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Essays on group decision-making with strategic interactions: an application to informal insurance

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À mon très cher papa,
À mes rayons de soleil, Arthur et Lili,
À Bertrand.

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Introduction

In developing countries, a large proportion of the population derives its income from agricultural activities. These activities are exposed to substantial risks due to large fluctuations in agricultural productions (due notably to weather shocks and diseases) and prices. In order to smooth their income and consumption, households would like to appeal to credit and insurance markets. Unfortunately, markets and state institutions are often missing or malfunctioning in these areas. The lack of credit and insurance markets lead individuals to form groups where members are both consumers and providers of insurance and/or credit services. Members are bound by a set of social norms and standards which economists consider as "institutions". The fact that these informal institutions are a response to missing markets explains their central role to promote development in rural areas. Indeed, not only do they have a direct role in terms of insurance or credit, but they also are a channel to share information and build social capital.

Institutions, legal or not, which are a set of standards and rules that govern community life (market, public authorities, social norms...) are an important ingredient of all societies. Even if we reduce the scope of economic development to a country's growth, empirical studies conclude that institutional factors are strong determinants of growth rates (Barro (1991), Barro (1998)). The importance of an appropriate set of institutions has also been stressed by North (1994) who has shown that, contrary to the general opinion, free market forces by themselves do not generate growth. Therefore, the economic development of a country will depend on its institutional capacities to cope with conflicts of interests which its agents face.

In this study, we address the question on which conditions and how informal institutions emerge. We investigate decision-making within institutions in the presence of conflicts of interest. In all three chapters of this thesis, an element of the problem itself is endogenous: the impact of the set of options from which the group can choose (Chapter 1), the formation of the group itself (Chapter 2) or how membership affects agents' behavior (Chapter 3).

In Chapter 1, drawn from an article co-written with Frédéric Gaspart, we look at the constitution of the set of alternatives to choose from. Given the extreme generality of the economic environment considered in this chapter, alternatives can have several representations and be applied to various contexts, including the one under study. Indeed, alternatives may be a set of individuals when a community gets to choose a local leader, or a set of platforms if the group gets to set the rules to apply to the insurance group, such as how much income members should commit to share, or who shouldn't be given access to the group. This chapter considers group decision-making in situations where the information structure of the conflict is very poor: only individuals' ordering on the set of alternatives matter. The tool for studying phenomena in such environments is the voting theory, also called ordinal multi-criteria analysis. As its name suggests, this method attempts at characterizing voting rules which satisfy a list of properties, or axioms. Among these desirable properties, this chapter focuses on the stability of the rule's outcome in the presence of variations in the set of alternatives. Our analysis defines this property in the following way: if a new alternative which does not bring about unfavorable information for the winner(s) enters an election, the outcome of the election should be unaltered. In other words, we hope, somehow, that only "relevant" information is used in a collective decision. We show that, if this property is formulated in a suitably weakened form, it is satisfied by many voting rules. We then study its compatibilities with other properties suggested in the literature and open the field of some possibilities.

In the last two chapters, the economic environment is more structured and is directly applied to the context of rural areas of developing countries. The setting is characterized by volatile incomes and informal risk sharing agreements as an alternative to formal insurance services. When such an informal insurance scheme is implemented, two types of moral hazard may occur: ex ante and ad interim moral hazard. The ex ante moral hazard results from an agent who becomes insured may adapt his behavior in order to increase his welfare at the expense of the other providers of insurance. Ex ante moral hazard may take the form of lower effort to limit risk or riskier production choices in order to increase expected income. The ad interim moral hazard appears in situations in which, after observing their income, individuals prefer to renege on their obligations. Since the lack of state intervention leads to imperfect contract enforceability, an informal insurance agreement must be designed in order to avoid defection, that is, its benefits must be, at any point in time, larger than the net benefit of defection. Chapter 2 addresses this issue by accounting for the existence of ad interim moral hazard of members, while Chapter 3 explores consequences of ex ante moral hazard through excessive risk-taking as a result of insurance availability.

In Chapter 2, we present a theory of endogenous formation of insurance groups which combines heterogeneity on agents' risk aversion under asymmetric information and lack of enforceability of contracts. Insurance services are repeated over time, and members reneging on their commitment are permanently excluded from the group. Income sharing inside the group is decided by majority voting and the size of the group adjusts to this decision through participation constraints. We show that in the long run, all group members agree on the same imperfect level of income sharing, which maximizes their utility subject to the participation constraint. This leads to a constrained-efficient equilibrium. A mean preserving spread of income implies more income sharing and a larger group size, which unambiguously improves the quality of insurance and allows more agents to become members. New members and possibly even old members may be better off, while non-members are worse-off. These results have relevant policy implications since they stress that one has to be careful about introducing risk-reducing policies without accounting for the adverse effect they might have on the stability of local communities.

Finally, in Chapter 3, which is a joint work with Matthieu Delpierre, we start from the twofold empirical observation that, on the one hand, the rate of technology adoption tends to be low among the poorest farm households and that, on the other hand, their consumption path seems to be more affected by idiosyncratic shocks. In order to explain these stylized facts, we study the interaction between a mutual insurance mechanism and the individual's risk-taking behavior, while accounting for the role of household wealth. Heterogeneously risk-averse agents non-cooperatively decide on their risk-coping strategies, namely their level of subscription to the mutual insurance scheme and their production choices. Since the participants may simultaneously choose their self- and mutual insurance strategies, the participation decision of all types and their respective risk-taking behavior are endogenous with the institutional context (the insurance mechanism). We highlight the inability of poor farmers to make an efficient use of community risk pooling mechanisms and their resulting need to smooth income ex ante. Indeed, thanks to their possibilities to rely on buffer stocks to smooth consumption, rich households tend to adopt riskier production strategies. Therefore, the income pool is more volatile and poorer farmers partially withdraw from mutual insurance by reducing their subscription to the income pool.

CHAPTER 1

Resistance to Strategic Candidacy: Axioms and Possibilities

In chapter 1, we study the stability of the voting outcome when variations in the set of candidates occur. Following Dutta et al. (2001)'s study of manipulation of voting procedures by a candidate, we suggest a weakened version of *Candidate Stability* in a multivalued environment. This weakening requires that if a new candidate who does not bring about unfavorable information for the winners enters the election, the outcome of the election is unaltered . We show that, taken alone, this property is satisfied by many voting rules. We study its compatibilities with other properties suggested in the literature and open the field of some possibilities.

1. Introduction

The key idea in a voting problem is that all the voters' preferences must be adequately taken into account by the voting rule. This meaning is attached to any implementability condition, exemplified by Strategy-proofness (Gibbard (1973), Satterthwaite (1975)), Maskin Monotonicity (Maskin (1985)) and other axioms. More specifically, the possibility of manipulating the outcome of voting rules by the voters, by the candidates or by an external authority, reflects a failure to process in an adequate fashion all the information contained in the problem.

In this chapter, we focus on the stability of the voting outcome when variations in the set of candidates occur. Indeed, there are circumstances in which the arrival or the withdrawal of a candidate in an election can produce unexpected effects. The French presidential election in 2002 was presented by the press as a case where the right-wing nationalist candidate, Jean-Marie Le Pen, disrupted a likely victory of the left-wing by eliminating the socialist candidate, Lionel Jospin, in the first run of the election. The iterative nature of this election was blamed for this possible bias, although it is of a much more general nature as we will see. Likewise, the question whether the outcome of the US presidential election in 1992 was influenced by the candidacy of Ross Perot has been put forward and studied in political science, in particular Brams & Merrill (1994). These two cases are very large elections but the impact of the addition or withdrawal of a candidate is just as relevant and more frequent in a small group if the election is tight between the other candidates and if everybody is well-informed about voters' preferences.

Dutta et al. (2001) initiate the study of manipulation of voting procedures by a candidate. They study single-valued voting procedure with the minor restriction on the domain of candidates' preferences that each candidate prefers to be elected. In this framework, they show that the outcome of every non-dictatorial voting procedure which satisfies unanimity is affected by the incentives of candidates to strategically influence the outcome by entering or exiting the election. Ehlers & Weymark (2003) provide an alternative, relatively simple, proof of Theorem 4 of Dutta et al. (2001) while Eraslan & McLennan (2004) generalize it in two directions. They first show that this theorem still holds when the outcome of the procedure is allowed to be a set of candidates. The second generalization is to allow the candidates and the voters to have weak preference orderings on the candidates' set. In an independent paper, Rodríguez-Álvarez (2006) also considers the extension to multi-valued voting procedures obtaining the special case of Eraslan & McLennan (2004) when both voters and candidates are assumed to have strict preferences. It also provides an extensive discussion of the connection between strategy-proofness and possible definitions of stability for multi-valued voting selections.

