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A Multicountry Model of the Term Structures of Interest Rates with a GVAR*

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Abstract

Extant multicountry affine term structure models (*ATSMs*) handle global financial interdependence at the cost of increasing model dimensionality. To address this challenge, we propose a novel no-arbitrage *ATSM* with risk factor dynamics following a global vector-autoregressive (*GVAR*). Compared to referenced benchmarks, the $GVAR-ATSM$ offers a more parsimonious representation, enables a faster estimation process, produces more precise model estimates, yields more plausible term premia dynamics, and improves bond yield out-of-sample forecasting. Furthermore, our empirical findings reveal the significant impact of China's economic stances on Latin American yield curve dynamics, underscoring its importance as a global economic player.

Keywords: global financial market, GVAR, term structure of interest rates

JEL classifications: C58, E44, G15

The interplay between the term structure of interest rates and economic unfolding is of key relevance to economic players. For central banks, the yield curve is an important tool of monetary policy transmission (Evans and Marshall 2007), whereas, for financial investors, the term structure reflects the expectations and risk-return assessments of the future macroeconomic outlook (Ang, Piazzesi, and Wei 2006). The linkage between macroeconomic fundamentals and the yield curve is at the core of affine term structure models (*ATSMs*).[1] While recent *ATSMs* have been developed to address the increasing financial

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1 This class of models includes, for instance, the works of Ang and Piazzesi (2003), Bikbov and Chernov (2010), and Wright (2011), among many others.

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interdependence among countries resulting from globalization, the substantial number of parameters in such models has made it challenging to capture comprehensively the transmission of cross-border shocks. This article sheds new light on this field by introducing a novel and parsimonious *ATSM* representation that effectively captures the impact of shocks in globalized economies. This new framework improves the tractability of large-scale *ATSM*s, deepens the understanding of global economic mechanisms driving yield curve movements, strengthens statistical inference, and improves the forecasting capabilities of *ATSM*s.

The backbone of our term structure framework is the influential work of Joslin, Priebsch, and Singleton (2014) (*JPS – ATSM*). In essence, *JPS – ATSM* assumes the absence of arbitrage opportunities and considers linear state space representations of the yield curve dynamics. Further, this model combines the spanned factors, representing the traditional yield curve factors such as level, slope, and curvature,[2] with the unspanned factors, representing macrofinancial variables. Within this framework, the spanned factors provide a good fit for the cross-section of bond yields, while the unspanned factors shape the time evolution of the term structure and contribute to explaining the relationship between bond yields and the real economy. Similarly to Joslin, Priebsch, and Singleton (2014), *ATSM*s have been extensively used in the past two decades to analyze the impact of domestic macroeconomic aggregates on the U.S. term structure.[3] More recently, these models have expanded to incorporate the impact of a globalized economy, considering both domestic and foreign factors in influencing domestic yield curve dynamics. For instance, Jotikasthira, Le, and Lundblad (2015) (*JLL – ATSM*) extended the *JPS – ATSM* framework to encompass multiple countries, allowing for an examination of the relative contributions of domestic and foreign shocks to yield curve movements.

As in the traditional *ATSM* literature, *JLL – ATSM* assumes that the joint dynamics of the risk factors evolve according to a first-order *VAR*. This feature of the model raises several important limitations. First, the inclusion of a few supplementary countries in the economic system could quickly render the model unreasonably large. This issue results, thus, in tractability issues leading to timely (when feasible) estimation. Second, the identification scheme of structural shocks requires a large set of zero restrictions on both the feedback and the variance-covariance matrices. As a result, the assessment of the relationship between any two variables of the system becomes undermined. Third, similar to standard *VAR* setups, the linkages among the units of an economic system are unspecified, resulting in these models being silent on the sources of transmission of cross-border shocks.

This article proposes a novel approach for modeling multicountry no-arbitrage *ATSM*s using a global vector-autoregressive (*GVAR*) framework to depict the dynamics of the risk factors. Based on Chudik and Pesaran (2016), our *GVAR* has two components: the marginal model, a standard *VAR* outlying global economic developments, and $VARX^*$ models, small-scale domestic *VAR*s augmented by weighted cross-section averages of foreign variables to capture the interdependence across countries. We improve our model estimates by integrating the bias-correction procedure introduced by Bauer, Rudebusch, and Wu (2012).

The transition matrix is a central pillar of *GVAR* specifications. It is precisely this peculiarity of the model that enables us to address the key shortcomings of the *JLL – ATSM*. First, the inclusion of an explicit transmission structure allows for transparent control of the mechanism of propagation of shocks, hence providing normative results. Second, imposing a structure of interdependence among countries greatly reduces the overall number of model parameters and thus boosts the accuracy of the estimates. In addition to

[2] See, for example, Hordahl, Tristani, and Vestin (2006), Rudebusch and Wu (2008), and Bekaert, Cho, and Moreno (2010) for structural interpretations for these factors.

[3] See, for instance, Ang and Piazzesi (2003), Rudebusch and Wu (2008), and Bauer, Rudebusch, and Wu (2012), among others.

improving econometric tractability, our $GVAR-ATSM$ avoids economic-based controversial restrictions, providing more robust economic interpretations. Moreover, due to its parsimonious feature, the required estimation time is approximately 2 to 4 times shorter than analog multicountry versions of the $JPS-ATSM$ and the $JLL-ATSM$.[4]

In addition to the approach employed by $JLL–ATSM$, two notable alternatives for addressing parameter proliferation in large-scale *ATSM*s are presented by Graveline and Joslin (2011) and Huang and Shi (2023).[5] Our study complements these works in several aspects. First, Graveline and Joslin (2011) propose a parsimonious no-arbitrage *ATSM* to jointly model interest rates and foreign exchange rates in the $G-10$ economies. They effectively reduce the number of parameters from 1406 to 243 by imposing restrictions primarily on the risk-neutral structure of their specification. In contrast, our work focuses on physical parameter restrictions and provides a more explicit approach to analyzing the interaction between domestic and foreign macroeconomic factors in influencing bond yield dynamics. Second, Huang and Shi (2023) employ a machine learning-enhanced *LASSO* model to construct a concise bond return predictor from a broad set of macro variables in the U.S. context. Their methodology primarily relies on a data-driven approach without explicitly modeling interdependencies across countries. Conversely, our framework integrates economically motivated cross-country interactions and can accommodate dynamically evolving interdependence.[6]

We apply our new $GVAR-ATSM$ to a group of four emerging economies: China, Brazil, Mexico, and Uruguay. Two key reasons underlie this choice. First, in contrast to advanced economies, emerging economies have not experienced constraints imposed by the nominal zero lower bound on short-term interest rates in the past decade.[7] This allows us to conduct a more up-to-date and relevant economic analysis, specifically examining the impact of China's emergence as a global economic player on the bond markets in Latin American countries.[8] Additionally, this approach enables us to consider the growing importance of these emerging economies in the current global financial-economic landscape. Second, selecting a group of Latin American countries ensures the presence of significant interconnections within the system. Specifically, since our preferred proxy measure of interconnectedness is based on trade flows[9] and geographical proximity consistently strengthens

[4] Arguably, the linear regression-based *ATSM* from Adrian, Crump, and Moench (2013) can yield nearly instantaneous model estimates. However, this framework is subject to criticism for overlooking the internal consistency condition of the pricing factors (see Joslin, Singleton, and Zhu 2011).

[5] Relatedly, $JPS-ATSM$ use a set of standard information criteria to rank all potential zero restrictions on the parameters governing risk premia. However, they show that performing this task for a single-country model with three spanned and two unspanned factors would involve estimating 2^{19} different specifications. Presumably, extending this procedure to multicountry models with larger dimensions is impractical.

[6] While one could enhance model simplicity by combining the *GVAR* approach with shrinkage methods such as LASSO or Ridge, our primary focus is on examining standard variables within macrofinance *ATSM*s. Future research may extend these techniques to the $GVAR–ATSM$, exploring their implications for variable selection and coefficient regularization.

[7] Besides Jotikasthira, Le, and Lundblad (2015), other macrofinance multicountry *ATSM*s based on advanced economies are Kaminska, Meldrum, and Smith (2013) Diez de los Rios (2017), Abbritti, Dell'Erba, and Sola (2018), Meldrum, Raczko, and Spencer (2018), and Bauer, Rudebusch, and Wu (2014). All of these works limit the period of analysis before 2008–2009 to avoid model estimation complications that turn up from the proximity of interest rates to the zero lower bound. It is widely documented that the performance of standard *ATSM*s in terms of model fit and forecasting is undermined when nominal short-term rates are constrained to the zero level (see, e.g., Christensen and Rudebusch 2016).

[8] In related work, Cesa-Bianchi et al. (2012) study the effects of the recent prominent insertion of China in the world trade on the transmission of international business cycle to Latin America.

[9] This choice aligns with referenced *GVAR* models, including the influential work of Pesaran, Schuermann, and Weiner (2004). Furthermore, we have conducted robustness tests by exploring alternative measures of interconnectivity, which are discussed in Section 5.

trading linkages (Disdier and Head 2008), a system including three regionally concentrated countries (Brazil, Mexico, and Uruguay), along with China, the world's leading exporter,[10] establishes a notable level of economic integration.[11] While previous studies have examined sovereign bond yields and risk premia in emerging markets, this work stands out as the first, to the best of our knowledge, to employ a flexible framework that enables the assessment of cross-country interactions.[12]

We evaluate our empirical findings from four main perspectives. First, we employ generalized impulse response functions (*GIRF*s) to analyze the yield curve responses of Latin American economies to an economic growth shock from China. This shock induces a steepening of the yield curves in Brazil and Mexico, characterized by decreased short-term yields (potentially signaling a positive economic outlook from increased exports to China) and increased long-term yields (reflecting inflation risk premia). In Uruguay, this same shock leads to a downward response across all bond maturities, possibly influenced by liquidity and debt sustainability factors. Additionally, we show that our model consistently produces narrower confidence intervals and is statistically indistinguishable from the $JLL-ATSM$.

Second, we use generalized forecast error variance decompositions (*GFEVD*s) to assess the impact of domestic and foreign shocks on yield curve volatility. Both $GVAR-ATSM$ and $JLL-ATSM$ reveal the significant role of cross-border developments in shaping domestic term structure dynamics, indicating a high level of integration into the global financial system. Notably, Uruguay consistently experiences a substantial proportion of foreign shocks, with their contribution exceeding 75% in key instances. These findings support the existence of a Global Financial Cycle, characterized by synchronized movements in risky asset prices worldwide, as documented by Miranda-Agrippino and Rey (2020).[13]

Third, we decompose bond yields into their expected short-term interest rate and term premia components. The results indicate that the $GVAR-ATSM$ estimates provide more plausible interest rate expectations and term premia from a macrofinance standpoint for all four countries. Specifically, in our sample, the short-term interest rates derived from the $GVAR-ATSM$ capture accommodative monetary policy and exhibit higher term premia during periods of economic slowdown. Conversely, the $JLL-ATSM$ yields an acyclical expected component in China and unrealistically large and volatile term premia in Brazil.

Finally, we evaluate the out-of-sample forecasting performance of bond yields for both the $GVAR-ATSM$ and $JLL-ATSM$ using fixed-smoothing asymptotics tests by Coroneo and Iacone 2020. Our results indicate that the $GVAR-ATSM$ demonstrates statistically superior forecast performance in over three times as many cases compared to the $JLL-ATSM$.

This article is organized as follows: Section 1 outlines the key features of the multicountry *ATSM*s investigated, including our $GVAR-ATSM$. Section 2 presents the main empirical findings, followed by a formal statistical test on the restrictions of our $GVAR$ in Section 3. Additionally, we conduct robustness checks in Section 4. Section 5 concludes.

[10] See the Direction of Trade Statistics (DTS) database released from the International Monetary Fund (IMF).

[11] Importantly, we do not include the U.S. economy in this economic system, since the sample period under investigation coincides, to a large extent, with the years of zero short-term rates. Instead, the model incorporates global economic factors which highly correlate with the stance of the U.S. economy. Furthermore, it is important to emphasize that the degree of interconnection between China and the Latin American economies has markedly strengthened in recent years. In terms of trade, China was the largest partner of Brazil and Uruguay and the second largest partner of Mexico in 2019 (DTS-IMF database).

