# a Geometric-based Method for the Perceptive Design of Hyperstatic Rigid Frames

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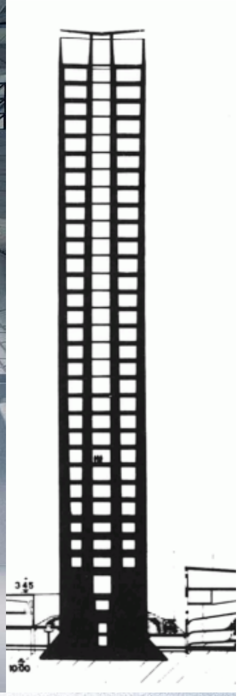
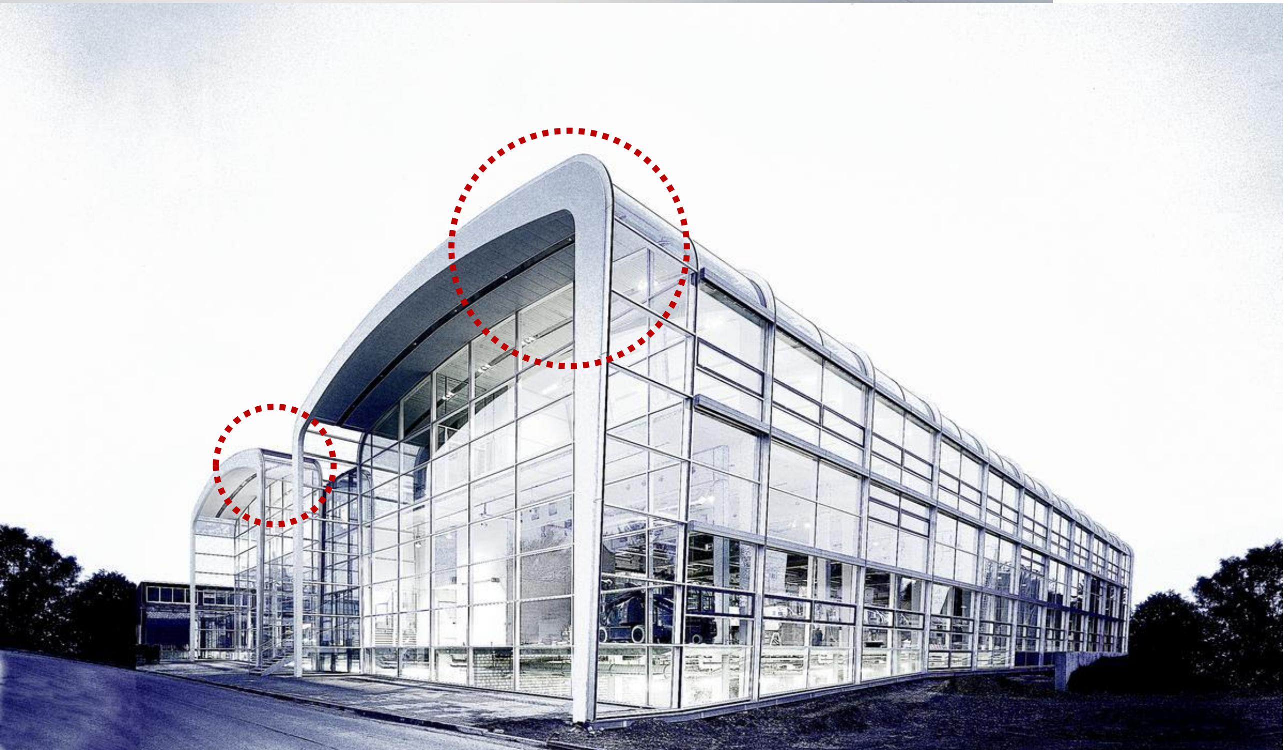
# hyperstatic rigid frames

Part I:
The history of
hyperstatic rigid frames
and
its analysis methods

Part II:
How to design
hyperstatic rigid frames
insightfully
with graphic statics ?

How to design <u>hyperstatic rigid frames</u> insightfully with graphic statics ?





Pirelli Tower (source:BMS)

GIRA Production Hall (Ingenhoven Architects, source:GIRA Giersiepen GmbH & Co. KG)

Rigid/moment-resisting connections and members

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statically indeterminate/ hyperstatic

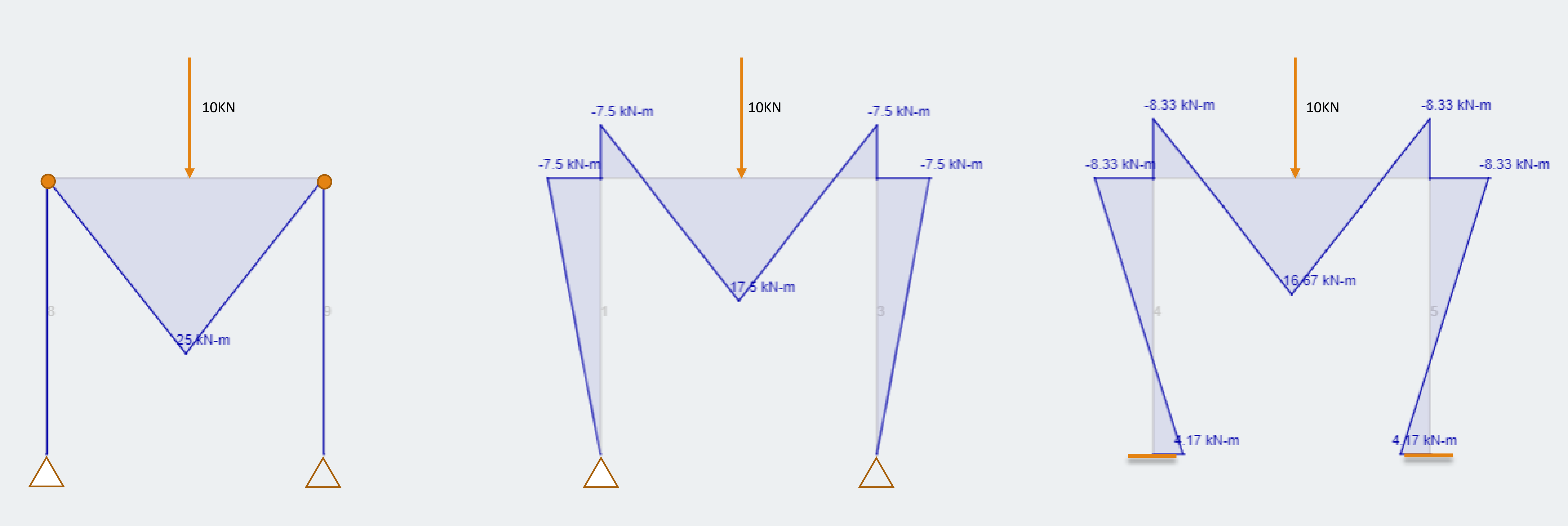
# The structural relevance of hyperstatic rigid frames

## 1. The structural relevance of rigid frames

- Geometrical relevance (force follow form)
- Structural integrity
- Lateral resistance capacity
- Constructability

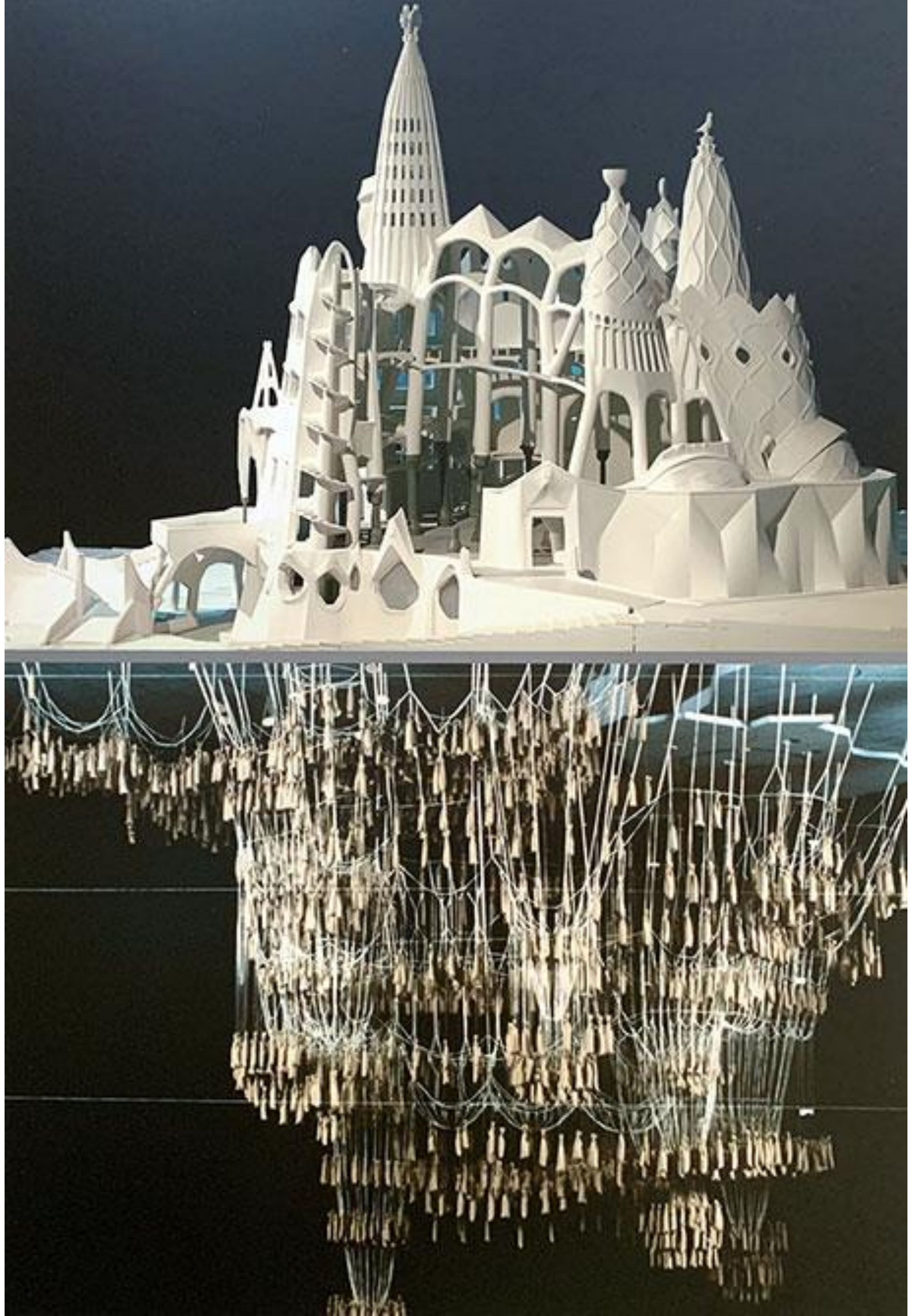
## 2. The structural relevance of statically indeterminacy

