A NATURAL STACKELBERG SITUATION UNDER DEMAND UNCERTAINTY

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Working Paper n° 8914

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D/1989/3082/14

A NATURAL STACKELBERG SITUATION UNDER DEMAND UNCERTAINTY.[*]

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June 1989

ABSTRACT: Within a symmetric differentiated duopoly under demand uncertainty framework, we give necessary and sufficient conditions for the existence of a Natural Stackelberg Situation. We show the importance of firms' risk aversion for the endogenisation of role setting in a leader-follower situation.

[*] Helpful comments from J. Drèze, O. Licandro and L. Lismont are acknowledged. The errors remain the author's responsability.

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INTRODUCTION

Oligopoly theory has traditionally analysed the economic leadership and follower role of firms within the Stackelberg framework. In such models, the asymmetry of firms' roles is imposed exogenously. There is no explanation as to why some of the firms should be leaders and others should be followers.

Two questions can then be posed: first, is it possible to make endogenous the attribution of asymmetrical roles; and second, once the roles are attributed, under what conditions will this situation be stable? [Jacquemin (1985), p.74, deals with the second question].

Recent papers have explored the attractiveness of the roles in a Stackelberg duopoly under various circumstances: Albaek (1988), Boyer and Moreaux (1983)(1987a)(1987b), Dowrick (1986), Gal-Or (1985)(1987). Most of the papers have been concerned with what can be called objective and strategic conditions for a leadership role. The only two papers taking into consideration uncertainty are Gal-Or (1987) and Albaek (1988).

The first one analyses a leader-follower game in a stochastic demand framework, and it demonstrates the reduced advantages of the leader. Further, the introduction of private information and the consideration of indirect inference from the follower may change the attractiveness of the roles in a Stackelberg duopoly. The main idea is that the leader's strategy reveals some information to the follower. However there is no consideration about how the Stackelberg situation has initially arisen

Svend Albaek (1988) faces up to this problem. Working within a differential duopoly model with cost uncertainty, he shows that under Cournot competition, a Natural Stackelberg Situation (NSS) is possible. A NSS is defined as "one where one firm prefers, in terms of expected profits, being leader to both being follower and to playing Nash, while the other prefers being follower to both being leader and to playing Nash".

The result at which Albaek arrives is that the firms' position will depend on the variance of costs and on the type of competition (Bertrand or Cournot competition). Due to his assumption of firms' risk-neutrality, Albaek is unable to analyse the influence of subjective conditions on the firms' position. Indeed, his result follows the traditional explanation of asymmetries based on "objective conditions" (financial power, technology, etc.). Another question thus remains open: is it possible to find "subjective" explanations to the leader-follower roles of the firms? Intuitively, one would say that the firms' risk-aversion could be a significant factor.

This paper attempts to explore this intuition, introducing demand uncertainty and a particular objective function of firms which is the same used by Pacheco, Perée and Torres (1988). Within an homogenous product framework, Pacheco, Perée and Torres analyse the duopoly equilibrium under demand uncertainty, but little attention is paied by them to the attribution of roles in Stackelberg equilibria.

The paper is organised as follows. In section 1, the basic certainty model (which we borrow extensively from Albaek) is analysed. Nash and Stackelberg equilibrium levels of output and profit are computed.

Section 2 contains the analysis of the basic differentiated duopoly model under demand uncertainty. The firms' decision criterion is the maximization of the cash equivalent or value of the random profit, which allows us to introduce their risk-aversion. Nash and Stackelberg equilibrium levels of output and net expected profit are computed.

Section 3 analyses Stackelberg equilibria. A Natural Stackelberg Situation (NSS) is explicitely derived within our model, and necessary and sufficient conditions for the existence of a NSS are given.

Finally, section 4 gives some final remarks, and proposes some possible extensions of the basic model.

Section 1. THE CERTAINTY MODEL

This section analyses a symmetric differentiated duopoly model with linear demand functions. The inverse demands may be presented as:

(1) $p_i = a - bq_i - cq_j$

$p_j = a - bq_j - cq_i$

and we assume: A1. $p_i > 0 \; \forall \; i,j$

A2. $b > 0$, $c > 0$, i.e., the goods are substitutes.

