Intertemporal general equilibrium and monetary theory [1]

Jacques H. Drèze [2] Heracles M. Polemarchakis [3]

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[2]CORE, Universitè Catholique de Louvain, Belgium.

[3]CORE, Universitè Catholique de Louvain, Belgium.

Abstract

The introduction of banks that issue money and supply balances and pay out their profits as dividends is the natural modification of the model of general competitive equilibrium that encompasses monetary economies.

Competitive equilibria exist. Nevertheless, eventhough there is a well defined money market, competitive equilibrium allocations are indeterminate. On an event tree with N nodes, of which S terminal, there are $N + S$ degrees of nominal and, possibly real, indeterminacy. Monetary policy removes some degrees of indeterminacy through a choice of instruments, set according to a state-contingent rule.

Interest rates are suitable instruments for the control of expected inflation but not of the variability of inflation.

Monetary policy is also effective due to redistributive effects and nominal rigidities.

Key words: money policy, equilibrium.

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Contents

1 PREVIEW

1.1 Conclusion

Theory is often at odds with the thinking and practice of monetary authorities. A confrontation of this situation is timely. General equilibrium theory serves to check the consistency of models and to bring out some of their properties. Surprisingly, an enquiry into monetary theory from the perspective of general equilibrium leads to conclusions that resemble views expressed recently by specialists in monetary theory and policy, and suggests an approach to policy that resembles the approach practised by monetary authorities. Also, it delineates theoretical issues more sharply than do less abstract formulations.

Conclusions, not original, though their emergence from a general equilibrium model is perhaps new, are that

1. monetary policy is defined by a choice of instruments and a state-contingent rule for adjusting these instruments as information is revealed;

2. the rule is bound to remain largely implicit, but greater transparency should help;

3. interest rates or, alternatively, money supplies are suitable instruments for the control of expected inflation, but not of the variability of inflation;

4. the real process of nominal price formation restricts, in an insufficiently understood manner, the ability of monetary policy to control nominal variables;

Effectiveness of monetary policy is synonymous with departures from monetary neutrality. Although neutrality is a theoretical possibility, it is associated with extreme assumptions and of little relevance.

The theory of monetary policy is concerned with the identification of significant departures from neutrality which have clearcut implications. Such departures include

1. the negative wealth effect of higher nominal interest rates, when ricardian equivalence is only partial, as it is bound to be under an incomplete market structure;

2. the partial translation of nominal interest rates into real rates, associated with nominal stickiness of prices and wages.

The extension of the formal analysis to incomplete asset markets and non-walrasian equilibria should add to this list.

These conclusions are extrapolative of theoretical results.

1.2 Organisation

The theoretical work underlying this paper, Drèze and Polemarchakis, (1994, 1997) consists in formulating an intertemporal general equilibrium model with money; spelling out reasonable assumptions that guarantee the existence of competitive equilibria; and characterising these equilibria, with particular attention to nominal indeterminacy and to neutrality. The presentation here is largely expository and partly speculative. Section 2 records concisely the essentials of the model of general equilibrium with time and uncertainty. Section 3 extends the model to monetary economies and states an existence result proved in Drèze and Polemarchakis (1997). Section 4 presents a basic result on nominal indeterminacy and draws implications for the definition of monetary policy. Section 5 discusses, somewhat heuristically, significant departures from neutrality. Section 6 illustrates the contents of sections 3, 4 and 5 with examples. Conclusions are outlined above.

2 INTERTEMPORAL GENERAL EQUILIBRIUM

2.1 An abstract model

The Arrow-Debreu (1954) model of general competitive equilibrium is structured on

1. commodities, $l \in \mathcal{L} = \{1, \ldots, L\}$; bundles of commodities are $x = (\ldots, x_l, \ldots)$;
2. firms, $j \in \mathcal{J} = \{1, \ldots, J\}$; each firm is described by Y^j, a set of feasible production plans, y^j;
3. individuals or households, $i \in \mathcal{I} = \{1, \ldots, I\}$; each individual is described by $\mathcal{Z}^i$, a set of feasible net trades in commodities, z^i, including prestations of labor; R^i, a preference ordering on Z^i; and θ^i, the endowment of shares in firms.

An *allocation* is an array $a = (\ldots, y^j, \ldots, z^i, \ldots)$, of production plans for firms and net trades for individuals; it is *feasible* when

$$\sum_{i \in \mathcal{I}} z^i - \sum_{j \in \mathcal{J}} y^j = 0.$$

Market decentralization is structured on prices of commodities, $p = (\ldots, p_l, \ldots)$, and the budget constraints of individuals

$$pz \leq \sum_{j \in \mathcal{J}} \theta^{i,j} v^j,$$

where

$$v^j = py^j$$

are the profits or market values of firms.

A feasible allocation and a price vector define a *market allocation* when the net trade of every individual satisfies the budget constraint.

A market allocation is a *competitive equilibrium allocation* and the associated prices are *competitive equilibrium prices* if

1. the net trade of every individual is maximal for his preference relation subject to the budget constraint;
2. the profit of every firm is maximal.

The structure is general and lends itself to other equilibrium concepts, such as equilibria with imperfectly competitive markets, game-theoretic equilibria, cooperative or non-cooperative, equilibria with quantity constraints or equilibria with incomplete markets.

2.2 Time and uncertainty

The relevance of the model of general equilibrium is substantially enhanced by the observation in Arrow (1953) and Debreu (1959, 1960) that a commodity can be defined, not only by its physical characteristics, including a time and place of delivery, but also by the "state of the world" prevailing at delivery.

The primitives of an economy, the list and characteristics of commodities, firms and households evolve over time under the influence of exogenous circumstances: the physical environment, discoveries and technological advances, population or culture... One may conceive of a description of such circumstances exhaustive enough that, for any date, primitives are defined uniquely up to that date, though contingently on developments over the remaining future. Applied to a terminal date, such a description defines a "state of the world;" applied to an intermediate date, it defines a subset of the states, an "event;" over a finite horizon, a "tree" describes the sequential resolution of uncertainty, technically, a filtration of the set of states.

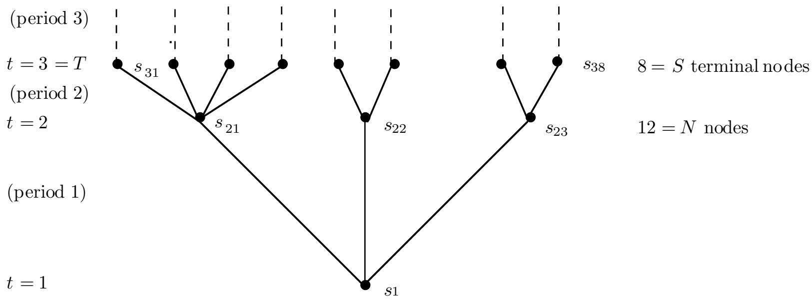

Figure 1. An event-tree.

Although the very idea of a finite tree is problematic, for practical purposes it is justified by the discreteness of measurements and the irrelevance at a date of branchings in a very distant future.

Dates are $t \in \mathcal{T} = \{1, \ldots, T\}$.

A finite tree, $\mathcal{F}$, contains nodes or date-events $n \in \mathcal{N} = \{1, \ldots, N\}$, of which S are terminal nodes or states of the world; evidently,

$$1 \leq S \leq N, \ 1 \leq T \leq N.$$

Events at date t are $s_t \in \mathcal{S}_t$, with $\mathcal{S}_1$ a singleton and $\mathcal{S}_T$ of cardinaliy S.

