Assets, Human Capital, and Growth*

P. Braila† and A. Turrini‡

November 1998.

Abstract

In this paper we illustrate the possible normative relevance of the links between human capital and financial assets via an example related to growth. When the financial structure is complete, growth is indeterminate because individual allocations between human capital and a tradable asset are indeterminate. When the financial structure is incomplete, the growth rate depends on the payoff structure of the assets. An issue of optimality for the structure of asset returns is raised.

Keywords: Incomplete Markets, Human Capital, Growth.
JEL Classification: D52, J24, O4.

*We would like to thank Professor Heracles Polemarchakis for insightfull conversastions and valuable advice. We also thank Gaetano Bloise and Luigi Ventura for their comments and useful suggestions. The usual disclaimer applies.

†CORE, Universite Catholique de Louvain; e-mail: braila@core.ucl.ac.be

‡University of Bergamo, Italy, and CORE, Universite Catholique de Louvain, 34 Voie du Roman Pays 1348, Louvain-la-Neuve,Belgium. e-mail turrini@core.ucl.ac.be.

1 Introduction

Because future labour incomes are uncertain, human capital bears risks.[1] Some risks are physical: death, accidents, illness. Other risks are economic, and result from the uncertainties affecting the effectiveness of education and training. In a sense, the formation of human capital corresponds to the accumulation of assets yielding uncertain outcomes. However, these assets have two distinguishing features. First, they are non-marketable and the investment in these assets must respect individual-specific feasibility constraints.[2] As a result, human capital can only be transferred onward and in finite amounts. Second, as opposed to financial assets, investments in human capital are likely to produce direct effects on productivity and output.

As other assets, human capital investments interact with the existing financial structure: they depend on the overall asset structure and simultaneously have an effect on the use of alternative financial instruments. The interaction of human capital with the financial structure has received some attention in positive analysis. In particular, it has been recently explored as a possible source of deviations of observed market outcomes from those predicted by the theory of financial markets.[3] The aim of this paper is rather that of showing how the mere fact that human capital does interfere with the existing asset structure may easily give raise to normative issues.

The core of our argument runs as follows. Human capital, though non-tradable, might act as substitute for some tradable asset in transferring wealth, thus causing possible indeterminacies when the financial structure corresponds to a full set of Arrow securities. Conversely, when financial markets are incomplete, the presence of human capital may lead to allocations that mimic those reached when financial markets are complete. In such cases, the structure of returns on financial assets may have real effects in that they shape the individual incentives towards undertaking education and training, activities that are likely to affect the production possibilities of the economy.

---

[1] See Drèze (1979) on the issue of human capital and risk-bearing.

[2] Given, for instance, by time availability and learning ability.

[3] It has been shown, for instance, that the presence of human capital-specific risk may explain both the equity premium (Weil (1992)) and the international portfolio puzzles (Bottazzi et al. (1996)).

We illustrate our argument via an example related to growth. We construct a simple OLG model in which the outcome of investments in human capital is subject to (aggregate) risk. When young, agents are uncertain about the future income they derive from their investment in human capital. In our example, the population is heterogeneous in terms of income, and this is sufficient to generate different attitutude towards risk for different agents. In such a framework, each individual gains from trading (ex-ante) financial assets with agents belonging to different income classes. If productivity is positively affected by aggregate human capital investments, a spillover is created that acts as the engine of growth (Lucas (1988)). We show that when the financial structure is complete, growth is indeterminate because *individual* allocations between human capital and a tradable asset are indeterminate. When the number of available financial assets is smaller than the dimension of the uncertainty, *the rate of growth depends upon the structure of asset returns*, thus leaving room for policy intervention.

It is well established from the theory of incomplete markets that Pareto efficiency of equilibrium allocations is not preserved when the asset market is incomplete. Moreover, the allocation is in this case affected by the asset structure (Geanakoplos and Polemarchakis (1986), Geanakoplos (1990)). Demange and Laroque (1995), among others, characterize the structure of optimal asset payoffs when markets are incomplete. They consider a static exchange economy where individual endowments are risky, information is symmetric, the distribution of the risk is known and individuals trade ex-ante. Our example is only loosely related to this literature. The introduction of human capital may allow for the effective completeness of the market, so that the asset structure may be irrelevant for consumption allocations even when a full set of Arrow securities is not available. We show, though, that dependence of consumption allocations on the asset structure is restored when it is introduced a process of endogenous growth generated by knowledge spillovers. The financial policy in this case matters for the determination of individual investment in human capital, and then for growth and consumption allocations. We study the optimality of the asset structure in such a framework.

