

**$\pi\pi$ Invariant mass spectrum in $V' \rightarrow V \pi\pi$
and the $f_0(600)$ pole.**

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We consider the phenomenological description of the two pion invariant mass spectrum in the $V' \rightarrow V\pi\pi$ decays. We study the parametrization of the amplitude involving both S and D wave contributions. From a fit to the two pion decays of the $\Upsilon(nS)$ and $\Psi(nS)$ we determine the $f_0(600)$ mass and width to be $m_{f_0} = 528 \pm 32$ MeV and $\Gamma_{f_0} = 413 \pm 45$ MeV. The mass and width values we report correspond to the real and imaginary part of the S matrix pole respectively.

The experimental identification of low mass scalar resonances is a long standing puzzle whose origin can be traced back to some of the following characteristics: a large decay width, possible mixing with multiquark or glue balls, overlap of resonances and the opening of channels, responsible for the appearance of cusp effects [1]. In particular the $f_0(600)$ has a long history, it has been included in some issues of the Review of Particle Properties but it has also been excluded for long periods by the Particle Data Group. Recently, experimental evidence from different corners of particle physics has accumulated confirming the existence of the $f_0(600)$ resonance. There have been attempts to interpret the low lying scalars as multiquark states [2, 3] or $\bar{K}K$ bound states [4, 5]. Also, models exist starting directly from chiral symmetry [6, 7, 8] or else unitarized models where the scalar nonet arises [9, 10]. However the understanding of actual processes involving scalar mesons for descriptions starting from first principles are not available, effective theories are not well suited to deal with these scalars, the use of sum rules is perhaps the best approach to the problem[11], however no predictions for all of the scalars have been advanced in that framework. Furthermore, the phenomenological description of broad resonance faces severe problems, the determination of the physical parameters -mass and width- is a non trivial problem that requires study of the non-resonant background dependence.

For small invariant mass of the pion pair, data is dominated by the phase shift [12, 13]. Studies have been carried in this kinematical region using chiral symmetry and Roy equations -solid theoretical tools- establishing thus a firm result to be considered by other analysis [14]. Inclusion of the di-pion low invariant mass favor a light and very broad $f_0(600)$ ($m \approx 470 \pm 30 \text{ MeV}$ and $\Gamma/2 \approx 295 \pm 20 \text{ MeV}$). Below 1 GeV, information on the $\pi\pi$ phase shift is obtained from πN scattering, $P\bar{P}$ annihilation at rest and central production, these data allow the existence of a broad ($\Gamma \approx 500 \text{ MeV}$) scalar meson resonance [15, 16]. Decay of pseudoscalar charmed mesons are also a source of valuable data involving scalar mesons, although the $f_0(600)$ has only been reported by the E-791 collaboration after the Dalitz plot analysis of $D \rightarrow \pi^+\pi^-\pi^+$ [17]. Another important source of information are the vector meson decays. The processes considered involve either the scalars themselves or pion pairs together with photons or vector mesons. Among these we can mention $(\rho, \omega, \Phi) \rightarrow P \bar{P} \gamma$ [18, 19, 20, 21, 22], $J/\Psi \rightarrow P \bar{P} V$, $\Psi' \rightarrow \Psi P \bar{P}$, and $\Upsilon(nS) \rightarrow \Upsilon(mS) P \bar{P}$ [23], where P stands for a pseudo scalar (π or K)

and V for a vector meson (ρ, ω) . Experimental evidence for the contribution of scalar resonances to some of these processes has been reported, and a recent analysis of the data for $\Upsilon(nS) \rightarrow \Upsilon(mS)P \bar{P}$ concludes the contribution of the $f_0(600)$ with a large uncertainty in the width ($m = 526^{+48}_{-37}$, $\Gamma = 301^{+145}_{-100} \text{ MeV}$) [26]. It should also be noticed that in the experimental data for $\psi(2S) \rightarrow \pi^+\pi^- J/\Psi$ reported by the BES collaboration [25], no evidence for $f_0(600)$ contribution is foreseen.

In this paper we concentrate on the decay of heavy quark vector meson resonances (Υ , and Ψ), where pair of pions are produced with invariant mass ranging below the 1 GeV region. The kind of processes we are interested in has been considered by a number of authors using techniques as diverse as pure chiral symmetry [27], non-relativistic theory assuming the existence of an Adler zero [28], the color field multipole expansion in the non-relativistic limit [29, 30], effective Lagrangian based on chiral symmetry and the heavy quark expansion [31, 32, 33], and also a purely phenomenological description based on a Breit-Wigner parametrization [26]. The framework so developed is used then to describe the two pion invariant mass spectrum and, in some cases, also the angular distribution. The latter is important since existing experimental data can discriminate among the proposed models. Furthermore, as far as we know, the complete parametrization of the amplitude has not been discussed and some confusion has arisen concerning the “S” and “D” wave contributions to the amplitude [33].

Our purpose is to perform an analysis as general as possible, including both S and D waves in the $(P\bar{P})$ system. We show that Lorentz covariance fixes the parametrization of the amplitude in terms of four form factors. Using the two gluon mechanism implied by the OZI rule, the identification of the spin zero (di-pion S wave) and spin two (di-pion D wave) are unambiguously identified. For simplicity we restrict our analysis to two form factors, $a_0(m_{\pi\pi})$ and $a_2(m_{\pi\pi})$ which are associated to S and D waves respectively. We then invoke the pole approach to parametrize a_0, a_2 . We propose a Breit-Wigner plus a background for a_0 and a soft background for a_2 . In this way we claim that crossed channel as well as higher scalar resonance contributions are taken into account. Thus, we expect that the amplitude associated to crossed channel and higher resonance contribution behave as a soft function in the 500-800 MeV range, where the $f_0(600)$ is expected to lie. Our strategy

is to perform a joint fit to data from different processes, involving 165 points and 33 parameters.

