Technology adoption, capital maintenance and the technological gap[*]

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November 21, 2001

Abstract

In this paper, we study the properties of optimal growth models à la Nelson and Phelps (1966) where the labor resources of an economy can be allocated freely either to production, technology adoption or capital maintenance. We first characterize the balanced growth paths of a benchmark model without maintenance services. Then we introduce the maintenance activity via the depreciation rate of capital. We characterize the optimal allocation of labor across the three activities. We prove that when technological shocks occur, equilibrium maintenance and adoption operate in opposite directions. The main prediction of the model is that though capital maintenance deepens the technological gap by diverting labor resources from adoption, it generally increases the long run output level at equilibrium.

Keywords: Technology adoption, Capital maintenance, Technological gap, Long run output level

Journal of Economic Literature: E22, E32, O40.

[*]The authors acknowledge the support of the Belgian research programmes "Poles d'Attraction inter-universitaires" PAI P4/01, and "Action de Recherches Concertée" 99/04-235.

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1 Introduction

Several contributions have stressed in the recent years the role of technology adoption costs in the growth process. While particular importance has been given to the study of technological innovation processes, the considerable weight of adoption costs (up-to twenty times higher than those for innovation in GDP, see Jovanovic, 1997) has recently re-directed interest in the profession towards the study of problems of diffusion and adoption of new technologies. There is a very extensive literature of adoption, stressing mostly the specific vintage physical and human capital required for implementing innovations (see among others, Parente, 1994, Benhabib and Spiegel, 1994, and Parente and Prescott, 1997). Basically, implementing a new technology requires capital and (skilled) labor resources. By diverting those resources from the productive sectors, the implementation costs are a real burden for the economies, which may either produce a slump in GDP and productivity growth (Greenwood and Yorukoglu, 1997), or even cause the economies to get into a poverty trap (Dimaria and Le Van, 2000). The problem of costly adoption is obviously much more crucial for developing countries which typically lack capital and skills. As a consequence, most developing countries have kept on investing in dominated technologies for long periods of time, which has induced a steady rise in the technological gap with respect to the richest countries. This trend is even sharper after the debt crisis of the early eighties.

As technology adoption is costly, inducing a significant delay in the implementation of technological improvements, the maintenance of the technologies in use and the associated stock of capital is a key issue. Surprisingly, the recent macroeconomic literature has almost disregarded it.[1] Among the very few exceptions dealing with maintenance in the macroeconomy, one can enumerate McGrattan and Schmitz (1999), Licandro and Puch (2000) and Collard and Kollintzas (2001).[2] All these papers are mostly concerned with the cyclical properties of maintenance and its implications for the business cycle. In particular, the connection between the adoption and maintenance decisions is not studied. This omission is certainly not motivated by quantitative

[1] Not so in the seventies with the notable contributions of Feldstein and Rothschild (1974), and Nickell (1975).

[2] Another contribution dealing with investment and capital maintenance is due to Boucekkine and Ruiz-Tamarit (2001), but it takes a deliberate microeconomic approach.

considerations. Actually, maintenance and adoption costs are comparable in terms of GDP. While the former are estimated to be around 6% of GDP (see McGrattan and Schmitz, 1999, in the Canadian case), the latter typically amount to about 10% in developed countries as reported in Jovanovic (1997). There are further reasons to believe that adoption and maintenance decisions are indeed connected, and should be treated as such. An obvious reason is that many firms have in mind the maintenance implications of their adoption decisions. A firm can disregard the adoption of a new technology if it anticipates a costly pace of maintenance costs both in physical and human capital. This is even more crucial when technological advances are embodied in capital.

There is another reason to treat maintenance and adoption within the same framework. Actually, just like adoption, maintenance diverts resources from the other sectors and activities, it consequently "competes" with adoption in this respect. Maintenance activities do require human and physical capital just like adoption, though one can reasonably think that adoption is more intensive in human capital. The labor opportunity cost of maintenance is stressed in Tiffen and Mortimore (1994) who studied the role of capital maintenance and technology adoption in the growth recovery of Kenya. Maintenance expenditures are indeed included in social cost-benefit analysis in this country case but "...it is not always realized that labor on maintenance may have a rising opportunity cost ...this requires a wise management of recurrent maintenance costs if the benefits of investment are to be lasting". In this paper, we study the outcomes of optimal growth models à la Nelson and Phelps (1966) where the labor resources of an economy can be allocated freely either to production, adoption or maintenance. Technological progress is disembodied, the central planner has to decide how much labor has to be devoted to increasing the stock of knowledge of the economy (adoption) and how much labor should be assigned to the maintenance of capital.

Labor on maintenance decreases the depreciation rate of capital as in McGrattan and Schmitz (1999). A fraction of labor resources is devoted to adopt the innovations coming from abroad, from a leading country for example. There is no R&D activity and labor resources are fixed. This is likely to generate a never lasting technological gap as in the original Nelson and Phelps' contribution. This configuration is not only suitable for the study of developing economies. It is very well-known for example that the largest part of R&D devoted to the information technologies is undertaken in the USA, and that in this specific field, the European countries are "adopters" rather than innovators. In such a specific case, there exists a technological gap between the USA and Europe, a persistent technological gap indeed given the increasing difference in R&D expenses between the two continents in this field. This gap can derive from a deliberate strategy, adoption may be preferred to R&D. This paper adds another policy option, and addresses a complementary issue: Could it be a case where capital maintenance is preferred to adoption? What is the optimal allocation of human capital resources across activities? If presumably the maintenance activity diverts labor from adoption and is therefore likely to deepen the technological gap, does it in counterpart rise the (detrended) level of income as it reduces the negative impact of capital depreciation? How do maintenance and adoption decisions shift under exogenous changes in the pace of technological progress, and how does this affect the economy? All these questions will be tackled along this paper. Since the short run dynamics of the considered models do not add much to the results obtained in the balanced growth paths, we restrict our analysis to the latter paths.

The paper is organized as follows. The next section is devoted to briefly present and characterize the steady state equilibrium of a benchmark model of adoption a la Nelson and Phelps (1966) without maintenance. Section 3 incorporates maintenance in the benchmark model and examines how this affects the properties of the model in the steady state. Section 4 concludes.