One identifies commodities and, hence, prices by the pair of subscripts (l, s_t), where l refers to physical characteristics and s_t to the date-event contingently on which delivery takes place; x_{l,s_t} is a quantity of the physical commodity l at date t, contingent on the event s_t. The index of commodities runs over $\mathcal{L} \times \mathcal{N}$.

In market economies, the price p_{l,s_t} is the price of the contract traded at date 1 that gives right to one unit of commodity l at date t if and only if the event s_t obtains; $\sum_{s_t \in \mathcal{S}_t} p_{l,s_t}$ is the price of an unconditional promise to deliver one unit of commodity l at date t.

This framework is sufficiently general to encompass as special cases most models or situations. This does not negate the technical advantages that may result from treating time as continuous, possibly open-ended, or from relying on continuous random variables to describe some contingencies, or from analysing aggregative or representative individual models.

2.3 Incomplete markets

Events and event trees are a useful conceptual abstraction which helps clarify a number of issues in the economics of uncertainty. The specification which requires that markets clear at some initial date for delivery of any physical commodity contingent on any date-event has made the well developed theory of general equilibrium directly applicable to uncertain environments. But it is difficult to conceive of such a complete market structure as a realistic possibility. Reasons for skepticism are that

1. to clear all markets at some initial date requires trading then by as yet unborn individuals;

2. to enter a complete description of states of the world is informationally and computationally forbidding;

3. to transact on that many markets would be too costly for individuals as well as for organised exchanges.

A more realistic specification recognises that markets are incomplete. Individuals and firms trade on a limited set of markets, knowing that additional markets – spot markets, but also markets for assets, futures or options – will open at future dates. Trading is thus of necessity based on expectations about the prices that will prevail on these additional markets, when they open. Strong assumptions on the sequential structure of markets and on the expectations of individuals are required to obtain competitive equilibria, following Radner (1972), as "equilibria of prices, plans and price expectations."

3 MONEY IN GENERAL EQUILIBRIUM

3.1 Modeling options

Modeling options are guided by the desire to capture as simply as possible the essential roles and institutional features of money.

Fiat money produced at no cost by banks serves as a medium of transactions. Banks lend money to households and firms against promise of reimbursement with interest or, equivalently, in exchange for interest-bearing bonds. All initial holdings of money are the counterpart of debts to banks[1]. In the vocabulary of monetary economics, this is a model of "inside money." It is the model appropriate for economies with properly functioning central banks that only issue money in exchange for offsetting nominal claims. There is no default and loan contracts are met. Interest earned by banks accrues to shareholders as dividends[2].

Banks also accept deposits and pay depositors the same rate of interest which they charge on loans; equivalently, there are interest bearing nominal assets, including savings accounts, which dominate money as a store of value. The only reason for holding non-interest-bearing money is its usefulness as a means of payment and the cost or inconvenience of withdrawals or deposits at banks. This is also, for most, the only reason for holding non-interest-bearing money[3]. It is a convincing reason and there is no need to look for another[4].

The demand for money at given commodity prices and interest rates results from the preference maximizing choices of individuals and the profit maximizing choices of firms.

An important modelling choice concerns the treatment of time. In line with the standard model of general equilibrium, there is a finite number of dates or elementary periods. Formally, a date is a point in time. For purposes of interpretation, the length of a time period is more naturally thought of as non-trivial, but rather short. In particular, within each period, (i) no new information accrues; (ii) prices and rates of interest are constant; (iii) the precise timing of transactions and the resulting time profile of consumption and production do not affect preferences or production possibilities, while it does affect money balances and interest accounting; and (iv) banks accomodate variations in money demand[5].

A realistic, detailed treatment of the transactions technology and of its implications for preferences and production possibilities should allow for non-convexities. The simplest example, the impossibility of trading without at least one transaction with the bank, introduces a non-convexity as does, more generally, the integer nature of the number of financial transactions. Disregarding the integer condition on financial transactions is neither more nor less serious than disregarding the integer condition on the purchase of most physical commodities. There, the usual disregard for these non-convexities is justified by the argument that they are eliminated by aggregation over a large number of transactions none of which affects the aggregate significantly. The same applies here.

3.2 Model and notation

There is one currency, with no specification of a place where the currency is held or used; but there is the distinction between date-events: on a tree with N nodes, there are, effectively, N distinct currencies.

The notation is that of section 2. Prices of commodities are nominal. Nominal interest rates are $r = (\ldots, r_{s_t}, \ldots)$. In the tradition of general equilibrium, prices are expressed as date 1 values, in units of money at the single event associated with date 1. This convention applies also to prices of balances held at other date-events and to interest rates. For interest rates, the convention means that r_{s_t} is the value at date 1 of the interest due for a unit loan at date t contingent on s_t and reimbursed at date $t+1$. Given the other components of the price system, r_{s_t} stands in one-to-one correspondence with the spot one-period nominal rate. These conventions simplify accounting drastically. Spot prices and interest rates at s_t are $\tilde{p}_{s_t}$ and $\tilde{r}_{s_t}$, respectively.

Balances held by individual are $m = (\ldots, m_{s_t}, \ldots) \geq 0$. The usefulness of balances for transactions is captured in two ways: through a generalizations of the consumption sets of individuals and /or through generalizations of the preferences of individuals .

It is easiest to introduce these generalizations through standard examples. A net trade in commodities, z, can be expressed as the sum of a non-negative part, z^+, where $z^+_{l,s_t} = \max\{z_{l,s_t}, 0\}$, and a non-positive part, z^-, where $z^-_{l,s_t} = \min\{z_{l,s_t}, 0\}$. The cash-in-advance model in Clower (1967) imposes the constraint that

$$m_{s_t} \geq \tilde{p}_{s_t} z^+_{s_t},$$

where m_{s_t} are interpreted as beginning-of-period money holdings. More generally, the exchange set correspondence, $\Phi^i(p, r)$, allows for more complex constraints involving nominal interest rates as well as prices.

Alternatively, the inventory model of Baumol (1952) and Tobin (1956) can be formulated in terms of a transaction cost consisting in time spent on "trips to the bank." During the period corresponding to the date-event s_t, the required number of trips is determined by the ratio

$$\frac{\tilde{p}_{s_t} z^+_{s_t}}{m_{s_t}},$$

where m_{s_t} is now interpreted as average money holdings within the period. For instance, with a constant rate of expenditure, the number of trips is equal to $2\tilde{p}_{s_t} z^+_{s_t}/m_{s_t}$. It is then natural to enter that ratio as an argument of the preference ordering, $R^i(z, m, p, r)$, which, as a consequence, is defined conditionally on prices and, more generally, on interest rates as well.

By analogy, the liquidity services of cash balances to firms are captured by generalising their production sets into a production correspondence $\Psi^j(p, r)$, which defines, conditionally on prices and rates of interest, the production plans, y, and balances n, which are feasible; in line with the convention of measuring outputs positively and inputs negatively, $n \leq 0$.

The profits of the firm are

$$v^j(y, n, p, r) = py + rn,$$

which it maximizes.

The bank, denoted by the index $j = J + 1$, extends loans $M \geq 0$, on which it collects interest in the amount $v^{J+1}(M, r) = rM$, paid out as dividends.

