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Topology optimization methods for the design of electromagnetic actuators

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Abstract

Topology optimization aims to design devices in an automated way and, as its name implies, especially focuses on finding their optimal topology. Indeed, in a classical design approach, the device topology is generally chosen by the designer based on its experience and the design process consists in determining the optimal device dimensions by a parametric optimization. However, some design problems may not have been studied yet or may be better addressed by a non conventional design. The ability of topology optimization to propose efficient topologies starting from scratch becomes then a serious asset. The method is based on a splitting of the design space in cells in which it aims to find the optimal material amongst a given library defined by the designer.

This thesis aims to develop topology optimization methods for the design of electromagnetic devices, based on deterministic algorithms. The latter are indeed characterized by a fast convergence compared to heuristic algorithms. However, in the electromagnetic domain, they are usually faced with two main challenges. The former is related to intermediate materials, i.e. when different materials are mixed in the same cell. Those materials cannot usually be manufactured and the method must thus ensure that they are removed in the final design. The second challenge concerns local minimizers which prevent deterministic methods from finding the optimal solution.

The methods proposed within this thesis especially focus on these two challenges. They are divided in two groups: isotropic and anisotropic methods. In addition, whereas methods proposed in the literature usually focus on simple problems, the thesis intends to increase the complexity by studying different kinds of constraints and objective functions and different combinations of the four main electromagnetic materials, i.e. windings, iron, permanent magnets and air, through three study cases: magnetic thrusts, a switched reluctance motor (SRM) and a permanent magnet synchronous motor (PMSM). The thesis besides aims to develop and studying a mesh refinement algorithm.

Isotropic methods are characterized by cells having isotropic properties. Each material is then described by a unique design variable. Several mechanisms are required to ensure the global convergence of the method. These mechanisms are progressively introduced by successively studying problems involving different material combinations.

In winding or permanent magnet optimizations, the method implementation is very simple and no particular attention must be paid to local minimizers or intermediate materials.

Convergence issues however arise in iron-winding optimizations. This thesis research led to the proposal of two mechanisms to overcome them (Labbé and Dehez, 2010a; Labbé et al., 2009a,b). First, the sensitivity mapping aims to define a specific relation between the iron proportions and the cell permeabilities, in order to ensure a smooth sensitivity of the objective function. A geometric mapping is then usually well adapted and leads to a fast convergence. Second, a convexity mapping is introduced in order to affect the intermediate material properties and thereby either penalize or promote them. Local minimizers are indeed avoided thanks to a promotion of intermediate materials during the first phase of the optimization. The removal of intermediate materials in the final design is then ensured by a progressive penalization of their properties.

The consideration of a mass constraint in iron-winding optimizations required the development of an additional mechanism: an iron microstructure aligned with the magnetic fields. The latter actually ensures defining realistic relations between the cell permeabilities, the cell masses and the iron proportions in order to avoid the apparition of local minimizers. Intermediate materials have then a physical meaning and two approaches for removing them were highlighted during this thesis work. They can indeed still be penalized, but they can also sometimes be interpreted and replaced by equivalent iron structures. The iron microstructure is besides a support for computing the permeability of intermediate materials when saturation is taken into account.

Magnet-iron optimizations raise new challenges to ensure an initial convergence avoiding local minimizers. This thesis shows that the sensitivity mapping must be modified (Labbé et al., 2011): the geometric mapping is replaced by a reluctivity mapping. Intermediate materials composed of permanent magnets and iron must besides be promoted by a specific convexity mapping.

The last studied material combination is related to the optimization of ferromagnetic parts. Because it only involves air and iron, the convexity mapping becomes ineffective and specific methods were thus proposed within this thesis. They are based on convexity constraints whose shape differs if the objective function relies on the difference of magnetic energy (Labbé and Dehez, 2010b) or on the Maxwell stress tensor (Labbé and Dehez, 2010c, 2011b).

Isotropic methods are in conclusion efficient for different optimization prob-

lems and generally easy to implement. However, they suffer from some limitations: a specific method based on convexity constraints is required for iron optimizations, intermediate materials cannot always be interpreted, and some problems such as the stator design of a PMSM cannot be handled by the method.

The second group of methods is referred to as the anisotropic methods. The latter are based on the definition of an iron microstructure whose dimensions and orientation are, in opposition with isotropic methods, included as additional design variables. The sensitivity and convexity mappings are still implemented to affect the objective function sensitivity and the problem convexity. The implementation of anisotropic methods requires thus more effort and the computation time is besides increased because of a more complex finite element model and of a slower convergence. These methods were developed within this thesis work (Labbé and Dehez, 2011b) initially to overcome the limitations of isotropic methods, but they actually turn out to be far more general than the latter. First, they can be applied to problems where isotropic methods are ineffective. Next, they do not require a specific method for the optimization of ferromagnetic parts. They can indeed be successfully applied to all material combinations.

There was a strong motivation for studying mesh refinement during this thesis, since it could improve the resolution of the produced solutions while limiting the computation time. The proposed mesh refinement algorithm aims to refine the mesh at the material interfaces (Labbé and Dehez, 2011a; Labbé and Dehez, 2010d). These interfaces are detected from the derivatives of the material distribution. Since the topology optimization and the finite element model are both based on the same mesh, refining the latter at the interfaces both increases the resolution of the solution and the accuracy of the finite element computations. Mesh refinement turns out to be effective for reducing the computation time compared to a one-shot optimization on a homogeneous mesh achieving the same resolution.

In order to assess their effectiveness, the optimization methods were applied to complex optimization problems. The complexity of those problems raises new challenges for topology optimization, which could lead to further research in the domain. The main challenge lies in the definition of the design spaces. Whereas topology optimization initially aims to completely determine the optimal device topology starting from scratch, the definition of the design space usually already requires making some choices that limit the range of achievable topologies. For instance, the number of pole pairs of a PMSM is from the start fixed when the designer defines the design space and the corresponding symmetry. Another challenge is related to the computation time, especially when a high-resolution topology is required with a complex objective function. However, the method was nevertheless successfully applied to the design of magnetic thrusts, of a switched reluctance motor and of a permanent magnet synchronous motor.

The magnetic thrust optimization remains quite simple since it only involves permanent magnets. The main discussion concerns the determination of the optimal length of the design space, which is solved by performing several optimizations with different lengths (Labbé and Dehez, 2009, 2011a).

The SRM optimization aims to determine the characteristics of the optimal torque-to-mass and torque-to-volume ratios according to the SRM mass and volume. Several optimizations are thus performed with different masses, but also with different air-gap radii, since the optimal value of the latter is a priori fixed with the definition of the rotor design space and cannot be optimized by the method. This study reveals that the optimal torque-to-mass characteristic seems to have a linear shape. In addition, it shows that the optimal topology evolves with the SRM mass or volume, which makes impossible to infer a scaling law for deriving the optimal topology for any mass or volume.

The PMSM study focuses on the same goal, but introduces a new challenge related to the specification of the maximum torque ripple of the motor. This specification is included as a penalty factor in the objective function. The study shows that the PMSM optimal topologies could likely be described by a scaling law which is obtained through a power regression.

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Introduction

1

The classical approach for designing an electromagnetic device starts with the definition of an initial geometry. In order to assess the device performances, a model is then developed based on this geometry, from which a parametric optimization is carried out to determine the optimal device dimensions.

The main weakness of this approach lies in the choice of the initial geometry. Indeed, the latter either comes from the literature or from the knowledge of the designer, who tends to rely on designs that have already proved to be efficient. Even if this approach is generally justified, it does not support novelty. A designer may indeed be faced with a non conventional problem for which no previous knowledge exists, or with a problem where an unexpected design may outperform the conventional one.

1.1 Topology optimization approach

Topology optimization aims to design devices in a fully automated way, so that the designer work is limited to correctly describing the design problem. This description involves the definition of the objective function, the design space, the available materials and, if necessary, some constraints or specifications (Dyck and Lowther, 1996). The choice of an initial topology, required in the conventional design approach, can thus be avoided.

Topology optimization focuses on finding the optimal topology for a given design problem starting from scratch, i.e. determining which material should be placed in each point of space in order to maximize the objective function while satisfying the problem specifications. To this end, the method relies on a raster representation of the device: the design space is split in cells and the optimization goal is formulated

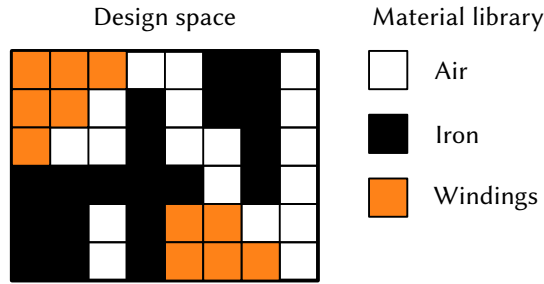


Fig. 1.1. Topology optimization.

as finding the optimal distribution of material inside these cells. Fig. 1.1 illustrates a topology optimization problem whose design space is split in square cells and whose material library includes three materials.

1.2 Scope of the work

Topology optimization is a wide topic and some choices had thus to be made to narrow the research scope, especially regarding the field of application, the computation tool, the optimization algorithm, the material distribution formalism and the considered objective functions and constraints.

1.2.1 Field of application

The thesis exclusively focuses on *quasi-static electromagnetic problems*. Therefore, the design of electrical motors is for instance limited to electromagnetic objectives, whereas thermal considerations are not taken into account. In addition, the studied optimization problems do not involve dynamic effects.

The thesis besides only considers 2D problems. Indeed, including the third dimension first requires increasing the number of cells to achieve a reasonable resolution, but also raises completely new challenges that are beyond the scope of this work. For instance, the continuity of windings must then be guaranteed for carrying a current density.

1.2.2 Computation tool

The optimization method requires a computation tool to evaluate the magnetic fields and the objective function of the optimized solutions. This evaluation is performed using a *finite element model* that was developed and implemented along with

the topology optimization method. The latter does thus not rely on a commercial finite element software. This decision of developing a home-made finite element model will be further justified in chapter 3.

1.2.3 Optimization algorithm

Optimization algorithms can be divided in two main groups: *deterministic* and *heuristic* algorithms. The thesis focuses on the former and specifically on the *steepest-descent method*, a choice that will also be justified in chapter 3. This kind of algorithms relies on the gradient of the objective function to update the design variables at each iteration. It is usually characterized by a fast convergence compared to heuristic algorithms but suffers from a high sensitivity to local minimizers and requires thus a proper formulation of the optimization problem. Heuristic algorithms are on the contrary very general and their application is quite straightforward on any optimization problem. They indeed do not require the computation of the objective function derivatives, but on the other hand involve a high computation time.

1.2.4 Material distribution formalism

The material distribution formalism describes the way materials are placed in the design space. With deterministic algorithms, a common approach is to use the elements of the finite element mesh as cells (Wang et al., 2004b). Indeed, the time consuming operations during the optimization are mainly the finite element computations and are thus not influenced by the number of design variables. There is thus no reason for decreasing the latter number and the highest possible resolution for a given finite element mesh is achieved when using the finite elements as cells. Other formalisms however exist, such as Voronoi cells whose positions can be optimized along with their composition, but those formalisms are usually implemented with heuristic optimization algorithms (Dehez et al., 2008).

1.2.5 Optimization problem complexity

Real optimization problems are usually characterized by complex objective functions and constraints. The design of an actuator for instance often involves specifications regarding the motor mass, the torque ripple, the heat transfers, the efficiency, the cost, the manufacturing constraints, etc. Since these specifications highly complicate the optimization problem, especially with deterministic methods, this thesis exclusively concentrates on two kinds of specifications. The first concerns the device mass: a special attention is indeed paid along the thesis on how this constraint must

be properly included. The second is related to the torque ripple and is addressed for the design of a permanent magnet synchronous motor.

Regarding the optimization objective functions, this thesis focuses on functions related either to the magnetic energy or to the Maxwell stress tensor, which is justified for the design of electromagnetic actuators. Indeed, they enable the computation of the actuator torques or forces.

1.3 Problem statement

Topology optimization was at first applied to mechanical problems. Much research has already been done in this domain and the method is now functional and efficient. The application of topology optimization to electromagnetic problems has in comparison been much less studied and is nowadays still faced with three main challenges. The first one is related to intermediate materials, i.e. when different materials are mixed in the same cell. The second one concerns local minimizers which prevent deterministic algorithms from finding the optimum design. The last challenge is related to the computation time required to perform the optimization which may become too long when a high-resolution solution is required. These three challenges are besides deepened by the fact that designers are willing to apply topology optimization to more and more complex problems.

1.3.1 Intermediate materials

Topology optimization is concerned with the distribution of predefined materials in cells. This kind of optimization is referred to as a 0-1 optimization, since each material is either present or not in a cell and its corresponding design variable is thus either equal to 0 or 1.

Whereas 0-1 optimization problems can easily be handled by heuristic algorithms, the deterministic gradient-based algorithm considered in this thesis requires continuous variables. The 0-1 problem must thus be relaxed so that the design variables can take intermediate values between 0 and 1. The design variables become then the proportions of each material in each cell. Different materials can thus be mixed in a given cell, the resulting material being referred to as an *intermediate material*, whereas the available basis materials for designing the device are called *solid materials*.

Even if intermediate materials are tolerated during the optimization process, the final design should only be composed of solid materials. Methods proposed in the literature usually remove these intermediate materials by a level-set approach: they are

simply replaced by the material having the highest proportion in the cell. However, this approach is generally not very efficient and the thesis therefore rather proposes two other approaches: either some mechanisms are introduced to make intermediate materials disappear in the final design, or effort is made to give realistic properties to these materials so that they can be interpreted by the designer and replaced by an equivalent solid structure.

1.3.2 Local minimizers

Performing topology optimization with gradient-based methods raises an issue regarding the presence of local minimizers. Indeed, whereas heuristic algorithms are well designed for trying to perform global optimizations, gradient-based algorithms are very sensitive to these local minimizers. If the algorithm converges to one of them, it is then stuck in this minimizer and the optimization stops. The presence of local minimizers is highlighted by the fact that the optimization result differs according to the initial material distribution, i.e. the initial value of the design variables.

The formulation of the optimization problem has a significant impact on the presence or not of local minimizers. In this thesis, the main effort in performing topology optimizations focuses thus on properly writing the optimization problems and on including mechanisms to avoid these local minimizers.

Whereas the local minimizer issue is widely studied in this work, leading to several propositions to overcome it, few references to local minimizer problems can be found in the literature. The consideration of this problematic constitutes thus one of the main points that distinguishes this work from previous research.

1.3.3 Computation time

The electromagnetic problems that were up to now considered in the literature remain quite simple and the computation time required for their optimization has never been highlighted as a serious issue. However, in order to handle more complex problems, especially when a high-resolution design is desired, researches to reduce the computation time will someday certainly become compulsory.

The reduction of the computation time involves improving the optimization algorithm and the problem formulation to increase the convergence speed and thereby reduce the number of iterations and of finite element evaluations. However, the research described in this thesis was based on a given optimization algorithm: the steepest-descent method. The reduction of the computation time through a comparison of different algorithms is thus beyond the scope of this work. In addition, the

use of home-made implementations of the optimization algorithm and of the finite element model results in higher computation times compared to efficient commercial tools, which explains why computation times are in this work accompanied by the number of finite element evaluations. The research however approaches the computation time issue through the formulation of the topology optimization problem and shows that in relative terms, a correct formulation may lead to significant improvements. The computation times reported in the thesis were obtained on an Intel Core 2 DUO 3.33 Ghz.

For the design of mechanical structures, mesh refinement has already been introduced to limit the computation time by only increasing the number of mesh elements where it is required. However, it has not yet been applied to the topology optimization of electromagnetic devices, which has until now been limited to low-resolution designs.

1.4 Objectives

The main objective of this thesis is the development of new methods that address the three current challenges of topology optimization in the electromagnetic domain, involving intermediate materials, local minimizers and, to a lesser extent, the computation time. This objective can be further detailed by the following requirements on the proposed methods.

1. They must always converge to the same solution whatever the initial material distribution, i.e. they must not be influenced by local minimizers. This objective is assessed by performing several optimizations with different initial conditions and verifying that the same result is always produced by the method.

The obtained solutions must besides achieve a high objective function. This objective is usually difficult to assess, since the optimal solution is obviously not known. The method results are thus rather assessed through a comparison with a large number of local minimizers, which will be further explained later.

2. The final result must only be composed of solid materials, or of intermediate materials that can be easily interpreted and replaced by an equivalent solid structure.

In order to limit the computation time to a reasonable value, the research also takes an interest in studying the implementation, the implications and the advantages of mesh refinement.

Whereas the present methods proposed in the literature usually focus on the optimization of ferromagnetic devices, this research besides aims to consider different combinations of the four main electromagnetic materials: air, iron, windings and permanent magnets.

The last thesis objective is the exploitation of the developed methods on real design problems, in order to assess their performances and limits.

1.5 Approach

The research in topology optimization in the electromagnetic domain is still at its beginning. Therefore, it would be premature to undertake the development a general method which should be effective for any optimization problem. The following approach was thus rather carried out during this thesis.

First, an optimization algorithm, the steepest-descent method, was selected notably for its simplicity (see section 3.2). It was next implemented along with a finite element model in a completely home-made Matlab program, in order to easily identify the sources of convergence problems. This program was then successively applied to different study cases, which highlighted the inherent challenges in their optimization.

The research aimed then to find solutions to overcome those specific challenges while expecting that these solutions could be successfully applied to a wide range of similar problems. It especially focused on the problem formulation: providing an efficient formulation to achieve a fast convergence and approach the global optimum. The fast convergence had to be observed in relative terms, since the choice of the optimization algorithm was probably not optimal and the home-made implementation of the finite element model could obviously not compete with efficient commercial tools.

The method developments within this document follow the same structure: the problems raised by the optimization of a particular study case are underlined and explained, and mechanisms are proposed to overcome them. The methods are thus progressively constructed by considering more and more complex problems.

Three study cases are successively studied: the design of magnetic thrusts, of a switched reluctance motor (SRM) and of a permanent magnet synchronous motor (PMSM). They enable the consideration of different types of optimization problems in terms of design variables, specifications and objective functions.

- All the combinations of materials, and thus of their related design variables, are involved within the three study cases. Winding-air and magnet-air opti-

mizations can be studied through the magnetic thrust design, winding-iron-air optimizations through the SRM and PMSM stator design, iron-air through the SRM rotor design and magnet-iron-air through the PMSM rotor design.

- The study cases are besides optimized both with and without a mass specification.
- The objective functions of the three problems are mainly based on the magnetic energy. The magnetic thrust optimization aims to maximize the latter whereas the SRM and PMSM optimization focuses on their torque, and thus on the maximization of a difference of magnetic energy. The Maxwell stress tensor is besides considered in a specific method dedicated to the SRM rotor optimization.

1.6 Thesis organization

The thesis is structured as follows.

Chapter 2 reviews the main topology optimization methods relying on deterministic algorithms that can be found in the literature.

Chapter 3 describes the optimization tool on which all the methods proposed in this thesis rely. It includes the general formulation of the topology optimization problem, the description of the optimization algorithm and the development of the finite element model.

Chapter 4 introduces the three considered study cases. It provides a complete description of their design spaces, of their objective functions and of the materials and constraints they involve.

Chapter 5 is dedicated to a first type of optimization methods based on isotropic materials. The complexity of the considered problems is progressively increased along the chapter through the involvement of different material combinations. Several mechanisms are proposed to handle these combinations and the method characteristics, performances and weaknesses are described in each case.

Chapter 6 focuses on the second method type involving anisotropic materials. These methods aim to overcome the limitations of isotropic methods thanks to the consideration of an iron microstructure. The method efficiency is again assessed for the different material combinations.

Chapter 7 is dedicated to mesh refinement. The refinement algorithm is described and illustrated on a study case. The resolution gain and the computation time reduction achieved by the refinement process are then assessed.

Chapter 8 intends to further exploit the proposed method. Whereas the previous chapters consider simplified versions of the study cases, this chapter shows how topology optimization can be applied to more complex problems.

Chapter 9 concludes the thesis and proposes some research directions for future work.

Topology optimization originates from the mechanical domain, where it was at first developed for the design of beams and other mechanical structures. The optimization aims then to find the optimal topology to minimize the compliance under some load conditions with a given quantity of material, knowing the support conditions. Fig. 2.1 shows for instance a simple problem considering two supports and a unique load.

According to a review paper by Rozvany (2001), important principles of topology optimization were already established in the beginning of the twentieth century by Michell (1904) in the context of the design of trusses. These principles were applied to grillages in the 1970s by Rozvany (1972) and were then extended and generalized by Prager and Rozvany (1977) to derive the topology optimization theory (Rozvany, 1992).

The first practical topology optimization method, based on finite elements, was only proposed in the late 1980s by Bendsøe and Kikuchi (1988). This method relies on the homogenization theory and was thus called the *homogenization method*. It was quickly recognized as a powerful tool for the design of mechanical structures and received a lot of attention in the following decades, notably by Kikuchi (Bend-

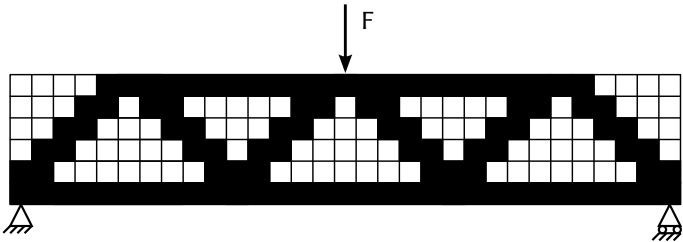


Fig. 2.1. Topology optimization of a simple beam with two supports.

søe et al., 1993) and Allaire (1997). An extensive overview of the homogenization method in the mechanical domain can be found in (Hassani and Hinton, 1998a,b,c).

In parallel, the *Solid Isotropic Microstructure with Penalization* (SIMP) method, also called the *density* method, was proposed by Bendsøe (1989), who originally called it the *direct method*. The method was later studied by Rozvany and Zhou (1991) who proposed the SIMP name. It aimed to tackle the technical difficulties of the homogenization approach, but however introduced other drawbacks related to mesh dependencies, checkerboard patterns and local minima (Sigmund and Petersson, 1998).

From the 1990s, topology optimization has been applied to various domains, such as the design of piezoelectric transducers (Carbonari et al., 2008; Lau et al., 2000; Silva et al., 1997), micro fluid mixers (Aage et al., 2008; Andreassen et al., 2009), ultrasonic wave transducers (W. Kim, 2007), acoustic devices (Dühring et al., 2008; Yoon et al., 2006), photonic bandgap fibers (Dühring et al., 2010) or photonic crystals (Borel et al., 2005; Têtu et al., 2005).

The electromagnetic domain received at first attention from Dyck and Lowther (1996), who proposed a method called the *optimized material distribution* (OMD) method. They besides highlighted the main challenges related to topology optimization in the electromagnetic domain and defined important rules for its application. However, ever since then, the matter only received attention from a few authors. Yoo et al. (2000) applied the homogenization method to an electromagnetic design problem through the definition of iron microstructures. However, it was quickly abandoned in favour of the SIMP method (Byun et al., 2002; Okamoto et al., 2006; Wang et al., 2004b). Later, Takahashi et al. (2009) proposed the ON/OFF method and Choi and Yoo (2008b) introduced a reluctivity-based method.

A major difference between topology optimization in the electromagnetic and in the mechanical domain lies in the consideration of the magnetic sources, i.e. the current density and the permanent magnetization (Dong-Hun et al., 2005). In the mechanical domain, these forcing terms correspond to the structure loads which are part of the problem definition.

In the following sections, the methods dedicated to the design of electromagnetic devices are briefly described, as well as the optimization algorithms on which they rely.

2.1 Optimized material distribution method

The first proposal of applying topology optimization to the design of electromagnetic devices was the OMD method proposed by Dyck and Lowther (1996). The

latter is based on the definition of one design variable per cell and per material describing its proportion (Lowther et al., 1998). The cell properties, which are exclusively isotropic, are then derived from these material proportions. The authors recommend a geometric mapping between the iron proportion and the permeability:

$$\mu(x) = \mu_0 \mu_r^{\rho(x)}, \quad (2.1)$$

with $\rho(x)$ and $\mu(x)$ the iron proportion and the permeability in position x and μ_0 and μ_r the air and iron permeabilities. The OMD method relies on a penalty-multiplier method for handling the constrained optimization problem.

2.1.1 Rules of intermediate materials

The first paper introducing the OMD method (Dyck and Lowther, 1996) has become a reference thanks to its thorough discussion about the challenges of performing topology optimization. Even if the authors do not address the question of the possible presence of local minimizers, they propose an in-depth analysis which actually has some connections with this latter issue.

They notably define two rules regarding the properties of intermediate materials. Since the final design must only be composed of solid materials, the properties of intermediate materials may seem unimportant. However, there are two situations that may arise which leads to a unique way of assigning their properties.

First, if intermediate materials have valuable properties that make them more efficient than solid materials, the optimization will result in a design mainly composed of these intermediate materials. The second situation is the opposite: if intermediate materials are expensive but the optimal design requires materials whose properties are different from the solid materials, the obtained design will be composed of different solid materials in adjacent cells. Indeed, the optimization attempts then to create a composite structure whose properties approach those of the required material. The problem is that this structure depends on the cell discretization, it becomes thinner as the cell sizes decrease.

From these observations, Dyck and Lowther suggested two rules for the definition of the intermediate material properties, in which a *composite material* refers to a true realistic material consisting of different solid materials arranged according to a given pattern on a microscopic scale.

1. Rule of intermediate materials

There must be a direct correspondence between intermediate materials and composite materials.

2. *Rule of composite structure*

The composite material corresponding to an intermediate material must be defined so as to make the most cost effective use of the solid material from which it is made.

The first rule guarantees that intermediate materials are not more efficient than solid materials so that they will only be found in the final design if the latter requires composite materials. In addition, it also implies that intermediate materials are valuable enough to avoid the situation in which composite structures are more efficiently produced at a cell level by distributing different materials in adjacent cells, which is not desirable.

The second rule ensures that intermediate materials have sufficiently good properties to avoid the creation of composite structures. Indeed, even if intermediate materials effectively correspond to composite materials, there could be a better arrangement of solid materials leading to more efficient properties. In this case, the second situation may still arise since the optimization method will rather attempt to create this arrangement on a cell scale.

The importance of these two rules will be emphasized along the thesis. Whereas Dyck and Lowther do not explicitly describe their application on a practical problem, the thesis will show that convergence problems are usually explained by the failure to comply with these rules and will describe how they can be exploited to develop a topology optimization method.

The authors also discuss the handling of intermediate materials in the final design. They can be interpreted by the designer if their properties effectively correspond to composite materials. The designer can then replace them by an equivalent macroscopic structure or keep them as they are if they are manufacturable. They can also be removed by penalizing their properties, but this may again raise the second situation.

2.1.2 Saturation of ferromagnetic materials

Dyck and Lowther (1997) also took an interest in taking into account the saturation of ferromagnetic materials. They again suggested considering composite materials defined by a microscopic structure aligned with the magnetic field, in order to realistically derive the permeability of intermediate materials. This microstructure is illustrated in Fig. 2.2 and is composed of two levels, the former being a succession of iron sheets aligned with the magnetic field, the latter introducing air voids in these sheets. By evaluating the magnetic fields in the microstructure, and especially in the ferromagnetic parts, an equivalent saturated permeability can be derived.

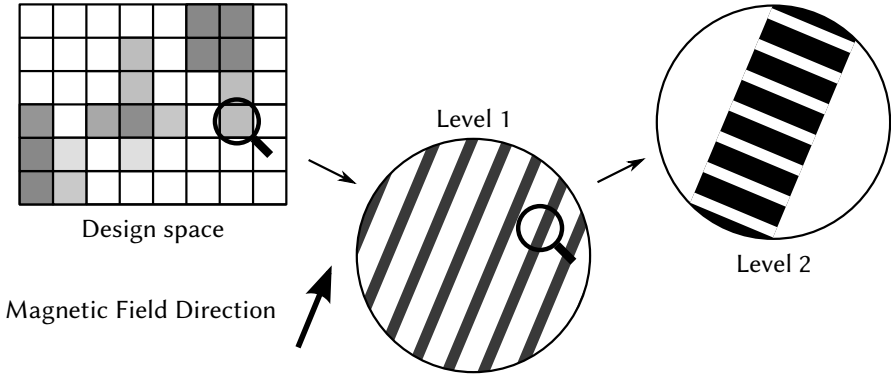


Fig. 2.2. Two-level iron microstructure for computing the saturated permeability of intermediate materials.

2.2 Homogenization method

The homogenization method attempts to go further than making a correspondence between intermediate and composite materials, by directly focusing on the optimization of the dimensions of the composite material structure, including its orientation. Fig. 2.3 illustrates a usual iron microstructure describing composite materials and proposed by Yoo et al. (2000). Each cell is thus characterized by three design variables: θ_e , x_e and y_e , with e the cell number. The macroscopic cell properties are then derived from a homogenization process: the equivalent properties of a large number of microstructure layouts are computed by a finite element method and are next interpolated during the optimization.

The homogenization method is supported by several authors in the mechanical domain (Bendsøe et al., 1993) since it avoids the creation of mesh-dependent structures, which corresponds to the second issue mentioned in the Dyck and Lowther's paper. Their two rules are indeed not always sufficient to guarantee that such structures do not appear. This has already been highlighted in the mechanical domain, and actually also applies to electromagnetic device optimizations, which will be shown in this thesis.

For the optimization of electromagnetic devices, the homogenization method only received attention from Yoo and Kikuchi (2000); Yoo et al. (2000). These papers basically focus on the transposition of the homogenization method to electromagnetic problems, and unfortunately do not emphasize its strengths and weaknesses.

The homogenization problem can be solved by different types of algorithms. Yoo et al. (2000) rely on the method of moving asymptotes (MMA) which was originally proposed by Svanberg (1987) and received then attention from several authors

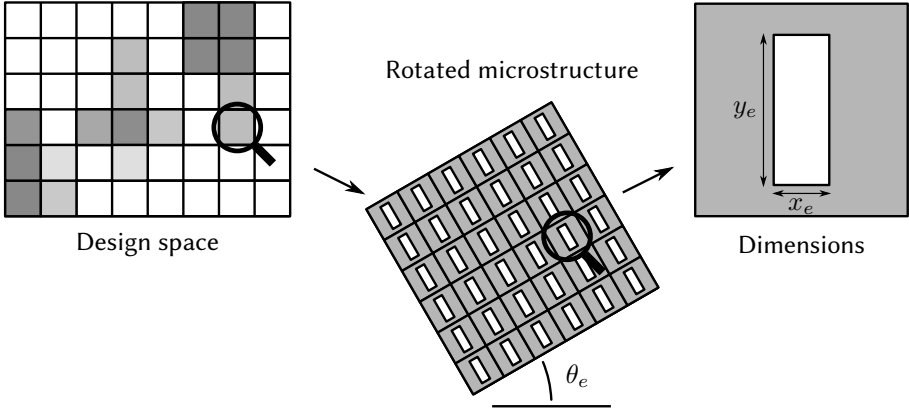


Fig. 2.3. Example of microstructure and design variables for the homogenization method.

(Bruyneel et al., 2002). In the mechanical domain, sequential linear programming is also usually applied in the homogenization approach (Nishiwaki et al., 1998), as well as methods using explicit optimality formulas based on the Karush-Kuhn-Tucker conditions (Allaire, 2005; Hassani and Hinton, 1998c).

2.3 Density method

In the mechanical domain, the density (or SIMP) method is a great concurrent to the homogenization method. Indeed, its implementation complexity is largely reduced since a unique design variable is considered to describe the proportion of a material in a cell. The method is characterized by a stiffness tensor of intermediate materials given by

$$E_{ijkl}(x) = \rho(x)^p E_{ijkl}^0, \quad (2.2)$$

with $\rho(x)$ the proportion of material in position x , p a penalty factor and E_{ijkl}^0 the stiffness matrix of the solid material. When p is equal to 1, the cell stiffness matrix is proportional to the material proportion. When p is increased, intermediate materials are penalized since their properties are reduced, which is used to ensure that they disappear in the final design. Several authors recommend using $2 < p < 4$.

With a gradient-based optimization method, the material proportions should be limited by a lower bound, such as

$$\rho(x) > 10^{-3}, \quad (2.3)$$

in order to avoid the singularity for $\rho(x) = 0$. Indeed, the gradient of the stiffness

matrix is given by

$$\frac{\partial E_{ijkl}(x)}{\partial \rho} = p\rho^{p-1}E_{ijkl}^0, \quad (2.4)$$

and is thus equal to zero when $\rho(x) = 0$ and $p \neq 1$. A gradient-based algorithm would be stuck in this case since it updates the design variables according to the objective function gradient.

This method was widely transposed for the design of electromagnetic devices, with the following relation between the iron proportion and the permeability:

$$\mu(x) = \mu_0(1 + (\mu_r - 1)\rho(x)^p), \quad (2.5)$$

with μ_0 and μ_r the air and iron permeabilities. It was first implemented by Byun et al. (2002) and Wang et al. (2004b).

With a penalty factor comprised between 2 and 4, they highlighted a convergence to solid designs without any intermediate material. A dependence of the results on the initial conditions has however been observed with the density method by e.g. Byun et al. (2002); Okamoto and Takahashi (2006). This convergence issue, related to the presence of local minima, has also been highlighted in the mechanical domain by Sigmund and Petersson (1998) along with other difficulties related to the creation of mesh-dependent structures and checkerboard patterns. The creation of mesh-dependent structures is then avoided by including constraints on the design perimeter (Bendsøe and Sigmund, 2009), whereas a continuation method has been proposed by Allaire and Kohn (1993) to tackle the local minima issue. This continuation method consists in making the p parameter evolve during the optimization in order to achieve a progressive penalization of intermediate materials. However, the continuation method has never been applied to electromagnetic problems.

Various optimization techniques have been implemented to solve the density problem such as sequential linear programming (SLP) (Carbonari and Silva, 2004; Shim et al., 2008, 2007; Wang et al., 2004a) or the method of moving asymptotes (MMA) (Kim et al., 2005a). The optimality criteria method (OCM), which is described in (Yin and Yang, 2001), has also been applied by Cho et al. (2003); Kim and Kim (2004); Yoo et al. (2008). Byun and Park (2007); Okamoto and Takahashi (2006) besides proposed using the steepest-descent method.

2.4 Reluctivity-based method

The reluctivity-based method was proposed by Choi and Yoo (2008b), who suggested computing the cell properties from a reluctivity relation:

$$\nu(x) = \nu_0(1 + (\nu_r - 1)\rho(x)), \quad (2.6)$$

with $\nu = \frac{1}{\mu}$. The latter is similar to the density relation, except that the penalization factor is removed. Choi indeed showed that intermediate materials naturally disappear thanks to this reluctivity relation when the material quantity is constrained, and that the penalization factor becomes thus unnecessary. This behaviour will be explained later in the thesis.

The reluctivity-based method relies on sequential linear programming for the optimization (Choi and Yoo, 2008b).

Choi and Yoo (2009) studied the simultaneous optimizations of iron and windings and of iron and permanent magnets, with the reluctivity-based method. The authors do not explicitly highlight a lack of convexity for these optimizations, but some results seem at first sight not optimal, which tends to show that the reluctivity-based method succeeds in removing intermediate materials in the final design, but is also negatively affected by local minimizers.

2.5 ON/OFF method

The ON/OFF method was mainly supported by Takahashi et al. (2009). The method only considers solid materials that cannot be mixed together. The cell compositions are thus switched from one material to another without any continuous evolution, which explains the method name. It however belongs to the deterministic method group since it relies on the objective function gradient to determine which cells should be modified (Okamoto et al., 2006; Takahashi et al., 2010).

For an iron-air optimization, the method is described by the following steps.

1. Definition of an initial material distribution.
2. Evaluation of the objective function and its gradient using the adjoint variable method.
3. Switching of the materials in cells where the gradient indicates that it would lead to a decrease of the objective function.
4. Topology smoothing.
 - a) Cut step: an iron cell surrounded only by air cells (or just one iron cell) is changed in air.
 - b) Attached step: an air cell surrounded by 4 or 5 iron cells is transformed in iron.
5. Evaluation of the objective function.

- If the latter is improved, repeat steps 2 to 5 with the new solution.
 - Otherwise, reduce the number of modified cells by a factor $\gamma = 0.9$ and modify only these N cells where the gradient indicates the highest objective function decrease. Then go back to step 4.
6. The optimization is terminated when $N < 1$.

The influence of the initial conditions and the optimality of the ON/OFF method have unfortunately never been studied. However, switching the cell compositions in an ON/OFF way does not address the issue related to the lack of convexity of the problem, since the topology is still updated according to the objective function gradient. Such study would thus certainly reveal an influence of the initial conditions on the method results.

A similar method was proposed by Dong-Hun et al. (2005) for the optimization of magnetic sources. The method also relies both on the objective function sensitivity to determine in which cells materials have to be switched, and on a mutation factor to adjust the number of modified cells and thereby ensure a reduction of the objective function.

2.6 Sensitivity analysis

The adjoint variable method (AVM) is a well-known method for the sensitivity analysis of the objective function required by gradient-based optimization algorithms. It is characterized by an easy implementation based on the existing finite element model, and by a low computation cost, the gradient of the objective function being obtained with only one additional finite element evaluation.

The AVM has been widely used in the mechanical domain (Haug et al., 1986) and was transposed by e.g. Dyck and Lowther (1994); Dyck et al. (1994); Igarashi et al. (2008) to the electromagnetic domain. It is described in section 3.3 with an approach that is slightly different: the latter focuses on the discrete form of the equations rather than on the weak form. However, both approaches lead to the same result: a method for efficiently computing the objective function gradient through a unique linear system resolution.

2.7 Applications

Topology optimization has already been applied to a wide range of problems, among all:

- jumping rings: produce a sufficient vertical force on a copper ring to levitate it while minimizing the power dissipation (Dyck and Lowther, 1994, 1996),
- linear motors: minimize the power required to generate a given force (Dyck and Lowther, 1997),
- inverse problems: find the shape and location of a dielectric material embedded in another material (Byun et al., 2000),
- magnetic resonance imaging (MRI) systems: achieve a uniform magnetic flux density in a given region (Byun et al., 2004a; Lee and Yoo, 2010),
- transformer cores: maximize the flux linkage between two coils (Byun et al., 1999),
- electrostatic actuators: maximize the motor torque (Byun et al., 2002; Raulli and Maute, 2005; Yoon and Sigmund, 2008),
- magnetic recording heads: maximize the recording field (Okamoto et al., 2006, 2005; Park et al., 2009a,b; Takahashi et al., 2008),
- H magnets (Yoo and Hong, 2004; Yoo and Kikuchi, 2000),
- C-core actuators (Choi and Yoo, 2008b, 2009; Kang et al., 2004; Wang and Kang, 2002; Wang et al., 2004b; Yoo, 2004; Yoo and Hong, 2004; Yoo et al., 2000; Yoo and Soh, 2005),
- Halbach arrays: maximize the attractive or repulsive force between two magnet layers (Choi and Yoo, 2008a; Choi et al., 2010),
- electromagnetic couplers: maximize the actuating force in a given direction while limiting the power consumption (Yoo et al., 2008),
- magnetostrictive sensors: increase the sensor output voltage for a given stress (Choi and Yoo, 2009),
- brushless DC motors (Wang et al., 2005),
- induction furnaces (Choi et al., 1998),
- HTS solenoids (Byun et al., 2004b),

- superconducting coils (Elhaut et al., 2011; Kim et al., 2005a,b; Park et al., 2003),
- magnetic rotation sensors (Kim et al., 2009),
- microwave devices (Chung et al., 2001a,b).

Optimization tool

3

This chapter focuses on the description of the basis optimization tool on which the methods proposed in this thesis rely. The implementation of a topology optimization method can be divided in three parts related to the formulation of the topology optimization problem, the optimization algorithm and the finite element model. In this thesis, the whole method has been implemented in a home-made Matlab program. This approach has the advantage that the source of convergence problems could be easier identified during the development of the optimization methods. However, this home-made implementation does not intend to be optimal: the finite element solver is far less efficient than commercial tools, which will increase the computation time. The same applies to the optimization algorithm, whose implementation could certainly be improved to speed up its execution.

The problem formulation is the main focus of this thesis with the goal of handling the local minimizer and intermediate material issues. It involves the description of the relations between the design variables and the cell properties, the definition of the objective function and the specification of the constraints. A general formulation is introduced in the first section of this chapter in order to provide a basis to describe the method proposed in the thesis.

The determination of the most efficient algorithm for performing topology optimization in the electromagnetic domain is beyond the scope of this work. The second section of this chapter describes the selected algorithm and justifies its implementation.

Eventually, the finite element model is developed in the third section.

3.1 General formulation of the topology optimization problem

A topology optimization problem is described by four main elements: a design space, a library of materials, an objective function and a set of constraints.

The *design space* defines the region in which the optimization is performed. For instance, the optimization of an electromechanical actuator may only focus on the design of the stator, whereas the rotor topology is fixed. Such optimization problem usually includes symmetry, which may also reduce the size of the design space. Indeed, all the stator poles are generally identical and the design space is then limited to one pole.

Since the optimization is supported by a finite element model, the whole problem must be meshed. Inside the design space, the elements of the finite element mesh are used as cells for performing the topology optimization, as already mentioned in chapter 1. The optimization goal is thus to determine the optimal material for each of these mesh elements, while the composition of the other elements is either given by the problem or obtained by symmetry from the design space.

The *library of materials* contains a description of all the materials that are available for the method to create the topology. The optimization problems considered in this thesis involve four materials: air, iron, windings (copper) and permanent magnets, that are described by three electromagnetic properties: the permeability, the current density and the remanent field, and one mechanical property: the material density. The design variables on which the optimization is performed are the proportions of each material in each cell, written ρ_α for material α , a vector whose size is equal to the number of cells. The air proportion is generally not included as a design variable and is rather deduced from the other material proportions:

$$\rho_a = 1 - \rho_\alpha - \rho_\beta - \dots \quad (3.1)$$

The *objective function* may depend on different quantities: the material proportions, the cell electromagnetic properties and/or the magnetic fields. Since optimization problems are usually written as minimization problems, quantities that must be maximized are inverted in the objective function.

The *set of constraints* must be expressed on the design variables and defines their feasible domain. They are maintained in this domain by orthogonal projections, which is further detailed in section 3.2.2. The set of constraints at least includes two constraints, referred to as the *cell composition constraints*, to ensure that the material

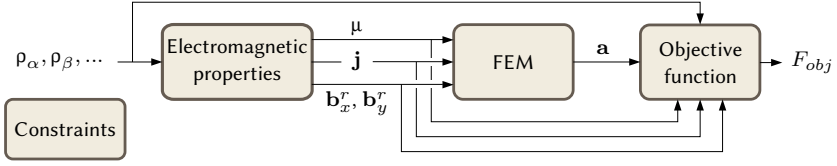


Fig. 3.1. General block diagram of topology optimization problems with isotropic materials.

proportions stay positive and usually that their sum is lower than 1 in each cell¹:

$$\rho_{\alpha,e}, \rho_{\beta,e}, \dots \geq 0, \quad (3.2)$$

$$\rho_{\alpha,e} + \rho_{\beta,e} + \dots \leq 1, \quad (3.3)$$

with e the cell number.

Whereas some constraints such as the maximum device mass are easily expressed as a function of the design variables, the constraint formulation can be more difficult regarding other specifications. The torque ripple of an electromechanical actuator is for instance provided by the finite element model and cannot be easily directly expressed in terms of the material proportions. Such kind of constraints is thus better included as a penalty term in the objective function. The challenge consists then in properly adjusting the penalty factor corresponding to each specification, but this issue will be rather addressed in chapter 8 for the permanent magnet synchronous motor optimization.

Fig. 3.1 represents the structure of a topology optimization problem as a block diagram. It includes the design variables, written here ρ_{α} and ρ_{β} . The electromagnetic properties of the cells are deduced from the latter, and are described by the following vectors:

- μ : the permeability,
- $\mathbf{j}$: the z component of the current density,
- $\mathbf{b}_x^r$: the x component of the remanent field,
- $\mathbf{b}_y^r$: the y component of the remanent field.

The z component of the remanent field and the x and y components of the current density are here not considered since this work only focuses on two-dimensional problems, in which the magnetic fields stay in the x - y plane.

¹This second constraint sometimes takes a more complex expression (section 5.3).

Knowing the cell properties, the magnetic fields can be computed using the finite element model. In Fig. 3.1, the output of the finite element model is represented by the vector potential $\mathbf{a}$, from which the B and H fields or the magnetic energy can easily be computed.

In a mathematical form, the optimization problem is written as follows:

$$\begin{aligned}
 \min_{\rho_\alpha, \rho_\beta, \dots} \quad & F_{obj}(\mathbf{a}, \mu, \mathbf{j}, \mathbf{b}_x^r, \mathbf{b}_y^r, \rho_\alpha, \rho_\beta, \dots), \\
 \mathbf{a} = \quad & f_{FEM}(\mu, \mathbf{j}, \mathbf{b}_x^r, \mathbf{b}_y^r), \\
 \mu = \quad & f_\mu(\rho_\alpha, \rho_\beta, \dots), \\
 \mathbf{j} = \quad & f_J(\rho_\alpha, \rho_\beta, \dots), \\
 \mathbf{b}_x^r = \quad & f_{B_x}(\rho_\alpha, \rho_\beta, \dots), \\
 \mathbf{b}_y^r = \quad & f_{B_y}(\rho_\alpha, \rho_\beta, \dots), \\
 c_i(\rho_\alpha, \rho_\beta, \dots) \leq 0, \quad & \forall i,
 \end{aligned} \tag{3.4}$$

with c_i the set of constraints.

The methods developed in this thesis basically focus on the blocks *electromagnetic properties* and *constraints* in Fig. 3.1. They indeed intend to propose an efficient formulation of the constraints and of the relations between the design variables and the cell electromagnetic properties, in order to tackle the local minimizer and intermediate material issues.

3.2 Optimization algorithm

Topology optimization is originally a 0-1 discrete problem: each material should be present or not in a given cell at the optimization end. A discrete formulation of the topology optimization problem has for instance been proposed by Stolpe and Svanberg (2003), which suggests implementing discrete optimization algorithms. However, the efficiency of these algorithms really varies from problem to problem, and topology optimization turns out to be very hard to solve. Indeed, an extremely slow convergence has been observed during this research with a classic branch-and-bound method.

In the literature, topology optimization methods rather rely on sequential linear programming (SLP) (Yoo et al., 2008) or on the method of moving asymptotes (MMA) (Svanberg, 1987), which proved to be efficient in the mechanical domain. These methods are applied to a relaxed formulation of the problem which allows the material proportions in the cells to continuously evolve between 0 and 1.

The algorithm implemented in this thesis is the steepest-descent method and is

based on the same relaxation. The choice of this basic method, which may at first seem questionable, is justified by multiple reasons. First, since a home-made Matlab program was desired in order to easily understand and analyze the optimization issues, its simple implementation was a serious asset. Second, in view of the problems that are nowadays considered in the literature, this algorithm is sufficient to maintain the computation time under reasonable limits. The method has besides already been applied to the topology optimization of electromagnetic devices by e.g. Byun and Park (2007); Okamoto and Takahashi (2006). Third, the thesis specifically focuses on the efficient formulation of the optimization problem to handle local minimizer and intermediate material issues. From this formulation, any gradient-based optimization algorithm may be applied, the determination of the optimal one being out of the scope of this work.

The steepest-descent method is the most common line-search method. At each iteration, a step size α is computed in order to minimize, at least approximately, the objective function in a given direction $\mathbf{d}^{(t)}$:

$$\min_{\alpha > 0} \Phi^{(t)}(\alpha) = F_{obj}(\rho^{(t)} + \alpha \mathbf{d}^{(t)}), \quad (3.5)$$

where t is the iteration number. $\Phi^{(t)}(\alpha)$ is the value of the objective function in the direction $\mathbf{d}^{(t)}$. The design variables are then updated by:

$$\rho^{(t+1)} = \rho^{(t)} + \alpha \mathbf{d}^{(t)}. \quad (3.6)$$

In the steepest-descent method, the line-search direction is given by the opposite of the gradient of the objective function:

$$\mathbf{d}^{(t)} = -\nabla F_{obj}(\mathbf{a}, \mu, \mathbf{j}, \mathbf{b}_x^r, \mathbf{b}_y^r, \rho). \quad (3.7)$$

The sensitivity analysis required to derive the objective function gradient is developed in section 3.3.

No stopping criterion is considered in this thesis. Indeed, the optimization is simply observed to be converged when all the cells contain a solid material. The material distribution then completely stops evolving as well as the objective function, which exactly stays at the same value.

The implementation of the steepest-descent method requires the consideration of two points. First, an algorithm is required to compute the step size α at each iteration. Second, the method must ensure that the optimization constraints are respected at each iteration. This second concern is here addressed through an orthogonal projection of the design variables on the constraints. Other methods exist, such as transforming the problem into an unconstrained problem using barrier or penalty functions (Bertsekas, 1999). However, with the simple constraints involved in this thesis, the orthogonal projection can be easily implemented and was

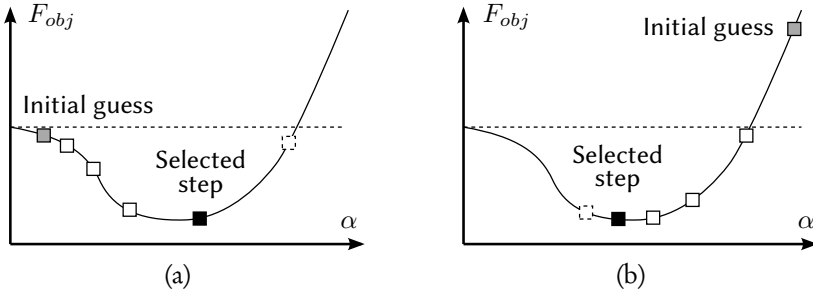


Fig. 3.2. Step computation algorithm.

observed to lead to higher convergence rates. A penalty function was nevertheless implemented in one case in chapter 8 in order to limit the torque ripple of a motor to a given fraction of its average torque. Indeed, this high-level constraint can hardly be directly expressed on the design variables.

3.2.1 Step computation algorithm

Computing the optimal step size is a time-consuming operation. Indeed, it involves the exact minimization of the objective function in the steepest-descent direction. However, a reasonable approximation is satisfactory to achieve quite good results.

Too short step sizes must be avoided in order to ensure a sufficient convergence speed. On the other hand, too large steps can lead to instability and could worsen the objective function. The step size is thus computed using the following algorithm.

- Make an initial guess of the step size α , for instance by taking the step size of the previous iteration.
- Compute the objective function $\Phi(\alpha)$ and compare it with its initial value $\Phi(0)$:
 - if $\Phi(\alpha) < \Phi(0)$, a larger step could be better: multiply α by a factor θ as long as it decreases the objective function;
 - if $\Phi(\alpha) > \Phi(0)$, the step size is too large: divide α by the factor θ until $\Phi(\alpha)$ becomes lower than $\Phi(0)$ and as long as it decreases the objective function.

The algorithm is illustrated in Fig. 3.2 for the two situations. It turns out to be effective in practice since too short as well as too large steps are avoided. In addition, the algorithm ensures a reduction of the objective function at each step, which guarantees a stable convergence.

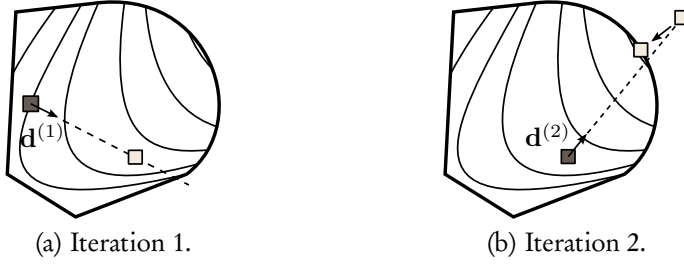


Fig. 3.3. Steepest-descent method: projection on the problem constraints.

3.2.2 Orthogonal projection on the constraints

When updating the design variables in the steepest-descent direction, they may come out of the feasible domain defined by the problem constraints. Fig. 3.3 shows for instance two iterations of the steepest-descent method for two design variables. They stay in the feasible domain during the first iteration, but they must be orthogonally projected on the constraints during the second iteration.

Without mass constraint, the orthogonal projection is straightforward since the problem constraints are defined cell by cell. They are indeed limited to the cell composition constraints (3.2) and (3.3). Even when the latter take more complex expressions, the orthogonal projection can still be easily performed cell by cell.

Including a mass constraint quite complicates the projection, since this constraint involves the material proportions of all the cells at the same time. Indeed, it has the following form:

$$\sum_e V_e (D_\alpha \rho_{\alpha,e} + D_\beta \rho_{\beta,e} + \dots) \leq m, \quad (3.8)$$

with V_e the volume of cell e , D_α and D_β the densities of materials α and β and m the device mass.

The orthogonal projection is not trivial anymore since it cannot be realized cell by cell. The algorithm used for the projection is then described by

Require: $\rho^{(t)}$, α , $\mathbf{d}^{(t)}$, m_{max}

- 1: $\rho^{(t+1)} \leftarrow \text{proj}(\rho^{(t)} + \alpha \mathbf{d}^{(t)})$
- 2: **if** $m(\rho^{(t+1)}) \leq m_{max}$ **then**
- 3: **return** $\rho^{(t+1)}$
- 4: **end if**
- 5: $\rho_0 \leftarrow \rho^{(t)}$
- 6: $\bar{\rho} \leftarrow \rho^{(t)} + \alpha \mathbf{d}^{(t)}$

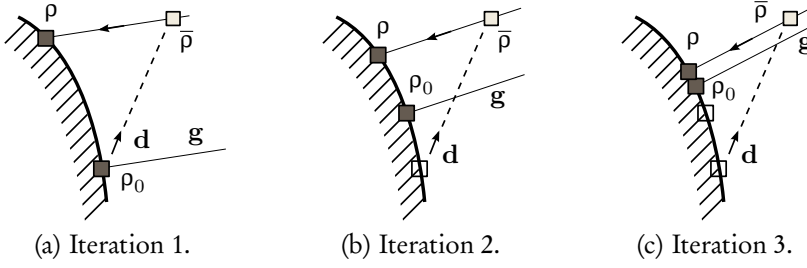


Fig. 3.4. Algorithm for the constraint orthogonal projection in mass-constrained optimizations.

```

7: repeat
8:    $\mathbf{g} \leftarrow \frac{\partial m(\rho)}{\partial \rho} \Big|_{\rho=\rho_0}$ 
9:    $\beta \leftarrow \text{solve } m(\text{proj}(\bar{\rho} - \beta \mathbf{g})) = m_{max}$ 
10:   $\rho^{(t+1)} \leftarrow \text{proj}(\bar{\rho} - \beta \mathbf{g})$ 
11:   $\rho_0 \leftarrow \gamma \rho_0 + (1 - \gamma) \rho^{(t+1)}$ 
12: until  $\sum \left| \mathbf{g} - \frac{\partial m(\rho)}{\partial \rho} \Big|_{\rho=\rho_0} \right| \leq tol$ 
13: return  $\rho^{(t+1)}$ 

```

where $m(\rho)$ provides the mass according to the material proportion ρ and $\text{proj}(\rho)$ projects ρ on the cell composition constraints.

