6.033	4.007	3.384	2.222	3.483	6.074	2.831	3.037	.441
40%bonds 60%equity	2	0.0	18.27	9.70	17.68	13.91	10.288	6.004	4.178	3.326	1.565	3.452	6.058	2.666	3.0288	.602
40%bonds 60%equity	2	3.35	18.25	9.68	17.66	13.89	10.270	5.987	4.182	3.350	1.551	3.436	6.057	2.667	3.0284	.588
40%bonds 60%equity	2	3.375	18.22	9.66	17.63	13.86	10.252	5.970	4.187	3.735	1.537	3.419	6.056	2.668	3.0279	.574
40%bonds 60%equity	1.8197	0	20.12	10.73	18.97	14.86	11.519	6.879	3.945	3.213	2.556	4.581	6.562	2.901	3.281	.518
60%bonds 40%equity	1.8197	0	20.28	10.74	19.12	14.81	11.531	6.888	3.795	3.268	3.241	4.592	6.566	3.047	3.283	.373
40%bonds 60%equity	1.8197	3.35	19.974	10.66	18.89	14.76	11.480	6.822	3.954	3.350	2.515	4.535	6.588	2.918	3.294	.427
40%bonds 60%equity	1.8197	3.375	19.967	10.64	18.88	14.75	11.473	6.811	3.956	3.375	2.507	4.526	6.593	2.921	3.297	.411

Table 1

Impact of Changes in the Portfolio Allocation of the Social Security Trust Fund

Other parameter values are: ", exponent of utility of consumption, =-2.5; <, exponent of utility of public goods, =-1.5 ; 2_1 = 3.0; 2_2 = 2.0 ;

$ = .25 ; t_{w1} = .08 L^1 = 20; L^2 = 10; * = .8;