In this chapter, we analyze multi-valued voting procedures when the voters have strict orderings on the set of options. The reason why we study correspondences stems from the fact that we require our voting rules to satisfy *anonymity* and *neutrality*. In this environment, with no hypothesis on candidates' preferences, we suggest a weakened property on Dutta et al. (2001)'s *Candidate Stability*: it requires that the outcome of an election be unaltered even if a new candidate who does not beat the former winner in a majority contest enters the election. Unlike Dutta et al. (2001)'s *Candidate Stability* which leads to an impossibility result, our axiom is satisfied by many voting rules while retaining much of the original appeal of the concept. Indeed, it can be argued against *Candidate Stability* that it requires too much independence with respect to the information brought about by the new candidate, i.e. all its pairwise majority contests and, in particular, its relative position in individual orderings compared with the former winner and its contenders. By adding a new clause, our axiom focuses on cases where irrelevant candidates arrive or depart. We show that this property provides a new justification for some Condorcet-consistent voting rules, in that our axiom and a weak technical requirement imply together a weakened version of the Condorcet principle. Also, in the case of three candidates, we provide a characterization of the Simpson-Kramer minmax rule, a rule which received previously little attention from the axiomatic literature.

The chapter is structured as follows. We devote Section 2 to the presentation of the setting. In Section 3, we present *Resistance to Strategic Candidacy*. We discuss its implications on the *Condorcet's Principle* in Section 4. Finally, in Section 5, we provide an illustration of the open possibilities when we combine *Resistance to Strategic Candidacy* with a *Consistency* property inspired by Young (1975).

2. Definitions and Notations

Let $X = \{a, b, c, d...\}$ be the non-empty set of candidates with cardinality $|X| = m$. We consider the case where $m \geq 3$. The case of two potential options is solved indisputably by selecting the majority winner(s) (May (1952)). The $3-$candidate case is the simplest case where issues of manipulation are really relevant and strategic candidacy may occur.

Let $N = \{1, 2, ...n\}$ be the non-empty set of voters, where n does not need to be odd. Assuming n to be odd would be difficult to defend when one is concerned with manipulation issues in small groups in which case tie may appear either in the true preference profile or as a result of strategic misrepresentation of preferences.

Each voter i has a linear preference ordering over X, that is a complete[1], transitive[2] and asymmetric[3] binary relation over X denoted R^i which belongs to $L(X)$, the set of linear orderings over X. A preference profile is a sequence $R^N = (R^1, R^2, ..., R^n)$. The set of all possible preference profiles of linear orderings over X is $L(X)^N$. Our axioms are related to the idea of manipulation in a context of complete information about R^N, for the candidates as well as for the voters.

A Social Choice Correspondence (SCC) is a mapping $S : L(X)^N \to 2^X$. It selects one or more candidates for each preference profile R^N. Indeed, as we assume from the outset that social choice rules are *anonymous*[4] and *neutral*[5] (names of the candidates and of the voters do not matter), the voting procedures have to allow for multi-valued choices. Furthermore, as the case with just two potential candidates is easily handled with a majority vote we will focus on social choice correspondences that coincide with majority voting in the two-candidate case.
We define a muti-valued environment as an environment where the collective choice mechanism is a SCC which selects different elements in X. We oppose such an environment to single-valued environments where the Social Choice Function (SCF) looks for a single

[1]A complete binary relation on X is such that $\forall a, b \in X$, we have aRb or bRa.

[2]A transitive binary relation on X is such that $\forall a, b, c \in X$, aRb and bRc implies aRc.

[3]An asymmetric binary relation on X is such that $\forall a, b \in X$, aRb implies $\neg(bRa)$.

[4]A social choice correspondence is *anonynous* if and only if $\forall\ \pi_1$, a permutation of $N; \forall R^N, R^{N\prime} \in L(X)^N : \left[\forall i \in N; \forall x, y \in X : xR^i y \Leftrightarrow xR^{\pi_1\prime} y\right] \Rightarrow S(R^N) = S(R^{N\prime})$

[5]A social choice correspondence is *neutral* if and only if $\forall\ \pi_2$, a permutation of $X; \forall R^N, R^{N\prime} \in L(X)^N : \left[\forall i \in N; \forall x, y \in X : xR^i y \Leftrightarrow \pi_2(x) R^{\pi_2\prime} \pi_2(y)\right] \Rightarrow \pi_2\left(S(R^N)\right) = S(R^{N\prime})$

winner of the election:$S : L(X)^N \to X$. There is a third class of collective choice mechanisms, Social Welfare Functions, which maps a set of individual linear orderings for everyone in the society to a social ordering: $S : L(X)^N \to L(X)$.

Finally, let us introduce a few notations that will be abundantly used throughout the paper. For any pair of candidates $\{y, z\}$, let $n_{zy}(R^N) = \#\{i \in N : zR^iy\} - \#\{i \in N : yR^iz\} = -n_{yz}(R^N)$, that is the difference between the number of voters in favor of candidate z against y and the number of voters in favor of candidate y against z. If z beats y in a majority tournament then $n_{zy}(R^N) > 0$.
We call a candidate $x \in X$ a *Condorcet winner* if $\forall y \in X, n_{xy}(R^N) \geq 0$.

3. Resistance to Strategic Candidacy

Dutta et al. (2001) introduce a property of *Candidate Stability* which requires that no candidate can strategically affect the outcome of a voting rule by withdrawing from an election. This means that it has to be a Nash equilibrium for all candidates to enter an election. They show that, in single-valued elections, a voting procedure satisfying *candidate stability* and *unanimity*[6] has to be dictatorial. Eraslan & McLennan (2004) generalize this result to multi-valued environments. To extend their theorem, they use a multi-valued variation of *Candidate Stability*:
$\forall x \in X$ and $\forall R^N \in L(X)^N$: (1) If $x \notin S(R^N, X)$ then $S(R^N, X) = S(R^N, X\backslash\{x\})$, and (2) $S(R^N, X) \subset S(R^N, X\backslash\{x\}) \cup \{x\}, \forall x \in X$.
However, as pointed out by Dutta et al. (2001), while (1) is a multivalued analog of their candidate stability property, (2) is a less clear implication of strategic candidacy. To focus on the possibility for candidates to manipulate the outcome of an election, we analyze

[6]A SCF S satisfies *unanimity* if $S(Y, R^N) = x$ for any $Y \subset X, R^N \in L(X)^N$, and $x \in Y$ such that $top(Y, R^i) = x$ for each $i \in N$.

part (1) of the condition. We rewrite this condition as an independence condition, candidacy-proofness: a SCC has to be immune to the manipulation by the candidates in a case where a candidate, almost sure to loose anyway, decides to withdraw in order to influence the final outcome of an election.

Candidacy-proofness[7]**:**
$\forall R^N \in L(X)^N, y \in S(R^N, X\backslash\{x\})$ *and* $x \notin S(R^N, X) \Longrightarrow y \in S(R^N, X)$.

If a SCC S satisfies this property, a winner in a $n-$candidate contest can only be overturned by a newcomer if the newcomer wins the $(n+1)-$candidate election. Even if some SCC (such as the top cycle correspondence) may satisfy this property, it is quite demanding. Indeed, in a case where $X = \{a, b, c\}$, this property can also be written:

$$\begin{matrix} a \notin S(R^N, \{a,b,c\}) \\ c \notin S(R^N, \{a,b,c\}) \end{matrix} \Longrightarrow n_{ba}(R^N) > 0$$

meaning that if neither a nor c are selected by a voting rule then b must beat a in majority contest; actually it must also beat c since a and c are interchangeable in the premise. Hence, in a $3-$candidate case, a candidate is selected alone only if she wins strictly all her majority contests.