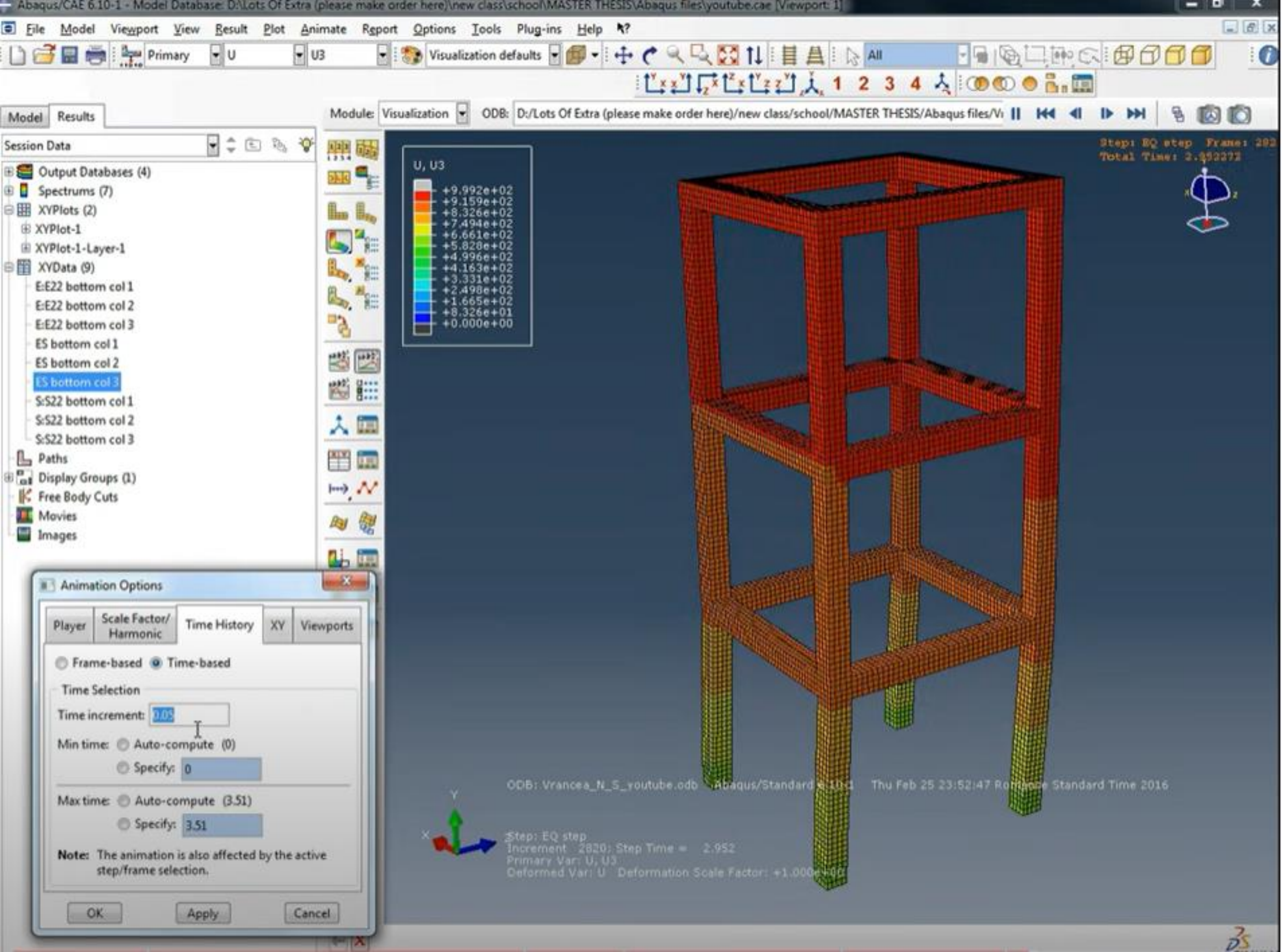
- Structural efficiency through hyperstability
- Structural redundancy for robustness and safety



How to design hyperstatic rigid frames insightfully
with graphic statics ?

*“**Creativity** is a result of the real **perception** of the problem.”*






# Limitation of the current design methods of FEM:

- Results are difficult to interpret
- Results are not interactive
- Demanding of mathematics knowledge

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- One way guess-and-check approach
- Not suitable for early stage of design
- Architects are deterred from in-depth form-finding of frames

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- Aesthetic value is often unexploited
- Often materially-inefficient

FEM analysis (source:youtube)

- Aesthetic value: Rigid frames an integrated element for architectural richness and elegance

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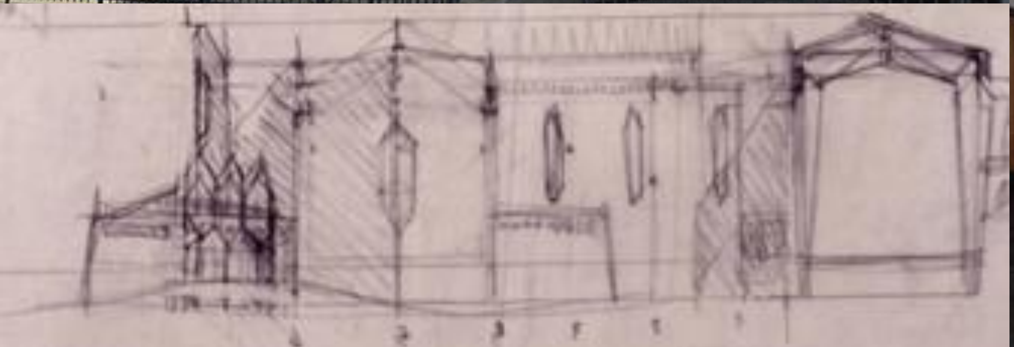
- Materially-efficient: varying sections according to bending moment

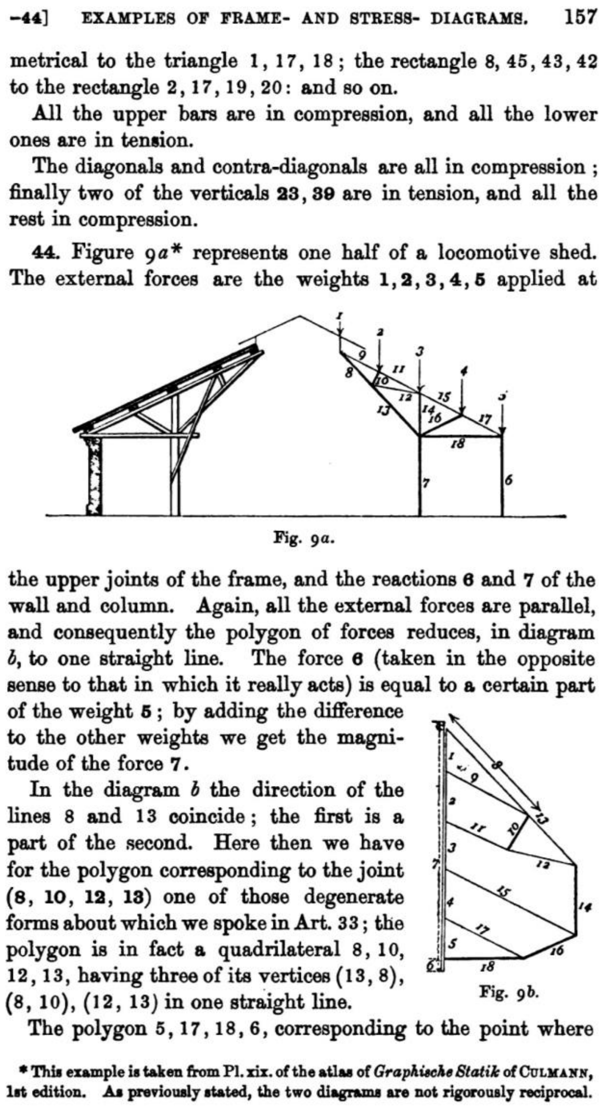


- Aesthetic value: Rigid frames an integrated element for architectural richness and elegance
- Materially-efficient: varying sections according to bending moment
- Spatial limitation


Sint-Jorisbrug bridge,
Gent,1907

How to design hyperstatic rigid frames insightfully with graphic statics ?

# Insight into the relation between form and forces



-44] EXAMPLES OF FRAME- AND STRESS- DIAGRAMS. 157

metrical to the triangle 1, 17, 18; the rectangle 8, 45, 43, 42 to the rectangle 2, 17, 19, 20: and so on.

All the upper bars are in compression, and all the lower ones are in tension.

The diagonals and contra-diagonals are all in compression; finally two of the verticals **23**, **39** are in tension, and all the rest in compression.

**44.** Figure 9*a** represents one half of a locomotive shed. The external forces are the weights **1**, **2**, **3**, **4**, **5** applied at

Fig. 9*a*.

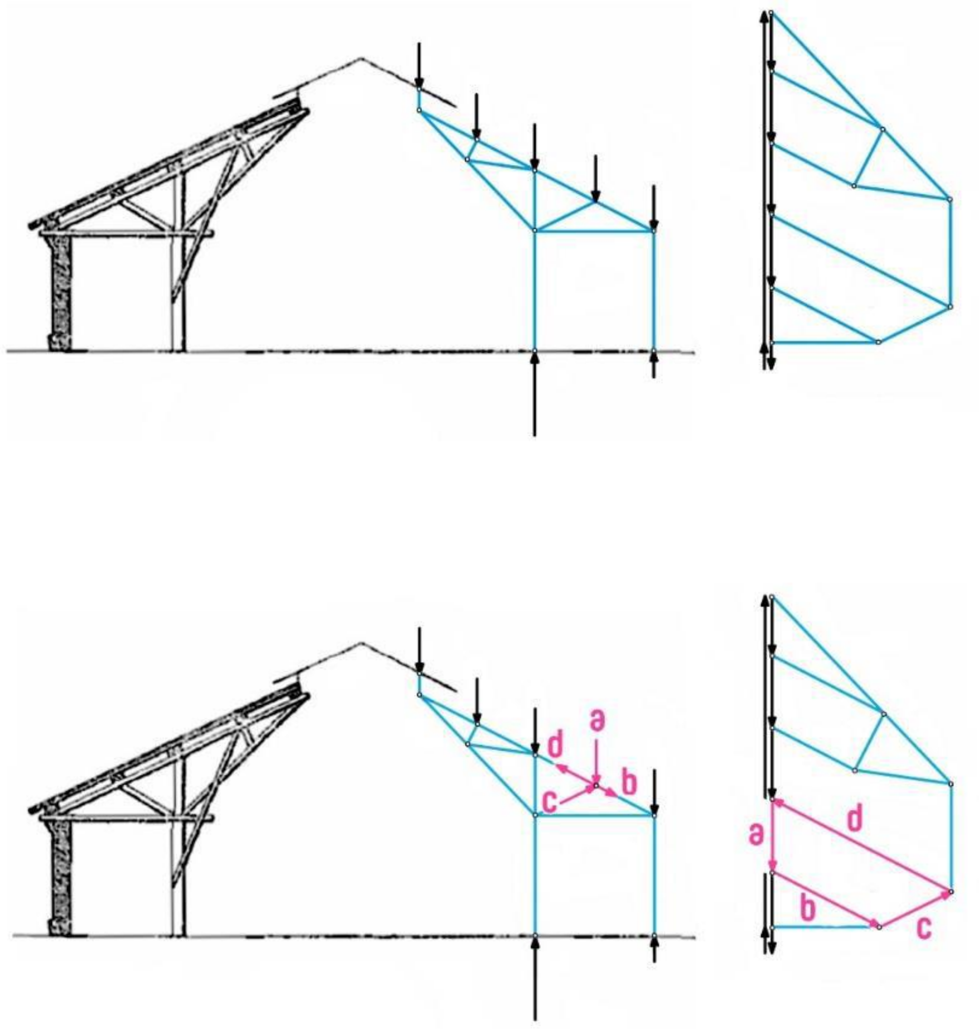
the upper joints of the frame, and the reactions **6** and **7** of the wall and column. Again, all the external forces are parallel, and consequently the polygon of forces reduces, in diagram *b*, to one straight line. The force **6** (taken in the opposite sense to that in which it really acts) is equal to a certain part of the weight **5**; by adding the difference to the other weights we get the magnitude of the force **7**.

