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APPLYING THE "MERIDIONAL STREAMFUNCTION" VISUALIZATION TECHNIQUE TO OPEN OCEAN DOMAINS

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ABSTRACT

It is common practice to represent by a streamfunction the transport field ensuing from the integration - over one horizontal coordinate - of the velocity field predicted by ocean models. This very useful visualization technique requires that the transport field be divergenceless. If this is not the case, it is suggested to extract the divergenceless transport field that is closest to that stemming from the model while verifying the impermeability conditions of the ocean surface and bottom. A variational approach is examined.

1. INTRODUCTION

Most of the quantities computed by ocean models are four-dimensional, i.e., they depend on time and three space coordinates. Since the vast majority of graphical tools are two- or three-dimensional, the graphical representation of the model results requires appropriate mappings onto a two- or three-dimensional space. This may be carried out in numerous ways by means of existing graphical packages.

One must however bear in mind that the role of computer graphics is not just to produce attractive pictures, but to help gain insight into the physics of the ocean flow under study. In other words, physical intuition and reasoning are also needed if profound understanding of the flow's mechanism is sought. Schematically, this might be expressed by the following relation: quality of the interpretation = (physical skill) × (computer graphics skill). This implies, for instance, that the best graphical software will probably prove useless if operated by someone having no idea of ocean dynamics!

An example of an interpretation technique based on little graphical skill but considerable physical skill is the "meridional streamfunction" approach. This technique, which has now become a standard for visualizing and discussing the results of OGCMs (Bryan, 1982; Bryan, 1987; Toggweiler et al., 1989; Killworth et al., 1991; Marotzke and Willebrand, 1991; England, 1992; Semtner and Chervin, 1992), consists in integrating the velocity in meridional planes over the longitudinal width of the ocean basin under study. Assuming that there is no water flux across the land-sea boundaries, the resulting two-dimensional vector field is divergenceless, implying that it may be represented with the help of a streamfunction - termed "meridional streamfunction" since the longitudinal velocity component is ignored. Contours of the streamfunction are drawn, and the difference between the streamfunction values associated with two given isolines corresponds to the water flux flowing between these isolines.

Of course, if it is preferable to integrate over the latitudinal width of the domain, a procedure similar to that described above may be utilized. In fact, the direction of integration does not matter. But, for the vector field to be properly represented by means of a streamfunction, it is crucial that it be divergenceless, meaning that the water velocity normal to the lateral boundaries of the domain of interest must be zero. Here, the "lateral boundaries" are defined to be the surfaces corresponding to the upper and lower values of the space variable over which the flow is integrated.

Thus, when the lateral boundaries are not impermeable, representing the integrated flow with the help of a streamfunction is, in general, impossible. This serious drawback may however be circumvented by resorting to the approximate technique established in the present note.

This study concentrates on the mathematical formulation of the method, leaving actual applications to a forthcoming, more extensive article.

Below, the following vocabulary is utilized: a quantity being measured in m s^{-1} is called "velocity", a "transport" is expressed in $m^2\ s^{-1}$ and $m^3\ s^{-1}$ (or, equivalently, Sverdrups) are used for "fluxes".

2. VARIATIONAL APPROACH

Consider the domain D depicted in Fig. 7. Let the "lateral coordinate" be $y - y_-(x) \leq y \leq y_+(x)$, which means that the flow in the (x, z) plane is to be represented. The vertical coordinate z is zero at the ocean surface, assumed flat, and is equal to $h(x, y)$ (≥ 0) at the bottom. The model velocity components along the (x, z) axes are (u^*, w^*), w^* being positive in the case of downwelling. The unit vectors ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$) are associated with the (x, y, z) coordinates.

The transport

$$\mathbf{U}^*(x,z) = U^*\,\mathbf{e}_x + W^*\,\mathbf{e}_z = \int_{y_-(x)}^{y_+(x)} (u^*\,\mathbf{e}_x + w^*\,\mathbf{e}_z)\,dy \tag{1}$$

is defined in the domain Ω, i.e., $x_- \leq x \leq x_+$ and

$$0 \leq z \leq H(x) \equiv \max_{y_- \leq y \leq y_+} h(x,y)$$

(Fig. 8). The vector field above obeys the following mass conservation equation

$$\frac{\partial U^*}{\partial x} + \frac{\partial W^*}{\partial z} = q^*, \tag{2}$$

where q^* is the sum of the normal components of the horizontal velocity entering D at (x, y_-) and (x, y_+).