[12] For instance, Chernov, Creal, and Hördahl (2020) studied five Asian-Pacific economies using a large-scale panel *VAR* that, nevertheless, requires a fair number of identification restrictions. Iania, Lyrio, and Moura (2021) employed a $JPS-ATSM$ to assess the role of global economic shocks on the risk premia dynamics of four major emerging markets from different regions of the world.

[13] In particular, Miranda-Agrippino and Rey (2020) show that a single global factor related to the global risk appetite can explain roughly 25% in asset price dynamics around the world.

1 The *GVAR–ATSM* and Early *ATSM*s With Unspanned Macroeconomic Risks

This section introduces our proposed $GVAR - ATSM$ and contrasts it with the established yield curve frameworks with unspanned macroeconomic risk of $JPS - ATSM$ and $JLL - ATSM$. One convenient aspect of these setups is that it enables the split of term structure models into their cross-sectional and time series dimensions in a clear manner. We use this opportune model feature to steer the presentation of our $GVAR - ATSM$ in Section 1.1. Next, we illustrate the parametrization difference between the $GVAR - ATSM$ and $JLL - ATSM$ based on a simplified economic system composed of the world and two domestic economies in Section 1.2. Subsequently, in Section 1.3, we underline the inherent limitations of extant frameworks to handle high-dimensional term structure systems and Section 1.4 underscores the computational advantages of $GVAR$ models.

1.1 The *GVAR–ATSM*

1.1.1 Cross-sectional dimension of the term structures

The cross-sectional setting of our $ATSM$ is based on two central equations. The first assumes that the country i short-term interest rate, $r_{i,t}$, is an affine function of N unobserved (latent) country-specific factors, $X_{i,t}$:

$$r_{i,t} = \delta_{i,0} + \delta'_{i,1} X_{i,t}, \tag{1}$$

where $\delta'_{i,1}$ is a N-dimensional vector with all entries equal to one. The second equation states that $X_{i,t}$ evolves according to a zero-mean maximally flexible $VAR(1)$ model under the risk-neutral measure (the Q-measure)

$$X_{i,t} = \Phi^{Q}_{\mathrm{i,X}} \mathrm{X_{i,t-1}} + \Sigma_{\mathrm{i,X}} \varepsilon^{Q}_{\mathrm{i,t}}, \quad \varepsilon^{Q}_{i,t} \sim \mathcal{N}(0, I_N), \tag{2}$$

where $\Phi^{Q}_{\mathrm{i,X}}$ is a diagonal matrix, the elements of which are real and distinct eigenvalues, λ^{Q}_{i}.

Based on Equations (1) and (2), Joslin, Singleton, and Zhu (2011) showed that a rotation from latent factors $X_{i,t}$ to portfolios of yields, the spanned factors $P_{i,t}$, leads to an observationally equivalent term structure representation. Specifically, the spanned factors are computed as $P_{i,t} = V_i Y_{i,t}$, where V_i is a full-rank matrix, and $Y_{i,t} = [y^{(1)}_{i,t}, \ldots, y^{(J)}_{i,t}]'$ is a J-dimensional vector that collects the set of zero-coupon bond yields with maturity of n periods, $y^{(n)}_{i,t}$. Accordingly, $Y_{i,t}$ is an affine function of $P_{i,t}$ as follows:

$$\mathrm{Y}_{i,t} = A_P(\delta_{i,0}, \lambda^{Q}_{i}, \Sigma_{i,X}, V_i) + B'_P(\lambda^{Q}_{i}, V_i) P_{i,t}, \tag{3}$$

where the forms of $A_P(\delta_{i,0}, \lambda^{Q}_{i}, \Sigma_{i,X})$ and $B_P(\lambda^{Q}_{i})$ are restricted to preclude arbitrage opportunities in the bond market of country i.

We follow the same structure present in $JLL-ATSM$ to build the cross-sectional dimension of our multicountry $ATSM$. Specifically, for an economic system composed of C countries, we encompass the sets of country-specific yields, spanned factors, and intercepts from Equation (3) into, respectively, $Y_t = [Y'_{1,t}, Y'_{2,t}, \ldots, Y'_{C,t}]'$, $P_t = [P'_{1,t}, P'_{2,t}, \ldots, P'_{C,t}]'$, and $A_P(\Theta) = [\mathrm{A}'_{\mathrm{P}}(\Theta_1), \mathrm{A}'_{\mathrm{P}}(\Theta_2), \ldots, \mathrm{A}'_{\mathrm{P}}(\Theta_{\mathrm{C}})]'$, where $\Theta_{\mathrm{i}} = \{\delta_{i,0}, \lambda^{Q}_{i}, \Sigma_{i,X}, V_i\}$ for each country i. Furthermore, we assume that the spanned factors of one country have no contemporaneous effect on the term structure of any other country. In this sense, $B_P(\Theta)$ is block diagonal such that $B_P(\Theta) = diag(\mathrm{B}'_{\mathrm{P}}(\Theta_1), \mathrm{B}'_{\mathrm{P}}(\Theta_2), \ldots, \mathrm{B}'_{\mathrm{P}}(\Theta_{\mathrm{C}}))$. Therefore, we represent the multicountry term structure for all C countries as

$$Y_t = A_P(\Theta) + \mathrm{B_P}(\Theta)\mathrm{P_t}. \tag{4}$$

1.1.2 State dynamics of the term structures: the *GVAR* model

In line with *JPS–ATSM* and *JLL–ATSM*, the country-specific model state dynamics incorporate N spanned factors and M financial-economic variables, also referred to as the domestic unspanned factors $M_{i,t}$. In this sense, we define the state vector of country i as $Z_{i,t} = [M'_{i,t}, P'_{i,t}]'$, where the dimension of $Z_{i,t}$ is given by $K = M+N$.

The $VARX^*$ builds on standard small-scale local VAR models augmented by foreign-specific variables, the *star* variables. These variables are a weakly exogenous weighted average of foreign variables. Formally, the star variables from country i are formed into a K-dimensional vector $Z^{*'}_{i,t} = \sum_{j=1}^{C} w_{i,j} Z'_{j,t}$, where the weight $w_{i,j}$ is a scalar that measures the degree of dependence of country i with each country j of the system. The $w_{i,j}$s are normalized by $\sum_{j=1}^{C} w_{i,j} = 1$ with $w_{ii} = 0$ for all i. The choice of $w_{i,j}$ is crucial since it determines the transmission channels among countries.

In addition to $Z^*_{i,t}$, the $VARX^*$ model for country i includes the K domestic (including both spanned and unspanned) risk factors and the G unspanned global factors, M^W_t. We assume that $Z_{i,t}$ follows a $VARX^*(1,1,1)$ under physical measure (the P-measure) of the form

$$Z_{i,t} = C^X_i + \Phi^X_{\mathrm{i}} \mathrm{Z_{i,t-1}} + \Phi^{X^*}_{\mathrm{i}} \mathrm{Z^*_{i,t-1}} + \Phi^{X^W}_{\mathrm{i}} \mathrm{M^W_{t-1}} + \Sigma^X_{\mathrm{i}} \varepsilon^X_{\mathrm{i,t}}, \quad \varepsilon^X_{i,t} \sim \mathcal{N}(0, I_K). \tag{5}$$

Without loss of generality, we do not include contemporaneous terms for ease of exposition. The marginal model describes the joint dynamics of M^W_t, which is assumed to evolve according to a standard unrestricted $VAR(1)$

$$M^W_t = C^W + \Phi^W M^W_{t-1} + \Sigma^W \varepsilon^W_t, \quad \varepsilon^W_t \sim \mathcal{N}(0, I_G). \tag{6}$$

In this arrangement, the dynamics of the global variables are limited to their own developments. To more closely match $JLL-ATSM$ (detailed further in Supplementary Appendix A.1), this current setting can be extended to accommodate the influence of a dominant country in the world economy. One possible way to pursue this implementation is to incorporate the risk factors from the dominant country into the system (6).[14]

Next, to form the $GVAR$, we connect the estimated $VARX^*$s through the country-specific link matrices, W_is. Specifically, to construct W_i, we collect country i's domestic and foreign variables in a $2K$–dimension vector $z_{i,t} = [Z'_{i,t}, Z^{*'}_{i,t}]'$ and all endogenous country-specific variables of the system from the $VARX^*$'s in a KC–dimension vector $z_t = [Z'_{1,t}, Z'_{2,t}, \dots Z'_{C,t}]'$. As such, W_i is a $2K{\times}KC$ matrix that satisfies the identity $z_{i,t} \equiv W_i z_t$.[15] To complete our setup, we add the global economic variables to the state vector as follows $Z_t = [M^{W'}_t, z'_t]'$. Based on these inputs, our first-order $GVAR$ representation under the physical measure has the following structure:

$$Z_t = C_y + \Phi_{\mathrm{y}} Z_{t-1} + \Sigma_y \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim \mathcal{N}(0, I_F), \tag{7}$$

where $C_y = [C^{W'}, C^{X'}_1, C^{X'}_2, \dots C^{X'}_C]'$, $\varepsilon_{y,t} = [\varepsilon^{W'}_t, \varepsilon^{X'}_{1,t}, \varepsilon^{X'}_{2,t} \ \dots \ \varepsilon^{X'}_{C,t}]'$, $\Sigma_y = diag(\Sigma^W, \Sigma^X_1, \Sigma^X_2, \dots \Sigma^X_C)$, and $F = CK + G$ is simply the dimension of Z_t. The matrix Φ_{y} is a composite of

[14] In the marginal model, any number of dominant countries can be included simultaneously. Clearly, the appropriate set of relevant dominant economies depends on the economic context under investigation.

[15] See Section 1.2 for the detailed structure of the link matrices.

the feedback matrices from both the marginal and the $VARX^{*'}$s models of all countries. More specifically,

$$\Phi_{y} = \begin{bmatrix} \Phi^{W} & 0_{G\times KC} \\ \Phi^{X^{W}} & G_1 \end{bmatrix},$$

where $\Phi^{X^{W}} = \begin{bmatrix} \Phi_1^{X^{W}} \\ \Phi_2^{X^{W}} \\ \vdots \\ \Phi_C^{X^{W}} \end{bmatrix}$ has dimensions $KC\times G$ and G_1 is a KC-dimensional squared matrix formed as $G_1 = \begin{bmatrix} \Phi_1 W_1 \\ \Phi_2 W_2 \\ \vdots \\ \Phi_C W_C \end{bmatrix}$ for $\Phi_i = [\Phi_i^{X}, \Phi_i^{X^*}]$ and $i = 1, 2, \ldots C$.

It is worth specifying the role played by unspanned factors in term structure developments. Although unspanned factors do not directly appear as an element of the pricing setting, they impinge on the dynamics of the spanned factors and ultimately affect bond yields through Equation (4).

1.1.3 Estimation procedure

Our estimation strategy builds on the single-country methodology of *JPS–ATSM*. This approach offers some important advantages compared to earlier reference macrofinance term structure models, such as that of Ang and Piazzesi (2003). In particular, the framework of Joslin, Priebsch, and Singleton (2014) allows for a simplified estimation procedure that, nevertheless, ensures efficient parameter estimates. The key reason is that the *JPS–ATSM* setting enables a convenient split of the likelihood function between the parameters governing the *P*- and *Q*-measures. This separation allows for relatively independent estimation of the physical and risk-neutral parameters, resulting in a significant reduction in computational burden during the estimation process.

Furthermore, to enhance the estimation of the *P*-dynamics parameters, we adopt the small sample bias-corrected estimation procedure proposed by Bauer, Rudebusch, and Wu (2012) (*BRW–BC*). Indeed, Bauer, Rudebusch, and Wu (2012) demonstrate some critical advantages of incorporating their method in *ATSM*s, including more persistent estimates of expected future short-term interest rates that closely align with the data, a more pronounced countercyclical pattern in term premium estimates and reduced forecasting errors in bond yield predictions. As a result, our estimation approach follows a two-step procedure.

In the first step, the parameters of the *GVAR* model are obtained based on the Chudik and Pesaran (2016) approach for stationary systems. For this purpose, we construct the star variables by calibrating the measurement of $w_{i,j}$ based on trade flow data (details of this measure are further elaborated in Section 2.1). Subsequently, we use ordinary least squares to separately estimate each country's $VARX^*(1,1,1)$ and the marginal model.[16] Next, we use the estimates of C_i^X, Φ_i^X, $\Phi_i^{X^*}$, $\Phi_i^{X^W}$, Σ_i^X, C^W, Φ^W, and Σ^W to build the *GVAR* parameters C_y and Φ_y presented in Equation (7). The *BRW–BC* is applied exclusively to refine the estimation of these latter parameters (C_y and Φ_y).

