

Toward a generic method for studying water renewal, with application to the epilimnion of Lake Tanganyika

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Abstract

We present a method, based on the concept of age and residence time, to study the water renewal in a semi-enclosed domain. We split the water of this domain into different water types. The initial water is the water initially present in the semi-enclosed domain. The renewing water is defined as the water entering the domain of interest. Several renewing water types may be considered depending on their origin. We present the equations for computing the age and the residence time of a certain water type. These timescales are of use to understand the rate at which the water renewal takes place. Computing these timescales can be achieved at an acceptable extra computer cost.

The above-mentioned method is applied to study the renewal of epilimnion (i.e. the surface layer) water in Lake Tanganyika. We have built a finite element reduced-gravity model modified to take into account the water exchange between the epilimnion and the hypolimnion (i.e. the bottom layer), the water supply from precipitation and incoming rivers, and the water loss from evaporation and the only outgoing river. With our water renewal diagnoses, we show that the only significant process in the renewal of epilimnion water in Lake Tanganyika is the water exchange between the epilimnion and the hypolimnion, other phenomena being negligible.

Key words:

water renewal, age, residence time, Lake Tanganyika, reduced-gravity model, finite elements

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1 Introduction

In semi-enclosed aquatic domains, most of the biomass, nutrients, contaminants, dissolved gasses and suspended particles are exchanged with their surrounding environment. Therefore, it is essential to understand hydrodynamic processes that transport water and its constituents. A first-order description of transport may be expressed as “age” and “residence time”, which we conceive as time scales of the water renewal of the semi-enclosed domain (Monsen et al., 2002). Several studies have been made about the water renewal in a semi-enclosed domain, using different definitions of renewal time scales to quantify the time that water remains inside the domain. Bolin and Rodhe (1973) defined the theoretical concepts of age and residence time, followed by Zimmerman (1976, 1988) who used the residence time to study the hydrodynamics of estuaries. Takeoka (1984) used the age and the residence time to study the water exchange in a coastal sea. Deleersnijder et al. (1997) and Tartinville et al. (1997) resorted to the residence time to study the transport of water and tracer from the Mururoa Lagoon to the Pacific Ocean. Braunschweig et al. (2003) defined different renewal time scales that were helpful to understand the hydrodynamics of the Tagus Estuary.

This article presents a method for studying water renewal that is based on the idea that the water can be viewed as a mixture of different water types. The latter are treated as passive tracers (Cox, 1989; Hirst, 1999; Goosse et al., 2001; White and Deleersnijder, 2006). The initial water is the water initially present in the domain of interest. The renewing water is the water entering the domain. We may define different renewing water types, depending on their origin. To quantify the rate at which water is renewed, we will estimate the residence time of the initial water and the age of the renewing water, in accordance with CART (Constituent-oriented Age and Residence time Theory, <http://www.climate.be/CART>).

The age of a water parcel is defined to be the time elapsed since this water parcel left the region in which its age is prescribed to be zero (Bolin and Rodhe, 1973; Takeoka, 1984; Delhez et al., 1999; Deleersnijder et al., 2001). The age is thus an interesting timescale for the different renewing water types. Furthermore, it only requires to solve well-known advection-diffusion equations (Delhez et al., 1999; Deleersnijder et al., 2001) to obtain a local renewal time scale, depending on space and time. The concept of age is very useful to quantify the time spent by renewing water parcels into the domain. But, by definition, the age of initial water parcels will always be the time elapsed since the initial instant. That is why we use the residence time for the initial water, but only for this water type. The residence time of a water parcel in a water body is defined as the time taken by this water parcel to leave the water body (Bolin and Rodhe, 1973; Takeoka, 1984; Delhez et al., 2004;

Delhez, 2006). There are two different approaches to compute the residence time. We can compute the residence time from the solution of an adjoint problem (Delhez et al., 2004). However, it is not easy to implement but it provides a local residence time, depending on space and time. We can also compute the residence time by means of a direct approach that is easier to implement (it only requires to solve advection-diffusion equations) but too expensive to get the same results as those of the adjoint problem. That is because the direct approach merely provides a global mean residence time, as an integral over space and time. A compromise can be achieved by dividing the semi-enclosed domain into a small number of regions. The mean residence time is then computed for each region. Moreover the mean residence time may be computed for a small number of different initial times. We have decided to adopt this approach.

The method that we use to study the water renewal in a semi-enclosed domain is as follows. Firstly, we define the different water types : the initial water and several renewing water types. Secondly, we compute the evolution of their concentration field. Then we compute the evolution of the renewing water age field. Finally, we divide the domain into different regions and we compute, for each region and for several initial times, the mean residence time of the initial water in this region. This approach requires no important additional numerical development once a hydrodynamic model of the problem is set up. These concepts are applied herein to the epilimnion (i.e. the surface layer) of Lake Tanganyika, for which a hydrodynamic model already exists (Gourgue et al., 2007). Note that this paper is just an illustration of a general theory to study the water renewal in semi-enclosed domains.

The present article is organized as follows. Section 2 describes the two-dimensional hydrodynamic model (Section 2.1) and the equations for the concentration of the different water types, the equations for the age of the different renewing water types and the equations for the residence time of the initial water (Section 2.2), as well as the numerical methods used to model them (Section 2.3). In Section 3, we present our results and we conclude with Section 4.

2 Model description

Lake Tanganyika is situated on the east of central Africa, and is shared by four developing countries : Democratic Republic of Congo, Burundi, Tanzania and Zambia (Figure 1). It lies between $3^{\circ}20'$ to $8^{\circ}45'$ S and $29^{\circ}05'$ to $31^{\circ}15'$ E. It is about 650 km long and 50 km wide on average. The mean depth of the lake is around 570 m, with a maximum depth of 1470 m. Thermal stratification in the lake is well marked so that we can define two distinct layers : the epilimnion and the hypolimnion. This two-layer regime is ubiquitous within

the lake. The epilimnion is the surface layer and is composed of relatively warm (24-28 °C), oxygenated water. The hypolimnion is the bottom layer and is composed by colder (23.5 °C), anoxic water. These two main layers are separated by a thin layer where temperature variations are more important, the thermocline. Overall, the thermocline lies deeper in the north than in the south with a mean depth of 50 m over the entire lake (Coulter and Spigel, 1991; Naithani et al., 2003). The region undergoes two main seasons : the dry season (approximately from May to August), characterized by strong winds blowing northwestward along the main axis of the lake, and the wet season (approximately from September to April), when the winds are generally weaker (Coulter and Spigel, 1991). During the dry season, the wind stress pushes the warmer epilimnion water away from the southern end of the lake toward the northern end. Upwelling occurs to replace the loss of water at the south and the epilimnion water accumulated at the north is pulled downward by gravity. The thermocline is then tilted downward toward the north and may totally disappear in the south. At the end of the dry season, when the southeasterly winds stop, the epilimnion and the hypolimnion slide over each other, and the thermocline oscillates to reach a new equilibrium. These waves get reflected at the lake boundaries and gradually transform into standing wave patterns, called internal seiches. By December, the thermocline reaches its mean level (around 50 m) but still oscillates throughout the wet season until the beginning of the next dry season and the onset of the southeasterly winds (Coulter and Spigel, 1991; Naithani et al., 2003). These observations have already been reproduced by a finite difference model (Naithani et al., 2002, 2003, 2004; Naithani and Deleersnijder, 2004). Similar results were obtained by means of the reduced-gravity finite element model used herein (Gourgue et al., 2007), so that there is no need for a detailed discussion of its validity in the present article.

2.1 The hydrodynamic model

It has already been shown that the two-dimensional reduced-gravity model is able to produce rather good results on the hydrodynamics of Lake Tanganyika (Naithani et al., 2002, 2003, 2004; Naithani and Deleersnijder, 2004). By using a reduced-gravity model, we assume that the density stratification is much more important in determining the internal oscillations than the underlying bottom topography. In Lake Tanganyika, it is appropriate to have recourse to such a model since stratification is present all year round and the epilimnion thickness is much smaller than the hypolimnion one (Coulter and Spigel, 1991; Naithani et al., 2003).

