

**Optimal Income Taxation  
and  
the Ability Distribution:  
Implications for Migration Equilibria**

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## **Abstract**

As recently argued by Diamond [1998], one of the key factors explaining the progressivity of an optimal non-linear income tax is the distribution of productivity among workers. Migration is one source of changes in the productivity distribution. How changes in the population's ability distribution affect optimal income tax schedules has received little attention. Changing the distribution generally changes both the objective function and the government budget constraint. We first consider the comparative statics of the fraction of highly-skilled workers with a Rawlsian welfare function (so that only the second effect is present) and a quasi-linear utility function. We perform the same analysis for a despotic social welfare function, and present some results for a utilitarian social welfare function.

We study the interaction between mobility and redistributive taxation. We consider mobility by either the skilled or unskilled population in both Rawlsian and majority voting frameworks where governments take the population as fixed. Our main result is that equal ability distributions across jurisdictions is a stable equilibrium when the unskilled are mobile, but only under certain conditions when the skilled are mobile.

## 1. Introduction

The optimal income tax model with a finite number of consumer types has proven to be a useful model to explore a great variety of issues in redistributive taxation. Stiglitz [1982] adapts Mirrlees's [1971] classic model by considering economies with only two ability levels. Brito *et al.* [1990] establish a number of basic results with many goods and many consumer types without relying on the single-crossing assumption for preferences, which is used in much of the literature.

Weymark [1987] derives a number of comparative statics results in the finite-type model by studying the case of a constant disutility of effort. He explores the effects of changing welfare weights and preference parameters. To date, the effect of the changes in the ability distribution in the population has received little attention, even though Diamond [1998] argues that this might be a key factor in tax progressivity. Changing the ability distribution is more complex than changing the welfare function because it generally affects both the objective function and the government budget constraint.<sup>1</sup> We first consider the special cases of Rawlsian and despotic welfare functions in order that only the second effect is present.

One application of such comparative statics is to examine the effect of migration on redistributive taxation. When individuals are free to migrate from one tax jurisdiction to another, they have the freedom to choose which income redistribution regime they will be subject to. This has long been an issue in the U.S. with regard to redistribution by state and local governments. A long tradition of research (Stigler [1957] and Wildasin

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<sup>1</sup> This is obviously true for a utilitarian social welfare function. For most other welfare functions, changing the distribution of types still affects the welfare function, although not necessarily in an obvious fashion.

[1994]) suggests that free mobility limits the amount of redistribution state governments can achieve. The effect of mobility on redistribution is becoming an issue in the European Union as barriers to labor mobility fall (along with barriers to capital mobility). National governments in the EU administer income redistribution policies, but they can no longer prevent immigration from EU member countries. We consider a variety of different forms of mobility between two jurisdictions which use optimal income taxes.

Most literature in tax competition and on the effect of mobility on redistributive policy focuses on linear tax-transfer schemes. Using nonlinear schemes is clearly more complex and calls for a number of simplifying assumptions. We thus assume that in each country there are only two types of skill or productivity and that national governments redistribute according to either a Rawlsian criterion or majority voting. We also study the behavior of a small open economy which takes as given the utilities obtained in the rest of the world. Only one type of worker can move at no cost: either the skilled or the unskilled. In this setting, we basically show that when the unskilled move tax harmonization results, and that this is not necessarily the case when the skilled are the mobile group.<sup>2</sup>

The remainder of this paper consist of four sections. In Section 2 we show the effect of changing the ratio of skilled to unskilled workers on the utility of both types when the social planner adopts either a Rawlsian or a despotic welfare objective. In Section 3 we consider the distributive incidence of mobility of either skilled or unskilled workers in a small open economy with a Rawlsian objective first and then with majority

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<sup>2</sup> Wilson [1992] looks at the case where both types of workers can move. In his paper, the two types of workers are not perfect substitutes in the production technology. See also Hindriks [2001].

voting. After having taken the utility of the mobile as given by the rest of the world, Section 4 shows how this reservation utility can be determined in a general equilibrium setting. Section 5 contains our conclusions.

## 2. The ability distribution and the utility profiles

### 2.1. The Rawlsian Outcomes

We consider a particular model which allows us to find a closed-form solution to the social planner's welfare maximization problem and to solve the comparative statics in detail. There are two types of consumers with earning abilities (or wages)  $w_1$  and  $w_2$ , where  $w_1 > w_2$ . There are  $n$  consumers of type 1 (the skilled) and  $1 - n$  consumers of type 2 (the unskilled). The government cannot identify individuals' abilities, but it knows the distribution of abilities. Except for their abilities, consumers are identical and have quasi-linear preferences:

$$U_i(c_i, L_i) = \ln(c_i) - L_i \quad i = 1, 2, \quad c_i \geq 0, \quad 0 \leq L_i \leq \bar{L}.$$

We use this particular example to develop closed-form solutions in order to explore the comparative statics in detail.<sup>3</sup>

The social planner maximizes a Rawlsian welfare function, subject to the information constraint that it cannot identify consumer types. The planner must base transfers on incomes, which he observes without error. Thus, the planner solves the problem:

$$\begin{array}{ll} \text{Max} & \ln(c_2) - \frac{y_2}{w_2} \\ c_1, c_2, y_1, y_2 & \end{array}$$

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<sup>3</sup> While this utility function may have some questionable properties, we conjecture that our main results can be qualitatively generalized. Some simulation results with more general utility functions indicate this to be true.

$$\begin{aligned}
\text{s.t.} \quad & n(y_1 - c_1) + (1 - n)(y_2 - c_2) \geq 0 & (\mu), \\
& \ln(c_1) - \frac{y_1}{w_1} - \ln(c_2) + \frac{y_2}{w_1} \geq 0 & (\lambda),
\end{aligned}$$

where  $\mu$  is the Lagrange multiplier on the budget constraint and  $\lambda$  the multiplier on the self-selection constraint.<sup>4</sup> Assuming an interior solution, the FOC for this problem are:

$$\begin{aligned}
-n\mu + \frac{\lambda}{c_1} &= 0, & n\mu - \frac{\lambda}{w_1} &= 0, \\
\frac{1}{c_2} - (1 - n)\mu - \frac{\lambda}{c_2} &= 0, & -\frac{1}{w_2} + (1 - n)\mu + \frac{\lambda}{w_1} &= 0,
\end{aligned}$$

The solution is:

$$\begin{aligned}
c_1^* &= w_1, & y_1^* &= w_1(1 - n) \ln \left( \frac{w_1 - nw_1}{w_2 - nw_1} \right) + w_2, \\
c_2^* &= \frac{w_2 - nw_1}{1 - n}, & \text{and} \quad y_2^* &= w_2 - nw_1 \ln \left( \frac{w_1 - nw_1}{w_2 - nw_1} \right).
\end{aligned}$$

Not surprisingly, the labor decision of the skilled is not distorted (no distortion at the top). Note that both aggregate consumption and income equal  $w_2$ , since this is an important feature in our results.

When  $w_2 - nw_1 \ln \left( \frac{w_1 - nw_1}{w_2 - nw_1} \right) < 0$ ,  $y_2 = 0$ .<sup>5</sup> Let  $\hat{n}$  denote the smallest value of  $n$  such that  $y_2 = 0$  ( $\hat{n}$  solves  $w_2 = \hat{n}w_1 \ln \left( \frac{w_1 - \hat{n}w_1}{w_2 - \hat{n}w_1} \right)$ ). We distinguish between these two

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<sup>4</sup> Clearly, both constraints will always bind with this objective. With the Rawlsian objective, the self-selection constraint that  $U_2(c_2, y_2) \geq U_2(c_1, y_1)$  can never bind. Similarly, in the despotic case below, the constraint that  $U_1(c_1, y_1) \geq U_1(c_2, y_2)$  never binds.