The algorithm first projects the design variables on the cell composition constraints. If the mass constraint is then respected, the projection is already done. Otherwise, the algorithm defines $\bar{\rho}$ the design variable values that must be projected and $\mathbf{g}$ the orthogonal direction to the mass constraint at the initial material proportion $\rho_0 = \rho^{(t)}$. Several iterations are then required to realise the projection. Fig. 3.4 illustrates the evolution of the design variables on the mass constraint after each iteration.

At each iteration, the algorithm seeks, by dichotomy, the right value of β for which the projection of $\bar{\rho} - \beta \mathbf{g}$ on the cell composition constraints meets the mass constraint. The obtained proportions $\rho^{(t+1)}$ are not yet correct, since the projection on the mass constraint was done with the direction $\mathbf{g}$ which is orthogonal for the initial proportions ρ_0 but not for the projected proportions $\rho^{(t+1)}$. A new projection direction is thus defined with a new value of ρ_0 derived from the old value and the projected proportions $\rho^{(t+1)}$. The parameter γ must be well adjusted to ensure a fast convergence of $\mathbf{g}$ while avoiding instability. The algorithm iterates until the right projection direction is found. The algorithm behaviour illustrated in Fig. 3.4 for two variables corresponds to $\gamma = 0.4$.

3.3 Finite element model

This section is dedicated to the finite element model on which all the optimizations performed in the thesis are based. The model is here only described considering isotropic materials in order to simplify the developments, but the latter can easily be extended to take anisotropy into account.

3.3.1 Finite element equation system

The finite element model is derived from the following Maxwell equations:

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad (3.9)$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (3.10)$$

$$\mathbf{B} = \mu \mathbf{H} + \mathbf{B}^r, \quad (3.11)$$

with $\mathbf{H}$ the magnetic field intensity, $\mathbf{B}$ the magnetic flux density, $\mathbf{A}$ the vector potential, $\mathbf{J}$ the current density, $\mathbf{B}^r$ the permanent magnet remanent field and μ the permeability.

These equations can be grouped into one:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J} - \frac{1}{\mu} \nabla \times \mathbf{B}^r. \quad (3.12)$$

The weak form is obtained by multiplying both sides of (3.12) by an arbitrary vector potential $\hat{\mathbf{A}}$ and by integrating them over the whole domain Ω :

$$\int_{\Omega} \hat{\mathbf{A}} \cdot \nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) d\Omega = \int_{\Omega} \left(\hat{\mathbf{A}} \cdot \mathbf{J} - \hat{\mathbf{A}} \cdot \frac{1}{\mu} \nabla \times \mathbf{B}^r \right) d\Omega. \quad (3.13)$$

This weak form remains equivalent to (3.12) since it must be verified for any $\hat{\mathbf{A}}$. The integration by parts gives

$$\begin{aligned} - \int_{\Omega} \nabla \cdot \left(\hat{\mathbf{A}} \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) \right) d\Omega + \int_{\Omega} (\nabla \times \hat{\mathbf{A}}) \cdot \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) d\Omega \\ = \int_{\Omega} \left(\hat{\mathbf{A}} \cdot \mathbf{J} - \hat{\mathbf{A}} \cdot \frac{1}{\mu} \nabla \times \mathbf{B}^r \right) d\Omega. \end{aligned} \quad (3.14)$$

The first term can be developed thanks to the divergence theorem:

$$- \int_{\Omega} \nabla \cdot \left(\hat{\mathbf{A}} \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) \right) d\Omega = \oint_{\partial\Omega} \mathbf{n} \cdot \left(\hat{\mathbf{A}} \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) \right) dS, \quad (3.15)$$

with $\mathbf{n}$ the normal unit vector. With usual boundary conditions, this integral is equal to zero. Indeed, homogeneous conditions are generally imposed on the vector

potential in order to cancel the normal component of the magnetic field on the boundaries. If the finite element model takes advantage of a plane of symmetry, the parallel component is then equal to zero on this plane which still cancels the integral.

The weak form reduces then to

$$\int_{\Omega} (\nabla \times \hat{\mathbf{A}}) \cdot \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) d\Omega = \int_{\Omega} \hat{\mathbf{A}} \cdot \mathbf{J} d\Omega - \int_{\Omega} \hat{\mathbf{A}} \cdot \frac{1}{\mu} \nabla \times \mathbf{B}^r d\Omega. \quad (3.16)$$

Since this thesis deals with 2D problems in the x - y plane, the vector potentials and the current density are aligned with the z axis while the remanent field is defined in the x - y plane:

$$\mathbf{A}(x, y) = \begin{bmatrix} 0 \\ 0 \\ A(x, y) \end{bmatrix}, \quad \hat{\mathbf{A}}(x, y) = \begin{bmatrix} 0 \\ 0 \\ \hat{A}(x, y) \end{bmatrix}, \quad (3.17)$$

$$\mathbf{J}(x, y) = \begin{bmatrix} 0 \\ 0 \\ J(x, y) \end{bmatrix}, \quad \mathbf{B}^r(x, y) = \begin{bmatrix} B_x^r(x, y) \\ B_y^r(x, y) \\ 0 \end{bmatrix}. \quad (3.18)$$

To derive the discrete form, the z components of the vector potentials A and $\hat{A}$ are approximated by discrete values in each node of the mesh:

$$A(x, y) = \sum_{i=1}^n a_i \tau_i(x, y), \quad (3.19)$$

$$\hat{A}(x, y) = \sum_{i=1}^n \hat{a}_i \tau_i(x, y), \quad (3.20)$$

with n the number of nodes and a_i the vector potential on node i . τ_i is a scalar shape function equal to 1 on node i and zero on the other nodes.

The permeability μ , the current density J and the remanent fields B_x^r and B_y^r

are approximated by discrete values in each cell¹:

$$\mu(x, y) = \sum_{e=1}^m \mu_e \phi_e(x, y), \quad (3.21)$$

$$J(x, y) = \sum_{e=1}^m j_e \phi_e(x, y), \quad (3.22)$$

$$B_x^r(x, y) = \sum_{e=1}^m b_{x,e}^r \phi_e(x, y), \quad (3.23)$$

$$B_y^r(x, y) = \sum_{e=1}^m b_{y,e}^r \phi_e(x, y), \quad (3.24)$$

with m the number of cells, ϕ_e a shape function equal to 1 on cell e and 0 everywhere else and μ_e , j_e , $b_{x,e}^r$ and $b_{y,e}^r$ the permeability, current density and remanent fields of cell e . The cell magnetizations are then given by:

$$m_{x,e} = \frac{b_{x,e}^r}{\mu_e}, \quad (3.25)$$

$$m_{y,e} = \frac{b_{y,e}^r}{\mu_e}. \quad (3.26)$$

These approximations are inserted in (3.16) to obtain

$$\begin{aligned} \sum_{i=1}^n \sum_{k=1}^n \sum_{e=1}^m \hat{a}_i a_k \frac{1}{\mu_e} \int_{\Omega} \phi_e \left(\frac{\partial \tau_i}{\partial x} \frac{\partial \tau_k}{\partial x} + \frac{\partial \tau_i}{\partial y} \frac{\partial \tau_k}{\partial y} \right) d\Omega &= \sum_{i=1}^n \sum_{e=1}^m \hat{a}_i j_e \int_{\Omega} \tau_i \phi_e d\Omega \\ &+ \sum_{i=1}^n \sum_{e=1}^m \hat{a}_i \left(m_{x,e} \int_{\Omega} \tau_i \frac{\partial \phi_e}{\partial y} d\Omega - m_{y,e} \int_{\Omega} \tau_i \frac{\partial \phi_e}{\partial x} d\Omega \right). \end{aligned} \quad (3.27)$$

This equation must be satisfied for any vector potential $\hat{A}$, and thus for any value of $\hat{a}_i$. We can then write:

$$\begin{aligned} \sum_{k=1}^n \sum_{e=1}^m a_k \frac{1}{\mu_e} \int_{\Omega} \phi_e \left(\frac{\partial \tau_i}{\partial x} \frac{\partial \tau_k}{\partial x} + \frac{\partial \tau_i}{\partial y} \frac{\partial \tau_k}{\partial y} \right) d\Omega &= \sum_{e=1}^m j_e \int_{\Omega} \tau_i \phi_e d\Omega \\ &+ \sum_{e=1}^m \left(m_{x,e} \int_{\Omega} \tau_i \frac{\partial \phi_e}{\partial y} d\Omega - m_{y,e} \int_{\Omega} \tau_i \frac{\partial \phi_e}{\partial x} d\Omega \right), \quad \forall i. \end{aligned} \quad (3.28)$$

Since the shape functions are known, the integral terms can be computed to

¹In this thesis, the topology optimization uses the elements of the finite element mesh as cells. That is the reason why the term *cell* can here be used instead of mesh element for describing the finite element model.

obtain the following matrices:

$$M_{ik}^{(e)} = \int_{\Omega} \phi_e \left(\frac{\partial \tau_i}{\partial x} \frac{\partial \tau_k}{\partial x} + \frac{\partial \tau_i}{\partial y} \frac{\partial \tau_k}{\partial y} \right) d\Omega, \quad (3.29)$$

$$C_{ie} = \int_{\Omega} \tau_i \phi_e d\Omega, \quad (3.30)$$

$$D_{x,ie} = \int_{\Omega} \tau_i \frac{\partial \tilde{\phi}_e}{\partial y} d\Omega, \quad (3.31)$$

$$D_{y,ie} = - \int_{\Omega} \tau_i \frac{\partial \tilde{\phi}_e}{\partial x} d\Omega. \quad (3.32)$$

Since the shape functions ϕ_e are equal to 1 on cell e and 0 on the other cells, they are not continuous and cannot be differentiated. In order to compute the matrices $D_{x,ie}$ and $D_{y,ie}$, they are thus replaced by $\tilde{\phi}_e$: a pyramidal function equal to 3 on the cell gravity centre and 0 on the cell boundaries. The quantity of material in the cell is unchanged with this new shape function since $\int_{\Omega} \phi_e d\Omega = \int_{\Omega} \tilde{\phi}_e d\Omega$.

From the vectors of the cell current densities $\mathbf{j}$, permeabilities μ and magnetizations $\mathbf{m}_x$ and $\mathbf{m}_y$, the vector potential is obtained by solving the following linear system:

$$\sum_{k=1}^n M_{ik}(\mu) a_k = \sum_{e=1}^m C_{ie} j_e + \sum_{e=1}^m D_{x,ie} m_x + \sum_{e=1}^m D_{y,ie} m_y, \quad (3.33)$$

$$\mathbf{M}(\mu) \mathbf{a} = \mathbf{C} \mathbf{j} + \mathbf{D}_x \mathbf{m}_x + \mathbf{D}_y \mathbf{m}_y, \quad (3.34)$$

with $\mathbf{M}(\mu)$ given by:

$$\mathbf{M}(\mu) = \sum_{e=1}^m \frac{1}{\mu_e} \mathbf{M}^{(e)}. \quad (3.35)$$

When saturation is taken into account, the cell permeabilities become a function of the magnetic fields. The finite element equation system (3.34) is then nonlinear and must be solved by an iterative method, such as the Newton-Raphson method.

3.3.2 Magnetic fields and coenergy

The magnetic flux density is derived from the potential vector by the following expression:

$$B_x(x, y) = \frac{\partial A(x, y)}{\partial y}, \quad (3.36)$$

$$B_y(x, y) = - \frac{\partial A(x, y)}{\partial x}. \quad (3.37)$$

Its average value in each cell provides the discretized vectors $\mathbf{b}_x$ and $\mathbf{b}_y$, the cell flux densities:

$$b_{x,e} = \frac{1}{p_e} \sum_{i=1}^n a_i \int_{\Omega} \phi_e \frac{\partial \tau_i}{\partial y} d\Omega, \quad (3.38)$$

$$b_{y,e} = -\frac{1}{p_e} \sum_{i=1}^n a_i \int_{\Omega} \phi_e \frac{\partial \tau_i}{\partial x} d\Omega, \quad (3.39)$$

with p_e the surface of cell e , given by

$$p_e = \int_{\Omega} \phi_e d\Omega. \quad (3.40)$$

The cell field intensities are besides given by

$$h_{x,e} = \frac{b_{x,e}}{\mu_e}, \quad (3.41)$$

$$h_{y,e} = \frac{b_{y,e}}{\mu_e}. \quad (3.42)$$

The magnetic coenergy is required to compute the objective functions of the optimization problems considered in the thesis. Without saturation, it is simply given by

$$W_{cm} = \sum_{e=1}^m p_e \left[\frac{(b_{x,e}^r + b_{x,e})h_{x,e}}{2} + \frac{(b_{y,e}^r + b_{y,e})h_{y,e}}{2} \right]. \quad (3.43)$$

With saturation, the iron microstructure described in section 5.3 must be considered:

$$W_{cm} = \sum_{e=1}^m p_e \left[t_e W_{cm,i} \left(\sqrt{h_{x,e}^2 + h_{y,e}^2} \right) + (1 - t_e) \left(\frac{(b_{x,e}^r + \mu_0 h_{x,e})h_{x,e}}{2} + \frac{(b_{y,e}^r + \mu_0 h_{y,e})h_{y,e}}{2} \right) \right], \quad (3.44)$$

with $W_{cm,i}(H)$ the function of the magnetic coenergy of iron according to the H field. The latter is derived from the iron B - H curve by

$$W_{cm,i}(H) = \int_0^H B(\tilde{H}) d\tilde{H}. \quad (3.45)$$

3.3.3 Sensitivity analysis

The gradient of an objective function F_{obj} with respect to the design variables ρ is required to compute the steepest-descent direction used in the optimization algorithm

(section 3.2). The developments provided in this section are equivalent to those of the adjoint variable method (Dyck and Lowther, 1994; Haug et al., 1986; Okamoto et al., 2006; Wang et al., 2004b), except that the latter are made based on a weak formulation of the objective function while this section rather considers its discrete form.

For a general objective function, the gradient with respect to the design variable $\rho_{\alpha,e}$ related to material α is given by:

$$\begin{aligned} \frac{dF_{obj}(\mathbf{a}, \mu, \mathbf{j}, \mathbf{b}_x^r, \mathbf{b}_y^r, \rho)}{d\rho_{\alpha,e}} &= \frac{\partial F_{obj}}{\partial \mathbf{a}}^T \frac{d\mathbf{a}}{d\rho_{\alpha,e}} + \frac{\partial F_{obj}}{\partial \mu_e} \frac{d\mu_e}{d\rho_{\alpha,e}} \\ &+ \frac{\partial F}{\partial j_e} \frac{dj_e}{d\rho_{\alpha,e}} + \frac{\partial F}{\partial b_{x,e}^r} \frac{db_{x,e}^r}{d\rho_{\alpha,e}} + \frac{\partial F}{\partial b_{y,e}^r} \frac{db_{y,e}^r}{d\rho_{\alpha,e}} + \frac{\partial F_{obj}}{\partial \rho_{\alpha,e}}. \end{aligned} \quad (3.46)$$

Only the first term of this expression requires a special attention. Indeed, the computation of the others is straightforward since the relations between the electromagnetic properties and the material proportions are known.

The finite element equation system (3.34) is differentiated as follows:

$$\frac{d\mathbf{M}(\mu)}{d\rho_{\alpha,e}} \mathbf{a} + \mathbf{M}(\mu) \frac{d\mathbf{a}}{d\rho_{\alpha,e}} = \mathbf{C} \frac{d\mathbf{j}}{d\rho_{\alpha,e}} + \mathbf{D}_x \frac{d\mathbf{m}_x}{d\rho_{\alpha,e}} + \mathbf{D}_y \frac{d\mathbf{m}_y}{d\rho_{\alpha,e}}. \quad (3.47)$$

The first term of (3.46) is thus given by

$$\begin{aligned} \frac{\partial F_{obj}}{\partial \mathbf{a}}^T \frac{d\mathbf{a}}{d\rho_{\alpha,e}} &= \\ \frac{\partial F_{obj}}{\partial \mathbf{a}}^T \mathbf{M}(\mu)^{-1} \left[-\frac{d\mathbf{M}(\mu)}{d\rho_{\alpha,e}} \mathbf{a} + \mathbf{C} \frac{d\mathbf{j}}{d\rho_{\alpha,e}} + \mathbf{D}_x \frac{d\mathbf{m}_x}{d\rho_{\alpha,e}} + \mathbf{D}_y \frac{d\mathbf{m}_y}{d\rho_{\alpha,e}} \right], \end{aligned} \quad (3.48)$$

with the differentiation of (3.35) giving:

$$\frac{d\mathbf{M}(\mu)}{d\rho_{\alpha,e}} = \frac{d\mathbf{M}(\mu)}{d\mu_e} \frac{d\mu_e}{d\rho_{\alpha,e}} = -\frac{1}{\mu_e^2} \mathbf{M}^{(e)} \frac{d\mu_e}{d\rho_{\alpha,e}}. \quad (3.49)$$

The efficiency of the sensitivity analysis relies on the computation of (3.48) by first solving the following system:

$$\mathbf{M}(\mu) \lambda = \frac{\partial F_{obj}}{\partial \mathbf{a}} \Leftrightarrow \lambda = \mathbf{M}(\mu)^{-1} \frac{\partial F_{obj}}{\partial \mathbf{a}}. \quad (3.50)$$

Since $\mathbf{M}(\mu)^{-1}$ is symmetric, the result λ can be introduced in (3.48). The derivative of the objective function with respect to any design variable can then be computed from a simple matrix multiplication:

$$\frac{\partial F_{obj}}{\partial \mathbf{a}}^T \frac{d\mathbf{a}}{d\rho_{\alpha,e}} = \lambda^T \left[-\frac{d\mathbf{M}(\mu)}{d\rho_{\alpha,e}} \mathbf{a} + \mathbf{C} \frac{d\mathbf{j}}{d\rho_{\alpha,e}} + \mathbf{D}_x \frac{d\mathbf{m}_x}{d\rho_{\alpha,e}} + \mathbf{D}_y \frac{d\mathbf{m}_y}{d\rho_{\alpha,e}} \right]. \quad (3.51)$$

Thanks to the prior λ computation, the sensitivity analysis is limited to a unique time-consuming linear system solving (3.50) and the computation of $\mathbf{M}(\mu)^{-1}$ could be avoided.

Eq. (3.51) can be further developed for the three materials considered in this thesis: iron, windings and permanent magnets, which gives

$$\frac{\partial F_{obj}}{\partial \mathbf{a}}^T \frac{d\mathbf{a}}{d\rho_{i,e}} = \lambda^T \left[\frac{1}{\mu_e^2} \frac{d\mu_e}{d\rho_{i,e}} \mathbf{M}^{(e)} \mathbf{a} - \frac{b_{x,e}^r}{\mu_e^2} \mathbf{D}_x \frac{d\mu}{d\rho_{i,e}} - \frac{b_{y,e}^r}{\mu_e^2} \mathbf{D}_y \frac{d\mu}{d\rho_{i,e}} \right], \quad (3.52)$$

$$\frac{\partial F_{obj}}{\partial \mathbf{a}}^T \frac{d\mathbf{a}}{d\rho_{w,e}} = \lambda^T \left[\mathbf{C} \frac{d\mathbf{j}}{d\rho_{w,e}} \right], \quad (3.53)$$

$$\frac{\partial F_{obj}}{\partial \mathbf{a}}^T \frac{d\mathbf{a}}{d\rho_{m,e}} = \lambda^T \left[\frac{1}{\mu_e} \mathbf{D}_x \frac{d\mathbf{b}_x^r}{d\rho_{m,e}} + \frac{1}{\mu_e} \mathbf{D}_y \frac{d\mathbf{b}_y^r}{d\rho_{m,e}} \right], \quad (3.54)$$

with $\rho_{i,e}$, $\rho_{w,e}$ and $\rho_{m,e}$ the proportions of iron, windings and permanent magnets in cell e .

3.3.4 Material distribution differentiation

The mesh refinement algorithm described in chapter 7 relies on the derivatives of the material distribution in the x and y axes. These derivatives can actually easily be obtained using the finite element matrices. The development is here illustrated for material α .

Its distribution is given by

$$\rho_\alpha(x, y) = \sum_{e=1}^m \rho_{\alpha,e} \phi_e(x, y), \quad (3.55)$$

which can be differentiated as follows:

$$\frac{\partial \rho_\alpha(x, y)}{\partial x} = \sum_{e=1}^m \rho_{\alpha,e} \frac{\partial \phi_e(x, y)}{\partial x}, \quad (3.56)$$

$$\frac{\partial \rho_\alpha(x, y)}{\partial y} = \sum_{e=1}^m \rho_{\alpha,e} \frac{\partial \phi_e(x, y)}{\partial y}. \quad (3.57)$$

The refinement algorithm actually requires that these derivatives are expressed on the mesh nodes, in the form of vectors $\mathbf{n}_{\alpha,x}$ and $\mathbf{n}_{\alpha,y}$. They are obtained from

the derivative average values around each node:

$$n_{\alpha,x,j} = \frac{1}{q_j} \int_{\Omega} \sum_{e=1}^m \rho_{\alpha,e} \tau_j \frac{\partial \tilde{\phi}_e}{\partial x} d\Omega = \frac{1}{q_j} \sum_{e=1}^m \rho_{\alpha,e} \int_{\Omega} \tau_j \frac{\partial \tilde{\phi}_e}{\partial x} d\Omega, \quad (3.58)$$

$$n_{\alpha,y,j} = \frac{1}{q_j} \int_{\Omega} \sum_{e=1}^m \rho_{\alpha,e} \tau_j \frac{\partial \tilde{\phi}_e}{\partial y} d\Omega = \frac{1}{q_j} \sum_{e=1}^m \rho_{\alpha,e} \int_{\Omega} \tau_j \frac{\partial \tilde{\phi}_e}{\partial y} d\Omega, \quad (3.59)$$

$$(3.60)$$

with

$$q_j = \int_{\Omega} \tau_j d\Omega. \quad (3.61)$$

The shape functions ϕ_e were replaced in (3.59) and (3.60) by $\tilde{\phi}_e$ in order to be able to differentiate them, as explained previously. It is actually possible to show that the derivatives computed by these expressions correspond to a material distribution that linearly evolves between the cell centres.

By defining the following matrices:

$$E_{x,je} = \frac{1}{q_j} \int_{\Omega} \tau_j \frac{\partial \tilde{\phi}_e}{\partial x} d\Omega, \quad (3.62)$$

$$E_{y,je} = \frac{1}{q_j} \int_{\Omega} \tau_j \frac{\partial \tilde{\phi}_e}{\partial y} d\Omega, \quad (3.63)$$

the vectors $\mathbf{n}_{\alpha,x}$ and $\mathbf{n}_{\alpha,y}$ are simply expressed by:

$$\mathbf{n}_{\alpha,x} = \mathbf{E}_x \mathbf{p}_{\alpha}, \quad (3.64)$$

$$\mathbf{n}_{\alpha,y} = \mathbf{E}_y \mathbf{p}_{\alpha}. \quad (3.65)$$

Study cases

4

This chapter is dedicated to the description of the study cases considered along the thesis. They involve in total four different materials that are here completely characterized in terms of electromagnetic and physical properties. For each study case, the objective function is described as well as the design space, the possible symmetry and the material library.

The material libraries of the considered study cases involve the following materials: air, iron, windings and permanent magnets. They are always characterized by the same properties which are given in Table 4.1.

The permeability of the iron material, given by $\mu_0\mu_r$, is in reality a non-linear function of the magnetic field because of the saturation phenomenon. However, the study cases are sometimes considered without saturation, in order to simplify the optimization. In this case, the relative permeability is set to

$$\mu_r = 200, \quad (4.1)$$

which is a quite low value that was chosen to avoid converging towards topologies having very thin iron structures. When saturation is taken into account, the iron permeability is rather characterized by the B - H curve given in Fig. 4.1, where the relative permeability is equal to 4000 for a zero magnetic field. This difference of permeability in comparison with the non saturated permeability value explains the fact that some optimizations lead to higher torques when saturation is taken into account.

Windings are characterized by a current density of 2.5×10^6 A/m² whereas permanent magnets have a remanent field of 1.09 T. For the sake of simplicity, the permeability of permanent magnet is assumed to be equal to the air permeability. In this way, the permeability of a cell is only determined by its proportion of iron, while the current density is related to the winding proportion and the remanent field to the permanent magnet proportion.

Table 4.1. Electromagnetic and physical properties of materials.

Material	Permeability (H/m)	Current density (A/m ²)	Remanent field (T)	Density (kg/m ³)
Air	μ_0	0	0	$D_a = 0$
Iron	$\mu_0 \mu_r$	0	0	$D_i = 7874$
Windings (copper)	μ_0	$J_w = 2.5 \times 10^6$	0	$D_w = 8940$
Sintered NdFeB magnets	μ_0	0	$B_m^r = 1.09$	$D_m = 7400$

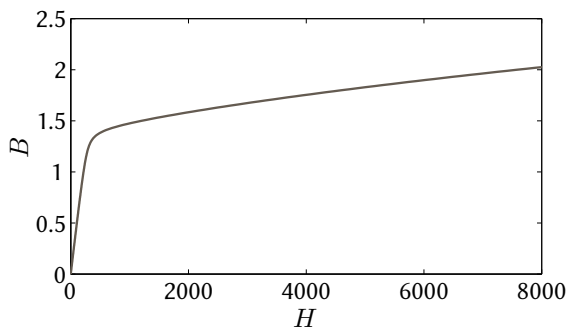


Fig. 4.1. B - H curve of iron.

4.1 Magnetic thrusts

This study case is related to the design of high-dynamics linear actuators for the manufacturing industry. Thrusts are indeed a fundamental component of such actuators in order to invert the movement direction. Using magnetic thrusts for this application is interesting because the shocks related to mechanical thrusts can then be avoided.

The system that must be optimized is illustrated in Fig. 4.2. The actuator moving part is traveling at a high speed between two magnetic thrusts. At each thrust, this moving part is sent back in the opposite direction thanks either to permanent magnets or to supplied windings. Both cases are considered in the thesis. The goal is to optimize their shape in order to maximize the speed of the moving part. Indeed, beyond a given speed, the repulsive force between the moving part and the thrust is not sufficient to prevent them to hit each other.

The optimization only considers the thrust part of the actuator, whereas the

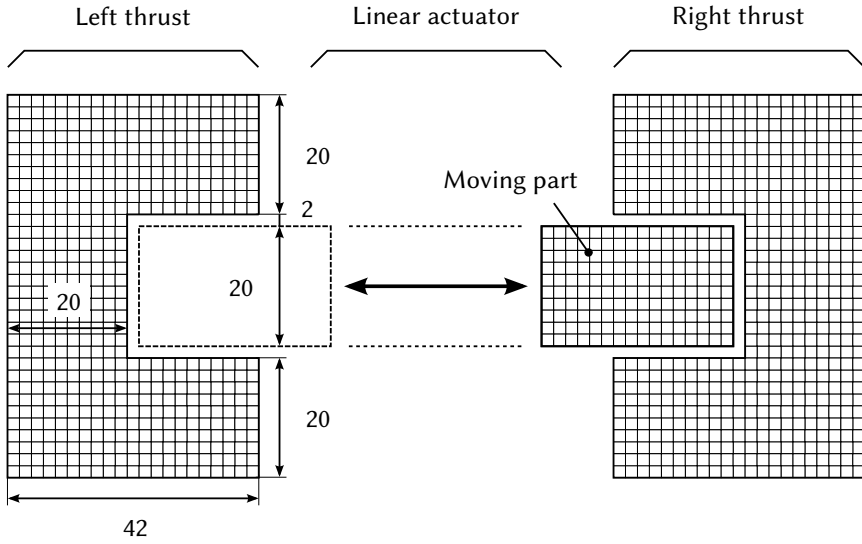


Fig. 4.2. Design space for the magnetic thrust optimization (dimensions in mm).

actuator components required to produce the oscillating movement are not taken into account and not illustrated in Fig. 4.2. Since the moving part is composed either of permanent magnets or windings, it would obviously be interesting to reuse them in the design of these components in order to avoid increasing the moving part mass and reducing the actuator performances.

The distance between the two thrusts represented in Fig. 4.2 is not realistic and is in fact much larger. This distance is assumed to be large enough so that the interactions between the moving part and each thrust can be considered individually (the effect of the left thrust is neglected when the moving part is close to the right thrust and vice-versa).

The design space of the moving part is a rectangle whereas those of the two thrusts have C-shapes. Their dimensions are given in Fig. 4.2 in millimetres. The problem is studied in two dimensions considering a length of 100 mm in the third dimension.

When the moving part approaches a thrust, it slows down and its kinetic energy is converted into magnetic energy. This conversion ends when the moving part stops at a given distance from the thrust, set to 2 mm. The maximum speed of the moving part can therefore be derived from the difference of magnetic energy between the positions in which the moving part is far and close to the thrust. This difference is actually equal to the magnetic energy due to the interaction between the moving part and the thrust when they are close to each other, which is given for the left

thrust by

$$W_{mag,lt-mp} = \int_{\Omega} A_{lt} (\nabla \times B_{mp}^r) d\Omega, \quad (4.2)$$

$$= - \sum_{i=1}^n \sum_{e=1}^m a_{lt,i} m_{x,mp,e} \int_{\Omega} \tau_i \frac{\partial \tilde{\phi}_e}{\partial y} d\Omega$$

$$+ \sum_{i=1}^n \sum_{e=1}^m a_{lt,i} m_{y,mp,e} \int_{\Omega} \tau_i \frac{\partial \tilde{\phi}_e}{\partial x} d\Omega,$$

$$= \mathbf{a}_{lt}^T (\mathbf{D}_x \mathbf{m}_{x,mp} + \mathbf{D}_y \mathbf{m}_{y,mp}),$$

$$= (\mathbf{D}_x \mathbf{m}_{x,lt} + \mathbf{D}_y \mathbf{m}_{y,lt})^T \mathbf{M}^{-1} (\mathbf{D}_x \mathbf{m}_{x,mp} + \mathbf{D}_y \mathbf{m}_{y,mp}). \quad (4.3)$$

with A_{lt} the potential vector generated by the left thrust and B_{mp}^r the remanent field of the moving part. The other terms are described in section 3.3 dedicated to the finite element model.

This magnetic energy is equal to the kinetic energy of the moving part when the latter is far from the thrusts:

$$W_{mag,lt-mp} = \frac{1}{2} m_{mp} v_l^2, \quad (4.4)$$

$$W_{mag,rt-mp} = \frac{1}{2} m_{mp} v_r^2, \quad (4.5)$$

which gives

$$v_l = \sqrt{2 \frac{W_{mag,lt-mp}}{m_{mp}}}, \quad (4.6)$$

$$v_r = \sqrt{2 \frac{W_{mag,rt-mp}}{m_{mp}}}, \quad (4.7)$$

with m_{mp} the moving part mass and v_l and v_r the maximum speeds at which the moving part can reach the left and right thrusts. The optimization aims thus to maximize the minimum of v_l and v_r . In order to write the problem as a minimization, the objective function is the opposite of the minimum of the two speeds:

$$F_{obj} = -\min(v_l, v_r). \quad (4.8)$$

4.2 Switched reluctance motor

This second study case focuses on the design of a switched reluctance motor (SRM) composed of 6 stator and 4 rotor poles. The goal is to maximize its average torque. The latter is computed from the difference of magnetic coenergy between the position in which the rotor is aligned with the supplied pole and the unaligned position

in which it is rotated of 30° :

$$F_{obj} = -\frac{W_{cm,a} - W_{cm,u}}{\pi/6}, \quad (4.9)$$

with $W_{cm,a}$ and $W_{cm,u}$ the magnetic coenergies corresponding to the aligned and unaligned positions.

A small exception is made in section 5.6.2 in which a method based on the Maxwell stress tensor is proposed. In this case, the optimization goal is the maximization of the torque produced when the rotor is in the unaligned position. Indeed, the Maxwell stress tensor provides the torque at a specific position and not an average torque. In the case of electromagnetic devices, it is given by:

$$T_{ij} = \frac{1}{\mu_0} B_i B_j - \frac{1}{2\mu_0} B^2 \delta_{ij}, \quad (4.10)$$

with δ_{ij} the Kronecker's delta. In two dimensions, the forces on a given element are computed by integrating on a closed boundary around this element the scalar product of the Maxwell tensor and the normal unit vector $\mathbf{n}$ to this boundary.

In the case of the SRM, the torque applied to the rotor is computed in polar coordinates on a circular boundary located at the air-gap centre:

$$F_{obj} = -\oint_0^{2\pi} R_a^2 \frac{1}{\mu_0} B_r(R_a, \theta) B_\theta(R_a, \theta) d\theta, \quad (4.11)$$

with R_a the air-gap centre radius and B_r and B_θ the radial and tangential flux densities.

The problem is again studied in two dimensions, a length of 1 meter is considered in the third dimension.

The SRM study case is divided in two subproblems related to the stator and the rotor optimizations. When the complete problem is studied in chapter 8, the stator and rotor design spaces are both considered at the same time.

The SRM stator optimization problem is described in Fig. 4.3. Since the stator is composed of 6 poles and each pole must be symmetrical to ensure obtaining the same torque in both rotation directions, the design space reduces to a twelfth of the stator. Three materials must be distributed in this design space: windings, iron and air. The rotor has a simple cross shape whose thickness is 30 mm.

The design space for the rotor optimization is represented in Fig. 4.4. It again reduces thanks to symmetry: only a eighth of the rotor must be considered. This optimization only deals with two materials since the rotor of a switched reluctance motor is only composed of iron and air.

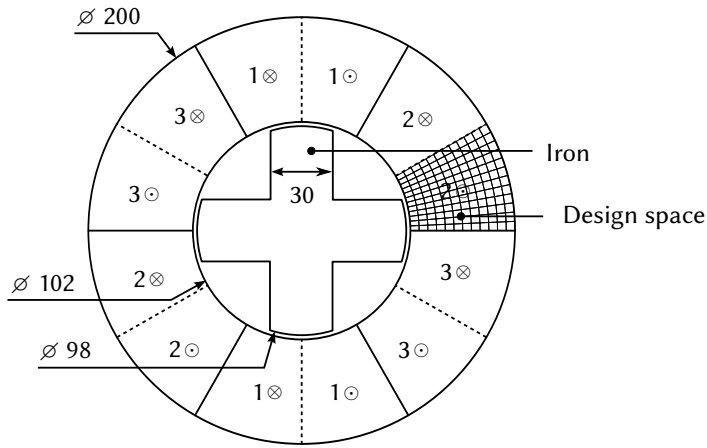


Fig. 4.3. Design space for the stator optimization of a switched reluctance motor (dimensions in mm).

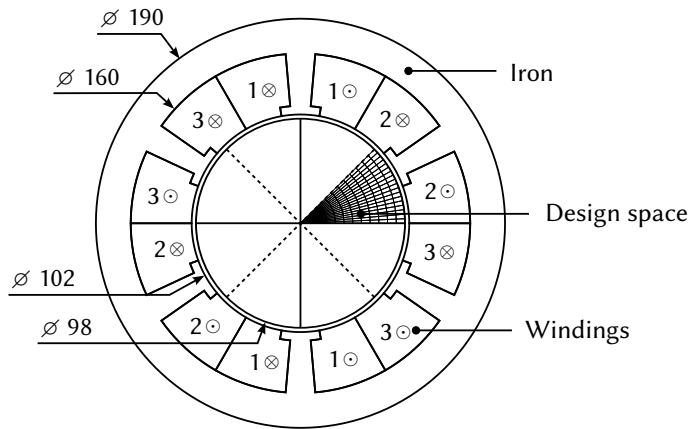


Fig. 4.4. Design space for the rotor optimization of a switched reluctance motor (dimensions in mm).

4.3 Permanent magnet synchronous motor

The optimization of the permanent magnet synchronous motor (PMSM) aims to maximize its average torque while limiting its torque ripple. The objective function is thus quite more complex compared to the SRM optimization. It indeed requires the evaluation of the magnetic fields in several rotor positions in order to compute the torque ripple and to take into account the continuous evolution of the winding currents. Two positions were in fact sufficient to evaluate the SRM average torque since it is supplied with switched currents, i.e. constant currents that switch from one phase to the next one at given rotor positions.

The PMSM is characterized by two pole pairs, parallel magnetized magnets and windings supplied with sinusoidal currents:

$$i_1(\theta_e) = J_w \sin\left(\theta_e + \frac{\pi}{3}\right), \quad (4.12)$$

$$i_2(\theta_e) = J_w \sin(\theta_e), \quad (4.13)$$

$$i_3(\theta_e) = J_w \sin\left(\theta_e - \frac{\pi}{3}\right), \quad (4.14)$$

with θ_e the electrical angle. The latter is equal to

$$\theta_e = p\theta_m, \quad (4.15)$$

with $p = 2$ the number of pole pairs and θ_m the mechanical position, i.e. the rotor angle.

The stator optimization problem is described in Fig. 4.5. The rotor is composed of surface-mounted permanent magnets and the stator of 12 symmetrical slots. The design space is thus reduced to one 24th of the stator. Three materials are involved: iron, windings and air.

The rotor optimization of the PMSM is related to three materials: iron, air and permanent magnets. The design space is represented in Fig. 4.6. All the cells have the same magnetization direction inside the design space, since permanent magnets have parallel magnetization directions. The latter are represented by arrows in Fig. 4.6.

In order to evaluate the objective function, the torque evolution according to the mechanical position must first be computed. Thanks to symmetry, this evolution must only be obtained in the interval $\theta_m \in [0; \pi/6)$. A given number of mechanical positions are chosen, written $\theta_1, \theta_2, \dots, \theta_n$, with $\theta_1 = 0$ and $\theta_n = \pi/6$. The average torque between these positions is then computed using the magnetic energy:

$$\tau_i = \frac{W_{cm}(\theta_m = \theta_{i+1}, \theta_e = p\bar{\theta}_i) - W_{cm}(\theta_m = \theta_i, \theta_e = p\bar{\theta}_i)}{\theta_{i+1} - \theta_i}, \quad (4.16)$$

with

$$\bar{\theta}_i = \frac{\theta_i + \theta_{i+1}}{2}. \quad (4.17)$$

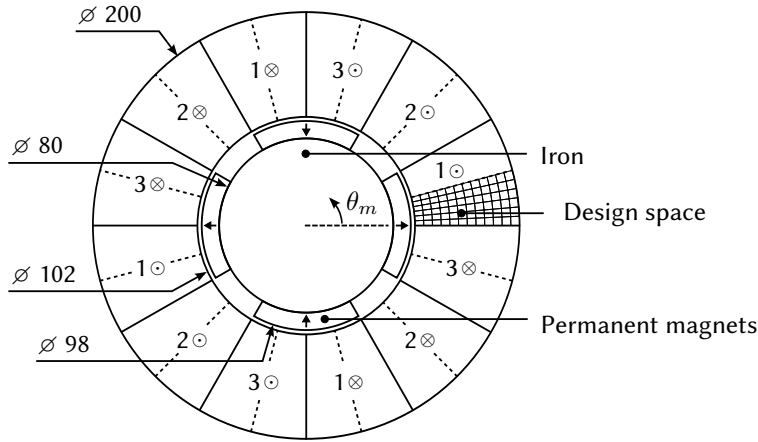


Fig. 4.5. Design space for the stator optimization of a permanent magnet synchronous motor (dimensions in mm).

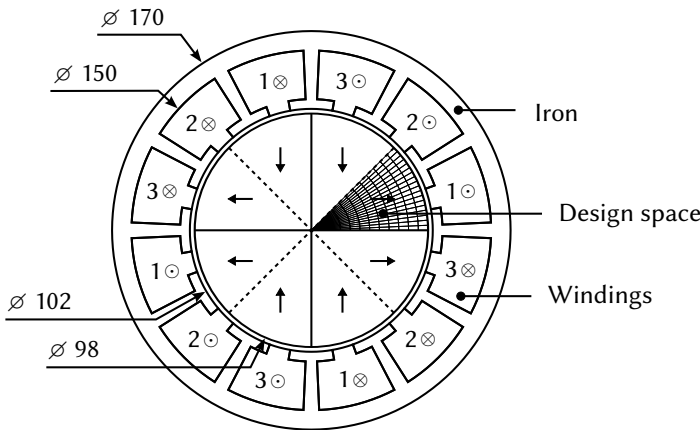


Fig. 4.6. Design space for the rotor optimization of a permanent magnet synchronous motor (dimensions in mm).

The PMSM average torque is then given by

$$\tau_{average} = \frac{1}{n-1} \sum_{i=1}^{n-1} \tau_i(\theta_{i+1} - \theta_i), \quad (4.18)$$

while the torque ripple is computed as the difference between the highest and the lowest torque:

$$\tau_{ripple} = \max_i(\tau_i) - \min_i(\tau_i). \quad (4.19)$$

The optimization aims to maximize the average torque:

$$F_{obj} = -\tau_{average}, \quad (4.20)$$

while limiting the torque ripple to 10 % of the latter:

$$\tau_{ripple} \leq 0.1\tau_{average}. \quad (4.21)$$

Since the PMSM objective function is quite time consuming, a simplified version is considered in chapters 5 and 6. Indeed, the study case is there only used for illustrating the proposed methods and considering a realistic objective function is then unnecessary.

The objective function is thus reduced to the average torque, computed roughly from two mechanical positions: $\theta_1 = 0$ and $\theta_2 = \pi/6$, which gives

$$F_{obj} = -\tau_{average} = \frac{W_{cm}(\theta_m = \pi/6, \theta_e = \pi/6) - W_{cm}(\theta_m = 0, \theta_e = \pi/6)}{\pi/6}. \quad (4.22)$$

The torque ripple specification is then not taken into account and windings are supplied with the following currents:

$$i_1 = J_w, \quad (4.23)$$

$$i_2 = 0.5J_w, \quad (4.24)$$

$$i_3 = -0.5J_w. \quad (4.25)$$

Isotropic materials

5

This chapter intends to describe the topology optimization of electromagnetic devices using methods based on isotropic materials. This category of methods is characterized by the definition of one design variable per material and per cell, in opposition with anisotropic methods, such as the homogenization method (Yoo et al., 2000), where several design variables are introduced to describe the dimensions of a given material microstructure. The isotropic approach is obviously simpler, but also has some limitations.

The chapter is mainly structured according to the materials involved in the optimization. The first studied problems are related to winding or permanent magnet optimizations, since those problems can be easily solved both with and without the introduction of a mass specification. Next, windings are considered in combination with iron, which leads to the apparition of local minimizer issues. These issues are discussed in the second section without mass specification, the latter being introduced in the third section. The next section focuses on the implication of saturation in the optimization method. The combination of iron and permanent magnets is then studied, since it requires some small modifications of the method. Finally, problems related to the optimization of ferromagnetic parts (combination of iron and air) are considered in the end of the chapter since they require a very specific mechanism to ensure a global convergence.

5.1 Windings or permanent magnets

The optimization problems related to windings or permanent magnets can usually be easily solved by a gradient-based algorithm, without being faced with local minimizers. This is explained by the fact that both the current density and the remanent field are in the right member of the finite element equation (3.34) and that the mag-

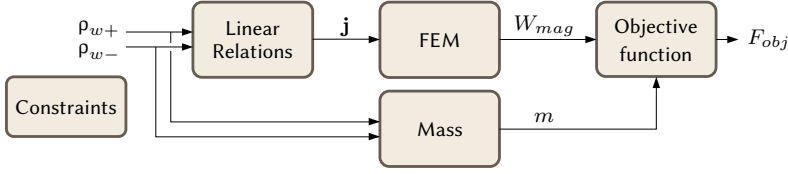


Fig. 5.1. Block diagram for winding optimizations.

netic fields are thus linearly dependent on these two properties. With the objective functions considered in this section, the steepest-descent algorithm easily succeeds in finding the global minimizer. This is not the case with the permeability, which will be discussed in the next section.

The section focuses on the study case described in section 4.1 to illustrate the method results. This study case is related to the design of two magnetic thrusts and of a moving part travelling at a high speed between them. The goal is to maximize the moving part speed while avoiding mechanical contacts with the thrusts. In this section, the optimization only focuses on the design of the moving part and one thrust, the complete system being studied in chapter 8.

5.1.1 Windings

The block diagram for the winding optimization is provided in Fig. 5.1. Two material proportions, $\rho_{w+,e}$ and $\rho_{w-,e}$, are considered to represent windings respectively with a z positive and z negative current density. The latter is derived from the material proportions using a simple linear relation:

$$j_e = (\rho_{w+,e} - \rho_{w-,e})J_w, \quad (5.1)$$

with J_w the current density of windings whose value is given in section 4.

The set of constraints is limited to the two usual cell composition constraints:

$$\rho_{w+,e} + \rho_{w-,e} \leq 1, \quad (5.2)$$

$$\rho_{w+,e}, \rho_{w-,e} \geq 0, \quad (5.3)$$

with the air proportion given by

$$\rho_{a,e} = 1 - \rho_{w+,e} - \rho_{w-,e}. \quad (5.4)$$

As described in section 4.1, the moving part maximum speed is computed from the ratio of the magnetic energy due to the interaction between the moving part and the thrust, to the moving part mass. The latter is computed by

$$m_{mp} = L + \sum_{e \in M} D_w V_e (\rho_{w+,e} + \rho_{w-,e}), \quad (5.5)$$

with L the fixed load of the moving part, D_w the density of windings, V_e the volume of cell e and M the set of cells corresponding to the moving part. Two fixed load values are considered here: $L = 0.1$ kg and $L = 1$ kg. Applying the steepest-descent algorithm described in section 3.2 to this optimization problem provides the results shown in Fig. 5.2, which raises several comments.

- The method converges in less than 20 iterations with a computation time lower than 1 minute, which underlines the efficiency of the steepest-descent algorithm on this simple problem.
- The same topology is always obtained, whatever the initial value of the material proportions (two similar results are actually possible, since inverting the current directions leads to the same performances). This problem is thus not faced with convexity issues.
- The problem does not either raise issues related to intermediate materials, i.e. mixes of windings and air, since they naturally disappear in the final design.
- The optimization algorithm creates a symmetry which was not imposed by the problem. The design space could therefore be reduced to the upper or lower part of the thrust. One winding proportion would then be sufficient with a positive (or negative) current density for the moving part and a negative (or positive) current density for the thrust.
- The thrust design space is completely filled with windings. Indeed, nothing limits the quantity of materials and adding windings in a thrust cell always leads to an increase of the objective function, even if this increase is very small.

In order to highlight which thrust cells provide the highest contribution to the maximization of the moving part speed, the optimization was performed again while limiting the thrust mass by the following constraint:

$$\sum_{e \in T} D_w V_e (\rho_{w+,e} + \rho_{w-,e}) \leq m_t, \quad (5.6)$$

with T the set of cells corresponding to the thrust and m_t the thrust maximum mass. The optimization results are given in Fig. 5.3 with $m_t = 1$ kg, where the dashed lines represent the design space limits. The introduction of the mass constraint does not raise any new problem regarding the method convergence. It only slightly complicates the orthogonal projection of the design variables on the constraints, as explained in section 3.2.

The obtained topologies achieve a moving part speed which only decreases of 15 % compared to the topologies of Fig. 5.2, for a reduction of almost 50 % of the mass and therefore of the power consumption. This observation demonstrates

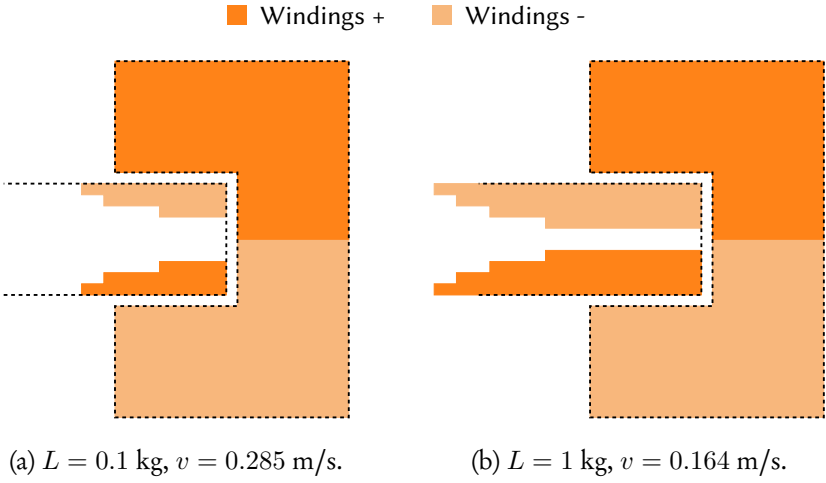


Fig. 5.2. Thrust optimization: winding topology.

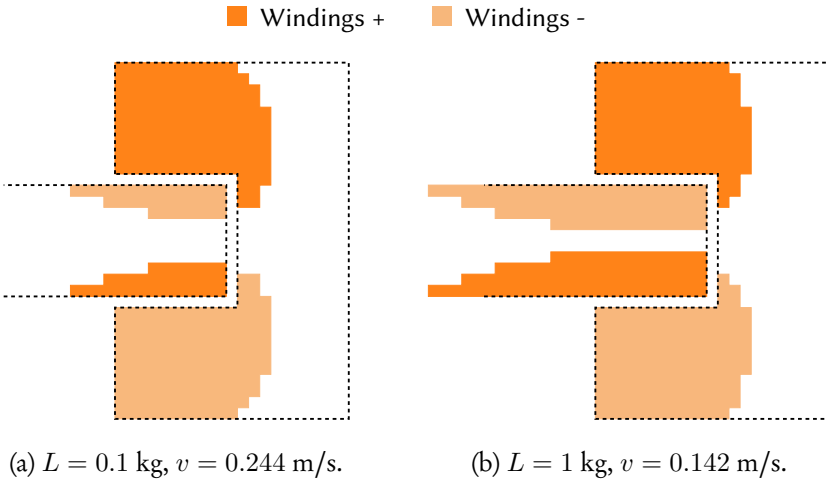


Fig. 5.3. Thrust optimization: winding topology with $m_t = 1 \text{ kg}.$

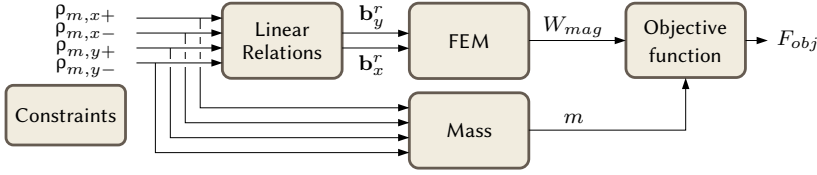


Fig. 5.4. Block diagram for permanent magnet optimizations with free magnetization directions.

the importance of the mass constraint on this particular problem to highlight cells having an important contribution to the objective function.

5.1.2 Permanent magnets

Permanent magnets are characterized by a remanent field, but also by a magnetization direction. According to the optimization problem, the latter may be free or limited to some given directions.

Free magnetization directions

In order to represent any magnetization direction, permanent magnets are described by four material proportions: $\rho_{m,x+}$ and $\rho_{m,x-}$ for the x positive and negative magnetization directions and $\rho_{m,y+}$ and $\rho_{m,y-}$ for the y directions.

The cell remanent fields in the x and y directions are then determined using linear relations:

$$b_{x,e}^r = (\rho_{m,x+,e} - \rho_{m,x-,e})B_m^r, \quad (5.7)$$

$$b_{y,e}^r = (\rho_{m,y+,e} - \rho_{m,y-,e})B_m^r, \quad (5.8)$$

with B_m^r the remanent field of permanent magnets. Fig. 5.4 shows the block diagram related to the permanent magnet optimization with free magnetization directions.

The design variables do not really represent a material proportion, since the permanent magnet proportion is rather given by

$$\rho_{m,e} = \sqrt{(\rho_{m,x+,e} - \rho_{m,x-,e})^2 + (\rho_{m,y+,e} - \rho_{m,y-,e})^2}, \quad (5.9)$$

while the magnetization direction is expressed by

$$\theta_{m,e} = \arctan \frac{\rho_{m,x+,e} - \rho_{m,x-,e}}{\rho_{m,y+,e} - \rho_{m,y-,e}}. \quad (5.10)$$

The cell composition constraints ensure that the permanent magnet proportions stay lower or equal than one:

$$\rho_{m,x+,e}^2 + \rho_{m,x-,e}^2 + \rho_{m,y+,e}^2 + \rho_{m,y-,e}^2 \leq 1, \quad (5.11)$$

$$\rho_{m,x+,e}, \rho_{m,x-,e}, \rho_{m,y+,e}, \rho_{m,y-,e} \geq 0. \quad (5.12)$$

They do not rely on the permanent magnet expression given in (5.9) in order to guarantee that all the design variables are comprised between 0 and 1. Indeed, using expression (5.9) would only constrain the differences $\rho_{m,x+,e} - \rho_{m,x-,e}$ and $\rho_{m,y+,e} - \rho_{m,y-,e}$ to be comprised between -1 and 1, while each design variable could take larger values.

Defining a set of four design variables to represent the permanent magnet distribution guarantees a fast convergence of the steepest-descent algorithm. Two other sets of variables could indeed be considered, but they both raise convergence issues.

The first alternative set of variables would be to consider the permanent magnet proportions ρ_m and the magnetization directions θ_m . But the convergence could then be stuck in a cell if its permanent magnet proportion is equal to zero and its magnetization direction is far from the optimal direction. Indeed, the steepest-descent algorithm updates, at each iteration, the design variables according to the derivative of the objective function. Therefore, the permanent magnet proportion cannot be increased since the wrong magnetization direction would lead to a deterioration of the objective function, while the magnetization direction cannot either be modified since it has no influence on the objective function as the cell magnetization is equal to zero. This kind of behaviour actually requires second-order methods which rely on the second-order derivatives in addition to the gradient.

The second alternative consists in reducing the number of design variables to two by considering $\rho_{m,x}$ and $\rho_{m,y}$, the permanent magnet proportions respectively in the x and y magnetization directions. These two variables would then be subject to the unique constraint:

$$\rho_{m,x,e}^2 + \rho_{m,y,e}^2 \leq 1, \quad (5.13)$$

which allows them to take negative values. This set actually makes impossible for the steepest-descent algorithm to remove completely permanent magnets from a cell. Indeed, the 4 design variable set enables the algorithm to decrease these variables until they come over constraint (5.12), in which case the orthogonal projection ensures that they are all exactly set to zero (see section 3.2 for the description of the steepest-descent algorithm). This mechanism is not possible anymore with two design variables since they can take negative values. They rather tend to oscillate around zero in cells where permanent magnets must be removed. A reduction of the optimization speed is then observed since the oscillations force the steepest-descent algorithm to make very small steps.

