

Distributed estimation of time-varying biases in relative state measurements

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Abstract—This note focuses on the distributed estimation of time-varying bias signals in relative state measurements of sensors, where each sensor measures the relative state of neighboring sensors and the measurements contain a time-varying bias signal that is generated from a linear exo-systems. Assume that sensors can communicate with others, we propose several distributed bias estimators, by which each sensor can reconstruct its own bias signal and the exo-system's state using local information.

I. INTRODUCTION

Recently, large-scale networks, like the robotic networks, traffic networks, and power systems, have been gaining rapid attention from researchers in various fields. To support the operation and maintain the performance of these networks, sufficiently accurate information of networks' condition should be collected from a large number of sensors, whose measurements may be coarse due to the low equality, and/or the working environment fluctuation. The errors in sensors measurements have been identified to impair networks, e.g., formation distortion, system state drift or performance degradation [1]-[4].

In this paper, we are interested in estimating the reading error of the sensor such that accurate measurement can be derived subsequently. In particular, we consider the scenario where the sensors in networks are able to measure the relative states of their neighbors and the measurement of each sensor may contain a time-varying bias signal. The problem of estimating bias or error in relative measurement has been studied in [2], [3], [5]-[8]. However, most of them focus on the bias in nonlinear measurement of relative state. For instance, [2], [5] aim at estimating and/or compensating the bias in the distance measurement (2-norm of the relative state), and [3], [6], [7] deal with biases in the orientations corresponding to relative states. Hence, their problem settings are dissimilar to ours. Moreover, the bias estimation methods proposed in these papers either cannot make the estimate of the bias converge to the actual value [5], [6], [7], or require complete or star measurement graphs [3]. The closest one is the paper [8], where the condition on the network topology and the number of biased sensors such that the biased sensors can be fully identified and all the biases can be estimated is characterized. Accordingly, several (semi) distributed bias estimation algorithms are proposed to reconstruct the biases based on the classes of the measurement graphs. Even though the problem formulation in [8] is the same as in this paper, it only considers constant bias. As a contrast, the bias signals in this paper are time-varying, a more general and common situation in real applications. We may also use the bias estimation methods in [8] for time-varying biases by running them for the measurements taken at each time instant. However, to provide timely accurate bias information for the usage in applications, the non-negligible estimation time would require that the bias signals vary slowly between two consecutive measurement times.

As the complementary problem of estimating error from incorrect relative state measurements, the problem of reconstructing accurate states has been extensively investigated by least-square regression-

based approaches [9]-[12] or norm regularization [13], [14]. However, these works assume that the statistical feature of measurement errors are known and/or the measurements are taken at one time instant, implying no consideration of time-varying bias signals.

Since biases can bring issues as mentioned before, there are works [1], [15] also studying how to mitigate the impact of constant relative state measurement biases in multi-agent consensus and formation without estimating the bias signals. The former addresses the problem of robust formation control of first-order agents with constant biases in relative state measurements. The latter proposes an adaptive compensator to suppress the problem of unbounded system state in second-order consensus brought by constant biases. The methods proposed in these two papers cannot fully eliminate the influence of biases, and achieve exact consensus [15] or formation [1].

In this paper, we assume that each node has one sensor and the relative measurements taken by the node are affected by the same bias, unlike [1], [2], [9]-[15] that have one different measurement bias or error per edge. We assume that each bias is a time-varying signal generated by a linear exo-system, which is widely used in formation control when the bias is a combination of constant and multiple sinusoidal signals [2]. This suits the scenarios where the working environmental variation is periodic, e.g., marine sensors perturbed by sea waves [16], [17]. Same as in [8], we propose bias estimation methods based on whether the measurement graph is bipartite or not. For non-bipartite graphs, we show that the ability to reconstruct the sensors biases and all exo-systems' states hinges on the observability of each exo-system. Then, for the case that each exo-system is observable and the sum of the associated dynamics matrix and the transpose is non positive, we propose a Luenberger observer, whose implementation only requires sensors to exchange local information, to estimate all bias signals. While for biases generated from general linear exo-systems, we proposed two distributed bias estimation approaches, inspired by the Luenberger observer and the bias estimator in [8]. We also show that these proposed bias estimation methods can be easily modified for the situation of bipartite measurement graphs, under the assumptions that some of the sensors are bias-free and also aware of their unbiased condition. Since the proposed estimators can deal with time-varying bias signals, they can be used in more scenarios compared with the bias estimators in [8], especially when non-constant relative states, e.g., satellite formations or vehicle platoons, are mixed with time-varying biases in the measurements.

The rest parts of the paper are organized as follows. In Section II, we introduce the notations, basic graph theory, and formulate the problem considered in this paper. In Section III, we characterize the condition under which the time-varying biases and the exo-systems' states can be reconstructed, and propose three distributed bias estimators. While in Section III, we show that the proposed estimators in Section III can also be used for bipartite graphs. Section V provides illustrative simulation and Section VI concludes the paper.

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II. PRELIMINARIES

A. basic notations and graph theory

For a collection of variables $\{z_i\}_{i=1,\dots,n}$, where $z_i \in \mathbb{R}^{m_i \times p_i}$ could be a scalar, vector and matrix, we let $\text{diag}\{z_i\}_{i=1,\dots,n} \in \mathbb{R}^{m \times p}$, with $m = \sum_{i=1}^n m_i$ and $p = \sum_{i=1}^n p_i$, represent the block diagonal matrix with the i th diagonal block equal to z_i . If all z_i are vectors, then $\text{col}\{z_i\}_{i=1,\dots,n}$ denotes the concatenation of all z_i . Let $\mathbf{1}_m$ and $\mathbf{0}_m$ denote the m -dimensional vectors with all elements equal to 1 and 0, respectively. Given a matrix A , A_i represents its i th row and a_{ij} represents its element in the i th row and j th column. If A is symmetric, then denote by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ the minimal and maximal eigenvalues of A , respectively. We let $A > (<=) 0$ represent that A is positive (non-positive) definite. The cardinality of a set \mathcal{S} is denoted by $|\mathcal{S}|$. For two sets \mathcal{S} and \mathcal{M} , we let $\mathcal{S} \setminus \mathcal{M}$ represent the complement of \mathcal{M} in \mathcal{S} .

Let the topology of an n nodes network be an undirected and connected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. Set $\mathcal{V} = \{1, \dots, n\}$ contains all the nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, where $e_{ij} := \{i, j\} \in \mathcal{E}$, or equivalently node i is a neighbor of j , means that node i can exchange information with j . Denote by \mathcal{N}_i the set of neighbors of node i and let $d_i = |\mathcal{N}_i|$. For a set $\mathcal{S} \subseteq \mathcal{V}$, let $\mathcal{V}_{\mathcal{S}}^c = \mathcal{V} \setminus \mathcal{S}$. The subgraph induced by \mathcal{S} is the graph, where the node set is \mathcal{S} and the edge set consists of all the edges in \mathcal{E} that have both endpoints in \mathcal{S} . The adjacency matrix A of \mathcal{G} is defined as $a_{ij} = 1$ if $\{i, j\} \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. For an undirected graph \mathcal{G} , arbitrary orientation can be assigned such that each edge $\{i, j\}$ has a head and a tail. The edge-node incidence matrix $B \in \mathbb{R}^{m \times n}$ of \mathcal{G} , with $m = |\mathcal{E}|$, is defined as $b_{ij} = 1$ if j is the head of edge $i \in \mathcal{E}$, $b_{ij} = -1$ if j is the tail, and $b_{ij} = 0$ otherwise. The Laplacian matrix L of \mathcal{G} can be obtained as $L = B^T B$. Matrix B can be decomposed as the head incidence matrix $B_+ \in \mathbb{R}^{m \times n}$ and the tail incidence matrix $B_- \in \mathbb{R}^{m \times n}$. The former is defined by letting $b_{+,ij} = 1$ if node j is the head of edge $i \in \mathcal{E}$ and 0 otherwise, while in the latter, $b_{-,ij} = -1$ if j is the tail, and 0 otherwise. We also let R denote the signless edge-node incidence matrix with $r_{ij} = |b_{ij}|$. Obviously, $B = B_+ + B_-$ and $R = B_+ - B_-$. Let $D = \text{diag}\{d_i\}_{i \in \mathcal{V}}$. The signless Laplacian matrix $A + D = R^T R$, and therefore, is positive semi-definite, with all the eigenvalues being real and nonnegative.