<sup>5</sup> Note that  $w_2 - nw_1 > 0$  holds everywhere in the region where  $y_2^* > 0$ .

regimes: one in which the unskilled do not work ( $n \geq \hat{n}$ ) and one in which they supply a positive amount of effort ( $n < \hat{n}$ ).

2.1.1. *The corner solution:  $L_2 = 0$ .*

For  $n > \hat{n}$ , the solution is:

$$\tilde{c}_1 = w_1, \quad \tilde{y}_1 = w_1 \ln(w_1) - w_1 \ln\left(\frac{n(\tilde{y}_1 - w_1)}{1-n}\right), \quad \text{and} \quad \tilde{c}_2 = \frac{n(\tilde{y}_1 - w_1)}{1-n}.$$

*Proposition 1:* For  $n > \hat{n}$ ,  $U_1 = U_2$  and they increase with  $n$ .

*Proof:* In this region,

$$\frac{\partial \tilde{y}_1}{\partial n} = \frac{-w_1(1-n)}{n(\tilde{y}_1 - w_1)} \left[ \frac{\tilde{y}_1 - w_1}{1-n} + \frac{n}{1-n} \frac{\partial \tilde{y}_1}{\partial n} + \frac{n(\tilde{y}_1 - w_1)}{(1-n)^2} \right] = \frac{-w_1}{n(1-n)} \frac{\tilde{y}_1 - w_1}{\tilde{y}_1} < 0.$$

In words, for  $n$  large enough that type 2 consumers do no work, type 1 labor supply falls with an increase in the fraction of type 1 consumers. As  $c_1 (= w_1)$  is independent of  $n$ , this implies that  $\frac{\partial U_1}{\partial n} > 0$ .

We also have:

$$\frac{\partial \tilde{c}_2}{\partial n} = \frac{\partial}{\partial n} \left[ \frac{n}{1-n} (\tilde{y}_1 - w_1) \right] = \frac{\tilde{y}_1 - w_1}{(1-n)^2} \left( 1 - \frac{w_1}{\tilde{y}_1} \right) > 0,$$

so that  $\frac{\partial U_2}{\partial n} > 0$ .

Examining the self-selection constraint with  $y_2 = L_2 = 0$ , it follows immediately that  $U_2 = U_1$ . **QED**

To sum up, in the regime with  $L_2 = 0$ , both utilities are equal and increase with  $n$ .

2.1.2. Positive labor supply for all types:  $L_1 > 0$  and  $L_2 > 0$

*Proposition 2:* For  $n < \hat{n}$ ,  $U_1 > U_2$ , and  $U_1$  decreases with  $n$ , while  $U_2$  increases with  $n$ .

*Proof:* Since the planner is maximizing  $U_2$  and an increase in  $n$  relaxes the budget

constraint, it is not surprising that  $U_2$  is increasing in  $n$ . Formally, when  $y_2^* > 0$ ,

$$U_2 = \ln\left(\frac{w_2 - nw_1}{1-n}\right) - 1 + n\left(\frac{w_1}{w_2}\right) \ln\left(\frac{w_1 - nw_1}{w_2 - nw_1}\right) = \frac{w_2 - nw_1}{w_2} \ln D - 1 + \ln w_1$$

$$\text{where } D = \frac{w_2 - nw_1}{w_1 - nw_1}.$$

$$\frac{\partial U_2}{\partial n} = -\frac{w_1}{w_2} \ln D + \frac{w_2 - nw_1}{w_2 D} \frac{\partial D}{\partial n}.$$

Substituting  $\frac{\partial D}{\partial n} = \frac{w_2 - w_1}{w_1(1-n)^2}$  into this, we obtain:

$$\frac{\partial U_2}{\partial n} = \frac{w_1}{w_2} \left( -\ln D + \frac{w_2 - w_1}{w_1(1-n)} \right) > 0,$$

since  $D - 1 = \frac{w_2 - w_1}{w_1(1-n)}$  and  $D - 1 - \ln D > 0$ .

In this region,  $U_1 = \ln(w_1) - (1-n)\ln(Z+1) - \frac{w_2}{w_1}$ , where  $Z =$

$\frac{w_1 - nw_1}{w_2 - nw_1} - 1$ . Differentiating with respect to  $n$ , we obtain

$$\frac{\partial U_1}{\partial n} = \ln(Z+1) - (1-n)w_1 \left[ \frac{1}{w_2 - nw_1} - \frac{1}{w_1 - nw_1} \right] =$$

$\ln(Z+1) - Z < 0$ . This last expression is negative since  $\ln(1+Z) < Z$ .

**QED**

We thus find that  $U_1$  falls as  $n$  increases, while  $U_2$  increases as  $n$  increases in the region where  $y_2 > 0$ . In contrast, when  $y_2 = 0$ ,  $U_1$  increases as  $n$  increases, as does  $U_2$ . See Figure 1 for an illustration.

To see the intuition behind this surprising nonmonotonicity, let us start with no skilled workers:  $n = 0$ . Unskilled workers must work to finance their consumption. As  $n$  increases, the unskilled receive a net benefit and they work less and less. With this particular utility function, labor supply absorbs all the income effects and their utility increases to the point that they eventually stop working at all. The utility of the unskilled workers increases as they work less and less and that of the skilled workers decreases as they work more and more. Aggregate consumption is constant.

At some point, the unskilled stop working and the distortion in their labor supply remains constant. Both utilities are identical and increases as “potential income”  $nw_1$  increases.

## 2.2. The Despotic Case

One justification for examining the Rawlsian solution is that, if the unskilled type form a majority, they may be able to impose the outcome most favorable to them. It is thus instructive to consider the opposite case where the able type impose their preferences. We refer to this as the despotic solution. Here the outcome would be the solution to:

$$\begin{aligned}
 & \text{Max}_{c_1, c_2, y_1, y_2} && \ln(c_1) - \frac{y_1}{w_1} \\
 & \text{s.t.} && n(y_1 - c_1) + (1 - n)(y_2 - c_2) \geq 0, \\
 & && \ln(c_2) - \frac{y_2}{w_2} - \ln(c_1) + \frac{y_1}{w_2} \geq 0.
 \end{aligned}$$

The FOC are:

$$\begin{aligned} \frac{1}{c_1} - n\mu - \frac{\lambda}{c_1} &= 0, & -\frac{1}{w_1} + n\mu + \frac{\lambda}{w_2} &= 0, \\ -(1-n)\mu + \frac{\lambda}{c_2} &= 0, & \text{and} \quad (1-n)\mu - \frac{\lambda}{w_2} &= 0. \end{aligned}$$

When the solution is an interior one<sup>6</sup>, it is:

$$\begin{aligned} c_1 &= \frac{w_1 - (1-n)w_2}{n}, & y_1 &= w_1 + (1-n)w_2 \ln\left(\frac{w_1 - (1-n)w_2}{nw_2}\right) \\ c_2 &= w_2, & \text{and} \quad y_2 &= w_1 - nw_2 \ln\left(\frac{w_1 - (1-n)w_2}{nw_2}\right). \end{aligned}$$

The resulting utility levels are:

$$\begin{aligned} U_1 &= \ln\left[\frac{w_1 - (1-n)w_2}{n}\right] - \left[\frac{w_1}{w_1} + \frac{(1-n)w_2}{w_1} \ln\left[\frac{w_1 - (1-n)w_2}{nw_2}\right]\right] \\ &= -1 + \left(\frac{w_1 - (1-n)w_2}{w_1}\right) \ln\left[\frac{w_1 - (1-n)w_2}{n}\right] + \frac{(1-n)w_2}{w_1} \ln w_2, \\ \text{and } U_2 &= \ln c_2 - \frac{y_2}{w_2} = \ln w_2 - \frac{w_1}{w_2} + n \ln\left[\frac{w_1 - (1-n)w_2}{nw_2}\right]. \end{aligned}$$

*Proposition 3:* In the despotic solution,  $\frac{\partial U_1}{\partial n} < 0$  and  $\frac{\partial U_2}{\partial n} > 0$ .