The set of data points we consider include the $\pi - \pi$ invariant mass distribution of the $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi\pi$ [23], $\Upsilon(3S) \rightarrow \Upsilon(2S) + \pi\pi$ [23], $\Upsilon(2S) \rightarrow \Upsilon(1S) + \pi\pi$ [23, 24], $\psi(2S) \rightarrow J/\psi + \pi\pi$ [25] decays. The following characteristics are worth-mentioning: *i*) we are considering flavor conserving processes, and all of them are expected to proceed through two gluons. This point will be relevant when discussing the parametrization of the form factors. *ii*) the smallness of the phase space available for the processes under consideration ($2m_\pi \leq \sqrt{s} \leq 0.9 \text{ GeV}$). Note that the expected central value and the large width of the $f_0(600)$ would imply non negligible resonance effects in these processes and *iii*) the invariant mass distribution for $s \rightarrow s_{th}$ -where s_{th} stands for the threshold value of the di-pion invariant mass- show a peculiar behavior to be contrasted with the typical $(s - 2m_\pi^2)$ expected in processes involving soft pions. Compare for example in Fig(1) the threshold behavior of the $\sqrt{s}$ distribution for $\Upsilon(2S) \rightarrow \Upsilon(1S) + \pi\pi$ or $\psi(2S) \rightarrow J/\psi + \pi\pi$ with $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi\pi$.

In order to understand the nature of the problem we face, remark that the more recent data [24, 25] have been analyzed in terms of the scale anomaly. Indeed, these processes can be fitted without problem, which brings the question if the full set of data we consider can be explained using the same formalism. Our results show that this is not the case and that inclusion of the $f_0(600)$ improves our understanding of the data. Thus, as far as we can see, any attempt to provide a successful description of the flavor conserving two pion decays of the Υ and ψ families should consider the full set of data, since the foreseen physical mechanisms (scale anomaly, scalar resonance exchange) are operative in all cases under consideration.

1 Parametrization of the amplitude

We are interested in the decay $V'(p', \eta') \rightarrow V(p, \eta)\pi(p_1)\pi(p_2)$ where the letters in parenthesis stand for the four-momenta and polarization of the corresponding particles. We introduce $q \equiv p_1 - p_2$ and $Q \equiv p_1 + p_2$. $Q^2 = s$,

and $p'^2 = m'^2, p^2 = m^2, p_1^2 = p_2^2 = m_\pi^2$. In order to obtain the general parametrization is convenient to consider the amplitude for the exchange of arbitrary spin zero and spin two meson like objects (although we do not consider the actual exchange of any particle). Let us first consider the S wave contribution. The amplitude for the $V' \rightarrow V + \text{scalar}$ can be written as:

$$\mathcal{M}_0 = \eta'^\mu \eta^\nu t_{\mu\nu}.$$

Covariance allow us to write $t_{\mu\nu}$ in terms of $g_{\mu\nu}$ and the independent (p', p) four-momenta. Imposing $p' \cdot \eta' = 0, p \cdot \eta = 0$ and using $p'^\mu t_{\mu\nu} = 0$, which follows from the fact that $V'(p')$ is produced through a virtual photon in an e^+e^- machine, we obtain:

$$\mathcal{M}_0 = a_0 \left(\eta \cdot \eta' - \frac{(p' \cdot \eta)(p \cdot \eta')}{p \cdot p'} \right) \quad (1)$$

Already at this point we encounter differences with the parametrizations used in the literature, where only the $\eta \cdot \eta'$ term is considered. Although sizable effects are not produced by the extra term it is important to work with the proper Lorentz invariant amplitude. In particular, differences could become relevant when polarization measurements are involved.¹ For the D wave contribution we write:

$$\mathcal{M}_2 = A^{\mu\nu} \Pi_{\mu\nu\rho\sigma} B^{\rho\sigma} \quad (2)$$

where $A^{\mu\nu}$ describes the $V'(p') \rightarrow V(p) + D(Q)$, D standing for a spin two meson like object, $B^{\rho\sigma}$ describes the $D \rightarrow \pi\pi$ amplitude and $\Pi_{\mu\nu\rho\sigma}$ is the spin two projector:

$$\Pi_{\mu\nu\rho\sigma} \equiv \sum_{\lambda=1}^5 h_{\mu\nu}(\lambda) h_{\rho\sigma}(\lambda). \quad (3)$$

The polarization tensor $h_{\mu\nu}$ has the following properties:

¹Note that $a_0^{-2} \sum_{pol} |M_0|^2 = 2 + \frac{m^2}{E^2}$, whereas $\sum_{pol} (\eta \cdot \eta') = 2 + \frac{E^2}{m^2}$, with m and E the outgoing vector meson mass and energy respectively.

$$h_{\mu\nu}(\lambda) = h_{\nu\mu}(\lambda), \quad Q^\mu h_{\mu\nu}(\lambda) = g^{\mu\nu} h_{\mu\nu}(\lambda) = 0 \quad (4)$$