2 The benchmark model

We consider an economy which comprises a continuum of infinite lived agents, indexed from 0 to 1. All individuals share the same preferences that are characterized by the lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t U(C_t);$$

where $0 < \beta < 1$ is a constant discount factor and C_t is consumption in period t. There is not desutility of labor and labor supply is exogenous and equal to one. We assume that labor is homogeneous, but it can be devoted either to production (L) or to adoption of new technologies (u), so that

$$1 = L_t + u_t.$$

The adoption of technologies is therefore costly since it requires a fraction of total labor supply. We consider the case of a small country with no research activity. A_t represents the state of technology in use in this country at date t. A^0_t represents the state of knowledge at time t, that is the best technological level achievable, which could be interpreted as the technological level of the rest of the world. A^0_t grows exogenously at a rate γ. The more labor is devoted to adoption, the smaller the technological gap between the small country and the rest of the world is. This modeling resembles Nelson and Phelps (1966). More precisely, we set the following adoption equation:

$$\begin{aligned} A_t &= A_{t-1} + d_t u_t^{\mu}\left[A^0_{t-1} - A_{t-1}\right] \\ 0 &< \mu < 1. \end{aligned}$$

A rise in the technology in use level, $A_t > A_{t-1}$, reflects either an increase in the labor fraction devoted to adoption, or an upward shift in the state of knowledge, A^0_t, or an exogenous shift in the productivity of this activity, d_t. An increase in the latter variable may for example reflect an exogenous improvement in the skills on the labor force. Note that adoption has decreasing returns to labor, ie. is concave with respect to u. Doubling the labor fraction devoted to adoption will increase the technology in practice level by a factor strictly lower than two. This assumption mimics the hypothesis usually done in the R&D literature according to which there exist decreasing returns to the research effort (for example, see Jones, 1995, or Caballero and Jaffe, 1993). We assume that just like research, technology adoption is subject to a crowding effect which mainly reflects redundancy in the adoption effort.

Finally the economy produces a final good with a Cobb-Douglas technology using capital and labor, with α the capital share. The technological progress, represented by the stock of knowledge A_t, is disembodied. Moreover, we hypothesize an implementation delay of one period, in other words it takes one period to incorporate the technological improvements in the productive sector:

$$Y_t = A_{t-1} K^{\alpha}_{t-1} L_t^{1-\alpha}.$$

2.1 The central planner problem

The fundamental decision to be taken by a central planner in such an economy is very simple: For a given stock of capital K_{-1}, and stock of knowledge A_{-1}, how much labor has to be devoted to increase the stock of knowledge and how much has to go to production, for the welfare of the economy to be maximized?

$$Max_{\{K_t; A_t; u_t; L_t; Y_t; C_t; I_t\}} \sum_{t=0} \beta^t U(C_t)$$

subject to

$$Y_t = A_{t-1} K_{t-1}^{\alpha} L_t^{1-\alpha}; \tag{1}$$

$$K_t = [1 - \delta] K_{t-1} + I_t; \tag{2}$$

$$A_t = A_{t-1} + d_t u_t^{\mu} \left[A^0_{t-1} - A_{t-1} \right]; \tag{3}$$

$$1 = L_t + u_t; \tag{4}$$

$$Y_t = C_t + I_t; \tag{5}$$

given A_{-1} and K_{-1} and the corresponding positivity conditions (notably $0 \leq u_t \leq 1$). δ is the capital depreciation rate and I_t is gross investment. The interior solution of this optimization problem is characterized by the following first order conditions:

$$U'(C_t) = \beta U'(C_{t+1}) \left[\alpha A_t K_t^{\alpha-1} L_{t+1}^{1-\alpha} + [1 - \delta] \right]; \tag{6}$$

$$\beta K_t^{\alpha} L_{t+1}^{1-\alpha} U'(C_{t+1}) = \lambda_t - \beta \lambda_{t+1} \left[1 - d_{t+1} u_{t+1}^{\mu} \right]; \tag{7}$$

$$(1 - \alpha) A_{t-1} K_{t-1}^{\alpha} L_t^{-\alpha} U'(C_t) = \omega_t; \tag{8}$$

$$\lambda_t d_t \mu u_t^{\mu-1} \left[A^0_{t-1} - A_{t-1} \right] = \omega_t; \tag{9}$$

where ω and λ are the multipliers associated with the labor market clearing condition and with the law of accumulation of knowledge respectively. ω_t can be interpreted as the shadow wage at t, and λ_t as the shadow price of knowledge at this date. Equation (6) is the standard Euler equation obtained from Ramsey growth models. Equation (7) provides the optimal rule for knowledge accumulation. The marginal productivity of knowledge (evaluated in terms of the marginal utility at $t+1$ because of the one-period implementation delay) should be equal to its shadow price at t, minus the potential gain in the value of knowledge from t to $t+1$. Equations (8)-(9) are the optimality conditions with respect to the labor variables. Since labor is homogeneous, the marginal productivity of labor devoted either to production or to adoption should be equal to the shadow wage.

An important issue that can be tackled very easily within our framework (à la Nelson and Phelps, 1966) concerns the behaviour of the technological gap over time and across countries (see also Benhabib and Spiegel, 1994). Indeed, the technological gap, TG_t, at t is by definition $\frac{A'_{t-1} - A_{t-1}}{A_{t-1}}$, which by equation (3) implies:

$$TG_t = \frac{1}{d_t u_t^{\mu}} \left(\frac{A_t}{A_{t-1}} - 1 \right):$$

The technological gap depends on the adoption effort u_t and on the exogenous productivity variable d_t. Together with the adoption equation (7), this simple expression permits an easy assessment of the dynamics of the technological gap and its long run properties. Note also that if the long run value of the adoption effort is nonzero, then the technological gap does not vanish in the the steady state, a remarkable property of Nelson and Phelps' adoption models which makes them most appropriate to study economic development problems. We now investigate the steady state properties of the dynamic system (1)-(9).