*Proof:* Differentiate  $U_1$  with respect to  $n$  to obtain:

$$\frac{\partial U_1}{\partial n} = \frac{w_2}{w_1} \ln\left(\frac{w_1 - w_2 + nw_2}{nw_2}\right) + \frac{w_2 - w_1}{nw_1} = \frac{w_2}{w_1} [\ln(1+A) - A] < 0$$

$$\text{where } A = \frac{w_1 - w_2}{nw_2}.$$

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<sup>6</sup> That is,  $0 \leq L_1 < \bar{L}$  and  $0 \leq L_2 < \bar{L}$  both hold.

$$\frac{\partial U_2}{\partial n} = \ln\left(\frac{w_1 - w_2 + nw_2}{nw_2}\right) + \frac{w_2 - w_1}{w_1 - w_2 + nw_2} = [-\ln(1+B) + B] > 0,$$

$$\text{where } B = \frac{w_2 - w_1}{w_1 - w_2 + nw_2} < 0.$$

**QED**

It is interesting to contrast the despotic solution with the Rawlsian solution. Here, there are no corner solutions: both types of workers always supply effort and therefore the profiles of  $U_1(n)$  and  $U_2(n)$  are monotonic and divergent. See Figure 2. As with the interior solution in the Rawlsian case, as  $n$  grows, the cost of the distortion (here it is skilled workers' labor supply that is distorted) grows, while the revenue collected as a result of the distortion (here it is raised from unskilled workers) falls.

Since both the Rawlsian and despotic solutions are polar cases, it is interesting to ask whether the possibility of  $U_1$  first decreasing and then increasing with  $n$  is specific to the Rawlsian criterion. In the appendix, we show that, under a utilitarian social welfare function with these preferences, the utility of the skilled initially falls as their proportion in the population increases, while the utility of the unskilled rises. Once  $n$  is large enough that the unskilled do no work, the utility of both types rises as the proportion of skilled workers increases.<sup>7</sup>

We now turn to the issue of optimal income tax competition. We distinguish the cases where the tax is chosen by a Rawlsian social planner and where it is chosen by the majority of citizens. We start by looking at the problem of a small economy with mobility of just the unskilled workers and then of just the skilled.

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<sup>7</sup> What we cannot show is that the utility of the skilled falls throughout the range where the unskilled work.

### 3. Optimal income tax competition: the small open economy case

Studying the equilibrium distribution of individuals of different abilities (or characteristics) can be a difficult task. Within the local public finance literature, there is the well-known “empty community problem.”<sup>8</sup> If all individuals are free to migrate, under some conditions, everyone prefers to live in one particular community even when everyone else does. Scale economies in public goods provision can give rise to such phenomena. Another problem is that such a model may not have an equilibrium. This may occur in our model which is purely one of redistribution. If all high-ability and all low-ability individuals locate in one community, for any positive level of redistribution from one type to another, any individual of the type which pays positive taxes would be better off to move to the empty community and pay zero taxes. If low-ability individuals are better off living in a jurisdiction with a higher proportion of high-ability individuals, then the low-ability individuals always want to move where there are relatively more high-ability individuals. Thus, low-ability individuals chase high-ability individuals. When high-ability individuals are also better off locating in a jurisdiction with relatively more high-ability individuals, then high-ability individuals always want to move away from low-ability individuals. Putting these two tendencies together, there is no equilibrium with free migration of all types.

Some of the literature considers general-equilibrium effects on wages in order to sustain equilibrium. Other papers, such as Brueckner and Zenou [1999], rely on changes in the price of land to prevent the temporary appearance of an empty community.

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<sup>8</sup> See, however, Wilson [1992].

Another approach is to consider differential mobility costs which allows only part of a group to migrate in response to a utility differential (for example, Leite-Monteiro [1997]).

We focus on a different mechanism which restores the existence of equilibrium in at least some cases. There is completely free mobility for one type of worker and zero mobility for the other type. In tax competition models such as Zodrow and Mieszkowski [1986] and Wilson [1991], mobile capital and immobile workers (or land) plays a similar role. Those models can also easily take the interpretation that the two factors of production are skilled and unskilled workers. One group is immobile and jurisdictions compete to attract the mobile factor (because the factors are complements in production, as well as because the mobile factor pays taxes).

The early work in the U.S. on how mobility could prevent redistribution by local governments focused on the case where high-ability workers were mobile. In contrast, the literature on welfare migration concerns itself with mobility of low-income households [see Brueckner, 2000]. Cremer and Pestieau [1998] have studied models of social insurance in which low-ability workers were the mobile group.

The essential difference between mobility by the less skilled and by the more skilled is that the less skilled move to a region carrying out more redistribution, while the more skilled would seek out a region undertaking little redistribution. The most basic question is whether migration by one group can lead to homogeneity in the skill distributions across regions.

We consider four different cases according to whether the skilled or unskilled move and whether the tax system results from a Rawlsian or majority voting outcome. Our model is one of a small country in a large common labor market (such as a small

U.S. state or a small country in the EU). Individuals decide to move in or out of the community by comparing the utility their type obtains in the small country to that available in the rest of the union (which may consist of small countries).

Let  $n_0$  denote the initial proportion of skilled workers in the small country. The reservation utility levels are  $\bar{U}_1$  and  $\bar{U}_2$ . The government is assumed to choose the tax rates given the observed ability distribution. We now consider each case in turn starting with the Rawlsian solution where the unskilled are the mobile group.

### **3.1. Rawlsian solution with mobile unskilled**

When the unskilled are the mobile type, migration of unskilled workers into the small country occurs whenever  $U_2 > \bar{U}_2$ . This in-migration lowers  $n$ . The reverse occurs whenever  $U_2 < \bar{U}_2$ . Since  $\frac{\partial U_2}{\partial n} > 0$ , free mobility of low-ability workers leads to the equalization of utilities of the unskilled between the small economy and the rest of the world. Furthermore, this process is dynamically stable in that, for any initial values of  $n$  in the small country,  $n$  converges to  $n_\alpha$  where  $n_\alpha$  is given by  $U_2(n_\alpha) = \bar{U}_2$ . In Section 4, we will see that  $n_\alpha$  is also the value of  $n$  in the rest of the world. Clearly if  $n_0 < n_\alpha$ , there will be out-migration of unskilled workers and the utility of those remaining increases; if  $n_0 > n_\alpha$ , there will be in-migration of unskilled workers with a loss of utility for those unskilled already residing there.

### **3.2. Rawlsian solution with mobile skilled**

When high-ability workers are the only ones who can migrate, the outcome is considerably more complex. First, over some range,  $\frac{\partial U_1}{\partial n} < 0$  for low values of  $n$  ( $n < \hat{n}$ ),

while  $\frac{\partial U_1}{\partial n} > 0$  always holds for high enough values of  $n$  ( $n > \hat{n}$ ). Second, because of the burden of redistribution, when all skilled workers are in a single community, skilled workers are better off in that community. As a consequence, when only skilled workers are mobile, there are two migration equilibria depending on the value of  $n$  and on the level of  $\bar{U}_1$ .