The reduced-gravity model consists of two layers of different constant temperatures and densities (Figure 2). Here, we also assume that the temperature is

constant inside these two layers. The shallow water equations are applied to both layers and two important hypotheses are made. First, the surface layer thickness ($H_1 = h_1 + \eta + \xi$) is considered to be much smaller than the bottom layer thickness ($H_2 = h_2 - \xi$), supposed to be infinitely thick. Second, the displacement of the free surface is assumed to be much smaller than that of the thermocline. These two hypotheses are well verified in Lake Tanganyika. This leads us to the equations of the reduced-gravity model (Naithani et al., 2003, 2004) from which the velocity in the epilimnion and the downward displacement of the thermocline may be obtained :

$$\frac{\partial \xi}{\partial t} + \nabla \cdot (H\mathbf{u}) = 0 \quad (1)$$

$$\begin{aligned} & \frac{\partial}{\partial t}(H\mathbf{u}) + \nabla \cdot (H\mathbf{u}\mathbf{u}) + f\mathbf{e}_z \times (H\mathbf{u}) \\ & = -(\epsilon g)H(\nabla \xi) + \nabla \cdot (H\mathbf{A} \cdot (\nabla \mathbf{u})) + \frac{\boldsymbol{\tau}}{\rho} , \end{aligned} \quad (2)$$

where we have dropped the subscripts “1” relative to the surface layer because we only deal with this layer. The variables are ξ , the downward displacement of the thermocline, and $\mathbf{u} = (u, v)$, the epilimnion depth-integrated velocity vector (the y direction is chosen to be along the main axis of the lake) ; f is the Coriolis parameter, (ϵg) is the reduced gravity, where g is the gravitational acceleration and $\epsilon = \frac{\rho_2 - \rho_1}{\rho_2}$ is the relative density difference ranging from 6.3×10^{-4} to 9.06×10^{-4} (Naithani et al., 2003); \mathbf{A} is the eddy viscosity tensor which is diagonal and positive, $\boldsymbol{\tau} = (\tau_x, \tau_y)$ is the surface wind stress, and ρ is the constant water density in the surface layer. To compute the surface wind stress, we use the data collected every six hours at Mpulungu (at the south end of the lake in Zambia) from 1st April 1993 to 31st March 1994 (Figures 3a and 3b) during the FAO/FINNIDA project “Research for the Management of the Fisheries on Lake Tanganyika”. We see that the wind stress is indeed more important along the main axis of the lake (the y direction) and that it is definitely stronger during the dry season.

The classical reduced-gravity model equations (1) and (2) do not take the different water exchanges into account. We want to add the effects of the water exchanges between the epilimnion and the hypolimnion, the effects of precipitation and evaporation, and the effects of the three most important incoming rivers (Malagarasi, Ruzizi and Lufubu) and the only outgoing river (Lukuga) (Figure 1). The mouths of these rivers are assumed to be too narrow to be resolved explicitly by the model. Therefore, the rivers are modelled by means of source/sink terms that are zero over most of the domain. Then, equation (1) becomes :

$$\frac{\partial \xi}{\partial t} + \nabla \cdot (H\mathbf{u}) = w + q_p + q_v + q_r , \quad (3)$$

where w is the entrainment velocity (i.e. the water flux crossing the thermocline) which is positive when hypolimnion water is entrained by turbulent processes into the epilimnion and negative otherwise, q_p is the water flux due to precipitation (always positive), q_v is the water flux due to evaporation (always negative), and q_r is the water flux due to rivers which is positive wherever a river brings water to the epilimnion and negative otherwise. These four functions have the dimension of a velocity. Assuming that only the water leaving the epilimnion carries non negligible momentum, equation (2) becomes :

$$\begin{aligned} & \frac{\partial}{\partial t}(H\mathbf{u}) + \nabla \cdot (H\mathbf{u}\mathbf{u}) + f\mathbf{e}_z \times (H\mathbf{u}) \\ &= -(\epsilon g)H(\nabla\xi) + \nabla \cdot (H\mathbf{A} \cdot (\nabla\mathbf{u})) + \frac{\boldsymbol{\tau}_w}{\rho} + (w^- + q_v + q_r^-) \mathbf{u} , \end{aligned} \quad (4)$$

where the last term is the momentum flux leaving the epilimnion, w^- and q_r^- denoting the negative part of w and q_r , respectively. The negative and positive parts of any function ψ are identified herein by subscripts “-” and “+”, respectively, with $\psi^\pm = \frac{\psi \pm |\psi|}{2}$. And since q_p is always positive and q_v is always negative, we have $q_p^+ = q_p$, $q_p^- = 0$, $q_v^+ = 0$ and $q_v^- = q_v$, explaining the last term in the right-hand side of equation (4).

To compute the entrainment velocity w we use the following parameterization (Naithani et al., 2007) :

$$w = \sqrt{\frac{3}{20} \frac{\sqrt{\tau_x^2 + \tau_y^2}}{\rho\sqrt{(\epsilon g)H}}} - w_d - \frac{\xi}{T_r} , \quad (5)$$

where the first term is the entrainment term inspired by Price (1979) and w_d is the detrainment velocity defined such that annual mean of the epilimnion volume remains approximately constant. There are large uncertainties in the parameterization of entrainment and detrainment. As a consequence, to avoid occasional occurrence of spurious values of ξ , a relaxation term (ξ/T_r) is needed, which slowly nudges the epilimnion thickness toward its equilibrium position. The relaxation timescale T_r is sufficiently long so that the relaxation term is generally much smaller than the entrainment and detrainment terms. For the problem studied in this paper, we used $w_d \simeq 10^{-5}$ m/s and $T_r = 45$ days. As it can be seen in equation (5), the entrainment velocity depends on the wind stress and the epilimnion thickness. But it is more sensitive to wind stress. The water renewal from hypolimnion to epilimnion ($w > 0$) occurs in the region where H (or ξ) is small but overall during important wind stress periods, i.e. during the dry season.

To compute the water fluxes due to precipitation, evaporation and rivers, we use monthly data collected by Bergonzini (1998) (Figures 3c and 3d). The

first two water fluxes are distributed uniformly over the surface of the lake. But the water flux due to a given river is distributed as a hyperbolic tangent decreasing with the distance from the mouth of this river. The characteristic lengthscale of the hyperbolic tangent is 3 km. Note that we had to slightly adjust the evaporation data in order to keep the annual mean of the epilimnion volume constant.

We concede that the treatment of rivers as sink/source terms is unusual. But their flow is about $10^2 \text{ m}^3/\text{s}$ and the flow crossing the thermocline is about $10^5 \text{ m}^3/\text{s}$. Therefore, the rivers carry negligible momentum throughout the lake compared with the water exchange between the epilimnion and the hypolimnion. It may not be true near the mouth of a river. But to take this local effect into account, we would have to make a drastic refinement of the mesh near the mouth of the four rivers. That would be useless since the river momentum is negligible.

The boundary conditions associated with equations (3) and (4) are presented in Appendix A.

2.2 Water renewal diagnoses

Different water types have to be traced in order to study the renewal of epilimnion water in Lake Tanganyika. We first define the initial water (with the subscript “*i*”) as the water initially in the epilimnion. We then define several renewing water types : the hypolimnion water (“*h*”) is the water entering the epilimnion from hypolimnion, the precipitation water (“*p*”) is the water entering the epilimnion from precipitation, and the river water (“*r*”) is the water entering the epilimnion from incoming rivers. We compute the concentration and the age of all of these water types. But the age is not a useful renewal timescale for the initial water because it is equal to the elapsed time. That is why we also compute the residence time for this water type.

2.2.1 Concentration

The concentration of every water type is denoted $C_s(t, \mathbf{x})$, where the subscript “*s*” can be equal to “*i*”, “*h*”, “*p*” or “*r*” for initial, hypolimnion, precipitation or river water, respectively. The following notations are of some use :

$$C_{tot}(t, \mathbf{x}) = C_i(t, \mathbf{x}) + C_h(t, \mathbf{x}) + C_p(t, \mathbf{x}) + C_r(t, \mathbf{x}) \quad (6)$$

$$C_{rw}(t, \mathbf{x}) = C_h(t, \mathbf{x}) + C_p(t, \mathbf{x}) + C_r(t, \mathbf{x}) , \quad (7)$$

where $C_{tot}(t, \mathbf{x})$ is the total concentration (i.e. the water concentration) and $C_{rw}(t, \mathbf{x})$ is the renewing water concentration.

The equations governing the concentration of any water type can be cast into the following generic conservative form :

$$\frac{\partial}{\partial t}(HC_s) + \nabla \cdot (H\mathbf{u}C_s) = \sigma_s + \nabla \cdot (H\mathbf{K} \cdot (\nabla C_s)) , \quad s = i, h, p, r , \quad (8)$$

where \mathbf{K} is the diffusivity tensor which must be symmetric and positive definite and σ_s is the source/sink term which takes on different values according to the water type :

$$\sigma_i = (w^- + q_v + q_r^-)C_i \quad (9)$$

$$\sigma_h = w^+ + (w^- + q_v + q_r^-)C_h \quad (10)$$

$$\sigma_p = q_p + (w^- + q_v + q_r^-)C_p \quad (11)$$

$$\sigma_r = q_r^+ + (w^- + q_v + q_r^-)C_r . \quad (12)$$

The initial conditions are

$$C_i(0, \mathbf{x}) = 1 \quad , \quad C_h(0, \mathbf{x}) = C_p(0, \mathbf{x}) = C_r(0, \mathbf{x}) = 0 , \quad (13)$$

and the boundary conditions are presented in Appendix A.