The budget constraint of an individual, a single constraint under a complete asset markets, takes the elementary form

$$pz^i + rm^i \leq \sum_{j \in \mathcal{J}} \theta^{i,j} v^j(y, n, p, r) + \theta^{i,J+1} v^{J+1}(M, r).$$

The interest accounting is implicit. In a deterministic world, all transactions on commodities are entered at values discounted to period 1. This is equivalent to settling them sequentially on the spot through credits and debits with the bank, and letting the bank pay or collect interest on the account's balance. The equivalence results from the assumption that the interest rate is the same for positive and negative balances. Similarly, profit shares may indifferently be collected initially at present value or period by period, as dividends credited to the bank account. There is however an exception to this accounting, namely cash balances on which no interest is earned. Accordingly, the interest foregone on cash balances must be added to the expenditures or deducted from the receipts.

The link between the overall budget constraint as written above and the spot accounting identities at individual nodes is readily illustrated for the case where $T = 2$. It is then economical to index the initial node, the date-event at date 1, as 0 and the different states of the world, date-events at date 2, as $s \in \mathcal{S} = \{1, \ldots, S\}$. Also, it helps to denote by $d^j = (d_0^j, \ldots, d_S^j\ )$ the net dividends paid by firm j across the nodes of the event-tree. The spot price at 0 of a contingent claim to commodity l in state s , $p_{l,s}$, may be expressed as the product of the spot price, q_s, of a contingent claim to one unit of money in state s and the spot price $\tilde{p}_{l,s}$ there. One writes f_s for the net purchases at 0 of nominal claims contingent on state s . Commodity prices and interest rates are constant within periods. Interest on initial or average balances is due at end of period, when dividends are also paid. The accounting identities, in beginning-of-period values, are then

$$p_0 z_0^i + \tfrac{\tilde{r}_0}{1+\tilde{r}_0} m_0^i + \textstyle\sum_{s\in\mathcal{S}} q_s f_s^i =$$

$$\tfrac{1}{1+\tilde{r}_0}\{\textstyle\sum_{j\in\mathcal{J}} \theta^{i,j} d_0^j + \theta^{i,J+1}\tilde{r}_0 M_0\},$$

and

$$\tilde{p}_s z_s^i + \tfrac{\tilde{r}_s}{1+\tilde{r}_s} m_s^i =$$

$$f_s^i + \tfrac{1}{1+\tilde{r}_s}\{\textstyle\sum_{j\in\mathcal{J}} \theta^{i,j} d_s^j + \tilde{r}_s M_s\}, \quad s \in \mathcal{S}.$$

If one multiplies the budget constraint at $s \in \mathcal{S}$ by q_s, adds it to the budget constraint at 0 and does so for all $s \in \mathcal{S}$, the terms in f_s^i cancel out, yielding

$$p_0 z_0^i + \textstyle\sum_{s\in\mathcal{S}} q_s \tilde{p}_s z_s^i + \tfrac{\tilde{r}_0}{1+\tilde{r}_0} m_0^i + \textstyle\sum_{s\in\mathcal{S}} q_s \tfrac{\tilde{r}_s}{1+\tilde{r}_s} m_s^i =$$

$$\textstyle\sum_{j\in\mathcal{J}} \theta^{i,j} (\tfrac{1}{1+\tilde{r}_0} d_0^j + \textstyle\sum_{s\in\mathcal{S}} q_s \tfrac{1}{1+\tilde{r}_s} d_s^j) + \theta^{i,J+1} (\tfrac{\tilde{r}_0}{1+\tilde{r}_0} M_0 + \textstyle\sum_{s\in\mathcal{S}} q_s \tfrac{\tilde{r}_s}{1+\tilde{r}_s} M_s).$$

With $q_s \tilde{p}_s = p_s$, $\tilde{r}_0/(1+\tilde{r}_0) = r_0$, $q_s \tilde{r}_s/(1+\tilde{r}_s) = r_s$, $v^j = d_0^j/(1+\tilde{r}_0) + \sum_{s\in\mathcal{S}} q_s d_s^j/(1+\tilde{r}_s)$, and $v^{J+1} = \tilde{r}_0 M_0/(1+\tilde{r}_0) + \sum_{s\in\mathcal{S}} q_s \tilde{r}_s M_s/(1+\tilde{r}_s)$, the last expression takes the simpler form written above, where $p = (p_0, \ldots, p_s, \ldots)$, $z^i = (z_0^i, \ldots, z_s^i, \ldots)$, and so on, as in section 2.

The equilibrium conditions must be augmented to require clearing of the market for balances

$$\sum_{i\in\mathcal{I}} m^i - \sum_{j\in\mathcal{J}} n^j \leq M.$$

3.3 Monetary equilibrium

A monetary economy is defined by

1. an event tree, $\mathcal{F}$, extending over dates $t \in \mathcal{T} = \{1, \ldots, T\}$, with nodes, date-events $s_t = n \in \mathcal{N} = \{1, \ldots, N\}$ of which $s = s_T \in \{1, \ldots, S\}$, terminal;

2. primary commodities, $l \in \mathcal{L} = \{1, \ldots, L\}$;

3. contingent commodities, associated with the L primary commodities and the N date-events , $(l, s_t) \in \mathcal{L} \times \mathcal{N}$;

4. prices of the contingent commodities, $p = (\ldots, p_{l,s_t}, \ldots)$ as of date 1 ;

5. a single currency, issued by a bank, indexed $J + 1$, charging or paying interest on account balances; the currency is not interest-bearing;

6. interest rates, $r = (\ldots, r_{s_t}, \ldots)$, one for each date-event, expressed as values of date 1;

7. firms, $j \in \{1, \ldots, J\}$; each firm is defined by its production correspondence $\Psi^j(p, r)$;

8. individuals or households, $i \in \mathcal{I} = \{1, \ldots, I\}$; each individual is defined by a triple, (Φ^i, R_i, e^i), where $\Phi^i(p, r)$ is the exchange set correspondence, R_i is a preference ordering on the graph of Φ^i, and e^i is the endowment of state-contingent commodities.

A private ownership monetary economy is defined by a monetary economy and an ownership matrix $\Theta = (\theta^{i,j}), i = 1, \ldots, I, j = 1, \ldots, J, J + 1$, where $\theta^{i,j}$ is the share of firm j , if $j \leq J$, or of the bank, if $j = J + 1$, owned by individual i.

The aggregate balances issued by the bank are M , an N-vector. At interest rates r, bank profits are rM.

The budget set of individual i is

$$B^i(p, r, y, M) = \{(x, m) \in \Phi^i(p, r) :$$

$$px + rm \leq pe^i + \textstyle\sum_{j \in \mathcal{J}} \theta^{i,j}(py^j + rn^j) + \theta^{i,J+1} rM\}.$$

A competitive equilibrium is defined by an allocation, $(\ldots, (x^i, m^i), \ldots, (y^j, n^j), \ldots)$, a price vector, p, a vector of interest rates, r, and aggregate balances, M, such that

1. for every individual, (x^i, m^i) belongs to $B^i(p, r, y, M)$ and is R_i-maximal;

2. for every firm, (y^j, n^j) belongs to $\Psi^j(p, r)$ and $py^j + rn^j \geq py + rn$, for all $(y, n) \in \Psi^j(p, r)$;

3. $\sum_{i\in\mathcal{I}} x^i \leq \sum_{j\in\mathcal{J}} y^j + \sum_{i\in\mathcal{I}} e^i$;

4. $\sum_{i\in\mathcal{I}} m^i - \sum_{j\in\mathcal{J}} n^j \leq M$.

