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Handling uncertainties in the seismic analysis using fuzzy theory

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ABSTRACT: This study deals with the handling of uncertainties in the seismic analysis of complex structures. As a case example of a complex structure, a long span suspension bridge is considered, whose main span is 3300 m long. This is, without doubt, a very complex structure for the nonlinearities, uncertainties and interactions involved in its behavior. This paper focuses the attention on the definition of the seismic input (artificial accelerograms) and the consequent dynamical analyses which are developed in presence of uncertainties. In order to treat these uncertainties, an analysis is developed based on the fuzzy theory. Therefore, the procedure performed in order to reproduce the fuzzy response of the bridge is illustrated.

1 INTRODUCTION

The structural behavior of a long suspension bridge, like the Messina Strait Bridge, has to be evaluated considering the uncertainties involved in the problem. The importance of the structure makes the deterministic analysis of the safety and performance inappropriate. Reliable analysis, using probabilistic or fuzzy model, have to be developed in order to increase the robustness of the evaluations.

Generally speaking, the seismic analysis of long suspension bridges involves a great amount of uncertainties related to seismic input parameters. In fact, the not complete knowledge about the domain of the problem introduces uncertainty on seismic wave propagation among the bridge foundations.

Having a not complete knowledge about the problem, the seismic analysis of long suspension bridges can be classified among *ill-structured problems* (Simon 1973). In these classes of problems, sensitivity and fuzzy approach to the analyses can be suitable to handle the imprecision knowledge.

Zadeh introduced the term fuzzy logic in 1965 (Zadeh 1965); from this date numerous applications with fuzzy approaches and fuzzy rules have been developed. The first applications of the fuzzy logic were in the engineering field of the control systems (Cirstea et al. 2002), in the systems of decisional support, in the modeling of the natural language and in the recognition of forms. In civil engineering, the fuzzy logic finds notable importance in problems that require analyses in presence of uncertainty. Some of the major applications in civil engineering concern the control techniques, the structural reliability and the treatment of uncertainties in the materials (Biondini et al. 2000, Provenzano and Bontempi 2000, Savoia 2002).

2 UNCERTAINTIES IN THE EARTQUAKE ANALYSIS

The seismic analysis of long suspension bridges is a characteristic problem of structural analysis with not complete knowledge about the domain conditions. Some of the more important uncertain parameters are:

- the position of the epicenter;
- the seismic intensity and the attenuation law;
- the velocity of the seismic waves;
- the frequencies of the seismic waves;
- the local site effects;
- the mass of the structure;
- the interaction with other loads;
- the soil-structure interaction.

It is important to note that some (but not all) of these parameters are referred to the composition of the soil. An accurate knowledge about the soil composition is very expensive in long bridges analyses. However, the complete knowledge of the soil does not make the problem well-structured because a great amount of uncertainties persists in the problem. In fact, it is impossible to define with precision the real position of the epicenter of the seismic event, or the mass of the structure (with trains and cars) during the earthquake.

The interaction phenomenon is also important to investigate accurateness the structural behavior. The dynamical interaction with wind or train can be important to define the performances (transversal slope, vertical acceleration etc.) during the earthquake event. At last, the soil-structure interaction can be relevant to characterize the local dissipation of the structure. About this specific aspect, a finite element model able to reproduce accurately this interaction is very expensive and simplified models are usually adopted. The complete knowledge of the soil topology decreases the degree of ill-structured of the problem, but it remains ill-structured and complex. In according with Newell (1968):

"*The nature of a problem determines the method of its resolution*".

It is clear that the seismic analysis of long suspension bridges, like the Messina Strait Bridge, cannot be performed with a modest number of deterministic analysis, but more reliable methods have to be considered.

3 THE FUZZY SET THEORY

The fuzzy logic allows reproducing the approximate reasoning of the human mind. Zadeh introduced the term fuzzy logic in 1965 (1965) but the idea of a third logical state that contrast the dichotomy logic (true or false) of Aristotle stems with Plato to arrive in modern times to the philosopher Lukasiewicz. The third logical state was named *possible* and one gives it a degree of truth variable between *0* (false) and *1* (true). The fuzzy logic provides then an opportunity to model some uncertain conditions and furnishes some techniques on which methods of approximate reasoning can be based.

