Contract design, credit markets and aggregate implications.

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Introduction

The primary concern of the thesis is to contribute to the analysis of the relationship between competition and incentives, when asymmetric information is explicitly taken into account.
The need for a theoretical understanding of the interplay between markets and contracts is at the center of many recent developments in contract theory. Furthermore, clarifying the role of the coordination mechanisms in markets where information is not perfectly distributed among agents is a crucial step to extend traditional macroeconomic schemes to situations where heterogeneity is considered:

> *Modifying the Brock-Mirman framework - which would eventually become the core for Real Business Cycle theory - to consider financial issues was a formidable task (and remains so today). Modelling imperfections in intertemporal trade obviously requires having an environment where there exists motivation for trade; this necessitates to introduce heterogeneity among agents [...]. In addition abiding strictly by the rules of the game requires endogenously deriving the financial system - after all financial institutions and financial contracts are endogenous variables and, except in the frictionless environment for which the MM theorem is relevant, determined jointly with real activity.*[1]

Our main focus will be the analysis of loan relationships: the research will face several traditional issues that stress the role of asymmetric information in credit markets as the main source of relevant economic phenomena.

First, the imperfect functioning of these markets originated by asymmetric information in lender-borrower relationships can be responsible for existence and propagation of aggregate fluctuations. Many different approaches have been recently undertaken to support the celebrated Gurley and Shaw (1955) conviction that the way in which entrepreneurs decide to finance their production activities and interact with financial institutions is to be conceived as the main source of economic fluctuations. Recent research emphasized the role of financial propagators of

[1] Gertler (1988), p.570.

exogenous shocks in economies where borrowers have private information over the realizations of investment projects (Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1999)) as well as the possibility of endogenous reversion mechanisms (Suarez and Sussman (1997), Kyiotaki and Moore (1997)). The main concern of this literature has therefore been to restate the traditional relationship between limited access to credit and economic fluctuations in a framework where the existence of borrowing constraints is derived from first principles. [2]

The thesis tries to contribute to these lines of research by extending and generalizing the basic representation of lender-borrower relationships involved. More precisely, the first two chapters aim at providing a further analysis of the relationship between borrowers' financial constraints and endogenous fluctuations. Our main target is to emphasize the role of optimal financial contracts choice in macroeconomic dynamics. That is, we follow a recent tendency in the study of credit markets:

> *A more recent and growing literature has investigated the role of financial market imperfections in the macroeconomy using models where agents are allowed to select the best contractual arrangements. Within this alternative research agenda, one is facing the challenge to provide an extension of the existing theories of incentives and financial contracting to a general equilibrium framework where the relation between business fluctuations and credit market imperfections can be analysed.*[3]

As a starting point, we reformulate several classical financial contracting problems under asymmetric information and argue that the so-called Costly State Verification (CSV) paradigm offers the most robust basis to rationalize the role of both debt and intermediaries in loan relationships. Chapter 1 tackles the relevant criticisms addressed to the CSV approach in the financial contracts literature and summarized by Hart in his 1995 book. Those criticisms involve first of all the superiority of stochastic

[2] Irving Fisher was one of the first contemporary scholars who tried to relate price deflations to reallocations of wealth from debtors to creditors leading to a fall in the overall demand. His work emphasized the high increase in the real burden of borrowers debt in the expansion preceding 1929: "The depression out of which we are now (I trust) emerging is an example of a debt-deflation depression of the most serious sort. The debts of 1929 were the greatest known, both nominally and really, up to that time" (Fisher (1933), p.345).

[3] Reichlin (2004), p. 862.

mechanisms with respect to deterministic ones in terms of payoffs and the lack of ex-post efficiency of the equilibrium debt contract. In addition, the optimal debt contract under CSV has the controversial property of identifying the auditing region with the bankruptcy one: in this respect, the CSV approach looks more like a theory of optimal auditing rather than a theory of debt (see Hart (1995)). We argue that the most recent reformulations of the basic CSV scheme overcome the first two difficulties and they also suggest some potential insights on how to address the more general critiques to the CSV approach. That is, we present some arguments in favor of the CSV framework to be used to examine the interaction between economic dynamics and firms' capital structure.

This analysis is explicitly carried out in Chapter 2 where the existence of a relationship between business cycle fluctuations and modifications in the structure of loan contracts is investigated. The aim is to provide a potential departure from the traditional corporate finance theories and to show that the characteristics of firms' capital structure (i.e. their debt-to-equity ratio) can be affected by macroeconomic conditions. We build on the Suarez and Sussman (1997) paper on endogenous fluctuations with asymmetric information in the market for loans by properly modelling a two-period financial contracting problem. The features of optimal securities issued at equilibrium are influenced by macroeconomic conditions, i.e. by equilibrium prices. In other words, asymmetric information in the market for loans is responsible for endogenous fluctuations to take place at equilibrium, but we can also argue that business fluctuations affect the nature of firms' financing over time. As a by-product, the debt-to-equity ratio in the overall economy will vary according to the dynamics of aggregate variables. This argument contrasts pecking-order based theories of financing (Myers, 1984) that explain the composition of firms' external funds by looking at the agency costs faced by entrepreneurs-borrowers under asymmetric information. The idea that macroeconomic conditions are an important component of the debt-equity choice is receiving an increasing support, both in applied and theoretical studies (see Levy (2001), for a survey). We consider a competitive economy populated by an infinite sequence of overlapping generations of borrowers-firms who live for three periods and produce only in the last two periods of their life. Borrowers hold a stochastic production technology but they do not have the resources to carry out their investment projects. The amount of external financing issued is determined by a financial contract signed by every generation of borrowers with a representative consumer-lender. If an optimal dynamic contract exists, then at equilibrium young firms will issue repayments that resemble equity, while old firms will have an incentive to finance through debt arrangements. These alternative securities will coexist in every period: the presence of a dynamic contracting issue is responsible for such a co-existence, while the emergence of standard debt as an optimal arrangement comes from the introduction of a CSV problem. The main feature of our economy turns out to be the interaction between the dynamic contracting problem and the existence of finitely-lived firms: the relevant implication will be the counter-cyclical behavior of the aggregate debt-to-equity ratio. Some econometric investigations developed on US data provide further support to this intuition.

One should note that the analysis developed in the first two chapters is entirely based on an extremely simplified way of looking at financial relationships under asymmetric information. That is, we restricted our investigation to situations where, at the stage of designing contractual arrangements, a single financier is interacting with a single borrower only. A main concern of the thesis is to provide new intuitions on the features of credit market equilibria in a scenario where the relationship between incentives and competition is more deeply examined.

With this perspective in mind, Chapter 3 focuses on the analysis of games where multiple principals compete over the contract offers they are making to multiple agents. Our aim is both to propose a unified framework to analyze the so-called literature on competing mechanisms and to provide some results on the characterization of equilibria in this class of games. By postulating that none of the principals can have the whole control of the communication among other players, the competing mechanisms literature falls outside the traditional applications of mechanism design to principal-agent relationships (Myerson (1982), Myerson (1991)). In such a scenario, several examples have been recently developed (starting from the seminal one of Peck (1997)) to show that whenever multiple principals are considered, it is then possible to construct communication schemes which corresponding equilibrium outcomes cannot be reproduced by simple and Incentive Compatible mechanisms, that are called "direct" by analogy with the single principal case. This finding is usually referred to as a *failure* of the Revelation Principle:

> *When contracting situations become more complex, the applicability and the usefulness of the revelation principle comes into question.*[4]

[4] Martimort and Stole (2002), p. 1659.

We develop this line of investigation in the specific context of Common Agency games.[5] These games have been extensively analyzed in recent theoretical and applied literature. [6] They involve the following timing: principals act as mechanism designers and simultaneously offer contracts to the single agent, given the messages they have received. In a next step, the agent chooses her preferred alternative, contracts are executed and the equilibrium allocation is implemented. The main rationale behind the so-called failure of the Revelation Principle can be summarized as follows: at the stage of contracting with any principal the agent's private information involves both her type and the mechanisms simultaneously offered by all the other principals. Direct mechanisms are too simple as principals cannot profit from the extra information held by the agent. The most recent literature offered several ways to face this difficulty.[7] Surprisingly, relative less attention has been devoted to define some domain conditions that guarantee the Revelation Principle to hold even when competition over contracts is allowed. We explore such a research direction identifying conditions on agent's preferences that are sufficient for the validity of the Revelation Principle when competing mechanisms are introduced. These conditions amount to a separability requirement on agent's preferences with respect to the contract offers made by every principal. The remarkable result is that no restriction on principals' preferences is introduced. Unfortunately, our results do not apply in a more general setting where an arbitrarily large number of agents is considered.[8] Nonetheless, we are able to show that even in these situations there is still some theoretical support to the restriction to the use of simple mechanisms where no communication is allowed. The proof of this last result concludes the chapter.

The last part of the thesis directly investigates the properties of credit market relationships when competing lenders are introduced in the analysis.

More in detail, Chapter 4 proposes a discussion of the traditional *credit view* of Monetary Policy. Asymmetric information in loan markets is

[5]Chapter 3 will discuss more in detail the possible conceptual inconsistencies related to the so-called failure of the Revelation Principle.

[6]Examples are given by: Biais, Martimort and Rochet (2000) who analyze a non-linear pricing environment, Parlour and Rajan (2001) who study the competition among lenders offering non-exclusive credit contracts, Kahn and Mookherje (1998) who present applications to the insurance literature and Dixit, Grossman, and Helpman (1997) in the context of Political Economy.

[7]These approaches will be discussed more in detail in Chapter 3.

[8]That is, when we deal with multi-principal multi-agents games: McAfee (1993), Prat and Rustichini (2003) and many others.

traditionally associated to the rise of a positive wedge between the entrepreneurial costs of external ad internal finance. Such a gap is referred to as the *external finance premium*.[9] Given the agency costs needed to induce either information revelation or effort performances by borrowers, the investment projects that were profitable by using internal funds may not be undertaken when the required funds should by raised by (uninformed) lenders. The so-called credit channel of monetary policy stresses the effect of monetary variables on the real economy that affect borrowers' access to credit. A crucial element of this transmission mechanism is the inverse relationship between borrowers' net worth and the external finance premium. Both theoretical and empirical literature emphasized several distinct effects at the basis of this mechanism.[10] Importantly, all these contributions share the reference to the principal-agent model as the relevant device to analyze the implications of asymmetric information in the credit market.
The chapter suggests a framework where financiers are modelled as multiple lenders who compete over the loan contracts they are simultaneously offering to a single borrower. Both the exclusive case (the borrower is constrained to accept only one contract at a time) and the non-exclusive case are developed. The standard models of the credit channel assume perfect competition among investors.[11] That assumption is usually translated into a zero-profit equilibrium condition and, in the specific context of principal-agent schemes, this calls for the restriction to a binding lender's participation constraint. Now, if lenders are strategically competing on their credit contract offers, positive-profit equilibria typically arise. In particular, it turns out that monetary factors may affect the real sector also modifying the structure of markets. The existence of a credit channel of monetary policy in such a context has not been analyzed so far in the literature.

In our setting, the market for loans typically involve multiple principals competing to offer contracts. When contracts are non-exclusive, each borrower can simultaneously accept the proposals of many financiers, giving rise to a contractual externality among competitors. The last chapter analyzes the aggregate implications of such externalities. That is, we consider a scenario where a Social Planner is subject to the same informational constraints faced by principals in a simple model of the credit market, and has complete control on the borrower's decisions of

[9]See Bernanke, Gertler, and Gilchrist (1996).

[10]See Bolton and Freixas (2000), Oliner and Rudebusch (1996) Holmstrom and Tirole (1997), Kashyap and Stein (1995), Repullo and Suarez (2000).

[11]See e.g. Holmstrom and Tirole (1997) and Repullo and Suarez (2000).

contracts acceptance. We identify conditions that sustain constrained-efficiency of credit market equilibria. We construct a simple competitive framework where the borrower could take a non-contractible effort decision: in every equilibrium the provision of credit will be the same as that attainable in a situation where exclusivity could be enforced. The result contrasts with the idea, advanced in some recent researches,[12] that credit rationing is a phenomenon associated to competition in contracts. Importantly, we argue that these differences are mainly due to alternative specifications of borrower's preferences, which suggests that the general welfare implications of competition in contracts have not yet completely been clarified in the literature.

[12]See Bisin and Guaitoli (2004) and Kahn and Mookherjee (1998).

Chapter 1

Costly State Verification and debt contracts: a critical resume

1.1 Introduction

This chapter provides an analysis of the most classical environment used to account for the widespread use of debt contracts in loan relationships.[1] Far from being a general review of the security design literature, the following paragraphs will be focused on debt as an optimal financial arrangement. The empirical relevance of such a contractual form together with its prominent role in bank financing is widely accepted: Harris and Raviv (1992) provide a very detailed exposition of the most relevant findings.

As will be clearer later on, the basic feature of a standard debt contract relies on the borrower's promise to offer a repayment constant over states, with the bank being allowed to seize the whole cash flow when the repayment cannot be guaranteed. Thus, a debt contract is a concave function of project's outcomes.

With the aim of defining an equilibrium where firms financing take a debt-like form, asymmetric information has been introduced in the lender-borrower relationship in several ways, from the pure adverse selection to the standard moral hazard case.

It is important to remark that the classical *hidden action* moral hazard framework cannot support per se the optimality of debt contracts

[1]This is a joint work with Eloisa Campioni and appeared as a published paper in the last 2003 issue of *Research in Economics*.

unless an external monotonicity constraint is imposed on the repayment structure (Innes, 1990). If we do not assume any monotonicity, the optimal repayment function under moral hazard R can be characterized as follows:

$$\begin{array}{lll} R(y) = 0 & for & y \geq y* \\ R(y) = y & for & y < y* \end{array}$$

where $y \in \mathbb{R}_+$ is the random return of the project. This is clearly different from a debt-like contract, since it provides opposite incentives for effort, giving the agent maximum payoff ($R(y) = 0$) when the result is *good* and maximum penalty ($R(y) = y$) when it's bad: this is usually referred to as a live-or-die contract. Debt can be predicted *only* if we require the repayment to be a non-decreasing function of the project return.[2]

We develop here the so-called Costly State Verification (CSV) paradigm, since according to our view it offers the most robust basis to rationalize the role of both debt and intermediaries in loan relationships. As a starting point, it is useful to remark that the CSV structure examines situations where symmetric information *at the time of contracting* is assumed: within this environment CSV identifies the case where agent's actions can be observed but the contingencies under which these have been taken cannot. [3]

One of the main goals of the present research is to tackle the relevant criticisms addressed to the CSV approach in the financial contracts literature and summarized by Hart in his 1995 book. Those criticisms involve first of all the superiority of stochastic mechanisms with respect to deterministic ones in terms of payoffs and the lack of *ex-post* efficiency of the equilibrium debt contract. In addition, the optimal debt contract under CSV has the controversial property of identifying the auditing region with the bankruptcy one: in this respect, the CSV approach looks more like a theory of optimal auditing rather than a theory of debt (Hart, 1995). Finally, while in financial relationships debt coexists with other claims, the basic CSV model leaves no room to alternative arrangements like equity

[2] Innes (1990), p. 33.

[3] In the particular terminology introduced by Arrow and restated in the popular Hart and Holmstrom (1987) work, this is a case of moral hazard with hidden information. Of course, the focus on this structure will restrict both the solution concepts we will employ and the nature of the implementation problem. Furthermore, under CSV a precise need for banking activity emerges: banks turn out to be essential in reducing monitoring costs (in Diamond (1984) terms they perform a delegated monitoring activity).

The point is that in non-bankruptcy states E (the borrower) always make a payment that is just enough to satisfy creditors; there is never anything left over for other claimants [...]. And, of course, in bankruptcy states outside shareholders receive nothing because there is not enough even to satisfy creditors. [4]

Our research attempts at showing how the most recent reformulations of the basic CSV scheme overcome the first two difficulties and suggests some potential insights on how to address the more general critiques to the CSV approach. The whole analysis is carried on within a *complete contracts* paradigm.

It is important to remark that our main focus is the optimal contracts design, so that we abstract from risk sharing issues and restrict the analysis to risk-neutral agents. To this extent, we can also disregard the distribution of bargaining power between agents since it does not affect the form of the optimal contract but only the division of the surplus.

Throughout the chapter we deal with one agent acting as a mechanism designer, who offers a financial contract to his counterpart under a well-defined set of constraints. When the borrower is such a mechanism designer, the lender's participation constraint can be interpreted as a zero-profit condition: since it is always binding, we can generally think of this lender as acting on perfectly competitive financial markets.

The discussion will hence be organized as follows: Section 1.2 will present the basic scheme, as it was developed in the classical Gale and Hellwig (1985) paper; Section 1.3 will examine the introduction of stochastic auditing rules in the *one period* contracting problem and Section 1.4 will present the recent work by Krasa and Villamil attempting to generalize the contracting problem when *commitment* is a strategic variable. Some potential rationales for the role of equity financing in a CSV environment are discussed in Section 1.5. Section 1.6 shows how a distinction between auditing and bankruptcy may arise either in the presence of a repeated interaction or when a collateral is introduced. Section 1.7 offers a discussion of the optimality of debt financing in a CSV with *ex-ante private information.* Finally, the relationship between CSV and the literature on financial intermediation is briefly sketched.

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[4] Hart (1995), p. 124.

1.2 The Gale and Hellwig scheme

An entrepreneur who is running a firm owns a project which requires an initial investment i at time zero and gives a random return y at time one. The entrepreneur-borrower wants to undertake the project but he has to rely on a lender to get external finance. The lender could alternatively access a competitive capital market with rate of return r. Both agents are risk-neutral.

At the time of contracting, agents have symmetric information on future outcomes. Once uncertainty over project's returns realizes, only the borrower can observe the state at no cost. Borrowers are assumed to report their private information through a message and lenders perform auditing activity conditional on the received messages. Auditing discloses the true state to the lender and its costs are a function of borrower's current assets.

Uncertainty is described by a set of states of nature $S = \mathbb{R}_+$ whose elements are indexed by s with relevant distribution function $H : \mathbb{R}_+ \to [0, 1]$. Let $i \in \mathbb{R}_+$ be the investment in the risky project, the returns from investment i in state s are given by $y(s, i)$.

Assumption 1: $y : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous function which is twice continuously differentiable on the interior of its domain. In addition, it is concave in i and the following boundary condition holds:

$$y(0, i) = y(s, 0) = 0 \tag{1.1}$$

Assumption 2: Auditing costs are defined by the continuous function $c(s, i)$; $c : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable on the interior of its domain. Furthermore:

$$\frac{\partial}{\partial i} c(s, i) \geq 0 \qquad \frac{\partial^2}{\partial i^2} c(s, i) \geq 0 \qquad \frac{\partial}{\partial s} c(s, i) \geq 0 \tag{1.2}$$

1.2.1 The contract problem

The total amount to be financed is i. The borrower acts as a mechanism designer: he offers a contract trying to induce lender's participation.

In the Gale-Hellwig (GH) framework the general mechanism is given by $\Gamma \equiv (i, M, C, R, B)$ where $M \subseteq \mathbb{R}_+$ is the borrower's message space, $C \geq 0$ is the entrepreneur's contribution (that is usually interpreted as the amount of *internal* equity), $R(m)$ is the repayment to the lender as a function of the declared state m. Finally, $B(m) : M \to \{0,1\}$ is the function defining the *auditing region*, with $B = 1$ identifying the set of states where auditing is actually taking place.[5]

The analysis of the symmetric information case is straightforward: the *first best* level of investment, say i^*, solves the following:

$$i^* = \arg\max_{i \geq 0} \int [y(s,i) - (1+r)i] dH \tag{1.3}$$

Uniqueness of the solution is guaranteed by Assumption 1.[6] Now, if $A \geq 0$ denotes the borrower's initial assets, in order for external financing to have a role we should also introduce the following:

Assumption 3: $\quad i^* \in (0, \infty)$ and $A < i^*$.
Moving to the private information case, we properly define the optimal contracting problem. The incomplete information game associated with this contract design problem is such that for every given mechanism, entrepreneur's strategies are messages $m \in M$ and the lender can accept or refuse the associated proposal. The equilibrium allocation is the result of implementation in Bayes-Nash equilibrium.

The mechanism design problem is to find an array (i, C, R, B) that maximizes borrower's expected utility under lender's zero profit (Individual Rationality constraint) and Incentive Compatibility (IC) constraints.

To define the role of the IC constraint, let us denote the entrepreneur's wealth in non-audited states as:

$$W(s,m) = y(s,i) + (1+r)(A - C) - R(m) \tag{1.4}$$

For every s realized state and m message such that $B(m) = 0$, the entrepreneur's wealth at the interim stage will depend both s and m. The Revelation Principle (Myerson, 1979) allows to restrict the attention to implementation through direct mechanisms, where the message space coincides with the state space $M = S = \mathbb{R}_+$. The notion of Incentive Compatibility in the CSV framework has been originally specified by Townsend.

Proposition 1 (Townsend, 1979) *A contract* Γ *is Incentive Compatible (IC) iff:*

[5]It should be noted that we are not including in the description of the general mechanism the principal's strategies "accept or refuse the mechanism", just for the sake of simplicity.

[6]The objective function in (1.3) represents the borrower's surplus given the lender is exactly guaranteed his reservation utility (lender's individual rationality constraint is binding).

- *there exists a constant* $R_1 \in \mathbb{R}_+$ *such that* $R(s) = R_1$ *for every* s *such that* $B(s) = 0$
- *for all* (s, s') *s.t.* $B(s') = 0$, $B(s) = 1$ *and* $W(s, s') \geq 0$, *then* $R_1 - R(s) \geq c(s, i)$.

The contract problem can be stated as follows:

Problem 1

$$\max_{i,C,R,B} \int [y(s,i) - (1+r)i - c(s,i)B(s)]dH \qquad (i)$$

$$(i, C, R, B) \quad \text{is} \quad IC \qquad (ii)$$

$$\int R(s)dH = (1+r)(i - C) \qquad (iii)$$

$$i \geq 0 \quad \text{and} \quad C \in [0, A] \qquad (iv)$$

$$R(s) \leq y(s,i) - c(s,i)B(s) + (1+r)(A - C) \qquad (v)$$

The role of constraint (iii) is that of a zero profit condition (Individual Rationality) for the lender, while (v) is a limited liability (LL) condition that prevents borrower from ending up with negative net wealth. We can now define a Standard Debt Contract (SDC henceforth).

Definition 1 *A contract* (i, C, R, B) *is a SDC iff* $\forall s$*:*

- *for some* $R_1 \in \mathbb{R}_+$, $(1 - B(s))(R(s) - R_1) = 0$ (*fixed repayment*)
- $B(s) = 1 \Leftrightarrow y(s, i) < R_1$ (*bankruptcy decision*)
- $B(s)R(s) = B(s)[y(s,i) - c(s,i) + (1+r)(A - C)]$ (*maximum recovery*)

It is worth noticing that in the definition above the set of states such that $B(s) = 1$ coincides with bankruptcy, while $y(s,i) - c(s,i)$ is the part of revenues the investor can recover under bankruptcy.

The argument advanced by GH to prove the optimality of SDC is presented in several steps. First of all, they restrict to the set of feasible contracts satisfying $(ii) - (v)$ and observe that without loss of generality it is possible to focus on the subset of contracts with *maximum equity participation* (MEP), that is contracts implying $C = A$. Given this conclusion, the degree of equity participation turns out to be indeterminate at equilibrium. [7] We prove hereafter that the SDC contract is the optimal mechanism.

[7] Gale and Hellwig (1985), p. 654.

Proposition 2 *If (i, C, R, B) is a solution to Problem 1 then it is a SDC, i.e. there exists $R_1 \in \mathbb{R}_+$ such that $R(s) = \min[y(s,i), R_1]$ and $B(s) = 1$ whenever $y(s,i) < R_1$.*

Proof. We organize the proof in several steps.

step I: Maximum Equity Participation (MEP)

Without loss of generality we can set $C = A$.

Assume (i, C, R, B) is an optimal contract. If we replace it with a new contract (i, C', R', B) where:

$$\begin{pmatrix} C' = A \\ R' = R - (1+r)(C - A) \end{pmatrix}$$

then the objective function and the relevant constraints are unchanged. In fact,

$\int R'(s)dH = (1+r)(i - C') \Leftrightarrow \int R(s)dH - (1+r)(C - A) = (1 + r)(i - A)$

and for every s and i

$R'(s) \leq y(s,i) - c(s,i)B(s) + (1+r)(C' - A) \Leftrightarrow R(s) - (1+r)(C - A) \leq y(s,i) - c(s,i)B(s)$

step II: Uniform Repayment

$R(s,i) = R(s',i) = R_1$ for all s, s' s.t. $B(s) = B(s') = 0$ is guaranteed by IC.

step III: Bankruptcy Rule

If (i, R, B) solves the principal-agent problem, then $B(s) = 1$ for every $s \in \mathcal{B}$ where $\mathcal{B} = \{s : y(s,i) < R_1\}$ where $R_1 = R(s,i)$ for $s \notin \mathcal{B}$

Observe that, if $y(s,i) < R_1$ then $R(s,i) \leq y(s,i) - c(s,i)B(s) \leq y(s,i) < R_1$ and therefore $R(s,i) < R_1$.

Now, assume by contradiction that $R(s,i) < R_1$ is compatible with $s \notin \mathcal{B} : y(s,i) \geq R_1$. The IC constraint will imply $R(s,i) \leq R_1$. There can be two cases:

a) if $R(s,i) = R_1$, it is possible to define a new contract $B' = B \backslash \{s\}$ that will be strictly preferred to the initial one, generating a contradiction;

b) if $R(s,i) < R_1$ we can define a new array $(R'(.), B')$ where $\mathcal{B}' = \mathcal{B} \backslash \{s\}$ and

$$R'(.) = \begin{cases} R_1 - d & \text{for} s = s' \\ R(s') & \text{otherwise.} \end{cases}$$

where d is a linear function of $[R_1 - R(s,i)]$ that increases the objective function and satisfies the IR and IC constraints.

Thus, the bankruptcy rule implies that if an array (i, R, B) solves the principal-agent problem, then $\mathcal{B} = \{s : y(s,i) < R_1\}$ defines the verification region as the bankruptcy one.

step IV: Maximum Recovery

$R(s) = y(s,i) - c(s,i)$ for all $s \in \mathcal{B}$

By contradiction we can argue that if $R(s) < y(s,i) - c(s,i)$ on some subset of $\mathcal{B}$, then it would be possible to raise $R(s)$ in that subset and to reduce R_1 so to keep the feasibility conditions unaltered and give the borrower higher payoff.

Therefore IR and LL constraints are binding and the lender appropriates all the available assets of the firm in bankruptcy states.

step V: Definition of R_1

If $(i, R(.), B)$ is an optimal solution of the principal-agent problem then $\exists R_1$ s.t. $R(s,i) = \min\{y(s,i) - c(s,i), R_1\}$ and $B(s) = 1$ whenever $s \in \mathcal{B} = \{s : y(s,i) < R_1\}$ thus (R_1, i) solves:

$$\max_{i,R_1} \int \min[y(s,i), R_1] dH - \int_{\mathcal{B}} c(s,i) dH$$

$$s.t. \int \max\{y(s,i) - R_1, 0\}\, dH = \overline{u}$$

■

Thus any optimal contract can be meaningfully represented by a SDC with MEP, that is by the array (i, R_1). Now, by previous assumptions we can write $R_1 \geq 0$[8] and without loss of generality we can also assume $R_1 \leq \sup\{y(s,i) \mid H(s) < 1\}$ for any optimal contract. The existence of upper and lower bounds for R_1 together with Assumption 2 ensure us about the existence of a state γ such that $y(\gamma, i) = R_1$. It follows that we can alternatively identify a contract with the pair (i, γ), where γ is defined to be the *bankruptcy point*, in the sense that bankruptcy occurs if and only if $s < \gamma$. [9]

In this resume of the GH incentive problem, we did not discuss the features of the second best level of investment i^{**}: the authors show that with a positive default probability and an increasing cost of bankruptcy with respect to i, the Pareto constrained level of investment is strictly lower than i^*. [10]

[8] Assume we cannot, then incentive compatibility implies that $R < 0$ and the zero-profit condition implies $i - A < 0$. It would follow that $i = i^*$ contradicting Assumption 3.

[9] Notice that γ is unique whenever $i > 0$, while if $i = 0$ (implying $A = 0$) such a γ does not exist, by Assumption 1.

[10] It is also important to mention that GH characterize their result as a form of credit rationing, where the size of the loan is the rationed variable. In such a perspective,

Interestingly, allowing for a risk-averse borrower together with a risk-neutral lender still enables us to find a sort of SDC at equilibrium. GH have shown that under these conditions bankruptcy states are associated to borrower's detention of a positive and constant amount of the asset constituting a form of insurance against unfavorable states. [11]

Two main criticisms have been addressed to this optimality result: first, Townsend (1979) himself suggested that allowing for stochastic auditing rules would have been Pareto improving and second, that there was an implicit assumption that both parties fully committed to the contractual obligations without any possibility of reneging or renegotiating the initial contract. New research started from these dissatisfactions.

1.3 Stochastic auditing rules

The first relevant extension of the basic CSV framework is to allow for random auditing following Mookherjee and Png (1989).[12] The basic finding we will discuss is that under stochastic verification the optimal contract may not exhibit the SDC feature, since it is not possible to identify a bankruptcy point in nontrivial optimal incentive schemes. Moreover, when auditing is not performed, repayments are not constant over states. We will just slightly modify our previous setting, in order to focus on the optimal contracts properties. We will consider a fixed investment level that generates n possible levels of realized income $Y_1, ..., Y_n$ with the correspondent probabilities $\lambda_1, ..., \lambda_n$. As before transfers $R_s = R(s)$ ($s = 1, 2, ...n$) from the agent to the principal depend on the agent's report on the realized state s, but now the report s is audited with probability p_s at the cost $c_s > 0$; if there has been untruthful report of state t when s occurred, then the agent must pay a penalty F_{ts}.[13] Finally, the analysis introduced here will keep the assumption of risk-

credit rationing is not due to indivisibility of investment projects, as in the classical Stigliz and Weiss (1981) work.

[11] A complete analysis of such a structure is developed in Garino and Simmons (1995). Their main finding is that at equilibrium the marginal utility of resources held by the borrower under bankruptcy should be equal to the expected marginal utility of consumption in non-bankruptcy states.

[12] In the original paper by Townsend (1979), the intuition on some possible improvement when we allow for stochastic auditing was already given.

[13] It's important to notice that this penalty was implicitly included in the discussion proposed by GH. In that setup, the penalty was to be such that consumption was set to zero in case of untruthful report: $Y_s - R_s - F_{ts} = 0$.

neutral agents[14]. In order to isolate the problem of designing the optimal mechanism, we leave aside the decision on the optimal level of investment in the risky project and we restrict to the case of a penniless borrower.

Therefore, the relevant *direct* mechanism is here represented by the array (p_s, R_s, F_{ss}) and the contractual problem can be written as:

Problem 2

$$\max_{p_s, R_s, F_{ss}} \sum_{s=1}^{n} \lambda_s [p_s(Y_s - R_s - F_{ss}) + (1 - p_s)(Y_s - R_s)] \qquad \forall s, t$$

s.t.

$$p_s(Y_s - R_s - F_{ss}) + (1 - p_s)(Y_s - R_s) \geq (1 - p_t)(Y_s - R_t) \qquad \forall s, t \quad (1.5)$$

$$\sum_{s=1}^{n} \lambda_s [R_s - p_s(c_s - F_{ss})] \geq (1 + r) \qquad \forall s, t \quad (1.6)$$

$$Y_s - R_s - F_{ss} \geq 0, \quad Y_s - R_s \geq 0, \quad 1 \geq p_s \geq 0 \qquad \forall s, t \quad (1.7)$$

where the objective is to choose a probability structure, a transfer and a penalty maximizing the borrower's expected utility under the incentive constraint (1.5) that guarantees agent's truthful report, the participation constraint (1.6) that gives the lender a minimum payoff $(1 + r)$ and the usual limited liability (1.7) constraints for both parties to avoid negative consumption.

The first result is that IR constraint has to be binding at the optimum, otherwise it would be possible to define a new contract that increases the agent's expected utility, reducing the penalty F_{ss}, without affecting the principal's payoff, but contradicting the optimality of the candidate solution. Then, it is shown that if the agent's report is verified and discovered to be truthful, she must not be punished, thus $F_{ss} \leq 0$. The main predictions, however, are stated in the following proposition:

Proposition 3 *(i) Optimal schemes exist.*

(ii) In any optimal scheme that provides the agent with positive consumption for every income realization, all income reports that are audited must be audited randomly: that is, $p_s < 1$ for all s .

(iii) Every optimal scheme has the property that if the agent's report is audited and verified to be truthful, the agent must be rewarded; that is, $F_{ss} < 0$ whenever $1 > p_s > 0$. [15]

[14] In the original Mookherjee and Png (1989) paper the borrower is risk-averse, but as we show the assumption is not crucial to obtain the result.

[15] See Mookherjee and Png (1989), p.406,for the equivalent statement.

This proposition deserves some comments since it is at the heart of the discussion on stochastic verification. The proof of part *(ii)* is given by contradiction assuming that there exist income returns that are always verified, i.e. $p_s = 1$ for some s, and then noticing that in such cases, it would always be profitable to slightly reduce these p_s without changing the direction of the IC inequalities but strictly diminishing the expected cost of auditing in IR. Thus any candidate mechanism with $p_s = 1$ for some s would not be optimal.

Up to this point only two components of the optimal mechanism have been characterized. The relationship between the optimal transfers and the optimal audit probabilities remains to be described:

Proposition 4 *In any optimal scheme, reports corresponding to the highest repayment will not be audited; all other reports must be audited with positive probability; reports corresponding to higher repayments will be audited with equal or lower probability, if $R_t > R_s$ and $1 > p_s$, then $p_t < p_s$.*

An analysis of the propositions above confirms that the SDC structure is fragile with respect to random auditing. The first proposition actually denies the existence of a threshold bankruptcy point. The second one states that the optimal repayments to the lender are state-contingent since p_s is everywhere lower than one. In other words, given that at the optimum the probability of no audit is never zero, a constant repayment across all states of solvency cannot be identified.

1.4 Contracts without commitment

The analysis we developed so far is based on the assumption of *full commitment* to the mechanism defined in the initial period. Whenever limited commitment is introduced, agents have an incentive to renegotiate the original agreement once the true state is known. In the GH framework in particular, the optimal debt contract is not *ex-post* efficient. [16]

[16] If renegotiation is allowed, then the entrepreneur's reward becomes a signal of the project's return: "As in many signalling models, there can be multiple *ex-post* equilibria, with the signal being interpreted in various ways. While refinements can eliminate some of these equilibria, the analysis of optimal *ex-ante* contracts quickly becomes complicated" (Hart, 1995, p.124).

The recent work by Krasa and Villamil (2000) provides a meaningful generalization of the GH set-up where limited commitment and stochastic contracts coexist. Their setup allows to define stochastic contracts or deterministic contracts as an equilibrium property of the relevant mechanism they introduce. When commitment is limited, deterministic mechanisms are optimal, hence the standard debt contract solves the security design problem; when the borrower fully commits to the initial proposal, stochastic mechanisms dominate deterministic ones.

The GH structure is modified in two respects: there exists limited commitment to the mechanism contracted initially and the lender decides whether to exercise enforcement or not on the basis of the observed repayment.

The interaction between two risk-neutral agents takes place over three periods: at date one nature chooses the project outcome in the discrete set $Y \subseteq \mathbb{R}_+$, at date two the borrower makes a voluntary payment R which is interpreted as a signal of the realized state by the lender and at date three the lender chooses whether or not to adhere the court to enforce the contractual promise from the borrower. His decision will be taken after having updated his beliefs on the realized outcome according to Bayes' rule. There is room for renegotiation. Enforcement is provided at no cost by a court whose technology will be specified later on. [17] Figure 1.1 provides a representation of the time structure of events.

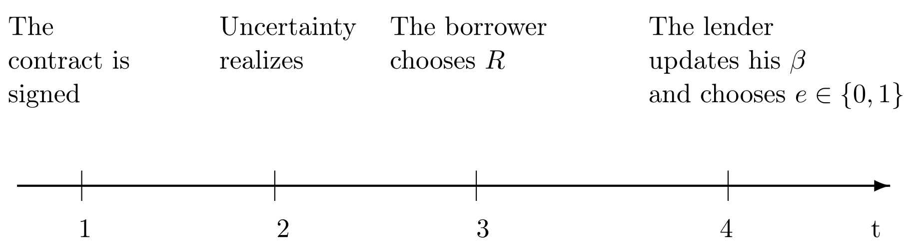


Figure 1.1: Time structure in Krasa and Villamil (2000)

The first departure from the basic framework can be found in the description of the contract: instead of defining the pair (R, B), a contract

[17] KV assume that the output of the project is observable but not "entirely" verifiable, in the sense that the entrepreneur can hide part of the realized return: we denote it with $\bar{y}$.

will be the array $(V, F, \sigma_B, \sigma_L)$, where V is the set of voluntary payments, F is a penalty payment, σ_B and σ_L are the associated mixed behavioral strategies.

In particular, $\sigma_B : Y \rightarrow \Delta(V)$ associates a probability to each payment $R \in V$ given the realization $y \in Y$ and $\sigma_L : V \rightarrow \Delta(\{0,1\})$ associates a probability to the choice of the enforcement e given the repayment R. If the enforcement is represented by the binary variable $e = \{0,1\}$, then agents ex-post payoffs can be defined as:

$$\pi_B(y, R, e) = y - R - e[F(x, R) + c_B]$$
$$\pi_L(y, R, e) = R + e[F(x, R) - c_L]$$

where c_B and c_L are the fixed cost of enforcement and $x = \max\{y - R - \bar{y}, 0\}$, $\bar{y}$ being the amount of investment return the entrepreneur can hide from enforcement.

The definition of the mechanism is completed by the choice Perfect Bayes Nash Equilibrium (PBNE) as the appropriate solution concept. [18]

1.4.1 Optimal deterministic contracts

To define the optimal contractual problem, we have to consider the event of renegotiation after the voluntary payment has been made: in such a case the lender could redefine new terms with respect to what was agreed in the initial period. Thus, the optimal problem to be solved in the initial period will have to be integrated by a second optimization problem that would give the continuation contract, once the lender's beliefs about the truly realized return will have been updated.

Thus, in such a dynamic environment, the revelation principle fails to apply since there exist future opportunities to modify the initial agreement.

In order to solve the renegotiation issue, a time consistency constraint will have to be imposed on the problem to guarantee that all possible

[18] We find useful to refer to the definition used by KV in order to be able to refer to the same conditions when stating theorems and giving proofs.

Definition 2 *A collection of strategies σ_B, σ_L and beliefs β, β' constitute a Perfect Bayesian Nash Equilibrium if and only if:*
(i) $\sigma_B \in \Sigma_B$ maximizes $E_{\sigma_B,\sigma_L}\pi_B(y, R, e)$ for every y
(ii) $\sigma_L \in \Sigma_L$ maximizes $\sum_{y\in Y}\beta'(R; y)E_{\sigma_L}\pi_L(y, R, e)$ for every R
(iii) β' is derived using Bayes' rule whenever possible.

incentives for altering the initial contract are foreseen in the first negotiation.[19]

Any optimal contract will hence maximize the investor's expected utility under the usual IR constraint, LL or feasible payments requirement, the PBNE *in lieu* of IC condition and a time consistency condition:

Problem 3 At $t = 0$ choose $\sigma_B(y;R), \sigma_L(R;e), V, F$ to maximize

$$\sum_y \beta(y) E_{\sigma_B,\sigma_L} \pi_L(y,R,e) \tag{1.8}$$

s.t.

$$\sum_{y\in Y} \beta(y) E_{\sigma_B \sigma_L} \pi_B(y,R,e) \geq \overline{U} \tag{1.9}$$

$$R \in [0,y] \text{ and } F(.) \in [0,x] \quad \forall y \in Y \tag{1.10}$$

$$\sigma_B(y;R), \sigma_L(R;e), \beta, \beta' \text{ is a PBNE at } t = 2 \tag{1.11}$$

$$R, F, \sigma_L(R;e) \text{ is time consistent} \tag{1.12}$$

Since there exist subsequent possibilities to alter the contract after is has been signed, we have to use indirect implementation through PBNE (1.11). In addition, ex-post efficiency implies time consistency (1.12).

This formulation of the contractual problem allows for stochastic mechanisms.

A debt contract is interpreted in this set-up as a set of bankruptcy states and a critical $y^* \in Y$ such that:

- $B = \{y \in Y : y < y^*\}$ and

$$e = \begin{cases} 0 & \text{for } y \notin B \\ 1 & \text{for } y \in B \end{cases}$$

- the entrepreneur pays

$$R = \begin{cases} R^* & \text{for } y \notin B \\ 0 & \text{for } y \in B \end{cases}$$

moreover,

$$F(y,R) = \begin{cases} 0 & \text{for } y \notin B \\ y & \text{for } y \in B \end{cases}$$

[19] If time consistency constraint is imposed, agents cannot recontract in future periods to increase their expected payoffs, (Dewatripont, 1989). In other terms, the time consistency constraint implies that in a future period of renegotiation agents will choose $R' = R, F' = F(x,R), \sigma'_L = \sigma_L$ where R', F', σ'_L is the continuation contract and R, F, σ_L is the initial contract.

Theorem 1 in Krasa and Villamil (2000) shows that under some restrictions lender's and borrower's minimal payoffs from enforcement activity, such as:

$$(\overline{y} - c) \in (0, x_0) \tag{*}$$

$$x_0 < \sum_{y<y^*} (y - c)\beta(y \mid y < y^*) \tag{**}$$

where $c = c_B + c_L$ is the total costs of auditing for the borrower and the lender, debt is optimal in a limited commitment environment with possibly stochastic mechanisms.[20] It can be stated in the following way:

Theorem 1 *Assume there exists a simple debt contract within the class of optimal CSV contracts which satisfies the conditions (*)-(**) and which gives the borrower the reservation utility $\overline{U}$, then this contract solves Problem 3.* [21]

In order to prove this theorem KV introduce three propositions showing first of all that time consistency implies a deterministic σ_L. Secondly, a one to one relationship between contracts in the CSV framework and in the KV model is proved also in terms of expected payoffs.[22] A debt contract in KV setup is optimal because it minimizes information revelation (debt is *informationally minimal*). The first conclusion is that the KV debt outcome turns out to be ex-post efficient and optimal even when random monitoring is allowed, extending therefore the standard CSV analysis.

1.4.2 Optimal stochastic contracts

In the same way, the optimality of stochastic contracts turns out to be an equilibrium outcome when we consider full commitment mechanisms.

[20] We refer to limited commitment because the voluntary repayment R cannot be retracted once it has been made, it is in fact "*money on the table*".

[21] The original statement has been corrected here to take into account the counterexample provided by Sharma (2003). Sharma's counterexample sheds light on the existence of a trade-off between minimization of auditing costs and time consistency constraints in the costly enforcement model. Nonetheless, it does not alter the main conclusion and the isomorphism between the CSV and the costly enforcement structure. See Krasa and Villamil (2003) for details.

[22] Without loss of generality, the analysis can be restricted to deterministic σ_B. See the Appendix of Krasa and Villamil (2000).

In this case, agents commit to the initial contract and have no opportunity to alter their decisions at any point in time. The time consistency constraint is therefore irrelevant here and the Subgame Perfect Nash Equilibrium concept allows to solve the sequential game.

The new problem to be solved is:

Problem 4
At $t = 0$ choose $\sigma_B(y; R), \sigma_L(R; e), V, F$ to maximize:

$$\sum_y \beta(y) E_{\sigma_B,\sigma_L} \pi_L(y, R, e) \tag{1.13}$$

s.t.

$$\sum_{y \in Y} \beta(y) E_{\sigma_B \sigma_L} \pi_B(y, R, e) \geq \overline{U} \tag{1.14}$$

$$R \in [0, y] \text{ and } F(.) \in [0, x] \tag{1.15}$$

$$\text{part (i) of PBNE definition} \tag{1.16}$$

KV show that Problem 4 is equivalent to the original problem defined by Gale and Hellwig (1985) and elaborated by Mookherjee and Png (1989), if we consider that the monitoring probability in the GH can be identified with the enforcement probability in the model with commitment. The following theorem holds:

Theorem 2 *Problem 2 and 4 are equivalent. Stochastic contracts are optimal.*

The most important conclusion that can be derived from the KV general framework is that the relevance of debt contracts emerges in environments where renegotiation is possible as in credit and loan contracts, while stochastic contracts emerge when there is the institutional possibility of full-commitment to auditing activity (public authorities or insurance companies).

1.5 Debt and outside equity under CSV

As far as the standard CSV framework accounts for the existence of debt claims, it cannot cope with firms' capital structures involving combination of debt and external equity. The only existing equity is owned entirely by borrowers while outside investors can only be debt claimants.[23]

[23]It was already noted by Townsend that "The [CSV] model as it stands may contribute to our understanding of closely held firms, but cannot explain the coexistence of publicly held shares and debt" (Townsend, 1979, p.289).

Some recent generalizations of the CSV aim at discussing firms' capital structure in a security design perspective where the optimal financial arrangement turns out to be a mix of debt and equity.

Boyd and Smith (1999) present a model where the role of equity comes from the availability for the borrower of a technology with observable returns. The optimal repayment has a constant component that looks like debt and a state-contingent one which is interpreted as equity.