In the diagram *b* the direction of the lines 8 and 13 coincide; the first is a part of the second. Here then we have for the polygon corresponding to the joint (**8**, **10**, **12**, **13**) one of those degenerate forms about which we spoke in Art. 33; the polygon is in fact a quadrilateral 8, 10, 12, 13, having three of its vertices (13, 8), (8, 10), (12, 13) in one straight line.

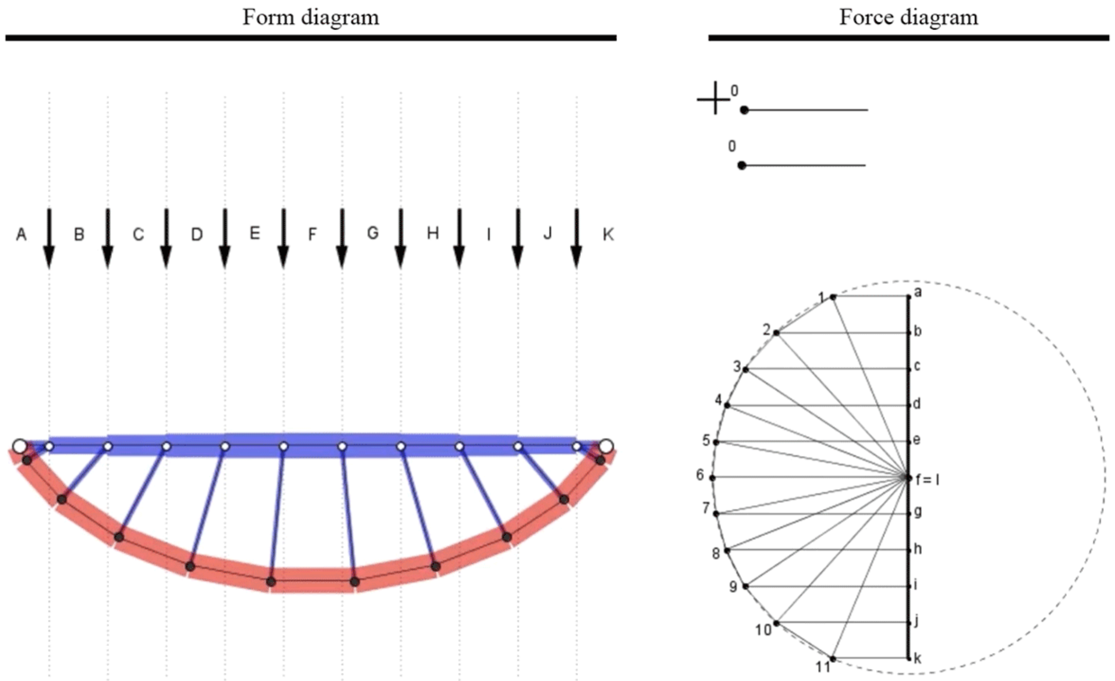
Fig. 9*b*.

The polygon 5, 17, 18, 6, corresponding to the point where

* This example is taken from Pl. xix. of the atlas of *Graphische Statik* of CULMANN, 1st edition. As previously stated, the two diagrams are not rigorously reciprocal.




Source: Cremona's Le figure reciproche nella statica grafica




Source: Block’s reasearh group

**Advantages:**

- insightful into the relation between form and forces.
- Two-way approach
- Less demanding of mathematics knowledge

**Disadvantage:**

- Limited applications

# How to design hyperstatic rigid frames insightfully with graphic statics ?

## RESEARCH GOAL:

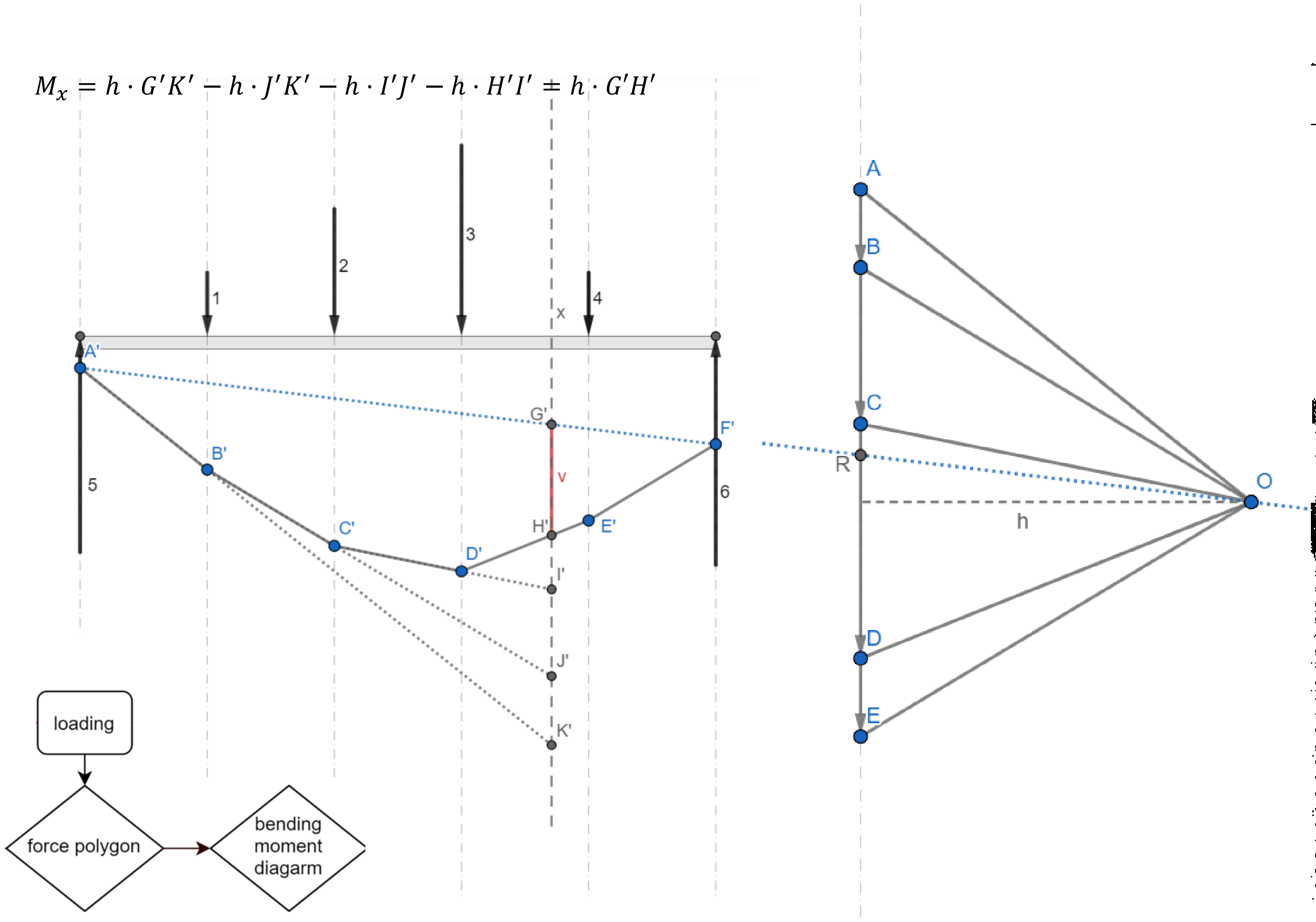
- A geometric-based method for exploring form of hyperstatic rigid frames perceptively for early stage of design
- From a guess-and-check approach to perceive-and-explore approach

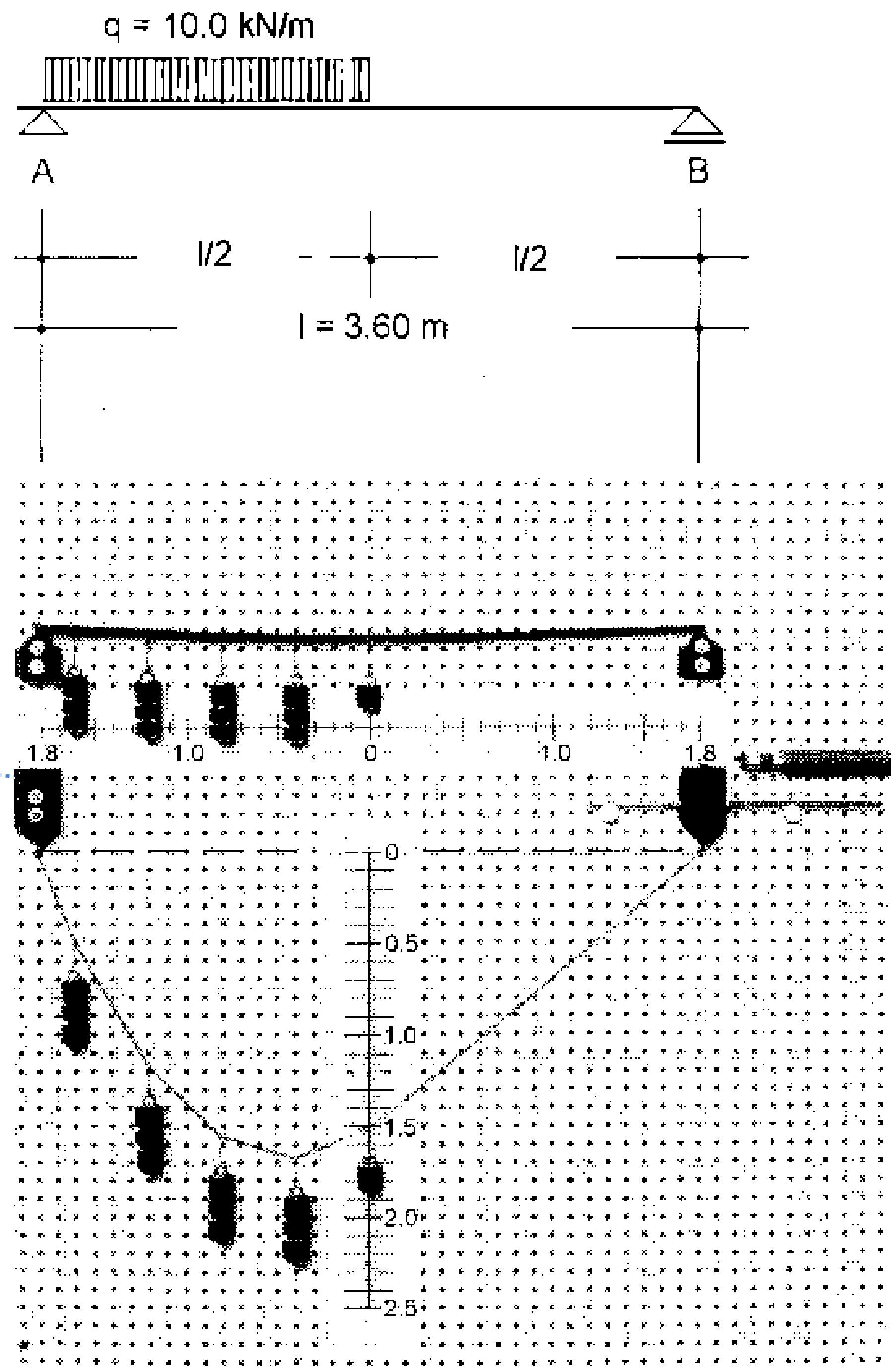
## REQUIREMENTS

- Insightful into the relation between form and forces
- Capable of guiding form exploration
- Two-way control: modify both form and forces
- Readily grasped by architects as well as engineers