The basis of fuzzy logic was developed by numerous authors: Zadeh (1965), Mamdani (1976), Takagi and Sugeno (1985). Different authors conceived slightly different fuzzy systems, particularly about the output evaluation. In this paper, one considers a fuzzy approach based on the followings three passages:

- Fuzzifucation of the input variables.
- Application of the inference fuzzy rules and composition of the fuzzy outputs.
- Defuzzification of the output variables.

3.1 *Fuzzifucation of the input variables.*

A deterministic input value is fuzzified using a membership function $\mu(x)$. A membership function assigns to each value of the input variable x a certain value from the interval [*0*, *1*].

Many different types of membership function are applied: triangular, trapezoidal, Gaussian, harmonic, polynomial, etc. Every type of membership function has some advantages and some disadvantages. For example, the triangular membership function needs a small amount of data to define its shape but it is not continuously differentiable.

If one introduces a logical variable w:

$$w = \begin{cases} 1 & \text{for } (e\text{-}a) \le x < (e+a) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

the symmetrical triangular function can be written in the following form (Fig. 1):

$$\mu(x) = w\left(\frac{a-|x-e|}{a}\right) \tag{2}$$

while, the Gaussian symmetrical function can be expressed as (Fig. 2):

$$\mu(x) = exp\left[-\left(\frac{x-b}{a}\right)^2\right] \tag{3}$$

In literature (Piegat 2001), one can also find expressions for asymmetrical function. The parameters e and a or b and a for the Gaussian function have to be identified using known data about the problem. One notices that asymmetrical Gaussian or polynomial functions require identifying more parameter to define the shape of the membership function.

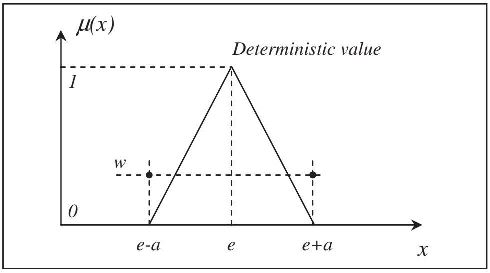

Figure 1. Triangular symmetrical function.

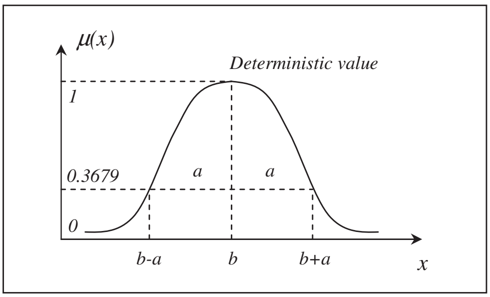

Figure 2. Gaussian symmetrical function.

The selection of membership function depends on the amount of information about the problem to be modeled. If the amount of information concerning the problem is limited, the most simple membership function should be used. Lack of information about the problem often makes impossible to identify all the parameters of nonlinear membership function (Piegat 2001).

3.2 *Application of the inference fuzzy rules and composition of the fuzzy outputs.*

The inference fuzzy rules map the input membership to the output membership function of the model. The inference rules are the kernel of the fuzzy model (Fig. 3). They involve the following steps:

- evaluation of fulfillment degrees of the rules;
- determination of the activated membership functions;
- determination of the resulting membership function of the conclusion of all rules.

The last step of the inference system is the evaluation of the output membership function as a result of all inference rules. This process is sometimes named accumulation and can be accomplished in many ways.

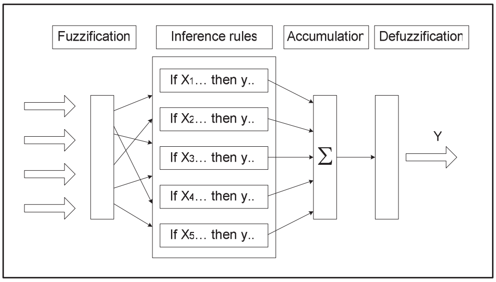

Figure 3. System inference rules.