A3. Cournot competition (quantity is the strategic variable)

A4. There are no fixed costs and the firms have constant and identical marginal costs normalized to zero.

If both firms maximise profits, under assumptions A1-A4, Nash and Stackelberg equilibrium levels of output can be computed. The profit function may be written as:

(2) $\Pi_i = (a - bq_i - cq_j)\, q_i$

The first-order condition of profit maximisation give us:

(3) $q_i(q_j) = \dfrac{(a - c\,q_j)}{2\,b}$

which is firm i's reaction function.

The derivation of Nash equilibrium levels of output and profit is standard and will not be made explicit here.

(4) $q^N = a / (2b + c)$

(5) $q^L = a(c-2b)/2(c^2-2b^2)$

(6) $q^F = [ac^2 + 2b(c-2b)] / 4b(c^2-2b^2)$

(7) $\Pi^N = a^2b / (2b+c)^2$

(8) $\Pi^L = a^2(2b-c)^2 / 8b(2b^2-c^2)$

(9) $\Pi^F = a^2(4b^2-2bc-c^2)^2 / 16b(2b^2-c^2)^2$

where N,L and F, correspond respectively to Cournot-Nash, Stackelberg leader and Stackelberg follower strategies.

Lemma 1 (Albaek): "In the certainty model the profits to the Stackelberg leader, the Stackelberg follower and to firms in Nash equilibrium can be ranked as follows":

$$\Pi^L > \Pi^N > \Pi^F$$

The lemma follows directly from a simple comparison of expressions (7), (8), and (9).

In this model, it is always better to be the leader if the other firm plays as follower. But there is no endogenous way of determining who will play which role.

Section 2. NASH AND STACKELBERG EQUILIBRIUM UNDER DEMAND UNCERTAINTY

Consider the random inverse demand:

$$(10)\quad \hat{p}_i = a - bq_i - cq_j + \hat{u}$$

where:

^ denotes a random variable;
$\hat{u}$ is a random variable with mean u and variance V .

Both firms are assumed to have the same objective function, which is assumed to be what Pratt (1964) calls the cash equivalent or value of random profit, that is, the expected profit net of the risk premium (henceforth net expected profit).

$$(11)\quad Ji = E(\hat{\Pi}_i) - \mu(\hat{\Pi}_i),$$

where J is the net expected profit, E(.) denotes the expectation operator, and $\mu(.)$ denotes the risk premium.

The risk premium $\mu(.)$ may be written [see Sandmo (1969) for a formal derivation]:

$$(12)\quad \mu(.) = \frac{r_i V(.)}{2}$$

where r_i mesures firm i's risk- aversion.

The optimisation problem becomes:

$$(13)\quad \text{Max } J_i = E(\hat{\Pi}_i) - \frac{r_i V(\hat{\Pi}_i)}{2}$$

$$= [a + u - b q_i - cq_j] q_i - \frac{r_i V q_i^2}{2}$$

The first and second order conditions give us:

$$(14)\quad q_i(2b + r_i V) = a + u - c q_j$$

$$(15)\quad 2b + r_i V > 0$$

From the first order condition we can derive firm i's reaction function under demand uncertainty:

$$(16)\quad q_i(q_j) = \frac{A - c q_j}{2b + r_i V}$$

where $A = a + u$

The Nash equilibrium levels of output can be derived by taking the intersection of firms' reaction functions, which gives us (see appendix):

$$(17)\quad q_i^N = \frac{A(x_j - c)}{x_i x_j - c^2}$$

where $x_i = 2b + r_i V$

And the firm i's net expected profit becomes (see app.):

$$(18)\quad J_i^N = \frac{A^2}{D^2}(x_j - c)^2\left(x_i - b - \frac{r_i V}{2}\right)$$

where

$$D = (2b + r_i V)(2b + r_j V) - c^2 = x_i x_j - c^2$$

The influence of increases in u and V is extensively treated by Pacheco, Perée, and Torres (1988), within a framework of a homogeneous product. Most of their results hold within our differentiated duopoly framework. We leave the discussion of the differences for the final section, and we now compute the Stackelberg equilibria. Still within a Cournot competition framework, we consider a firm as a Stackelberg follower when it has a zero output conjectural variation. In our model, this means that it simply sets output to maximise net expected profit, taking its rival's output as given. A leader, by contrast, assumes that its rival will adopt the followership pattern, and maximizes its net expected profit given this assumption. Mathematically this means that the leader will maximise subject to the follower's reaction function.