Using these relations together with the projector property of $\Pi_{\mu\nu\rho\sigma}$ one finds:

$$\Pi_{\mu\nu\rho\sigma} \equiv \frac{1}{2}P_{\mu\rho}P_{\nu\sigma} + \frac{1}{2}P_{\mu\sigma}P_{\nu\rho} - \frac{1}{3}P_{\mu\nu}P_{\rho\sigma}, \quad (5)$$

with

$$P_{\mu\nu} = g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2}. \quad (6)$$

On the other hand, by Lorentz covariance $B_{\rho\sigma} \propto q_\rho q_\sigma$. It is convenient to introduce:

$$\Pi_{\mu\nu} \equiv \Pi_{\mu\nu\rho\sigma} B^{\rho\sigma} = q_\mu q_\nu - \frac{1}{3}P_{\mu\nu}Q^2. \quad (7)$$

Furthermore, using Lorentz covariance and imposing again the conditions $p' \cdot \eta' = 0$, $p \cdot \eta = 0$ and $p'^\mu A_{\mu\nu} = 0$, due to the coupling of V' with a virtual photon, we find the general structure for $A_{\mu\nu}$, and through Eq. (2) the di-pion D wave amplitude (Note that $Q^\mu \Pi_{\mu\nu} = Q^\nu \Pi_{\mu\nu} = 0$, therefore $p'^\mu \Pi_{\mu\nu} = p^\mu \Pi_{\mu\nu}$).

$$\begin{aligned} \mathcal{M}_2 = & b \left(\eta'^\mu \eta^\mu - \frac{\eta'^\nu p^\mu}{p \cdot p'} (\eta' \cdot p) \right) \Pi_{\mu\nu} + c \left(\eta'^\mu (\eta \cdot p') - p'^\mu \frac{(\eta \cdot p')(\eta' \cdot p)}{p \cdot p'} \right) p'^\nu \Pi_{\mu\nu} \\ & + \frac{a_2}{m'^2 - m^2} \left((\eta \cdot \eta') - \frac{(\eta \cdot p')(\eta' \cdot p)}{p \cdot p'} \right) p'^\mu p'^\nu \Pi_{\mu\nu}. \end{aligned} \quad (8)$$

Where we introduced the $m'^2 - m^2$ factor to work with a dimensionless a_2 .

In order to obtain the decay rate, we carry out the integration over the p_1, p_2 Lorentz invariant phase space in the two pion center of mass reference frame, *i.e.* $\vec{Q} = 0$, $q_0 = 0$. We obtain:

$$\begin{aligned}
\Gamma_0 &\equiv \frac{1}{2m'} \frac{1}{3} \int \sum_{pol} |\mathcal{M}_0|^2 \frac{d^3 p}{(2\pi)^3 2p_0} \frac{d^3 p_1}{(2\pi)^3 2p_{10}} \frac{d^3 p_2}{(2\pi)^3 2p_{20}} (2\pi)^4 \delta^4(Q - p_1 - p_2) \\
&= \frac{1}{48\pi m'} \int \sum_{pol} |\mathcal{M}_0|^2 \frac{d^3 p}{(2\pi)^3 2p_0} \left(1 - \frac{4m_\pi^2}{Q^2}\right)^{\frac{1}{2}}.
\end{aligned}$$

The S-D wave interference vanishes upon integration. Indeed covariance imply

$$\begin{aligned}
\Gamma_{int} &\sim \frac{1}{3m'} \int \sum_{pol} (R_e \mathcal{M}_0 A^{\mu\nu} \Pi_{\mu\nu}) \frac{d^3 p_1}{(2\pi)^3 2p_{10}} \frac{d^3 p_2}{(2\pi)^3 2p_{20}} (2\pi)^4 \delta^4(Q - p_1 - p_2) \\
&= a g_{\mu\nu} + b Q_\mu Q_\nu
\end{aligned}$$

but using $Q^\mu \Pi_{\mu\nu} = g^{\mu\nu} \Pi_{\mu\nu} = 0$, it follows that $a = b = 0$.

For the D wave we proceed along the same lines.

$$\Gamma_2 = \frac{1}{2m'} \frac{1}{3} \sum_{pol} \int \frac{d^3 p}{(2\pi)^3 (2p_0)} (A^{\mu\nu} A^{*\rho\sigma}) \bar{\Pi}_{\mu\nu\rho\sigma}$$

where

$$\begin{aligned}
\bar{\Pi}_{\mu\nu\rho\sigma} &= \int \sum_{pol} (\Pi_{\mu\nu} \Pi_{\rho\sigma}) \frac{d^3 p_1}{(2\pi)^3 (2p_{10})} \frac{d^3 p_2}{(2\pi)^3 (2p_{20})} (2\pi)^4 \delta^4(Q - p_1 - p_2) \\
&= \mathbf{x} \Pi_{\mu\nu\rho\sigma}.
\end{aligned}$$

$\mathbf{x}$ is determined using the reference frame where $\vec{Q} = 0$, $q_0 = 0$. For example

$$\bar{\Pi}_{3333} = \frac{2}{3} \mathbf{x} = \int \sum_{pol} (\Pi_{33} \Pi_{33}) \frac{d^3 p_1}{(2\pi)^3 (2p_{10})} \frac{d^3 p_2}{(2\pi)^3 (2p_{20})} (2\pi)^4 \delta^4(Q - p_1 - p_2),$$

in this way we obtain

$$\mathbf{x} = \frac{Q^4}{60\pi} \left(1 - \frac{4m_\pi^2}{Q^2}\right)^{\frac{5}{2}}$$

We observe that using the $q_0 = 0, \vec{Q} = 0$ reference frame, it is easy to show that $\Pi_{\mu\nu} \sim Y_2(\theta, \phi)$, *i.e.* it is associated to the di-pion D wave. This together with the fact that a_2, b and c can only depend upon $s \equiv (p_1 + p_2)^2$, allow us to conclude that $\mathcal{M}_2$ describes the di-pion spin 2 wave. In the following we consider the particular case where only one Lorentz invariant amplitude is included in $A_{\mu\nu}$. To this end we set $b = c = 0$ in Eq(8). This is the simplest way to consistently introduce D wave effects in the invariant amplitude. Within this approximation we obtain for the di-pion invariant mass distribution ($s \equiv (p_1 + p_2)^2 = Q^2$).