Let $\overline{c} = \sum_{\ell,s_t} p_{\ell,s_t} > 0$ denote the general price level.

Proposition 1 *Under standard assumptions, competitive equilibria exist, for all $\overline{r} \geq 0$, all general price levels $\overline{c} > 0$, and some associated M.*

One assumption is not standard: in order to allow for homogeneity of degree zero, of the exchange set correspondence and the preference ordering, in prices of commodities and money balances, it is logically required, and it is analytically possible, to weaken the standard continuity assumption when *both* $z^+_{s_t}$ and m_{s_t} vanish.

4 NOMINAL INDETERMINACY AND MONETARY POLICY

4.1 Nominal indeterminacy

The complication introduced by money in general equilibrium analysis is increased nominal indeterminacy. The "degrees of nominal indeterminacy" or d.o.n.i.'s are the dimension of a set of nominal variables that may be specified exogenously, while maintaining the existence of a competitive equilibrium. In the Arrow-Debreu model, there is a single d.o.n.i., the overall price level. It is indeed easy to give examples of economies that admit a unique competitive equilibrium in quantities and relative prices. In contrast, proposition reveals that monetary economies with N date-events admit at least $N + 1$ d.o.n.i.'s – N nominal interest rates and one overall price level. A more precise statement is possible.

Proposition 2 *Competitive equilibria for a monetary economy on an event-tree with N date-events, of which S terminal, display $N + S$ degrees of nominal indeterminacy. A similar property holds on each subtree, conditionally on any initial date-event and compatible terminal date*[6].

At each date-event, three nominal variables must be set: a short-run, within-period, nominal interest rate, a price level and money balances. Given two of these, the third follows through money demand by firms and individuals, at equilibrium. This leaves two free nominal variables per node.

When $N = S = 1$, which corresponds to one period under certainty, $2 = N + S$ variables are indeed free. For instance, the price level could be inherited, with the interest rate or money supply set by the bank.

Nominal interest rates carry real effects – up to a negligible set of primitives – because (i) higher rates lead agents to economise on money balances, at real costs; (ii) higher rates widen the wedge between effective buying prices, which include liquidity costs, and selling prices; (iii) higher rates entail redistributive effects between borrowers, lenders and bank shareholders. On the other hand, because preferences and constraints are assumed homogeneous of degree zero in nominal prices and money balances, there is absence of money illusion, the price level and the associated supply of inside money have no real implications. This last property is lost when nominal claims and liabilities appear in the initial endowments.

When $N > S = 1$, which corresponds to multiple periods certainty, the two nominal variables per period are constrained by $N - 1$ no-arbitrage conditions, the "Fisher conditions." At each non-terminal node, the nominal short rate, $\tilde{r}_t$, the real short rate, $\tilde{\rho}_t$ and the one-period prospective inflation rate, $\tilde{\varphi}_t$, must satisfy, at equilibrium, the relation $1 + \tilde{r}_t = (1 + \tilde{\rho}_t)(1 + \tilde{\varphi}_t)$, often approximated by the statement that the real rate is equal to the nominal rate minus inflation. There remain $2N - (N - 1) = N + 1 = N + S$ degrees of nominal indeterminacy. If all nominal rates are set by the bank, all inflation rates are determined, at equilibrium; but the overall price level remains free. If all price levels are determined, through a process of nominal price formation, initialised, the terminal interest rate, $\tilde{r}_T$ is free. For a long horizon, $\tilde{r}_T$ is of little consequence to initial decisions, whereas the overall price level has immediate import. There is an inescapable asymmetry between the forward-looking implications of interest rates for inflation, and the backward-looking implications of inflation for interest rates.

When $N > S > 1$, which corresponds to multiple periods under uncertainty, there are $2N$ nominal variables to be determined, and $N - S$ non-terminal nodes. At each of these, a no-arbitrage condition links the short nominal rate to the present value, at prices for contingent nominal claims, of real rates and inflation rates one period hence. This property is conveniently illustrated on the standard two-period, single physical commodity model where $N = S + 1 > 2, L = 1$. Nominal, *spot* prices of the commodity are $\tilde{p} = (p_0, \tilde{p}_1, \ldots, \tilde{p}_s, \ldots, \tilde{p}_S)$, and, $q = (q_1, \cdots, q_S)$ are the nominal prices at 0 of claims to one unit of money contingent on states $s = 1, \ldots, S$. The nominal rate $\tilde{r}_0$ and the vector q are related by $\sum_{s \in \mathcal{S}} q_s = (1 + \tilde{r}_0)^{-1}$: one unit of money in each state can be acquired either by buying S unit claims at prices q_s , or by buying a one-period nominal bond at price $(1 + \tilde{r}_0)^{-1}$. This is the single no-arbitrage condition. It leaves free both the second period nominal rates $\tilde{r}_s$ and the second period relative prices $\tilde{p}_s / \sum_{s \in \mathcal{S}} \tilde{p}_s$, with associated relative prices for contingent claims, $q_s / \sum_{s \in \mathcal{S}} q_s$. In particular, if a competitive equilibrium specifies the interest rates $\tilde{r} = (\tilde{r}_0, \tilde{r}_1, \ldots, \tilde{r}_S)$ and spot prices $(\tilde{p}, q)$, the same physical allocation would also be sustained by $(r, \tilde{p}', q')$, where $p'_0 = p_0$, $\tilde{p}'_s = \lambda_s \tilde{p}_s, q'_s = \lambda_s^{-1} q_s,$, for each s, and $\lambda = (\lambda, \ldots, \lambda_S) \gg 0$ satisfies $\sum_{s \in \mathcal{S}} \lambda_s^{-1} q_s = (1 + \tilde{r}_0)^{-1}$. One concise way to capture that property is to note that, at equilibria consistent with given nominal rates of interest, expected inflation is well-defined, but *the variability of inflation is unrestricted*[7].

Similarly, if the money supply at each node were fixed, but interest rates adjusted flexibly to clear the money market, there would remain S degrees of price level indeterminacy. With p_0 inherited from the past, r_0 would follow from (p_0, M_0) through money demand. Once again, expected inflation would be determined by a no-arbitrage condition. But the variability of price levels and of the associated $\tilde{r}_s'$ s would be unrestricted.

The twin property just exhibited for the two-period, one commodity model is fully general and worth stating as a corollary to proposition 2.

Corollary 1 *In a monetary economy on an event-tree with N date-events, of which S terminal, competitive equilibria at given nominal interest rates $\overline{r}$ or, alternatively, at given money supplies $\overline{M}$ display S degrees of nominal indeterminacy, corresponding to the price levels at the terminal nodes. Looking at inflation rates between any non-terminal node and its successors, expected inflation rates, suitably defined, are well-defined, but the variability of inflation is unrestricted.*

This corollary has important implications for monetary policy. The S d.o.n.i.'s associated with the price levels have no real consequences, in the formulation with complete markets and no nominal claims in the initial endowments[8].

4.2 Monetary policy

Monetary policy removes the degrees of nominal indeterminacy.

"*Comprehensive*" monetary policy removes all $N + S$ degrees of indeterminacy.

A monetary policy is a choice of instruments, at most $N + S$, and a pre-announced, state-contingent rule for setting these instruments on the basis of sequential information about the resolution of uncertainty. It is neither a fixed, state-independent rule, nor a purely discretionary policy. If the policy is announced and followed, there are no "surprises:" adjustments in the level of the instruments may be regarded as "anticipated," though in a contingent sense. Changes in the rule, on the other hand, may be labeled unanticipated[9].