There is substantial agreement on the relevance of financial markets for the growth performance of economies. Financial systems permit a more efficient allocation of resources and mobilize a higher amount of savings through better hedging, diversification and pooling of risk. However, while an abundant theoretical literature exists studying different channels through which the existence of financial markets may have a positive effect on growth, there is not much work highlighting the relation between the financial structure and the growth performance of an economy (Levine (1997)). In this paper, we take a small step in this direction. In our model, financial markets enhance growth because they allow the *specialization of heterogenous agents in the use of different assets*, as alternative tools to transfer wealth over time. Relatively rich agents specialize in financial assets, relatively poor agents specialize in human capital investments, and aggregate investments in human capital increase with the extent of this specialization. We show that gains from specialization can be fully exploited provided that the financial structure is *not complete* and that asset returns are sufficiently high in states where human capital investments are effective. Moreover, in the presence of an optimal structure of asset returns, the individual constraints that characterize human capital investments must be binding for some agent. Depending on the distribution of income, either poor agents could not invest more than desired, or rich agents could not invest less.

The remainder of the paper is organized as follows. In section 2 we present the structure of our argument in a static framework. In section 3, we develop an overlapping generations model and study the impact of the financial structure on consumption allocations and investments in human capital. In section 4 we find conditions that must be fulfilled by an optimal asset structure. The concluding remarks follow.

2 Structure of the problem

We present here the basic argument that will be used in the application to growth in the next sections. There are one good, two types of individuals (differentiated by their endowments) and two periods. In the second period, individual endowments are subject to aggregate uncertainty: they may assume two different values, according to the state that is realized. Information is perfect. Individuals choose consumption plans maximizing a well-behaved Von Neumann-Morgenstern utility function. Denoting individuals by a superscript, $i, i = 1, 2$, and time and states of nature (respectively, $t, t = 1, 2$ and $s, s = 1, 2$) by subscripts, the problem of individual i writes as follows

$$\max_{c_1^i, c_{2,1}^i, c_{2,2}^i} E(u^i) = u^i(c_1^i) + \pi u^i(c_{2,1}^i) + (1-\pi) u^i(c_{2,2}^i), \tag{1}$$

where π is the probability of state 1.

In the first period, agent $i, i = 1,2$ is born with endowment ω_1^i. Endowments in the second period are 0 and $\omega_{2,2}$, respectively, in the first and second state (no heterogeneity in the state 2). First period endowments can be transferred to state 1 of period 2 via a 1 to 1 storage technology, z. The set of constraints for the problem of individual i writes as follows:

$$c_1^i = \omega_1^i - z^i \tag{2}$$

$$c_{2,1}^i = z^i \tag{3}$$

$$c_{2,2}^i = \omega_{2,2} \tag{4}$$

$$0 \leq z^i \leq \omega^i. \tag{5}$$

The storage technology works as a non-marketable asset in transferring wealth across time. In such a framework, at most two marketable inside assets (y_s, with price q_s, $s = 1,2$) can be non-redundant (as many as the number of states of nature), modifying the set of constraints (2)-(5) in the following way:

$$c_1^i = \omega_1^i - z^i - q_1 y_1^i - q_2 y_2^i \tag{6}$$

$$c_{2,1}^i = z^i + \omega_{2,1}^i + y_1^i \tag{7}$$

$$c_{2,2}^i = \omega_{2,2} + y_2^i. \tag{8}$$

The market-clearing conditions impose

$$y_1^1 + y_1^2 = 0 \tag{9}$$

$$y_2^1 + y_2^2 = 0. \tag{10}$$

When the market for both assets is open, an equilibrium for problem (1),(6)-(10) requires $q_1 = 1$. A different price would entail both agents simultaneously willing to sell or buy indefinitely asset 1, compensating with equivalent transfers of z. However, as long as $q_1 > 1$, individuals are willing to sell z, and arbitrage is made impossible by the constraint $z^i \geq 0$. Conversely, when $q_1 < 1$, both agents are willing to buy asset 1, and this violates market-clearing. At equilibrium, each individual finds the storage technology z^i and asset y_1^i to be perfect substitute in transferring resources for consumption in state 1: the payoffs of the storage technology and that of asset 1 are collinear.[4] The individual allocation between z^i and y_1^i is therefore indeterminate. Since markets are complete, individual consumption allocations are instead determinate and Pareto optimal.