The graph \mathcal{G} is bipartite if \mathcal{V} can be partitioned into two sets \mathcal{V}_+ and \mathcal{V}_- such that no two nodes from the same set are neighbors. The following results related to the matrix property of non-bipartite graphs are used in the papers.

Lemma 1: [18] An undirected connected graph \mathcal{G} is bipartite if and only if its signless incidence matrix R does not have full column rank.

Lemma 2: [19] Consider a non-bipartite \mathcal{G} . For any subset \mathcal{S} of \mathcal{V} , let $\kappa(\mathcal{S})$ be the minimum number of edges whose removal from \mathcal{E} disconnects \mathcal{S} from $\mathcal{V} \setminus \mathcal{S}$ and results in a bipartite subgraph induced by \mathcal{S} , and denote $\psi = \min_{\mathcal{S} \subseteq \mathcal{V}} \frac{\kappa(\mathcal{S})}{|\mathcal{S}|}$, then $\lambda_{\min}(R^T R) \geq \frac{\psi^2}{4d_{\max}}$, where $d_{\max} = \max_{i \in \mathcal{V}} d_i$.

B. Problem formulation

We consider a sensor network where each sensor is identified with a node in a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $|\mathcal{V}| = n \geq 2$ and $m = |\mathcal{E}|$. Each sensor $i \in \mathcal{V}$ is associated with a state variable $q_i(t) : \mathbb{R} \rightarrow \mathbb{R}$ and able to measure the information $q_j(t) - q_i(t)$ for all $j \in \mathcal{N}_i$. We assume that each node can communicate with its neighbors, and are interested in a scenario where the measurements taken by sensors may be subjected to biases. As a result of the bias, the relative position read by sensor i at time t is given as

$$z_{ij}(t) = q_j(t) - q_i(t) + w_i(t), \quad \forall j \in \mathcal{N}_i \quad (1)$$

where $w_i(t) : \mathbb{R} \rightarrow \mathbb{R}$ is an unknown term representing the bias of sensor i at time t . For each $i \in \mathcal{V}$, we assume that $w_i(t)$ is a time-varying signal generated by the following exo-system Σ_i ,

$$\begin{aligned} \dot{v}_i(t) &= S_i v_i(t) \\ w_i &= C_i v_i(t) \end{aligned} \quad (2)$$

where $v_i(t) \in \mathbb{R}^{p_i}$, $S_i \in \mathbb{R}^{p_i \times p_i}$ and $C_i \in \mathbb{R}^{1 \times p_i}$ are the internal state, the system matrix and the output matrix of Σ_i , respectively. In this paper, we assume that S_i and C_i are known to each sensor i . For a periodic bias signal, this can be achieved by letting the sensor measure the relative state of some neighbor for a sufficiently long time when the relative state is constant, then using the Fourier analysis to estimate the frequency of the bias signal. As for the bias generated from a general linear exo-system, the subspace method [20] can be used to identify the dynamic model of the bias. We note that this assumption is widely accepted in formation control of robots in the environment with periodic background noises [2].

The presence of biases can degrade the network performance or even devastate the network. To counteract these impacts, we are interested in estimating the biases. Note that in this paper, we only consider scalar sensor states and bias signals. However, the results of the paper can be extended to the case that the sensor state and the bias signal are vectors without much effort.

To formulate the problem, we first rearrange the equality in (1) in a vector form. After assigning arbitrary orientation to \mathcal{G} , we collect in the vector $\zeta \in \mathbb{R}^m$ all the measurements z_{ij} for which node $i \in \mathcal{V}$ is the head of the edge $\{i, j\} \in \mathcal{E}$, which gives $\zeta = -Bx + B_+w$, with $B_+ \in \mathbb{R}^{m \times n}$ denoting the head incidence matrix. Similarly, we collect in the vector $\eta \in \mathbb{R}^m$ all the measurements z_{ij} for which node $i \in \mathcal{V}$ is the tail of the edge $\{i, j\} \in \mathcal{E}$, and obtain $\eta = Bx - B_-w$, where $B_- \in \mathbb{R}^{m \times n}$ is the tail incidence matrix. Hence,

$$z := \begin{bmatrix} \zeta \\ \eta \end{bmatrix} = \begin{bmatrix} -B \\ B \end{bmatrix} x + \begin{bmatrix} B_+ \\ -B_- \end{bmatrix} w \quad (3)$$

Given the measurement vector z and the exo-system matrices S_i, C_i , $i \in \mathcal{V}$, we aim to estimate the bias w_i for all $i \in \mathcal{V}$. A similar problem has been considered for constant biases in [8], where the authors showed that a vector w is the unique solution (3) in an admissible set of bias if and only if it is the unique solution in the same set to the following equation

$$\bar{w} = [I_m \ I_m] z = R w \quad (4)$$

where I_m is the identity matrix with m rows and $\bar{w} = \text{col}\{\bar{z}_e\}_{e \in \mathcal{E}}^T \in \mathbb{R}^m$. Note for each edge (i, j) , $\bar{w}_{ij} = \bar{w}_{ji}$. Based on this equivalent equation, the paper [8] showed that the ability to estimate all the constant biases depends on if the measurement graph \mathcal{G} is bipartite or not. It also proposed several distributed algorithms to estimate the constant biases.

In this paper, we focus on time-varying bias signals, hence the bias estimation approaches used in [8] do not apply readily. We will propose new methods to estimate the biases. In the later, we will first consider the situation that the measurement graph \mathcal{G} is non-bipartite, then discuss the case of bipartite graphs shortly.

III. BIAS ESTIMATORS FOR NON-BIPARTITE GRAPHS

The following assumption holds throughout this section.

Assumption 1: The measurement graph \mathcal{G} is non-bipartite.

Let $v = \text{col}\{v_i\}_{i \in \mathcal{V}} \in \mathbb{R}^p$, $S = \text{diag}\{S_i\}_{i \in \mathcal{V}} \in \mathbb{R}^{p \times p}$, with $p = \sum_{i \in \mathcal{V}} p_i$, and $C = \text{diag}\{C_i\}_{i \in \mathcal{V}} \in \mathbb{R}^{n \times p}$, then all the exo-systems and measurements can be written compactly as

$$\begin{aligned} \dot{v} &= S v \\ w &= C v \end{aligned} \quad (5)$$

$$\bar{w} = Rv = RCv \quad (6)$$

where \bar{w} is the output of the system and accessible to nodes. One way to reconstruct $w(t)$ is to transform it into the problem of observing v from \bar{w} , for which we first need to examine whether v is observable. The following result gives the sufficient and necessary condition for the observability of system (5), (6).