From the leader's maximisation problem we can derive (see app.):

$$(19) \quad q_i^L = \frac{A(x_j - c)}{(D - c^2)}$$

The substitution of this value in the follower's reaction funtion gives us:

$$(20) \quad q_j^F = \frac{A(D - c x_j)}{(D - c^2) x_j}$$

Once the output levels are derived we are able to derive the firms' net expected profit (see app.):

$$(21) \quad J_i^L = \frac{(x_j - c)^2 A^2}{2(D - c^2) x_j} = \frac{(q_i^L)^2 (D - c^2)}{2 x_j}$$

$$(22)\quad J_i^F = \frac{(D - c x_j)^2 A^2}{2(D - c^2)^2 x_j} = \frac{(q_j^H)^2 x_j}{2}$$

From these results we can now face up to the problem of the attractiveness of different roles and the influence on it of risk-aversion (a subjective condition) and information.

Section 3. A NATURAL STACKELBERG SITUATION UNDER DEMAND UNCERTAINTY

A Natural Stackelberg Situation (NSS) is defined by Albaek as "one where one firm prefers, in terms of expected profits, being leader to both being follower and to playing Nash, while the other prefers being follower to both being leader and to playing Nash".

Since we are working within a net expected profit maximisation framework, we redefine a NSS as one where one firm (henceforth firm i) prefers ***-in terms of net expected profits*** - being leader to both being follower and to playing Nash, while the other (in our case firm j) prefers being follower to both being leader and to playing Nash. The question is whether such a situation can exist.

Formally, the problem can be expressed as a comparison of net expected profits in the following terms.

A NSS exists if and only if all of the following conditions are satisfied:

(C_1) $J^L_i > J^N_i$

(C_2) $J^L_i > J^F_i$

(C3) $J^F_j > J^L_j$

(C4) $J^F_j > J^N_j$

The first result from net expected profit comparisons is expressed in the following proposition:

PROPOSITION 1. It is always better -in terms of net expected profits- to play Stackelberg leader than to play Nash. (see app.)

Since (C_1) holds for all i,j; conditions 3 and 4 can be summed up as follows:

$$J^F_j > J^L_j > J^N_j,$$

so the two necessary and sufficient conditions for the existence of a NSS are (C_2) and (C_3).

Manipulating (C_2) and (C_3), we derive the following

PROPOSITION 2. A NSS can exist if and only if

$$(23)\quad \left(\frac{q_i^L}{q_i^F}\right)^2 > \frac{D + c^2}{D - c^2} \quad \text{and}$$

$$(24)\quad \left(\frac{q_j^L}{q_j^F}\right)^2 < \frac{D + c^2}{D - c^2}$$

These necessary and sufficient conditions depend on firms' risk-aversion. Indeed, two inequations in r_i, r_j can be derived from (23) and (24):

$$\text{I)}\quad (2b+r_jV)\left[-(2b+r_iV)^2 + 2b+r_iv - c^2\right] > 2c$$

$$\text{II)}\quad (2b+r_iV)\left[-(2b+r_jV)^2 + 2b+r_jv - c^2\right] < 2c$$

Solving this inequation system (see app) yields the following necessary conditions which limit the range within which a NSS can exist.

$$\mathbf{NC_1.-}\quad r_j V > r_i V$$

$$\mathbf{NC_2.-}\quad c < {}^1/_2$$

Let $x_i = 2b+r_iV$, and $x_j = 2b+r_jV$.