For  $n_0 < n_\gamma$ , where  $n_\gamma$  is the larger value of  $n$  such that  $U_1(n_\gamma) = \bar{U}_1$ :  $n$  converges to  $n_\beta$ , which is the smallest  $n$  such that  $U_1(n_\beta) = \bar{U}_1$ .

For  $n_0 > n_\gamma$ :  $n$  converges to 1.

This is illustrated in Figure 3. For  $n_\beta < n_0 < n_\gamma$ , skilled workers exit up to the point B where  $\bar{U}_1 = U_1(n_\beta)$ . For  $n_0 < n_\beta$ , they enter until the point B is reached. Under those initial conditions, we have a stable migration equilibrium.

If  $\bar{U}_1$  is high enough that there is only a single value of  $n$  ( $n_\delta$ ) such that  $\bar{U}_1 = U_1(n_\delta)$  and  $n_0 > n_\delta$ , we witness an inflow of skilled workers from the rest of the world and this only stops when  $n = 1$ . In the next section, we discuss the relevance of such an equilibrium. At this point, it suffices to say that our small open economy has become a large open economy. If  $n_0 < n_\delta$ , the skilled leave the jurisdiction and  $n$  converges to 0.

### **3.3. Majority voting with mobile unskilled**

In many democratic federations with free mobility of workers, new residents are eligible to vote after only a brief period (typically one month in the U.S.). It is thus possible that mobility of voters could quickly affect the consensus on income

redistribution. Here we consider an extreme version of this position where the social decision is the Rawlsian redistribution outcome whenever  $n < \frac{1}{2}$  and the despotic outcome whenever  $n > \frac{1}{2}$ . Again, we consider the cases of unskilled and skilled worker mobility in isolation and we start with the first.

We have to distinguish four cases depending on whether  $n_0 \leq \frac{1}{2}$  and on the level of  $\bar{U}_2$ . Recall that  $U_2(n)$  increases with  $n$  both for  $n < \frac{1}{2}$  and  $n > \frac{1}{2}$ , but the profile for  $n < \frac{1}{2}$  lies everywhere above the profile for  $n > \frac{1}{2}$ . Again we define  $n_\alpha$  as such that  $\bar{U}_2 = U_2(n_\alpha)$ ;  $n_\alpha$  can be smaller or larger than  $\frac{1}{2}$ . See Figure 4.

For  $n_\alpha < \frac{1}{2}$  and  $n_0 < \frac{1}{2}$ : It is clear that  $n$  converges towards  $n_\alpha$  and  $U_2$  towards  $\bar{U}_2$ . We have a stable equilibrium.

For  $n_\alpha < \frac{1}{2}$  and  $n_0 > \frac{1}{2}$ : Here,  $n$  converges to 1; all unskilled leave the country and obtain  $\bar{U}_2$ .

For  $n_\alpha > \frac{1}{2}$  and  $n_0 > \frac{1}{2}$ : Again we have a stable equilibrium with  $n$  converging to  $n_\alpha$  and  $U_2$  to  $\bar{U}_2$ . Here, the unskilled do not receive transfers, but pay to fund transfers for the skilled.

For  $n_\alpha > \frac{1}{2}$  and  $n_0 < \frac{1}{2}$ : In this case, all the unskilled workers will immigrate to this country, so  $n$  converges to 0. The relevance of such an equilibrium is dealt with in the next section.

### **3.4. Majority voting with mobile skilled**

We again combine the Rawlsian solution for  $n < \frac{1}{2}$  and the despotic solution for  $n > \frac{1}{2}$ . To keep this analysis simple, we assume that  $\hat{n}$  is larger than  $\frac{1}{2}$ . This reduces the number of cases and does not affect the qualitative nature of our results (if  $\hat{n} < \frac{1}{2}$ , no

value of  $\hat{n} < n < \frac{1}{2}$  is a stable equilibrium). The profile of  $U_1(n)$  is therefore decreasing for  $n < \frac{1}{2}$  and for  $n > \frac{1}{2}$ , with the profile for  $n > \frac{1}{2}$  lying everywhere above the profile for  $n < \frac{1}{2}$ . See Figure 5.

As before,  $n_0$  can be above or below  $\frac{1}{2}$  and  $n_\alpha$  (where  $U_1(n_\alpha) = \bar{U}_1$ ) can be above or below  $\frac{1}{2}$ . We thus distinguish 4 cases.

For  $n_\alpha < \frac{1}{2}$  and  $n_0 < \frac{1}{2}$ : Here  $n$  converges to  $n_\alpha$  and  $U_1$  to  $\bar{U}_1$ .

For  $n_\alpha < \frac{1}{2}$  and  $n_0 > \frac{1}{2}$ : Here all skilled workers from the rest of the world immigrate to the small country and  $U_1$  converges to  $U_1(1)$ . Again there is the issue of the relevance of such an equilibrium.

For  $n_\alpha > \frac{1}{2}$  and  $n_0 < \frac{1}{2}$ : Skilled workers leave the small country and obtain  $\bar{U}_1$  elsewhere, so  $n$  converges to 0.

For  $n_\alpha > \frac{1}{2}$  and  $n_0 > \frac{1}{2}$ : Here  $n_\alpha$  is a stable equilibrium with  $U_1$  converging to  $\bar{U}_1$ .

In contrast to the case where  $n_\alpha < \frac{1}{2}$  and  $n_0 < \frac{1}{2}$ , the skilled receive transfers in the migration equilibrium.

#### 4. Optimal income tax competition: the world equilibrium

Up to now, we have considered the case of a small open economy assuming that the rest of the world offered either  $\bar{U}_1$  or  $\bar{U}_2$  to the skilled and unskilled workers respectively.<sup>9</sup> Throughout, we assumed free mobility for either the skilled or the unskilled. Some of the results we obtained raise serious questions of sustainability. For example, can we believe that a small open economy can welcome all the skilled workers

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<sup>9</sup> In confederations of countries, the rest of the world is the set of all other country members.

from the rest of the world? What would that mean? Clearly, one factor yielding such an outcome is our assumption of constant productivities  $w_1$  and  $w_2$ .

In this section, we assume that the rest of the world is made of a large number of small countries. Our interest is in evaluating the possibility and stability of symmetric equilibria in which all countries have the same proportions of skilled and unskilled workers. We approach this question by assuming that all other countries have an identical proportion of skilled workers (equal to  $\bar{n}$ ) and that they all have the same tax system in place. We also assume that the inflow of workers from our small open economy does not change that value.

With all these assumption we now review the different cases studied in the previous section and see the kind of world equilibrium they lead to.

#### **4.1. *Rawlsian solution with mobile unskilled***

In this case, in the rest of the world,  $U_2(\bar{n}) = \bar{U}_2$ . Then, the equilibrium in the small open economy has  $n = n_\alpha = \bar{n}$ . In other words, equal proportions of low ability workers in all countries and equal utility for both types of workers everywhere is a stable equilibrium.

#### **4.2. *Rawlsian solution with mobile skilled***

Here we have to distinguish two possibilities depending on whether  $\bar{n} \gtrless \hat{n}$ . If  $\bar{n} < \hat{n}$ ,  $\bar{n}$  corresponds to  $n_\beta$  in the previous section. If  $n_0 < n_\gamma$ , then we have a stable equilibrium at  $\bar{n}$  with  $\bar{U}_1 = U_1(\bar{n})$ .

If  $n_0 > n_\gamma$  while  $\bar{n} < \hat{n}$ , there does not exist an equilibrium. Having all the skilled workers converging to a single community with all the other communities left with just unskilled workers is a questionable outcome.

Consider now the other possibility:  $\bar{n} > \hat{n}$ . We find the same questionable outcome: all the skilled workers migrate into a single country.