Lemma 3: Under assumption 1, system (5) with output (6) is observable if and only if each exo-system Σ_i with w_i as the output is observable.

Proof. First, since S and C are block diagonal matrices, system (5) with output w is observable if and only if each exo-system Σ_i with the output w_i is observable. Then by Lemma 1 and Assumption 1, R has full column rank and (4) has a unique solution. This implies that the map from w to \bar{z} defined by (6) is bijective. By these, we arrive at the conclusion. ■

This lemma implies that as long as each exo-system Σ_i that generates the bias signal w_i is observable, we are able to reconstruct the internal state vector $v(t)$ and the bias vector $w(t)$, which may be realized by using the Luenberger observer as

$$\begin{aligned} \dot{\hat{v}} &= S\hat{v} + L(\bar{w} - R\hat{w}) \\ \hat{w} &= C\hat{v} \end{aligned} \quad (7)$$

where $\hat{v} = \text{col}\{\hat{v}_i\}_{i \in \mathcal{V}} \in \mathbb{R}^p$, with $\hat{v}_i \in \mathbb{R}^{p_i}$ for $i \in \mathcal{V}$, and $\hat{w} = \text{col}\{\hat{w}_i\}_{i \in \mathcal{V}} \in \mathbb{R}^n$ are the estimate of v and w , respectively, and matrix $L \in \mathbb{R}^{p \times n}$ is selected to make $S - LRC$ Hurwitz. However, a problem may arise: for an arbitrarily selected L that satisfies the condition, the Luenberger observer may not be implemented in a distributed manner.

A. A distributed bias estimator for stable exo-systems

In the case that each exo-system Σ_i belongs to a special class of stable systems, the matrix L in observer (7) can be selected such that each sensor only needs local information. In detail, for each sensor $i \in \mathcal{V}$, we let \hat{v}_i and \hat{w}_i mentioned in the previous subsection be given as

$$\begin{aligned} \dot{\hat{v}}_i &= S_i \hat{v}_i + kC_i^\top \sum_{j \in \mathcal{N}_i} (\bar{w}_{ij} - \hat{w}_i - \hat{w}_j) \\ \hat{w}_i &= C_i \hat{v}_i \end{aligned} \quad (8)$$

where k is an arbitrary positive value. Note that each sensor only needs to exchange the measurement related variable \bar{w}_{ij} and the estimate \hat{w}_i, \hat{w}_j with its neighbors to update the internal state estimate and the bias estimate.

For each edge $e_{ij} = \{i, j\} \in \mathcal{E}$, it is easy to know $Re_{ij}\hat{w} = \hat{w}_i + \hat{w}_j$, where Re_{ij} is the e_{ij} th row of R . By this equality and the definition of R , the first differential equation in (8) can be written as

$$\begin{aligned} \dot{\hat{v}}_i &= S_i \hat{v}_i + k_i C_i^\top \sum_{\mathcal{N}_i} (\bar{w}_{ij} - Re_{ij}\hat{w}) \\ &= S_i \hat{v}_i + kC_i^\top [R^\top]_i (\bar{w} - R\hat{w}) \end{aligned}$$

Collecting the equation above for all $i \in \mathcal{V}$ yields

$$\dot{\hat{v}} = S\hat{v} + kC^\top R^\top (\bar{w} - R\hat{w}) \quad (9)$$

from which, we know that estimator (8) is indeed a Luenberger observer, with $L = kC^\top R^\top$.

For this observer, we have the following result:

Theorem 1: Consider the graph \mathcal{G} with Assumption 1 and assume that each bias w_i , with $i \in \mathcal{V}$, is generated by exo-system (2), where the pair (S_i, C_i) is observable. Let each sensor $i \in \mathcal{V}$ uses estimator (8). If each exo-system Σ_i satisfies $S_i + S_i^\top \leq 0$, then \hat{v} and \hat{w} converges exponentially fast to v and w , respectively.

Proof. Let $e_v = \hat{v} - v$ be the estimate error of v . Subtracting equation (9) by (5) and noticing $\bar{w} = Rv = RCv$ and $\hat{w} = C\hat{v}$, we have

$$\dot{e}_v = Se_v - kC^\top R^\top RCe_v \quad (10)$$

Consider a potential function $V = \frac{1}{2}e_v^\top e_v$, it is easy to have

$$\begin{aligned} \dot{V}(t) &= \frac{1}{2}e_v^\top (S^\top + S)e_v - ke_v^\top C^\top R^\top RCe_v \\ &\leq -k\|RCe_v\|_2^2 \leq 0 \end{aligned}$$

where the first inequality follows from $S^\top + S \leq 0$, since each diagonal block $S_i^\top + S_i \leq 0$ by assumption.

It is trivial that $V(e_v)$ is positive definite and radially unbounded. By the LaSalle's invariance principle, e_v should converge to the maximal invariant set \mathcal{M} in

$$\mathcal{H} := \{e_v \in \mathbb{R}^p : \dot{V} = 0\} \subseteq \{e_v \in \mathbb{R}^p : RCe_v = 0\},$$

where the \subseteq relation holds since neither of the two terms in $\dot{V}(t)$ is positive.

By Assumption 1 and Lemma 1, $RCe_v = 0$ implies $Ce_v = 0$. Putting it into (10), we have

$$\mathcal{M} \subseteq \{e_v \in \mathbb{R}^p : \dot{e}_v = Se_v \cap Ce_v = 0\}$$

By the condition that each pair (S_i, C_i) is observable, the pair (S, C) should also be observable. Hence, the maximal invariant set \mathcal{M} in \mathcal{H} is a subset of $\{e_v = 0_p\}$. Since $e_v(t) = 0_p \in \mathcal{H}$, $\mathcal{M} = \{e_v = 0_p\}$. This shows that system (10) is asymptotically stable. Since the system is linear, it is also exponentially stable. Thus we have that \hat{v} and \hat{w} converges exponentially fast to the actual internal state v and bias w , respectively. ■

Remark 1: Exo-systems Σ_i with the system matrices S_i satisfying $S_i + S_i^\top$ are stable and can be decomposed as the exponentially stable modes and the neutral stable modes. Hence, the bias signal w_i can be regarded as the composition of a constant, exponentially decay signals and multiple sinusoidal signals with different frequencies. This suits the scenario where the sensor bias comes from periodic environmental changes, as mentioned in [2].

B. Distributed bias estimation for general exo-systems

For general observable exo-systems Σ_i , it may be difficult to find a proper matrix L for observer (7) such that each sensor only exchanges local information with its neighbors. For this situation, we propose a different distributed bias estimation method. Note that if each sensor $i \in \mathcal{V}$ can access its bias signal w_i , then it can implement the Luenberger observer

$$\begin{aligned} \dot{\hat{v}}_i &= S_i \hat{v}_i + L_i(w_i - \hat{w}_i) \\ \hat{w}_i &= C_i \hat{v}_i \end{aligned} \quad (11)$$

where $L_i \in \mathbb{R}^{m_i}$ makes $S_i - L_i C_i$ Hurwitz. On the other hand, if w_i is constant for each $i \in \mathcal{V}$, it has been shown in [8] that if each sensor i adopts the estimator

$$\dot{\hat{w}}_i = k \sum_{j \in \mathcal{N}_i} (\bar{w}_{ij} - \tilde{w}_i - \tilde{w}_j) \quad (12)$$

with k an arbitrary positive value, each variable \hat{w}_i converges to the actual value of w_i exponentially fast. However, this is not the case in this paper, hence the two approaches mentioned above cannot work any more. Inspired by these two methods, we connect them together to derive a new one hereafter.