When considering free magnetization directions, the optimization should thus definitely be performed on four design variables. There is besides no extra computation time caused by the two additional variables, since it is mainly related to the finite element computations which are anyway based on the two vectors $\mathbf{b}_x^r$ and $\mathbf{b}_y^r$.

The moving part mass is computed by

$$m_{mp} = L + \sum_{e \in M} D_m V_e \sqrt{\rho_{m,x+,e}^2 + \rho_{m,x-,e}^2 + \rho_{m,y+,e}^2 + \rho_{m,y-,e}^2}, \quad (5.14)$$

with L the fixed load of the moving part, D_m the density of permanent magnets, V_e the volume of cell e and M the set of cells corresponding to the moving part. The mass is not expressed as a function of the permanent magnet proportion (5.9) in order to ensure that cells are emptied of magnets by decreasing all the design variables to zero against constraint (5.12). As explained before, this behaviour guarantees a fast convergence since the design variables are exactly set to zero by the orthogonal projection. If the mass was computed from the permanent magnet proportion, it could actually be equal to zero even if the design variables are different of zero, when $\rho_{m,x+,e} = \rho_{m,x-,e}$ and $\rho_{m,y+,e} = \rho_{m,y-,e}$. In this case, the convergence speed reduction due to an oscillation of the design variables around a given value again arises. Expressing the mass with (5.14) ensure avoiding this situation since cells in which all the design variables are different of zero are never produced because their mass is then unnecessarily increased.

The optimization results for two fixed loads of the moving part are given in Fig. 5.5. The comments raised by these results are similar to those made for the winding optimization.

- The method converges very quickly (about 15 iterations, less than 1 minute).
- Intermediate materials naturally disappear in the final design.
- Four different topologies can be obtained. They correspond to an x positive (represented in Fig. 5.5), x negative, y positive and y negative magnetization of the moving part. Whatever the initial material distribution, the method always converges to one of these four topologies, which is further discussed in chapter 8.
- A symmetry or antisymmetry is observed depending on the moving part magnetization direction.
- The thrust design space is completely filled with permanent magnets.

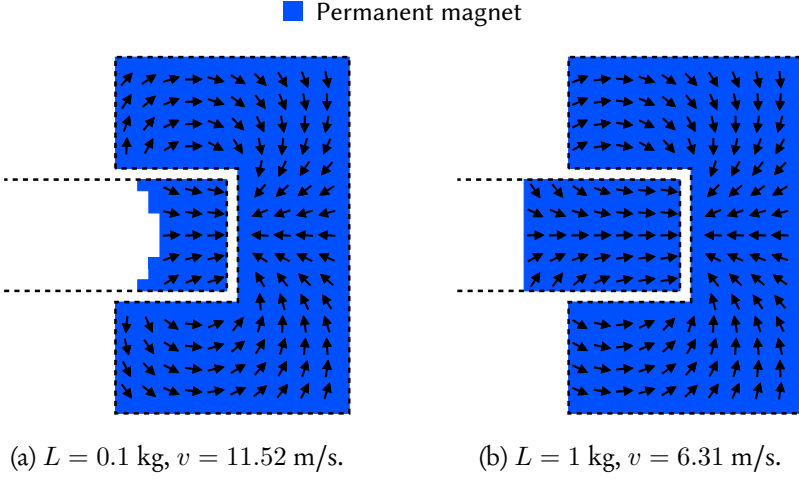


Fig. 5.5. Thrust optimization: permanent magnet topology with free magnetization directions.

Regarding the latter comment, a mass constraint on the fixed thrust can again be introduced:

$$\sum_{e \in T} D_m V_e \sqrt{\rho_{m,x+,e}^2 + \rho_{m,x-,e}^2 + \rho_{m,y+,e}^2 + \rho_{m,y-,e}^2} \leq m_t. \quad (5.15)$$

As for the winding optimization, the mass constraint does not affect the method convergence. Fig. 5.6 shows the optimizations results which underline the low speed reduction (10 %) compared with the thrust mass decrease (almost 50 %).

Fixed magnetization directions

The solutions obtained with free magnetization directions may be difficult to manufacture. In order to limit permanent magnets to a finite number of magnetization directions, each direction is actually represented by a distinct design variable. The optimization problem is then written in its usual form with several materials and the cell magnetizations are given by

$$b_{x,e}^r = (\rho_{m1,e} \cos(\theta_{m,1}) + \rho_{m2,e} \cos(\theta_{m,2}) + \dots) B_m^r, \quad (5.16)$$

$$b_{y,e}^r = (\rho_{m1,e} \sin(\theta_{m,1}) + \rho_{m2,e} \sin(\theta_{m,2}) + \dots) B_m^r, \quad (5.17)$$

with $\rho_{m1}, \rho_{m2}, \dots$ the permanent magnet proportions corresponding to the magnetization directions $\theta_{m,1}, \theta_{m,2}, \dots$. These magnetization directions are a priori fixed by the designer and are therefore not considered as design variables. The block diagram is represented in Fig. 5.7 considering two magnetization directions.

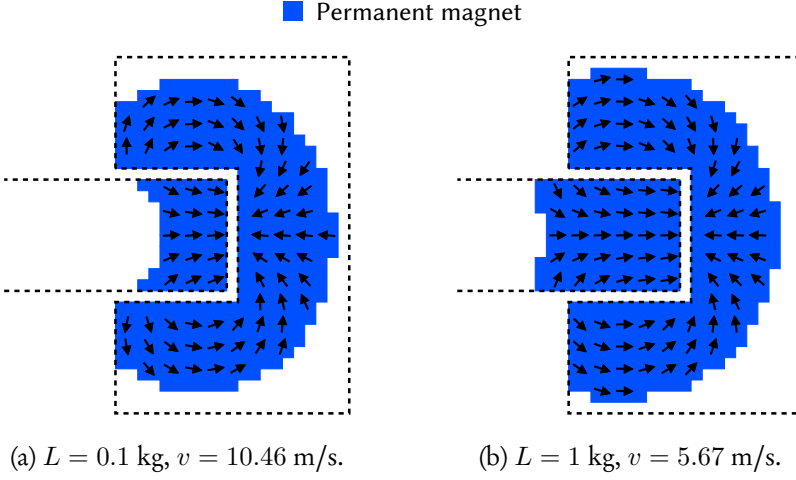


Fig. 5.6. Thrust optimization: permanent magnet topology with free magnetization directions and $m_t = 1 \text{ kg}$.

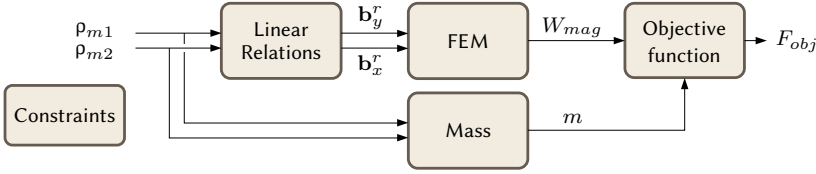


Fig. 5.7. Block diagram for permanent magnet optimizations with fixed magnetization directions.

Regarding the cell composition constraints, they are simply expressed in their usual form:

$$\rho_{m1,e} + \rho_{m2,e} + \dots \leq 1, \quad (5.18)$$

$$\rho_{m1,e}, \rho_{m2,e}, \dots \geq 0. \quad (5.19)$$

Compared with the free magnetization direction problem where the design variables were the components of the magnetization direction, the material proportions ρ_{m1} and ρ_{m2} do here represent real materials and not a component of the magnetization direction. That is the reason why the first constraint can simply be written as a sum of the permanent magnet proportions in each cell, whereas the norm of the magnetization components was previously considered in (5.11). The moving part mass is similarly expressed as follows:

$$m_{mp} = \sum_{e \in M} D_m V_e (\rho_{m1,e} + \rho_{m2,e} + \dots), \quad (5.20)$$

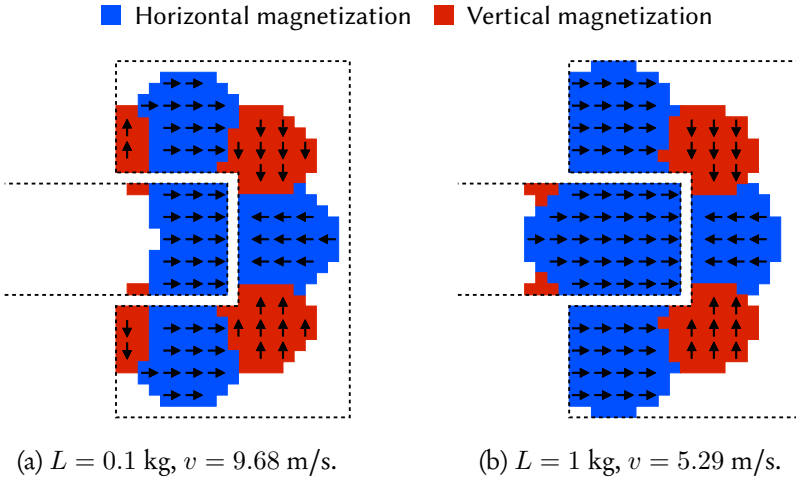


Fig. 5.8. Thrust optimization: permanent magnet topology with fixed magnetization directions and $m_t = 1 \text{ kg}$.

while the constraint on the thrust mass is given by

$$\sum_{e \in T} D_m V_e (\rho_{m1,e} + \rho_{m2,e} + \dots) \leq m_t. \quad (5.21)$$

Fig. 5.8 provides the optimization results considering four fixed magnetization directions: the positive and negative x and y directions. The optimization again converges very quickly to a solution without any intermediate material. Four solutions can be generated as well, and are also characterized by a symmetry or antisymmetry.

Compared to the free magnetization direction results, the speed reduction is limited to about 7 % whereas the manufacturability is greatly improved.

5.2 Iron and windings

Whereas the previous section shows that windings can easily be handled by a topology optimization method, iron actually requires more attention. Therefore, this section only focuses on *volume-constrained* problems, i.e. problems that do not include a mass specification and for which the design space defines the maximum volume of the device. Two mechanisms must be implemented: the sensitivity mapping for ensuring a fast convergence of the steepest-descent algorithm and the convexity mapping for avoiding local minimizers.

The block diagram is used as a support to illustrate where those mechanisms are introduced in the optimization method structure. In view of the study cases con-

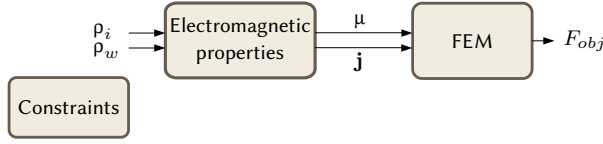


Fig. 5.9. Simplified block diagram for iron-winding optimizations.

sidered in this section, it can actually be reduced to the block diagram represented in Fig. 5.9. The objective functions indeed do not involve the material proportions so that the "FEM" and "Objective function" blocks could be grouped into a unique block. Iron is here assumed to have linear properties, saturation is introduced in section 5.4.

The section focuses on the study case described in section 4.2: the stator design of a switched reluctance motor (SRM). The design variables are the iron and winding proportions ρ_i and ρ_w and the cell composition constraints take the usual expressions:

$$\rho_{i,e} + \rho_{w,e} \leq 1, \quad (5.22)$$

$$\rho_{i,e}, \rho_{w,e} \geq 0, \quad (5.23)$$

with the air proportion deduced from

$$\rho_{a,e} = 1 - \rho_{i,e} - \rho_{w,e}. \quad (5.24)$$

5.2.1 Sensitivity mapping

The iron proportion in a cell determines its permeability, in the same way that the winding proportion specifies its current density. Whereas linear relations are suitable to link the winding and magnet proportions to the cell current densities and remanent fields, several authors recommend using other relations regarding the permeability. Indeed, the sensitivity of the objective function with respect to the latter is generally much higher at low permeability values: increasing the permeability in a low permeability cell has a greater impact on the objective function than in a high permeability cell. This difference of sensitivity may actually reduce the convergence speed, since the descent direction has then different magnitudes leading to cells converging quickly and other slowly.

The relation between the iron proportion and the permeability is here called the *sensitivity mapping*, since it aims to ensure a regular sensitivity of the objective function with respect to the iron proportions to speed up the optimization.

For this purpose, three mappings are discussed and compared. The *permeability mapping* is the simplest one, it establishes a linear relation between the permeability and the iron proportion:

$$\mu_e = \mu_0(1 + (\mu_r - 1)\rho_{i,e}), \quad (5.25)$$

with μ_r the relative permeability of iron. The inefficiency of this mapping has already been mentioned, but it is anyway considered in order to highlight the high sensitivity of the objective function at low permeabilities and to make comparisons with the other mappings.

Choi and Yoo (2008b) recommended defining linear relations between the reluctivity and the iron proportion, which is here referred to as the *reluctivity mapping*:

$$\nu_e = \nu_0 + (1 - (\nu_r - 1)\rho_{i,e}), \quad (5.26)$$

$$\mu_e = \frac{\mu_0}{1 - (\frac{1}{\mu_r} - 1)\rho_{i,e}}. \quad (5.27)$$

Eventually, a *geometric mapping* was suggested in the OMD method (Dyck and Lowther, 1996):

$$\mu_e = \mu_0 \mu_r^{\rho_{i,e}}. \quad (5.28)$$

These three mappings are plotted in Fig. 5.10 and the block diagram is modified to introduce the sensitivity mapping, as shown in Fig. 5.11.

The SIMP polynomial mapping (Byun et al., 2000) given by

$$\mu_e = \mu_0(1 + (\mu_r - 1)\rho_{i,e}^n), \quad (5.29)$$

with usually $2 < n < 4$, is here not considered since its derivative with respect to $\rho_{i,e}$ is equal to zero when $\rho_{i,e} = 0$. An additional constraint must thus generally be introduced to ensure that the iron proportion never reaches zero, i.e. for instance $\rho_{i,e} > 10^{-3}$.

The optimization of the SRM stator with these three mappings provides interesting results that are shown in Fig. 5.12, Fig. 5.13 and Fig. 5.14, respectively for the permeability, reluctivity and geometric mappings. Two kinds of behaviour can be distinguished from these optimizations.

- With the permeability mapping, the optimization always results in the same topology for any initial conditions. However, the latter contains some iron-winding composites.
- With the reluctivity and geometric mappings, the produced topologies differ according to the initial conditions, but iron-winding composites always naturally disappear in the final design.

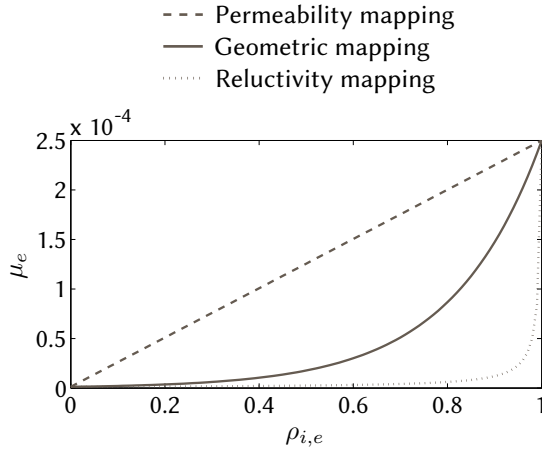


Fig. 5.10. Illustration of the three sensitivity mappings with $\mu_r = 200$.

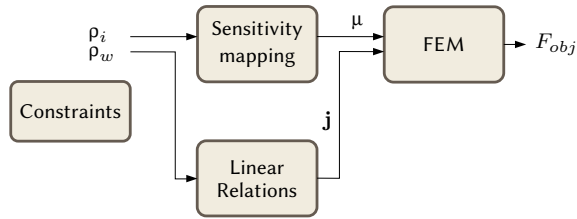


Fig. 5.11. Block diagram for iron-winding optimizations: introduction of the sensitivity mapping.

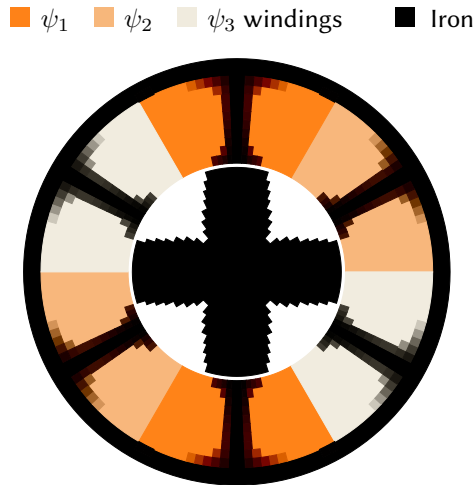


Fig. 5.12. SRM stator design with the permeability mapping: unique topology; $\tau = 87.92$ Nm.

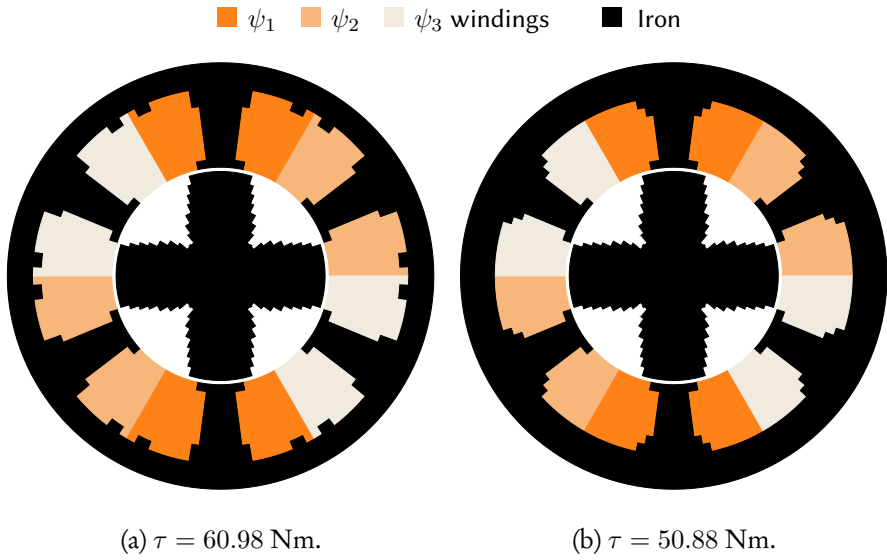


Fig. 5.13. SRM stator design with the relativity mapping: different topologies according to the initial conditions.

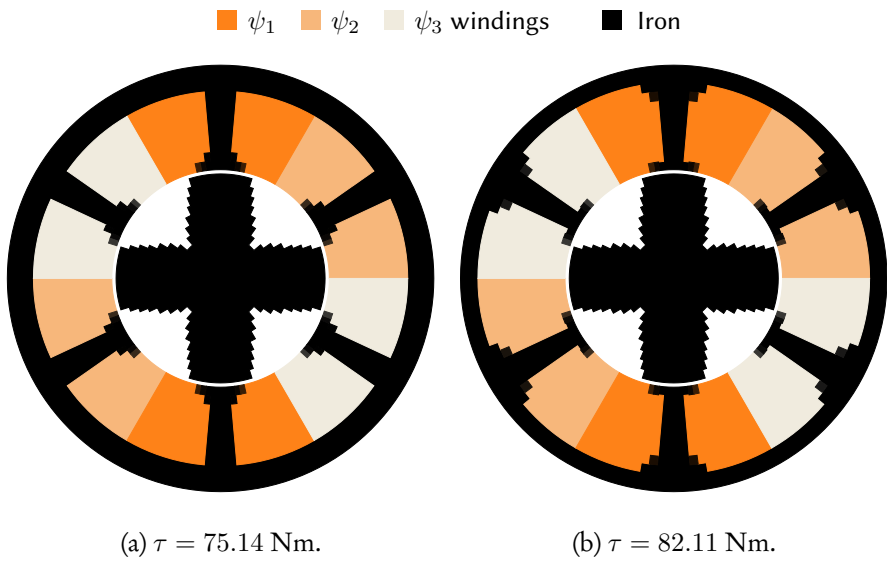


Fig. 5.14. SRM stator design with the geometric mapping: different topologies according to the initial conditions.

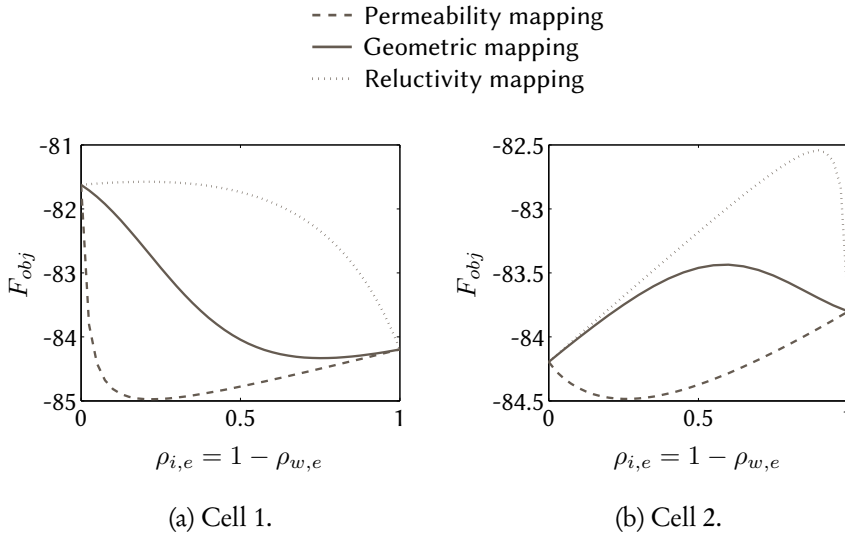


Fig. 5.15. Evolution of the objective function in two different cells.

These behaviours are related to the convexity of the optimization problem which is actually affected by the sensitivity mapping. However, this question is discussed later since the sensitivity mapping rather focuses on improving the regularity of the objective function sensitivity. In order to assess this sensitivity, two cells of a given topology were selected and their composition was progressively modified from windings to iron. According to the sensitivity mapping, a different objective function evolution is observed, as shown in Fig. 5.15.

Even if these curves were obtained on two cells chosen arbitrarily, they reflect the typical objective function shapes corresponding to the three sensitivity mappings. The permeability mapping actually leads to a high sensitivity for low permeabilities, as illustrated in Fig. 5.15(a), while the reluctance mapping results in high sensitivities for high permeabilities, as shown in Fig. 5.15(b).

The geometric mapping is therefore the wiser choice since it guarantees a regular sensitivity. The resulting improvement of the convergence speed cannot actually be highlighted on the SRM stator study case, since all the optimizations converge in less than 10 iterations. However, it becomes significant in more complex problems which is illustrated in section 5.3.3.

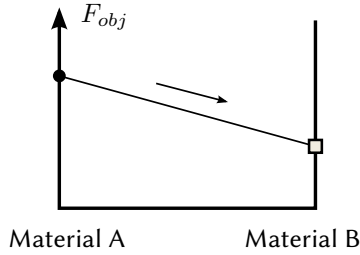


Fig. 5.16. Linear objective function.

5.2.2 Convexity mapping

None of the results obtained with the three sensitivity mappings is satisfactory, since the method converges either to local minimizers or to a topology containing intermediate materials. These two behaviours actually reflect the convexity of the optimization problem. Fig. 5.16 and Fig. 5.17 illustrate this question: they represent three different possible shapes of the objective function for a cell evolving from a material A to a material B.

- If this evolution is linear (Fig. 5.16), or at least *monotonically increasing or decreasing*, the steepest-descent algorithm easily converges to the optimal material. This behaviour was observed for the optimization of windings or permanent magnets in the previous section.
- In Fig. 5.17(a), the objective function has a *convex* shape. In this case, the algorithm always converges to the same point whatever the initial cell composition, but this solution is in fact an intermediate material. This situation arises when the permeability mapping is used, which is confirmed both by the optimization results and by the two objective function shapes shown in Fig. 5.15.
- The behaviour observed with the reluctivity and the geometric mappings actually corresponds to the third case shown in Fig. 5.17(b): a *concave* objective function. According to the starting point, the algorithm converges to a different material, but this material is always solid, i.e. not intermediate. This behaviour is also observed in Fig. 5.15, except in the first cell for the geometric mapping. The geometric mapping leads thus to a behaviour which is convex in some cells and concave in others. A careful observation of the topologies shown in Fig. 5.14 reveals that they in fact contain some composites at the end of the stator teeth, even if it is not really obvious in the figure.

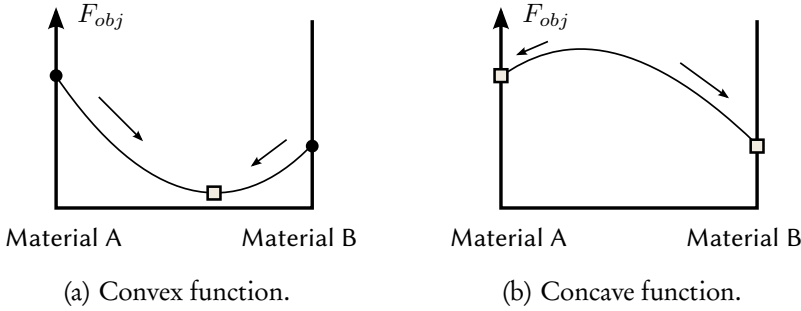


Fig. 5.17. Convex and concave objective functions.

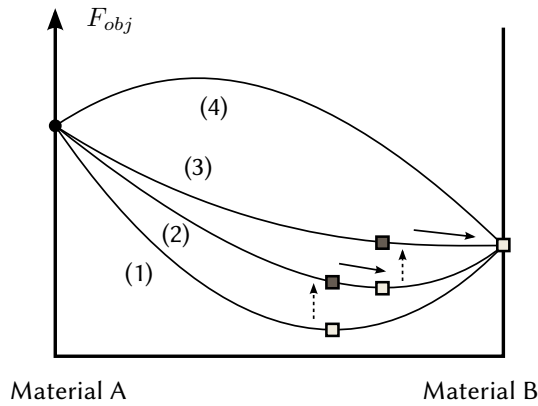


Fig. 5.18. Convergence to the optimal material by progressively modifying the objective function shape.

In order both to always converge to the same solution and to remove intermediate materials, the proposed approach is to progressively change the objective function shape during the optimization, by modifying the electromagnetic properties of intermediate materials. The goal of this process is illustrated in Fig. 5.18. The optimization begins with a convex shape (curve 1), which guarantees that the same topology is produced for any initial conditions and that a continuous increasing or decreasing objective function can be expected when modifying thereafter the objective function shape. The latter evolves then towards a concave shape (curve 2 to 4) in order to remove composite materials. This progressive evolution supports the convergence towards the optimal material.

The objective function shape can be adjusted by modifying the electromagnetic properties of intermediate materials. Giving poor properties to mixes of material A and B indeed results in a concave shape, while a convex shape can be obtained by

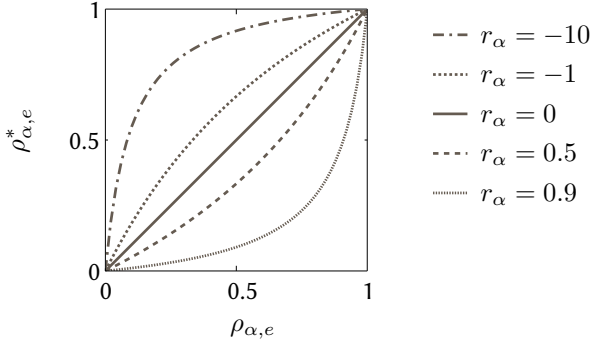


Fig. 5.19. Convexity mapping.

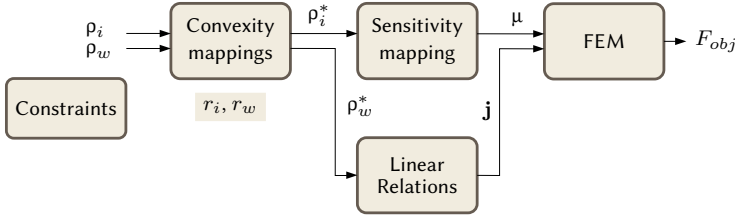


Fig. 5.20. Block diagram for iron-winding optimizations: introduction of the convexity mapping.

conferring them valuable properties.

This promotion/penalization of intermediate materials is realized by a *convexity mapping*, which is one of the main contributions of this research:

$$\rho_{\alpha,e}^* = \frac{1 - r_\alpha}{1 - r_\alpha \rho_{\alpha,e}} \rho_{\alpha,e}, \quad (5.30)$$

with ρ_α the proportion of material α , ρ_α^* the promoted/penalized proportion and $r_\alpha \in (-\inf, 1)$ the convexity mapping parameter related to material α . This convexity mapping is plotted in Fig. 5.19. When r_α is equal to zero, the mapping has no influence. However, intermediate proportions of material α are promoted for $r_\alpha < 0$ and are penalized for $r_\alpha > 0$. The mapping equation was selected for its property to become tangent to the borders of the box $[0; 1] \times [0; 1]$ for the extreme values of r_α . Fig. 5.20 shows the new block diagram including the convexity mapping.

In order to penalize/promote intermediate materials, the convexity mapping can be applied either to the iron or winding proportions, or even to both. Here, it will only be used with the iron proportion while the parameter r_w related to the winding proportions is maintained at zero.

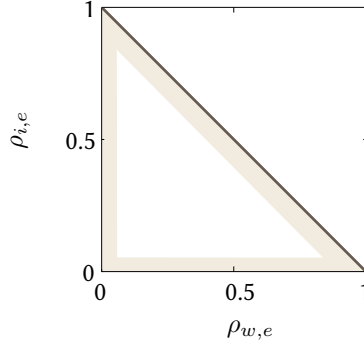


Fig. 5.21. Feasible domain defined by the cell composition constraints on the material proportions.

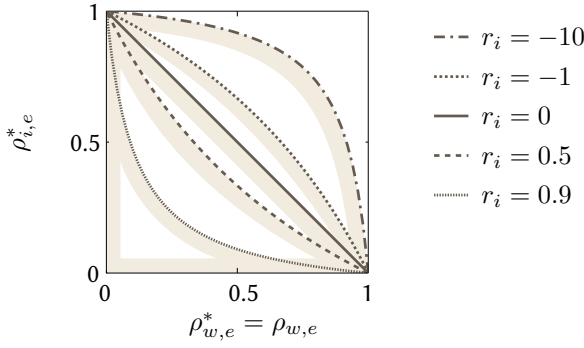


Fig. 5.22. Feasible domain on the penalized material proportions.

The influence of the convexity mapping is clearly illustrated by the feasible domain of the penalized proportions ρ_i^* and ρ_w^* . The cell composition constraints lead to a triangular domain on the iron and winding proportions, as shown in Fig. 5.21. According to the value of r_i , the iron penalization parameter, the feasible domain for the penalized proportions takes different shapes that are illustrated in Fig. 5.22.

- When r_i is negative, the penalized proportions of iron and windings can both be large at the same time. Cells containing an iron-winding composite achieve thus a high permeability and a high current density. A convex objective function shape can therefore be expected.
- On the contrary, the penalized proportions are limited to low values when r_i is positive, which leads to intermediate materials having poor electromagnetic properties. The objective function thus rather takes a concave shape.

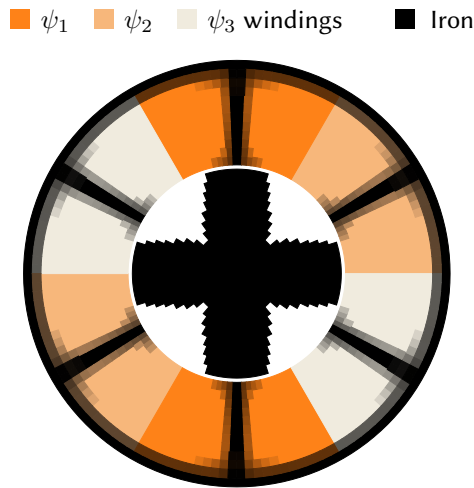


Fig. 5.23. SRM stator design with the geometric mapping and $r_i = -10$; $\tau = 91.75$ Nm.

In order to achieve the objective function evolution shown in Fig. 5.18, the parameter r_i is initially set to a negative value. After a given number of iterations required for the initial convergence, its value is progressively increased until all the composite materials disappear.

5.2.3 Application

Fig. 5.23 shows the optimization result obtained with the geometric mapping and $r_i = -10$. Thanks to the promotion of intermediate materials, the objective function takes a convex shape and the same solution is obtained with any initial conditions.

In order to remove intermediate materials from this topology, the value of r_i is progressively increased during the optimization. Fig. 5.24(b) provides the r_i evolution: 10 iterations are first realized to converge to the topology shown in Fig. 5.23 and r_i is next linearly increased to 0.9 during 30 iterations. Fig. 5.24(a) shows that the objective function initially decreases when r_i is maintained at $r_i = -10$. Then it slightly increases because of the penalization of intermediate materials. The whole optimization requires a computation time of about 5 minutes and 722 finite element evaluations.

The final optimization result produces a torque of 84.20 Nm and is shown in Fig. 5.25. As previously explained, topologies are assessed in this thesis through a comparison with the solutions of a method specifically designed to converge towards

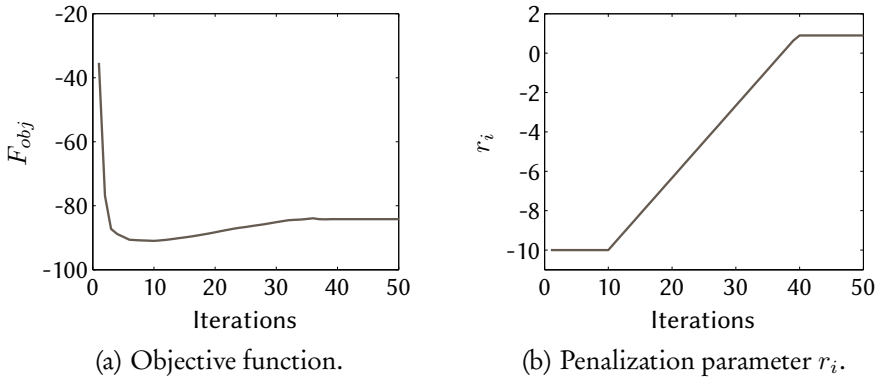


Fig. 5.24. SRM stator optimization with the convexity mapping.

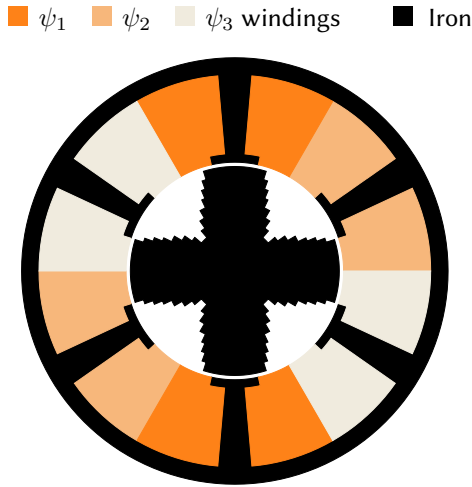


Fig. 5.25. Final design of the SRM stator optimization; $\tau = 84.20$ Nm.

local minimizers containing only solid materials. This method and its results are presented in appendix A. The best local minimizer for the SRM stator design found in appendix A actually corresponds to the same topology as the solution found here. Therefore, we could most probably assume that this topology is in fact the global minimizer of this optimization problem.

The penalization of intermediate materials must obviously be performed sufficiently slowly. Indeed, the steepest-descent algorithm requires a given number of iterations to converge while intermediate materials are penalized. Fig. 5.26 shows for instance two topologies obtained with fast penalizations. In order to produce the topology shown in Fig. 5.25, the penalization must at least span on 20 iter-

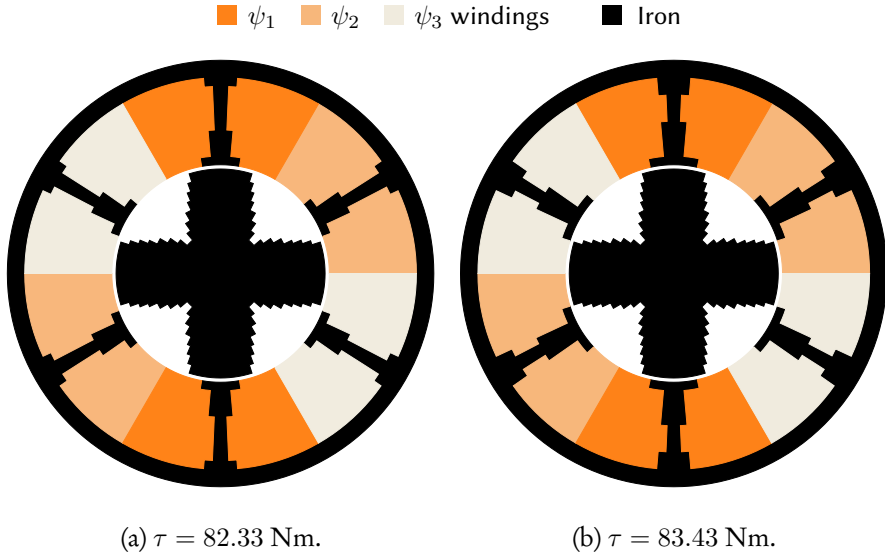


Fig. 5.26. SRM stator designs obtained with too fast penalizations: (a) 5 iterations, (b) 15 iterations.

ations. The determination of the required number of iterations may a priori be difficult. However, it can usually be quickly found by trial and error: the penalization is sufficiently slow if the produced solution remains unchanged for even slower penalizations.

5.2.4 Limitations

The method based on the convexity mapping is unfortunately not effective for all iron-winding optimization problems. For instance, it fails to optimize the study case described in section 4.3 that is related to the stator design of a permanent magnet synchronous motor (PMSM). The initial convergence towards a unique solution cannot be ensured, even with large negative values of r_i . Fig. 5.27 shows two different topologies obtained both with $r_i = -10$ but with different initial conditions.

The ineffectiveness of the convexity mapping method is in this case explained by the fact that local minimizers are not always due to poor electromagnetic properties of intermediate materials. The method here actually attempts to create an anisotropic structure whose permeability is high in the radial direction and low in the tangential one. This structure aims to direct the magnetic fields produced by the permanent magnets towards the outer part of the stator. The method results in a lot of thin teeth whose positions differ according to the initial conditions. It is then stuck in a local minimizer, since reducing the iron proportion in a tooth decreases

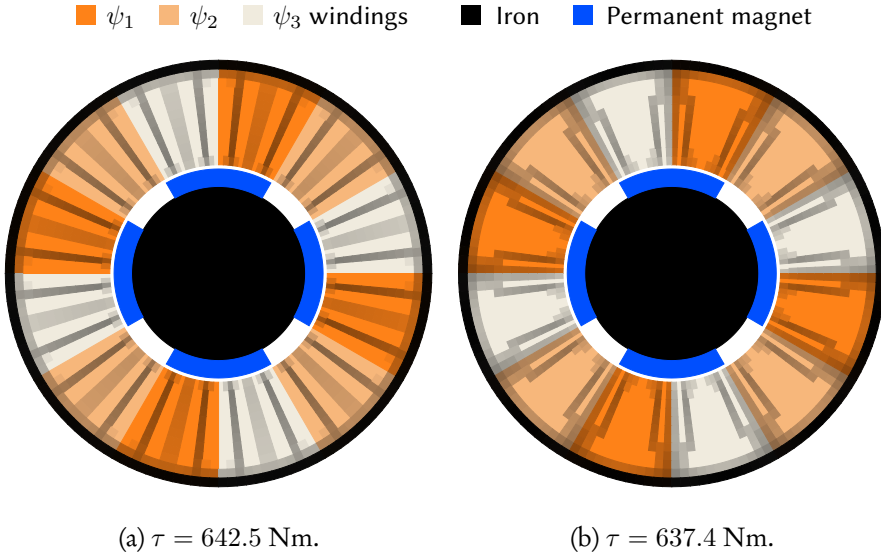


Fig. 5.27. Different topologies obtained for the PMSM stator design with the geometric mapping and $r_i = -10$.

the permeability in the radial direction, while increasing its proportion between two teeth raises the permeability in the tangential direction. This kind of convexity issue actually requires the introduction of anisotropic materials and is thus discussed in chapter 6.

Applying penalization by increasing r_i to 0.9 leads to the solutions shown in Fig. 5.28, which are characterized by thin iron teeth and a large torque difference.

5.3 Mass constraint

In the previous section, the quantity of materials that composes the obtained topologies is only limited by the size of the design space. Therefore, the method always completely fills the latter in order to maximize the motor torque. This kind of optimization is referred to as a *volume-constrained optimization* since defining the stator design space leads to constraining the outer diameter of the motor. This section rather focuses on *mass-constrained optimizations*, in which the material quantity is limited by a constraint on the device mass¹.

¹In the literature, the constraint limiting the quantity of material is often called a *volume fraction constraint*, specifying for instance that 50 % of the design space can be filled with materials. This constraint should not be confused with what is here referred to as *volume-constrained optimizations*, where such constraint is definitely not applied.

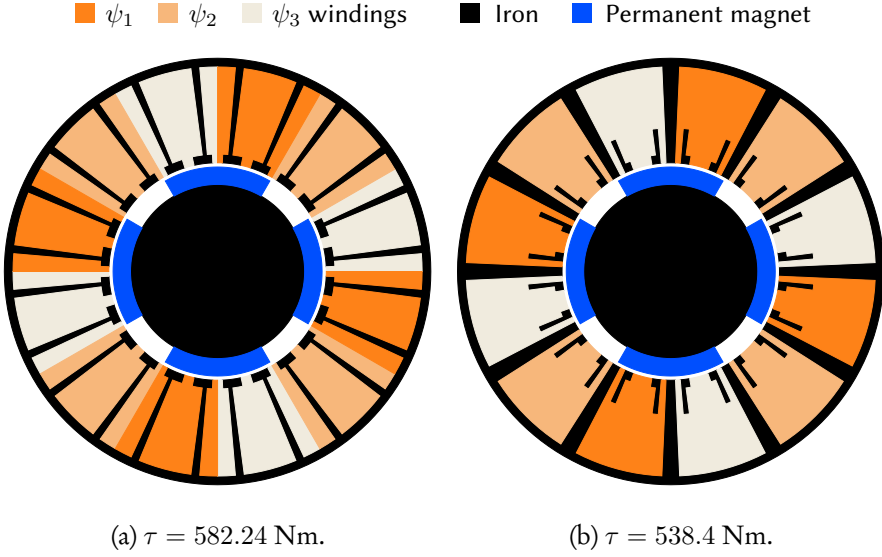


Fig. 5.28. PMSM stator design: penalization of intermediate materials in the topologies obtained with the convexity mapping.

In section 5.1, a mass constraint has already been introduced without any difficulty for the design of magnetic thrusts. However, this optimization problem only involves windings or permanent magnets for which the relations between the material proportions and the electromagnetic properties are linear. Because of the sensitivity mapping, adding a constraint on the iron mass is not so trivial and may raise convergence issues.

The main challenge of mass-constrained optimizations is to ensure the initial convergence towards a unique solution for any initial conditions. With this purpose, this section introduces an iron microstructure which is used as a support to compute the cell permeabilities and the topology mass from the iron proportions. Then, the handling of intermediate materials is addressed: they can either be interpreted in some cases or be removed using penalization.

5.3.1 Convexity issues

In order to highlight these issues, the SRM stator optimization problem is considered with the following mass constraint:

$$\sum_e V_e (D_i \rho_{i,e} + D_w \rho_{w,e}) \leq m_s, \quad (5.31)$$

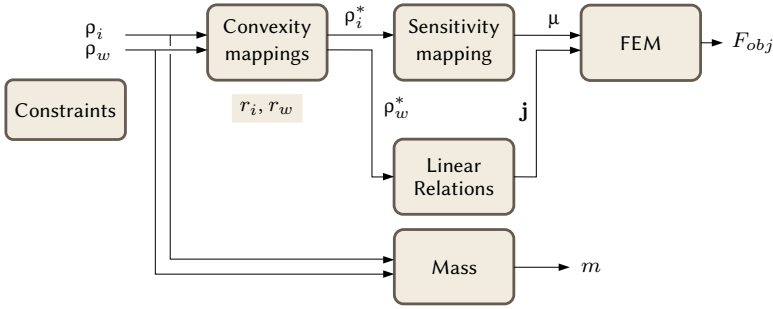


Fig. 5.29. Block diagram for iron-winding optimizations: introduction of the mass computation.

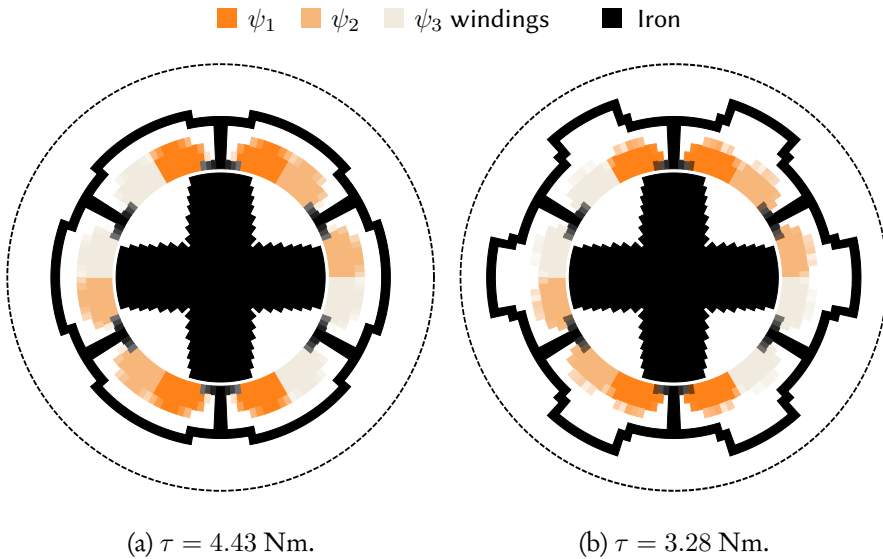


Fig. 5.30. Mass-constrained SRM stator design with no convexity mapping.

with D_i and D_w the densities of iron and windings and V_e the volume of cell e . The maximum stator mass m_s is set to 60 kg. Fig. 5.29 shows the block diagram including the mass computation.

Without any penalization or promotion of intermediate materials, the optimization converges to local minimizers such as those represented in Fig. 5.30, which underlines the concavity of the objective function. The dashed circle represents the outer limit of the design space. Large air holes are observed in the stator slots: the concave shape of the objective function indeed prevents the iron distribution from evolving and adapting to the winding distribution.

With the parameter r_i set to -10, a great improvement can already be noticed

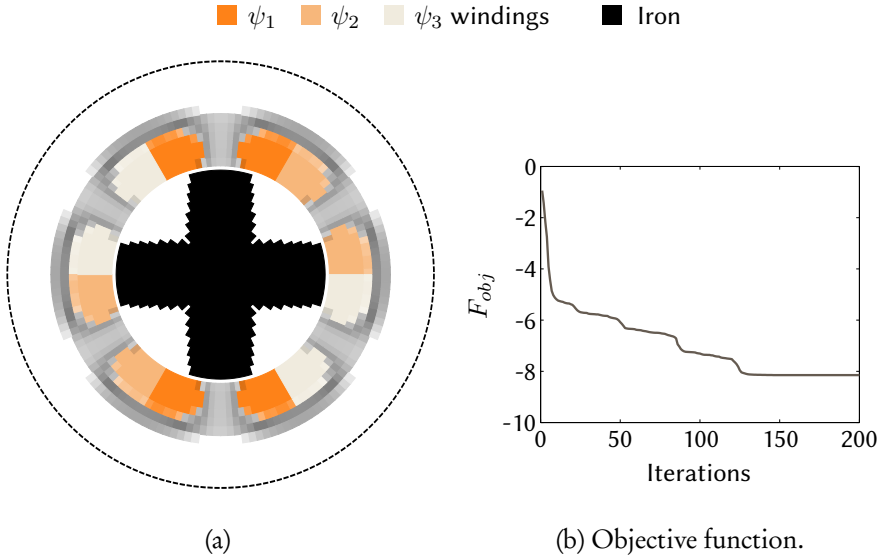


Fig. 5.31. Mass-constrained SRM stator optimization with $r_i = -10$; $\tau = 8.15 \text{ Nm}$.

since the optimization leads to the topology represented in Fig. 5.31(a). The iron distribution now fits with the winding distribution and the topology contains a lot of air-iron composites, which implies a convex objective function between these two materials. A reduction of the convergence speed is however observed because of the mass constraint: the optimization requires about 140 iterations to converge whereas 10 iterations were sufficient for the volume-constrained problem.

With $r_i = -10$, the problem actually still includes local minimizers. Indeed, Fig. 5.32 shows a topology obtained with other initial conditions having wider teeth. The fact that iron-winding composites are almost not produced in the two topologies shows that this lack of convexity is due to a concave behaviour between these two materials.

The value of the parameter r_i can hardly be further decreased, since it greatly affects the convergence. Fig. 5.33 shows for instance an optimization result which has not yet converged after 200 iterations with $r_i = -100$. Using too large value of r_i indeed leads to sensitivity issues: Fig. 5.19 in fact shows that a high sensitivity is expected for low iron proportions when the parameter r_i takes a large negative value.

In addition, decreasing the value of r_i both improves the properties of iron-winding and iron-air composites. Whereas the iron distribution well surrounds the winding distribution with $r_i = -10$, it is completely spread in the topology shown

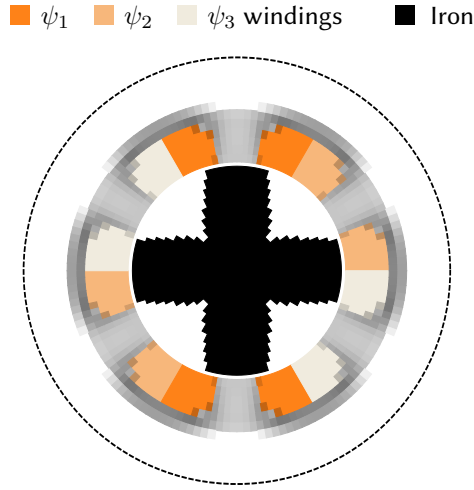


Fig. 5.32. Other topology obtained for the mass-constrained SRM stator optimization with $r_i = -10$; $\tau = 8.40$ Nm.

in Fig. 5.33 since iron-air composites are too valuable.

5.3.2 Iron microstructure

Mass-constrained optimizations can be realized in a significantly more efficient way by considering an *iron microstructure* in each cell. Its goal is to provide a realistic basis for defining the relations between, on one hand, the iron proportions and, on the other hand, the cell permeabilities and the topology mass. Even if a microstructure is here introduced, the cell permeabilities are still assumed to be isotropic, methods based on anisotropic materials being indeed covered in chapter 6.

The need for realistic properties of intermediate materials has already been pointed out by Dyck and Lowther (1996), who established the intermediate material rules quoted in chapter 2. These rules state that the properties of an intermediate material must be derived from a corresponding real composite material, whose pattern is optimally defined to achieve the highest performances.

In order to reach the highest permeability for the lowest mass, iron should be distributed according to the microstructure represented in Fig. 5.34. The latter is actually an iron sheet aligned with the magnetic field and whose thickness is given by t_e , the cell being completely filled with iron for $t_e = 1$. It leads to a permeability

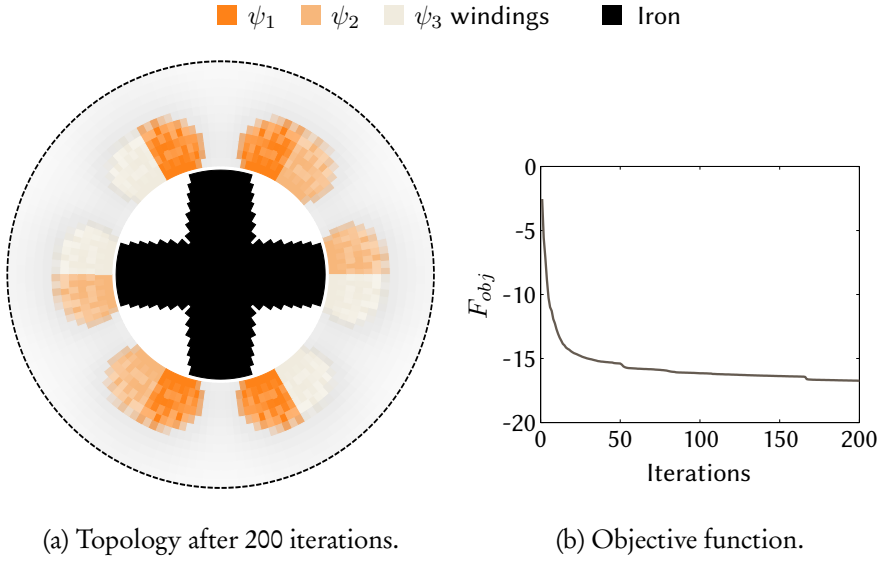


Fig. 5.33. Mass-constrained SRM stator optimization with $r_i = -100$.

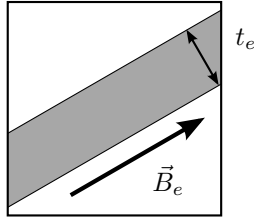


Fig. 5.34. Iron microstructure aligned with the magnetic field.

and a mass that are both linearly proportional to t_e :

$$\mu_e = \mu_0(1 + (\mu_r - 1)t_e), \quad (5.32)$$

$$m_{i,e} = V_e D_i t_e, \quad (5.33)$$

with μ_e and $m_{i,e}$ the equivalent permeability and mass of the microstructure in cell e and V_e the cell volume.

In the previous optimizations, the iron mass of a cell was linearly deduced from its iron proportion while its permeability was computed through the geometric mapping. Compared with the iron microstructure relations, the permeability was thus penalized since the geometric mapping leads to lower permeability values than a linear relation, as shows Fig. 5.10. From this observation, the topologies obtained with different values of the parameter r_i can be easily explained.

- Without the convexity mapping (Fig. 5.30), the penalization, due to the geometric mapping, of the cell permeabilities in comparison with their mass results in topologies in which almost all the iron cells are solid. Intermediate iron materials indeed have poor properties, which leads to concave objective function shapes between iron and air or iron and windings. The optimization creates thus solid iron circuits and the iron distribution is then stuck and cannot evolve to surround the winding distribution.
- With $r_i = -10$ (Fig. 5.31), the convexity mapping actually compensates more or less the effect of the geometric mapping. Since intermediate iron cells have then quite realistic properties, the cell evolution between iron and air becomes possible which results in a suitable distribution of iron around windings.
- If iron composites are too much promoted (Fig. 5.33), the method spreads the iron distribution on the whole design space, since it can thereby achieve a higher permeability with a lower mass.

These behaviours were already partly described by Dyck and Lowther (1996) when they established the intermediate material rules. They indeed foresaw the spreading of intermediate materials in case their properties rise above those of the corresponding composites. But their reasoning only focuses on the influence of the optimization problem formulation on the optimal design, and misses thus the question of local minimizers which may prevent the method from finding this optimal design. In the case of poor intermediate properties, they indeed only predict the creation of composite structures through the distribution of different solid materials in adjacent cells, in case the optimization problem in fact requires composite materials. This is not the situation that occurs in Fig. 5.30, where the method ineffectiveness is due to the presence of local minimizers. However, the Dyck and Lowther's intermediate material rules also applies to tackle convexity issues: even if they didn't raise this question, they provided its suitable resolution.