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{(m'p)\sqrt{s}}{3(4m'\pi)^3} \sum_{pol} \left| (\eta \cdot \eta') - \frac{(\eta \cdot p')(\eta' \cdot p)}{p \cdot p'} \right|^2 (SW + DW) \quad (9)$$

where p stands for the three momentum ($p = |\vec{p}|$) and

$$SW = |a_0(s)|^2 \left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{1}{2}} \quad (10)$$

$$DW = \frac{\left|\frac{a_2(s)}{m'^2 - m^2}\right|^2}{180} \left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{5}{2}} \left[(s - (m'^2 + m^2))^2 - 4m'^2 m^2 \right]^2 \quad (11)$$

with

$$2m'p = \left[(s - m'^2 - m^2)^2 - 4m'^2 m^2 \right]^{\frac{1}{2}}. \quad (12)$$

$$\sum_{pol} \left| \eta \cdot \eta' - \frac{(\eta \cdot p')(\eta' \cdot p)}{p \cdot p'} \right|^2 = 2 + \frac{4m^2 m'^2}{(m^2 + m'^2 - s)^2} \quad (13)$$

We have now a general expression describing the decay $V' \rightarrow V\pi\pi$, which involves two invariant amplitudes, associated to the S and D wave respectively. We assume, when $\sqrt{s} \leq 0.9 \text{ GeV}$, the S wave is composed of the $f_0(600)$ and a non resonant background. This is an approximation since other resonances could contribute to the amplitude in the kinematical region considered. However, it is reasonable to expect that the contribution of the $f_0(980)$ ($\Gamma_{f_0(980)} \approx 100 \text{ MeV}$) and higher states decaying in two pions behave softly in the neighborhood of the $f_0(600)$, even if the latter is a broad resonance. Crossed channel contributions are treated in a similar way, *i.e.* considered as soft functions of the di-pion invariant mass. Therefore we parametrize the form factor associated to the S wave in the following way.

$$a_0^{(i)} = \left(\frac{a^{(i)}m_0^2}{D(s)} + \frac{b^{(i)}}{1 - \frac{c^{(i)}s}{m_0^2}} \right), \quad (14)$$

m_0 is introduced for dimensional reasons and is fixed to $m_0 = 0.5 \text{ GeV}$. For $D(s)$ we used two different expressions:

$$\begin{aligned} D(s) &= s - m^2 + \Pi(s) \\ D(s) &= s - m_p^2 + im_p\Gamma_p \end{aligned} \quad (15)$$

where $\Pi(s)$ stands for the $f_0(600)$ self energy which involves loop of kaons and pions. Note that Eq(15) defines the mass m_p and width Γ_p of the resonance in terms of the real and imaginary part of the pole of the S matrix. If a pole exist, it should be independent of the process where it is observed. However neither the residue nor the background have to be the same for different processes. For this reason we include the index i which is associated to the physical decay under consideration. In the kinematical region of interest the D channel is non-resonant, thus we can approximate it by a soft background:

$$a_2^{(i)} = \left(\frac{f^{(i)}}{1 - \frac{g^{(i)}s}{m_0^2}} \right) \quad (16)$$

It should be clear that our approach is a phenomenological, and that we have not attempted to explicitly incorporate the scale anomaly. In fact, it can be argued that such a contribution is included within the background.

2 The fit

We consider the following decays: $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi\pi$ [23], $\Upsilon(3S) \rightarrow \Upsilon(2S) + \pi\pi$ [23], $\Upsilon(2S) \rightarrow \Upsilon(1S) + \pi\pi$ [23, 24], $\psi(2S) \rightarrow J/\psi + \pi\pi$ [25], resulting in a total of 165 points. All the data points but the last have been gotten from plots since no listings of the data are available. It is important to mention that previous fits [26] of the $\psi(2S) \rightarrow J/\psi + \pi\pi$ included only a subset of the experimental data.

Following the analysis in [24, 25] we first attempted a fit in terms of the scale anomaly [29, 30]. The fit produces a $\chi^2_{d.o.f.} > 2$. This is not an unexpected result, the amplitude associated to the scale anomaly is derived under the assumption that the pions are soft, and this is not the case for all the data under consideration. In fact, according to [26] and previous analysis [1], the $f_0(600)$ is expected to contribute to these processes. This is the motivation to try a fit in terms of the pole approach, as discussed in the previous section.

The fit using the pole approach is based on Eqs(9,10, 11,14,15,16). The pole can be parametrized in terms of the full propagator, including the self-energy $\Pi(s)$, which involves loops of kaons and pions. In such case, as a result of the fit, we get a mass and the g_{fKK} and $g_{f\pi\pi}$ coupling constants, and from these the pole position is determined. Unfortunately, the fit turns out to be insensitive to the values of the coupling constants, *i.e.* different values of the g 's lead to essentially the same fit, but correspond to pole positions far from each other. For this reason our analysis is restricted to $D(s)$ given by Eq(15) where the mass (m_p) and width (Γ_p) are obtained directly from the fit.