Since w_i is not directly accessible to sensor i , we replace w_i in (11) by \tilde{w}_i generated from (12). This leads to the following equation

$$\begin{aligned} \dot{\hat{v}}_i &= S_i \hat{v}_i + L_i(\tilde{w}_i - \hat{w}_i) \\ \hat{w}_i &= C_i \hat{v}_i \end{aligned} \quad (13)$$

As for tracking the time-varying signal $w_i(t)$, one can check that the subtraction of (12) by (2) leads to a term related to the bias estimation error and the extra term $-C_i S_i v_i$. Hence, in a naive way, we can add $C_i S_i v_i$ to the right-handed side of (12). However, since sensor i cannot access v_i , we replace v_i by \hat{v}_i . This yields

$$\dot{\hat{w}}_i = k \sum_{j \in \mathcal{N}_i} (\bar{w}_{ij} - \tilde{w}_i - \tilde{w}_j) + C_i S_i \hat{v}_i \quad (14)$$

Let $\hat{w} = \text{col}\{\hat{w}_i\}_{i \in \mathcal{V}}$, $\tilde{w} = \text{col}\{\tilde{w}_i\}_{i \in \mathcal{V}}$ and $L = \text{diag}\{L_i\}_{i \in \mathcal{V}} \in \mathbb{R}^{p \times n}$. Collecting equation (13) and (14) for each $i \in \mathcal{V}$ together and noticing $\hat{w}_i = C_i \hat{v}_i$, we have

$$\begin{aligned} \dot{\hat{v}} &= S\hat{v} + L(\tilde{w} - \hat{w}) = S\hat{v} + L(\tilde{w} - C\hat{v}) \\ \dot{\tilde{w}} &= -kR^\top R(\tilde{w} - w) + CS\hat{v} \end{aligned} \quad (15)$$

The following result shows that these two simple replacements lead to a locally implementable estimator of the internal state and the bias.

Theorem 2: Consider the sensor network with Assumption 1 and assume that each bias w_i , with $i \in \mathcal{V}$, is generated by exo-system (2), where the pair (S_i, C_i) is observable. Associate each sensor $i \in \mathcal{V}$ with two variables $\hat{v}_i \in \mathbb{R}^{p_i}$, $\tilde{w}_i \in \mathbb{R}$ and let them evolve as (13) and (14), respectively. If $k \geq k_i$, where k_i for each $i \in \mathcal{V}$ satisfies

$$k_i > \frac{(L_i^\top P_i + C_i S_i) Q_i^{-1} (L_i^\top P_i + C_i S_i)^\top}{2\lambda_{\min}(R^\top R)} \quad (16)$$

then \hat{v}_i and \tilde{w}_i converge exponentially fast to v_i and w_i for each $i \in \mathcal{V}$.

Proof. Let $e_v = \hat{v} - v$ and $e_w = \tilde{w} - w$ be the estimate error of v and w , respectively. Subtracting the first equation of (15) by (5), we have

$$\begin{aligned} \dot{e}_v &= S e_v + L(\tilde{w} - w + w - C\hat{v}) \\ &= (S - LC) e_v + L e_w \end{aligned}$$

Then subtracting the second equation of (15) by \dot{w} leads to

$$\begin{aligned} \dot{e}_w &= -kR^\top R e_w + CS\hat{v} - \dot{w} \\ &= -kR^\top R e_w + CS(\hat{v} - v) \\ &= -kR^\top R e_w + CS e_v \end{aligned}$$

For each $i \in \mathcal{V}$, since $S_i - L_i C_i$ is Hurwitz, there should exist two positive definite matrices $P_i, Q_i \in \mathbb{R}^{p_i \times p_i}$ satisfying the Lyapunov equation

$$(S_i - L_i C_i)^\top P_i + P_i (S_i - L_i C_i) = -Q_i$$

Let $P = \text{diag}\{P_i\}_{i \in \mathcal{V}} \in \mathbb{R}^{p \times p}$ and $Q = \text{diag}\{Q_i\}_{i \in \mathcal{V}} \in \mathbb{R}^{p \times p}$. Select the Lyapunov function as $V(t) = \frac{1}{2} e_v^\top P e_v + \frac{1}{2} e_w^\top e_w$. Apparently, V is radially unbounded and positive for all (e_v, e_w) excluding the origin. We have

$$\begin{aligned} \dot{V}(t) &= \frac{1}{2} \dot{e}_v^\top P e_v + \frac{1}{2} e_v^\top P \dot{e}_v + e_w^\top \dot{e}_w \\ &= \sum_{i \in \mathcal{V}} \left(\frac{1}{2} \dot{e}_{v_i}^\top P_i e_{v_i} + \frac{1}{2} e_{v_i}^\top P_i \dot{e}_{v_i} \right) + e_w^\top \dot{e}_w \\ &= \frac{1}{2} \sum_{i \in \mathcal{V}} e_{v_i}^\top ((S_i - L_i C_i)^\top P_i + P_i (S_i - L_i C_i)) e_{v_i} \\ &\quad + \sum_{i \in \mathcal{V}} e_{w_i} L_i^\top P_i e_{v_i} + e_w^\top CS e_v - k e_w^\top R^\top R e_w \\ &= -\frac{1}{2} \sum_{i \in \mathcal{V}} e_{v_i}^\top Q_i e_{v_i} + \sum_{i \in \mathcal{V}} e_{w_i}^\top L_i^\top P_i e_{v_i} \\ &\quad + \sum_{i \in \mathcal{V}} e_{w_i}^\top C_i S_i e_{v_i} - k e_w^\top R^\top R e_w \\ &\leq -\frac{1}{2} \sum_{i \in \mathcal{V}} (e_{v_i}^\top Q_i e_{v_i} + 2k \lambda_{\min}(R^\top R) e_{w_i}^\top e_{w_i}) \end{aligned}$$

$$\begin{aligned} &- 2 \sum_{i \in \mathcal{V}} e_{w_i}^\top (L_i^\top P_i + C_i S_i) e_{v_i} \\ &\leq -\frac{1}{2} \sum_{i \in \mathcal{V}} \begin{bmatrix} e_{v_i} \\ e_{w_i} \end{bmatrix}^\top O_i \begin{bmatrix} e_{v_i} \\ e_{w_i} \end{bmatrix} \end{aligned} \quad (17)$$

where $e_{v_i} \in \mathbb{R}^{p_i}$ is the sub-vector of e_v corresponding to sensor i , e_{w_i} is the i th entry of e_w , the fourth equality holds since CS is a block diagonal matrix, and

$$O_i = \begin{bmatrix} Q_i & -P_i L_i - S_i^\top C_i^\top \\ -L_i^\top P_i - C_i S_i & 2k \lambda_{\min}(R^\top R) \end{bmatrix}$$