Four aspects of these definitions will be discussed.

Current trends in monetary thinking set the stage.

Regarding choice of instruments, the trend is to privilege nominal short rates. This seems to reflect, at least in part, growing reservations about the meaningfulness of controlling the money supply, in a world of money-market accounts and credit card transactions. Simultaneously, the targetting of monetary aggregates is loosing ground to inflation targetting. Central bank independence is increasingly seen as necessary, and perhaps also sufficient, for an efficient conduct of monetary policy, namely one that contains inflation. Regarding the cost of inflation, levels and variability both matter, and are seen as positively correlated.

(i) Interest policies and comprehensiveness.

In an event-tree with N date-events, of which S terminal, there are $N+S$ degrees of nominal indeterminacy, but only N short nominal rates, one per node. A monetary policy relying on nominal short rates as instruments is not comprehensive. As stated in corollary 1, competitive equilibria associated with such a policy have at each node well-defined expected rates of inflation, compatible with the announced interest rates, but the variability of inflation is unrestricted. As a consequence, the long run realised inflation, or even the conditional expected inflation given a future date-event, are uncontrolled.

These considerations suggest that *nominal rates do not provide suitable instruments for inflation targetting, and, in particular, for containing the variability of inflation, at equilibrium.*

Our incursions into the monetary literature have not led to any discussion of this difficulty; and discussions with better[22] read colleagues and informed practitioners have not dispelled the mystery. This raises the following

Question: *Is the inability of interest policies to control inflation variability, at equilibrium, recognised by monetary theory ? If so, why is it not in the foreground of the discussions on inflation targetting through interest policies ? If not, is it because the models of monetary theory hide the fact, or lead to different conclusions ? Is it because the control of inflation variability is deemed to follow from the control of expected inflation ? Or is it because monetary theory is concerned with out-of-equilibrium situations ?*

(ii) Additional instruments.

What instruments, other than nominal short rates, could be used to implement a comprehensive monetary policy ? If the price level were treated as adjusting passively to a comprehensive monetary policy, at equilibrium, then control of the money supply would remain desirable *as a complement, not a substitute*, to interest policies. The fact that there are $N+S<2N$ d.o.n.i.'s reveals that the two sets of instruments should be used in a coordinated manner. That is, contingent money growth at $t+1$ must be compatible with the expected inflation associated with the nominal rate at t. Subject to that constraint, a state-independent, though time-dependent, growth of the money supply would deserve consideration as a "rough" approach to containing inflation variability at equilibrium[10].

In the framework of complete asset markets, an interesting alternative set of instruments is provided by open-market operations on contingent nominal claims, or "Arrow securities," after Arrow (1953). Indeed, in a monetary economy, complete markets mean markets for both real and nominal state-contingent claims. Given the real allocation, hence the real rates, the price at node s_t of a unit nominal claim contingent on any successor event s_{t+1} is subject to a no-arbitrage condition, akin to a "contingent Fisher condition," yielding the inflation factor $1+\tilde{\varphi}_{s_{t+1}}$ as the ratio of the price of a real contingent claim to the price of a nominal contingent claim. Given the real price, controlling the nominal price means controlling the state-contingent inflation, at equilibrium. With options markets and derivative pricing, that unexplored avenue is not outlandish. The sobering fact is that markets for options are developing mostly for price-contingent options, with mixed success for options contingent on economic indicators, as the fate of the Chicago exchange for options on the CPI and the absence of macro-markets[11] indicate, and with no markets in sight for options contingent on exogenous events.

(iii) Autonomous inflationary pressures.

The root of the inflation problem, also as seen by policy analysts, is that the price level does not adjust passively to a comprehensive monetary policy, but is instead subject to autonomous impulses. Prices are set by agents, firms and unions, or emerge on organised markets, for oil or agricultural crops. Some degrees of nominal indeterminacy are de facto lifted by the process of nominal price formation. Hence they are not available to conduct a comprehensive monetary policy.

In order to assess realistically the prospects for controlling nominal variables through monetary policy, it is imperative to understand the process of nominal price formation. Current understanding is far from adequate[12]. Example 6.2 illustrates the extent to which nominal price formation curtails the scope of monetary policies. It introduces as a property of nominal price formation the stipulation that nominal prices of individual commodities are non-decreasing over time. The nominal interest rate has a uniquely determined floor. If the bank sets a lower rate, it forces the economy out of competitive equilibrium; if it sets a higher rate, it induces unnecessary excess inflation.

When the price level does not respond passively to a comprehensive monetary policy, a number of intriguing questions arise. First, one would like to know whether the process of nominal price formation claims fewer or more than S d.o.n.i.'s. If fewer, one could still think about interest rate policies as part of a definition of equilibrium; if more, one should consider such policies outright as calling for out-of-equilibrium analysis, unless the range of admissible policies be suitably curtailed. Second, should one think about out-of-equilibrium situations as arising within a non-tâtonement process of adjustment towards equilibrium; or as successive elements in a sequence of short-run equilibria; or possibly as situations of persistent disequilibrium ? There is clearly room for alternative, perhaps partly complementary, approaches to out-of-equilibrium analysis of monetary policy, with qualitatively different implications.

The qualitative differences are suggestively illustrated by a simple remark. Staggered contracts are supposed to define a sequence of short-run equilibria, constrained by the rigidity of the prices which are not currently up for revision, but were previously set under rational expectations. Positive inflation implies continuously changing relative prices, and the dispersion of prices is a source of misallocation[13]. In contrast, along an adjustment process, prices move to correct disequilibria, which, incidentally, is more convincing than moving according to a fixed or random sequence; speedier price adjustments, possibly accompanied by higher inflation, if there is asymmetry between upward and downward flexibility, entail less misallocation[14]. It is, thus, important to understand the process of nominal price formation.

An intriguing question about the process of nominal price formation is whether states of the world can be defined so exhaustively that nominal price levels are uniquely determined given the state. This would call for introducing such qualitative considerations as union militancy or oil cartel effectiveness in the definition of the states, along with crop sizes, indirect tax rates and the like. That question arises in non-monetary general equilibrium theory as well[15]. The monetary context is somewhat more complex, due to the nominal indeterminacies. One should rather expect the nominal price level to be fully determined given the state and the preannounced monetary policy. But this also calls for a very extensive concept of states, implying grossly incomplete markets. The situation is even more problematic than it appears at first, due to the uncertainty about the very process of nominal price formation; should the alternative theories, and learning about their plausibility, also appear in the definition of the states ?

(iv) Transparency.

Explicit, state-contingent monetary policy is not easy. The difficulty is the same for a central bank as for private agents drawing state-contingent contracts. In practice, one would expect the rule to be defined with reference to a few economic indicators, like inflation rates, unemployment rates, capacity utilization rates, and the like. These are not primitive exogenous events, like a political decision about EMU or the outcome of an election, as the logic underlying event-trees requires.

It is possible that sophisticated central bankers do in fact follow a state-contingent rule, but choose to keep the rule implicit. They prefer to evaluate the state of the economy on the basis of all the evidence in their possession and draw conclusions, without tying their hands to an explicit rule involving a limited set of indicators. A formal discussion of the relative merits of these two approaches lies beyond the scope of this paper. Yet, advocates of transparency as a virtue in the conduct of monetary policy would plead for a gradual shift of emphasis towards more explicit disclosure of the indicators used by central bankers, and of their influence on the adjustments of instruments.