Consider now the case where only one marketable asset is present. It is well-known (e.g., Geanakoplos (1990)) that in this setup the allocation is generally no more independent of asset payoffs. Denoting by, respectively, r_1 and r_2 the payoff of the asset in state 1 and state 2, individual budget constraints rewrite as[5]

$$c_1^i = \omega_1^i - z^i - qy^i \tag{11}$$

$$c_{2,1}^i = z^i + r_1 y^i \tag{12}$$

$$c_{2,2}^i = \omega_{2,2} + r_2 y^i \tag{13}$$

$$y^1 + y^2 = 0. \tag{14}$$

[4] The equilibrium (augmented) payoff matrix of this economy is

$$R = \begin{bmatrix} \text{-1} & \text{-1} & -q_2^* \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By mere inspection of the above matrix we can see that the first two columns are linearly dependent.

[5] This asset will not be redundant as long as $r_2 \neq 0$. In all the exposition that follows, we assume $r_1 > 0$, $r_2 > 0$.

If any agent happens to be constrained in its choice of z by the requirement that $0 \leq z^i \leq \omega^i$, consumption allocations would not be Pareto efficient because of market incompleteness. Only allowing for another asset agents could be in the position to efficiently smooth their consumption. In this case, the incompleteness of the financial structure has real effects. We say then that markets are *effectively incomplete*. However, if all individuals maximize their utility by choosing a positive level of z compatible with their first period endowment, then the storage technology perfectly substitutes the missing tradable asset in transferring wealth over time. We say in this case that markets are *effectively complete*, because consumption allocations would be unchanged compared with the case where a full set of Arrow securities is available. In spite of this, individual choices do not replicate in all respects the equilibrium realized under complete markets. In particular, individual allocations between z^i and y^i need not be indeterminate. Furthermore, even if asset returns are irrelevant for consumption allocations, they may matter for the individual choice between z^i and y^i as alternative tools for consumption smoothing.

The above considerations may have consequences in common economic problems characterized by uncertainty. In the following sections, we give an intuition of this through an application to a standard growth framework. In an overlapping generation economy, investments in the storage technology are interpreted as (risky) human capital investments. Allowing for an endogenous growth process generated by knowledge spillovers from human capital, growth increases as aggregate human capital investments increase. However, the pace of human capital accumulation is determinate only if *individual* investments in human capital are determinate. As a result, starting from a complete asset markets case, only eliminating one asset can restore economic policy.

3 Financial structure, human capital and growth

3.1 The model

A continuum of agents of measure 1 live for two periods, 1 and 2, and at each time t, $t = 0, 1, ..., \infty$, a new generation is born. Population is constant. In the economy there is only one (non-storable) consumption good produced by a continuum of perfectly competitive firms using one production factor, H, human capital. Production, Y, is linear in H. Individuals' human capital differs in terms of its productivity. Output at time t also depends upon a time-dependent, common productivity term F_t.

When *young*, agents are born with one unit of human capital and one unit of available time. Agents may be either of type 1 or of type 2, according with the productivity of their human capital, δ^1 or δ^2. Without loss of generality, we assume that $\delta^1 > \delta^2$ and we call, consistently, type-1 individuals "rich", and individuals of type 2 "poor". In each generation, a share n, $n \in (0,1)$, of agents is born of type 1, and a share $1-n$ is born of type 2. So, agents differ in their "genetic abilities", which are handed down, from generation to generation, along the same dynasty as, for instance, in Grandstein *et al.* (1996).

When *old*, individuals' human capital endowments depend upon how much they invested in education when young; *i.e.*, on the fraction of time they devoted to learning. Human capital investments, however, yield uncertain outcomes. Uncertainty in our model originates from the possibility that human capital investments may *fail on aggregate*.[6] Each generation, at period t, faces some probability of receiving a type of education which ends up being unproductive. In such a case, education does not affect the amount of human capital available to individuals when old, at time $t+1$. So, two states may be realized in each period, state 1 or state 2, according as human capital investments are successful or not. The probability at which state 1 (resp. 2) occurs is known, constant across time, and equal to π (resp., $1-\pi$). When the investment in human capital fails (state 2), productivity differences across agents are also likely to be smoothed. We consistently assume that, in case of success (state 1), human capital endowments of old agents of type i at time t equal the fraction of time invested in education when young, s^i_{t-1}, and their productivity is given by δ^i. In case of failure (state 2), instead, human capital endowments are independent of the investment undertaken when young, equal across agents, and equally productive. We denote by δ the amount of output that can be produced out of this endowment. Aggregate output at time t in expected terms, Y^e_t, is therefore given by

$$\begin{aligned} Y^e_t \;&=\; F_t\left[n\delta^1(1-s^1_t)+(1-n)\delta^2(1-s^2_t)+\right. \\ &\left.+\pi(n\delta^1 s^1_{t-1}+(1-n)\delta^2 s^2_{t-1})+(1-\pi)\delta\right]. \end{aligned} \tag{15}$$

Assume, for the moment, that the common productivity term F_t is increased between time t and time $t+1$ by a commonly known, exogenous factor

[6] There is evidence that aggregate uncertainty may matter for human capital accumulation. Jacoby and Skoufias (1997), for instance, show that unanticipated aggregate shocks (rainfalls) significantly affect children' s school attendance in rural India.