2.2 The balanced growth paths

We assume a logarithmic utility function. Along the balanced growth path u_t and L_t are constant, and the remaining variables grow at constant rates. Denoting by g_X the long-run growth factor of a variable X_t and $\overline{X}$ its long run level, we have the following properties:

Proposition 1 If A'_t growth at rate $\gamma > 1$, all the other variables growth at strictly positive rates with

$$g_A = \gamma$$

$$g_C = g_K = g_I = g_Y = g_A^{\frac{1}{1-\alpha}} = \gamma^{\frac{1}{1-\alpha}}$$

These properties are trivially checked by writing the restrictions among the different growth rates that the system (1)-(9) impose. To compute the long run levels, the same approach has to be followed. In order, to simplify a little bit, we use equation (9) to eliminate the multiplier λ. The resulting eight restrictions are:

$$
\begin{aligned}
\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} &= \alpha A K^{\alpha-1} L^{1-\alpha} + 1 - \delta \\
\frac{\beta K^{\alpha} L^{1-\alpha} \gamma^{\frac{-1}{1-\alpha}}}{C} &= \frac{\omega \left[\gamma - \beta (1 - d u^{\theta}) \right]}{d \theta u^{\theta-1} (1 - A)} \\
(1-\alpha) A K^{\alpha} L^{-\alpha} &= C \omega \gamma^{\frac{1}{1-\alpha}} \\
A(\gamma - 1) &= d u^{\theta} (1 - A) \\
K \gamma^{\frac{1}{1-\alpha}} &= (1-\delta) K + I \gamma^{\frac{1}{1-\alpha}} \\
Y &= C + I \\
L + u &= 1 \\
Y \gamma^{\frac{1}{1-\alpha}} &= A K^{\alpha} L^{1-\alpha}.
\end{aligned}
$$

It is then quite easy to prove that the previous stationary system has a solution if the growth rate of technological progress γ is big enough.

Proposition 2 If $\gamma > 1$:

(i) a unique stationary equilibrium exists for our economy,

(ii) the long run technological gap being $TG = \frac{\gamma - 1}{d u^{\theta}}$, we have the following comparative statics properties:

$$
\begin{aligned}
\frac{\partial u}{\partial d} < 0; \quad \frac{\partial TG}{\partial d} < 0 \\
\frac{\partial u}{\partial \gamma} > 0; \quad \frac{\partial TG}{\partial \gamma} > 0
\end{aligned}
$$

The proof is in the appendix. The comparative statics results are important to understand the basic mechanisms at work with our adoption specification. The same mechanisms will work in the extensions considered afterwards. First note that the productivity of the adoption activity increases, ie. d rises, the fraction of labor devoted to adoption goes down but the technological gap goes down too. Given the expression of the long run technological gap, this means that the product $d u^{\theta}$ rises when d goes up despite the reduction in the adoption effort. It is not hard to understand this outcome. Productivity improvements in adoption allow to increase the stock of knowledge even with a lower labor contribution to adoption. In such a case, more labor is assigned to production. If the increase in d compensates the decrease in u, ie. if $d\,u^{\mu}$ increases, so that the stock of knowledge keeps on rising, the economy gains a double advantage: More production and lower technological gap.

When γ increases, the technological gap is likely to rise sharply if the economy does not increase substantially its adoption effort. However if the adoption effort increment is too big, a very low fraction of labor will be left for production, and the consumption level and welfare of the economy will fall dramatically. There is a clear trade-off here. In our model, this trade-off is settled as follows: While the labor allocation to adoption will rise, it will not rise enough to offset the negative effect of the exogenous technological acceleration on the technological gap.

Let us introduce maintenance now.

3 Incorporating capital maintenance in the adoption model

We introduce maintenance of capital as a labor service. Labor can be devoted to a third activity, maintenance, and we denote by m. The clearing condition of the labor market becomes:

$$1 = L_t + u_t + m_t. \tag{10}$$

Maintenance services allow to reduce the physical depreciation of capital, as in McGrattan and Schmitz (1999), Licandro and Puch (2000) and Boucekkine and Ruiz-Tamarit (2001). In such a framework, capital evolves over time according to the following law of motion:

$$K_t = [1 - \delta(m_t)]K_{t-1} + I_t. \tag{11}$$

By choosing m, the firms or the central planner determine the depreciation rate $\delta(m)$. The depreciation rate should fulfill the following requirements (see for example, Boucekkine and Ruiz-Tamarit, 2001):

(i) $\delta(m) > 0$, $\delta'(m) < 0$ $\delta''(m) > 0$

(ii) $\lim_{m \to \infty} \delta(m) = \bar{\delta}$

Conditions (i) are the expected positivity, monotonicity and convexity requirements. Condition (ii) ensures that the economy can not go below a minimal value corresponding to the "natural" depreciation of capital, $\bar{\delta}$.

This is the unique deviation considered in this section with respect to the benchmark model. In particular, the adoption side of the model is unchanged, ie. equation 3) is not altered. This means that the adoption decision is only connected to the maintenance decision through the labor resources constraint (10). In certain cases, the pace of adoption can slow down because of the expected induced maintenance costs, as we mentioned in the introduction. This is especially the case when technological advances are embodied in capital goods. If technological progress is disembodied, the link between capital accumulation and the implementation of innovations is broken down, and one can perfectly disconnect the two decisions. In such a case, adoption and maintenance interact via labor resources competition. This is the approach followed in this paper. Technological progress is disembodied, the maintenance services affect the pace of capital accumulation and the adoption efforts shape the pace of technological progress within the country.

Both activities affect the production function, via the capital input for maintenance, and directly through total factor productivity A_t for adoption. Actually there is a third (indirect) effect on production coming from these two decisions: As u and /or m rise, there is less labor left for production. Hence, despite its simple structure, our model presents enough interaction channels to allow for a non-trivial discussion. Beside the derivation of the optimal allocation of resources across activities, a very interesting issue arises in our set-up, as mentioned in the introduction: If maintenance diverts labor from adoption and is therefore likely to deepen the technological gap, does it in counterpart rise the level of income since it tends to increase the stock of capital? The above stated three effects of maintenance and adoption on output suggest that this question may not be settled in a simple analytical fashion. We tackle it below after characterizing the central planner problem.