Common sense has it that the the following properties must be satisfied :

$$0 \leq C_s(t, \mathbf{x}) \leq 1 , \quad s = i, h, p, r \quad (14)$$

$$C_{tot}(t, \mathbf{x}) = 1 \quad (15)$$

$$\lim_{t \rightarrow \infty} C_i(t, \mathbf{x}) = 0 \quad (16)$$

$$\lim_{t \rightarrow \infty} C_{rw}(t, \mathbf{x}) = 1 . \quad (17)$$

We demonstrate that the properties (14) to (17) hold true in appendix B.

Let us now define \bar{C}_s the global mean concentration of the water type “s” as

$$\bar{C}_s(t) = \frac{\int_{\Omega} H(\mathbf{x}, t)C_s(\mathbf{x}, t)d\mathbf{x}}{\int_{\Omega} H(\mathbf{x}, t)d\mathbf{x}} \quad s = i, h, p, r , \quad (18)$$

where Ω is the domain of interest (i.e. the lake surface since we work with a two-dimensional model). Then, if we divide Ω into different regions $\Omega_1, \dots, \Omega_n$,

we may also define $\bar{C}_{s,j}$ the mean concentration of the water type “s” from the region j as

$$\bar{C}_{s,j}(t) = \frac{\int_{\Omega_j} H(\mathbf{x}, t) C_s(\mathbf{x}, t) d\mathbf{x}}{\int_{\Omega_j} H(\mathbf{x}, t) d\mathbf{x}} \quad s = i, h, p, r \quad ; \quad j = 1 \dots n . \quad (19)$$

2.2.2 Age

As introduced in section 1, the age of a water parcel is defined to be the time elapsed since this water parcel left the region in which its age is prescribed to be zero (Bolin and Rodhe, 1973; Takeoka, 1984; Delhez et al., 1999; Deleersnijder et al., 2001).

Let us define the age concentration and the age of every water type as $\alpha_s(t, \mathbf{x})$ and $a_s(t, \mathbf{x})$, respectively. They are linked together as follows (Delhez et al., 1999; Deleersnijder et al., 2001) :

$$a_s(t, \mathbf{x}) = \frac{\alpha_s(t, \mathbf{x})}{C_s(t, \mathbf{x})} , \quad s = i, h, p, r . \quad (20)$$

Assuming that the age of a water parcel is zero at the moment it enters the epilimnion and that every water parcel leaves the epilimnion *along with its age*, then the equations governing the evolution of the age concentration of every water type can be cast into the following generic conservative form :

$$\begin{aligned} \frac{\partial}{\partial t} (H\alpha_s) + \nabla \cdot (H\mathbf{u}\alpha_s) &= HC_s + (w^- + q_v + q_r^-)\alpha_s \\ &+ \nabla \cdot (H\mathbf{K} \cdot (\nabla\alpha_s)) , \quad s = i, h, p, r . \end{aligned} \quad (21)$$

The initial conditions are

$$\alpha_s(0, \mathbf{x}) = 0 , \quad s = i, h, p, r , \quad (22)$$

and the boundary conditions are presented in Appendix A.

We claim that the following property must be satisfied :

$$0 \leq a_s(t, \mathbf{x}) \leq t , \quad s = i, h, p, r . \quad (23)$$

And because the initial water can only leave the domain of interest (i.e. there is no source of *new* initial water), its age should be equal to the elapsed time :

$$a_i(t, \mathbf{x}) = t . \quad (24)$$

Properties (23) and (24) are demonstrated in appendix C.

Let us now define \bar{a}_s the global mean age of the water type “s” as

$$\bar{a}_s(t) = \frac{\bar{\alpha}_s(t)}{C_s(t)} \quad s = i, h, p, r , \quad (25)$$

where $\bar{\alpha}_s$ is the global mean age concentration of the water type “s”, and is defined as

$$\bar{\alpha}_s(t) = \frac{\int_{\Omega} H(\mathbf{x}, t) \alpha_s(\mathbf{x}, t) d\mathbf{x}}{\int_{\Omega} H(\mathbf{x}, t) d\mathbf{x}} \quad s = i, h, p, r , \quad (26)$$

where Ω is the domain of interest. Then, if we divide Ω into different regions $\Omega_1, \dots, \Omega_n$, we may also define $\bar{a}_{s,j}$ the mean age of the water type “s” from the region j as

$$\bar{a}_{s,j}(t) = \frac{\bar{\alpha}_{s,j}(t)}{C_{s,j}(t)} \quad s = i, h, p, r \quad ; \quad j = 1 \dots n . \quad (27)$$

where $\bar{\alpha}_{s,j}$ is the mean age concentration of the water type “s” from the region j , and is defined as

$$\bar{\alpha}_{s,j}(t) = \frac{\int_{\Omega_j} H(\mathbf{x}, t) \alpha_s(\mathbf{x}, t) d\mathbf{x}}{\int_{\Omega_j} H(\mathbf{x}, t) d\mathbf{x}} \quad s = i, h, p, r \quad ; \quad j = 1 \dots n . \quad (28)$$

2.2.3 Residence time

A partial differential problem has been established for estimating the age of the different renewing water types, which is the time that has elapsed since entering the epilimnion. The solutions to this problem are well behaved, i.e. the age of every renewing water type is larger than zero and smaller than the

elapsed time. The age of the initial water appears as the solution to a similar partial differential problem and is equal to the elapsed time, which makes it uninteresting. The age of the initial water is equal to the elapsed time because the initial water can only leave the domain of interest. Therefore, it would be more relevant to focus on the time needed for the initial water to leave this domain. To that end, the residence time is a proper diagnosis, because, as introduced in Section 1, the residence time of a water parcel in a water body is defined as the time taken by this water parcel to leave the water body (Bolin and Rodhe, 1973; Takeoka, 1984; Delhez, 2006; Delhez and Deleersnijder, 2006).

To compute mean residence times from different regions of the lake, we first have to divide the domain Ω into n different regions $\Omega_1, \dots, \Omega_n$. Then we can define n different initial water types, i.e. the initial water from region 1, \dots , the initial water from region n . The concentration of the initial water from region j is denoted $\theta_j(t, \mathbf{x})$. The equation governing the concentration of initial water from region j is the same as that of the concentration of initial water, i.e. equation (8) with $s = i$, and thus can be cast into the following generic conservative form :

$$\frac{\partial}{\partial t}(H\theta_j) + \nabla \cdot (H\mathbf{u}\theta_j) = (w^- + q_v + q_r^-)\theta_j + \nabla \cdot (H\mathbf{K} \cdot (\nabla\theta_j))$$

$$j = 1, \dots, n . \quad (29)$$

The equation governing the initial water from region j and its boundary condition (see Appendix A) are the same for each region j . The only difference lies in the initial condition :

$$\theta_j(t_0, \mathbf{x} \in \Omega_j) = 1 \quad \text{and} \quad \theta_j(t_0, \mathbf{x} \in \Omega \setminus \Omega_j) = 0 , \quad j = 1, \dots, n . \quad (30)$$

Finally, we can compute the mean residence time of the initial water from region j at time t_0 as

$$\bar{\Theta}_j(t_0) = \frac{\int_{t_0}^{\infty} \int_{\Omega} H\theta_j(t, \mathbf{x}) d\mathbf{x} dt}{\int_{\Omega} H\theta_j(t_0, \mathbf{x}) d\mathbf{x}} , \quad j = 1, \dots, n . \quad (31)$$

That is the time taken by a water parcel initially in region j to leave the epilimnion. That is why the surface integrals must extend on the entire lake surface Ω .

Note that if $n = 1$, expression (31) yields the global mean residence time of initial water at time t_0 :

$$\bar{\Theta}(t_0) = \frac{\int_{t_0}^{\infty} \int_{\Omega} HC_i(t, \mathbf{x}) d\mathbf{x} dt}{\int_{\Omega} HC_i(t_0, \mathbf{x}) d\mathbf{x}} . \quad (32)$$

It should be pointed out that we have an additional system of equations to solve for each region and for each initial time. For this study we have defined three regions (from south to north) and twelve initial times (the beginning of each month). We thus had to solve $3 \times 12 = 36$ additional systems of equations at each time step. This is a very important amount compared with the two systems for the hydrodynamic model (to compute ξ and \mathbf{u}), the four systems to compute the concentration of the different water types (C_i , C_h , C_p and C_r), and the three systems to compute the age concentration (and thus the age) of the different renewing water types (α_h , α_p and α_r). The mean residence time is thus very expensive compared with the age and it is moreover much less accurate since it is a global variable.

2.3 Numerical implementation

The finite element method is used to discretize both the hydrodynamic model equations and the concentration and age concentration equations. We have discretized the non conservative form of these equations on the mesh displayed in Figure 4.