This unusual outcome where all skilled workers conglomerate in a single country is a consequence of our assumptions. This demonstrates the necessity to adopt a more realistic model for future analysis, for example, with some complementarity in production between the two types of labor. Such factors would prevent all skilled workers from moving to a single country. The fact would remain that mobility would work against equalizing the ability distribution across countries.

#### **4.3. Majority voting with mobile unskilled workers**

In this case, set  $\bar{U}_2 = U_2(\bar{n})$ . For any  $\bar{n}$ , there is a stable equilibrium with  $\bar{n}$  as the ability distribution everywhere. Whether redistribution is toward or away from the unskilled depends on which type has a majority in the overall labor market ( $\bar{n} \gtrless \frac{1}{2}$ ).

#### **4.4. Majority voting with mobile skilled workers**

For  $\bar{n}$  and  $n_0 < \frac{1}{2}$  we have a stable equilibrium with  $\bar{n}$  and  $\bar{U}_1$ . For  $\bar{n}$  and  $n_0 > \frac{1}{2}$ , we also have a stable equilibrium.

However when  $\bar{n} < \frac{1}{2}$  and  $n_0 > \frac{1}{2}$  or when  $\bar{n} > \frac{1}{2}$  and  $n_0 < \frac{1}{2}$ , we find that no stable equilibrium exists.

### **5. Conclusions**

The prevailing view in public finance is that, in an economic union with labor mobility, decentralized redistribution policy causes some form of adverse selection and, hence, is rather ineffective. Poor households when they are mobile are attracted by welfare programs. Rich households when they are mobile are repelled by the prospect of having to pay for these programs.

Most studies of redistribution policy with tax competition consider linear tax and transfer instruments. The contribution of this paper is to cast the problem of decentralized redistribution policy within a framework of optimal nonlinear income taxation. We distinguish four settings which are presented in Table 1. We consider cases where either the skilled or unskilled are mobile. We also study two types of collective decision-making: the Rawlsian criterion and majority voting.

What appears clearly is that mobility of the unskilled raises fewer problems than mobility of the skilled. Recall that  $n$  is the fraction of skilled workers in the small open economy,  $n_0$  is the initial fraction and  $\bar{n}$  is the fraction prevailing in the rest of the world that we assume adopts the same objectives as the small economy. When the unskilled are mobile,  $n$  tends to  $\bar{n}$  and utility of the unskilled converges to  $\bar{U}_2$  (the utility of the unskilled in the rest of the world). Under majority voting, when  $n_0 < \frac{1}{2}$  and  $\bar{n} > \frac{1}{2}$  (or the other way around), then the process of harmonization involves switching from one social criterion to another.

When the skilled are mobile and the government has a Rawlsian objective function, then  $n$  tends to  $\bar{n} = n_\beta$  for sufficiently low values of both  $n_0$  and  $\bar{n}$ . Otherwise, there is no harmonization nor a stable equilibrium. When the skilled are mobile and the majority chooses the tax policy, then there is harmonization only if both  $n_0$  and  $\bar{n}$  both lie above or below  $\frac{1}{2}$ .

These contrasting findings can be explained by the utility profiles that emerge from changing  $n$ . Regardless of the welfare criterion, the ability of the unskilled always increases as  $n$  increases (with the sole exception of a jump down as  $n$  crosses  $\frac{1}{2}$ ), whereas

the utility of the skilled has a U-shaped profile under the Rawlsian and majority voting criteria.

To obtain these results, we had to resort to a number of assumptions: two types of individuals and a utility function with linear disutility of effort. Would a larger number of types or a more general utility function yield different results? This is an open question. We believe that our results would be more complicated but qualitatively similar to those presented here.<sup>10</sup>

On the issue of whether redistribution would be better administered by some central (supranational) government, what can we say? When tax harmonization is the outcome, it seems clear that such a central government would not do better than our decentralized governments. The reason is simple: in this paper, our small open economy takes policies in the rest of the world as given and does not behave strategically. Furthermore, when harmonization occurs, each economy is a clone of the others. A centralized tax policy would then have the same outcome.<sup>11</sup>

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<sup>10</sup> In numerical simulations with utilitarian objectives and utility functions such as Cobb-Douglas and CES, the nonmonotonicity of utility of the skilled as  $n$  changes is often found. We found the utility of the unskilled always to be monotonic with respect to changes in  $n$ .

<sup>11</sup> Hamilton, Lozachmeur, and Pestieau [2001] show that, with Rawlsian objectives and mobile unskilled, the autarkic and strategic outcomes are identical with identical communities *ex ante*.

## Appendix

### Comparative Statics with a Utilitarian Social Welfare Function

We now consider the case of a utilitarian social welfare function. Here, a change in the ability distribution changes both the objective function and the resource constraint. In the Rawlsian problem, the ability distribution only affects the resource constraint. Thus, the comparative statics are more complex than those considered by Weymark [1987]. We use the same utility function as before, so the planner's problem is now:

$$\begin{aligned} \text{Max}_{c_1, c_2, y_1, y_2} \quad & n \left( \ln(c_1) - \frac{y_1}{w_1} \right) + (1-n) \left( \ln(c_2) - \frac{y_2}{w_2} \right) \\ \text{s.t.} \quad & n(y_1 - c_1) + (1-n)(y_2 - c_2) \geq 0 \\ & \ln(c_1) - \frac{y_1}{w_1} - \ln(c_2) + \frac{y_2}{w_2} \geq 0. \end{aligned}$$

The FOC for this problem are:

$$\begin{aligned} \frac{n}{c_1} - n\mu + \frac{\lambda}{c_1} &= 0, & -\frac{n}{w_1} + n\mu - \frac{\lambda}{w_1} &= 0 \\ \frac{(1-n)}{c_2} - (1-n)\mu - \frac{\lambda}{c_2} &= 0, & -\frac{(1-n)}{w_2} + (1-n)\mu + \frac{\lambda}{w_2} &= 0, \end{aligned}$$

and the constraints.

The solution is:

$$c_1 = w_1$$

$$c_2 = w_1 \left( \frac{w_2(1+n) - nw_1}{w_2n + w_1(1-n)} \right)$$

$$y_1 = nw_1 - (1-n)w_1 \ln \left( \frac{w_2(1+n) - nw_1}{nw_2 + (1-n)w_1} \right) + (1-n)w_1 \left( \frac{w_2(1+n) - nw_1}{nw_2 + (1-n)w_1} \right)$$

$$y_2 = nw_1 + nw_1 \ln \left( \frac{w_2(1+n) - nw_1}{nw_2 + (1-n)w_1} \right) + (1-n)w_1 \left( \frac{w_2(1+n) - nw_1}{nw_2 + (1-n)w_1} \right).$$

Let  $C = \frac{w_2(1+n) - nw_1}{nw_2 + (1-n)w_1}$  which is less than one for all  $n$ .

For  $n$  large enough,  $C < 0$ . In this range, the solution has  $y_2 = 0$ . The outcome is then identical to that under the Rawlsian objective function.