3.1 The central planner problem

The central planner problem is the same as the one considered in the benchmark model with two differences, the accumulation law of capital (equation (11) instead of (2)) and the clearing condition of the labor market (equation

(10) instead of (4)). The interior solution of this optimization problem is characterized by the following first order optimality conditions:

$$U'(C_t) = \beta U'(C_{t+1})\left[\alpha A_t K_t^{\alpha-1} L_{t+1}^{1-\alpha} + [1-\delta(m_{t+1})]\right]; \qquad (12)$$

$$\beta K_t^{\alpha} L_{t+1}^{1-\alpha} U'(C_{t+1}) = \lambda_t - \beta\lambda_{t+1}\left[1 - d\,u_{t+1}^{\theta}\right];$$

$$(1-\alpha)A_{t-1}K_{t-1}^{\alpha}L_t^{-\alpha}U'(C_t) = \omega_t;$$

$$\lambda_t d\,\theta\, u_t^{\theta-1}\left[A'_{t-1} - A_{t-1}\right] = \omega_t;$$

$$K_{t-1}[-\delta'(m_t)]U'(C_t) = \omega_t; \qquad (13)$$

where ω and λ are defined as in subsection 2.1. The necessary conditions with respect to knowledge, A_t, labor, L_t, and adoption, u_t, are not altered. As one can check, the second, third and fourth equation of the system just above, corresponding to the latter conditions, are the optimality conditions (7), (8) and (9) of the central planner problem in the benchmark model. The Euler equation (12) now incorporates the impact of maintenance on capital accumulation. A change in the maintenance path over time affects the accumulation of capital and the paths of production and consumption. A new optimality condition has to be taken into account. Equation (13) characterizes indeed the optimal maintenance decision. The marginal benefit from maintaining the stock of capital (evaluated in terms of the marginal utility of consumption) should be equal to the shadow wage, which is the marginal cost of maintaining capital. We now provide a characterization of the balanced growth paths.

3.2 Balanced growth paths

The balanced growth paths are defined as in subsection 2.2. In particular, we are seeking paths where L_t, m_t and u_t are constant and comprised between 0 and 1, and where the other variables grow at a constant rate. It is not difficult to check that Proposition 1 still applies in our extension, that is $g_A = \gamma$, and $g_C = g_K = g_I = g_Y = \gamma^{\frac{1}{1-\alpha}}$. In order to come with an analytical characterization as simple as possible of the existence and uniqueness issues, we parameterize the depreciation function as follows:

$$\begin{aligned} \delta(m) &= a - cm^b \\ 0 &< b < 1 \\ a &> c > 0. \end{aligned}$$

The restrictions on the values of the parameters are set to fulfill the requirements (i)-(ii) to be satisfied by an admissible depreciation function. $0 < b < 1$ is required for depreciation function be convex, and $a > c$ means that the "natural" depreciation rate, $\delta = a - c$ is positive. Parameter a is the capital depreciation if the planner does not devote any labor to maintain capital and $a - c$ is the natural depreciation rate. With this specification, the model implies the following restrictions of the levels of variables along a balanced growth path:

$$
\begin{aligned}
\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} &= \alpha A K^{\alpha-1} L^{1-\alpha} + 1 - a + c m^{b} \\
\frac{\beta K^{\alpha} L^{1-\alpha}}{\gamma^{\frac{1}{1-\alpha}} C} &= \frac{\omega \left[\gamma - \beta(1 - d u^{\theta})\right]}{d \theta u^{\theta - 1}(1 - A)} \\
b c m^{b-1} K &= C \omega \gamma^{\frac{1}{1-\alpha}} \\
(1-\alpha) A K^{\alpha} L^{-\alpha} &= C \omega \gamma^{\frac{1}{1-\alpha}} \\
A(\gamma - 1) &= d u^{\theta} (1 - A) \\
\gamma^{\frac{1}{1-\alpha}} C &= A K^{\alpha} L^{1-\alpha} + \left[1 - a + c m^{b}\right] K - \gamma^{\frac{1}{1-\alpha}} K \\
L + m + u &= 1 \\
\gamma^{\frac{1}{1-\alpha}} Y &= A K^{\alpha} L^{1-\alpha}
\end{aligned}
$$

The long-run system involves eight variables: L, u, m, C, Y, K, A and ω. It is possible to find a set of sufficient conditions for an admissible solution to exist and to be unique, and to derive some comparative statics.

Proposition 3 *Assume that the parameters of the model check the following condition*

$$
c < (1 - b)\left[\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a\right]
$$

Then:

(i) a unique steady solution exists with $0 < u < 1$, $0 < m < 1$ and $0 < L < 1$,

(ii) the following comparative statics apply:

(v) with respect to d, $\frac{\partial u}{\partial d} < 0$, $\frac{\partial m}{\partial d} > 0$, $\frac{\partial L}{\partial d} > 0$, and $\frac{\partial TG}{\partial d} < 0$.

(w) with respect to γ, $\frac{\partial u}{\partial \gamma} > 0$ and $\frac{\partial m}{\partial d} < 0$. The sign of $\frac{\partial L}{\partial \gamma}$ and $\frac{\partial TG}{\partial \gamma}$ is ambiguous.

The proof of the proposition is extremely heavy, it is reported in the appendix. First let us comment on the assumption made in the proposition. Quite straightforwardly, this assumption imposes a lower bound for the technological progress factor, γ. This lower bound depends mainly on the maintenance parameters a, b and c. Since $\beta < 1$ and $\gamma > 1$, and since the parameters a and c are directly related to the capital depreciation rates, and thus are small real numbers, our assumption should be very easily checked except when b tends to 1. This never happens in our numerical experiments as we will see later in this subsection.

