CONSTRUCTING AND ADJUSTING BENDING MOMENT DIAGRAMS AND ELASTIC CURVES WITH GRAPHIC STATICS

29/08/2019 RESEARCH SEMINAR

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CONSTRUCTING AND ADJUSTING BENDING MOMENT DIAGRAMS AND ELASTIC CURVES WITH GRAPHIC STATICS

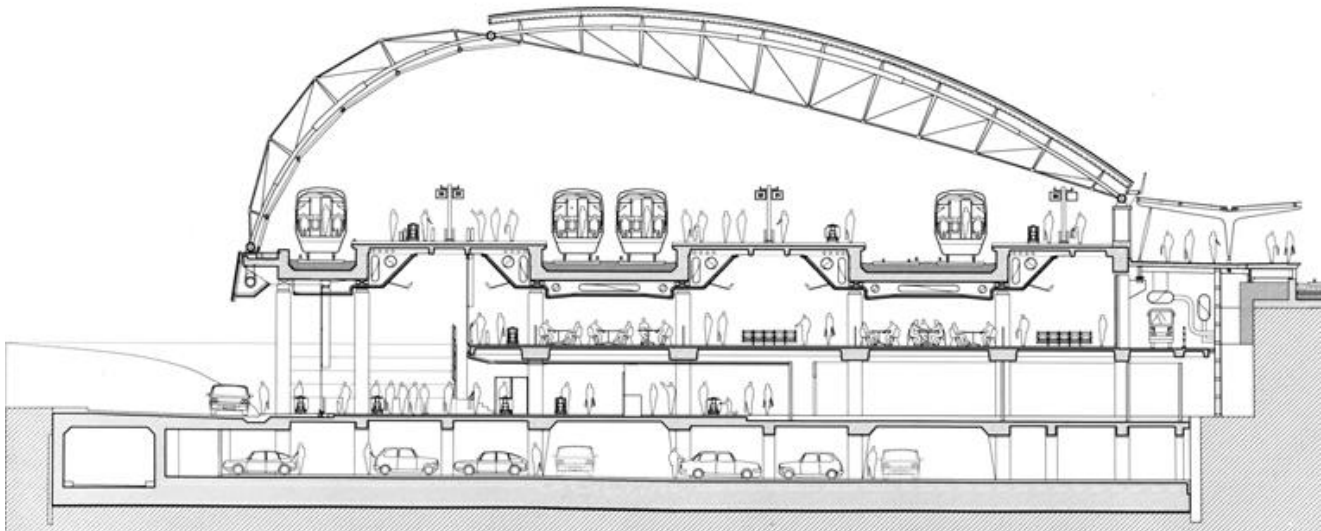
Meaning of the topics:

- A proved powerful tool for structural design
- Of the very few tools shared by architects and civil engineers

What is the essence of bending moment diagrams and elastic curves ?

- Modified function graphs - (commonly practiced)
- “Integration machine “ - (graphic statics)
- Interactive design tool - (research objectives)

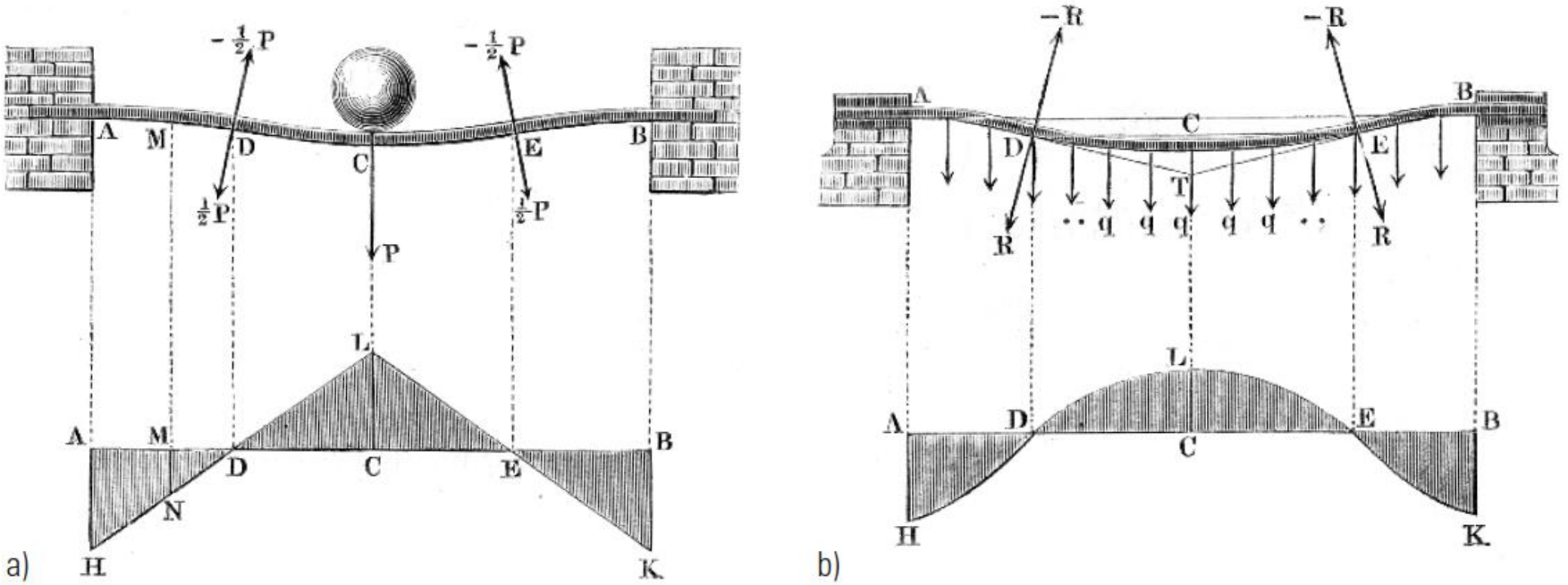
$$\frac{d^2M(x)}{dx^2} = -q(x) \qquad \frac{d^2\omega(x)}{dx^2} = \frac{M(x)}{EI}$$



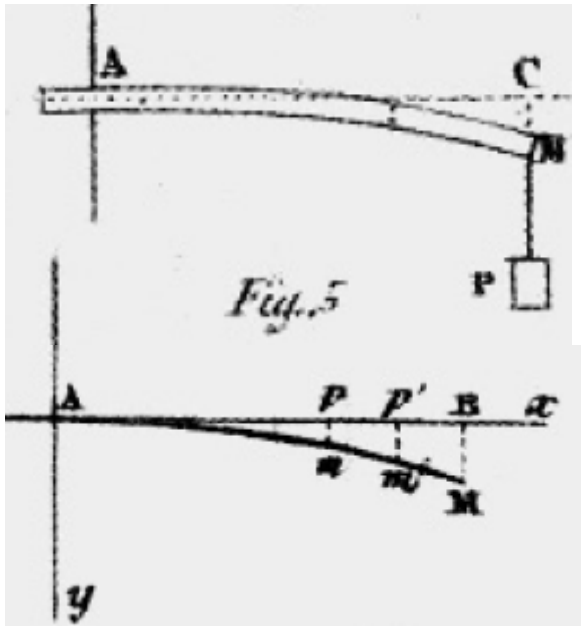
Section of the Waterloo International railway station

1. Bending moment diagrams and elastic curves as modified function graphs
   - Weisbach' graphical presentation of bending moments
   - Navier's graphical analysis of elastic curves

Character: the diagrams are only the graphical presentation of result of calculation

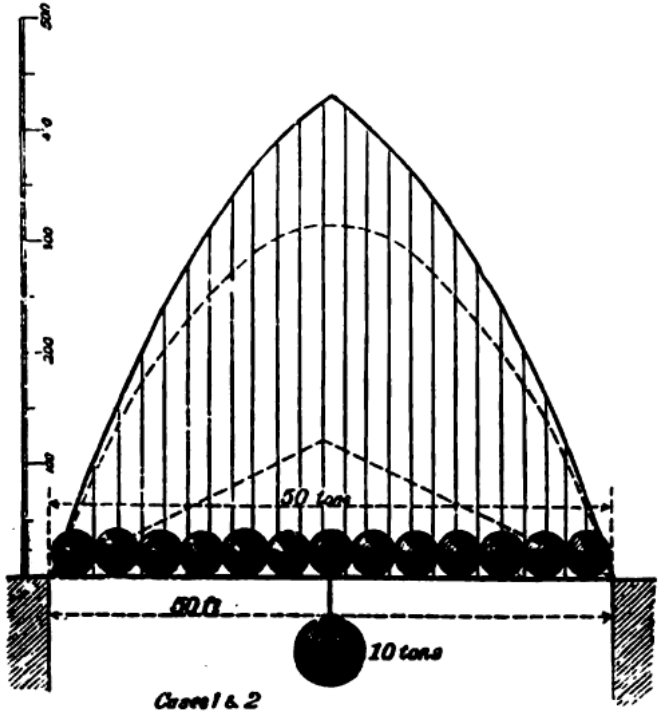


(Weisbach, 1855)