[16] Notably, least squares estimation is feasible in this context since the pricing factors $P_{i,t}$ are directly observable and given by $P_{i,t} = V_i Y_{I,t}$ (Section 1.1.1). The entries of V_i are computed using loadings from principal component analysis, as in Joslin, Singleton, and Zhu (2011).

In the second step, we use maximum likelihood to convene the country-specific risk-neutral parameters $\Phi^{\mathbb{Q}}_{i,X}$, $\delta_{i,0}$, and $\Sigma_{i,X}$, present in $A_P(\Theta)$ and $B_P(\Theta)$ of Equation (4).[17] Joslin, Priebsch, and Singleton (2014) showed that the estimation of the P-dynamics parameters by least squares generates variance-covariance estimates close to the global optimum. As such, based on the estimates from the first estimation step, we employ the $N \times N$ matrices at the bottom right of Σ^X_i as the starting values of $\Sigma_{i,X}$ in the second stage.[18] Due to the absence of closed-form expressions, $\Phi^{\mathbb{Q}}_{i,X}$ and $\Sigma_{i,X}$ must be evaluated numerically.

1.2 *GVAR–ATSM* and *JLL–ATSM*: A Two-Country Example

Similarly to our *GVAR–ATSM*, *JLL–ATSM*'s cross-section representation is given by Equation (4) and the risk factors dynamics follow a (restricted) $VAR(1)$ process comprising G global and K country-specific factors for each of the C countries.

In contrast, *JLL–ATSM* differs from the *GVAR–ATSM* in three key aspects. First, in the former, the economic system designates one worldwide large (dominant) economy which, by assumption, is the sole one that can impact the developments of the global and the other economies of the cohort. Second, Jotikasthira, Le, and Lundblad (2015) resort to a series of projections to build the risk factors of interest. In particular, this approach aims at forming domestic variables, the dynamics of which are trimmed from the influence of either other countries, the global economy, or both. Third, it follows from the *VAR* structure of the model that the feedback and the variance-covariance matrices are formed by $(KC+G)^2$ entries each. In this sense, it is straightforward to note that the inclusion of supplementary countries and/or domestic factors can dramatically increase the number of model parameters. To mitigate this over-parametrization issue, *JLL–ATSM* imposed a set of zero restrictions on both the feedback and the Cholesky factor of the variance-covariance matrices. In Supplementary Appendix A.1, we detail the general design of these two *VAR* matrices and the method used in the construction of the orthogonalized risk factors.

To emphasize the parametrization contrast between the two approaches, we present a reduced two-country ($C=2$) model example. For the sake of simplicity, we assume that this economic cohort contains one global factor ($G=1$), in addition to one spanned ($N=1$) and one unspanned factor ($M=1$) per country. For both models, we denote the global factor by $M^W_t = m^W_t$. For the *GVAR–ATSM*, we set $P_{i,t} = p_{i,t}$, $M_{i,t} = m_{i,t}$ for $i = 1,2$, so the model state vector is $Z_t = [m^W_t, m_{1,t}, p_{1,t}, m_{2,t}, p_{2,t}]'$. Analogously, for the *JLL–ATSM*, we index the variables with the superscript e to denote the orthogonalized version of the risk factors, namely: $P_{i,t} = p^e_{i,t}$ and $M_{i,t} = m^e_{i,t}$ where the variables from the dominant economy are indexed by $i=1$. Therefore, in *JLL–ATSM*, we have $Z_t = [m^W_t, m^e_{1,t}, p^e_{1,t}, m^e_{2,t}, p^e_{2,t}]'$. As such, we cast both model P-dynamics in terms of a generic $VAR(1)$ representation,

$$Z_t = \Phi_0 + \Phi_1 Z_{t-1} + \Sigma\varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, I_5).$$

Next, we partition the feedback, Φ_1, and the Cholesky factor, Σ, matrices in four segments to distinguish the different sources of interactions across and among domestic and global factors. The first two relationships relate to the responses of the global variables to their own lagged values and country-specific developments. As we set $G=1$, the model coefficients that capture these effects are present in the first rows of both matrices: while the elements of the first columns jointly provide the impact from the global variables, the remaining parameters apprehend the influence from the domestic risk factors. The two

[17] *JPS–ATSM* also requires the estimation of the variance of the errors from the country-specific J–N portfolio of yields observed with errors. Like Jotikasthira, Le, and Lundblad (2015), we assume that these parameters are equal at the country level, but they can differ across countries.

[18] Note that, by definition, the bottom right $N \times N$ part of Σ^X_i is equivalent to $\Sigma_{i,X}$.

other relationships involve the sensitivity of the set of domestic variables. In particular, the lower-left 4×1 (lower-right 4×4) corner accounts for the effect of the global (country-specific) factor dynamics. We present below a detailed design of both Φ_1 and Σ according to this four-square division.[19] For the $GVAR-ATSM$, we present the composition of the matrices in terms of the model parameters of Equations (5) and (6) as follows:

$$\Phi_1^{GVAR} = \left[\begin{array}{c|c} \Phi^W_{(1\times1)} & 0_{(1\times4)} \\ \hline \begin{array}{c}\Phi^{X^W}_{1(2\times1)} \\ \Phi^{X^W}_{2(2\times1)}\end{array} & G_{1(4\times4)} \end{array}\right]_{(5\times5)}, \text{ where } G_1 = \left[\begin{array}{c} \left[\Phi_1^X \; \Phi_1^{X^*}\right]_{(2\times4)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ w_{1,1} & 0 & w_{1,2} & 0 \\ 0 & w_{1,1} & 0 & w_{1,2} \end{bmatrix}_{(4\times4)} \\ \\ \left[\Phi_2^X \; \Phi_2^{X^*}\right]_{(2\times4)} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ w_{2,1} & 0 & w_{2,2} & 0 \\ 0 & w_{2,1} & 0 & w_{2,2} \end{bmatrix}_{(4\times4)} \end{array}\right]_{(4\times4)}, \text{ and } \Sigma^{GVAR} = \left[\begin{array}{c|cc} \Sigma^W_{(1\times1)} & 0 & 0 \\ \hline 0 & \Sigma^X_{1(2\times2)} & 0 \\ 0 & 0 & \Sigma^X_{2(2\times2)} \end{array}\right]_{(5\times5)}.$$

For the $JLL-ATSM$, we mark with x the entries indicating the absence of zero-restrictions,

$$\Phi_1^{JLL} = \left(\begin{array}{c|cccc} x & x & x & 0 & 0 \\ \hline x & x & x & 0 & 0 \\ x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \end{array}\right)_{(5\times5)} \text{ and } \Sigma^{JLL} = \left[\begin{array}{c|cccc} x & 0 & 0 & 0 & 0 \\ \hline x & x & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 \\ x & x & 0 & x & 0 \\ 0 & 0 & x & 0 & x \end{array}\right]_{(5\times5)}.$$

In this example, both the $GVAR-ATSM$ and $JLL-ATSM$ entail estimating 21 parameters in the feedback matrix. However, the $GVAR-ATSM$ exhibits a more parsimonious structure with fewer parameters in the Cholesky factor—7 compared to 9 in the $JLL-ATSM$. While the advantage of parameter reduction is not pronounced in this simplified framework with only two countries, its significance becomes evident in larger systems accommodating multiple economies, as we discuss next.

1.3 Limitations of $JPS-ATSM$ and $JLL-ATSM$

A well-known shortcoming of VAR models is that the dimensionality of the system can largely increase with the inclusion of a few additional factors. This drawback becomes more pronounced when considering $ATSM$s estimated following Joslin, Priebsch, and Singleton (2014), as the estimation of the variance-covariance matrix involves numerical optimization techniques (see Section 1.1.3).

To illustrate the required estimation computational burden of several $ATSM$s, we present in Table 1 the number of parameters in the variance-covariance matrix that require estimation for specifications containing different numbers of countries. The entries of this same table are computed for models featuring two global unspanned factors ($G = 2$), two domestic unspanned factors ($M = 2$), and three domestic spanned factors ($N = 3$). The

[19] We omit the display of the intercept terms, Φ_0, as they play little role in the dynamics and the empirical results of interest from both models.

Table 1. Number of estimated parameters in the variance-covariance matrix

	Number of countries			
Model label	**3**	**4**	**5**	**10**
JPS-ATSM	153	253	378	1378
JLL-ATSM	81	133	198	835
GVAR-ATSM	48	63	78	153

Note: The entries in the table are computed for an economic system containing two global unspanned factors ($G = 2$), two domestic unspanned factors ($M = 2$), and three domestic spanned factors ($N = 3$).

label $JPS - ATSM$ refers to a model in which the risk factor dynamics are completely unrestricted, whereas $JLL - ATSM$ contemplates a restricted VAR along the lines of the original paper of Jotikasthira, Le, and Lundblad (2015). The $GVAR - ATSM$ is our setup detailed in Section 1.1.

It is possible to verify that $JPS - ATSM$ and $JLL - ATSM$ quickly become intractable as the number of countries in the economic system increases.[20] The construction of the variance-covariance of a five-country model requires the (numerical) estimation of 378 parameters in the $JPS - ATSM$ and almost 200 in the $JLL - ATSM$, more than double of the $GVAR - ATSM$. It is also worth noting that a ten-country $GVAR - ATSM$ depends upon the estimation of as many parameters as a $JPS - ATSM$ formed by three economies.

In addition to building on a heavily parameterized VAR system, we consider that $JLL - ATSM$'s set of restrictions imposed in this same VAR is open to criticism in two further dimensions. First, the authors resort to Cholesky decomposition as the identification strategy of the structural shocks of the model. This approach has been widely criticized because of the dependence of the results on the ordering of the countries in the VAR (see, e.g., Bikbov and Chernov 2010). Moreover, this choice implies the unrealistic assumption that all of the variables of the dominant economy are more exogenous than any other factor from the other economies in the system. Second, the structure of the feedback and variance-covariance matrices fully disrupts the transmission channel of cross-border shocks among non-dominant economies—an undesirable feature of models designed to study multicountry specifications.

1.4 *GVAR*: Source of Computational Advantages

The primary sources of computational advantage from the $GVAR - ATSM$ are twofold. The first relates to the role played by the star variables in $GVAR$ setups. Intuitively, these variables produce a significant data shrinkage since they synthesize the foreign economic influences on the domestic risk factors dynamics. Second, $GVAR$ frameworks rely on low-dimensional systems (each country's $VARX^*$ and the marginal models), which can be individually estimated. Accordingly, the required estimation complexity of each of these setups depends, independently, on the number of global variables (G) in the marginal model and each country's risk factors (K) in the individual $VARX^*$s. In contrast, in traditional VAR-based frameworks such as the ones from $JPS - ATSM$ and $JLL - ATSM$, model estimation needs to be carried out at once to the entire and large model dimension ($F = CK + G$). In this sense, the $GVAR - ATSM$ bears substantial estimation efficiency gains over the $JLL-ATSM$ once the number of countries in the system becomes sizeable.

[20] We further illustrate the degree of complexity of these models by presenting the required estimation time for a four-country system in Supplementary Appendix A.2.

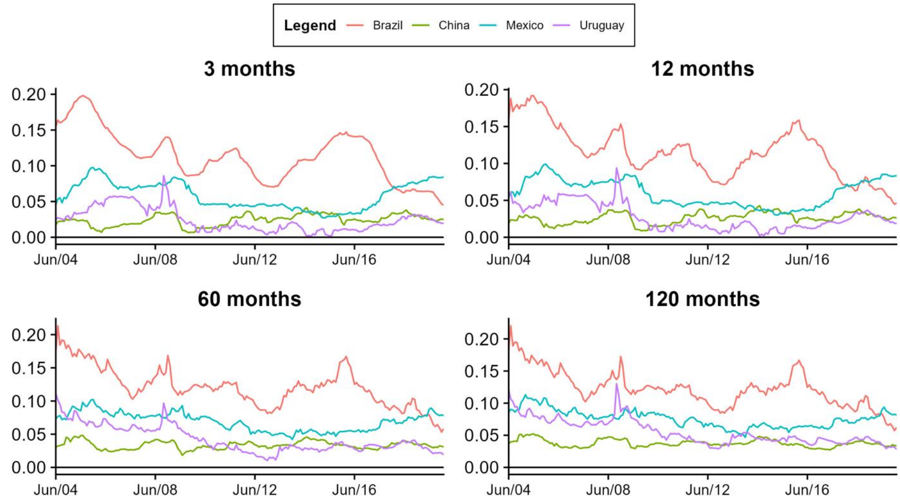

Figure 1. Time series of bond yields.