Furthermore, it makes sense that a new candidate, not too bad, may modify the result of an election because she brings some complementary information helpful for a comparison of the other candidates. However, the complementary information should be irrelevant if the additional candidate is no match for the previous winner, namely she performs poorly in a majority contest against the winner of the $n-$candidate election. In such instances, the voting

[7]It is obvious that this property is implied by part (1) of the multivalued variation of *Candidate Stability*.

results shouldn't be modified by the addition of a new candidate. Formally, if a candidate y is selected by a SCC, then y must still be selected whenever a new candidate x, beaten by y in a majority contest and sure to loose anyway, enters the election. We call this property *Resistance to Strategic Candidacy.*

Resistance to strategic candidacy (RSC):
$\forall R^N \in L(X)^N, \text{if}\, y \in S(R^N, X\backslash\{x\}) : [x \notin S(R^N, X) \text{ } and \text{ } n_{yx}(R^N) \geq 0] \Longrightarrow y \in S(R^N, X)$

RSC implies that the outcome selected by a SCC is not too sensitive to "small" changes in the preference profile. We examine an example to illustrate this fact.
Let $X = \{a, b, c\}$, $\#N = 4$. Consider the profile of voters' preferences R^N where

$$\begin{aligned} aR^i bR^i c \text{ for } i &= 1, 2; \\ bR^i aR^i c \text{ for } i &= 3, 4. \end{aligned}$$

The preference profile becomes $R^{N\prime}$ when voter $i = 4$ shifts its preferences to $bR^4 cR^4 a$. By anonymity, the outcome selected by the voting procedure when only a and b run for the election has to be $S(R^N, \{a, b\}) = S(R^{N\prime}, \{a, b\}) = \{a, b\}$. Furthermore, a and b still have to be selected in the presence of c for R^N and $R^{N\prime}$. Indeed, in order to satisfy *RSC*, a SCC selecting c in the preference profile R^N has to select a and b as well because if $b \notin S(R^N, X)$, since $a \in S(R^N, X\backslash\{b\})$ (the SCC has to select the majority winner in the $2-$candidate case) and $n_{ab}(R^N) \geq 0$, RSC implies that $a \in S(R^N, X)$. The same is true if $a \notin S(R^N, X)$ since $b \in S(R^N, X\backslash\{a\})$ and $n_{ab}(R^N) \geq 0$, it implies that $b \in S(R^N, X)$. The same argument applies in the preference profile $R^{N\prime}$. An immediate conclusion of the axiom can also be drawn in the $3-$candidate case: in the presence of a Condorcet winner, another candidate cannot be selected alone. This just amounts to rewriting the axiom by contraposition.

Let us examine some examples of voting procedures which do not satisfy this criterion.

1) The Copeland rule:

We denote by $Cop(R^N)$ the Copeland solution set (Copeland (1951)):

$\forall R^N \in L(X)^N, x \in Cop(R^N)$
$\Longleftrightarrow x \in \arg\max[\#\{y \in X | n_{xy}(R^N) \geq 0\} - \#\{y \in X | n_{yx}(R^N) \geq 0\}]$

Let us illustrate this rule with the following example. Let $X = \{a, b, c\}, \#N = 4$ and a preference profile R^N such that

$$\begin{aligned} aR^ibR^ic \text{ for } i &= 1; \\ bR^iaR^ic \text{ for } i &= 2; \\ bR^icR^ia \text{ for } i &= 3; \\ cR^iaR^ib \text{ for } i &= 4. \end{aligned}$$

This example shows that the Copeland rule does not satisfy the RSC property. Indeed, $Cop(R^N, X) = \{b\}$ and $Cop(R^N, \{a, b\}) = \{a, b\}$ while $n_{ac}(R^N) \geq 0$. Candidate a is not elected anymore because of the arrival of c.

2) Scoring Rules

It can also be shown that all scoring rules don't satisfy this property. In the case of three candidates, all scoring rules may be distinguished from each other on the basis of a single parameter $\lambda \in [0, 1]$. Each voter gives 1 point to her preferred candidate and λ point to the candidate in second position. The rule picks up the candidate(s) with the highest total score.

With the same preference profile R^N as for the Copeland rule, we show that none of the scoring rules with $\lambda < 1$ satisfy this criterion. Indeed, $SC_\lambda(R^N, \{a, b\}) = \{a, b\}$and $SC_\lambda(R^N, X) = \{b\}$ because the score of b, $s_\lambda(b)$, is higher than the scores of a and c when $\lambda < 1$.

The profile R^N used to show that the Copeland and scoring rules ($\lambda < 1$) are vulnerable to strategic candidacy is interesting in itself. It is composed of a Condorcet-cycle triplet plus a fourth preference ordering different from the other three. RSC says that the top of the fourth preference order should not be selected alone. This seemingly surprising phenomenon stems from the fact that the second candidate is a Condorcet winner too. In the particular profile R^N, our axiom requires that both Condorcet winners be selected. For example, antiplurality (i.e. scoring with $\lambda = 1$) selects both, although a six-voter profile can be constructed to show this rule is vulnerable to strategic candidacy.

But there are voting rules which satisfy this criterion: for example, all voting rules which select all the candidates who are not strictly beaten in any majority contest.

4. Deduction of the Weak Condorcet Principle

The Condorcet principle has been widely studied in the voting literature and imposes that a voting procedure always selects the Condorcet winner when such a candidate exists. In a multivalued environment, such a criterion may be generalized in several ways. Indeed, ties may occur and several Condorcet winners may coexist. A first version of this criterion requires that a voting procedure selects all Condorcet winners when they exist and only them. A weaker version would only impose that a voting procedure selects all Condorcet winners but with no restriction on other alternatives to be part of the solution set. In this paper, we focus on the weaker version of the Condorcet principle:

Weak Condorcet Principle (WCP):
$\forall R^N \in L(X)^N$, *if* $\exists\ x \in X : n_{xy}(R^N) \geq 0, \forall\ y \in X$ *then* $x \in S(R^N, X)$

There is a clear link between *RSC* and the weak version of the Condorcet Principle. Indeed, our property, combined with another, very weak, property implies that a Condorcet winner has to be selected by a voting procedure when it exists. To show this result, let us first introduce the idea that a solution may never select all candidates in the presence of a Condorcet winner:

Not Everybody (NotEV):
if $\exists\ x \in X : n_{xy}(R^N) \geq 0, \forall y \in X$ *then* $\big[\ \exists\ z \in X : n_{xz}(R^N) > 0\big]$
$\Longrightarrow \big[\ \exists\ z \in X : n_{xz}(R^N) > 0$ and $z \notin S(R^N, X)\big]$

This property says that in the presence of a Condorcet winner, at least one candidate, strictly defeated by the Condorcet winner in a majority contest, has to be excluded of the outcome set. This property is a weak generalization of the majority principle when more than two candidates are present. Indeed, in a 2−candidate case, the axiom imposes that the majority winner is the only selected outcome.
Of course, all Condorcet methods satisfy this axiom. It is also satisfied by the Borda scores as a Borda loser is never a Condorcet winner. [8]

Let us now study the link between *RSC* and the *Weak Condorcet Principle.* The relationship is revealed by a mechanism of induction on the number of candidates. Applying repeatedly *RSC*, we can

[8]The plurality and the anti-plurality do not satisfy *Not Everybody* since it is possible to find preference profiles for which there exists a Condorcet winner and all candidates but the Condorcet winner are selected as an outcome.

reduce the economy to a case where *NotEV* occurs to be a binding constraint. This enables us to state a first theorem demonstrating an axiomatic justification to the Condorcet principle:

THEOREM 1. *If a voting procedure $S(R^N, X)$ satisfies Not Everybody and Resistance to Strategic Candidacy then it satisfies the Weak Condorcet Principle.*

PROOF. Let $\#X = n$. Suppose there exists $a \in X$ such that $n_{ax}(R^N) \geq 0$ for all $x \in X$ and let's assume that $a \notin S(R^N, X)$. Two possible cases may occur. (1) In the first case, all candidates are weak Condorcet winners: For all $x, y \in X$; $n_{xy}(R^N) = 0$. Then, since $S(R^N, X) \neq \varnothing$, there exists a $z \in X$ such that $n_{zx}(R^N) \geq 0$ for all $x \in X$ and $z \in S(R^N, X)$. We consider $Y \equiv X \backslash \{z\}$. If $a \in S(R^N, Y)$ then by *RSC* $a \in S(R^N, X)$ which is a contradiction. If $a \notin S(R^N, Y)$ then let $\#Y = (n-1)$ and $a \notin S(R^N, Y)$ and we proceed by induction until $\#Y = 2$. (2) In the second case, there exists at least one candidate which is not a Condorcet winner: there exists a pair $x, z \in X$ such that $n_{xz}(R^N) > 0$. By *NotEV*, $z \notin S(R^N, X)$. Consider $Y \equiv X \backslash \{z\}$. If $a \in S(R^N, Y)$ then by *RSC* $a \in S(R^N, X)$ which is a contradiction. If $a \notin S(R^N, Y)$ then let $\#Y = (n-1)$ and $a \notin S(R^N, Y)$ and we proceed by induction until $\#Y = 2$.
For $\#Y = 2$, if $a \notin S(R^N, Y)$, S is not consistent with *NotEV*: a contradiction. □

5. An illustration of RSC with three candidates

To illustrate the compatibility of *RSC* with other properties of the literature, let us add some weak and general axiom which is not directly related to *RSC* to have a better understanding of the scope of possibilities of RSC combined with existing criteria. Young

(1975) suggests a consistency axiom[9] which expresses the idea that when the population is divided in two electoral circumscriptions, a candidate elected in both should be selected in the whole population too. Indeed, a voting rule has to satisfy such a property so that central authorities would have no incentive to divide a population into several circumscriptions (in order to divide a group of voters) or to merge some circumscriptions (in order to dilute a coalition).