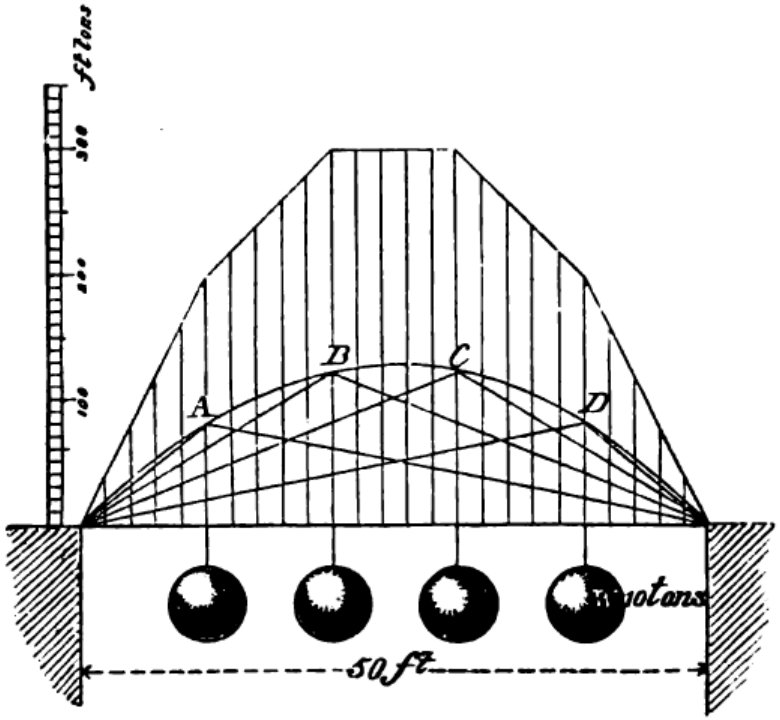


(Navier's 1833)

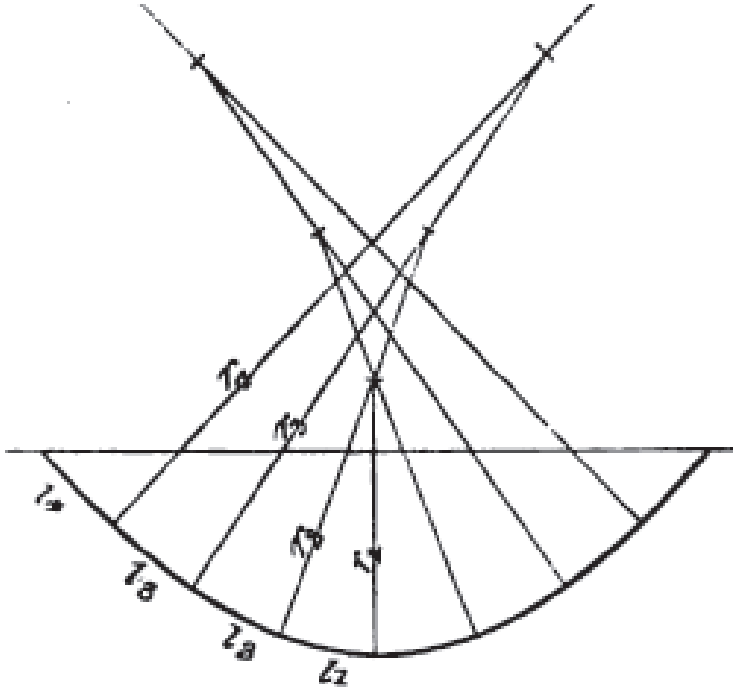
Special methods



Superposition method



Superposition method



Radii method

(Jenkins, 1867)

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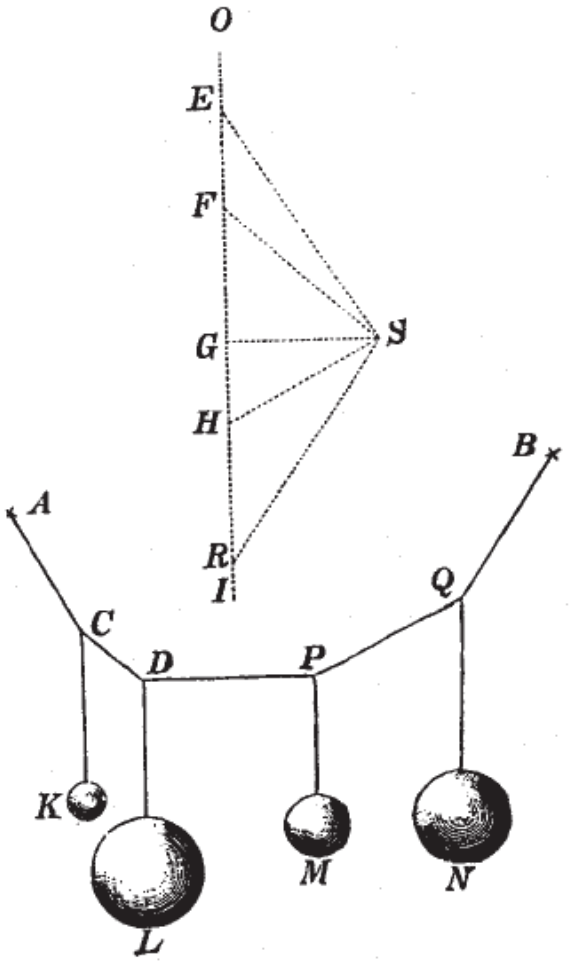
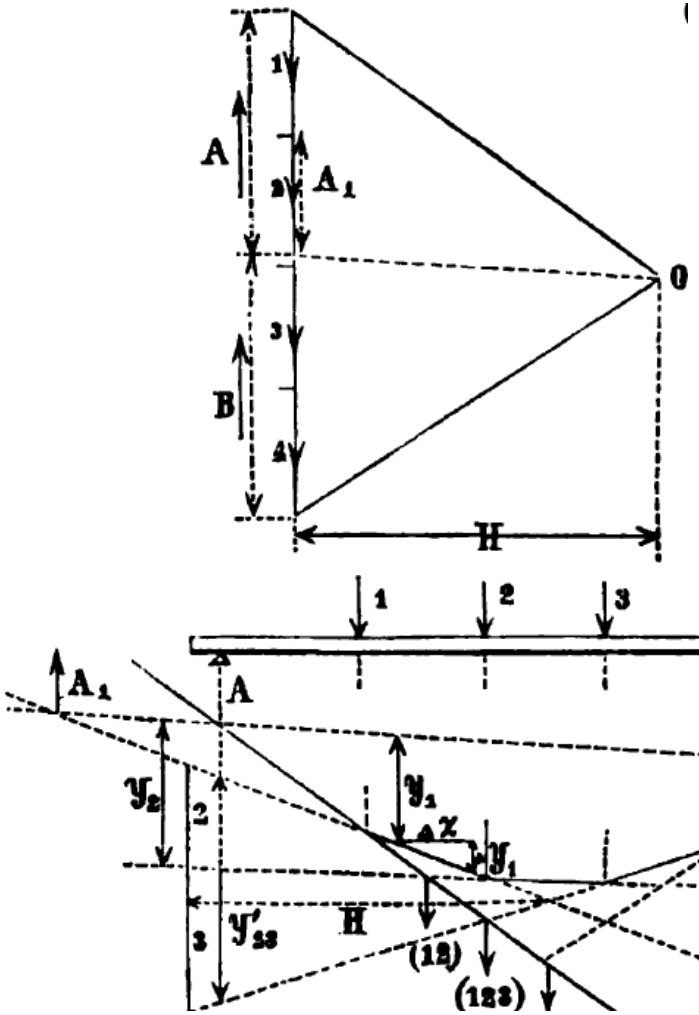
2. Bending moment diagrams and elastic curves as “integration machine”



Force diagram

Form diagram

(Varignon 1825)