5 NON-NEUTRALITY

5.1 Liquidity costs

Nominal rates of interest almost always carry real effects, when money balances do not earn interest. In the simplest possible framework, an Edgeworth box extended to monetary exchange, as in example 6.1 below, money demand results from a cash-in-advance constraint. The bank sets the rate of interest, and the price level via money supply. Alternatively, the price level, which is inconsequential, is exogenous and the bank accomodates money demand.

Higher rates of interest restrict trade and, up to redistributive aspects which favor the shareholders of the bank, they lower utility. This is the general argument, due to Vickrey (1959) and Friedman (1968) that it is inefficient to economise on costless balances.

This particular departure from neutrality is not very significant, in a world of electronic and plastic money, where demand deposits or money-market accounts pay interest[16]. But the cost is largely independent of nominal rates; rather, it is due to the difference between two rates: the rate on demand deposits versus the rate on savings deposits. Anyhow, the cost is small compared to the real cost of credit card transactions.

With reference to current debates on monetary policy, real effects associated with monetary transactions are not in the foreground. The concern lies with the impact of monetary policy on inflation and activity levels. Monetary theory deals with the identification of such impact or absence thereof and with the transmission channels along which they operate.

The theory of competitive equilibria in economies with complete markets is ill-suited for that task. Incomplete asset markets and non-Walrasian equilibria must be brought in. There are good independent reasons for doing so anyhow. But it is technically more demanding. It is necessary to rely on examples to illustrate the possibility of fitting empirically relevant departures from neutrality into intertemporal general equilibrium theory, suitably broadened.

5.2 Wealth effects

A significant source of effectiveness of monetary policy lies in the wealth effects of interest rates when there is outstanding public debt and "ricardian equivalence" is only partial; meaning that the households do not match increases in public debt by increases in private savings just adequate to service the debt and retire it at maturity. The empirical record[17], suggests indeed a match by private savings well below 100% . The econometric work on the transmission mechanism[18], similarly points to the effect on consumer wealth, hence on savings, as the main transmission channel from nominal interest rates to consumer spending. For the general equilibrium theorist, incomplete ricardian equivalence and incomplete markets go hand-in-hand. The fact that many households involved in market clearing at future dates, when debt will be retired, are not participating in transactions today is one reason for inexistence of long term contingent or futures markets. The suggestion that they may be represented by dynastic ancestors is well-taken but the representation is only partial, and the practical impossibility of borrowing today on the strength of future earnings of descendants puts many households at corner solutions. Hence the partial equivalence.

This significant component of the thinking of policy analysts and monetary authorities is easily embodied in formal models of intertemporal general equilibrium; example 6.3 illustrates the point.

5.3 Nominal rigidities

Nominal wage and price stickiness is another major source of monetary policy effectiveness. This is natural, since downward nominal rigidities of wages and prices, the more relevant case, breed individually involuntary unemployment and excess capacities. Excess supply of labour or goods generates income effects which reduce aggregate spending thereby reducing the need for price increases on other goods. Policy analysts seem to agree that contractionary monetary policies are effective in curbing inflation through reduced activity, in the short run. A suitably flexible general equilibrium model allows for that feature. The real challenge to general equilibrium theorists is to model the price formation process, allowing for quantity-constrained allocations during the adjustment process, which now interacts with the monetary policy.

Models with many goods – but no auctioneer – are needed to develop relevant theories of price formation and inflation. When some prices, oil or wages of high-skilled employees, rise for exogenous reasons – a cartel, an energy tax, or skill-biased technological progress – there is no mechanism whereby the myriad of prices for goods and services not directly affected by this development should fall by the amount required to keep the price level constant.

An incomplete asset market, especially in economies with production and asymmetric information, provides additional and distinct, if related, arguments for the effectiveness of monetary policy; interestingly, some formalize earlier arguments and rely solely on the role of money as a unit of account.

Assets serve to transfer revenue across dates and realizations of uncertainty; a transfer of revenue is attainable if it is the payoff of a portfolio of marketed assets; the allocation of resources at equilibrium depends essentially on the attainable transfers of revenue. When the asset market is complete, all transfers of revenue are attainable; when the asset market is incomplete, policies that alter the span of the payoffs of marketed assets affect the allocation of resources at equilibrium. If assets are real or indexed, changes in the price level across dates or realizations of uncertainty fail to alter the span of the payoff of marketed assets and are neutral. If assets are nominal, changes in the price level, the reciprocal of the purchasing power of units of account, alter the span of the payoffs of marketed assets and have real effects[19].

A limitation of several previous results is the fact that they establish the existence of real effects of price level variability; typically also demonstrate the suboptimality of state-independent rates of inflation; but fail to characterise in operational terms second-best efficient state-profiles of inflation[20].

6 EXAMPLES

6.1 Liquidity costs

Money provides liquidity and balances are issued by a bank that accomodates demand and pays out its profit as dividend to shareholders. There exists a continuum of competitive equilibria indexed by the rate of interest. Up to redistributive effects, competitive allocations are pareto ranked, and a vanishing rate of interest yields a pareto optimal allocation[21].

There is a one time period and no uncertainty − the indices of dates and date-events or states of the world are unnecessary.

Commodities are $l \in \mathcal{L} = \{1, 2\}$.

A bundle of commodities is $z = (z_1, z_2)$, and prices of commodities are $p = (p_1, p_2)$.

There is one medium of exchange, and balances are m.

A monetary authority controls the rate of interest, r, and the supply of balances, M.

There is no production.

Individuals are $i \in \mathcal{I} = \{1, 2\}$.

The exchange set correspondence of each individual is defined by $\Phi^i(p, r) = \{(z, m) : pz^+ \leq m\}$: a cash-in-advance constraint is operative. The preference correspondence of individual 1 is represented by the utility function $u^1(z) = \ln(1 + z_1) + \ln z_2$, and of individual 2 by the utility function $u^2(z) = \ln z_1 + \ln(1 + z_2)$; the $(z_l)'$ s are net trades summing, at equilibrium, to zero for each commodity. Equivalently, individuals 1 and 2 have identical preferences represented by logarithmic utility functions over consumption, and endowments $(1, 0)$ and $(0, 1)$ respectively. The share of individual 1 in the bank is $\theta^1 \geq 0$, and that of individual 2 is $\theta^2 = 1 - \theta^1 \geq 0$.

At rate of interest $\overline{r}$, competitive equilibrium prices of commodities are

$$p_1 = \overline{M}\tfrac{2+2\overline{r}+\theta^1\overline{r}^2}{2+\overline{r}},$$

$$p_2 = \overline{M}\tfrac{2+2\overline{r}+\theta^2\overline{r}^2}{2+\overline{r}}.$$

The net trade of commodities of individual 1 is

$$z_1^1 = \tfrac{(1+\overline{r})(1+\theta^1\overline{r})}{2+2\overline{r}+\theta^1\overline{r}^2} - 1,$$

$$z_2^1 = \tfrac{1+\theta^1\overline{r}}{2+2\overline{r}+\theta^2\overline{r}^2},$$

and the utility of either individual at equilibrium is

$$u^i = \ln(1 + \overline{r}) + 2\ln(1 + \theta^i\overline{r}) - \ln(2 + 2\overline{r} + \theta^1\overline{r}^2) - \ln(2 + 2\overline{r} + \theta^2\overline{r}^2).$$

There is a continuum of distinct equilibrium allocations indexed by the nominal rate of interest, $\overline{r}$.