The fit involves 33 parameters. We consider four processes ($i = 1$ to 4 in Eqs(14, 16)) and for each of these we require seven parameters (b and f which are complex, and a, c, g). Three normalization factors are also parameters of the fit -because some of the reported data refers to number of events, not to a differential decay rate- and finally the mass and width of the resonance. The fit leads to a pole located at $m_p = 528 \pm 32$ MeV and $\Gamma_p = 413 \pm 45$ MeV together with the parameters reported in Table(1), and a $\chi^2_{d.o.f.} = 1.12$. The result of the joint fit are shown as the solid lines in Fig(1). We do not report the result of the fit for S wave alone, however we should mention

that including D wave effects improves the total χ^2 but the $\chi_{d.o.f.}^2$ remains unchanged. Note in this respect that the fit parameters are obtained from the di-pion invariant mass spectrum, the angular distributions are not involved. Once the fit parameters are fixed, numerical integration over the di-pion invariant mass is performed and we obtain -up to normalization again- the following angular distributions (in GeV units) and the curves shown in Fig(3):

$$\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi[24] : \quad \frac{d\Gamma}{d(\cos\theta)} = 0.07(0.0674(3\cos^2\theta - 1)^2 + 1.09) \quad (17)$$

$$\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi[23] : \quad \frac{d\Gamma}{d(\cos\theta)} = 2.2(0.0009(3\cos^2\theta - 1)^2 + 0.511) \quad (18)$$

$$\Psi(2S) \rightarrow J/\Psi\pi\pi[25] : \quad \frac{d\Gamma}{d(\cos\theta)} = 6.5(5.49(3\cos^2\theta - 1)^2 + 359) \quad (19)$$

Besides the pole position and the quality measure through the $\chi_{d.o.f.}^2$, it is instructive to analyze the different contributions to the decay rate. These are shown in Fig(2) where the dots corresponds to the pole contribution, the dashed line to the background and resulting fit is represented by the continuous line. From these plots one can see that the pole and background contributions must interfere destructively in all cases, except the $\Upsilon(3S) \rightarrow \Upsilon(1S)$ transition for which the kinematical region includes the pole position and both, constructive (below the pole-mass) and destructive (above the pole) interferences occur leading thus to the structure around 650 MeV. In table(2) we quote the contributions to the χ^2 from the different processes.

3 Summary

In this paper we consider the phenomenological description of the following decays: $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi + \pi$, $\Upsilon(3S) \rightarrow \Upsilon(2S) + \pi + \pi$, $\Upsilon(2S) \rightarrow \Upsilon(1S) + \pi + \pi$, $\psi(2S) \rightarrow J/\psi + \pi + \pi$. As far as we can see, it is not possible to obtain a good quality fit in terms of the scale anomaly alone.

Using general arguments, we derived an expression for the invariant amplitude describing flavor conserving processes of the type $V' \rightarrow V + \pi + \pi$, including both S and D waves, which involves two invariant amplitudes. We parametrized the S wave form factor with a pole plus a soft background and the D wave form factor by pure soft background. Fitting the data yields a pole in $m_p = 528 \pm 32 \text{ MeV}$ and $\Gamma_p = 413 \pm 45 \text{ MeV}$ with $\chi^2_{d.o.f.} = 1.12$. We remark that a strong interference among the pole and the background is required to fit the data. Thus, our analysis seems to indicate that physics in S wave pion-pion interaction below 800 MeV is governed by a subtle interplay between $f_0(600)$ meson contributions and a big background, difficult to understand in terms of conventional physics and which could be associated to the scale anomaly.

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FIGURE CAPTION.

Figure 1. Data points used in the analysis [23, 24, 25] and the resulting fit (solid line). In the horizontal axes we plot $m_{\pi\pi} = \sqrt{s}$ whereas the vertical refers to the differential decay rate $\frac{d\Gamma}{d\sqrt{s}}$, or number of events, as shown in the plots and discussed in the quoted references. For the same reaction, open circles and solid triangles refer to data obtained from exclusive and inclusive processes respectively.