When  $y_2 > 0$ , the utilities of the two types are:

$$U_1 = \ln w_1 - n + (1-n) \ln(C) - (1-n)C,$$

$$\text{and } U_2 = \ln w_1 + \ln C - \frac{nw_1}{w_2} - \frac{nw_1}{w_2} \ln C - \frac{(1-n)w_1}{w_2} C.$$

Differentiating  $U_1$  with respect to  $n$ , we obtain:

$$\frac{\partial U_1}{\partial n} = -1 + \frac{(1-n)}{C} \frac{\partial C}{\partial n} + C - (1-n) \frac{\partial C}{\partial n} - \ln C,$$

$$\text{where } \frac{\partial C}{\partial n} = - \frac{(w_2 - w_1)^2}{(nw_2 + (1-n)w_1)^2} < 0.$$

Rewriting, we obtain:

$$\frac{\partial U_1}{\partial n} = \{C - 1 - \ln C\} + (1-n) \left[ \frac{1-C}{C} \right] \frac{\partial C}{\partial n}.$$

The term in brackets is always positive, while the second term is negative. At  $n = 0$ ,

taking a Taylor series for  $\ln C$  around  $C = \frac{w_2}{w_1}$ , we obtain

$$\begin{aligned} \frac{\partial U_1}{\partial n} \Big|_{n=0} &= -1 + \frac{w_2}{w_1} + \frac{w_2 - w_1}{w_2} - \frac{(w_2 - w_1)^2}{2w_2^2} + \frac{(w_2 - w_1)^3}{w_1^2 w_2} + R \\ &= \left( -1 + \frac{w_2}{w_1} \right) + \frac{w_2 - w_1}{w_2} \left( 1 - \frac{w_2 - w_1}{2w_2} + \frac{(w_2 - w_1)^2}{w_1^2} \right) + R, \end{aligned}$$

where  $R < 0$  is the remainder term in the Taylor expansion. Since  $w_2 < w_1$ , the first two

terms are negative. Thus,  $\frac{\partial U_1}{\partial n} < 0$  at  $n = 0$ .

Differentiating  $U_2$  with respect to  $n$ , we obtain:

$$\begin{aligned}\frac{\partial U_2}{\partial n} &= \frac{1}{C} \frac{\partial C}{\partial n} - \frac{w_1}{w_2} - \frac{w_1}{w_2} \ln C + \frac{w_1}{w_2} C - \frac{nw_1}{w_2 C} \frac{\partial C}{\partial n} - \frac{(1-n)w_1}{w_2 C} \frac{\partial C}{\partial n} \\ &= \frac{w_1}{w_2} (C - 1 - \ln C) + \frac{1}{C} \frac{\partial C}{\partial n} \left( 1 - \frac{w_1}{w_2} \right) > 0\end{aligned}$$

since both terms are positive.

Thus, we see that, under more general SWFs, the utility of the skilled can first decline and then increase as their proportion in the population increases. Whether the unskilled work or not, their utility increases as the proportion of skilled increases.

**Table 1**  
**Equilibria with Different Tax Systems**

| Tax Criterion          | Mobile Group                                                              |                                                                                                                                                                  |
|------------------------|---------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                        | Unskilled                                                                 | Skilled                                                                                                                                                          |
| <b>Rawlsian</b>        | $n \rightarrow \bar{n}$<br>$U_2 \rightarrow \bar{U}_2$<br>(harmonization) | if $\bar{n} < \hat{n}$ and $n_0 < n_\gamma$ , $n \rightarrow \bar{n}$<br>and $U_1 \rightarrow \bar{U}_1$ (harmonization)<br>otherwise, unsustainable equilibrium |
| <b>Majority voting</b> | $n \rightarrow \bar{n}$<br>$U_2 \rightarrow \bar{U}_2$<br>(harmonization) | if $n_0$ and $\bar{n}$ both $> 1/2$ or $< 1/2$ ,<br>harmonization<br>otherwise, unsustainable equilibrium                                                        |

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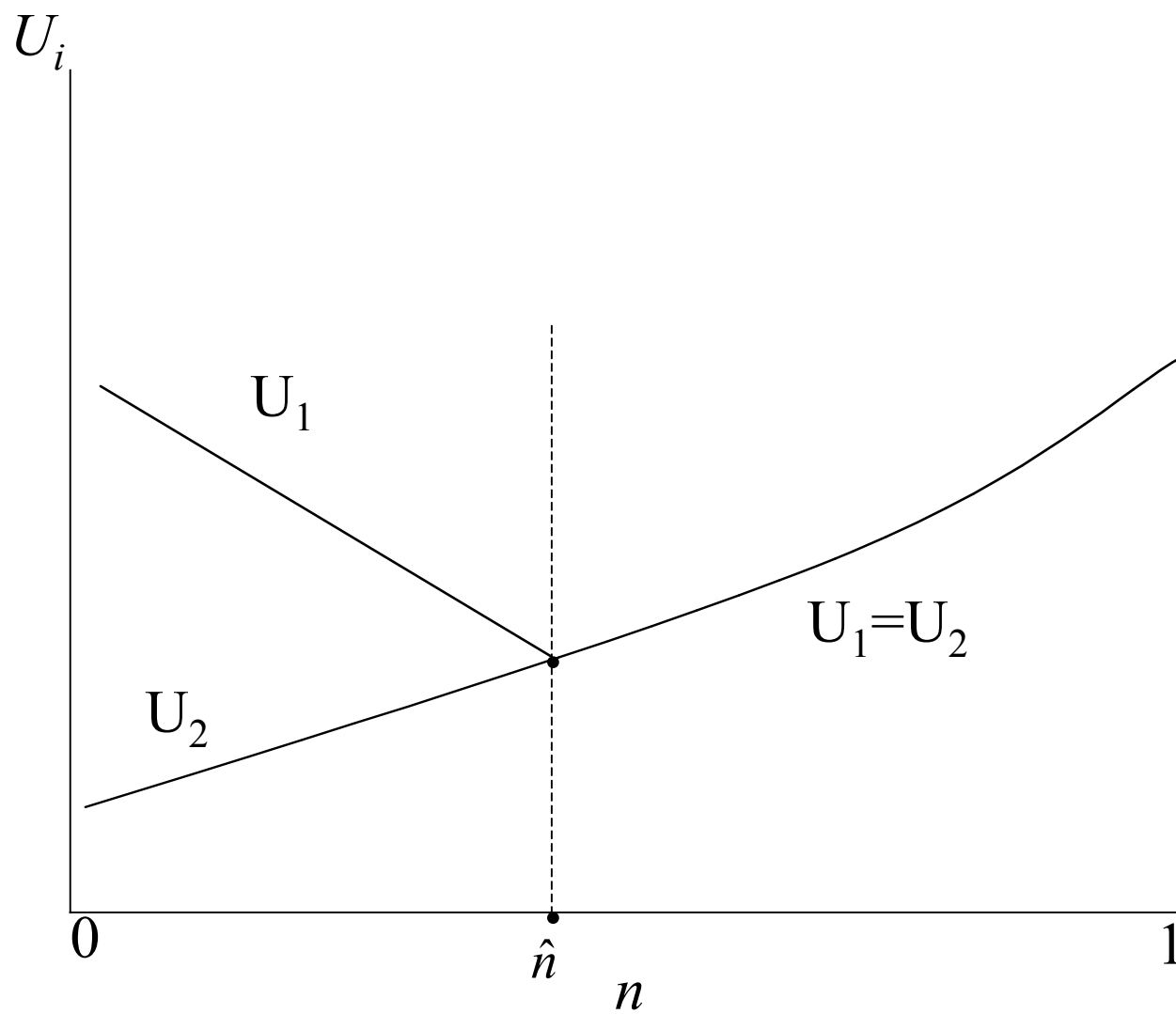