Force diagram

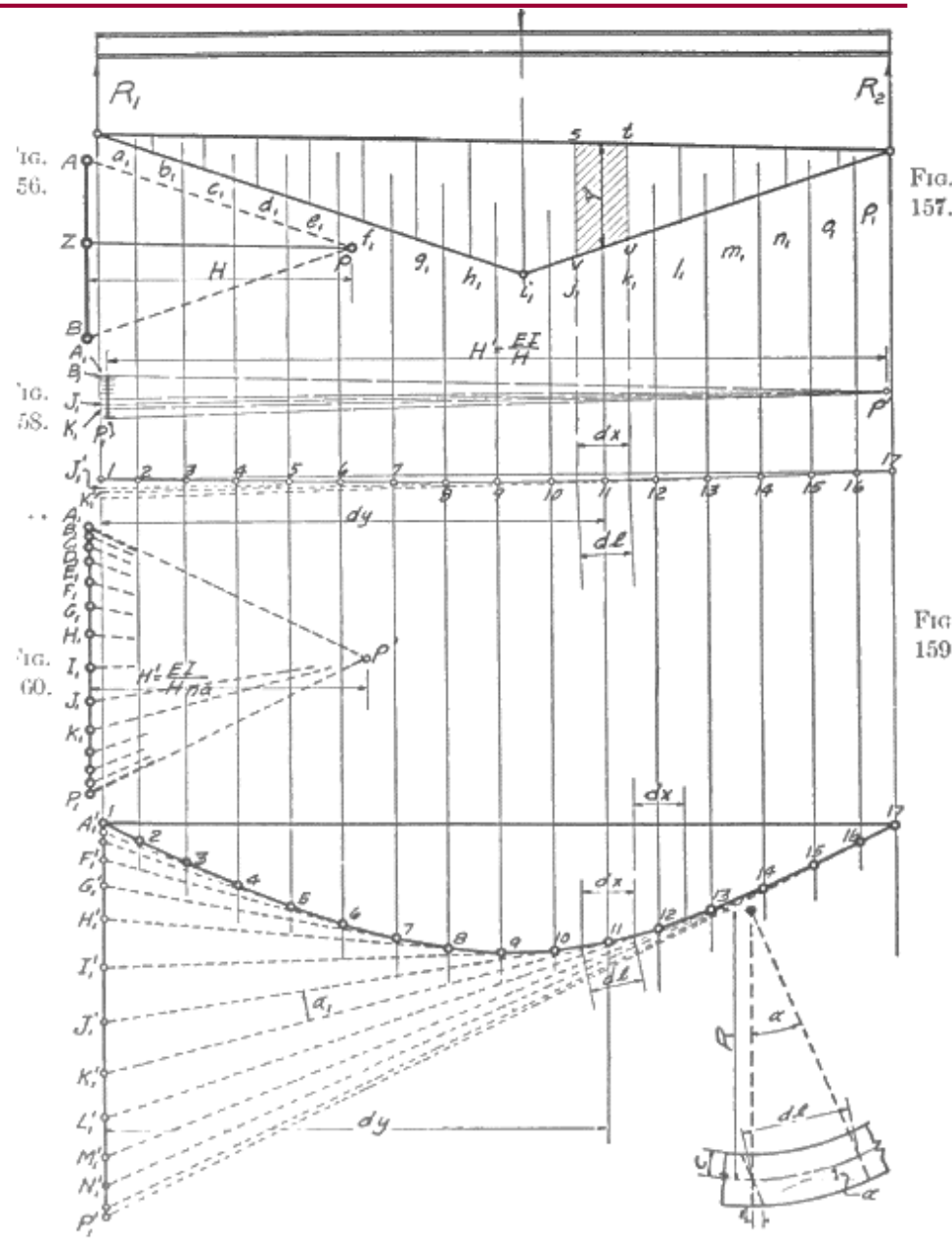
Moment diagram

(Culmann 1866)

the deflection curve could be represented by a funicular polygon analogically taking the bending moment diagram M as the “loading” instead of the loads. [Mohr, 1868]

$$\frac{d^2M(x)}{dx^2} = -q(x) \qquad \frac{d^2\omega(x)}{dx^2} = \frac{M(x)}{EI}$$

$$H' = \frac{EI}{n * H} \qquad V'(x) = n * \omega(x)$$



(Wolfe 1921)

Beams with varying moment of inertia

$$H' = \frac{EI}{n * H}$$

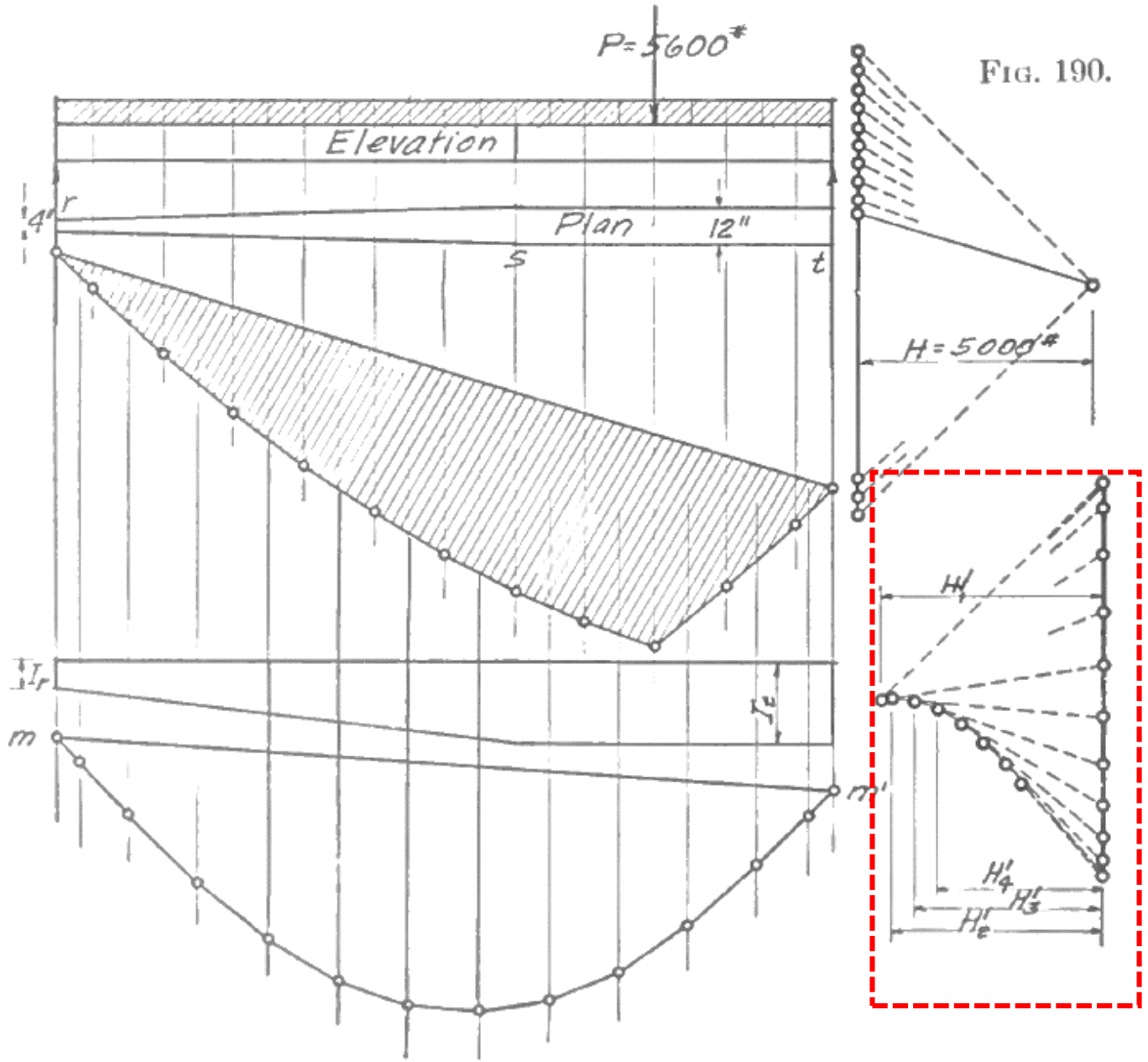


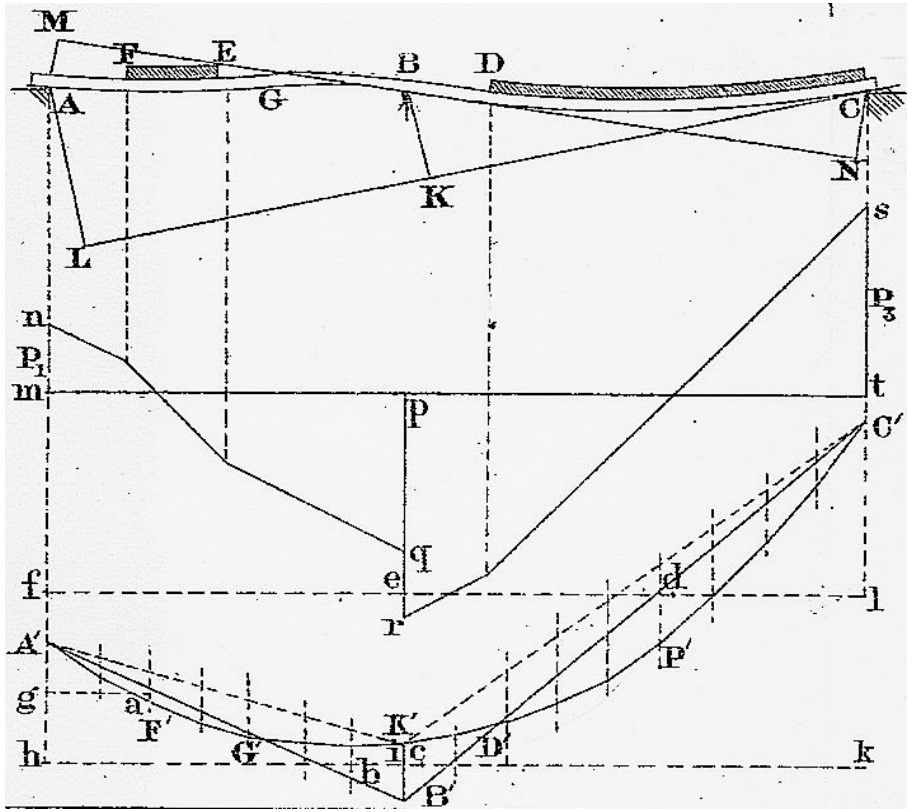
Fig. 190.