Note: The graphs show the monthly historical evolution of domestic bond yields corresponding to maturities of 3, 12, 60, and 120 months. Rates are annualized. The sample period is from June 2004 to January 2020.

2 *GVAR – ATSM*: An Empirical Illustration For Latin American Countries

We study an economic system consisting of China and three Latin American economies: Brazil, Mexico, and Uruguay. This selection offers significant advantages for our study. First, unlike advanced economies, the short-term interest rates of each country in our sample remained notably distant from zero throughout the entire study period, as evidenced by Figure 1. Accordingly, this characteristic of the dataset makes it particularly well-suited for implementing a term structure model aligned with the framework proposed by Joslin, Priebsch, and Singleton (2014). Second, the presence of substantial financial and economic linkages among these countries allows for effective consideration of spillover effects. We select China as the dominant economy for estimating the *JLL – ATSM* due to its recent prominent economic role in the region.[21] However, for the sake of maintaining the generality of our model, we do not assign a dominant country for the *GVAR – ATSM*.

We organize the presentation of our empirical application into three distinct parts. Firstly, in Section 2.1, we introduce the treated dataset used throughout the analysis. Subsequently, we compare the empirical results generated by the *GVAR – ATSM* and *JLL – ATSM* from two distinct perspectives. In Section 2.2, we present the outcomes related to the in-sample analysis, including *GIRF*s, *GFEVD*s, and the decomposition of bond yields into their expected future short-term interest rate and risk compensation components. Secondly, considering the substantial differences in physical dynamics between the *GVAR – ATSM* and *JLL – ATSM*, we evaluate the out-of-sample performance of bond yields for both specifications in Section 2.3. Additionally, we provide an overview of the

[21] While it is common in many studies to designate the U.S. economy as the dominant one (e.g., Canova 2005), our decision to choose China as the dominant economy stems from its more preponderant bilateral trade relations with two of the three Latin American economies in our sample. Based on data from the DTS-IMF database for the year 2019, China's (U.S.'s) combined sum of exports and imports accounted for approximately 10% (62%), 25% (15%), and 22% (8%) of the total trade of Mexico, Brazil, and Uruguay, respectively. However, we also conducted an alternative version of the model (not reported) using global risk factors based on U.S. domestic variable. Our key empirical findings remained unchanged, indicating a strong correlation between the global factor and the U.S. economy.

required estimation burden for the various multicountry specifications under study in Supplementary Appendix A.2. It is shown that the estimation time for the $GVAR-ATSM$ is materially shorter compared to the corresponding multicountry versions of both the $JPS-ATSM$ and the $JLL-ATSM$. Replication of all the results showcased in the article can be done using the *R*-package *MultiATSM* (Moura 2023).

2.1 Data

We use monthly data ranging from June 2004 to January 2020 (188 observations). While data are available for a slightly longer period, we have chosen to begin our sample in 2004 and end it before the emergence of the COVID-19 pandemic. This decision strikes a balance between maximizing the sample size and mitigating the potential impact of a significant structural break in the data. Our dataset contains four categories of series: (i) global economic indicators; (ii) domestic economic indices; (iii) country-specific term structure of interest rates; and (iv) bilateral trade flows.

In line with the prevailing practice in the macrofinance term structure literature, our model incorporates unspanned factors that encompass domestic rates of inflation ($\pi_{i,t}$) and economic activity growth ($g_{i,t}$), along with their global counterparts, π_t^W and g_t^W. To quantify π_t^W, we use the year-over-year variation in the Organisation for Economic Cooperation and Development (OECD) aggregated Consumer Price Index (CPI) containing all its member countries. As a proxy variable of g_t^W, we employ Lutz Kilian's index of global real economic activity, available on the author's webpage.[22] At the country level, we retrieve the country-specific CPIs from the IMF. We then calculate the year-on-year changes to build $\pi_{i,t}$. For the composition of $g_{i,t}$, we collect the monthly GDP leading indicator compiled by the OECD.[23] The domestic spanned factors are computed as the three first principal components of each country's bond yield set, and, can thus be interpreted as level ($L_{i,t}$), slope ($S_{i,t}$), and curvature ($C_{i,t}$) (see, for instance, Litterman and Scheinkman 1991).

Consequently, our country-specific state vector ($Z_{i,t}$) consists of three spanned factors ($N = 3$) and two unspanned domestic factors ($M = 2$). Thus, the complete state vector Z_t is formed by $Z_t = [M_t^{W'}, M_{1,t}^{'}, P_{1,t}^{'}, M_{2,t}^{'}, P_{2,t}^{'} \ \dots \ M_{C,t}^{'}, P_{C,t}^{'}]^{'}$, where $M_t^W = [g_t^W, \pi_t^W]^{'}$, $M_{i,t} = [g_{i,t}, \pi_{i,t}]^{'}$, and $P_{i,t} = [L_{i,t}, S_{i,t}, C_{i,t}]^{'}$. Notably, for the $JLL-ATSM$, the variables with the index $i = 1$ pertain to the dominant economy. Supplementary Figure A.1 presents a graphical representation of the time series for all the built-up risk factors used in our analysis.

The term structure of each country comprises end-of-month zero-coupon yields from maturities of 3, 6, 12, 36, 60, and 120 months. To broaden the sample span, we adopt different sorts of datasets. For China, Mexico, and Uruguay, we use government bond yields. For the two former countries, data are denominated in domestic currency and are collected from Bloomberg, whereas for the latter, the yields are U.S. dollars denominated and are retrieved from the Uruguayan stock exchange.[24] For Brazil, we collect swap fixed-DI contracts, quoted in domestic currency from Brazil's Stock Exchange and Over-the-Counter Market (B3).[25]

We build the transition matrix, W, from the trade flows between each pairwise combination of countries in the sample. Specifically, we define the strength of dependence of

[22] See Kilian (2009) and Kilian and Zhou (2018) for a detailed methodological explanation of the compilation of this index.

[23] The latest index is not available for Uruguay. In such a case, we construct a monthly time series from the cubic interpolation of quarterly GDP data, to which we apply the Hodrick–Prescott filter to remove the GDP trend (see Hodrick and Prescott 1997).

[24] We use U.S. dollar-denominated bonds for Uruguay due to the absence of domestic currency-denominated yields with maturities over 5 years before September 2017.

[25] Swap fixed-DI contracts are derivative securities indexed to the overnight rate of Brazil's interbank loan market. The 10-year yield of this indicator is unavailable before September/05. To address this, we estimated and fitted a Nelson-Siegel model for the missing data period.

country i concerning country j, w_{ij}, as $w_{ij} = \frac{I_{ij}+E_{ij}}{\sum_{j=1}^{C} I_{ij}+E_{ij}}$, where I_{ij} (E_{ij}) corresponds to the value of goods imports (exports) from economy i to j. Further, to compute I_{ij} and E_{ij}, we aggregate the yearly series of bilateral trade flows from 2004 to 2019 over the entire sample period.[26] These trade flow values, including goods, imports, and exports, are sourced from the IMF and are free on board and denominated in U.S. dollars. Supplementary Appendix Table A.6 reports the complete transition matrix. It is worth noting that $W = [w_1^{'}, w_2^{'}, \ldots w_C^{'}]^{'}$, where $w_i = [w_{i,1}, w_{i,2}, \ldots w_{i,C}]$ for $i = 1, 2 \ldots C$.

The use of trade flow weights as a proxy measure for capturing interconnectivity between countries is a standard approach in the *GVAR* literature (Pesaran, Schuermann, and Weiner 2004; Chudik and Fratzscher 2011; Eickmeier and Ng 2015). Chudik and Pesaran (2016) endorse the use of trade weights, as they are able to capture not only economic but also technological, political, and cultural interlinkages among countries. In addition to its support in the literature, we adopt trade flows as our benchmark measure due to their data availability across the entire period under study. While measures based on bilateral financial flows could potentially capture bond market linkages more accurately, the existing databases do not provide an extensive time series for all combinations of countries in our sample. Nevertheless, for robustness, we also test alternative model specifications using different linkage measures based on capital flows. These results are discussed in Section 4.

2.2 In-Sample Evaluation

2.2.1 GIRFs and GFEDVs analyses

We employ the *GIRF*s and *GFEDV*s following the approach outlined in Pesaran and Shin (1998).[27] These methods do not require the identification of shocks according to some canonical procedure. Rather, they account for correlated shocks using the historically observed distribution of the errors. As such, in contrast to the Cholesky identification restrictions employed in *JLL–ATSM*, *GIRF*s and *GFEDV*s from Pesaran and Shin (1998) provide empirical results that are invariable to the ordering of the variables. This feature of *GIRF*s and *GFEDV*s is imperative in a fairly large system like the one under study, in which the risk factors can be ordered in more than 10^{21} alternative forms.

The current setting enables the appraisal of 22 risk factor shocks. Due to space constraints, we focus on the term structure responses of Brazil, Mexico, and Uruguay to an economic growth shock in China, presented in Figure 2.[28] It is worth emphasizing that *GIRF*s do not aim to identify the specific sources of changes in output growth, such as technology or monetary policy shocks. Instead, they depict the outcomes when a specific variable experiences changes, without attributing causality. Given this limitation, we can conjecture about the potential economic channels underlying the observed responses, considering the mechanisms incorporated in our model. As such, based on Cesa-Bianchi et al. (2012), we interpret our findings as evidence that a Chinese economic growth shock has a substantial synchronized impact on the business cycle of Latin America.

Figure 2 depicts that the bond yields of Brazil and Mexico show a similar response to a shock in Chinese output growth. Short-term yields initially decrease, while long-term yields (above 36-month maturity in Brazil and 120-month maturity in Mexico) respond positively, with the response in Brazil being immediate and in Mexico occurring after 6 months. This dual pattern could be attributed to different factors. The decline in short-term yields

[26] We do not include the observations from the year 2020 to ensure a more representative measure of interconnectedness for the period under study. Two reasons underlie this choice. First, we aim to mitigate potential distortions arising from the disruptive effects of the COVID-19 pandemic on global trade. Second, since the time series of the risk factors only includes the observation for January in 2020, it would not be realistic to include a measure of trade integration that spans the entire year.

[27] We present in Supplementary Appendix A.1.3 the analytical derivations of the *GIRF*s and *GFEDV*s.

[28] The remaining outcomes of the other shocks can be obtained by using the *MultiATSM* package.

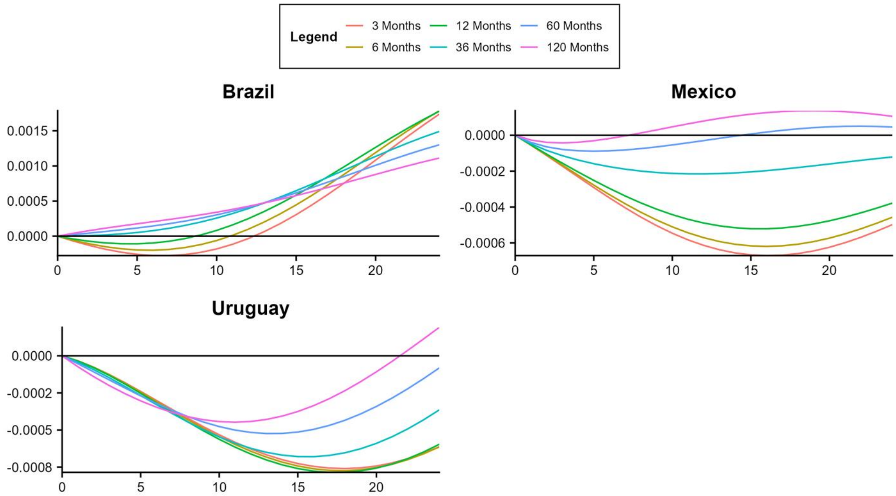

Figure 2. Generalized impulse responses from the $GVAR - ATSM$—shock to economic growth in China.
Note: The *x*-axes are expressed in months and the bond yield responses (*y*-axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

could reflect a more positive economic outlook in Latin American economies, potentially driven by increased exports to China. This, in turn, contributes to boosting output growth in Brazil and Mexico, enhancing their ability to meet debt repayment obligations and reducing short-term default risk (Reinhart, Rogoff, and Savastano 2003). On the other hand, the increase in long-term yields could be explained by expectations of future monetary policy and the need for higher compensation due to potential inflationary dynamics. More specifically, higher economic growth often leads to expectations of future inflation, putting upward pressure on future policy rates. Additionally, an inflationary environment may require bondholders to demand higher compensation for holding long-term nominal bonds (Wright 2011).