Much more importantly, the comparative statics results already give some insight into the complex interactions at work in our model. In the case of an exogenous improvement in the adoption technology, ie. when d rises, the registered comparative statics are quite similar to those of the benchmark model. For the same reason as in the latter model, despite the adoption effort is reduced, the technological gap goes down as the product $d\,u^{\mu}$ decreases. The reduction in the labor allocation for adoption permits the assignment of more labor resources to maintenance. Maintenance and adoption work in opposite directions, and this property seems to be one of the most salient outcomes of the model in several situations. Indeed, the same property arises in the case where a technological acceleration occurs, ie. γ rises. The labor allocation to adoption increases while labor on maintenance goes down. The fact that adoption and maintenance move in opposite directions in both cases reflects mainly the arbitrage between "productive" and "non-productive" labor, namely between L and the sum $u + m$. Suppose that both adoption and maintenance go up in response to a technological acceleration. Then labor allocation to production and so to consumption is likely to decrease sharply. When only adoption or maintenance labor shifts upward, this guarantees that in the worse case labor allocation to production will only decrease slightly. Actually, the reaction of variable L when γ changes is analytically ambiguous while it increases clearly when d goes up. There is an easy explanation to this contrast. When d varies, it has a unique direct effect, namely on the adoption technology. It results in a change in labor on adoption, and it only affects the maintenance labor via the labor resources constraint. In contrast, when γ moves, it does not only affect directly the adoption technology, it also enters explicitly the optimal capital accumulation decision,[3]

[3] Which is given by the first equation of the steady state system of equations listed at the beginning of this subsection.

which depends on m. Hence, when γ moves, there are definitely more economic interactions and mechanisms at work in comparison with the shock on d. As a consequence, while the latter case permits a complete analytical characterization, the former does not.

The ambiguity in labor on production when γ shifts upward is responsible for another ambiguity to arise: The technological gap can a priori increase or decrease under a technological acceleration, while it clearly goes up in the benchmark model. Recall that $TG = \frac{\gamma - 1}{d\,\theta}$ as in the benchmark model. In the latter model, a technological acceleration stimulates a bigger labor allocation to adoption. However, this rise in labor on adoption is not sufficient to keep as close to the technological frontier as before the acceleration. When maintenance is included, labor on adoption is additionally favored by the decrease in maintenance, unless the labor resources freed by this reduction in maintenance are ultimately assigned to production. Since the effect of a technological acceleration on L is ambiguous, so is its effect on the magnitude of adoption rise. This explains in turn the ambiguity of the technological gap response.

Since some comparative statics are analytically ambiguous, we resort to numerical experiments. First of all, we calibrate our model in order to meet some precise criteria. Given the objectives of this article, three principal indicators have been kept in mind along the way: The fraction of adoption costs in terms of GDP, the fraction of maintenance costs in terms of GDP and the ratio investment to output. The table just below gives all the details about the benchmark parameterization we choose.

a	b	c	θ	d	α	β	γ
0.12	0.24	0.119	0.7	1	0.4	0.97	1.033

The parameters β, α and γ are fixed a priori in line with the range of values usually considered in the literature. The parameter a has been fixed in order to have a capital depreciation rate equal to 12% in the absence of any maintenance effort. Again this value is in the range of values considered in the literature. The parameter c has been fixed to 0.119 in order to have a capital depreciation rate of 0.1% when all labor resources are devoted to maintenance. The remaining parameters b, θ and d have been chosen in order to fit three statistics: a maintenance cost around 6%, an adoption cost around 10% and an investment ratio around one third. Here are the characteristics of the steady state equilibria generated with this parameterization for the benchmark and extended models:[4]

	Benchmark model	Extended model
m	{	0.0522723
u	0.1197704	0.113502 (#)
L	0.882296	0.834226 (#)
A	0.871416	0.868538: (#)
TG	0.147557	0.15136 (#)
K	2.08184	3.39566 (")
I	0.346321	0.37636 (")
Y	1.02677	1.20345 (")
C	0.680448	0.827091 (")
$\frac{wu}{Y}$	0.117634	0.1098699
$\frac{wm}{Y}$	{	0.046
$\frac{I}{Y}$	0.33	0.31

The comparison between the benchmark and extended models shows clearly that the maintenance services induce a decreasing labor allocation to either adoption or production, which in turn deepens the technological gap. We will come back to this issue in the next section with an analytical characterization of the latter e®ects. At the minute, we will investigate numerically the long run implications of a technological acceleration since some analytical results are ambiguous, namely those on labor input in production and on the technological gap. Recall that a technological acceleration stimulates adoption (u increases) but does not necessarily imply a lower technological gap. The e®ect on the technological gap depends on the optimal reallocation of labor, and in particular on the size of adoption e®ort's increment. To investigate this issue, we conduct the following numerical experiment. We increase $°$ by 1% and compute the induced increments in m, u, L and TG relative to the increment in $°$. We perform this computation for several values of $µ$ since this parameter captures precisely the e±ciency of the adoption e®ort. A higher $µ$'s value should induce higher adoption e®ort and possibly a decrease in the technological gap. The results (given in %) of the experiment are given just below.

[4] Obviously, the parameters b and c are not relevant for the benchmark model, while $± = a$.

	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 0.95$
$\frac{\Delta u}{\Delta \gamma}$	23.7	21.4	19.5	17.9	16.5	15.36	14.34	13.45	13
$\frac{\Delta m}{\Delta \gamma}$	-14.8	-15.5	-15.49	-15.83	-16.15	-16.45	-16.73	-16.99	-17.11
$\frac{\Delta L}{\Delta \gamma}$	0.47	0.16	0.14	-0.46	-0.76	-1	-1.33	-1.59	-1.72
$\frac{\Delta TG}{\Delta \gamma}$	24.8	22.9	21.33	20	18.86	17.89	17	16.31	15.9

It should be noted that while the numerical derivatives with respect to u (Resp. m) are always positive (Resp. negative), the relative variation of L is positive for sufficiently small θ's values and negative for θ big enough. This confirms our analytical results for u and m on one hand, and reflects the ambiguity of production labor's response to a technological acceleration on the other hand. An increase in θ rises the profitability of doing adoption. As the value of this parameter increases, the labor reallocation is more and more favorable to adoption, which explains why production labor decreases from $\theta = 0.5$. Finally, note that even for θ very high, the effect on technological gap is always positive, which means that the technological gap does indeed worsen after a technological acceleration even in this parametric situation. Though the value of u increases as θ rises, this is not sufficient to decrease the technological gap as the increments in u when γ rises get smaller and smaller. This seems to be a robust property of the models à la Nelson and Phelps.