Fig. 191.

Fig. 193. (Wolfe 1921)

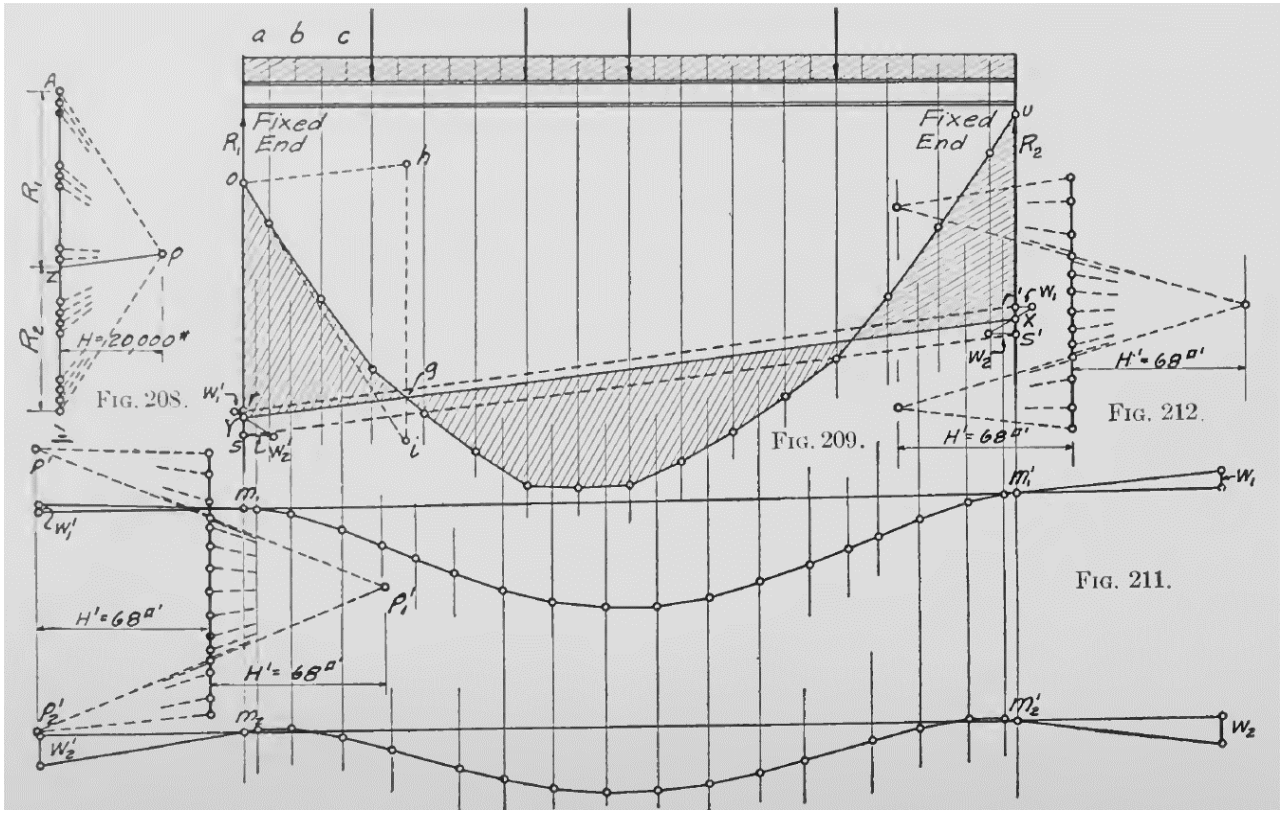
Statically indeterminate Beams
Key problem: determine the closing string

Greene's method
the moment-area method
trial and error



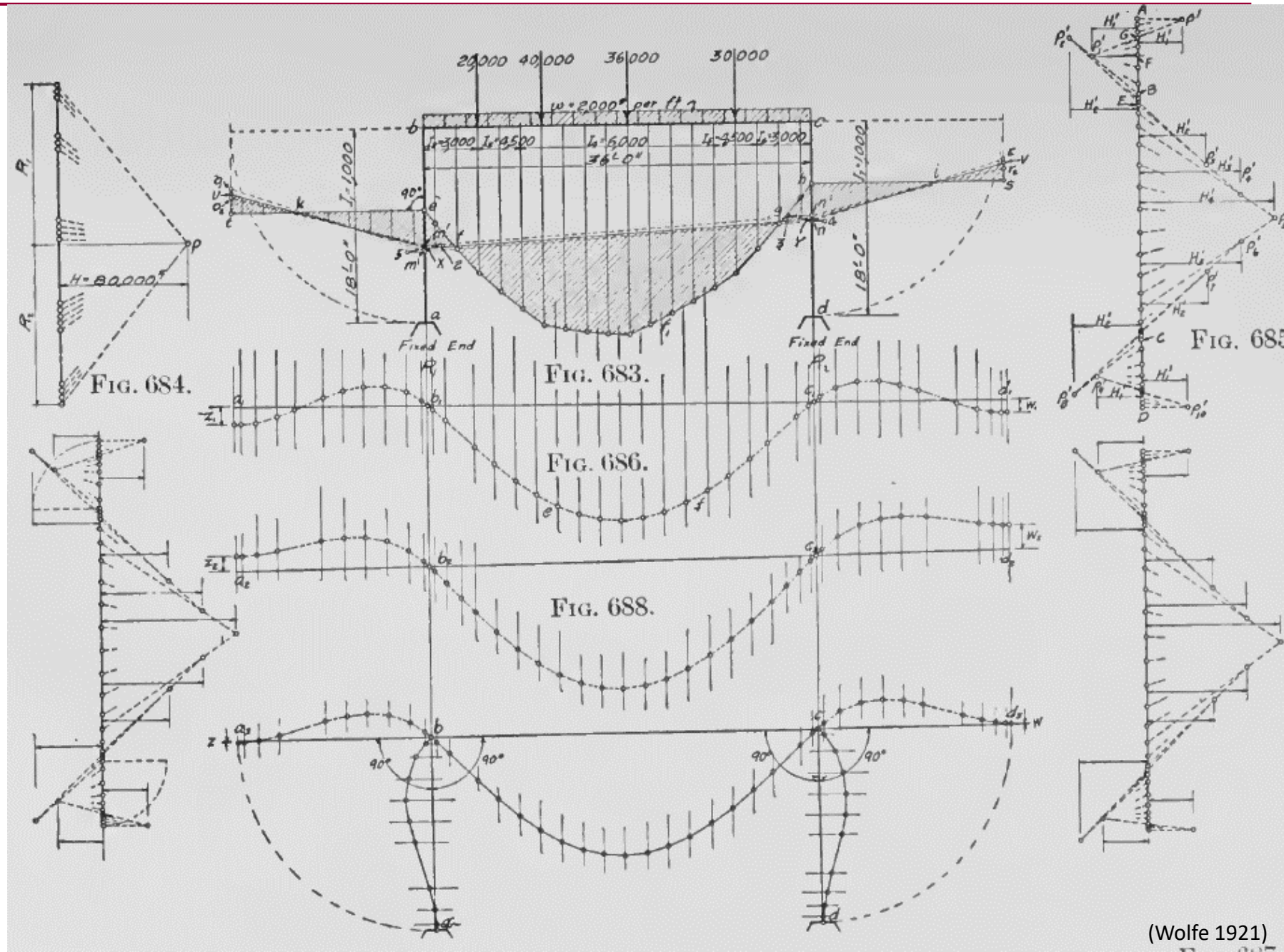
(Greene 1873)

Wolfe's method
the elastic curve
Systematic trial and error



(Wolfe 1921)

Rigid hyperstatic portal frame



(Wolfe 1921)

EVALUATION

- STRENGTHS

provides insights of the relation between variable (load, length and I) and bending moments/elastic curves

- WEAKNESSES

Not adjustable provided no problem-solving means
Cumbersome by hand-drawing
Inaccuracy

POSSIBILITY OF FURTHER DEVELOPMENT

- feasible to backtrack on according variables of diagrams adjusted in certain ways
- Parameter design platform