More formally, there exists three technologies: a storage one available to all agents offering a safe return r and two productive technologies available to the entrepreneur-borrower only. The first one is a production technology with observable and discrete random returns $x = \{x_1, ..., x_n\}$, the second provides returns represented by means of a continuous random variable $y \in [0, \bar{y}]$ subject to CSV. [24] The following relationship holds:

$$\hat{y} = \int_0^{\bar{y}} y dH(y) > \hat{x} = \sum_n p_n x_n > r$$

i.e., there is no incentive to invest in the commonly available technology, since it provides the lowest possible return. The choice between the observable and the unobservable one will depend on the amount of verification costs c that is only paid investing on y.

A contract is defined by the triple $(B, R(.), \theta)$: B is the binary variable associated to the monitoring decision,[25], $R(y, x_n)$ is a repayment function and θ denotes the ratio between resources invested in the observable technology and those invested in the unobservable one.

The set-up of the problem is standard: the borrower offers a contract trying to induce lender's participation under IC and LL constraints. The objective function is

$$\theta\hat{x} + (1-\theta)\hat{y} - \sum_n p_n \int_{\mathcal{B}} [R(y, x_n) - c] dH(y) - \sum_n p_n \int_{\mathcal{B}^c} R(x_n) dH(y)$$

where the IC guarantees that the repayment over $\mathcal{B}^c = [0, \bar{y}] \backslash \mathcal{B}$ will be $R(x_n)$, hence independent of y. The solution is derived in two steps. In a first stage θ is taken as given and it is shown that:

$$\mathcal{B}(x_n) = \left[0, \frac{R_1(x_n) - \theta x_n}{1-\theta}\right) \tag{1.17}$$

[24] We follow the authors in introducing a discrete returns distribution; this is done to avoid some technical difficulties.

[25] As usual, we will use $\mathcal{B}$ to indicate the set of states where monitoring takes place, hence given a certain realization $y \in [0, \bar{y}]$, if $B(y) = 1$ then $y \in \mathcal{B}$.

$$R(y, x_n) = \theta x_n + (1-\theta)y \quad \forall y \in \mathcal{B} \tag{1.18}$$

That is, the verification region coincides with the bankruptcy region and is a decreasing function of θ. In case of bankruptcy, the lender seizes all available cash flows. In the second step the upper bound of the bankruptcy region $z_n \equiv \dfrac{R_1(x_n) - \theta x_n}{1-\theta}$ and θ are determined.

At equilibrium, z_n is independent of n, i.e. the extension of the bankruptcy region is not influenced by the realization of the observable technology: there exists a sort of smoothing of the monitoring costs across states. Furthermore, the equilibrium value θ^* turns out to be in the interior of $[0, 1]$, i.e. the entrepreneur always invests in both technologies, and (1.17) implies that the optimal repayment in the non-monitoring states is:

$$R_1(x_n) = (1-\theta^*)z(\theta^*; c) + \theta^* x_n$$

where $z(\theta^*; c)$ is the equilibrium value for the bankruptcy threshold given θ^*. In terms of interpretation, $R_1(x_n)$ includes both elements of equity and debt contracts. The equity component is the term that depends on the realization of the observable technology, $\theta^* x_n$. The payment to debt-holders is identified with $(1-\theta^*)z(\theta^*; c)$ which is not contingent on firms' performance. Of course, due to the peculiar specification of the productive technology, bankruptcy is defined as the event of firms unable to repay their *debt* obligation. There can be cases when only equity-holders perform monitoring, but still they cannot start a bankruptcy procedure. These cases correspond to firms experiencing low returns, which nevertheless avoid bankruptcy repaying their debt-holders. Outside-equity holders cannot force bankruptcy even if they can use other instruments to uncover information about the firm.

As a first attempt to extend the CSV approach towards mixed financial structures involving debt and equity, this is still limited by the strong assumption that equity emerges only when technological returns are observable. In addition, the discussion about the positive effect of combining alternative claims, in particular the fact that equity helps reducing verification costs acting as a "collateral", is based on the comparison between a costly claim with another one which is not subject to any cost. If a cost could be associated to equity, for example following the dilution cost paradigm, [26] then a more satisfactory theory of complex financial structures could emerge.

[26] See Myers and Majluf (1984).

Such a dissatisfaction pushed towards alternative approaches, where the cost of equity is explicitly considered. Hvide and Leite (2002) give a possible explanation to the co-existence of debt and outside equity inside an ex-post efficient mechanism, where both security-holders are subject to an asymmetry of information with respect to the project realizations and holding each claim is costly.

Debt is identified by the face value $R \in \mathbb{R}_{++}$ and the right to perform auditing; equity is taken as a linear proportion of firms' revenues: $E = \beta y$ with $\beta \in (0,1]$ together with the associated right to intervene. They focus on the game at the renegotiation stage, when the borrower proposes an $\tilde{R}$ to debt-holders and an $\tilde{E}$ to shareholders.

Each investor has the choice between accepting or rejecting the proposal, with independent probabilities $p(\tilde{R}) \in [0,1]$ and $q(\tilde{E}) \in [0,1]$ respectively. The possibility to play mixed strategies at equilibrium is crucial to guarantee that the initial contract constitutes a SPNE. We restate the main Hvide and Leite (2002) proposition, which defines the optimal security:

Proposition 5 *In equilibrium,* $\tilde{R} = R$ *and* $\tilde{E} = \beta(y - R) - c_E$ *with* $p(.) = 1$*,* $q(.) > 0$ *whenever* $y \geq R^*$ *and* $\tilde{R} = y - c_D$*,* $\tilde{E} = E = \beta c_D - c_E$*, with* $p(.) > 0$ *and* $q(.)$ *constant whenever* $y < R^*$*.*

Equilibrium defines a threshold level R^* above which debt obligations are entirely fulfilled, equity-holders are residual claimant and accept the new proposal with a positive probability, which increases with y. For realizations of y below R^*, debt holders refuse the proposed $\tilde{R}$ and monitor the realized state, equity holders will only participate for the proportion βc_D of revenues.

A separation of control on entrepreneur's report emerges at equilibrium: for high realization states the equity-holders discipline the firm, while for low realizations the auditing mechanism of the CSV applies.

This new set-up accounts both for *strategic default* at equilibrium, by which even if there are enough resources to satisfy debt obligations the entrepreneur defaults, and for violation of the *absolute priority rule*, which appears in the low-realizations region where in fact even if debt obligations are not satisfied in full nonetheless equity-holders receive some positive repayment.

It is important to notice that this approach concentrates on the analysis of a subgame perfect equilibrium contract and therefore it focuses on the robustness property of some given initial proposal with respect to subsequent profitable deviations. Of course, this method wanders from the security design approach we followed throughout these notes and it does not provide an answer to the question of why should firms strategically choose mixed financial structures.

1.6 Auditing costs and bankruptcy

The main feature of the CSV framework is the idea that the ex post asymmetry of information can be eliminated at some cost: the cost of verification. This was originally interpreted by Townsend (1979) and GH (1985) as a pecuniary cost in terms of utility losses that could depend on states realizations and on the amount invested, no matter which of the party had to bear it.

The role played by verification costs in this framework has been subject to two main remarks: auditing does not necessarily imply bankruptcy, in the sense that a firm could in principle be inspected several times before terminating her operations. Furthermore, empirical studies show that auditing costs represent only a fraction of overall default costs. [27]

The present section is devoted to the discussion of these two issues. The first part aims at showing that repetition of the interaction could overcome the identification of monitoring with bankruptcy and the second suggests that it is possible to think of bankruptcy procedure as involving the loss of valuable assets for the entrepreneur.

1.6.1 Optimal debt contracts in dynamic contexts

This paragraph will explicitly summarize the main attempts to represent a dynamic interaction within the CSV literature, with the idea that repetition can introduce a separation between auditing and bankruptcy.

We start referring to Webb's (1992) work, where the basic distinction between *short term* and *long term* contracts is in Hart and Holmstrom (1987) we argue that long-term contracts derive from an inability to costlessly verify contingencies. In particular, if output realizations are not costlessly verifiable, a long-term contract may then be used to induce truthful revelations that cannot be supported by a sequence of short-term contracts. This leads to a saving of verification costs" (Webb (1992), p.1113). He argues that a sequence of two *short term* standard debt contracts cannot be the solution of a twice repeated CSV problem: it is

[27] See again Hart (1995), p. 125.

in fact Pareto dominated by a contract with contingent repayments in the first period.

The main differences with the basic one-stage environment are the following: each entrepreneur can select a project in each of the two periods $t = 1, 2$; the project requires an initial investment K_t and yields random return y_t.

A long term contract can be signed at $t = 1$. The entrepreneur is free to sign contracts with competitive outside lenders at any point in time. If a sequence of standard debt contracts is selected, then there exists a threshold repayment level R_t which the investor repays when she is solvent and identifies the bankruptcy region in any period. Now, a long-term contract implying contingent first period repayments together with debt in the second period, improves upon a short-term sequence. The reason is that the amount the entrepreneur will borrow in the second period is inversely related to his first period net assets; therefore, since the second period contract is contingent upon first period reported states, having variable repayments in the first period can reduce the risk of bankruptcy.

We want to remark that Webb's result strongly relies upon the assumption that the relevant IR constraints are recursive, that is they can be written period by period.[28] Actually, long term contracts require a weaker IR constraint to hold: it imposes the lender to earn non-negative expected profits across states. There exists an explicit improvement when such a weaker IR holds: transferring utility across states is a further source of reduction of the probability of bankruptcy. A numerical example along these lines has been recently provided by Snyder (2001).

Chang (1990) considers a two-period CSV interaction subject to an ex-ante IR constraint: we resume here his main assumptions and results.

At time zero the entrepreneur needs funds to finance an investment of 1, which provides random returns y_1 at time one and y_2 at time two. y_t is distributed over $[0, Y_t) \subseteq \mathbb{R}_+$ according to a distribution function H_t for $t = 1, 2$, y_1 and y_2 are by assumption independent. Both agents are risk-neutral.

The verification cost function $c_t(x_t)$ for $t = 1, 2$ is non decreasing and smooth and x_t represents the total value of the asset of the firm at time t.

In this context a contract is simply a list $K = [B_1(y_1),\ R_1(y_1),\ B_2(y_1, y_2),\ R_2(y_1, y_2)]$, which prescribes a verification decision and a

[28]The author interprets this as a way to model market competition in each relevant period.

repayment for each period as function of the reported realizations for y_1 and y_2. [29]

Since agents are risk-neutral, we can refer to the dual problem and minimize the expected verification costs under incentive compatibility, individual rationality and limited liability constraints. The solution of the problem is provided using optimal control techniques and when $c_2(.)$ is strictly increasing, can be described as follows:[30]

- when $y_1 > R_1(y_1) + R_2(y_1)$ all debt is repaid at date $t = 1$;
- when $m < y_1 \leq R_1(y_1) + R_2(y_1) = M$ then $B_1(y_1) = 0$ and the borrower pays $R_1(y_1) = y_1$ and $R_2(y_1) = M - y_1$;
- when $y_1 \leq m$ then $B_1(y_1) = 1$ audit takes place and $r_1(y_1) = y_1$ and $r_2(y_1) = y_2 = k < R_2(y_1)$

where $R_2(y_1)$ and $r_2(y_1)$ represent the threshold values for the second-period repayment as a function of first-period project's realization respectively when $B_1(y_1) = 0$ and $B_1(y_1) = 1$; m, k and M are positive constants.

The optimal contract is therefore a standard debt contract with the additional features that the firm is required to repurchase a minimum amount of her liability before maturity (m) and she has the option to pay up to M in the intermediate date. These can be interpreted as a coupon and a call option , where the higher the repayment in the first period the smaller the repayment in the second period with a consequent reduction in the risk of bankruptcy. In terms of interpretation we also want to underline that the optimal contract defines a region where despite auditing takes place at $t = 1$ the firm will continue operating until the final period, it is not clear here. This is a first intuition to distinguish between auditing and bankruptcy and it sheds light on the need for further investigation in this direction.

[29] Specifically, $B_t(y_t, y_s)$ is a binary variable representing the probability that verification takes place after the announcement(s) y_t (and y_s). Since no randomization is allowed, $B_t(.)$ can only take values $\{0, 1\}$ for every $t = 1, 2$. In particular, it takes value one when bankruptcy occurs, where $R_t(y_t, y_s)$ is the payment to the lender at t given the announcement(s) y_t (and y_s).

[30] We will follow Chang (1990)) notation in writing the repayment functions with capital letters when there is no verification and in lower-case letters when verification occurs. Thus, $R_1(y_1)$ represents the payment to be given the investor in case of no verification, $B_1(y_1) = 0$, while $r_1(y_1)$ is the repayment when verification occurs $B_1(y_1) = 1$. The same holds for $R_2(y_1, y_2)$ and $r_2(y_1, y_2)$.

1.6.2 Collateralized debt contracts

The literature on collateralized debt has recently provided an interpretation of the role of collateral as a potential substitute for verification cost within the hidden information moral hazard framework: we will briefly present the basic framework proposed by Lacker (2001) in the specification with a perfectly divisible collateral good. The approach formalizes the notion that collateral requirements act as repayment incentives, in the sense that the borrower will have to surrender his collateral in the event that payment cannot be made as promised. [31] The crucial condition for the optimality of debt contract is that the borrower and the lender have different relative valuations of the collateral good, in particular it is required that the borrower values this good more than the lender.

The model is very simple: there are two dates $t = 1, 2$, two agents $j = b, l$ and two goods $i = 1, 2$ where good 1 is the payment good and good 2 is the collateral good. Let us assume, without loss of generality, that the risk-averse borrower (agent b) is endowed with a random quantity of good 2 ($\widetilde{y}$) and owns $k > 0$ units of good 1, while the risk-neutral lender (agent l) has a known and constant endowment e of good 1. Agents meet in the first period and contract upon the repayment due in exchange for a loan advance, in the second period the random variable realizes and agent b observes the realization: he can truthfully report it to agent l or lie and claim a lower y. The lender cannot observe the realized y but she wants to induce truthful revelation from the borrower. Here is the role of the collateral: the contract prescribes that if the promised repayment cannot be made, the lender can appropriate the collateral of the borrower. Since the borrower values the collateral good more than the lender, the optimal contract will minimize the expected value of the collateral transfer subject to incentive compatibility and individual rationality constraints. More precisely, a contract is defined as a pair of functions of the states $[R_1(y), R_2(y)]$ that represent the transfers of good 1 and 2, respectively. The optimal contract can be found solving the following constrained expected utility maximization problem:

[31] With this respect the collateral good has the same role as the "non-pecuniary penalties" in the CSV model developed by Diamond (1984). He introduced the possibility that even if it could not be possible for the lender to verify due to the extremely high verification costs, there existed some "non-pecuniary penalties", in terms of loss of reputation of the entrepreneur for example, that could induce the borrower to truthfully reveal the realized state. We will briefly discuss this item in Section 1.8.

Problem 5

$$\max_{R_1(y),R_2(y)} \int [u_1(y - R_1(y)) + u_2(k - R_2(y))]\, h(y)dy$$

$$s.t.\ u_1(y-R_1(y))+u_2(k-R_2(y)) \geq u_1(y-R_1(y'))+u_2(k-R_2(y')) \quad (IC)$$

$$\forall (y, y') \in \Omega \times \Omega \ \ s.t.\ \ y' < y$$

$$\int [e + R_1(y) + \mu_l R_2(y)]\, h(y)dy \geq \overline{v_l} \qquad (IR)$$

$$R_1\,(y) \in [-e, y] \qquad (LL)$$

$$R_2\,(y) \in [0, k] \qquad (LL)$$

where we have assumed that u_i with $i = 1, 2$ are the continuous, strictly increasing and concave utility functions of the borrower for good 1 and 2; that the lender is a risk-neutral agent and μ_l is her marginal rate of substitution between collateral and payment goods; $h(y)$ is the density function for the random variable $\widetilde{y}$ which is defined on the support $\Omega = [y_0, y_1]$. The constraints on Problem 5 are the incentive compatibility, the individual rationality for the lender and the limited liabilities constraints, respectively.

Definition 3 *A* collateralized debt contract *is defined as a pair* $[R_1^*(y, R), R_2^*(y, R)]$ *such that:*

$$\begin{cases} R_1^*(y, R) = \min\{y, R\} \quad \forall y \in [y_0, y_1] \\ R_2^*(y, R) = 0 \quad \forall y \in [R, y_1] \\ R_2^*(y, R) = k - \phi_2\left[u_2(k) - u_1'(0)(R - y)\right] \quad \forall y \in [y_0, R) \end{cases}$$

where R is the contractual repayment of good 1. This contract implies, for realizations of y greater than R, a constant transfer of good 1, R, and none of good 2, while if $y < R$ then the borrower will have to transfer all of good 1 and part of good 2. It is also evident that the solution to the examined maximization problem will provide the minimum amount of transfer in collateral admissible under incentive and participation constraints. Let $\rho^*(y)$ be the coefficient of absolute risk aversion of the borrower for good 1 and $\mu_b^*(y)$ and μ_l be the borrower's and lender's marginal rate of substitutions between the collateral and the consumption good, the main proposition for the optimality of collateralized debt states the following:

Proposition 6 *If a collateralized debt contract* $[R_1^*(y,R), R_2^*(y,R)]$ *satisfies IR with equality and*

$$\mu_l < \mu_b^*(y) - \rho^*(y)\mu_b^*(y)\phi(y) \quad \forall y \in [y_0, y_1]$$

then R is the unique optimal contract.

When the borrower is risk-averse,[32] the second term on the right hand side of the previous inequality is positive, optimality of the debt contract requires that the gap between the borrower's and the lender's valuations of the collateral to be greater than $\rho^*(y)\mu_b^*(y)\phi(y)$, which represents the improvement in risk-sharing that can be obtained increasing the collateral payment for state y while reducing the payment of good 1. [33] In other words, Proposition 6 can be interpreted as a lower bound on $\mu_b^*(y) - \mu_l$, that is the gap between the relative valuations of the collateral to the two agents, and since it depends on the coefficient of absolute risk aversion, it shows that the smaller $\rho^*(y)$ the smaller the effect of incentive constraint and the smaller the gain of giving the borrower less collateral. When the condition on marginal rates of substitutions does not hold, in each state there exists an allocation at which the marginal rates of substitution of the borrower and the lender are the same and the contract resembles a riskless debt. The incentive compatibility constraint binds and the value of the repayment is constant over states.

1.7 Optimal contracting under ex-ante and ex-post private information

This section discusses the optimality of debt financing when a more complex structure of information is introduced.

There have been relatively few works suggesting a generalization of the CSV framework to allow for ex-ante private information. The first attempt has probably been provided by **?** who built on the Stigliz and Weiss (1981) intuitions, but the clearest analysis is in our view the one

[32]In the special case when the borrower is also risk-neutral, the relevant proposition simply requires that $\mu_l < \mu_b$, i.e. that the borrower simply values the collateral good more highly than the lender. The optimal contract therefore is the one that minimizes the expected deadweight loss due to the transfer of the collateral from the high-value user to the low-value user.

[33]In particular, $\phi(y)$ is a term which emerges in the optimization problem and measures the value of resulting improvement in risk sharing. See Lacker (2001) for details.

recently suggested by Choe (1998), who formally introduces a mechanism design set up. We will therefore briefly discuss his main results.

Let us start from the GH framework assuming that both the investment level and the auditing cost c_1 are fixed. Ex ante private information is represented by the random variable $x : \Omega \longrightarrow \Theta = \{\theta_1, \theta_2\}$ and it is obviously correlated with $y : \Omega \longrightarrow \mathbb{R}_+$ describing the returns from the project: the conditional distribution function will be given by H_i. [34] The following mechanism fully characterizes the lender-borrower relationship.

Definition 4 *A mechanism is a collection* (D, S, C, F_L, M, B, R) *where each element represents:*

$D = \{0, 1\}$ *the borrower's decision of accepting (1) or rejecting (0) the mechanism;*

S *the borrower's space of signals;*

$C : S \rightarrow \{0, 1\}$ *the decision of accepting (1) or rejecting (0) the project;*

$F_L : \{0, 1\}$ *the lender's decision of financing (1) or not (0) the project;*

M *the borrower's space of messages (reports);*

$B : S \times M \rightarrow \{0, 1\}$ *the (deterministic) auditing region;*

$R : S \times M \times B(.,.) \times Y \rightarrow \mathbb{R}_+$ *the transfer to the lender.*

An important assumption for the following analysis is:

Assumption 4: H_1 dominates H_2 in the sense of first order stochastic dominance; moreover $\int ydH_1 > K + c_1$, $\int ydH_2 < K$ and $p \int ydH_1 + (1-p) \int ydH_2 \geq K + c_1$.

In other words, the borrower gets higher profits undertaking the project only when θ_1 is observed rather than undertaking it regardless of the observed signal. In the first case the borrower gets $\int ydH_1 - K$, while in the second one $p \int ydH_1 + (1-p) \int ydH_2 - K$ and the latter identifies the optimal Pareto-constrained allocation we want to implement.

Two general classes of mechanisms are considered:

- mechanisms with precommitment (MWP) where the lender chooses his strategy over F_L after the borrower has sent his signal on ex ante private information, thus $f_L : S \rightarrow F_L$ is the contingent strategy;

- mechanisms without precommitment (MNP) where the lender moves before receiving that message and his uncontingent strategy is given by $f_L \in F_L$. [35]

[34] H_i is the conditional distribution of Y given $X = \theta_i$ with support $[y_1, y_2] \subset \mathbb{R}_+$.

[35] Notice that the borrower's strategies remain the same in the two cases: (d, R', R'') where $d \in D$, $R' : \Theta \rightarrow S$ and $R'' : \Theta \times X \rightarrow M$.

For the MNP the relevant solution concept is Perfect Bayesian equilibrium suitable for signaling games, while in the MWP a complete information game is played and the (Bayes) Nash equilibrium implementation applies.

The principal's expected utility maximization problem is subject to the usual participation and limited liability constraints:

Condition 1 *(LL) A mechanism satisfies limited liability if, for all* $(s,m) \in S \times M$ *and* $x \in X$*, there is* $v \in [y_1, y_2]$ *such that:*

$$R(s,m,B(s,m),y) \leq B(s,m)(y-c_1) + [1-B(s,m)]v \quad \text{if } \ C(s)=1;$$

$$R(s,m,B(s,m),y) \leq K \quad \text{if } \ C(s)=0.$$

Individual Rationality guarantees that the borrower will always accepts the mechanism, i.e. $d=1$, the lender will always finance the project in case of precommitment, i.e. $f_L = 1$. The MNP will reduce to the array (S, C, F_L, M, B, R) excluding D and the MWP to (S, C, M, B, R) where both D and F_L have been excluded.

However, given the structure of the problem there is an advantage for both the lender and the borrower to play the game with precommitment. In fact, if the lender could take the financing decision after receiving the borrower's signal and if he could correctly infer the type of borrower from the received signal, he would never finance a project which belongs to type θ_2, by Assumption 4, while the borrower would always prefer the project to be started. In MNP it is not optimal for the borrower to send a signal which allows the lender to separate between types: only pooling equilibria can thus be sustained and the optimal Pareto constrained allocation can never be implemented. [36] In order to reach this allocation we will consider MWP and within this class of mechanisms, only those supporting separating equilibria.

Lemma 1 *For any mechanism with precommitment that has a pooling equilibrium in which the project is chosen, there exists a mechanism with precommitment that has a separating equilibrium and dominates it.*

Applying the Revelation Principle (Myerson, 1979), we can restrict our attention within the class of mechanisms with precommitment to those direct mechanisms that allow for truthful implementation, in other words we restrict to Incentive Compatible (IC) mechanisms.

[36] See *Lemma 1* in Choe (1998), p.244.

Lemma 2 *For any mechanism with precommitment that has a separating equilibrium, there exists a direct mechanism with $S = \Theta$, $M = Y$ which has an equilibrium in which the borrower truthfully reports the observed information signal as well as the return from the project. The equilibrium payoffs for both agents in the direct mechanism are the same as the equilibrium payoffs of the original mechanisms.*

Given the two sources of asymmetric information, there will be two different IC constraints: one for the truthful report of the information signal and the other for the report of the project return. The restriction to direct mechanisms allows us to describe the MWP with a triple (B, R, r) where B represents the verification region, R the transfer to the lender and r, with $r \leq K$ is defined as $R(\theta_2, ., ., .) = r$.

The incentive compatibility constraints (IC) are specified as:

- (IC_1) for truthful report of the information signal:

$$\int \{y - R[y, B(y), y] - c_1 B(y)\} dH_1 \geq K - r$$

$$K - r \geq \int \{y - R[y, B(y), y] - c_1 B(y)\} dH_2$$

- (IC_2) for truthful report of the return on the project:

  there exists $v \in [y_1, y_2]$ such that $\forall y, y'$:

$$R[y', B(y'), y] = v \qquad \text{if} \;\; B(y') = 0$$

$$R[y, B(y), y] + c_1 \leq v \quad \text{if} \;\; B(y) = 1, B(y') = 0 \quad \text{and} \quad y \geq R[y', B(y'), y].$$

Lemma 3 *Being the mechanism a triple (B, R, r) we have to redefine the corresponding IR and LL constraints, for the borrower (B) and the lender (L) respectively:*

$$p \int \{y - R[y, B(y), y] - c_1 B(y)\} dH_1 + (1-p)(K - r) \geq 0 \qquad (IR_B)$$

$$p \int R[y, B(y), y] dH_1 + (1-p) r - K \geq 0 \qquad (IR_L)$$

$$\forall y, y' \qquad R[y, B(y), y'] \leq [1 - B(y)] v + B(y)(y' - c_1) \qquad (LL)$$

for some $v \in [y_1, y_2]$

The optimal mechanism is obtained maximizing the lender's objective function under IC_1, IC_2, the IR and LL constraints.

Given this structure, the IR_B is implied by LL. This is clearly an extension of the original GH framework and Choe's result consists in showing that whenever the project is undertaken, the optimal mechanism has the SDC features, in particular:

$$\begin{array}{lll} R[y', B(y'), y] = y - c_1 & \text{and} \quad B(y') = 1 & \text{if } y' \leq v \\ R[y', B(y'), y] = v & \text{and} \quad B(y') = 0 & \text{if } y' > v \end{array}$$

Notice that any such mechanism having the SDC form satisfies LL and IC_2. An optimal mechanism (B, R, r) can be classified as a standard debt contract whenever r can support (B, R) to meet the additional IC_1 constraint.

The following proposition states that the standard debt contract (v, r) is a feasible mechanism: [37]

Proposition 7 *There exists a pair* $(v, r) \in [y_1, y_2] \times [0, K]$ *satisfying both* IC_1 *and* IR_L.

The argument suggested by Choe to show the optimality of debt relies on the property that a SDC minimizes the auditing region, the existence of an r satisfying IC_1 is basically guaranteed by Assumption 4.

Proposition 8 *If (B,R,r) is a mechanism with precommitment satisfying IC, IR and LL then there exists a mechanism with a standard debt contract that satisfies the same constraints and dominates it.*

Looking at managerial relationships, this setting prescribes that a positive compensation should be guaranteed to the manager even when a project is not undertaken. This type of compensation can be interpreted as a *golden parachute* giving the manager an incentive to make an efficient use of ex-ante private information.

1.8 Debt contracts and financial intermediation

Up to now we tried to argue that the CSV framework is the most appropriate way to analyze the optimality of debt contracts. Many important

[37] See Choe (1998), p.248.

contributions tried to evidence how the basic predictions of CSV can be compatible with the existence of financial intermediaries, too. Such intermediaries perform different activities:

> "(1) they issue securities which have payoff characteristics which are different from those of the securities they hold (2) they write debt contracts with borrowers (3) they hold diversified portfolio (4) they process information. Also, they ration credit in equilibrium which some would characterize as an empirical fact, as do Stigliz and Weiss (1981)". [38]

We resume in what follows the main results obtained by Williamson (1986). The basic aim of his work is to show that an equilibrium allocation where financial intermediation activity is performed Pareto dominates the direct lending allocation, under the assumption of so called *large scale investment project*. The reason for a financial intermediary to exist is to eliminate the duplication of monitoring activity which would emerge in case of direct lending.

More formally, let us assume that every lender is endowed with one unit of consumption good and thus the fixed outlay $i > 1$ to start an investment can only be provided by i lenders. The first step is to consider the usual CSV environment when financial intermediation is not allowed. The optimal allocation implemented by the standard debt contract will guarantee the following expected utility for the direct lender:

$$U_D(R) = \int_0^R y\ h(y)\ dy + R(1 - H(R)) - cH(R) \tag{1.19}$$

where $y \in [0, \overline{y}]$, is the random return from the project, $h(.)$ and $H(.)$ are the density and distribution function of y, respectively, c is the monitoring cost and R is the usual fixed payment in the debt contract. The second step is to introduce financial intermediaries. It should be noticed that intermediaries are here represented by groups of lenders who write an identical contract with every entrepreneur they finance. Each intermediary aims at maximizing his expected utility function issuing financial claims to other lenders and lending i units of the unique consumption good to every entrepreneur she is contracting with. [39] If an intermediary is contracting with m borrowers, she will herself participate

[38] Williamson (1986), p.161.

[39] This way of characterizing financial intermediaries is quite different from the traditional Diamond (1984) one where intermediaries are conceived as specific agents performing the classical delegated monitoring activity. It should also be clear that Williamson is not dealing with coalitions of intermediaries *à la* Boyd-Prescott, either.

with her endowment and the amount $mi - 1$ will be raised from other lenders. Under some regularity conditions on the relevant distribution functions, each of the m optimal contracts signed between the intermediary and the m borrowers is a standard debt contract. It can be shown that the constant repayment R received by j-th borrower ($j = 1, ...m$) solves the following problem:

Problem 6

$$\max_R i \int_R^{\overline{y}} y_j h(y_j) dy_j$$

s.t.

$$\int_0^R y_j h(y_j) dy_j + R(1 - H(R)) - (c/i)H(R) = r$$

In this problem, we let r to be the riskless interest rate paid on the capital market. It turns out that the lender's expected utility will be $U(R)$:

$$U(R) = \int_0^R yh(y)dy + R(1 - H(R)) - (c/i)H(R)$$

which is clearly greater than $U_D(R)$ and leaves unchanged the utility of the borrower. In other words, at any equilibrium (R^*, r^*, q^*)[40] direct lending is dominated by financial intermediation: lenders who act as intermediaries are somehow producing information in a more efficient way. This is clearly a positive result for the existence of financial intermediaries. [41]

It should be remarked that the most popular work on financial intermediation under asymmetric information is probably the one due to Diamond (1984), developed inside a CSV structure. As well known, the main departure from Williamson's framework stays in the introduction of nonpecuniary bankruptcy penalties which replace the auditing activity in inducing truthful revelation of the realized state; they are deadweight

[40] Notice that r^* and q^* are the market risk-less interest rate and the aggregate loan quantity at equilibrium, respectively, and R^* is the constant repayment in the debt contract. An equilibrium with intermediation is defined by a triple (R^*, r^*, q^*) which satisfies:

(i) R^* solves Problem 6

(ii) $q^* = \alpha \int_{\underline{t}}^{r^*} h(t)dt$

(iii) either (a) $q^* = (1 - \alpha)i$
or (b) $q^* < (1 - \alpha)i$ and $1 - H(R^*) - (c/i)h(R^*) = 0$.

[41] It is important to remark that we're not dealing here with the issue of credit rationing, that is a central part of Williamson's analysis.

losses at social level since the lender cannot appropriate them. However, the optimality of debt obtained by Diamond is not robust to the introduction of borrower's risk aversion, as Hellwig has recently shown:

> *The nonlinearity of the borrower's utility function implies that the nonpecuniary bankruptcy penalty that is required to discourage the borrower from underreporting his ability to pay will itself be given by a nonlinear function of the amount of underreporting.*[42]

This result provides a very strong argument in favor of the standard CSV framework as the only foundation for the coexistence of debt and intermediation. [43]

1.9 Conclusion

The main objective of our discussion has been to isolate the conditions under which the fact that it is costly for external investors to observe project returns implies that debt is the optimal financial arrangement. The argument has been developed in such a way to address several recent qualifications to the Costly State Verification framework as presented in Hart (1995). Our conclusion is that the simple one-period Gale and Hellwig (1985) scheme has been significantly generalized to cope with the fundamental critiques raised in the literature.

The last section of the work remarks that under some definite circumstances it is optimal for debt issues to be intermediated by institutions identified with banks: Hellwig (2001) provides a careful restatement of the coexistence of debt and intermediation.

Starting from these results, two streams of research can further be explored.

The first one deals with a generalization of the logical structure we've been describing up to now through the introduction of multi-lender analysis, that is a potential requisite for decentralizing the contractual allocation. We just mention here the very recent work by Khalil, Martimort and Parigi (2002) building on *common agency* literature: one of the most relevant predictions of this work is to recover optimality of debt

[42]Hellwig (2001), p. 417.

[43]It should also be stressed that the *collateralized debt* approach presented in the Section 1.6.2 is unaffected by Hellwig's remark.

contracts as a consequence of principals competing both in transfers and in monitoring.

The second one is definitely more related to macroeconomic issues. Given that under this asymmetric information structure Modigliani-Miller's theorem fails to hold, there is room for the CSV framework to be used to analyze the interaction between economic dynamics and firms' capital structure.

This issue will be tackled in the next chapter.

Chapter 2

Financial contracting along the business cycle

Over the last few decades there has been a considerable resurgence of interest in the hypothesis that a deeper understanding of the nature of credit market imperfections may be helpful in characterizing economic cycles. In this regard there have been several different approaches to developing the original intuition of Gurley and Shaw (1955) that the way in which agents finance their activities, interact with financial institutions, and choose their contractual arrangements is the main source of economic fluctuations. A majority of these studies have followed the approach most common in business cycle theories, where the economic system is assumed to be intrinsically stable, and its dynamics might become erratic and cyclical because of the impact of exogenous stochastic shocks. These shocks, either monetary or real, may in turn crucially alter firms *balance sheets*, thus limiting their access to financial markets.[1] In contrast, a somewhat less popular research direction has analyzed competitive economies where the presence of asymmetric information in the market for loans is responsible for the existence of endogenous business fluctuations. From this perspective the nature of financial markets is a source of instability that prevents economic dynamics from gravitating around its stationary position.[2]

[1]This is the reference framework of very influential works like Greenwald and Stigliz (1986), Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996) and Kiyiotaki and Moore (1997).

[2]In a series of contributions in the late 80s M. Woodford argued that the explicit consideration of the imperfect working of financial markets could alleviate the lack of empirical relevance of the cases of competitive economies exhibiting endogenous fluctuations.

In more recent years, a deeper representation of loan relationships, i.e., an analysis of the allocations implemented by financial contracts, has been conceived as a natural generalization and reinforcement of the latter approach. One of the most important works in this direction has been provided by Suarez and Sussman (1997), who show that, given a lender-borrower relationship that is subject to moral hazard, liquidity effects turn out to be the source of endogenous fluctuations in an economy where financing takes place through both external and internal revenues. Since firms' efforts to subscribe to good projects is a decreasing function of the amount of external financing, cycles may arise because of the dependence of internal liquidity on prices. Hence during booms there will be an increasing demand for external financing, with the main consequence of amplifying moral hazard effects and this will induce recessions.[3]

The present essay adds to this literature, and hence departs from traditional corporate finance theories, by suggesting the existence of a relationship between business cycle fluctuations and modifications in the structure of loan relationships. More specifically, we consider the case where the *form* of the financial contract is dependent on macroeconomic conditions, i.e., on equilibrium prices. In other words, asymmetric information in the market for loans is still responsible for endogenous fluctuations taking place in equilibrium, but we further argue that the business cycle affects the nature of firms' financing over time. As a result we show that the debt-to-equity ratio in the overall economy will vary according to the dynamics of aggregate variables. One should note that this argument lies in contrast to pecking-order based theories of financing (Myers, 1984) that explain the composition of firms' external indebtness by looking at the agency costs faced by entrepreneurs-borrowers under asymmetric information.

For the task at hand we embed a simple two-period contracting problem into an economy á la Suarez and Sussman. More precisely, we consider a competitive economy populated by an infinite sequence of overlapping generations of borrowers-firms which live for three periods and produce only in the last two periods of their life. Borrowers hold a stochastic production technology but do not have the resources to carry out their investment projects. The amount of external financing issuance is determined by a financial contract signed by each generation of borrowers with a representative consumer-lender. If an optimal dynamic contract exists, then at equilibrium young firms will issue repayments that resemble equity, while old firms will have an incentive to finance through debt arrangements. These alternative forms of financing will

[3]See also Reichlin and Siconolfi (2004).

coexist in every period: the presence of a dynamic contracting issue is responsible for such a coexistence, while the emergence of standard debt as an optimal security comes from the introduction of a Costly State Verification (CSV) problem. The main feature of our economy turns out to be the interaction between the dynamic contracting problem and the existence of finitely-lived firms. Such an interaction is the key to understanding the role of liquidity and the potential switching between debt and equity financing in our economy.

The research closest to our theoretical model has been carried out in several works by Boyd and Smith.[4] These authors defined a framework that can allow for the co-existence of debt and equity financing in a one-stage borrowing relationship. Additionally, they embedded such a contractual structure into an optimal growth framework in order to explain how there can exist an incentive for equity issuing in economic development. The growth process is then entirely driven by investments in the commonly available and observable technology given that the amount invested in non-verifiable projects is kept constant at its physical maximum.

One should note that there are already a number of theoretical studies that argue that macroeconomic conditions are an important component of the debt-equity choice (see Levy (2001), for a survey). Moreover, a growing mass of empirical studies is suggestive of the possibility that the aggregate debt-to-equity ratio may exhibit counter-cyclical behavior. For example, Choe, Masulis, and Nanda (1993) provide evidence that equity is issued in an anti-cyclical manner, while Korajczyk and Levy (2002) find that firms opt for debt rather than equity financing the lower the rate of market return is. We set out to seek more explicit empirical support for our theoretical result of a counter-cyclically moving aggregate debt-to-equity ratio by examining historical data for the US. We indeed find evidence strongly supportive of this.

The remainder of the chapter is organized as follows. Section 2.2 discusses the basic Suarez and Sussman framework. Section 2.3 presents our extension of this model. Empirical support for our main finding is found in Section 2.4. Concluding remarks are provided in the final section.

2.1 The Suarez and Sussman economy

Suarez and Sussman (SS) consider a competitive economy evolving in discrete time $[t = 0, 1, 2, ..., \infty]$ and populated by two types of agents:

[4] See Boyd and Smith (1998) and Boyd and Smith (1999).

entrepreneurs-borrowers and consumers-lenders. Two goods are exchanged. The first one is the numeraire good, while the second is referred to as coffee. Only the former can be invested. Commodities are exchanged in a competitive market where the relative spot price of coffee p_t is determined. Coffee is the only produced good. Consumers are identical and infinitely lived. In contrast, there exists an infinite sequence of three-period living overlapping generations of borrowers. Entrepreneurs have linear preferences with respect to the numeraire and cannot consume coffee. Each generation of entrepreneurs consists of a continuum of agents with unit mass; they have no physical endowment but they hold a stochastic technology for the production of coffee. Production can be activated by paying a cost consisting of one unit of numeraire and the production process takes place during two periods: in the first one $Y > 0$ is produced, while second period output is identified by the binary random variable $\tilde{Y}$ that can take the two values $\{0, Y\}$ with probabilities $1 - \pi$ and π, respectively. Production is subject to a standard moral hazard problem: borrowers can take a costly hidden action (effort) that affects the output probability distribution.[5] More precisely, borrowers' effort is set equal to π, the probability of getting Y. The disutility of effort is given by the continuous function $\psi(\pi) : [0, 1] \to \mathbb{R}_+$ which is increasing, strictly convex and satisfies standard boundary assumptions. The lender is allowed to exchange with every generation of borrowers one unit of the asset he holds (the numeraire) with an activity that will give him a repayment R_{t+1} in period $t + 1$ for every financial arrangement signed in $t-1$. The entrepreneur born in $t-1$ offers a contract that maximizes his expected payoff under the standard Incentive Compatibility (IC), Individual Rationality (IR), and Limited Liability (LL) constraints. If the feasible set is non-empty (so that an optimal contract exists), then $\pi^{**} = \pi^{**}(p_t, p_{t+1})$ will be the optimal level of effort. Existence of an infinite population of borrowers of measure one is postulated, so that the law of large numbers applies. Thus, the market clearing condition for the coffee market in $t + 1$ turns out to be:

$$[1 + \pi^{**}(p_t, p_{t+1})]\, Y = D(p_{t+1}) \tag{2.1}$$

where $D(p_{t+1})$ is the aggregate demand. Supply is provided both by entrepreneurs born in $t - 1$ and in t. At time $t + 1$ the young entrepreneurs (i.e. the ones born in t) offer the amount Y of coffee, while the incumbents' supply is given by $\pi^{**}(p_t, p_{t+1})Y$. Equation (2.1) implicitly defines a non-linear first order difference equation in p, that is there ex-

[5]That is, it has the effect of stochastically improving project's returns.

ists a continuously differentiable function $g : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$ such that: $p_{t+1} = g(p_t)$.

SS show that g is monotonically decreasing on a definite region, the necessary condition for existence of two-period endogenous fluctuations is satisfied. Furthermore, in order for endogenous fluctuations to take place it should be that $g'(p^*) < -1$, where p^* denotes the steady state price level. This condition holding, the system will be oscillating between *recession* periods, when the price is low and the moral hazard problem is more severe, and *booms* periods when borrowers have additional incentives to exercise effort and reduce the probability of a distress.

This prediction fully explains the role of liquidity in affecting the incentive problem: a higher p_t implies that firms become more liquid, and therefore less dependent on external finance. Thus, they will raise π and the supply of coffee in period $t + 1$, which creates the potential for lowering p_{t+1} increasing next period's need of borrowing and so on. As a consequence borrowers' liquidity turns out to be a counter-cyclical variable, and several scholars have strongly questioned the empirical relevance of such a prediction.[6]

2.2 Debt, equity and the business cycle

The structure we discussed thus far constitutes one of the first attempts to provide a general explanation of how asymmetric information in credit markets may affect the dynamics of economic activity. In addition, the presence of an incentive conflict is unambiguously responsible for the existence of an endogenous reversion mechanism. Liquidity affects entrepreneurs-borrowers economic decisions in sharp contrast with the Modigliani-Miller argument that assigns a central role to net present values.

We provide a further departure from this argument by characterizing the debt-to-equity ratio as an endogenous variable and relating its dynamic behavior to business fluctuations. In order to account for this, we allow for a more appropriate design of financial institutions, trying to generalize the security design analysis. In other words, we restate the finding that asymmetric information in credit markets is a source of endogenous fluctuations, but introduce an argument in favor of endogenous cycle theories: the nature and the amplitude of business cycle dynamics affects the nature of financial arrangements.

Importantly, SS build their contract problem on three crucial assumptions: there exists moral hazard with hidden action, both lenders and

[6]See Reichlin (2004) among many others.

borrowers are risk-neutral and subject to Limited Liability and the stochastic realizations of output are identified by the binary variable $\{0, Y\}$. The implications of these need to be considered carefully.

First, one should observe that the Limited Liability constraint for the generation of borrowers born in $t-1$ forces the repayment R_{t+1} to take the form $\{0, x\}$ where x is the unknown to be determined by the contract problem itself. This kind of contract cannot be interpreted as a well defined financial instrument: it is neither a debt, nor an equity, given that it turns out to be (weakly) monotone over states, but is defined over two states only. In other words, referring to such a simplified binary structure prevents the proper design of a financial arrangement, the form of the optimal contract being de facto determined by the working of Limited Liability alone.

Moreover, even though Limited Liability is a sufficient condition to induce monotonicity of the repayment functions when output is binary, monotonicity cannot be easily predicted in a more general setting. The result has been shown by Innes (1990) and the basic intuition is straightforward: whenever lenders cannot observe effort, then the outcome realization will be interpreted as a signal of borrowers' action. The repayment will therefore be high (taking the form of a punishment) when output is low and viceversa. That is, we get a *live or die* contract. Under bilateral risk neutrality the only way to avoid such an uncomfortable result and to predict a debt-like optimal contract is to explicitly assume monotonicity through the introduction of a monotonicity constraint on repayments.[7]

In order to allow for more robust predictions on the nature of financial relationships, we hence modify the basic representation of asymmetric information. The most recent research on financial contracting has made significant steps in showing how the so called Costly State Verification (CSV) paradigm can constitute a solid foundation for debt contracts. The basic, one-stage CSV structure refers to situations where *symmetric* information is assumed at the time of contracting: stochastic returns from a project can be observed at no cost by borrowers only, that is, we have *ex-post* private information. Borrowers are assumed to report their private information through a message. Lenders can verify the state only through the performance of a costly auditing activity.

In an influential paper, Gale and Hellwig (1985) showed the optimality of debt contracts in this framework. More recently, Krasa and Villamil (2000) provided general conditions for the optimality of debt when

[7]The role of this constraint is of course played by Limited Liability in the $\{0, Y\}$ case. Some rationales for the introduction of this constraint are briefly discussed in Innes (1990), p.50.

limited commitment and stochastic auditing are allowed.[8] At the same time, Hellwig (2001) has restated the conditions ensuring co-existence of optimal debt contracts and intermediaries performing a delegated monitoring activity under CSV. In other words, he defined a set of sufficient conditions that ensure at the same time optimality of debt contracting and optimality for debt to be intermediated by banks.