Nevertheless, this criterion is strong and is sufficient to characterize the family of all scoring rules (Young (1975)) when it is combined with continuity, anonymity, and neutrality. Let us therefore introduce a weakened version of consistency. A voting rule should select the same candidate in the whole population as in two sub-populations if the reason why this candidate is selected in these two populations is the same, namely the results of tournaments involving this candidate in the sub-populations are not contradictory.

Weakened Reinforcement (WR):
$\forall N_1, N_2; \forall R^{N_1} \in L(X)^{N_1}; \forall R^{N_2} \in L(X)^{N_2};$
$\forall S(R^{N_1}, X) \cap S(R^{N_2}, X); \forall i, j \in \{1, 2\};$
If $\forall y, z \in X : n_{zy}(R^{N_i})\ n_{yz}(R^{N_j}) \geq 0$ *then* $x \in S(R^{N_1}, R^{N_2}, X)$.

Because this axiom is a weakening of Young's, it is obvious that all scoring rules satisfy this axiom. Actually, the question is to understand to what extent this weakening widens the scope for possibilities. It can be shown that no iterative scoring rules satisfy this criterion (see in the appendix). However, in the three-candidate case, some Condorcet consistent methods satisfy this property. The Simpson-Kramer (Simpson (1969), Kramer (1977)) voting rule (also called minmax voting rule) is one example (see the appendix).

[9]Formally, let N_1 and N_2, two distinct groups of voters. Let R^N be decomposed into R^{N_1}, the profile group of N_1, and R^{N_2}, the profile group of N_2. If $S(R^{N_1}, X) \cap S(R^{N_2}, X) \neq \emptyset$ then $S(R^N, X) = S(R^{N_1}, X) \cap S(R^{N_2}, X)$.

There are plenty of SCC which satisfy *RSC*, *WR* or *NotEV* but not the three of them. For example, the Borda method satisfies *WR* and *NotEV* but violates *RSC* while the Pareto set violates *NotEV*. Also, any Condorcet method which does not select the Borda loser (as the Nanson or the Black methods) satisfies *RSC* or *NotEV* but may fail to satisfy *WR*. In the three-candidate case, we may imagine several SCC which satisfy *RSC*, *WR* and *NotEV* but one is the Simpson-Kramer rule, that reacts in an intuitive manner to the weighted outcomes of majority tournaments. We conclude this paper with a characterization of the Simpson-Kramer rule in the three-candidate case:

THEOREM 2. *If a voting rule satisfies Not Everybody, weakened reinforcement and resistance to strategic candidacy, then in the three candidate case with an odd number of voters this voting rule contains the Simpson-Kramer solution.*

PROOF. We will show that if $SK(R^N) \not\subseteq S(R^N)$ then one of the three axioms is not satisfied. Without loss of generality, let $a \notin S(R^N), a \in SK(R^N)$ and $n_{ab}(R^N) \geq 0$.
There are two cases :
(1) $n_{ac}(R^N) \geq 0$. By RSC, $S(R^N) = \{b, c\}$ violating *NotEV*.
(2) $n_{ac}(R^N) < 0$. We first show that $n_{ab}(R^N) > 0$. Because $a \in SK(R^N)$, $0 > n_{ac}(R^N) \geq -n_{ab}(R^N)$. So, $n_{ab}(R^N) > 0$. It also follows that $0 > n_{ac}(R^N) \geq n_{cb}(R^N)$. So, $n_{bc}(R^N) > 0$. This case corresponds to a Condorcet cycle.
We may split this preference profile, R^N, into (R^{N_1}, R^{N_2}) with R^{N_1} such that

$$aR^ibR^ic \text{ for } -n_{ac}(R^N) \text{ voters}$$
$$bR^icR^ia \text{ for } -n_{ac}(R^N) \text{ voters}$$
$$cR^iaR^ib \text{ for } -n_{ac}(R^N) \text{ voters}$$

Notice that, by neutrality, $S(R^{N_1}) = \{a, b, c\}$ and $n_{ab}(R^{N_1}) > 0$. If $a \in S(R^{N_2})$, WR implies that $a \in S(R^N)$ because $n_{ac}(R^{N_2}) = n_{ac}(R^N) - n_{ac}(R^{N_1}) = 0$ and $n_{ab}(R^{N_2}) = n_{ab}(R^N) - n_{ab}(R^{N_1}) > 0$. Since by construction, $a \notin S(R^N)$, we can infer that $a \notin S(R^{N_2})$. But, then by RSC, $S(R^N) = \{b, c\}$ violating *NotEV*. $\square$

Young & Levenglick (1978) show that the Condorcet's principle together with a "consistency" and a neutrality property determine a unique rule known as Kemeny's rule. This rule coincides with the Simpson-Kramer's in the three-candidate case. Our theorem may therefore be viewed as a special case of the contribution of Young & Levenglick (1978). Indeed, the proof is based on the same mechanism. However, they have to define the outcome set of a voting procedure in an unusual way: the outcome set of a voting rule has to be one or more rankings of all candidates (set of preference orders). The literature usually defines a voting procedure as a function or a correspondence associating one or several alternatives to each preference profile or which associates a unique collective preference order to each preference profile. The definition of Young and Levenglick's solution concept plays an important role in their proof. As an illustration of our axiom, this theorem shows in the same time an application of the idea Young and Levenglick in the limited case of three candidates but with another concept of voting procedure.

6. Conclusion

Taking notice of the impossibility results initiated by Dutta et al. (2001) on the topic of candidate stability, we study an axiom that retains a significant part of the original concept while leaving some possibilities open. This axiom, that we call *Resistance to Strategic Candidacy*, says that the winner of a n-candidate election should not be overturned if a newcomer, loosing in the $(n+1)$-candidate election, does not beat her in a pairwise comparison. We show that a voting rule that is not encompassing all candidates and satisfies RSC must select a Condorcet winner or more when one exists, a weak version of the Condorcet Principle.

Nonetheless, not all the Condorcet-consistent rules are resistant to Strategic Candidacy, as we show with a counter-example for the Copeland solution. Our axiom can be weakened to allow for more possibilities, but it is unclear how to do so without loosing much of the original meaning.

We conclude by characterizing the Simpson-Kramer rule in the 3-candidate case, showing some robustness of an argument developed by Young & Levenglick (1978) on the basis of a slightly tailor-made solution concept. This construction continues an axiomatic attempt at synthesizing the conflicting appeals of Condorcet-consistent rules and scoring rules.

7. Appendix

7.1. No iterative scoring rule satisfy WR. It can be shown that no iterative scoring rule satisfy WR. Indeed, with the following example, we show that the axiom is violated by the iterative scoring rule for which $0 < \lambda < 1/2$:
Let $X = \{a, b, c\}$; two sub-populations $N_1 = \{1, 2, 3\}$ and $N_2 = \{4, 5, 6, 7\}$ and a preference profile $\underline{R}$:

$$\begin{aligned} aR^ibR^ic \text{ for } i &= 1, 4; \\ bR^icR^ia \text{ for } i &= 2, 5; \\ cR^iaR^ib \text{ for } i &= 3, 6; \\ aR^icR^ib \text{ for } i &= 7. \end{aligned}$$

In population N_1, the scores of a, b and c are identical so $a \in S^2_{\lambda>0}(R^{N_1})$. In population N_2, the worse score is the score of b, $s_\lambda(b)$. Then, $a \in S^2_{\lambda>0}(R^{N_2})$. Therefore, the candidate $a \in S^2_{\lambda>0}(R^{N_1}) \cap S^2_{\lambda>0}(R^{N_2})$. Moreover the majority contests between all pairs of candidates doesn't change of sign.
But, in the whole population $(N_1 \cup N_2)$, the score of b is the lowest one if $\lambda > 0$ and so b should be eliminated after the first round. The rule will than select c : $S^2_{\lambda>0}(R^{N_1}, R^{N_2}, X) = \{c\}$. Therefore, the positional iterative methods violate the criterion when $\lambda > 0$.