The Chinese economic growth shock has a distinct impact on the Uruguayan term structure, with yields across the maturity spectrum decreasing. As Uruguay's yields are denominated in U.S. dollars, this reaction is less likely to be influenced by the monetary authority's response to aggregate demand shocks. Instead, U.S. dollar inflows can push bond yields downward through the risk compensation channel in two ways. First, even modest capital inflows can significantly reduce default risk in Uruguay, given its smaller economy compared to Brazil and Mexico. Second, the relatively illiquid nature of Uruguay's bond market may lead financial investors to require a smaller liquidity premium when the country experiences U.S. dollar inflows.[29]

In Figure 3, we present a qualitative comparison of the *GIRF*s obtained from the $GVAR - ATSM$ and the $JLL - ATSM$ across different maturity ranges. To calculate 95% confidence bounds, we employ a residual-based bootstrap procedure (explained in detail in Supplementary Appendix A.3). In the case of the $JLL - ATSM$, the *GIRF*s are computed based on the shock to orthogonalized Chinese economic growth. Further, to ensure a consistent comparison between the outputs produced by the two models, we also apply the

[29] For the sake of illustration, in 2013, Uruguay was ranked 98th out of 183 countries on the IMF's Financial Market Depth index, which measures the development level of domestic financial markets in terms of size and liquidity (see Svirydzenka 2016).

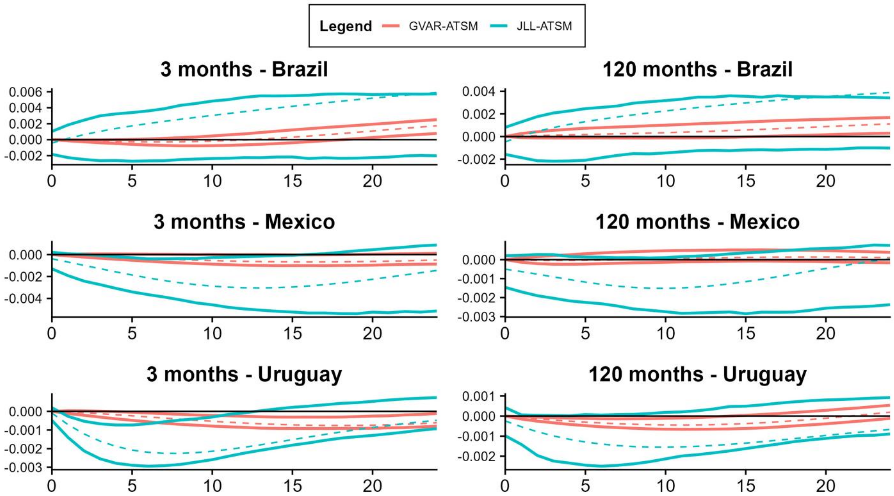

Figure 3. Generalized impulse responses of the $GVAR-ATSM$ and the $JLL-ATSM$—shock to economic growth in China.

Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping with 1000 draws. For $JLL-ATSM$, the yield responses are computed for the orthogonalized version of the Chinese economic growth shock. The *x*-axes are expressed in months and the bond yield responses (*y*-axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

$BRW-BC$ to the $JLL-ATSM$. This allows us to mitigate potential biases or distortions that may arise during the estimation process.

Three points deserve attention. First, we find that the $GIRF$s obtained with the $GVAR-ATSM$ are statistically indistinguishable from those of the $JLL-ATSM$ at this level of significance. Second, we show that the confidence intervals of $GVAR-ATSM$ are substantially narrower than those obtained from $JLL-ATSM$. This finding is consistent with the patterns observed in the $GIRF$s of the three other domestic economic growth shocks reported in Supplementary Appendix A. These results emphasize an additional improvement of the $GVAR-ATSM$ over the $JLL-ATSM$. Once both models are adjusted for distortionary biases, our more parsimonious representation consistently provides more precise and reliable statistical inferences. In contrast, the $JLL-ATSM'$s extensive model parametrization generally tends to lead to larger statistical inferences regarding variable responses to shocks.

Third, $GVAR-ATSM$ indicates that the Chinese output growth shock is, at times, statistically significant for the three Latin American economies. This finding constitutes an interesting result as it offers new insights into the transmission channels of shocks among these countries. Previous studies have extensively documented U.S. monetary policy shocks as a significant driver of economic fluctuations in Emerging Markets (Mackowiak 2007; Iacoviello and Navarro 2019), particularly in Latin America (Canova 2005), as well as on the dynamics of risky financial assets globally (Miranda-Agrippino and Rey 2020). However, less attention has been given to China as a meaningful source of international spillover transmission. While some studies have explored the increased synchronization of business cycles between China and Latin America (Cesa-Bianchi et al. 2012), our research extends beyond the macroeconomic dimension by showing the nontrivial effects of China's economic conditions on the asset price dynamics of Latin American economies. This result

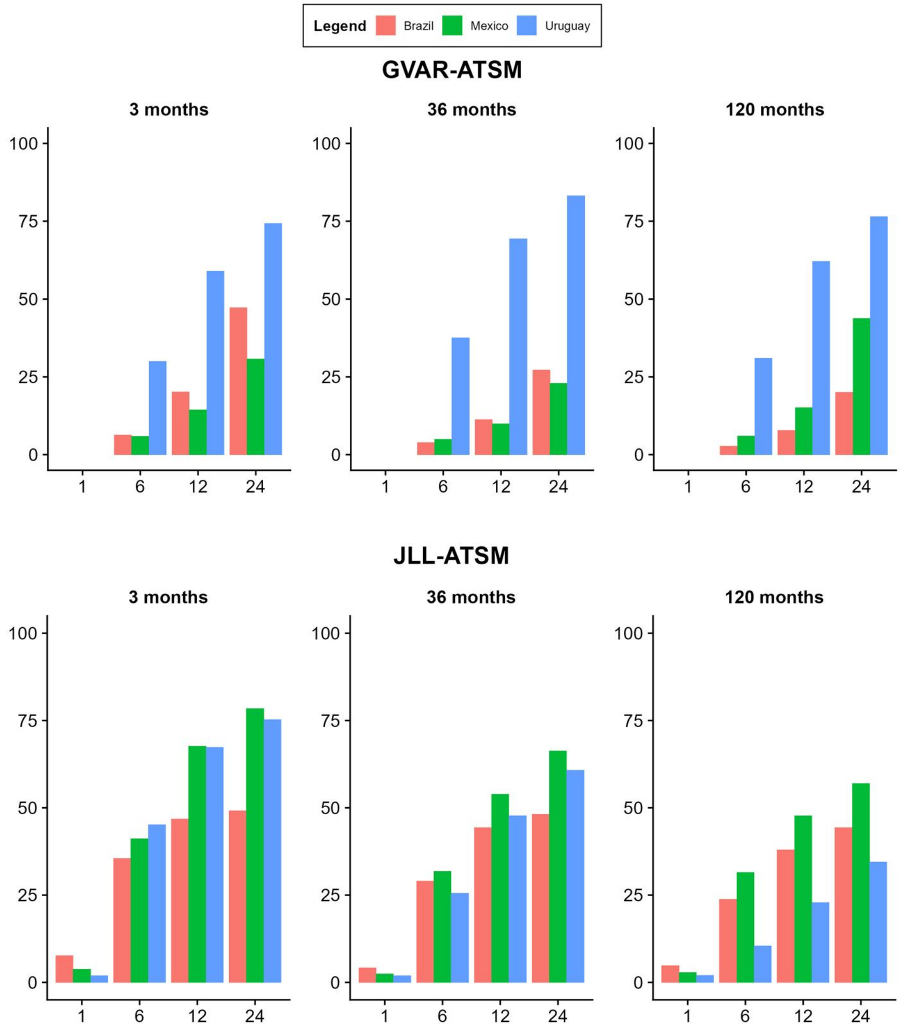

Figure 4. Generalized forecast error variance decomposition of bond yields—share of foreign shocks.

Note: The bars represent the sum of the proportion of the cross-border shocks (global and foreign economies' spanned and unspanned factors) for each economy. The *x*-axes show the number of forecast periods ahead in months. The *y*-axes are expressed in percentages. For $JLL-ATSM$, the outputs are computed based on the orthogonal version of the risk factors.

is particularly surprising given the relatively recent strengthening of trade relations and increased interdependence among these countries.[30]

Figure 4 displays the *GFEVD*s, illustrating the respective contributions of domestic and foreign shocks to the volatility of bond yield shocks across various maturity segments. Two

[30] In a related study, Kearns, Schrimpf, and Xia (2022) found evidence of monetary policy spillovers from major central banks, such as the Federal Reserve and the European Central Bank, affecting bond yields around the globe. They observed that these spillovers occur partly through the macroeconomic link channel, where countries with higher trade integration and synchronized GDP growth or inflation dynamics experience stronger transmission effects.

aspects are worth emphasizing. First, our analysis reveals that foreign shocks serve as the primary drivers of interest rates volatility in the $JLL-ATSM$ for the 3- and 36-month maturities bonds across the three economies, as well as in the $GVAR-ATSM$ for all maturities in Uruguay. This emphasizes the important role of cross-border developments in shaping the dynamics of domestic term structures, indicating the significant integration of these economies into the global financial system. The presence of foreign shocks as key influencers underscores the relevance of considering the broader international context when analyzing domestic bond markets. Second, we observe that the significance of foreign shocks grows as the forecast horizon extends for both models and across all bond yields in the examined countries. Notably, Uruguay stands out as the economy that is consistently influenced by the highest shares of foreign shocks. This effect is particularly evident after a 24-month horizon, where the proportion of foreign shocks exceeds 75% for both models.

2.2.2 Bond yields decomposition

In this section, we focus on decomposing country-specific yield curves into their expected and term premia components. This decomposition provides policymakers with valuable insights and inputs for monetary policy decisions. The expected component of the yield curve reflects market-based expectations about the future path of monetary policy, allowing policymakers to gauge inflation expectations. On the other hand, the term premia captures the additional compensation investors demand for holding longer-term bonds relative to shorter-term ones. This component provides valuable information about market participants' risk appetite and can help identify potential vulnerabilities in financial markets.

In line with Joslin, Priebsch, and Singleton (2014), we conduct the decomposition at the level of forward rates. Specifically, we focus on the rate at which an investor can lock in today for a one-year loan starting in 24 months in country i, denoted as $f_{i,t}^{(24,36)}$. Additionally, we define $\tilde{f}_{i,t}^{(24,36)}$ as the model-implied expected short-term interest rate (the risk-neutral forward rate), for the expected one-year yield prevailing 24 months into the future in country i. Correspondingly, $fp_{i,t}^{(24,36)}$ represents the respective term (forward) premium component. Accordingly, the decomposition of the forward rates of interest can be expressed as $f_{i,t}^{(24,36)} = \tilde{f}_{i,t}^{(24,36)} + fp_{i,t}^{(24,36)}$ for each economy within the system.

In Figure 5, we present the breakdown of forward rates for each economy in our sample using both the $GVAR-ATSM$ and $JLL-ATSM$. To provide insights into the relationship between forward rate components and the state of the economy, we incorporate a measure of economic activity expansion and slowdown for each economy in the figure. For this purpose, we adopt the business cycle dating procedure introduced by Harding and Pagan (2002). This method is specifically designed to identify turning points, such as peaks and troughs, in the business cycle. In the figure, we highlight the periods of economic contraction (from peaks to troughs) with gray-shaded areas.