In order to include the iron microstructure in the optimization method, a rewriting of the sensitivity mappings, of the mass constraint and of the cell composition constraint is required. Fig. 5.35 shows the resulting block diagram.

Sensitivity mappings

As explained in section 5.2, the sensitivity mapping intends to define a specific relation between the design variable p_i and the permeability in order to ensure a smooth sensitivity of the objective function and improve thereby the convergence speed of the steepest-descent algorithm. This relation must thus be kept unchanged with the iron microstructure proposed here.

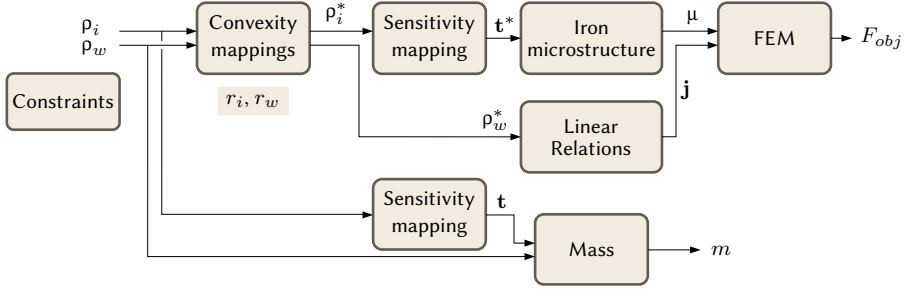


Fig. 5.35. Block diagram for iron-winding optimizations: introduction of the iron microstructure.

With this purpose, the sensitivity mappings, instead of providing the permeabilities of the cells, become a change of variables between ρ_i and t or ρ_i^* and t^* , as illustrated in Fig. 5.35:

$$t_e = f_{sens}(\rho_{i,e}), \quad (5.34)$$

while the cell permeabilities are next derived from (5.32).

The permeability, reluctivity and geometric mappings are thus respectively rewritten as follows:

$$t_e = \rho_{i,e}, \quad (5.35)$$

$$t_e = \frac{\nu_r \rho_{i,e}}{1 + (\nu_r - 1) \rho_{i,e}}, \quad (5.36)$$

$$t_e = \frac{\mu_r^{\rho_{i,e}} - 1}{\mu_r - 1}. \quad (5.37)$$

Replacing these new sensitivity mappings in (5.32) indeed leads to their original expressions introduced in section 5.2. The introduction of the iron microstructure has thus no effect on the relations between the material proportions and the cell magnetic properties. It rather establishes physical mass relations, thanks to the introduction of the intermediate variable t from which the device mass is computed.

Constraints

With the introduction of the iron microstructure, the mass is computed based on the variable t . The mass constraint is indeed given by

$$\sum_e V_e (D_i t_e + D_w \rho_{w,e}) \leq m_s. \quad (5.38)$$

Since constraints must be expressed on the design variables, it is written

$$\sum_e V_e (D_i f_{sens}(\rho_{i,e}) + D_w \rho_{w,e}) \leq m_s. \quad (5.39)$$

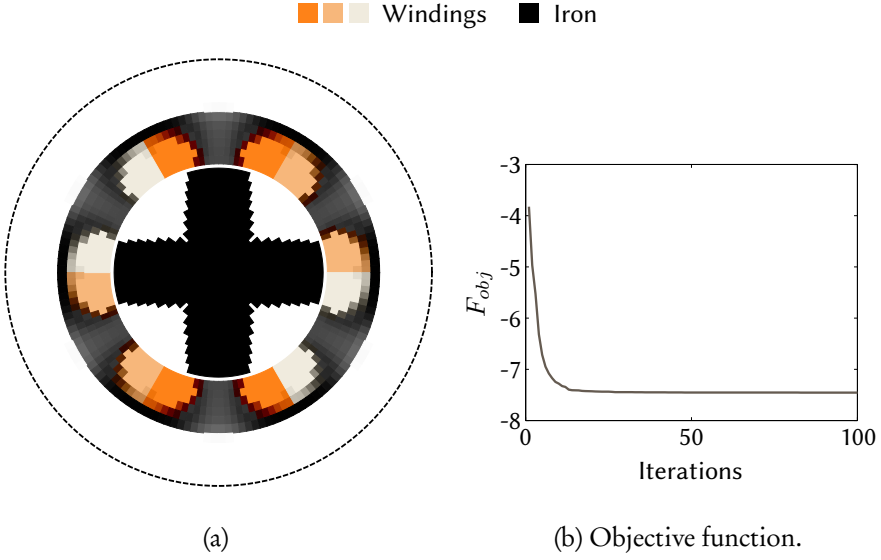


Fig. 5.36. Mass-constrained SRM stator optimization with the iron microstructure; $\tau = 7.46$ Nm.

The cell composition constraint is also expressed taking the microstructure into account:

$$t_e + \rho_{w,e} \leq 1, \quad (5.40)$$

which leads to

$$f_{sens}(\rho_{i,e}) + \rho_{w,e} \leq 1. \quad (5.41)$$

The optimization constraints take thus non linear expressions, which slightly complicates the orthogonal projection algorithm, as explained in section 3.2.

5.3.3 Application

With the iron microstructure, the optimization converges without any convexity mapping to the topology shown in Fig. 5.36. Thanks to the realistic relations between the cell permeabilities and masses, iron is properly distributed. A lack of convexity is however still observed, since the topology shown in Fig 5.37 could be obtained with other initial conditions. The introduction of the iron microstructure nevertheless led to interesting improvements.

- The initial convergence is ensured without any convexity mapping. The search for a proper value of r_i which compensates the effect of the sensitivity mapping can thus be avoided.

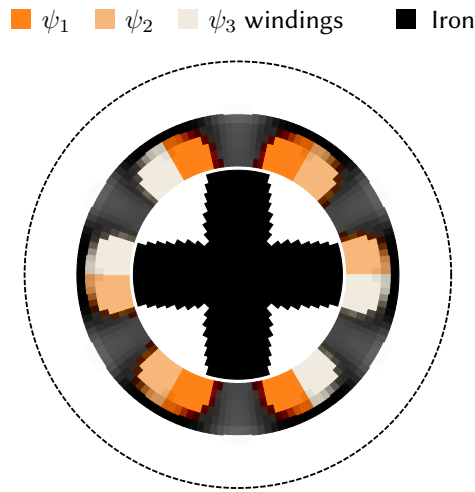


Fig. 5.37. Other topology obtained for the mass-constrained SRM stator design with the iron microstructure; $\tau = 7.45$ Nm.

- The convergence speed is greatly improved: the topology is found after less than 30 iterations, which requires about 5 minutes.
- The differences observed between the topologies produced by the method are very limited. The torques of the two designs shown in Fig. 5.36 and Fig. 5.37 indeed only differ of about 1 %.

The volume-constrained SRM stator optimization could also be realized with the iron microstructure, which removes the necessity of initially promoting iron composites with the convexity mapping. However, except if intermediate materials must be interpreted, the optimization is easier performed without the microstructure since the latter introduces non linear cell composition constraints that complicate the orthogonal projection of the design variables.

This optimization problem is complex enough to highlight the convergence speed difference between the three sensitivity mappings. Thanks to the iron microstructure, the sensitivity mapping has no more effect on the optimization problem since it reduces to a simple change of variables. The objective function evolutions can thus effectively be compared. Fig. 5.38 underlines the efficiency of the geometric mapping: the convergence is about 5 times faster in comparison with the permeability mapping while the optimization has not yet converged after 100 iterations with the reluctivity mapping.

In the same way as volume-constrained optimizations, mass-constrained optimizations are performed in two steps. First, the initial convergence towards a unique

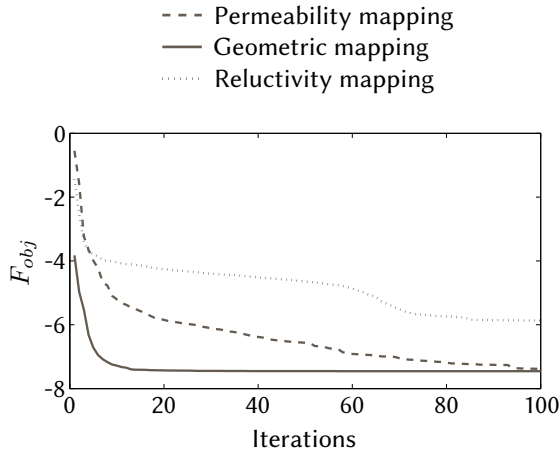


Fig. 5.38. Convergence speed comparison between the three sensitivity mappings.

solution is ensured by giving valuable properties to intermediate materials. Second, the latter must be eliminated from the obtained design.

The first step was achieved by the introduction of the iron microstructure, even if some small differences can still be observed between the produced topologies. Regarding the second step, two possibilities can here be considered: whereas intermediate materials were always removed by *penalization* in volume-constrained optimizations, they can sometimes also be *interpreted* in mass-constrained optimizations and replaced by equivalent solid structures.

Intermediate material penalization

The common approach for removing intermediate materials is to progressively penalize their electromagnetic properties thanks to the convexity mapping. Starting with the result shown in Fig. 5.36, this penalization provides the topology represented in Fig. 5.39. The evolutions of the objective function and of parameter r_i are plotted in Fig. 5.40.

Whereas penalization can easily be applied to volume-constrained optimizations, it may sometimes lead to poor results in mass-constrained problems. Here, it actually forces cells composed of intermediate iron proportions to become solid. In order to maintain a ferromagnetic circuit around the windings, the global iron proportion of the topology must thus be increased. Because of the mass constraint, the global winding proportion must in compensation be decreased. However, since the optimization problem becomes progressively concave, the iron distribution cannot evolve and fit to the decreasing winding distribution. The penalization creates thus

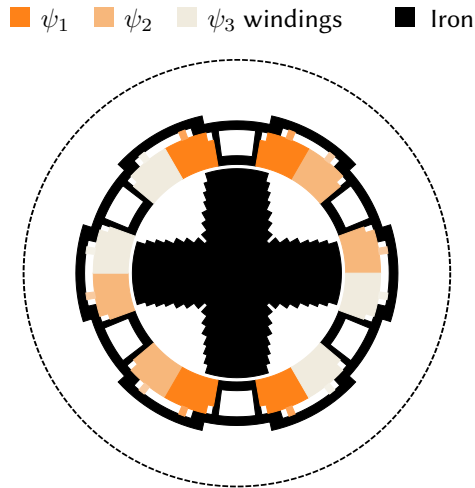


Fig. 5.39. Final design of the mass-constrained SRM stator optimization obtained by penalization; $\tau = 5.38$ Nm.

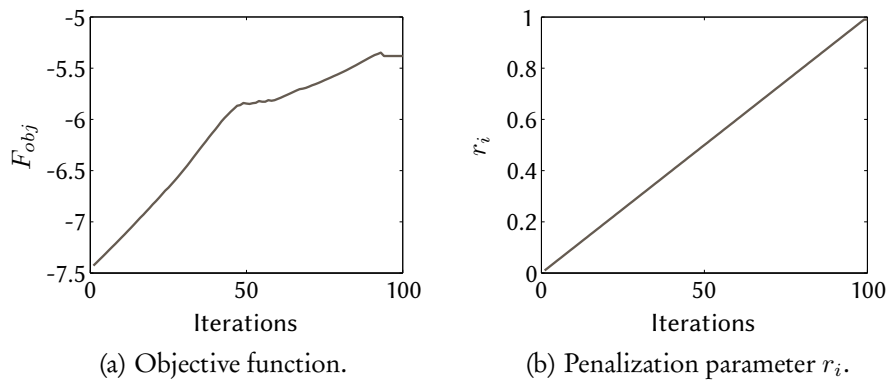


Fig. 5.40. Penalisation of intermediate materials for the mass-constrained SRM stator optimization.

air holes in the stator slots. This behaviour is here limited but can still be observed in the topology shown in Fig. 5.39.

Compared to the best local minimizer found in appendix A which produces a torque of 6.01 Nm, the topology found by penalization is about 10 % below. This difference is mainly explained by the air holes in the stator slots.

The penalization is performed in 28 minutes, which must be added to the computation time required for the initial topology, leading to a total time of 33 minutes and 2238 finite element evaluations.

Intermediate material interpretation

Since a topology may be deteriorated by the penalization because of the concavity of the objective function imposed during this process, it is generally more efficient to attempt to interpret intermediate materials. Indeed, thanks to the definition of the iron microstructure, their properties correspond to those of real composite materials.

The interpretation of intermediate materials may nevertheless be impossible in some cases. Indeed, the iron microstructures give to the cells the equivalent permeabilities and masses of sheets of iron aligned with the magnetic field. Their interpretation is thus only valid if the magnetic field always keeps the same direction in the concerned cells, since the iron sheet orientations would otherwise be undetermined. Two situations may lead to different magnetic field directions in the same cell.

- The objective function evaluation often requires several field computations: the maximization of the SRM mean torque for instance involves the magnetic energy in two rotor positions.
- Optimization problems usually include symmetry: the same field direction must thus also be observed in symmetric cells.

Fig. 5.41 highlights the iron cells for which interpretation is possible according to the two previous conditions. Unfortunately, only a few cells of this topology are concerned since the stator teeth have different magnetic field directions.

In order to clearly describe the interpretation process, the latter is illustrated along the axis shown in Fig. 5.41(b) that is perpendicular to the magnetic field direction. Fig. 5.42(a) shows the cell compositions on this axis and, in the upper strip, the corresponding interpreted material distribution¹. The latter is here almost equivalent to the original distribution.

¹Contrary to the topology representation in Fig. 5.41, the iron proportion is here given according

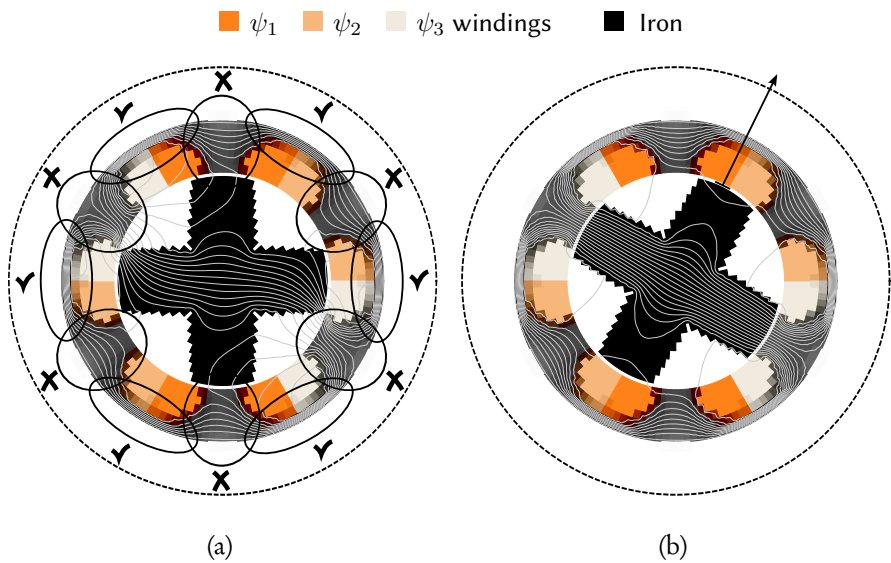


Fig. 5.41. Mass-constrained SRM stator design: highlight of the cells in which the interpretation of intermediate materials is possible.

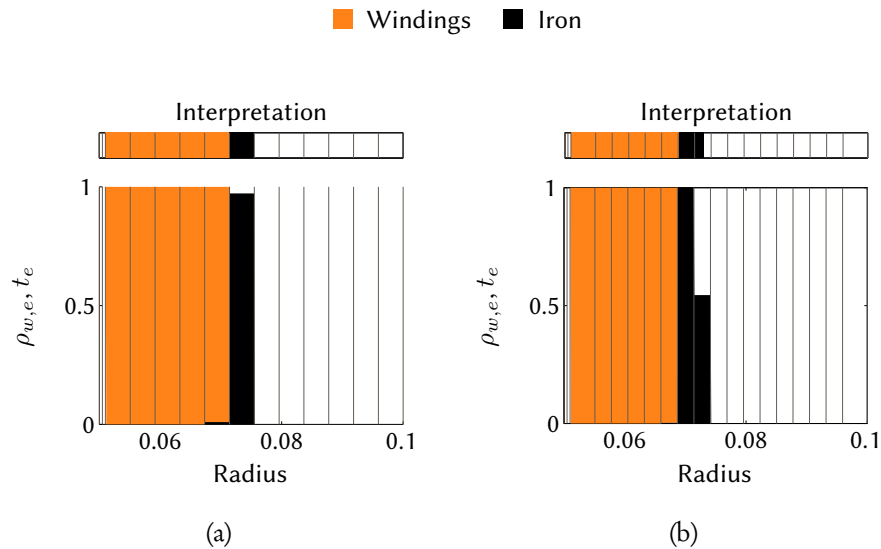


Fig. 5.42. Interpretation of composite materials for two mesh resolutions.

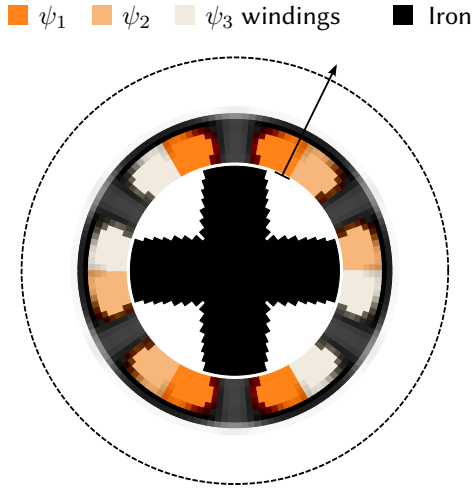


Fig. 5.43. Mass-constrained SRM stator design on a finer mesh.

Along the considered axis, intermediate materials are actually due to the cell representation of the solution: the desired iron circuit thickness cannot be achieved with the cell resolution and intermediate materials thus appear. In order to highlight this phenomenon and besides obtain a more interesting topology to illustrate the intermediate material interpretation, the SRM stator optimization was performed on a slightly finer mesh. The result is shown in Fig. 5.43 whereas the cell compositions along the same axis are plotted in Fig. 5.42(b). In this case, intermediate materials must really be taken into account to derive the interpreted material distribution, each cell is in fact filled with a t_e fraction of iron and a $\rho_{w,e}$ fraction of windings.

The presence of intermediate materials due to the cell-representation resolution is confirmed by the fact that the interpreted iron distributions exactly have the same thickness for both meshes, which shows that the method attempts to converge to the same iron circuit. In the first case, the cell sizes actually correspond to the desired thickness whereas intermediate materials are required in the second case. These results emphasize the interest for interpreting intermediate materials when it is possible.

to the variable t corresponding to the microstructure thickness instead of ρ_i , which is more consistent to derive the interpreted distribution. That is the reason why some intermediate materials that are well visible in Fig. 5.41 become negligible in Fig. 5.42(a).

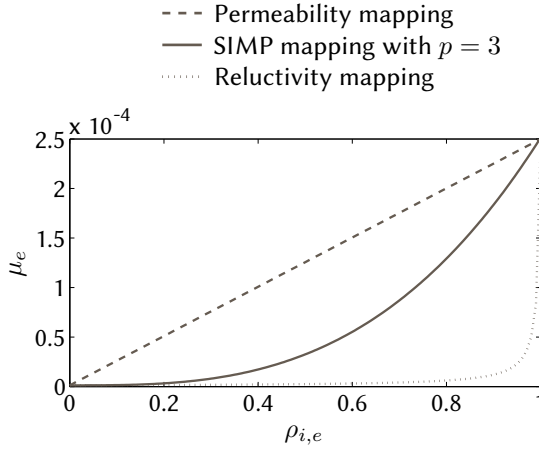


Fig. 5.44. Illustration of the SIMP and reluctivity mappings.

5.3.4 Comparison with the density and reluctivity-based methods

The considerations developed in this section make possible a discussion about the results of the density and reluctivity-based methods proposed in the literature and briefly described in chapter 2. The density method is based on a polynomial mapping including a penalty factor (with usually $2 \leq p \leq 4$) whereas the reluctivity-based method relies on the reluctivity mapping. These two mappings are represented in Fig. 5.44.

The two methods were promoted for their convergence towards solid materials. This property can now be easily explained: they both penalize the permeability of intermediate materials in comparison with the linear relation, whereas the material quantity constraint is linearly expressed on the material proportions. In addition, the fact that the reluctivity mapping does not require any penalization factor (Choi and Yoo, 2009) is explained by its inherent strong penalization compared with the density mapping.

However, those methods fail in applying progressively the penalization and their results thus depend on the initial material distribution. They besides do not include the geometric mapping, which leads to a slower convergence of the steepest-descent algorithm.

5.4 Saturation

In the previous sections, iron was assumed to have linear properties. Since this hypothesis is in fact rarely verified, this section intends to describe how saturation can be introduced in the optimization.

5.4.1 Iron composite permeability

Taking saturation into account raises a question about how the permeability of intermediate iron cells must be computed. Indeed, whereas the B-H curve of solid iron is a known intrinsic property, the determination of the permeability of an intermediate iron cell requires some attention.

In order to maintain the correspondence between intermediate and composite materials, and thus keep the possibility to interpret the former, the iron microstructure should again support the permeability computation. Since the microstructure is aligned with the magnetic field, the H field is constant inside the cell. Indeed, the parallel value of H remains unchanged at boundaries between different materials. The saturated relative permeability of iron is thus computed from the H field of the cell:

$$\mu_{r,e} = f_{\mu_r} \left(\sqrt{h_{x,e}^2 + h_{y,e}^2} \right), \quad (5.42)$$

which leads to the following cell permeability:

$$\mu_e = \mu_0 \left(1 + \left(f_{\mu_r} \left(\sqrt{h_{x,e}^2 + h_{y,e}^2} \right) - 1 \right) t_e \right). \quad (5.43)$$

The B-H curve of iron used in this thesis is given in chapter 4.

Fig. 5.45 shows the block digram in which the H fields in each cell, written $\mathbf{h}_x$ and $\mathbf{h}_y$, are provided by the finite element model to the iron microstructure block. Deriving the saturated permeability of intermediate iron materials using a microstructure was originally proposed by Dyck and Lowther (1997).

5.4.2 Sensitivity mapping

The sensitivity mappings must be slightly modified when introducing saturation. Indeed, the geometric (5.37) and reluctivity (5.36) mappings involve the relative permeability of iron, and become thus also functions of the H field. Introducing the saturated value of the permeability in a sensitivity mapping is actually complex and useless, since the latter only intends to improve the convergence speed by a change of variables. The permeability can thus be replaced in the sensitivity mapping expression by an arbitrary value, written $\hat{\mu}_r$, which should be chosen between 1 and

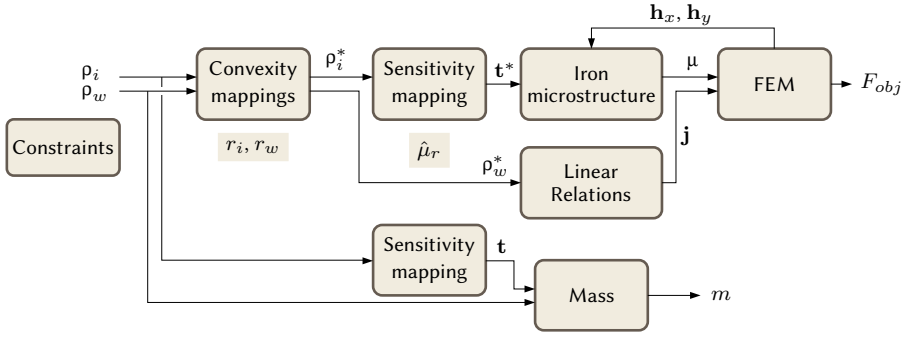


Fig. 5.45. Block diagram for iron-winding optimizations: introduction of saturation.

the maximum iron permeability. Its value does not affect the optimization results, it only influences the sensitivity mapping expression and thus the convergence speed of the steepest-descent algorithm. The geometric and reluctivity mappings become respectively

$$t_e = \frac{\hat{\mu}_r^{\rho_{i,e}} - 1}{\hat{\mu}_r - 1}, \quad (5.44)$$

$$t_e = \frac{\hat{\nu}_r \rho_{i,e}}{1 + (\hat{\nu}_r - 1) \rho_{i,e}}. \quad (5.45)$$

The parameter $\hat{\mu}_r$ is also included in the block diagram shown in Fig. 5.45 to underline the fact that the sensitivity mapping is computed with an arbitrary value of the permeability.

5.4.3 Application

The effect of saturation is here illustrated on the same study case as in the previous section: the mass-constrained stator design of a SRM. Fig. 5.46 shows the obtained topology with the dashed line representing the outer limit of the stator design space and Fig. 5.47 the evolutions of the objective function and of the penalization parameter r_i . The arbitrary permeability value $\hat{\mu}_r$ was set to 200.

Saturation has a double negative effect on the computation time. The finite element computations are more time consuming since an iterative process is required to compute the cell permeabilities. In addition, a slower convergence of the steepest-descent algorithm is observed, which leads to an increase of the required number of iterations. The computation time here raises to 2 hours and 35 minutes while the number of finite element evaluations remains limited to 2270.

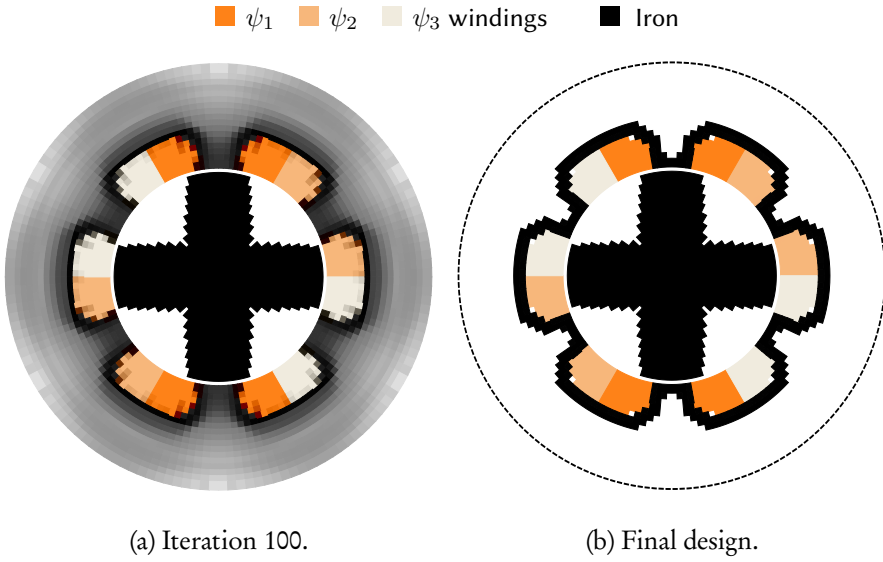


Fig. 5.46. Mass-constrained SRM stator design with saturation; $\tau = 11.27$ Nm.

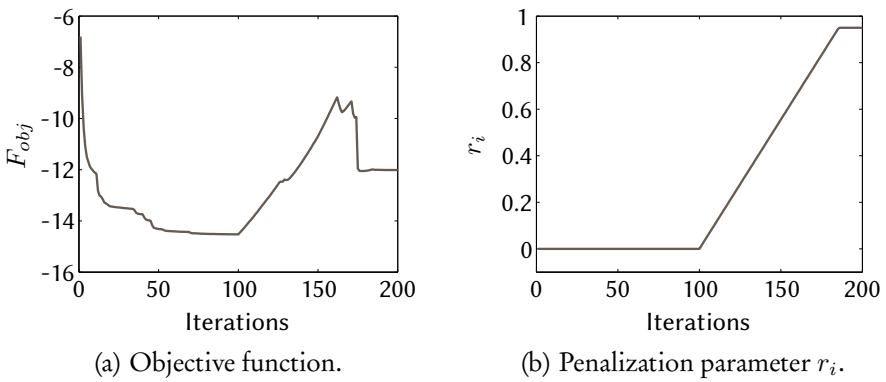


Fig. 5.47. Mass-constrained SRM stator optimization with saturation.

The best solution found in appendix A amongst all the local minimizers produces a torque of 10.90 Nm, which is 3 % below the design obtained in this section. This comparison highlights the efficiency of introducing the iron microstructure, especially when taking saturation into account.

5.5 Iron and permanent magnets

The definition of realistic relations between the iron mass and permeability, thanks to the iron microstructure, led to a significant improvement of the initial convergence for iron-winding optimizations. Unfortunately, similar results cannot be achieved for problems related to iron and permanent magnets.

The study case considered in this section concerns the design of the rotor of a PMSM, described in section 4.3. This optimization problem only involves one magnetization direction. A unique design variable ρ_m is thus sufficient to represent the permanent magnet proportions whereas the cell remanent fields are obtained by the following linear functions:

$$b_{x,e}^r = \rho_{m,e} \cos(\theta_m) B_m^r, \quad (5.46)$$

$$b_{y,e}^r = \rho_{m,e} \sin(\theta_m) B_m^r, \quad (5.47)$$

with B_m^r the permanent magnet remanent field. The magnetization direction can here be represented by the scalar θ_m since the PMSM is composed of permanent magnets having parallel magnetization directions. The latter are thus identical in all the cells of the design space. Other optimization problems may however involve permanent magnets whose magnetization direction differs in each cell, such as radial or tangential magnets. The parameter θ_m must then simply be replaced by a vector $\theta_{m,e}$.

The iron proportion is still given by ρ_i and the permeability deduced from the iron microstructure, taking saturation into account.

If the rotor mass is not constrained, the PMSM optimization actually results in a rotor completely filled with permanent magnets. In order to produce interesting results, the following mass constraint is thus included:

$$\sum_e V_e (D_i f_{sens}(\rho_{i,e}) + D_m \rho_{m,e}) \leq m_r, \quad (5.48)$$

with D_i and D_m the densities of iron and permanent magnets, V_e the volume of cell e and m_r the maximum rotor mass set to 30 kg.

Without any kind of penalization, the method converges to local minimizers, such as those presented in Fig. 5.48. These results show that the lack of convexity is

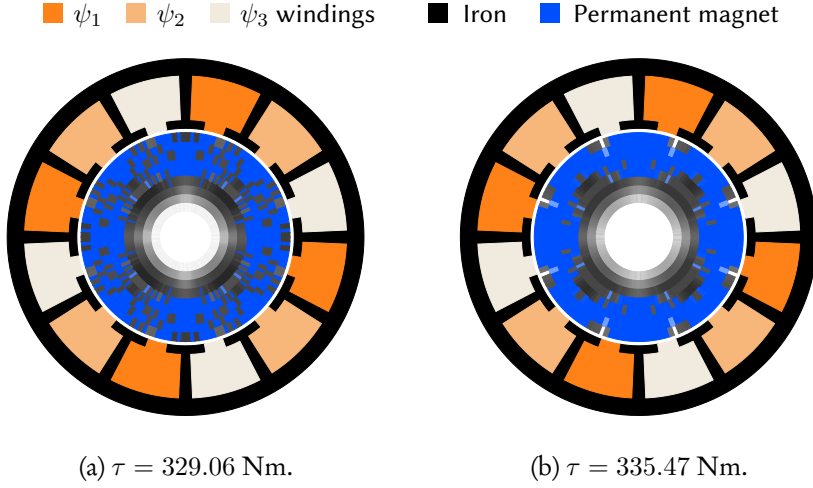


Fig. 5.48. Lack of convexity for the rotor design of the PMSM with no convexity mapping.

due to iron-magnet composites having poor properties, since none of these composites is produced. In order to promote them, a new *convexity mapping* is required.

5.5.1 Convexity mapping

The convexity mapping described in section 5.2.2 applies to a unique material in order to promote/penalize its properties. In this section, a new mapping focusing specifically on mixes of two materials is introduced. This mapping is given by:

$$\rho_{\alpha,e}^* = \frac{1 - r_{\alpha\beta}}{1 - r_{\alpha\beta} \frac{\rho_{\alpha,e}}{\rho_{\alpha,e} + \rho_{\beta,e}}} \rho_{\alpha,e}, \quad (5.49)$$

with the subscripts α and β referring respectively to materials α and β and $r_{\alpha\beta}$ the penalization factor on material α related to α - β composites.

The mapping only affects intermediate materials since it reduces to $\rho_{\alpha,e}^* = \rho_{\alpha,e}$ when $\rho_{\beta,e} = 0$. Its effect is illustrated in Fig. 5.49, in comparison with the former convexity mapping (5.30). The figure shows curves of the proportions $\rho_{\alpha,e}^*$ and $\rho_{\beta,e}$ for a cell filled with 50 % of material and evolving between materials α and β . It actually highlights the effect of both mappings on the line $\rho_{\alpha,e} + \rho_{\beta,e} = 0.5$ and emphasizes the fact that the new convexity mapping only promotes/penalizes mixes of materials whereas the former mapping affects intermediate proportions of $\rho_{\alpha,e}$, whatever the value of $\rho_{\beta,e}$.

The block diagram shown in Fig. 5.50 includes the new penalization parameters r_{im} and r_{mi} for iron-magnet optimizations. The two convexity mappings are

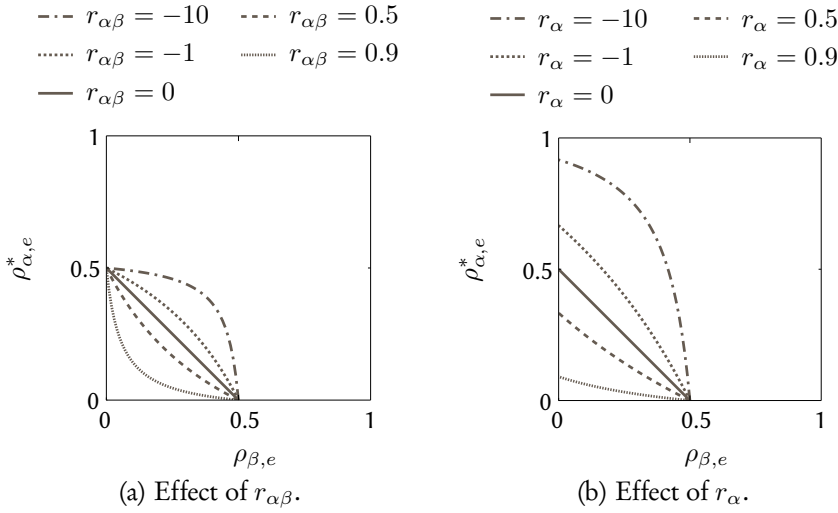


Fig. 5.49. Comparison of the two convexity mappings.

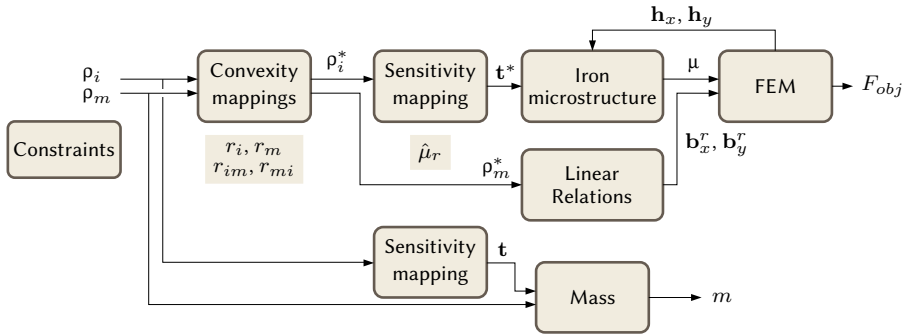


Fig. 5.50. Block diagram for iron-magnet optimizations.

implemented simultaneously with the following expressions:

$$\rho_{i,e}^* = \frac{1 - r_{im}}{1 - r_{im} \frac{\rho_{i,e}}{\rho_{i,e} + \rho_{m,e}}} \frac{1 - r_i}{1 - r_i \rho_{i,e}} \rho_{i,e}, \quad (5.50)$$

$$\rho_{m,e}^* = \frac{1 - r_{mi}}{1 - r_{mi} \frac{\rho_{m,e}}{\rho_{i,e} + \rho_{m,e}}} \frac{1 - r_m}{1 - r_m \rho_{m,e}} \rho_{m,e}. \quad (5.51)$$

5.5.2 Sensitivity mapping

In the previous sections, the study of the sensitivity mapping influence on the convergence speed led to the choice of a geometric mapping. However, the sensitivity mapping also has an impact, that has not yet been pointed out, on the convexity mapping action. Indeed, since it is applied after the latter, it affects the penalization

or promotion of the electromagnetic properties in comparison with the mass, for the same value of the penalization parameters.

Let's take an example to illustrate this effect. In order to obtain an iron micro-structure thickness of $t_e = 0.1$, the iron proportions must be respectively equal to

$$\rho_{i,e,geo} = 0.57, \quad (5.52)$$

$$\rho_{i,e,rel} = 0.96, \quad (5.53)$$

with the geometric and the reluctivity mappings. With a permanent magnet proportion equal to 0.2 and the penalization factor r_{mi} set to -10, the penalized magnet proportions are then respectively

$$\rho_{m,e,geo}^* = 0.61, \quad (5.54)$$

$$\rho_{m,e,rel}^* = 0.81. \quad (5.55)$$

For the same iron mass, the magnet proportions are thus more promoted with the reluctivity mapping.

The optimization problems considered in the previous sections did not require a special attention regarding this sensitivity mapping influence since, on one hand, mass-constrained optimizations are initially performed without any promotion or penalization and, on the other hand, iron-winding composites could be well promoted and penalized in volume-constrained optimizations with the geometric mapping. However, the promotion of iron-magnet composites is here insufficient with a geometric mapping for ensuring an initial convergence towards a unique result. Iron-magnet optimizations actually require the implementation of the reluctivity mapping.

In order to still achieve a fast convergence, it is recommended to switch to the geometric mapping after the penalization process once iron-magnet composites have disappeared. The iron proportions must then be adjusted in order to keep the value of t unchanged:

$$\begin{aligned} \rho_{i,e,geo} &= f_{geo}^{-1}(f_{rel}(\rho_{i,e,rel})), \\ &= \frac{\log\left(1 + \frac{(\hat{\mu}_r - 1)\hat{\nu}_r \rho_{i,e,rel}}{1 + (\hat{\nu}_r - 1)\rho_{i,e,rel}}\right)}{\log(\hat{\mu}_r)}. \end{aligned} \quad (5.56)$$

5.5.3 Application

The convergence to a unique solution for any initial conditions could be verified with the reluctivity mapping and $r_{mi} = -10$. Fig. 5.51(a) highlights a fast initial convergence which shows that the reluctivity mapping is in fact efficient when

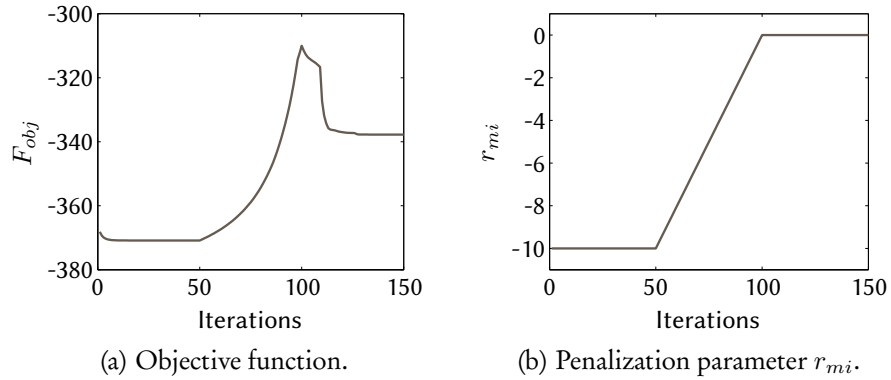


Fig. 5.51. Mass-constrained PMSM rotor optimization.

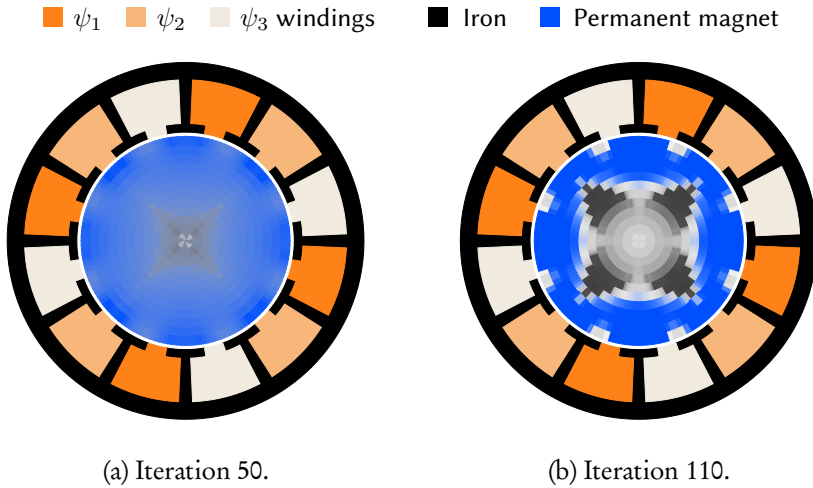


Fig. 5.52. Mass-constrained PMSM rotor design.

iron-magnet composites are promoted. The obtained topology is represented in Fig. 5.52(a) and contains a large fraction of iron-magnet composites. Because of the convexity mapping, these composites cannot be interpreted and must thus be removed by penalization.

After 50 iterations, the value of r_{mi} is progressively increased to 0. Iron-magnet composites totally disappear during this penalization process and the topology shown in Fig. 5.52(b) is produced. The reluctivity mapping is then replaced by a geometric mapping at iteration 110, which leads to a significant improvement of the convergence speed, as shown in Fig. 5.51(a). The obtained design represented in Fig. 5.53 contains iron-air composites which, contrary to iron-magnet composites,

■ ψ_1
■ ψ_2
■ ψ_3 windings
 ■ Iron
 ■ Permanent magnet

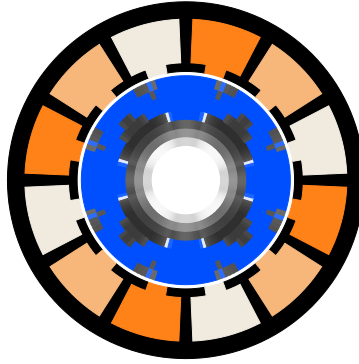


Fig. 5.53. Mass-constrained PMSM rotor design; $\tau = 337.5$ Nm.

can either be removed by penalization or interpretation. The computation time required for this optimization is 2 hours and 40 minutes.

Intermediate material penalization

The iron-air composite penalization is simply performed by increasing the value of r_i , as shown in Fig. 5.54. The obtained design is represented in Fig. 5.55 and produces a torque of 325.91 Nm. Compared to the 323.64 Nm achieved by the best local minimizer found in appendix A, it represents a gain of about 1 %, which highlights the efficiency of the proposed method.

The penalization is performed in 50 minutes, which leads to a total computation time of 3 hours and 30 minutes for the whole optimization and 2070 finite element evaluations.

Intermediate material interpretation

This optimization is actually a interesting example to illustrate the interpretation of composite materials. Indeed, the magnetic field in the iron-air composites is here mainly determined by the permanent magnets and remains thus almost unchanged in the two rotor positions, as shown in Fig. 5.56. Iron-air composites can thus be interpreted as iron sheets aligned with the magnetic field and whose thickness is given by t . The interpreted topology is shown in Fig. 5.57 and is very close to the topologies that can be obtained with finer meshes, as shown in section 8.2.

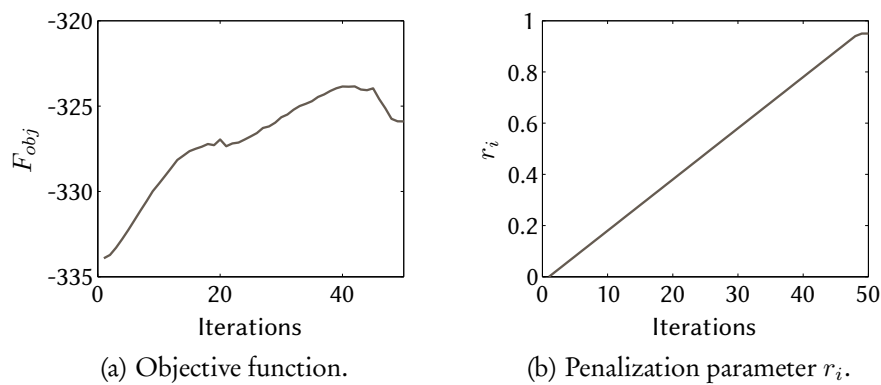


Fig. 5.54. Mass-constrained PMSM rotor optimization: penalization of iron-air composites.

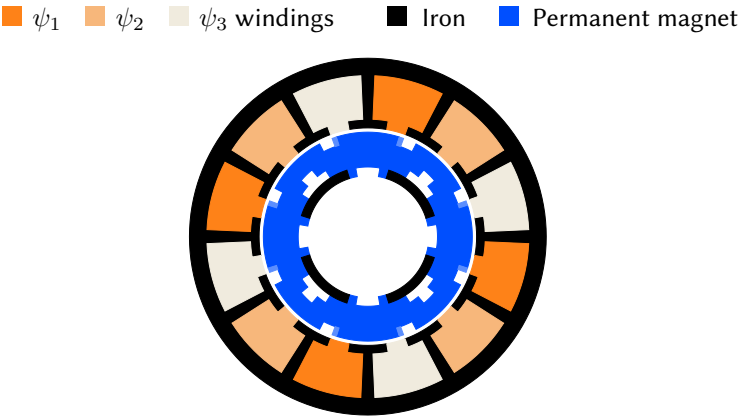


Fig. 5.55. Final design for the mass-constrained PMSM rotor optimization obtained by penalization; $\tau = 325.91$ Nm.

5.6 Iron

This section is dedicated to the design of ferromagnetic parts. This kind of optimization only involves air and iron through a unique design variable per cell, the iron proportion ρ_i , which actually raises new issues regarding the convexity. Indeed, it will be shown that the optimization problem lacks convexity by default and the convexity mappings are unable to improve it, both in volume-constrained and mass-constrained optimizations.

In volume-constrained optimizations, the method naturally converges to solid materials, but the produced solutions differ according to the initial conditions. This

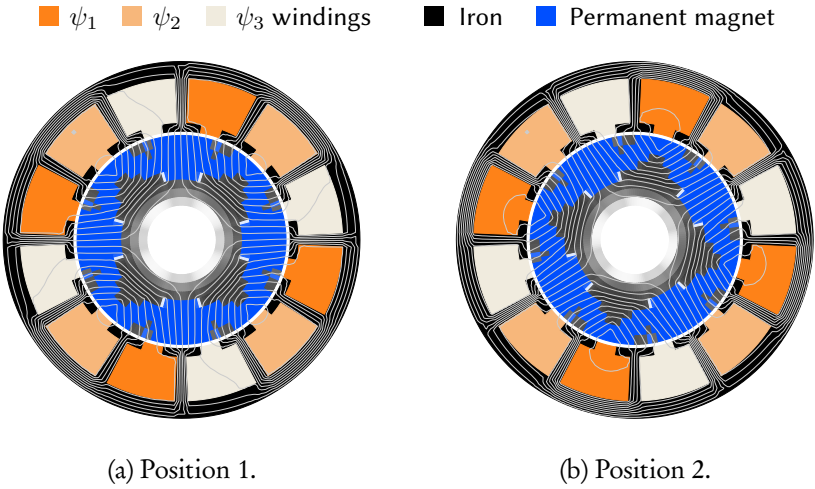
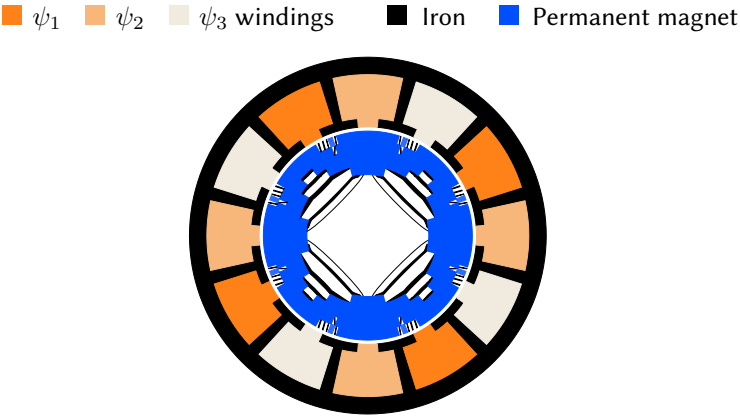


Fig. 5.56. Mass-constrained PMSM rotor design: magnetic field lines.



is the typical behaviour with a concave objective function. However, the convexity mappings are here ineffective for modifying the objective function shape. Indeed, they actually affect the quantity of material that an intermediate cell can contain: while the cell composition constraints define a triangular domain on the material proportions, the feasible domain on the penalized proportions can be increased or decreased as explained in section 5.2.2. They become thus useless for the design of ferromagnetic parts since the cell composition constraints are reduced to

$$0 \leq \rho_{i,e} \leq 1, \quad (5.57)$$

which leads to the same feasible domain for the penalized material proportions:

$$0 \leq \rho_{i,e}^* \leq 1. \quad (5.58)$$

The convexity mappings have thus no effect on volume-constrained optimizations involving a unique design variable.

New mechanisms are thus required to guarantee the problem convexity. With this purpose, two methods based on *convexity constraints* are described in this section. The former relies on the magnetic energy for the force computation whereas the latter is based on the Maxwell stress tensor. The methods are illustrated on the study case related to the design of the rotor of a SRM and described in section 4.2.

In mass-constrained optimizations, convexity mappings mainly aim to modify the electromagnetic properties of intermediate materials in comparison with their mass. However, they turn out to be ineffective for the design of ferromagnetic parts since they cannot guarantee the convergence towards a unique solution. Fig. 5.58 indeed shows two different solutions obtained with $r_i = -10$ for the maximization of the SRM average torque computed with the magnetic energy, with the maximum rotor mass set to 20 kg. These two solutions may seem to have different masses in view of the size of their rotor, but this is actually due to the fact that intermediate materials are not really obvious in the figure since their iron proportions are close to 1.

The convexity constraints introduced in this section are exclusively effective for volume-constrained optimizations. Mass-constrained problems are thus not covered but will be addressed in the next chapter dedicated to anisotropic methods.

5.6.1 Torque computation with the magnetic energy

The first approach to overcome convexity issues for the design of ferromagnetic parts is based on the magnetic energy. The force or torque of an actuator can indeed be calculated by evaluating the magnetic energy in two positions of its moving part and computing the ratio of the magnetic energy difference to the displacement.

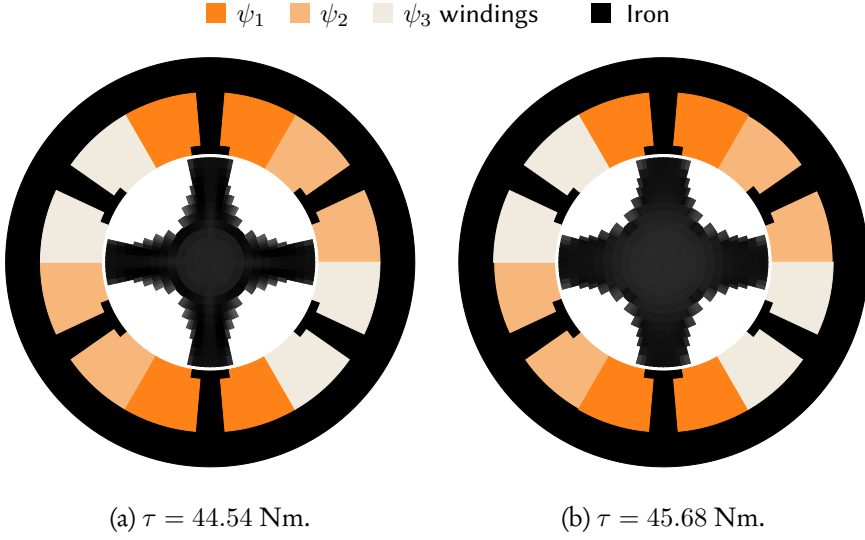


Fig. 5.58. Lack of convexity for the mass-constrained SRM rotor design with $r_i = -10$.

The optimization problem related to the maximization of the SRM average torque computed with the magnetic energy lacks of convexity. Fig. 5.59 shows for instance two rotor topologies obtained with different initial conditions.

Convexity constraints

The proposed approach relies on the optimization of two separate topologies corresponding to the two rotor positions in which the magnetic energy is evaluated. The design variables describing the iron proportions are thus divided in ρ_{i1} and ρ_{i2} , respectively for the first and the second position. Since the optimization method must, in the end, provide a unique topology, new constraints called *convexity constraints* are defined in order to progressively force the two topologies to become identical¹. These constraints are actually expressed on the microstructure thicknesses t_1 and t_2 :

$$t_{1,e} \leq \frac{(1 - s_W)t_{2,e}}{1 - s_W t_{2,e}}, \quad (5.59)$$

$$t_{2,e} \leq \frac{(1 - s_W)t_{1,e}}{1 - s_W t_{1,e}}. \quad (5.60)$$

¹The fact that the method cannot handle mass-constrained problems is explained by the use of convexity constraints. Indeed, since the optimization is based on two topologies, the mass can hardly be computed. Besides, adding a mass constraint, for instance using the mean material proportions, actually cancels the effectiveness of the convexity constraints.

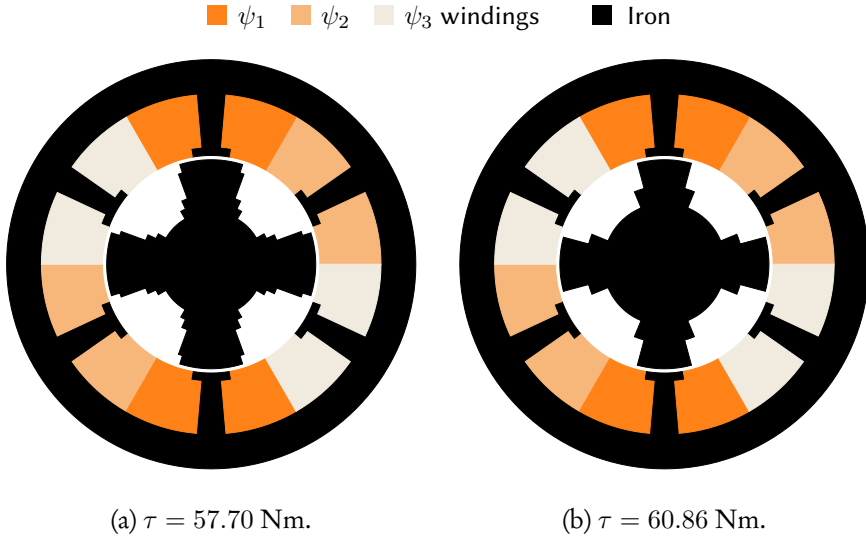


Fig. 5.59. Lack of convexity for the volume-constrained SRM rotor design with the magnetic energy.