When q^* is zero, $\mathbf{U}^*$ is divergence-free. Hence, this transport may be represented by means of a streamfunction, which is no longer possible when $q^* \neq 0$. In this case, only an appropriate divergenceless part of $\mathbf{U}^*$, hereafter denoted $\mathbf{U}$, may be represented by a streamfunction Ψ, i.e., (Fig. 8)

$$\mathbf{U} = U\,\mathbf{e}_x + W\,\mathbf{e}_z = \frac{\partial \Psi}{\partial z}\,\mathbf{e}_x - \frac{\partial \Psi}{\partial x}\,\mathbf{e}_z = -\nabla \times \Psi\,\mathbf{e}_y, \tag{3}$$

with $\nabla = \mathbf{e}_x\,\partial/\partial x + \mathbf{e}_z\,\partial/\partial z$. The transport field $\mathbf{U}$ identically satisfies the mass conservation equation $\nabla \bullet \mathbf{U} = 0$. It remains to define what is meant by "an appropriate divergenceless part of $\mathbf{U}^*$".

It seems quite natural to compute $\mathbf{U}$ as the divergence-free transport field that is closest to $\mathbf{U}^*$. Thus, $\mathbf{U}$ may be obtained as the vector field corresponding to the minimum of the functional

$$I = \int_\Omega |\,\mathbf{U} - \mathbf{U}^*\,|^2 \; d\Omega\,, \tag{4}$$

with $d\Omega = dx\;dz$.

Many other functionals would have been equally valid, but we opted for a quadratic form, because determining the function rendering such a functional minimum is usually straightforward.

To allow meaningful physical interpretations, it is clear that $\mathbf{U}$ must respect the impermeability of the upper and lower boundaries of Ω. In other words, it is required that Ψ satisfy the following essential conditions:

$$[\;\Psi(x, z=0),\; \Psi(x, z=H)\;] = (0, \Psi_b = const.), \tag{5}$$

where Ψ_b represents the constant, horizontal flux flowing through the domain from x_- to x_+.

By expressing that the first order variation of I must be zero, the Euler-Lagrange equations of the minimum principle are obtained, i.e.,

$$\nabla^2\,\Psi = \frac{\partial U^*}{\partial z} - \frac{\partial W^*}{\partial x}\,,\ \text{in}\ \Omega\,, \tag{6}$$

$$\begin{aligned}\{\,[\frac{\partial \Psi}{\partial x}]_{x=x_-}\,,\,[\frac{\partial \Psi}{\partial x}]_{x=x_+}\} &= -\,[\;W(x_-, z),\; W(x_+, z)\;] \\ &= -\,[\;W^*(x_-, z), W^*(x_+, z)\;]\,.\end{aligned} \tag{7}$$

Since $\nabla^2\,\Psi = \partial U/\partial z - \partial W/\partial x$, Eq. (6) means that $\mathbf{U}$ and $\mathbf{U}^*$ must have the same "vorticity". The boundary conditions (7) appear as "natural conditions" of the minimum principle, while the impermeability of the surface and the bottom are imposed as "essential conditions" (Fig. 8).

Relations (5) – (7) form a well-posed, elliptic, partial differential problem, the solution of which is easily obtained. However, Ψ_b is unkwown. In fact, Ψ_b must be such that I be minimum.

The value of Ψ_b might be determined by a "brute force", iterative method. Let k represent an integer index associated with the iterations. If I_k, I_{k+1} and I_{k+2} denote successive values of I corresponding to $\Psi_{b,k}$, $\Psi_{b,k+1}$ and $\Psi_{b,k+2}$, respectively, it is possible to obtain an approximation to the value of Ψ_b rendering I minimum. Indeed, assuming that I may be quadratically interpolated over $\Psi_{b,k}$, $\Psi_{b,k+1}$ and $\Psi_{b,k+2}$, expressing that $\partial I/\partial \Psi_b = 0$, one gets

$$\Psi_{b,k+3} = -\frac{A}{2B}\,,$$

where

$$\begin{aligned} A &= (I_{k+1} - I_k)(\Psi^2_{b,k+2} - \Psi^2_{b,k+1}) - \\ &\quad (I_{k+2} - I_{k+1})(\Psi^2_{b,k+1} - \Psi^2_{b,k}) \\ B &= (I_{k+2} - I_{k+1})(\Psi_{b,k+1} - \Psi_{b,k}) - \\ &\quad (I_{k+1} - I_k)(\Psi_{b,k+2} - \Psi_{b,k+1}). \end{aligned} \tag{8}$$

This process may be repeated until the minimum of I is sufficiently approached.

Another iterative method may be equally valid for seeking the minimum of I over Ψ_b. Whatever the method selected, it will be necessary to solve the problem (5) – (7) several times. This may be computationally expensive, especially if the number of grid points placed in Ω is large.