7.2. The Simpson-Kramer voting rule. In the 3-candidate case, we show that the Simpson-Kramer voting procedure, $SK(R^N)$, is an example of rule satisfying WR. We first introduce the definition of the Simpson-Kramer solution:
$\forall R^N \in L(X)^N, x \in SK(R^N) \Longleftrightarrow x \in \arg\max_{z\in X}[\min_{\forall y\in X} n_{zy}(R^N)]$
with $n_{zy}(R^N) = \#\{i \in N : zR^iy\} - \#\{i \in N : yR^iz\}$

Proof. Let $a \in SK(R^{N_1}) \cap SK(R^{N_2})$.
By WR, $a \in SK(R^{N_1}, R^{N_2})$.
There are three cases :

(1) Candidate a is a Condorcet winner in population N_1 and in population N_2. By definition, a is a condorcet winner in the whole population too and so, a is selected by Simpson-Kramer in the whole population.
(2) Candidate a is a Condorcet winner in N_1 but not in N_2. Let, without loss of generality, $n_{ab}(R^{N_2}) > 0, n_{bc}(R^{N_2}) > 0$ and $n_{ca}(R^{N_2}) > 0$. And by the conditions of the axiom:$n_{ab}(R^{N_1}) \geq 0, n_{bc}(R^{N_1}) \geq 0$ and $n_{ca}(R^{N_1}) \geq 0$. So, $n_{ac}(R^{N_1}) = 0$.
We have:

$$\begin{array}{rcl}
\min_x n_{ax}(R^N) & = & 0 + n_{ac}(R^{N_2}) \\
\min_x n_{bx}(R^N) & \leq & n_{ba}(R^{N_1}) + n_{ba}(R^{N_2}) \leq n_{ba}(R^{N_2}) \leq n_{ac}(R^{N_2}) \\
\text{because } a & \in & SK(R^{N_2}) \\
\min_x n_{cx}(R^N) & \leq & n_{cb}(R^{N_1}) + n_{cb}(R^{N_2}) \leq n_{cb}(R^{N_2}) \leq n_{ac}(R^{N_2}) \\
\text{because } a & \in & SK(R^{N_2})
\end{array}$$

$\Longrightarrow a \in SK(R^N)$
(3) Candidate a is neither a Condorcet winner in N_1 and in N_2. Without loss of generality, we assume that $n_{ab}(R^{N_1}) > 0, n_{bc}(R^{N_1}) > 0$ and $n_{ca}(R^{N_1}) > 0$. And by the conditions of the axiom : $n_{ab}(R^{N_2}) > 0, n_{bc}(R^{N_2}) > 0$ and $n_{ca}(R^{N_2}) > 0$.
So, $\forall i \in \{1, 2\} : a \in SK(R^{N_i}) \Longrightarrow 0 > n_{ac}(R^{N_i}) > n_{ba}(R^{N_i}), n_{cb}(R^{N_i})$
$\Longrightarrow 0 > n_{ac}(R^N) > n_{ba}(R^N), n_{cb}(R^N)$
$\Longrightarrow a \in SK(R^N)$. $\square$

CHAPTER 2

Informal Insurance with Endogenous Group Size

We present a theory of endogenous formation of insurance groups which combines heterogeneity on agents' risk aversion under asymmetric information and lack of enforceability of contracts. Income sharing inside the group is decided by majority voting and the size of the group adjusts to this decision through participation constraints. At equilibrium, all group members agree on the same imperfect level of income sharing, which yields a constrained-efficient equilibrium. Comparative statics on the risk faced by the community provide interesting results. A mean preserving spread of income implies more income sharing and a larger group size. New members, and possibly even old members may be better off, while non-members are worse-off. These results have relevant policy implications.

1. Introduction

In rural areas of developing countries, people are exposed to substantial income risks due to large fluctuations of agricultural productions and prices, weather shocks, incomplete markets, ... The lack of access to formal insurance and credit markets leads individuals to create informal insurance agreements. These agreements may be bilateral or take the form of larger groups where households are both consumers and providers of insurance. Since no legal framework exists to enforce these agreements, they must be designed to be self-enforceable, that is, the expected benefits from becoming a group member must be, at any point in time, larger than the gains from defection.

Empirical studies of risk-sharing find evidence of partial insurance in rural communities but uniformly reject the hypothesis of full insurance at the community level (Deaton (1992), Townsend (1994), Udry (1994), Grimard (1997), Jalan & Ravallion (1999), Ligon et al. (2002), Gertler & Gruber (2002)). It appears that the major constraint explaining the failure of full insurance arises from the lack of effective enforcement mechanism to support these insurance arrangements.

In this paper, we present a theory of endogenous formation of insurance groups which reproduces the above mentioned stylized facts and discuss its normative properties. The model combines heterogeneity on agents' risk aversion under asymmetric information and lack of enforceability of contracts involving group members. Insurance services are repeated over time, and members reneging on their commitment are permanently excluded from the group. This framework gives rise to an ad interim participation constraint, first introduced by Coate & Ravallion (1993): when incomes are observed, members drawing a high income face a trade-off between the current gain of keeping the integrality of this income and the loss of all future insurance services. This constraint is more likely to be binding for agents whose risk aversion is low. Since agents' types are unobservable, and all members of the community want to benefit from insurance ex ante, the only variable on which the group has to agree is the extent of insurance provided by the group, that is, the income sharing rule. The decision-making process is modeled by majority voting. The size of the insurance group decreases since members whose participation constraint is not satisfied progressively leave the group as they draw high incomes. This change in the group composition in the early periods of the group calls for a new vote at each period as long as the group size is not stable.

We show that in the long run, all group members agree on the same imperfect level of income sharing, which maximizes their utility subject to the participation constraint. This equilibrium is constrained efficient, that is, given the participation constraint, no agent can be made better-off without reducing the utility of one agent. Indeed, decreasing income sharing in order to ensure that the participation constraint of new members would be satisfied reduces the utility of current members. A comparative statics analysis with respect to an increase in the risk faced by the community is also provided. Interestingly, a mean preserving spread (MPS further) has a positive effect on both the level of the sharing rule and the size of the group, which unambiguously improves the quality of insurance and allows more agents to become members. Since a mean preserving spread increases the group size, three groups of agents emerge and are affected in different ways. First, the effect of a MPS has an ambiguous impact on the welfare of the agents who were already members prior to its introduction. Second, the "new members", who are the agents who enter the group thanks to the MPS, enjoy a higher welfare thanks to the increase in risk. Finally, the third group of agents who are still out of the insurance group is worse-off after the introduction of the MPS. Therefore, while the income risk increases, some agents' welfare may be improved. These results stress that one has to be careful about introducing risk-reducing policies without accounting for the adverse effect they might have on the stability of local communities.

Related literature

Important contributions have studied risk-sharing agreements in rural societies by accounting for the lack of contract enforceability, starting with Coate & Ravallion (1993). The space of possible efficient contracts limited to stationary transfers considered by Coate & Ravallion (1993) has been expanded to dynamic efficient arrangements (Kocherlakota (1996), Ligon et al. (2002)), maintaining the crucial and detrimental role of ad interim participation constraints.