In terms of the risk-neutral rates, two key points highlight the divergence between the two models. Firstly, in the majority of cases, the $GVAR-ATSM$ effectively captures the implementation of accommodative monetary policy by central banks during periods of economic slowdown. For instance, during the Global Financial Crisis of 2008, the $GVAR-ATSM$ aligns with the observed decline in short rates across all countries. In contrast, the $JLL-ATSM$ exhibits a rather unresponsive behavior in China and displays an erratic pattern in Brazil. Another notable discrepancy occurs in China, where the $JLL-ATSM$ appears to be acyclical to the stage of the business cycle throughout the sample period.

Second, the $JLL-ATSM$ generates largely implausible risk-neutral rates in Brazil across various periods. Specifically, in 2004, the $JLL-ATSM$ estimates risk-neutral rate above

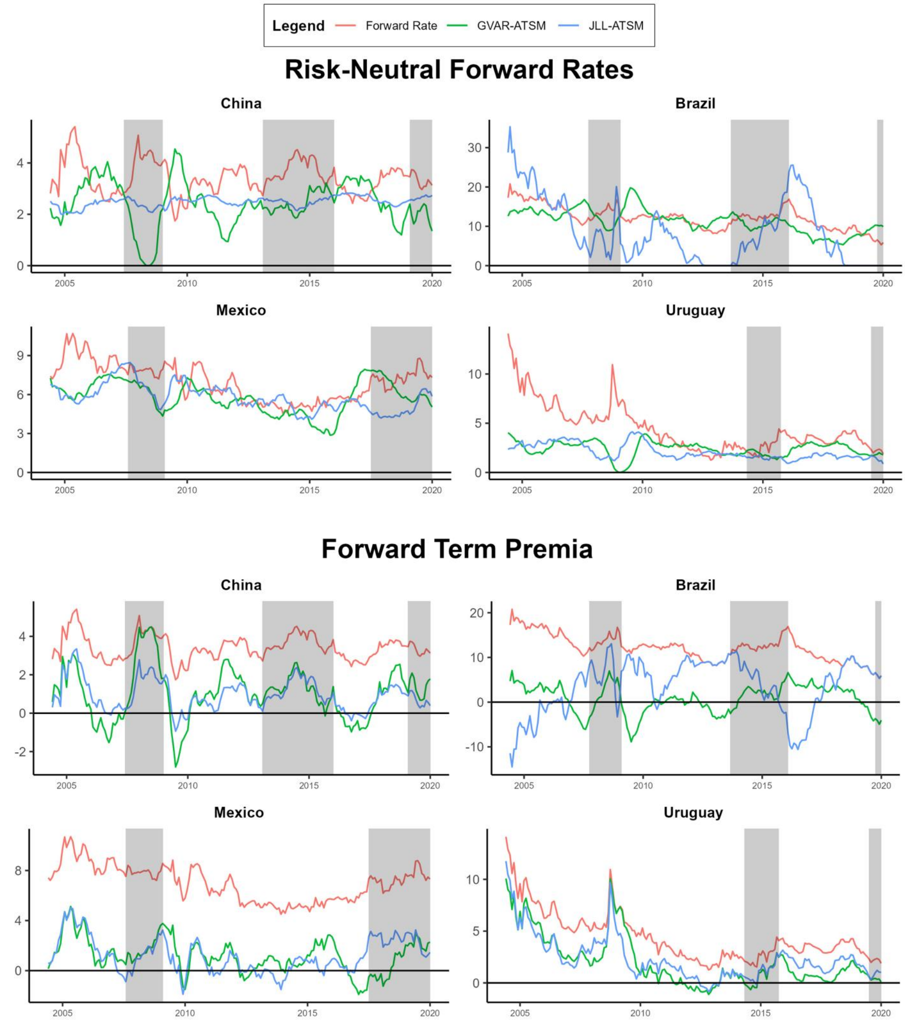

Figure 5. Forward rate decomposition.

Note: This figure displays the decomposition of country-specific one-year forward rates with a maturity of 36 months. The graphs illustrate the breakdown into risk-neutral forward rates and forward term premia for both the $GVAR-ATSM$ and the $JLL-ATSM$. The gray-shaded areas indicate periods of domestic economic contraction. The sample period for the analysis covers June 2004 to January 2020.

30% despite the prevailing forward rate being around 20%. In 2012, it indicates near-zero risk-neutral rates while the current forward rate was approximately 10%. Furthermore, in 2017, after the economic crisis had subsided, the model indicates a highly unrealistic risk-neutral rate above 20%.

Regarding the dynamics of term premia, we find that both the $GVAR-ATSM$ and $JLL-ATSM$ yield rather similar results. Consistent with the existing literature (e.g., Ludvigson and Ng 2009; Bauer, Rudebusch, and Wu 2012), both models demonstrate a countercyclical response of term premia to the business cycle. This pattern is particularly evident in China, Mexico, and Uruguay, where both models exhibit local spikes in term

Table 2. Unconditional moments—forward rates

	China	Brazil	Mexico	Uruguay
Mean				
Forward rates	3.430	12.176	6.980	4.429
Risk-neutral rates				
GVAR	2.400	11.335	5.798	2.383
JLL	2.485	8.710	5.779	2.080
Term premia				
GVAR	1.030	0.840	1.181	2.046
JLL	0.945	3.515	1.203	2.347
Variance				
Forward rates	0.430	8.550	2.063	5.844
Risk-neutral rates				
GVAR	0.807	8.917	1.527	0.591
JLL	0.038	66.834	1.075	0.610
Term premia				
GVAR	1.926	10.091	1.877	6.139
JLL	0.663	36.711	1.918	4.607

Note: Summary statistics (mean and variance) of one-year forward rates with a maturity of 36 months (denoted as $f_{i,t}^{24,36}$) for the four economies in the sample. The table displays model-implied quantities derived from the *GVAR–ATSM* and *JLL–ATSM*, segmented into risk-neutral rates and term premia components.

premia during periods of economic slowdowns. However, for Brazil, the *GVAR–ATSM* demonstrates more plausible dynamics compared to the *JLL – ATSM*, as the latter indicates unrealistic negative risk compensation values below −10% in the years 2004 and 2015.

In Table 2, we extend our evaluation of bond yield decomposition accuracy by examining the unconditional moments, precisely the means and variances, generated by each model. Given the similarity in model fit between the two frameworks, the mean and volatility of the forward rate display little distinction. As a result, we present the results only once to avoid redundancy.

In line with the findings discussed in Figure 5, we observe that the volatility of the risk-neutral forward rate is notably higher for the *GVAR – ATSM* in the cases of China and Mexico. In particular, the *JLL – ATSM* shows relatively low volatility in China, suggesting limited sensitivity to changes in the state of the economy. As pointed out by Bauer, Rudebusch, and Wu (2012), this result can be attributed to the bias-correction procedure implemented in the *GVAR – ATSM*, which leads to slower mean reversion and consequently increases the volatility of risk-neutral rates.

For Brazil, the table further reinforces the disparities between the two models, emphasizing the challenges faced by the *JLL – ATSM* in accurately capturing the dynamics of the risk-neutral rate and term premium. Notably, the variance of the risk-neutral rate is approximately eight times larger than that of the forward rate, and the mean of the risk-neutral rate deviates significantly from the forward rate. In contrast, the *GVAR – ATSM* produces results that closely resemble the unconditional moments of the forward rates. As for Uruguay, both models demonstrate overall similar features.

As a final evaluation, we conduct the linear projection of yields (LPY henceforth) tests proposed by Dai and Singleton (2002). These tests aim to formally assess the model's capacity to accurately capture the historical behavior of yields under both historical and risk-neutral measures. In Supplementary Appendix A.4, we provide a detailed explanation of the methodology employed in conducting these tests and discuss the results comprehensively. Overall, the results of the tests indicate that our *GVAR* model effectively aligns with the observed bond yield dynamics for all four countries of our sample.

2.3 Out-of-Sample Forecasting Performance

In this section, we examine the out-of-sample forecasting performance of the *GVAR – ATSM* compared to the *JLL – ATSM*. We acknowledge that the inclusion of no-arbitrage restrictions in *ATSM*s may not necessarily improve bond yield forecasts compared to standard factor models (Duffee 2011). In this sense, we do not intend to claim that the *GVAR – ATSM* forecasts outperform any other model. Rather, our primary goal is to evaluate whether our less parameterized framework can provide more accurate predictions in contrast to the competing multicountry *JLL – ATSM*.

To generate out-of-sample forecasts, we first estimate the *GVAR – ATSM* and *JLL – ATSM* using a restricted information set from June 2004 to September 2012 (100 observations). Next, we forecast bond yields for horizons of 1, 3, 6, and 12 periods for each country and calculate the forecast errors. This process is iteratively repeated by including the next month's observation and removing the oldest to maintain a constant sample length for each forecast set (rolling window estimator).[31] Table 3 displays the root-mean-square errors (*RMSE*s) for bond yield maturities of 3, 12, 60, and 120 months from this procedure. We also include the *RMSE*s of the random walk model (*RW*) as a benchmark for model-free comparison. The table values are expressed in percentage points.

An overall analysis of Table 3 reveals that the model-based forecasts from both *ATSM*s generally exhibit larger *RMSE*s compared to the *RW* model. In fact, the literature extensively documents the challenge of outperforming random walk forecasts. This challenge is particularly attributed to the high persistence observed in yield curve data (see e.g., Golinski and Zaffaroni 2016), which solidifies the *RW* model as a strong competitor in this context.[32] Moreover, Table 3 highlights two common features about the forecasts properties of *GVAR – ATSM* and *JLL – ATSM*. First, both models tend to capture more accurately future bond yield fluctuations in China and less accurately in Brazil. Second, unsurprisingly, the forecast precision of both frameworks worsens with the forecast horizons. More specifically, for the one-month ahead predictions, the *RMSE*s vary between 0.15% and 1.07%, whereas, for the twelve-period forecast, the errors exceed 5% in the worst cases.

In many instances, the *GVAR – ATSM* outperforms the *JLL – ATSM* with smaller forecast errors. To formally assess predictive accuracy differences between these frameworks, we employ the fixed-*b* and fixed-*m* tests by Coroneo and Iacone (2020). These tests employ fixed-smoothing asymptotics for the Diebold–Mariano test (Diebold and Mariano 1995) to correct for potential large-size distortions in small out-of-sample datasets.[33] Proposed and based on Giacomini and White (2006), these tests also account for the influence of uncertainty in model parameter estimation on test distributions. Specifically, Giacomini and White (2006) and Coroneo and Iacone (2020) establish a forecast environment with

[31] To ensure that the global number of forecast errors is identical across all forecast horizons, the final array of forecasts is generated using the information set containing data from October 2010 to January 2019. Moreover, to ensure stability in the estimation process in each forecasting sub-sample, we impose the restriction that all the risk-neutral eigenvalues must be strictly smaller than 1.

[32] Indeed, studies such as Bauer, Rudebusch, and Wu (2012) have demonstrated that even their bias-corrected framework face challenges in surpassing the forecasting performance of the *RW* model. Furthermore, compelling evidence presented by Diebold and Li (2006) indicates that the *RW* model frequently produces more accurate yield curve forecasts compared to various standard benchmark models.

[33] The fixed-*b* and fixed-*m* tests are based on different asymptotic procedures for the limit distribution of the loss differential between two alternative forecast models. Specifically, the fixed-*b* relies on the assumption that the bandwidth-to-sample size ratio is constant to build the weighted autocovariance estimate of the long-run variance. The fixed-*m* estimates the long-run variance using a weighted periodogram estimate with a constant truncation parameter *m* that is independent of the sample size.