Though the present work does not discuss in detail the properties of the CSV apparatus, it tries to develop a simple dynamic contracting structure allowing for debt and equity financing to co-exist at equilibrium.[9] It is important to recall that the basic CSV framework is already allowing for a form of equity financing: insiders hold all the equity, with the consequence that only *internal* equity is considered.[10] A departure from the benchmark CSV environment becomes necessary if we want to account for the role of outside equity investors:

> *This sort of capital structure does not look much like that employed in the United States or most other developed nations. In particular, major corporations typically have a large class of outside equity holders who are not necessarily any better informed than debt holders.*[11]

We try to develop these intuitions in the following pages.

2.2.1 Technology and information

We keep the focus on the interaction between a representative lender and an infinite population of measure one of three-period living overlapping generations of entrepreneurs-borrowers. In order to avoid considering risk sharing issues, bilateral risk neutrality is assumed. We also parallel SS in constructing a discrete time, two-good economy, where only one good is produced; we let p_t for $t = [0, 1, 2, ...]$ be the price of this good. Borrowers have exclusive access to a stochastic technology that is productive for two periods. Technology has the following characteristics:

[8] Krasa and Villamil results may overcome the issue raised by Mookherjee and Png (1989), among others: whenever stochastic auditing is preformed, then debt will not emerge as the optimal security.

[9] A more general evaluation of the most recent developments in CSV schemes in the light of the traditional critiques moved to those constructions (and summarized in the Hart's 1995 book) is proposed in Attar and Campioni (2003).

[10] That is, funds owned by borrowers and directly invested in the project are exchanged for a residual claim that has an equity form.

[11] Boyd and Smith (1999), p. 271.

Assumption 1: Production takes place with a *stochastic linear* technique in each period. One unit of numeraire at $t-1$ is transformed into y units of the final good at period t and x units of the final good at $t+1$. Both y and x are continuous random variables with the continuous distribution functions $G(y) : [0, \bar{y}] \to [0, 1]$ and $F(x) : [0, \bar{x}] \to [0, 1]$. In addition, we have that x and y are independently distributed and $\hat{y} \equiv \int_0^{\bar{y}} y dG(y)$ is greater than $\hat{x} \equiv \int_0^{\bar{x}} x dF(x)$.

Assumption 2: The investment is perfectly divisible. Borrowers born, say in $t-1$, are allowed to choose their amount of investment in the project $i_{t-1} \in [0, \bar{\imath}] \subset \mathbb{R}_+$.

Assumption 3: Both agents have access to a deterministic (storage) technology, giving one unit of the consumption good for any unit of the numeraire invested.

We consider penniless entrepreneurs. In order for production to be activated, borrowers should hence rely on external loans. For every borrower born in $t-1$ external finance is obtained through a financial contract that specifies the investment made in $t-1$ and the amount of numeraire repaid to the lender in t and in $t+1$, respectively. The next two assumptions define the characteristics of information.

Assumption 4: The information structure is such that y is fully observable by the two parties, whereas second period realizations are observable by borrowers only. The lender can become informed of the true state through the performance of a costly auditing activity. Thus, a standard Costly State Verification problem (CSV) arises in the second production period.

Assumption 5: Auditing costs are a well behaved, increasing, and strictly convex function of borrower's investments. We define the auditing cost function as $\gamma(i) : [0, \bar{\imath}] \to \mathbb{R}_+$, with $\gamma' > 0$ and $\gamma'' > 0$. In particular, we restrict our analysis to the quadratic case $\gamma(i) = ai^2$, with $a > 0$.

Assumptions 4 and 5 imply that both the lender-borrower relationship and the nature of the implementation problem are different from those tackled in the SS economy.

2.2.2 Consumers

In order to focus on the alternative representation of financial arrangements that we are suggesting here, we adopt the same representation of consumers as SS. Two types of consumers are introduced in the analysis.

First, there is an infinitely-lived, representative consumer-lender with quasi-linear preferences with respect to the numeraire good. He demands the amount c_t of the consumption good and the amount b_t of the intermediate good in each period t. At the same time, he offers the loan amount i_t.
His behavior can be hence stated as follows:

$$\max_{\{c_t, b_t\}_{t=0,1,2,...}} E_t \left[\sum_{s=0}^{\infty} (b_{t+s} + u(c_{t+s})) \right] \tag{2.2}$$

$s.t.$

$$b_t + p_t c_t + i_t = e + R_{t-1} i_{t-1} \qquad \text{for} \quad t = 1, 2, ... \tag{2.3}$$

$$b_t \geq 0, \;\; c_t \geq 0 \tag{2.4}$$

where R_{t-1} is repayment that the consumer received at time $t-1$ acting as a lender for each unit of credit issued, and e is his endowment of the numeraire good.

We will henceforth assume that $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is strictly increasing and strictly concave.[12] We also follow SS in restricting our attention to interior solutions of (2.2)-(2.4), so that lenders demand for the consumption good in period t will be given by $c_t = D(p_t)$, with $D'(p_t) \leq 0$ for every t.

Borrowers are assumed to have linear preferences with respect to the numeraire good and their discount factor is set equal to one. This, together with the risk-neutrality assumption, makes the time of their consumption decisions irrelevant; we will hence focus on borrowers who only consume in the last period of their life. One should notice that the aggregate demand for consumption good in each period t is raised by lenders only.

Thus, each lender can exchange with borrowers born in $t-1$ one unit of the safe asset he holds (namely, the numeraire) with an activity that will give him a repayment in any of the two production periods t and $t+1$. The nature of this exchange is properly described by a financial contract.

[12] For simplicity reasons, we also assume that $u(.)$ is concave enough to guarantee convergence of the infinite sum in (2.2).

2.2.3 Financial contracting

A financial contract signed by a borrower born in $t-1$ defines the allocation of resources that will be delivered as repayments to the lender in periods t and $t+1$, denoted R_t and R_{t+1}, respectively, together with the amount invested in the stochastic technology i_{t-1}. Moreover, a contract should specify a set of states where auditing is performed. Observability of y allows to write contracts that are conditional on first period realizations, while lenders strategies should be specified conditionally on the message $m \in M \subset \mathbb{R}_+$ sent by borrowers once they have observed x.

The general mechanism involving a representative consumer-lender and an entrepreneur-borrower born in $t-1$ is given by:

$$\Gamma = [i_{t-1}, R_t(y), M, R_{t+1}(m, y), A_{t+1}(y)] \tag{2.5}$$

where M is the borrower's message space, $A_{t+1}(y) \subseteq M$ is the set of audited reports and $B_{t+1}(y)$ is its complement. Observe that we are restricting to the class of deterministic auditing schemes: for any given message $m \in M$ auditing is either performed with probability one or not performed at all. We do this on the grounds that we believe that the most recent work in the CSV literature has shown how this hypothesis can constitute a reasonable simplification.[13]

Events take place in the sequence described in Figure 2.1.

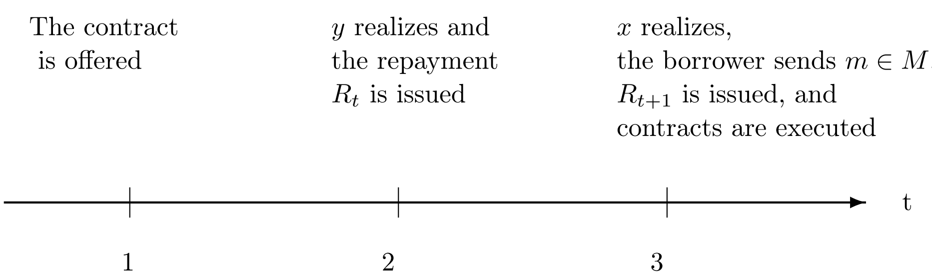


Figure 2.1: Timing of the lender-borrower interaction

The simple nature of the dynamic interaction enables us to restrict our attention to the subset of *direct* mechanisms, that is to the set of mechanisms where the message space and the state space coincide. We are thus able to set $M = [0, \bar{x}]$ without loss of generality.[14]

[13] See again the discussion in Attar and Campioni (2003).

[14] Given that a contract can be written conditionally to first period realizations, then there are no incentives for renegotiation. In order to ensure that the Revelation

The security design problem is defined by the maximization of the expected utility of the borrower (who designs the mechanism) over the set of feasible mechanisms. Given Assumptions (1) – (6), every optimal contract chosen by a borrower born in $t-1$ is a solution to the following problem:

$$\underset{\Gamma}{Max} \;\; i_{t-1}\int_0^{\bar{y}}\left[p_t y - R_t(y) + \int_{A_{t+1}} \left(p_{t+1}x - R_{t+1}(x,y)\right) dF(x)\right] dG(y) +$$

$$+ i_{t-1}\int_0^{\bar{y}}\int_{B_{t+1}} \left(p_{t+1}x - R_{t+1}(x,y)\right) dF(x)dG(y)$$

s.t.

$$R_t(y) \in [0, p_t y] \qquad \text{for each } y \in [0,\bar{y}] \tag{2.6}$$

$$R_{t+1}(x,y) \in [0, p_{t+1}x + p_t y - R_t(y)] \qquad \text{for each } y \in [0,\bar{y}] \tag{2.7}$$

$$R_{t+1}(x,y) = R_{t+1}(y) \qquad \text{for each } y \in [0,\bar{y}], \forall x \in B_{t+1} \tag{2.8}$$

$$R_{t+1}(y) \geq R_{t+1}(x,y) \qquad \text{for each } y \in [0,\bar{y}], \forall x \in A_{t+1} \tag{2.9}$$

$$\int_0^{\bar{y}}\left[R_t(y) + \int_{A_{t+1}} \left(R_{t+1}(x,y) - \frac{\gamma(i_{t-1})}{i_{t-1}}\right) dF(x)dG(y)\right] +$$

$$+ \int_0^{\bar{y}}\int_{B_{t+1}} R_{t+1}(x,y_n)dF(x)dG(y) \geq 1 \tag{2.10}$$

The Limited Liability constraints (2.6) and (2.7) rule out the possibility of negative consumption for borrowers in both periods. In particular, the second period constraint (2.7) embeds the additional liquidity coming from retained earnings. The same caveat holds for the second period Incentive Compatibility constraints (2.8) and (2.9). The former imposes that when verification does not happen, then repayments of date two will

Principle holds it is sufficient to assume that the parties are committed to contractual prescriptions.

be independent of the reported cash flow, while the latter states that in verified states it is unfeasible for the borrower to give back the fixed repayment $R_{t+1}(y_n)$. Constraints (2.8) and (2.9) taken together constitute a necessary and sufficient condition to induce borrower's truthful revelation at equilibrium.[15]

Auditing costs are paid by the lender who has access to a competitive capital market where the interest rate is normalized to zero. The Individual Rationality constraint guaranteeing lender's participation to the mechanism is given by (2.10) and has been written down in per-unit terms.[16] The magnitude $\dfrac{\gamma(i_{t-1})}{i_{t-1}} = ai_{t-1}$ defines the amount of auditing costs per unit of consumption good invested in the stochastic technology.

We characterize the solution through backward induction, applying the results of the one-period CSV to the last stage of the game.[17] We first show that the repayment issued in the second period takes the form of a standard debt contract (SDC), that is:

- Repayment is fixed over non bankruptcy states.
- When bankruptcy takes place, then the lender recovers the whole amount of the project and the borrower becomes a residual claimant.

Proposition 9 *Any feasible contract* Γ *is dominated by a contract implying:* $R_{t+1}(x,y) = p_{t+1}x + p_t y - R_t(y)$ $\forall x \in A_{t+1}$, $\forall y$ *and* $R_{t+1}(x,y) = R_{t+1}(y)$ $\forall x \in B_{t+1}$, $\forall y$.

Proof. The second part of the proposition is implied by (2.8). In order to complete the proof we proceed in three steps.

First we argue that the optimal contract predicts the auditing region to coincide with the bankruptcy region. That is, if the array $[R_{t+1}(y), R_{t+1}(x,y), A_{t+1}(y)]$ is part of an optimal contract, then we should have:

$$A_{t+1}(y) = \{x : p_{t+1}x + p_t y - R_t(y) \leq R_{t+1}(y)\}$$

Now, if $p_{t+1}x + p_t y - R_t(y) \leq R_{t+1}(y)$, then $R_{t+1}(x,y) \leq R_{t+1}(y)$ by (2.9).

[15] A formal derivation of Incentive Compatibility constraints in the form of (2.8) and (2.9) is proposed in Townsend (1979).

[16] That is, both sides of (2.10) have been divided by i_{t-1}.

[17] Existence of economically meaningful solutions for a more general version of our Problem 1 has been already shown by Chang (1990).

The other direction is proved by contradiction: if $R_{t+1}(x,y) \leq R_{t+1}(y)$ implied $p_{t+1}x + p_t y - R_t(y) > R_{t+1}(y)$ on an interval $C_t \subset A_{t+1}$, then two cases would be possible:

- $R_{t+1}(x,y) = R_{t+1}(y)$; then $p_{t+1}x + p_t y - R_t(y) > R_{t+1}(x,y)$, violating (2.9).
- $R_{t+1}(x,y) < R_{t+1}(y)$. In this case we can define a new feasible triple $\left[R'_{t+1}(y), R'_{t+1}(x,y), A'_{t+1}(y)\right]$ such that $A'_{t+1} = A_{t+1}/\left\{C_t\right\}$ and $R'_{t+1}(x,y) < R_{t+1}(x,y)$, so that borrower's payoff is strictly greater.[18]

The last step is to show that *maximum recovery* takes place in bankruptcy states: for every $x \in A_{t+1}$ optimal contracting implies that $R_{t+1}(y) = p_{t+1}x + p_t y - R_t(y)$. That is, the lender gets the entire proceeds from the investment.
Finally, in order to complete the proof, we remark that the lender's Individual Rationality constraint is binding at the solution. As a consequence, the borrower's objective function becomes:

$$i_{t-1}\left[(p_t\hat{y} + p_{t+1}\hat{x} - 1) - \int_0^{\bar{y}}\int_{A_{t+1}} \frac{\gamma(i_{t-1})}{i_{t-1}} dF(x)dG(y)\right]$$

Then, every feasible contract $\{R_{t+1}(y), A_{t+1}\}$ is dominated by the (feasible) contract $\Gamma' = \left\{R'_{t+1}(y), A'_{t+1}\right\}$ with $R'_{t+1}(y) = p_{t+1}x + p_t y - R_t(y)\ \forall x \in A'_{t+1}$. Γ' minimizes the extension of the bankruptcy region[19] and, at the same time, it increases borrower's payoff. ■

Proposition 9 implies that:

$$A_{t+1}(y) = [0, \phi(y)], \qquad \phi(y) = \frac{R_{t+1}(y) + R_t(y) - p_t y}{p_{t+1}} \tag{2.11}$$

The extension of the auditing (bankruptcy) region is negatively correlated with consumption good prices. This defines a *liquidity effect*: a rise either in p_t or in p_{t+1} makes the borrower less dependent on external finance reducing the probability of a default.

Relying on second period equilibrium and plugging the Individual Rationality constraint into the borrower's objective function, the original problem can be rewritten as:

[18] A precise formulation for R' has been for instance suggested in Gale (2001).

[19] Given that borrower's consumption is protected by Limited Liability, the bankruptcy region cannot be reduced further.

$$\underset{i_{t-1},R_t(y),R_{t+1}(y)}{Max} i_{t-1}\left[(p_t\hat{y}+p_{t+1}\hat{x}-1)-\int_0^{\bar{y}}\int_{A_{t+1}} ai_{t-1}dF(x)dG(y)\right] \tag{2.12}$$

s.t.

$$R_t(y)\in[0,p_ty] \qquad \text{for } y\in[0,\bar{y}] \tag{2.13}$$

$$\int_0^{\bar{y}} R_t(y)+\int_0^{\bar{y}}\int_0^{\phi(y)}\left(p_{t+1}x+p_ty-R_t(y)-ai_{t-1}^2\right)dF(x)dG(y)+$$
$$+\int_0^{\bar{y}}\int_{\phi(y)}^{\bar{x}} R_{t+1}(y)dF(x)dG(y)=1 \tag{2.14}$$

The problem is reformulated as the minimization of last period bankruptcy (auditing) costs under the Individual Rationality and the first period Limited Liability constraints. We are hence left with a concave program which solution can be characterized in a standard way. The relevant Lagrangian will therefore be:

$$L=i_{t-1}\left[(p_t\hat{y}+p_{t+1}\hat{x}-1)-\int_0^{\phi(y)} ai_{t-1}dF(x)dG(y)\right]+$$

$$+\lambda\int_0^{\bar{y}}\left[R_t(y)+\int_0^{\phi(y)}\left(p_{t+1}x+p_ty-R_t(y)-ai_{t-1}\right)dF(x)\right]dG(y)+$$

$$+\lambda\left[\int_0^{\bar{y}}\int_{\phi(y)}^{\bar{x}} R_{t+1}(y)dF(x)dG(y)-1\right]+\mu\left[p_ty-R_t(y)\right]$$

For every t, necessary conditions for optimality are the following:

$$\frac{\partial L}{\partial i_{t-1}}=0\Rightarrow\lambda=i_{t-1}^2\frac{NPV-2ai_{t-1}^2\int_0^{\bar{y}}\int_0^{\phi(y)}\left[p_{t+1}x+p_ty-R_ty\right]dF(x)dG(y)}{ai_{t-1}^2\int_0^{\bar{y}}\int_0^{\phi(y)}dF(x)dG(y)} \tag{2.15}$$

$$\frac{\partial L}{\partial R_t}=0\Rightarrow(\lambda+i_{t-1})\int_0^{\phi(y)} 2ai_{t-1}dF(x)=\mu \tag{2.16}$$

$$\frac{\partial L}{\partial R_{t+1}} = 0 \Rightarrow (\lambda + i_{t-1}) \left[(-\frac{1}{p_{t+1}}) a i_{t-1} f(\phi(y)) \right] + \lambda \left[1 - F(\phi(y)) \right] = 0 \tag{2.17}$$

$$\mu \geq 0, \qquad \mu \left[p_t y - R_t(y) \right] = 0 \tag{2.18}$$

$$\int_0^{\bar{y}} R_t(y) dG(y) + \int_0^{\bar{y}} \int_0^{\phi(y)} \left(p_{t+1} x + p_t y - R_t(y) - a i_{t-1} \right) dF(x) dG(y) + \\ + \int_0^{\bar{y}} \int_{\phi(y)}^{\bar{x}} R_{t+1}(y) dF(x) dG(y) = 1 \tag{2.19}$$

where $NPV = p_t \hat{y} + p_{t+1} \hat{x}$ is the net present value of the investment project.

Optimal contracts are hence identified by a system of five non-linear equations in the five unknowns: λ, μ, i_{t-1}, $R_t(y)$, $R_{t+1}(y)$.

One should observe that $\lambda > 0$ holds, given that the lender's Individual Rationality binds. As a by-product, we have:

$$NPV - \int_0^{\bar{y}} \int_0^{\phi(y)} 2 i_{t-1} \left[p_{t+1} x + p_t y - R_t y \right] a i_{t-1} dF(x) dG(y) \geq 0 \tag{2.20}$$

and $i_{t-1} > 0$. From (2.17) we also get:

$$1 - F(\phi(y)) - \frac{a i_{t-1} f(\phi(y))}{p_{t+1}} > 0 \tag{2.21}$$

Given equation (2.16), we get that $\mu > 0$: we are hence concerned with a corner solution for $R_t(y)$.

In the present setting the choice over the dynamic structure of repayments is therefore rather trivial since the borrower always has the incentive to give back the entire first period outcome (with the lender becoming the owner of the firm). This prediction is a direct implication of having assumed *symmetric information* in the first period: as a consequence, there is no room for an incentive cost of repaying in the first period. We also remark that (2.16) implies:

$$R_t(y) = p_t y \qquad \forall y \in [0, \bar{y}] \tag{2.22}$$

That is, the repayment issued at period t by borrowers born in $t-1$ will take an *equity* form. The decision faced by a borrower born in $t-1$

whether to repay in the first or in the second period defines the amount of (*external*) equity and debt he issues in t and $t+1$, respectively. Given the overlapping generations structure of firms, debt and equity will co-exist as alternative forms of external financing in each period.[20]

The relative composition of securities issued at equilibrium will be affected by the behavior of prices and costs over time.

A first step in understanding the properties of equilibrium dynamics is the analysis of the responsiveness of investments to consumption's good prices. Implicit differentiation enables us to state the following:

Proposition 10 *For every t, optimal contracts predict i_{t-1} to positively depend both on p_t and on p_{t+1}. Furthermore, we get $\dfrac{\partial i_{t-1}}{\partial p_{t+1}} > \dfrac{\partial i_{t-1}}{\partial p_t}$ for every t.*

Proof. The proof is given in the Appendix. ■

A *profitability effect* takes place: higher prices induce a raise in the marginal profitability of investments in the random project. Beyond this, the proposition implies that entrepreneurs' investments are more sensible to last period prices. This happens because a raising p_{t+1} induces a contraction in the bankruptcy region on top of the increase in monetary revenues.[21]

2.2.4 Intertemporal equilibrium and dynamics

At the stage of defining a market clearing condition for the consumption good in $t+1$, we should take into account that in each period two generations of producers are actually active: those born in t and those born in $t-1$. They offer the amount $\hat{y}$ and $\hat{x}$ of the consumption good, respectively. The consumption good market clearing condition for $t+1$ will therefore be:

$$i_{t-1}(p_t, p_{t+1})\hat{x} + i_t(p_{t+1}, p_{t+2})\hat{y} = D(p_{t+1}) \tag{2.23}$$

This equation implicitly defines a relationship among equilibrium prices. That is, there exist a continuously differentiable function $q(p_t, p_{t+1})$: $\mathbb{R}_{++} \times \mathbb{R}_{++} \to \mathbb{R}_{++}$ such that:

[20]We remark that equity financing could take the form of either *internal equity*, that coincides with the entrepreneurial wealth invested in the project, or of *external equity* identified by the amount of repayment that turns out to be strictly monotone in output levels.

[21]Given (2.22), the extension of the bankruptcy region will depend on p_{t+1} only.

$$p_{t+2} = q(p_t, p_{t+1}) \qquad \textit{for every } t \tag{2.24}$$

In other words, the sequence of market clearing prices corresponds to the solution of a second order and non-linear difference equation.

The first step in the analysis of the dynamical system (2.24) is to check whether a stationary solution exists. That is, we should verify the existence of a fixed point $\bar{p}$ of the q function such that $\bar{p} = q(\bar{p})$. If there exist such a $\bar{p}$, then:

$$i_{t-1}(\bar{p}, \bar{p})\hat{x} + i_t(\bar{p}, \bar{p})\hat{y} = D(\bar{p}) \tag{2.25}$$

The right hand side of (2.25) is increasing in $\bar{p}$ and the left hand side is decreasing in $\bar{p}$, then, once the boundary value $D(0)$ has been properly defined, there must exist a $\bar{p} \in \mathbb{R}_{++}$ that satisfies (2.25). One should also notice that, for the steady-state equilibrium to be economically meaningful, it should be:

$$p_t\hat{y} + p_{t+1}\hat{x} - 1 > 0$$

for $p_t = p_{t+1} = \bar{p}$.

That is, the investment project should have a positive net present value.

Given two initial conditions on final good prices, the behavior of the solution of the dynamical system (2.24) can be investigated considering the correspondent planar system of first order difference equations. We represent (2.24) with the family of dynamical systems $v_{t+1} = Q(v_t, k)$, where $v_t \equiv (z_t, p_t)$ and $z_t \equiv p_{t+1}$. Also, $k \equiv (\hat{x}, \hat{y}) \in {\mathbb{R}_+}^2$ is a vector of parameters indexing the system. Under our assumptions it is possible to state that a period-two cycle exists:

Proposition 11 *The system $v_{t+1} = Q(v_t, k)$ with $Q : {\mathbb{R}_+}^2 \times K \rightarrow {\mathbb{R}_+}^2$ with assigned initial conditions admits the existence of period doubling (flip) bifurcations in a sufficiently smooth neighborhood of the non-hyperbolic equilibrium $(\bar{p}, \bar{k})$ where $\bar{p}$ is the stationary solution of the system parameterized in $\bar{k}$.*

Proof. We explain here how period doubling fluctuations may take place in the economy.

We start from the system:

$$i_{t-1}(p_t, p_{t+1})\hat{x} + i_t(p_{t+1}, p_{t+2})\hat{y} = D(p_{t+1})$$

We transform this second order non linear equation into a planar system. We define $z_t = p_{t+1}$ and we are left with:

$$\begin{cases} i_{t-1}(p_t, z_t)\hat{x} + i_t(z_t, z_{t+1})\hat{y} &= D(z_t) \\ p_{t+1} &= z_t \end{cases}$$

Observe that the first equation implicitly defines the function m : $z_{t+1} = m(z_t, p_t)$ and, given our previous results, we have that $\dfrac{\partial m}{\partial p_t} < 0$ and $\dfrac{\partial m}{\partial z_t} < 0$. Then, we can approximate the dynamics of the system around its stationary solution looking at the corresponding Jacobian Matrix:

$$J = \begin{bmatrix} \dfrac{\partial m}{\partial z_t} & \dfrac{\partial m}{\partial p_t} \\ 1 & 0 \end{bmatrix}$$

and evaluating it at $p = \bar{p}$. Thus, we get that the trace of matrix is negative and its determinant is positive. That is, J has two negative eigenvalues; they are:

$$\lambda_1 = \frac{m_z + \sqrt{m_z^2 + 4m_p}}{2}, \quad \lambda_2 = \frac{m_z - \sqrt{m_z^2 + 4m_p}}{2}$$

with

$$\lambda_1, \lambda_2 < 0 \qquad \text{and} \qquad \lambda_1 > \lambda_2$$

We also introduce the following notation:

$$S(\hat{y}) \equiv \frac{\partial i_{t-1}}{\partial p_t} = \frac{\partial i_t}{\partial p_{t+1}}\ , \ T(\hat{x}) \equiv \frac{\partial i_t}{\partial p_{t+2}} = \frac{\partial i_{t-1}}{\partial p_{t+1}}$$

with $T > S$ for every $(\hat{x}, \hat{y})$ array, from Proposition 1. It follows that:

$$m_z = -\frac{\hat{x}T + \hat{y}S - D'(p_{t+1})}{\hat{y}T} \qquad \text{and} \qquad m_p = -\frac{\hat{x}S}{\hat{y}T}$$

In order for a period-two cycle to take place the parameters $\hat{x}$ and $\hat{y}$ should be such that the eigenvalue λ_2 lies on the boundary of the unitary circle. That is,

$$\lambda_2 = -1 \qquad \Longleftrightarrow \qquad \hat{x} = \hat{y} - \frac{D'(p_{t+1})}{S - T}$$

Furthermore, the eigenvalue λ_2 should cross -1 for small changes in the array of parameters. We consider small variations in $\hat{x}$ to get:

$$\frac{\partial \lambda_2}{\partial \hat{x}} = \frac{1}{2}\left[\frac{\partial m_z}{\partial \hat{x}} - \left(m_z\frac{\partial m_z}{\partial \hat{x}} + 2\frac{\partial m_p}{\partial \hat{x}}\right)\right] = (S-T)\hat{y} - \hat{x}T + D'(p_{t+1}) < 0$$

The eigenvalue is hence a monotone function of $\hat{x}$ and this completes the proof. ■

We have restated the well known finding that the presence of cyclical dynamics is induced by asymmetric information in loan relationships. This can be easily shown if we think of the first best as the allocation implemented when auditing costs are set equal to zero. In this case the investment level will be set to $\bar{\imath}$, independently of the price structure. Then, (2.23) will be an equation in p_{t+1} only and the market clearing equilibrium will be associated to a stationary outcome. Thus, existence of credit market imperfections turns out to be a *necessary* condition for endogenous fluctuations to take place at equilibrium.

The liquidity effects emphasized in Proposition 1 are the basic sources of business fluctuations. Consider the entrepreneurs born in $t-1$ and those born in t; if p_{t+1} increases, then both generations will raise their investment levels. On the other hand, the final good demand $D(p_{t+1})$ is declining. To guarantee equilibrium in the final good market for period $t+1$, a sharp decline in p_{t+2} is therefore needed. This finally brings about a net contraction in the amount of credit i_t issued to "young" entrepreneurs.

A relevant feature of business fluctuations is therefore that whenever the relative price of the final good is low, then aggregate supply is mostly provided by young entrepreneurs, while the opposite happens in a high price regime. In other words, the following corollary of Proposition 11 holds:

Corollary 1 *Assume that a period-two cycle takes place. Then, the $t+1$ final good supply $O_{t+1} = i_{t-1}(p_t, p_{t+1})\hat{x} + i_t(p_{t+1}, p_{t+2})\hat{y}$ will be lower than the $t+2$ supply $O_{t+2} = i_t(p_{t+1}, p_{t+2})\hat{x} + i_{t+1}(p_{t+2}, p_{t+3})\hat{y}$ whenever $p_{t+1} > p_t$.*

Proof. It is enough to observe that in a period-two cycle $i_{t-1} = i_{t+1}$ and $i_t = i_{t+2}$. We can therefore rewrite:

$$O_{t+1} - O_{t+2} = (i_{t-1} - i_t)\,\hat{x} + (i_t - i_{t-1})\,\hat{y} = (i_{t-1} - i_t)\,(\hat{x} - \hat{y}) < 0$$

■

We therefore have that a high price regime will be accompanied by expansions in the availability of loanable funds and it will be followed, given the time-articulated structure of production, by an expansion of the consumption good supply.

2.2.5 The debt-to-equity ratio

When period-two cycles occur, we can further characterize the repayment dynamics and make explicit the role of financial institutions in economic fluctuations. This allows us to examine the behavior over time of the aggregate debt-to-equity ratio. In particular, we will relate the behavior of the debt-to-equity ratio to the dynamic evolution of final good output. This follows from the fact that external financing is needed in the production of final good only.

Keeping the focus on period $t+1$, we let V_{t+1}^{d} be the amount of credit that is issued in the form of debt by the generation of borrowers born in $t-1$ and V_{t+1}^{e} be the amount of numeraire used to finance investments through equities in the same period that in our framework is entirely provided by entrepreneurs born in t. The debt-to-equity ratio defines the proportion of firms that issue equity with respect to those that are issuing debt in the same period.

In particular, we have:

$$V_{t+1}^{d} = i_{t-1}\left(1 - p_t \hat{y}\right)$$

where $i_{t-1}p_t\hat{y}$ is the amount of credit repaid in the form of equity by firms born in $t-1$ when they are in the first period of their life.[22] Observe that V^d turns out to be a counter-cyclical variable: if $t+1$ is a low-price, expansionary period, then debt financing declines for two reasons. First, because of the contraction in the demand for credit raised in the deflationary period $t-1$ and, second, given that the per-unit amount of investment that is going to be debt-financed, $(1 - p_t\hat{y})$, will also be low.

Let us now consider equity financing for period $t+1$. We have that:

$$V_{t+1}^{e} = i_t p_{t+1} \hat{y}$$

The dynamic behavior of V^e cannot be defined in a straightforward way. Suppose there is an expansion in $t+1$. Then, those firms that are issuing equities in $t+1$ must have raised a high credit demand in $t-1$. On the other hand, the amount of credit that is going to be equity financed in $t+1$ depends on current period consumption good prices, that turn out to be low. Thus, for the overall amount of equity financing to be pro-cyclical we should have $V_{t+1}^{e} > V_{t}^{e}$. That is:

$$\frac{i_t}{i_{t+1}} > \frac{p_t}{p_{t+1}}$$

[22] We remark here that V_{t+1}^{d} is defined net of bankruptcy costs that are assumed to be paid by lenders.

If the volume of credit fluctuates more than the composition of credit, then equity financing should positively co-vary with final output.

Finally, the aggregate leverage for period $t+1$ will be:

$$\sigma_{t+1} = \frac{V_{t+1}^{d}}{V_{t+1}^{e}} = \frac{i_{t-1}\left(1-p_t\hat{y}\right)}{i_t p_{t+1}\hat{y}}$$

Whatever the path followed by V^e, the debt-to-equity ratio exhibits a counter-cyclical dynamics. In particular, it turns out that the component of equity financing is more responsive to price fluctuations than the debt one. The following corollary describes the behavior of the aggregate leverage σ:

Corollary 2 *Suppose that a period-two cycle takes place and consider every period $t+1$ such that $p_{t+1} > p_t$. Then, the $t+1$ aggregate leverage $\sigma_{t+1} = \frac{V_{t+1}^{d}}{V_{t+1}^{e}} = \frac{i_{t-1}\left(1-p_t\hat{y}\right)}{i_t p_{t+1}\hat{y}}$ will be higher than the period t leverage $\sigma_t = \frac{V_t^d}{V_t^e}$.*

Proof. To have $\sigma_{t+1} > \sigma_t$ it should be:

$$\left(\frac{i_{t+1}}{i_t}\right)^2 > \frac{\left(1-p_{t+1}\hat{y}\right)p_{t+1}}{\left(1-p_t\hat{y}\right)p_t}$$

The inequality is satisfied if:

$$\left(1-p_{t+1}\hat{y}\right)p_{t+1} < \left(1-p_t\hat{y}\right)p_t$$

which corresponds to

$$\left(p_t + p_{t+1}\right)\hat{y} > 1 \tag{2.26}$$

Given that we are restricting to steady state equilibria where the Net Present Value of the investment project is always positive, thus guaranteeing that (2.26) always holds. ■

One should note that period t leverage turns out to be low in the aggregate because the equity financed young firms raise funds when prices and profitability are high, whereas the opposite happens in $t+1$. This confirms several recent findings about the impact of "equity market timing" on capital structure (Baker and Wurgler, 2002).

2.3 Empirical evidence

Our theoretical model predicts that the aggregate debt-to-equity ratio should move counter-cyclically. As noted in the introduction there is already some existent tentative empirical evidence that may support this fact. For instance, in his survey of the literature, Levy (2001) notes that debt issuing tends to be counter-cyclical, while financing through equity is generally pro-cyclical for firms. Also, Choe, Masulis, and Nanda (1993) provide evidence that equity is issued in an anti-cyclical manner. Perhaps the strongest support comes from the study of Korajczyk and Levy (2002). Estimating a probit model of debt versus equity choice of financing using firm level data, the authors discover that firms opt the more for debt financing the lower the rate of market return is.

In this section we set out to seek explicit empirical support for a negative relationship between the aggregate debt-to-equity ratio and the business cycle using data from the US. To this end we use the quarterly series of the debt-to-equity ratio for the non-farm non-financial corporate business sector available since 1952 taken from the Board of Governors of the Federal Reserve System "Flow of Funds Accounts", where debt refers to total value of credit market instruments and equity is measured at its total market value. The non-farm non-financial corporate business sector includes all private domestic corporations as well as holding companies, S-corporation and real estate management corporations, but excludes corporate farms and financial institutions. One should note that this branch of economic activity accounts for over half of all net private investment in the US economy and at least two thirds of business borrowing; see BoG (2000) and Tyler (2001). The information used by the Fed to construct this data are taken from securities markets reports, industry trade association releases, commercial bank reports of conditions, finance company surveys, and the Quarterly Financial Report for Manufacturing, Mining, and Trade Corporations Survey published by the Bureau of Census. Finally, as an indicator of the business cycle we construct real GDP growth rates from quarterly Real GDP series obtained from the US Bureau of Economic Analysis.

We plot the quarterly debt-to-equity ratio and GDP growth rate series simultaneously in Table 2.1 where the Grey curve denotes GDP growth rate and the debt-to-equity ratio is depicted in the black curve. Accordingly, the debt-to-equity ratio was relatively low from the start of our sample period until the early 1970s, but then rose substantially to peak in the mid 1980s. While it has been since then mostly on steady decline, more recently there appears to be an increasing trend towards

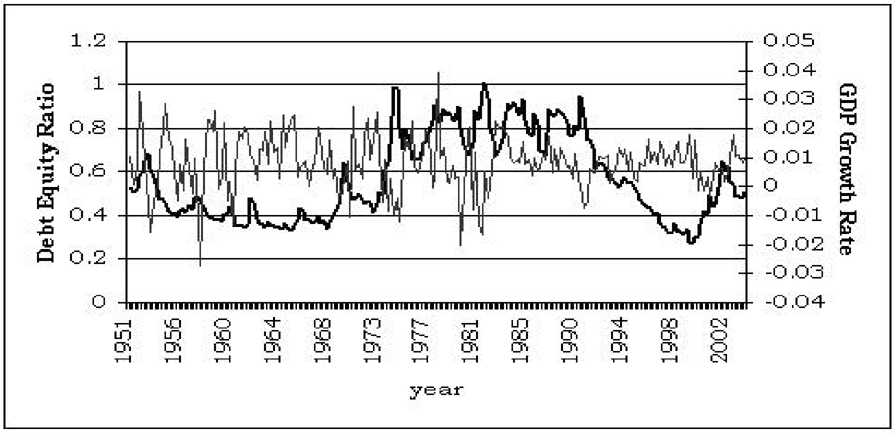


Table 2.1: The US Debt to Equity Ratio and GDP Growth Rate

utilizing debt-to-equity financing in the non-farm non-financial corporate business sector. More importantly from the perspective of the current essay, however, one needs to consider whether the movements in the debt-to-equity ratio may be related to the business cycle as measured by the GDP growth rate. In this regard the patterns in Table 2.1 seem to suggest that the aggregate debt-to-equity ratio indeed moves negatively with the GDP growth rate, both in terms of individual peaks and general movements.

While the visual examination of the data is indicative of a counter-cyclical nature of the aggregate debt-to-equity ratio, clearly this needs to be subject to tests of a more rigorous nature. We thus proceeded to estimate the following empirical equation:

$$\sigma_t = \alpha + \sum_{i=1}^{4} \beta_i GDP\_GRATE_{t-i\rightarrow t} + \sum_{i=2}^{4} \delta_i Q_i + \lambda TIME\_TREND_t + \epsilon_t \tag{2.27}$$

where σ_t is the aggregate debt-to-equity ratio at t, GDP_GRATE is the GDP growth including potential lags to allow for non-contemporaneous effects, $TIME\_TREND$ is a time trend, Q consists of a set of dummies to allow for seasonal effects, and is a random error term with the usual properties. An important aspect for proper inference from (2.27) is the underlying assumption that σ_t follows a stationary process and we thus proceeded to test this using a number of standard Dickey-Fuller and augmented Dickey-Fuller tests. However, in all cases there was no evidence to the contrary and we thus proceeded to estimate (2.27) using standard OLS.

	(1)	(2)	(3)	(4)	(5)
GDP_GRATE	-3.735***	-2.945**	-2.872*	-2.961**	-3.327**
	(1.376)	(1.452)	(1.463)	(1.476)	(1.517)
GDP_GRATE(-1)		-2.439*	-2.049	-1.972	-1.985
		(1.453)	(1.524)	(1.539)	(1.548)
GDP_GRATE(-2)			-1.273	-0.993	-0.763
			(1.460)	(1.536)	(1.551)
GDP_GRATE(-3)				-0.898	-0.581
				(1.473)	(1.543)
GDP_GRATE(-4)					-1.243
					(1.481)
TIME_TREND	0.001***	0.001***	0.001***	0.001***	0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q1	-0.000	0.005	0.004	0.003	0.003
	(0.037)	(0.037)	(0.037)	(0.037)	(0.038)
Q2	0.010	0.015	0.016	0.014	0.012
	(0.037)	(0.037)	(0.037)	(0.038)	(0.038)
Q3	-0.006	-0.003	-0.002	-0.002	-0.008
	(0.037)	(0.037)	(0.037)	(0.037)	(0.038)
Obs.	212	211	210	209	208
F-Test	3.65***	3.52***	3.09***	2.72***	2.56***
R-squared	0.08	0.09	0.10	0.10	0.10

Notes: (1) Standard Errors in Parantheses; (2) ***, **, and * signify one, five, and ten per cent significance levels.

Table 2.2: Regression Results

The results of estimating (2.27) are provided in Table 2.2. As the lack of significance on the coefficients on the quarter dummies indicates, there is no systematic seasonal component to movements in the debt-to-equity ratio. In contrast, the positive and statistically significant estimate on the time trend suggests that since the early 1950s the debt-to-equity ratio has been rising over time. More importantly, however, we find that the

GDP growth rate enters negatively and significantly into the estimated equation, suggesting, in support of our theoretical prediction, that the use of debt relative to equity as a means of financing in the non-financial private sector rises in economic downturns, but falls when the economy recovers. For instance, the estimated coefficient indicates that a one percentage point fall in the GDP growth rate will reduce the debt-to-equity ratio by 0.037. We also investigated whether there where lagged effects arising from business cycle movements on the debt-to-equity ratio by systematically including lagged terms of the GDP growth rate in the remaining columns of Table 2.2.[23] As suggested by these, the counter-cyclical effect persists beyond the contemporaneous effect. However, firms' financing choice between equity and debt responds relatively quickly, so that adjustments to economic shocks are usually completed within a quarter of occurrence.

2.4 Conclusion

We provide new support to the idea that the structural malfunctioning of the credit market might cause in persistent fluctuations in the level of economic activity. The main feature of the economy turns out to be the interaction between the dynamic contracting problem and the existence of finitely-lived firms and this interaction provides the key to understanding the role of liquidity and the potential switching between debt and equity financing in our economy. Specifically, we find that the debt-to-equity ratio in the overall economy will vary counter-cyclically. Historical Data from the US does indeed provide empirical support for this result.

Nonetheless, an important issue still arguably needs to be tackled: the present framework does not allow yet for any sort of equity issuing cost.[24] This induces two unpleasant consequences: equities are issued up to their physical maximum and, furthermore, we cannot explain the use of equity financing in projects that generate unobservable cash flows. A formal development of this issue can be conceived as the next step of the present research.

[23]We also experimented with including various lags of the dependent variable, but these turned out to be statistically insignificant.

[24]It is well known that in the Myers and Majluf (1984) study the informative cost of equity plays a determinant role in defining the *pecking order* of financial choices.

2.5 Appendix

Proof of Proposition 10 in the text.

We first have to show that at every optimal contract:

$$\frac{\partial i_{t-1}}{\partial p_{t+1}} > 0, \qquad \frac{\partial i_{t-1}}{\partial p_t} > 0 \qquad \text{for } t = 1, 2, ... \tag{A2}$$

We consider the system (2.15)-(2.19) recalling that $\mu > 0$ implies $p_t y = R_t(y)$. Then, we have that $\phi(y) = \dfrac{R_{t+1}(y)}{p_{t+1}}$.

By considering (2.15) and (2.17) together we get:

$$\left[NPV - 2ai_{t-1}F(\frac{R_{t+1}}{p_{t+1}})\right]\left[1 - F(\frac{R_{t+1}}{p_{t+1}}) - ai_{t-1}f(\frac{R_{t+1}}{p_{t+1}})\right] - \frac{(ai)^2}{p_{t+1}}F(\frac{R_{t+1}}{p_{t+1}})f(\frac{R_{t+1}}{p_{t+1}}) = 0 \tag{2.28}$$

Let us denote $K_1 = NPV - 2ai_{t-1}F(\dfrac{R_{t+1}}{p_{t+1}})$ and $K_2 = 1 - F(\dfrac{R_{t+1}}{p_{t+1}}) - ai_{t-1}f(\dfrac{R_{t+1}}{p_{t+1}})$. From (2.20) and (2.21) we have that $K_1 > 0$ and $K_2 > 0$; equation (2.28) can be hence rewritten as

$$K_1 K_2 \frac{(ai)^2}{p_{t+1}} F(\frac{R_{t+1}}{p_{t+1}}) f(\frac{R_{t+1}}{p_{t+1}}) = 0$$

The Individual Rationality constraint turns out to be:

$$p_t\hat{y} + p_{t+1}\int_0^{\frac{R_{t+1}}{p_{t+1}}} (p_{t+1}x - p_{t+1}ai_{t-1})\, dF(x) + R_{t+1}\left[1 - F(\frac{R_{t+1}}{p_{t+1}})\right] = 0 \tag{2.29}$$

The optimal values of i_{t-1} and of R_{t+1} are the solutions of the system of non-linear equations (2.28)-(2.29). The system has the form:

$$\begin{cases} g_1(i_{t-1}, R_{t+1}; p_t, p_{t+1}) = 0 \\ g_2(i_{t-1}, R_{t+1}; p_t, p_{t+1}) = 0 \end{cases}$$

where $g_1(.)$ and $g_2(.)$ denote equations (2.28) and (2.29), respectively. Implicit differentiation allows us to write:

$$\frac{\partial i_{t-1}}{\partial p_t} = (-\frac{1}{\Delta})\left[\frac{\partial g_2}{\partial R_{t+1}}\frac{\partial g_1}{\partial p_t} - \frac{\partial g_1}{\partial R_{t+1}}\frac{\partial g_2}{\partial p_t}\right] \tag{2.30}$$

$$\frac{\partial i_{t-1}}{\partial p_{t+1}} = (-\frac{1}{\Delta}) \left[\frac{\partial g_2}{\partial R_{t+1}} \frac{\partial g_1}{\partial p_{t+1}} - \frac{\partial g_1}{\partial R_{t+1}} \frac{\partial g_2}{\partial p_{t+1}} \right] \tag{2.31}$$

where Δ is the determinant of the Jacobian matrix of our non-linear system. We can straightforwardly conclude that:

$$(-\frac{1}{\Delta}) > 0, \frac{\partial g_2}{\partial R_{t+1}} > 0, \frac{\partial g_2}{\partial p_t} > 0, \frac{\partial g_2}{\partial p_{t+1}} > 0$$

and

$$\frac{\partial g_1}{\partial R_{t+1}} < 0, \frac{\partial g_1}{\partial p_t} > 0, \frac{\partial g_1}{\partial p_{t+1}} > 0$$

As a consequence, $A2$ holds.