3.3 Maintenance, technological gap and income level

Since the maintenance activity diverts labor from adoption, it is clear that it tends to increase the technological gap as adoption is the unique activity which permits the control of this gap. As we argue in the introduction, one can then ask: What is the trade-off? Maintenance affects directly the capital stock, and via the optimal allocation of labor resources, it is also a determinant of the adoption effort (u) and the amount of labor devoted to production (L). Therefore, it affects output through three distinct channels, the capital stock (K), the stock of knowledge (A) and the labor input (L). The first effect is output-augmenting but the two others are likely to be negative. Therefore, the effect on output level is hardly unambiguous. It seems however clear that a possible benefit from maintaining the stock of capital is an output increment. In this situation, maintenance activities deepen the technological gap but at the same time, they rise the output level. This very important issue is treated in details in this subsection.

To this end, we will compare the benchmark model with the model with maintenance (assuming obviously that the parameters common to the two models have the same values). How does the incorporation of the maintenance activity change the equilibrium values of adoption labor and labor input in production, and what is the total effect on (detrended) output? We denote by X_A (Resp. X_M) the value of a variable X in the benchmark model (Resp. in the model with maintenance). We have the following simple intermediate results.

Proposition 4 *Maintenance activities have a negative effect on the labor allocation to either adoption and production. Hence they tend to increase the equilibrium technological gap:*

$$u_A > u_M;\ L_A > L_M;\ TG_A < TG_M:$$

The proof is quite trivial, it is briefly sketched in the appendix. It follows that maintenance has three effects on output level, each of them with an unambiguous sign. A nonzero labor on maintenance increases the stock of capital (positive effect) but decreases adoption effort (lower A, negative effect) and labor input in production. After some trivial algebraic manipulations, one can write the ratio $\frac{Y_M}{Y_A}$ as follows:

$$\frac{Y_M}{Y_A} = \left[\frac{\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a}{\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a - cm^b}\right]^{\frac{\alpha}{1-\alpha}} \cdot \left[\left(\frac{A_M}{A_A}\right)^{\frac{1}{1-\alpha}} \cdot \frac{L_M}{L_A}\right]: \qquad (14)$$

The first term of the left hand side of (14) reflects the positive effect of maintenance on output through capital accumulation. The second and third multiplicative terms reflect the two negative effects of maintenance on output, lower stock of knowledge and lower labor in the production sector. Overall the effect is ambiguous, and unsurprisingly, there is no analytical way to get rid of this ambiguity. So we resort to numerical experiments as in the last subsection. We keep on working on the same benchmark parameterization. One would naturally think that the positive effect of maintenance on output can dominate if the size of labor on maintenance is enough big and /or the size of labor on adoption is enough small. We analyze this kind of conjectures through sensitivity studies with respect to the parameters of the model. We display below the most interesting sensitivity study results, namely those for the parameters c and μ. A lower value for c (Resp. μ) induces a lower maintenance (Resp. adoption) equilibrium value. The table just below gives the results of the experiments on c.

	c = 0.03	c = 0.05	c = 0.07	c = 0.09	c = 0.11	c = 0.119
m	0.00623	0.0128422	0.02124	0.03178	0.0451	0.05227
u	0.117207	0.11668	0.116	0.115159	0.114083	0.1135
$\frac{Y_M}{Y_A}$	1.022	1.045	1.073	1.1	1.15	1.172

As c goes up, m increases, u goes down and the ratio $\frac{Y_M}{Y_A}$ rises.[5] Even very small values of labor on maintenance have a positive effect on output equilibrium level. The trade-off is therefore clear: More maintenance means a bigger technological gap but also a bigger output level. The same result is obtained from the sensitivity analysis with respect to μ as one can check in the table below.

		μ = 0.2	μ = 0.35	μ = 0.5	μ = 0.65	μ = 0.8	μ = 0.95
Benchmark model	u	0.02	0.045	0.075	0.1068	0.1393	0.1715
	Y	1.277	1.196	1.12015	1.049	0.9842	0.9253
Maintenance model	u	0.019	0.043	0.079	0.1029	0.1347	0.1663
	m	0.06	0.058	0.0558	0.05318	0.05	0.047
	Y	1.5	1.41	1.317	1.23	1.151	1.08
	$\frac{Y_M}{Y_A}$	1.18	1.178	1.175	1.173	1.17	1.16

This suggests the following. If one is primarily concerned with the long run output level of a country, the maintenance decision should be placed at the heart of the analysis. Indeed our theoretical and numerical findings suggest that even if the size of maintenance services in terms of GDP is very small, it can affect notably the long run level of output through capital accumulation. In this respect, less adoption and more maintenance may be an optimal decision. This should of interest for those who, like Parente (1997), argue of the development theories which have the property that differences in policies translate into differences in steady state income levels rather than in growth rates.

[5] It is trivial to prove that the ratio $\frac{Y_M}{Y_A}$ goes to 1 when c tends to 0 since in this limit case the maintenance variable disappears.

4 Conclusion

In this paper, we have provided a simple theory of capital maintenance and technology adoption using optimal growth models a la Nelson and Phelps where the labor resources of an economy can be allocated freely either to production, adoption or maintenance. There are very few papers dealing with maintenance, and a fortiori with the role of capital maintenance in technological choices. In this paper, we analyze a situation where adoption and maintenance "compete" for labor resources. This is only one of the channels through which the two activities interact, as we explain in the introduction. Though this is certainly the easiest way to relate adoption to maintenance, the considered model proves very rich in terms of induced economic mechanisms and interactions, and sheds light on some important properties of maintenance.

Beside mathematically characterizing the optimal allocation of labor across the three activities, we prove for example that equilibrium maintenance and adoption operate in opposite directions when technological shocks occur. This is in particular true when there is an exogenous technological change. Maintenance is a kind of substitute to adoption is such cases. Much more importantly, we find that though capital maintenance deepens the technological gap by diverting labor resources from adoption, it generally increases the long run output level at equilibrium. As we claim in the last section, this is a very interesting result for the proponents of a development theory aimed at deriving the policies which yield differences in steady state income levels rather than in growth rates.