# The analysis of the bending moment of a beam with graphic statics

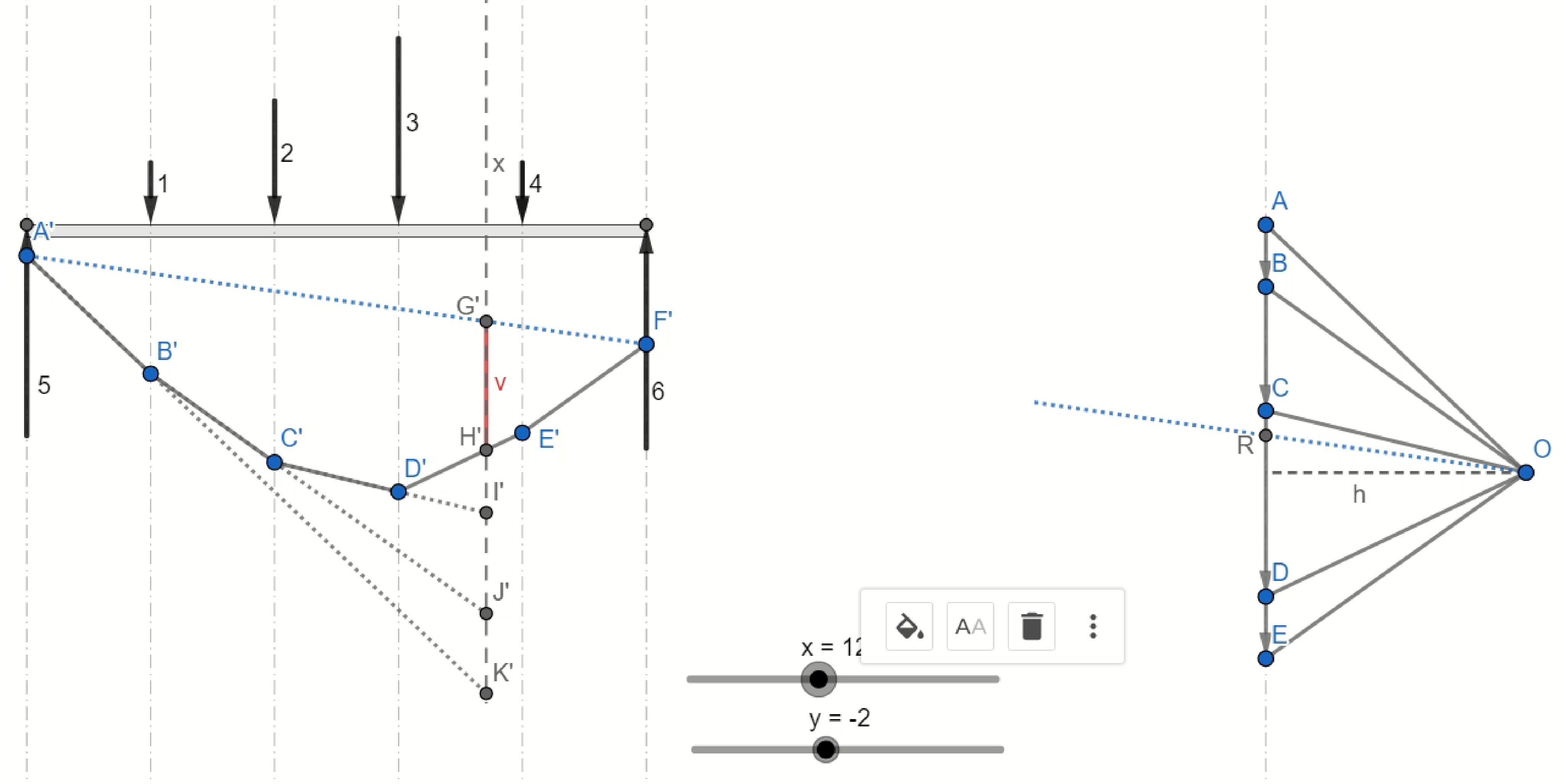
$$M_x = h \cdot G'K' - h \cdot J'K' - h \cdot I'J' - h \cdot H'I' = h \cdot G'H'$$




First funicular polygon/bending moment diagram

The analysis of the bending moment of a statically determinate beam

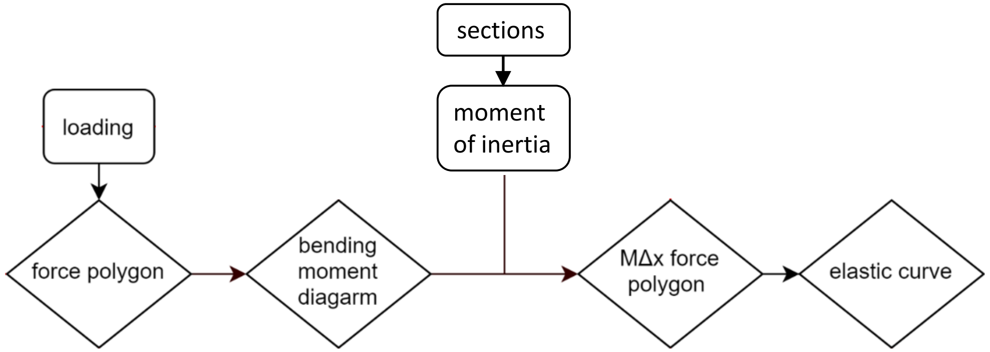


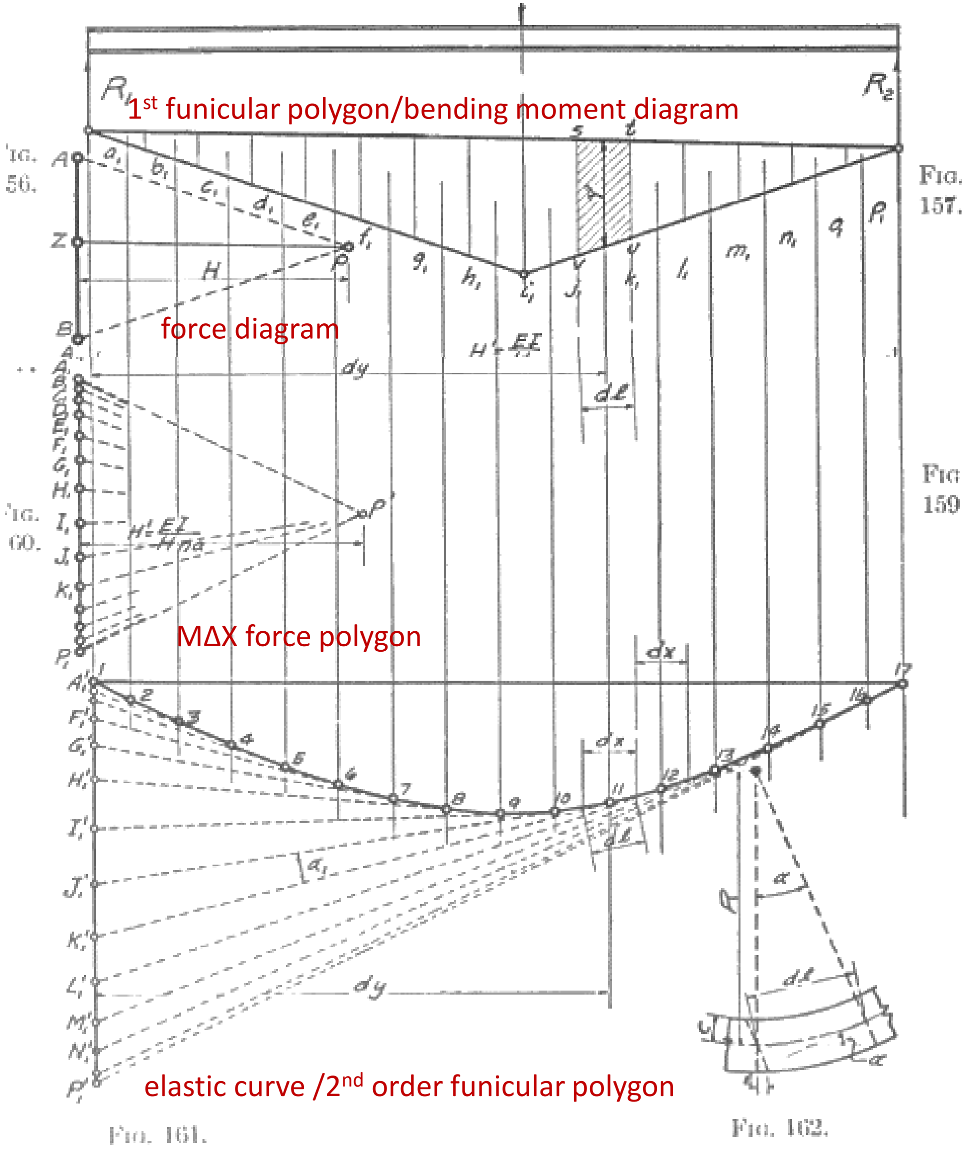

The position of the pole won’t affect the value of bending moment

"The deflection curve could (also) be represented by a funicular polygon analogically taking the bending moment as the "loading" instead of loads."[Mohr, 1868]