$\kappa_t > 1:$

$$F_{t+1} = \kappa_t F_t. \tag{16}$$

At the beginning of their youth, agents choose their consumption plans, facing an uncertain environment in period 2. In the following analysis, we use the subscripts j, t, s, to distinguish, respectively, the life period (age) of each agent $j = 1, 2$, the time period t, and the state $s = 1, 2$. We assume further that preferences are equal across agents and logarithmic in consumption.[7] The problem for an individual of type $i, i = 1, 2$ born at time t is the following

$$\max_{c^i_{1t}, c^i_{2t+1,1}, c^i_{2t+1,2}} \ln(c^i_{1t}) + \pi \ln(c^i_{2t+1,1}) + (1 - \pi)\ln(c^i_{2t+1,2}) \qquad i = 1, 2 \tag{17}$$

subject to

$$c^i_{1t} = F_t \delta^i (1 - s^i_t) \tag{18}$$

$$c^i_{2t+1,1} = \kappa_t F_t \delta^i s^i_t \tag{19}$$

$$c^i_{2t+1,2} = \kappa_t F_t \delta \tag{20}$$

$$0 \leq s^i_t \leq 1 \tag{21}$$

The individual problem collapses to that of choosing how much to invest in human capital, (chosing s^i_t), under constraint (21), namely, the requirement that this investment cannot be negative and cannot exceed the amount of available time (fixed to unity for all agents). In other terms, constraint (21) summarizes the features that distinguish human capital investments with respect to financial investments: human capital is non-marketable and its accumulation must satisfy individual-specific feasibility requirements.

The solution of problem (17)-(21) yields an equal fraction of time devoted in accumulating human capital for each agent in each period: $s^i_t = \frac{\pi}{1+\pi}, i =$

[7] All the following results hold under a general CRRA representation for utility functions. The logarithmic specification is adopted for analytical convenience.

$1, 2$, for all t. Aggregate human capital in expected terms is thus constant over time, and aggregate expected output, as expressed in (15), grows at rate κ_t. Investing in human capital is in this case the only way to allocate consumption across time. Due to income heterogeneity, the introduction of asset markets would permit agents to better smooth their consumption. As it will become clear in the next sections, rich agents, when young, would gain from lending to poor agents.

3.2 Asset markets, human capital investments and consumption allocations

3.2.1 Two assets case

Consider the introduction of a full set of Arrow securities: y_1 and y_2. Individual budget constraints rewrite in analogy with (6)-(8) and market-clearing imposes

$$ny_{1t}^1 + (1-n)y_{1t}^2 = 0 \tag{22}$$

$$ny_{2t}^1 + (1-n)y_{2t}^2 = 0. \tag{23}$$

The first order conditions for utility maximization with respect to s_t^i and y_{1t}^i are mutually compatible if and only if the following relations are satisfied together

$$q_{1t} = \frac{1}{\kappa_t} \tag{24}$$

$$F_t\delta^i\left(1-s_t^i\right) - q_{1t}y_{1t}^i - q_{2t}y_{2t}^i = \frac{\kappa_t F_t \delta^i s_t^i + y_{1t}^i}{\kappa_t \pi}. \tag{25}$$

No equilibrium with $q_{1t} \neq \frac{1}{\kappa_t}$ is possible: both type of agents would be willing to exploit arbitrage opportunities between asset 1 and human capital. Summing-up first-order conditions across agents and using market clearing, the price for asset 2 is obtained as

$$q_{2t} = \frac{(1-\pi)}{1+\pi}\frac{\overline{\delta}}{\kappa_t\delta}, \tag{26}$$

where $\overline{\delta} \equiv n\delta^1 + (1-n)\delta^2$ is average individual-specific productivity. After substitution of (26) into first-order conditions, the following equilibrium relation between s_t^i and y_{1t}^i is derived

$$s_t^i = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}}{2\delta^i} - \frac{y_{1t}^i}{\kappa_t F_t \delta^i}. \tag{27}$$