Obviously, much work remains to do for a much comprehensive appraisal of the role of maintenance as a determinant of technological choices. Beside endogenizing growth, which is not a very hard task, more fundamental refinements have to be undertaken. For example, one may think that maintenance plays a more crucial role if technological advances are embodied in capital goods, and this may reinforce the conclusions of this paper. On the other hand, one may find questionable our assumption according to which adoption and maintenance use the same input (labor), it would be then useful to examine the properties of alternative models where adoption and maintenance do not use exactly the same combination of inputs. All these issues are in our research agenda.

5 References

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6 Appendix

1. Proof of Proposition 2: The proof is simple. Indeed, by the means of successive substitutions, one can reduce the system of eight equilibrium restrictions above to a single implicit equation involving only u:

$$H(u) = \beta\theta(\gamma-1)(1-u) - (1-\alpha)u\left[\gamma-\beta(1-du^{\theta})\right] = 0$$

It could be easily checked that $H(u)$ is a decreasing and concave function which tends to $\beta\theta(\gamma-1)$ when u tends to zero, and to $-(1-\alpha)[\gamma-\beta(1-d)]$ when u tends to one. Since $\gamma > 1$, $\beta < 1$ and $d > 0$, we have $[\gamma-\beta(1-d)] > 0$. Thus there exists an unique $u^* \in (0;1)$ which satisfies $H(u) = 0$. This establishes property (i). The comparative statics are derived explicitly. Denote by $R = \beta(\gamma-1)\theta + (1-\alpha)[\gamma-\beta(1-du^{\theta}(1-\theta))] > 0$. We have:

$$\frac{\partial u}{\partial d} = \frac{-(1-\alpha)\beta u^{\theta+1}}{R} < 0$$

$$\frac{\partial TG}{\partial d} = \frac{-(\gamma-1)[\beta\theta(\gamma-1)+(1-\alpha)(\gamma-\beta(1-du^{\theta}))]}{d^2u^{\theta}R} > 0$$

$$\frac{\partial u}{\partial \gamma} = \frac{(1-\alpha)u[1-\beta(1-du^{\theta})]}{(\gamma-1)R} > 0$$

$$\frac{\partial TG}{\partial \gamma} = \frac{1}{du^{\theta}}\left[1-\frac{(1-\alpha)\theta[1-\beta(1-du^{\theta})]}{R}\right] > 0$$

Since $(1-\alpha)\theta[1-\beta(1-du^{\theta})] < R$, $\frac{\partial TG}{\partial \gamma} < 0$.

2. Proof of Proposition 3: We can reduce the steady state equilibrium conditions to two equations in $(m;u)$.

$$m = g(u) = 1-u-\frac{(1-\alpha)u[\gamma-\beta(1-du^{\theta})]}{\beta\theta(\gamma-1)} \tag{15}$$

$$F(m;u) = 1-a+cm^{b}+\frac{\alpha}{1-\alpha}bcm^{b-1}(1-m-u)-\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} = 0 \tag{16}$$

The negative slope of function g in (15) is obvious. Concerning (16), one can apply the implicit function theorem. $F(m;u)$ defines m as a differentiable function of u, $(m = f(u))$ and $f'(u) = -\frac{F_u}{F_m}$. Since F_u and F_m are negative ($F_m < 0$ is assured by the restriction on the parameters imposed in proposition 3), the slope of the implicit function f is also negative. We now check that g is above f when u tends to zero, and g is below f when u tends to one. Let us prove that $g(0) > f(0)$. Note that $g(0) = 1$. On the other hand,

$$F(m;0) = \left[cm^b \left(1 - \frac{\alpha b}{1-b} \right) + \frac{\alpha b c m^{b-1}}{1-\alpha} \right] - \frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a = 0.$$

The first term of the previous equation defines a decreasing function of m which tends to infinity as m tends to zero and is equal to c for $m = 1$. Since $c < \frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a$; it follows that $0 < f(0) < 1$, so $g(0) > f(0)$.

Now observe that $g(1) < 0$ and $f(1) > 0$ since $F(m;1) = 1 - a + cm^b \left(1 - \frac{\alpha b}{1-\alpha} \right) - \frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} = 0$ and as by assumption $c < (1-b)\left(\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a \right)$. So $g(1) < f(1)$. Therefore, the system (15)-(16) defines two $(m;u)$-curves which intersect only once when both m and u vary in the interval $(0;1)$. Which establishes property (i).

As for the comparative statics, we consider the following system of equations:

$$\begin{array}{lrcl}
(F) & 1 - a + cm^b + \dfrac{\alpha b c m^{b-1}(1 - u - m)}{1-\alpha} - \dfrac{\gamma^{\frac{1}{1-\alpha}}}{\beta} & = & 0; \\
(G) & \beta(1 - u - m)\theta(\gamma - 1) - (1-\alpha)u\left[\gamma - \beta(1 - du^{\theta})\right] & = & 0; \\
(H) & TG - \dfrac{\gamma - 1}{du^{\theta}} & = & 0.
\end{array}$$

The Jacobian matrix of this system can be expressed by:

$$J = \begin{bmatrix} F_u & F_m & 0 \\ G_u & G_m & 0 \\ H_u & 0 & 1 \end{bmatrix}$$

where the first, second and third columns of J refer to the partial derivatives of the left hand sides of the equations of the system with respect to u, m, and TG. It is easy to check that F_u, F_m, G_u, G_m and H_u are all strictly negative. The Jacobian determinant $(F_u G_m - F_m G_u)$ is thus strictly negative:

$$\begin{aligned}
\det J &= F_u G_m - F_m G_u \\
&= \left[\frac{\beta\theta(\gamma - 1)}{m} \left(cm^b - (1-b)\left(\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a \right) \right) \right. \\
&\quad \left. + \left[\gamma - \beta(1 - du^{\theta}) + \beta d\theta u^{\theta} \right](1-\alpha)F_m \right] < 0. \qquad (17)
\end{aligned}$$