Since they must be rewritten on the design variables, i.e the material proportions, their expression is different according to the sensitivity mapping. With the permeability and the reluctivity mappings, they are written

$$\rho_{i2,e} \leq \frac{(1 - s_W)\rho_{i1,e}}{1 - s_W\rho_{i1,e}}, \quad (5.61)$$

$$\rho_{i1,e} \leq \frac{(1 - s_W)\rho_{i2,e}}{1 - s_W\rho_{i2,e}}. \quad (5.62)$$

With the geometric mapping, they are given by

$$\frac{\hat{\mu}_r^{\rho_{i2,e}} - 1}{\hat{\mu}_r - 1} \leq \frac{(1 - s_W)(\hat{\mu}_r^{\rho_{i1,e}} - 1)}{(\hat{\mu}_r - 1) - s(\hat{\mu}_r^{\rho_{e,1}} - 1)}, \quad (5.63)$$

$$\frac{\hat{\mu}_r^{\rho_{e,1}} - 1}{\hat{\mu}_r - 1} \leq \frac{(1 - s)(\hat{\mu}_r^{\rho_{e,2}} - 1)}{(\hat{\mu}_r - 1) - s(\hat{\mu}_r^{\rho_{e,2}} - 1)}. \quad (5.64)$$

The convexity constraints involve a parameter s_W which determines their shape and thereby the tolerated difference between the two topologies. Fig. 5.60 represents the constraints for different value of this parameter. The latter is actually used in the same way as the penalization parameters of the convexity mappings: its value is progressively increased during the optimization in order to reduce the differences between the two topologies. The convexity constraints indeed reduces to $\rho_{i1,e} = \rho_{i2,e}$ when $s_W = 0$.

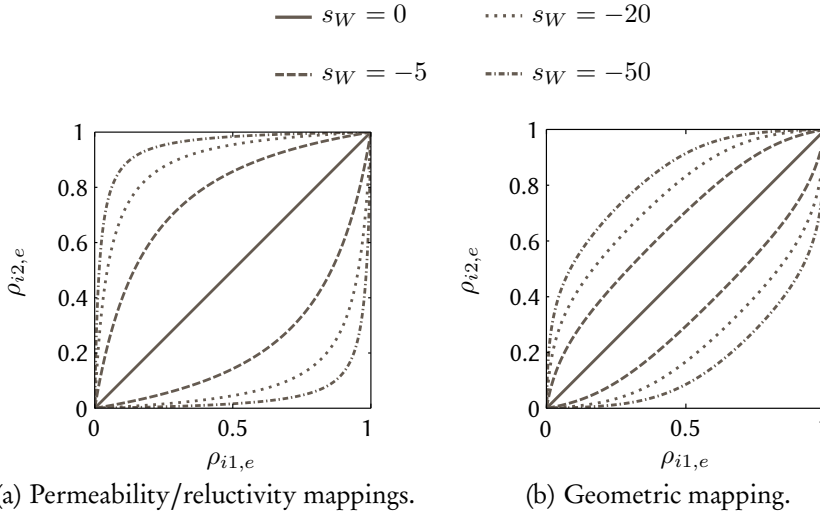


Fig. 5.60. Convexity constraints for iron optimizations based on the magnetic energy.

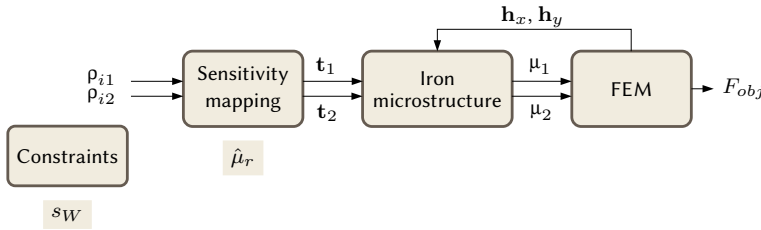


Fig. 5.61. Block diagram for iron optimizations using the magnetic energy.

Fig 5.61 provides the block diagram for iron optimizations including the s_W parameter in the constraint block.

Application

Thanks to this approach, the initial convergence towards a unique solution can be guaranteed with $s_W = -100$ for the SRM rotor design. The optimization was performed in 200 iterations and s_W was maintained at this value during the 30 first iterations. Fig. 5.62 shows the evolutions of the objective function and of the parameter s_W .

The latter is modified using an exponential relation, since its impact on the constraint shape is more important at low values. Using a linear evolution leads then to a slow penalization initially and a strong penalization when s_W approaches zero. As

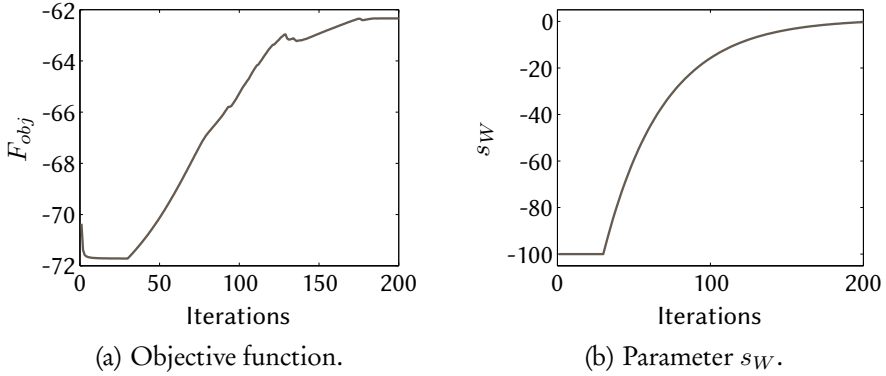


Fig. 5.62. Volume-constrained SRM rotor optimization with the magnetic energy.

explained in section 5.2.3, the penalization must be sufficiently slow so that slowing down further this process still results in the same solution. Thanks to the exponential evolution of the parameter s_w , the penalization can here be performed in 170 iterations. With a linear evolution, this number would have to be increased to ensure a sufficiently slow penalization when s_W comes close to zero.

Fig. 5.63, Fig. 5.64 and Fig. 5.65 illustrate the topologies obtained at different points of the optimization and highlight the reduction of the differences between them thanks to the convexity constraints. The final design is represented in Fig. 5.66 and achieves a torque of 62.34 Nm. The optimization requires a computation time of 3 hours and 2542 finite element evaluations.

The efficiency of the solution obtained with the convexity constraints is confirmed through a comparison with the best local minimizer found in appendix A which produces a torque of 62.66 Nm. The torque difference is indeed lower than 0.5 % and the chance of finding a better solution without the convexity constraints is limited to 0.8 %.

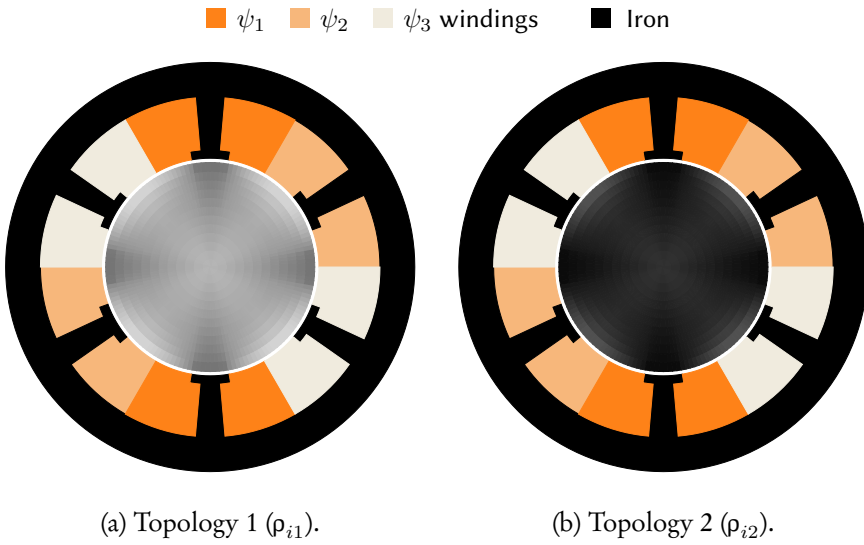


Fig. 5.63. Volume-constrained SRM rotor design with the magnetic energy: iteration 30.

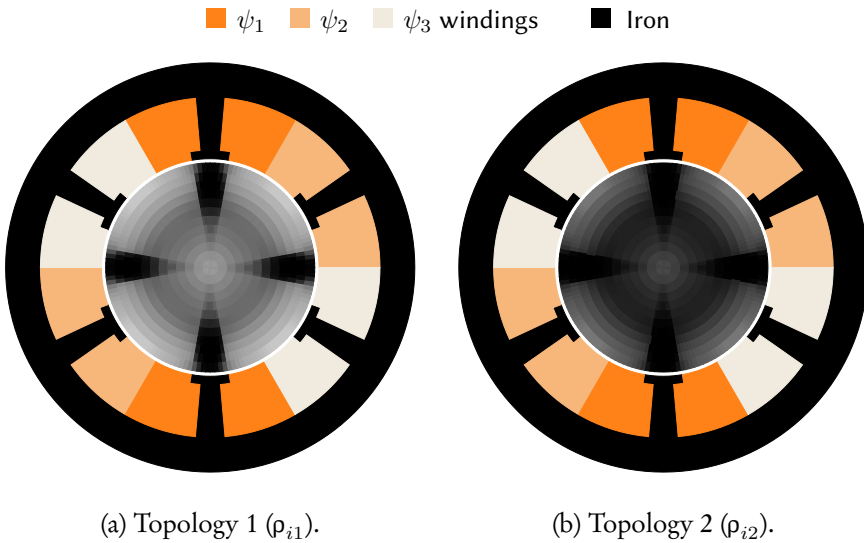


Fig. 5.64. Volume-constrained SRM rotor design with the magnetic energy: iteration 100.

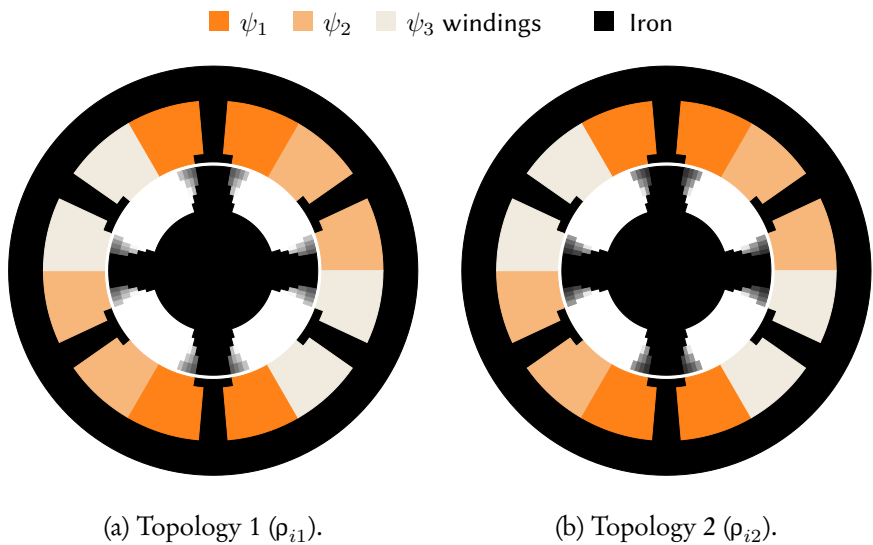


Fig. 5.65. Volume-constrained SRM rotor design with the magnetic energy: iteration 150.

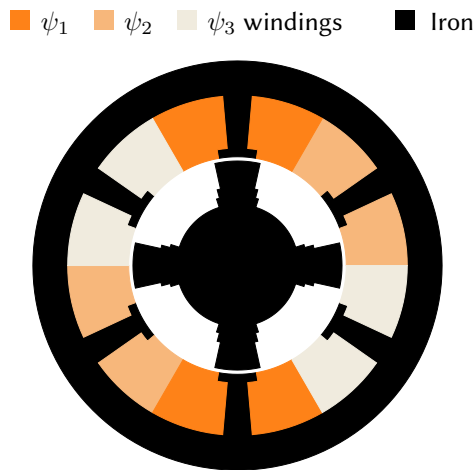


Fig. 5.66. Final design for the volume-constrained SRM rotor optimization with the magnetic energy; $\tau = 62.34 \text{ Nm}$.

5.6.2 Torque computation with the Maxwell stress tensor

The torque computation with the Maxwell stress tensor leads as well to the convergence to local minimizers for the SRM rotor design. Fig. 5.67 shows for instance two topologies obtained with different initial conditions.

Convexity constraints

The approach proposed with the magnetic energy can actually be transposed for the Maxwell stress tensor. Two topologies are again considered and the optimization aims to maximize the virtual torque that would be produced by combining in the Maxwell tensor (4.11) the radial (or horizontal) magnetic field of the first topology with the tangential (or vertical) field of the second one.

The two topologies, still defined by ρ_{i1} and ρ_{i2} , are however subject to different convexity constraints. The optimization is actually realized in two steps. First, the iron proportions of the two topologies are subject to

$$(\rho_{i1,e} - 0.5)^2 + (\rho_{i2,e} - 0.5)^2 \leq s_1^2, \quad (5.65)$$

with s_1 increasing from 0 to 0.5. This constraint is actually a circle whose radius is s_1 , as illustrated in Fig. 5.68(a). The optimization begins with all the material

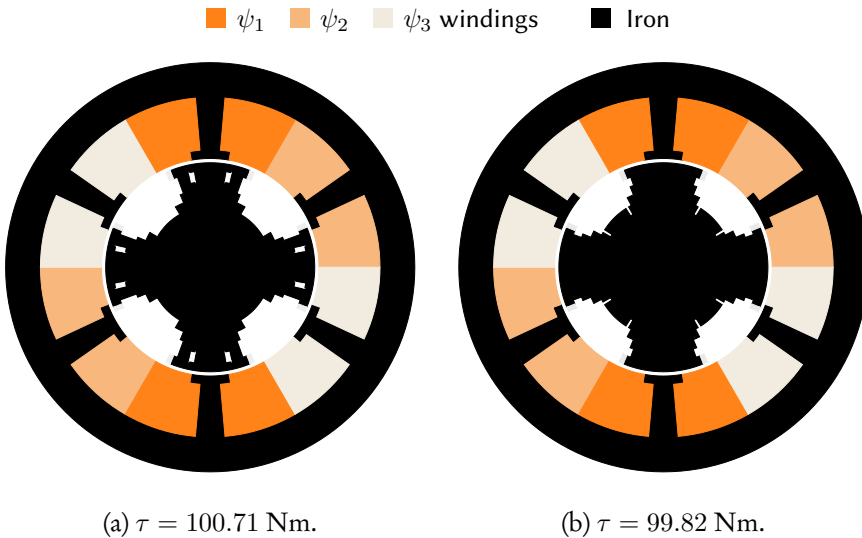


Fig. 5.67. Lack of convexity for the volume-constrained SRM rotor optimization with the Maxwell stress tensor.

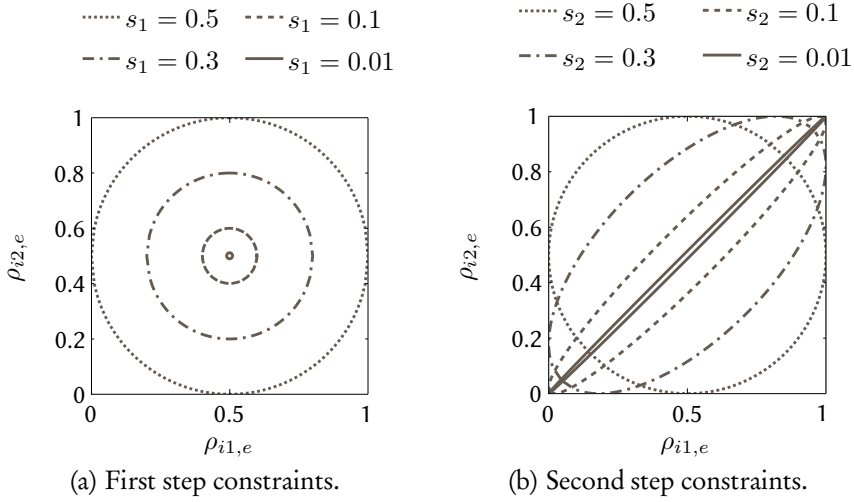


Fig. 5.68. Convexity constraints for iron optimizations based on the Maxwell stress tensor.

proportions equal to 0.5, when $s_1 = 0$. The parameter s_1 is then increased, which gives progressively freedom to the algorithm for designing the two topologies.

In the second step, constraint (5.65) is replaced by:

$$\frac{(\rho_{i1,e} - \rho_{i2,e})^2}{2s_2^2} + \frac{(\rho_{i1,e} + \rho_{i2,e} - 1)^2}{2(0.5 - s_2^2)} \leq 1, \quad (5.66)$$

with s_2 decreasing from 0.5 to 0, as shown in Fig. 5.68(b). When $s_1 = s_2 = 0.5$, the constraints have the same shape, which guarantees a smooth transition between the first and the second step. At the optimization end, $s_2 = 0$ induces $\rho_{i1,e} = \rho_{i2,e}$ which ensures obtaining a unique topology. This second step constraint is thus used to force progressively the two topologies to be identical. The block diagram including the parameters s_1 and s_2 is represented in Fig. 5.69.

The convexity constraints introduced for iron optimizations based on the magnetic energy are rather obvious: their shape consistently evolves from a complete square feasible domain for large negative values of s_W to the unique line $\rho_{i1} = \rho_{i2}$. The justification of the constraint shapes for the Maxwell tensor is in contrast not straightforward. They were actually derived from an observation of the objective function evolution in the ρ_{i1} - ρ_{i2} plane, which usually reveals a "hill" shape with four minima in the square corners¹ and a maximum in the middle. The first-step circular constraint allows thus the optimization algorithm to "turn" around the "hill" without falling in one of the four local minima. The second-step constraint represents then the obvious evolution from a circular domain to a straight line.

¹ $(\rho_{i1,e}; \rho_{i2,e}) = (0; 0), (0; 1), (1; 0), (1; 1)$.

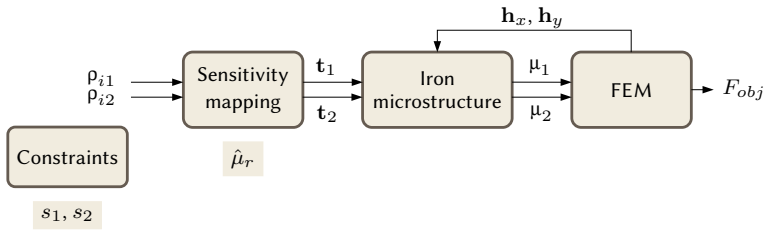


Fig. 5.69. Block diagram for iron optimizations using the Maxwell stress tensor.

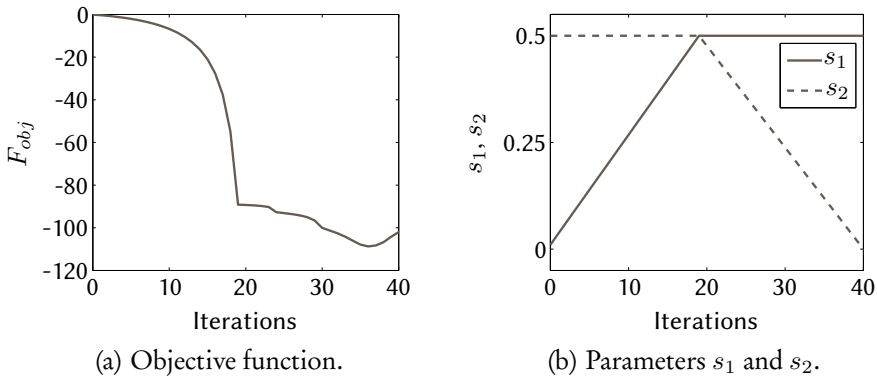


Fig. 5.70. Volume-constrained SRM rotor optimization with the Maxwell stress tensor.

Application

The method was applied to the design of the SRM rotor with the evolution of the parameters s_1 and s_2 given in Fig. 5.70(b). The optimization required 475 finite element evaluations and a computation time of 38 minutes. Fig. 5.71, Fig. 5.72 and Fig. 5.73 show the topologies produced during the optimization whereas the final design is represented in Fig. 5.74. The latter achieves a torque of 101.98 Nm while the best local minimizer found with a basic steepest-descent algorithm in appendix A produces a torque of 103.05 Nm. The torque difference is here limited to 1 % and the chance of finding a better solution without the convexity constraints is 4 %. On this specific study case, the method based on the Maxwell tensor is thus not as efficient as the method based on the magnetic energy, but still achieves good results. Attention must be paid to the fact that the torques achieved by the two methods cannot be compared with each other, since they correspond to different objective functions.

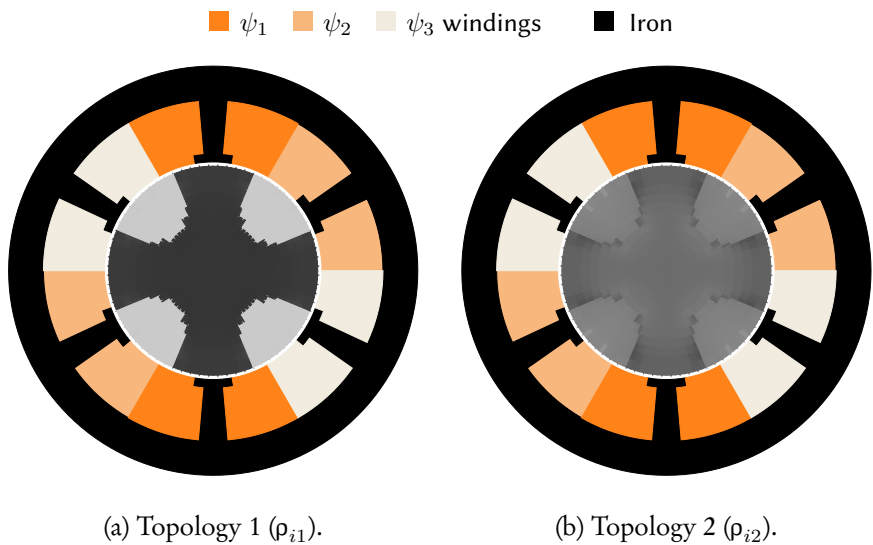


Fig. 5.71. SRM rotor design with the Maxwell stress tensor: iteration 10.

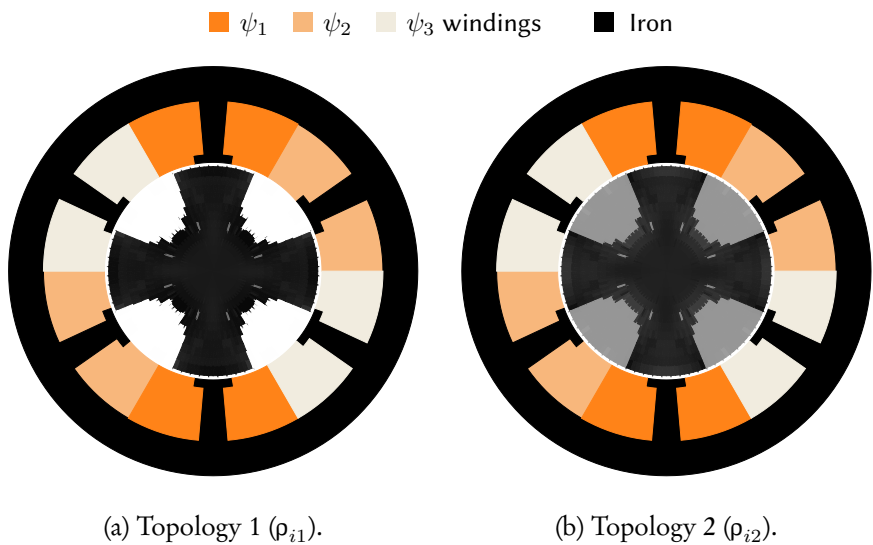


Fig. 5.72. SRM rotor design with the Maxwell stress tensor: iteration 20.

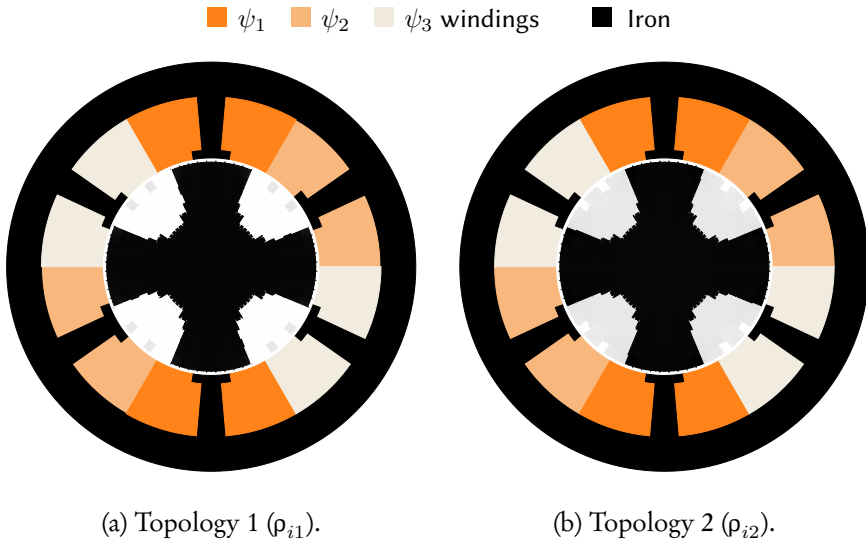


Fig. 5.73. SRM rotor design with the Maxwell stress tensor: iteration 30.

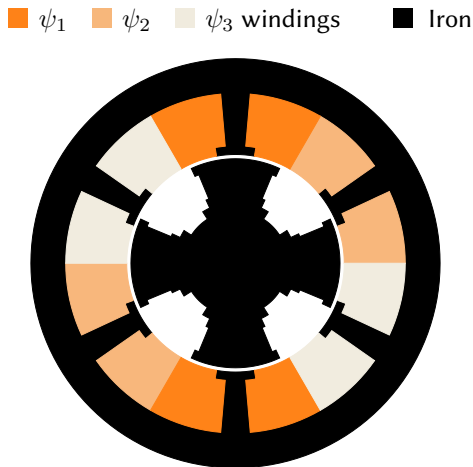


Fig. 5.74. Final design for the SRM rotor optimization with the Maxwell stress tensor; $\tau = 101.98 \text{ Nm}$

5.7 Conclusion

This conclusion intends to provide a global view of the methods proposed in this chapter. Indeed, even if each combination of materials has its own specificities, main common principles can nevertheless be highlighted.

Fig. 5.75 represents the complete block diagram including all the mechanisms that were introduced in this chapter.

- The *convexity mappings* are used to penalize/promote intermediate materials and are divided in two groups. The former mappings focus on a unique material and are characterized by a single subscript parameter whereas the latter exclusively affect composites of two materials and are characterized by a double subscript parameter.
- The *sensitivity mapping* is a change of variables intending to improve the convergence speed of the steepest-descent algorithm.
- The *iron microstructure* aims to define realistic relations between the iron proportion, the permeability and the mass of cells so that intermediate material properties correspond to those of real composite materials. The microstructure is important to guarantee a convex behaviour and besides gives the possibility, under some conditions, to interpret intermediate materials.
- The *constraint* block includes different types of constraints, such as the cell composition constraints and an eventual mass constraint. According to the considered problem, additional constraints may be required. The optimization of ferromagnetic parts is for instance based on specific convexity constraints.

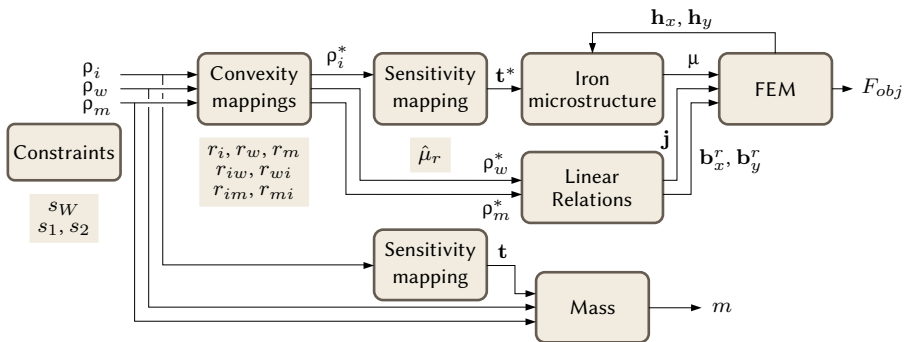


Fig. 5.75. Complete block diagram of the isotropic methods.

Based on this method structure, problems are optimized in two steps. The first one aims to ensure the convergence towards a unique topology for any initial conditions, whereas the second one focuses on the handling of intermediate materials in this topology.

The most efficient approach to achieve global convergence in the first optimization step consists in establishing a correspondence between intermediate materials and real composite materials, as suggested by Dyck and Lowther (1996). This correspondence is obtained thanks to the definition of an iron microstructure which describes how iron is distributed inside intermediate iron cells. Thereby, the apparition of local minimizers can be avoided as well as the spreading of intermediate materials in the whole design space in mass-constrained optimizations, which occurs respectively when intermediate materials have too poor and too valuable properties. This was successfully applied to winding, magnet¹ and iron-winding optimizations.

However, defining such realistic properties is not always sufficient. Iron and iron-magnet optimizations indeed still lack of convexity with the iron microstructure. The properties of intermediate materials must then be further improved thanks either to the convexity mappings or to convexity constraints.

The first step usually results in a topology containing intermediate materials, except for winding or permanent magnet optimizations. According to the approach followed in this first step, different strategies are possible for handling these intermediate materials. The first one consists in progressively penalizing their properties so that they naturally disappear during the optimization. This penalization process however inherently creates local minimizers which may lead to convergence problems and non optimal results. The second strategy can only be considered when the initial global convergence was achieved by the definition of realistic properties for intermediate materials. Thanks to their physical meaning, the latter can then be interpreted under some conditions, and replaced by equivalent solid structures.

As explained in section 3.2, the optimization is performed without any stopping criterion, since the convergence is simply observed when all the cells have converged to a unique material. Actually, the development of a stopping criterion would nevertheless be interesting for the first optimization step, to determine from which iteration the penalization or the interpretation could be carried out. However, as already mentioned before, this thesis especially focuses on the efficient formulation of the optimization problem to handle local minimizer and intermediate material issues, while the determination of most efficient optimization algorithm is beyond the scope of the work.

¹Winding and magnet optimizations do obviously not require the introduction of an iron microstructure. However, the linear relations between the material proportions, the electromagnetic properties and the masses in fact ensure the direct correspondence between intermediate and composite materials.

Although methods based on isotropic materials provide satisfactory results on most of the study cases considered in this thesis, they suffer from two limitations. The former is related to the optimization of ferromagnetic parts, and the latter to the design of the PMSM stator.

Ferromagnetic parts indeed require a specific attention compared to the other study cases: convexity constraints must be included in the optimization in order to obtain a convex behaviour. Having such a specific case is not desirable, a global method working in all the situations is obviously preferable. In addition, these convexity constraints make impossible the introduction of a mass constraint since the latter inactivates the former in some cells and cancels therefore their effect.

Isotropic methods also turn out to be ineffective for the design of the stator of a PMSM. The lack of convexity is there explained by the fact that an anisotropic structure is required for maximizing the motor torque. Such structure cannot be efficiently produced with isotropic materials, which leads to the convergence towards local minimizers. This situation is actually related to the second rule of intermediate materials proposed by Dyck and Lowther (1996): besides defining a direct correspondence between intermediate and real composite materials, the latter must make the most cost effective use of the solid materials from which it is made. In the PMSM case, the next chapter will show that the most cost effective composite pattern is actually an anisotropic iron microstructure oriented in the radial direction. This is the reason why this study case cannot be handled with isotropic materials.

Methods based on isotropic materials are nevertheless effective in most topology optimization problems. They besides benefit from a simple implementation and display fast convergence. Even if they suffer from some limitations, they generally remain an interesting and efficient approach for solving topology optimization problems.

Anisotropic materials

6

The development of methods based on anisotropic materials initially aimed to overcome the limitations of isotropic methods. The PMSM stator optimization, for instance, specifically underlines the need for such methods, since the creation of anisotropic structures cannot be effectively handled with isotropic materials. In fact, the development of anisotropic methods did not only lead to an alternative approach for solving those difficult problems, it besides resulted in an efficient tool for a much wider range of problems. All the study cases considered in this thesis can indeed be effectively handled thanks to the introduction of anisotropic materials. Compared to isotropic methods, the introduction of anisotropic materials brings thus a major simplification since it leads to a unique and general approach for solving a large range of problems.

When speaking about anisotropic materials for the topology optimization of electromagnetic devices, it actually only concerns the iron material. In addition to its proportion, the method also considers the way it is laid out inside the cells. Therefore, an iron microstructure, whose dimensions are optimized along with the material proportions, is introduced and the anisotropic cell permeabilities are then derived from this microstructure.

The anisotropic method shares similarities with the homogenization method, initially introduced for the design of mechanical structures and later applied by Yoo et al. (2000) to electromagnetic problems. It is however based on a different definition of the design variables, and still takes advantage of the effectiveness of the sensitivity and convexity mappings. In addition, the homogenization process requires the interpolation of finite element computations to derive the equivalent permeability of the microstructure, whereas the anisotropic method simply considers a network of reluctances in series and parallel.

This chapter is divided in four parts. The first one focuses on the definition of the anisotropic iron microstructure and of all its characteristic parameters. The

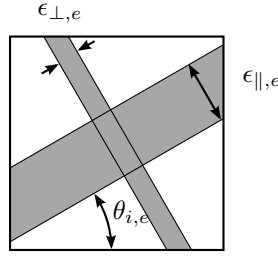


Fig. 6.1. Anisotropic iron microstructure.

second part deals with the optimization constraints which must be slightly adapted for anisotropic optimizations. The method behaviour is then illustrated on a simple application case in the third part: the PMSM stator design without saturation. Eventually, the method is applied to the four main study cases considered in the thesis in order to assess its results, i.e the stator and rotor optimizations of the PMSM and SRM.

6.1 Anisotropic iron microstructure

This section intends to describe all the parameters that characterize the *anisotropic iron microstructure*. The latter is represented in Fig. 6.1 and differs from the isotropic microstructure in two ways: its orientation is defined by a variable $\theta_{i,e}$, with the subscript i referring to iron, instead of being aligned with the magnetic field and it is composed of two iron sheets whose thicknesses are given by $\epsilon_{||,e}$ and $\epsilon_{\perp,e}$, respectively in the $\theta_{i,e}$ and in its perpendicular direction.

The microstructure is actually equivalent to the classic microstructure used in the homogenization method and shown in Fig. 2.3. Indeed, considering rectangular air holes or cross-shaped iron structures leads to the same composite material, with $y_e = 1 - \epsilon_{||,e}$ and $x_e = 1 - \epsilon_{\perp,e}$. In the electromagnetic domain, composite materials are however generally represented as layers of different materials and not as perforated structures, which suggests rather using the cross-shaped representation. Compared to the homogenization method, the design variables are besides defined differently in order to tackle convergence issues, as it will be explained later.

The cell permeability is computed considering a network of reluctances in series

and parallel:

$$\mu_{\parallel,e} = \mu_0 \mu_r \epsilon_{\parallel,e} + (1 - \epsilon_{\parallel,e}) \frac{\mu_0}{1 + \left(\frac{1}{\mu_r} - 1 \right) \epsilon_{\perp,e}}, \quad (6.1)$$

$$\mu_{\perp,e} = \mu_0 \mu_r \epsilon_{\perp,e} + (1 - \epsilon_{\perp,e}) \frac{\mu_0}{1 + \left(\frac{1}{\mu_r} - 1 \right) \epsilon_{\parallel,e}}, \quad (6.2)$$

with μ_r the relative permeability of iron and $\mu_{\parallel,e}$ and $\mu_{\perp,e}$ the cell permeabilities respectively in the $\theta_{i,e}$ and in its perpendicular direction.

In this chapter, saturation is taken into account by computing the values of the relative permeability μ_r in (6.1) and (6.2) from the norms of the H field in the cells. This simplified approach is actually only valid in solid cells or in cells where the iron microstructure is aligned with the magnetic field. This is the case for most of the results presented in this chapter since intermediate materials are removed by penalization. However, in order to derive valid results from an interpretation of composite materials, a more complex approach should be investigated, taking realistically into account the microstructure for the computation of the saturated cell permeabilities.

6.1.1 Microstructure orientation

The discussion developed in section 5.1.2 regarding the design variables for permanent magnet optimizations also applies to the microstructure design variables. Indeed, considering the microstructure orientation as a design variable also leads to convergence issues when the cell is emptied of iron. The method may then be stuck in a situation where, on one hand, the iron proportion cannot be increased because of a wrong orientation and, on the other hand, the orientation cannot be modified since it does not affect the objective function when the iron proportion is zero.

The microstructure is thus more effectively described by four variables $\rho_{i,x+}$, $\rho_{i,x-}$, $\rho_{i,y+}$ and $\rho_{i,y-}$, with

$$\rho_{i,x} = \rho_{i,x+} - \rho_{i,x-}, \quad (6.3)$$

$$\rho_{i,y} = \rho_{i,y+} - \rho_{i,y-}, \quad (6.4)$$

$$\theta_{i,e} = \arctan \frac{\rho_{i,y,e}}{\rho_{i,x,e}}. \quad (6.5)$$

The microstructure orientation is only defined for an angle interval of π , since turning the microstructure of 180° leads to the same anisotropic permeability. One of the four design variables could thus be removed: it would still be possible to represent any microstructure orientation. However, it would create a discontinuity:

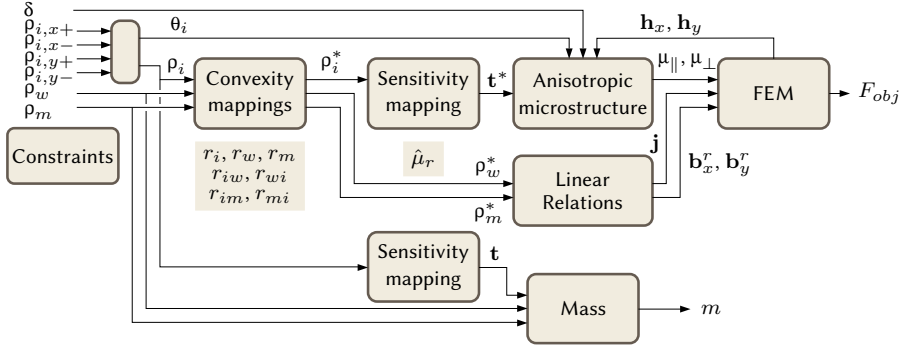


Fig. 6.2. Block diagram for anisotropic material optimizations: introduction of the shape factor.

removing the variable $\rho_{i,x-}$ would for instance make impossible to turn the microstructure in the counterclockwise direction when its orientation is equal to $\pi/2$. In order to avoid the resulting convergence issue, the optimization is rather performed with four variables, which has in fact no effect on the computation time since the finite element model anyway relies on the anisotropic permeability defined by $\mu_{||,e}$ and $\mu_{\perp,e}$.

6.1.2 Shape factor

In addition to the microstructure orientation, the four design variables also define the iron proportion, written ρ_i :

$$\rho_{i,e} = \sqrt{(\rho_{i,x+,e} - \rho_{i,x-,e})^2 + (\rho_{i,y+,e} - \rho_{i,y-,e})^2}, \quad (6.6)$$

$$= \sqrt{\rho_{i,x,e}^2 + \rho_{i,y,e}^2}. \quad (6.7)$$

As in isotropic methods, the iron proportion is subject to the convexity and sensitivity mappings in order to derive t and t^* , the proportions related to the microstructure, which is illustrated in Fig. 6.2. An additional design variable δ , called the *shape factor*, is then introduced to describe how the iron proportion t^* is distributed along the two microstructure axes.

As shown in Fig. 6.3, the shape factor is defined in the interval $[-1; 1]$, with the whole iron proportion being in the parallel direction for $\delta_e = 1$ and in the perpendicular direction for $\delta_e = -1$. The microstructure sheet thicknesses are in

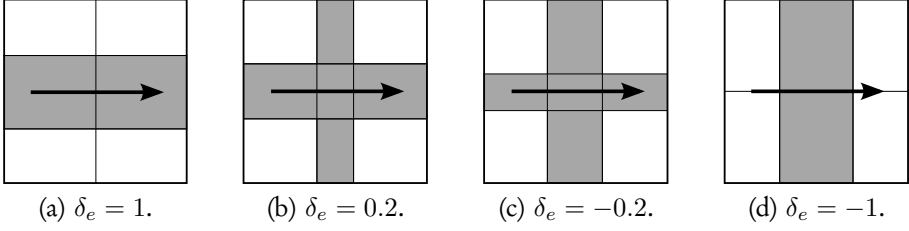


Fig. 6.3. Distribution of iron along the two microstructure axes according to the shape factor.

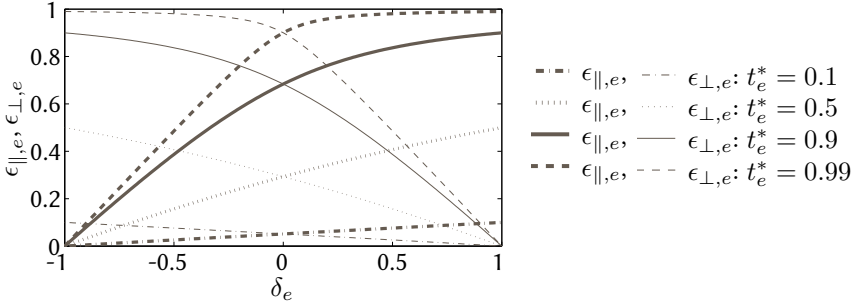


Fig. 6.4. Microstructure dimension evolutions according to the shape factor for different values of t_e^* .

fact given by the following expressions:

$$\epsilon_{\parallel,e} = 1 + \frac{\delta_e t_e^*}{2} - \sqrt{1 + \left(\frac{\delta_e t_e^*}{2}\right)^2} - t_e^*, \quad (6.8)$$

$$\epsilon_{\perp,e} = 1 - \frac{\delta_e t_e^*}{2} - \sqrt{1 + \left(\frac{\delta_e t_e^*}{2}\right)^2} - t_e^*. \quad (6.9)$$

These two functions are represented in Fig. 6.4. They ensure that the proportion of iron in the microstructure is always equal to t_e^* since they verify the following relation:

$$t_e^* = \epsilon_{\parallel,e} + \epsilon_{\perp,e} - \epsilon_{\parallel,e} \epsilon_{\perp,e}. \quad (6.10)$$

The importance of considering a cross-shaped microstructure including a shape factor can be highlighted from Fig. 6.3. Let's imagine a cell in which the microstructure orientation is initially horizontal (Fig. 6.3(a)), but should optimally be vertical (Fig. 6.3(d)). With an I-shaped microstructure, the evolution towards the vertical direction would actually be impossible (or at least very slow): the gradient of the objective function according to the microstructure angle is indeed equal to zero since the microstructure can be equally turned clockwise or counterclockwise

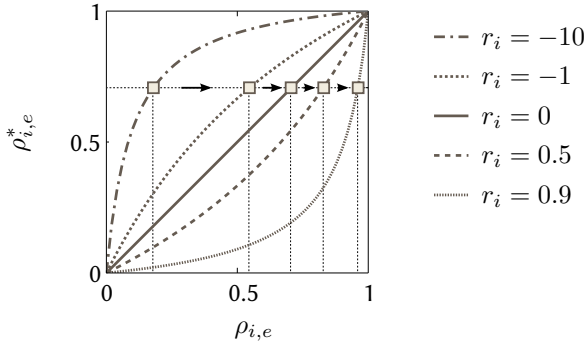


Fig. 6.5. Ineffectiveness of the convexity mapping for removing iron composites in cells where anisotropy is required.

to improve the objective function. With a cross-shaped microstructure, the perpendicular thickness can simply be increased while the parallel thickness is decreased (Fig. 6.3(a) to (d)), by reducing the shape factor value.

6.1.3 Anisotropy penalization

With isotropic materials, the penalization of iron composites is exclusively performed using the convexity mappings. Indeed, their presence is then explained by two reasons. In volume-constrained optimizations, the method mixes iron with windings (or permanent magnets) in order to increase both the permeability and the current density (or the remanent field) of the cell. In mass-constrained optimizations, composites are besides due to a balance between the cell electromagnetic properties and the cell masses. In both case, a penalization of their electromagnetic properties makes them disappear in the final design.

The introduction of anisotropy in iron composites creates a third possible reason explaining their presence: precisely their anisotropic property. Indeed, anisotropic cells may be very efficient in some optimization problems; an intermediate anisotropic cell can be more valuable than a solid but isotropic cell. In this case, even if the method could increase the cell permeability, it rather keeps it intermediate to guarantee a high anisotropy. The convexity mappings become then totally ineffective to remove these intermediate cells: whatever the value of the penalization parameters r_i , r_{iw} or r_{im} , the method will still fill them with intermediate iron proportions in order to keep their anisotropy. When the penalization parameters are increased, the iron proportions $\rho_{i,e}$ simply raise and become very close to 1, but never equal to 1, in order to compensate the penalization effect and achieve the same value of $\rho_{i,e}^*$ and t^* , as illustrated in Fig. 6.5.

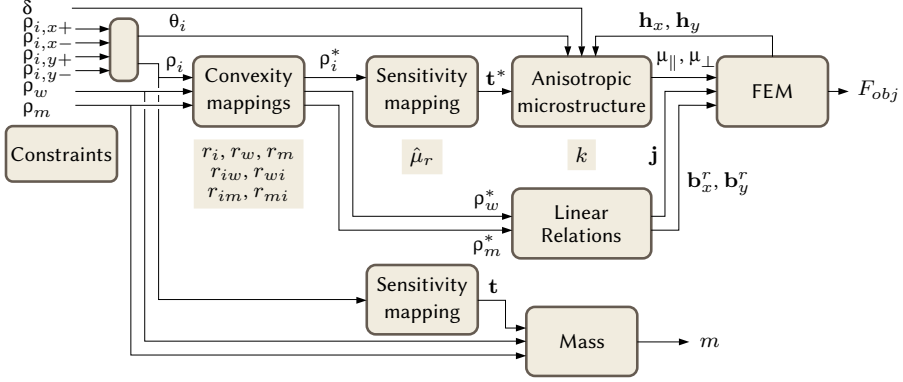


Fig. 6.6. Block diagram for anisotropic material optimizations: introduction of the anisotropy penalization parameter.

In order to remove these iron composites, the penalization must be performed on their anisotropy. With this purpose, a new *anisotropy penalization parameter* k is introduced in the cell permeability expressions:

$$\mu_{||,e} = \left(\mu_0 \mu_r \epsilon_{||,e} + (1 - \epsilon_{||,e}) \frac{\mu_0}{1 + \left(\frac{1}{\mu_r} - 1 \right) \epsilon_{\perp,e}} \right) (1 - k) + (\mu_0 + (\mu_0 \mu_r - \mu_0) t_e) k, \quad (6.11)$$

$$\mu_{\perp,e} = \left(\mu_0 \mu_r \epsilon_{\perp,e} + (1 - \epsilon_{\perp,e}) \frac{\mu_0}{1 + \left(\frac{1}{\mu_r} - 1 \right) \epsilon_{||,e}} \right) (1 - k) + (\mu_0 + (\mu_0 \mu_r - \mu_0) t_e) k. \quad (6.12)$$

The latter reduce to their original expressions (6.8) and (6.9) when $k = 0$ and to the isotropic permeability expression (5.32) when $k = 1$. The parameter k is thus set to 0 except during the penalization process, it may then be increased to 1 for removing iron composites. It is included in the block diagram under the "Anisotropic microstructure" block in Fig. 6.6.

6.2 Constraints

This section deals with two points regarding the optimization constraints. First, a new mechanism is introduced in the cell composition constraints for penalizing composite materials. Next, a new constraint is included in order to prevent convergence issues related to solid iron cells.

6.2.1 Cell composition constraints

The cell composition constraints are given by

$$f_{sens}(\sqrt{\rho_{i,x+,e}^2 + \rho_{i,x-,e}^2 + \rho_{i,y+,e}^2 + \rho_{i,y-,e}^2}) + \rho_{w,e} + \rho_{m,e} \leq 1, \quad (6.13)$$

$$\rho_{i,x+,e}, \rho_{i,x-,e}, \rho_{i,y+,e}, \rho_{i,y-,e}, \rho_{w,e}, \rho_{m,e} \geq 0. \quad (6.14)$$

As for permanent magnet optimizations, they do not rely on the iron proportion expression (6.6) in order to ensure that all the design variables are comprised between 0 and 1. They besides guarantee that the composition of intermediate cells respects the following relation:

$$f_{sens}(\rho_{i,e}) + \rho_{w,e} + \rho_{m,e} \leq 1 \Leftrightarrow t_e + \rho_{w,e} + \rho_{m,e} \leq 1. \quad (6.15)$$

The penalization of intermediate materials can also be realized by modifying the latter relation. Indeed, the following linear constraint actually defines a smaller feasible domain on the material proportions:

$$\rho_{i,e} + \rho_{w,e} + \rho_{m,e} \leq 1. \quad (6.16)$$

Iron-winding or iron-magnet composites can thus be lowered by progressively reducing constraint (6.15) to (6.16). This evolution is performed using a penalized version of the sensitivity mapping:

$$f_{sens}^*(\rho_{i,e}) = \frac{\hat{\mu}_r^* \rho_{i,e} - 1}{\hat{\mu}_r^* - 1}, \quad (6.17)$$

with $\hat{\mu}_r^*$ given by:

$$\hat{\mu}_r^* = 1 + (\hat{\mu}_r - 1)s. \quad (6.18)$$

The sensitivity mapping is equivalent to its original expression with $s = 1$ and it reduces, by taking the limit, to $f_{sens}^*(\rho_{i,e}) = \rho_{i,e}$ for $s = 0$.

The first cell composition constraint is thus rewritten as:

$$f_{sens}^*(\sqrt{\rho_{i,x+,e}^2 + \rho_{i,x-,e}^2 + \rho_{i,y+,e}^2 + \rho_{i,y-,e}^2}) + \rho_{w,e} + \rho_{m,e} \leq 1, \quad (6.19)$$

which ensures respecting (6.15) when $s = 1$ and (6.16) when $s = 0$.

There may seem to be a redundancy between this penalization through the parameter s and the convexity mappings. However, the latter turn out in practice to be much more efficient when they are preceded by the former. In addition, the reduction of the cell composition constraint to a linear expression when $s = 0$ simplifies the orthogonal projection of the design variables for the rest of the optimization, and thereby reduces the computation time.

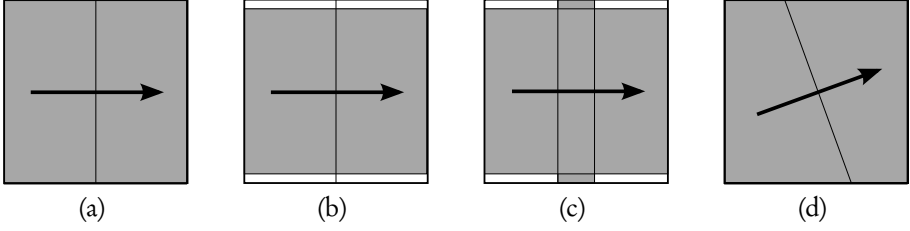


Fig. 6.7. Convergence issue in solid iron cells.

6.2.2 Maximum iron proportion constraint

An additional constraint must be included in the optimization problem in order to help the algorithm convergence. This constraint aims to avoid the convergence problem illustrated in Fig. 6.7(a). The microstructure should again optimally take a vertical orientation as shown in Fig. 6.3(d) and the cell is initially totally filled with iron. The convergence problem comes from the fact that none of the three microstructures represented in Fig. 6.7(b) to Fig. 6.7(d) can be obtained starting from this initial situation. Indeed, the iron proportion (Fig. 6.7(b)) and the shape factor (Fig. 6.7(c)) cannot be reduced since it would induce a large decrease of the permeability in the vertical direction. In addition, the microstructure direction has no influence on the cell permeability, neither has it on the objective function. The steepest-descent algorithm will thus never converge to a vertical magnetization direction.

This situation can be avoided by preventing the iron proportion to reach 1 using the following constraint:

$$\sqrt{\rho_{i,x+,e}^2 + \rho_{i,x-,e}^2 + \rho_{i,y+,e}^2 + \rho_{i,y-,e}^2} \leq \rho_{i,max}. \quad (6.20)$$

This constraint is applied during the first iterations of the optimization and ensures that the microstructure directions are properly optimized. It is then released ($\rho_{i,max}$ is set to 1) in order to continue the optimization with the usual set of constraints.

Fig. 6.8 provides the complete block diagram of the anisotropic method, including the parameters s and $\rho_{i,max}$.

6.3 Method illustration on a simple problem

In this section, the effectiveness of all the mechanisms introduced in this chapter is illustrated on a simple optimization problem: the volume-constrained PMSM stator design without saturation. First, the initial convergence towards a unique design is

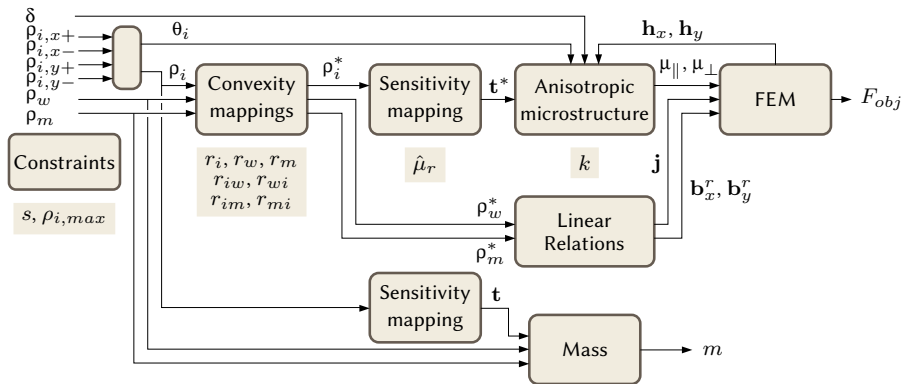


Fig. 6.8. Block diagram for anisotropic material optimizations: introduction of the penalization elements.

ensured thanks to the shape factor, the cell composition constraint and the maximum iron proportion constraint. Then, anisotropic composite materials can either be interpreted or penalized using the parameters k and s and the convexity mappings.

6.3.1 Initial convergence

The optimization result is given in Fig. 6.9, with the maximum iron proportion $\rho_{i,max}$ set to 0.9 during the 30 first iterations. The iron repartition along the two microstructure axes is represented by white lines: a simple line indicates that the shape factor is either equal to 1 or -1 and two lines of equal size that the latter is equal to 0. This result is always obtained whatever the initial conditions.