For the hydrodynamic model, we use the finite element model SLIM (Second-generation Louvain-la-Neuve Ice-ocean Model, <http://www.climate.be/SLIM>) which is essentially built upon the work by Hanert et al. (2004, 2005) for the shallow water equations. The elevation and velocity variables are approximated by linear conforming (P_1) and linear non-conforming (P_1^{NC}) shape functions, respectively. Therefore, the elevation nodes are situated at the vertices of each element of the mesh, and the velocity nodes at the middle of their edges. To enhance robustness, advection terms are computed with an upwind-biased scheme (Hanert et al., 2004).

For the equations of concentration (8) and (29) and age concentration (21), given that they are very similar to those of momentum conservation (4), we use a similar method. That makes our approach very interesting : we do not have to implement a completely new model to compute ages and mean residence

times once the hydrodynamic model is built. Therefore, the concentration and age concentration variables are approximated by linear non-conforming shape functions (P_1^{NC}) and their nodes are situated at the middle of the edges of each element of the mesh. We also compute an upwind-biased scheme to compute advection terms (Hanert et al., 2004).

The solution technique is sequential. At each time step, we first compute the elevation field ξ , then we compute simultaneously the horizontal depth-integrated velocity fields u and v , next we compute the different concentration fields and finally the different age concentration fields. The mean residence times are only computed at the end of the simulation because we have to integrate in time from t_0 for a sufficiently long time. By sufficiently long time, we mean the time to allow θ_j to be much smaller than 1 throughout the lake. In this study we compute these equations over a period of three years.

3 Results

First, a four-year run of the hydrodynamic model was carried out to get the regime solution. Then, a four-year run was carried out to compute both the concentration and age distribution of the different water types. Finally, twelve runs were carried out to compute the mean residence time with different initial times. We used homogeneous constant eddy viscosity and diffusivity tensors, which means that \mathbf{A} and \mathbf{K} are diagonal identity tensors multiplied by a constant $A = 3 \text{ m}^2/\text{s}$ and $K = 10 \text{ m}^2/\text{s}$, respectively. The time step is 12 minutes.

Before analyzing the renewal of the epilimnion water, we take a brief look at its hydrodynamic cycle, which is done in more detail by Gourgue et al. (2007). The epilimnion volume budget varies with the supply and the loss of water from hypolimnion, precipitation, evaporation and rivers. Figure 5a shows the evolution of the global mean epilimnion thickness

$$\bar{H}(t) = \frac{\int_{\Omega} H(\mathbf{x}, t) d\mathbf{x}}{\int_{\Omega} d\mathbf{x}}, \quad (33)$$

which is proportional to the epilimnion volume. It grows from the beginning of the cycle (1st April) to approximately the end of the dry season (end of August). Then it decreases to reach its minimum value at the end of the cycle (end of March). It is approximately the same shape as the wind stress cycle on Figure 3b, which is the most important factor of the exchange of water between the epilimnion and the hypolimnion. We may thus expect that

this latter phenomenon be more important than the three others considered (precipitation, evaporation and rivers).

Figure 5b shows the evolution of the mean epilimnion thickness from different regions :

$$\bar{H}_j(t) = \frac{\int_{\Omega_j} H(\mathbf{x}, t) d\mathbf{x}}{\int_{\Omega_j} d\mathbf{x}} \quad j = 1, 2, 3 . \quad (34)$$

In this paper, we always divide the domain in three regions, $j = 1, 2, 3$ referring to the south, the center and the north of the lake, respectively. We see that the mean epilimnion thickness is greater in the north than in the south, especially during the dry season. This is due to the important wind stress along the main direction of the lake during this season. The epilimnion water is pushed from south to north where it accumulates. Following equation (5), the water supply from hypolimnion to epilimnion is more important in the south. This is why we may expect the age of the hypolimnion water to be smaller in the south than in the north. This would be caused by a larger supply of *new* hypolimnion water in the south, making it *younger*. On the other hand, the water loss from epilimnion to hypolimnion is more important in the north. This is why we may expect the residence time of the initial water to be smaller in the north than in the south.

We see on Figure 6a that the global mean concentrations of precipitation water and river water never exceed 2% and 0.5%, respectively. Thus, we have

$$\bar{C}_{rw} = \bar{C}_h + \bar{C}_p + \bar{C}_r \simeq \bar{C}_h . \quad (35)$$

As expected, while the renewing water replaces the initial water, the global mean initial water concentration \bar{C}_i decreases toward 0, and the global mean renewing water concentration $\bar{C}_{rw} \simeq \bar{C}_h$ increases toward 1. These trends are well marked during the dry season, when the winds are stronger and thus when the supply of hypolimnion water is more important. It is worth stressing that it takes only one year to replace 90% of the initial water. Figure 6b shows that the renewing water concentration is larger in the south than in the north. That is because the supply of hypolimnion water is more important in this region. We see the same features in Movie 1 (available online). The renewing water concentration increases everywhere in the lake, but faster during the dry season and also faster in the south than in the north of the lake. This confirms that the water exchange between the epilimnion and the hypolimnion seems to be the only important process driving the renewal of epilimnion water in Lake Tanganyika.

We see in Figure 7a that the global mean hypolimnion water age increases at the beginning of the simulation. But just before the second dry season, it begins to decrease. That is because the wind stress begins to be significant (see Figure 3). The supply of *new* hypolimnion water is important at this moment and hypolimnion water in the epilimnion becomes *younger*. Then, around the end of the dry season, the global mean hypolimnion water age begins to increase again, until the next rise of the wind stress, and so on. It is the same process for the global mean age of precipitation water and river water, except that the most important supply of *new* renewing water occurs during the wet season for these water types. We also see in this figure that the global mean ages of precipitation water and river water are on the same order of the global mean initial water age :

$$\bar{a}_h \sim \bar{a}_p \sim \bar{a}_r , \quad (36)$$

Since $\bar{a}_s = \bar{\alpha}_s / \bar{C}_s$ and because \bar{C}_p and \bar{C}_r are very small compared with \bar{C}_h , equation (36) implies that $\bar{\alpha}_p$ and $\bar{\alpha}_r$ are also very small compared with $\bar{\alpha}_h$. Following the definition of the age, we have

$$\bar{a}_{rw} = \frac{\bar{\alpha}_{rw}}{\bar{C}_{rw}} = \frac{\bar{\alpha}_h + \bar{\alpha}_p + \bar{\alpha}_r}{\bar{C}_h + \bar{C}_p + \bar{C}_r} \simeq \frac{\bar{\alpha}_h}{\bar{C}_h} = \bar{a}_h , \quad (37)$$

That is what we see in Figure 8. This is another confirmation that the water exchange between the epilimnion and the hypolimnion seems to be the only important process driving the renewal of epilimnion water in Lake Tanganyika. Figure 7b shows, as expected, that the renewing water age is less important in the south than in the north. That is because the mean epilimnion thickness is smaller in the south, which implies, following equation (5), that a more significant supply of *new* hypolimnion water occurs in the south. We see the same features in Movie 2 (available online). The renewing water age decreases everywhere in the lake from the beginning of the year until the end of the dry season. It then increases until the end of the year. But the renewing water is always *younger* in the south than in the north of the lake.

Now, consider the residence time. The latter depends on the value of $(w^- + q_v + q_r^-)$, the water flux leaving the epilimnion. Since $w^- \sim 10^{-5}$ m/s , $q_v \sim 10^{-8}$ m/s and q_r^- is zero almost everywhere, we may assume that the residence time only depends on the value of w^- . Once again, it confirms that the renewal of epilimnion water in Lake Tanganyika is largely dominated by the water exchange between the epilimnion and the hypolimnion. The residence time increases when $|w^-| = 0$, i.e. when water stays into the epilimnion, and it decreases when $|w^-| > 0$, i.e. when water leaves the epilimnion. The first term of equation (5) is always positive, but is on the same order of magnitude as the second one, by definition of the detrainment velocity w_d . So, to have $|w^-| > 0$,

the relaxation term (ξ/T_r) has to be important. Thus, overall, the residence time depends on the value of ξ , i.e. the value of the epilimnion thickness H : the residence time increases or decreases when the epilimnion is thin or thick, respectively. That explains why the evolution of the global mean residence time (Figure 9a) seems to be inversely proportional to the global mean epilimnion thickness (Figure 5), and why, as expected, the mean residence time is larger in the south than in the north (Figure 9b). Coulter and Spigel (1991) find a residence time of about one year, defining it as the mass of a certain nutrient present in the epilimnion dividing by the rate of addition of this nutrient in the epilimnion. Following our more accurate definition, the residence time is much smaller. The mean residence time of the initial water integrated over the whole year is equal to 127 days.