In a next step, we emphasize that for every $\hat{x} \in [0, \bar{x})$ there exist an open neighborhood of $\hat{y}$ centered on $\hat{x}$ where $\dfrac{\partial i_{t-1}}{\partial p_{t+1}} > \dfrac{\partial i_{t-1}}{\partial p_t}$. If we consider $\hat{x} = \hat{y}$ implicit differentiation gives us:

$$\left(\frac{\partial i_{t-1}}{\partial p_{t+1}} - \frac{\partial i_{t-1}}{\partial p_t} \right) = \frac{\partial g_2}{\partial R_{t+1}} \left[\frac{\partial g_1}{\partial p_{t+1}} - \frac{\partial g_1}{\partial p_t} \right] - \frac{\partial g_1}{\partial R_{t+1}} \left[\frac{\partial g_2}{\partial p_{t+1}} - \frac{\partial g_2}{\partial p_t} \right] > 0 \tag{2.32}$$

A continuity argument guarantees that the result goes through for all $\{\hat{y} : |\hat{y} - \hat{x}| < \epsilon\}$, with $\epsilon > 0$. ∎

Chapter 3

Incentive schemes with multiple principals

3.1 Introduction

The literature on competing mechanisms studies strategic interactions where a set of players, referred to as principals, compete in the design of incentive schemes offered to another set of players, called agents.[1] A crucial assumption of the analysis is that the mechanisms offered by each player cannot be contingent on those offered by the others. This makes this set-up different from the traditional applications of mechanism design to standard generalized principal-agent models (see Myerson (1982)).

There has been a great interest in the recent years for a theoretical assessment of such situations. Understanding the relationship between incentives and competition has become a major issue in contract theory and it appears to be a necessary step to extend traditional macroeconomic schemes to markets where information is not perfectly distributed among agents. [2]

The first aim of the present chapter is to present a simple set-up that could encompass the most recent studies in the literature on games with

[1]This essay elaborates on the three former works "Common agency games with separable preferences" (coauthored with D. Majumdar, G. Piaser and N. Porteiro), CORE-DP 2003-102, "Pure strategy and no-externality with multiple agents: a comment" (coauthored with E. Campioni, G. Piaser and U. Rajan), CORE-DP 2004-50, and "Negotiation and take-it-or-leave-it in common agency: a comment" (coauthored with G. Piaser and N. Porteiro), mimeo University of Roma.

[2]In the same way, the relationship between markets and contracts has been intensively investigated in recent studies in General Equilibrium theory (for a discussion see Bisin and Gottardi (1999)).

many principals and many agents. In the particular situation where a number of principals is competing in the presence of a single agent, the framework developed in this chapter delivers a general version of a common agency game.

Common agency games refer to a scenario where multiple principals compete over the contract offers they make to a single agent. It involves the following timing: principals act as mechanism designers and simultaneously offer contracts to the single agent. In a next step, the agent chooses her preferred alternative, contracts are executed and the equilibrium allocation is implemented. The study of such contractual situations is receiving great attention in the recent literature. Examples include: Biais, Martimort, and Rochet (2000) who analyze non-linear pricing interactions in financial markets, Parlour and Rajan (2001) who study the competition among lenders offering non-exclusive credit contracts. Applications to lobbying are presented in Dixit, Grossman, and Helpman (1997), while Bernheim and Whinston (1998a) develop an analysis of exclusive dealing agreements.[3]

The situations where principals compete in the presence of many agents have not yet been intensively investigated from a theoretical perspective. In an influential paper, McAfee (1993) introduced a framework to study auctions where multiple sellers compete over the mechanisms they are offering to buyers. More recently, Prat and Rustichini (2003) have stressed the theoretical relevance of multi-principal multi-agent games in political economy, labor economics and industrial organization.

In very general terms, we can say that the literature on competing mechanisms has focused on two core issues: the characterization of the set of equilibria of multi-principal multi-agent games and the study of their efficiency properties.[4]

This chapter tries to contribute to the first research direction. Defining a general device to characterize equilibrium outcomes of games where principals compete through mechanisms in the presence of many agents is still an open issue.

Given that none of the principals has the whole control of the communication among other players, we are outside the domain of application

[3]In the terminology introduced by Segal and Whinston (2003), these contributions can be seen as different *bidding* games.

[4]Surprisingly, there are still very few results concerning existence in the literature: Laussel and Le Breton (2001) and Berkok (1990) study existence of Nash equilibria in common agency games with complete and incomplete information, respectively, while Prat and Rustichini (2003) show existence of pure strategy equilibria in a multi-principal multi-agent game with complete information and contractible actions.

of the standard Revelation Principle (Myerson (1979), Myerson (1982)). Furthermore, several examples have been recently developed (starting from the seminal one of Peck (1997)) to show that whenever multiple principals are considered, it is then possible to construct communication schemes which corresponding equilibrium outcomes cannot be reproduced by simple Incentive Compatible mechanisms, that are called "direct" by analogy with the single principal case. With some possible conceptual inconsistency, this finding is usually referred to as a *failure* of the Revelation Principle.

Section 3.3 will examine the issue in greater detail, trying to contribute to this research direction in the specific context of common agency games.

Yet, in order to provide clearer intuitions about equilibrium characterization in multi-principal games, a further topic should be investigated: what loss of generality is involved in the restriction to simple mechanisms? In other words, is it possible to say that equilibria in the restricted class of "direct mechanisms" survive to the possibility that principals might adopt more sophisticated communication mechanisms? Equilibrium outcomes satisfying such a requirement have been called *regular* (Epstein and Peters (1999), Martimort and Stole (2002), Peters (2003)). Some contributions managed to show that every pure strategy equilibrium of a common agency game where principals are restricted to use direct mechanisms is in fact a regular equilibrium.[5] This argument has usually been used to provide some support for restricting the analysis to equilibria of simple "direct" mechanism games (Martimort and Stole, 2001).

When multiple agents are introduced in the framework, then the literature has not offered yet any insight in terms of equilibrium characterization. The last section of this chapter provides a detailed discussion and suggests some new results.

3.2 Multi-principal multi-agent games

We consider a scenario where there are N principals (indexed by $i \in N = \{1, 2, ..., N\}$) contracting with J agents (indexed by $j \in J = \{1, 2, ..., J\}$).

Agent j's type θ^j is drawn from the set Θ^j according to the probability distribution F^j that is common knowledge.[6] Individual types are

[5] See Peters (2003).

[6] At this point we would like to stress that no assumption is needed on the finiteness or the non-finiteness of Θ^j. This is to emphasize the fact that an agent's type represent only her idiosyncratic nature. In particular, Θ^j can be considered finite for every j.

assumed to be independent random variables: we denote $\Theta{=}\times_{j=1}^{J}\ \Theta^j$ and we let $F{=}\times_{j=1}^{J}\ F^j$ be the relevant probability distribution over Θ.[7]

Each principal i is allowed to choose an action in the set $\Delta(Y_i)$, i.e. the set of lotteries that can be generated over the set of deterministic allocations Y_i. An allocation can be, for example, monetary transfers, tax rates, prices, or quantities, depending on the particular interpretation of the model. Every agent j can take the unobservable effort decision $e^j \in E^j$ and the action $h^j \in H^j$ that all principals can contract upon. We denote the vector of efforts as $e = (e^1, e^2, ..., e^J) \in E = \times_{j=1}^{J} E^j$ and the vector of actions as $h = (h^1, h^2, ..., h^J) \in H = \times_{j=1}^{J} H^j$. For simplicity, we take the sets H^j and E^j to be finite for all i and j.
Each principal can hence write contracts that are contingent on the array of actions $h \in H$ chosen by agents. In addition, non-contractibility of e introduces a standard moral hazard problem.

Players have expected utility preferences. Agent j's payoff is given by the von Neumann–Morgenstern utility function $U^j(y, e, h, \theta^j)$, and for each principal i the payoff is given by $V_i(y, e, h, \theta)$.

We consider a simple structure for communication games under incomplete information. Each principal independently communicates with agents selecting a communication process or mechanism. A mechanism chosen by principal i is constituted by a message space M_i and an allocation rule that associates a (stochastic) outcome to every array of messages and of contractible actions chosen by agents.
Allocations are determined in the following way. First, every principal selects and offers a mechanism; mechanisms are public, in the sense that they are observed by all agents. In a next step, each agent simultaneously sends a message to each principal and takes her action decisions. More formally, principal i chooses a relevant message space $M_i^j \in \mathcal{M}_i^j$ for each agent j, where $\mathcal{M}_i^j$ is the set, that we assume to be finite, of all feasible message spaces he could choose to receive communications from the $j{-}th$ agent. We also define $M_i = \times_{j=1}^{J} M_i^j$.
Given M_i, an allocation rule chosen by principal i is the map:

$$\pi_i : M_i \times H \rightarrow \Delta(Y_i)$$

This assumption, in fact, would rule out the possibility of including private market information in the agent's type space as it is done in Epstein and Peters (1999).

[7]The assumption of independent types has been made for expositional purposes. Correlated types can be embedded in the analysis as long as the distribution of Θ^j conditional on a realized Θ^{-j} varies continuously with Θ^{-j}(Epstein and Peters (1999), p. 123).

With some abuse of notation, we call $\pi_i(y_i|m_i, h)$ the probability distribution of y_i conditional on the received array of messages $m_i \in M_i$ and on the observed vector of agents' actions $h \in H$.
A pure strategy for principal i is therefore to design the stochastic mechanism (M_i, π_i). We often write $\sigma_i = (M_i, \pi_i) \in \Sigma_i$ where Σ_i is the (finite) set of feasible mechanisms for principal i. That is, Σ_i denotes the i-th principal strategy set and we let $\Sigma_N = \times(\Sigma_i)_{i=1}^N$ be the collection of all these sets. One should observe that a pure strategy for the principal can involve the choice of lotteries over allocations.[8]

In multi-principal multi-agent games, the behavior of each agent j depends on both her type θ^j and on the public mechanisms $(\sigma_i)_{i=1}^N$ offered by all principals and observed by all agents. The dependence of the message m_i^j sent to principal i on other principals mechanisms differentiates this setting from the standard single principal one where only valuations matter. That is, at the stage of contracting with the $i-th$ principal, agents possess private information about their own preferences *and* about the contract offers they are simultaneously receiving from all the others $-i$ principals. The latter is usually referred to as *market information*:

> *When firms compete by choosing mechanisms, followed by consumers choosing firms, the type refers to the initial information about each others' characteristics, which we call the initial type, and the new information consumers have about the mechanisms chosen by firms.*[9]

Thus, a pure strategy for agent j is given by the map:

$$\sigma^j : \Theta^j \times \Sigma_N \longrightarrow M^j \times E^j \times H^j$$

where $M^j = \times_{i=1}^N(M_i^j)$. That is, after observing the whole array of offered mechanisms, each agent j is simultaneously choosing an effort level $e^j \in E^j$, an observable action $h^j \in H^j$ and an array of messages $m^j \in M^j$, one for each principal. One should note that each $j-th$ agent is choosing one contractible action h^j for all principals. We also denote Σ^j the (finite) strategy set for agent j and we let $\Sigma^J = \times_{j=1}^J \Sigma^j$.
Agents' participation decisions can be directly introduced in the analysis. One possible way would be to consider the choice of every $j-th$ agent to accept the contract offer of each $i-th$ principal as a binary non-contractible action.[10]

[8] There is a subtle distinction between mixed strategies and random mechanisms for principals, as clarified by Peters (2001).

[9] Peck (1997), p.2

[10] See, for instance, Epstein and Peters (1999).

The time structure of the interaction is provided in Figure 3.1.

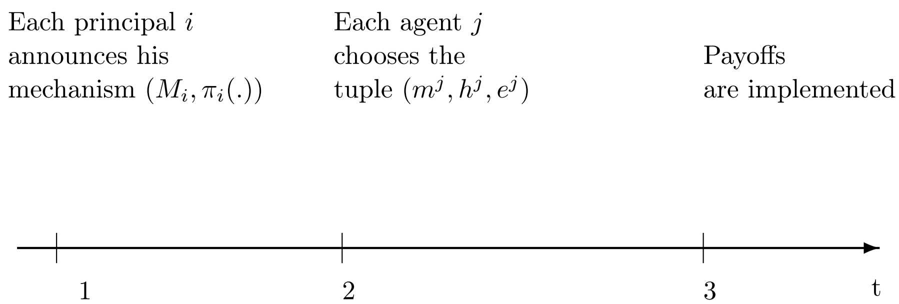


Figure 3.1: Timing of the generalized game

We can hence define the multi-principal multi-agent game with incomplete information G:

$$G = \left[\Sigma_N, \Sigma^J, V_i(y, e, h, \theta), U^j(y, e, h, \theta^j), F, \Theta\right] \tag{3.1}$$

Our analysis focuses on the set of pure strategy Perfect Bayesian equilibria of the game G.

We first consider agents behavior. For every given array of mechanisms chosen by principals $(\sigma_1, \sigma_2, \dots, \sigma_N) \in \Sigma_N$, the strategy profile $\left(\sigma^{j*}\right)_{j=1}^{J}$ constitutes a continuation equilibrium in the subgame played among agents if none of the agents has an incentive to unilaterally deviate from either the reporting decision m^{j*} or the action decisions e^{j*} and h^{j*}. That is, σ^{j*} is part of a continuation equilibrium if for each $\theta^i \in \Theta^i$, for each $(\sigma_i)_{i=1}^{N}$, and for each M it maximizes:

$$\int_Y \int_{\Theta^{-j}} U^j \left(y, e^j, e^{-j*}, h^j, h^{-j*}, \theta^j\right) \left[\times_{i=1}^{n} \pi_i(y_i | m_i^j, m_i^{-j*}, h^j, h^{-j*})\right] dF^{-j}(\theta^{-j})\, dy$$

for every $j = 1, 2, ..., N$.

More precisely, we say that the array $\left[(\sigma^{j*})_{j=1}^{J}, (\sigma_i^*)_{i=1}^{N}\right]$ is a Perfect Bayesian equilibrium (PBE) of the game G if:

- $(\sigma^{j*})_{j=1}^{J}$ is a continuation equilibrium of the game played among agents for every given array of mechanisms chosen by principals;

- Given that all principals but the $i-th$ one are choosing the mechanism array $\sigma^*_{-i} = \left(M^*_{-i}, \pi^*_{-i}\right)$ and that agents behave according to the continuation strategy $\left(\sigma^{j*}\right)_{j=1}^{J}$, then the mechanism σ_i^* chosen by every principal i maximizes his expected utility:

$$\int_Y \int_\Theta V_i\left(y_i, y_{-i}, e^*(.), h^*(.), \theta\right) \pi_i(y_i|m_i^*(.), h^*(.))\pi^*_{-i}(y_{-i}|m^*_{-i}(.), h^*(.))dF(\theta)\, dy$$

where $m^*(.)$, $e^*(.)$, and $h^*(.)$ denote the agents' best replies to the array of offered mechanisms.

At this point we would like to remark that the main assumption implicit in the literature on competing mechanisms is that only one-sided communication is considered. In other words, principals are not allowed to provide agents with any private information over allocations. As a consequence, none of the players (and, in particular, none of the principals) is able to generate a communication system that could coordinate the decisions of the others. This makes the relationship between the competing mechanism literature and the traditional mechanism design approach (Myerson (1982), Myerson (1991)) quite puzzling. Some recent researches have also emphasized this point, explicitly considering the need to introduce an additional contracting stage before the game G is played.[11]

As a matter of fact, the framework we have introduced here encompasses most of the multi-principal multi-agent schemes recently introduced in the literature. In particular, Prat and Rustichini (2003) study a game where many principals compete on simple transfer schemes they offer to a set of agents who are allowed to take a fully contractible action (that is, $E = \{\emptyset\}$) and should pay some additively separable cost of exerting it. Peters (2004) considers a complete information scenario where agents are only allowed to take unobservable actions and have to participate with all principals. A pure incomplete information game is analyzed by Yamashita (2004). Once again, all these works do not consider the possibility for principals to send messages (recommendations) privately observed by agents.

Importantly, for an arbitrary collection of strategy spaces $\left(\Sigma_N, \Sigma^J\right)$, the multi-principal multi-agent game G may be intractable. Many theoretical works have tried to define some devices useful to characterize the

[11] See for instance Jackson and Wilkie (2005).

(possibly very large) set of equilibria of the game G. In analogy with the single principal analysis, a natural starting point has been to consider the notion of "direct mechanisms." It is important to remark that this notion has originally been introduced having in mind a single principal scenario,[12] and the extension to the multi-principal case turns out not to be straightforward. Once again, we adopt here a definition of direct mechanisms that embeds those used in the literature.

In direct mechanism games the messages received by every principal i from the $j-th$ agent must belong to the type set Θ^j. That is, in a direct mechanism $M_i^j = \Theta^j$ for every $i = 1, \ldots, N$ and for every $j = 1, \ldots, J$. An allocation rule for principal i in a such a game is defined by the map $\tilde{\pi}_i$:

$$\tilde{\pi}_i : \Theta \times H \rightarrow \Delta(Y_i)$$

In a direct mechanism game, we can hence denote $\tilde{\sigma}_i = \tilde{\pi}_i$ a pure strategy for principal i.

Agent i's behavior is described by the pure strategy:

$$\tilde{\sigma}^j : \left(\tilde{\Sigma}_i\right)_{i=1}^N \times \Theta^j \rightarrow \Theta^j \times E^j \times H^j$$

Given these restrictions on players' strategy sets, we can define the corresponding direct mechanism game G_D in the following way:

$$G_D = \left[(\tilde{\Sigma}_i)_{i=1}^N, (\tilde{\Sigma}^j)_{j=1}^J, V_i(y,e,h,\theta), U^j(y,e,h,\theta^j), F, \Theta\right] \tag{3.2}$$

where $(\tilde{\Sigma}_i, \tilde{\Sigma}^j)_{i \in N, j \in J}$ are players' strategy spaces.

When games with *pure incomplete information* are considered (i.e. when both H^j and E^j are empty sets for all $j = 1, 2, ...J$), the notion of Incentive Compatible direct mechanism is a straightforward extension of the notion developed for the single principal case. A direct mechanism is Incentive Compatible if $m_i^j = \theta^j$ for every i, that is, if agents' message strategies consist in truthfully revealing their type to all Principals.

If one considers games with *complete information* where agents are assumed to take actions, then the standard notion of Incentive Compatibility is more cumbersome to apply, since the concept of "obedience" is not well defined when there are many principals. Most of the literature on competing mechanisms simply ignores this issue by assuming that principals are not allowed to send any recommendations to agents.

[12]See for instance Dasgupta, Hammond, and Maskin (1979).

A relevant number of studies have recently tried to investigate the relationship between the set of equilibria of the game G and that of a simpler game where principals' strategies are restricted in the way defined above. Most of the theoretical work in common agency models emphasized the existence of equilibrium outcomes in the general game that cannot be reproduced in the corresponding direct mechanism game. As we already said, these results are usually presented as *failures* of the Revelation Principle in common agency games.

Several devices have therefore been suggested to get a full characterization of equilibria in that class of games. The next section tries to contribute to this line of research.

3.3 Common Agency games with separable preferences

3.3.1 Introduction

This section studies the applicability of the Revelation Principle in common agency games.

To give a picture of the scenario we refer to, let us consider the example of a single borrower entering into contractual agreements with more than one lender, say lender 1 and lender 2. The payoff to the borrower depends on the contracts for both lenders, that the borrower accepts. As a consequence, at the stage of acceptance, the borrower while contracting with one lender takes into account the contracts offered by the other one. Examples of such contractual situations are many in the literature.

One important issue in this class of problems is the study of communication mechanisms between the principals and the agent. In the classical one-principal one-agent (or multi-agent) problem, the study of such communication mechanisms is simplified by the celebrated Revelation Principle (Gibbard (1973); Myerson (1979)). By the Revelation Principle, every state-contingent contract induced by the Bayesian Nash Equilibrium of a communication game is generated by a Bayesian Incentive Compatible direct mechanism. In such scenario, the relevant private information for the principal is only the agent's "type". When more than one principal contracts with one agent, generally, such simple direct mechanisms *do not* reproduce all the equilibria of the underlying communication game.

The main rationale behind the failure of the Revelation Principle can be summarized as follows: at the stage of contracting with any principal the agent's private information involves both her type and the mechanisms simultaneously offered by all the other principals. Direct mechanisms are too simple as principals cannot profit from the extra information held by the agent.[13]

The most recent literature offers two alternative ways to face this difficulty. On the one hand, Epstein and Peters (1999) show that in multi-principal-multi-agent problems it is possible to construct a *universal* type space that embeds all the agents' private information: with respect to this space the Revelation Principle is still valid. In the same line, the recent work by Pavan and Calzolari (2003) introduces a *Markovian* Revelation Principle for common agency games. It is possible to restrict without loss of generality to mechanisms implying agent's truthful revelation of her *extended* type, including agent's private information, as well as all the payoff-relevant decisions contracted with the multiple principals. The main difficulty with these general solutions is the analytical complexity of the relevant mechanisms and types that limits their applicability to common contracting problems.

On the other hand, Peters (2001) and Martimort and Stole (2002), instead of looking at the relationship between indirect and direct mechanisms when multiple principals compete, emphasize how the payoffs supported by general indirect mechanisms can be as well supported as equilibria in the class of games where principals strategies are menus over the relevant alternatives. That is, they suggest an extension of the so-called Taxation Principle (see Rochet (1986)) to a multi-principal framework: this amounts to substituting a potentially rich set of mechanisms with a simpler one, where the agent is not required to strategically reveal her private information.

Relatively less attention has been devoted to the definition of domain conditions that guarantee the Revelation Principle to hold in common agency games.[14] The present section explores such a research direction trying to identify conditions on agent's preferences that are sufficient for the Revelation Principle to hold when principals compete through mechanisms. Our condition is a separability requirement on the agent's

[13]For examples of multiprincipal games in which the Revelation Principle fail, see Peck (1997); Epstein and Peters (1999); Peters (2001) and Martimort and Stole (2002).
[14]This line of research has originally been suggested by Martimort (1992).

preferences with respect to the contract offers made by the various principals. To be more precise, what we require is that the agent's preference ordering over the contract offers by one principal should not depend on those made by the others. If the requirement is satisfied, then every message sent by the agent to principal i will not depend on the contract offer she is receiving from principal j. This enables us to reestablish the Revelation Principle. When the relevant environment is one of incomplete information (pure adverse selection), the above condition is sufficient per se. The situation differs when the agent takes a non-contractible effort choice (i.e., if we add to the problem moral hazard considerations). In this case we need to strengthen the condition by requiring that the agent's preference ordering over the contract offers of any principal is independent of the non-contractible actions taken by the agent herself.

Finally, the case in which the agent's action is contractible is particulary interesting. Restricting to a scenario where agent's actions are fully observable allows us to isolate the external effects induced by competition over contracts. A relevant portion of the common agency literature has been built in a setting of *complete* information where the action taken by the agent is *fully contractible* for every principal. This is, in fact, the reference framework of traditional works, like Bernheim and Whinston (1986b); Dixit, Grossman, and Helpman (1997); Bernheim and Whinston (1998a); Laussel and Le Breton (1998) and Laussel and Le Breton (2001). With respect to this class of games, we are able to show that a separability requirement is a sufficient condition to restore the Revelation Principle. Importantly, most of common agency models belongig to this class satisfy our separability condition and thus any equilibrium supported by indirect mechanisms can be supported through direct mechanisms as well.

It worth noting that no restriction on principals' preferences is introduced throughout the analysis. That is, the Revelation Principle can be supported even if we allowed for direct externalities among principals.

To our best knowledge, an approach close to ours has only been followed by Peters (2003), who developed a similar intuition. His work is however focused on common agency games where the agent's private information does not affect her preference order. As a consequence, incomplete information is not an issue in his analysis. When considering contractible actions, Peters (2003) obtains a condition for the validity of the Revelation Principle that requires both the absence of direct externalities among principals, and a separability condition on the agent's preferences. Our analysis shows that, in fact, only the second requirement is needed and the re-establishment of the Revelation Principle does not depend on any restriction over principals' preferences. Thus, our contribution, by generalizing Peters' results, can be applied to a wider range of games where the validity of the Revelation Principle could not be previously assessed.

3.3.2 The model

We consider a scenario where there are a number of principals (indexed by $i \in N = \{1, ..., n\}$) contracting with one agent (denoted by index, 0). The agent's type is drawn from a set Θ having a probability distribution $F(.)$ that is common knowledge. We maintain the assumption that the agent takes the observable action h from the set H and the non-contractible effort e from the set E. Each principal i controls allocations in the set Y_i.

The payoff to principal $i \in \{1, \dots N\}$ is represented by the vNM utility function $V_i : \times_{k \in N} Y_k \times E \times H \times \Theta \to \mathbb{R}_+$. For the agent the payoff is represented by the function $U : \times_{k \in N} Y_k \times E \times H \times \Theta \to \mathbb{R}_+$.

For the sake of expositional clarity, we follow the literature in common agency games and assume that principal i's set of feasible message spaces $\mathcal{M}_i$ is a singleton for $i = 1, 2, ..., N$. This allows us to consider the array of message spaces $(M_1, M_2, \dots, M_N)$ as given. As a consequence, a mechanism σ_i chosen and offered by principal i will be only identified by an allocation rule, that is, a map from $M_i \times H$ to Y_i, where M_i is the i-th principal's message space.
We also make a standard richness assumption regarding M: we should have $\Theta \subset M_i$ for any i.[15]

As before, we denote Σ_i the set of all strategies for principal i, and Σ_N the collection of all the Σ_i sets. The agent's strategy is to simultaneously choose a message m_i for each principal and the two actions h and e. The strategy for the agent is hence the map $\sigma_0 : \Theta \times \Sigma_N \to M \times H \times E$ and Σ_0 is the collection of all the σ_0 maps.

[15] The assumption is introduced in the benchmark theoretical works in common agency (see Peters (2001) and Martimort and Stole (2002)).

One should notice that we are introducing a further simplification by assuming that each principal i is restricted to the choice of deterministic allocations in the set Y_i.[16]
Hence, given the array of message spaces $M = M_1, M_2, ..., M_N$ the common agency game Γ_M is:

$$\Gamma_M = \{\Theta, \Sigma_N, \Sigma_0, U(y, e, h, \theta), (V_i(y, e, h, \theta))_{i\in N}, F(.)\}.$$

It is important to remark that, given our previous assumptions, Γ_M turns out to be a finite game.

We focus on the *pure strategy* Perfect Bayesian Equilibria (PBE) of the game Γ_M.

Definition 5 *A strategy profile $\sigma^* = ((\sigma_i^*)_{i=1}^n, \sigma_0^*) \in \Sigma_N \times \Sigma_0$ is a pure strategy PBE of the game Γ_M if:*

$\forall \theta \in \Theta$, $\forall (\sigma_i)_{i\in N} \in \Sigma_N(M)$,

$$\sigma_0^* = (m^*, e^*, h^*) \in \underset{(m,h,e)\in M\times H\times E}{\arg\max} U((\sigma_i(m_i, h))_{i\in N}, h, e, \theta)$$

$\forall i \in N$,

$$\sigma_i^* \in \underset{\sigma_i\in\Sigma_i}{\arg\max} \int_{\theta\in\Theta} V_i(\sigma_i, \sigma_{-i}^*, e\left(\sigma_i, \sigma_{-i}^*, \theta\right), h\left(\sigma_i, \sigma_{-i}^*, \theta\right), \theta) dF(\theta)$$

Observe that in the game Γ_M the message spaces can be quite complex. One simple mechanism is one where the message space M_i coincides with the agent's type space Θ, for any i. In this context, such mechanisms can be called "direct mechanisms".

When restricting to direct mechanisms, we define the strategy of principal i as the map $\tilde{\sigma}_i : \Theta \times H \to Y_i$; we let $\tilde{\Sigma}_i$ be the strategy space for principal i and $\tilde{\Sigma}_N$ be the collection of all such strategy profiles.

The strategy of the agent is then a map $\tilde{\sigma}_0 : \Theta \times \tilde{\Sigma}_N \to \Theta^N \times H \times E$, and $\tilde{\Sigma}_0$ denotes the collection of all such maps.

Given Θ, the common agency game induced by direct mechanisms is the tuple:

$$\Gamma_{\Theta^N} = \left\{\Theta, \tilde{\Sigma_N}, \tilde{\Sigma}_0, U(y, h, e, \theta), (V_i(y, h, e, \theta))_{i\in N}, F(.)\right\}.$$

[16]This is a standard assumption in the common agency literature. Our main results will not be affected by the explicit consideration of stochastic allocation rules (see Martimort and Stole (2001), p. 20).

Direct mechanisms have an obvious appeal - the message spaces are simple. The question then is, when can one restrict attention to direct mechanisms? In other words, is it always the case that for any equilibrium σ^* of the game Γ_M there is an equilibrium $\tilde{\sigma}^*$ of the game Γ_{Θ^N} such that the two equilibria are outcome equivalent? For common agency games this question is usually answered in the negative (see, for instance, Martimort and Stole (2002); Peters (2001)). In games with multiple principals, there exist equilibria whose outcomes cannot be supported in equilibrium in the corresponding direct mechanism game.

A main aim of this essay is to introduce restrictions on the fundamentals of the game Γ_M that allow us to restore the Revelation Principle.

First of all, we define the class $\mathcal{G}$.

Definition 6 *A common agency game Γ_M belongs to the class $\mathcal{G}$ if, for every $i \in N$, for every y_{-i}, and for every $\theta \in \Theta$:*

$$U(y_i, y_{-i}, e, h, \theta) \neq U(y_i', y_{-i}, e', h', \theta), \forall y_i, y_i', e, e', h, h' \textit{for } (y_i, e, h) \neq (y_i', e', h').$$

That is to say, everything else equal, the agent is never indifferent to a change in the contract offer made by any of the principals. Furthermore, it is required that the best reply of the agent in terms of both the effort e and the action h should be unique for every given array of allocations offered by principals.
Given that we are considering finite games, this condition is generically satisfied in the payoff space of the agent.

In the remaining of the section we consider common agency games that belong to the class $\mathcal{G}$ and we try to show how the introduction of a separability requirement on agent's preferences enable us to restore the Revelation Principle.[17]

3.3.3 The pure Incomplete Information case

In the next pages we focus on incomplete information games in which the agent takes neither an effort choice nor a contractible action. We define this scenario by assuming $H = E = \{\emptyset\}$. With a slight abuse of notation, we denote a principal's strategy in this pure incomplete information game by $\sigma_i : M_i \to Y_i$. As before, the set of strategies for

[17]The need for restricting the analysis to games in the class $\mathcal{G}$ will be clarified in the Appendix.

principal i is given by Σ_i and $\Sigma_N = \times_{i\in N}\Sigma_i$. The strategy for the agent is a map $\sigma_0 : \Theta \times \Sigma_N \to M$. The game Γ_M is then the tuple:

$$\Gamma_M = \{\Theta, \Sigma_N, \Sigma_0, U(.,\theta), (V_i(.,\theta))_{i\in N}, F(.)\}.$$

Analogously, we can define the direct revelation game Γ_{Θ^N} as the tuple:

$$\Gamma_{\Theta^N} = \left\{\Theta, \tilde{\Sigma}_N, \tilde{\Sigma}_0, U(y,\theta), (V_i(y,\theta))_{i\in N}, F(.)\right\},$$

where $\tilde{\Sigma}_i$ is the set of strategies for principal i with $\tilde{\sigma}_i \in \tilde{\Sigma}_i : \Theta \to Y_i$. The strategy for the agent is $\tilde{\sigma}_0 \in \tilde{\Sigma}_0 : \Theta \times \tilde{\Sigma} \to \Theta^N$.

Observe that the agent's preferences are defined over the product set of allocations offered by the principals. Given the product set structure of alternatives we shall assume that the agent's preferences are separable over each component of the set of alternatives in an appropriate sense. By "separability" we mean that the agent is able to define unambiguously his preference over the allocations offered by principal i that does not depend on the contract offers made by the other principals. More formally, we state the following:

Definition 7 *Weak Separability (WS):*

The agent's utility function U is weakly separable in the principals' decisions for all $\theta \in \Theta$, if for all $y_i, y_i' \in Y_i$ and for all y_{-i}, $y_{-i}' \in Y_{-i}$,

$$\left[U(y_i, y_{-i}, \theta) > U(y_i', y_{-i}, \theta)\right] \Longrightarrow \left[U(y_i, y_{-i}', \theta) > U(y_i', y_{-i}', \theta)\right] \quad (3.3)$$

Examples of weakly separable utility functions include additively separable utility functions. However, a weakly separable utility function is not necessarily additively separable (see Fishburn (1970) for a counterexample). We note here that the separable utility assumption is common in many applied analysis (see Deaton and Muellbauer (1980)).

If the agent's utility function $U(.)$ is weakly separable in the allocations offered by the principal, then the *partial utility* for the agent over the contracts offers Y_i by principal i can be defined in a natural way. Given that the offers made by the other principals are fixed at the level $\hat{y}_{-i} \in Y_{-i}$, the partial utility over Y_i is:

$$\hat{U}_{\hat{y}_{-i}}(y_i, \theta) = U(y_i, \hat{y}_{-i}, \theta)$$

We start by showing some properties satisfied by equilibrium strategies when the agent has separable preferences. In particular, we show that for each i and for any $\hat{y}_{-i} \in Y_{-i}$, the i-th component of the agent's optimal strategy maximizes her partial utility at the level $\hat{y}_{-i}$. More formally, denoting m_i as the projection of σ_0 on M_i, we have the following:

Claim 1 *If* $\sigma_0^* = \left(m_i^*, m_{-i}^*\right)$ *is a best reply to strategies* (σ_i, σ_{-i})*, then for all* $\hat{y}_{-i} \in Y_{-i}$:

$$m_i^* \in \arg\max_{m_i \in M_i} \hat{U}_{\hat{y}_{-i}}(\sigma_i(m_i), \theta)$$

Proof. Suppose the claim is false. This implies that there exists a $m_i' \in M_i$ such that $\hat{U}_{\hat{y}_{-i}}(\sigma_i(m_i'), \theta) > \hat{U}_{\hat{y}_{-i}}(\sigma_i(m_i^*), \theta)$. Now, from Weak Separability it follows that:

$$U\left(\sigma_i\left(m_i'\right), \sigma_{-i}\left(m_{-i}^*\right), \theta\right) > U\left(\sigma_i\left(m_i^*\right), \sigma_{-i}\left(m_{-i}^*\right), \theta\right).$$

This contradicts the assertion that σ_0^* is a best reply to (σ_i, σ_{-i}). ■

A direct consequence of Claim 1 is that:

$$\sigma_i^*\left(m_i^*(\sigma^*, \theta)\right) = \sigma_i^*\left(m_i^*(\sigma_i^*, \theta)\right)$$

That is, every PBE exhibits the following property: principal i's optimal strategy is independent of principal j's mechanism. It is worth noting that this property on principals' behavior is just an implication of the separability restriction on agent's preferences.

We also remark that given the mechanism offered by principal i, then the message he receives from the agent depends on the agent's type θ only. At this point we are in a position to state the main theorem of this section.

Theorem 3 *Consider a pure incomplete information common agency game* Γ_M *in the class* $\mathcal{G}$*. If the preferences of the agent satisfy the Weak Separability condition, then for every outcome that can be supported as a* PBE *of* $\Gamma_M \in \mathcal{G}$*, there is an equilibrium of the direct mechanism game* Γ_{Θ^N} *that implements the same outcome.*

Proof. Let us consider a PBE σ^* of the indirect mechanism game Γ_M. We will now demonstrate that the distribution over outcomes generated by the strategy profile σ^* can be replicated in a direct mechanism game.

Consider the game Γ_{Θ^N} where principal i's strategy $\tilde{\sigma}_i : \Theta \to Y_i$ is defined as $\tilde{\sigma}_i(\theta) = \sigma_i^*(m_i^*(\sigma_i^*, \theta))$.[18] The agent's strategy is a map $\tilde{\sigma}_0$:

[18]Observe that the direct mechanism we are constructing is incentive compatible. In a pure incomplete information scenario, we say that a direct mechanism is Incentive Compatible if and only if it is an optimal choice for the agent to report her type thruthfully.

$\Theta \times \tilde{\Sigma}_N \to \Theta^N$. Notice that the agent's best reply coincides with the one defined in Γ_M when principals were using indirect mechanisms. We can then rewrite principal i's payoff as

$$\int_{\theta\in\Theta} V_i(\tilde{\sigma}_i(\theta), \tilde{\sigma}_{-i}(\theta), \theta)dF(\theta) = \int_{\theta\in\Theta} V_i(\sigma_i^*(m_i^*(.,\theta)), \sigma_{-i}^*(m_{-i}^*), \theta)dF(\theta)$$

Specifically,

$$\int_{\theta\in\Theta} V_i(\tilde{\sigma}_i(\theta), \sigma_{-i}^*(m_{-i}^*), \theta)dF(\theta) = \int_{\theta\in\Theta} V_i(\sigma_i^*(m_i^*), \sigma_{-i}^*(m_{-i}^*), \theta)dF(\theta)$$

We now show that no principal has a unilateral profitable deviation. Suppose that this is not true. Then, let us suppose that there exists a principal i and a strategy $\sigma_i^{'}(.)$ such that:

$$\int_{\theta\in\Theta} V_i(\sigma_i'\left(\theta_i'(\theta)\right), \tilde{\sigma}_{-i}\left(\theta_{-i}'(\theta)\right), \theta)dF(\theta) > \int_{\theta\in\Theta} V_i(\tilde{\sigma}_i(\theta), \tilde{\sigma}_{-i}(\theta), \theta)dF(\theta)$$

where θ_i' and θ_{-i}' are the agent's messages to principals i and $-i$ when principal i is deviating. Obviously, if principal i changes his offered mechanism, then the agent's reply to this deviation will be different. But Weak Separability implies that the agent's reply to the other principals will be unchanged (i.e; $\theta_{-i}' = \theta$). Therefore, it follows that:

$$\int_{\theta\in\Theta} V_i\left(\sigma_i'(\theta_i'(\theta)), \sigma_{-i}^*(m_{-i}^*), \theta\right) dF(\theta) > \int_{\theta\in\Theta} V_i(\tilde{\sigma}_i(\theta), \sigma_{-i}^*(m_{-i}^*), \theta)dF(\theta)$$

$\Longrightarrow$

$$\int_{\theta\in\Theta} V_i(\sigma_i'(\theta_i'(\theta)), \sigma_{-i}^*(m_{-i}^*), \theta)dF(\theta) > \int_{\theta\in\Theta} V_i(\sigma_i^*(m_i^*), \sigma_{-i}^*(m_{-i}^*), \theta)dF(\theta)$$

However this contradicts the fact that σ^* is a PBE. This concludes the proof of Theorem 3. ■

From Claim 1 it follows that at the stage of considering the contract between the agent and principal i we can restrict our attention to the agent's partial utility for any fixed $\hat{y}_{-i}$, and that principal i's optimizing behavior is independent of the agent's contract with other principals. This implies that for any fixed level $\hat{y}_{-i}$, we can break the general common agency game into N games, each of which is a single principal-agent problem where the Revelation Principle applies. Since the optimizing behavior for the two conceived partners remain unchanged in the original problem, we can apply the Revelation Principle taking one principal at a time and retrieve the Revelation Principle from the original problem. The general implication of the theorem is emphasized in the following remark:

Remark 1 *Theorem 3 says the following: given the collection of message spaces $M = \{M_1, ..., M_N\}$ for any equilibrium σ^* in the game Γ_M there exists an equilibrium $\tilde{\sigma}^*$ in the game Γ_{Θ^N} such that the distributions over outcomes in σ^* and $\tilde{\sigma}^*$ are the same. Since the choice of M has been arbitrary, we get back to the Revelation Principle.*

We can now illustrate the consequences the implications of assuming separability in agent's preferences.

First consider the following game, borrowed from Martimort and Stole (2002), where the type of the agent is fixed and known.

Example 1: The agent's type space is degenerate and there are two principals: each of them can offer the three allocations A, B and C. That is, we have: $Y_1 = Y_2 = \{A, B, C\}$. The payoffs of principal 1, principal 2 and the agent, $\left\{V^1, V^2, U\right\}$, are given by the following matrix:

	$y_2 = A$	$y_2 = B$	$y_2 = C$
$y_1 = A$	$(1, 1, a)$	$(2, 0, 2)$	$(-1, 5, b)$
$y_1 = B$	$(1, 2, 2)$	$(1, 1, 1)$	$(0, 0, c)$
$y_1 = C$	$(5, -1, b)$	$(0, 0, c)$	$(0, 0, 0)$

In the original Martimort and Stole's example we have $a = 1$, $b = 10$ and $c = 0$. If we consider the direct mechanism game, then there is no strategic role for the agent and the equilibrium outcomes are those that are sustained as Nash equilibria of the game played among the two principals. In particular, there is only one pure strategy Nash equilibrium: both principals play $\{C\}$ and the corresponding outcome is $(0, 0, 0)$.
However, if principals are allowed to communicate with the agent, then the outcome $(1, 1, 1)$ can also be supported at equilibrium. In particular, consider the indirect mechanism game where each principal is endowed with the same message space $M_1 = M_2 = \{m_1, m_2\}$. Now, assume that each principal chooses the following allocation rule:

$$\sigma_i : M_i \to Y_i$$
$$\sigma_i (m_1) = B$$
$$\sigma_i (m_2) = C$$

for $i = 1, 2$.

The allocation rules σ_1, σ_2 are part of a Nash equilibrium. At equilibrium the agent will send the message m_1 to both principals and the outcome $(1, 1, 1)$ will be implemented. Principal 1 cannot profitably deviate by offering any alternative indirect mechanism because the agent is acting as a coordinating device. If he deviates by offering the mechanism:

$$\begin{aligned}
&\sigma_1 : M_1 \to Y_1 \\
&\sigma_1 (m_1) = A \\
&\sigma_1 (m_2) = A
\end{aligned}$$

then the agent will send the message m_2 to principal 2 making the deviation unprofitable. The example is therefore emphasizing that there can exist outcomes implementable through indirect mechanism that are not implementable through direct mechanisms.

It is important to remark that under alternative assumptions on agent's preferences the outcome $(1, 1, 1)$ cannot be implemented through indirect mechanisms neither. To show that, let us assume $a > 2$, $b < 2$ and $c < 1$. In such a case, allowing for indirect mechanisms does not enlarge the set of equilibria sustainable trough direct mechanisms. For $(1, 1, 1)$ to be sustained as equilibrium payoffs, the utility of the agent should satisfy the following:

$$U (y_i = B, y_j = B) \geq U (y_i = C, y_j = B)$$

$$U (y_i = B, y_j = A) < U (y_i = C, y_j = A)$$

for every $i \neq j$.

The conditions above cannot be simultaneously fulfilled if the agent has separable preferences. Whenever the preferences of the agent are separable and he could gain by choosing $y_i = B$ instead of $y_i = C$ (given $y_j = B$), then the same has to hold when $y_j = A$. That is, the marginal impact on the agent's utility of a change in principal i's action has to be independent of principal j's action. The allocation $\{B, B\}$ cannot therefore be supported when preferences are separable, even if we allow principals to offer indirect mechanisms to the agent.

This first example hence provides an important hint for the analysis: existence of a *preference reversal* order for the agent over principals' actions is needed for indirect mechanisms to improve over direct mechanisms. This is not possible whenever preferences are separable.

The important contribution of the Martimort and Stole's work consisted in showing that in order to characterize the whole the set equilibria of common agency games it is sufficient to restrict prinicipals to offer menus of allocation. Notably, if preferences are separable then every allocation implentable through menus can as well implemented by direct mechanisms. We emphasize this point in a second example, still taken from Martimort and Stole (2002).

Example 2: There are two principals and one agent with type $\theta \in \{-1, 1\}$, drawn with equal probability. Each principal has two actions $y_i \in \{0, 1\}$. The utilities are $V_1 = \theta\,(y_1 - y_2)$ for principal 1, $V_2 = \theta\,(y_2 - y_1)$ for principal 2 and $U = \theta\,(y_1 + y_2)$ for the agent. Equilibrium strategies are given by the menus $\{0, 1\}$ for both principals. The outcome will always be $y_1 = y_2 = 0$ (when $\theta = -1$) and $y_1 = y_2 = 1$ (when $\theta = 1$).

Observe that agent's preferences are separable in the actions of the principals and, for this reason, the same outcome can be achieved as an equilibrium of a direct mechanism game. Simply consider the following contract offers: each principal proposes the agent a direct mechanism T_i such that:

$$T_i = \begin{cases} y_i = 0 & \text{if} \quad \theta = -1 \\ y_i = 1 & \text{if} \quad \theta = 1. \end{cases}$$

This mechanism implements the same outcome that was supported by menus.

These two examples considered together emphasize the relationship between the set of outcomes implementable by indirect mechanisms and by direct mechanisms if the agent has separable preferences. Whenever separability is assumed, then every equilibrium associated to an indirect mechanism (or menu) game can be as well supported by direct mechanisms.

The next paragraphs will deal with situations where the agent is allowed in addition to take unobservable actions.

3.3.4 Moral Hazard

The main purpose of this section is to extend the Revelation Principle to a situation where there are moral hazard considerations in addition to adverse selection factors. We assume that the single agent only takes an effort e from the set E (i.e. $H = \{\emptyset\}$), and that this effort choice is not contractible.