The output membership function $\mu(y)$ represents the fuzzy output of the complete rule base. If one needs a crisp output value, a defuzzification process has to be carried out.

3.3 *Defuzzification of the output variables.*

The defuzzification is the process to get a numerical value representing the output membership function. Different techniques of defuzzification are proposed in literature: the center of gravity or the maxima methods are the most used (Driankov et al. 1993).

In the center of gravity method, the output value corresponds to the position of the center of gravity of the output membership function. If $\mu(y)$ is the output membership function of the output variable y, and S is the support of the output variable, the position of the center of gravity is given by:

$$y' = \frac{\int_S y_i \mu(y) dy}{\int_S \mu(y) dy} \tag{4}$$

It is important to notice that in the practice the membership function are often represented by polygonal figures (sum of triangles and/or trapezes). In this case, the calculation of the center of gravity is notably simplified. The membership function can be decomposed in simple figures whose center of gravity is known and there is not the necessity to perform the integral calculation.

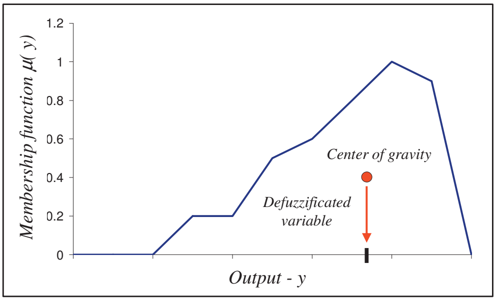

Figure 4. Defuzzification using the center of gravity method.

In the center of gravity method, all activated membership functions take part in the defuzzification process. In this way the sensitivity of the fuzzy model to changes in the inputs values is guaranteed.

However, in some applications, one can be interested only in the maximum value of the output variable. In this case, a maxima method can be assumed to defuzzifier the output membership function. The maxima method assumes the greatest value of the output variable having a grade of membership different to zero as the defuzzificated value (Fig. 5). Clearly, the maximum method is faster than the center of gravity method.

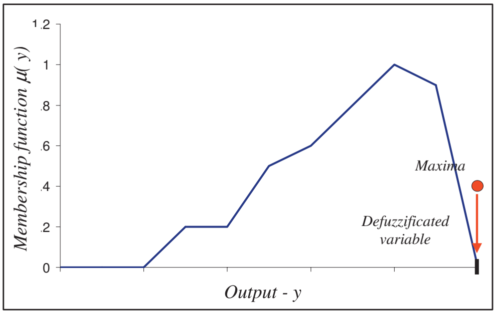

Figure 5. Defuzzification by maxima method.

4 DEFINITION OF THE SEISMIC PROBLEM

In this study, the seismic safety of a bridge like the Messina Strait Crossing (see the official site of Stretto di Messina S.p.A.) is considered. This is a long suspension bridge with a main span 3300 m long: the total length of the deck, 60 m wide, is 3666 m (including side spans). The deck is formed by three box sections, the outer ones for the roadways and the central one for the railway. The two towers are 383 m high and the deck, in the middle of the bridge, is 77 m high in order to provide a minimum vertical clearance for navigation of 60 m – with the most unfavorable static load conditions – over a width of 600 m (Catallo et al. 2003). In the tower zones (points *5* and *8* in Figure 8) nonlinear links are present between the tower and the railway box section both in the transversal and longitudinal directions. These nonlinear links have a material and contact nonlinearities because a gap is present in which the displacement is free. The free displacement for the nonlinear device is 0.03 m in transversal direction while it is 0.50 m along the longitudinal direction.

The numerical model was developed taking in account the three-dimensionality of the structure to evaluate the transversal and torsional behaviors. The structural code used for this study is ADINA (however, the complexity and the importance of the structure makes the use of different codes necessary to govern the numerical hypotheses). The choice of ADINA has been performed considering the possibility to use this code coupled with a FORTRAN program that drives the analyses in automatic way (Sgambi and Bontempi 2004).