Hence if k is such that each O_i is positive definite,

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2} \sum_{i \in \mathcal{V}} \lambda_{\min}(O_i) \left\| \begin{bmatrix} e_{v_i} \\ e_{w_i} \end{bmatrix} \right\|_2^2 \\ &\leq -\frac{1}{2} \min_{i \in \mathcal{V}} \lambda_{\min}(O_i) \left\| \begin{bmatrix} e_v \\ e_w \end{bmatrix} \right\|_2^2 \\ &\leq -c_1 V(t) \end{aligned}$$

with $c_1 = \min_{i \in \mathcal{V}} \lambda_{\min}(O_i) / \max\{\lambda_{\max}(P), 1\}$. Then the estimate error $[e_v^\top e_w^\top]^\top$ converges to zero exponentially fast. Finally, by the Schur complement condition, $O_i > 0$ is equivalent to $Q_i > 0$ and

$$2k \lambda_{\min}(R^\top R) I - (L_i^\top P_i + C_i S_i) Q_i^{-1} (L_i^\top P_i + C_i S_i)^\top > 0$$

from which, it is readily to have (16). \blacksquare

The parameter k can be selected based on (16) centralizedly. It can also be computed by sensors in a distributed way. By S_i and C_i , each sensor can easily obtain L_i, Q_i, P_i . Hence, each sensor can calculate the lower bound k_i locally by (16) if $\lambda_{\min}(R^\top R)$ is accessible. One can verify that ψ in Lemma 2 satisfies $\psi \geq \frac{1}{n}$, hence $\lambda_{\min}(R^\top R) \geq 1/(4d_{\max} n^2)$. Then $k_i > 2n^2 d_{\max} (L_i^\top P_i + C_i S_i) Q_i^{-1} (L_i^\top P_i + C_i S_i)^\top$ is sufficient to make (16) hold for each $i \in \mathcal{V}$. Hence, k_i can be computed locally if each sensor knows the network size and the maximum degree, which can be obtained by the average consensus and the max consensus algorithms, respectively [21], [22]. To find $k \geq k_i, i \in \mathcal{V}$, the sensors can calculate $\max_{i \in \mathcal{V}} k_i$ also by the max consensus algorithm.

C. Fully distributed bias estimation algorithm

In the bias estimation method proposed in the previous subsection, the parameter k is selected based on the global information $\lambda_{\min}(R^\top R)$. Although it is discussed at the end that this information can be derived distributedly, the sensors still need to run three rounds of consensus algorithms. In this subsection, we modify the estimator (13), (14) to a new one, which guarantees that each sensor can estimate the internal state v_i and bias w_i without knowing any global information of the network.

In detail, for each sensor $i \in \mathcal{V}$, the bias estimator is given as

$$\begin{aligned} \dot{\hat{v}}_i &= S_i \hat{v}_i + L_i (\bar{w}_i - C \hat{v}_i) \\ \dot{\hat{w}}_i &= \sum_{j \in \mathcal{N}_i} k_{ij}(t) (\bar{w}_{ij} - \tilde{w}_i - \tilde{w}_j) + C_i S_i \hat{v}_i \\ \dot{k}_{ij}(t) &= h (\bar{w}_{ij} - \tilde{w}_i - \tilde{w}_j)^2, \quad j \in \mathcal{N}_i \end{aligned} \quad (18)$$

where $h > 0$ and k_{ij} is the edge gain for edge $\{i, j\} \in \mathcal{E}$. As shown in the following result, convergence of this algorithm requires $k_{ij}(t) = k_{ji}(t)$ for each edge $\{i, j\} \in \mathcal{E}$, which is satisfied by setting $k_{ij}(0) = k_{ji}(0)$.

Theorem 3: Consider the same setting as in Theorem 2. Associate each sensor $i \in \mathcal{V}$ with two variables $\hat{v}_i \in \mathbb{R}^{p_i}$, $\tilde{w}_i \in \mathbb{R}$ and let them evolve as (18), with $k_{ij}(0) = k_{ji}(0)$ for each $\{i, j\} \in \mathcal{E}$, then \hat{v} and \tilde{w} converge to v and w , respectively, and each $k_{ij}(t)$ converges to some finite value.

Proof. Let $k(t) = \text{col}\{k_{ij}(t)\}_{\{i,j\} \in \mathcal{E}} \in \mathbb{R}^m$, $K(t) = \text{diag}\{k_{ij}(t)\}_{\{i,j\} \in \mathcal{E}}$, and $[Re_w]^2 = \text{col}\{(Re_w e_w)^2\}_{e \in \mathcal{E}}$. From (18) and (5), the estimate error e_v and e_w satisfy

$$\begin{aligned} \dot{e}_v &= (S - LC)e_v + Le_w \\ \dot{e}_w &= CSe_v - R^T K(t) Re_w \\ \dot{k} &= h[Re_w]^2 \end{aligned} \quad (19)$$

Select the Lyapunov function as

$$V(t) = \frac{1}{2} e_v^T P e_v + \frac{1}{2} e_w^T e_w + \frac{1}{2h} (k(t) - \bar{k} \mathbf{1}_m)^T (k(t) - \bar{k} \mathbf{1}_m)$$

where \bar{k} is a positive value that will be determined later. Then similarly as (17), we have

$$\begin{aligned} \dot{V}(t) &= \frac{1}{2} \dot{e}_v^T P e_v + \frac{1}{2} e_v^T P \dot{e}_v + e_w^T \dot{e}_w + \frac{1}{h} (k(t) - \bar{k} \mathbf{1}_m)^T \dot{k}(t) \\ &= -\frac{1}{2} e_v^T Q e_v + e_v^T (PL + S^T C^T) e_w - e_w^T R^T K(t) Re_w \\ &\quad + (k(t) - \bar{k} \mathbf{1}_m)^T [Re_w]^2 \\ &= -\frac{1}{2} e_v^T Q e_v + e_v^T (PL + S^T C^T) e_w - e_w^T R^T K(t) Re_w \\ &\quad + e_w^T R^T K(t) Re_w - \bar{k} e_w^T R^T Re_w \\ &= -\frac{1}{2} e_v^T Q e_v + e_v^T (PL + S^T C^T) e_w - \bar{k} e_w^T R^T Re_w \end{aligned}$$

where the third equality comes from $k(t)^T [Re_w]^2 = e_w^T R^T K(t) Re_w$ and $\bar{k} \mathbf{1}_m^T [Re_w]^2 = \bar{k} e_w^T R^T Re_w$. Select $\bar{k} \geq k_i$ for all $i \in \mathcal{V}$, with k_i given as in Theorem 2, then as shown in Theorem 2, we have

$$\dot{V}(t) \leq -\frac{1}{2} \sum_{i \in \mathcal{V}} \lambda_{\min}(O_i) \left\| \begin{bmatrix} e_{v_i} \\ e_{w_i} \end{bmatrix} \right\|_2^2$$

This implies that $V(t)$ is bounded and so is each k_{ij} . Note that k_{ij} for each $\{i, j\} \in \mathcal{E}$ is monotonically increasing by (18), hence it converges to some finite value. Finally, since $\dot{V}(t) \equiv 0$ implies that $e_v = 0$ and $e_w = 0$, we have \hat{v} and \hat{w} converges to the actual value of v and w by the LaSalle invariance principal. ■

IV. BIAS ESTIMATION FOR BIPARTITE GRAPHS

In this section, we discuss the case that the measurement graph \mathcal{G} is bipartite. For constant biases, [8] showed that the biases can be estimated if less than half of the sensors in the network are biased. A linear-programming based distributed bias estimator has also been proposed. For time-varying biases, we can also impose the condition on the number of biased sensors as in [8]. However, the estimator in [8] is not easy to be adapted for time-varying biases. To circumvent this difficulty, we assume that in the network, there exist at least one bias-free sensors and these sensors know that their measurements are correct. We denote by \mathcal{F} the set of these bias-free sensors, then $w_i(t) = 0$ for all $t \geq 0$ and $i \in \mathcal{F}$. In the following, we only present a bias estimation method for the case that each nonzero bias is generated by a general linear exo-system.