4 Conclusion

We have suggested a theoretical method based on the concepts of age and residence time to study the renewal of epilimnion water in Lake Tanganyika. This method is easy to implement when the hydrodynamic model is already set up and it may be easily adapted to the water renewal of a more general (even three-dimensional) semi-enclosed domain. Moreover, the method presented to compute the age and the residence time uses all the information of the solution obtained by the hydrodynamic model. The goal is always to separate the water of the domain into different water types : the initial water and the various renewing water types. We then compute the age of the different renewing water types and the residence time of the initial water.

This method allowed us to show that the effects of precipitation, evaporation and river input are negligible in the renewal of epilimnion water in Lake Tanganyika. The only significant effect is the water exchange by turbulent processes between the epilimnion and the hypolimnion. Moreover we showed that it takes only one year to replace about 90 % of the water in the epilimnion by water from hypolimnion. This is particularly important considering that the hypolimnion contains the major part of the nutrients but is also anoxic. The residence time of the water in the epilimnion is indeed equal to 127 days on average.

Because the water exchange between the epilimnion and the hypolimnion depends on the wind stress and the epilimnion thickness, these two parameters regulate the water renewal. The renewing water age decreases when the water exchange from hypolimnion to epilimnion is important, i.e. when the wind stress is important. That is why the renewing water age decreases during the dry season and increases during the wet season. Moreover, the water exchange from hypolimnion to epilimnion is particularly important where the epilimnion

is thin, leading to a smaller renewing water age in the south than in the north. The initial water residence time decreases when the water exchange from epilimnion to hypolimnion is important. We have shown in section 3 that this water exchange is roughly inversely proportional to the epilimnion thickness. Hence the initial water residence time decreases during the dry season (when the epilimnion thickness increases) and increases during the wet season (when the epilimnion thickness decreases). Finally, since the epilimnion is always the thinnest in the south, the initial water residence time is systematically larger there.

The next steps of this work will be to build a three-dimensional model of Lake Tanganyika including the epilimnion and the hypolimnion. It will allow us to study the water exchange between these two layers without the parameterization (5).

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A Boundary conditions

Since the rivers are taken into account by means of source/sink terms, the boundary of the domain of interest (Γ) can be assumed to be impermeable to both advective and diffusive matter fluxes. Therefore, if \mathbf{n} denotes the outward unit normal to the boundary, and u_t is the velocity component tangential to the boundary, the boundary conditions associated with equations (3), (4), (8), (21) and (29), respectively, are as follows :

$$\left[\mathbf{u} \cdot \mathbf{n} \right]_{\mathbf{x} \in \Gamma} = 0 \quad (38)$$

$$\left[H \mathbf{n} \cdot \mathbf{A} \cdot (\nabla u_t) \right]_{\mathbf{x} \in \Gamma} = 0 \quad (39)$$

$$\left[H \mathbf{n} \cdot \mathbf{K} \cdot (\nabla C_s) \right]_{\mathbf{x} \in \Gamma} = 0, \quad s = i, h, p, r \quad (40)$$

$$\left[H \mathbf{n} \cdot \mathbf{K} \cdot (\nabla \alpha_s) \right]_{\mathbf{x} \in \Gamma} = 0, \quad s = i, h, p, r \quad (41)$$

$$\left[H \mathbf{n} \cdot \mathbf{K} \cdot (\nabla \theta_j) \right]_{\mathbf{x} \in \Gamma} = 0, \quad j = 1, \dots, n. \quad (42)$$

B Concentration properties

We want to demonstrate that the properties (14) to (17) hold true.

By adding the equations governing the concentration of each water type, the equation satisfied by the water concentration C_{tot} is readily seen to be

$$\begin{aligned} & \frac{\partial}{\partial t}(HC_{tot}) + \nabla \cdot (H\mathbf{u}C_{tot}) \\ &= (w^+ + q_p + q_r^+) + (w^- + q_v + q_r^-)C_{tot} + \nabla \cdot (H\mathbf{K} \cdot (\nabla C_{tot})) . \end{aligned} \quad (43)$$

Taking into account continuity equation (3), the boundary conditions (38) and (40) and the initial conditions (13), the solution of equation (43) is

$$C_{tot}(t, \mathbf{x}) = 1 . \quad (44)$$

Therefore, property (15) holds true.

If Ω represents the domain of interest (i.e. the lake surface), elementary manipulations of the equation (8) applied to the initial water and the continuity equation (3) yield

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\Omega} HC_i^2 d\mathbf{x} &= - \int_{\Omega} \left((|w| + q_p - q_v + |q_r|) C_i^2 \right. \\ &\quad \left. + 2H(\nabla C_i) \cdot \mathbf{K} \cdot (\nabla C_i) \right) d\mathbf{x} . \end{aligned} \quad (45)$$

As \mathbf{K} is positive definite, q_p is positive and q_v is negative, the integrand in the right-hand side member of equation (45) is positive unless C_i is zero at every location of the domain of interest. As a result, the integral over Ω of (HC_i^2) , which is a measure of the magnitude of the initial water concentration, will

decrease until the initial water concentration is zero at every point. Therefore, property (16) holds true.

Equations (6) and (7) and properties (15) and (16) imply that property (17) holds true, which is trivial.

Demonstrating that property (14) holds true is equivalent to show that the concentration C_s and the overshooting of the concentration $\hat{C}_s = 1 - C_s$ are positive, or that their negative parts are zero, i.e.

$$C_s^-(t, \mathbf{x}) = 0, \quad s = i, h, p, r \quad (46)$$

$$\hat{C}_s^-(t, \mathbf{x}) = 0, \quad s = i, h, p, r. \quad (47)$$

Intricate manipulations of the equations (8) and (3) are needed to show that the negative part of every concentration C_s obeys an expression of the form

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\Omega} H(C_s^-)^2 d\mathbf{x} = - \int_{\Omega} \left(\omega(C_s^-) |C_s^-| + 2H(\nabla C_s^-) \cdot \mathbf{K} \cdot (\nabla C_s^-) \right) d\mathbf{x} \\ s = i, h, p, r. \end{aligned} \quad (48)$$

A similar expression holds valid for the negative part of \hat{C}_s , i.e.

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\Omega} H(\hat{C}_s^-)^2 d\mathbf{x} = - \int_{\Omega} \left(\omega(\hat{C}_s^-) |\hat{C}_s^-| + 2H(\nabla \hat{C}_s^-) \cdot \mathbf{K} \cdot (\nabla \hat{C}_s^-) \right) d\mathbf{x} \\ s = i, h, p, r. \end{aligned} \quad (49)$$

The Table 1 provides $\omega(C_s^-)$ and $\omega(\hat{C}_s^-)$.

Clearly, the integrands in the right-hand side of equations (48) and (49) are positive, unless C_s^- and \hat{C}_s^- are zero at every point. As a consequence, the measures of the undershootings and the overshootings of the different water type concentrations cannot increase. But, at the initial time, they are already zero, by virtue of the initial conditions (13). Thus, C_s^- and \hat{C}_s^- will be zero at any time and location, implying that property (14) holds true.

C Age properties

We want to demonstrate that the properties (23) and (24) hold valid.

Using the equation of the initial water concentration, i.e. the equation (8) with $s = i$, it is readily seen that $\alpha_i(t, \mathbf{x}) = C_i(t, \mathbf{x})t$ is the solution of the partial

differential problem of equations (21) to (22). Therefore, property (24) holds true.

Since property (24) holds true, the age of the initial water satisfies the property (23) for $s = i$. But for the sake of generality and completeness, the demonstration below will keep dealing with the initial water.

Since the different water type concentrations $C_s(t, \mathbf{x})$ are positive, showing the validity of property (23) is equivalent to showing that the age concentration α_s and the overshooting of the age concentration $\hat{\alpha}_s = C_s t - \alpha_s$ are positive, or that their negative parts are zero, i.e.

$$\alpha_s^-(t, \mathbf{x}) = 0, \quad s = i, h, p, r \quad (50)$$

$$\hat{\alpha}_s^-(t, \mathbf{x}) = 0, \quad s = i, h, p, r. \quad (51)$$

Intricate manipulations of the equations (21) and (3) are needed to show that the negative parts of every age concentration α_s obeys an expression of the form

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\Omega} H(\alpha_s^-)^2 d\mathbf{x} = - \int_{\Omega} \left(\omega(\alpha_s^-) |\alpha_s^-| + 2H(\nabla \alpha_s^-) \cdot \mathbf{K} \cdot (\nabla \alpha_s^-) \right) d\mathbf{x} \\ s = i, h, p, r. \end{aligned} \quad (52)$$

A similar expression holds valid for the negative part of $\hat{\alpha}_s$, i.e.