Although the dynamic analyses are performed with a direct integration method, a modal analysis of the structure is always suitable to characterize its behavior and to define the sensibility to dynamic actions (Cook 1995). In Figure 6 the modal participating mass ratio versus the vibration mode number are plotted for the three Cartesian directions.

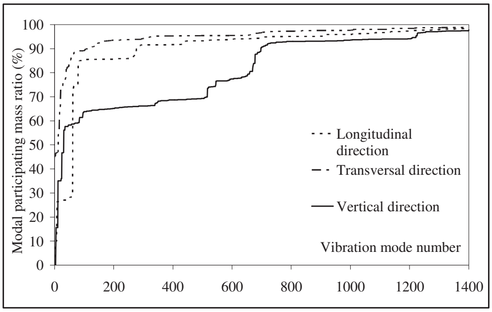

Figure 6. Modal participating mass ratio.

One notice from Figure 6 that a very high number of modes have to be considered in the dynamic analyses to reach an adequate accuracy. From these considerations, one can set the numerical step of the time integration at 0.04 sec (about 1/6 of the thousandth vibration mode).

The Figure 7 (not in scale along the vertical axis) shows the increment of the modal participating mass ratios on the range of period. It is interesting to notice that only a limited mass of the bridge (30%) is strongly excited by the seismic motion.

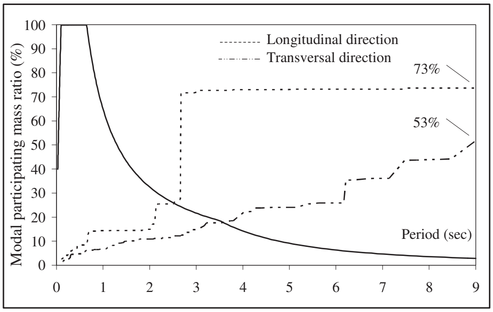

Figure 7. Modal participating mass ratio - response spectrum.

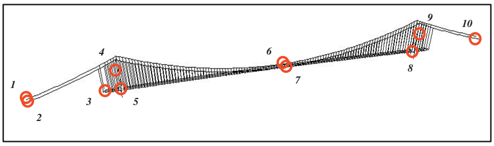

Figure 8. Numerical model and measurement points.

In Figure 8, the measure points where the structural response is analyzed are reported. In particular, the response parameters listed in the following table are considered.

Table 1. Measurement points considered in the analyses.

Label	Parameter
1 – TC	Tension in the main cable
2 – TC	Tension in the main cable
3 – LD	Longitudinal displacement
4 – TH	Tension in the left side hanging
5 – TD	Transversal displacement
5 – VD	Vertical displacement
6 – TC	Tension in the main cable
7 – TD	Transversal displacement
7 – VD	Vertical displacement
8 – TD	Transversal displacement
8 – VD	Vertical displacement
9 – TH	Tension in the left side hanging
10 – TC	Tension in the main cable

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5 FUZZY APPROACH OF THE SEISMIC ANALYSIS

In the analysis of a complex bridge, the handling of uncertainties is extremely important to understand the real behavior of the structure. The seismic analysis of the long suspension bridge, like Messina Strait Bridge, involves different uncertainties.

Principally the uncertainties concern the seismic input definition, for the impossibility to define the seismic action with accuracy. Sensitivity analysis and more reliable approach based on modern soft-computing method are suitable (Sgambi 2005-B).

In this paper, a fuzzy approach to consider the uncertainty on seismic intensity is adopted. The seismic action is defined using spectrum compatible accelerograms with different seismic intensity (PGA), along each Cartesian direction. Using a fuzzy approach, one can define a fuzzy peak ground acceleration considering a maximum uncertainty of *±30%* on the deterministic value (Figure 9).