Denote $\mathcal{V}_{\mathcal{F}}^c = \mathcal{V} \setminus \mathcal{F}$. For each sensor $i \in \mathcal{V}_{\mathcal{F}}^c$, its internal state estimator \hat{v}_i and bias estimator \hat{w}_i are given as

$$\begin{aligned} \dot{\hat{v}}_i &= S_i \hat{v}_i + L_i (\hat{w}_i - C \hat{v}_i) \\ \dot{\hat{w}}_i &= k \sum_{j \in \mathcal{N}_i \setminus \mathcal{F}} (\hat{w}_{ij} - \hat{w}_i - \hat{w}_j) + k \sum_{j \in \mathcal{N}_i \cap \mathcal{F}} (\hat{w}_{ij} - \hat{w}_i) + C_i S_i \hat{v}_i, \end{aligned} \quad (20)$$

where L_i is the same as in Subsection III-B. Let $\hat{v} = \text{col}\{\hat{v}_i\}_{i \in \mathcal{V}_{\mathcal{F}}^c}$, $\hat{w} = \text{col}\{\hat{w}_i\}_{i \in \mathcal{V}_{\mathcal{F}}^c}$, $S = \text{diag}\{S_i\}_{i \in \mathcal{V}_{\mathcal{F}}^c}$, $L = \text{diag}\{L_i\}_{i \in \mathcal{V}_{\mathcal{F}}^c}$, $C = \text{diag}\{C_i\}_{i \in \mathcal{V}_{\mathcal{F}}^c}$, $P = \text{diag}\{P_i\}_{i \in \mathcal{V}_{\mathcal{F}}^c}$ and $Q = \text{diag}\{Q_i\}_{i \in \mathcal{V}_{\mathcal{F}}^c}$. Then collecting the estimator (20) for all $i \in \mathcal{V}_{\mathcal{F}}^c$, we have

$$\dot{\hat{v}} = S \hat{v} + L(\hat{w} - C \hat{v})$$

$$\dot{\hat{w}} = k R_{\mathcal{V}_{\mathcal{F}}^c}^T (\hat{w} - R_{\mathcal{V}_{\mathcal{F}}^c} \hat{w}) + CS \hat{v},$$

where $R_{\mathcal{V}_{\mathcal{F}}^c} \in \mathbb{R}^{m \times (n - |\mathcal{F}|)}$ is the sub-matrix of R obtained by deleting the columns whose indices are in \mathcal{F} .

To show the convergence of this bias estimator, we need the following property of matrix $R_{\mathcal{V}_{\mathcal{F}}^c}$.

Lemma 4: If \mathcal{G} is bipartite and $|\mathcal{F}| \geq 1$, then $R_{\mathcal{V}_{\mathcal{F}}^c}$ has full column rank.

proof. The statement is equivalent to that any vector $\mu \in \mathbb{R}^{n - |\mathcal{F}|}$ satisfying $R_{\mathcal{V}_{\mathcal{F}}^c} \mu = 0$ is zero. To show this, we let $\mu' = [\mu^T \ 0_{|\mathcal{F}|}^T]^T$. Since $R_{\mathcal{V}_{\mathcal{F}}^c} \mu = 0$, $R \mu' = 0$ should also holds. Then by [8, Lemma 2], each element of μ' has the same absolute value. Hence for $i \in \mathcal{V}_{\mathcal{F}}^c$, $|\mu'_i| = |\mu'_j| = 0$ with $j \in \mathcal{F}$. This shows $\mu = 0$. ■

We can show the following result.

Theorem 4: Consider a bipartite sensor network and assume that each bias w_i , with $i \in \mathcal{V}_{\mathcal{F}}^c$, is generated by the exo-system (2), where the pair (S_i, C_i) is observable. Associate each sensor $i \in \mathcal{V}$ with two variables $\hat{v}_i \in \mathbb{R}^{p_i}$, $\hat{w}_i \in \mathbb{R}$ and let them evolve as (20). If $k \geq k_i$, where k_i for each $i \in \mathcal{V}_{\mathcal{F}}^c$ satisfies

$$k_i > \frac{(L_i^T P_i + C_i S_i) Q_i^{-1} (L_i^T P_i + C_i S_i)^T}{2 \lambda_{\min}(R_{\mathcal{V}_{\mathcal{F}}^c}^T R_{\mathcal{V}_{\mathcal{F}}^c})} \quad (21)$$

then \hat{v} and \hat{w} converges exponentially fast to v and w , respectively.

Proof. Similar as in the proof of Theorem 2, the estimation error satisfies

$$\begin{bmatrix} \dot{e}_v \\ \dot{e}_w \end{bmatrix} = \begin{bmatrix} S - LC & L \\ CS & -k R_{\mathcal{V}_{\mathcal{F}}^c}^T R_{\mathcal{V}_{\mathcal{F}}^c} \end{bmatrix} \begin{bmatrix} e_v \\ e_w \end{bmatrix}$$

Let $V(t) = \frac{1}{2} e_v^T P e_v + \frac{1}{2} e_w^T e_w$, then one can check that

$$\begin{aligned} \dot{V}(t) &= -\frac{1}{2} e_v^T Q e_v + e_v^T (PL + S^T C^T) e_w - e_w^T R_{\mathcal{V}_{\mathcal{F}}^c}^T R_{\mathcal{V}_{\mathcal{F}}^c} e_w \\ &\leq -\frac{1}{2} \sum_{i \in \mathcal{V}_{\mathcal{F}}^c} (e_{v_i}^T Q_i e_{v_i} + 2k \lambda_{\min}(R_{\mathcal{V}_{\mathcal{F}}^c}^T R_{\mathcal{V}_{\mathcal{F}}^c}) e_{w_i}^2 \\ &\quad - 2e_{v_i}^T (P_i L_i + S_i^T C_i^T) e_{w_i}) \end{aligned}$$

Note that $R_{\mathcal{V}_{\mathcal{F}}^c}$ has full column rank as shown by Lemma 4, $\lambda_{\min}(R_{\mathcal{V}_{\mathcal{F}}^c}^T R_{\mathcal{V}_{\mathcal{F}}^c}) > 0$. Then analogous to the proof of Theorem 2, if k satisfies condition (21), $\dot{V}(t) \leq -c_2 V(t)$ for some positive c_2 , and the estimate error converges exponentially fast to zero. ■

Remark 2: Estimator (8) proposed for biases generated from stable exo-systems and the adaptive estimator (18) for general exo-systems can be modified straightforwardly to work for the case of bipartite measurement graphs with some sensors being bias-free and aware of the unbiasedness of their measurements.

Remark 3: All the proposed bias estimators ((8), (15), (18), (20)) can also be used in other applications, like consensus [8] and formation [23], where measurements of agents relative positions are used and are subject to time-varying biases.

V. SIMULATION

In this section, we validate the proposed estimators by simulation.