$$M(x) = \iint q(x)\, d^2x$$

$$y(x) = \frac{1}{EI} \iint M(x)\, d^2x \qquad h = \frac{EI}{n \cdot H}$$



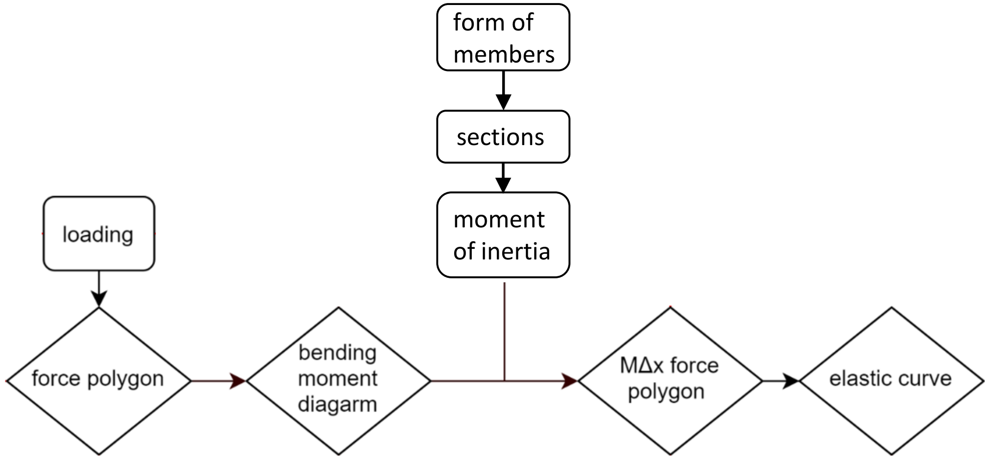


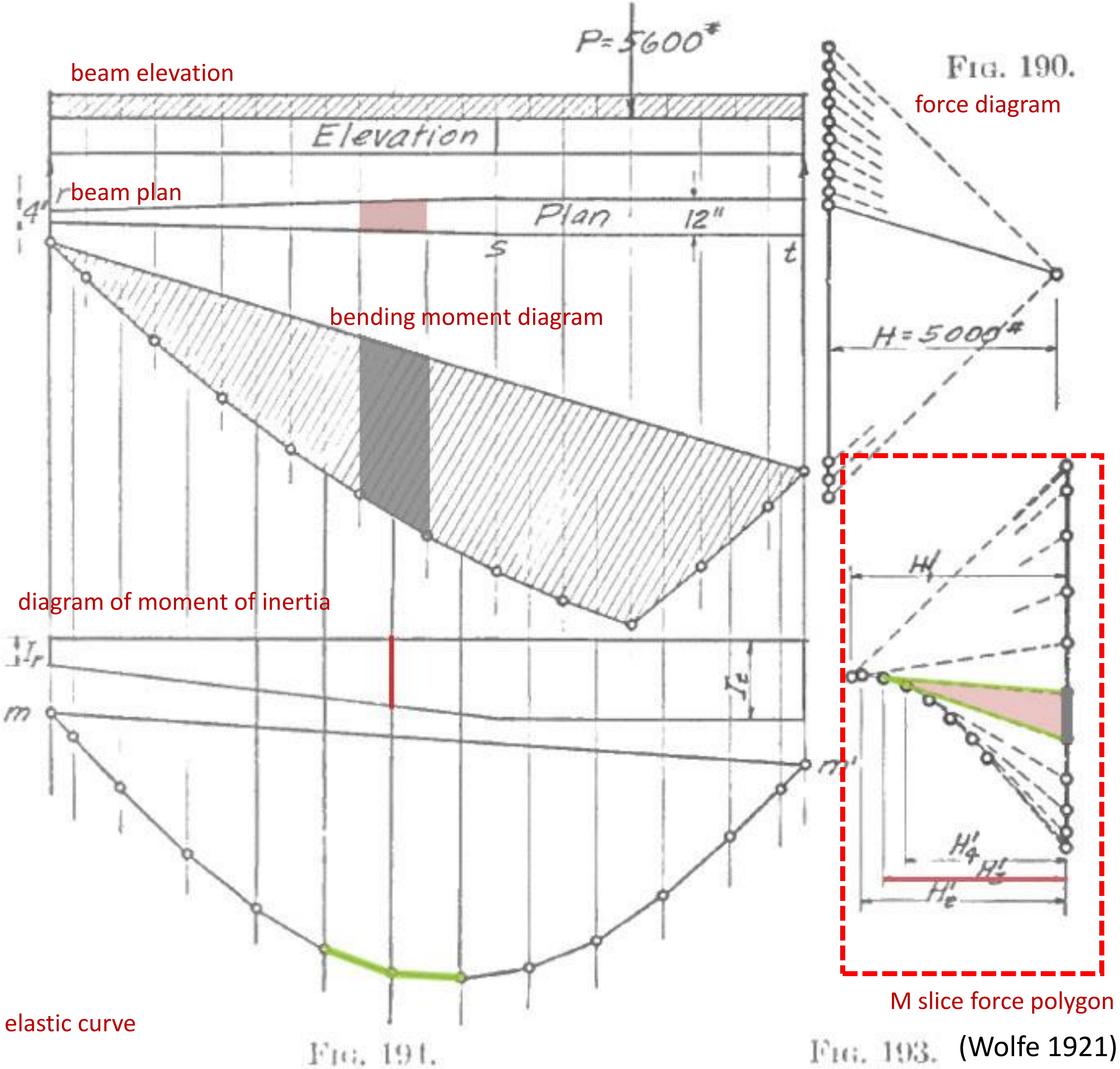

(Wolfe 1921)

## Beams with varying **moment of inertia**

a) the pole distance for each segment in proportional to its corresponding inertia and equals $\frac{EI}{n \cdot H}$ ,

b) Each ray intersects the previous one or its extension on the pole.

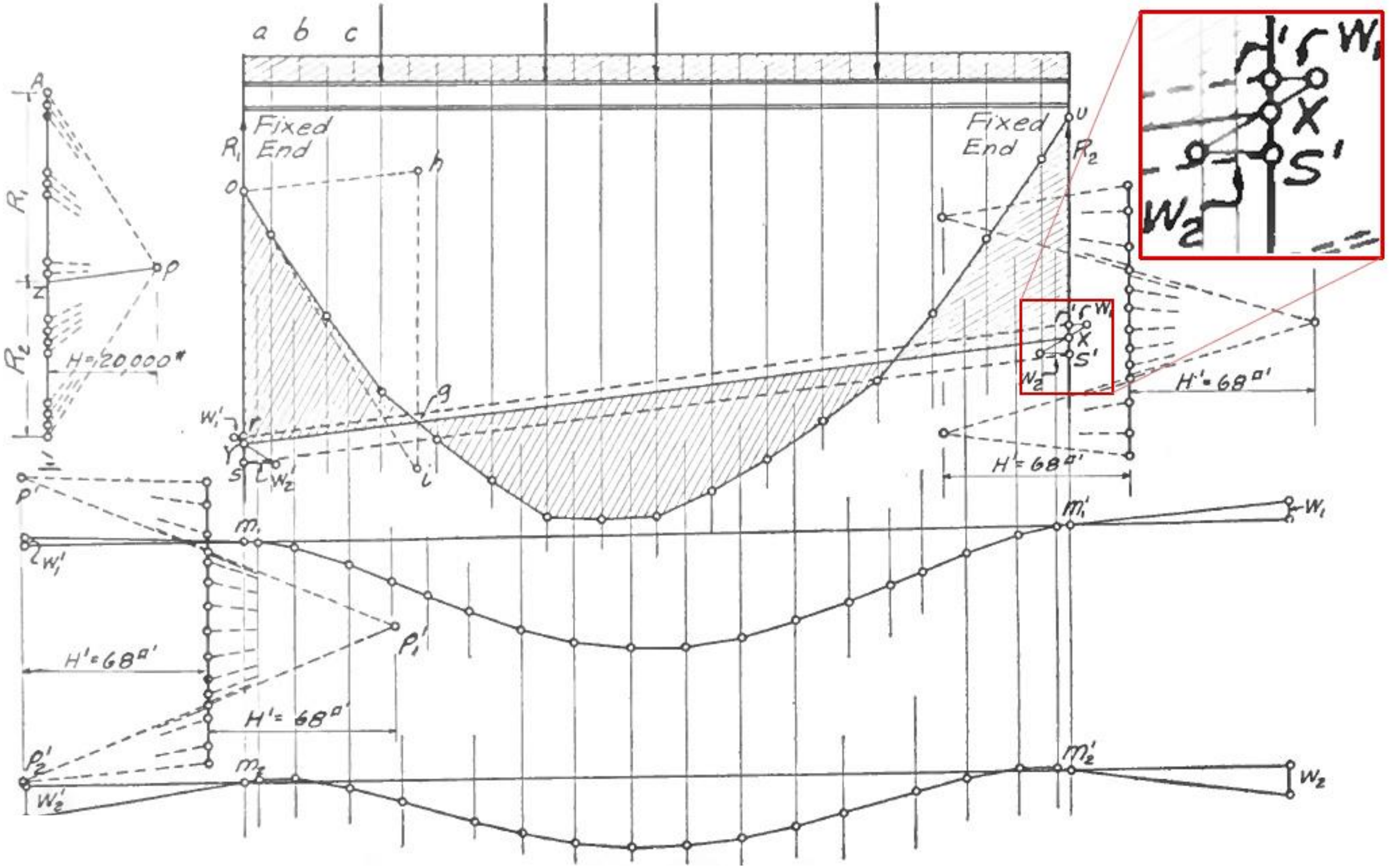


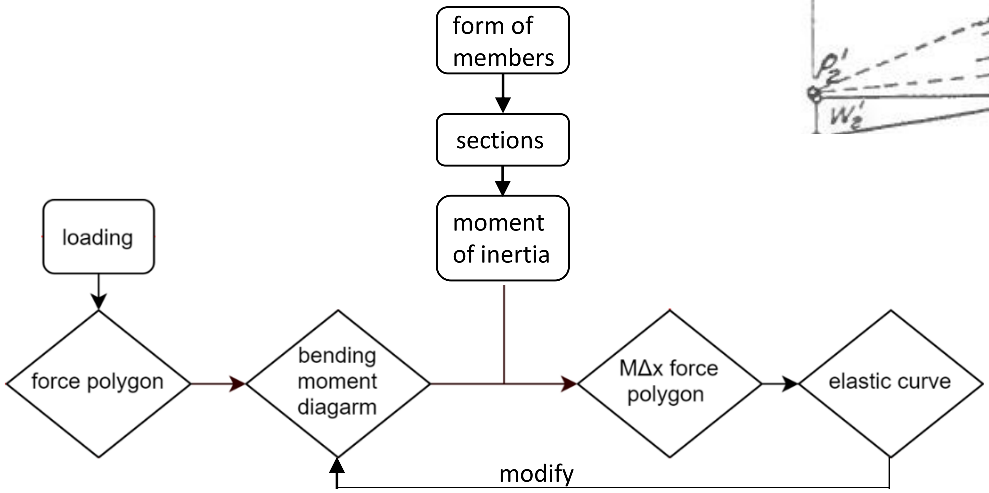



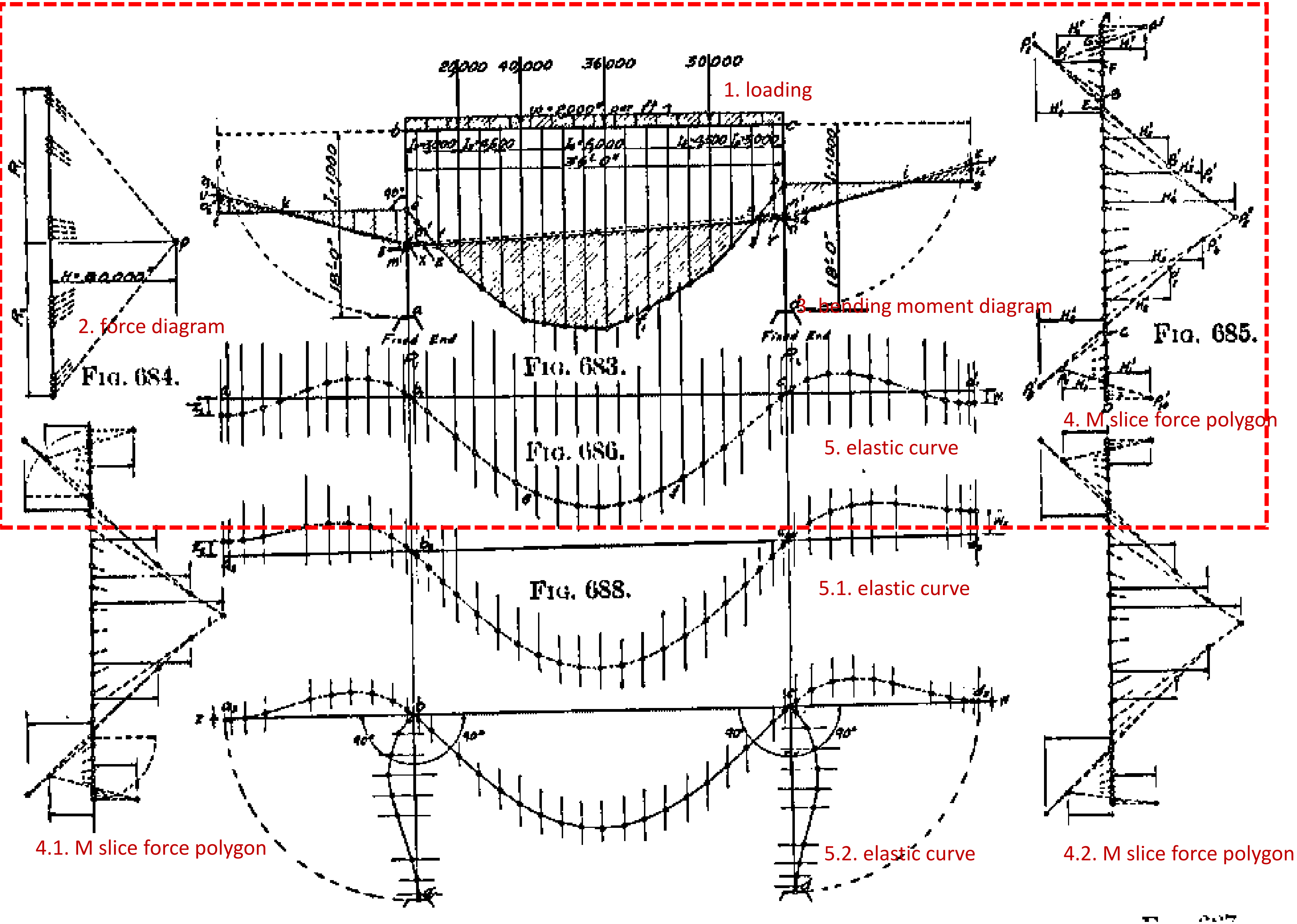

(Wolfe 1921)