Figure 1 – The Rawlsian Outcome

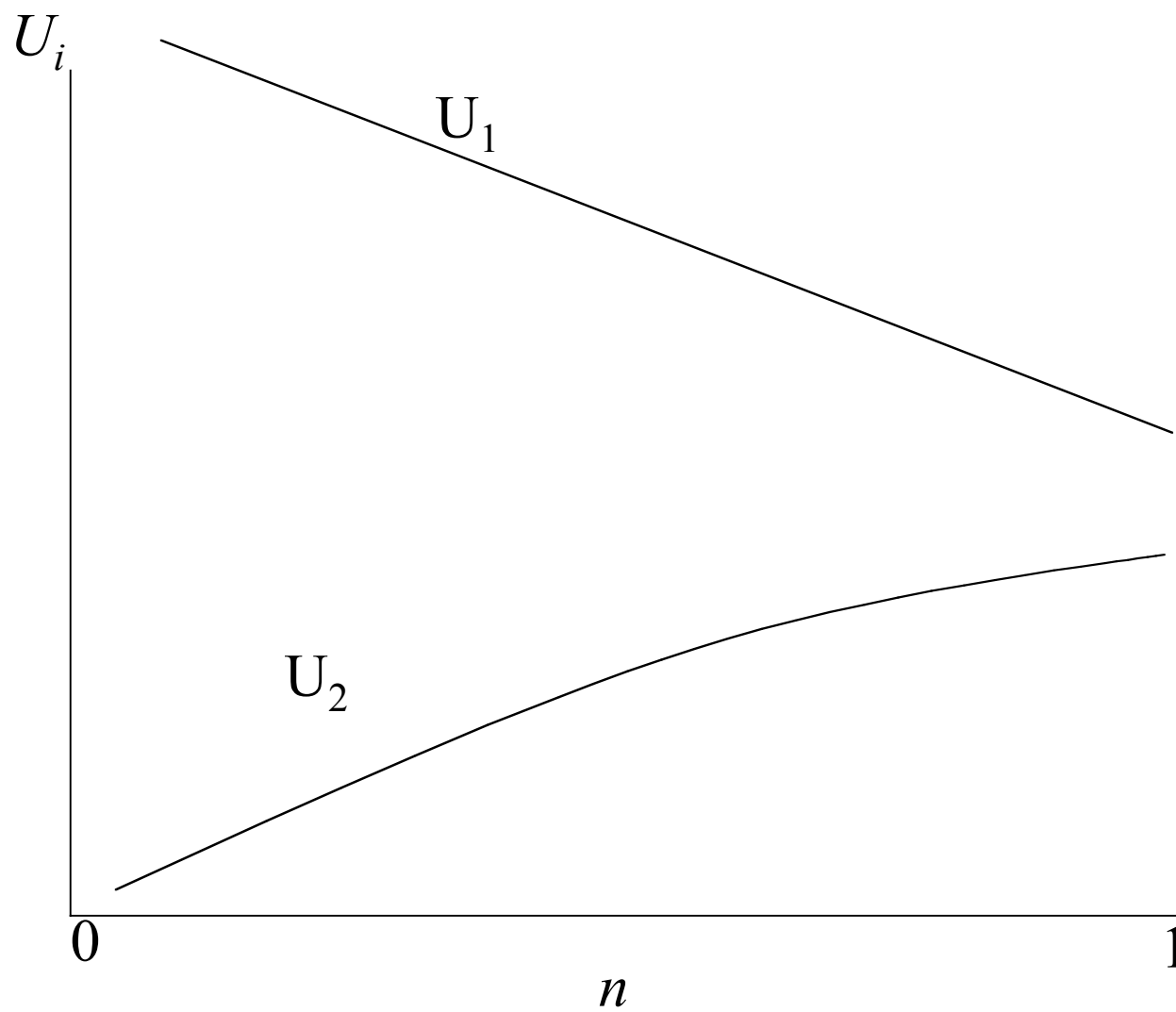


Figure 2 – The Despotic Outcome

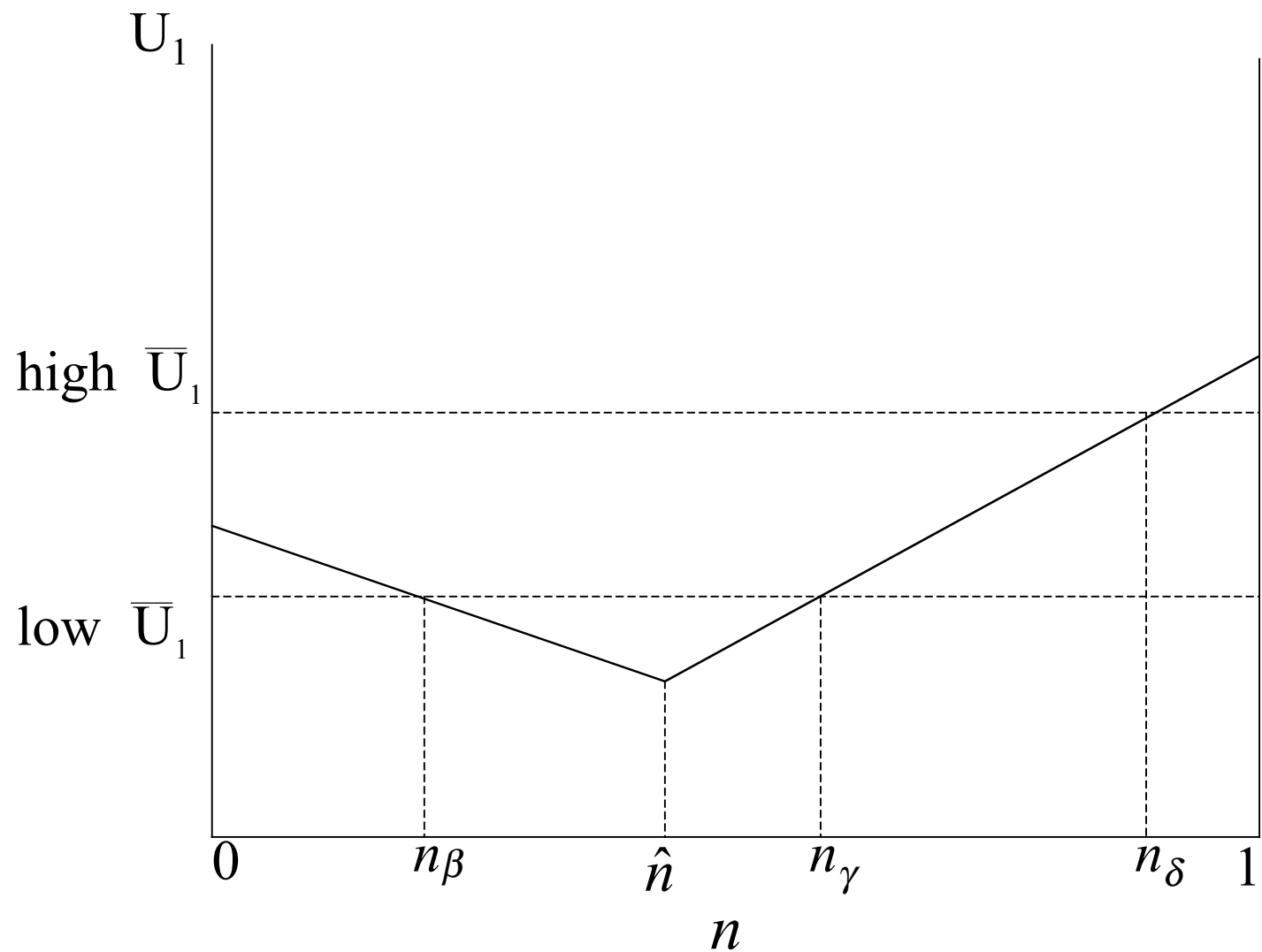


Figure 3 – The Rawlsian Outcome With Mobile Skilled Workers