As mentioned before, when effort is not contractible, the principal's choice is only contingent on the message he receives. The strategy for principal i in a game with moral hazard is a map $\sigma_i : M_i \to Y_i$. The agent's strategy is to choose a message m_i for each principal and an effort e. The strategy for the agent is a map $\sigma_0 : \Theta \times \Sigma_N \to M \times E$.

In the corresponding direct mechanism game, the strategy for principal i is the mapping $\tilde{\sigma}_i : \Theta \to Y_i$. The strategy of the agent is $\tilde{\sigma}_0 : \Theta \times \tilde{\Sigma}_N \to \Theta^N \times E$.

In such a scenario the separability condition on the agent utility function $U(.)$ will not in general be a sufficient condition to get the Revelation Principle. The reason is that the optimal action chosen by the agent depends on all the offers she is receiving from principals; as a consequence assuming separability in $U(.)$ does not guarantee that the agent's utility function will remain separable if we evaluate it at the optimal chosen action. Although the Revelation Principle fails for the class of separable games already defined, a strengthening of Condition WS retrieves the Revelation Principle.

We refer to this condition as Strong Separability (SS).

Definition 8 Strong Separability (SS)*:*

The agent's utility function U is strongly separable in the principals' decisions y_i's and effort e if for all $\theta \in \Theta$, for all $y_i, y_i' \in Y_i$, for all y_{-i}, $y_{-i}' \in Y_{-i}$ and for all $e', e'' \in E$

$$\left[U(y_i, y_{-i}, e', \theta) > U(y_i', y_{-i}, e', \theta)\right] \Longrightarrow \left[U(y_i, y_{-i}', e'', \theta) > U(y_i', y_{-i}', e'', \theta)\right].$$

The SS condition implies that, for each principal i, the agent has an unambiguous preference ordering over the contract offers made by i that does not depend either on the contract offers by other principals or the effort choice of the principal.

As before, we define a notion of partial utility for the agent over principal i's offers for fixed offers made by the other principals and for a fixed level of effort. The partial utility for the agent over principal i's offers y_i for a fixed level of offers $\hat{y}_{-i}$ made by the other principals and for a fixed level of effort $\hat{e}$ is defined as:

$$\hat{U}_{\hat{y}_{-i},\hat{e}}(y_i, \theta) = U(y_i, \hat{y}_{-i}, \hat{e}, \theta)$$

In analogy with Claim 1 we now state the following:

Claim 2 *If $\sigma_0^* = \left(m_i^*, m_{-i}^*, e^*\right)$ is a best reply to strategies (σ_i, σ_{-i}), then for all $\hat{y}_{-i} \in Y_{-i}$ and for all $\hat{e} \in E$:*

$$m_i^* \in \underset{m_i \in M_i}{\arg\max} \hat{U}_{\hat{y}_{-i},\hat{e}}(\sigma_i(m_i), \theta)$$

Proof. The proof is similar to that of Claim 1. ■

We can now introduce the main result of this section.

Theorem 4 *Consider a common agency game* Γ_M *in the class* $\mathcal{G}$*. If the preferences of the agent satisfy the Strong Separability condition, then for every outcome that can be supported as a PBE of* $\Gamma_M \in \mathcal{G}$*, there is an equilibrium of the direct mechanism game* Γ_{Θ^N} *that implements the same outcome.*

Proof. We have to show that every payoff profile associated to the equilibrium strategies σ^* of the game Γ_M can be replicated in the game Γ_{Θ^N}.

We can define the function $\tilde{V}_i$ such that:

$$\tilde{V}_i(\tilde{\sigma}_i(.), \tilde{\sigma}_{-i}(.), \theta) = V_i(\tilde{\sigma}_i(.), \tilde{\sigma}_{-i}(.), e^*(\tilde{\sigma}_i(.), \tilde{\sigma}_{-i}(.)), \theta)$$

We consider the following strategies: $\tilde{\sigma}_i(\theta) = \sigma_i^*(m_i^*(\sigma_i^*, \theta)$ for principal $i \in N$ and

$$\tilde{\sigma}_0 = \left[m_i^*(\theta, \tilde{\sigma}) = \theta, e^*(\sigma_i^*(m_i^*(\sigma_i^*, \theta), \sigma_{-i}^*(m_i^*(\sigma_{-i}^*, \theta))\right]$$

for the agent. Then, considering $\tilde{V}_i$ for $i \in N$, we can replicate the argument already given in Theorem 3. ■

The theorem shows how, under moral hazard, a strengthening of our separability condition is required to retrieve the Revelation Principle in common agency games. Separability in agent's preferences for every level of effort is a sufficient condition for the agent's indirect utility to be separable in the contract offers.

A second issue of interest is a situation where the agent's actions are observable and principals can therefore offer contracts that are contingent on agent's choices. We deal with this issue in the next pages.

3.3.5 Contractible Action

We start by analyzing the extreme case where no unobservable action can be chosen; that is, $E = \{\emptyset\}$. For each message m_i received from the agent, principal i decides which allocation y_i will be offered conditionally on the chosen h.

We first consider the simple scenario where the action h is fully contractible for every principal and the preference ordering of the agent is independent of her type. A particular setting that fits this framework is that in which there is complete information about the agent's preferences. We already emphasized that this is the reference framework of traditional works in common agency. In this section we show how, whenever we consider such a set-up, our separability requirement is a sufficient condition to characterize the whole set of equilibria associated to indirect mechanisms through simple direct mechanisms.

In order to adapt our notion of separability to a case where the agent's preference ordering is independent of her type, we introduce a new notion of separability. For the sake of clarity, we will refer to this condition as θ-separability:

Definition 9 *θ-Separability:*

The utility function U is θ-separable in the principals' decisions y_i, for all $h \in H$, if for all $y_i, y_i' \in Y_i$, for all y_{-i}, $y_{-i}' \in Y_{-i}$ and for all $\theta, \theta' \in \Theta$:

$$\left[U(y_i, y_{-i}, h, \theta) > U(y_i', y_{-i}, h, \theta)\right] \Longrightarrow \left[U(y_i, y_{-i}', h, \theta') > U(y_i', y_{-i}', h, \theta')\right].$$

In this setting we now restate our Theorem 3.

Theorem 5 *Consider a common agency game Γ_M in the class $\mathcal{G}$. If the preferences of the agent satisfy the θ-Separability condition, then for every outcome that can be supported as a PBE of $\Gamma_M \in \mathcal{G}$, there is an equilibrium of the direct mechanism game Γ_{Θ^N} that implements the same outcome.*

Proof. Let us consider the PBE $(\sigma_i^*, h^*, m_i^*)_{i \in N}$ of $\Gamma_M \in \mathcal{G}$. We will show that the outcome implemented by this equilibrium can be reproduced as an equilibrium of a game where principals' strategies are restricted to take-it or leave-it offers.

We start by introducing the following definition:

$$\tilde{m}_i\left(\sigma_i^*, \sigma_{-i}^*, h, \theta\right) \in \arg\max_{m_i \in M_i} U\left(\sigma_i^*\left(m_i, h\right), \sigma_{-i}^*(m_{-i}^*, h), h, \theta\right), \forall i \in N, \forall h \in H.$$

Since the preferences of the agent satisfy the θ-separability condition, $\tilde{m}_i\left(.\right)$ does not depend on θ. It is only contingent on σ_i^*, σ_{-i}^* and h. Formally: $\tilde{m}_i\left(\sigma_i^*, \sigma_{-i}^*, h\right)$.

The candidate for equilibrium in the take-it or leave-it offer game is the strategy profile $\left(\tilde{\sigma}_i\left(h\right), \tilde{\sigma}_{-i}\left(h\right), \tilde{h}\right)$, where:

$$\begin{aligned} \tilde{\sigma}_i\left(h\right) &= \sigma_i^*\left(\tilde{m}_i\left(\sigma_i^*, \sigma_{-i}^*, h\right), h\right), \ \forall i \in N, \forall h \in H \\ \tilde{h} &\in \arg\max_h U((\tilde{\sigma}_i(h))_{i \in N}, h, \theta). \end{aligned}$$

The proof of the Theorem is organized in two steps. We first show that in the take-it or leave-it offer game the agent's best reply to the strategies $\tilde{\sigma}_i(h)$ is h^*. Then, we show that, given $\tilde{\sigma}(h)$, no principal has a unilateral deviation.

Claim 3 *If* $(h^*, m^*) \in \arg\max_{h,m} U\left((\sigma_i^*(m_i, h))_{i\in N}, h, \theta\right)$*, then:*

$$h^* \in \arg\max_{h} U\left((\tilde{\sigma}_i(h))_{i\in N}, h, \theta\right)$$

Proof. The proof is straightforward, given the definition of $\tilde{\sigma}_i(h)$. ■

Consider now a possible deviation in the take-it or leave-it game: Principal i is deviating to $\sigma_i'(h) \neq \tilde{\sigma}_i(h)$. A take-it or leave-it offer is, in fact, a particular indirect mechanism. The deviation $\sigma_i'(.)$ was, therefore, available in the original game Γ_M. We first show that faced with the same deviation in Γ_M and Γ_Θ the optimal response of the agent in terms of action will be the same.

Claim 4 *If*

$$\left(h', m_{-i}'\right) \in \arg\max_{h,m} U\left(\sigma_i'(h), \sigma_{-i}^*(m_{-i}, h), h, \theta\right).$$

then

$$h' \in \arg\max_{h} U\left(\sigma_i'(h), \tilde{\sigma}_{-i}(h), h, \theta\right),\ \forall i \in N.$$

Proof. The proof is again given by contradiction. If this is not true, then there must exist a level $\bar{h}$ such that:

$$U\left(\sigma_i'(\bar{h}), \tilde{\sigma}_{-i}(\bar{h}), \bar{h}, \theta\right) > U\left(\sigma_i'(h'), \tilde{\sigma}_{-i}(h'), h', \theta\right). \tag{3.4}$$

We know that

$$U\left(\sigma_i'(h'), \tilde{\sigma}_{-i}(h'), h', \theta\right) = U\left(\sigma_i'(h'), \sigma_{-i}^*(\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, h'\right), h'), h', \theta\right),$$

and

$$U\left(\sigma_i'(\bar{h}), \tilde{\sigma}_{-i}(\bar{h}), \bar{h}, \theta\right) = U\left(\sigma_i'(\bar{h}), \sigma_{-i}^*(\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, \bar{h}\right), \bar{h}), \bar{h}, \theta\right).$$

Given the definition of $\tilde{m}_i(.)$ and applying the θ-separability condition we get that:

$$\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, h'\right) \in \arg\max_{m_{-i}\in M_{-i}} U\left(\sigma_i'\left(h'\right), \sigma_{-i}^*\left(m_{-i}, h'\right), h', \theta\right).$$

As a result we have that $\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, h'\right)$ and m'_{-i} have to be payoff equivalent for the agent (both are optimal responses). Therefore:

$$U\left(\sigma_i'(h'), \sigma_{-i}^*(\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, h'\right), h'), h', \theta\right) = U\left(\sigma_i'(h'), \sigma_{-i}^*(m'_{-i}, h'), h', \theta\right), \tag{3.5}$$

Hence, we get:

$$U\left(\sigma_i'\left(\bar{h}\right), \sigma_{-i}^*\left(\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, \bar{h}\right)\right), \bar{h}, \theta\right) > U\left(\sigma_i'(h'), \sigma_{-i}^*\left(m'_{-i}, h'\right), h', \theta\right). \tag{3.6}$$

But this contradicts the fact that:

$$\left(h', m'_{-i}\right) \in \arg\max_{h,m} U\left(\sigma_i'(h), \sigma_{-i}^*(m_{-i}, h), h, \theta\right)$$

■

The analysis showed that the agent's best reply in terms of the action choice to principal i's deviating behaviour $\sigma_i'(.)$ will be the same in the game where take-it or leave-it strategies are played, and in the indirect mechanism game.

If σ_i' was indeed a profitable deviation for principal i, we have;

$$\int_{\theta\in\Theta} V_i\left(\sigma_i'\left(h'\right), \tilde{\sigma}_{-i}\left(h'\right), h', \theta\right) dF(\theta) > \int_{\theta\in\Theta} V_i\left(\left(\tilde{\sigma}_i\left(h^*\right)\right)_{i\in N}, h^*, \theta\right) dF(\theta)$$

$\Longrightarrow$

$$\begin{aligned} &\textstyle\int_{\theta\in\Theta} V_i\left(\sigma_i'\left(h'\right), \sigma_{-i}\left(\tilde{m}_i\left(\sigma_i^*, \sigma_{-i}^*, h'\right)\right), h', \theta\right) dF(\theta) \\ &\qquad > \textstyle\int_{\theta\in\Theta} V_i\left(\left(\sigma_i^*\left(m_i^*, h^*\right)\right)_{i\in N}, h^*, \theta\right) dF(\theta) \end{aligned} \tag{3.7}$$

Now, applying the θ-separability condition we get that:

$$\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, h'\right) \in \arg\max_{m_{-i}\in M_{-i}} U\left(\sigma_i'\left(h'\right), \sigma_{-i}\left(m_{-i}, h'\right), h', \theta\right)$$

Observe now, that $\left(\sigma_i^*\left(m_i^*\right)_{i\in N}, h^*, m^*\right)$ is a PBE, therefore we have:

$$\begin{aligned} &\textstyle\int_{\theta\in\Theta} V_i\left(\sigma_i^*\left(m_i^*, h^*\right), \sigma_{-i}^*\left(m_{-i}^*, h^*\right), h^*, \theta\right) dF(\theta) \geq \\ &\qquad \textstyle\int_{\theta\in\Theta} V_i\left(\sigma_i'\left(h'\right), \sigma_{-i}^*\left(m'_{-i}, h'\right), h', \theta\right) dF(\theta) \end{aligned}$$

where

$$\left(h', m'_{-i}\right) \in \arg\max_{m_{-i}\in M_{-i}, h\in H} U\left(\sigma_i'(h), \sigma_{-i}^*(m_{-i}, h), h, \theta\right) \tag{3.8}$$

Since $\tilde{m}_{-i}\left(\sigma_i^*, \sigma_{-i}^*, h'\right)$ belongs to the arg max set in equation (3.6), we have a contradiction. ■

The intuition behind this result is the following. Under separability, every deviation taken by principal i in the indirect mechanism game can affect the message sent to principal j only through a change in the action choice of the agent. Since the optimal response in terms of effort is the same in the two relevant games *and it is contractible (observable)*, then there is no real informational loss in restricting to direct mechanisms. Note that this argument cannot be reproduced in a moral hazard setting. The reason is that the agent's effort choice is non-contractible and, hence, cannot be observed by the principals. As a result, the principal cannot extract any relevant information from the agent's action. Thus, if we restrict our attention to the externalities induced by multiple contracting,[19] then, whenever agent's preferences are separable, direct mechanisms are enough to characterize all equilibria of the game.

As stated in the introduction, many common agency models considered in the literature satisfy this separability condition. Examples are given by Bernheim and Whinston (1986b); Grossman and Helpman (1994); Dixit, Grossman, and Helpman (1997); Laussel and Le Breton (1998).[20] Our θ-separability condition is also satisfied in some games of incomplete information (e.g. Le Breton and Salanie (2003)).

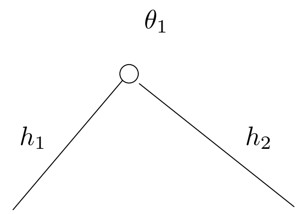
A key assumption of all these models is the absence of direct externalities among principals. That is, if we think of multiple sellers competing on the prices they are offering to a single agent, then the price charged by seller i can be affected by the quantity the agent is buying from him, but not by the quantity bought from seller j. On the contrary, Bernheim and Whinston (1998a) introduced a framework with direct externalities among principals in order to study exclusive dealing issues. Similarly, de Villemeur and Versaevel (2003) considered a model of Research and Devolpement where every principal (a self interested firm) is directly affected by the decisions taken by the other principals. As our condition only involves agent's preferences, we can show that the Revelation Principle fullly applies also in these papers.

[19] That is, we disregard asymmetric information issues.

[20] Many models have been built on these seminal works and most of them satify our separability condition. The reader can refer to Aidt (1998); Rama and Tabellini (1998) and Laussel and Le Breton (2001), among many others. The same separability condition is also introduced in recent theoretical works on multi-principal multi-agent games (see Prat and Rustichini (2003)).

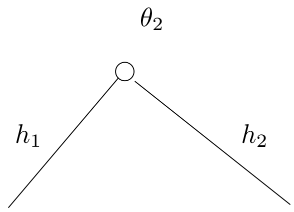
We have only considered environments with θ-separable preferences because allowing for the action h to be contractible introduces a new source of externality among principals. When the agent's preferences are type-dependent, a principal by changing his mechanism can induce a change, not only in the agent's effort choice, but also in the report of her private information to other principals. This interaction between the contractible action choice and the agent's private type, introduces an essential problem of non-separability.

We now present an example showing that, in situations where agent's preferences are type-dependent, even the Strong Separability condition does not retrieve the Revelation Principle.



A	B
$(3,3,1)$	$(2,0,2)$
$(0,2,\frac{3}{2})$	$(1,-1,\frac{5}{2})$
$(0,0,\frac{1}{2})$	$(0,0,1)$

	A	B
A	$(0,0,2)$	$(0,-1,3)$
B	$(0,0,\frac{21}{10})$	$(\frac{3}{2},\frac{3}{2},4)$
C	$(0,0,\frac{1}{2})$	$(0,0,1)$



A	B		A	B
$(1,1,5)$	$(0,0,\frac{3}{4})$	A	$(0,0,\frac{1}{2})$	$(-1,-1,0)$
$(0,0,6)$	$(1,-1,1)$	B	$(-1,-1,0)$	$(-2,2,\frac{5}{2})$
$(0,0,7)$	$(0,0,\frac{3}{2})$	C	$(-1,-1,4)$	$(-1,-1,\frac{7}{2})$

In the example above the agent's type space is $\Theta = \{\theta_1, \theta_2\}$ and she is allowed to take the action $h \in H = \{h_1, h_2\}$, which is assumed to be contractible. There are two principals: principal 1 has the three available actions $\{A, B, C\}$, while principal 2 can choose between A and B only. The outcome $\left\{\left(\frac{3}{2}, \frac{3}{2}, 4\right) \text{ if } \theta = \theta_1,\ (1,1,5) \text{ if } \theta = \theta_2\right\}$ cannot be sustained as an equilibrium in the direct mechanism game. If it were an equilibrium, then principals' strategies should involve the following behaviors: whenever the message θ_1 is sent and h_2 is performed, then both principals play B; whenever the message θ_2 is sent and h_1 is performed, they both play A.

Principal 2 has a profitable deviation in playing B irrespectively of both the received message and the performed action. If he takes such a strategy, then the agent has an incentive to misreport her type and choose h_2 when she is θ_2. In such a case principal 1 is playing H and principal 2 earns a payoff of 2 instead of 1.

We stress here that the same outcome can indeed be supported by allowing principals to offer menus. Let us consider the following strategy profiles: both principals play A if h_1 is chosen; when the agent plays h_2, then principal 2 plays B and principal 1 offers the menu $\{B, C\}$. If principal 2 deviates to a strategy that prescribes to play B irrespectively of h, then the θ_2 agent will play h_2 and ask for C from principal 1, that makes the deviation unprofitable.

3.3.6 Discussion

We provided a sufficient condition that determines a class of common agency games in which the Revelation Principle retains all its power. Every allocation supported as an equilibrium inside an arbitrarily huge class of indirect mechanisms can therefore be implemented through simple direct mechanisms. The condition does not put any restriction on principals' preferences.

We require the utility of the agent to satisfy *separability properties* with respect to the actions taken by all the principals and to every *non-contractible* action taken by herself.

In a pure incomplete information framework, the requirement is that of *separability.* This property holding, messages sent by the agent to every principal will only depend on her type. Thus, a direct mechanism is a sufficiently powerful tool, and nothing is gained from allowing the principals to design more complex indirect mechanisms. In a more general setting where moral hazard is considered, we have shown that the same result holds, provided the condition of separability is appropriately redefined. Agent's preferences are then required to satisfy a *strong separability condition*. The result could therefore potentially matter in auctions where the agent has an unknown reservation value for the object he earns or produces, and the principals make offers for the object (or quantity) that the agent accepts or rejects. Extending our result to this framework ensures us that the Revelation Principle holds if there is no limitation in the amount that can be produced. If only one unit is produced, then changing the offer of one principal might break the agent's ranking over the others and the separability requirement is violated.

Our simple separability condition is also satisfied in several traditional common agency models, where incomplete information is ruled out and the agent is assumed to take contractible actions. We emphasized how separability allows us to restrict our attention to take-it or leave-it offers by principals without any loss of generality. The allowance for direct externalities among principals enables us to significantly extend the domain of applicability of our condition.

In concluding this section it is important to remark that, as long as common agency games are considered, there still exist some support to the use of direct mechanisms independently of the existence of separable preferences. In other words, it is possible to show that every equilibrium outcome in a direct mechanism is "robust" to the possibility that principals might use more complex communication structures. That is:

Theorem 6 *For every Perfect Bayesian Equilibrium $\tilde{\sigma}^*$ of every direct mechanism game Γ_{Θ^N} there exist a strategy profile σ^* in the indirect mechanism game Γ_M such that the two equilibria are outcome equivalent.*

Proof. See the proof of Theorem 1 and 2 in Peters (2003). ■

The equilibrium $\tilde{\sigma}^*$ is usually referred to as *regular*.

When multiple agents come into the picture, then no general result about equilibrium characterization is currently available. In particular, the recent work of Peters (2004) provides some examples suggesting that the equilibria of multi-principal multi-agents games where principals are restricted to offer direct mechanisms do not even satisfy the requirement of "regularity", i.e. Theorem 6 fails to hold.

Despite this apparent lack of foundations, the literature on competing mechanisms with multiple agents has always adopted the simplifying assumption of restricting principals to the use of simple mechanisms (see McAfee (1993) and Prat and Rustichini (2003)).

The next section will properly deal with multi-principal multi-agent games and will provide some results about the regularity of the pure strategy equilibria of direct mechanism games.

3.3.7 Appendix

The main aim of this Appendix is to show that the separability requirements on the preferences of the agent are not sufficient per se to guarantee the Revelation Principle in common agency games.

We propose here a slightly modified version of our Example 1. The agent's type space is degenerate and there are two principals: each of them can offer the three allocations A, B and C. That is, we have: $Y_1 = Y_2 = \{A, B, C\}$. The payoffs to principal 1, principal 2 and to the agent, $\{V^1, V^2, U\}$, are given by the following matrix:

	$y_2 = A$	$y_2 = B$	$y_2 = C$
$y_1 = A$	$(1, 1, 0)$	$(2, 1, 0)$	$(-1, 5, 0)$
$y_1 = B$	$(1, 2, 0)$	$(1, 1, 0)$	$(0, 0, 0)$
$y_1 = C$	$(5, -1, 0)$	$(0, 0, 0)$	$(0, 0, 0)$

In this game, the preferences of the agent satisfy the Weak Separability condition in a trivial sense. The Revelation Principle is, however, violated.

First, one should notice that the outcome $(1, 1, 1)$ cannot be supported as an equilibrium in the direct mechanism game: both principals can profitably deviate by playing $\{A\}$.

Then, we consider the situation where the principals offer the menus $\{B, C\}$, $\{B, C\}$ and the agent chooses the contract B from both menus. The reader can check that given these menus the agent does not have a strictly profitable deviation. Moreover, consider any principal $i \in \{1, 2\}$. Given that the other principal is offering the menu $\{B, C\}$, a potential profitable deviation for principal i is to the degenerate menu $\{A\}$. However, if principal i offers $d_i = A$, and principal j offers the menu $\{B, C\}$, a best reply for the agent would be to choose C from the non-deviating principal, and this makes principal i strictly worse off. Since the game is symmetric in the principals' payoffs, the menu offers $\{B, C\}$ by both principals constitute a Nash equilibrium that supports the outcome $\{1, 1, 1\}$.

We remark that the game presented in this example does not belong to the class $\mathcal{G}$. It is enough to observe that if Principal 2 were playing B, then the agent's payoff would always be equal to one, irrespectively of Principal 1's choice.

When this is the case, indirect mechanisms (and, in particular, menus) can be used to sustain outcomes that are not achievable by using direct mechanisms. Menus allow principals to strategically profit from the agent's indifference.

3.4 Pure strategy equilibria with multiple agents

Characterizing the whole set of equilibria of multi-principal multi-agent games is a very difficult task. The standard approach developed in the common agency literature was mainly trying to restore the role of the agent as a correlating device by considering menu theorems (Peters, 2001; Martimort and Stole, 2002). An alternative strategy is to define conditions on the fundamentals of the game that can be sufficient for the Revelation Principle to hold. None of these strategies can be applied when multiple principals interact with multiple agents: Han (2004) and Yamashita (2004) have recently emphasized several difficulties in extending menu theorems to games with multiple agents. Furthermore, Peters (2004) has proposed an example of a complete information game with 2 principals and 2 agents where, though agents' preferences are separable in principals' offers, the Revelation Principle fails to apply. A possible intuition for such failures can be expressed in the following way: by using some complex communication mechanisms, principals can induce agents to coordinate on outcomes that are not reachable through simple mechanisms. In other words, the punishment for a deviating principal is induced by a change in the joint efforts of all agents. Every agent changes her effort choice because she expects other agents to send a signal that changes the non-deviator's action in a way that makes the new effort level optimal. This change in effort is preventing possible deviations.

The present section does not introduce any device to characterize equilibria in multi-principal, multi-agent settings;[21] we show that in this class of games there exist equilibria of direct mechanism games that turn out to be "regular". There could therefore be some rationale for the study of applications where principals are restricted to use simple contract offers.

After discussing an example recently provided by Peters (2004), we prove a theorem for multi-principal multi-agent games with complete information, stating that there can be equilibrium outcomes of direct mechanism games that survive when principals can choose more complicated indirect mechanisms.

Peters argues that when considering complete information multi-principal multi-agent games, there is not much support for the arbitrary restriction to the use of simple mechanisms for principals. If principals

[21] A particular two-sided communication device is suggested in Yamashita (2004), while Epstein and Peters (1999) propose a *universal* notion of type for multi-principal multi-agent games. With specific reference to this extended agents' type space the Revelation Principle is still valid.

are allowed to choose mechanisms involving some communication with agents, then:

> *In a multiple agency environment [...] pure strategy equilibria are not robust against the possibility that principals might deviate to more complex indirect mechanisms.*[22]

In other words, the direct mechanisms equilibria of multi-principal, multi-agent games will not in general be regular in the sense clarified in Theorem 6. We devote the next paragraphs to the discussion of the Peters' example.

We begin by describing example 1 in Peters (2004). There are two principals and two agents in an environment with pure moral hazard: agents take private actions that principals cannot observe. Hence, the contracts proposed by the latter cannot depend on the effort of the agents.

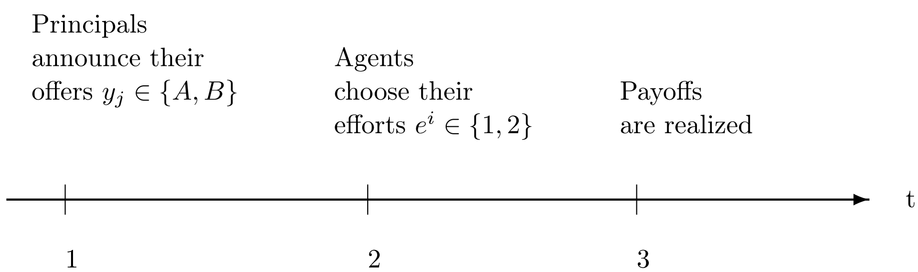


Figure 3.2: Timing 1, direct mechanism interaction

As shown in Figure 3.2, the direct mechanism game begins with Principals P_1 and P_2 simultaneously choosing actions in the space $\{A, B\}$. Then, Agents A_1 and A_2 observe the principals' choices, and simultaneously choose a level of effort in the set $\{1, 2\}$. The normal form of the game is exhibited in Table 3.1. The principals' choices affect which cell of the larger matrix is chosen. The agents then play the 2×2 subgame in that cell. Payoffs should be interpreted in the following way: the first payoff is the payoff to principal 1 who chooses the row in the outer table. The second payoff is the payoff to principal 2 who chooses the column in

[22] Peters (2004), p. 184.

the outer table. The third payoff is to agent 1 who chooses the row in the inner table, and similarly for the last payoff.

	A	B
A	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & * & * \\ e=2 & * & * \end{array}$	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & \left(\frac{7}{8},\frac{7}{8},-\frac{3}{2},\frac{5}{4}\right) & \left(\frac{7}{8},\frac{7}{8},\frac{5}{4},-\frac{3}{2}\right) \\ e=2 & \left(\frac{7}{8},\frac{7}{8},\frac{5}{4},-\frac{3}{2}\right) & \left(\frac{7}{8},\frac{7}{8},-\frac{3}{2},\frac{5}{4}\right) \end{array}$
B	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & \left(0,0,-\frac{3}{2},\frac{5}{4}\right) & \left(0,0,\frac{5}{4},-\frac{3}{2}\right) \\ e=2 & \left(0,0,\frac{5}{4},-\frac{3}{2}\right) & \left(0,0,-\frac{3}{2},\frac{5}{4}\right) \end{array}$	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & (3,0,-1,-1) & (1,1,1,1) \\ e=2 & (1,1,1,1) & (0,3,-1,-1) \end{array}$

Table 3.1: Normal form of the example in Peters (2004)

Observe that principals' decisions are restricted to be deterministic offers. That is, principals are not allowed to use lotteries over the actions $\{A, B\}$. Given this restriction, in the direct mechanism game in Figure 3.2, there exist three subgame perfect equilibria in which both principals play the pure strategy B. These equilibria are:

- P_1 and P_2 both play B; A_1 plays 1 and A_2 plays 2; each player gets a payoff of 1.
- P_1 and P_2 both play B; A_1 plays 2 and A_2 plays 1; each player gets a payoff of 1.
- P_1 and P_2 both play B; A_1 and A_2 randomize and play 1 with probability 1/2; P_1 and P_2 both get a payoff of 5/4, A_1 and A_2 get a payoff of 0.

If a principal can communicate with the agents before choosing his mechanism, the first two pure strategy equilibria do not survive. More specifically, Peters considers the case where P_1 asks each agent $i = 1, 2$ to send a message $m_1^i \in \{A, B\}$ and designs the following mechanism:

$$\sigma_1(m_1^1, m_1^2) = \begin{cases} B & \text{if } m_1^1 = m_1^2, \\ A & \text{otherwise.} \end{cases}$$

Whenever the agents do not coordinate on their messages, P_1 chooses A. There is full commitment in this game on the part of the principals. Thus, in the indirect mechanism game, when agents send their messages, they anticipate the resultant allocations. However, neither agent observes allocations before choosing effort. Since the allocation of principal 1 depends on messages sent by both agents, each agent has beliefs over the resulting allocations. Hence, we can think about agents choosing messages and efforts simultaneously.

If principal 2 continues to play B, the deviation σ_1 is profitable for principal 1. The continuation game associated with the strategies $\{\sigma_1, B\}$ has multiple equilibria. In the first class of equilibria (referred to as E_1), agents both report B (alternatively, both report A) and randomize equally over efforts, inducing a payoff $5/4$ for P_1.[23] In the second equilibrium (E_2) agents randomize equally over *both* messages and actions. That is, they select each message–action pair with probability 1/4. These behaviors induce a payoff of $17/16$ for P_1 and of $-1/16$ for each agent.

Finally, the strategy profile E_3 has each agent randomizing equally over the message–action pairs $(m_1, 1)$ and $(m_2, 2)$. The corresponding payoffs are $19/16$ for P_1, and $1/8$ and $-5/4$ for the agents. However, E_3 turns out not to be an equilibrium, since the best-replies of agent 2 to the strategy of agent 1 are $(m_2, 1)$ and $(m_1, 2)$, instead of $(m_1, 1)$ and $(m_2, 2)$.

Nevertheless, the strategy σ_1 is a profitable deviation in Peters' example given that, in every continuation equilibrium, the deviating principal earns a payoff strictly greater than 1. In particular, the example emphasizes that the payoffs $(1, 1, 1, 1)$ cannot be sustained at equilibrium when P_1 is allowed to communicate with agents.

3.4.1 The role of non-deterministic direct mechanisms

We now consider the original set-up of Peters' example 1 and assume that principals can use non-deterministic direct mechanisms. That is, a pure strategy for P_1 may also involve the choice of a lottery over A and B. We show that, in such a scenario, all the payoffs associated with principal 1's deviation to σ_1 can be supported using direct mechanisms.

To develop this argument, we start with a simple remark: in the equilibria discussed above, principal 2 always chooses the strategy B. Thus, we can restrict our analysis to the second column of table 1. We can therefore focus on the optimal action of principal 1, and interpret this example as a single principal, multi-agent game, without loss of generality.

[23]For every player, this equilibrium is payoff-equivalent to the mixed strategy equilibrium of the take-it or leave-it offer game.

Suppose, in particular, that principal 1 randomizes equally between A and B (with principal 2 playing B). Then, the agents are effectively faced with the following subgame, where the first number in each cell is the principal's expected payoff, and the remaining two numbers are the expected payoffs to agents 1 and 2, respectively.[24]

	$e = 1$	$e = 2$
$e = 1$	$(31/16, -5/4, 1/8)$	$(15/16, 9/8, -1/4)$
$e = 2$	$(15/16, 9/8, -1/4)$	$(7/16, -5/4, 1/8)$

The agents' subgame exhibits only a mixed strategy equilibrium, with both agents equally randomizing over actions. This randomization yields the principal a payoff of $\frac{17}{16}$, and the agents a payoff $-\frac{1}{16}$.[25] Assigning a higher probability to the choice of B in the direct mechanism increases the principal's payoff. In general, if the principal plays A with probability p and B with probability $(1-p)$, where $p \in (0,1)$, the unique equilibrium in the agents' subgame still has the agents randomizing equally over actions, and the principal gets an expected payoff $\frac{5}{4} - \frac{8}{3}p$.

Hence, if the principal is allowed to randomize over contracts, the outcome $(1, 1, 1)$ cannot be supported as an equilibrium. The same argument can be reproduced if we consider another principal, say principal 2, playing a fixed strategy B as in Peters' original example, and examine the best response of principal 1. This suggests that the restriction to deterministic contracts in the direct mechanism game is critical. [26]

To sum-up, by explicitly considering stochastic direct mechanism in the Peters' example, it is possible to show that:

- every outcome associated to the deviating behavior σ_1 can be as well sustained by a direct mechanism,

- the outcome (1,1,1,1) turns out not to be an equilibrium in the direct mechanism game either.

[24]Since principal 2's strategy is fixed at B, his payoffs are ignored. As mentioned above, this is now equivalent to a single principal, two-agent game.

[25]If participation constraints (e.g., a reservation of utility of zero for the agents) are a factor, note that the agents payoffs in each cell of the original game can be increased by $\frac{1}{16}$ without affecting the equilibria.

[26]For pure incomplete information games with one principal contracting with several agents, Strausz (2003) has shown that it is no longer true that any payoff implementable by deterministic indirect mechanisms can be matched with a *deterministic* direct mechanism. However, direct mechanisms retains their power if we allow principals to use non-deterministic schemes.

One should notice that the role of stochastic mechanisms is to allow principals to exactly reproduce agents' random behavior in the effort subgame.

Out of the specific Peters' example, though, allowing for stochastic mechanisms does not eliminate the existence of profitable deviations for principals. If we focus on a complete information scenario, where agents take a non-contractible effort, it is then possible to construct several situations where the equilibrium outcomes of games where no communication is allowed are not robust to unilateral deviations of a single principal to a more complex communication mechanism.
We remark here that this result is independent of the existence of many principals. It is well known that, even if we were considering only one principal who is interacting with many agents who take unobservable actions, there can be communication mechanisms where players coordinate on outcomes that do not belong to the convex hull of the set of Nash equilibria that are achievable in our simple "direct mechanism" game. The main rationale behind this result is that the principal is not allowed to use any sort of correlating device.
In the traditional perspective of Aumann (1974) and Myerson (1982) such a task was performed by the single principal through recommendations privately sent to each agent over her actions. In order to show the relevance of principal's recommendations, the following paragraphs provide a simple example of a complete information game with one principal and two agents whose direct mechanisms equilibria survive *only* if the principal can recommend a particular behavior to agents. Importantly, the example assumes the same temporal structure as Peters' original one, that is, agents choose an action at the same time they send messages to the principal.

3.4.2 On the role of recommendations: an example

We consider a complete information game with one principal and two agents. The principal chooses an allocation in the set ΔY, where $Y = \{y_1, y_2\}$. Agents 1 and 2 choose efforts from the sets $E_1 = \{a_1, a_2\}$ and $E_2 = \{b_1, b_2\}$, respectively. Payoffs are given by the utility functions $V(y, a, b)$, $U_1(y, a, b)$ and $U_2(y, a, b)$. The relevant subgames are depicted in the following cells:

$y = y_1$

	b_1	b_2
a_1	$(0, 0, 10)$	$(50, 6, 6)$
a_2	$(0, -10, -10)$	$(-10, 0, 10)$

$y = y_2$

	b_1	b_2
a_1	$(0, 0, -10)$	$(-200, 0, 0)$
a_2	$(0, 10, 0)$	$(4, 1, 6)$

The three elements in each cell denote the utilities of the principal, agent 1, and agent 2, respectively. Each subgame has a unique equilibrium, (a_1, b_1) when $y = y_1$ and (a_2, b_2) when $y = y_2$. With deterministic allocations and take-it or leave-it offers, the principal's expected utility is 4, since the equilibrium is (y_2, a_2, b_2).

Suppose the principal chooses a lottery over allocations, so that $y = py_1 + (1 - p)\, y_2$. Then, the agents' efforts subgame is

	b_1	b_2
a_1	$(0,\ 0,\ 20\,p - 10)$	$(250\,p - 200,\ 6\,p,\ 6\,p)$
a_2	$(0,\ 10 - 20\,p,\ -10\,p)$	$(4 - 14\,p,\ 1 - p,\ 4\,p + 6)$

For $p < \frac{5}{7}$, b_2 strictly dominates b_1. For $p < \frac{1}{7}$, agent 1's best response is a_2, with resultant utility for the principal $4 - 14p$, which is maximized at $p = 0$ and gives a value of 4. For $p \in (\frac{1}{7}, \frac{5}{7})$, agent 1's best response is a_1, which results in a utility of $250p - 200$ for the principal. This has a supremum at $p = \frac{5}{7}$, and delivers a value of $-\frac{150}{7}$. For $p > \frac{5}{7}$, the unique equilibrium of the agents' subgame is (a_1, b_1), with principal utility being 0. Hence, even with lotteries, (y_2, a_2, b_2) remains the equilibrium of the game, and the principal's utility is 4.

Now, consider the following situation. The allocations are not publicly observed. The principal is allowed to communicate with agent 1, with the message space being $M_1 = \{m_1, m_2\}$. Suppose principal 1 chooses allocations according to the strategy:

$$\begin{aligned} \sigma(m_1) &= y_1, \\ \sigma(m_2) &= y_2. \end{aligned}$$

In this situation, given σ and his own message, agent 1 knows the allocation, whereas agent 2 does not. Hence, the agents' effort game can be represented by the following matrix

	b_1	b_2
(m_1, a_1)	$(0, 0, 10)$	$(50, 6, 6)$
(m_2, a_1)	$(0, 0, -10)$	$(-200, 0, 0)$
(m_1, a_2)	$(0, -10, -10)$	$(-10, 0, 10)$
(m_2, a_2)	$(0, 10, 0)$	$(4, 1, 6)$

The game now exhibits the following unique equilibrium:

- Agent 1 mixes between (a_1, m_1) and (a_2, m_2), with probabilities $3/5$ and $2/5$.
- Agent 2 mixes between b_1 and b_2 with probabilities, $1/3$ and $2/3$.

The principal's expected payoff is $21\,1/15$, that was not achievable in the initial game. This is then the unique equilibrium of the indirect mechanism, which represents a profitable deviation by the principal from take-it-or-leave-it offers.

Following Myerson (1982) and Harris and Townsend (1981), we can resuscitate the equilibrium of the indirect mechanism game in a direct mechanism game. In our specific example with complete information, in a direct mechanism the principal offers an allocation and makes a recommendation to each agent about the effort he should choose. Agents observe the recommendations, but do not directly observe allocations. A direct mechanism in this example may therefore be characterized as a probability function $\pi : \{y_1, y_2\} \times \{a_1, a_2\} \times \{b_1, b_2\} \to [0, 1]$, where $\pi(y, a, b)$ is the probability the principal chooses allocation y and recommends effort a to agent 1 and b to agent 2. Note that a stochastic allocation can be represented as a randomization over y, keeping a and b fixed.

In the equilibrium of the indirect mechanism above, the resultant distribution over allocations and efforts is $\pi(y_1, a_1, b_1) = 1/5$, $\pi(y_1, a_1, b_2) = 2/5$, $\pi(y_2, a_2, b_1) = 2/15$ and $\pi(y_2, a_2, b_2) = 4/15$. Suppose the principal plays this strategy in the direct mechanism. That is, the principals chooses allocations and efforts according to $\pi(\cdot)$, and announces the resulting recommendations to the agents.

It is straightforward to check that neither agent has an incentive to deviate, so the mechanism is Incentive Compatible. For example, when agent 2 is told "b_2", his posterior beliefs place probability $3/5$ on (y_1, a_1) and $2/5$ on (y_2, a_2). Given these beliefs, b_2 is a (weak) best response. The principal obtains the utility $21\frac{1}{15}$, as before.

This example stresses that, when agents have private information about allocations, it is important to model principals' recommendations in characterizing the set of equilibria. That is, the introduction of some correlation device is a necessary step to provide a general characterization of equilibria of multi-principal multi-agent games even when the analysis is restricted to complete information.[27]

3.4.3 A general result

We now try to show that there could be some foundation for the restriction to the use of simple incentive schemes in multi-principal multi-agent games. We clarified here that this literature is mainly concerned with situations where only one-sided communication between principals and agents is considered. Our contribution will be to show that there exist a class of direct mechanism games which equilibrium outcomes survive to the introduction of general communication mechanisms such as those described in Section 3.2.
We remark here that our analysis will be restricted to a complete information framework and to a specific time structure. More precisely, agents should take their effort decisions after observing the allocations chosen and offered by principals.
In complete information games without communication, principals do not solicit messages from agents. That is, $M_i^j = \emptyset$ for every $i = 1, \ldots, n$ and for every $j = 1, \ldots, k$. Thus, given the definition of a direct mechanism game proposed in Section 3.2, we restrict here to a particular game G_D where no communication among players is allowed. We denote this game without communication G_{WC}.

A pure strategy for principal i in G_{WC} is given by $\tilde{\sigma}_i \in \Delta(Y_i)$. Agents move after observing the realized allocations $(y_i)_{i=1}^N \in Y$. Agent j's behavior is described by the map $\tilde{\sigma}^j : Y \to E^j$. We will also use the notation $\tilde{\delta}^j$ to denote a mixed strategy played by agent j.

Our main aim is to remark that, as long as all allocations are publicly observed, every equilibrium outcome of a complete information game without communication remains an equilibrium outcome when principals can use communication schemes.[28] We let G_{WC} denote a game without communication, and $G_{\mathcal{M}}$ the more general indirect mechanism game.[29] The relevant timing for the game $G_{\mathcal{M}}$ is depicted in Figure 3.3.

[27] Aumann (1974) introduced the concept of correlated equilibrium for analyzing noncooperative games with pre-play communication; the relevance of two-way communication to provide a general version of the Revelation Principle for Bayesian games was further elaborated by Forges (1986).

[28] We remark here that the space of feasible indirect mechanisms $\mathcal{Z}$ is also restricted. That is, we do not allow for any sort of communications from principals to agents.

[29] Note that $M_i^j = \emptyset$ is a feasible choice for principal j in game $G_{\mathcal{M}}$.

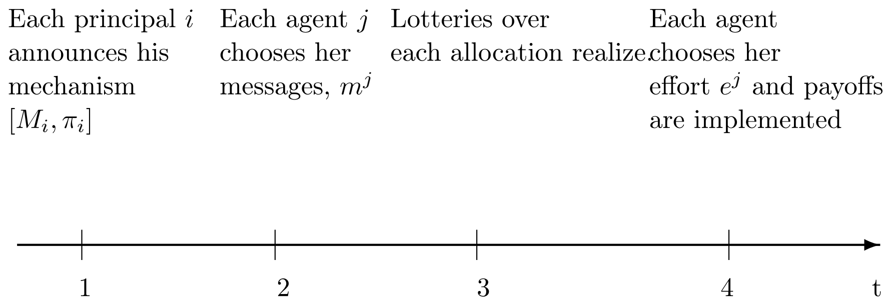


Figure 3.3: Timing of the game $G_{\mathcal{M}}$

Consider any game G_{WC}, and any subgame perfect equilibrium $\left(\{\tilde{\sigma}_i^*\}_{i=1}^n, \{\tilde{\delta}^{j*}\}_{j=1}^k\right)$ in which principals are playing pure strategies and agents are allowed to randomize. We show that no principal has an incentive to deviate unilaterally to a more complex indirect mechanism. That is, in the game $G_{\mathcal{M}}$, in which principals can offer message spaces to agents, it is an equilibrium for every principal to behave as he was doing in the game G_{WC}. Therefore, there is an outcome of $G_{\mathcal{M}}$ that generates the same outcome (i.e., probability distribution over allocations and efforts, (y, e)) as the subgame perfect equilibrium of G_{WC}. We formally state this intuition in the following theorem:

Theorem 7 *Consider the game G_{WC}. Suppose it has a subgame perfect equilibrium in which each principal i plays a pure strategy $\tilde{\sigma}_i^* \in \Delta Y_i$, and each agent j plays the strategy $\tilde{\delta}^{j*} \in \Delta(E^j)$. Then, there exists a subgame perfect equilibrium of the indirect game with public allocations $G_{\mathcal{M}}$ that generates the same outcome.*

Proof. Consider the game $G_{\mathcal{M}}$. Suppose that, in this game, every principal i offers the mechanism $(M_i, \sigma_i) = (\emptyset, \tilde{\sigma}_i^*)$; that is, every message space is an empty set and the allocations are the same as in the subgame perfect equilibrium of G_{WC}.