A. Non-bipartite graphs

A non-bipartite graph with 6 sensors in Fig. 1 is considered. *Stable exo-systems.* In the first case, we let the bias of each sensor be generated by a stable exo-system as in Section III-A. In detail, for each sensor $i \in \mathcal{V}$, the matrices of the exo-system Σ_i are given as

$$S_i = \begin{bmatrix} 0 & \frac{i\pi}{3} \\ -\frac{i\pi}{3} & 0 \end{bmatrix}, \quad C_i = [1 \ 0] \quad (22)$$

The initial condition of the internal state of exo-system Σ_i is set as $v_i(0) = [\cos \frac{i\pi}{6} \quad -\sin \frac{i\pi}{6}]^T$. For this case, we use estimator (8).

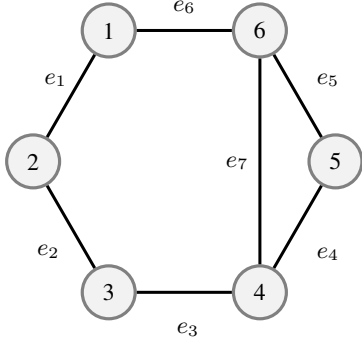


Fig. 1: The non-bipartite graph in the simulation

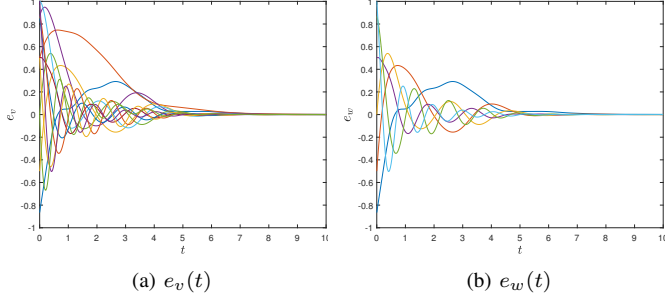


Fig. 2: The estimation error of biases generated by stable exo-systems with the non-bipartite graph in Fig. 1

The simulation result is shown in Fig. 2, from which one can see that the estimate errors of the internal state and the bias both converge to zero.

Unstable exo-systems. In this case, the matrices of the exo-system Σ_i for node $i = 1, 3, 5$ are same as that for the case of stable exo-systems. While for sensor 2, 4, 6, the matrices of the exo-systems are given as

$$S_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad S_4 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad S_6 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ C_2 = C_4 = C_6 = [1 \ 0]^\top \quad (23)$$

For this case, we first use estimator (13), (14). The matrix L_i is selected as

$$L_1 = L_3 = L_5 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, L_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, L_4 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}, L_6 = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \quad (24)$$

Let $Q_i = I$, P_i can be solved accordingly,

$$P_1 = \begin{bmatrix} 0.333 & -0.477 \\ -0.477 & 1.701 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1 \end{bmatrix} \\ P_3 = \begin{bmatrix} 0.333 & -0.159 \\ -0.159 & 0.485 \end{bmatrix}, \quad P_4 = \begin{bmatrix} 3.250 & 2.083 \\ 2.083 & 1.583 \end{bmatrix} \\ P_5 = \begin{bmatrix} 0.333 & -0.095 \\ -0.095 & 0.388 \end{bmatrix}, \quad P_6 = \begin{bmatrix} 3.250 & -2.083 \\ -2.083 & 1.583 \end{bmatrix}$$

Then, one can check that the lower bound on k implied by (16) is 35.063. This bound may be conservative, as we observe from Fig. 3 that the estimate errors also converge for $k = 15$.

We then use the fully distributed estimator (18) and select $h = 50$. The simulation results are shown in Fig. 4(c). One can see that the estimation error indeed converges to zero and the edge gain k_{ij} increases monotonically to some finite value.

B. Bipartite graphs

For the situation of bipartite graphs, we consider a 10 vehicles platoon [24]. The dynamics of each vehicle $i = 1, \dots, n$ is a second-

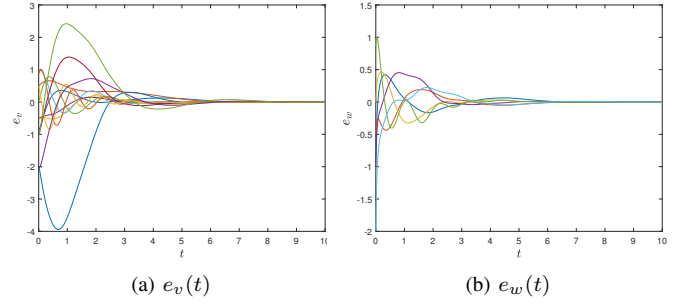


Fig. 3: The estimation error of biases generated by unstable exo-systems with the non-bipartite graph in Fig. 1 by the estimator (13) and (14) with $k = 15$.

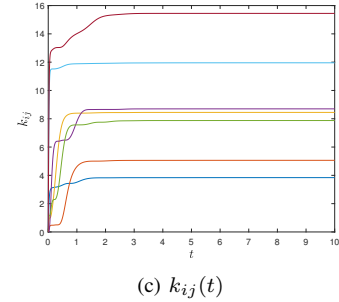
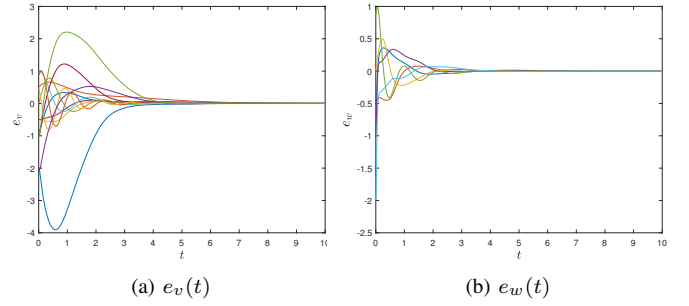


Fig. 4: The estimation error of biases generated by unstable exo-systems with the non-bipartite graph in Fig. 1 by the estimator (18).

order integrator,

$$\ddot{Z}_i = U_i \quad (25)$$

where $Z_i, U_i \in \mathbb{R}$ are the position and acceleration of vehicle i . The objective is to make each vehicle follow a constant velocity V_d and maintain the inter-vehicle distance Δ ($\delta_{i-1,i} = Z_{i-1} - Z_i - \Delta = 0$ for $i = 2, \dots, 10$). The measurement network of vehicles is a line graph: vehicle i is connected with $i + 1$ for $i = 1, \dots, 9$. We use the control architecture of scenario I in [24] and assume that the relative state measurements taken by vehicle 1 and 10 are bias-free. For other vehicles, the measurement biases w_i are generated by the exo-systems in (2), with S_i, C_i given as (23) if $i = 2, 4, 6$ and (22) if $i = 3, 5, 7, 8, 9$. Due to the measurement biases, the inter-vehicle distances diverge from the desired value Δ as shown in Fig. 5(d).

To estimate the biases, we use estimator (20) with $k = 15$. The matrix L_i is selected as in (24) if $i = 2, 4, 6$, and $L_i = [3 \ 0]^\top$ if $i = 3, 5, 7, 8, 9$. We also compensate the biases in the control inputs by their estimates $w_i(t)$ for $i = 2, \dots, 9$. Fig. 5(a), 5(b) and 5(c) show the simulation results when using the bias estimator and compensator. From these figures, one can see that the estimate errors for the internal state and the bias converge to zero. Moreover, the inter-vehicle distances converge to Δ .