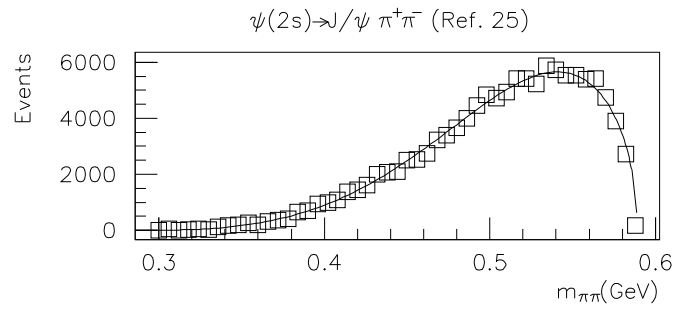
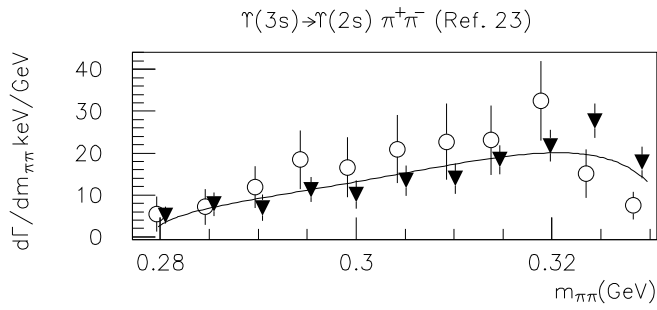
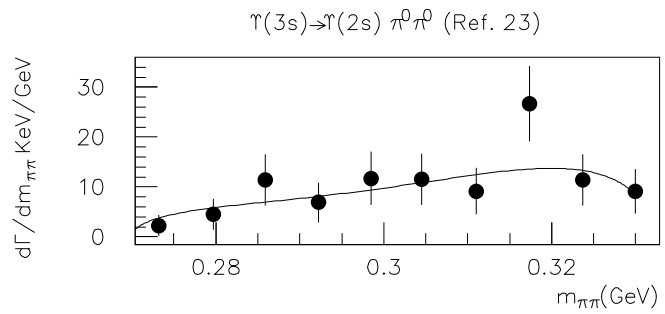
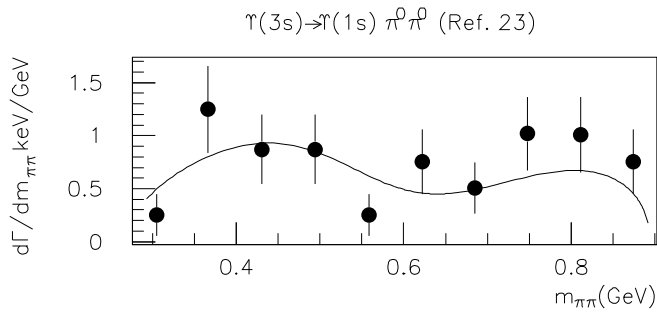
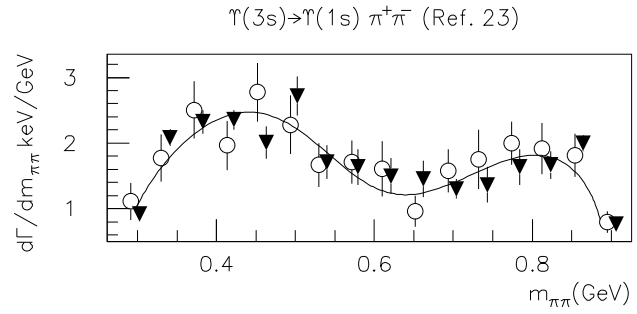
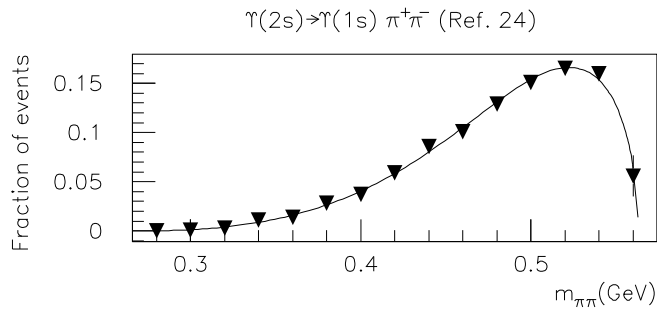
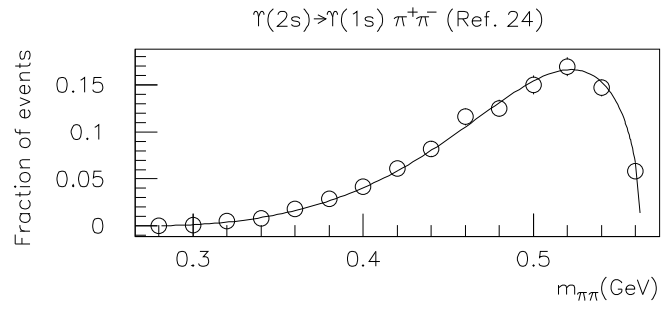
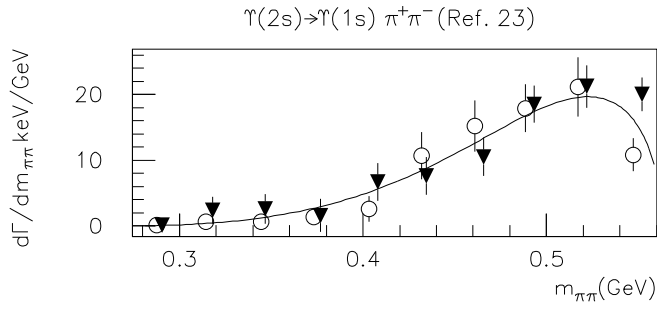
Figure 2. The curve resulting from the fit to the data (solid line), and for comparison the pole (dashed line) and background (dotted line) contributions. Note that for the $\Upsilon(3S) \rightarrow \Upsilon(2S) + \pi\pi$ the fit-curve is close to the axes, which implies a strong destructive interference among the pole and the background contributions.