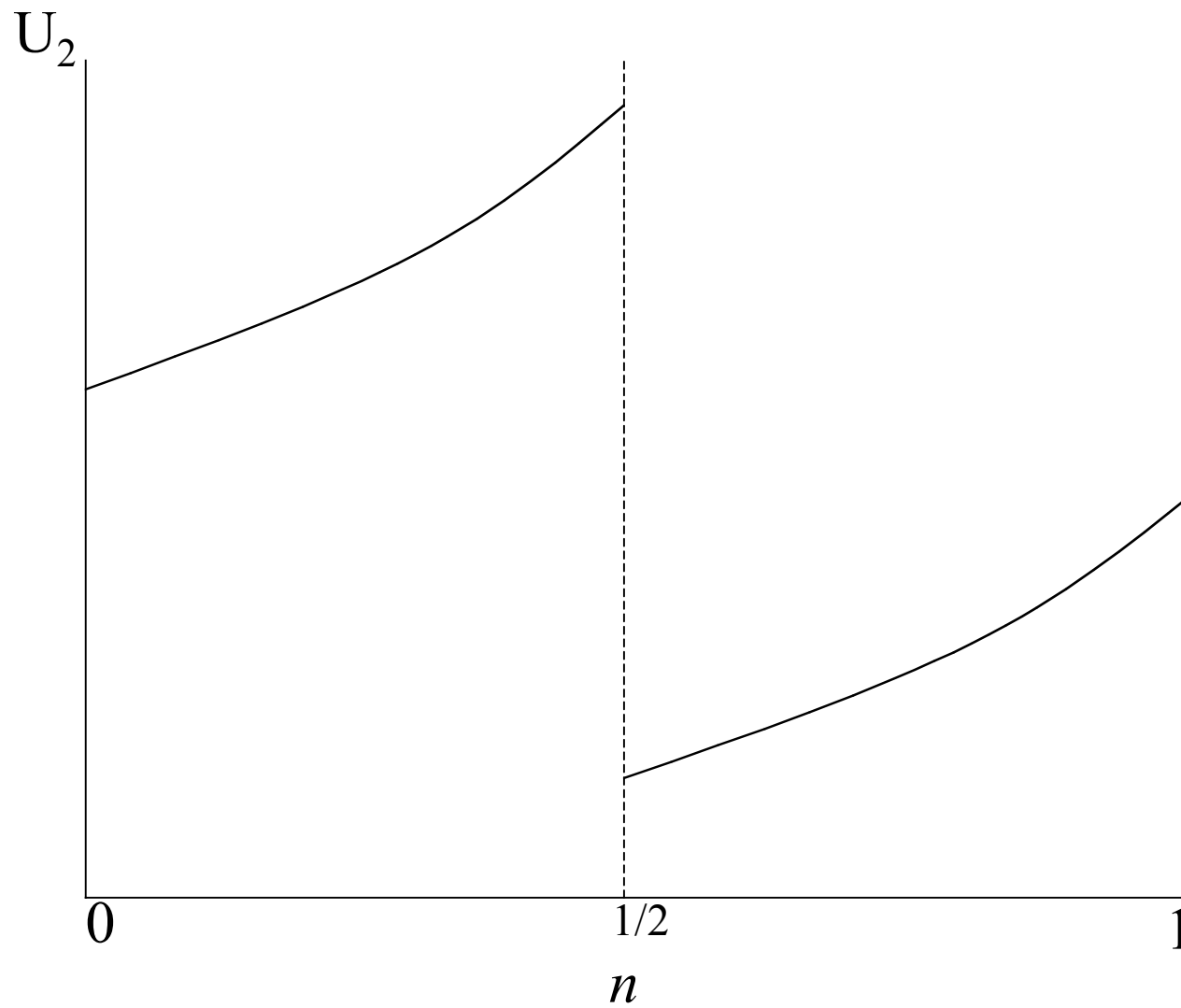


Figure 4 – Majority Voting When Unskilled Are Mobile

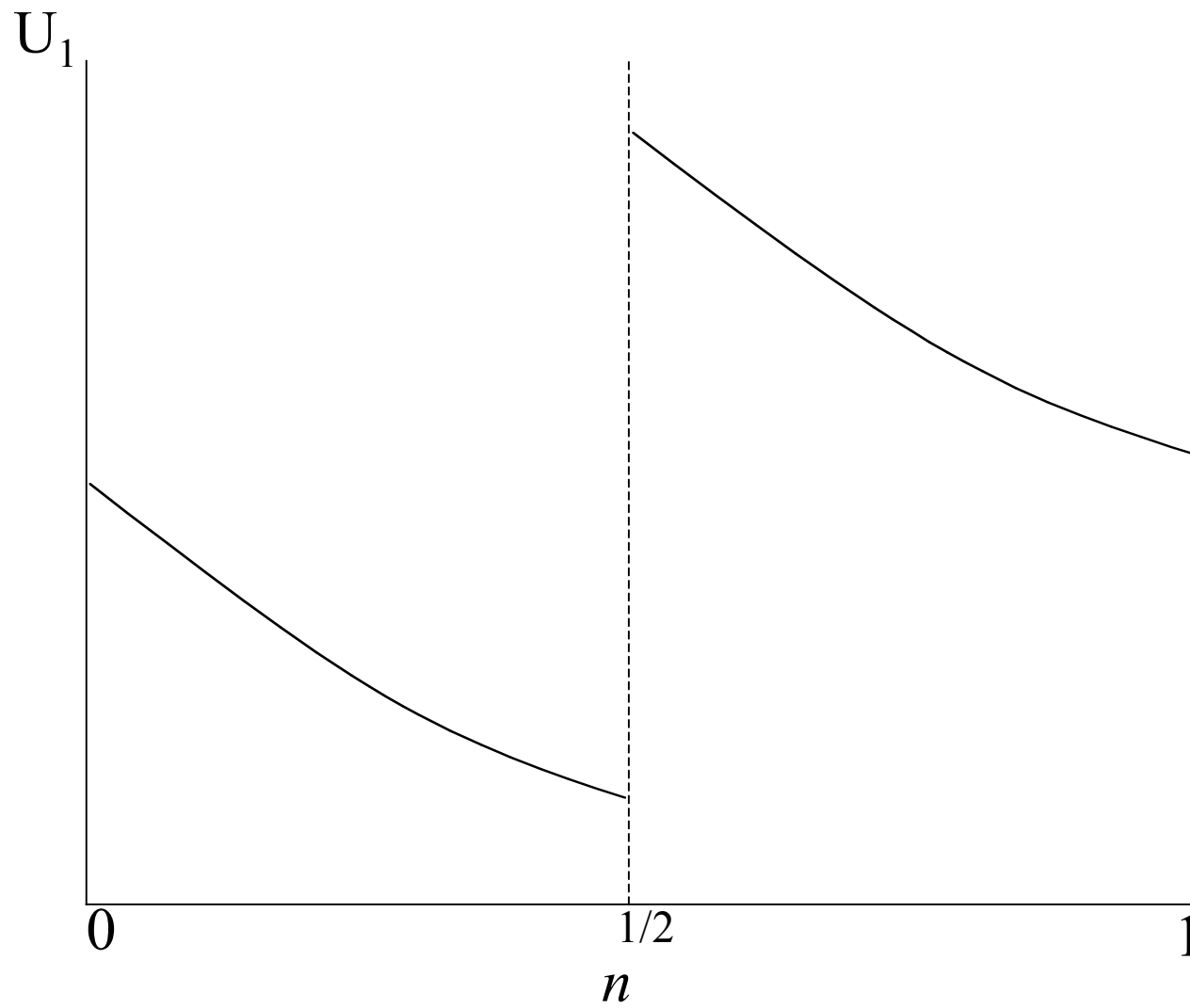


Figure 5 – Majority Voting When Skilled Are Mobile