Agent i, $i = 1, 2$, is indifferent between any combination between s_t^i and y_{1t}^i which is compatible with (27). From an individual viewpoint, human capital and asset y_1 are equally efficient tools in transferring wealth. The *individual* distribution between human capital and financial investments is therefore *indeterminate* when a full set of Arrow securities is present. The market for asset y_1 must clear, and this creates the following relation concerning human capital investments across agents

$$n\delta^1 s_t^1 + (1-n)\,\delta^2 s_t^2 = \overline{\delta}\frac{\pi}{1+\pi}, \tag{28}$$

which must hold together with the requirement that $0 \leq s_t^i \leq 1$, $i = 1, 2$.[8] Condition (28) ensures that the amount of human capital supplied at each period is constant. Consequently, output in expected terms (15) only grows because common productivity, F_t, grows, as in the case where financial assets are not available. However, it is important to note that any sequence $\left\{s_t^1, s_t^2 : 0 \leq s_t^i \leq 1,\ i = 1, 2,\ t \geq 0\right\}$ which is compatible with (28) may realize. Hence, in the presence of a full set of Arrow securities, *human capital investments are indeterminate* not only at the individual level, but also *in the aggregate*, expressed as $ns_t^1 + (1-n)\,s_t^2$.

Individual consumption allocations are instead determinate and given by:

$$c_{1t}^{i*} = \frac{F_t}{2}\left[\delta^i + \frac{(1-\pi)}{1+\pi}\overline{\delta}\right] \qquad i = 1, 2 \tag{29}$$

$$c_{2t+1,1}^{i*} = \frac{\kappa_t F_t \pi}{2}\left[\delta^i + \frac{(1-\pi)}{1+\pi}\overline{\delta}\right] \qquad i = 1, 2 \tag{30}$$

$$c_{2t+1,2}^{i*} = \kappa_t F_t \delta\left[\frac{(1+\pi)\delta^i + (1-\pi)\overline{\delta}}{2\overline{\delta}}\right] \qquad i = 1, 2 \tag{31}$$

Since markets are complete, allocations (29)-(31) are necessarily independent of asset payoffs.

[8] It is checked that when (38) is satisfied at most one type of agent can be constrained by the requirement that $0 \leq s_t^i \leq 1$.

3.2.2 One asset case

Consider now the case where only one marketable asset is available. This asset, y, pays off r_1 in state 1 and $r_2 \neq 0$ if state 2 is realized. Again, individual budget constraints are modified in analogy with (11)-(14). The fact that one marketable asset is missing from a full set of Arrow securities matters for consumption allocations only if any agent is constrained in his (her) choice by the requirement that $0 \leq s_t^i \leq 1$. In this case, markets are effectively incomplete. However, if agents happen not to be constrained in their maximization problem, markets are effectively complete even if one financial asset is missing. Human capital is in this case an efficient substitute for the missing tradable asset. Equilibrium consumption allocations are independent of asset payoffs, and expressed as in (29)-(31). What we want to remark is that in this case, as opposed with the case where two assets are present, *individual* human capital investments are determinate.

The interior solution for the individual problem is characterized as follows

$$q_t^* = \frac{1}{\kappa_t} r_1 + \frac{\overline{\delta}\,(1-\pi)}{\delta \kappa_t\,(1+\pi)} r_2 \tag{32}$$

$$s_t^{1*} = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}}{2\delta^1(1+\pi)} - \frac{(1-n)\,(1+\pi)\left(\delta^1-\delta^2\right)\delta}{2\delta^1\overline{\delta}} \frac{r_1}{r_2} \tag{33}$$

$$s_t^{2*} = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}}{2\delta^2(1+\pi)} + \frac{n\,(1+\pi)\left(\delta^1-\delta^2\right)\delta}{2\delta^2\overline{\delta}} \frac{r_1}{r_2} \tag{34}$$

$$y_t^{1*} = \frac{\delta \kappa_t F_t (1-n)\left(\delta^1-\delta^2\right)(1+\pi)}{2\overline{\delta} r_2} \tag{35}$$

$$y_t^{2*} = -\frac{\delta \kappa_t F_t n\left(\delta^1-\delta^2\right)(1+\pi)}{2\overline{\delta} r_2}. \tag{36}$$