The subgames induced by these mechanisms coincide with those induced by the mechanisms $(\tilde{\sigma}_i^*)_{i=1}^N$ in the game G_{WC}. Hence, the strategy profile $\tilde{\delta}^{j*}$ for every agent j must remain an equilibrium strategy in the continuation game played among agents.

Suppose now that principal i deviates unilaterally to a mechanism (M_i', σ_i'), with $M_j' \neq \emptyset$. The continuation game induced by such a deviation is described as follows. First, every agent observes the mechanism chosen by the deviating principal and sends him a message. Given the received messages m, the deviating principal chooses probability distributions over allocations according to σ_i', while all other principals choose their allocations σ_{-i}^*. After observing the allocations induced by $\left(\sigma_i', \sigma_{-i}^*\right)$, agents choose efforts.

We analyze the continuation game through backward induction; that is, we first discuss the equilibria of the subgame played over efforts. Let $\tilde{\delta}$ denote the induced equilibrium efforts at this stage.

Note that the allocation strategy of principal i is committed to at the first stage of the game. Hence, using backward induction, the next stage to consider is the one at which agents send messages. Every agent chooses a message m_i^j to be sent to principal i fully anticipating the resulting allocations and equilibrium outcomes in the efforts game. Finiteness of message spaces M_i^j for $j = 1, \ldots, J$, guarantees existence of an equilibrium array of messages. Let μ' denote the (possibly mixed) strategy equilibrium in messages, with $\mu'(m_i)$ being the probability that the deviating principal i receives the message array $m_i = (m_i^1, \ldots, m_i^J)$.

Given the message array m_i, principal i offers the allocation $\sigma_j'(m_j)$. Let $supp(\mu'(\cdot))$ denote the support of μ'. Every realization $m_i \in supp(\mu'(\cdot))$ induces a payoff for principal i given by $\tilde{V}_i(\sigma_i'(m_i), \tilde{\sigma}_{-i}^*), \tilde{\delta}(\sigma_i'(m_i), \sigma_{-i}^*)$. Denote this payoff as $\tilde{V}_i(m_i)$. Since the message space is finite, the set of induced payoffs $\{\tilde{V}_i(m_i)\}_{m_i \in supp(\mu'(\cdot))}$ has a maximum, with some corresponding allocation $\hat{\sigma}_i$.

For the deviation to be profitable, the maximum must be strictly greater than $V_i(\sigma^*, \delta^*)$ (else it must remain an equilibrium for principal i to offer σ_i^* for every $m_i \in M_i$). Further, it must not be an equilibrium for agents to play the strategy $\tilde{\delta}^*(\hat{\sigma}_i, \sigma_{-i}^*)$ in the continuation game, else it trivially remains an equilibrium for principal i to offer $(\emptyset, \sigma_i^*)$ in game $G_{\mathcal{M}}$.[30]

We now remark that for every $m_i \in supp\,(\mu'\,(\cdot))$, the corresponding allocation offered by the deviating principal, $\tilde{\sigma}_i(m_i)$, was already a feasible choice in the original game. If principal i had chosen $\hat{\sigma}_i$ in the game G_{WC}, he would have once again induced an effort profile given by $\tilde{\delta}^*(\hat{\sigma}_i, \sigma_{-i}^*)$ in the corresponding subgame.

[30]Note the distinction here between single and multiple principal models. With a single principal, it is typical to assume the principal can induce his most preferred equilibrium in the agents' continuation game. With multiple principals, this may not hold.

As a consequence, if the strategy (M_i', σ_i') is a profitable deviation in the game $G_{\mathcal{M}}$, then σ_i^* could not have been a best response for principal i in the game G_{WC}, breaking the conjectured equilibrium. ■

The theorem tries to contribute to the literature of multi-principal multi-agent games with complete information in which allocations are publicly observed.[31] It applies to settings in which it may be reasonable to think that private offers from principals to agents are not feasible.

[31] See e.g. McAfee (1993), Prat and Rustichini (2003) among others.

Chapter 4

Financial structure and monetary policy with competing lenders

4.1 Introduction

The credit view of monetary policy relates the effects of a monetary intervention to the difficulties for borrowers to access the credit market.[1] Fundamental imperfections in credit relationships, mainly due to asymmetric information problems, constitute the main channel for monetary transmission.

This research constitutes a first attempt in understanding how the credit channel works when strategic competition among different lenders is explicitly considered. As a matter of fact, we depart here from the principal-agent representation of loan relationships to consider external financiers who compete by simultaneously offering alternative loan contracts to a single borrower.

Both theoretical and empirical works have emphasized two distinct effects at the basis of the transmission mechanism. The first one is referred to as *the broad credit channel*. The increased cost of external financing relative to internal funds induced by a tighter policy adversely affects borrowers balance sheets and thus the amount of collateral available for lenders. Hence, a restrictive monetary policy raising open-market interest rates reduces the amount of lending supply in the market for funds. The *bank lending channel* instead recognizes that bank financing is for

[1]This chapter elaborates on a joint research with Eloisa Campioni.

somewhat reasons special to firms and puts at the center-stage the high dependence of banks credit supply on the conduct of monetary policy.[2]

The contributions stressing the distinction between these two effects have been many. All share the reference to the principal-agent model as the relevant framework to study credit relationships under asymmetric information.[3] The main target of this chapter is to investigate to what extent the restriction to such a simple strategic interaction affects the characterization of the transmission mechanism. More precisely, the standard models of the credit channel assume perfect competition among investors in the financial market; thus, they restrict the analysis to zero-profit equilibria for lenders in an economy subject to asymmetric information. In the optimal contracting approach, this is represented by a binding *Individual Rationality* constraint for the (single) financier. Now, if lenders compete on the loan contracts they offer to borrowers, then they could strategically exploit each other's presence and positive-profit equilibria may appear. The existence of a credit channel for monetary policy in such a context has not been analyzed in the literature, so far.

Our aim is to develop this research line. The present chapter builds on the Repullo and Suarez (2000) work that introduced a microeconomic model of the choice between bank and non-bank finance using a moral hazard setting in the credit market. We model competition among lenders over the financial contracts they offer to a single entrepreneur/borrower. We analyze the situation where exclusivity clauses are imposed as well as the one where the borrower is not restricted on the amount of contracts she could accept. It turns out that the main predictions of the credit channel literature are robust to the introduction of multiple financiers who strategically interact over loan contracts. Furthermore, when the relationship between competition and incentives is explicitly considered, monetary interventions can also drastically affect the structure of the market for loans.

The discussion is organized as follows: Section 4.2 recalls the structure of the Repullo-Suarez benchmark economy, Section 4.3 presents the multi-lender interaction under exclusivity and Section 4.4 deals with the non-exclusive case. Finally, we devote Section 4.5 to the monetary policy transmission mechanism in the multi-lender framework.

[2]"As highlighted by Kashyap, Stein, and Wilcox (1993), the bank lending channel theory makes a key prediction: after a monetary tightening, the supply of bank loans should decline by more than the supply of other types of debt"(Oliner and Rudebusch (1996), p. 13).

[3]As a reference, see for example Besanko and Kanatas (1993), Holmstrom and Tirole (1997), Repullo and Suarez (2000) who study a moral hazard scenario, and Bolton and Freixas (2000), who focus on an incomplete information set-up.

4.2 The Repullo-Suarez economy

Our reference point will be the discussion on monetary policy measures in a simple one-borrower-one-lender economy as the Repullo-Suarez (RS) one. RS consider a one-good, two periods ($t = 0, 1$) economy populated by two categories of risk-neutral agents: lenders-investors and borrowers-entrepreneurs. Every lender is supposed to hold an infinite amount of the single good while the borrower has a limited endowment.

Production of the final good takes place with a linear and stochastic technology; I units of the good invested at time 0 are transformed at time 1 into:

$$\begin{cases} \text{AI} & \text{with probability} \quad \text{p} \\ 0 & \text{with probability} \quad \text{1-p} \end{cases}$$

If we denote W the borrower's endowment and θ the proportion of internal resources invested in the project, so that $\theta \equiv \dfrac{W}{I}$, then the relevant set of economies is the one with $\theta < 1$: borrowers hold the production technology but have to rely on external financing to carry out their investment project. One should observe that the amount I invested in the stochastic project is fixed.

The economy is subject to a moral hazard problem. Borrowers can affect the probability distribution of cash flows through the performance of some effort e. RS adopt the simplifying assumption $e = p \in [0, 1]$. As usual, lenders cannot observe borrowers' action, i.e. effort is not verifiable. To complete the description, notice that for a given effort choice, $e = p$, the borrowers receive the amount of private benefits $\varphi(p)AI$, where the function φ is decreasing and strictly concave.[4]

The characteristics of financing issued at equilibrium are defined by a financial contract signed at time $t = 0$. The design of the optimal contract between a lender and a borrower endowed with a given θ is very simple: the borrower acts as a mechanism designer and the optimal unitary repayment $R \in \mathbb{R}_+$ solves the following problem:

$$\max_{R,p} \quad u_E(R, p; \theta) = \quad I \left\{ p \left[A - R(1 - \theta) \right] + \varphi(p) A \right\}$$

s.t.

[4]We emphasize here that the basic structure of the RS economy is the same as that of Holmstrom and Tirole (1997), who considered a discrete effort case.

$$A - R(1-\theta) = -\varphi'(p) \tag{4.1}$$

$$pR \geq r \tag{4.2}$$

That is, the borrower chooses the pair (p, R) that maximizes her expected profits under the Incentive Compatibility constraint (4.1)[5] and the lender's Individual Rationality constraint (4.2) guaranteeing to the investor an expected payoff at least equal to the risk-free return, $r > 0$.[6] Using a standard argument we can argue that (4.2) always binds at every solution. Thus, we can restrict to equilibria implying zero profits for lenders without any loss of generality.

RS set-up the problem with specific functional forms so to get closed form solutions. The point we want to make here does not tackle this feature of the model, hence we will stick to their assumptions on technologies and preferences as close as possible. Let us hence introduce the necessary assumptions to state their results:

Assumption 1 $\varphi(p) = \dfrac{1}{2\lambda}(1-p^2)$

Assumption 2 $\dfrac{8}{7} < \dfrac{\lambda A}{r} < 4.$

with $\lambda \in (0,1)$. That is, the entrepreneur's private benefit is a quadratic function of p. These two assumptions holding, it is straightforward to show that there exists a critical value $\bar{\theta} \in (0,1)$ such that borrowing is not feasible for entrepreneurs with net worth ratio $\theta < \bar{\theta}$. In particular, we get:

$$\bar{\theta} = 1 - \frac{\lambda A^2}{4r}$$

That is, higher values of r (that RS associate to a restrictive monetary policy) correspond to a contraction in the aggregate feasible set. This defines the *broad credit channel* of monetary policy.

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[5]The Incentive Compatibility constraint takes into account the effort choice by entrepreneurs-borrowers. The simple nature of this contract problem allows RS to directly consider the first order condition of the problem faced by borrowers in their effort choice.

[6]Observe that assuming $R \in \mathbb{R}_+$ corresponds to the introduction of a *Limited Liability* constraint.

To emphasize the driving force of the transmission mechanism of monetary policy we have to consider the feasible set of the problem as parameterized in θ. There exists a unique value $\theta = \bar{\theta}$ such that (4.1) and (4.2), considered as an equality, are tangent in the (p, R) plane. As a consequence, whenever $\theta < \bar{\theta}$, the feasible set will be empty: economies with such a low endowment will not admit the possibility of external finance. The feasible set is depicted in Figure 4.1: it is identified by the segment between the two points A and B.[7] Given the structure of the problem, it is possible to show that borrowers maximize their payoff by choosing the allocation corresponding to point A.

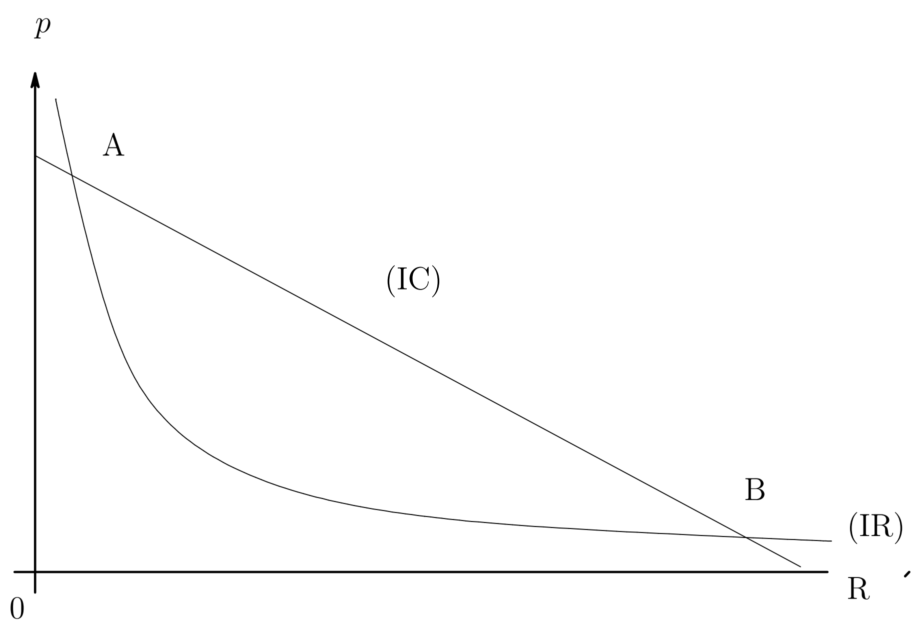

Figure 4.1: The feasible set of a principal-agent problem in a RS economy.

In a next step, RS study the relationship between the composition of external financing and the transmission mechanism. Thus, they assume availability of two alternative modes of funding the investment project, i.e. market finance and bank finance. Importantly, these financial assets

[7]One should note that in the specific quadratic case studied by RS the Incentive Compatibility constraint turns out to be a line.

are identified here by alternative techniques. In other words, borrowers have access to two alternative options for the production of the final good implying two different private benefit functions (with different λs):

$$\varphi_b(p) = \frac{1}{2\lambda_b}(1 - p^2)$$

and

$$\varphi_m(p) = \frac{1}{2\lambda_m}(1 - p^2)$$

with $0 < \lambda_m < \lambda_b < 1$. Bank financing is supposed to be more costly to the entrepreneur since it is associated to higher monitoring intensity:

$$\varphi_b(p) < \varphi_m(p) \qquad \forall p < 1.$$

To have a non-trivial choice among the techniques, it is also assumed that the effect on the incentive constraint is opposite: reducing the probability of success (lowering p) entails a smaller sacrifice of private benefits under bank finance than under market finance:

$$\varphi_m'(p) < \varphi_b'(p) < 0.$$

Solving the principal-agent game does not significantly change if the borrower is endowed with two alternatives for the production of the final good, since it can be seen as two separate games. Comparing the two modes of financing, it can be shown that there exist two threshold levels of θ which can be ordered as follows:

$$0 < \bar{\theta}_b < \bar{\theta}_m < 1$$

The two forms of financing are effectively feasible only in the set of economies where $\theta \in \left[\bar{\theta}_m, 1\right]$. Finally, RS deduce that in such a region there can exist a critical value $\theta^* \in \left[\bar{\theta}_m, 1\right)$ such that:

$$u_m(\theta) \geq u_b(\theta) \qquad \text{iff} \quad \theta \geq \theta^* \tag{4.3}$$

Thus, the highly collateralized borrowers[8] will prefer market finance, those with intermediate net worth $\left(\bar{\theta}_b < \theta < \theta^*\right)$ will get bank finance and those with little net worth $\left(\theta < \bar{\theta}_b\right)$ will not receive any funding. In such a scenario, RS are able to provide a formal assessment of the so-called bank lending channel of monetary policy.

[8]That is, the borrowers whose endowment is such that $\theta \geq \theta^*$.

To summarize: RS consider a standard model of the credit channel of monetary policy, where credit relationships are represented as a principal-agent interaction with hidden effort (moral hazard). Two features deserve further attention:

- The transmission mechanism is analyzed in a scenario where lenders are always earning zero profits. Existence of a binding zero-profit constraint (IR constraint) is interpreted as a shortcut for competition in financial markets.
- Alternative sources of funding (i.e. bank financing and market financing) are associated to different techniques available to borrower-firms. No strategic interaction among financiers is considered. In particular, existence of mixed financial structures for a single firm is disregarded.

The next two sections will try to understand to what extent these sort of predictions depend on the particular restriction to a principal-agent model for the credit market.

4.3 Multiple lenders and exclusive contracts

The co-existence of several lenders deeply modifies the nature of the game if we consider them as strategic players who compete in financing an investment project. To discuss the issue of mixed financial structures and of firms' choices among alternative funding, we explicitly introduce a framework with heterogeneous lenders. Each of them proposes a credit line and a repayment structure taking into account its technological features and the existence of a competitor. Importantly, competition takes place here under an exclusivity assumption: each borrower is allowed to accept only one financial contract at a time.

We will first show that competition among a finite number of diversified lenders supports zero-profit equilibria only as a non-generic case. Positive profits equilibria naturally emerge in this context; hence, considering a zero-profit condition simply as a short-cut for competition in the lending market cannot be appropriate.

More specifically, we modify the RS set-up in the following way: the two sources of external financing are modelled now as two different lenders. Lenders are therefore differentiated with respect to their monitoring technology, which is specific to the particular form of funding they offer. We maintain the simplification that RS propose and we summarize all the differences among alternative financial contracts in different monitoring intensities. Hence, there exist two lenders m, b offering market and bank financing, respectively.

Lenders compete by simultaneously offering a contract to the unique entrepreneur. At this stage, we maintain the assumption of fixed investment scale, that is, a given amount I is needed for the project. We concentrate on the asymmetric lenders case so to allow for mixed financial structures. The interaction among the contractual parties is modelled as a 3-stage game:

- at stage 0, each lender $i = m, b$ simultaneously proposes a contract, i.e. a repayment R_i;[9]
- at stage 1, the entrepreneur selects among the offered contracts the one she accepts;
- at stage 2, the entrepreneur exerts the effort p induced by the contract he signed and the contractual obligations are executed.

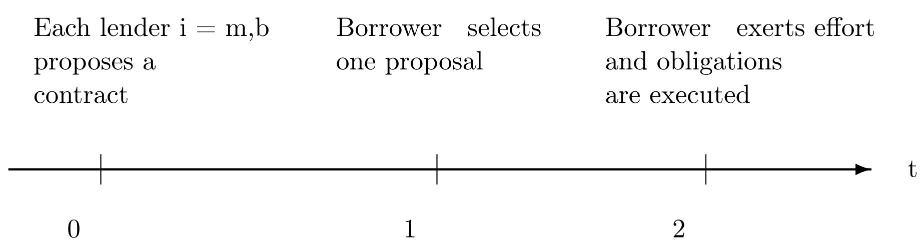

Figure 4.2: Time structure of the three stage-game

We abstract here from any partial commitment and renegotiation problem to concentrate on the effects of competition among lenders on the equilibrium outcomes. Using a backward induction argument, we analyze the reduced form game associated to the Incentive Compatible level of effort. To define this game, let us describe players strategies and payoffs. Each lender i's strategy is given by the unitary repayment:

$$R_i \in \mathcal{S}_i = \left[0, \frac{AI}{I-W}\right] \quad \text{for} \quad i = m, b$$

Let $\mathcal{S}_i$ define lender i's strategy space, so that $R_i \in \mathcal{S}_i$ for $i = m, b$. A (pure) strategy for the entrepreneur is given by the map:

[9] In this case with fixed investment scale and exclusive lending, the contract reduces to a repayment R.

$$s_E : \mathcal{S}_m \times \mathcal{S}_b \rightarrow \{0, m, b\} \times [0, 1]$$

where we denoted $\{0, m, b\}$ the decision to either accept none of the contracts, or to accept contract m or b, respectively. We let $\mathcal{S}_E$ be the strategy space of the entrepreneur, i.e. $s_e \in \mathcal{S}_E$ and we assume that this set also includes mixed strategies.

Under the exclusivity assumption, only one of the two contracts will be accepted and the inactive investor would simply get his reservation utility.

The following paragraphs provide a complete characterization of credit market equilibria under exclusivity.

We focus, for simplicity reasons, on a reduced form of the multi-lender game that embeds the optimal effort choice by entrepreneurs-borrowers. Hence, we denote:

$$u_E^{IC}(R_i) = \begin{cases} \dfrac{\lambda_i}{2A}(A - R_i(1-\theta))^2 - \dfrac{A}{2\lambda_i} & \text{for } i = m, b \\ r\theta & \text{if } u_E^{IC}(R_i) < \bar{u} \end{cases}$$

the borrower's Incentive Compatible payoff. That is, when the borrower exerts the optimal level of effort, she will get a payoff of $u_E^{IC}(R_i)$ accepting the lender i's contract R_i. In a similar way, Incentive Compatible payoffs for lenders are:

$$\pi_m^{IC} = \begin{cases} R_m(1-\theta)\left[\dfrac{\lambda_m}{A}(A - R_m(1-\theta))\right] & \text{if } u_E^{IC}(R_m) \geq u_E IC(R_b) \\ r(1-\theta) & \text{otherwise} \end{cases}$$

$$\pi_b^{IC} = \begin{cases} R_b(1-\theta)\left[\dfrac{\lambda_b}{A}(A - R_b(1-\theta))\right] & \text{if } u_E^{IC}(R_b) > u_E^{IC}(R_m) \\ r(1-\theta) & \text{otherwise} \end{cases}$$

where π_m^{IC} (π_b^{IC}) denotes the payoff (in unitary terms) of the lender offering a market-type (bank-type) of financing evaluated at the optimal effort level. The Incentive Compatible game associated to the optimal effort choice $p^*(R_i)$ for $i = m, b$ is thus given by the following collection of strategies and payoffs:

$$\Gamma^{IC} = \left\{\pi_m^{IC}, \pi_b^{IC}, u_E^{IC}(.), S_m, S_b, S_E\right\}$$

The relevant equilibrium concept for Γ^{IC} is Subgame Perfection, where each of the players maximizes his/her payoff given the behavior of the others and their sequential strategy choices. An equilibrium is a triple (R^*_m, R^*_b, s^*_E) such that each lender solves the optimal contractual problem, given the contract offered by her opponent, the players' participation constraints and the IC level of effort.

In particular, a Subgame Perfect Equilibrium of Γ^{IC} where the contract m is bought is the pair of repayments (R^*_m, R^*_b) together with the decision of the borrower to accept m and implement the induced IC level of effort p^*, i.e.:

$$R^*_m \in \underset{R_m}{argmax} \quad p^*(R_m)R_m$$

s.t.

$$p^*(R_m)[A - R_m(1-\theta)] + \varphi_i(p^*(R_m))A \geq \max\left\{r\theta, u_E^{IC}(R^*_b)\right\} \quad (IR_E)$$

$$p^*(R_m)R_m \geq r \qquad (IR_m)$$

where the contract offer of lender b, R^*_b, is such that:

$$p^*(R^*_m)R^*_b = r$$

that is, it delivers him zero extra-profits. With respect to the RS framework, the existence of competition among lenders translates into an endogenous reservation utility for the entrepreneur who can always accept the contract offered by b if the conditions of m were too strict (IR_E). Finally, the participation constraint of Lender m defines the minimum level of equity below which the external financier denies the necessary financing, i.e. $\bar{\theta}_m$.

One should note that $p^*(R_m)$ is the optimal effort choice corresponding to the decision of accepting the contract offer of lender m. The relationship $p^*(R_m)$ is implicitly defined from the Incentive Compatibility constraint. That is:

$$p^*(R_m) = \{p : A - R_m(1-\theta) = -\varphi'_i(p)A\}$$

Given $p^*(R_m)$ and $p^*(R_b)$, Figure 4.3 represents the Incentive Compatible unitary payoffs $p^*(R_i)R_i$ for lender $i = \{m, b\}$ as concave functions of the repayments R_m and R_b. Consider lender b: for all the repayment values lower than R_{1b} and higher than R_{2b}, the payoff curve lies below the line $r(1-\theta)$, that is, his participation constraint is violated.

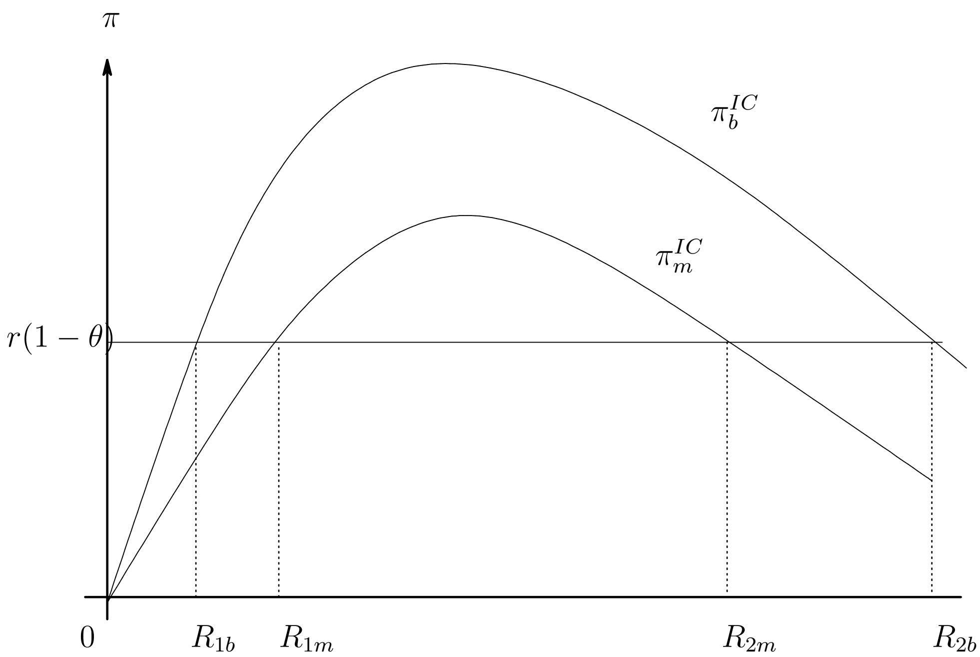

Figure 4.3: Incentive Compatible payoffs

4.3.1 Competitive equilibria

To show that we can recover all the standard results of the credit channel literature, we will first focus on an economy where lenders earn zero extra-profits at equilibrium.[10] A zero-profit equilibrium of Γ^{IC} where the borrower is indifferent between buying contract m and contract b is such that:

- $\pi_m^{IC}(R_m^*) = \pi_b^{IC}(R_b^*) = r(1-\theta)$,
- $u_E^{IC}(R_m^*) = u_E^{IC}(R_b^*)$ for $\theta \in (\bar{\theta}_m, 1)$,

with none of the players having a profitable deviation. As such, this equilibrium is characterized by the borrower appropriating the entire surplus and each lender's participation constraint binding at equilibrium.

[10] This is the main feature of our reference works, like Oliner and Rudebusch (1996), Repullo and Suarez (2000), Holmstrom and Tirole (1997) and Bolton and Freixas (2000).

We will follow the literature on Bertrand-type of equilibrium and focus on cases where the lenders cannot randomize, but the borrower can.[11]

We first observe that the zero-profit equilibrium is a non-generic outcome in the space of endowments. That is, there is only one $\theta = \theta^*$ which satisfies the borrower indifference condition between the choice of market and bank financing:

Proposition 12 *In the exclusive contracting framework, there exists one and only one economy where a zero-profit equilibrium can be sustained, corresponding to a single value of $\theta = \theta^*$*

.

Proof.

Plugging the IC level of effort into the participation constraint of each lender, we get the values $R_m(\theta)$ and $R_b(\theta)$, which guarantee zero-profits to the two lenders:

$$R_m(\theta) = \frac{\lambda_m \pm \sqrt{\lambda_m^2 - \dfrac{4r\lambda_m(1-\theta)}{A}}}{2(1-\theta)\dfrac{\lambda_m}{A}} \tag{4.4}$$

$$R_b(\theta) = \frac{\lambda_b \pm \sqrt{\lambda_b^2 - \dfrac{4r\lambda_b(1-\theta)}{A}}}{2(1-\theta)\dfrac{\lambda_b}{A}} \tag{4.5}$$

The lenders payoff functions are depicted in Figure 4.3, and R_1 (respectively R_2) denotes the positive (respectively negative) root.[12] Each pair of repayments $R_i(\theta)$ for $i = m, b$ on the decreasing part of the IC payoff functions cannot describe an equilibrium strategy. It would always be profitable for the lender to reduce the repayment, increasing both her own profits and the probability of her contract being accepted. For the values of $R_i(.)$ on the increasing part of the payoff functions, the lenders engage in some Bertrand-type of competition.

The pair $\{R_{1m}(\theta), R_{1b}(\theta)\}$ is the only candidate which is robust to profitable deviations by any of the two lenders. Hence, for it to characterize a zero-profit equilibrium, we should finally check whether there exist a non-empty set of endowments $\Omega = \{\theta \in [0,1] : u_E^{IC}(R_{1m}(\theta)) =$

[11] In Parlour and Rajan (2001) these equilibria are referred to as *deterministic*.

[12] We again follow RS in restricting to the values of A such that $R_m(\theta)$ and $R_b(\theta)$ are real roots (see Repullo and Suarez (2000), pp. 1942.)

$u_E^{IC}(R_{1b}(,\theta))\}$ that makes the borrower indifferent between buying b and m. Let

$$\Delta u_E^{IC} = u_E^{IC}(R_{1m}(\theta)) - u_E^{IC}(R_{1b}(\theta)$$

be the difference in borrower's utility between buying the m and the b zero-profit contracts. We get:

$$\begin{aligned} \Delta u_E^{IC}(\theta) &= \frac{A}{2}\left[\frac{1}{\lambda_m} - \frac{1}{\lambda_b}\right] + \frac{A}{4}\left[\lambda_m - \lambda_b\right] \\ &- \frac{A}{4}\left[\sqrt{\lambda_m^2 - \frac{4r\lambda_m(1-\theta)}{A}} - \sqrt{\lambda_b^2 - \frac{4r\lambda_b(1-\theta)}{A}}\right] \end{aligned}$$

Assumption 3 guarantees that if we evaluate $\Delta u_E^{IC}(\theta)$ at $\theta = \bar{\theta}_m$ we have:

$$\Delta u_E^{IC}(\bar{\theta}_m) = 2(\lambda_b - \lambda_m) + \lambda_b\lambda_m(2\lambda_b - \lambda_m) < 0$$

That is, in a neighborhood of $\bar{\theta}_m$, bank finance is strictly preferred to market finance. In addition, we have that:

$$\lim_{\theta \to 1} \Delta u_E^{IC}(\theta) = \frac{A}{2}\left[\frac{1}{\lambda_m} - \frac{1}{\lambda_b}\right] > 0$$

i.e., there will be a neighborhood of $\theta = 1$ where market finance is strictly preferred to bank finance.

Since $\Delta u_E^{IC}(\theta)$ is a continuous and increasing function of θ, there exists only one $\theta^* \in \Omega$ such that $\Delta u_E^{IC}(\theta^*) = 0$. If θ is equal to θ^*, then the borrower is indifferent between accepting contract m or b; in particular, accepting m is a best reply for her. Hence, the values R_{1m}, R_{1b} and the borrower accepting m with positive probability completely describe the zero-profit equilibrium.

■

Solving the IC constraint for the effort choice and the binding participation constraint for each lender, we can characterize the equilibrium repayments. Then, considering IR_E, we can identify the net worth ratio θ^* that sustains the zero-profit equilibrium (see Figure 4.4).

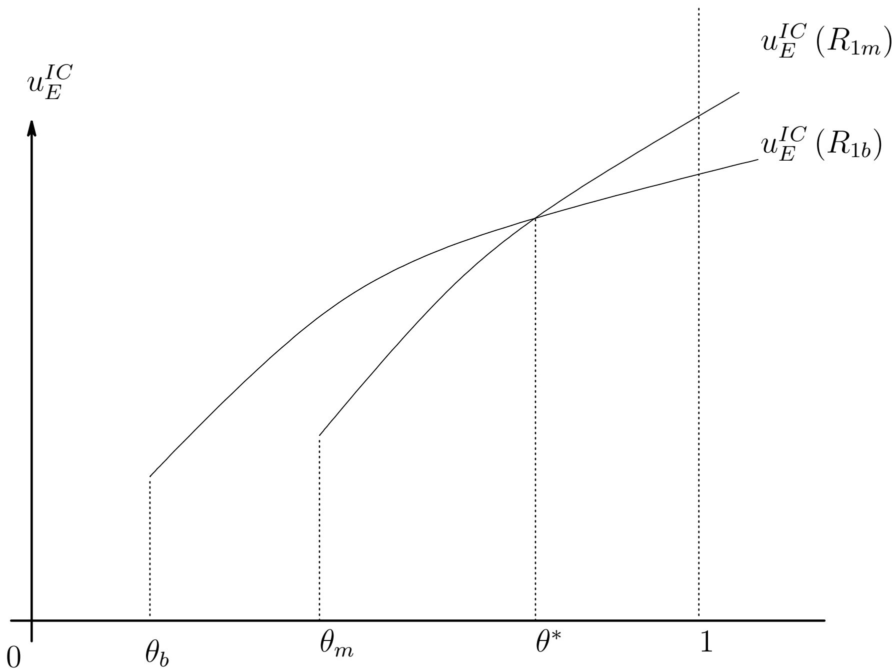

Figure 4.4: Borrower's indirect utility

At equilibrium, the entrepreneur gets the whole surplus and she randomly chooses which of the two contracts to accept. None of the lenders can profitably deviate: to increase the probability of his contract being chosen he should lower his repayment and hence his profits.

We also remark that, given the optimal entrepreneurial effort choice, it can be shown that lenders' participation constraints will not be satisfied whenever:

$$\theta < \bar{\theta}_b = 1 - \frac{A\lambda_b}{4r}$$

for lender b and

$$\theta < \bar{\theta}_m = 1 - \frac{A\lambda_m}{4r}$$

for lender m.

Finally, using Assumption 1-3 on the monitoring technologies we can rank the relevant thresholds, as it is shown in Figure 4.5.

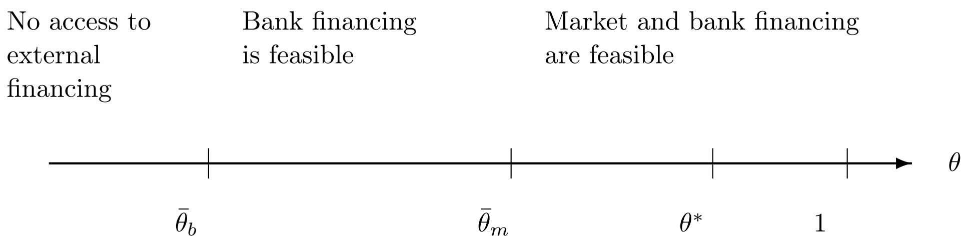

Figure 4.5: Ranking of the net worth ratios and access to external funds

The next paragraph argues that for every θ different from θ^*, zero-profit equilibria will not survive.

4.3.2 Non-competitive equilibria

The whole set of equilibria of the economy can be characterized if we consider the borrower's utility for an m-type or a b-type of contract. More precisely, Proposition 12 has already emphasized that there exist an open neighborhood of $\theta = 1$ where market finance is strictly preferred to bank finance $\left(\Delta u_E^{IC}(\theta) > 0\right)$. If the two lenders were offering zero-profit contracts, then a borrower with θ close enough to one, would get a *strictly* higher payoff accepting m. We also introduce the following assumption:

Assumption 3 $\lambda_b > 2\lambda_m$.

This is a technical condition to exclude that for values of θ_m very close to one, the lender m will ever become a monopolist. In such a case the equilibrium with zero profits cannot be sustained and lender m will enjoy market power:

Proposition 13 *For every $\theta \in [\theta^*, 1)$, there exists an equilibrium with positive profits for lender m.*

Proof. The candidate for equilibrium is the array (R_m^*, R_{1b}^*, m) such that, for every $\theta \in [\theta^*, 1)$:

- $\pi_m^{IC}(R_m^*) - r(1-\theta) > 0$

- $\pi_b^{IC}(R_{1b}^*) - r(1-\theta) = 0$
- $u_E^{IC}(R_m^*) = u_E^{IC}(R_{1b}^*) > r\theta$

and the borrower chooses m. That is, the borrower buys contract m, lender m makes positive profits and lender b offers a zero-profit contract threatening entrance.

First, consider lender m. Given the contract of lender b, R_{1b}^*, he must guarantee the entrepreneur at least $u_E^{IC}(R_{1b}^*)$ if his contract is to be accepted. Our Assumption 2 guarantees that for every θ in the interval $[\bar{\theta}_b, 1]$, we have: $u_E^{IC}(R_{1b}^*) > r\theta$. It follows that the relevant participation constraint for the borrower will bind at the endogenous reservation utility $u_E^{IC}(R_{1b}^*)$. In particular, we have that:

$$u_E^{IC}(R_{1b}^*) = \frac{A}{4}\left(\lambda_b + \sqrt{\lambda_b^2 - \frac{4r\lambda_b(1-\theta)}{A}}\right) + \frac{A}{2\lambda_b} - \frac{r(1-\theta)}{2}$$

which is an increasing and convex function of θ. Solving the IR_E constraint for the optimal R_m, we get a quadratic expression:

$$\frac{(1-\theta)^2\lambda_m}{2A}R_m^2 - (1-\theta)\lambda_m R_m + \frac{A^2(1+\lambda_m^2) - 2A\lambda_m u_e(R_{1b}^*)}{2A\lambda_m} = 0 \quad (4.6)$$

which for every θ has two real and distinct solutions if the discriminant Δ is non-negative. We also get:

$$\Delta = A^2(1-\theta)^2\lambda_m^4 - (1-\theta)^2\lambda_m^2\left[A^2(1+\lambda_m^2) - 2A\lambda_m u_E^{IC}(R_{1b}^*)\right] \quad (4.7)$$

If λ_b is large enough, i.e. $\lambda_b > 2\lambda_m$, both solutions to (4.7) are greater than one, hence for all $\theta \in [\theta^*, 1]$ the discriminant is non-negative. Hence, there exists a $R_m^* \in \mathbb{R}_+$ which constitutes a best reply to R_{1b}^*.
The values of R_m, say R_{1m} and R_{2m}, solving (4.6) are:

$$R_{1m,2m} = \frac{A\lambda_m \pm \sqrt{A[2\lambda_m u_E(R_{1b}^*) - A]}}{\lambda_m(1-\theta)}$$

with $R_{1m} \leq R_{2m}$. One should notice that R_{1m} is associated to a higher payoff for lender m, i.e. $R_m^* = R_{1m}$.
Consider now the behavior of lender b: if m is offering R_m^* and he plays R_{1b}^*, the entrepreneur will choose m. In such a context there is no profitable deviation for b since he can neither make an offer lower than R_{1b}^*

without running into negative profits nor raise the repayment since the borrower will always buy m.
The entrepreneur participation constraint is binding at the utility level $u_E^{IC}(R_{1b}^*)$. In particular, buying the market-type of contract is a best reply. ■

At equilibrium the dominant lender m offers a contract at a positive interest rate, the entrepreneur accepts it and the second lender b is shut out of the market. The contract guarantees the dominant lender positive profits. The entrepreneur is indifferent between accepting contract m and contract b, however it is not an equilibrium to buy the b-contract because the dominant lender can reduce her repayment just enough to make the lender strictly prefer her offer.

Given the structure of this problem with the lenders offering contracts under exclusivity, it is possible that the most efficient financier keeps her opponent out of the market enjoying some extra-profits.
If θ is greater than θ^*, then $u_E^{IC}(R_{1m}^*)$ will also be greater than $u_E^{IC}(R_{1b})$, i.e. lender m can profitably deviate towards a positive profit equilibrium where the borrower will accept his contract for sure. The SPNE equilibrium will therefore be a triple $\{R_m^*, R_{1b}^*, m\}$ such that accepting lender m's contract is a best reply. To characterize R_m^* we solve lender m's maximization problem under an IR_E constraint binding at the endogenous reservation utility $u_E^{IC}(R_{1b}^*)$, that is the utility the borrower would get accepting b's offer. In other words, competition threat has an effect on the division of the surplus, it implies that lender m leaves some surplus to the entrepreneur. The threat to accept lender b's proposal destroys the monopolistic profits of the principal-agent model.

Analogously, it can be argued that when we consider values of θ in the interval $[\bar{\theta}_m, \theta^*)$, the bank type of contract will be chosen:

Proposition 14 *Whenever $\theta \in [\bar{\theta}_m, \theta^*)$, at equilibrium lender b's contract is chosen and it ensures him positive profits.*

Proof. See the Proof of Proposition 13. ■

We still have to consider values of θ below $\bar{\theta}_m$, where the market-type of financing is not feasible, but the bank-type is. For all the initial endowment levels θ belonging to the interval $[\bar{\theta}_b, \bar{\theta}_m)$, the only lender which could offer a contract is lender b, hence the entrepreneur can only choose bank financing. Lender b behaves as a monopolist: he has all the bargaining power and at equilibrium appropriates all the extra-surplus, the entrepreneur's utility is binding at her reservation level $r\theta$. Hence, the active lender enjoys positive profits at equilibrium even though profits are lower than those earned in a monopolistic regime. As a consequence, we have the following:

Corollary 3 *If $\theta \in [\bar{\theta}_b, \bar{\theta}_m)$, then the only feasible choice for the entrepreneur is given by the bank financing option. At equilibrium lender b enjoys monopoly profits.*

Finally, those entrepreneurs whose initial amount of resources is below $\bar{\theta}_b$, are credit rationed. None of the financiers accepts to invest in their production activity, hence they do not have access to the credit market in any form.

We therefore confirm the role of entrepreneur's endowment as to access different financial resources that Repullo and Suarez identify, but we are also able to characterize the different conditions of access to the same form of credit that borrowers' experience. In particular, firms with very low endowment have access to bank financing at the worst conditions. As the amount of initial assets increases, the borrower still recurs to bank-financing which is more costly for the firm but the bank applies more favorable conditions. Finally, only those borrowers with high equity participation enjoy the better contractual terms of the market-type of contracts.

4.4 The non-exclusive scenario

When a borrower can accept more than one contract at a time, we enter a common agency set-up. There, multiple principals attempt at influencing the action taken by the single agent (effort choice) and by doing so they produce an external effect on her investment choice. Every contractual relationship is characterized by a moral hazard problem. Lender i's actions can affect other lenders' payoffs either directly or indirectly through the entrepreneur's effort choice and the decision on which contracts to accept. Such an externality from financial contracts can lead to a contraction in the aggregate volume of lending.

To develop this line of investigation, we slightly modify our original set-up to properly account for non-exclusive interactions. In particular, we construct a simple common agency model for the credit market, where bank and market financing are offered sequentially. The main target of the analysis is to allow for existence of a mixed financial structure for the single firm at equilibrium, a situation that is not usually considered in the standard credit channel literature. In the set-up we consider, the borrower, after observing the contract offers she is receiving from the two lenders, is allowed to either accept the two contracts together or to refuse both of them. In the terminology introduced by Bernheim and Whinston (1986a), we focus on an *intrinsic* common agency scheme. The relevant timing is depicted in Figure 4.6 and the contractual interaction is sequential: lender b acts first, then lender m offers his contract to the borrower. Finally, the borrower decides whether to accept or reject the proposals, she exerts effort and payoffs are distributed. For each lender, a contract is given by an investment level and a repayment line, i.e. $K_i = (I_i, R_i)$ with $i = \{b, m\}$.

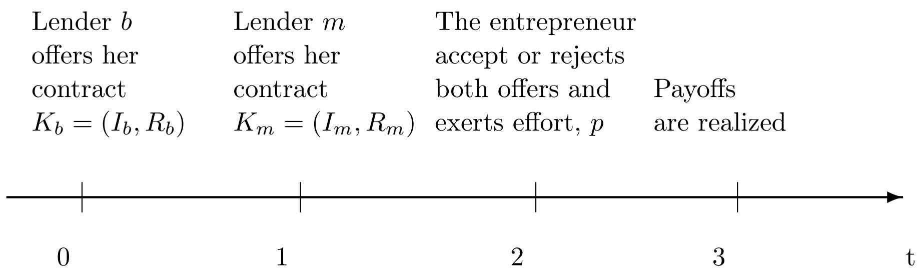

Figure 4.6: Timing of the contractual interaction

Credit relationships are therefore represented as environments with sequential bilateral contracting. A typical example of such interactions is given by those venture capital problems where an upstream principal (the venture capitalist) offers a financial contract to an agent (the start-up firm) anticipating that in a next step he will contract with suppliers, retailers and, possibly, other investors (downstream principals). In these situations the upstream principal takes advantage of his Stackelberg position by designing the upstream relationship in such a way to take into account the implications of downstream contracting. In particular, we chose to identify the upstream principal with Lender b and the downstream principal with Lender m. The sequential nature of the interaction allows lender b to offer allocations that are contingent on the contractual offers made by lender m. This enables him to internalize the effects of his behavior on other players' choices. Nonetheless, a form of contractual externality is still present: the borrower's decision on which contracts to accept (either both or none of them) is not contractible by any of the two lenders.