All these models conclude that the extent of insurance provided by the group is limited. However, they consider the size of the group and the characteristics of its members as exogenous. This paper contributes to the literature by endogenizing both the extent of insurance provided by the group and the size of the group itself under asymmetric information. The model takes account that the benefits from risk-sharing derive not only from the extent of risk-sharing but also from the overall size of the insurance arrangement while maintaining the role of the lack of contract enforceability. More precisely, there is a trade-off to the quality of insurance: it can be improved by increasing both the group size and the extent of risk-sharing. However, due to the lack of contract enforceability, an increase in the extent of risk-sharing results in a smaller size of the insurance group. Only few theoretical papers have studied the endogenous formation of insurance groups. Relying upon an argument of coalition-proofness, Genicot & Ray (2003) prove the existence of an upper bound on the size of insurance groups and show that the degree of risk-sharing is non-monotonic in the level of uncertainty or need for insurance in the community. Bold (2009) focuses on the optimal contract on the set of history-dependant contracts that are robust with respect to coalition deviations. In this paper, the problem of deviations by coalitions is not present since at equilibrium, all group members agree on the same sharing rule level, which maximizes their utility.[1]

Other considerations may also be advanced to explain the group size limits as cast, kinship, or informational problems. Murgai et al. (2002) suggest another explanation to the boundaries of mutual insurance groups and the quality of insurance within these groups based on costs of group formation and transaction costs.

This paper provides relevant insights for policy makers since it shows that policies reducing income variance at the community level have ambiguous impacts on welfare. Such policies should therefore be accompanied by measures protecting non-members of insurance groups, since their numbers are likely to increase.

Section 2 presents the model and describes the equilibrium. Comparative statics are provided in Section 3 and Section 4 concludes.

2. The model

The economy is composed of a finite number of agents i where $i \in \{1, ...N\}$ with $N \geq 2$. Each agent lives for an infinity of periods $t \in \{1, 2, ..., +\infty\}$ and draws at each period a random income Y_{it} that is high h with probability p and low l with probability $(1-p)$. Y_{it} is independently and identically distributed across agents and periods.[2] For all $i \in \{1, ...N\}$ and $t \in \{1, 2, ..., +\infty\}$, the mean and

[1] This simplification arises from the assumption on agents' mean-variance preferences, which introduces separability and leads all members to only focus on minimizing the income variance despite their differences in risk aversion.

[2] The zero correlation assumed here is just a convenient simplification. Indeed, correlation needs not be nil for the surplus of mutual insurance to exists.

the variance of the income are

$$E(Y_{it}) = ph + (1-p)l \equiv \mu, \tag{2.1}$$

$$Var(Y_{it}) = p(1-p)(h-l)^2 \equiv \sigma^2. \tag{2.2}$$

Agents are heterogeneous in risk aversion. This captures various sources of heterogeneity in terms of preferences but also in terms of capacities to mitigate risk, for example by diversifying production, or to face shocks thanks to consumption smoothing mechanisms. Agents' coefficient of absolute risk aversion a is private information and follows a distribution $F(a)$ inside the population. Agents' instantaneous preferences are defined over the mean and the variance of their income. Agent i at period t has instantaneous utility:

$$u_{it}(C_{it}) = E(C_{it}) - a_i Var(C_{it}), \tag{2.3}$$

where C_{it} is the agent's disposable income which is entirely consumed at each period.[3] In other words, if this agent is not a member of the insurance group, $C_{it} = Y_{it}$, whereas if he/she is a member of the group, $C_{it} = I_{it}$, where I_{it} is the post-transfer -*insured*- income defined by the following sharing rule. At each period t, each member i concedes a proportion $\alpha \in [0,1]$ of his/her income Y_i and the sum of all contributions is equally split. Without loss of generality, this technology is assumed costless. Formally,

$$I_{it} = (1-\alpha)\, Y_{it} + \frac{1}{n}\sum_{j=1}^{n} \alpha Y_{jt} = Y_{it}\left(1 - \alpha\left(1 - \frac{1}{n}\right)\right) + \alpha \sum_{j \neq i} \frac{Y_{jt}}{n},$$

where $i, j \in \{1, ...n\}$ and n is the number of agents committing to the insurance group.[4] The mean income under mutual insurance is

[3] This utility function can be interpreted in the following way. Preferences are defined over one good, i.e. the average income, and one bad, i.e. its variance. These two arguments are perfect substitutes, and the marginal rate of substitution between mean and variance equals the risk aversion coefficient a_i.

[4] In this model, n is endogenous.

unchanged, while the variance of income is smaller than $Var(Y_{it})$:

$$\begin{aligned} E(I_{it}) &= ph + (1-p)l \equiv \mu \\ Var(I_{it}) &= \Phi(\alpha, n)\,\sigma^2, \end{aligned}$$

where

$$\begin{aligned} \Phi(\alpha, n) &= \left(1 - \alpha\left(1 - \frac{1}{n}\right)\right)^2 + (n-1)\frac{\alpha^2}{n^2} \\ &= 1 - \frac{n-1}{n}\alpha(2-\alpha) \\ &\in [0;1]\,. \end{aligned} \tag{2.4}$$

Note that for any $\alpha \in [0,1]$, the larger the size of the insurance group, the lower the variance.[5] This stems from the fact that the transfer received by each contributor, $\frac{1}{n}\sum_{j=1}^{n} \alpha Y_{jt}$, is less variable when the community size increases:

$$\begin{aligned} \frac{\partial \Phi}{\partial n} &= -\frac{\alpha(2-\alpha)}{n^2} \\ &< 0. \end{aligned} \tag{2.5}$$

Also note that the variance of the post-transfer income is decreasing in α and is minimized when $\alpha = 1$:

$$\begin{aligned} \frac{\partial \Phi}{\partial \alpha} &= -2\frac{n-1}{n}(1-\alpha) \\ &\leq 0. \end{aligned} \tag{2.6}$$

Since both arguments unambiguously decrease the income variance and therefore increase utility, one might wonder why, in the absence of ex ante moral hazard, the insurance network is not composed of the whole community and does not apply perfect sharing: $n = N$,

[5] Note that there is no reduction in the variance ($\Phi = 1$) if the insurance group is composed of a single individual ($n = 1$) or no income is shared ($\alpha = 0$). Also, if perfect sharing is applied, $\Phi(1, n) = \frac{1}{n}$.

$\alpha = 1$.[6] The reason thereof is due to the fact that the insurance scheme is repeated over time. This repetition gives rise to the so-called ad interim constraint: some agents might have an incentive to renege on their membership obligations once they observe their income draw. Indeed, this case will emerge if the current gain of not sharing income exceeds all the future gains of mutual insurance. Let us analyze this problem in detail by first describing the ad interim stage.

2.1. The ad interim participation constraint. At the ad interim stage, an agent observes the income he/she has drawn, but not the other agents' income, which he/she considers as random variables. Formally, agent i observes the drawn income $y \in \{l, h\}$, so that the post-transfer income at the ad interim stage if i respects the agreement is:

$$I_{it}^{AI} = (1-\alpha)\, y + \frac{\alpha}{n} y + \frac{\alpha}{n} \sum_{j \neq i} Y_{jt},$$

where the first two terms are constant and the last one is a sum of random variables. Therefore, the agent's contemporaneous utility at the ad interim stage is determined by

$$E\left(I_{it}^{AI}\right) = y + \left(1 - \frac{1}{n}\right) \alpha \left(\mu - y\right), \tag{2.7}$$

$$Var\left(I_{it}^{AI}\right) = (n-1) \frac{\alpha^2}{n^2} \sigma^2. \tag{2.8}$$

[6] Under ex ante moral hazard, the distribution of incomes is endogenous and insurance might have a negative effect on agents' effort to obtain a high income.