$$\mathbf{NC_3.-}\quad (2x_i-1)^2 < 1-4c^2$$

$\mathbf{NC_4.-}$ **If** $(2x_j-1)^2 >_< 1-4c^2$, **then it is necessary that**

$$x_i >_< \frac{2c}{(-x_j^2+x_j-c^2)}$$

OBSERVATION 1. The first necessary condition (NC_1) shows us -a contrario- an impossibility proposition: if the distribution function on the stochastic term ($\hat{u}$) is common knowledge to both firms (so that V is the same to both firms), and if they have the same attitude towards risk ($r_i=r_j$), then a NSS cannot exist. There must be an asymmetry either in firms'risk-aversion ($r_.$) or in information held by firms (V), or in the firms' perception of risk (V), to ensure the possibility of existence of a NSS. This means that under demand uncertainty we find that there must exist "subjective" conditions for the setting of

roles in a leader-follower situation. To show this is the main purpose of this paper.

OBSERVATION 2. "Subjective" conditions do not suffice for the existence of a NSS; objective and strategic conditions (on demand elasticities and private information parameters) are also required. Indeed, NC_2, NC_3, and NC_4 appear as a bound to cross effects (c) and a range of risk-aversion values bounded by own effects (b). It is worth noting that the domain delimited by (23) and (24) depends in our assumption on costs functions of the firms. Further, the introduction of cost functions with differentiation costs ought to allow us to analyse the interdependence between "subjective", "strategic", and "objective" conditions.

The major original result of our model is that there exists a set of values of firms'risk-aversion, demand elasticities, and information parameters within which a Natural Stackelberg Situation is possible.

The importance of this original result is that:

First, there is an endogenous way of explaining the setting of roles in a leader - follower situation.

Second, the economic dominance on a market (usually explained in terms of objective and strategic conditions) is explained also by subjective conditions. Under uncertainty, a risk-averse firm may prefer to be dominated.

Section 4. FINAL REMARKS

We have limited our analysis to the NSS. The main reason for this is that the analysis of Nash and Stackelberg equilibrium has already been developed by Pacheco, Perée and Torres within a homogeneous product framework, and their results hold within our model. The only difference between the two models concerns the effect of a variation in firms' risk-aversion on the Stackelberg equilibrium. In the homogeneous product framework, an increase in the follower's risk-aversion leads to an ambiguous effect on the leader's output (Pacheco, Perée and Torres, p.17), while in our model the effect is an increase (reduction) in the leader's output, if and only if the leader's risk premium is greater (smaller) than the difference -in terms of the demand elasticities- between cross effects and own effects [Porto (1989)]. Under the usual assumption that own effects are greater than cross effects, this difference is negative and the leader's risk premium is positive, so that an increase in the follower's risk aversion leads to an increase in the leader's output.

Concerning section 3, the major result which we hope could be further developed is the importance of firms' attitude towards risk on economic dominance. The relevant idea is that a dominated situation can be altered by changing the firm's risk aversion. Traditionally, the firms' risk aversion is considered as a parameter, but -for example- a change of ownership could modify this parameter and so could alter a dominated situation. Collusion and mergers will then alter market structure because the change in firms' risk-aversion. This would be an exogenous change.

An interesting possible extension of the model would be to endogenise this change.

It is worth noting that interdependence of "subjective", "objective", and "strategic" conditions of a Natural Stackelberg Situation could be developed introducing a cost function with differentiation costs.

Finally, a dynamic version of the model could complement Gal-Or's analysis concerning how the Stackelberg situation arises.

Appendix

Derivation of (17)

A1) $q_i(q_j) = \frac{A - c\,q_j}{x_i}$ and $q_j(q_i) = \frac{A - c\,q_i}{x_j}$ (16)

where $x_i = 2b + r_i V$ and $A = a + \mu$

A2) $q_i^N = \frac{A - c\left(\frac{A - c\,q_i}{x_j}\right)}{x_i} = \frac{A\,x_j - c\,A + c^2 q_i}{x_i\,x_j}$

A3) so $q_i^N = \frac{A(x_j - c)}{x_i x_j - c^2}$ (17)

Derivation of (18)