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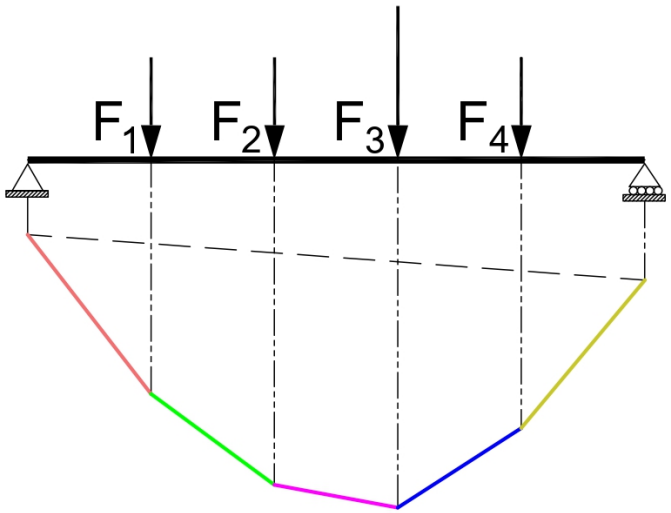
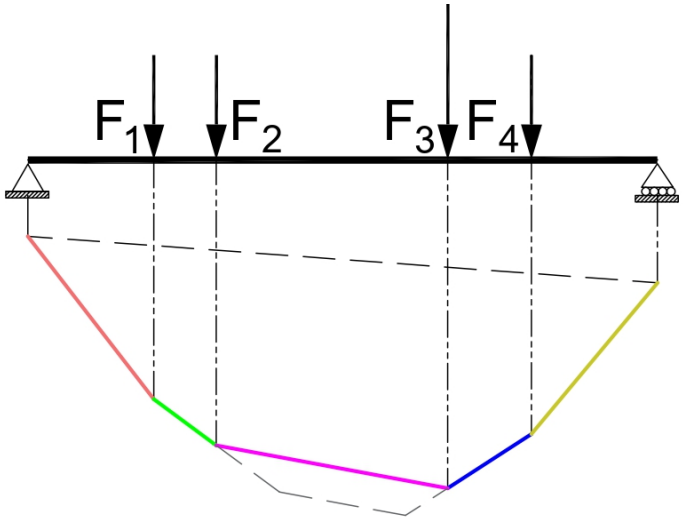
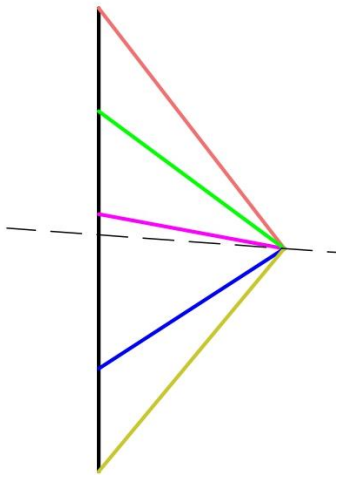


Form diagram

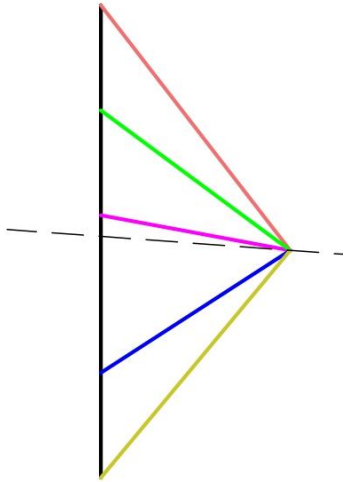
translating



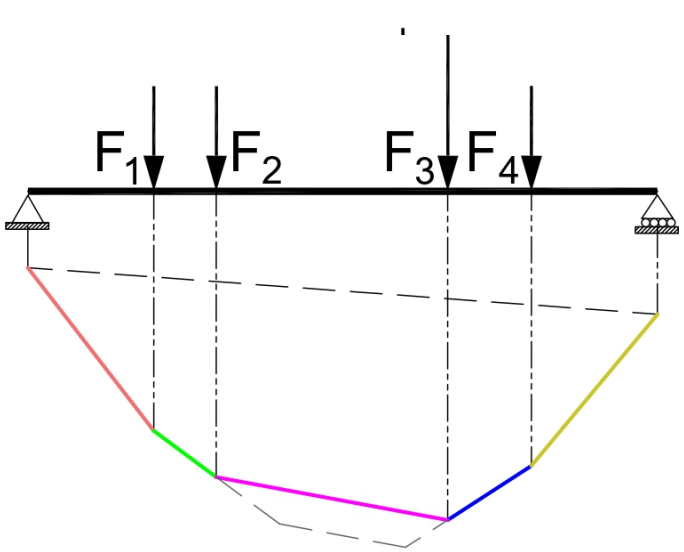
Form diagram



Force diagram

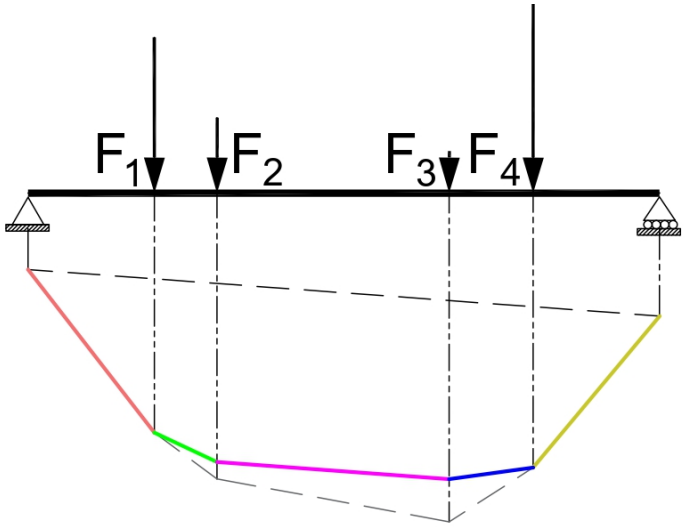


Force diagram



Form diagram

pivoting



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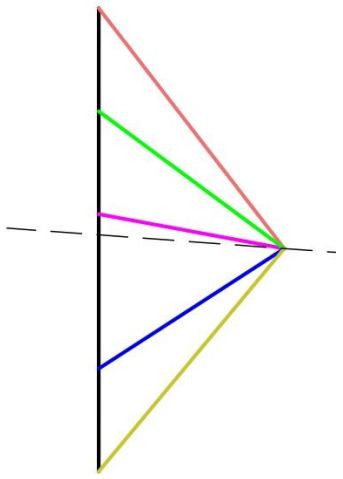
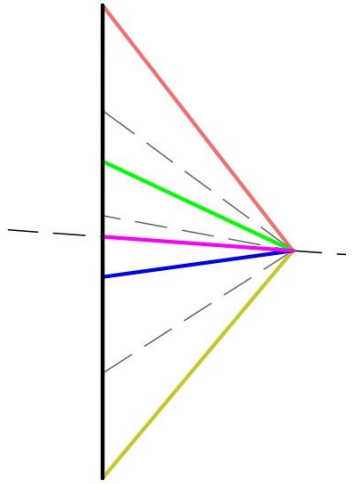
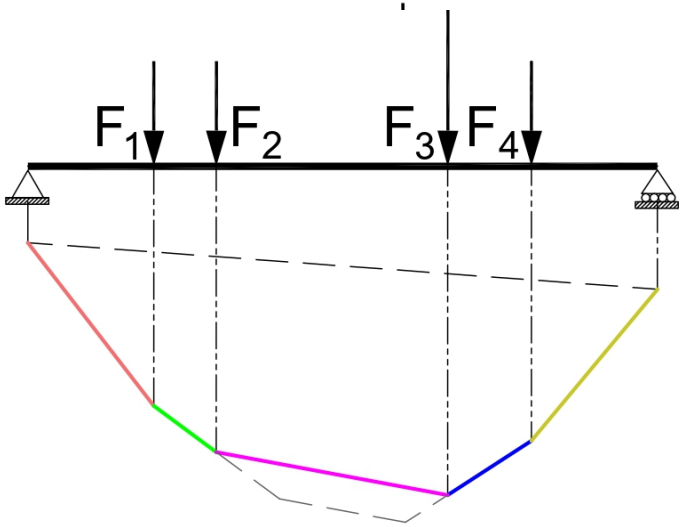