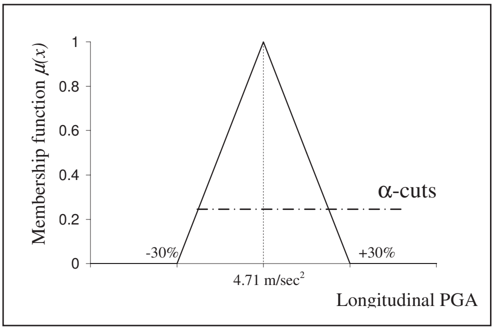

Figure 9. Membership function of the longitudinal peak ground acceleration (PGA).

In Table 2 the different PGA assumed along the three Cartesian directions are reported. In this approach, each PGA is fuzzified using triangular membership functions with a support of *±30%.*

Table 2. Peak ground acceleration along the three Cartesian directions.

Axis direction	PGA/g	PGA (*m/sec²*)
Vertical	*0.45*	*4.41*
Transversal	*0.60*	*5.88*
Longitudinal	*0.48*	*4.71*

With the procedure explained in the third paragraph, it is possible to investigate the bridge behavior for every parameter point out in Table 1. Clearly, the *if-then* rules indicated in the third paragraph do not subsist in explicit way. The structural behavior is the inference system that associates the input fuzzy variables with the output ones.

Sectioning the input membership functions for specified membership degree (α-cuts) and considering the structural behavior, it is possible to transform the input interval in the output interval (Sgambi 2005-B). The output membership function is then built repeating this procedure for different membership degree of the input membership function (different α-cuts).

However, a specific value of uncertainty along the longitudinal direction can be associate to different values of uncertainty along the other directions. In these cases, the membership function becomes a hyper-surface (Piegat 2001) and the α-cut intervals become hyper-intervals. In the specific case considered the hyper-intervals are the volumes defined in Figure 10.

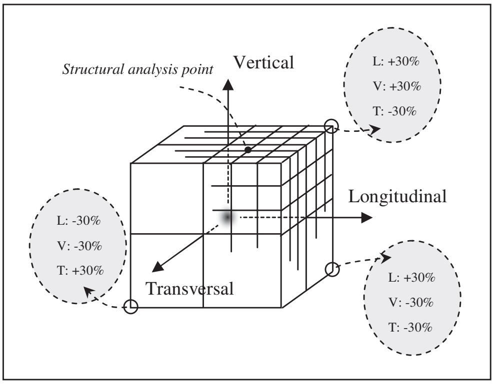

Figure 10. The hyper-interval of the input membership function.

Considering four α-cut sections, one can put in evidence the *343* (*7x7x7*) analyses necessary to perform the fuzzy analysis. In Figure 11, an example of output membership function is reported, at the instant $\bar{t}$ of the seismic analysis. The distortion of the function is due to the existence of a nonlinear relation between the input and the output variables.

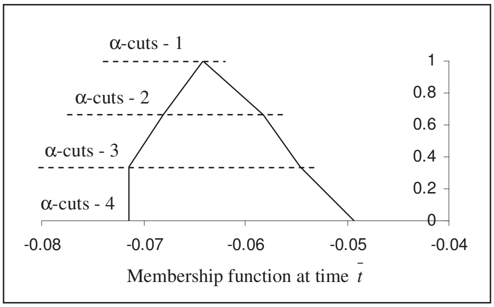

Figure 11. Example of output membership function.

For every output parameter, it is possible to represent the temporal variability for all the performed analyses. In Figure 12 the groups of seismic responses associated to the transversal displacement of the bridge deck (*5-TD*) and the axial tension in the main cable (*1-TC*) are plotted.

The boundaries of the response curves shown in Figure 12 are put in evidence in Figure 13. One can assign to these curves the name of envelope curves. In fact, they represent the area in which the seismic response is present (in particular, also the deterministic and the fuzzy response).