The effectiveness of the shape factor, the cell composition constraint and the maximum iron proportion constraint can be demonstrated by highlighting the convergence issues that arise without these mechanisms.

Shape factor

Fig. 6.10 shows a solution obtained without the shape factor, i.e. with δ constrained to 1. This optimization result highlights some cells that were stuck in a direction perpendicular to their optimal direction, since the evolution shown in Fig. 6.3 is not possible anymore. In order to obtain a sufficient permeability in the radial direction, those cells are completely filled with iron instead of taking the right orientation.

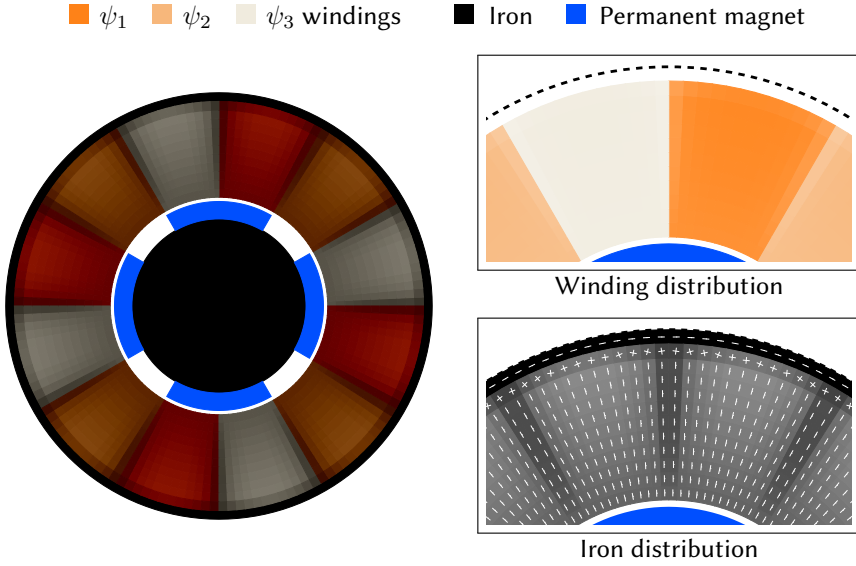


Fig. 6.9. Volume-constrained PMSM stator design with the anisotropic method; $\tau = 672.16$ Nm.

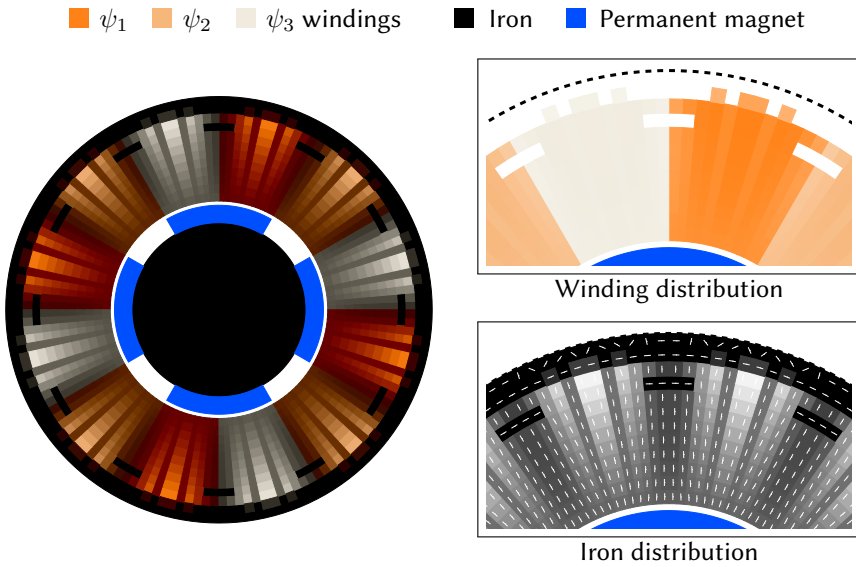


Fig. 6.10. Optimization result with I-shaped microstructures (without shape factor); $\tau = 633.25$ Nm.

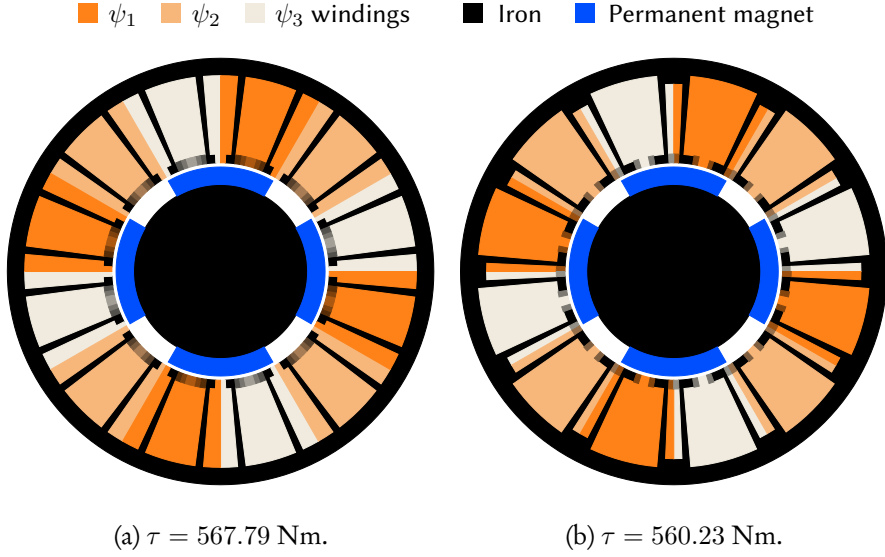


Fig. 6.11. Different optimization results with a linear cell composition constraint.

Cell composition constraint

The same optimization was also performed with a linear cell composition constraint, i.e. with $s = 0$. Fig. 6.11 underlines the importance of taking the sensitivity mapping into account in the cell composition constraint. Indeed, the linear constraint strongly penalizes composite materials, which prevents cells from moving from one material to the other and leads to solutions containing almost only solid materials.

Maximum iron proportion constraint

Fig. 6.12 shows a result obtained without limiting the iron proportion during the first iterations. In the second to last row, some cells are completely filled with iron and stuck in this configuration since their orientation is not optimal.

6.3.2 Intermediate material interpretation

Since the cell properties are derived from a physically realistic anisotropic microstructure, intermediate materials can easily be interpreted. Indeed, the two conditions on the magnetic field direction introduced in section 5.3.3 for isotropic materials are not required anymore, since the microstructure direction is not determined

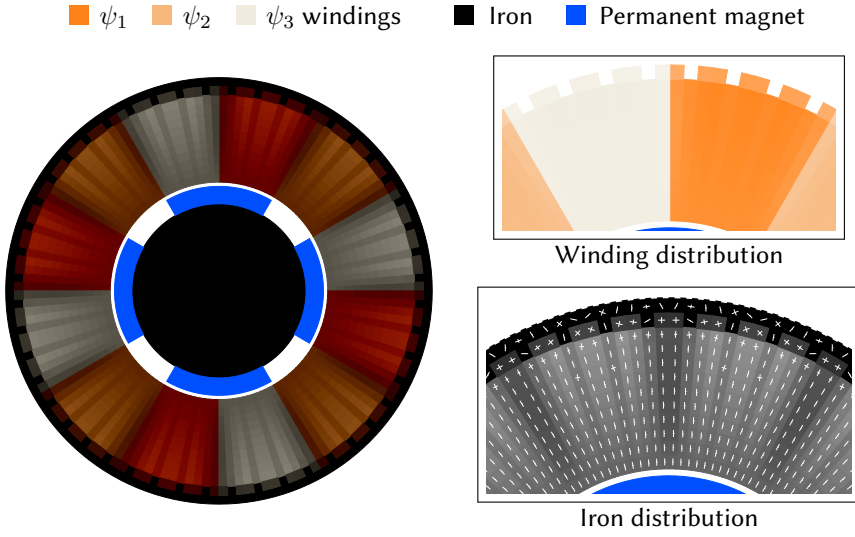


Fig. 6.12. Optimization result without applying the maximum iron proportion constraint during the first iterations; $\tau = 659.61$ Nm.

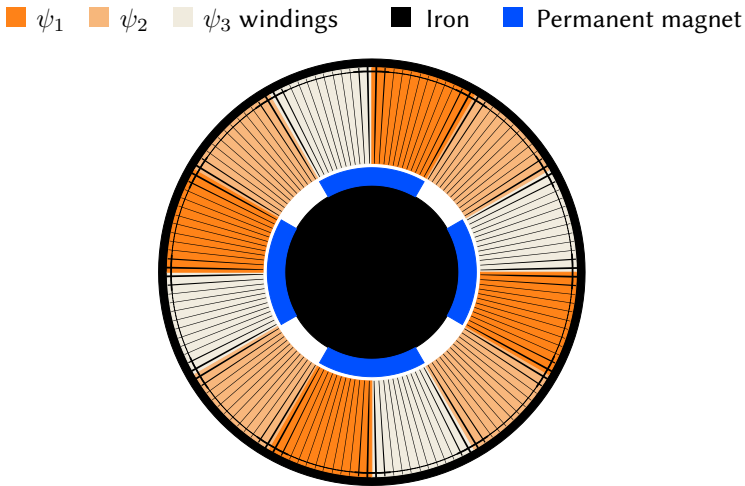


Fig. 6.13. Volume-constrained PMSM stator design: interpretation of intermediate materials.

by the magnetic field. The interpretation of the solution shown in Fig. 6.9 results in the topology represented in Fig. 6.13. The latter is composed of a large number of very small teeth.

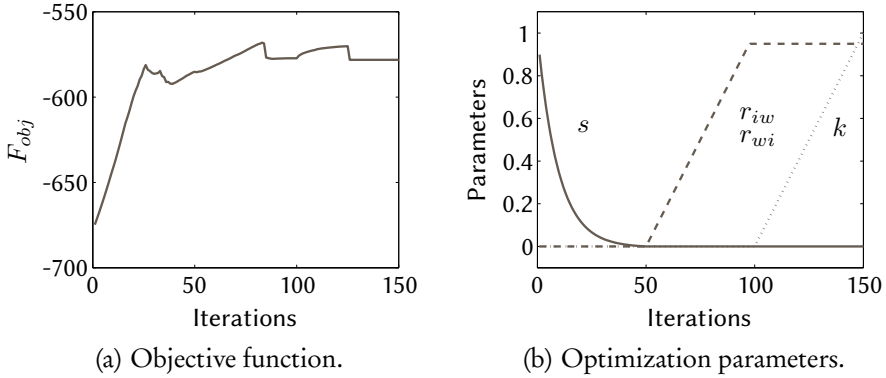


Fig. 6.14. Volume-constrained PMSM stator design: penalization of intermediate materials.

6.3.3 Intermediate material penalization

The alternative approach for removing intermediate materials is the penalization of their electromagnetic properties. Three penalizations were here successively carried out, as shown in Fig. 6.14(b). First, the cell composition constraint was modified by reducing the value of the parameter s . Then, iron-winding composites were penalized by the parameters r_{iw} and r_{wi} of the convexity mappings. Finally, anisotropic iron cells were removed by increasing the parameter k . The evolution of the objective function is plotted in Fig. 6.14(a).

As it was already the case in section 5.6.2, the parameter s is modified using an exponential evolution, since it has a greater impact at low values than at high values. A linear evolution would then lead to a slow penalization initially and a too fast penalization when s comes close to zero.

Fig. 6.15 represents the topologies produced at iteration 50 and 100, respectively after the cell composition constraint and the convexity mapping penalizations. The final design, shown in Fig. 6.16, produces a torque of 578.23 Nm. Compared to the initial topology leading to a torque of 672.16 Nm, a reduction of 14 % is observed. However, the manufacturability was greatly improved compared with the interpreted solution.

The penalized topology however achieves a torque that is 3.8 % better than the best local minimizer found in appendix A. This result highlights the fact that this optimization problem can hardly be handled using isotropic materials, as already shown in section 5.2.4.

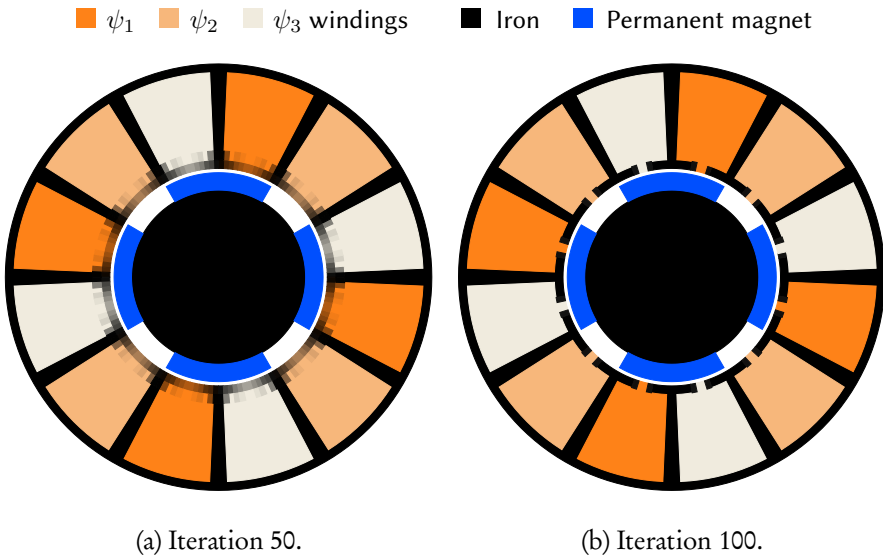


Fig. 6.15. Volume-constrained PMSM stator design: penalization of intermediate materials.

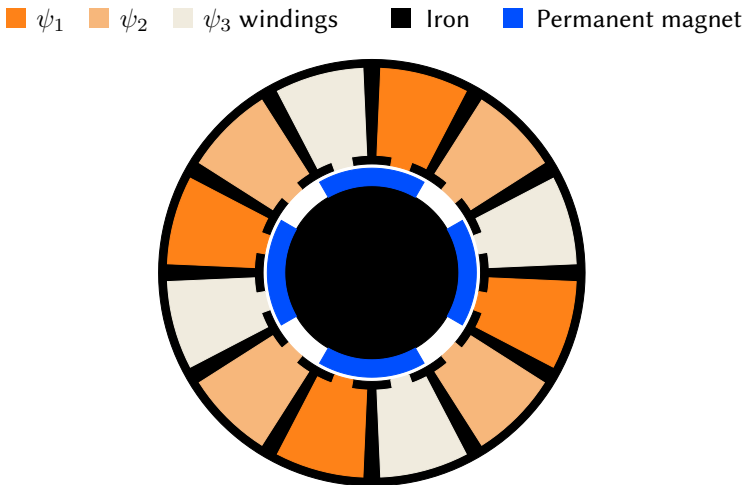


Fig. 6.16. Final design of the volume-constrained PMSM stator optimization;
 $\tau = 578.23 \text{ Nm}$.

6.4 Application and assessment

The anisotropic method can efficiently be applied to all the study cases covered in this thesis. This section shows how the penalization is carried out in each case and highlights the convincing results obtained by the method, taking saturation into account.

6.4.1 Iron and windings

Two study cases involve iron-winding optimizations: the stator design of the SRM and of the PMSM. Both cases are here optimized with and without a mass constraint.

Volume-constrained PMSM stator design

The optimization was performed in 350 iterations. Fig. 6.17(b) shows the evolution of the optimization parameters. First, the iron proportion is limited to $\rho_{i,max} = 0.8$ during 50 iterations. The $\rho_{i,max}$ value actually does not really matter, the convergence issue described in section 6.2 would equally be solved with for instance $\rho_{i,max} = 0.9$. Regarding the number of iterations, it turns out here in practice to be sufficient for the iron microstructures to take the right orientation. The constraint is next released and 150 iterations are realized to ensure the initial convergence. Fig. 6.18 shows the obtained design.

The penalization starts with the decrease of s , which reduces mixes of iron and windings by modifying the cell composition constraint. This penalization is not sufficient to remove iron-winding composites, which explains the use of the convexity-

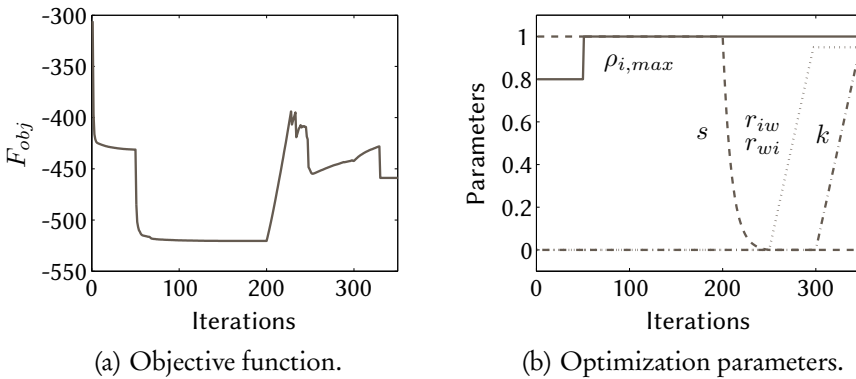


Fig. 6.17. Volume-constrained PMSM stator optimization.

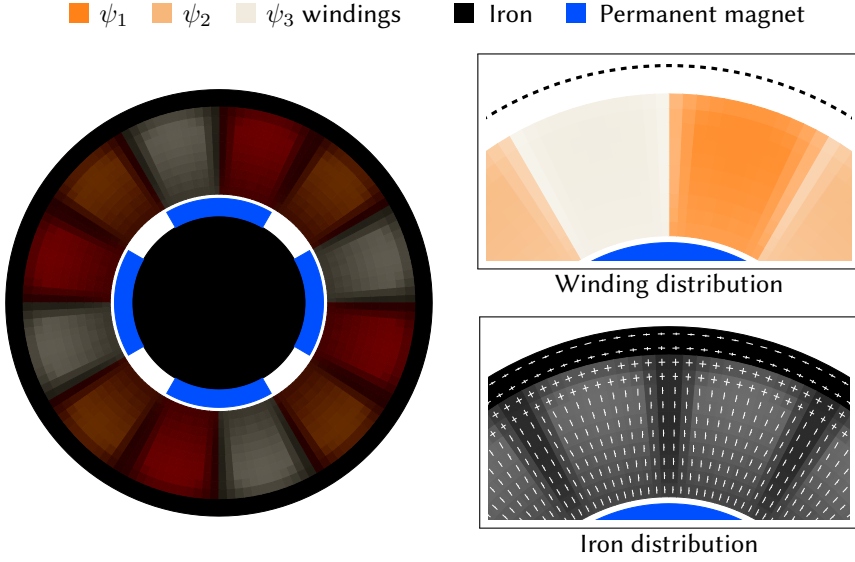


Fig. 6.18. Volume-constrained PMSM stator design: non-penalized solution (iteration 200).

mapping through the penalization parameters r_{iw} and r_{wi} . Fig. 6.19 shows the obtained topology after the iron-winding composite penalization. This result may at first seem strange, because of the closed slots. The latter are due to the fact that iron cells still contain a small proportion of air, even if this is not obvious in the figure. The anisotropy of these cells directs the field in the right direction which leads to an efficient topology.

The convexity mappings are ineffective for removing these iron-air composites. They will thus rather be removed by progressively making those composite isotropic, which is realized using the parameter k . The final design is then represented in Fig. 6.20.

This design achieves a torque of 458.89 Nm, which is 2.3 % better than the best local minimizers found by the isotropic algorithm in appendix A. In addition, the latter generally results in much worse solutions.

The computation time required for the whole optimization is 9 hours, which corresponds to 3802 finite element evaluations.

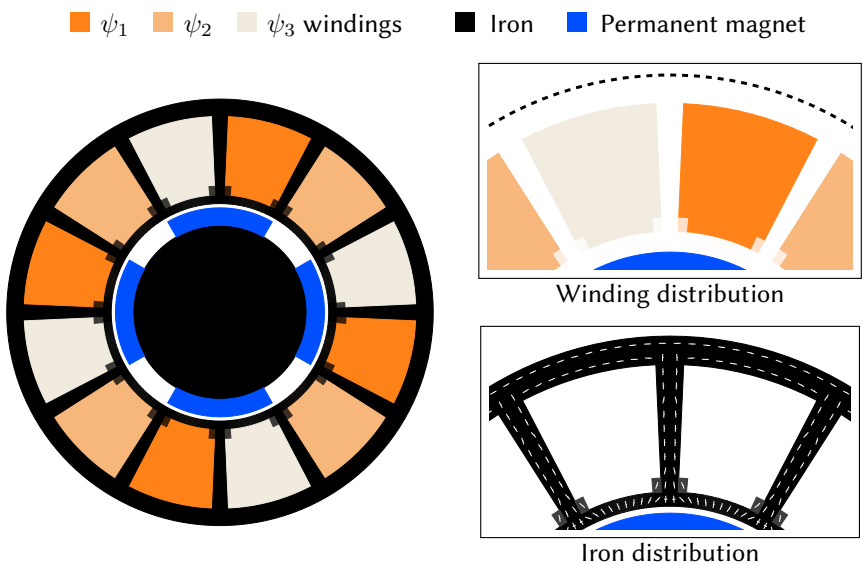


Fig. 6.19. Volume-constrained PMSM stator design: iron-winding composite penalization (iteration 300).

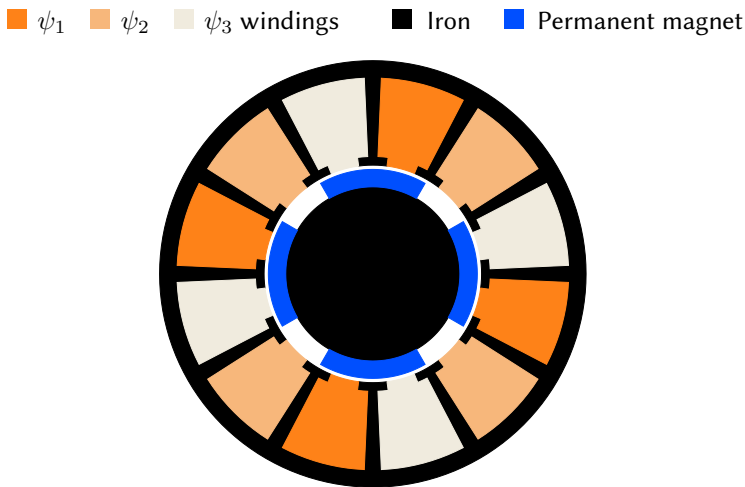


Fig. 6.20. Final design of the volume-constrained PMSM stator optimization;
 $\tau = 458.89 \text{ Nm}$.

Mass-constrained PMSM stator design

With a mass constraint, the initial topology obtained before the penalization is shown in Fig. 6.21. The evolutions of the objective function and of the optimization parameters are plotted in Fig. 6.22.

The penalizations of iron-winding composites and of the anisotropy are not sufficient to remove all composite materials in mass-constrained optimizations as shown in Fig. 6.23. Whereas the method never leaves isotropic iron-air composites in volume-constrained optimizations, since a solid iron cell always leads to a better objective function, those composites are here created to satisfy the mass constraints. 50 additional iterations are thus realized in order to penalize them with the parameter r_i . The computation is however limited to 5 hours and 30 minutes for 4963 finite element evaluations.

The final design is shown in Fig. 6.23(b). It produces a torque of 119.63 Nm, which is 1.5 % lower than the best local minimizer found in appendix A. However, only two of those local minimizers produce a higher torque, which corresponds to a probability of 0.2 %.

The solution obtained here is characterized by the same kind of air holes as those noticed in section 5.3. They are again due to the penalization process which makes the shape of the objective function concave and thereby prevents the iron circuit

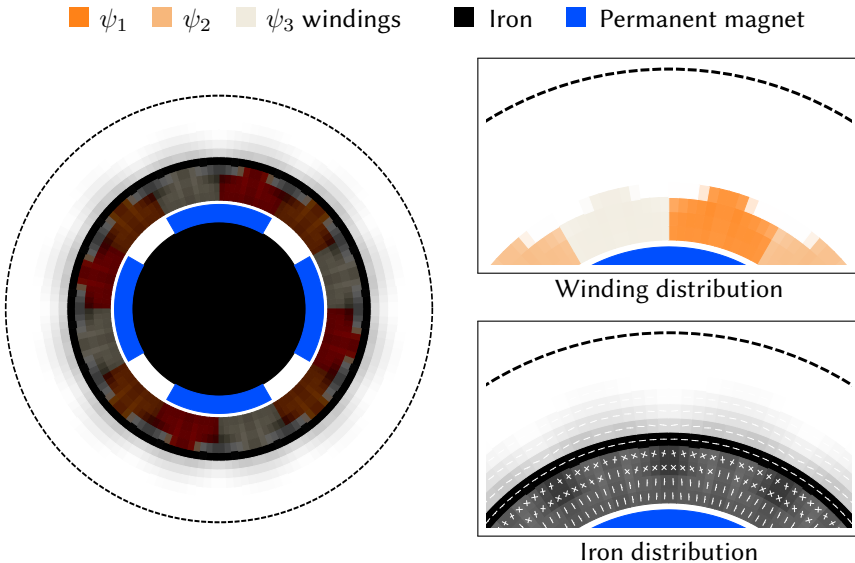


Fig. 6.21. Mass-constrained PMSM stator design: non-penalized solution (iteration 200).

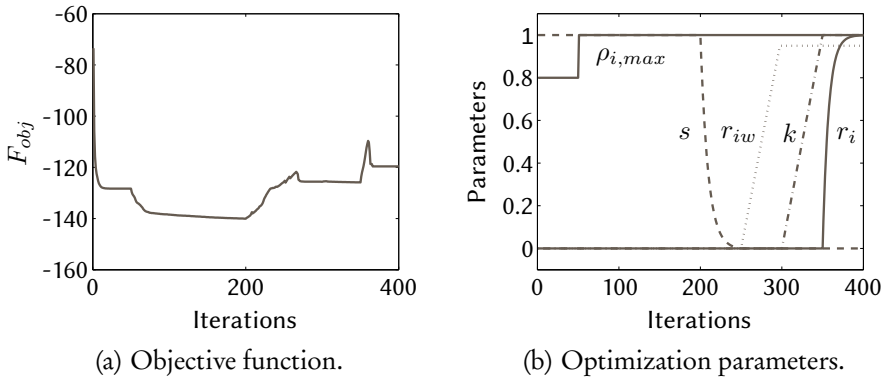


Fig. 6.22. Mass-constrained PMSM stator optimization.

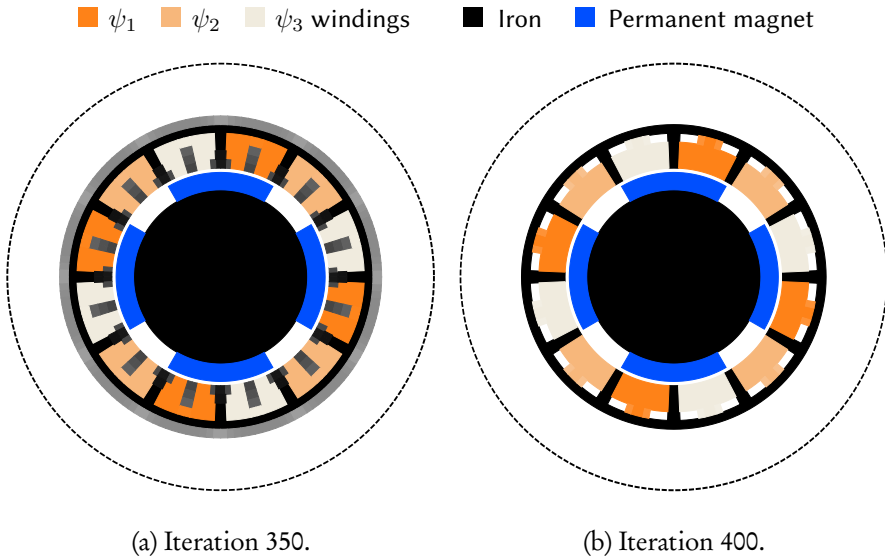


Fig. 6.23. Mass-constrained PMSM stator design: (a) iron-winding composite and anisotropy penalizations, (b) final design; $\tau = 119.63$ Nm.

from moving and fitting to the winding distribution.

Volume-constrained SRM stator design

The optimization of the SRM is performed exactly in the same way as the PMSM optimization, which underlines the effectiveness of the method in being applicable to different optimization problems.

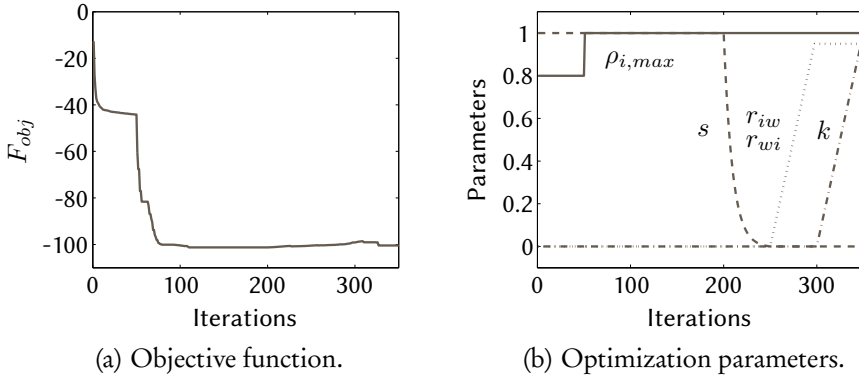


Fig. 6.24. Volume-constrained SRM stator optimization.

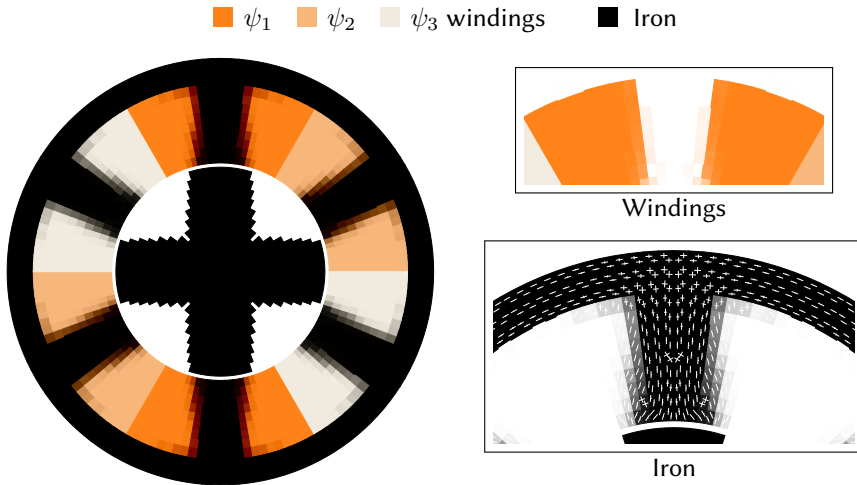


Fig. 6.25. Volume-constrained SRM stator design: non-penalized solution (iteration 200).

The evolutions of the objective function and of the optimization parameters are shown in Fig. 6.24. Fig. 6.25 represents the design obtained before penalization and Fig. 6.26 the final design. The method actually achieves to produce the same design as the best local minimizer found in appendix A, which tends to show that this is probably the optimal solution. The optimization is performed in 14 hours and 30 minutes and 4580 finite element evaluations.

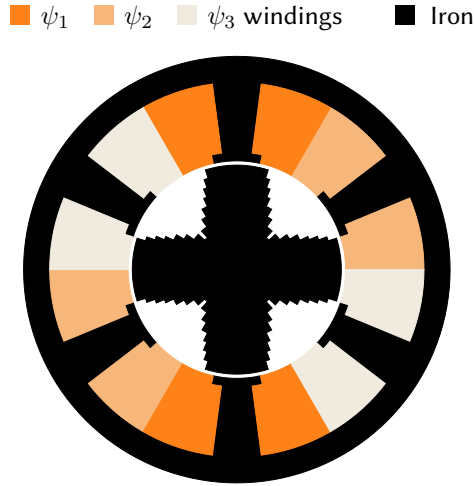


Fig. 6.26. Final design of the volume-constrained SRM stator optimization; $\tau = 100.42$ Nm.

Mass-constrained SRM stator design

The mass-constrained SRM stator optimization, with the same penalization parameter evolutions than for the PMSM optimization, leads to the objective function evolution shown in Fig. 6.27 and the non penalized design represented in Fig. 6.28. The final topology is shown in Fig. 6.29 and achieves a torque of 11.56 Nm. Compared to the best local minimizer found in appendix A, a gain of 6 % can be noticed. It is besides better than the solution obtained with the isotropic method in section 5.4 which achieves a torque of 11.27 Nm. The optimization requires 4564 finite element evaluations and a computation of 3 hours.

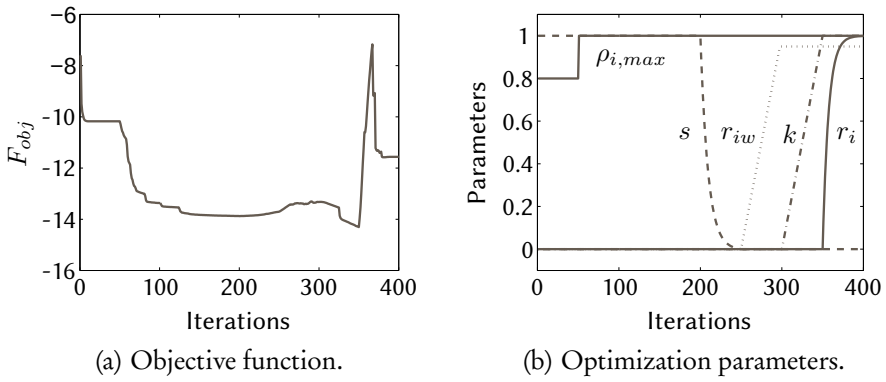


Fig. 6.27. Mass-constrained SRM stator optimization.

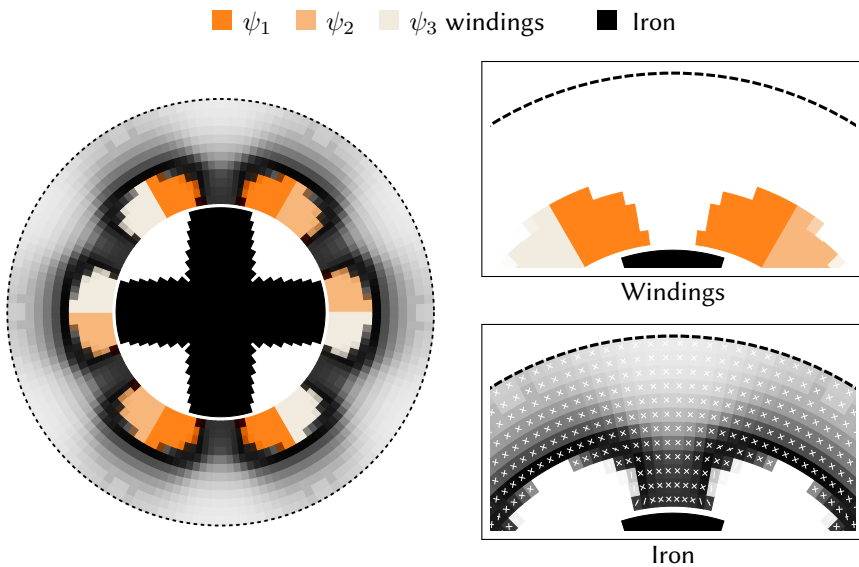


Fig. 6.28. Mass-constrained SRM stator design: non-penalized solution (iteration 200).

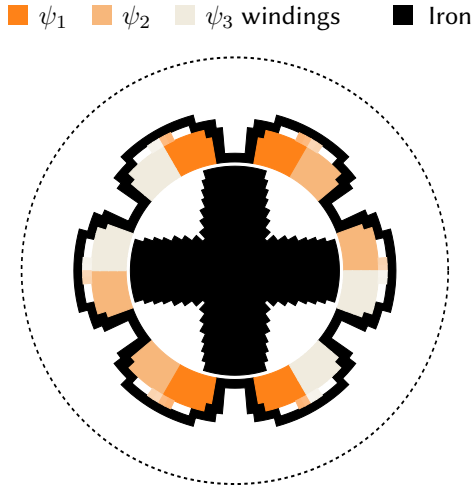


Fig. 6.29. Final design of the mass-constrained SRM stator optimization; $\tau = 11.56$ Nm.

6.4.2 Iron and permanent magnets

The study case related to the distribution of iron and permanent magnets is the design of the PMSM rotor. This optimization is always performed with a mass constraint since the method otherwise fills the whole design space with permanent magnets.

In section 5.5, the development of the isotropic method for iron-magnet optimizations highlights two main differences in comparison with the other optimizations.

- Iron-magnet composites must be promoted in order to ensure a global initial convergence, these composites having otherwise poor properties.
- The geometric mapping must be replaced by a reluctivity mapping. The sensitivity mapping has indeed an impact on the promotion of iron-magnet composites and this promotion is not sufficient with the geometric mapping.

The introduction of anisotropic materials has no influence on iron-magnet composites. They must thus again be promoted by setting the convexity mapping parameter r_{mi} to -10 and by implementing a reluctivity mapping. The obtained solution is then represented in Fig. 6.30.

Fig. 6.31 shows the evolutions of the objective function and of the penalization parameters. The penalization again starts after 200 iterations, first with the param-

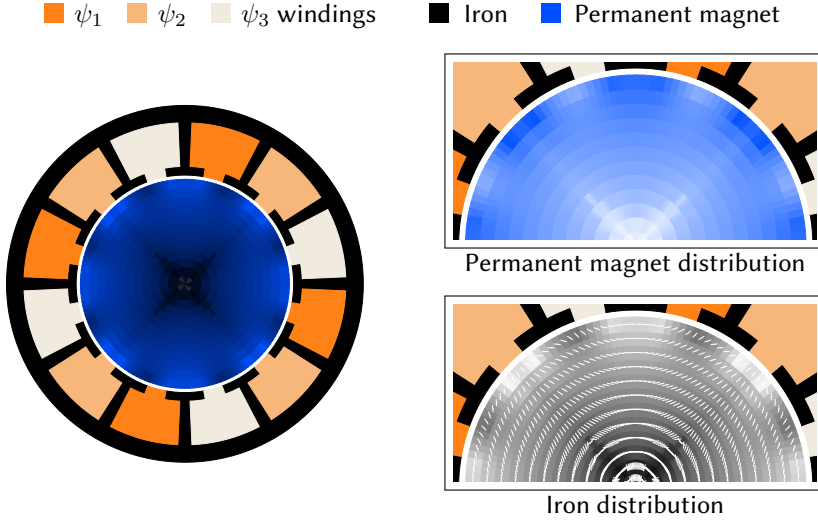


Fig. 6.30. Mass-constrained PMSM rotor design: initial promotion of iron-magnet composites (iteration 200).

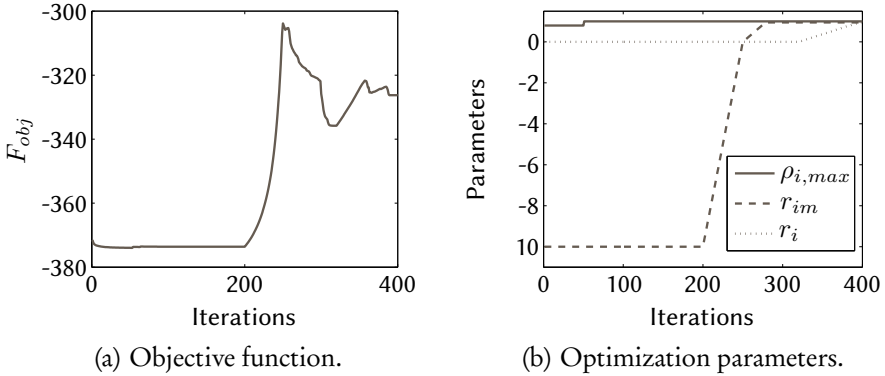


Fig. 6.31. Mass-constrained PMSM rotor optimization.

eter r_{im} . The latter is increased to 0.95 in 80 iterations. It is maintained at 0.95 until iteration 300, and the reluctivity mapping is then replaced by the geometric mapping. Fig. 6.31(a) highlights the convergence speed improvement achieved by the geometric mapping at iteration 300.

Fig. 6.32 shows the topology obtained at iteration 320, i.e. when iron-magnet composites have disappeared and when the solution has correctly converged thanks to the geometric mapping. In order to remove iron-air composites, the convexity-mapping parameter r_i is eventually increased, which leads to the final design shown

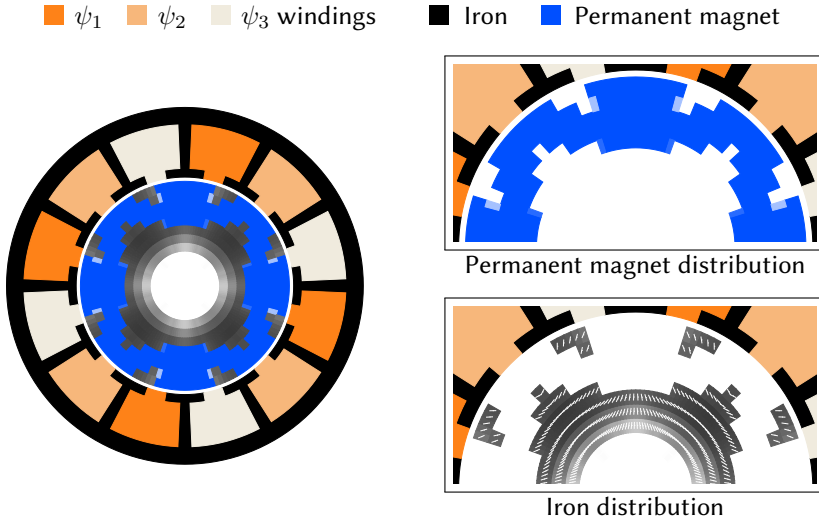


Fig. 6.32. Mass-constrained PMSM rotor design: penalization of iron-magnet composites (iteration 320).

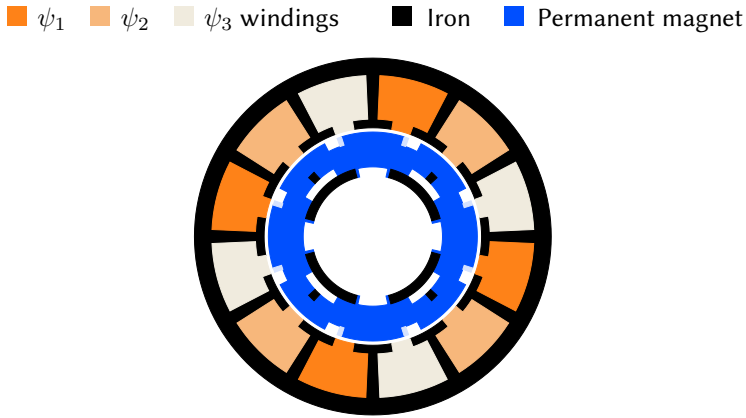


Fig. 6.33. Final design of the mass-constrained PMSM rotor optimization; $\tau = 326.7$ Nm.

in Fig. 6.33 after a total computation time of 24 hours and 30 minutes and 7498 finite element evaluations. Note that the anisotropy penalization is not required for this optimization problem.

The final design produces a torque of 326.7 Nm. Compared to the best local minimizer found in appendix A, it represents a gain of 1 %, which again highlights the efficiency of the anisotropic method. It is besides better than the solution obtained with the isotropic method which achieves a torque of 325.91 Nm.

6.4.3 Iron

This section shows that the anisotropic method can directly be applied to the design of ferromagnetic parts, without any special attention. This is a major improvement in comparison with the isotropic method, for which two topologies subject to convexity-constraints must be considered. In addition, the anisotropic method is able to handle mass-constrained iron optimizations.

Volume-constrained SRM rotor design

Without mass constraint, the optimization is actually very simple since penalization is not required to make iron-air composite disappear. The optimization is realized in 400 iterations, the proportion of iron being limited to 0.8 during the 200 first iterations, which leads to a computation time of 6 hours corresponding to 4236 finite element evaluations.

Fig. 6.35 shows the topology produced when the iron proportion is still limited by the maximum iron proportion constraint. The final design is represented in Fig. 6.36. It achieves a torque of 62.15 Nm, which is less than 1 % behind the best local minimizer found in appendix A. The result obtained with isotropic materials is actually 0.5 % better.

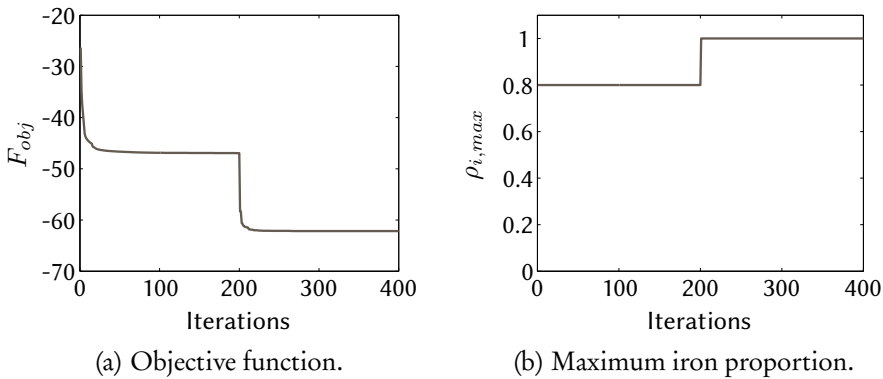


Fig. 6.34. Volume-constrained SRM rotor optimization.

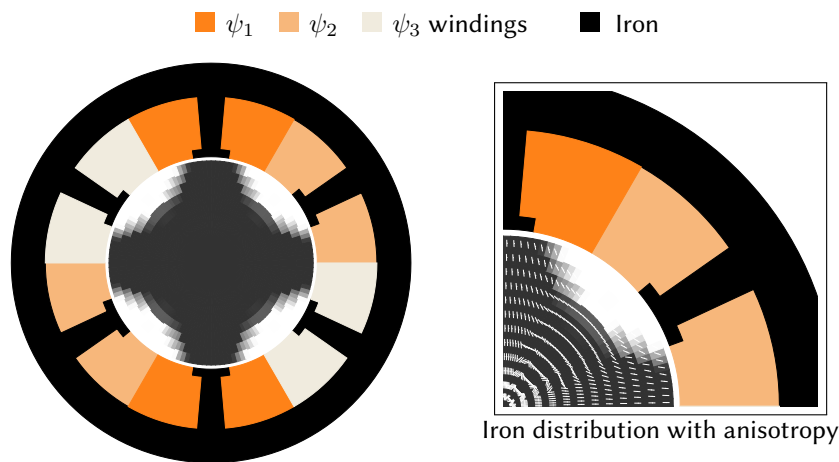


Fig. 6.35. Volume-constrained SRM rotor design: maximum iron proportion constrained to 0.8 (iteration 200).

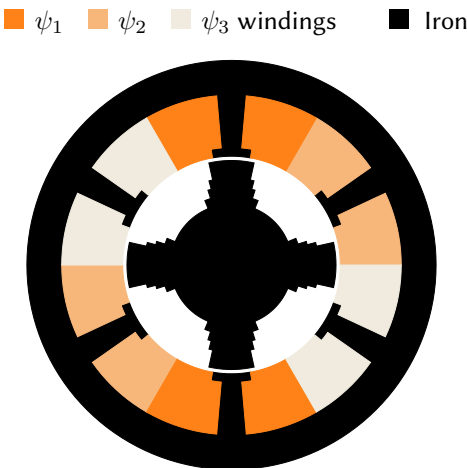


Fig. 6.36. Final design of the volume-constrained SRM rotor optimization; $\tau = 62.15$ Nm.

Mass-constrained SRM rotor design

The mass constraint actually has no effect during the first part of the optimization in which the iron proportion is limited to 0.8. Indeed, it is not activated because of this maximum iron proportion constraint: the mass of the topology shown in Fig. 6.35 is indeed lower than 20 kg and this topology is thus again obtained at iteration 200.

The mass constraint becomes effective from iteration 200, when the maximum iron proportion constraint is released. The topology obtained at iteration 250 is represented in Fig. 6.38. Contrary to the volume-constrained optimization, penalization must be carried out to remove iron-air composites. First, the iron anisotropy is progressively reduced using the parameter k , which leads to the topology shown in Fig. 6.39(a). Then, the remaining iron-air composites are eliminated using the convexity-mapping parameter r_i . The whole optimization is performed in 10 hours and required 7768 finite element evaluations.

The final design is represented in Fig. 6.39(b). It achieves a torque of 60.78 Nm, which is 1 % better than the best local minimizer found in appendix A. The anisotropic method efficiency is thus again underlined on this last study case.

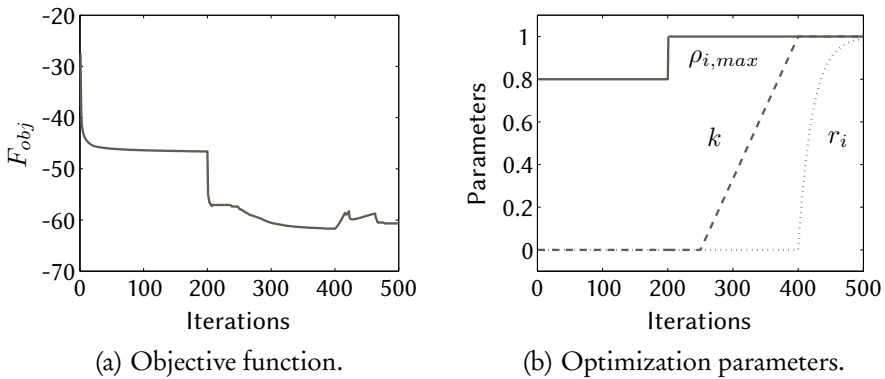


Fig. 6.37. Mass-constrained SRM rotor optimization.

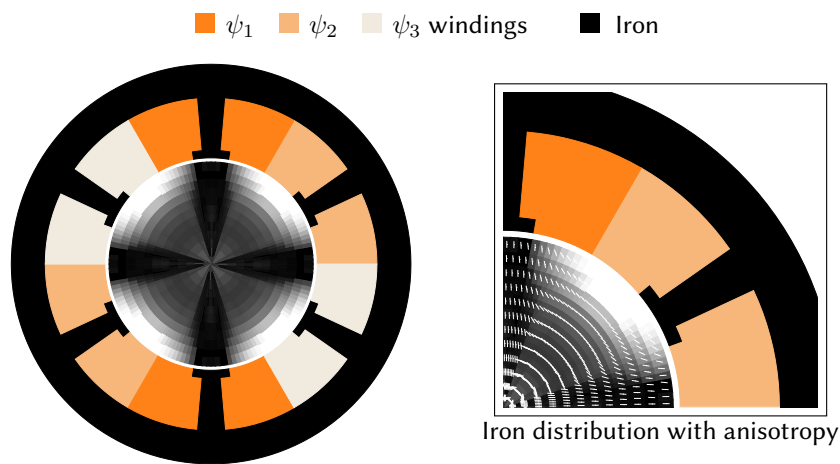


Fig. 6.38. Mass-constrained SRM rotor design: non-penalized solution (iteration 250).

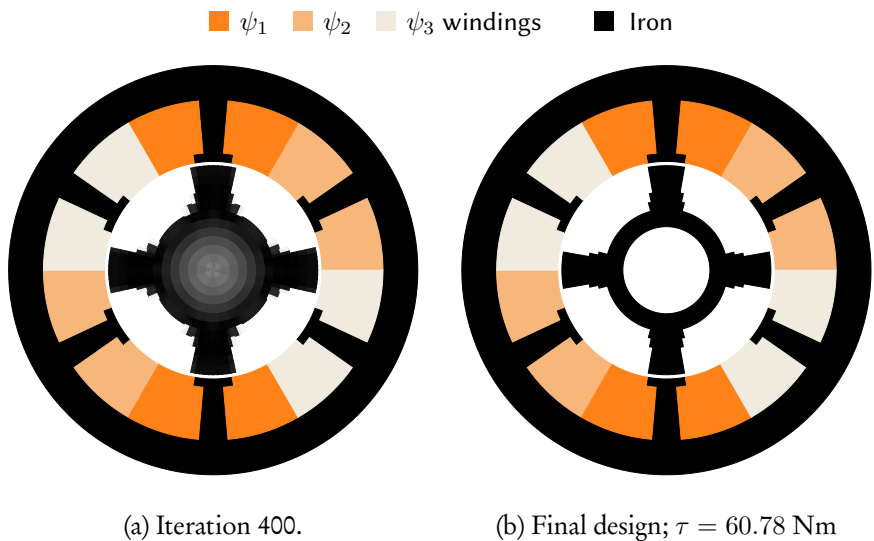


Fig. 6.39. Mass-constrained SRM rotor design.

6.5 Conclusion

In conclusion, the anisotropic method is based on an anisotropic iron microstructure whose differences with the isotropic microstructure lie in its cross shape and in the optimization of its orientation. Five design variables are required to represent the iron distribution. Four of them are the components of a vector whose direction is the microstructure orientation and whose amplitude is the iron proportion. A shape factor besides determines the repartition of iron along the two microstructure axes.

Compared to the isotropic method, the implementation of the anisotropic method is more complex, especially because anisotropy must be introduced in the finite element model. In addition, the computation time is increased: more iterations are required for the convergence because of the additional design variables and the finite element computations are besides slower in order to take anisotropy into account.

The anisotropic method however benefits from two main assets. First, it is able to successfully handle the seven optimization problems related to the SRM and PMSM study cases, whereas the isotropic method was only effective for four of them¹. Indeed, the method succeeds in problems where the lack of convexity is related to the necessity of anisotropic materials, which results with isotropic methods in the apparition of anisotropic structures (distribution of different solid materials in adjacent cells). The second asset concerns the interpretation of composite materials. The latter was only possible under some conditions with the isotropic method. Indeed, since the microstructure orientation is then aligned with the magnetic field, the latter must remain constant in a given cell, the microstructure orientation being otherwise undetermined. This is not the case with anisotropic materials: all the intermediate materials can be interpreted as real composite materials.

The efficiency of the anisotropic method is highlighted through its application to the seven problems related to the SRM and PMSM study cases in the last section. The methods provides results that are better than all the local minimizers found in appendix A in five problems on seven. In addition, when a better topology could be found in appendix A, the torque difference observed is lower than 1.5 %. This difference is not significant in view of the low mesh resolution which clearly limits the accuracy both in the solution representation and in the finite element computations.

¹The volume-constrained and mass-constrained optimizations of PMSM stator and the mass-constrained optimization of the SRM rotor could not be performed with the isotropic method.

The anisotropic method is thus an interesting alternative to the isotropic method when the latter is ineffective on the considered problem, or if a more general approach is preferred.