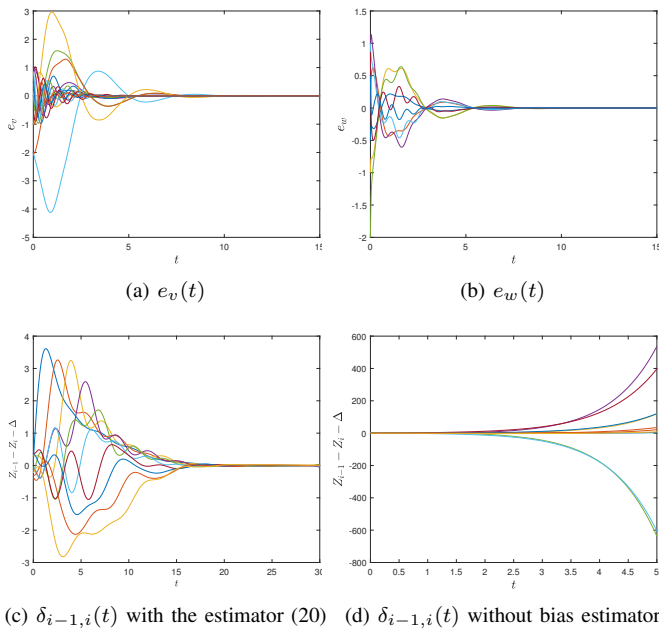


Fig. 5: The estimation error of biases and the inter-vehicle distances for the 10 vehicles platoon. In the simulation, $V_d = 5$ m/s, $\Delta = 1$ m. The initial position Z_i and velocity \dot{Z}_i of vehicle i is generated randomly within $[-i - 0.5, -i + 0.5]$ and $[-0.5, 0.5]$, respectively.

VI. CONCLUSION

In this paper, we investigated the problem of estimating time-varying bias signals in relative state measurements taken by connected sensors, that are generated from linear exo-systems. For non-bipartite sensor networks, three distributed bias estimators have been proposed to reconstruct the exo-systems and the bias signals. The first estimator is suitable for a class of stable observable exo-systems. The second works for general observable exo-systems, but requires global information of the network. The last one removes such requirement. We also showed that the proposed estimators can be applied for bipartite networks after introducing some bias-free sensors. Future works will focus on estimating time-varying bias signals generated from non-linear exo-systems, removing the requirement that the exo-systems' matrices are known, and including the dynamics of nodes.

REFERENCES

- [1] F. Liu, A. S. Morse, and B. D. O. Anderson, "Robust control of undirected rigid formations with constant measurement bias in relative positions," *2016 American Control Conference (ACC)*, pp. 143–147, 2016.
- [2] H. G. De Marina, M. Cao, and B. Jayawardhana, "Controlling rigid formations of mobile agents under inconsistent measurements," *IEEE Transactions on Robotics*, vol. 31, no. 1, pp. 31–39, 2014.
- [3] Z. Meng, B. D. Anderson, and S. Hirche, "Formation control with mismatched compasses," *Automatica*, vol. 69, pp. 232–241, 2016.
- [4] A. Martinelli, A. Renzaglia, and A. Oliva, "Cooperative visual-inertial sensor fusion: fundamental equations and state determination in closed-form," *Autonomous Robots*, vol. 44, no. 3, pp. 339–357, 2020.
- [5] S. Bolognani, S. Del Favero, L. Schenato, and D. Varagnolo, "Consensus-based distributed sensor calibration and least-square parameter identification in wsns," *International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal*, vol. 20, no. 2, pp. 176–193, 2010.
- [6] B.-H. Lee and H.-S. Ahn, "Distributed formation control via global orientation estimation," *Automatica*, vol. 73, pp. 125–129, 2016.
- [7] K.-K. Oh and H.-S. Ahn, "Formation control and network localization via orientation alignment," *IEEE Transactions on Automatic Control*, vol. 59, pp. 540–545, 2014.
- [8] M. Shi, C. De Persis, P. Tesi, and N. Monshizadeh, "Bias estimation in sensor networks," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 3, pp. 1534–1546, 2020.

- [9] A. Carron, M. Todescato, R. Carli, and L. Schenato, "An asynchronous consensus-based algorithm for estimation from noisy relative measurements," *IEEE Transactions on Control of Network Systems*, vol. 1, pp. 283–295, 2014.
- [10] I. Shames, A. N. Bishop, and B. D. O. Anderson, "Analysis of noisy bearing-only network localization," *IEEE Transactions on Automatic Control*, vol. 58, pp. 247–252, 2013.
- [11] B. Wang and Y.-P. Tian, "Distributed network localization: Accurate estimation with noisy measurement and communication information," *IEEE Transactions on Signal Processing*, vol. 66, pp. 5927–5940, 2018.
- [12] C. Ravazzi, N. P. K. Chan, and P. Frasca, "Distributed estimation from relative measurements of heterogeneous and uncertain quality," *CoRR*, vol. abs/1710.08632, 2017.
- [13] P. Barooah and J. Hespanha, "Estimation on graphs from relative measurements," *IEEE Control Systems*, vol. 27, pp. 57–74, 2007.
- [14] N. Bof, M. Todescato, R. Carli, and L. Schenato, "Robust distributed estimation for localization in lossy sensor networks," *IFAC-PapersOnLine*, vol. 49, no. 22, pp. 250–255, 2016.
- [15] S. Sukumar, E. Panteley, A. Loría, and W. Pasillas, "On consensus of double integrators over directed graphs and with relative measurement bias," *2018 IEEE Conference on Decision and Control (CDC)*, pp. 4147–4152, 2018.
- [16] A. Caiti, A. Garulli, F. Livide, and D. Prattichizzo, "Localization of autonomous underwater vehicles by floating acoustic buoys: a set-membership approach," *IEEE Journal of Oceanic Engineering*, vol. 30, no. 1, pp. 140–152, 2005.
- [17] M. Belmont, J. Baker, and J. Horwood, "Avoidance of phase shift errors in short term deterministic sea wave prediction," *Journal of Marine Engineering & Technology*, vol. 2, no. 2, pp. 21–26, 2003.
- [18] D. Cvetković, P. Rowlinson, and S. K. Simić, "Signless laplacians of finite graphs," *Linear Algebra and its applications*, vol. 423, no. 1, pp. 155–171, 2007.
- [19] M. Desai and V. Rao, "A characterization of the smallest eigenvalue of a graph," *Journal of Graph Theory*, vol. 18, no. 2, pp. 181–194, 1994.
- [20] M. Viberg, "Subspace methods in system identification," *IFAC Proceedings Volumes*, vol. 27, no. 8, pp. 1–12, 1994.
- [21] I. Shames, T. Charalambous, C. N. Hadjicostis, and M. Johansson, "Distributed network size estimation and average degree estimation and control in networks isomorphic to directed graphs," in *2012 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*. IEEE, 2012, pp. 1885–1892.
- [22] F. Iutzeler, P. Ciblat, and J. Jakubowicz, "Analysis of max-consensus algorithms in wireless channels," *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 6103–6107, 2012.
- [23] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.
- [24] P. Barooah, P. G. Mehta, and J. P. Hespanha, "Mistuning-based control design to improve closed-loop stability margin of vehicular platoons," *IEEE Transactions on Automatic Control*, vol. 54, no. 9, pp. 2100–2113, 2009.