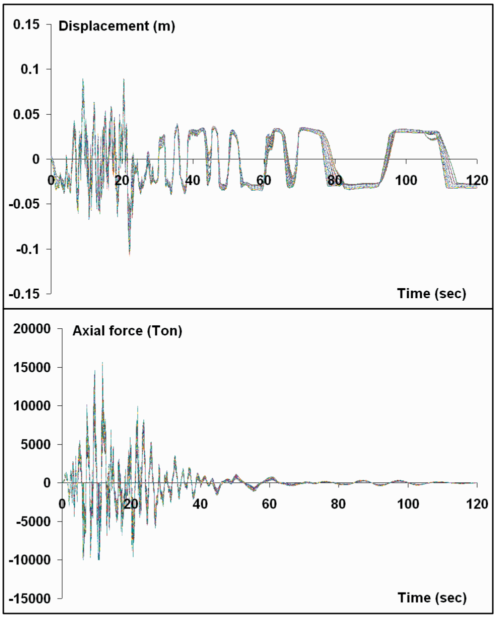

Figure 12. Group of the seismic responses for the output parameters *5-TD* (top) and *1-TC* (bottom).

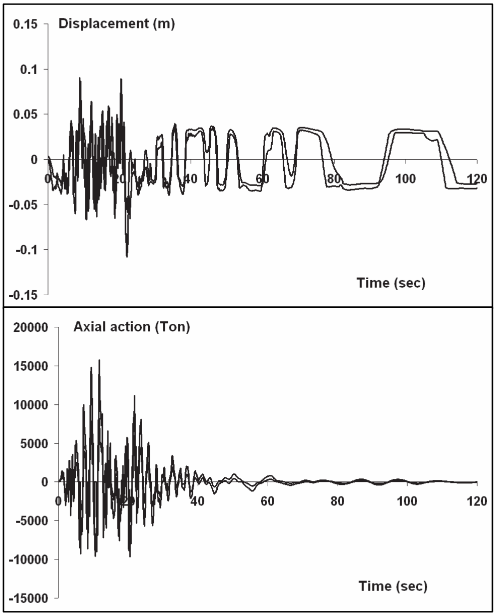

Figure 13. Envelope diagram for the transversal displacement for the output parameters *5-TD* (top) and *1-TC* (bottom).

The Figure 12-top present some interesting aspects. In fact, it is the reproduction of the transversal displacement of the bridge deck close the tower. This displacement is influenced by the presence and the characteristics of the nonlinear devices that linked the railway box section to the tower. Observing this figure, it is possible to notice that during the first part of the seismic event (0 – 30 sec) this device is not working; in fact, the maximum transversal displacement is about 0.10 m versus the 0.03 m of free displacement permitted by the device. In the second part of the seismic event, the effect of the nonlinear device is evident. During the first part of the seismic event, the relative displacement between the tower and the bridge deck remain close to zero even if the absolute displacement increases. The nonlinear device not works because there is no relative displacement. Relative displacement becomes important after the seismic violent phase, when the tower tends to stop faster than the bridge deck. Only during this second phase, the nonlinear device controls the relative (and absolute) displacement of the bridge deck.

To get the fuzzy response, one has to consider the temporal dependence of the response curves. At every output time, it is possible to evaluate the output membership function and defuzzy the response (Figure 14). It is important to defuzzy the response curve before evaluating the maximum and the minimum values because they can happen in different temporal moments.

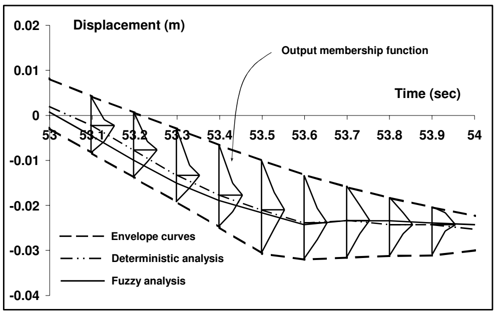

Figure 14. Qualitative image of the fuzzy response construction.

Figure 15 shows an example of the comparison between the fuzzy and the envelope curves while Figure 16 shows the comparison between the fuzzy and the deterministic analysis. The fuzzy response is evaluated with the explained procedure and using a center of gravity as defuzzification rule.