A4) $J_i = (A - b\,q_i - c\,q_j)\,q_i - \frac{r_i V}{2} q_i^2$

A5) $J_i^N = \left[A - b\left(\frac{A(x_j - c)}{D}\right) - c\left(\frac{A(x_i - c)}{D}\right)\right] \frac{A(x_j - c)}{D} - \frac{r_i V}{2}\left(\frac{A(x_j - c)}{D}\right)^2$

where $D = x_i x_j - c^2$

A6) $J_i^N = \frac{\left[D\,A - b\,A(x_j - c) - c\,A(x_i - c)\right] A(x_j - c) - \frac{r_i V}{2}(A(x_j - c))^2}{D^2}$

A7) $J_i^N = \frac{A^2}{D^2}(x_j - c)\left[D - b(x_j - c) - c(x_i - c)\right] - \frac{r_i V}{2}(x_j - c)$

A8) $J_i^N = \frac{A^2}{D^2}(x_j - c)^2\left(x_i - b - \frac{r_i V}{2}\right)$ (18)

Derivation of (19)

Firm *i* has a Stackelberg leader strategy, so it maximaizes net expected profit subject to *j*'s reaction function.

A26) $$\underset{q_i}{\text{Max}}\ J_i = \left(A - bq_i - \frac{cA}{x_j} + \frac{c^2 q_i}{x_j}\right) q_i - \frac{r_i V}{2} q_i^2$$

(A27) $$\frac{\partial J_i}{\partial q_i} = A - bq_i - \frac{cA}{x_j} + \frac{c^2 q_i}{x_j} - bq_i + \frac{c^2 q_i}{x_j} - r_i V q_i = 0$$

(A28) $$\frac{\partial J_i}{\partial q_i} = q_i \frac{\left(-2bx_j + 2c^2 - r_i V x_j\right)}{x_j} + \frac{A(x_j - c)}{x_j} = 0$$

(A29) $$q_i (D - c^2) = A(x_j - c)$$

(A30) $$q_i^L = \frac{A(x_j - c)}{(D - c^2)} \quad (19)$$

Derivation of (21)

Substitution of (19) and (20) for (11) gives us

(A31) $$J_i^L = \frac{A^2(x_j - c)}{(D - c^2)} - b\frac{A^2(x_j - c)^2}{(D - c^2)^2} - c\frac{A^2(x_j - c)(D - c x_j)}{(D - c^2)^2 x_j} - \frac{r_i V}{2}\frac{A^2(x_j - c)^2}{(D - c^2)^2}$$

(A32) $$J_i^L = \frac{A^2}{(D - c^2)^2 x_j}\left\{(x_j - c)x_j(D - c^2) - b(x_j - c)^2 x_j - c(x_j - c)(D - c x_j) - \frac{r_i V x_j (x_j - c)^2}{2}\right\}$$

(A33) $$J_i^L = \frac{A^2}{(D - c^2)^2 x_j}(x_j - c)\left(D(x_j - c) - (x_j - c)x_j\left(b + r_i\frac{V}{2}\right)\right)$$

(A34) $$J_i^L = \frac{A^2}{2(D - c^2)x_j}(x_j - c)^2$$

Derivation of (22)

$$(A35)\quad J_j^F = \frac{A^2(D - c x_j)}{(D - c^2) x_j} - b\frac{A^2(D - c x_j)^2}{(D - c^2)^2 x_j^2} - c\frac{A^2(x_j - c)(D - c x_j)}{(D - c^2)^2 x_j} - \frac{r_i V}{2}\frac{A^2(D - c x_j)^2}{(D - c^2)^2 x_j^2}$$

$$(A36)\quad J_j^F = \frac{A^2}{(D - c^2)^2 x_j^2}\left\{(D - c x_j)(D - c^2) x_j - b(D - c x_j)^2 - c(x_j - c)(D - c x_j) x_j - \frac{r_j V (D - c x_j)^2}{2}\right\}$$

$$(A37)\quad J_j^F = \frac{A^2(D - c x_j)^2}{(D - c^2)^2 x_j^2}\left(x_j - b - \frac{r_j V}{2}\right)$$

$$(A38)\quad J_j^F = \frac{A^2(D - c x_j)^2}{2(D - c^2)^2 x_j}$$

Proof of Proposition 1

$$(A39)\quad J_i^L = \frac{A^2}{2(D - c^2) x_j}(x_j - c)^2 > \frac{A^2}{2D^2}(x_j - c)^2 x_i = J_i^N$$

$$(A40)\quad \frac{D^2}{D - c^2} > x_i x_j$$

$$(A41)\quad \frac{D^2}{D - c^2} > D + c^2$$

$$(A42)\quad D^2 > D^2 - c^4.$$

Idem for firm j.