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\Omega} H(\hat{\alpha}_s^-)^2 d\mathbf{x} = - \int_{\Omega} \left(\omega(\hat{\alpha}_s^-) |\hat{\alpha}_s^-| + 2H(\nabla \hat{\alpha}_s^-) \cdot \mathbf{K} \cdot (\nabla \hat{\alpha}_s^-) \right) d\mathbf{x} \\ s = i, h, p, r. \end{aligned} \quad (53)$$

The Table 2 provides $\omega(\alpha_s^-)$ and $\omega(\hat{\alpha}_s^-)$.

Clearly, the integrands in the right-hand side of equations (52) and (53) are positive, unless α_s^- and $\hat{\alpha}_s^-$ are zero at every point. As a consequence, the measures of the undershootings and the overshootings of the different water type age concentrations cannot increase. But, at the initial time, they are already zero, by virtue of the initial conditions (22). Thus, α_s^- and $\hat{\alpha}_s^-$ will be zero at any time and location, implying that property (23) holds true.

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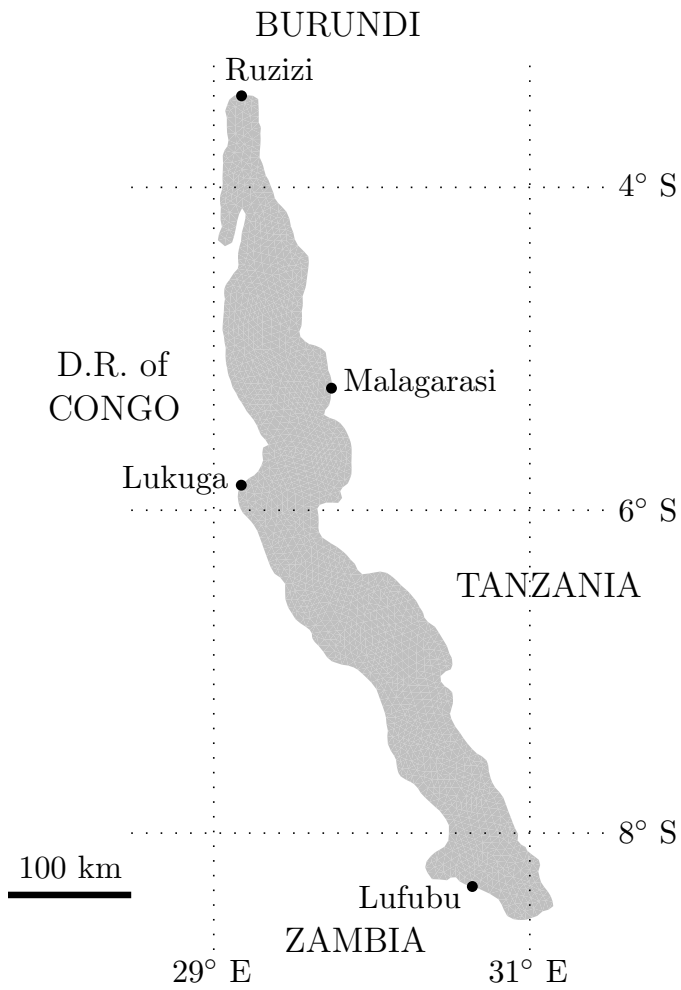


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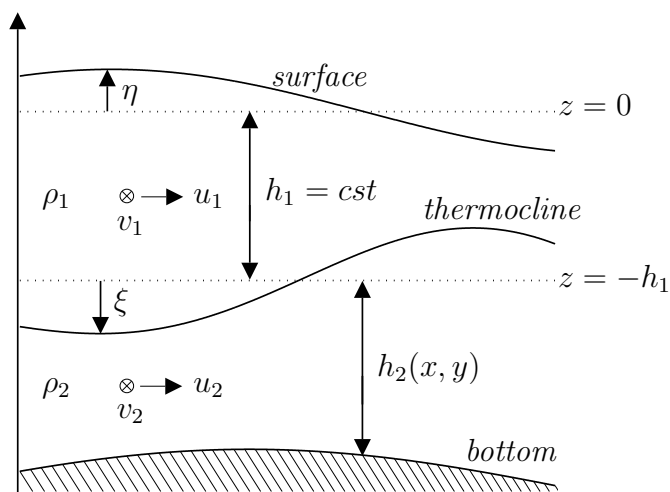


Fig. 2. The parameters and variables of a two-layer model : η is the upward displacement of the surface, ξ is the downward displacement of the thermocline, ρ_i is the density of the i^{th} layer, u_i and v_i are the depth-integrated velocity components of the i^{th} layer in the x and y directions, h_1 is the undisturbed thickness of the top layer, or the mean thermocline depth, and h_2 is the distance between the mean thermocline depth and the bottom.

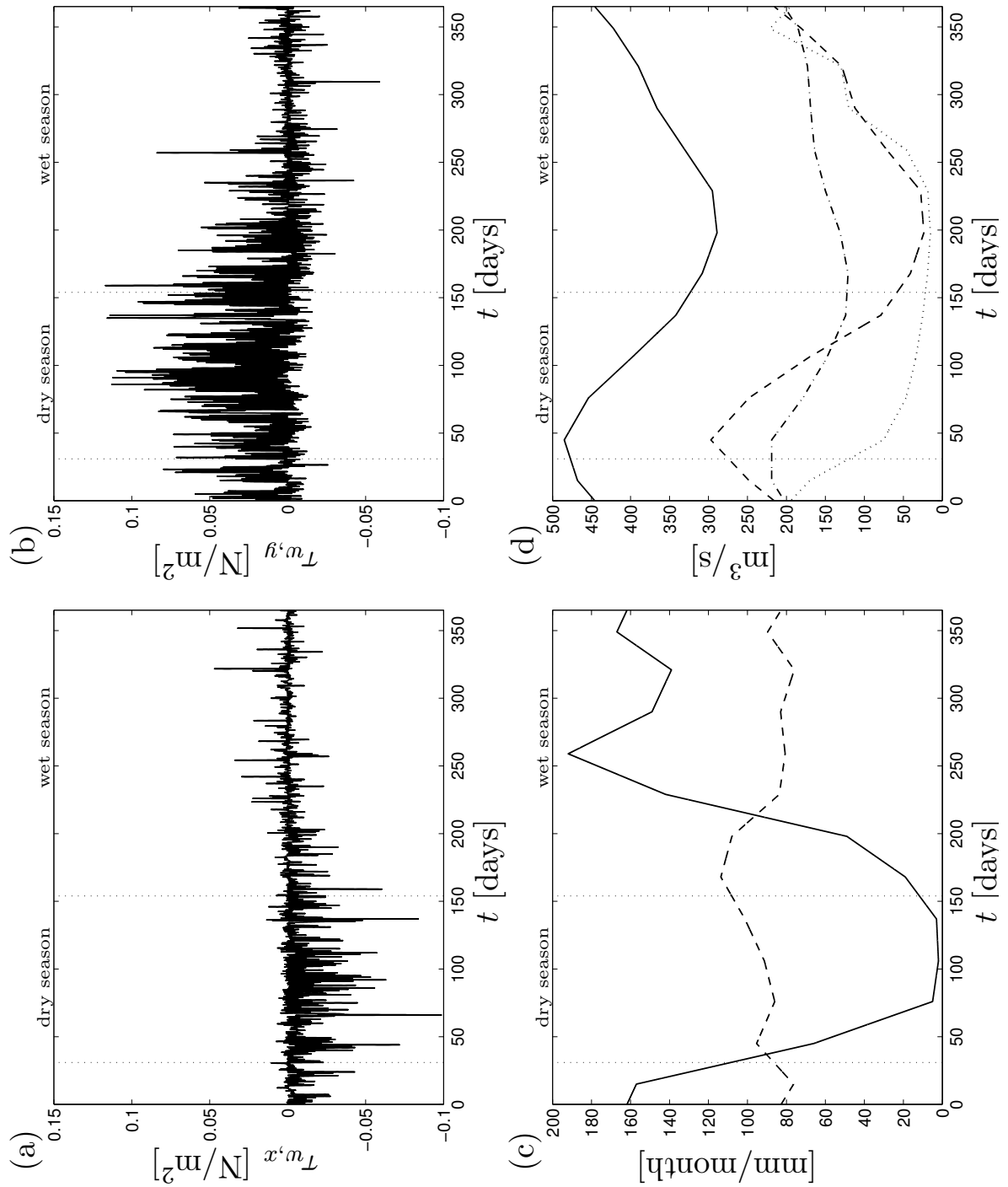


Fig. 3. The various model forcings. (a-b) Wind stress in the x and y directions, respectively, measured at Mpulungu in Zambia, every six hours from 1st March 1993 to 31st April 1994. (c) Absolute value of the monthly lake-averaged water fluxes due to precipitation (solid) and to evaporation (dashed) (Bergonzini, 1998). These water fluxes are always positive and negative, respectively. (d) Monthly water flow of the most important rivers : Malagarasi (dashed), Ruzizi (dash-dotted) and Lufubu (dotted) are incoming rivers, Lukuga (solid) is an outgoing river (Bergonzini, 1998).