Lenders are heterogeneous with respect to their monitoring technologies. Lenders b represent the bank financing which is more costly to the entrepreneur since it can be associated to higher monitoring intensity by external financiers. Market financing is offered by the m-type of lenders, and being associated to anonymous stake-holders can be characterized by lower auditing, hence higher private benefit for the entrepreneur:

> [...] *We assume that the difference between them relates to the extent to which the corresponding lenders monitor their borrowers in order to ameliorate the moral hazard problem. Thus monitoring in this context means the application by the lender of some external control mechanism that makes the diversion of project resources towards private uses less profitable to the entrepreneur.*[13]

Lender b cannot therefore fully exploit the market power coming from being the first mover. If the amount of credit issued by b gets larger and larger as compared with that offered by m, monitoring costs become very high as well. The borrower might then end up rejecting both contracts. This is a main trade-off of the non-exclusive interaction.

In addition, we assume that both lenders, b and m, have access to a deposit market where they collect the resources to finance the entrepreneurial sector. Deposits are costly for the banking sector as well as for non-bank lenders. The cost function we consider in our exercise is:

$$\psi(I_i) = 8I_i^2(1+r)$$

with $i = \{m, b\}$. The cost of collecting funds is increasing and convex in the amount of investment (I_i) supplied by each financier, and it is increasing in the risk-free rate of return r, which the depositors earn when investing on the bond market.

The aggregate investment level $I = I_m + I_b + W$ is still assumed to be fixed. Also, with probability p the production is successful, and, without loss of generality, we focus on the situation where the net product AI is equal to one, with $A > 1$. Hence, it must be that $I < 1$ throughout our discussion. The definition of the borrower's payoff is also unchanged with respect to the exclusive game. Importantly, in this non-exclusive scenario, the entrepreneurial private benefit $\varphi(p)$ depends on the composition of firms' external finance:

$$\varphi(p; I_b) = \frac{1-p^2}{16I_b}$$

for every $I_b \in (0, I - W]$. Given the effort p, the private benefit is a decreasing function of the amount of financing provided by the banking sector. This functional form has been chosen to represent the fact that

[13] Repullo and Suarez (2000), p. 1937.

bank financing is more costly for firms due to the higher and better monitoring intensity of the banking sector relatively to the market one.

We now define players' strategies and payoffs. For the first-mover, lender b, the strategy space is simply the set of all possible pairs of investment and repayment:

$$K_b = (I_b, R_b) \in \mathcal{K}_b \ \subset \mathcal{R}_+^2$$

For the second-mover, lender m, the decision on which contract to offer depends on the choice of her predecessor,

$$K_m = (I_m, R_m) : \mathcal{K}_b \to \mathcal{R}_+^2.$$

Let $\mathcal{K}_m$ be the strategy space of lender m. Having received the contract offers, the entrepreneur's strategy is to choose whether to accept or reject the contracts and which effort to exert:

$$\sigma_E : \mathcal{K}_m \times \mathcal{K}_b \quad \to \quad \{0,1\} \times \ [0,1].$$

We denote the entrepreneur strategy set as Σ_E, with $\sigma_E \in \Sigma_E$. We also let R_m and R_b be the total repayments for lender m and lender b, respectively.[14]

Hence, the payoff of lender $i = \{m, b\}$ is:

$$\pi_i = \begin{cases} pR_i - 8I_i^2(1+r) & \text{if the borrower buys both contracts} \\ 0 & \text{otherwise} \end{cases}$$

and

$$u_E = p\,(AI - R_b - R_m) + \varphi(p; I_b).$$

for the single borrower. We also recall that $AI = 1$.

The intrinsic common agency game is given by $\Gamma^c = \{\mathcal{K}_b, \mathcal{K}_m, \Sigma_E, \pi_b, \pi_m, u_E\}$. The relevant equilibrium concept for Γ^c is Subgame Perfection. The next paragraphs provide a characterization of the set of positive-profit Subgame Perfect Nash Equilibria (SPNE) where both contracts are accepted.

[14]This is a main difference with the previous section where R_i was taken to be the unitary repayment earned by lender i. Given that the final output is set equal to one, the distinction between unitary and total repayments turns out not to be crucial in this section.

4.4.1 Credit market equilibria

Definition 10 *Given the borrower's endowment W, we say that the strategy profile $\{K_b^* = (I_b^*, R_b^*) \in \mathcal{K}_b, K_m^* = (I_m^*, R_m^*) \in \mathcal{K}_m\}$ together with the borrower's decision of accepting both contracts and choose the associated level of effort p^* constitute a (pure strategy) SPNE for the game Γ^c if K_m^* and K_b^* are solutions to the following two problems:*

- *Problem M*

$\forall K_b \in \mathcal{K}_b,$

$$(R_m^*(K_b), p^*(K_b, K_m(K_b)), I_m^*(K_b)) \in \underset{R_m,p,I_m}{argmax} \quad pR_m - 8(1+r)I_m^2$$

s.t.

$$I = I_m + I_b + W$$

$$p = 8I_b\left[1 - R_m - R_b\right] \tag{IC}$$

$$p[1 - R_m - R_b] + \frac{1-p^2}{16I_b} \geq W(1+r) \tag{IR_E}$$

- *Problem B*

$$K_b^* = (R_b^*, I_b^*) \in \underset{R_b,I_b}{argmax} \quad p^*(I_b, R_b)R_b - 8(1+r)I_b^2$$

s.t.

$$I_b \leq I - W \tag{4.8}$$

$$\pi_m^* \geq 0 \tag{IR_M}$$

where $\pi_m^* = p^*(I_b, R_b)R_m^*(I_b, R_b) - 8(1+r)I_m^{*2}(I_b, R_b)$ is the value function of Problem M.
That is, we consider the optimal choices of m that induce an Incentive Compatible level of effort as well as the borrower's decision of buying both contracts. Given the endowment W and the investment level of the upstream principal I_b, I_m is just residually determined: $I_m = I - W - I_b$. The optimal choice of b must respect feasibility (4.8) and should guarantee the participation of lender m.

We provide here a simple characterization of equilibria in terms of the borrower's endowment W. To this extent, we first investigate the properties of the solution to Problem M and to Problem B in the following lemmas.

Lemma 4 *There exists an open set of endowments* $\{W : W \in (\phi_1, \phi_2) \subset [0,1]\}$ *such that there is a candidate SPNE* (K_b^*, K_m^*, p^*) *where none of the constraints (4.8),* (IR_E) *and* (IR_M) *is binding.*

Proof. We first solve Problem M with an (IR_E) which is slack, and then verify that at the solution it is indeed the case. This gives us:

$$R_m^*(R_b) = \frac{1-R_b}{2} \quad \text{and} \quad p^*(I_b, R_b) = 4I_b(1-R_b)$$

We then solve the design of the optimal contract for b, taking into account that lender m must make positive profits at the candidate equilibrium and he must provide a positive amount of credit (i.e. (4.8) and (IR_M) should not bind). Solving Problem B we get a Stackelberg result for the repayments:

$$R_b^* = \frac{1}{2} \quad \text{and} \quad R_m^* = \frac{1}{4}$$

and an interior solution for I_b:

$$I_b^* = \frac{1}{16(1+r)}.$$

The investment choice of lender m is residual:

$$I_m^* = I - W - I_b^*.$$

Finally, the Incentive Compatible level of effort is:

$$p^* = 4\frac{1}{16(1+r)}\frac{1}{2} = \frac{1}{8(1+r)}.$$

Now, if the borrower's participation constraint (IR_E) were binding, the repayment and the IC effort choice of lender m would have looked like:

$$R_m(I_b, R_b) = 1 - R_b - \frac{1}{8I_b}\sqrt{16I_bW(1+r)-1}$$

and

$$p(I_b, R_b) = \sqrt{16I_bW(1+r)-1}.$$

For the square root to be well defined it should be $16I_bW(1+r)-1 \geq 0$. Plugging $p(I_b, R_b)$ into Problem B, we get:

$$\max_{I_b, R_b} \quad \left(\sqrt{16I_bW(1+r)-1}\right) R_b - 8(1+r)I_b^2$$

The objective function is now linear in R_b, hence by limited liability we get to the corner solution $R_b = 1$. No matter what the effort level would be and what proportion of investment the lender m would finance, he would get a zero repayment hence a net loss in the game. Thus, there is no incentive for lender m in tightening the borrower's participation constraint whenever (4.8) and (IR_M) are slack.

Finally, we observe that the interval of relevant entrepreneur's endowment that sustain this solution is, in fact, delimited by the followings:

- lender b cannot be a monopolist in the loan offer. Hence, using the expression obtained at equilibrium for I_b we obtain that: $I_b \leq I - W$ for all
$$W < I - \frac{1}{16(1+r)} \equiv \phi_2$$
- lender m has to be willing to participate, hence he can at most be left at zero-profits. Hence, it should be:
$$\pi_m^* = p^* R_m^* - 8(1+r)I_m^{*2} \geq 0$$
that corresponds to:
$$W \geq I - \frac{1}{8(1+r)} \equiv \phi_1$$

with $\phi_1 < \phi_2$. ■

The Lemma stresses that for every $W \in (\phi_1, \phi_2)$ a candidate for equilibrium is a repayment and investment pair that is associated to a mixed financial structure for firms. As a by-product, it is emphasized that the allocations where I_b is set equal to $I - W$ and (IR_M) does not bind are not compatible with a binding participation constraint for the borrower. With a similar argument, it can be shown that the allocation that delivers the whole surplus to Lender b (i.e., (IR_E) and (IR_M) bind simultaneously) turns out not to be feasible. One should note that in this particular solution the total revenues earned by each lender, say $p^*R_b^*$ and $p^*R_m^*$, are independent of the borrower's net worth. Alternative values of W only affect the distribution of monitoring costs paid by financiers. More precisely, as W gets lower, the fraction of the aggregate demand for loans $I-W$ that should by financed by the m lender becomes larger and larger. If the entrepreneur's endowment is lower than ϕ_1, then lender m does not have any incentive to participate, since revenues do not compensate monitoring costs.

We now investigate the relevance of situations where lender b compensates lender m for not offering any credit, i.e. $I_m = 0$. Lender b does not enjoy monopolistic profits, but has to sacrifice part of his rent due to the competition of the opponent: it should hence be $R_b > 0$. Such a scenario is described in the following:

Lemma 5 *There exists a level of endowment $\theta_2 \in (\phi_1, \phi_2)$ such that: for every $W \geq \theta_2$ there is a candidate SPNE where (4.8) binds and both (IR_M) and (IR_E) do not. We refer to this allocation as a "compensating outcome".*

Proof. The solution to Problem M will be unchanged. We only have to consider Problem B in the particular case where lender b finances the entrepreneur alone, i.e. $I_b = I - W$. The optimal repayments will not change:

$$R_b = \frac{1}{2} \quad \text{and} \quad R_m = \frac{1}{4}$$

On the other hand, the Incentive Compatible effort level will be different and given by:

$$p = 8I_b(1 - R_b - R_m) = 8(I - W)\frac{1}{4} = 2(I - W).$$

Plugging these expressions into the participation constraint of the entrepreneur, we verify whether there exists an interval of endowments that guarantees feasibility of this solution. The (IR_E) constraint becomes:

$$\frac{I-W}{2} + \frac{1 - 4(I-W)^2}{16(I-W)} \geq W(1+r)$$

which leads to a second-order polynomial in W:

$$4\left[1 + 4(1+r)\right] W^2 - 8I\left[1 + 2(1+r)\right] W + 1 + 4I^2 \geq 0 \tag{4.9}$$

The corresponding equation admits the two real and distinct roots:

$$W_{1,2} = \frac{4I\left[1 + 2(1+r)\right] + -\sqrt{\Delta}}{4\left[1 + 4(1+r)\right]}$$

where

$$\Delta = 64I^2(1+r)^2 - 16(1+r) - 4.$$

Both W_1 and W_2 turn out to be smaller than I. The inequality (4.9) is satisfied for all $W \geq W_2$ and for all $W \leq W_1$. Computation shows that for all values of the parameters we have:

$$\theta_2 \equiv W_2 < \phi_2 = I - \frac{1}{16(1+r)}$$

and for $I > \dfrac{1}{8(1+r)}$, i.e. for values of investment not too close to zero,

$$\theta_2 > \phi_1 = I - \frac{1}{8(1+r)}.$$

Hence, the negative root W_1 will be smaller that ϕ_1. Since, for values of endowment below ϕ_1, lender m is not participating to the investment we can concentrate on the positive root, which defines the threshold θ_2 above which compensating solutions can be sustained. ■

Finally, we consider situations where the leader b acquires a monopolistic position in lending and pushes m out of the credit market. This solution has the following features: lender b proposes $\tilde{K}_b = (I - W, \tilde{R}_b)$, lender m proposes $\tilde{K}_m = (0, 0)$, the borrower buys both contracts and exerts the induced level of effort. Lemma 6 establishes the feasibility of this solution:

Lemma 6 *There exist two critical endowment levels $\gamma_1 \in (0, \phi_1)$ and $\gamma_2 \in (\theta_2, 1)$ such that for every $W < \gamma_1$ and for every $W > \gamma_2$ there exists a candidate SPNE $\left(\tilde{K}_b = \left(I - W, \tilde{R}_b\right), \tilde{K}_m = (0,0), \tilde{p}\right)$ where the borrower accepts both offers and lender m earns monopolistic profits.*

Proof. Lender b is financing the entire amount and solves her profit-maximization problem taking $\tilde{K}_m = (0, 0)$ as given:

$$\max_{R_b, p} \quad pR_b - 8(I - W)^2(1+r)$$

s.t.

$$p = 8(I - W)(1 - R_b)$$

$$8(I - W)(1 - R_b)^2 + \frac{1 - 64(I - W)^2(1 - R_b)^2}{16(I - W)} \geq W(1+r)$$

If the borrower's participation constraint is slack, then the repayment for lender b remain still equal to one half of the entire production, i.e. $R_b = \frac{1}{2}$. In this case, considering the borrower's participation constraint, we get the following second-order degree inequality in W:

$$16(2+r)W^2 - 16(3+r)IW + 16I^2 + 1 \geq 0$$

The two real and distinct roots are:

$$W = \gamma_{1,2} = \frac{8I(3+r) \pm \sqrt{\tilde{\Delta}}}{16(2+r)}$$

where

$$\tilde{\Delta} = 64I^2(3+r)^2 - (16I^2+1)16(2+r).$$

Hence, the relevant values of the endowment are those in the set $\{W : W > \gamma_2, W < \gamma_1\}$: in particular, both γ_1 and γ_2 are smaller than I. We also remark that in order to get real roots we have to assume that I is large enough; more precisely, a sufficient condition turns out to be $I > \frac{1}{\sqrt{2}}$.[15] Furthermore, it is straightforward to verify that

$$\gamma_2 \in (\phi_1, \phi_2)$$

We can also show that $\gamma_2 > \theta_2$.
We denote:

$$dif = \theta_2 - \gamma_2 = dif_1 + dif_2$$

where

$$dif_1 = \frac{2I(3+2r)}{2(5+4r)} - \frac{2I(3+r)}{2(4+2r)}$$

and

$$dif_2 = \sqrt{\frac{16I^2(1+r)^2 - 4(1+r) - 1}{4(5+4r)^2}} - \sqrt{\frac{4I^2(1+r)^2 - (2+r)}{4(4+2r)^2}}$$

It can be shown, after some algebra, that

$$\theta_2 > \gamma_2 \quad \Longleftrightarrow \quad dif_1 > -dif_2$$

whenever

[15] After few calculations, we get that $\tilde{\Delta}$ is positive whenever $I^2(2r+2)^2 > (2+r)$ that is true for every $r > 0$ if $I > \frac{1}{\sqrt{2}}$.

$$36I^2(1+r)^2 < (dif_2)^2$$

This condition corresponds to:

$$(18+9r)(16+4r^2+16r)-3 < 0 \quad \rightarrow \quad 36r^3+216r^2+432r+285 < 0 \quad (4.10)$$

that is a third-degree polynomial in r, with two conjugate complex roots and one negative real root, let it be $\bar{r}$; importantly (4.10) is verified for every rate of interest r below $\bar{r}$.[16] Thus, for every non-negative rate of interest dif has a negative sign, and $\gamma_2 > \theta_2$.

Finally, we emphasize that for every positive r we have that $\gamma_1 < \phi_1$. This can be shown if one notices that considering $r = 0$ we get:

$$\phi_1 - \gamma_1 = \frac{1}{4}I - \frac{1}{8} + \frac{1}{4}\sqrt{I^2 - \frac{1}{2}}$$

that is strictly positive provided that $I > \frac{1}{\sqrt{2}}$. We also have that:

$$\frac{d\gamma_1}{dr} < 0 \quad and \quad \frac{d\phi_1}{dr} > 0$$

Thus, the sign of the inequality will not be reversed if we considered any r greater than zero. ∎

Lemma 4, 5 and 6 analyze the properties of the feasible sets of Problem m and Problem b for alternative values of the borrowers' endowment W. Their main implications are summarized in Figure 4.7.

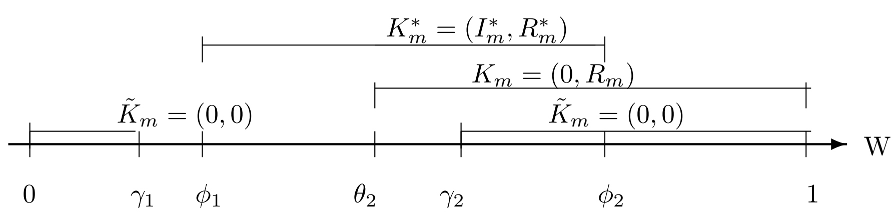

Figure 4.7: Complete description of the endowment space

[16] The expression for the unique negative real root is: $\bar{r} = -\frac{1}{6}\sqrt[3]{45+3\sqrt{222}} - \frac{1}{2\sqrt[3]{45+3\sqrt{222}}} - 2$.

Figure 4.7 represents the thresholds in the endowment space.
For very low values of W (i.e. for $W < \gamma_1$) the only outcome that satisfies Incentive Compatibility and participation constraints for all players is one where lender b has a monopolistic position. If, on the contrary, one considers borrowers with a high net worth W, then the monopolistic solution, the compensating solution and the "mixed" one are both feasible.

The next step is to provide a characterization of credit market equilibria corresponding to different intervals of W.

First, if one considers a W greater than γ_2 then, given the offer $K_m \in \mathcal{K}_m$ of lender m, lender b will always have an incentive to choose $K_m = (0, R_m)$ instead of $\tilde{K}_m = (0,0)$ that gives him a payoff of zero. As a consequence, we have that:

Remark 2 *Zero-profit contracts for lender m, $\tilde{K}_m = (0,0)$, can never be supported at a SPNE equilibrium for every W greater than γ_2.*

We now look at the payoffs earned by lender b at each feasible solution. We have that:

$$\pi_b = \begin{cases} \pi_b^1 = \dfrac{1}{32(1+r)} & \textit{mixed solution} \\ \pi_b^2 = (I-W) - 8(1+r)(I-W)^2 & \textit{compensating solution} \\ \pi_b^3 = 4(I-W) - 8(1+r)(I-W)^2 & \textit{monopolistic solution} \end{cases}$$

It is straightforward to show that if lender m offers a positive repayment and the borrower exerts the optimal level of effort, then lender b will always have an incentive to implement the "mixed" solution, whenever it is feasible to do so.
We have that:

$$\pi_b^1 - \pi_b^2 \geq 0 \quad \text{if and only if} \quad 8(1+r)(I-W)^2 - (I-W) + \frac{1}{32(1+r)} \geq 0$$

This second order equation in W has two real and identical roots given by:

$$W = \bar{W} = I - \frac{1}{16(1+r)} = \phi_2$$

That is, when contracting with a borrower endowed with $W = \bar{W}$, Lender b would get the same payoff in the mixed solution and in the compensating one. Moreover, observe that:

$$\pi_b^2 = (I - W) - 8(1 + r)(I - W)^2$$

is increasing in W up to $\bar{W}$ and decreasing afterwards, and the second-order derivative of this function is always negative, so that $\bar{W}$ is the (unique) maximum.

For every investment project of any entrepreneur having an initial endowment smaller than ϕ_2, both kind of loans will hence be used to finance production and the firm will have mixed financial structure.

Remark 3 *For every $W \in [\phi_1, \phi_2]$, firms will exhibit at equilibrium a mixed financial structure. For $W > \phi_2$, compensating equilibria will emerge.*

In the example we have constructed mixed financial structure turns out to be a widespread feature of firms' balance sheets. While entrepreneurs with high amount of internal funds can safely accrue to bank financing minimizing the costs of external loans, firms with lower private resources are offered to differentiate their sources of funds.

It remains to discuss what happens to entrepreneurs with very low endowment. We know that on the left hand side of ϕ_1, the lenders offering market-type of contracts do not have any incentive to finance the investment. Hence, we could look for contracts in which the only active financier is the banking sector. In this region though we do not expect the presence of the non-bank financing to be that relevant, and it is likely that the banks can acquire here a monopolistic position.

The result we provide is that there does not exist an equilibrium in which lender b and lender m coexist in the game and lender m issues a positive amount of credit.

Proposition 15 *For every $W \in [0, \phi_1)$, it is not feasible for lender m to offer a positive amount of investment in the project.*

Proof. Take the proof of existence of an equilibrium with mixed financial structure. From that we know that the sequential game guarantees to lender m positive profits up to a lower-bound level of endowment, ϕ_1. For every level of endowment below ϕ_1, lender m always makes negative profit when offering some positive investment. Hence, ϕ_1 represents for the lender m the lowest possible amount of internal resources such that the moral hazard problem in the common lending context is still bearable. Let us check whether it is possible to find another pair of contracts both for lender b and lender m which guarantee lender m's participation even below ϕ_1.
Given the optimization problem of lender m, we know how will the repayment and the Incentive Compatible effort depend on the contract that b is going to offer:

$$R_m(R_b) = \frac{1 - R_b}{2} \quad \text{and} \quad p(I_b, R_b) = 4I_b(1 - R_b)$$

Lender b has an even stronger position in the upper tail of Ws, since he can exploit his leading position and keep lender m at zero-profit. The problem she will face is the following:

$$\max_{R_b, I_b} \quad 4I_b(1 - R_b)R_b - 8(1 + r)I_b^2$$

s.t.

$$4I_b\frac{(1 - R_b)^2}{2} - 8(1 + r)(I - W - I_b)^2 = 0$$

Using the constraint to derive an expression for the repayment, we get:

$$R_b = 1 - \sqrt{\frac{4(I - W - I_b)^2(1 + r)}{I_b}}$$

and substituting it in the objective function, we can rewrite the problem as:

$$\max_{I_b} \ (I - W - I_b)\sqrt{4I_b(1 + r)} - 4(1 + r)\left[(I - W)^2 + I_b^2 - 2(I - W)I_b\right] - (1 + r)I_b^2$$

Taking the first order derivative and setting it equal to zero, we obtain a third-order polynomial in I_b:

$$100(1+r)I_b^3 - [160(1+r)(I-W) - 9]\, I_b^2 +$$
$$+ \left[64(1+r)(I-W)^2 - 6(I-W)\right] I_b^2 - (I-W)^2 = 0$$

which exhibits two complex conjugate roots and one real negative root. Hence, within the relevant set of parameters there is no level of investment that lender b could select to sustain any equilibrium featuring also the presence of lender m. ■

That is, when dealing with borrowers with "intermediate" net worth (i.e. when W is between ϕ_1 and ϕ_2), both financiers actively participate in the project. If W gets larger and larger, then the mixed financing equilibrium cannot be sustained anymore: the demand for loans $I-W$ becomes very small as compared to the (inelastic) supply I_b^* provided by the "leading" lender b. As a consequence, the only way to implement credit market equilibria is to allow b to provide the entire amount of liquidity $I-W$. In such a situation, the m lender should get a positive reward for not supplying any form of credit. This defines the compensating solution.

If, on the contrary, borrowers' internal funds are very low, then the amount of financing that should by provided by the "follower" becomes large. It is then possible that the associated rise in monitoring costs prevents m from participating. This happens whenever W is lower than ϕ_1. The only situation we can therefore associate to this set of lower endowments is that of a purely monopolistic position of the banking sector with the m lender providing no credit at all and getting a zero repayment.

The composition of firms' external financing at equilibrium is depicted in Figure 4.8.

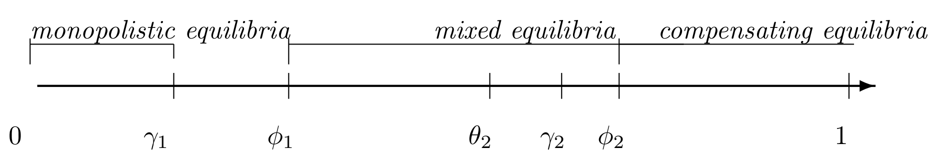

Figure 4.8: Different equilibria in the endowment's space

4.5 The credit channel of Monetary Policy with competing lenders

We now try to use the previous results to discuss the transmission mechanism of monetary policy as it affects banks and other financial intermediaries.

Our set-up shares with most of the credit channel literature the reference to a one-period partial equilibrium, analysis of the credit market. As a consequence, we deduce the effects of policy interventions from a simple comparative statics exercise by considering alternative values of the risk-free interest rate r that we assume to be controlled by a monetary authority: as a matter of fact, this is a major limit of the analysis.

We first consider the simple framework of competition under exclusivity developed in Section 4.3. In such a scenario, we can restate the main prediction of the so-called broad credit channel of monetary policy: small and poorly-capitalized firms are hit the most by a tightening monetary policy. The result is stated in the following:

Proposition 16 *Every increase in the risk-free interest rate r implies a reduction in the net worth ratios at which both bank and market finance are feasible, namely $\bar{\theta}_b$ and $\bar{\theta}_m$.*

Proof. The result follows from the definition of

$$\bar{\theta}_i = 1 - \frac{A\lambda_i}{4r}$$

for $i = m, b$. ■

A tight monetary policy weakens borrowers' financial positions and reduces therefore the set of borrowers with whom lenders can contract without incurring losses. The same framework can also cope with another standard finding of the credit view literature: when the interest rate raises, a greater spread between repayments issued by different financiers is typically observed.[17] That is:

Proposition 17 *Consider again a scenario of competition under exclusivity among b and m. Whenever both market finance and bank finance are available (i.e. for every $\theta \geq \bar{\theta}_m$), then a tightening of monetary policy will shift upwards both R_b and R_m. Furthermore, the equilibrium spread between the two borrowing rates will get larger.*

[17] See, for instance, Oliner and Rudebusch (1996).

Proof. We just have to consider the equilibrium expressions of R_b and R_m derived in Proposition 13 and 14. ■

Let us finally remark that in a model with fixed investment scale, the definition of aggregate credit demand is of limited relevance. We chose to adopt the simplification that Repullo and Suarez suggest in their paper, where they define:

$$L_m = \int_{\theta^*}^{1} (1-\theta) I dF(\theta)$$

$$L_b = \int_{\bar{\theta}_b}^{\theta^*} (1-\theta) I dF(\theta)$$

$$L = L_m + L_b = \int_{\bar{\theta}_b}^{1} (1-\theta) I dF(\theta)$$

as the market lending, the bank lending and the total lending amounts, respectively. Following a monetary contraction, the market and the aggregate level of lending decline as an implication of the so-called *balance sheet* channel.[18]

To complete the discussion of the exclusive case, we focus on the equilibria with positive profits for m, where b is offering a zero-profit (latent) contract, as they were introduced in Proposition 13. The main effect of such an offer is to threaten the lender m from charging the monopoly price. The presence of a latent competitor leaves to market-financed entrepreneurs a payoff that is higher than their (exogenous) reservation utility.

Restrictive monetary policies in these cases have a direct effect on the supply of loans by banks. As r rises, banks should ask for a higher repayment to satisfy the zero-profit equilibrium requirement. This will have a clear redistributive effect reducing the borrowers' share of the surplus. If the interest rate is set high enough, then a monopolistic outcome, where lender m appropriates all the extra-surplus, could be implemented.

The set-up proposed in Section 4.3, though, is very specific to analyze the market power of lenders in those particular cases when it is possible to control for exclusive dealing. A natural step forward would be to discuss more effective competition, i.e. situations where the borrower

[18]See Kashyap, Stein, and Wilcox (1993).

is allowed to contract simultaneously with several lenders, and to analyze the properties of the equilibria emerging with this kind of strategic interaction.

The intrinsic common agency model studied in Section 4.4 predicts existence of mixed financial structures at equilibrium. In particular, highly capitalized firms choose mixed financing due to the fact that the benefit they get from exerting higher efforts are inversely related to the amount of bank loans they choose. Hence, all things being equal, an entrepreneur will always try to finance the lowest possible share of investment through bank contracts. Finally, there are firms for which the banks can exercise a dominant role in financing the investment projects, nonetheless non-bank investors should be compensated to stay out of the market. This happens to those firms with the highest amount of private resources, which may sustain compensating equilibria.

In this scenario, monetary policy has also important effects on the structure of the credit markets. In particular, a restrictive monetary policy will reduce the relevance of mixed financing and favor monopolistic outcomes. An increase in the interest rate r will raise both ϕ_1 and ϕ_2, but ϕ_1 will react more sensitively, representing that the effects of a tightening policy are more pronounced on the less endowed firms. As a consequence, there will be a contraction of the region of equilibria where mixed financing is issued. Thus, the market-type of credit will decline:

Proposition 18 *A restrictive monetary policy reduces firm's access to market-type of financing.*

Proof. To see that the entire mixed financing region is shifted upwards, just observe that:

$$\frac{d\phi_1}{dr} = \frac{1}{8(1+r)^2} > 0$$

and

$$\frac{d\phi_2}{dr} = \frac{1}{16(1+r)^2} > 0$$

In addition, consider the magnitude of the reactions: for every change in risk-free interest rate, ϕ_1 shifts rightwards more than ϕ_2.

Consider now the aggregate market lending:

$$L_m^* = \int_{\phi_1}^{\phi_2} I_m^* dF(W) = \int_{\phi_1}^{\phi_2} \left(I - W - \frac{1}{16(1+r)} \right) dF(W)$$

The reaction of the aggregate market-type of financing following a restrictive monetary policy can be measured by:

$$\frac{dL_m^*}{dr} = \int_{\phi_1}^{\phi_2} \frac{dI_m^*}{dr} dF(W) + I_m^*(\phi_2, r)\frac{d\phi_2}{dr} - I_m^*(\phi_1, r)\frac{d\phi_1}{dr}$$

that is to say:

$$\frac{dL_m^*}{dr} = \frac{1}{16(1+r)^2}\left[F(\phi_2) - F(\phi_1)\right] - \frac{1}{128(1+r)^3}$$

Given the uniform distribution of borrowers on the $[0, 1]$ interval, we can evaluate the sign of the following expression:

$$\frac{dL_m^*}{dr} = \frac{1}{16(1+r)^2}\left[(\phi_2 - \phi_1) - \frac{1}{8(1+r)}\right]$$

which given $(\phi_2 - \phi_1) = \dfrac{1}{16(1+r)}$ will always be negative.

■

If r rises, then the region of mixed financing shifts rightwards and shrinks, i.e. the conditions to access external financing are more stringent and in particular, the economy experiments a contraction in the market-type of contracts.[19] Consider now the region where compensating equilibria are implemented. At equilibrium large firms are bank-financed; yet, banks are offering a positive repayment to the non-bank intermediaries. The main effect of such an offer is to keep the lender m out of the direct investment phase. In addition, the presence of competition leaves the firm a payoff that is higher than her (exogenous) reservation utility: the entrepreneur is getting some positive extra-surplus.

Restrictive monetary policies in these cases have a direct effect on the profits of banks given that all contractual decisions are unaffected. As r rises, the allocation (p, I_b, R_b, R_m) as given in Lemma 5 remains unchanged, hence a higher cost of funds' provision for the bank immediately affects his profits.

At the same time, tightening policies reduce the firm's share of the surplus. More precisely, the gap between the payoff that the borrower would get by buying b's contract and her reservation utility becomes smaller.

There exists a critical value of the risk-less interest rate $\bar{r}$ such that for every $r > \bar{r}$, the profits of lender b will become negative, and the credit market will collapse. The expression for $\bar{r}$ is given by: $\bar{r} = \dfrac{1}{8(I - W)} -$

[19] We also remark that the threshold γ_1 will also decrease, so that the aggregate feasible set will get smaller.

1.[20] Thus, whenever restrictive monetary policies pushed r above this threshold $\bar{r}$, banks would not lend and the financial market would break down.

At the level of the individual firm, movements in the interest rate change the composition of the financial structure for those firms which have access to both:

Proposition 19 *Whenever both market-type of finance and bank finance are available (i.e. for every $W \in [\phi_1, \phi_2]$), a tightening monetary policy changes the composition of firms' financial structure. It reduces the amount of investment financed through bank loans, and as a by-product, increases the amount of investment provided by non-bank financiers.*

Proof. We just have to consider the equilibrium expressions of I_b^* and I_m^* as defined in the proof of Lemma 4:

$$I_b^* = \frac{1}{16(1+r)}$$

and by complementarity,

$$I_m^* = I - W - \frac{1}{16(1+r)}.$$

■

Hence, a tightening monetary policy (i.e. a higher r) implies:

(1) a reduction in the aggregate quantity of lending, due to the contraction in the aggregate feasible set;

(2) a shift towards higher net worth borrowers in the allocation of credit: only highly collateralized firms will survive to recessionary policies, (*flight to quality*);

(3) at the level of individual firms, bank financing is substituted with market financing;

(4) finally, a decline in the aggregate supply of market financing.

With respect to previous literature, we emphasize here that the predictions of the credit channel integrate with strategic competition among lenders and equilibria with positive profits.

Modelling the competition between bank and non-bank intermediaries as a multi-principal game provides us with new possible characterizations of the credit channel. In particular, it turns out that the

[20] There is also a critical level of r with respect to the participation constraint of the borrower, but in this economy $\bar{r}$ is the lowest, hence the credit market crisis generates on the supply side.

structure of credit markets might be deeply affected by monetary interventions. If exclusive competition is considered, then tough policies may implement monopolistic outcomes. If non-exclusivity is explicitly introduced, then the amount of externalities among players become higher. As a consequence, restrictive measures could even induce a breakdown of the credit market.

Understanding the welfare properties of the equilibria of multi-principal games is regarded as a relevant issue in the recent developments in contract theory. The next chapter tries to contribute to this research direction further investigating the relationship between competition and incentives in credit markets.

Chapter 5

Multiple lending and constrained efficiency in the credit market

5.1 Introduction

The chapter is devoted to the analysis of credit markets where several lenders strategically compete over the contracts they offer to entrepreneur-borrowers. At the stage of contracting, the decision of the unique borrower crucially depends on the loans she is simultaneously receiving from all the lenders of the economy. We consider a set-up where the contracts are *non-exclusive*, i.e. the borrower is allowed to accept more than one contract at a time. The main aim of the work is to emphasize the interplay of contractual externalities in determining the welfare properties of the equilibria arising in such framework.

Exclusivity clauses are not explicitly imposed in several financial relationships. It is recognized that many firms have access to multiple credit sources, as shown by Petersen and Rajan (1995) for the small firms in the U.S., or by **?** for the Italian case. The credit card market has been proposed as an example of non-exclusive dealings, as Bizer and DeMarzo (1992) and Parlour and Rajan (2001) have already explained. Examples of financial interactions with non-exclusive contracting which aim at clarifying the relationship between incentives and competition are very recent: the main general results and implications are discussed by Segal and Whinston (2003).

The relevant finding of the literature on non-exclusivity can be summarized as follows: the contractual externalities emerging when several principals interact with one common agent, can be responsible for existence of *second-best* inefficient equilibria. In other words, a social planner who is subject to incentive constraints and feasibility can achieve outcomes that Pareto dominate the equilibrium outcomes of players' interactions.

Constrained inefficient equilibria have mostly been analyzed in insurance set-ups (Kahn and Mookherjee (1998), Bisin and Guaitoli (2004)). The present essay proposes an investigation on the welfare properties of equilibria in the credit market, where strategic competition is over financial contracts.

The theoretical contributions relating credit market imperfections and monetary policy have mostly used the principal-agent model as a tool to represent credit relationships. In that set-up, the agent designs the optimal mechanism, in general the contract, which is then proposed to the principal. The contract solving the agent's maximization problem has to satisfy incentive compatibility and participation constraints. The solution of the principal-agent model is equivalent to the outcome of a planner's maximization problem under exclusive contracting and subject to the same asymmetry of information, given appropriate welfare weights. Hence, all the equilibria of the contracting game are second-best efficient.

Having enriched the analysis of competition in the credit market using the common agency approach to financial contracting, the step we make with this essay is to analyze the welfare properties of the equilibria of the new game. This work could then be regarded as part of a research project on welfare foundations for policy intervention, in particular along the lines of the credit channel of monetary policy.

We propose a simple, static and partial equilibrium model of the credit market to study multiple credit relationships. We model competition between an arbitrarily large and finite number of lenders who offer credit lines to a single borrower. The borrower has to take some non-observable action; she can choose not to perform the hidden action and to divert resources for private use, in which case she will appropriate a fixed proportion of the received liquidity.[1] By exerting effort in the investment project, she will get some stochastic returns from the production technology. None of the lender is irrelevant, in the sense that every proposed loan affects the entrepreneur on the effort she will choose.

The possibility to sign several contracts simultaneously generates externalities among the financiers. These externalities are responsible for

[1] This model is indeed compatible with the literature on strategic default. For a detailed justification of such an approach to strategic default, refer to Parlour and Rajan (2001).

the emergence of equilibrium results which are not in line with Bertrand theory of competition. If the agency costs are high enough, competition among financiers delivers non-competitive results in the forms of credit rationing and of positive extra-profits at equilibrium.

The model is closely related to the Parlour and Rajan (2001)'s work on the credit cards market, which we regard as a useful benchmark to provide insights on the emergence of positive profit equilibria. We modified its structure to make it closer to a moral hazard framework, with binary effort choice. Therefore, the characterization of the equilibria mirrors the Parlour and Rajan (2001) results, and the contribution of this work relies on the welfare analysis of the credit market equilibria.

Examining this issue, we show that every positive-profit equilibrium is constrained Pareto efficient, i.e. it would be the outcome of the decision of a central authority subject to the same informational constraints. The result is not in line with the main findings of the existing literature, which has always emphasized that non-exclusive contracting generates those externalities which sustain constrained inefficient outcomes.[2]

In this example, we clarify how could the incentive structure eliminate the (constrained) inefficiency result. The crucial element turns out to be the preference structure of the borrower in case of default. Having linear preferences in case of default, the entrepreneur will tend to buy all contracts available when choosing to shirk. In an equilibrium where all existing lenders are active financiers in production and the incentive compatibility constraint is binding, there is no difference in the allocation implemented by the market interaction and by the social planner. The feasible sets of the two problems result to be the same.

Surprisingly, the same argument applies to the insurance literature. Whenever the assumption of risk-averse agents is removed in Bisin and Guaitoli (2004) and Kahn and Mookherjee (1998), those positive profits equilibria, which were the inefficient ones, collapse to allocations on the second-best frontier. While in insurance contexts it is widely accepted that the customer be risk-averse, in a lender-borrower interaction this assumption has often been relaxed.[3]

The discussion is organized in the following way: Section 5.2 presents the features of the set-up, which we elaborated on Parlour and Rajan (2001)'s model. Section 5.3 discusses the credit market equilibria. Section 5.4 provides the welfare analysis and the results on the efficiency of the positive-profit equilibria as compared to those obtained in the in-

[2] See Arnott and Stiglitz (1993), Kahn and Mookherjee (1998), Bisin and Guaitoli (2004).

[3] See for example, Parlour and Rajan (2001) or Holmstrom and Tirole (1997).

surance set-ups. Section 5.5 concludes. The Appendix contains all the proofs.

5.2 The model

Credit relationships are represented in a very simple way. The borrower is penniless, though she has access to the technology for the production of the only existing good. The production process is subject to random realizations: if the amount I is invested, with probability p the production successfully yields $G(I) > 0$, while with probability $(1-p)$ the outcome will be 0. The production function $G(I)$ is assumed to be continuous, increasing and strictly concave in I:

$$G'(I) > 0, G''(I) < 0$$

Inada conditions hold: $lim_{I \to 0} G'(I) = \infty$ and $lim_{I \to \infty} G'(I) = 0$.

There are $N \geq 2$ lenders (indexed by $i \in N = \{1, 2, ..., N\}$) who compete over the loan contracts simultaneously offered to a single borrower.

Having received all the contracts' proposals, the borrower decides which of them to sign, taking into account that she can accept any subset of them.[4] She must also take a non-contractible action (effort), that affects the set-up of the production activity and the successful repayment of the obtained loans. The effort can take only the two values e^H and e^L, with $e^H > e^L$. If the high effort e^H is chosen, production takes place and the borrower gets $G(I) - R$ with probability p and 0 otherwise (i.e. with probability $1-p$). If, on the contrary, the low effort e^L is taken she earns the private benefit BI with probability 1 and loans are not repaid.[5]

Let us describe the normal form of the game we are considering. Lenders strategically compete over financial contracts. The strategy of lender i is given by the choice of the contract C_i. The contract offer of lender i is defined by a repayment line R_i and a loan amount I_i, i.e.

[4]This defines a scenario of *delegated* common agency Martimort and Stole (2003)

[5]For the sake of precision, we are assuming that the space of events in this economy is made out of three elements: i.e. $Y = \{G(I), 0, BI\}$. The private choice of effort affects the probability distribution of these realizations. In particular, if the entrepreneur exerts high effort e^H, the probability vector is given by the triple: $\{p, 1-p, 0\}$ with $p > 0$. If instead the entrepreneur shirks, i.e. chooses e^L, the lottery is degenerate and equal to $\{0, 0, 1\}$.

$$C_i = (R_i, I_i) \in \mathcal{C}_i \subseteq \mathbb{R}^2{}_+$$

Given the space of feasible contract offers for each lender i, i.e. $\mathcal{C}_i$, we define the aggregate space of contracts in the loan sector as $\mathcal{C} = \times_i \mathcal{C}_i$. The borrower's strategy is therefore given by the map:

$$s_b : \mathcal{C} \to \{0,1\}^N \times \left\{e^H, e^L\right\}$$

With a small abuse of notation, we also define the generic element of the set $\{0,1\}^N$ as the array $a_b = \left(a_b^1, a_b^2, ..., a_b^N\right)$ where $a_b^i = \{0,1\}$ is the borrower's decision of rejecting or accepting lender i's offer.

The choice of the array a_b defines the set of accepted contracts $\mathcal{A}$:

$$\mathcal{A} = \left\{i \in N : a_b^i = 1\right\}$$

that is, borrower's decisions are identified by the choice of the effort level and by the definition of the relevant set $\mathcal{A}$. Her strategy set will be denoted as $\mathcal{S}_b$, that is: $s_b \in \mathcal{S}_b$.

Let us now consider the payoffs. The borrower's payoff is defined by:

$$\pi_b = \begin{cases} p\left[G(I) - R\right] & \text{if } e^H \text{ is chosen} \\ BI & \text{if } e^L \text{ is chosen} \end{cases}$$

where R and I denote the aggregate repayment and investment, respectively:

$$R = \sum_{i \in \mathcal{A}} R_i \text{ and } I = \sum_{i \in \mathcal{A}} I_i.$$

Lender i's payoff for every $i \in \{1, 2, ..., N\}$ is given by:

$$\pi_i = \begin{cases} pR_i - (1+r)I_i & \text{if } e^H \text{ is chosen} \\ -(1+r)I_i & \text{if } e^L \text{ is chosen} \end{cases}$$

whenever his contract is accepted and zero otherwise: $r \in \mathbb{R}_+$ is the lender's cost of collecting deposits.[6] Observe that lender i's payoff will not directly depend on lender j's strategies. Existence of contractual externalities among lenders is originated by the borrower's behavior only: at the stage of contracting with lender i, the action chosen by the borrower depends on the contractual offer she is receiving from lender j.[7]

[6]Lenders here do not have infinite endowment. They rely on the deposit market to finance entrepreneurial activity.

[7]This is usually referred to as the absence of *direct externalities* among principals. Most common agency models have been developed in such a simplified scenario. Examples of recent researches where direct externalities among principals are considered include Bernheim and Whinston (1998b) and Martimort and Stole (2003).

We can therefore model credit market interactions as a sequential game, with a first stage where several lenders play a simultaneous move game and a second stage where the borrower decides on acceptance/rejection of each offer and finally exerts effort. In formal terms, loan relationships are represented by the following complete information common agency game Γ:

$$\Gamma = \{(\pi_i)_{i \in N}, \pi_b, \mathcal{C}, \mathcal{S}_b\}$$

5.3 Credit market equilibria

This section discusses the properties of the Subgame Perfect Nash Equilibria of the game Γ. The characterization of these equilibria is preliminary to examining their welfare properties. The equilibria match those discussed by Parlour and Rajan (2001). Having presented the equilibria of the credit market we pass at characterizing the constrained Pareto frontier of this economy, and at showing that competition among lenders sustains here only second-best efficient allocations (equilibria).

We start by introducing the following assumption.