Some remarks concerning expressions (32)-(36) are in order. First, the price of the financial asset, and equilibrium values for s_t^i and y_t^i depend upon the returns on the financial asset. While the asset price and equilibrium values for trades in y depend upon growth rates $\kappa_t - 1$, investments in human capital are independent of them, and constant over time. In this case, we can consistently drop the time index and write $s_t^{i*} = s^{i*}$, $i = 1, 2$. Second, rich agents always buy asset y, while the poor are sellers. Since in case of failure (state 2) human capital productivity and then income is made equal for both, the rich are relatively more averse to state 2 compared with the poor. They are therefore those that are willing to buy asset y, in that this is the unique means for transferring wealth to the second period of life in case state 2 realizes. Finally, it is to note that an interior solution for the individual problem of both type of agents (namely, $0 < s^{i*} < 1$, $i = 1, 2$) exists as long as r^1/r^2 and δ^1/δ^2 are not too high.[9]

3.3 Endogenous growth

Assume now that the common productivity term F_t is not anymore determined by an exogenous process. Imagine that, as is often assumed in the growth literature, productivity at time t depends upon the stock of accumulated knowledge. Accordingly, productivity increases over time in proportion with aggregate investments in human capital undertaken by the old generation. The growth rate of the economy at period t, g_t, is thus given by

$$g_t = \frac{F_{t+1} - F_t}{F_t} = s_t, \tag{37}$$

where

$$s_t = ns_t^1 + (1 - n)\, s_t^2 \tag{38}$$

Human capital investments of young agents of generation t, on aggregate, produce an externality on the productivity realized at time $t+1$ (knowledge spillover).[10] Each young agent, though, being atomistic, does not take into account the influence of his human capital on the productivity that will be realized during his second period of life. The individual problem writes therefore as in the previous cases, but the term $\kappa_t F_t$ is now to be replaced by F_{t+1}^e, where the superscript e denotes expectations. Productivity realized at time $t+1$ depends on aggregate human capital investments undertaken

[9] Positive values for r_1 exist such that the solution is in the interior whenever the "income gap" $\frac{\delta^1}{\delta^2}$ is lower than a given threshold, i.e., whenever $\frac{\delta^1}{\delta^2} < 1 + \frac{2}{\pi n(1-\pi)}$.

[10] It is commonly argued in modern growth theory that human capital accumulation may produce externalities that matter for growth (see, for instance, Lucas (1988)).

at period t. At a perfect foresight equilibrium, plans and conjectures are mutually compatible, so that $F^e_{t+1} = F_{t+1} = F_t(1 + s_t)$.

When knowledge spillovers operate, the growth rate of the economy is driven by investments in human capital. The financial structure of the economy, when shaping individual choices between alternative forms of investment, also produces additional real effects through the growth rate. Consider the case where a full set of Arrow securities is available. Under this scenario, since human capital and financial assets are perfect substitutes, individual, and then aggregate investments in human capital are indeterminate. In such a case, the growth rate between each time period will also be indeterminate. Conversely, if only one asset is present, human capital investments need not be indeterminate. Consider the case where agents are not constrained by the requirement that $0 \leq s^{i*} \leq 1$. In such a case, the expression for total human capital investments is obtained as[11]

$$s^* = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}\widetilde{\delta}}{2\delta^1\delta^2\,(1+\pi)} + \frac{n\,(1-n)\,\delta\,(1+\pi)\left(\delta^1-\delta^2\right)^2}{2\delta^1\delta^2\overline{\delta}}\frac{r_1}{r_2}, \tag{39}$$

where $\widetilde{\delta} \equiv n\delta^2 + (1-n)\delta^2$. Aggregate human capital investments are affected by asset returns. Notice that what matters for human capital accumulation is only the payoff ratio, r^1/r^2, which gives a measure of the relative efficiency of the asset in transferring wealth in the two states. Consistently, henceforth we normalize $r_2 = 1$, and set $r \equiv r_1$. By inspection of expressions (33) and (34), one checks that, as r increases, rich (poor) individuals use less (more) human capital in transferring wealth to the second period. Human capital investments react in opposite way to r according as agents happen to be short or long of asset y. The higher is r, the higher is the amount of output that poor agents have to pay back to the rich in period 2, when state 1 realizes. This brings about a lower supply of y on the part of the poor, a higher asset price q, and a more intense use of human capital as a tool to redistribute wealth from the first to the second life period. As for the rich, their behaviour is symmetric: when r increases, their demand for y is raised, and their use of human capital is reduced, because it has became a less efficient asset in comparison with y. So, as r becomes higher, the use of human capital increases at an aggregate level because *agents become more specialized in their use of alternative assets*. The higher r, the higher the relative use of human capital on the part of the poor. Since poor agents always invest a

[11] The economy is constantly on a determinate balanced growth path (no transition dynamics), so that the time index can be omitted.

higher fraction of available time in accumulating human capital compared with the rich, aggregate human capital investments increase.