Table 3. Out-of-sample RMSE

	China				Brazil				Mexico				Uruguay			
	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M
$h = 1$																
GVAR-ATSM	0.375	0.294	0.179	0.167	0.237	0.463	0.651	0.707	0.442	0.327	0.360	0.376	1.076	0.501	0.382	0.356
JLL-ATSM	0.349	0.280	0.161	0.151	0.562	0.702	0.839	0.901	0.435	0.365	0.390	0.389	0.872	0.545	0.497	0.554
RW	0.243	0.227	0.146	0.143	0.314	0.460	0.602	0.626	0.213	0.221	0.290	0.284	0.275	0.278	0.299	0.292
$h = 3$																
GVAR-ATSM	0.548	0.540	0.371	0.341	0.633	0.986	1.206	1.277	0.527	0.540	0.718	0.740	1.357	0.726	0.679	0.663
JLL-ATSM	0.482	0.506	0.339	0.310	1.549	1.603	1.703	1.769	0.613	0.653	0.806	0.797	1.039	0.835	0.923	0.912
RW	0.438	0.418	0.281	0.283	0.851	0.978	1.173	1.203	0.429	0.423	0.461	0.512	0.490	0.510	0.486	0.486
$h = 6$																
GVAR-ATSM	0.692	0.716	0.561	0.510	1.398	2.006	2.112	2.069	0.763	0.847	1.073	1.022	1.762	1.138	1.019	0.983
JLL-ATSM	0.651	0.719	0.541	0.480	3.003	2.810	2.521	2.480	0.909	1.065	1.242	1.154	1.488	1.274	1.431	1.430
RW	0.563	0.544	0.436	0.460	1.619	1.670	1.791	1.818	0.717	0.684	0.608	0.702	0.706	0.743	0.744	0.710
$h = 12$																
GVAR-ATSM	0.961	0.941	0.709	0.585	3.406	4.372	4.102	3.834	1.380	1.390	1.342	1.150	2.164	1.440	1.184	1.131
JLL-ATSM	1.082	1.039	0.769	0.616	5.737	5.094	3.914	3.601	1.953	2.038	1.932	1.550	2.584	1.878	2.276	2.447
RW	0.793	0.792	0.687	0.696	2.938	2.889	2.682	2.644	1.301	1.210	0.816	0.864	0.876	0.936	0.946	0.921

Note: The table presents the out-of-sample root mean square errors (*RMSEs*) for the country-specific bond yields with maturities 3, 12, 60, and 120 months (*M*). The forecast horizons (h) are 1, 3, 6, and 12 months. The line denoted by *RW* represents the forecast errors generated by the random walk model. Forecasts are generated using rolling windows of 100 observations, with the initial forecast covering the period from June/2004 to September/2012. The values of the *RMSE* are expressed in percentage points.

asymptotically non-vanishing estimation uncertainty, achieved through rolling windows forecasts, as applied in our analysis.

In Table 4, we report the outcomes for the fixed-b and fixed-m tests for the same set of maturities, forecast horizons, and evaluation periods as those in Table 3. We assume quadratic losses for forecast errors, and the loss differentials are computed by subtracting the (quadratic) forecast errors of the $JLL-ATSM$ from the ones of the $GVAR-ATSM$. As such, negative (positive) values of the test statistics indicate evidence in favor of the superior accuracy performance of the $GVAR-ATSM$ ($JLL-ATSM$). The null hypotheses of the tests are that the predictive ability of the two methods is equal. The entries marked with one (two) asterisk(s) indicate two-sided significance at a 10% (5%) significance level.

The $GVAR-ATSM$ shows superior forecasting performance across the entire maturity spectrum in Brazil and Mexico, and the 120-month maturity bond in Uruguay. The superiority of the $GVAR-ATSM$ forecasts is statistically significant at the 5% level in several cases. Conversely, the $JLL-ATSM$ outperforms in short-term forecasts for some maturities in China, with statistical significance at 5% for 12-month and 60-month bonds, and at 10% for 120-month bonds. Out of all 128 cases reported, the fixed-m and fixed-b tests favor the $GVAR-ATSM$ in approximately 26% of the cases, while the $JLL-ATSM$ is superior in only 7%.[34] When considering additional observed yields (6-month and 36-month maturities), the percentage of cases favoring the $GVAR-ATSM$ remains constant, while the proportion of favorable cases for the $JLL-ATSM$ slightly increases to 8% (not reported).

Having observed the superior performance of the $GVAR-ATSM$ in forecasting yield curves for Brazil, Mexico, and Uruguay, we delve deeper into understanding the specific dynamics that contribute to this advantage. In Figure 6, we present the forecast errors for both the $GVAR-ATSM$ and $JLL-ATSM$ focusing on the same bond maturities as before for all four countries. To save space, we only report the results for the three-month ahead forecast, which exhibits the most contrasting differences between the two models as shown in Table 4. Figure 1 gathers the actual time series of bond yields.

For Brazil, the $GVAR-ATSM$ exhibits significantly smaller forecast errors for all maturities, particularly during two distinct periods: from late 2014 to late 2015 and from early 2016 to early 2017. During the former period, both models tend to underestimate prevailing bond yields (positive forecast errors) as interest rates rise, while during the latter period, they frequently overestimate (negative forecast errors) future yield curve movements as rates trend downward. However, the $GVAR-ATSM$ more accurately captures the changing trends, displaying smaller errors. A similar pattern emerges in Mexico and Uruguay, notably in the mid-2010s. Despite a disrupted downward trend since 2008, the $JLL-ATSM$ persistently forecasts declining rates in Mexico and Uruguay during 2014–2015. These findings suggest that the $GVAR-ATSM$, with its greater flexibility resulting from the more parsimonious representation compared to the $JLL-ATSM$, is more suitable for capturing turning points in the data.

3 Testing the *GVAR* Restrictions

As discussed in Section 1, our $GVAR$-based framework offers a more parsimonious model representation compared to the alternative VAR setups. This improved parsimony is primarily attributed to the inclusion of star variables, which effectively reduce the number of parameters driving the dynamics among the risk factors. In this sense, it can be of interest

[34] As a robustness check, we restrict the evaluation period to the first half of the original forecast period (from September 2012 to November 2015). The main results, as presented in Supplementary Appendix Table A.4, exhibit an improvement in favor of the $GVAR-ATSM$. Specifically, approximately 27% of the cases favor the $GVAR-ATSM$, while less than 2% favor the $JLL-ATSM$.

Table 4. Comparative predictive accuracy test (full sample)

	China				Brazil				Mexico				Uruguay			
	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M
$h=1$																
Fixed-b	0.976	2.737**	2.550**	2.06*	**−3.579****	**−3.165****	**−2.611****	**−2.570****	0.278	−1.628	−1.227	−0.498	1.742	−1.315	−1.612	−1.406
Fixed-m	0.900	3.313**	2.222*	1.936*	**−2.996****	**−2.720****	**−2.502****	**−2.419****	0.283	−1.743	−1.187	−0.473	1.508	−1.153	−1.541	−1.412
$h=3$																
Fixed-b	1.482	2.779**	2.014*	1.695	**−2.745****	**−2.419****	**−1.941***	**−1.891***	−1.493	**−2.172***	−1.203	−0.702	1.314	−1.312	−1.609	**−2.626****
Fixed-m	1.258	2.813**	1.701	1.543	**−2.338****	**−2.034***	−1.761	−1.747	−1.416	**−2.177***	−1.108	−0.66	1.177	−1.101	−1.43	**−2.705****
$h=6$																
Fixed-b	0.527	−0.083	0.585	0.865	**−2.400****	−1.663	−0.890	−0.887	−1.574	**−1.923***	−1.064	−0.891	0.794	−0.916	−1.581	**−2.181***
Fixed-m	0.483	−0.082	0.549	0.829	**−2.065***	−1.415	−0.752	−0.745	−1.363	−1.714	−0.956	−0.829	0.676	−0.788	−1.342	−1.842
$h=12$																
Fixed-b	−1.169	−1.033	−0.727	−0.400	−1.684	−0.741	0.264	0.351	**−2.454****	**−2.618****	**−2.829****	**−2.540****	−0.851	−1.200	**−1.886***	**−2.524****
Fixed-m	−1.210	−0.938	−0.642	−0.342	−1.498	−0.702	0.261	0.344	**−1.895***	**−2.092***	**−2.374****	**−2.240***	−0.729	−1.027	−1.636	**−2.231***

Note: Fixed-*m* and fixed-*b* test statistics of comparative predictive accuracy from Coroneo and Iacone (2020). The forecast error losses are quadratic. Positive entries indicate a higher average loss for the forecast produced by the *GVAR–ATSM*. The null hypotheses are that both models have equal predictive accuracy. Entries with one (two) asterisk(s) show two-sided significance at a 10% (5%) significance level. Cells highlighted in bold indicate a statistically superior forecasting performance in favor of the *GVAR–ATSM*. The evaluation period of the tests is from September 2012 to January 2019.

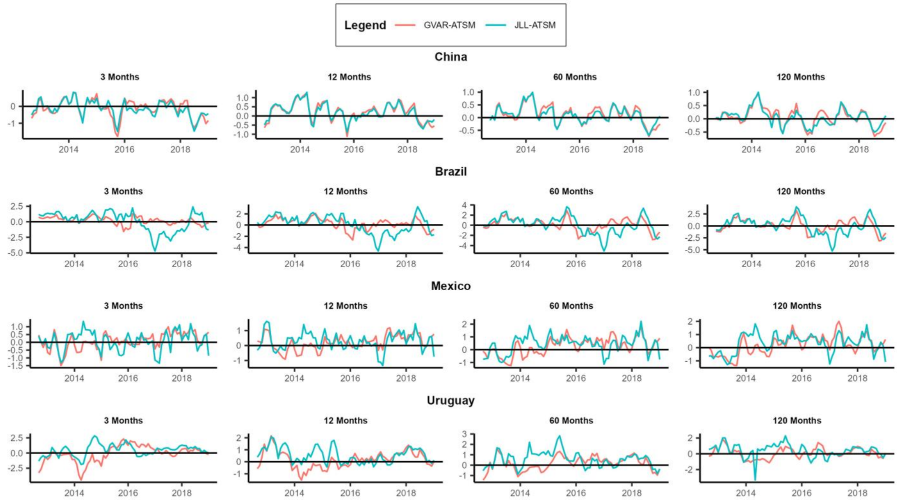

Figure 6. Forecast errors for 3-month ahead projections.

Note: Forecast errors of the country-specific bond yields with maturities 3, 12, 60, and 120 months. The forecast horizon is set to three months ahead ($h = 3$). The information set of the first (last) forecasts contains data from June/2004 (October/2010) to September/2012 (January/2019).

to investigate whether the imposed restrictions through the star variables are effectively supported by the data. In essence, testing the validity of these restrictions involves evaluating the suitability of trade flows as a measure of interconnectedness within our *GVAR* setup. To achieve this, we present a formal statistical test that assesses the validity of the restrictions imposed in our *GVAR* in comparison with an unrestricted *VAR* model.[35] This test is conducted within a four-country framework, consistent with our empirical analysis developed in Section 2.[36]

To begin, we construct an unrestricted reduced form $VAR(1)$ model that captures the dynamics of the risk factor in country i. This model incorporates lagged values of the risk factor in country i, as well as variables from the three other countries in the economic system (indexed by j, k, and l), and the world economy (indexed by W):

$$Z_{i,t} = C_i + \Phi_{\mathrm{i}} Z_{i,t-1} + \Phi_{\mathrm{j}} z_{j,t-1} + \Phi_{\mathrm{k}} z_{k,t-1} + \Phi_{\mathrm{l}} Z_{l,t-1} + \Phi_{\mathrm{W}} Z^{W}_{t-1} + \Sigma_i \varepsilon_{i,t}. \quad (8)$$

Next, we note that the restricted *VAR* version of our model for country i corresponds to the $VARX^{*}(1,1,1)$ presented in Equation (5). We reformulate it as follows:

$$Z_{i,t} = C^{X}_{i} + \Phi^{\mathrm{X}}_{\mathrm{i}} \mathrm{Z}_{\mathrm{i,t-1}} + \Phi^{*}_{\mathrm{i}} (\mathrm{w}_{\mathrm{ij}} \mathrm{Z}_{\mathrm{j,t-1}} + \mathrm{w}_{\mathrm{ik}} \mathrm{Z}_{\mathrm{k,t-1}} + \mathrm{w}_{\mathrm{il}} \mathrm{Z}_{\mathrm{l,t-1}}) + \Phi^{\mathrm{X}^{\mathrm{W}}}_{\mathrm{i}} \mathrm{M}^{\mathrm{W}}_{\mathrm{t-1}} + \Sigma^{\mathrm{X}}_{\mathrm{i}} \varepsilon^{\mathrm{X}}_{\mathrm{i,t}}. \quad (9)$$

Thus, the unrestricted *VAR* in Equation (8) encompasses our restricted model through the following relationship:

[35] In a general *ATSM* with unspanned risk, jointly testing the validity of restrictions for P and Q dynamics parameters is challenging. This complexity arises because, within this model category, market prices of macroeconomic risk factors are not identified under the null hypothesis of unspanned macro risk, potentially leading to a non-standard distribution of the overidentifying test statistic (see, e.g., Dufour 1997). Thus, our focus is exclusively on testing restrictions within the physical dynamics.