On the other hand, if the agent does not respect the agreement, his/her consumption does not depend on the other members' income drawings and is therefore certain:

$$\begin{aligned} Y_{it}^{AI} &= y, \\ E\left(Y_{it}^{AI}\right) &= y, \\ Var\left(Y_{it}^{AI}\right) &= 0. \end{aligned}$$

In other words, because at the ad interim stage agents only observe their own income, respecting the insurance agreement instead of breaking it increases a member's income variance by $(n-1)\frac{\alpha^2}{n^2}\sigma^2$. Also, respecting the agreement allows the agent to benefit from a net transfer of $\left(1-\frac{1}{n}\right)\alpha\left(\mu-y\right)$, which is positive if the agent had drawn a low income, and negative if he/she had drawn a high income. Since agents drawing a high income both face a higher variance and are net contributors to the group, they might be tempted to renege on their obligations. In order to limit the extent of this phenomenon the group applies a trigger strategy, which consists in permanently excluding transgressors from all future insurance possibilities. Therefore, the actualized total utility at the ad interim stage of period t_0 for an agent who observes an income y and decides to break the agreement (B) writes:

$$\begin{aligned} U_i^{AI}\left(B;y\right) &= u(Y_{it_0}^{AI}) + \sum_{t=t_0+1}^{\infty} \delta^t u_{it}\left(Y_{it}\right) \\ &= y + \frac{\delta}{1-\delta}\left(\mu - a_i\sigma^2\right), \end{aligned}$$

where $\delta \in [0,1]$ is a discount factor. Conversely, the actualized total utility of an agent who always *respects* the informal agreement (R)

writes:

$$
\begin{aligned}
U_i^{AI}(R;y) &= u\left(I_{it_0}^{AI}\right) + \sum_{t=t_0+1}^{\infty} \delta^t u_{it}(I_{it}) \\
&= y + \frac{\delta}{1-\delta}\left(\mu - a_i \Phi \sigma^2\right) - a_i (n-1)\frac{\alpha^2}{n^2}\sigma^2 \\
&\quad + \frac{n-1}{n}\alpha(\mu - y).
\end{aligned} \tag{2.9}
$$

We can now formally define the ad interim participation constraint, which states that the utility of respecting the agreement is larger than the utility of breaking it:

$$
\begin{aligned}
U_i^{AI}(R;y) &\geq U_i^{AI}(B;y) \\
&\Longleftrightarrow \\
a_i\sigma^2\left(\frac{\delta}{1-\delta}(1-\Phi) - (n-1)\frac{\alpha^2}{n^2}\right) &\geq \frac{n-1}{n}\alpha(y-\mu).
\end{aligned}
$$

This condition highlights the existence of three terms. The two terms on the left hand side pertain to the income variance and are independent of the income drawn. They indicate that respecting the agreement implies a trade-off between reducing the income variance of all future periods and increasing the contemporaneous one. It can be shown that the first effect dominates the second for all n and α under the sufficient condition that agents exhibit a reasonable level of patience ($\delta > 1/3$). The sign of the right hand side depends on the income drawn and represents the expected transfer that the agent offers to the group. Clearly, if $y = l$, this transfer is negative, so that the participation constraint is always satisfied (under the above mentioned restriction on δ). Having described all the effects, one can simplify and rewrite the participation constraint under the form of a lower bound on risk aversion, for the only relevant case where $y = h$:

$$
a_i \geq \Theta \equiv \left[p(h-l)\left(\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n}\right)\right]^{-1}. \tag{2.10}
$$

Therefore, given n and α, the number of agents whose ad interim constraint is respected, n_R, is:

$$n_R = 1 + (N-1)(1 - F(\Theta)). \tag{2.11}$$

As we have seen from the previous equations, agents who are the most eager to enter the insurance group and respect the agreement are those who are the most averse to risk. We also know that the larger the group size and the larger the sharing rule, the better for all agents, provided that all members respect the transfer rule. However, setting a large level of income sharing has a negative impact on the number of agents who can credibly commit to always respect the group's transfer rule. The next subsection describes the mechanism behind the group formation.

2.2. Group formation. We have seen that a large group size is desirable, since this improves risk spreading. However, depending on the level of the sharing rule, some agents might choose to renege on their obligations towards the group when they draw a high income. When this is the case, the sanction they undergo is permanent exclusion from the group. The key to group formation is therefore to find the optimal trade-off between the group's size and its sharing rule.

Preferences of agents are unobservable, so that it is not possible to exclude agents from the group before they choose not to respect the insurance contract. Therefore, for any degree of α at the first period, the whole community enters the group. However, as soon as incomes are drawn at the ad interim stage, a fraction of the community leaves the group. On average, the number of agents who don't respect the contract and leave the group at the ad interim stage of the first period equals $p(N - n_R)$, that is, the fraction of agents who receive a high income p times the $N - n_R$ agents whose ad interim participation constraint is not satisfied. More generally, the expected number of agents who leave the group at period $t \geq 1$ is $p\,(1-p)^{t-1}\,(N - n_R)$. Conversely, the number of agents remaining in the group at period t is $n_R + (\ N - n_R)\,(1-p)^t$ on average. As t becomes sufficiently large, the size of the group converges towards its stable level n_R with probability 1.[7] Since n_R itself depends on n, the equilibrium size, which is noted n^*, is defined formally by the following implicit function:

$$n^* = n_R\,(\alpha, n^*)\,. \tag{2.12}$$

Since preferences are unobservable, and the composition and size of the group follows a process of successive exits, the group's only decision variable is the sharing rule α that is imposed to all its members. The effect of the sharing rule on the steady state group size is obtained by applying the implicit function theorem to equation (2.12):

$$\frac{\partial n^*}{\partial \alpha} = \frac{\partial n_R}{\partial \alpha} / \left(1 - \frac{\partial n_R}{\partial n}\right).$$

An increase in α decreases n_R, but the net effect on n^* is negative only if the increase in n_R due to an increase in n is not too strong. Indeed, a low level of sharing decreases the incentives to participate in the insurance group, but on the other hand, the participation constraint, which defines n_R, is more severely binding if α is high. The actual group size is a result of this tension. This is stated in Lemma 1:

Lemma 1. *The steady state size of the insurance group n^* decreases when α increases if and only if $\frac{\partial n_R}{\partial n} < 1$.*

Proof. The proof and a discussion of the condition that $\frac{\partial n_R}{\partial n} < 1$ are provided in the appendix. □

[7] Note that the higher p, the higher the speed of convergence.

Prior to defining the equilibrium characteristics of the group, let us describe how preferences of members are aggregated. Since all agents of the community are willing to benefit from the insurance provided by the group, the whole community at the first stage has to agree on the level of α which will be applied. The aggregation of preferences is modeled by majority voting. Since the composition of the group is likely to change over the first periods of the group's life, a new vote is organized at the beginning of each period, until the group composition is stable.

Proposition 1 defines the characteristics of the group once it is stable, that is, when all members of the group satisfy the participation constraint.

PROPOSITION 1. *Under the necessary condition that* $\frac{\partial n_R}{\partial n} < 1$*, the steady-state group characteristics are determined by* α^* *and* $n^*(\alpha^*)$*, where* α^* *is such that*

$$\frac{\partial \Phi}{\partial \alpha} + \frac{\partial \Phi}{\partial n}\frac{\partial n^*}{\partial \alpha} = 0. \tag{2.13}$$

PROOF. We proceed by first solving the voting game at the first period and show how the changes in group composition affect the votes of the subsequent periods. To solve the voting game at period 1, one has to define the preferred α of each agent. Let us start with the most risk averse agent of the community. He/she will choose a level of sharing which maximizes his/her utility as a permanent member of the insurance group, taking into account both the direct effect of α on variance reduction and its indirect effect on the reduction of the steady-state/stable group size.[8] Let us define this formally. The utility of an agent who always respects

[8]For simplicity, we neglect the dynamics of group size in the initial stages and focus on the steady-state group size n^*.

the sharing rule (R) is

$$U_i(R) = \frac{\mu - a_i \Phi(\alpha, n^*) \sigma^2}{1-\delta}.$$

Since we focus on the steady-state group size (2.12) and agents anticipate the effect of α on n^*, the preferred sharing level of the most risk averse agent α^* must be such that

$$\begin{aligned} \frac{dU_i(R)}{d\alpha} &= \frac{\partial U_i(R)}{\partial \alpha} + \frac{\partial U_i(R)}{\partial n}\frac{\partial n^*}{\partial \alpha} \\ &= \frac{\partial U_i(R)}{\partial \Phi}\left(\frac{\partial \Phi}{\partial \alpha} + \frac{\partial \Phi}{\partial n}\frac{\partial n^*}{\partial \alpha}\right) \qquad (2.14) \\ &= 0. \end{aligned}$$

Note that the second order condition is satisfied as shown in the appendix. The preferred level α^* defines a lower bound on risk aversion $\Theta(\alpha^*)$ above which the participation constraint is satisfied and a stable group size defined by (2.12). Interestingly, all the agents whose risk aversion is larger than $\Theta(\alpha^*)$ also have a preferred α equal to α^*. Indeed, for all i such that $a_i \geq \Theta(\alpha^*)$, the preferred α does not depend on risk aversion since α^* is given by equation (2.13), which does not depend on risk aversion. In other words, there is a mass n_R whose preferred α is α^*. If the distribution of risk aversion in the community is such that these $n^*(\alpha^*)$ agents form a majority (that is $n^*(\alpha^*)/N \geq 1/2$), the group's sharing rule is set at α^* at period 1.