Assumption 4 *We consider $B < 1$.*

Observe that given Assumption 4, the choice of low action determines a social loss of $BI - I(1 + r)$, where I is the aggregate level of investment. Hence, there cannot be any equilibrium in which this low action is implemented. We hence focus our attention on those situations where e^H is implemented. That is, at equilibrium the borrower will be given incentives not to undertake the low action. The relevant Incentive Compatibility constraint will therefore be:

$$p\left[G\left(\sum_{i \in \mathcal{A}} I_i\right) - \sum_{i \in \mathcal{A}} R_i\right] \geq B \sum_{i=1}^{N} I_i \tag{5.1}$$

Notice that if the low action is chosen, the borrower always has the incentive to accept the whole array of offered contracts.[8] This greatly simplifies the incentive analysis.

[8]The incentive compatibility constraint controls for the incentive on aggregate default, which is the only relevant case due to the monotonicity of the borrower's payoff in this case.

The investment level that maximizes the aggregate surplus defines the *first-best* I^*. If the aggregate surplus is given by $S = \pi_b + \sum_i^N \pi_i$, then

$$I^* = \underset{I}{argmax} \;\; S \equiv \;\; \underset{I}{argmax} \;\; pG(I) - (1+r)I$$

where I^* is such that $pG'(I^*) = 1 + r$ and that the corresponding surplus is positive.[9]

If there were no incentive problem (i.e. if the borrower were not taking any hidden action), then every equilibrium would involve the *first-best* amount of lending I^*. When strategic behavior of the borrower is considered under the additional assumption of exclusive contracting, i.e. when we explicitly consider incentive constraints but we further assume that the borrower can only accept one contract at a time, then lenders compete à la Bertrand over contracts; at equilibrium they get zero profits and the borrower appropriates the whole surplus.[10]

If we allow for non-exclusive contracting, then we formally enter into a common agency set-up. Given the high degree of externalities involved in the analysis, positive profits equilibria and low levels of aggregate investment are a typical feature, in general. In our model we can sustain zero-profit equilibria with competition among lenders offering non-exclusive contracts for those parameter values which make the moral hazard problem very mild. Before going further, let us define the relevant equilibrium concept for the game Γ.

Definition 11 *A (pure strategy) equilibrium of the game* Γ *is an array* $[\left(\tilde{R}_i, \tilde{I}_i\right)_{i\in N}, (\tilde{a}_b^i)_{i\in N}, p]$ *such that:*

- *the borrower is optimally choosing the set of accepted contracts* $\mathcal{A}$ *(i.e. she is choosing her optimal array* $a_b \in \{0,1\}^N$ *) and implementing the high level of effort;*
- *for every lender* $i \in \{1, 2, ..., N\}$*, the pair* $\left(\tilde{R}_i, \tilde{I}_i\right)$ *is a solution to the following problem:*

$$\max_{R_i, I_i} \; p\, R_i - (1+r)I_i$$

s.t.

$$p\left[G\left(\sum_{j\neq i} \tilde{I}_j \tilde{a}_b^j + I_i \tilde{a}_b^i\right) - \left(\sum_{j\neq i} \tilde{R}_j \tilde{a}_b^j + R_i \tilde{a}_b^i\right)\right] \geq B\left(\sum_{j=1}^{N} \tilde{I}_j \tilde{a}_b^j + I_i \tilde{a}_b^i\right)$$

[9] Inada conditions guarantee the existence of such an I^*.

[10] The mechanism of undercutting on each opponent's proposal squeezes any possible rent for the lenders. There cannot be an equilibrium with positive profits for the lenders, given they are all identical and the entrepreneur chooses only one proposal.

This inequality is the borrower's Incentive Compatibility constraint and it is formulated in terms of aggregate investment and aggregate revenues. The borrower has no endowment, and her exogenous reservation utility is zero so that her participation decision will always be satisfied. This constraint defines lender i's set of feasible contracts under non-exclusivity.

We can characterize equilibrium allocations in terms of the incentive parameter B. More precisely, we introduce the threshold value B_z, which defines the lowest level of incentives compatible with the *first-best* level of investment:

$$B_z = \frac{pG(I^*) - (1+r)I^*}{I^*} \tag{5.2}$$

Fig.5.1 identifies B_z using the total surplus hump-shaped curve, $S = pG(I) - (1+r)I$, and straight lines starting from the origin with slope equal to B. Equation (5.2) implies that B_z is the slope of the ray that cuts the surplus' curve at its maximum.

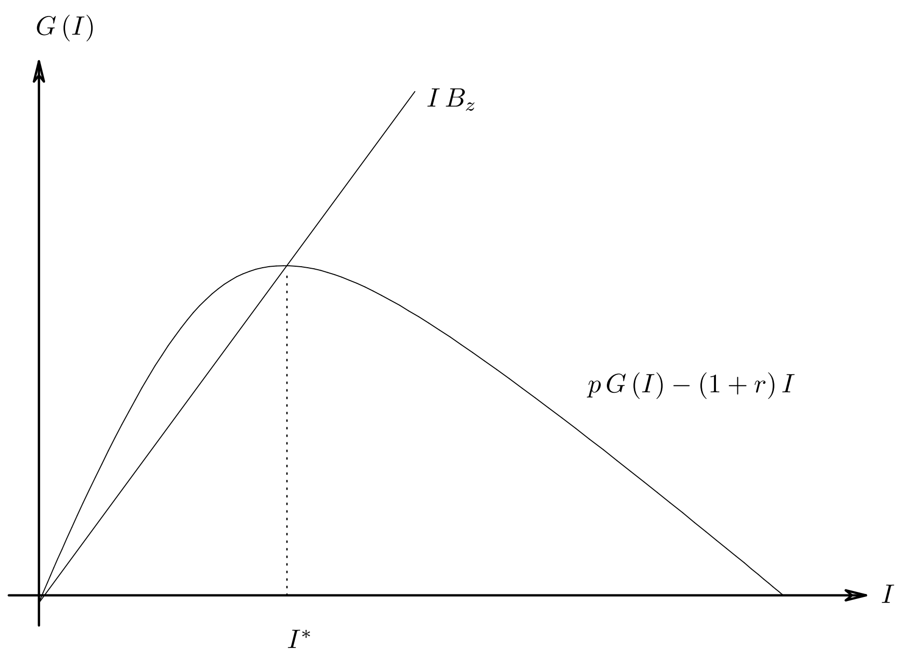

Figure 5.1: Graphical representation of B_z

If $B = B_z$, then the first-best investment I^* is feasible and the Incentive Compatibility constraint is binding. From equation (5.2) it follows straightforwardly that if I^* is implemented, then the borrower gets the entire surplus. Lenders' profits are equal to zero in the aggregate and the corresponding aggregate repayment will be R^* such that $pR^* - (1+r)\, I^* = 0$.[11]

Whenever $B > B_z$ allocations giving zero-profits to lenders can be sustained only with a level of debt lower than I^*.[12] We denote this level of aggregate investment $\bar{I}(B) < I^*$. On the contrary, if $B < B_z$, it is then possible to achieve I^* and to leave at the same time to leave some extra-surplus to lenders.

We denote $I_B = \min \left\{ \bar{I}(B), I^* \right\}$ the highest level of investment that is at the same time feasible and such to guarantee to the borrower the full appropriation of the social surplus.

If $B > B_z$, then $\bar{I}(B)$ is the maximum incentive compatible level of aggregate investment. If $B < B_z$, the intersection of the corresponding straight line from the origin with the curve S will be on the right-hand side of the first-best level of investment. Hence, I^* will always be feasible and the maximum social surplus will be achieved.

As will be clear from the following discussion, equilibria exhibit different features according to the magnitude of the parameter B relative to the threshold B_z.

5.3.1 Equilibria with zero profits for the financiers

When the incentive to undertake a low action is small enough, the impact of the asymmetric information problem is reduced and it is possible to show that only a Bertrand outcome can be sustained.[13] In such a situation, there are at least to lenders $i, j \in \{1, 2, ..., N\}$ with $i \neq j$ offering the contract $\mathcal{C}_i = \mathcal{C}_j = (R^*, I^*)$ where $R^* = \frac{I^*(1+r)}{p}$ and gives them zero extra profits. The first best level of investment is achieved. This is stated in the following:

Proposition 20 *Denote $B_c = \frac{p\,G(I^*) - I^*(1+r)}{2I^*}$. Whenever $B \leq B_c$, then the outcome (R^*, I^*) can be supported as a (pure strategy) equilibrium of the game Γ.*

Proof. We consider the following array of offered contracts:

[11] This of course implies that every lender earns zero profit, given that their condition is symmetric and limited liability holds.

[12] That is, the first-best investment level I^* cannot be implemented.

[13] We refer to a Bertrand outcome as to a perfectly competitive outcome.

$$\{(R_i, I_i) = (R_j, I_j) = (R^*, I^*) \quad \text{for } i \neq j; \quad (R_k, I_k) = (0,0) \;\; \forall\, k \neq i, j\} \tag{A.1}$$

That is, there are two lenders, say lender i and lender j who offer the first-best allocation, while all other lenders are offering the null contract $(0,0)$. The borrower is indifferent between accepting the $i-th$ and the $j-th$ contract; given that $B \leq B_c$, accepting all contracts and choosing low action is never a best reply.
In such a scenario, no lender has a profitable deviation given that the first-best outcome is implemented and the borrower's profit is maximized. ∎

The intuition for the result is the following: consider a scenario where $(N-2)$ lenders are not active, while each of the remaining two can offer a contract associated to a level of debt equal to I^*. Notice that by definition, $2B_c = B_z$: this guarantees that two lenders are enough to sustain the zero-profit equilibrium. If $B = B_c$, then the borrower is indifferent between accepting any of the two contracts exerting the desired effort, and accepting both of them taking zero effort. As long as any single lender $k = i, j$ offers a contract different from the zero-profit one $\left(R_k = \frac{I^*(1+r)}{p}, I_k = I^*\right)$, a Bertrand argument applies: the two-lenders competition generates undercutting on each other's offers until the marginal cost of funds meets marginal revenues.
One should note that the competitive result emerging in a scenario of exclusivity can hence be implemented even without imposing exclusivity clauses. If the incentive to shirk is low, then, despite the high amount of externalities associated to competition under non-exclusivity, the first best outcome can be reached.
If the incentive to take the low action falls between B_c and B_z, then zero-profits equilibria may arise only if N is large enough. The intuition is the following: consider a scenario where $B < B_z$ and $(N-1)$ lenders are offering the contract (R_i, I_i), where $I_i = \frac{I^*}{N-1}$ and $R_i = \frac{I^*(1+r)}{(N-1)p}$ is the repayment level that guarantees zero-profits to the i-th lender when offering the loan amount $\frac{I^*}{N-1}$. The borrower is accepting all of them and implementing the high level of effort. There is therefore room for the n-th lender to offer the zero profits contract (R_n, I_n); if this offer is accepted, then I^* can in principle be implemented. The closer is B to B_z, the higher the number of lenders $(N-1)$ that is necessary to guarantee that the offer of the n-th lender will be feasible. Formally, the same is stated in the following proposition:

Proposition 21 *If $B \in (B_c, B_z)$, then there exists a critical number of lenders N_B such that for all $N > N_B$ the aggregate allocation (R^*, I^*) is an equilibrium outcome.*

Proof. The proof is discussed in the Appendix. ■

5.3.2 Equilibria with positive profits

If we consider the case $B > B_z$, then positive profits equilibria are a general feature of the analysis. These equilibria are such that every lender is active in the market, though the aggregate investment level turns out to be strictly lower than I^*. Competition over financial contracts and moral hazard determine rationing in credit supply and redistribution towards the intermediation sector.

Whenever $B > B_z$, we are in the increasing part of the social surplus function $S = p\,G\,(I) - (1+r)I$ represented in Fig. 5.1. As a consequence, a single lender i offering a zero-profit contract can profitably deviate if all the others are playing a zero-profit strategy: a Bertrand outcome cannot be sustained at equilibrium.

In particular, we are able to show the existence of a (symmetric) positive-profit equilibrium where all lenders are active: each of them offers the same amount of loan and the same repayment, $\tilde{\mathcal{C}} = \left(\tilde{I}, \tilde{R}\right)$. Existence of this equilibrium is established in the following proposition:

Proposition 22 *If $B \in [B_z, B_m)$, then there is a critical number of lenders N_B such that for every $N \geq N_B$, there exist a positive profit equilibrium. The equilibrium outcome $\left(N\tilde{R}, N\tilde{I}\right)$ is characterized by the following set of equations:*

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = p\left[G\left((N-1)\tilde{I}\right) - (N-1)\tilde{R}\right] \tag{5.3}$$

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = BN\tilde{I} \tag{5.4}$$

and exhibits the feature that:

$$(N-1)\tilde{I} > I_m \tag{5.5}$$

Proof. The proof is given in the Appendix. ■

Notice that B_m and I_m are the threshold level of the incentive parameter above which a monopolistic equilibrium can be sustained, and the monopolistic level of investment, respectively.[14]

At equilibrium, all the N existing lenders are active and the borrower is indifferent between accepting $(N-1)$ or N contracts while exerting high effort, as shown by (5.3). This *no-side-contracting* condition is crucial to establish existence of equilibria with positive profits in several works on moral hazard in insurance economies because it prevents additional purchases of insurance.[15] Furthermore, when the borrower accepts N contracts, her Incentive Compatibility constraint (5.4) will bind. Finally, the aggregate level of credit issued by $(N-1)$ lenders is strictly greater than I_m that corresponds to the investment chosen by one monopolistic lender (5.5).

Equilibria with positive profit may also emerge when the incentive to take low action is relatively small. In such a case, the first-best level of investment I^* will be achieved but the distribution of the total surplus will be rather favorable to the lenders. These equilibria can be shown to exist when B is below the threshold:

$$B_l = \frac{p\,G\,(I_m) - I_m(1+r)}{I^* + I_m}$$

that is smaller than B_z.[16] They are sustained by latent contracts, i.e. contracts which are not bought at equilibrium and are used to deter entry. Existence of such equilibria is stated in the following:

Proposition 23 *For every $B \in (B_c, B_l]$, there exists a pure strategy equilibrium where only one contract (say, $\mathcal{C}_i$) is bought. The contract guarantees a positive profit to lender i. Furthermore, there is a second lender (say, lender j) who offers a zero profit contract that is not accepted. All other lenders are not active.*

Proof. The proof is given in the Appendix. ■

The equilibrium of Proposition 23 is sustained by a latent contract, i.e. a contract not traded at equilibrium but used as a device to deter

[14]In fact, every monopolistic level of investment is an implicit function of the incentive parameter of the corresponding problem. In this case, $I_m(B)$ for every $B \in [B_z, B_m)$ is the relevant set of monopolistic investment levels.

[15]See Bisin and Guaitoli (2004) and Kahn and Mookherjee (1998)

[16]This does not contradict what we said before. it simply means that for some parameter's values, there is multiplicity of equilibria.

potential entrants. The analysis of these sort of equilibria has been introduced in **?** and developed by Bisin and Guaitoli (2004).

The main concern of this note is to characterize the welfare properties of credit market equilibria when multiple lenders compete over loan contracts. The next section will therefore provide a welfare analysis of the equilibrium outcomes associated to the game Γ.

5.4 Welfare analysis

We will provide here a description of the economy's feasible set, that is the set of players' payoffs corresponding to the allocations implementable by a social planner. We introduce the notion of social planner and the related concept of *constrained* efficiency in the same way as it is done in the literature on incentives in competitive markets (see for instance Bisin and Guaitoli (2004)), but we manage to characterize the whole constrained Pareto frontier. The social planner will choose the aggregate investment level I and the aggregate repayment R to maximize his preferences over the aggregate feasible set that is usually denoted the *utility possibility set*.[17]

We will henceforth denote π_L the payoff earned by lenders in the aggregate credit sector and π_b the corresponding borrower's payoff. Let us start considering the first-best situation, where the relevant constraints faced by the planner are those imposed by technology and resources (together with limited liability requirements). The corresponding utility possibility set is:

$$\mathcal{F}(\pi_L, \pi_b) = \left\{(\pi_L, \pi_b) \in \mathbb{R}^2_+ : \pi_L + \pi_b \leq pG(I^*) - I^*(1+r)\right\} \tag{5.6}$$

The frontier of the set $\mathcal{F}$ is referred to as the first-best Pareto frontier. All the arrays (π_L, π_b) belonging to this Pareto frontier are such that there does not exist a pair $(\pi'_L, \pi'_b) \in \mathcal{F}$ with $\pi'_L \geq \pi_L$ and $\pi'_b > \pi_b$ or $\pi'_L > \pi_L$ and $\pi'_b \geq \pi_b$.

Observe that the payoffs functions $\pi_L(R, I)$ and $\pi_b(R, I)$ evaluated at the high level of effort are both linear in the aggregate repayment R. As a consequence, the first-best Pareto frontier will be a downward-sloping

[17] In particular, given that the N lenders are homogeneous, the *social welfare function* will be a weighted sum of the payoffs of the N lenders and of the borrower.

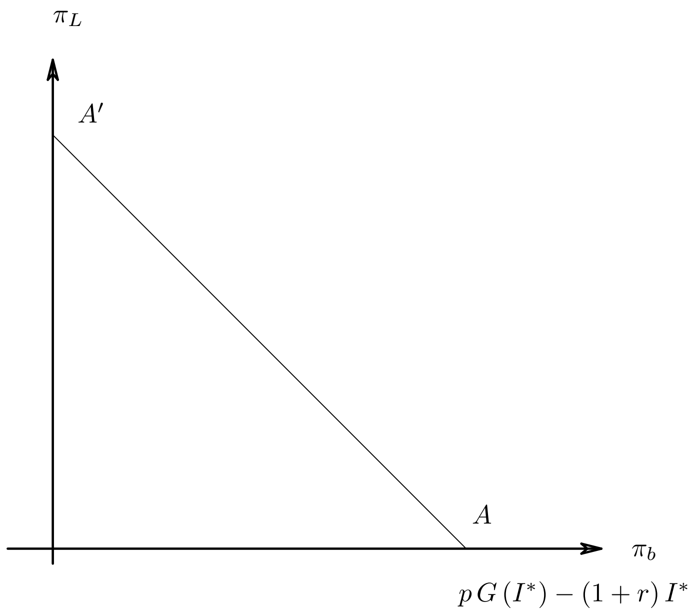

Figure 5.2: The first-best Pareto frontier

45-degree line. By using the variable pR as a transfer, we can draw the first-best Pareto frontier $\pi_L^*(\pi_b^*)$ as the line depicted in Figure 5.2.

Every point on the frontier corresponds to the optimal investment level I^*. In particular, point A identifies a situation where the whole surplus is distributed to the borrower, $\pi_b^* = pG\,(I^*) - (1+r)I^*$, so that $p\,R = (1+r)I^*$, i.e. $\pi_L^* = 0$. On the contrary, if $\pi_b^* = 0$ then from (5.6) we get $pR = pG\,(I^*)$, i.e. lenders are receiving everything and the borrower is left at her reservation utility of zero (point A').

Let us now define the second-best allocations, i.e. the set of allocations implementable by a planner who is facing informational constraints. The *constrained utility possibility set* is the set of outcomes (π_L, π_b) such that:

$$\mathcal{F}'\,(\pi_L, \pi_b) = \left\{(\pi_L, \pi_b) \in \mathbb{R}_+^2 : \pi_L \leq \pi_L^{**}\,(\pi_b^{**}, B)\,, \quad \pi_b \leq \pi_b^{**}\right\}$$

for each $\pi_b^{**} \in [0, pG\,(I^*) - I^*(1+r)]$. One should note that $\pi_L^{**}\,((\pi_b)^{**}, B)$ is such that:

$$\pi_L^{**}(\pi_b^{**}, B) = \max_{R,I} pR - (1+r)I \tag{5.7}$$

s.t.

$$pR - (1+r)I + \pi_b^{**} \leq pG(I) - (1+r)I \tag{5.8}$$

$$\pi_b^{**} \geq BI \tag{5.9}$$

With respect to the first-best problem, we have introduced here the Incentive Compatibility requirement appearing in equation (5.9). Observe that for a given π_b^{**}, the lender's objective function is monotone in R, hence equation (5.8) will bind at the optimum. We can therefore substitute the expression for pR obtained in (5.8), in the objective function. The system (5.7)-(5.9) can be rewritten as:

$$\pi_L^{**}(\pi_b^{**}, B) = \max_I p\,G(I) - \pi_b^{**} - (1+r)I \tag{5.10}$$

s.t.

$$\pi_b^{**} \geq BI \tag{5.11}$$

One should notice that the *constrained utility possibility set* and the second-best Pareto frontier are parameterized in the incentive structure B. Recall that we defined B_z as the level of the incentive parameter such that:

$$pG(I^*) - (1+r)I^* = B_z I^*$$

implying that $pR = (1+r)I^*$, i.e. lenders make zero profits. Hence,

$$\forall\, B < B_z \quad pG(I^*) - (1+r)I^* < BI^*$$

That is, constraint (5.11) is slack and the first-best is feasible in the second-best problem. The point $(\pi_b^{**}, \pi_L^{**}) = (pG(I^*) - (1+r)I^*, 0)$ belongs to the second-best Pareto frontier (point A in Figure 5.3). Hence given $B < B_z$, there is room to reduce π_b^{**} without making the constraint (5.11) binding. There will therefore be an interval of entrepreneur's utilities, i.e. $\pi_b^{**} \in [BI^*, p\,G(I^*) - I^*(1+r)]$, such that the second-best Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$ coincides with the first-best one $\pi_L^*(\pi_b^*)$ (Figure 5.3). By further reducing the entrepreneur's payoff we get to $\pi_b^{**} = BI^*$ and $\pi_L^{**} = p\,G(I^*) - I^*(1+r) - BI^*$. Every further reduction in π_b^{**} will imply a decrease in the investment level.

If we consider the case $B > B_z$, equation (5.11) will always be binding at the optimum level of investment, hence it is not possible to sustain the first-best investment level I^*. As a consequence, for every $B > B_z$

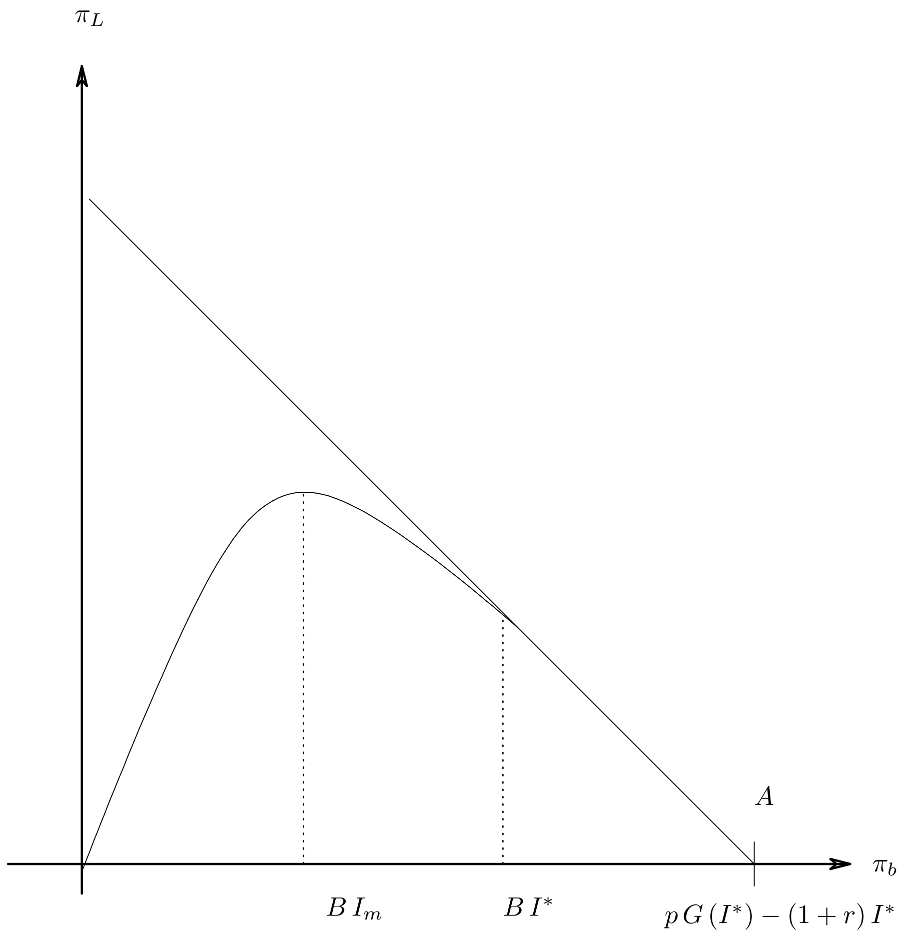

Figure 5.3: The first and second-best Pareto frontiers for $B < B_z$

the second-best frontier $\pi_L^{**}(\pi_b^{**}, B)$ will always lie below the first-best one, as it is depicted in Figure 5.4.

Hence, while for the cases of relatively mild incentive problem the second-best frontier has a linear part, that corresponds to the implementation of the first-best level of investment, when the moral hazard becomes harsher the frontier contracts inwards.

No matter the value of B, the highest possible payoff for the lending sector corresponds to the monopolistic allocation, when the entrepreneur is squeezed to a payoff of $\pi_b^{**} = BI_m$ and the lenders appropriate all the

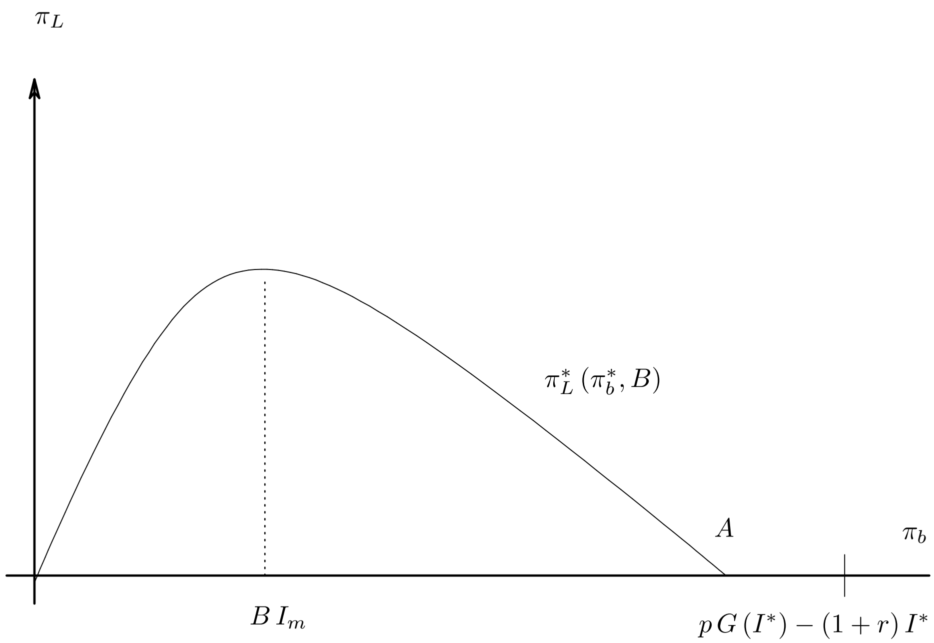

Figure 5.4: The second-best Pareto frontier for $B > B_z$

rest.[18] Whenever $\pi_b^{**} < BI_m$ every reduction in π_b^{**} calls for a reduction in π_L^{**}. In the limit the only way to set $\pi_b^{**} = 0$ is to fix an investment level equal to zero, so that there will not be anything left for the lenders, either. Finally, we argue that the concavity of $G(I)$ induces a concavity in the second-best Pareto frontier (Figure 5.3 and Figure 5.4).

Lemma 7 *Take any $B \in [0,1]$ then for every $\pi_b^{**} \leq BI^*$ the frontier $\pi_L^{**}(\pi_b^{**}, B)$ is a concave curve. In particular, $\pi_L^{**}(\pi_b^{**}, B)$ has a maximum in $\pi_b^{**} = BI_m$. For every $\pi_b^{**} < BI_m$, $\pi_L^{**}(\pi_b^{**}, B)$ is monotonically increasing.*

Proof. If (5.11) is not binding, we are back to the linear part of the frontier, which is trivially concave. The interesting case is that of a binding incentive compatibility constraint (5.11). Given π_b^{**} and B, then I is uniquely determined and given by $I = \frac{\pi_b^{**}}{B}$. As a consequence, we get:

[18] Recall that every monopolistic investment depends on the value of the incentive parameter, hence it should be written $I_m(B)$.

$$\pi_L^{**}(\pi_b^{**}, B) = pG\left(\frac{\pi_b^{**}}{B}\right) - \frac{\pi_b^{**}}{B}(1+r) - \pi_b^{**}$$

that is a strictly concave function of π_b^{**}. In particular, for $B > B_z$ the second-best Pareto frontier is strictly concave. ■

Defining the constrained Pareto frontier of the economy gives us more intuitions about the welfare implications of competition over loan contracts. The existence of positive profits equilibria and some form of rationing in credit markets where an arbitrarily large number of homogeneous lenders is competing, turn out to be the by-product of the competitive process itself under asymmetric information. In such circumstances, a planner facing the same informational constraints as the lenders, cannot implement Pareto-dominant allocations with respect to the equilibrium outcomes of the strategic interactions between N lenders and a single borrower.

The equilibria with positive profits and latent contracts described in Proposition 23 fall in the region where the incentive levels satisfy $B < B_z$ and it is always possible to sustain the first-best level of investment I^* together with $\pi_b^{**} > BI^*$. Hence, the latent contracts are just a device for a different sharing of the surplus. The equilibrium level of investment would be the same that a social planner would choose when solving (5.10)-(5.11) with a slack incentive compatibility constraint. This equilibrium allocation would correspond to a point on the linear part of the second-best Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$ where it coincides with the first-best one.

With respect to the efficiency properties of the equilibria described in Proposition 22 we state the following:

Proposition 24 *Take a $B > B_z$ and consider the positive profits equilibrium defined in Proposition 22. Then, if we denote as $\tilde{\pi}_b$ and $\tilde{\pi}_L$ the payoffs earned by the single borrower and by all the lenders, respectively, we have that the pair $(\tilde{\pi}_b, \tilde{\pi}_L)$ belongs to the constrained Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$.*

Proof. We first introduce a useful definition. Assume that the borrower earns $\tilde{\pi}_b$ in the positive-profits equilibrium, we denote $\tilde{\pi}_L(\tilde{\pi}_b)$ the lenders' payoff induced by $\tilde{\pi}_b$ at equilibrium.
Let us now take $\pi_b^{**} = \tilde{\pi}_b$ and construct the equilibrium relationship $\tilde{\pi}_L(\tilde{\pi}_b)$. In the positive-profits equilibrium defined by (5.3)-(5.5) each lender offers the same contract $(\tilde{I}, \tilde{R})$ and in the aggregate the borrower buys all contracts and exerts high effort. The borrower is indifferent

between accepting N or $(N-1)$ contracts. Let us call I^A the amount of credit issued and pR^A the expected revenues of the lenders. Given that the Incentive Compatibility constraint is binding in this equilibrium, we then have $\pi_b^{**} = \tilde{\pi}_b = BN\tilde{I}$, that implies:

$$I^A = \frac{\tilde{\pi}_b}{B} = \frac{\pi_b^{**}}{B} \tag{5.12}$$

where we denoted $I^A = N\tilde{I}$.
Given the borrower payoff $\tilde{\pi}_b$ and the incentive parameter B, the aggregate investment level I^A that supports $\tilde{\pi}_b$ at equilibrium is uniquely determined. In particular, the Incentive Compatibility constraint of the equilibrium defines the same level of aggregate investment of the second-best problem. This investment level I^A determines the aggregate surplus of the economy as:

$$S^A = pG(I^A) - (1+r)I^A \tag{5.13}$$

and the lenders' payoff once deduced the borrower's utility π_b^{**}:

$$\pi_L^{**}\left(\pi_b^{**}\right) = S^A - \pi_b^{**} = p\,G\left(I^A\right) - (1+r)I^A - BI^A \tag{5.14}$$

Notice that the payoff the credit sector earns is strictly positive:

$$\pi_L^{**}\left(\pi_b^{**}\right) = pR^A - (1+r)I^A > 0 \tag{5.15}$$

In particular, the system of equations (5.12)-(5.14) identifies a pair (π_L^{**}, π_b^{**}) belonging to the frontier of the *constrained utility possibility set* $\mathcal{F}'\left(\pi_b, \pi_L\right)$.
■

The result can be summarized as follows: if the agent's preferences exhibit risk neutrality, even for just one of the effort choices, the common agency framework delivers equilibria which are constrained Pareto efficient. Despite the externalities due to strategic competition over financial contracts, the incentive structure remains unaffected when the preferences of borrower exhibits some risk neutrality.

The equilibrium described in Proposition 22 represents the case in which all the existing lenders of the economy are actively financing production, and are making positive profits. Equation (5.3) and (5.5) inhibit individually profitable deviations of every active lender by keeping the borrower indifferent between buying N or $N-1$ contracts and by assuming that the aggregate level of investment be greater than the monopolistic one with just $N-1$ contracts.

In equilibrium the Incentive Compatibility constraint (5.4) is binding, and constrained-inefficient outcomes cannot be sustained. In particular, (5.4) determines the aggregate level of investment for every given payoff of the entrepreneur, and (5.5) only serves as a redistributive rule within the lending sector.

With respect to the existing literature, it sheds light on some implicit relationship between the welfare considerations on competition under asymmetric information (inefficiency) and preference structures.[19]

5.5 Conclusion

This work aimed at analyzing the relationship between incentives and competition in a context with one agent interacting with several principals. With respect to the previous literature, we found out that there exists a link between non-exclusive contracts, strictly concave preferences and the very definition of constrained inefficiency. The work suggests that there are still directions to explore when dealing with common agency games.[20]

The main objective was to characterize the welfare properties of equilibria in which several lenders were simultaneously financing an entrepreneurial activity, and making positive profits. We analyzed two different type of positive-profit equilibria: one supported by inactive lenders (*latent contracts*), and another one where all existing lenders result active financiers at equilibrium. We have been able to show that all these positive-profit equilibria are constrained efficient.

Future research should deal with realized default, excluded by assumption in this model, so to understand completely the link between the literature on constrained inefficiency in insurance context and in strategic default models. It would also be nice for policy analysis, to observe how the co-existence of several external financiers changes the incentive to strategic default in case of contraction in the credit supply.

[19]One should notice that the strict concavity of the customer's preferences is crucial to sustain the inefficient result in Bisin and Guaitoli (2004), so that weakening concavity in that model would make the inefficient equilibrium collapse to the second-best. Though risk aversion a natural assumption in insurance models, most of the banking literature typically considers risk neutral entrepreneurs/borrowers.

[20]Notice that this research does not tackle at all the issue equilibria in direct versus indirect mechanism games, in the lines of Martimort and Stole (2002), Peters (2001).

5.6 Appendix

Proof of Proposition 21 in the text.
Consider the case of $B \in (B_c, B_z)$ and a given number of lenders N. If every lender offers the contract $(R', I') = \left(\frac{R^*}{N-1}, \frac{I^*}{N-1}\right)$, it is Incentive Compatible for the borrower to accept $(N-1)$ contracts and exert high effort. We want to show that these prescriptions (strategies) for each lender and for the borrower constitute an equilibrium of the game Γ.
Notice that in the case we described, the borrower obtains the first-best aggregate level of investment buying $(N-1)$ contracts, attaining her maximum expected payoff and each single lender gets zero profits. Let us evaluate if there exist profitable deviations.
Given what his opponents offer, lender i can never propose a loan that the borrower will accept and guarantees herself positive profits. When all $j \neq i$ lenders offer (R', I'), whatever lender i proposes, the borrower can always buy the remaining $(N-1)$ contracts and achieve her maximum payoff. Hence, it is a best response for lender i to offer (R', I') when all other lenders offer (R', I').
To guarantee that the borrower has no profitable deviations, we have to eliminate incentives to shirk:

$$pG\left(\frac{N-1}{N-1}I^*\right) - (N-1)\frac{I^*(1+r)}{N-1} \geq BN\frac{I^*}{N-1} \tag{A.3}$$

that is, we want that the utility she gets from buying all N contracts and exerting low action be lower than the first-best payoff:

$$pG\left(I^*\right) - I^*(1+r) \geq BN\frac{I^*}{N-1} \tag{A.4}$$

This translates onto an incentive parameter that satisfies the following:

$$B \leq \frac{pG\left(I^*\right) - I^*(1+r)}{I^*\frac{N}{N-1}} \tag{A.5}$$

Let us define $B_N = \frac{pG(I^*)-I^*}{I^*\frac{N}{N-1}}$. As N increases, $B_N \to B_z$. Hence for every $B \in (B_c, B_z)$ there exists a N_B such that for every $N > N_B$, $B \leq B_N$ and the borrower has no incentive to deviate from buying $(N-1)$ contracts and choosing p. There does not exist any contract for any lender i that gives him positive profits and is accepted by the borrower. Hence, (R', I') for each lender and the borrower accepting $(N-1)$ contracts and exerting high effort constitute an equilibrium. ■

Proof of Proposition 22 in the text.

The proof is organized in two steps. First, we show that there is an aggregate contract $\left(N\tilde{R}, N\tilde{I}\right)$ which is a solution of the system (5.3)-(5.4) and satisfies (5.5). In a next step we show that the strategy profile $(R_i, I_i) = \left(\tilde{R}, \tilde{I}\right)$ for every lender $i = 1, 2, ...N$ together with the borrower decision of accepting all contracts and choosing the high level effort is a subgame perfect equilibrium of the game Γ.
Considering (5.3) and (5.4) together we get:

$$p\left[G\left((N-1)\tilde{I}\right) - \left(1 - \frac{1}{N}\right) G\left(N\tilde{I}\right)\right] - B\tilde{I} = 0 \tag{A.6}$$

We define $f\left(I\,|N, B\right) = p\left[G\left((N-1)I\right) - \left(1 - \frac{1}{N}\right) G(NI)\right] - BI$.
Observe that given the system (5.3)-(5.5), we are considering aggregate investment level that should belong to the interval $[I_B, I_m]$, where $I_B = \min\left\{\bar{I}(B), I^*\right\}$ and I_m represents the monopoly level of investment for the appropriate interval. Now, we also denote $I_B^o \equiv \frac{I_B}{N}$. Evaluating the function $f(I\,|N, B)$ at I_B^o, we obtain:

$$f\left(I_B^o\right) = pG\left(\frac{(N-1)}{N} I_B\right) - pG\left(I_B\right) - \frac{1+r}{N} I_B \tag{A.7}$$

Given that the function G(.) is concave and recalling that $G'\left(I_B\right) > 1 + r$, we have that

$$pG(I_B) - \frac{(1+r)}{N} I_B > p\left[G\left(\frac{(N-1)I_B}{N}\right)\right] \tag{A.8}$$

and $f\left(I_B^o\right) < 0$.
Using a similar argument, and recalling the definition of B_m we can check that for every $B < B_m$ there exist an N_B large enough such that $\forall N \geq N_B$ we get:

$$f\left(\frac{I_m}{N-1}\right) > 0 \tag{A.9}$$

By continuity of $f(.)$, for every $N \geq N_B$ there exists a value $\tilde{I}(B, N)$ such that $f\left(\tilde{I}\right) = 0$. Given $\tilde{I}$, the value of R satisfying (5.3)-(5.4) can be defined in a direct way.
Now, we have to show that at equilibrium every lender will offer the contract $\left(\tilde{R}, \tilde{I}\right)$ and that the borrower will always have an incentive to accept all contracts and to implement the high action.
Let us start with the borrower's behavior: if each lender is playing $(\tilde{R}, \tilde{I})$, then the borrower's strategy of accepting N contracts and exerting effort

is a best reply. Equations (5.3) and (5.4) guarantee that when $\left(N\tilde{R}, N\tilde{I}\right)$ is offered in the aggregate, then the borrower cannot deviate by accepting $(N-1)$ contracts and shirking. This means that she is not in the decreasing part of her payoff function, so that no deviation involving reductions in the number of accepted contracts will be profitable. In particular, accepting N contracts will be a best reply.
Let us consider now the behavior of the N lenders. Suppose all $(N-1)$ lenders except lender i offer $\left(\tilde{R}, \tilde{I}\right)$ and consider lender i's best response. Assume lender i offers (R_i, I_i), his best payoff can be measured with respect to the aggregate amount of loans the borrower takes up:

$$\begin{aligned} \pi_i &= pR_i - (1+r)I_i = \\ &= pG(k\tilde{I} + I_i) - (1+r)I_i - kp\tilde{R} - \\ &\quad - \max\left\{pG((N-1)\tilde{I}) - p(N-1)\tilde{R}, B\left((N-1)\tilde{I} + I_i\right)\right\} \end{aligned} \tag{A.10}$$

where π_i is lender i's payoff as a function of (R_i, I_i) and $k = \{0, 1, 2, ..., N-1\}$ is the number of contracts the entrepreneur buys together with the i-th. On the right hand side of the equation we represented the surplus at the aggregate amount of investment $(k\tilde{I} + I_i)$ net of the reimbursements of the k lenders offering $\left(\tilde{R}, \tilde{I}\right)$ and of the entrepreneur's utility. The entrepreneur can obtain $pG\left((N-1)\tilde{I}\right) - p(N-1)\tilde{R}$ accepting the $(N-1)$ contracts and exerting effort or $B\left((N-1)\tilde{I} + I_i\right)$ accepting all the contracts offered and shirking.
There can be two cases: either $I_i \leq \tilde{I}$ or $I_i > \tilde{I}$. Let us consider first the case when $I_i \leq \tilde{I}$. From the definition of the equilibrium, it is clear that the borrower will always prefer to accept at least $(N-1)$ contracts and to exert effort. In this case, the individual revenue of each of the $(N-1)$ lenders obtained by using (4) and (5) will be:

$$p\tilde{R} = p\left[G\left(N\tilde{I}\right) - G\left((N-1)\tilde{I}\right)\right] \tag{A.11}$$

In addition, given $I_i \leq \tilde{I}$ and the concavity of $G(.)$ the entrepreneur will buy all the $(N-1)$ contracts together with the i-th, hence lender i's payoff can be written as:

$$\pi_i = pG\left((N-1)\tilde{I} + I_i\right) - (1+r)I_i - pG\left((N-1)\tilde{I}\right) \tag{A.12}$$

which is maximized setting $I_i = \tilde{I}$ and guarantees a payoff of $p\tilde{R} - (1+r)\tilde{I}$.

Consider now the case of $I_i > \tilde{I}$, which induces the low action and a payoff such as:

$$\pi_i = pG\left(k\tilde{I} + I_i\right) - (1+r)I_i - pkR - B(N-1)\tilde{I} - BI_i \tag{A.13}$$

which is increasing in k and takes into account that the contract offered by the lender i could affect the number of contracts the borrower would accept together with exerting high effort.
The first order condition for a maximal π_i with respect to I_i gives:

$$pG'\left(k\tilde{I} + I_i\right) - (1+r) - B = 0 \tag{A.14}$$

which implies $k\tilde{I} + I_i = I_m$.
Hence, $I_i = I_m - k\tilde{I}$ and $(N-1)\tilde{I} > I_m$ imply that the highest number of contracts which can be accepted together with the i-th is $k = N-2$. Hence, the optimal value of k is such that:

$$k = max_{k' \in \{1,2,\dots,N-2\}}\{k' \Big| I_m - k\tilde{I} > 0\}$$

It follows that I_i cannot be greater than $\tilde{I}$, which contradicts the initial assumption.
Therefore, the optimal choice of lender i can only be $I_i \leq \tilde{I}$ which implies that his best response will be to offer a contract $\left(\tilde{R}, \tilde{I}\right)$.
Hence, the specific contracts $\left(\tilde{R}, \tilde{I}\right)$ exist and they are robust to individually profitable deviations when the number of lenders is sufficiently high and $B \in [B_z, B_m)$. ■

Proof of Proposition 23 in the text.
Given the definition of I_m and I^* and the continuity of $G(.)$ there exists a B_l such that:

$$pG\left(I_m\right) - I_m(1+r) = B_l\left(I_m + I^*\right) \tag{A.15}$$

Now, for every $B \in (B_c, B_l]$ we consider the function $x(I) = pG(I) - I(1+r) - B\left(I + I^*\right)$; by continuity there exists an investment level I' such that $x\left(I'\right) = 0$.
The equilibrium is defined by one lender, say lender i, offering the investment $I_i = I^*$ and the repayment R_i s.t. $pR_i = pG\left(I^*\right) - I^*(1+r) - \left(pG\left(I'\right) - I'(1+r)\right)$, hence making positive profits. A second lender, say lender j offers the zero-profit contract with $I_j = I'$ and $pR'_j = (1+r)I'$. All other lenders $k \neq i, j$ are offering the null contract $(0,0)$. The borrower is accepting contract i, only.

Given the behavior of the other players, lender i must offer the borrower at least a payoff of $pG(I') - I'(1+r)$ in order for his contract to be bought. Hence, he has the incentive to set the investment level at I^* so to realize the maximum amount of profits $p\,R_i$.
Let us now consider lender j: he cannot profitably deviate from the level of investment $I_j = I'$ and be guaranteed that his offer is accepted, without inducing the borrower to shirk to low action.
Given the existence of the latent contract j, no contract offering positive investment level proposed by any of the inactive lenders will be accepted at equilibrium.
Finally, the borrower is indifferent between accepting either contract i or j in isolation and choosing high effort, and buying both contracts and choosing low action. That is, accepting i only is a best reply. ■

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