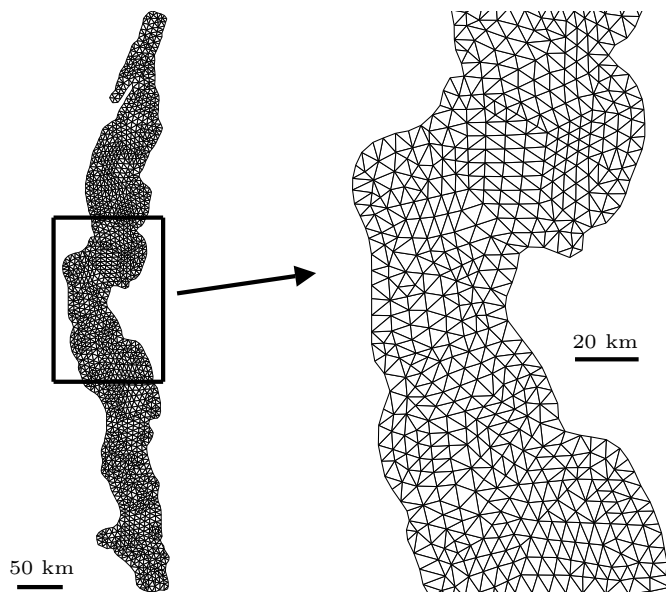


Fig. 4. Unstructured mesh of Lake Tanganyika used for the simulations of this paper. There are 2997 triangular elements. The mean mesh size is about 5 km.

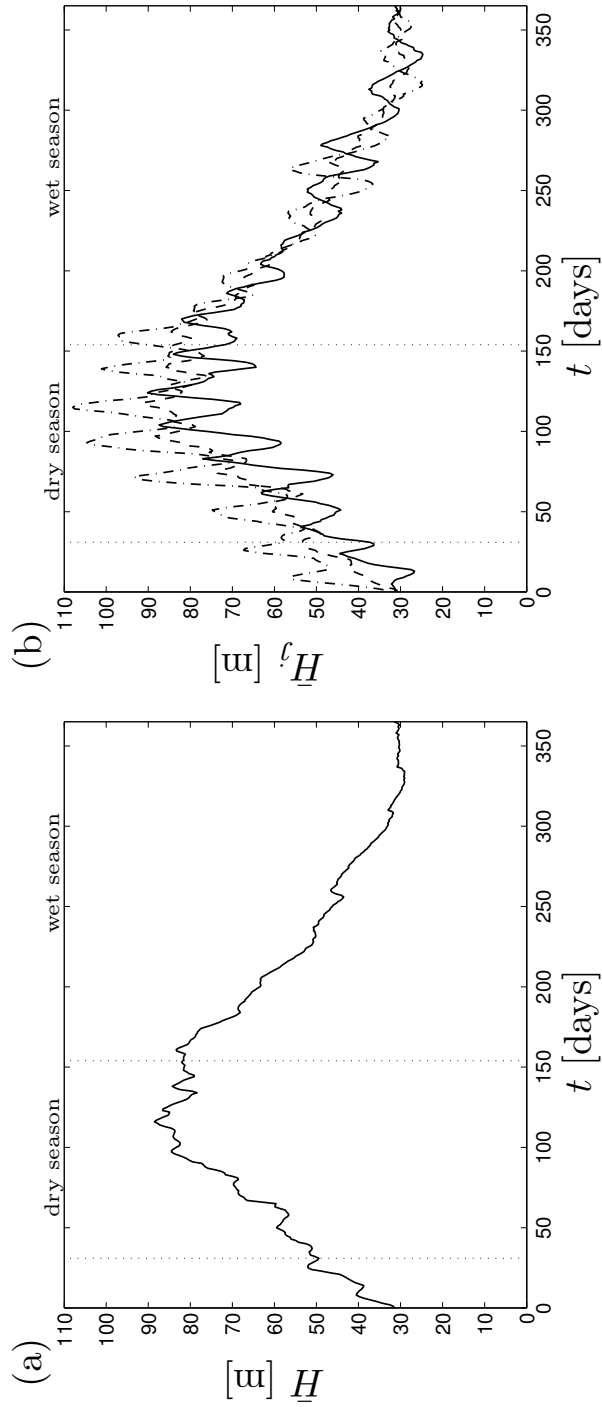


Fig. 5. (a) Evolution of the global mean epilimnion thickness \bar{H} , at regime state. (b) Evolution of the mean epilimnion thickness \bar{H}_j ($j = 1, 2, 3$ from south to north) in the south (solid), in the center (dashed) and in the north (dash-dotted), at regime state.

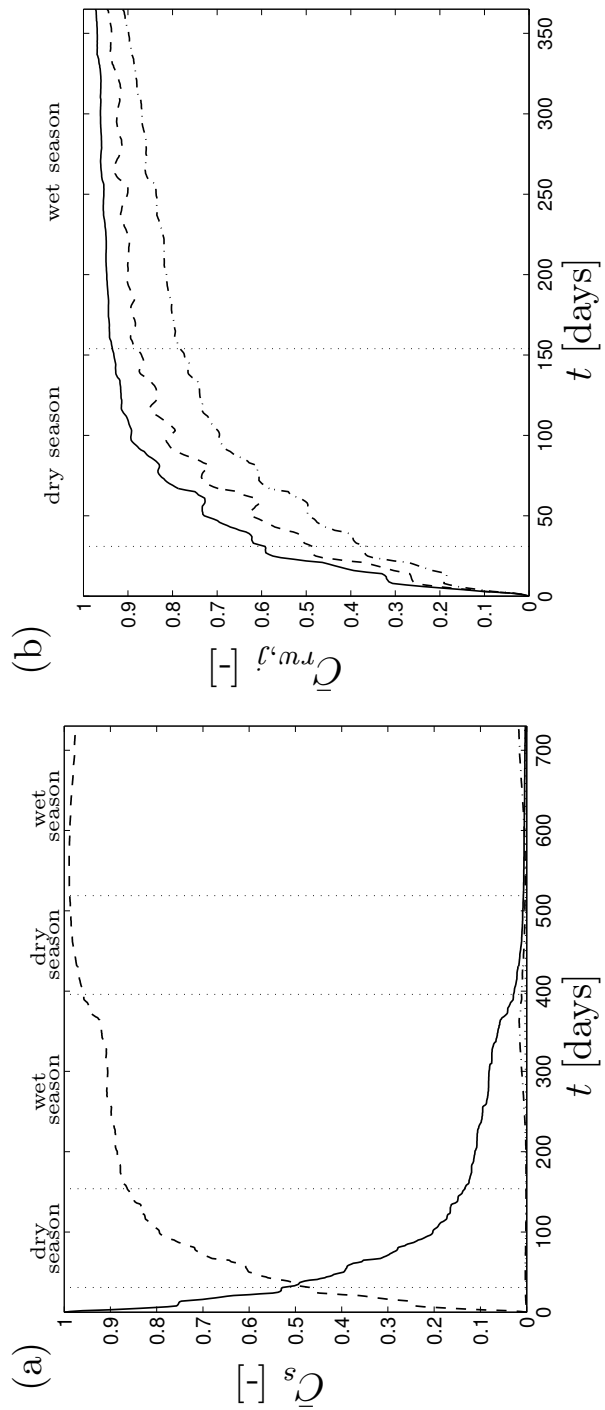


Fig. 6. (a) Evolution of the global mean initial (solid), hypolimnion (dashed), precipitation (dash-dotted) and river (dotted) water concentrations \bar{C}_s ($s = i, h, p, r$), during the first two years. (b) Evolution of the mean renewing water concentration $\bar{C}_{rw,j}$ ($j = 1, 2, 3$ from south to north) in the south (solid), in the center (dashed) and in the north (dash-dotted), during the first year.