Force diagram

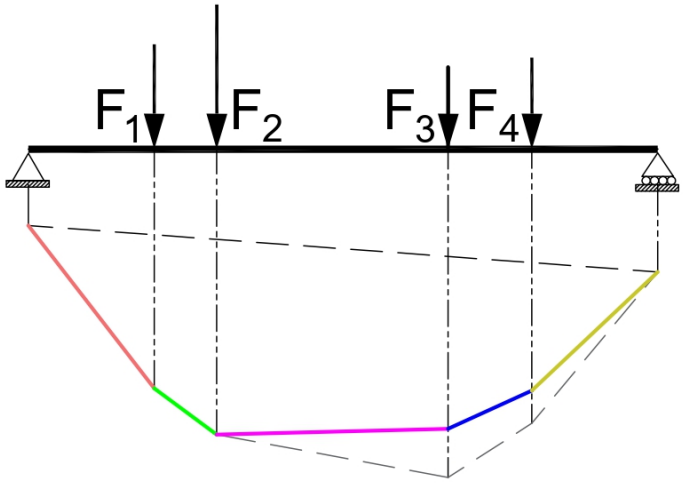


Force diagram

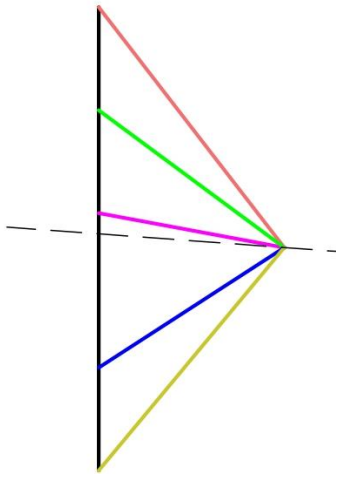


Form diagram

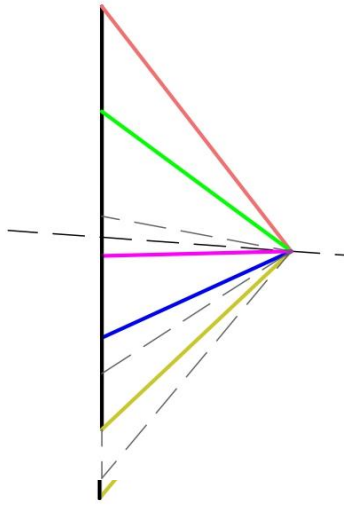
pivoting



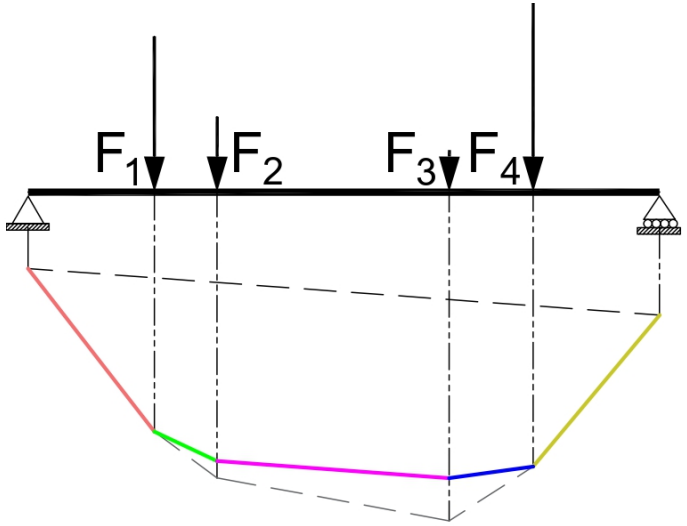
Form diagram



Force diagram

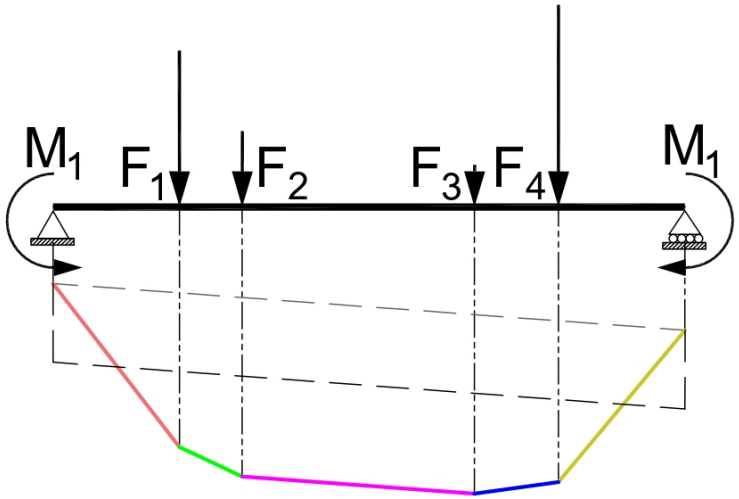


Force diagram



Form diagram

Moving baseline



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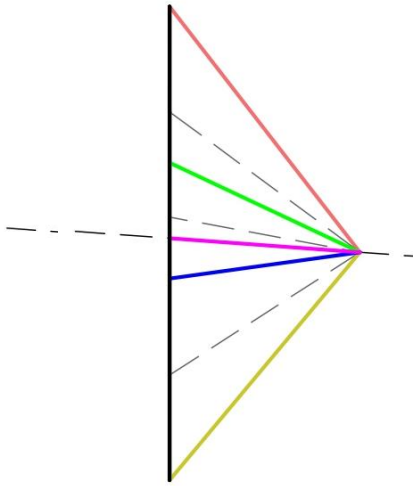
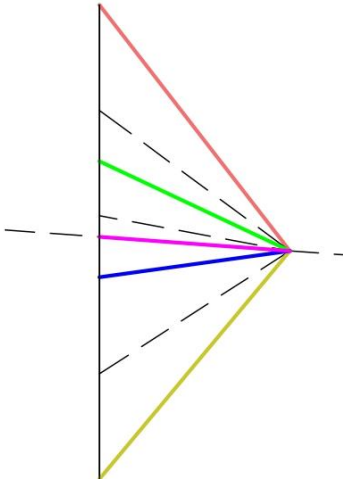
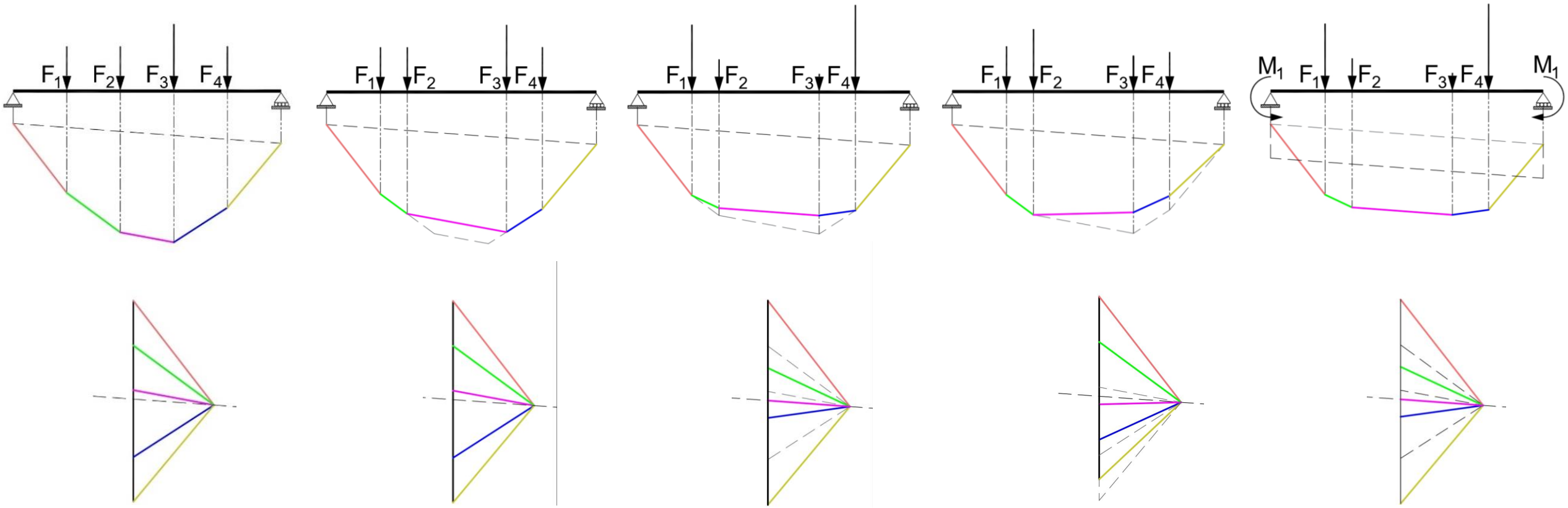