Key techniques for hyperstatic beams:

1. Using trial bending moment diagrams for hyperstaticity
2. Using elastic curves to check and adjust



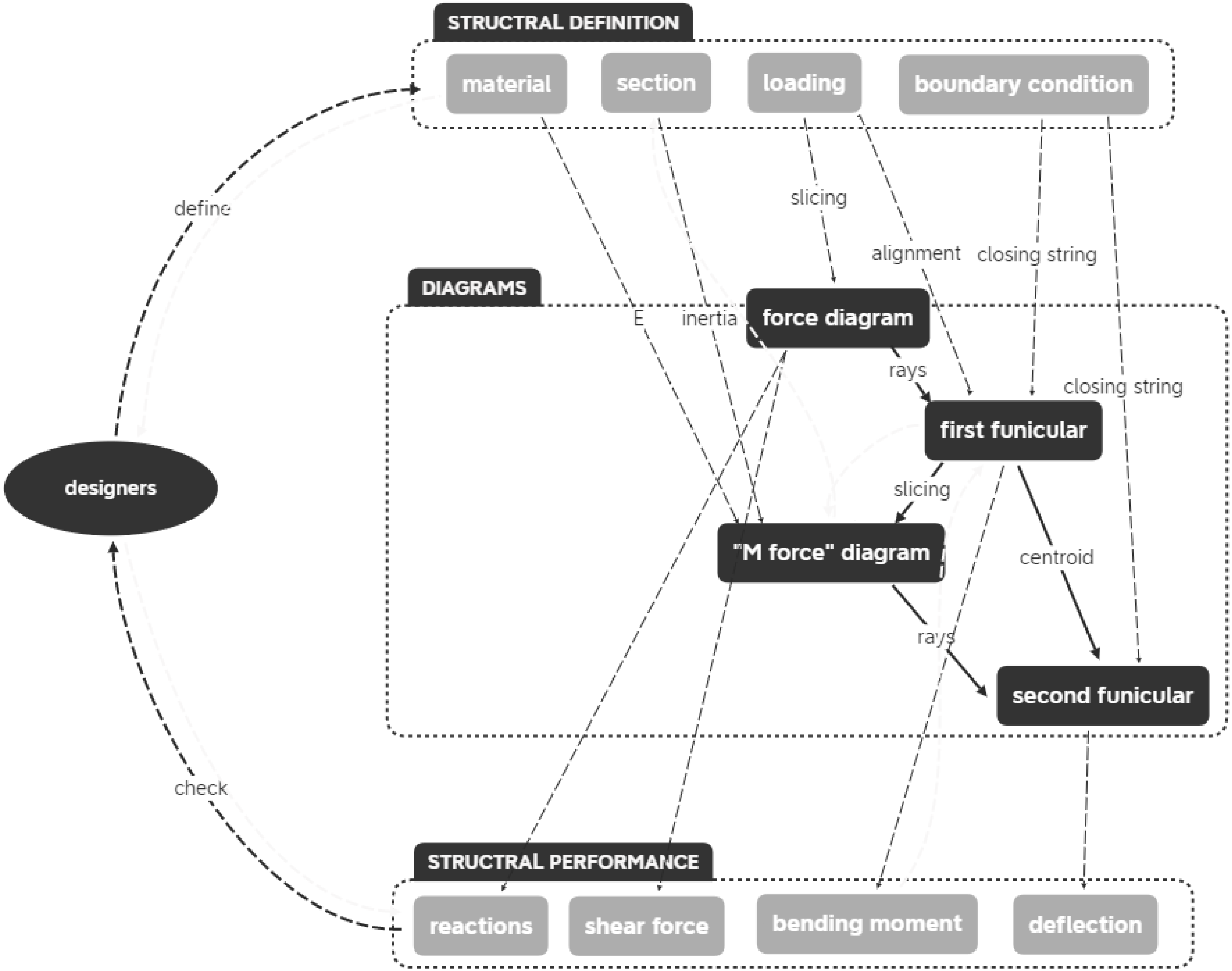






## Key techniques for hyperstatic rigid frames:

1. Frames are rotated into continuous beams
2. Using trial bending moment diagrams for hyperstaticity
3. Using elastic curves to check and adjust

(Wolfe 1921)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second funicular
check
STRUCTRAL PERFORMANCE
reactions
shear force
bending moment
deflection

**CURRENT LIMITATIONS:**

A. Still only an analysis method

B. Slow and cumbersome with hand-drawing

C .Difficult to modify

D. The relation between Moment and Inertia still not lucid for hyperstatic frames

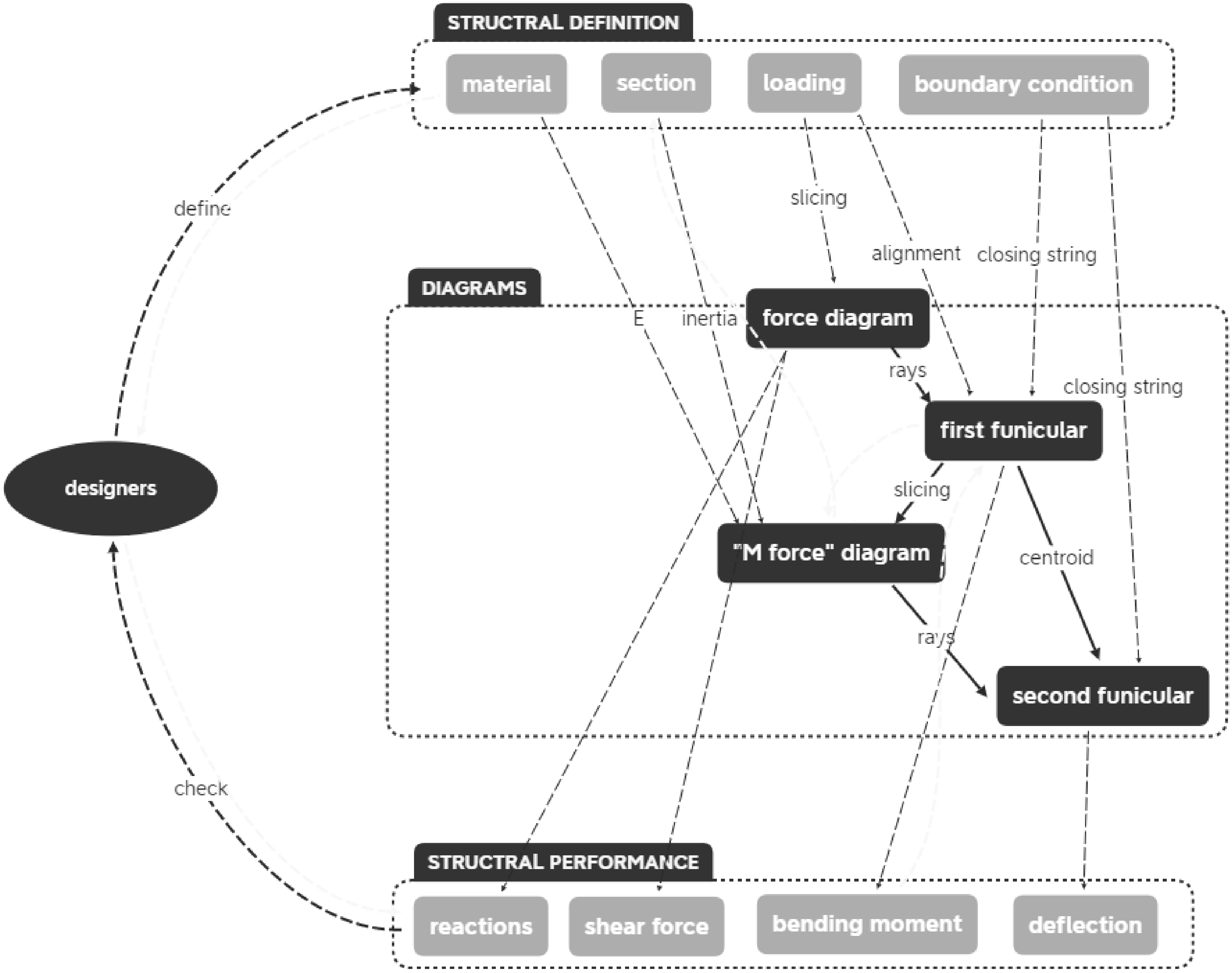
E. Limited to orthogonal portal frames under vertical loading

**TORWARDS AN INSIGHTFUL DESIGN TOOL:**

ABC. Combining with parameterization

D. Combining virtual displacement method for hyperstatic frames

E1.Frames under horizontal loading
E2.The portal frames with oblique members
E3. Multi-story and multi-span frames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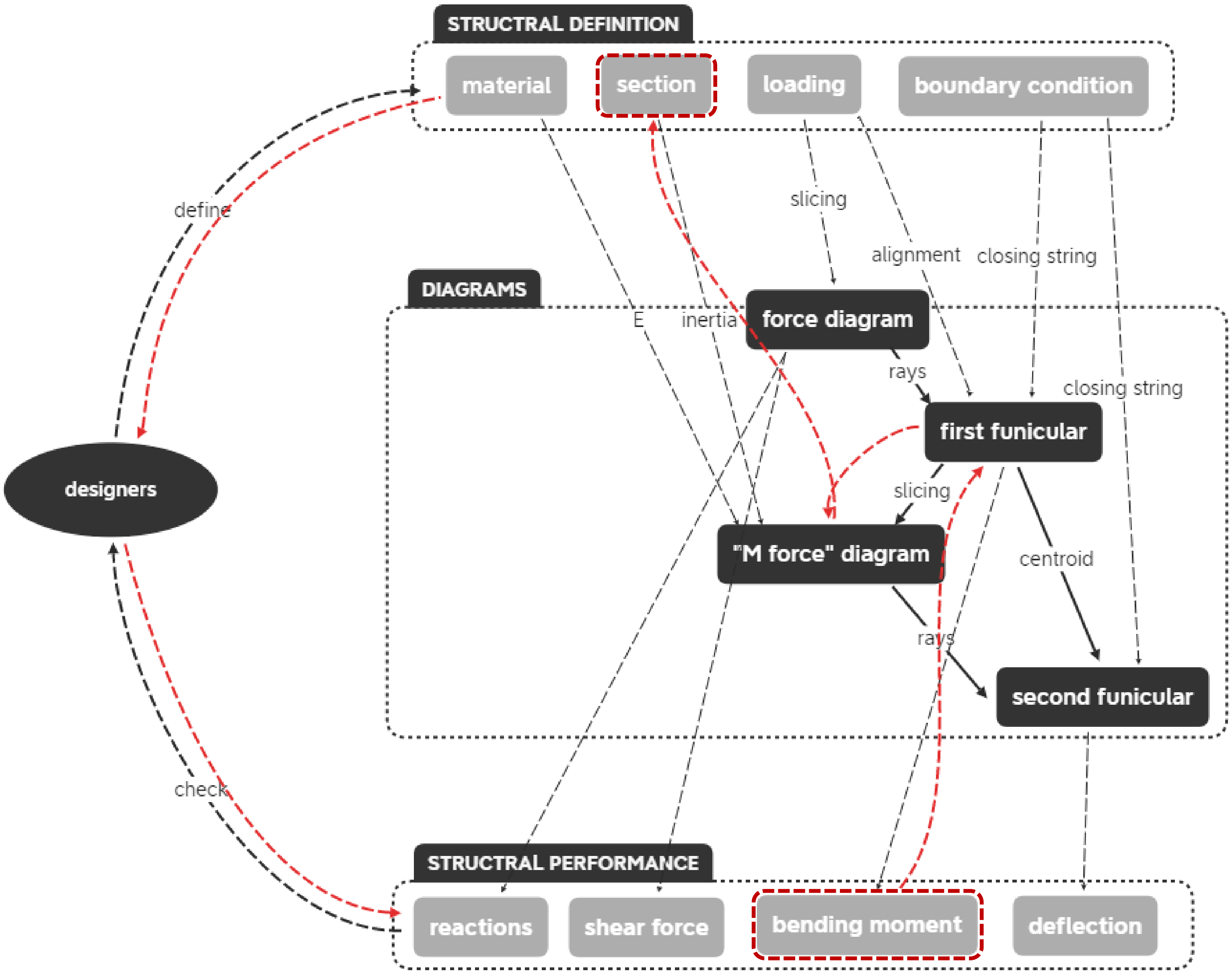