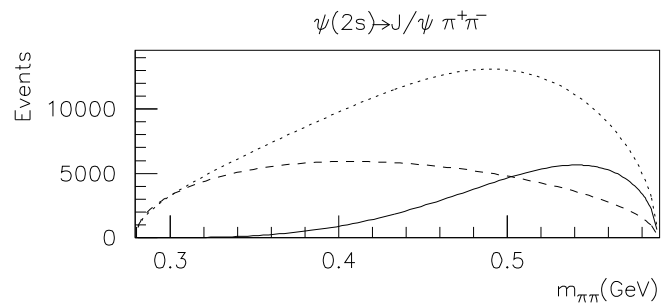
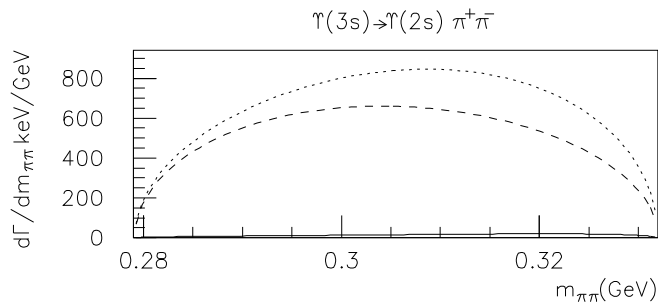
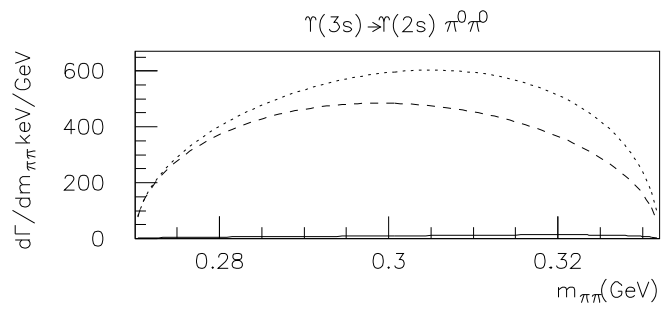
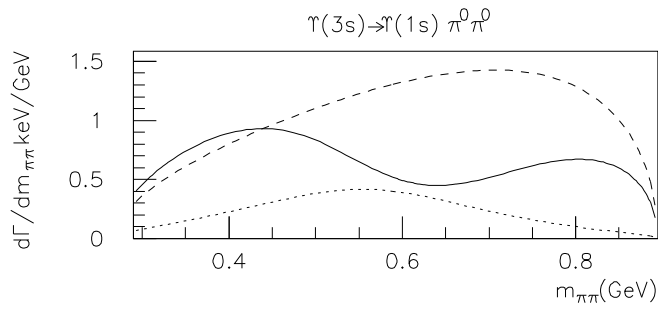
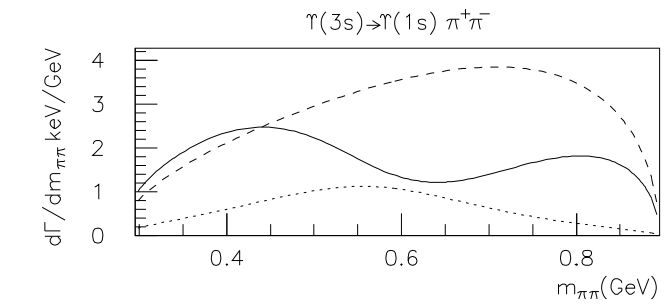
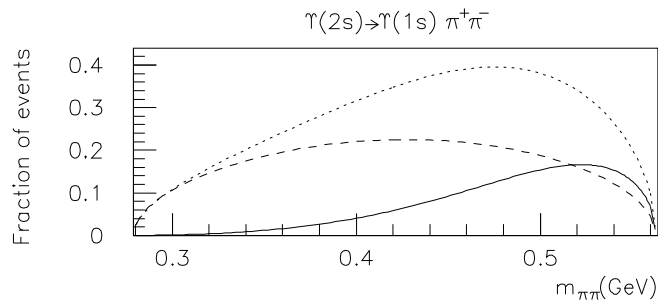
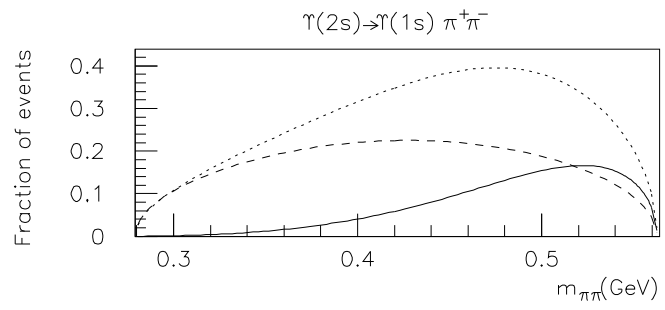
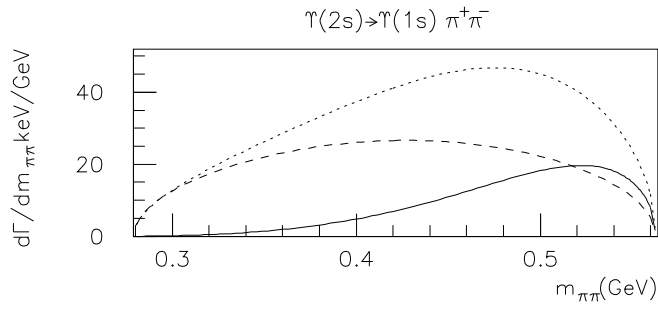
Figure 3. Angular distribution as obtained from Eqs(17,18,19) and compared to data from Ref([24, 23, 25]). Open circles and solid triangles refer to data obtained from exclusive and inclusive processes respectively.

TABLE CAPTION.

Table 1. Parameters resulting from the fit. The normalization factors refer to: N_a data from Ref([24]), N_2 to the $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi^0\pi^0$ and N_3 to $\Upsilon(3S) \rightarrow \Upsilon(2S) + \pi^0\pi^0$. The parameters are defined by Eqs(14, 16). Processes are labeled 1: $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi$, 2: $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ 3: $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi\pi$ and 4: $\Psi(2S) \rightarrow J/\Psi\pi\pi$.