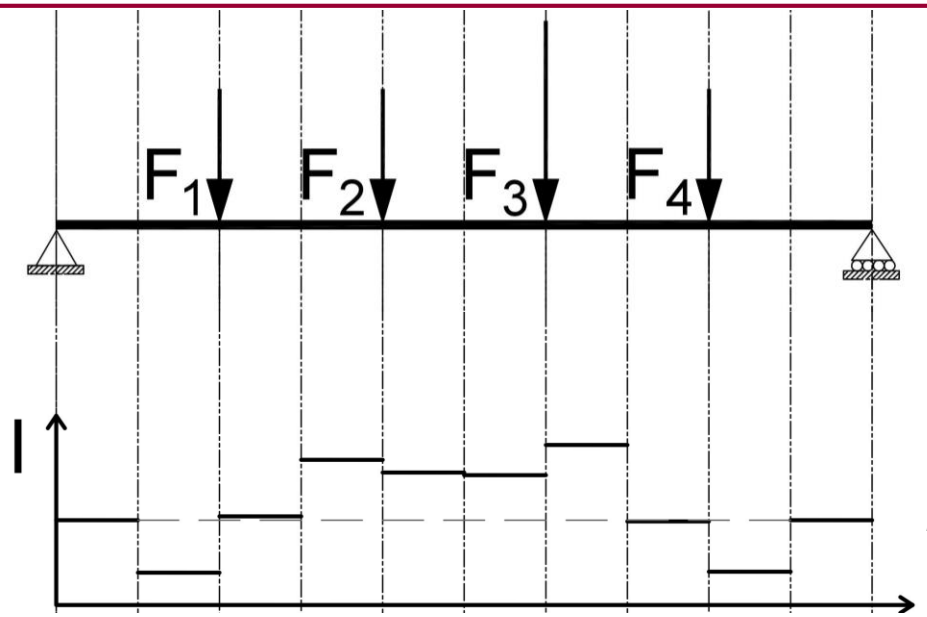
Force diagram



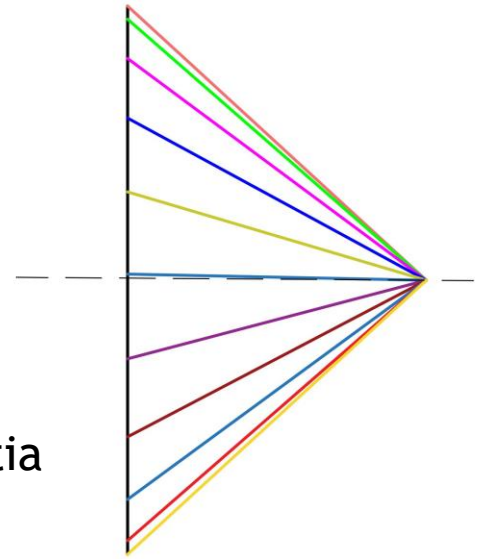
Force diagram



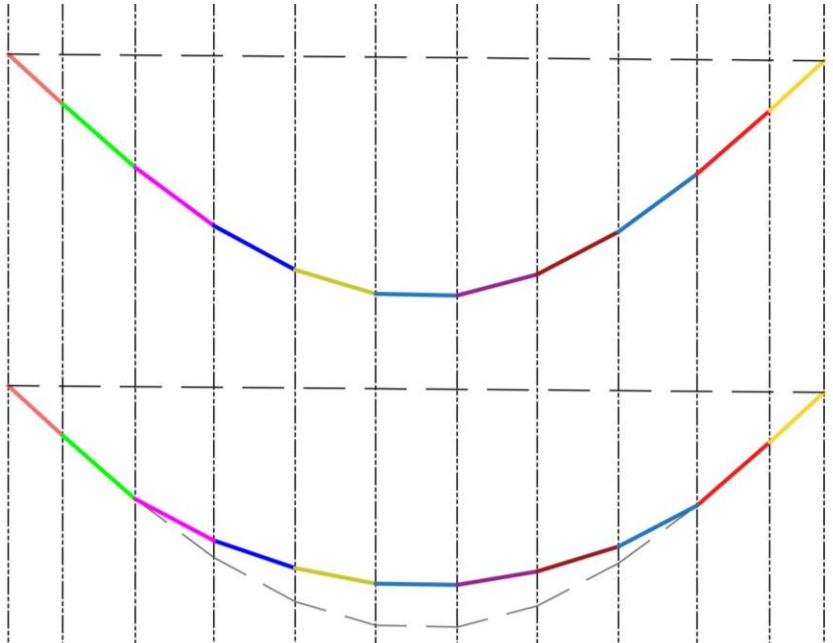
original translating pivoting pivoting Moving baseline



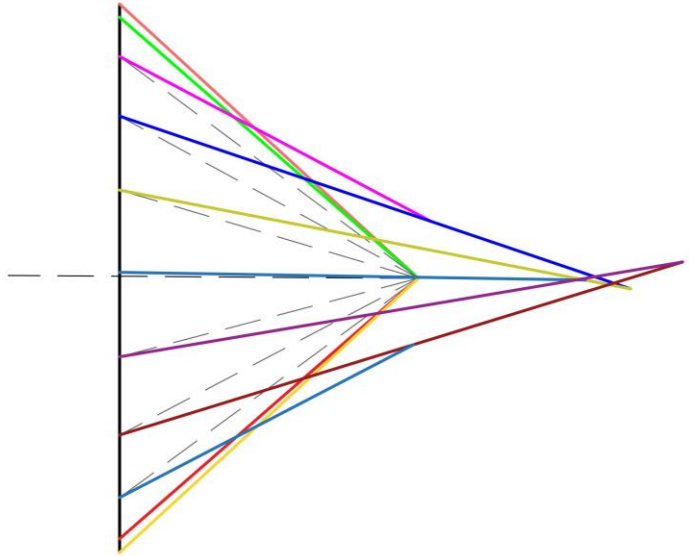
Moment of inertia



Moment force diagram



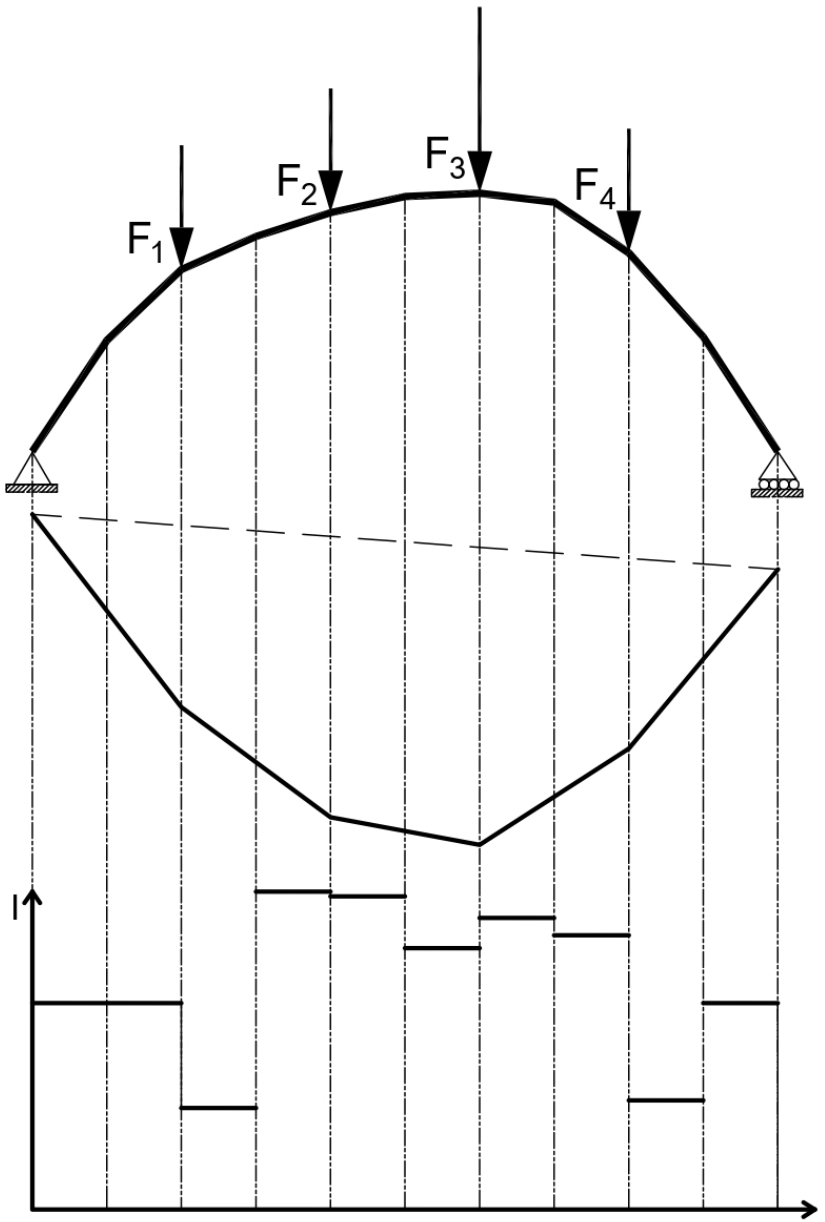
Elastic curves



TO BE CONTINUED…

1. Limited usage
2. Other adjustment possibilities
3. More complex structures
4. Parameterizing
5. Combining with other theories?