Mesh refinement

7

In the topology optimization methods developed in this thesis, the finite element model and the distribution of materials are both based on the same mesh. Introducing mesh refinement is thus particularly promising. Indeed, it would both improve the graphical resolution of the topology and the accuracy of the finite element model. In addition, a reduction of the mesh element sizes around the material interfaces would increase the resolution while limiting the number of elements and thus the computation time. With a mesh refinement process, a topology optimization method becomes therefore able to determine accurately the optimal shape and dimensions of a device, in addition to its topology. Mesh refinement has already been proposed for the design of mechanical structures by e.g. Maute and Ramm (1995), but has never been applied in the electromagnetic domain.

In order to progressively refine a solution, the optimization is performed several times between which the mesh refinement algorithm is applied. The problem is thus first optimized on an initial coarse mesh. Based on the obtained solution, the mesh refinement algorithm provides a new mesh on which the optimization is again realized. The process continues until a sufficient resolution is achieved, which usually requires one or two refinement steps.

The chapter is divided in four parts. First, the mesh refinement algorithm is described. Then, it is applied to the design of the SRM stator. It is next assessed in terms of the computation time reduction and of the gain of resolution. Eventually, a correspondence is highlighted between solutions obtained by mesh refinement and those derived by an interpretation of intermediate materials.

7.1 Mesh refinement algorithm

In order to determine which zones have to be refined, the algorithm is based on the derivatives of the material distribution. This distribution is actually differentiated in the x and y directions using the finite element model, which provides the derivatives on the mesh nodes as explained in section 3.3.4.

The algorithm is here illustrated on the 1D example shown in Fig. 7.1(a). It is composed of four cells and involves an interface between windings and iron, with the black dots representing the mesh nodes. It does not involve intermediate materials, since mesh refinement is performed based on final results to which penalization has been applied. The derivatives provided by the finite element model actually correspond to a material distribution in which the material proportions evolve linearly between the cell centres, as represented by the dashed line in Fig. 7.1(a) for the winding distribution. These derivatives are written $\mathbf{n}_{i,x}$, $\mathbf{n}_{i,y}$, $\mathbf{n}_{w,x}$, $\mathbf{n}_{w,y}$, $\mathbf{n}_{m,x}$, $\mathbf{n}_{m,y}$, $\mathbf{n}_{a,x}$ and $\mathbf{n}_{a,y}$, with the subscripts i , w , m and a referring respectively to iron, windings, magnets and air, and the subscripts x and y to the axis of the differentiation.

The norms of the derivatives corresponding to each material are then summed:

$$n_j = \sqrt{n_{i,x,j}^2 + n_{i,y,j}^2} + \sqrt{n_{w,x,j}^2 + n_{w,y,j}^2} + \sqrt{n_{a,x,j}^2 + n_{a,y,j}^2}, \quad (7.1)$$

with j the node number. This sum is represented in Fig. 7.1(b): $\mathbf{n}$ is only different of zero at the interface node. The computation of the element sizes for the refined mesh is then realized in two steps.

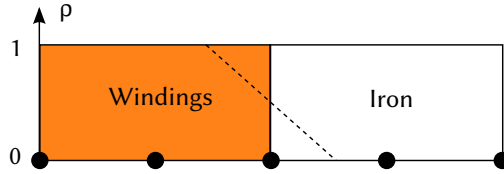
First, they are deduced from the inverse of $\mathbf{n}$ which actually corresponds to a division by two of the cell sizes around the interfaces. Indeed, the material distribution is assumed to evolve linearly between the cell centres for the derivative computation, as explained in section 3.3.4. The inverse of the derivative of one material around an interface corresponds thus to the local cell size. Since an interface is composed of two materials, the inverse of their derivative sum leads to a cell size divided by two.

The element sizes of the first step are written $\hat{s}_j$ and are bounded by s_{min} and s_{max} , respectively the minimum and maximum tolerated element sizes:

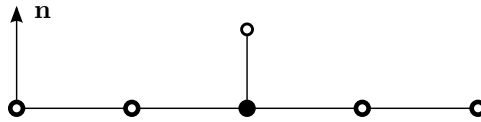
$$\hat{s}_j = \min \left(\max \left(\frac{1}{n_j}, s_{min} \right), s_{max} \right). \quad (7.2)$$

Fig. 7.1(c) illustrates this computation for the 1D example.

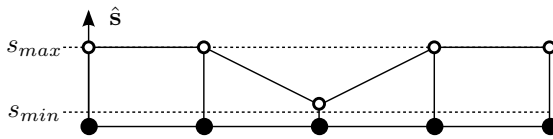
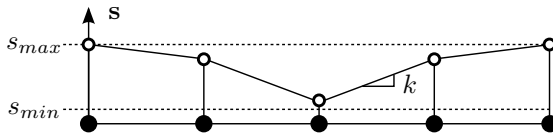
The second step consists in imposing a maximum slope k to the element size distribution in order to avoid obtaining small elements close to large elements and



(a) 1D material distribution.



(b) Sum of the derivative norms on the mesh nodes.

(c) New element sizes $\hat{s}$: first step.(d) New element sizes s : second step.**Fig. 7.1.** Mesh refinement algorithm: illustration on a 1D interface.

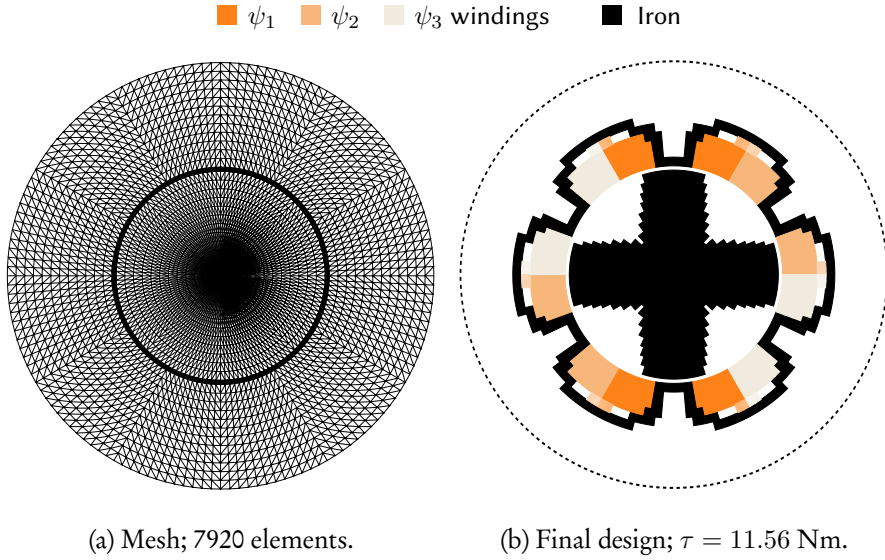


Fig. 7.2. Initial topology on a coarse mesh.

therefore ensure a good mesh quality:

$$s_j = \min_l (\hat{s}_l + k d_{jl}), \quad (7.3)$$

with d_{jl} the distance between node j and node l and the parameter k set to 0.2. The distribution of the element sizes is represented in Fig. 7.1(d). Eventually, the refined mesh is generated using GMSH, a finite element mesh generator developed by Geuzaine and Remacle (2009). The latter supports the definition of the element size distribution based on the previous mesh and assumes linear evolutions of between the mesh nodes.

7.2 Application

The mesh refinement algorithm is here illustrated for the mass-constrained stator design of the SRM. The study case is optimized using the anisotropic method and the initial design has already been obtained in section 6.4.1. The latter is represented with its mesh in Fig. 7.2 and its optimization required a computation time of 3 hours.

The material distribution is first differentiated in the x and y directions. Fig. 7.3 shows the obtained distribution of $\mathbf{n}$ and Fig. 7.4 the resulting element sizes $\hat{s}$ and s .

Based on the latter, GMSH generates the refined mesh represented in Fig. 7.5(a).

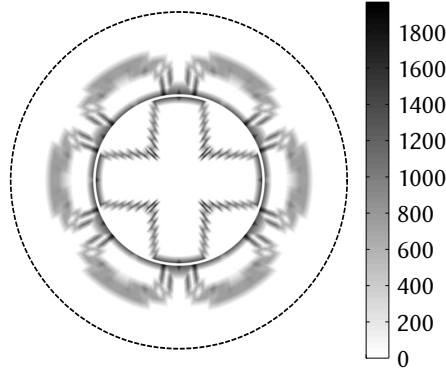


Fig. 7.3. Sum of the derivative norms for the mass-constrained SRM stator design.

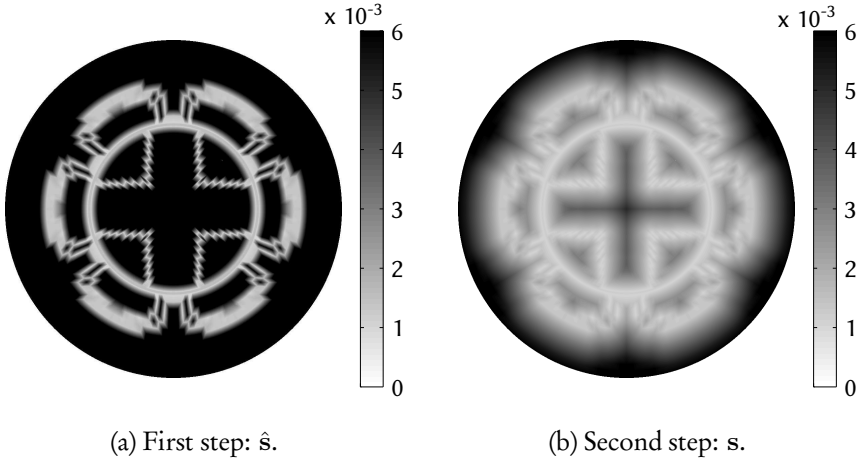


Fig. 7.4. Element sizes for the mass-constrained SRM stator design.

With this new mesh, the optimization leads to the design shown in Fig. 7.5(b) and requires a computation time of 13 hours and 45 minutes. A significant improvement of the graphical resolution of the solution can be highlighted, which besides results in an increase of the objective function.

A second refinement step can be performed from this solution, which leads to the mesh and the design shown in Fig. 7.6. The topology resolution is then further improved and a gain on the objective function is again noticed. The computation time however raises to 3 days and 2 hours.

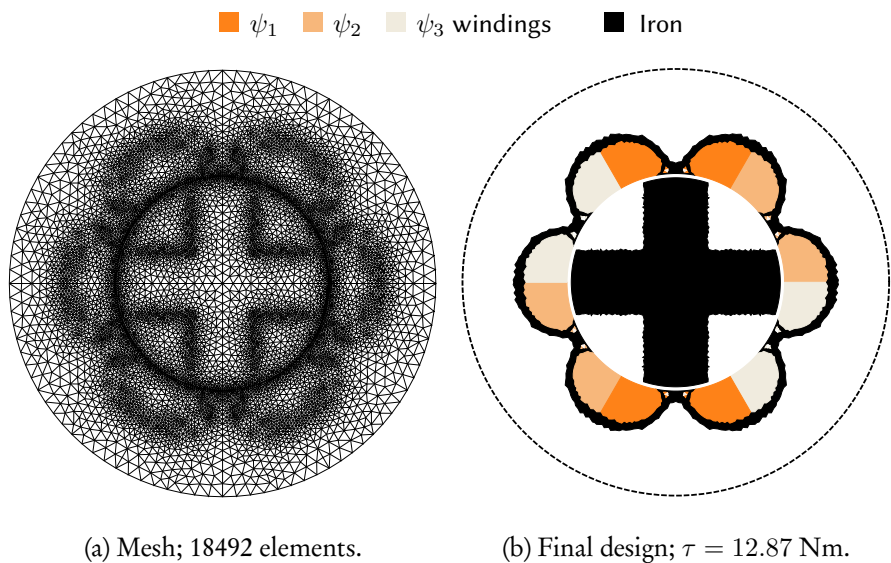


Fig. 7.5. Topology produced after the first mesh refinement.

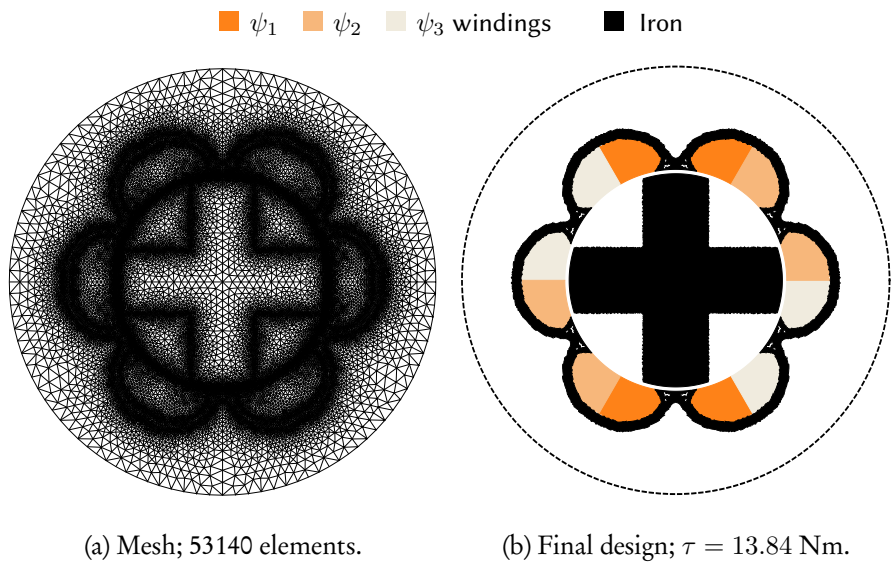


Fig. 7.6. Topology produced after the second mesh refinement.

7.3 Mesh refinement assessment

In this section, the mesh refinement algorithm results are assessed from two points of view. First, the reduction of the computation time is estimated in comparison with a one-step optimization on a homogeneous fine mesh corresponding to the same resolution. Next, the gain of resolution is examined through a comparison with a design obtained on a homogeneous mesh whose element sizes were adjusted to achieve the same computation time.

7.3.1 Computation time reduction

The assessment of the computation time reduction can only be conducted considering the first refinement step, i.e. in comparison with the topology shown in Fig. 7.5. Indeed, it requires obtaining the same resolution on a homogeneous mesh, which is impossible for the second step design since it leads to a very large number of mesh elements and a huge computation time.

The mesh used for the assessment cannot actually be represented because of its very small elements. The latter were set everywhere to the s_{min} value used for the first refinement step, which corresponds thus to the size of the smallest elements of the mesh shown in Fig. 7.5(a). The latter had 18492 elements and this number here raises to 72680, which already highlights the inefficiency of using homogeneous fine meshes.

The optimization results in a similar topology, as shown in Fig. 7.7, which has about the same resolution since the mesh element sizes are roughly equivalent at the material interfaces¹. However, the computation time raises to 3 days and 17 hours, which is about 5 times more than the sum of the computation times required with the mesh refinement algorithm, i.e 16 hours and 45 minutes (3h + 13h45).

In addition, two research directions could be explored to further reduce the computation time required with the mesh refinement algorithm. Indeed, the optimization is at each mesh refinement step restarted from scratch, which suggests that some iterations could be saved if the previous solution was reused. A first approach could be to map the previous non-penalized result on the new mesh and use it as starting point for the next optimization, which would certainly reduce the number of iterations required for obtaining the new non-penalized solution. Another approach would be to limit the optimization to the interface regions and fix the material distribution in the other regions.

¹The resolution is actually slightly better, since the cell size does not reach s_{min} at all the interfaces in Fig. 7.5.

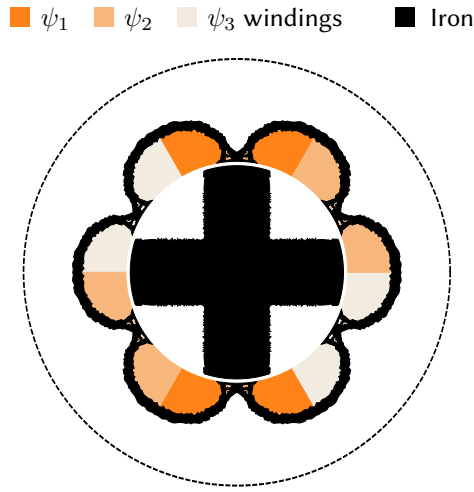


Fig. 7.7. Topology with an homogeneous fine mesh; $\tau = 13.48$ Nm; 72680 elements.

7.3.2 Gain of resolution

The optimization was also realized on a homogeneous mesh whose resolution was adjusted in order to reach about the same computation time as the refinement algorithm. This mesh is shown in Fig. 7.8 and has 23220 elements, which corresponds to cells having a mean surface of 1.3 mm^2 , whereas this surface goes down to 0.25 mm^2 in the air gap and around the interfaces for the refined mesh in Fig. 7.5.

The optimization on the homogeneous mesh leads to the design represented in Fig. 7.8, with a computation time of 16 hours and 23 minutes. A reduction of resolution is noticed in comparison with the topology obtained after the first refinement step.

The assessment of the computation time reduction led to a solution which required, by chance, the same computation time as the complete two-step refinement process. Therefore, a comparison between Fig. 7.6 and Fig. 7.7 is also relevant and besides underlines an improvement of the resolution and an increase of the objective function thanks to the refinement process.

7.4 Correspondence with interpreted solutions

This section intends to show the correspondence between results derived from the interpretation of intermediate materials and those obtained by mesh refinement and

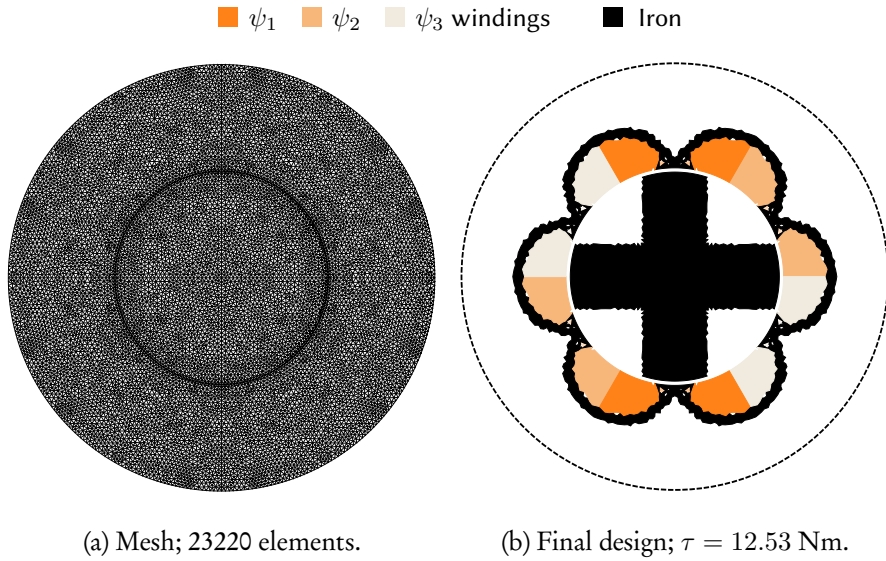


Fig. 7.8. Topology produced with the same computation time.

penalization. Indeed, refining the mesh enables the method to create the thin iron structures characterizing the interpreted solutions.

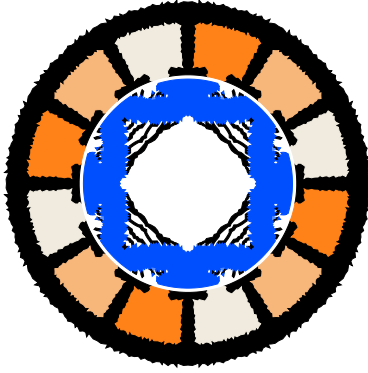
The mesh refinement algorithm was applied to the optimization of the PMSM rotor using the anisotropic method. The result is shown in Fig. 7.9(a) for one refinement step. The latter must be compared with the topology obtained in chapter 6 in Fig. 6.32, for which the interpretation of intermediate materials gives the solution shown in Fig. 7.9(b)¹. A striking correspondence is noticed between the two topologies, which shows that both approaches lead in the end to the same results. The interpretation approach is faster in terms of computation time, but requires some analysis from the designer. The mesh refinement approach is on the contrary much longer but directly provides finished results.

7.5 Conclusion

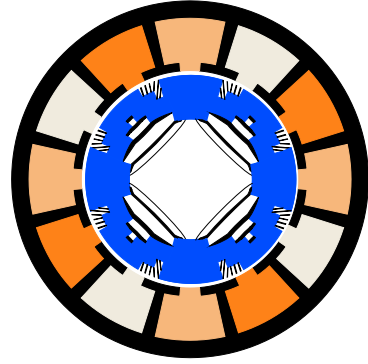
Thanks to the mesh refinement process, the topology optimization method is able to provide finished results that don't need further adjustments. Indeed, the process ensures both the graphical resolution of the topology and the accuracy of the finite element model. In the case iron composite structures are required in the optimal de-

¹The torque announced for this interpreted solution is actually the torque of the original one, since there is a physical correspondence between these two topologies. In practice, a small difference would be expected if the interpreted solution torque was evaluated based on a fine finite element mesh.

■ ψ_1
■ ψ_2
■ ψ_3 windings
 ■ Iron
 ■ Permanent magnet



(a) $\tau = 329.5$ Nm.



(b) $\tau \approx 335.8$ Nm.

Fig. 7.9. Correspondence between solutions obtained by penalization and mesh refinement and solutions derived by interpretation. (a) Penalized result with one refinement step; (b) interpreted result without mesh refinement.

sign, it besides allows the method to create those structures, whereas they had to be derived by an interpretation of intermediate materials without mesh refinement. In addition, it leads to a significant reduction of the computation time compared with the use of homogeneous fine meshes. The efficiency and the interest for implementing topology optimization methods it thus greatly improved, since they become able to accurately determine both the shape and dimensions of a device, in addition to its topology.

Method exploitation

8

This chapter focuses on the study of three optimization problems. The first one concerns the design of switched reluctance motors for which the optimization aims to determine the curves of the optimal torque-to-mass and torque-to-volume ratios according to the motor mass and volume. The second problem is related to the design of permanent magnet synchronous motors and has the same objective, but involves a specification regarding the torque ripple of the motor. The last problem concerns the optimization of two magnetic thrusts composed of permanent magnets in order to maximize the speed of a moving part travelling between them. All the results presented in this chapter take saturation into account.

8.1 Design of a switched reluctance motor

This section intends to study the optimal topologies maximizing, on one hand, the SRM torque-to-mass ratio and, on the other hand, its torque-to-volume ratio. The goal is to derive the characteristics of the maximum torque-to-mass and torque-to-volume ratios according to the SRM mass and volume. In both cases, the optimization is faced with an issue regarding the determination of the optimal air-gap radius, written R_a . Indeed, the latter is initially fixed with the definition of the rotor design space and therefore cannot be optimized by the algorithm.

The SRM study is divided in two parts. The former is dedicated to the description of the optimization method, the latter to the determination of the optimal topologies maximizing the SRM torque-to-mass and torque-to-volume ratios.

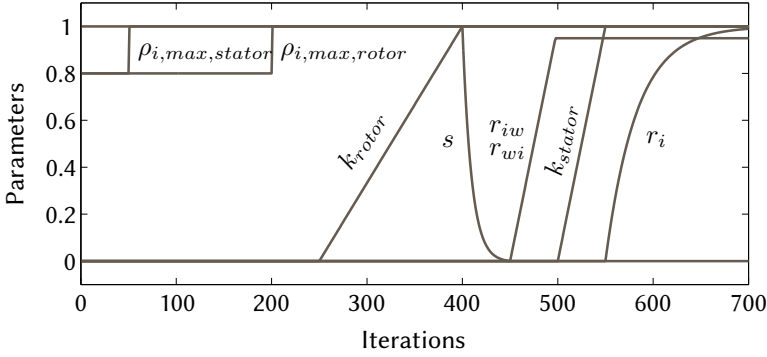


Fig. 8.1. Parameter evolutions for the mass-constrained SRM optimization.

8.1.1 Optimization method

The SRM is optimized using the anisotropic method and one refinement step. The optimization process is here illustrated for the mass- and volume-constrained problems. In the first case, the mass is limited to $m = 45$ kg whereas the motor outer radius is restricted to $R_m = 0.1$ m in the second case. The air-gap radius is set to $R_a = 0.05$ m in both cases, which corresponds to the original description of the SRM study case that can be found in chapter 4.

Mass-constrained problem

The evolutions of the optimization parameters are represented in Fig. 8.1. The maximum iron proportion is limited to 0.8 during 50 iterations for the stator ($\rho_{i,max,stator}$) and during 200 iterations for the rotor ($\rho_{i,max,rotor}$). 50 iterations are then realized to converge to the non penalized design shown in Fig. 8.2.

At iteration 250, the rotor anisotropy is penalized by increasing k_{rotor} during 150 iterations. The solution obtained after this penalization is represented in Fig. 8.3(a). The iron-winding composites are then removed by a penalization first on the cell composition constraint and next with the mapping parameters r_{wi} and r_{iw} , which leads to the solution shown in Fig. 8.3(b). Eventually, the penalization of the stator anisotropy and the increase of the mapping parameter r_i makes the remaining iron-air composites disappear. The final solution is represented in Fig. 8.4 and Fig. 8.5 shows the evolution of the objective function during the optimization. The parameter evolutions were derived in practice by trial and error, all penalizations being first executed in 50 iterations and next lengthened if necessary. Exponential evolutions are used for some parameters when their impact is observed to be too low

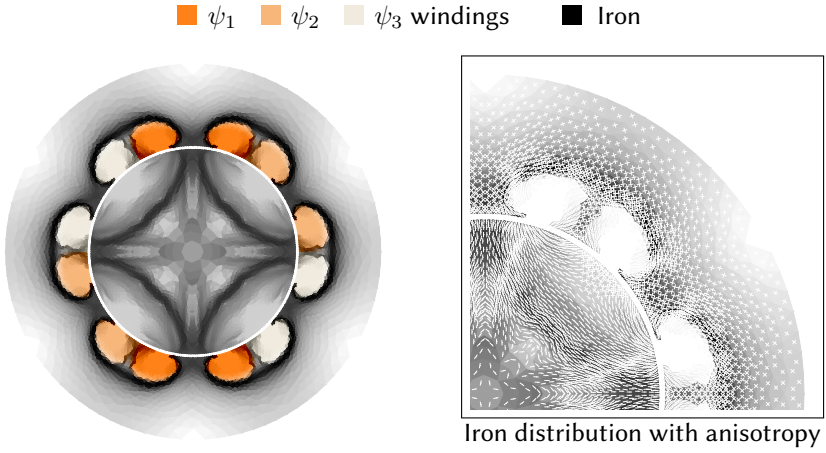


Fig. 8.2. Mass-constrained SRM design with $R_a = 0.05$ m and $m = 45$ kg: non penalized solution (iteration 250).

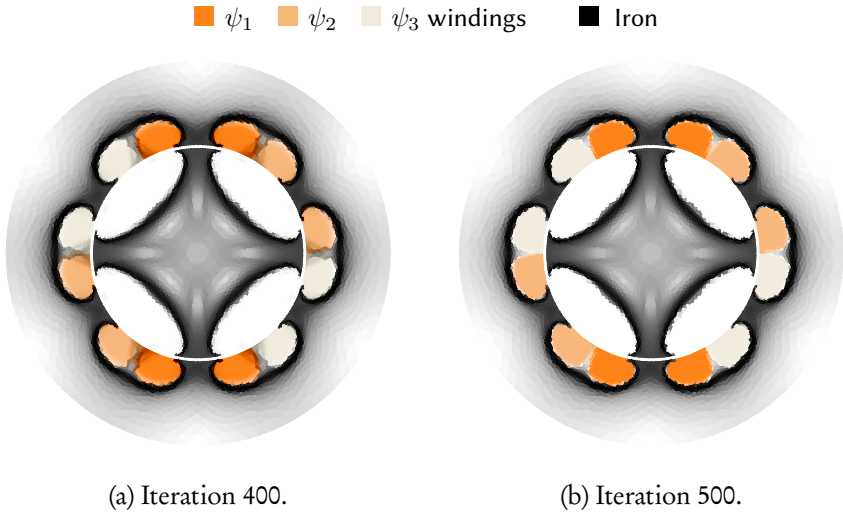


Fig. 8.3. Mass-constrained SRM design with $R_a = 0.05$ m and $m = 45$ kg: penalization.

initially and too strong eventually. The computation time for the mass-constrained SRM optimization ranges from 2 to 3 days.

The order in which the penalization of intermediate materials is carried out on the stator and the rotor proved to provide efficient results. However, there may be other effective penalization orders.

The topology shown in Fig. 8.3 is characterized by thin iron structures, which

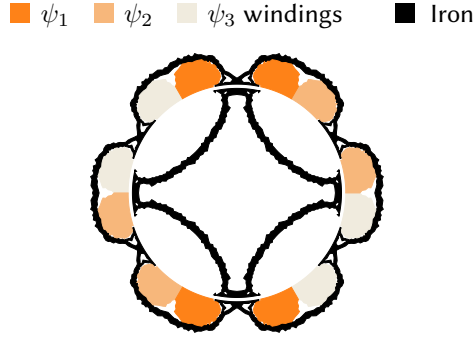


Fig. 8.4. Final design of the mass-constrained SRM optimization with $R_a = 0.05$ m and $m = 45$ kg; $\tau = 5.83$ Nm.

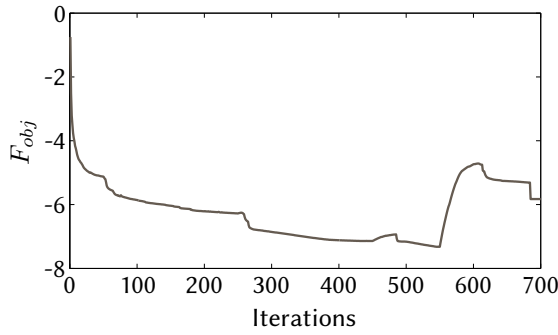


Fig. 8.5. Mass-constrained SRM optimization with $R_a = 0.05$ m and $m = 45$ kg.

suggests that the saturation of iron could be important. The magnetic flux density B is represented in Fig. 8.6 for the two rotor positions. It reaches 2 T in some cells, which is quite high for a saturable ferromagnetic material. This is due to the saturation curve given in chapter 4 that actually overestimates the properties of iron.

Volume-constrained problem

The volume of the SRM is constrained by the definition of the stator design space, and particularly its outer radius written R_m . Indeed, maximizing the motor torque leads the method to completely fill the stator design space, since there is no mass constraint limiting the quantity of material. The optimization method is here illustrated with $R_m = 0.1$ m, which is the original value given in the study case description in chapter 4.

The volume-constrained problem is handled by the parameter evolutions shown

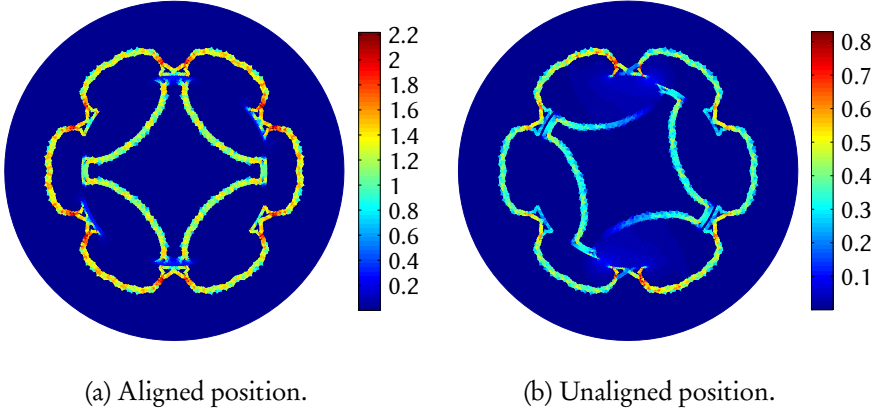


Fig. 8.6. Magnetic flux density B in the optimal SRM topology.

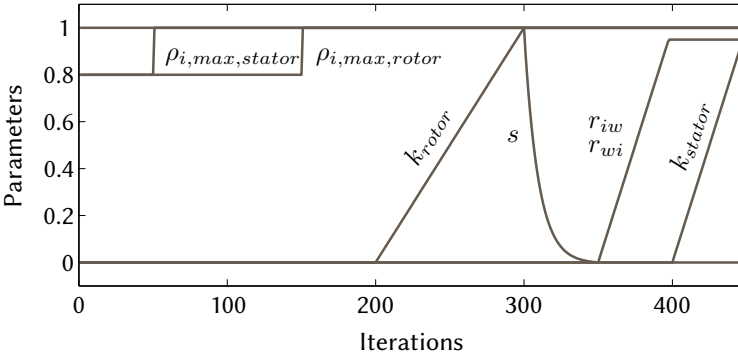


Fig. 8.7. Parameter evolutions for the volume-constrained SRM optimization.

in Fig. 8.7. Compared to mass-constrained optimizations in which the mass constraint reduces the convergence speed, the optimization is here performed in 450 iterations. The maximum iron proportion constraint is released after 50 iterations for the stator and 150 for the rotor. The non penalized design, obtained at iteration 200, is represented in Fig. 8.8.

The rotor anisotropy is then penalized which leads to the solidification of the rotor, as shown in Fig. 8.9(a). Intermediate materials on the stator are eventually removed using successively the cell composition constraint, the convexity mapping and the anisotropy penalization. The final design is represented in Fig. 8.9(b).

The evolution of the objective function during the optimization is plotted in Fig. 8.10. Even if the number of iterations was reduced compared to mass-

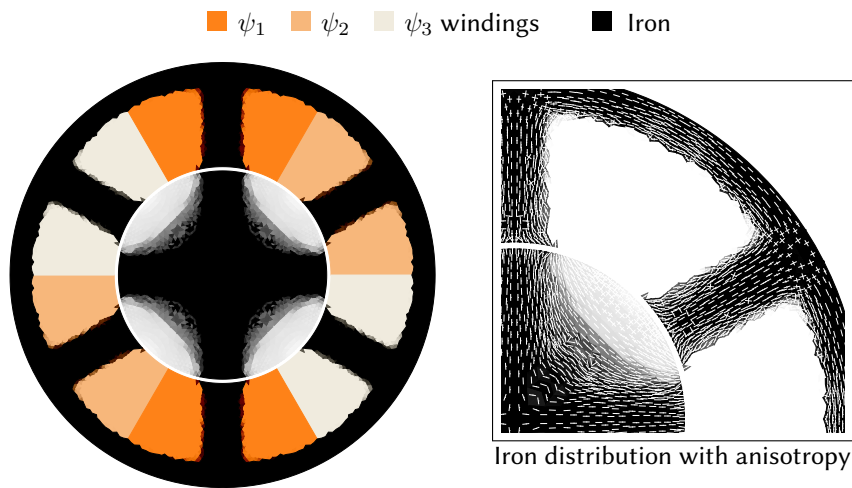


Fig. 8.8. Volume-constrained SRM design with $R_a = 0.05$ m and $R_m = 0.1$ kg: non penalized design.

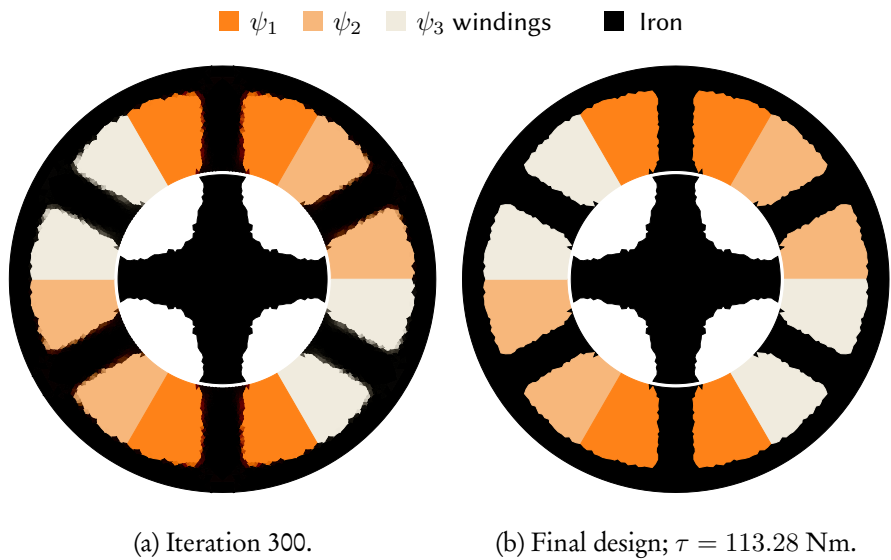


Fig. 8.9. Volume-constrained SRM design with $R_a = 0.05$ and $R_m = 0.1$.

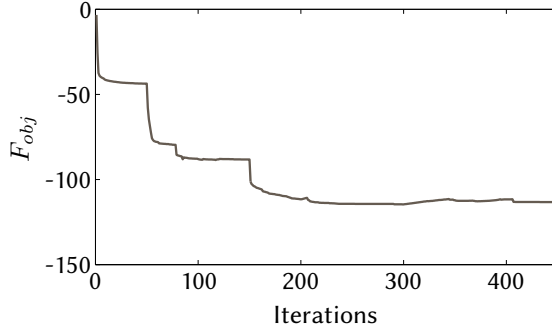


Fig. 8.10. Volume-constrained SRM optimization with $R_a = 0.05$ m and $R_m = 0.1$ kg.

constrained optimizations, the computation time still averages 3 days.

8.1.2 Torque-to-mass and torque-to-volume ratios

This section aims to determine the characteristics of maximum torque-to-mass and torque-to-volume ratios according to the SRM mass and volume. Several optimizations are thus performed with different masses and volumes. In addition, for each mass or volume, different air-gap radii R_a must be considered in order to derive its optimal value. These optimizations result thus in two-entry tables providing the torque according to the air-gap radius R_a and the mass m or the volume V .

Torque-to-mass ratio

Table 8.1 provides the optimization results of the torque-to-mass ratio study. For each mass, the highest torque is emphasized in bold, which highlights the obvious fact that the optimal rotor radius becomes higher as the motor mass increases. The torque-to-mass ratios corresponding to Table 8.1 are besides plotted in Fig. 8.11, except for $R_a = 0.025$ m since none of the obtained solutions is optimal. The figure shows for instance that an air-gap radius of 50 mm is better than 40 mm or 60 mm for masses comprised between about 40 kg and 75 kg.

Fig. 8.12 shows the evolution of the optimal torque-to-mass ratio according to the mass. This figure highlights a roughly linear evolution which can be approximated by the linear regression:

$$\frac{\tau_{opt}}{m} = 0.0028 m. \quad (8.1)$$

Table 8.1. SRM torque according to the mass m and the air-gap radius R_a .

$m \backslash R_a$	0.025	0.04	0.05	0.06	0.075
0	0	0	0	0	0
30	2.08	2.55	2.04	1.84	1.6
45	4.31	5.46	5.83	4.83	3.68
60	7.87	10.01	10.5	10.16	9.05
90	15.31	22.47	22.96	23.19	21.6
120	24.63	35.82	38	39.78	38.45
150	33.71	51.74	55	57.66	60.07

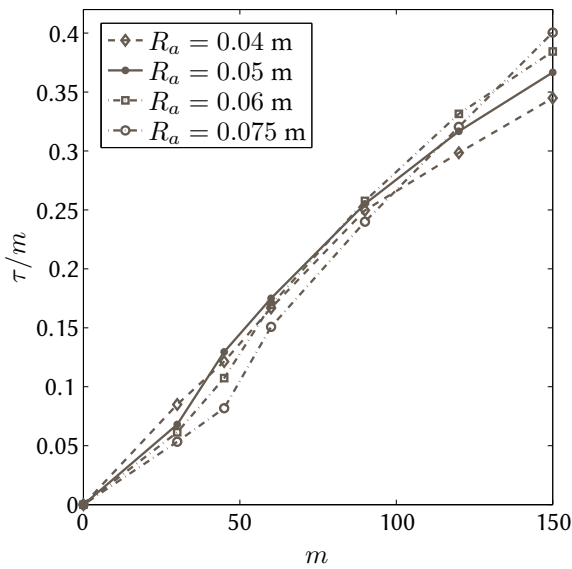


Fig. 8.11. Evolutions of the SRM torque-to-mass ratio for four air-gap radii.

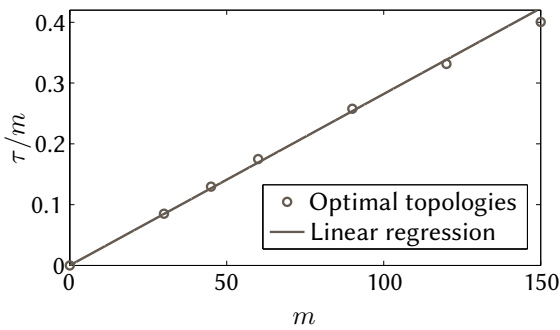


Fig. 8.12. Evolution of the SRM optimal torque-to-mass ratio.

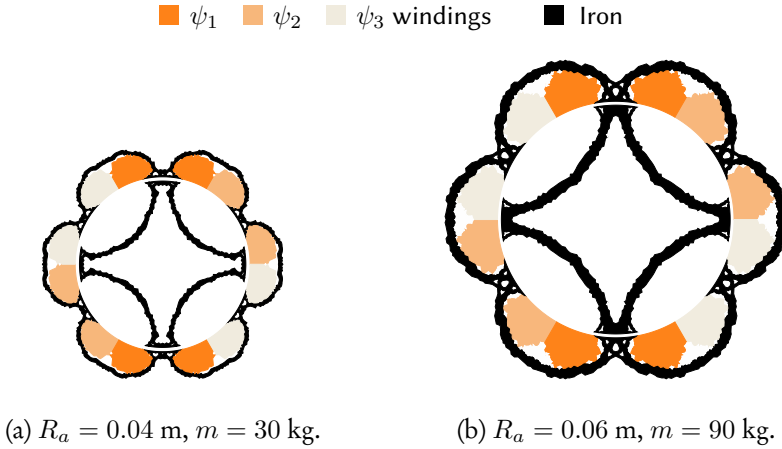


Fig. 8.13. Optimal SRM topologies for $m = 30$ kg and $m = 90$ kg.

It is besides interesting to study how the optimal topologies evolve with the SRM mass, in order to determine whether it would be possible to derive a scaling law describing the optimal solution for any motor mass. The similarities between the solutions represented in Fig. 8.4 and Fig. 8.13 (especially solution (a)) indeed suggest that they could be simply scaled to infer the optimal solutions for other masses.

However, this supposition turns out to be wrong. Fig. 8.14 indeed shows the torque evolutions when scaling these three topologies to achieve masses from 0 to 150 kg. The figure highlights non linear evolutions that reach the optimal torque-to-mass ratio line only for the mass at which the corresponding optimal topology was found. It is thus unfortunately impossible to deduce a scaling law for the SRM problem, the optimization process is always required to obtain the optimal topology for a given mass.

Torque-to-volume ratio

The same approach was followed to determine the optimal torque-to-volume ratio characteristic. The number of optimizations was however reduced to 3 air-gap radii and 5 volumes because of the long computation time required for all the problems handled in this chapter. The results are presented in Table 8.2 and the torque-to-volume ratio curves are plotted in Fig. 8.15. The latter shows that an air-gap radius of 50 mm is optimal for volumes comprised between 0.025 and 0.045 m³.

Fig. 8.16 shows that the topologies achieving the optimal torque-to-volume ratios cannot either be described by a scaling law. Three topologies are indeed scaled leading to different torque evolutions that are similar to those obtained in the previ-

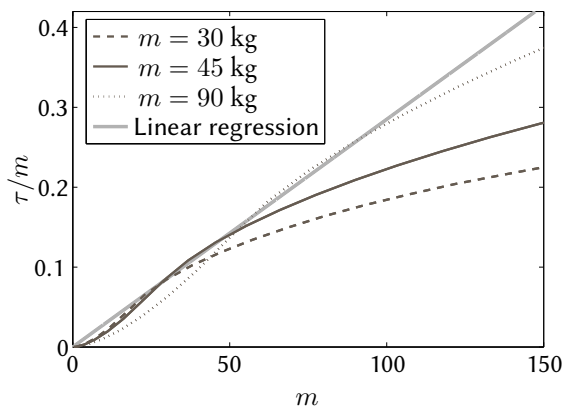


Fig. 8.14. Torque-to-mass ratio evolutions of the scaled optimal topologies for the mass-constrained SRM optimization.

Table 8.2. SRM torque according to the volume V and the air-gap radius R_a .

V	$R_m \setminus R_a$	0.04	0.05	0.06
0.015	0.07	25.51	15.84	x
0.02	0.08	41.81	39.07	21.9
0.031	0.1	105.35	113.28	98.14
0.045	0.12	186.75	219.96	219.21
0.062	0.14	x	367.58	385.85

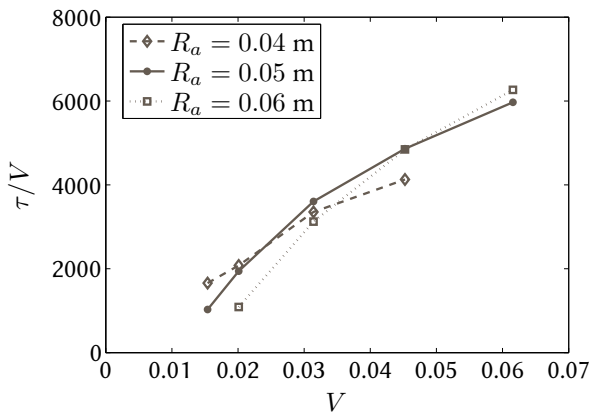


Fig. 8.15. Evolutions of the SRM torque-to-volume ratio for four air-gap radii.

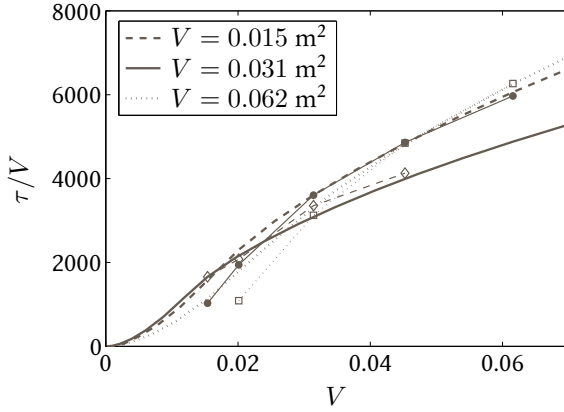


Fig. 8.16. Torque evolutions of the scaled optimal topologies for the volume-constrained SRM optimization.

ous section, which shows that the optimal topology actually evolves with the motor volume.

8.2 Design of a permanent magnet synchronous motor

The main difference between the PMSM and the SRM optimizations lies in the consideration of the torque ripple. Indeed, as described in chapter 4, the PMSM optimization is subject to a specification stating that it must be limited to 10 % of the mean torque. In order to evaluate this torque ripple and to take into account the variation of the stator currents with the rotor position, the magnetic energy must be computed in several rotor positions, which highly increases the computation time.

The PMSM is here only studied with a mass constraint since volume-constrained optimizations lead the method to completely fill the rotor with permanent magnets.

8.2.1 Optimization method

The maximum torque ripple specification can hardly be included as a constraint in the optimization problem, since constraints must be expressed on the design variables. It is thus rather handled by a penalty in the objective function. The latter is written:

$$F_{obj} = -\tau_{average} + \alpha \text{Var}(\tau_i), \quad (8.2)$$

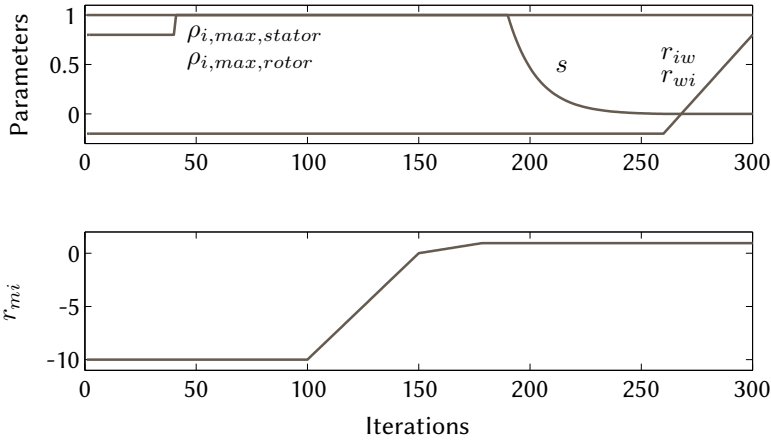


Fig. 8.17. Parameter evolutions for the mass-constrained PMSM optimization.

with α the penalization factor and $\text{Var}(\tau_i)$ the variance of the torque evolution. As explained in chapter 4, τ_i represents the mean torque between the mechanical positions θ_i and θ_{i+1} .

In order to achieve a torque ripple of 10 %, the value of α is regulated as follows during the optimization:

- if $\tau_{\text{ripple}} > 0.1 \tau_{\text{average}}$, then $\alpha^{(t+1)} = 1.1 \alpha^{(t)}$,
- otherwise, $\alpha^{(t+1)} = 0.97 \alpha^{(t)}$,

with $\alpha^{(t)}$ the value of the penalization factor at iteration t . The PMSM optimization is also realized using the anisotropic method. However, because of the complex objective function, mesh refinement could not be carried out in order to limit the computation time. The mesh was nevertheless generated by GMSH which leads to a better resolution, since regular square meshes concentrate uselessly a lot of cells in the rotor centre. In addition, iron-air composites are interpreted rather than being penalized, since the low mesh resolution prevents the method from accurately adjusting the thickness of the iron structures. These composites are thus tolerated in the final design.

The optimization method is here illustrated for an air-gap radius set to $R_a = 0.05$ m and a motor mass limited to 75 kg. The evolution of the optimization parameters is plotted in Fig. 8.17. The maximum iron proportion constraint is released after 40 iterations. The solution obtained at iteration 100, in which iron-magnet composites are highly promoted, is represented in Fig. 8.18. The latter composites

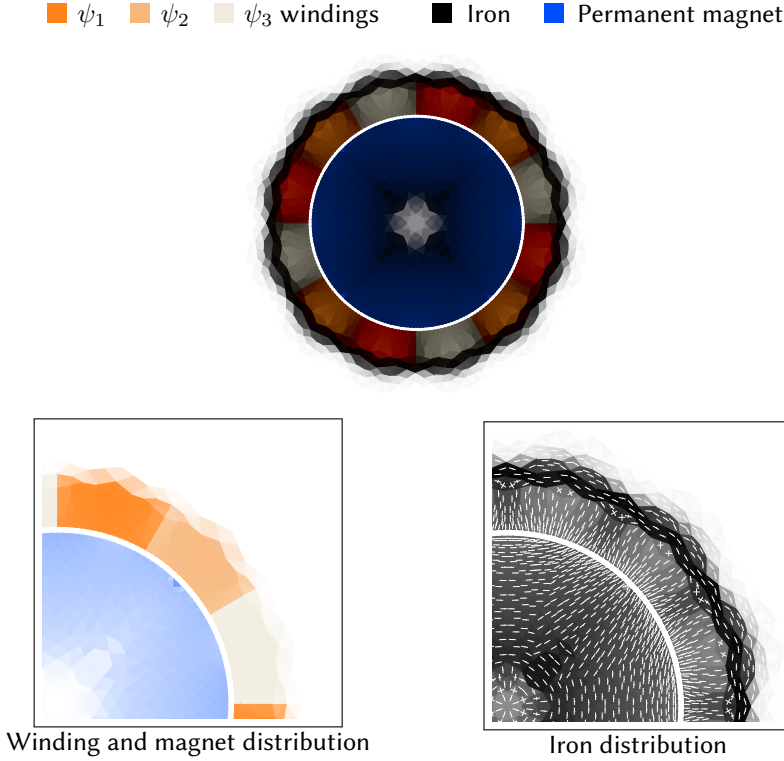


Fig. 8.18. Mass-constrained PMSM design with $R_e = 0.05$ m and $m = 75$ kg: promotion of iron-magnet composites (iteration 100).

are then penalized using the r_{mi} parameter, and Fig. 8.19 shows the solution at iteration 200. The reluctivity mapping is then replaced by a geometric mapping to increase the convergence speed. Eventually, iron-winding composites are removed thanks to the cell composition constraint parameter s and the convexity mapping parameters r_{iw} and r_{wi} . The final design is represented in Fig. 8.20. The evolutions of the objective function and of the penalization parameter α related to the torque ripple are plotted in Fig. 8.21. The PMSM optimization requires a computation time comprised between 1 and 3 days.

8.2.2 Torque ripple issue

The consideration of the torque ripple specification actually raises two issues. The former is related to the formulation of the specification inside the objective function, and the latter to the discretization in the computation of the torque evolution according to the rotor position.

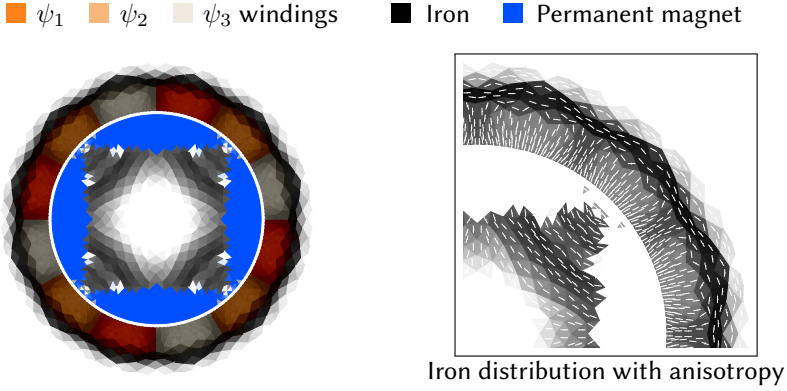


Fig. 8.19. Mass-constrained PMSM design with $R_e = 0.05$ m and $m = 75$ kg (iteration 200).

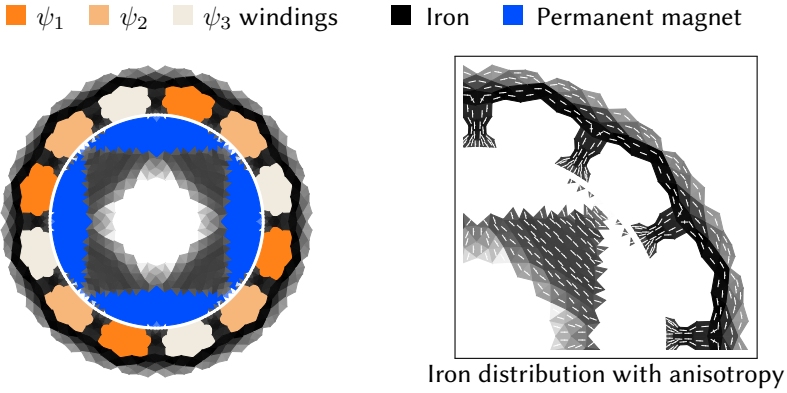


Fig. 8.20. Final result of the mass-constrained PMSM optimization with $R_e = 0.05$ m and $m = 75$ kg; $\tau_{average} = 137.64$ Nm.