Table 2. $\chi^2_{d.o.f.}$ for each separate process as obtained from the fit to the di-pion invariant mass distribution.





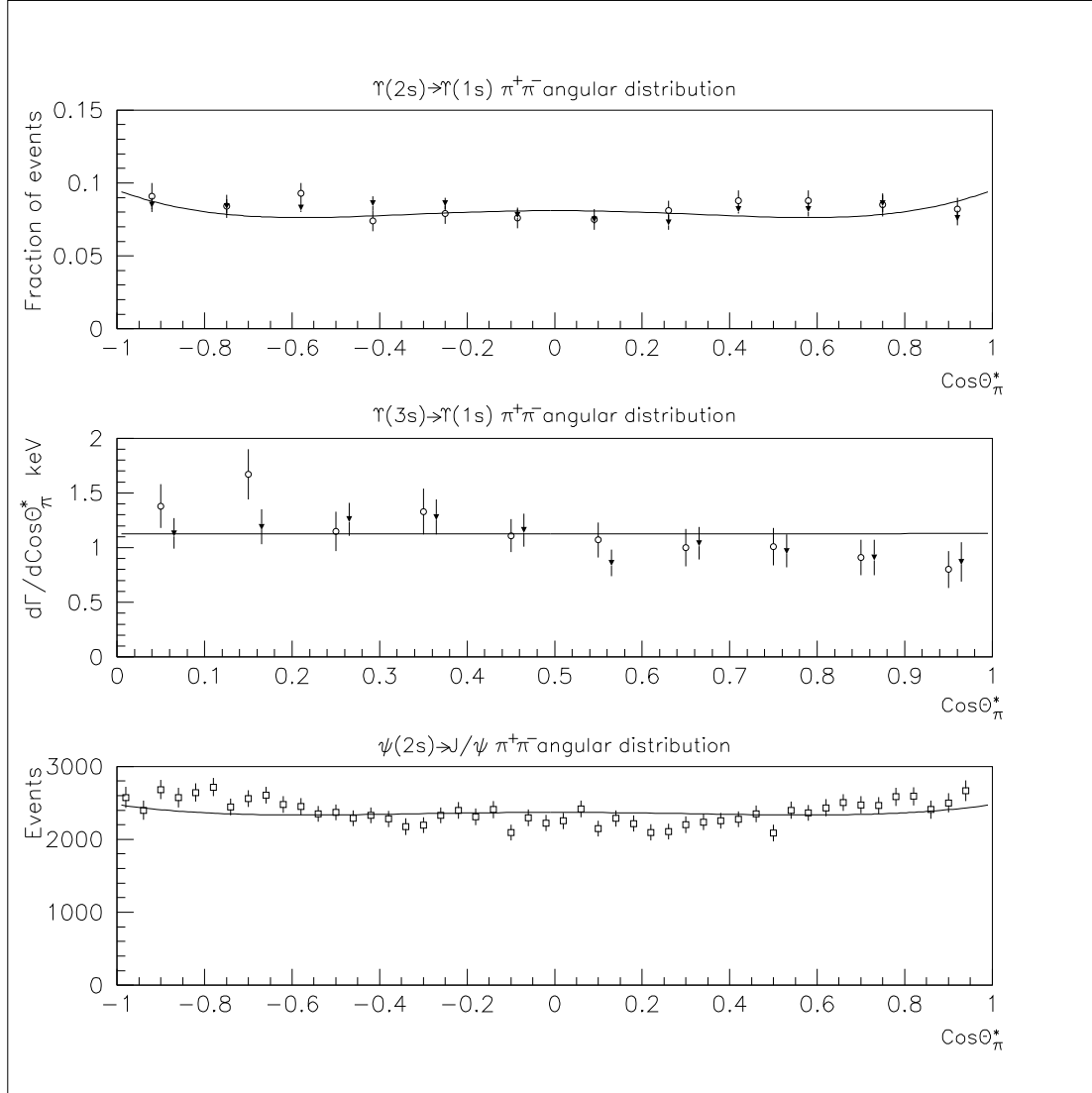


Figure 3.

N_a	8.4×10^{-3}
N_2	0.36
N_3	0.62
a_1	8.2×10^3
b_1	$5.3 \times 10^3 + i5.1 \times 10^3$
c_1	-0.14
f_1	$-1.6 \times 10^3 + i3.8 \times 10^3$
g_1	0.4
a_2	7.6×10^2
b_2	$-1.1 \times 10^3 + i8.4 \times 10^2$
c_2	5.4×10^{-2}
f_2	$2.3 + i2.8$
g_2	0.33
a_3	-1.2×10^5
b_3	$-7.8 \times 10^4 - i1.0 \times 10^5$
c_3	-1.1
f_3	$-4.8 \times 10^3 - i9.0 \times 10^5$
g_3	-5.0×10^4
a_4	4.8×10^4
b_4	$3.2 \times 10^4 + i3.7 \times 10^4$
c_4	-0.48
f_4	$3.2 \times 10^4 - i3.2 \times 10^4$
g_4	-4.9×10^{-2}

Table 1.

Process	N_{data}	$\chi^2_{d.o.f.}$
$\Upsilon(2s) \rightarrow \Upsilon(1s)\pi\pi$	48	0.74
$\Upsilon(3s) \rightarrow \Upsilon(1s)\pi\pi$	40	1.3
$\Upsilon(3s) \rightarrow \Upsilon(2s)\pi\pi$	32	1.1
$\psi(2s) \rightarrow J/\psi\pi\pi$	45	1.5

Table 2.

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