THANK YOU !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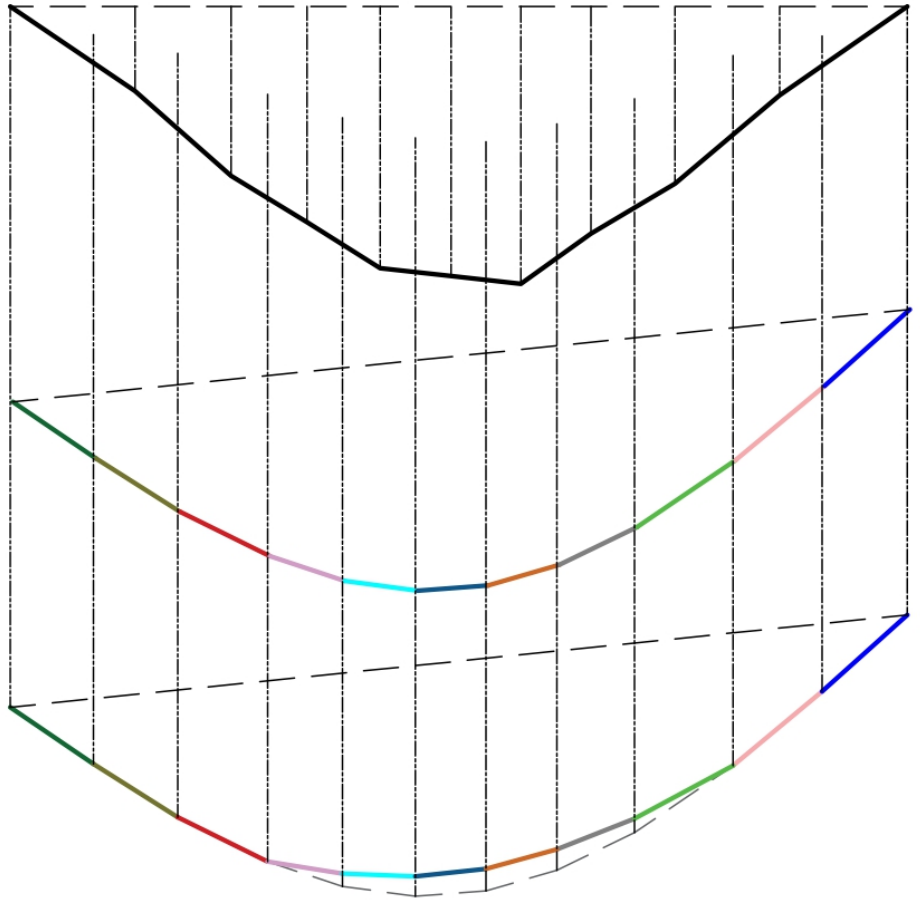
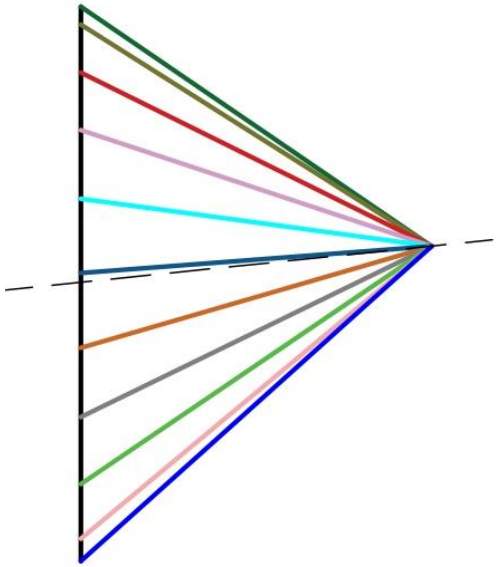
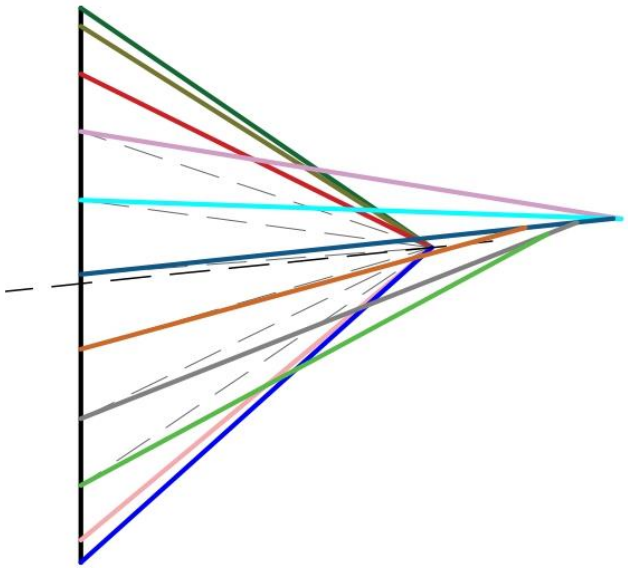






B. HISTORICAL REVIEW

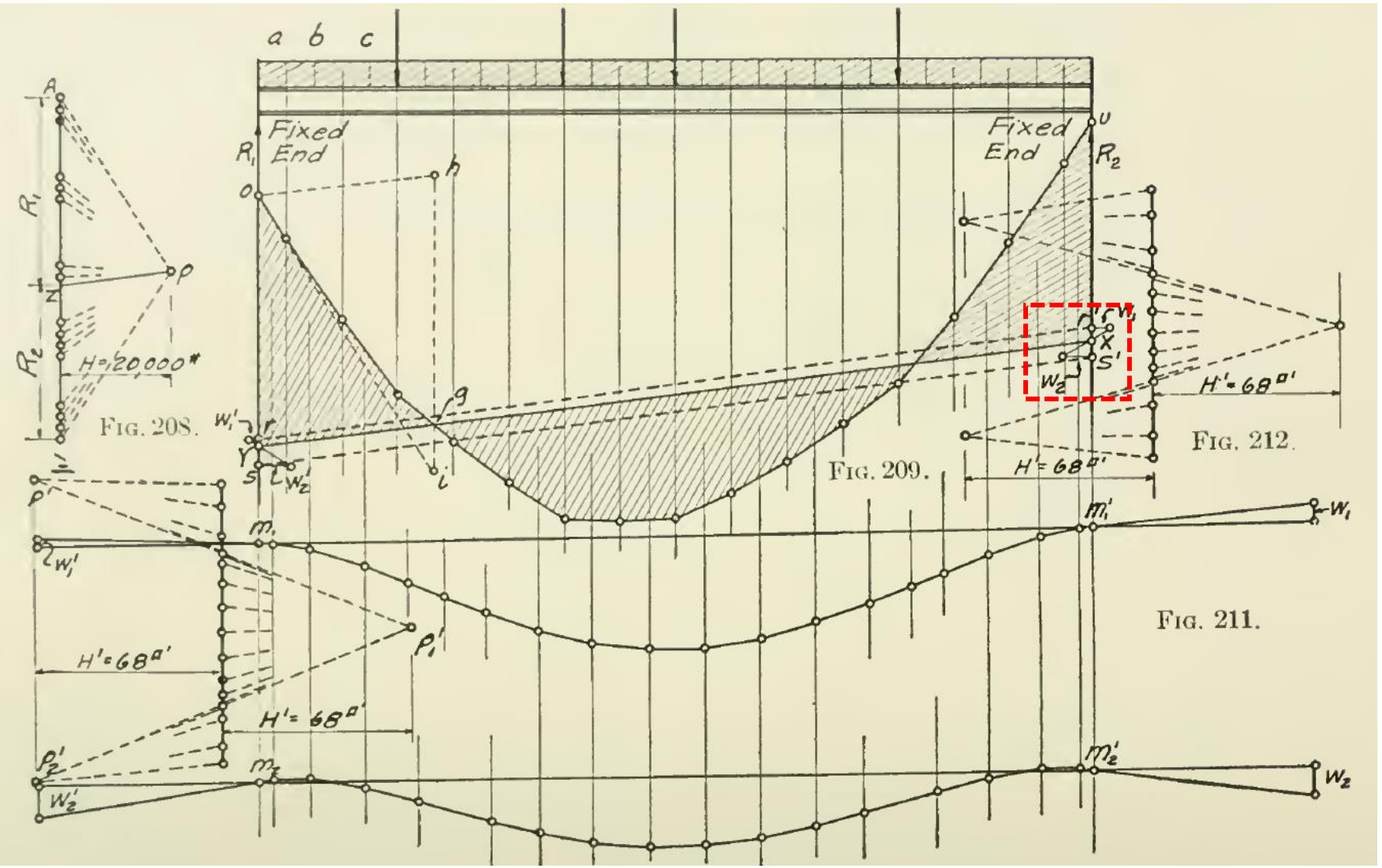
I. graphical analysis of beams

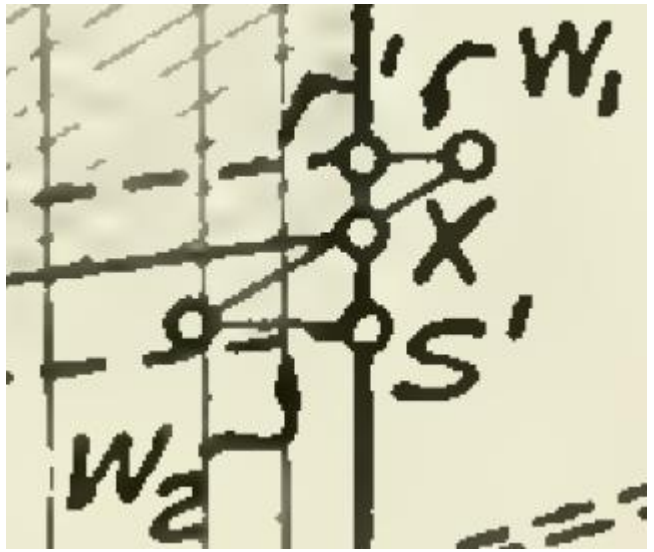
II. graphical analysis of hyperstatic beams

III. graphical analysis for beams with varying moment of inertia

IV. graphical analysis for rigid frames

II. GRAPHICAL ANALYSIS OF HYPERSTATIC BEAMS





(Wolfe 1921)

FORM DIAGRAM → FORCE POLYGON → M CURVES → 'M POLYGON' → ELASTIC CURVE

D. FOLLOWING TASK

- Programming in grasshopper , make it automatic and interactive
- complicated frames: multi-storey, multi-span, or even 3 dimension