[36] We thank here an anonymous referee for advising the implementation of this testing procedure.

Table 5. Test of model restrictions

	China	Brazil	Mexico	Uruguay
LR statistic	33.839	66.238	34.953	34.788
p-value	(0.961)	(0.062)	(0.947)	(0.950)

Note: The table displays the results of the statistical test conducted to evaluate the validity of the imposed restrictions in the *GVAR* model. The *LR statistic* entries correspond to the value of the statistic from a standard likelihood ratio test. The null hypothesis posits that the restrictions are supported by the data. The critical value of the test, given a significance level of 5% and 50 restrictions, is $\chi^2_{50} = 67.5$.

$$\Phi_j = \Phi_i^* w_{ij}, \quad \Phi_k = \Phi_i^* w_{ik}, \quad \Phi_l = \Phi_i^* w_{il}.$$

These interrelations enable us to compare the restricted and unrestricted models and evaluate the validity of the imposed restrictions. To conduct this assessment, we perform a standard likelihood ratio test for each of the four economies in the sample. This test involves estimating Equations (8) and (9) and employing the maximum likelihood estimates from these setups to construct the test statistic. Specifically, since both the restricted and unrestricted models are based on conditionally Gaussian distributions, the test statistic for country i can be written as:

$$LR_i = T\, ln\left(\frac{\hat{V}_{i,r}}{\hat{V}_{i,u}}\right),$$

where T is the time-series dimension of both models and $\hat{V}_{i,u}$ and $\hat{V}_{i,r}$ denote the variance-covariance matrices of the unrestricted and restricted models, respectively, given by $\hat{V}_{i,u} = \frac{1}{T}\sum_{t=1}^{T}(\varepsilon_{i,t}^{'}\varepsilon_{i,t})$ and $\hat{V}_{i,r} = \frac{1}{T}\sum_{t=1}^{T}(\varepsilon_{i,t}^{X'}\varepsilon_{i,t}^{X})$.

Under the null hypothesis, it is assumed that the set of restrictions imposed on the physical parameters holds. Intuitively, a small value of LR_i suggests no significant loss of information from the imposed restrictions, indicating consistency with the data. In contrast, a large value of LR_i indicates a substantial loss of information, suggesting inconsistency between the imposed restrictions and the data. Asymptotically, under the null, the test statistic LR_i follows a $\chi^2_{(q)}$ distribution, where q represents the number of restrictions imposed in the restricted model. In the context under study, $Z_{i,t}$ is a 5-dimensional vector, leading to the estimation of 115 (65) parameters for the unrestricted (restricted) model, thus yielding 50 restrictions.

Table 5 displays the outcomes of the likelihood ratio test conducted for the four countries of our sample, with a significance level set at 5%. The findings suggest that no significant evidence exists to reject the null hypothesis in any of the cases. Notably, the evidence is particularly strong for China, Mexico, and Uruguay, where the p-values are approximately 95%. While the evidence for Brazil is less robust, it remains statistically significant at the 5% level. Overall, these test results provide robust support for the selection of trade weights as an appropriate measure of interconnectedness.

4 Robustness Checks

As previously stated, the transition matrix holds a critical position in *GVAR* models, as it determines the level and nature of interdependence among countries. As a result, using different measures of interconnectedness can lead to varied in-sample and out-of-sample forecasting outcomes. Accordingly, an essential question arises concerning the robustness of our primary findings to plausible modifications of our *GVAR* benchmark specification presented in Section 2.

In our baseline model, we assume that the interconnectedness between any two economies is represented by the average bilateral trade flows from 2004 to 2019, implying a stable degree of dependence over time. To enhance generality, we estimate two models that allow these weights to vary over time.[37] Specifically, we modify the benchmark model in the two dimensions where the interconnectedness measure, $w_{i,j}$, are involved: the formation of the star variables, $Z^*_{i,t}$, and the link matrices, W_i. In both alternative models, we build the star factors using specific trade flows corresponding to the year of the risk factor observation. Next, we use trade weights from 2004 in one model version and weights from 2019 in the other model version to construct the link matrices. By setting W_i at the two extreme years of our sample, we can capture the effect of the increased dependence of Latin American economies on China over the recent years and assess how it influences our empirical results.[38]

Additionally, considering our primary focus on bond markets, we investigate alternative setups using financial flows as a proxy measure of interconnectedness. To construct these measures, we use the yearly portfolio flows data from the IMF's Coordinated Portfolio Investment Survey. However, due to data availability constraints, the sample span for the four countries in our sample is limited to start from 2015. As a result, we set $w_{i,j}$ to be fixed, specifically taking the average between the years 2015 and 2019, for both the star factors and the link matrices. Based on this measure, we estimate two models. One setup is exclusively based on financial flows, while the other uses a hybrid approach, associating the strength of dependence of the unspanned factors with the average of the full sample fixed trade flows, and the spanned factors with portfolio flows.

To save space, we present only the *GIRF*s as an in-sample comparison metric and the forecasting performance as an out-of-sample comparison. The labels *Trade 2004* and *Trade 2019* refer to models with time-varying trade weights, where the year in the labels corresponds to the link matrices used. Additionally, we designate *Financial* for the model using financial flows as the sole measure of interconnectedness, and *Trade-Financial* for the setup combining both trade and portfolio flows. For the out-of-sample analysis, we also include the outputs from the model labeled *Trade*, which represents our benchmark specification. We report the alternative measures of transition matrices in Supplementary Appendix Table A.6 in the .

Figure 7 presents the responses of all bond yield maturities to an economic growth shock in China, as reported in Figure 2. Remarkably, our analysis demonstrates a high degree of consistency with our baseline model's results. Specifically, for Brazil and Mexico, the growth shock from China leads to the yield curve steepening in all four specifications, with long maturity bonds responding positively in most cases. For Uruguay, we observe a consistent downward response of yields in all three models that use trade flows as an interdependence proxy, mirroring the results of our benchmark model. The sole exception is the *Financial* model, where a shift upwards is observed.

[37] Arguably, nominal exchange rate movements likely influence bond investor decisions in global financial markets. Although we do not explicitly include exchange rate volatility as a separate variable in the model, using time-varying trade flows as a measure of interconnectedness may indirectly capture the impact of exchange rate fluctuations. In particular, as trade flows change over time, they reflect shifts in currency exchange rates and economic activity, providing a way to account for potential exchange rate effects in our analysis.

[38] Our approach aligns with the methodology used by Cesa-Bianchi et al. (2012), who employed a *GVAR* with a time-varying $w_{i,j}$s and year-specific W_i matrices to study how structural changes in trade patterns in recent decades, attributed to China's emergence as a global economic force, have impacted the business cycle in Latin America. Supplementary Appendix Table A.6 confirms that the strength of dependence of Brazil, Mexico, and Uruguay on China's trade has substantially increased in these 15 years, with China accounting for the largest shares in these three countries in 2019.

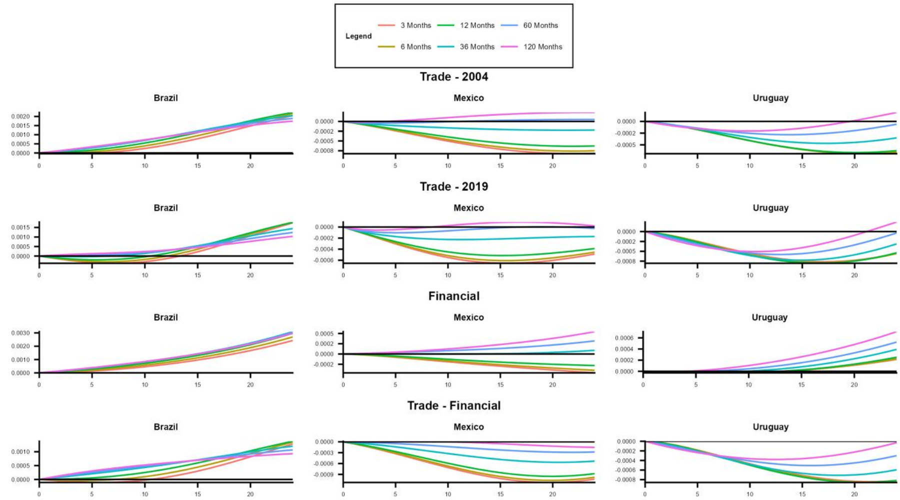

Figure 7. Generalized impulse responses from the *GVAR–ATSM*—shock to economic growth in China (several models).

Note: The labels *Trade 2004* and *Trade 2019* refer to models where interconnectedness measures are solely based on trade flows. In these models, star variables are computed using yearly time-varying weights, and link matrices are determined by the trade flows of the years 2004 in *Trade 2004* and 2019 in *Trade 2019*. The label *Financial* refers to a setup based exclusively on the fixed-weight average of portfolio flows from 2015 to 2019, used in the computation of both the star factors and the link matrix. *Trade-Financial* is a fixed-weight model with interconnectedness measures based on the average of trade flows from 2004 to 2019 and the average of portfolio flows from 2015 to 2019, associated with the unspanned and spanned factors, respectively. The *x*-axes are expressed in months and the bond yield responses (*y*-axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

To compare the out-of-sample forecasting performance of the different models, we present the results in Supplementary Appendix Table A.5. To facilitate comparison with the results discussed in Section 2.3, we report the same forecast horizons, bond maturities, and forecast assessment period used in Table 3.[39]

In general, we observe that all models produce *RMSE*s of comparable magnitudes across different forecast horizons, maturities, and countries. However, the benchmark model stands out by outperforming the *Trade-Financial* model with smaller *RMSE*s in approximately 64% of cases. Regarding the *Trade 2004* and *Financial* models, the baseline's performance is relatively similar, with around 48% and 44% of superior cases, respectively. Notably, in the case of the *Trade 2019* specification, the baseline setup exhibits superior performance in only 17% of instances. This finding suggests that considering the most recent strength of interdependence among the countries, where China plays a dominant role as a leading economy within this cohort, can lead to improved explanatory power in the forecasting performance of *GVAR* models.

5 Conclusions

This article builds a multicountry *ATSM* in which the dynamics of the risk factors evolve according to a *GVAR* model. Compared to extant multicountry frameworks, *GVAR – ATSM* captures the interdependence among countries within economic systems since it requires the specification of a cross-border transmission channel. Our setting also

[39] Facing challenges in estimating the *Financial* model at seven information sets using the *BRW–BC* procedure, we resort to forecasting at these points without implementing the bias correction procedure.

offers a more parsimonious term structure model representation, which grants a faster estimation process while providing more economically meaningful empirical results and better predictive power.

Our *GVAR* – *ATSM* is estimated on a system composed of four emerging markets (Brazil, China, Mexico, and Uruguay) for which the trading linkages are relevant. Based on the analysis of impulse responses and variance decompositions, we show that the Chinese output growth shock, alongside other cross-border developments, has an important influence on the yield curve dynamics of these three Latin American countries. This result underscores the important dependence of Latin America on worldwide economic events. Furthermore, our findings support that China has recently assumed a major role in the emerging markets of Latin America. While earlier studies, pointed to the U.S. economy as an important source of shock transmission to Latin America, we present evidence that financial investors seem to account for economic developments in China as a relevant factor of risk. Moreover, the *GVAR* – *ATSM* produces more economically plausible risk premia dynamics, further supporting the robustness and effectiveness of our approach. Our results also demonstrate that *GVAR* – *ATSM* outperforms the referenced multicountry *JLL* – *ATSM* in terms of the out-of-sample forecasting performance of bond yields.

The *GVAR* – *ATSM* paves the way for a broad range of empirical questions related to the interactions among term structures and the macroeconomy around the globe. An interesting extension of our setting is to integrate economies whose interest rates are constrained by an effective lower bound. Such a framework certainly raises major estimation challenges and thus is left for future research.

Supplemental Material

Supplemental material is available at *Journal of Financial Econometrics* online.

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