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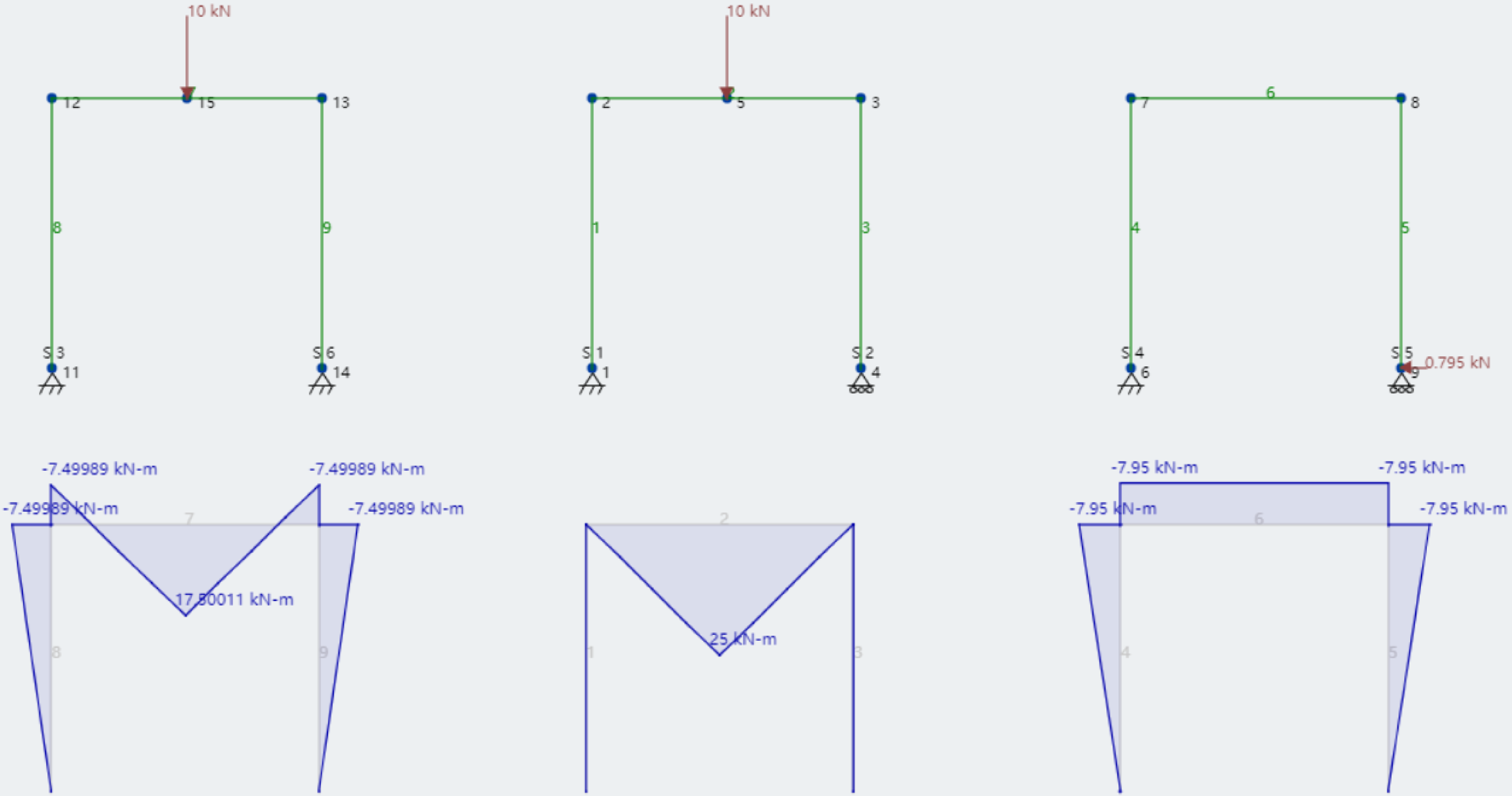
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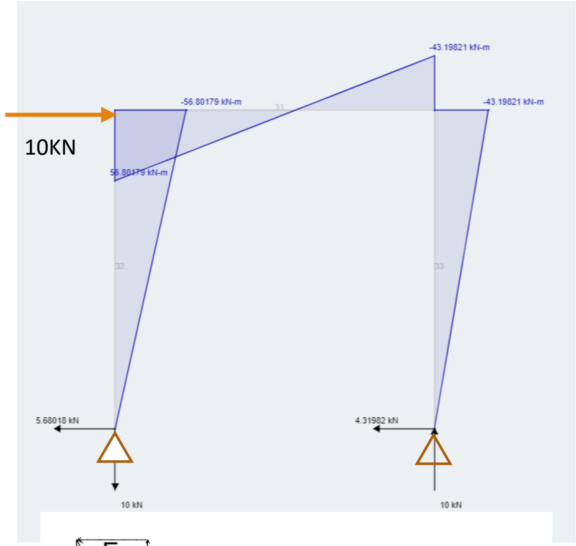
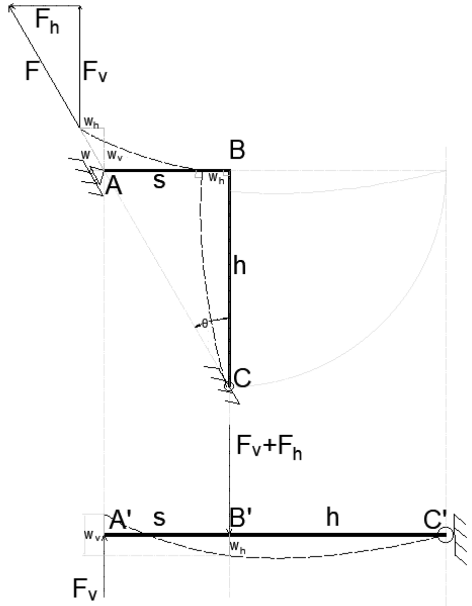
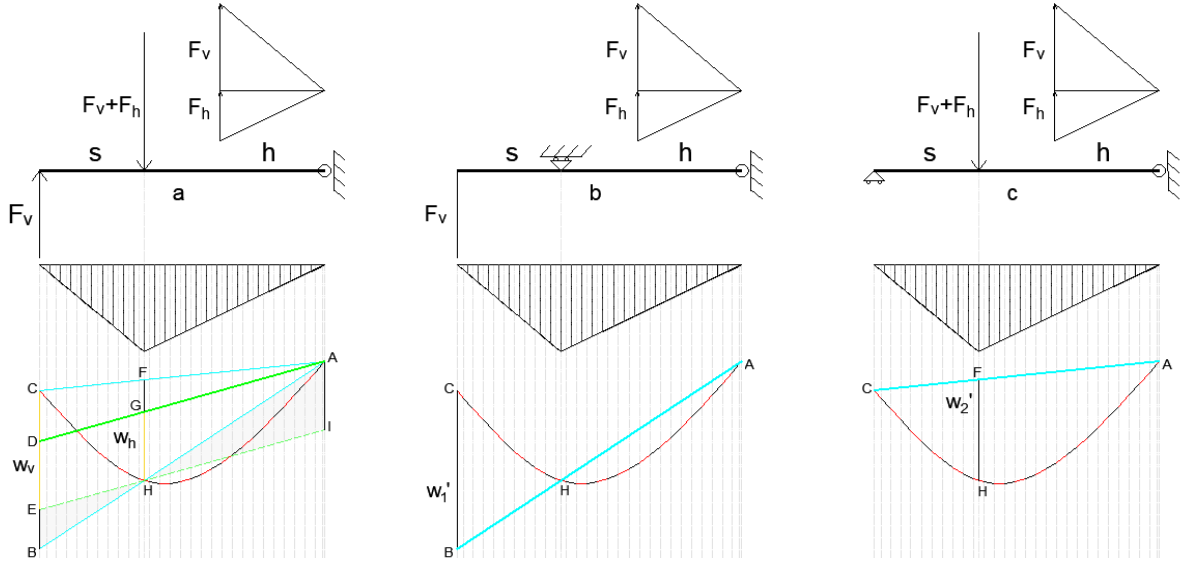
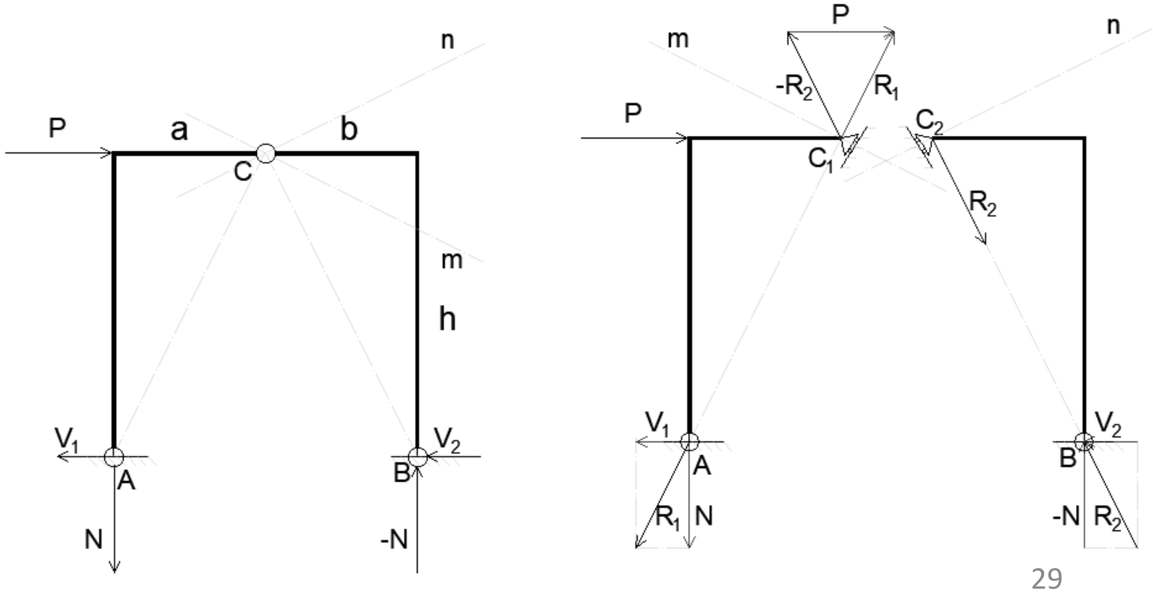
# E1. Portal frames under horizontal loading