For $\theta^1 = \theta^2 = 1/2$, the competitive equilibrium allocation associated with $\overline{r}$ is pareto superior to that associated with $\overline{r}' > \overline{r}$; and the competitive equilibrium allocation associated with $\overline{r} = 0$ is pareto superior to all others. But this is not necessarily the case when $\theta^1 \neq \theta^2$; in general, transfers of revenue may be required to compensate individuals for the loss in dividend income from their shares in banks.

6.2 Nominal rigidities

Price rigidities affect equilibrium nominal interest rates.

Dates or time periods are $t \in \{1, 2\}$. There is no uncertainty – the indices of date-events or states of the world are unnecessary.

Commodities are $l \in \mathcal{L} = \{1, 2\}$.

A bundle of commodities is $z = (z_{1,1}, z_{2,1}, z_{1,2}, z_{2,2},)$, and spot prices of commodities are $\tilde{p} = (\tilde{p}_{1,1}, \tilde{p}_{2,1}, \tilde{p}_{1,2}, \tilde{p}_{2,2})$.

Transactions do not require the use of a medium of exchange and there is no bank and no balances.

There is no production.

Individuals are $i \in \mathcal{I} = \{1, 2\}$.

The exchange set and the preferences of individuals are invariant with respect to the prices of commodities and the interest rate; the exchange set coincides with the set of net trades that yield strictly positive consumption. The preferences of individual 1 are represented by the utility function $u^1(z^1) = \ln((1/2) + z^1_{1,1}) + \ln z^1_{2,1} + \ln(1 + z^1_{1,2}) + \ln z^1_{2,2}$, and of individual 2 by the utility function $u^2(z^2) = \ln z^1_{1,1} + \ln((1/2) + z^1_{2,1}) + \ln z^2_{1,2} + \ln(1 + z^2_{2,2})$; the $(z_l)'$ s are net trades summing, at equilibrium, to zero for each commodity. Equivalently, individuals 1 and 2 have identical preferences represented by intertemporal logarithmic utility functions over consumption and endowments $(1/2, 0, 1, 0)$ and $(0, 1/2, 0, 1)$ respectively.

Nominal spot prices are downward rigid, with

$$\tilde{p}_{1,1} \geq 1, \; \tilde{p}_{2,1} \geq 1,$$

and

$$\tilde{p}_{1,2} \geq \tilde{p}_{1,1}, \; \tilde{p}_{2,2} \geq \tilde{p}_{2,1}.$$

Equilibrium obtains at

$$x^i_{l,1} = \frac{1}{4}, \; x^i_{l,2} = \frac{1}{2}, \quad l \in \mathcal{L}, i \in \mathcal{I},$$

provided

$$\tilde{p}_{l,1} = \frac{2\tilde{p}_{l,2}}{1 + r}, \quad l \in \mathcal{L}.$$

Thus, $\tilde{p}_{\ell,2} \geq \tilde{p}_{\ell,1}$ requires $r \geq 1$.

For $r < 1$, only equilibria with quantity constraints exist.

6.3 Wealth effects

Interest rates have wealth effects when ricardian equivalence fails, due to staggered generations.

Dates or periods are $t \in \mathcal{T} = \{1, 2\}$; there is no uncertainty – the indices of date-events or states of the world are unnecessary.

Commodities are $\{n, c\}$, labour and a produced consumption good, respectively.

A bundle of commodities is $z = (n_1, c_1, n_2, c_2)$ and spot prices of commodities are spot wages, $\tilde{w}_t$, and $\tilde{p}_t$, spot prices for the consumption good: $\tilde{p} = (\tilde{w}_1, \tilde{p}_1, \tilde{w}_2, \tilde{p}_2)$; the overall price level is defined by $\tilde{w}_1 = 1$, probably inherited.

Transactions do not require the use of a medium of exchange and there is no bank and no balances. However, individuals have, possibly, non-zero initial holdings of "inside" money or purchasing power.

There is a single firm. Variables without superscript refer to the firm's activity.

The production set of the firm is in variant with respect to prices and the rate of interest. The production set of the firm is $Y = \{y : c_1 = \theta(-n_1)^{1/2}, n_1 \leq 0, \theta > 0, c_2 = -n_2, n_2 \leq 0\}$. Equilibrium requires $\tilde{w}_2 = \tilde{p}_2$.

At each date, labour is employed as input in order to produce the consumption good, as output. There is no storage.

Individuals are $i \in \mathcal{I} = \{1, 2, 3\}$.

The exchange set and the preferences of individuals are invariant with respect to the prices of commodities and the interest rate; the exchange set coincides with the set of net trades that yield non-negative consumption. The demographic structure of the economy corresponds to a "slice" from an economy with overlapping generations.

Individual 1 lives through period 1; his preferences are represented by the utility function $u^1(z^1) = c_1^1$; the individual has no endowment in commodities; but he is endowed with a bond promising d units of money or purchasing power at the beginning of period 2, with value $d/(1 + r)$ at period 1; he consumes $c_1^1 = d/(1 + r)p_1$, in period 1.

Individual 2 lives through periods 1 and 2; his preferences are represented by the utility function $u^2(z^2) = c_1^2 c_2^2 (2 + n_1^2)$; equivalently, the individual is endowed with $\bar{n}_1^2 = 2$ units of labor in period 1; in addition, the individual is endowed with the ownership of the firm. Consequently, the budget constraint of the individual is

$$\tilde{w}_1 n_1^2 + \tilde{p}_1 c_1^2 + \frac{\tilde{p}_2 c_2^2}{1 + r} \leq v,$$

where v are the profits of the firm; $v = \tilde{p}_1 \theta(-n_1)^{1/2} + n_1$ is maximal at

$$-n_1 = (\frac{\theta \tilde{p}_1}{2})^2,\ c_1 = \frac{\theta^2 \tilde{p}_1}{2}, v = \frac{\theta^2 (\tilde{p}_1)^2}{4}.$$

Utility maximization by individual 2 implies that

$$\tilde{p}_1 c_1^2 \;=\; \frac{\tilde{p}_2 c_2^2}{1+r} \;=\; 2 + n_1^2.$$

The budget constraint imposes that

$$2\tilde{p}_1 c_1^2 \;=\; -n_1^2 + v \;=\; -n_1^2 + \frac{\theta^2 (\tilde{p}_1)^2}{4}.$$

Individual 3 lives through period 2; his preferences are represented by the utility function $u^3(z^3) = c_2^3$; the individual is endowed with labor at date 2, $\bar{n}_2^3 = 1$, which he supplies inelastically: $n_2^3 = -1$; in addition, he is subject to a levy of d, in order to retire the bond; he consumes $c_2^3 = (\tilde{w}_2 - d)/\tilde{p}_2$, in period 2.

With the good not storable, equilibrium at date 1 requires that

$$c_1 \;=\; c_1^1 + c_1^2 \text{ with } -n_1 = -n_1^2 = (\frac{\theta \tilde{p}_1}{2})^2, c_1^2 \;=\; \frac{\theta^2 \tilde{p}_1}{4}.$$

Hence,

$$c_1 \;=\; \frac{\theta^2 \tilde{p}_1}{2} \;=\; \frac{d}{(1+r)\tilde{p}_1} + \frac{\theta^2 \tilde{p}_1}{4},\ \tilde{p}_1 \;=\; \frac{2}{\theta}(\frac{d}{1+r})^{1/2}$$

so that

$$c_1 \;=\; \theta(\frac{d}{1+r})^{1/2},\ -n_1 \;=\; \frac{d}{1+r} \;=\; \tilde{p}_1 c_1^2 = v.$$

This equilibrium has the property that $\tilde{p}_1, c_1$ and $|n_1|$ are decreasing in r : there is an input-inflation trade off.