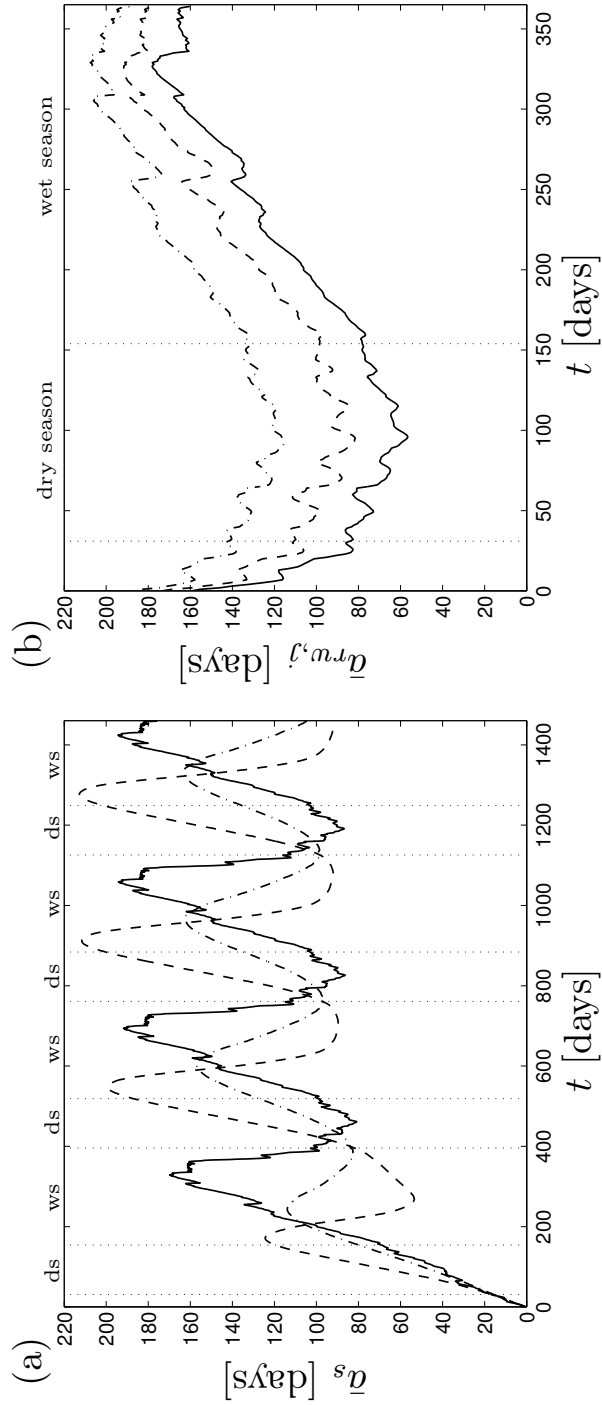


Fig. 7. (a) Evolution of the global mean hypolimnion (solid), precipitation (dashed) and river (dash-dotted) water ages \bar{a}_s ($s = h, p, r$), during the first four years. (b) Evolution of the mean renewing water age in the south (solid), in the center (dashed) and in the north (dash-dotted) $\bar{a}_{rw,j}$ ($j = 1, 2, 3$ from south to north), at regime state.

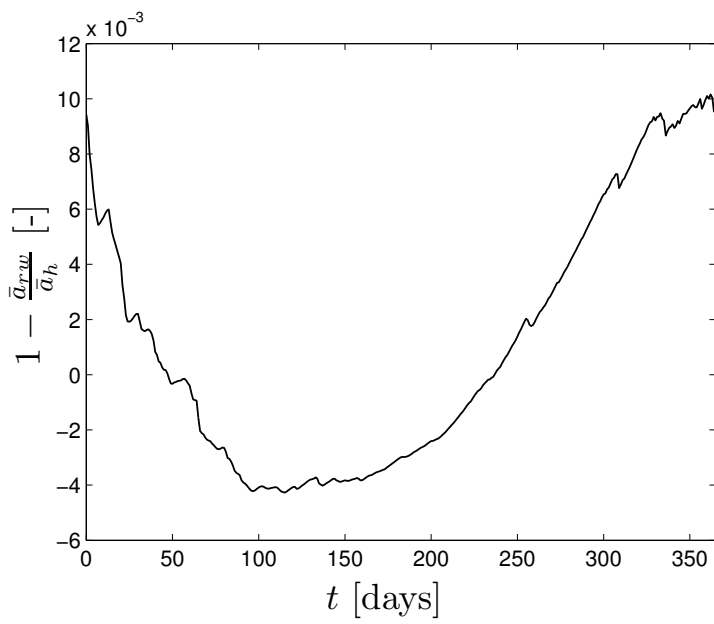


Fig. 8. Evolution of the relative difference between the global mean renewing water age and the global mean hypolimnion water age, at equilibrium state.

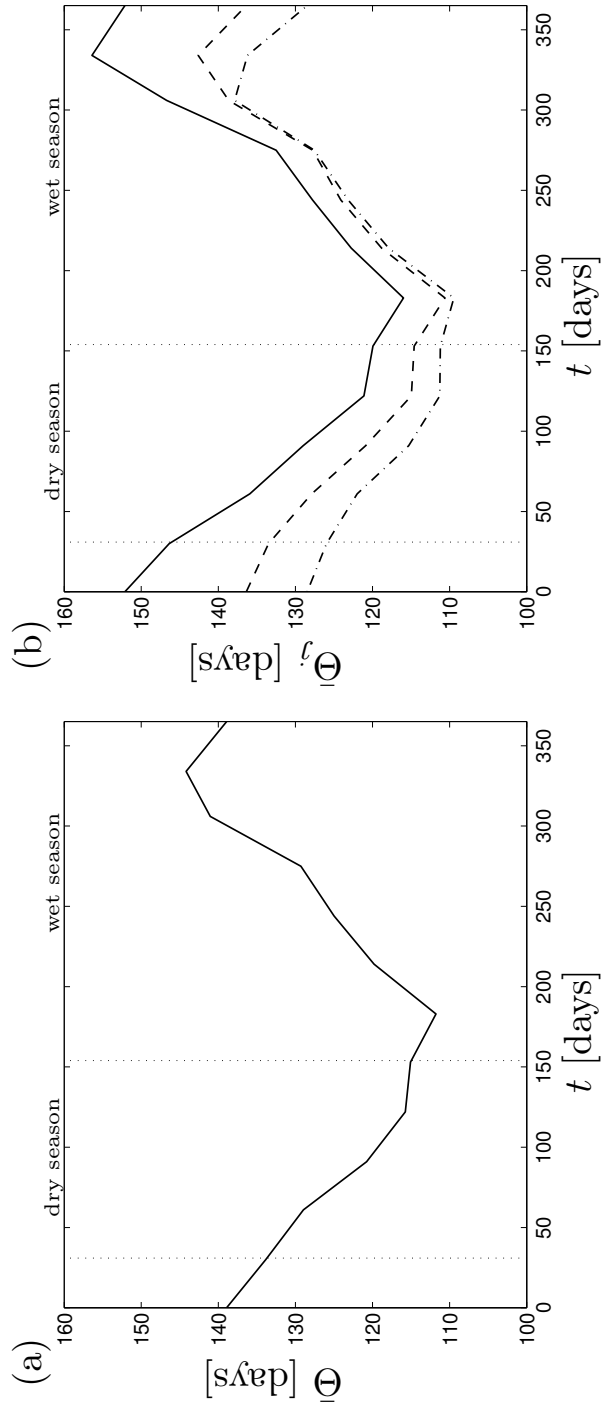


Fig. 9. (a) Evolution of the global mean initial water residence time $\bar{\Theta}$. (b) Evolution of the mean initial water residence time in the south (solid), in the center (dashed) and in the north (dash-dotted) $\bar{\Theta}_j$ ($j = 1, 2, 3$ from south to north).

	$\omega(C_s^-)$	$\omega(\hat{C}_s^-)$
$s = i$	$(w + q_p - q_v + q_r) C_i^- $	$(w + q_p - q_v + q_r) C_i^- + 2(w^+ + q_p + q_r^+)$
$s = h$	$(w + q_p - q_v + q_r) C_h^- + 2w^+$	$(w + q_p - q_v + q_r) C_h^- + 2(q_p + q_r^+)$
$s = p$	$(w + q_p - q_v + q_r) C_p^- + 2q_p^+$	$(w + q_p - q_v + q_r) C_p^- + 2(w^+ + q_r^+)$
$s = r$	$(w + q_p - q_v + q_r) C_r^- + 2q_r^+$	$(w + q_p - q_v + q_r) C_r^- + 2(w^+ + q_p)$

Table 1

Values of $\omega(C_s^-)$ and $\omega(\hat{C}_s^-)$ from equations (48) and (49), respectively, depending on the water type.

	$\omega(\alpha_s^-)$	$\omega(\hat{\alpha}_s^-)$
$s = i$	$(w + q_p - q_v + q_r) \alpha_i^- + 2HC_i$	$(w + q_p - q_v + q_r) \alpha_i^- $
$s = h$	$(w + q_p - q_v + q_r) \alpha_h^- + 2HC_h$	$(w + q_p - q_v + q_r) \alpha_h^- + 2w^+t$
$s = p$	$(w + q_p - q_v + q_r) \alpha_p^- + 2HC_p$	$(w + q_p - q_v + q_r) \alpha_p^- + 2q_p t$
$s = r$	$(w + q_p - q_v + q_r) \alpha_r^- + 2HC_r$	$(w + q_p - q_v + q_r) \alpha_r^- + 2q_r^+ t$

Table 2

Values of $\omega(\alpha_s^-)$ and $\omega(\hat{\alpha}_s^-)$ from equations (52) and (53), respectively, depending on the water type.