Fortunately, by somewhat reworking the partial differential problem (5) – (7), it may be seen that a direct, non-iterative determination of Ψ_b is possible.

3. NON-ITERATIVE METHOD

As the problem to be solved is linear, its solution may be considered a linear combination of several functions, i.e.,

$$\Psi(x, y) = \Psi^0(x, y) + \Psi_b\Psi^1(x, y)\,. \tag{9}$$

The function Ψ^0 is the solution of (5) – (7), except that

$$\Psi^0(x, z=H) = 0\,, \tag{10}$$

which means that no water flux through the domain is associated with Ψ^0. On the other hand, Ψ^1 is obtained from the following partial differential problem:

$$\nabla^2\,\Psi^1 = 0,\ \ \text{in}\ \Omega\,, \tag{11}$$

$$[\;\Psi^1(x, z=0),\; \Psi^1(x, z=H)\;] = (0, 1)\,, \tag{12}$$

$$\{\ [\frac{\partial \Psi^1}{\partial x}]_{x=x_-}\ ,\ [\frac{\partial \Psi^1}{\partial x}]_{x=x_+}\ \}\ =\ (0,\ 0)\ \cdot \qquad (13)$$

For a given domain Ω, Ψ^1 is independent of the flow regime. Thus, Ψ^1 should be calculated once and for all.

It is easily seen that $\Psi^0\ +\ \Psi_b\ \Psi^1$ is actually the solution to (5) – (7). To determine Ψ_b, we introduce (9) into (4) and seek the minimum of I over Ψ_b. Accordingly, Ψ_b is the solution of $\partial I/\partial \Psi_b = 0$, i.e.,

$$\Psi_b\ = \\ -\ \frac{\int_\Omega (\nabla\Psi^0 \bullet \nabla\Psi^1\ +\ \mathbf{e}_y \bullet (\mathbf{U}^* \times \nabla\Psi^1)\ )\ d\,\Omega}{\int_\Omega\ |\ \nabla\Psi^1\ |^2\ d\Omega} \qquad (14)$$

which indeed corresponds to the minimum of I, because

$$\frac{\partial^2 I}{\partial \Psi_b^2}\ =\ \int_\Omega\ |\ \nabla\Psi^1\ |^2\ d\Omega\ \geq\ 0\ \cdot \qquad (15)$$

4. CONCLUSION

The mathematical foundations of a technique aiming at adapting the classical "meridional streamfunction" method to open ocean domains has been discussed. It must be stressed that the approach suggested in the present note does not simply amount to taking the rotational part of a given two-dimensional transport field: the transport field we determine is divergenceless and has the same curl as that produced by the model, but, in addition, it is required that it satisfy the impermeability of the upper and lower boundaries of the domain.

Whether or not the technique discussed here will turn out to be useful is presently not clear. It has yet to be applied to model results, which will then allow assessing the physical interpretations it will lead to. It is however believed that our method will prove appropriate for visualizing the results of large- and small-scale ocean models, as well as those of atmospheric models – after minor modifications.

Finally, it must be stressed that determining Ψ only is not sufficient, since it does not permit us to evaluate to what extent $\mathbf{U}$ is a good approximation of the modelled transport field $\mathbf{U}^*$. As a result, it will be necessary to systematically estimate the "distance" between $\mathbf{U}$ and $\mathbf{U}^*$, for example, by computing

$$\varepsilon\ =\ [\frac{\int_\Omega\ |\ \mathbf{U} - \mathbf{U}^*\ |^2\ \ d\Omega}{\int_\Omega\ |\ \mathbf{U}^*\ |^2\ d\Omega}]^{\frac{1}{2}}\ \cdot \qquad (16)$$

Clearly, for large ε, the greatest precaution will be required when discussing the field $\mathbf{U}$– or Ψ. In such a case, it might turn out preferable to modify the domain of integration D so as to obtain a smaller value of ε, allowing more relevant discussion of the resulting streamfunction Ψ.