By Cramer's rule we obtain

$$\frac{\partial m}{\partial d} = \frac{1}{|\det J|}[G_d F_u]$$

as $G_d = -(1-\alpha)\beta d u^{\theta+1} < 0$, $\frac{\partial m}{\partial d} > 0$

$$\frac{\partial u}{\partial d} = \frac{1}{|\det J|}[-G_d F_m] < 0 \tag{18}$$

$$\frac{\partial TG}{\partial d} = \frac{H_d(F_u G_m - F_m G_u) + H_u F_m G_d}{|\det J|}$$

where $H_u = \frac{\gamma-1}{d^2 u^\theta} > 0$ and $H_d = \frac{(\gamma-1)\theta}{d u^{\theta+1}}$. Taking into account (17) and (18), we can express $\frac{\partial TG}{\partial d}$ as:

$$\frac{\partial TG}{\partial d} = -\frac{(\gamma-1)}{d^2 u^\theta}\left[1+\frac{\theta d}{u}\frac{\partial u}{\partial d}\right]$$

$$\left|\frac{\theta d}{u}\frac{\partial u}{\partial d}\right| = \frac{\theta d}{u}\frac{|G_d F_m|}{|\det J|}$$

Developing the previous expression, we can easily check that $\frac{\partial TG}{\partial d} < 0$ since $\left|\frac{\theta d}{u}\frac{\partial u}{\partial d}\right| < 1$. Indeed :

$$\left|\frac{\theta d}{u}\frac{\partial u}{\partial d}\right| = \frac{\theta d}{u}|G_d F_m| = |F_m|(1-\alpha)\beta d\theta u^\theta < |\det J| =$$

$$\beta\theta(\gamma-1)\frac{}{m\left[\sigma m^b - (1-b)\left(\frac{\gamma}{\beta}\right)^{\frac{1}{1-\alpha}} - 1 + a\right] + \left[\gamma - \beta(1-du^\theta) + \beta d\theta u^\theta\right](1-\alpha)F_m} :$$

As for the comparative statics with respect to γ, we use again Cramer's rule to obtain:

$$\frac{\partial m}{\partial \gamma} = \frac{1}{|\det J|}[G_\gamma F_u - G_u F_\gamma] < 0$$

where $G_\gamma = \frac{(1-\alpha)u[1-\beta(1-du^\theta)]}{(\gamma-1)} > 0$, and $F_\gamma = -\frac{\gamma^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)\beta} < 0$

$$\frac{\partial u}{\partial \gamma} = \frac{1}{|\det J|}[G_m F_\gamma - G_\gamma F_m] > 0$$

$$\frac{\partial TG}{\partial \gamma} = \frac{1}{du^\theta}\left[1 - \frac{(\gamma-1)\theta}{u}\frac{\partial u}{\partial \gamma}\right] :$$

The sign of $\left[1 - \frac{(\gamma-1)\theta}{u}\frac{\partial u}{\partial \gamma}\right]$ being ambiguous, so is the sign of $\frac{\partial TG}{\partial \gamma}$.

Let us finally study the comparative statics concerning L, the labor input in production. First, it should be noted that $\frac{\partial L}{\partial d}$ depends on the size of $\frac{\partial m}{\partial d}$ and $\frac{\partial u}{\partial d}$:

$$\frac{\partial L}{\partial d} = -\frac{\partial m}{\partial d} - \frac{\partial u}{\partial d} = \frac{-G_d\,(F_u - F_m)}{|\det J|}.$$

Developing a bit more the numerator of the previous expression, it is easy to check that the negative effect of d on adoption labor is not compensated by the increment in m. In order to satisfy the labor restriction ($1 = L + m + u$), L should therefore increase.

$$\frac{(1-\alpha)\beta d\, u^{\theta+1}\left[\frac{(1-b)}{m}\left(\frac{\gamma^{\frac{1}{1-\alpha}}}{\beta} - 1 + a\right) - \text{œ}m^{b-1}\right]}{|\det J|} > 0.$$

As for the effect of a change in the parameter γ on L, which is equal to:

$$\frac{\partial L}{\partial \gamma} = -\frac{\partial m}{\partial \gamma} - \frac{\partial u}{\partial \gamma},$$

it is ambiguous. A quick look at the expressions of $\frac{\partial m}{\partial \gamma}$ and $\frac{\partial u}{\partial \gamma}$ is sufficient to understand this result. The sign of the difference of the two latter expressions depends on the values of the parameters of the maintenance function with respect to the those of the adoption technology, and there is no a priori relationship between the two sets of parameters. The numerical results reported in the main text make clear that the sign of $\frac{\partial L}{\partial \gamma}$ does effectively depend on the parameterization.

3. **Proof of Proposition 4**: Assume that the values of the parameters common to the two models are fixed. From the previous proofs of Propositions 2 and 3, one can see that adoption is determined by the following conditions in the benchmark model and in the extended model respectively

$$\beta\theta(\gamma - 1)(1 - u_A) = (1-\alpha)u_A[\gamma - \beta(1 - d\,u_A^{\theta})];$$

$$\beta\theta(\gamma - 1)(1 - u_M - m) = (1-\alpha)u_M[\gamma - \beta(1 - d\,u_M^{\theta})].$$

Algebraically speaking, the first equation is a particular case of the second with $m = 0$. Since the second equation defines a negative relationship between u_M and m, u_A is bigger than u_M, as long as $m > 0$. As the technological gap is decreasing in adoption labor $TG_A < TG_M$ is directly checked.

We can rewrite the two conditions just above in terms of labor devoted to adoption and labor devoted to production:

$$\beta\theta(\gamma - 1)L_A = (1 - \alpha)u_A[\gamma - \beta(1 - d\, u_A^{\theta})];$$

$$\beta\theta(\gamma - 1)L_M = (1 - \alpha)u_M[\gamma - \beta(1 - d\, u_M^{\theta})]:$$

The latter equations define labor in production as an strictly increasing function of adoption labor. Since $u_A > u_M$, it follows that $L_A > L_M$.