The Table 3 shows the maximum value of the output parameters investigated both by the center of gravity and by the maxima method. It is clear that if one use a maxima method one get more severe response.

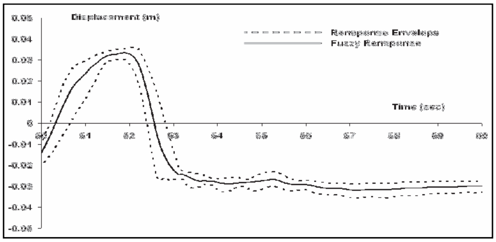

Figure 15. Comparison between the fuzzy response and the response envelope.

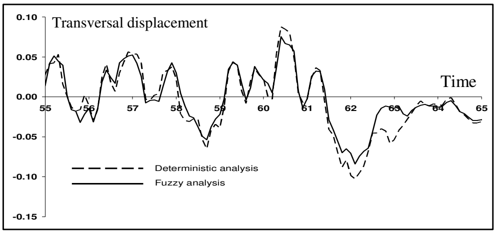

Figure 16. Comparison between the fuzzy response and the deterministic response.

Table 3. Maximum value of the parameters investigated.

Measurement point	Center of gravity method	Maxima method
1 – TC	12020 Ton	15780 Ton
2 – TC	11990 Ton	15680 Ton
3 – LD	0.81 m	1.0 m
4 – TH	213 Ton	286 Ton
5 – TD	0.081 m	0.090 m
5 – VD	0.053 m	0.070 m
6 – TC	3815 Ton	5097 Ton
7 – TD	0.64 m	0.83 m
7 – VD	0.62 m	0.84 m
8 – TD	0.093 m	0.110 m
8 – VD	0.065 m	0.085 m
9 – TH	262 Ton	341 Ton
10 – TC	11930 Ton	15325 Ton

If one takes into account the increment of the output parameter due to the adoption of the maxima method, one obtain the percentage of increment reported in Table 4.

Table 4. Increment of the output parameters defuzzyfied with maxima method.

Measurement point	Percentage of increment
1 – TC	31%
2 – TC	31%
3 – LD	23%
4 – TH	34%
5 – TD	11%
5 – VD	32%
6 – TC	34%
7 – TD	30%
7 – VD	35%
8 – TD	18%
8 – VD	31%
9 – TH	30%
10 – TC	28%

It is interesting to compare these increment values with the initial uncertainties (*±30%*). If the structural problem were linear, the deterministic response would correspond to the response defuzzyfied with the center of gravity method, because no distortion occurs on the output membership function. In this case, one would get percentage of increments equal to initial uncertainties (*±30%*). The values reported in Table 4 show a little influence of the geometrical nonlinearity on the uncertainties propagation but a strong influence of the mechanical nonlinearity. For this reason, a fuzzy analysis is suitable to get a more reliable design of the nonlinear devices.

6 CONCLUSIONS

In the seismic analyses of long suspension bridge, like Messina Strait Bridge, one cannot forget that the numerical model is affected by uncertainties. These uncertainties can influence the structural behavior and invalidate the results of the analyses.

In order to estimate the uncertainties importance on the seismic input definition, many analyses have been developed. Using a fuzzy theory, a nonlinear-fuzzy response of the bridge has been performed. The fuzzy analysis has a particular significance close the zone where the nonlinear behavior is heavy. In fact, as shown in the text, near these zones the output membership functions are distorted by the nonlinear behavior. The significance of this distortion is a *non-proportional* uncertainty sensibility of the structure. It is evident that in these cases, many analyses have to be developed to evaluate the uncertainties importance.

The response of the bridge can be evaluated using different defuzzification methods. In this case two independent defuzzification procedures have been considered: the center of gravity and the maxima method. The comparison between the results obtained from the two procedures can facilitate the understanding of the weight of the uncertainties in the model.

In this study, the seismic intensity was assumed as the uncertain input variable but the procedure is extendible to other variables as frequencies of the seismic waves or the mass of the bridge.

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