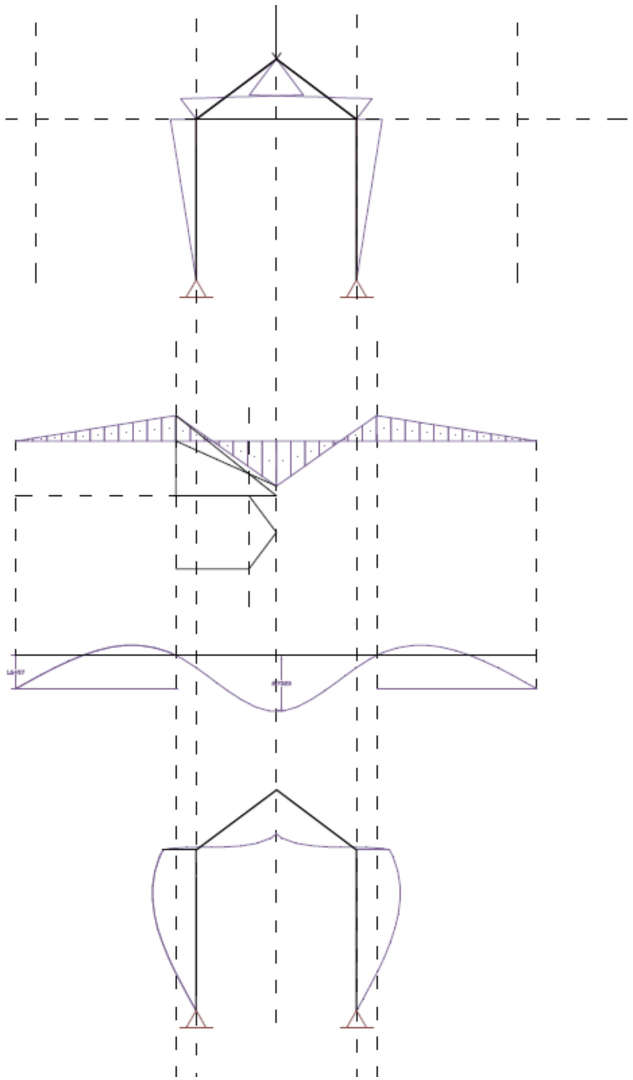


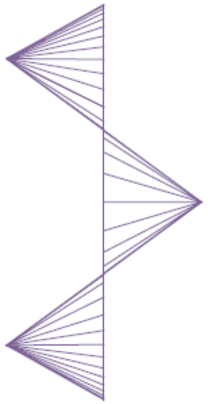



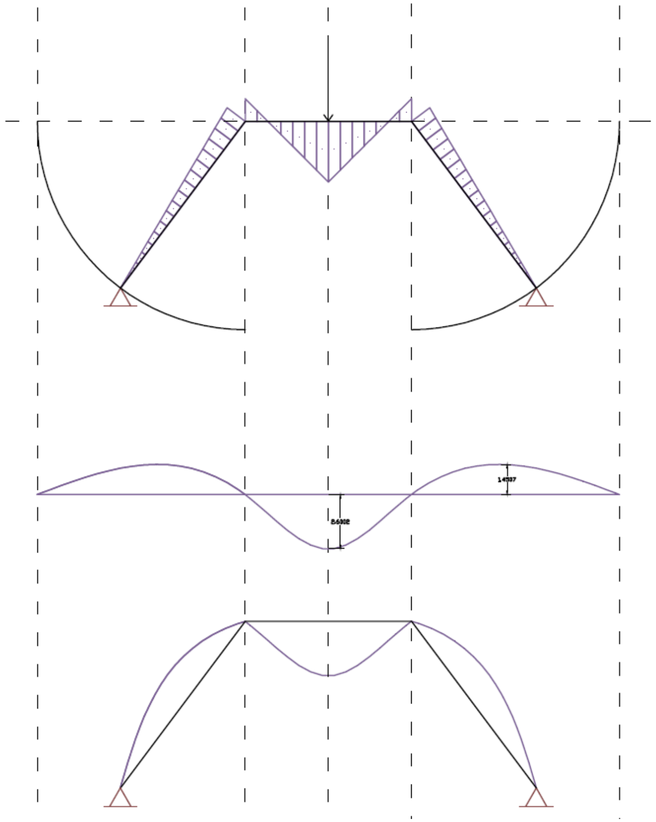



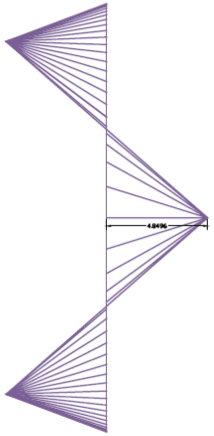



# E2. Portal frames with oblique members

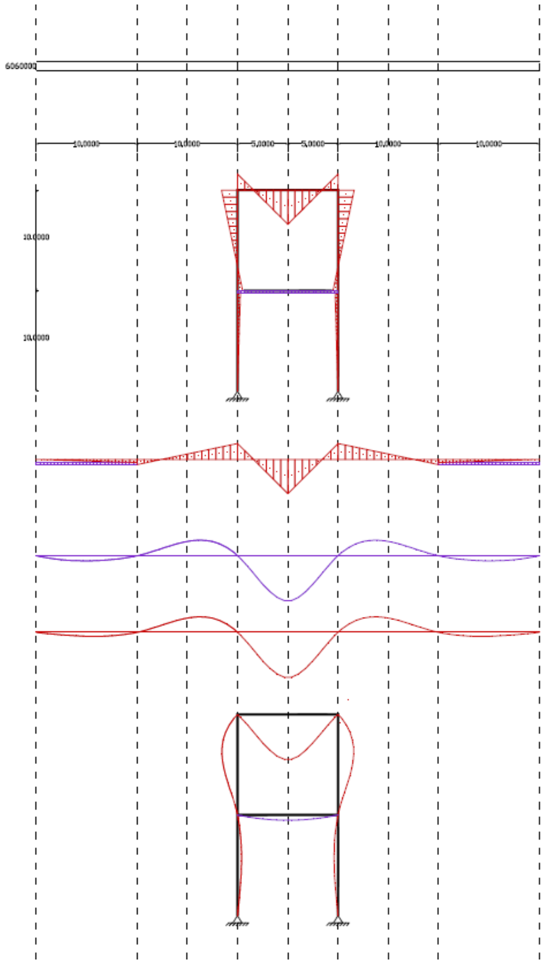


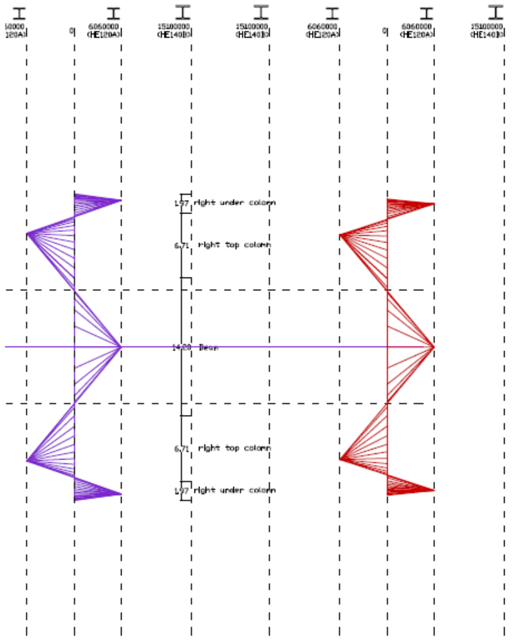


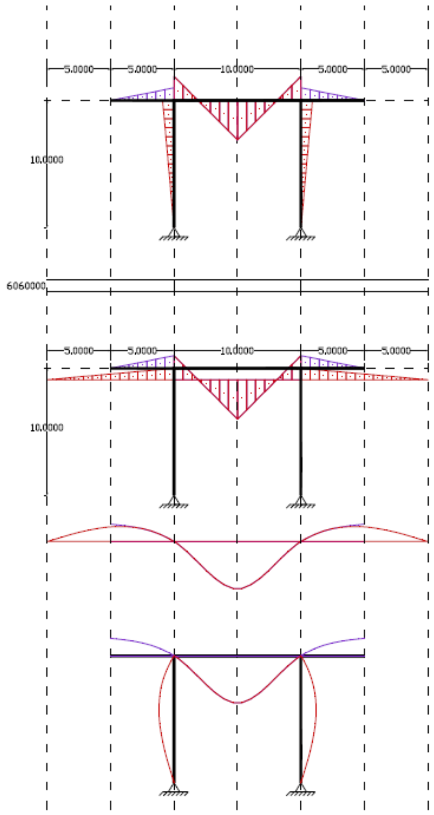


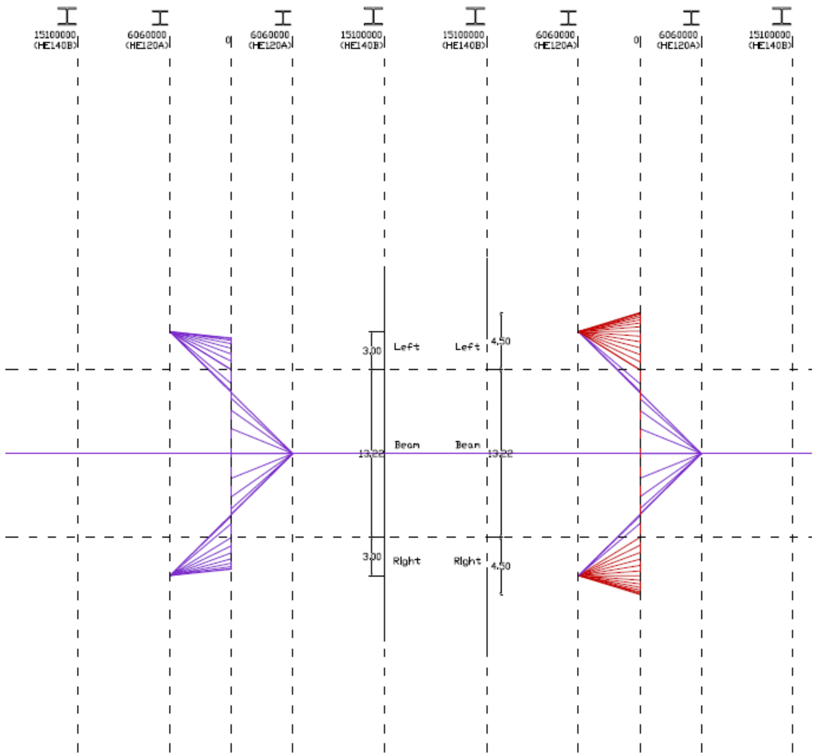


# E3. Multi-story and multi-span frames









## POTENTIAL APPLICATION:

- Early stage of architectural structure design
- Engineering analysis
- Facilitating engineering teaching and learning

# DISCUSSION

**Advantages**

- Insightful
- Chronology-free
- Less demanding for mathematics
- Educational

**Limitation**

- Difficulty with high degree of indeterminacy
- Interactivity is still limited
- Still quite theoretical (material)

THANK YOU !