Proof of Proposition 2

(A43) $J_i^L > J_i^F$

From (19) and (21), and (20) and (22), we can rewrite:

(A44) $J_i^L = \frac{(D - c^2)}{2 x_j}(q_i^L)^2 > \frac{x_i}{2}(q_i^F)^2 = J_i^F$

So $J_i^L > J_i^F$ if and only if

$$\frac{q_i^{L^2}}{q_i^F} > \frac{D + c^2}{D - c^2}$$

Similarly, $J_j^F > J_j^L$, if and only if

$$\frac{q_j^{L^2}}{q_j^F} \leq \frac{D + c^2}{D - c^2}$$

A NATURAL STACKELBERG SITUATION

$J_i^L > J_i^F$

Let $x_i x_j = P$,

(A45) $J_i^L = \frac{A^2}{2(D - c^2)x_j}(x_j - c)^2 > \frac{A^2(D - c x_j)^2}{2 x_i (D - c^2)^2} = J_i^F$

(A46) $\frac{(x_j - c)^2}{x_j} > \frac{(P - c^2 - c x_i)^2}{x_i(P - 2c^2)}$

(A47) $(x_j - c)^2 x_i^2 (P - 2c^2) > (P - c^2 - c x_i)^2 P$

(A48) $(P^2 - 2 c x_i P + c^2 x_i)(P - 2c^2) > (P - c^2 - c x_i)^2 P$

(A49) $(c^2 x_i - c^2 x_i^2 - c^4) P > 2 c^3 x_i$

(A50) $(- x_i^2 + x_i - c^2) x_j > 2 c$ (I)

Similarly, $J_j^F > J_j^L$, if and only if

(A51) $\left(-x_j^2 + x_j - c^2\right)x_i \leq 2c$ (II)

Derivation of Necessary Conditions

(I) and (II) can be rewritten

(I') $\left(-x_i^2 + x_i - c^2\right)x_j - 2c > 0$

(II') $\left(x_j^2 - x_j + c^2\right)x_i + 2c > 0$

From (I') and (II'), we know that a necessary condition is given by

(I') + (II') > 0, so

(A52) $-x_i^2x_j + x_ix_j - c^2x_j + x_ix_j^2 - x_ix_j + c^2x_i > 0$

(A53) $x_ix_j(x_j - x_i) - c^2(x_j - x_i) > 0$

(A54) $D(x_j - x_i) > 0$

as $D > 0$, so

(NC_1) $(x_j - x_i) > 0$

From (I) we have $\left(-x_i^2 + x_i - c^2\right)x_j > 2c > 0$ so

$\left(-x_i^2 + x_i - c^2\right)$ must be positive. For that there are two necessary conditions:

(A55) $\Delta = \sqrt{1 - 4c^2} \in R$, so $c \leq \frac{1}{2}$, which is (NC_2)

(A56) $\frac{1 - \sqrt{1 - 4c^2}}{2} < x_i < \frac{1 + \sqrt{1 - 4c^2}}{2}$ which is (NC_3)

Finally, from (II) we have $\left(-x_j^2 + x_j - c^2\right)x_i < 2c$

so, if $\left(-x_j^2 + x_j - c^2\right) > 0$, then

(A57) $x_i < \dfrac{2c}{\left(-x_j^2 + x_j - c^2\right)}$

$\left(-x_j^2 + x_j - c^2\right) > 0$ if $c \leq \frac{1}{2}$, and if

$(2x_j - 1)^2 < 1 - 4c^2$

Similarly, if

$(2x_j - 1)^2 > 1 - 4c^2$

then the necessary condition is

(A58) $x_i > \dfrac{2c}{\left(-x_j^2 + x_j - c^2\right)}$ which completes (NC_4)

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