From inspection of expressions (29)-(31) for individual consumptions, after substituting κ_t with $1+s^*$, it is immediate that asset payoffs affect consumption allocations through the growth rate. Once a process of endogenous growth is introduced through knowledge spillovers, consumption allocations are no more independent of asset returns even when agents are not constrained by the requirement that $0 \leq s_t^i \leq 1$, $i=1,2$. The introduction of an externality generated by human capital poses a general issue of optimality of the asset structure.

4 Optimal asset structure

Assume that the asset structure is under the control of economic policy. How an efficient asset structure would be charaterized for this economy? When all assets are present, the growth rate is indeterminate. An authority whose aim is the control of the growth rate of the economy would then first get rid of an asset. Recall also that, as long as all individuals are maximizing at the interior (i.e., $0 < s^{i*} < 1$, $i=1,2$), higher growth rates are clearly Pareto improving, because entailing higher future consumption for all agents without compromising present consumption possibilities for anyone. When knowledge spillovers are taken into account, growth rates depend upon human capital investments and then on asset payoffs. As a result, as long as asset returns are compatible with individual human capital investments not being constrained by (21), higher values for r entail Pareto improvements (check from (39)). Recall that whenever income distribution is not too unequal (i.e., whenever $\frac{\delta^1}{\delta^2} < 1 + \frac{2}{\pi n(1-\pi)}$) values for r, $r \geq 0$, exist such that the requirement $0 \leq s^{i*} \leq 1$, $i=1,2$ is not binding. As r increases, rich (poor) agents reduce (increase) their investment in human capital, until one type of agent become costrained at either $s_t^1 = 0$ or $s_t^2 = 1$. Denote by r^1 and r^2 the highest payoff ratio compatible with, respectively, s_t^1 and s_t^1 not being constrained by (21), i.e: $r^1 \equiv \left\{\max r \colon s^{1*} \geq 0\right\}$, $r^2 \equiv \left\{\max r : s^{2*} \leq 1\right\}$. Denote by r^c the asset return ratio prevailing at a candidate optimum. It follows form the argument sketched above that $r^c \geq \min\left[r^1, r^2\right]$; the reverse would mean that the asset structure is Pareto dominated by either r^1 or r^2.

If $r^1 \leq r^2$, then at an optimal asset structure rich agents would make investments in human capital that are null, but would finance abundant use of human capital on the part of the poor. At the opposite, if $r^1 \geq r^2$, poor agents at the optimum would spend all their youth in accumulating human capital, and rich agents would make a "small", but positive investment in human capital.[12] The values for r^1 and r^2 are obtained, respectively, by setting $s^{1*} = 0$ and $s^{2*} = 1$ from (33) and (34)

$$r^1 = \frac{\overline{\delta}\pi\left[\delta^1(1+\pi) + \overline{\delta}(1-\pi)\right]}{\delta(\delta^1 - \delta^2)(1-n)(1+\pi)^2} \tag{40}$$

$$r^2 = \frac{\overline{\delta}\left[2\delta^2 + \pi\left(1-\pi\right)\left(\delta^2 - \overline{\delta}\right)\right]}{\delta(\delta^1 - \delta^2)n((1+\pi)^2} \tag{41}$$

Which of these two cases is relevant to characterize. Optimal asset returns depend on the fundamentals of the economy. It is immediate that

$$r^1 \gtreqless r^2 \Leftrightarrow \frac{\delta^1}{\delta^2} \gtreqless \frac{(1-n)}{n\pi}. \tag{42}$$

Thus, if the economy is characterized by a sufficiently small income-gap δ^1/δ^2, at the optimum rich agents would not invest in human capital, while, if the income-gap is large enough, poor agents would make the maximum investment in human capital. We then call the first type of economy a "relatively equal" economy, and the second type a "relatively unequal" economy.

So, an optimal asset structure requires the payoff ratio to be at least equal to $\min\left[r^1, r^2\right]$. Will it be higher? If yes, at the prevailing asset payoffs, some agents will find themselves constrained by the requirement $0 \leq s_t^i \leq 1$. For the static economy, this case is clearly sub-optimal because markets become effectively incomplete. However, in the presence of endogenous growth generated by knowledge spillovers, this case cannot be excluded. The reason is that when knowledge spillovers are present, individuals do not take into account the effect of their human capital investment on productivity growth. As a consequence, agents always underinvest in human capital. So, if setting $r > \min\left[r^1, r^2\right]$ leads to higher aggregate human capital investments, faster growth may compensate for the static inefficiency and increase welfare for all agents.[13] The basic findings of this section are summarized in the following proposition.