Also, $\tilde{p}_2 c_2 = (1+r)\tilde{p}_1 c_1^2 = d$.

Equilibrium at date 2 requires that $c_2 = -n_2 = -n_2^3 - \frac{d}{\tilde{p}_2} + \frac{d}{\tilde{p}_2} = -n_2^3$: it obtains as an identity. The price $\tilde{p}_2$ is indeterminate, but affects the allocation of output between individuals 2 and 3. A unique $\tilde{p}_2$ is easily obtained, for instance by changing the preferences of individual 2, so that $u^2(z^2) = c_1^2(b + c_2^2)(2 + n_1^2)$.

REMARK

If the productivity parameter, θ, which is knwon at the beginning of period 1, is stochastic, as is the case in real business cycle models, the interest rate, r, can be chosen, as a function of θ,

1. to stabilise the price of the consumption good, $\tilde{p}_1$, which calls for $1 + r \propto \theta^{-2}$;

2. to stabilise output, c_1, which calls for $1 + r \propto \theta^2$;
3. to stabilise employment, $-n_1$, which calls for constant r.

Naturally, one cannot stabilise 3 variables with a single instrument.

NOTES

1. Initial debts place implicit restrictions on price levels if survival of debtors is to be guaranteed. That complication is eschewed in Drèze and Polemarchakis (1997) by assuming zero initial holdings of nominal claims or liabilities. Footnote 8, below, elaborates on this.

2. This distinguishes the formulation from that of Dubey and Geanakoplos (1992) and accounts for the indeterminacy of interest rates.

3. It was pointed out by Jean-Michel Grandmont that a non-interest-bearing money could be regarded as a form of price rigidity, which tilts the scale against neutrality. The remark is well-taken, but the specification remains empirically relevant; also, all markets, including the markets for balances, do clear.

4. It is natural that some authors, among others, Ostroy and Starr (1974) and more recently Kiyotaki and Wright (1989), have looked upstream of simple market institutions in order to explain why most transactions take the form of exchanges of goods for money. Here, this commonplace observation is a factual starting point.

5. For specific purposes of financial analysis, like cash management, detailed attention is paid to the day-to-day profile of money demand and overnight interest rates, Hellwig (1992). At the same time, it is legitimate to disregard these aspects when investigating the theoretical foundations of monetary policy. It is, after all, an intrinsic feature of fiat money that its supply can be adjusted from day to day at no real cost, and it is a property of a well-functioning banking system that it accomodates smoothly short run variations in money demand.

6. The precise meaning of "similar" is explained in footnote 8 below.

7. Technically, the expectation is defined, not with respect to the probabilities of future states, which may differ across individuals, but with respect to the prices q_s for unit nominal claims contingent on the states. This expectation is well defined because $(\sum_{s\in\mathcal{S}} q_s(\tilde{p}_s - p_0)/p_0)/\sum_{s\in\mathcal{S}} q_s = (\sum_{s\in\mathcal{S}} q'_s(\tilde{p}'_s - p_0)/p_0)/\sum_{s\in\mathcal{S}} q'_s$. The variability of inflation is unrestricted, because the vectors $(\ldots, \tilde{p}_s/\sum_{s\in\mathcal{S}} \tilde{p}_s, \ldots)$ and $(\ldots, \tilde{p}'_s/\sum_{s\in\mathcal{S}} \tilde{p}'_s) = (\ldots, \lambda_s\tilde{p}_s/\sum_{s\in\mathcal{S}} \lambda_s\tilde{p}_s, \ldots)$ are variation-free, up to the constraint $\sum_{s\in\mathcal{S}} \lambda_s^{-1} q_s = (1+r_0)^{-1}$.

8. When proposition 4.1 is applied to the subtree starting at some node $s_t, t > 1$, a competitive equilibrium defined over the entire tree starting at s_1 typically entails non-zero nominal claims or liabilities at s_t. The indeterminacy of nominal variables over the subtree follows from the indeterminacy over the entire tree, but is accompanied with variations in the

nominal claims and liabilities at s_t matching the variations in the price level there. Survival of consumers is not an issue, because the nominal liabilities at s_t are part of a feasible allocation over the entire tree.

9. Interestingly, Eichenbaum (1997), also quoting Chari, Christiano and Eichenbaum (1996), advocates an "institutional innovation," whereby monetary policy would take the form of a commitment to one-period-ahead, state-contingent policy actions.

10. The approach is "rough," because different states and events may entail (i) different growth rates of real output or (ii) different levels of nominal interest rates, in addition to (iii) different inflationary pressures. Differences (i) and (ii) entail differences in money demand that could be matched by adjustment of money supply at unchanged price levels.

11. Shiller (1993).

12. Several simple models have been proposed, for instance staggered contracts by Taylor (1980) and Calvo (1983), or menu-costs by Mankiw (1985) and Blanchard and Kiyotaki (1987).

13. As in Goodfriend and King (1997).

14. As in Drèze (1993,b).

15. Drèze (1993,a).

16. Feldstein (1996) estimates liquidity cost at .01% of GDP per percentage point of nominal interest. This estimate is consistent with the inventory-theoretic equality between interest foregone on balances and the value of real resources devoted to economise on balances. Lucas (1997) obtains slightly higher values, which are however sensitive to modeling assumptions at very low levels of inflation, or of interest differentials.

17. For example, Tobin (1980).

18. For example, the FMP model – Modigliani (1971).

19. This argument was developed by Detemple, Gottardi and Polemarchakis (1993) and Magill and Quinzii (1992), exploiting the argument of Balasko and Cass (1989), Cass (1985) and Geanakoplos and Mas-Colell (1989) for the indeterminacy of competitive equilibrium allocations when assets are nominal and the asset market is incomplete.

20. The most constructive result in this vein, due to Magill and Quinzii (1996), establishes that adding a price-level variability within exogenous real events can only do harm; this is a plausible, useful result – but it does

not bear on the issue of improving risk-sharing through suitably characterised second-best state-profiles of inflation rates. Another constructive result is that, in an economy with risky assets and a defaultless nominal bond, where the CAPM assumptions on endowments and preferences are satisfied, state-independent inflation is efficient.

21. Drèze and Polemarchakis (1994).

22. (Note added in print.) After completing this paper, we have come to realise that our corollary 1 bears some analogy (though the substantive conclusions are distinct) to the discussion of price-level indeterminacy in such papers as Sargent, T.N. and N. Wallace (1975), "'Rational' expectations, the optimal monetary instrument, and the optimal money supply rule", *Journal of Political Economy,* vol. 83, 241-254; Woodford, M. (1994), "Monetary policy and price level determinacy in a cash-in-advance economy", *Economic Theory,* vol. 4, 345-380; or Sims, C.A. (1994)-, "A simple model of the determination of the price level and the interaction of monetary and fiscal policy", *Economic Theory,* vol. 4, 381-399. Recasting their specification and assumptions into our framework, to reconcile conclusions, is beyond the scope of this last-minute addition.

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