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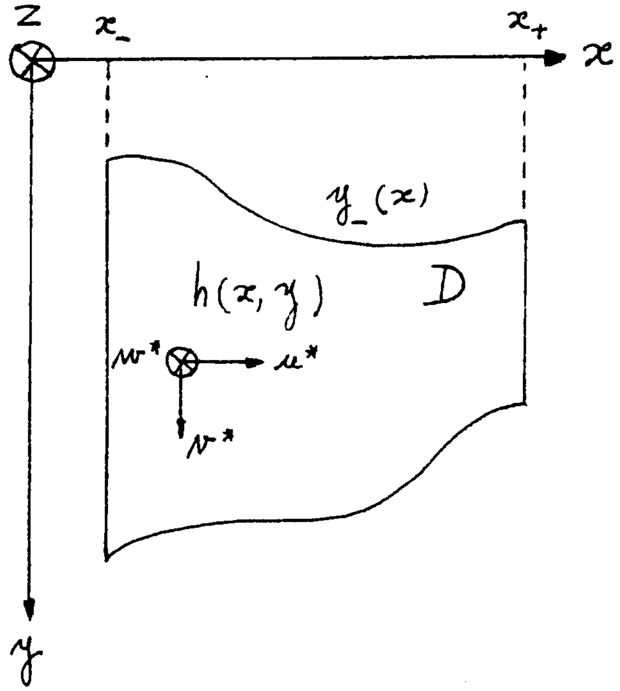

FIGURE 7
(Deleersnijder and Beckers)
The domain of interest D, with the direction of the coordinate axes and the velocity components.

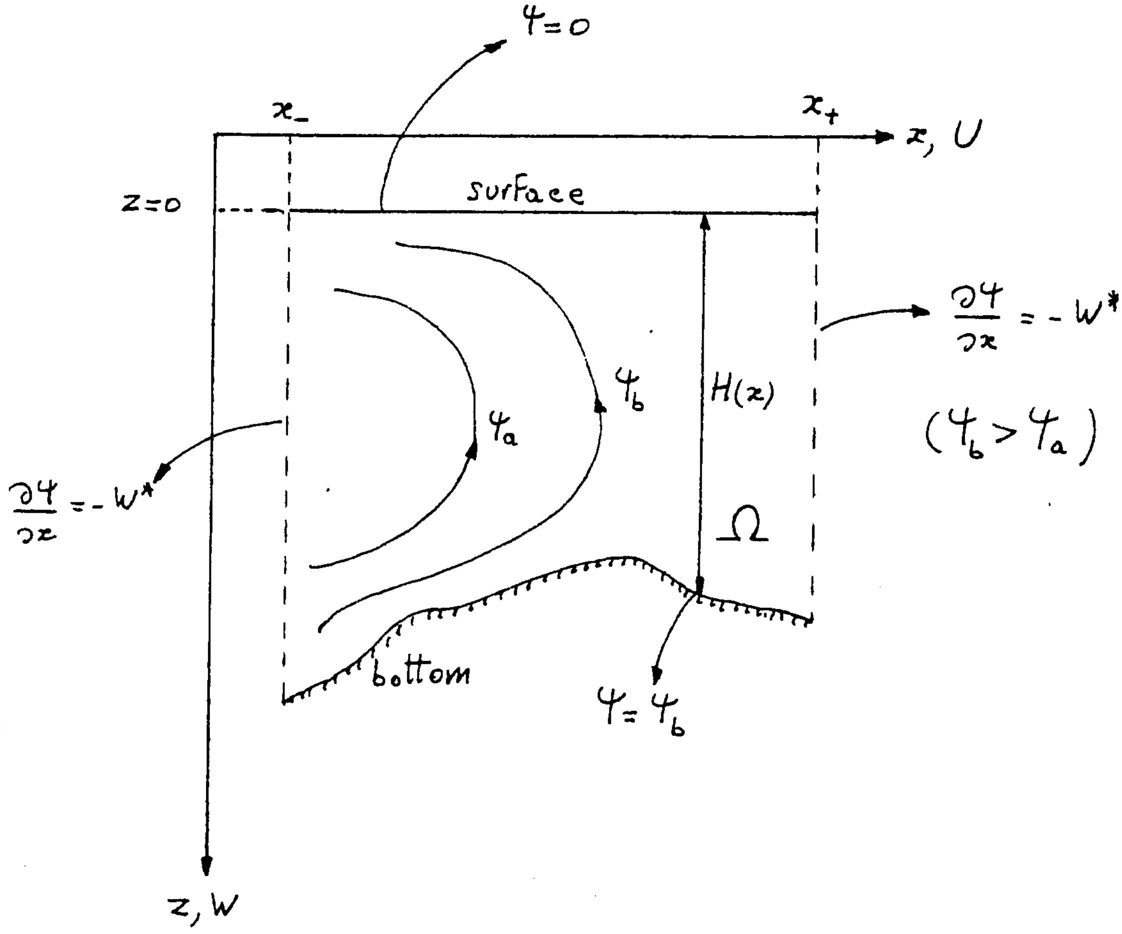

FIGURE 8

(Deleersnijder and Beckers)

The domain Ω in the vertical plane (x, z), with the boundary conditions to be applied to the streamfunction Ψ.