[12] It is a matter of simple algebra to show that $s^1 > 0$ when poor agents are constrained at $s^2 = 1$.

[13] Few can be said concerning aggregate investments in human capital when $r > \min\left[r^1, r^2\right]$. Unreported simulations show that, depending on parameters values, investments for the unconstrained agents may increase or decrease compared with the case where $r = \min\left[r^1, r^2\right]$. Furthermore, since investments in human capital depend in this case upon expected growth rates, multiple equilibria may arise.

Proposition *At an optimal asset structure, if the economy is relatively equal* ($\frac{\delta^1}{\delta^2} < \frac{(1-n)}{n\pi}$), *then* $r \geq r^1$, *and type 1 (rich) agents invest* $s^1 = 0$; *if the economy is relatively unequal* ($\frac{\delta^1}{\delta^2} > \frac{(1-n)}{n\pi}$), *then* $r \geq r^2$ *and type 2 (poor) agents invest* $s^2 = 1$.

5 Conclusions

In this paper we raise a point concerning the normative consequences of the interaction between human capital and financial assets as alternative tools to transfer wealth in a risky environment. In our example, when a full set of Arrow securities is present, individual, and then aggregate investments in human capital are indeterminate. When one financial asset is missing, yet, the investment in human capital is determinate and is affected by asset payoffs. Allowing for externalities generated by aggregate human capital investment, growth rates are necessarily indeterminate when a full set of Arrow securities is present, while it is determinate if one financial asset is missing. The asset payoffs in this second case affects the rate of growth, and then consumption allocations. Optimality of the asset structure arises as a natural issue. Policy action requires the absence of one financial assets from a full set of Arrow securities: only in this case the growth reate can be under control. Asset payoffs should then be structured in such a way that a group of agents is constrained in their human capital investment decisions. Relatively poor agents, at the optimum, should specialize in human capital investments, rich agents would finance human capital of the poor through investments in the financial asset. Depending on the initial distribution of income, either the poor may be willing to invest more than feasible in human capital, or the rich may be willing to make a negative investment. Of course, this contrast with the result that, in absence of externalities, the unconstrained equilibrium is Pareto superior to cases where agents are constrained.

This paper may contribute to fill some gaps existing in two unrelated fields of literature. On the one hand, it gives an illustration of the possible applied relevance of the issue concerning the optimality of the asset structure. In our opinion, this issue has been so far too much confined abstract analysis (see, e.g., Demange and Laroque (1995)). On the other, it poses some new issues in the analysis of the relation between financial markets and growth. There is substatial agreement that few work has been done so far trying to relate the characteristics of the asset structure to the growth performance of countries (Levine (1997)).

References

[1] Bottazzi, L., P. Pesenti, and E. van Wincoop, (1996), "Wages, profits, and the international portfolio puzzle", *European Economic Review*, 40, 219-254.

[2] Demange, G., and G. Laroque, (1995), "Optimality of incomplete markets", *Journal of Economic Theory* 65, 218-32.

[3] Drèze, J., (1990), "Human capital and risk-bearing", in J. Drèze: *Essays on Economic Decisions under Uncertainty*, Cambridge: Cambridge University Press

[4] Geanakoplos, J., (1990), "An introduction to general equilibrium with incomplete asset markets", *Journal of Mathematical Economics*, 19, 1-38.

[5] Geanakoplos, J. and H. Polemarchakis, (1986), "Existence, regularity and constrained suboptimality of competitive allocations when markets are incomplete", in *Essays in Honor of Kenneth Arrow*, Volume 3, ed. by W. Heller, R. Starr, and D. Starrett. Cambridge: Cambridge University Press.

[6] Grandstein, M. and M. Justman, (1996), "The political dynamics of education,endogenous growth and inequality in a mixed economy", *Journal of Economic Growth*, last issue.

[7] Jacoby, H., G., and E. Skoufias, (1997), "Risk, financial markets, and human capital in a developing country", *Review of Economic Studies*, 64, 311-35.

[8] Levine, R., (1997), "Financial development and economic growth: views and agenda", *Journal of Economic Literature*, 35, 668-726.

[9] Lucas, R., (1988), "On the mechanics of economic development", *Journal of Monetary Economics* 22, 3-42.

[10] Weil, P., (1992), "Equilibrium asset prices with undiversifiable income risk", *Journal of Economic Dynamics and Control*, 16, 769-790.