The approach followed here for considering the torque ripple specification is not optimal. Indeed, this specification is expressed based on the difference between the highest and the lowest torque, whereas the objective function involves the torque variance. However, including a maximum and a minimum expression inside the objective function is not recommended, since the steepest-descent direction for reducing the torque ripple would be only derived from the maximum and minimum torque positions. A convergence issue would then arise when about the same maximum (or minimum) torque is produced in two positions. The optimization would only focus on reducing the torque in the highest torque position, which in fact has a very limited impact since the second position becomes quickly in turn the highest torque position. Both positions should actually be considered in this situation to avoid the steepest-descent algorithm to make very small steps. In order to easily

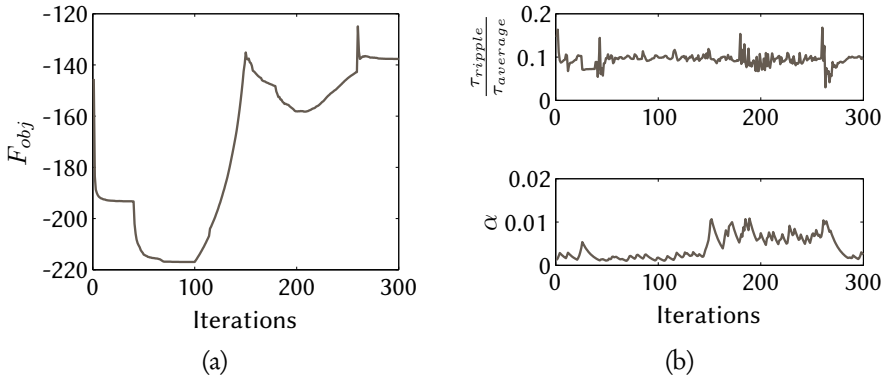


Fig. 8.21. Mass-constrained PMSM design with $R_a = 0.05$ m and $m = 75$ kg. (a) Objective function, (b) torque ripple and penalization factor.

solve this convergence issue, the variance is here rather used. However, the penalization factor α is nevertheless regulated based on the real value of the torque ripple.

The second issue is related to the number of rotor positions in which the torque is evaluated. In this section, the PMSM optimization is performed considering 5 positions for a rotor displacement of $\frac{\pi}{6}$. The solution shown in Fig. 8.20 leads for instance to the torque evolution plotted in Fig. 8.22(a), in which the torque ripple is indeed limited to 10 % of the mean torque. However, when evaluating the torque in more rotor positions, higher harmonics are taken into account and they may exceed the specification. This is for instance illustrated in Fig. 8.22(b) with 20 rotor positions. Therefore, the specification on the torque ripple must mention the harmonics that have to be considered. The higher the frequency of these harmonics, the larger the number of positions required for evaluating the torque is, which leads to a longer computation time.

8.2.3 Torque-to-mass ratio

The same analysis as for the SRM optimization can be made regarding the torque-to-mass ratio. Table 8.3 provides the torques obtained with three air-gap radii and five motor masses. Fig. 8.23 represents the corresponding torque-to-mass ratios. It shows that a radius of 50 mm is optimal for masses between around 60 kg and 110 kg. The two optimal topologies for this air-gap radius are represented in Fig. 8.20 and Fig. 8.24.

Contrary to the SRM problem, proposing a scaling law seems here possible. Indeed, Fig. 8.25 shows the torque evolution obtained when scaling the topology

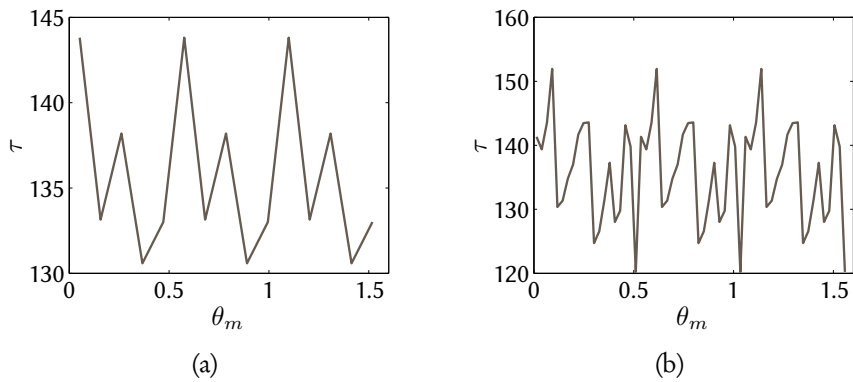


Fig. 8.22. Torque evolution according to the mechanical angle for $R_a = 0.05$ m and $m = 75$ kg. (a) 5 point evaluation; (b) 20 point evaluation.

Table 8.3. PMSM torque according to the mass m and the air-gap radius R_a .

$m \setminus R_a$	0.04	0.05	0.06
25	21.63	11.97	x
50	70.78	66.5	46.29
75	132.54	137.64	126.24
100	204.4	212.71	208.19
150	x	374.25	390.72

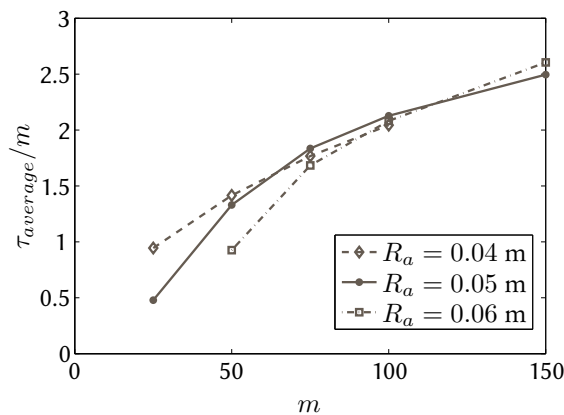


Fig. 8.23. Evolutions of the PMSM torque-to-mass ratio for three air-gap radii.

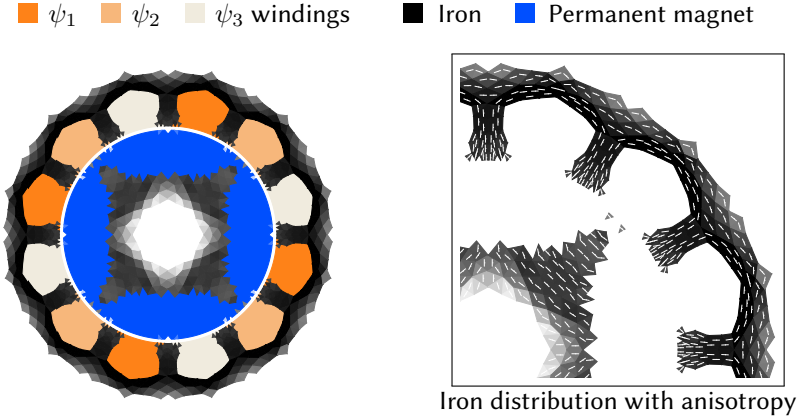


Fig. 8.24. Mass-constrained PMSM design with $R_e = 0.05$ m and $m = 100$ kg; $\tau_{average} = 212.71$ Nm.

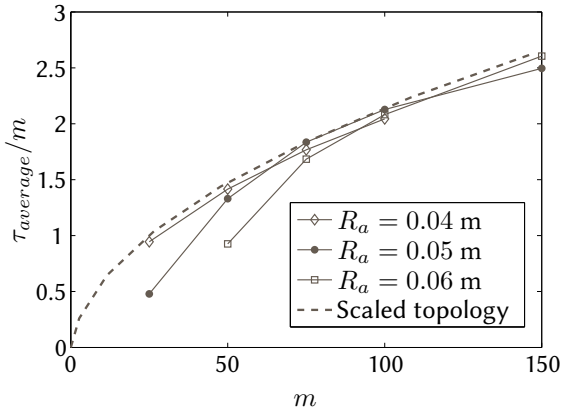


Fig. 8.25. Fitting between the torque-to-mass ratios achieved when scaling an optimal topology and the ones obtained by optimization.

shown in Fig. 8.20. This evolution seems to properly fit the optimal torque-to-mass ratios achieved with the other air-gap radii. More optimization results would however be required to validate this observation, but this kind of scaling law has already been proposed for PMSM by Labbé et al. (2010). This latter study though did not take saturation into account.

The torque evolution of the scaled topology can be described by a power regression, as shown in Fig. 8.26:

$$\frac{\tau_{average,opt}}{m} = 0.1579 m^{0.563}. \quad (8.3)$$

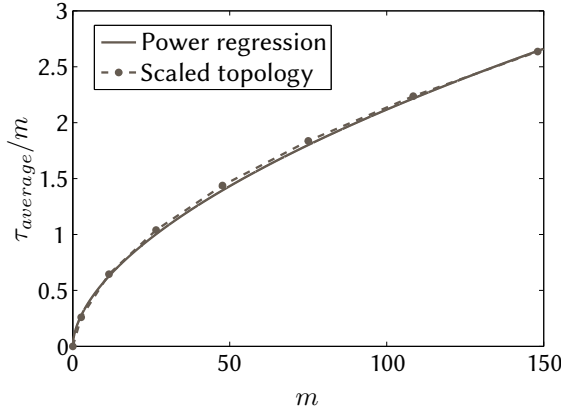


Fig. 8.26. Torque evolution obtained when scaling the optimal topology.

8.3 Design of magnetic thrusts

The magnetic thrust design problem has already received attention in chapter 5, in order to illustrate the topology optimization of windings and permanent magnets. The study case was however simplified by considering only one of the two thrusts. Including the second thrust in the optimization problem has two consequences.

First, the optimization objective becomes the maximization of the minimum of v_l and v_r , the maximum moving part speeds when reaching respectively the left and the right thrust. The maximization of the minimum of two functions cannot effectively be performed by a steepest-descent algorithm, since the line-search direction is then only derived from the derivative of the function that has the minimum value. However, the objective function can here simply be replaced by the mean of v_l and v_r :

$$F_{obj} = -\frac{v_l + v_r}{2}, \quad (8.4)$$

since the method converges then to symmetrical topologies and equal speeds in the left and right directions.

The second consequence is related to the length of the moving part design space. Indeed, considering two thrusts raises an issue regarding the definition of this length, which is discussed in the first part of this section.

Whereas several kinds of materials are considered in chapter 5, this section only focuses on permanent magnets having free magnetization directions. In addition, the optimization is performed on a fixed mesh without mesh refinement.

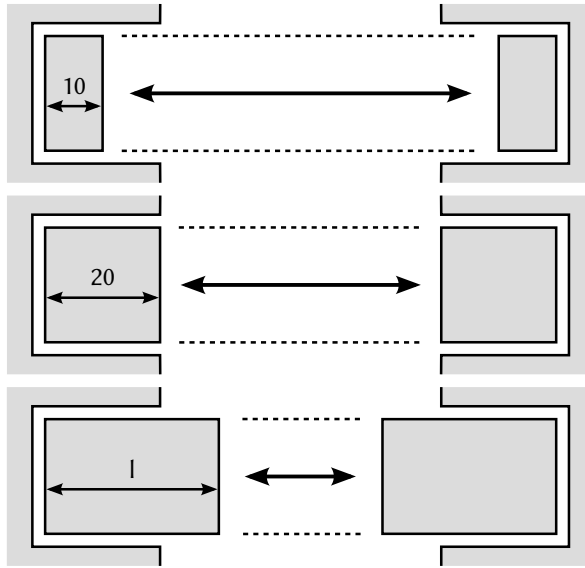


Fig. 8.27. Three design space lengths for the moving part.

8.3.1 Length of the moving part design space

In order to compute the moving part speeds in the two directions, the magnetic energy is evaluated successively when placing the moving part close to the left and to right thrust. That raises an issue regarding the length of the moving part design space, written l . This length must be defined initially before starting the optimization in order to be able to place the moving part in front of the two thrusts, as illustrated in Fig. 8.27. However, it has an influence on the maximum speed that can be achieved.

This issue is illustrated by performing optimizations with four design space lengths: 10 mm, 20 mm, 40 mm and 60 mm. Fig. 8.28 and Fig. 8.29 show respectively the results for a moving part load $L = 1$ kg and $L = 0.1$ kg. Only the left thrust is represented since the right thrust can be obtained by symmetry. With $L = 1$ kg, the design space is almost always completely filled with permanent magnets. However, in Fig. 8.29, some cells are filled with air when the length becomes larger than 20 mm. As the design space length increases, permanent magnets are first placed in the centre of the design space, the distance between the moving part and the thrusts being then greater than 2 mm, which is obviously not optimal for maximizing the speed. When the design space length becomes even larger, the moving part magnets are rather placed at the two design space ends, which actually corresponds to two distinct thrusts.

In order to get around this issue, the study case is solved by performing several

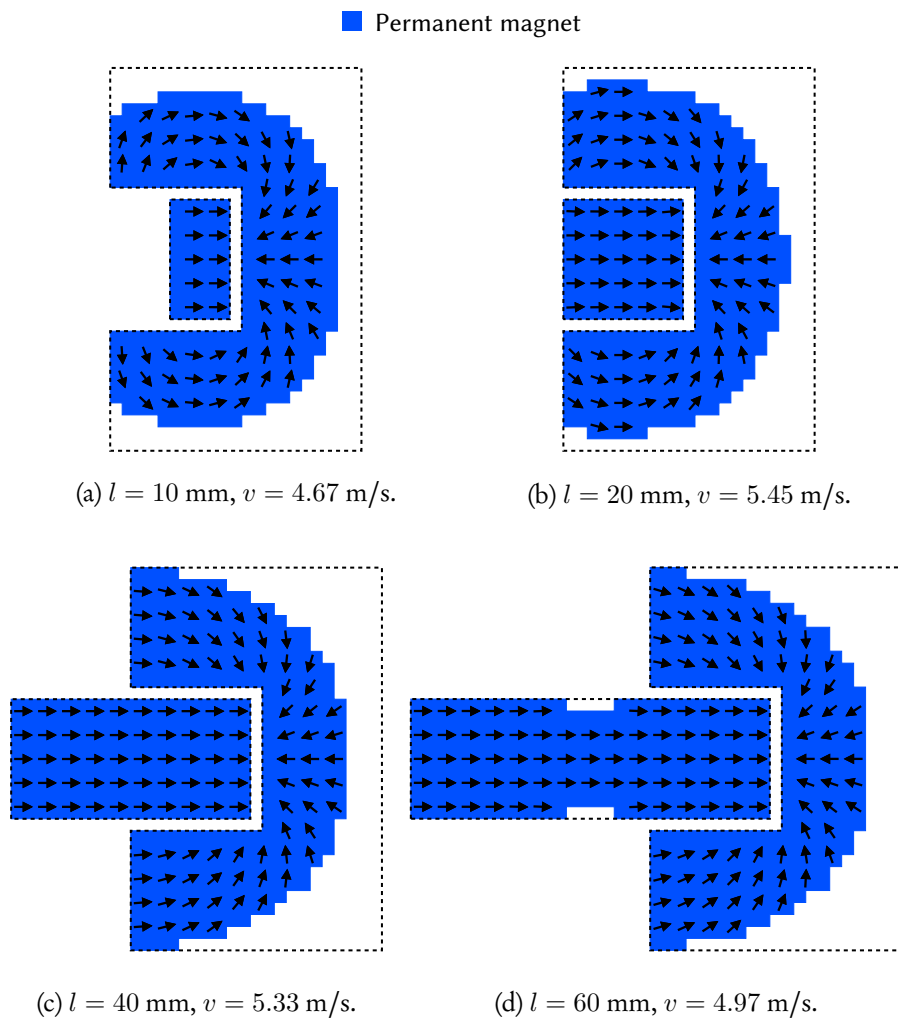


Fig. 8.28. Design of magnetic thrusts with $L = 1 \text{ kg}$ and different moving part lengths.

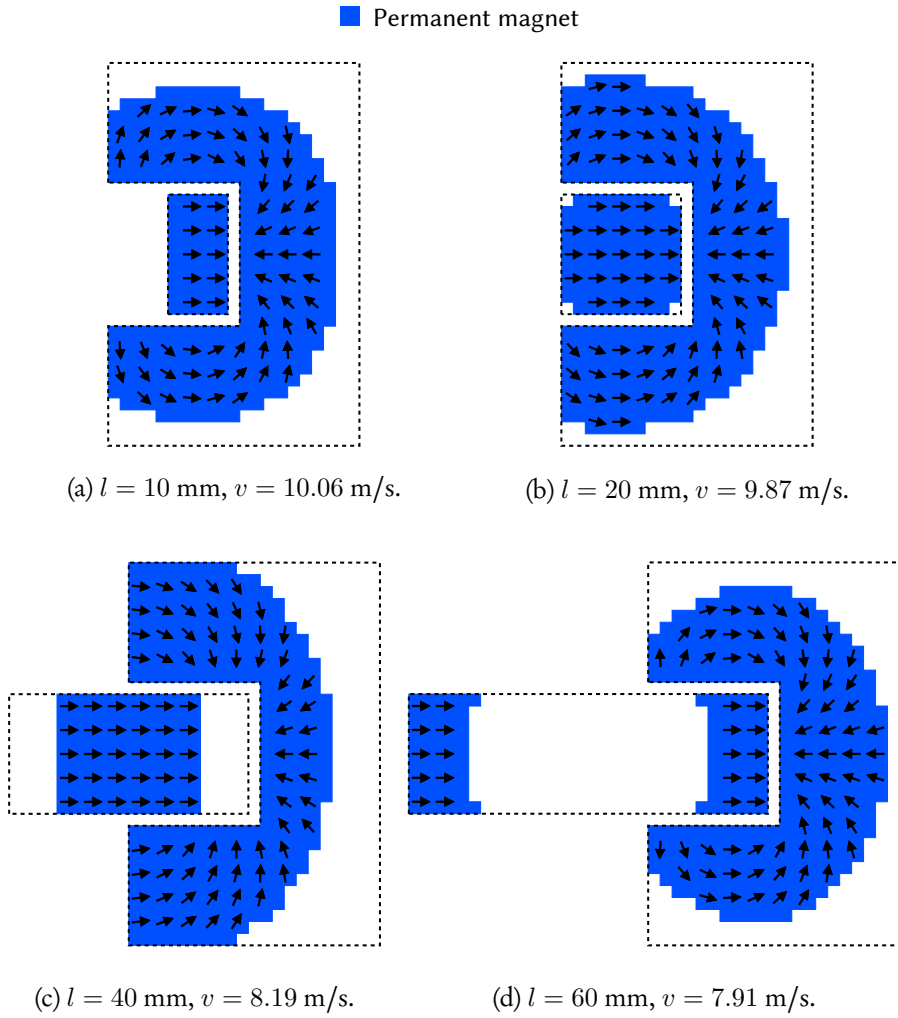


Fig. 8.29. Design of magnetic thrusts with $L = 0.1 \text{ kg}$ and different moving part lengths.

optimizations considering different lengths. Since the problem only involves permanent magnets and thus a linear finite element model, the computation time is indeed quite short and the optimization can easily be repeated.

8.3.2 Local minimizers

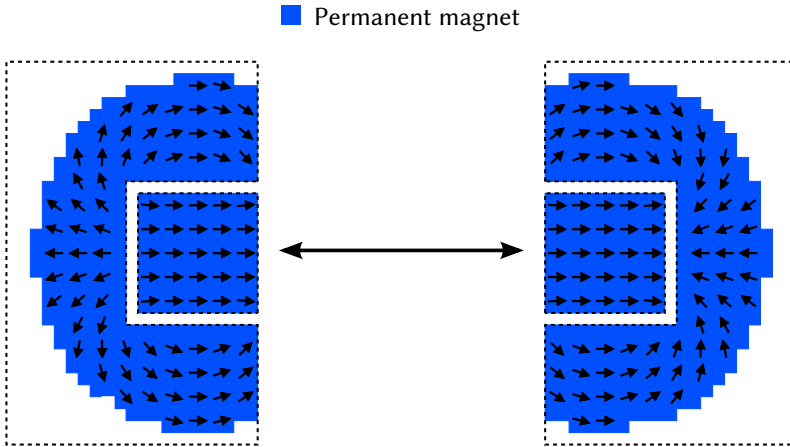
The magnetic thrust optimization problem is characterized by the presence of four local minimizers. Indeed, according to the initial conditions, the optimization may result in a moving part having either a horizontal or vertical magnetization direction, as shown in Fig. 8.30. The two other local minimizers are simply obtained by inverting the magnetization directions of these two solutions and they obviously lead to the same moving part speeds.

The optimization problem lacks thus convexity, which means that there is no guarantee that the steepest-descent algorithm will find its global optimum. However, this is not a major issue with only four local minimizers.

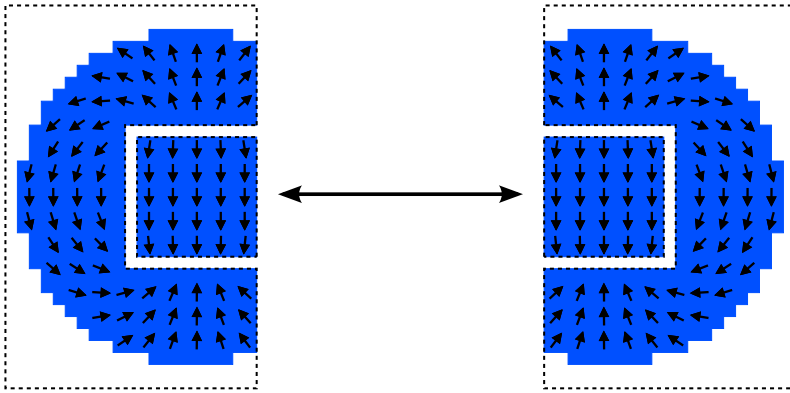
Amongst the four local minimizers, the horizontal magnetization direction should be preferred. Besides leading to a higher maximum speed, this topology still produces a repulsive force when the distance between the moving part and the thrust is lower than 2 mm. This is not the case with the vertical magnetization solution for which the moving part is actually in an unstable position in Fig. 8.30(b) where there are no horizontal forces. When the distance between the thrust and the moving part is reduced, an attractive force is thus produced, which involves that the moving part may be stuck against the thrust.

8.3.3 Optimal designs

In order to determine the optimal moving part length, the optimization was performed 16 times with lengths from 5 mm to 80 mm. Fig. 8.31 and Fig. 8.32 show the achieved speeds respectively for $L = 1$ kg and $L = 0.1$ kg with the horizontal and vertical magnetizations. These figures confirm that a higher speed is achieved in the first case. The optimal design space length is 15 mm for a fixed load of 0.1 kg and 25 mm for 1 kg. The optimal topologies are represented in Fig. 8.33.



(a) Horizontal magnetization of the thrust; $v = 5.45$ m/s.



(b) Vertical magnetization of the thrust; $v = 5.39$ m/s.

Fig. 8.30. Local minimizers for the thrust optimization with $L = 1$ kg.

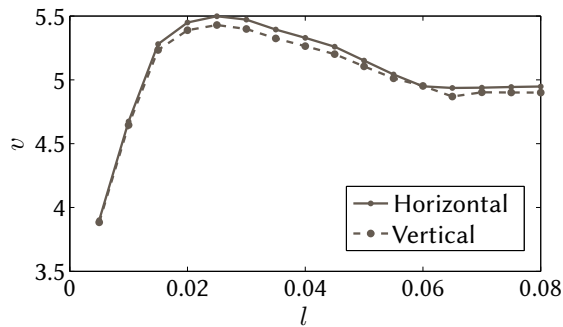


Fig. 8.31. Moving part speed according to the moving part length with $L = 1$ kg for the two moving part magnetization directions.

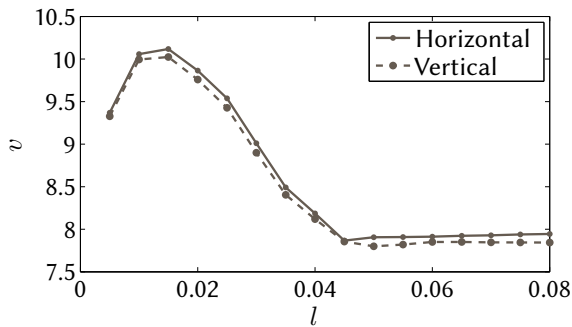
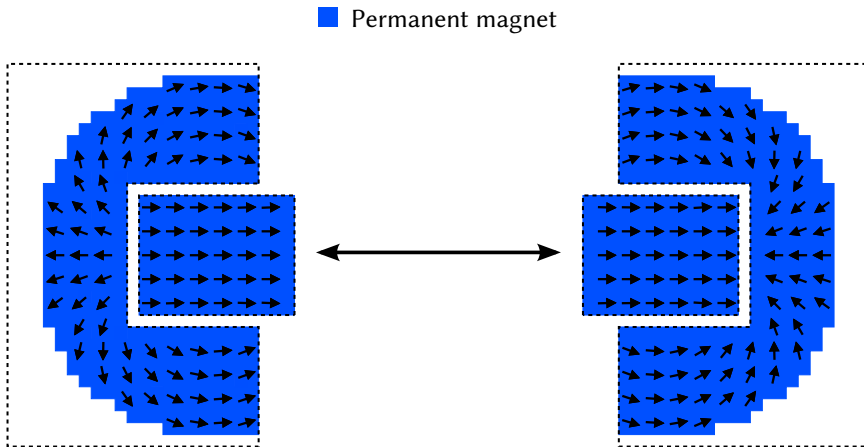
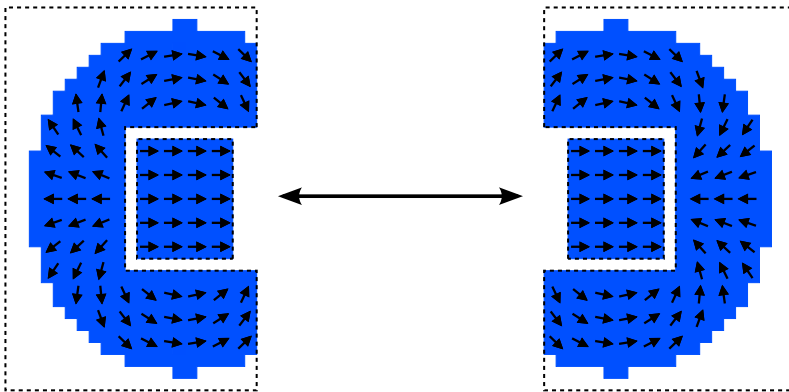


Fig. 8.32. Moving part speed according to the moving part length with $L = 0.1$ kg for the two moving part magnetization directions.

(a) $L = 1 \text{ kg}$; $v = 5.50 \text{ m/s}$.(b) $L = 0.1 \text{ kg}$; $v = 10.12 \text{ m/s}$.**Fig. 8.33.** Final design for the magnetic thrust optimization.

Conclusion

9

The research presented in this thesis aimed to develop and implement topology optimization methods dedicated to the design of electromagnetic devices. The proposed methods rely on a deterministic algorithm: the steepest-descent algorithm. In comparison with heuristic methods, deterministic algorithms are indeed characterized by a fast convergence thanks to the use of the objective function gradient. However, their application to topology optimization raises two main challenges related to the handling of intermediate materials and local minimizers. The thesis specifically focused on overcoming these two challenges. Intermediate materials are indeed tolerated during the optimization in order to apply a gradient-based algorithm, but have to be removed in the final design. Regarding the local minimizers, they actually prevent the algorithm from finding the optimal solution of the optimization problem.

In addition, the research intended to increase the complexity of the considered problems in comparison with the study cases that can be found in the literature. Different combinations of the four electromagnetic materials: air, iron, windings and permanent magnets, are studied, as well as different objective functions and constraints, with a particular attention to the involvement of a mass constraint. Three study cases were considered with this purpose: the design of magnetic thrusts, of a switched reluctance motor and of a permanent magnet synchronous motor. This work besides took an interest in developing a mesh refinement algorithm, with the objective of increasing the resolution of the produced solutions while limiting the computation time.

Two groups of methods were proposed to reach these objectives: the isotropic and anisotropic methods. The former have the advantage of their simplicity and their fast convergence. Indeed, each material is represented by a unique design variable and considering isotropic magnetic properties keeps the finite element model simple and fast. This work led to three main contributions regarding this group of

methods. First, the sensitivity mapping focuses on ensuring a smooth sensitivity of the objective function in order to improve the convergence speed of the steepest-descent algorithm. Second, the convexity mapping was proposed for modifying the problem convexity and thereby removing intermediate materials while avoiding local minimizers. Third, an iron microstructure was introduced in mass-constrained optimizations in order to realistically derive the properties of intermediate materials. In addition, a specific mechanism was developed for the optimization of ferromagnetic parts, based on the simultaneous optimization of two topologies and on convexity constraints. Even if these methods were successfully applied to different optimization problems, they still suffer from several limitations. Some problems, such as the optimization of ferromagnetic parts, indeed require a specific method and others, such as the stator optimization of a PMSM, cannot be handled at all with isotropic materials.

Those limitations motivated the study of anisotropic methods. The latter involve anisotropic permeabilities deduced from iron microstructures whose dimensions and orientations are, contrary to isotropic methods, optimized along with the material proportions. A specific microstructure characterized by a cross shape and described by an orientation and a shape factor was proposed in this thesis work. In comparison with isotropic methods, anisotropic methods are more complex since anisotropic permeabilities must be handled by the finite element model and two additional design variables must be included. However, they are far more general and can be applied to all the study cases considered in the thesis.

In parallel with the development of the optimization methods, the research also got onto the implementation of a mesh refinement algorithm. The latter proved to be effective for reducing the computation time required for achieving a given design resolution. It besides ensures the accuracy of the finite element model. In addition, in problems requiring composite iron materials, it actually highlights the creation of sheets of iron that become thinner as the mesh size decreases, in order to obtain magnetic properties that approach those of the required composite materials.

The research work was completed by the application of the proposed methods to more advanced problems. The latter highlighted new challenges of topology optimization, especially regarding the definition of the design space which is actually an a priori decision that limits the range of solutions that can be produced. Several optimizations had indeed to be performed to determine some device dimensions that cannot be handled by the optimization algorithm.

Arrived at the end of this thesis, it is interesting to compare the initial expectations with the actual achievements. The objectives and research directions were obviously progressively defined as long as the research progressed. Indeed, its initial goal was very general: exploring the domain of topology optimization for the

design of electromagnetic devices, considering deterministic algorithms. In view of the results recalled above, we can today say that this objective has been met in various ways. First, the thesis in fact considers practical problems that are well-known and interesting for the electromagnetic actuator design community. Next, problems that are inherent in their optimization were highlighted. The issues related to local minimizers and intermediate materials were especially studied and illustrated. Solutions were then proposed to overcome these issues, resulting in the development of several functional methods. Finally, these methods were applied in order to assess their performances and limits.

This research was conducted with the perspective of a motor designer, whose main expectation is to obtain designs that meet the specifications. That is the reason why the study and the comparison of different optimization algorithms was beyond in the scope of the thesis. In addition, little work energy has been provided in some mathematical aspects such as for instance the implementation of a stopping criterion, the observation of the convex property of a problem, or the proof of the optimality of a method or a result. These concepts were rather considered in a more practical or intuitive approach, which was sufficient for fulfilling the expectations of a motor designer, but would maybe require a more rigorous study in a mathematical point of view.

Several choices had to be done in the initial phase of the thesis, with a limited knowledge of the research domain. Retrospectively, it's interesting to analyze their impact on the thesis results, more particularly regarding the development of a home-made finite element model and optimization algorithm, the consideration of the steepest-descent algorithm and the selection of the three study cases.

The decision of coding everything in Matlab, including the finite element model and the optimization algorithm, turned out to be profitable. It allowed us to really pointed out where the optimization issues arose and to better analyze them. It led to an in-depth understanding of these issues which was helpful for the method development and could also become a valuable support for future developments or for considering more complex problems. This decision however had its drawbacks: the home-made implementation cannot compete with efficient commercial tools in terms of computation time and flexibility.

The selection of the steepest-descent algorithm has already been justified in the thesis by its easy implementation and its sufficient convergence speed to handle the considered problems. The thesis actually quickly concentrated on the issues related to local minimizers and intermediate materials which are totally independent of the optimization algorithm as long as the latter relies on the gradient and works with continuous material proportions. The thesis indeed primarily focused on developing a functional method, which pulled the study of the optimization algorithms out

the scope of the work.

The three study cases were at first selected in order to consider all the possible material combinations. Their optimization actually highlighted other interesting difficulties, such as the fact that the stator optimization of the PMSM which includes the same materials as the SRM, i.e. windings, air and iron, actually requires the consideration of anisotropic materials. However, it would also be interesting to study to which extent the proposed methods are effective, by the consideration of other study cases.

The results that can be expected with the proposed methods require some comments as well. Of course, these methods are not optimal in the sense that they do not necessary converge to the global optimum and that better solutions could be found for a few problems with other methods. In addition, the produced topologies sometimes present some obvious defects, such as air holes in stator slots. These weaknesses could perhaps be reduced by simple modifications of the methods, such as a different use of the mappings for the penalization, or maybe require further in-depth improvements. They could also be merely unavoidable. However, from the point of view of a motor designer, the optimality is not always required, as long as the design meets the specifications. In addition, topology optimization is usually viewed as a tool for designing unconventional devices, for which the designer has no a priori idea of the required topology. In this framework, the designer again expects an efficient design, not necessarily the optimal one. Finally, a parametric optimization will usually be also performed in order to accurately determine the devices dimensions, even if mesh refinement could in some cases replace this two-step approach.

Compared to the literature, the obtained results are a success in the sense that different actuators were completely optimized. The results presented in the "method exploitation" chapter are besides promising. Even if they have not yet been enough studied, they announce the possibility of applying topology optimization to the real design of motors.

A lot of work has however still to be done. Even if the thesis covers a wide range of aspects of topology optimization, it also opens up new ways of research.

First, the computation time could become a limiting factor for the topology optimization of complex problems. It could foremost be reduced by replacing the home-made implementation of the finite element model by a commercial and efficient one. Great improvements could also be achieved regarding the optimization algorithm. Therefore, comparing the convergence speeds achieved with other algorithms, such as the method of moving asymptotes or sequential linear programming, would be an interesting study. In addition, the automation of the parameter evolutions, through for instance the implementation of stopping criteria, would be

very attractive, since it could replace the trial-and-error approach and besides maybe reduce the number of required iterations to what is strictly necessary. The mesh refinement algorithm could also be further improved: using the solution obtained at the previous step as a starting point for performing the optimization on the refined mesh would certainly reduce the optimization length.

Second, in parallel with the method improvement, researches could also be initiated for adding new method functionalities. As already mentioned, the definition of the design space and of the design symmetry limits the range of possible solutions. For instance, the air-gap radius of a PMSM or a SRM, or their number of poles, cannot be optimized by the method. Those problems may thus be better studied either by adding the possibility of moving the design space boundaries, or by considering a unique design space for both the stator and the rotor.

Third, the manufacturability of the solutions produced by topology optimization cannot be guaranteed. Formulating such constraint seems arduous, but it would be interesting to highlight an optimization problem for which the manufacturability becomes an issue and study how this could be handled.

Fourth, the proposed methods proved to be effective on three study cases. Their application to other optimization problems would be interesting to study in which extent they remain effective. The optimization could for instance also determine the optimal current density of windings, which would suppose taking thermal considerations into account. Applying topology optimization to 3D problems would also raise interesting questions, such as how to handle the winding material since its continuity must be guaranteed to carry a current density.

Fifth, the performances of the proposed methods could be further assessed through a comparison between a real motor and a redesign of the latter using these methods. Such comparison is not yet possible based on the results obtained in this thesis, since it requires working with exactly the same properties of materials and including thermal considerations, since the latter are taken into account in the design of real motors. It would also be interesting to validate experimentally the topology optimization results, for instance by manufacturing one of the designs obtained in the optimization of the permanent magnet synchronous motor.

Result comparison



This appendix intends to provide a summary of the results presented along the thesis in order to easily compare them, in terms of objective function, finite element evaluations and computation time. In addition to the solutions provided by the isotropic and anisotropic methods, this appendix also includes optimization results used to assess them. Indeed, these methods rarely achieve to provide the global optimum and even if they do, that cannot be proven. It is indeed impossible in practice to find for certain the optimal solution of a topology optimization problem since it requires the complete enumeration of all the possible solutions, which is obviously infeasible in view of the large number of design variables. In order to nevertheless assess the method results, they are compared with the results of a classic steepest-descent algorithm.

As explained in section 5.2, optimization problems can be separated in two groups according to the shape of the objective function, which can either be concave or convex. In the first case, the steepest-descent algorithm converges towards different local minimizers according to the initial conditions and the produced topologies are only composed of solid materials. This appendix precisely takes advantage of this behaviour to generate a large number of good candidates for being the optimal topology by simply running the classic steepest-descent algorithm with random initial conditions.

If the objective function rather has a convex shape, the steepest-descent algorithm always converges towards the same topology which besides contains intermediate materials. In this case, the problem must be modified by a penalization of intermediate materials to achieve a concave behaviour. For each study case, this penalization is explicitly described when implemented.

A method result can then be assessed at two levels. First, it can be compared with the best local minimizer regarding the objective function. Second, it is also in-

teresting to evaluate the chance of finding a better solution with the classic steepest-descent algorithm, especially when the objective function is by default concave which makes the penalization unnecessary and a classic steepest-descent algorithm efficient for producing solid solutions. This probability is simply given by the fraction of local minimizers that achieve a better objective function.

The appendix is structured as follows. The first section presents comparison tables for assessing the different method results. The second section describes how local minimizers were generated with the classic steepest-descent algorithm for each study case. In addition, it provides a more thorough presentation of the results: the distribution of the local minimizer objective functions are given using histograms and the best solution amongst all the local minimizers is plotted for each study case.

A.1 Comparison tables

Three tables are provided in this section, giving for each case the objective function achieved in Nm, the computation time and the number of finite element evaluations (FEE). Table. A.1 concerns problems that were optimized without taking saturation of iron into account. It only includes the SRM stator problem, with and without a mass constraint. Table. A.2 provides the results obtained using the three methods on the four mass-constrained problems considered along the thesis. Table. A.3 is dedicated to the three volume-constrained problems, the optimization of the SRM rotor being divided in two since the objective function was either based on the magnetic energy or on the Maxwell tensor.

Table A.1. Comparison table for the SRM stator problems without saturation.

	Isotropic materials	Anisotropic materials	Classic algorithm	
			1000 designs	1 design
Volume-constrained	84.20 Nm 5 min 722 FEE	Not applied	84.20 Nm 9h46	35 s/design
Mass-constrained	5.38 Nm 33 min 2238 FEE	Not applied	6.01 Nm 23h19	$P(\tau > 5.38) = 0.15$ 90 s/design

Table A.2. Comparison table for the mass-constrained SRM and PMSM problems with saturation.

Mass	Isotropic materials	Anisotropic materials	Classic algorithm	
			1000 designs	1 design
SRM stator	11.27 Nm 2h35 2270 FEE	11.56 Nm 3h 4564 FEE	10.90 Nm 14j 1h	20 min/design
SRM rotor	Not effective	60.78 Nm 10h 7768 FEE	59.99 Nm 16j 6h	23 min/design
PMSM stator	Not effective	119.63 Nm 5h30 4963 FEE	121.41 Nm 10j 12h	$P(\tau > 119.63) = 0.002$ 15 min/design
PMSM rotor	325.91 Nm 3h30 2070 FEE	326.7 Nm 24h30 7498 FEE	323.64 Nm 6j 10h	9 min/design

Table A.3. Comparison table for the volume-constrained SRM and PMSM problems with saturation.

Volume	Isotropic materials	Anisotropic materials	Classic algorithm	
			1000 designs	1 design
SRM stator	98.08 Nm 1h22 706 FEE	100.42 Nm 14h30 4580 FEE	100.42 Nm 12j 16h	$P(\tau > 98.08) = 0.002$ 18 min/design
SRM rotor W_{mag}	62.34 Nm 3h 2542 FEE	62.15 Nm 6h 4236 FEE	62.66 Nm 13j 4h	$P(\tau > 62.34) = 0.008$ $P(\tau > 62.15) = 0.023$ 19 min/design
SRM rotor M. T.	101.98 Nm 40min 475 FEE	Not applied	103.05 Nm 21h20	$P(\tau > 101.98) = 0.04$ 80 s/design
PMSM stator	Not effective	458.89 Nm 9h 3802 FEE	448.65 Nm 11j 21h	17 min/design

A.2 Local minimizer generation

This section intends to describe the generation of local minimizers through the application of a classic steepest-descent algorithm. Only the SRM and PMSM are here considered. Indeed, as explained in section 5.1, the magnetic thrust problem is actually quite simple and is well handled by a steepest-descent algorithm. There is thus no need for assessing these method results.

A.2.1 Switched reluctance motor

The SRM optimization problem is divided in two subproblems where either the stator or rotor is optimized.

Stator optimization

Since the optimization problem is not concave by default, the local minimizers were here obtained with the isotropic method. The latter was implemented exactly in the same way as it is described in chapter 5, but the penalization parameter r_i was set to 0.95 after only three iterations without saturation and after 6 iterations with saturation. Thereby, the problem takes very quickly a concave behaviour which ensures the convergence towards local minimizers containing no intermediate material. The penalization could have been applied directly when starting the optimization, but it leads then to really poor results.

Four cases are considered along the thesis, depending whether saturation is considered or not and if a mass constraint is included or not. For each case, 1000 local minimizers were generated. Fig. A.1 and Fig. A.2 shows the distribution of their objective functions, as well as the best local minimizer, for the volume-constrained optimization respectively without and with saturation. The optimizations were also performed with a mass constraint limiting the stator mass to 60 kg, which led to the results shown in Fig. A.3 and Fig. A.4.

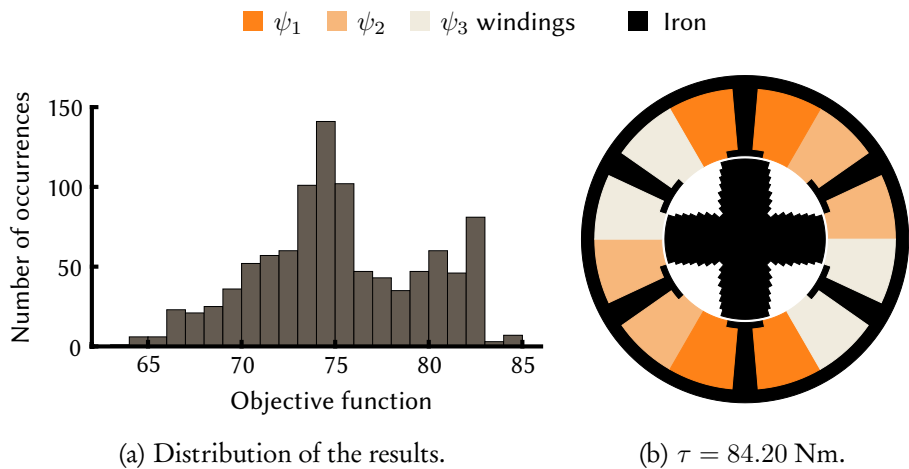


Fig. A.1. Stator design of the SRM.

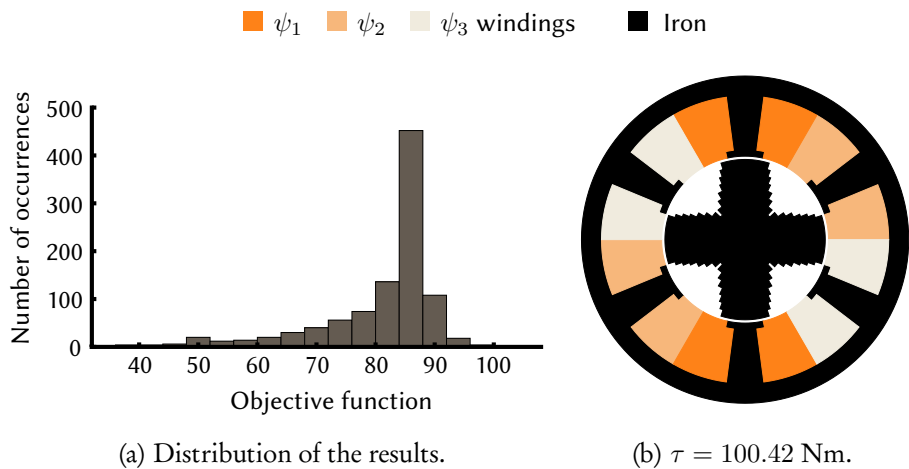


Fig. A.2. Stator design of the SRM with saturation.

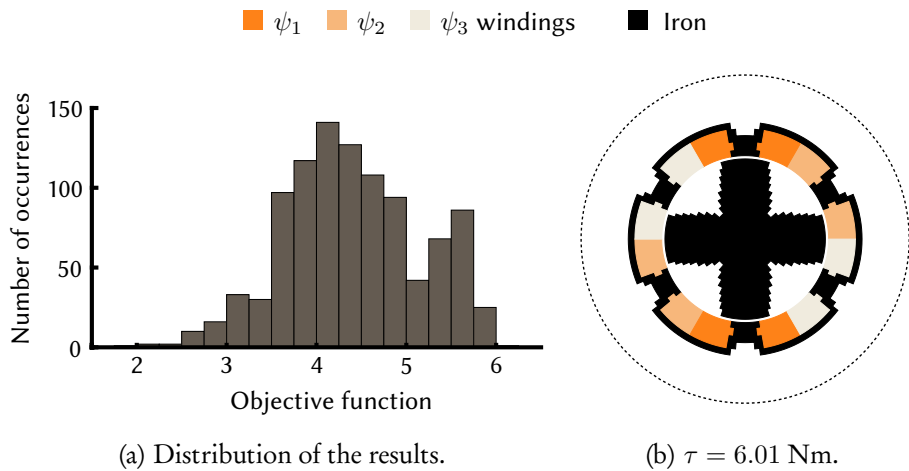


Fig. A.3. Stator design of the SRM with $m_s = 60$.

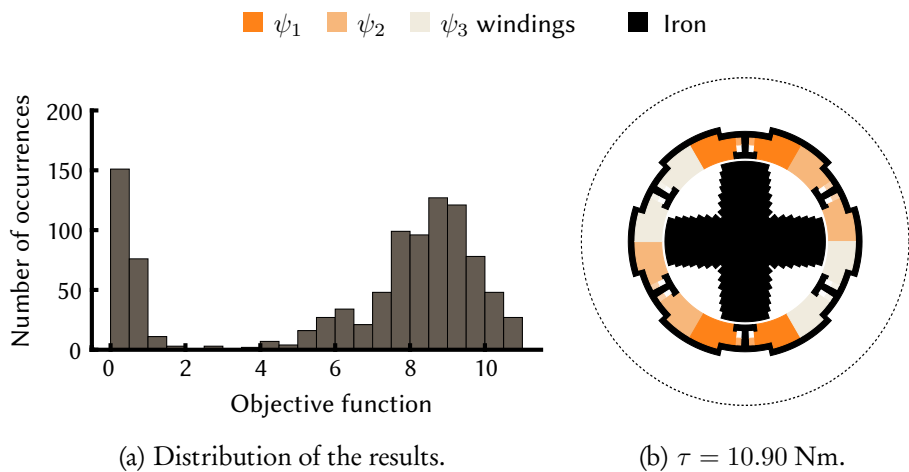


Fig. A.4. Stator design of the SRM with $m_s = 60$ and saturation.

Rotor optimization

For the volume-constrained optimizations, the steepest-descent naturally converges to local minimizers. Fig. A.5 provides the optimization results with saturation for the maximization of the average torque computed with the magnetic energy. When the rotor mass is limited to 20 kg, the penalization parameter r_i was set to 0.95 after 3 iterations to remove iron-air composites. The results are then shown in Fig. A.6.

Local minimizers were also generated for the maximization of the torque obtained with the Maxwell stress tensor in order to assess the method proposed in section 5.6. Only one case is then considered: a volume-constrained optimization taking saturation into account. The result is presented in Fig. A.7.

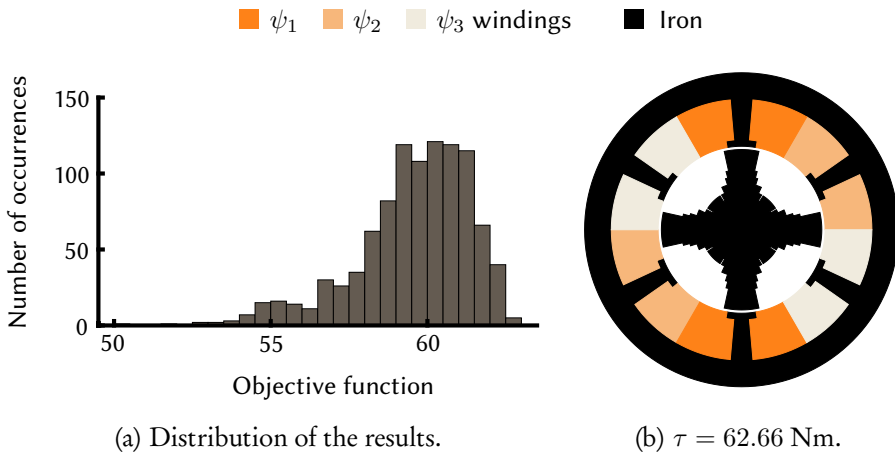


Fig. A.5. Rotor design of the SRM with saturation.

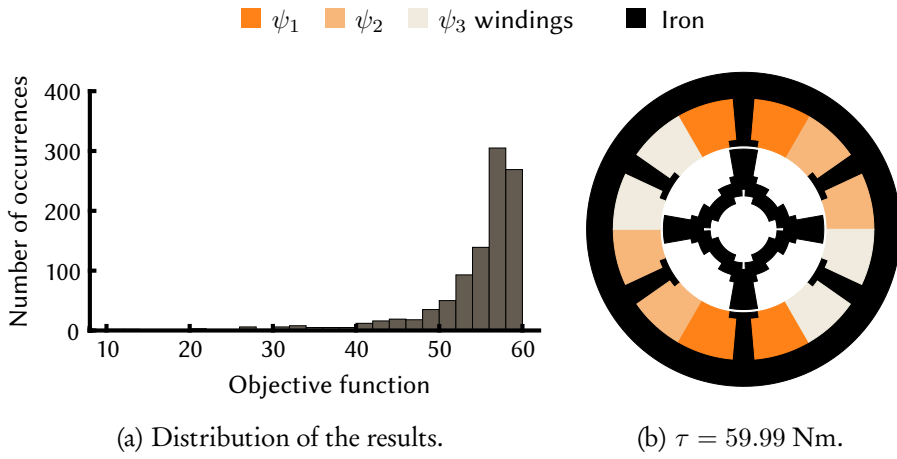


Fig. A.6. Rotor design of the SRM with $m_r = 20$ and saturation.

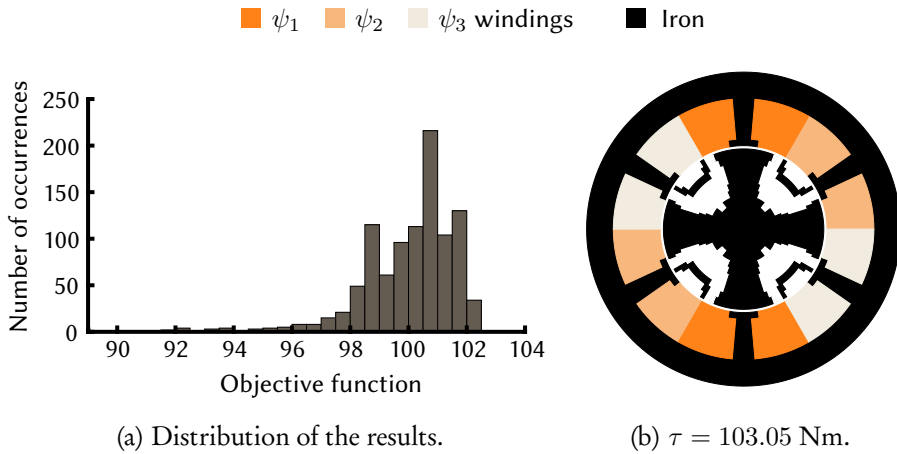


Fig. A.7. Rotor design of the SRM with saturation with the objective function based on the Maxwell tensor.

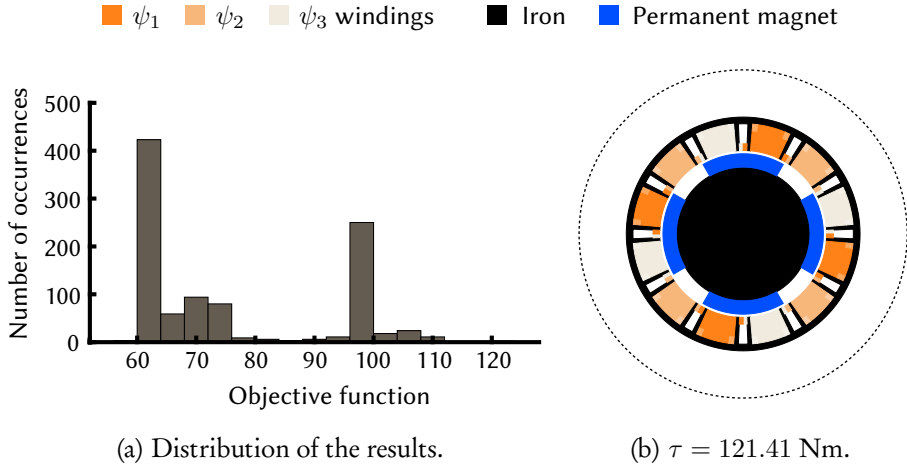


Fig. A.9. Stator design of the PMSM with $m_s = 60$ and saturation.

Rotor optimization

The rotor optimization of the PMSM always includes a mass constraint. Indeed, the method otherwise completely fills the design space with permanent magnets. Fig. A.10 provides the local minimizers obtained with the isotropic method by keeping r_{mi} at zero and setting r_i to 0.95 after 3 iterations.

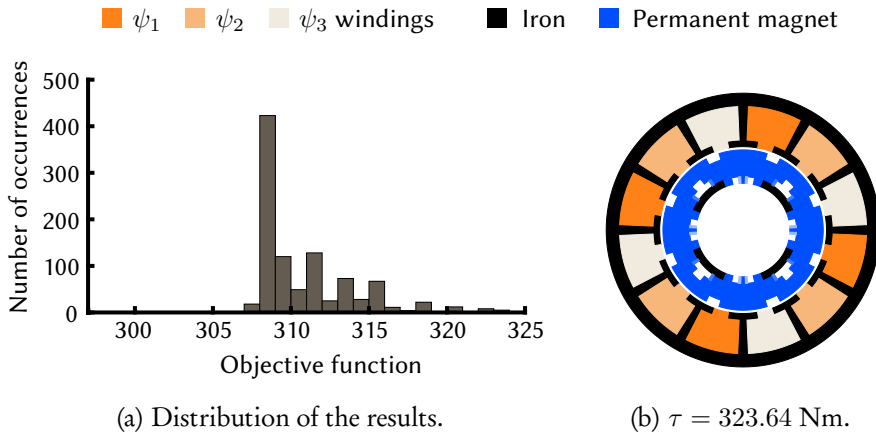


Fig. A.10. Rotor design of the PMSM with $m_r = 30$ and saturation.

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