What happens if these agents don't form a majority at period 1? We then have to analyze the preferred α of agents $j \in \{n^*(\alpha^*)+1, ..., N\}$, that is, agents whose a_j is lower than $\Theta(\alpha^*)$. These agents know that at α^*, they will exclude themselves from the group as soon as they draw a high income. If the discount factor is sufficiently large, so that the future benefits of mutual insurance are sufficiently important, the preferred α of these agents is lower than α^* and is set so as to make their ad interim participation constraint binding. By doing so, they will always benefit from the

insurance provided by the group.[9] The voting problem at stage 1 is therefore solved by the median voter in the profile of preferred α's $\left\{\alpha^*, \alpha^*, ...\alpha^*, \alpha^*_{n^*(\alpha^*)+1}, ..., \alpha^*_N\right\}$.
Having solved the voting problem at period 1, let us now analyze the subsequent periods. As time passes, agents whose a is lower than $\Theta\left(\alpha^*_m\right)$ leave the group. This change in group composition calls for a new vote on the level of the sharing rule. Since only the least risk averse agents leave the group, the profile of preferred α's shrinks at the benefit of those who prefer α^*, which eventually form a majority. In other words, if voting is repeated, as agents leave the group, the steady-state group characteristics are determined by α^* and n^*. □

Note that the implicit equation defining α^* which is presented in Proposition 1 can be equivalently stated in the following way

$$\frac{\partial n^*}{\partial \alpha} = \frac{\partial n}{\partial \alpha}|_{Var(I_{it})\text{ cst}}.$$

This equation has a clear interpretation: at α^*, the marginal effect of α on the group size at the steady state must be equal to the marginal rate of substitution of α for n which keeps the variance of the insured income constant.

Another important remark concerns possible deviations by coalitions. Clearly, since all group members unanimously have the same preferred alpha, the group is not subject to such deviations. This result is a direct implication of the assumption on agents' mean-variance preferences, which introduces separability and leads all members to only focus on minimizing the income variance despite their differences in risk aversion.

Agents who left the group are not part of our focus, but it is nonetheless interesting to notice that they may form another group among themselves. The information revealed by the exclusion is

[9] The preferred α of these agents is discussed formally in the Appendix.

limited: they all know that leaving the first group shows a type (risk-aversion) below the level required by the participation constraint at the time of the exit. Since the participation constraint of the first group evolves monotonically, re-entry is never an issue. The problem faced by those who are excluded in this process is very similar to the original problem of the population, but with a strictly smaller interval of types. This interval being composed of low levels of risk-aversion, the equilibrium sharing rate will be lower in the second group than in the first one. Society as a whole ends up being stratified in groups ordered by insurance coverages and attitudes towards risk.

Let us conclude this section by normatively assessing the equilibrium.

PROPOSITION 2. *The steady-state equilibrium is constrained-efficient.*

PROOF. We have to show that α^* is such that the welfare of an agent cannot be improved without decreasing another agent's utility given the enforceability and informational constraints. First, nobody inside the group can be made better-off as all members agree on the same preferred level α^*. Non-members could clearly be better-off by entering the group, but this would require to decrease α to make their participation constraint satisfied. In that case, the $n^*(\alpha^*)$ members would be worse off since α^* maximizes $U(R)$. □

3. Comparative statics

The aim of this section is to highlight the effects of the parameters on the degree of income pooling and the size of the insurance group. Indeed, the quality of insurance that is achieved in a given group is characterized by both characteristics of the group.

3.1. Mean Preserving Spread. In order to isolate the effect of an increase in the uninsured income risk on the quality of insurance while holding the other parameters of the distribution of Y, namely μ and p, fixed, we apply the following transformation to h and l:

$$\begin{aligned} h &= \mu + \sigma\sqrt{\frac{1-p}{p}}, \\ l &= \mu - \sigma\sqrt{\frac{p}{1-p}}. \end{aligned}$$

By doing so, the distribution of Y depends on the parameters μ, σ^2, p, which are independent of each other. In other words, we can now analyze the effect of a mean preserving spread, that is, an increase in σ while holding the mean μ (and the distribution of income probabilities) constant. Proposition 3 describes the effect of a MPS on the characteristics of the group at the steady-state.

Proposition 3. *A mean preserving spread of Y has a positive impact on both α^* and $n^*(\alpha^*)$:*

$$\begin{aligned} \frac{d\alpha^*}{d\sigma}|_{\mu\ cst} &> 0 \\ \frac{dn^*}{d\sigma}|_{\mu\ cst} &> 0. \end{aligned}$$

Proof. The proof is provided in the appendix. □

Therefore, when the pre-transfer income risk increases, the size of the insurance group increases and group members share a higher proportion of income. The impact of a higher risk therefore implies a reinforcement of communities.

Let us discuss why the MPS leads to an increase in both α^* and n^*, which may seem surprising at first since n^* is decreasing in α. To see this, first recall that an increase in the risk of the pre-transfer income incites group members to increase the quality of insurance, that is, to decrease Φ. Increasing income sharing α is one way to reach this objective. However, we know that increasing α tightens the participation constraint, which may lead to a decrease in the number of members. However, an increase in σ also has the effect of loosening the participation constraint, since non-members are more reluctant to face a larger uninsured risk. At equilibrium, the net effect on the group size is positive.

The following corollary states that since both effects go in the same direction at equilibrium, the quality of insurance is unambiguously improved. This does not mean however that the variance of the insured income I_{it} is lower.

COROLLARY 1. *A mean preserving spread of Y has a positive impact on the quality of the insurance, that is,* $\frac{d\Phi(\alpha^*, n^*(\alpha^*))}{d\sigma} < 0$. *The net effect on the variance of the insured income, $\Phi\sigma^2$, is ambiguous.*

PROOF. Applying Proposition 3, the proof of the corollary is straightforward since Φ is decreasing in both α and n. A formal proof is provided in the Appendix. □

The reason why the net effect on the variance of the insured income is ambiguous is the following. In addition to the reaction of members through α, some of the agents whose participation constraint was not satisfied now enter the group and contribute to improve the insurance quality. It is therefore unclear whether the insured income variance should increase or not.

Let us conclude this section on the mean preserving spread by discussing its normative implications.

3.2. Normative analysis of the impact of a mean preserving spread. Since a mean preserving spread increases the group size, three groups of agents are relevant for this normative analysis.

First, the agents who were already members prior to the introduction of the MPS. A direct implication of Corollary 1 is that the effect of a mean preserving has an ambiguous impact on the welfare of these "old members".

Second, the "new members" are the agents who entered the group thanks to the MPS. Interestingly, the MPS may have a positive impact on the new members' welfare.

Third, agents who are still out of the insurance group. Obviously, these agents are worse-off after the introduction of the MPS.

PROPOSITION 4. *The effect on welfare of a marginal mean preserving spread of Y*

(1) *is ambiguous for "old members" of the group*
(2) *is positive for "new members"*
(3) *is negative for agents who remain out of the group.*

PROOF. The proof of part 1 is a direct implication of Corollary 1. Part 2 comes from the fact that new members benefit from a discrete decrease in risk since they now benefit from insurance. This discrete decrease always dominates the marginal increase in variance.[10] Part three is straightforward. □

Let us conclude this normative analysis by a numerical simulation showing that the MPS can effectively increase the utility of "old members". Let us define the following parameters' values: $N = 120$, $\mu = 1$, $h = 2$, $\delta = 0.8$, and the support of the uniform distribution for risk aversion is $[a, b] = [1, 2]$. The following table provides the most salient results for these parameter values.

For an agent $a_i = 1,81$;	when $\sigma^2 = 0,15$;	$U_i = 4,97$;
	when $\sigma^2 = 0,12$;	$U_i = 3,64$.
For an agent $a_i = 2$;	when $\sigma^2 = 0,15$;	$U_i = 4,96$;
	when $\sigma^2 = 0,12$;	$U_i = 4,92$.

First note that in this example, a new member is clearly better off with the increase in σ. Indeed, for $\sigma^2 = 0.12$, an agent with a

[10] A formal proof of this argument is provided in the appendix.

risk aversion coefficient of 1.81 is out of the group and has a utility level of 3.64. By increasing σ^2 to 0.15, this agent's participation constraint is now satisfied and he enters the group, which provides him with a utility level of 4.97.

Second, and most interesting is the case of the agent with a risk aversion level of 2, that is the most risk averse agent of the community, who already was a member for $\sigma^2 = 0.12$ and had a utility level of 4.92. This level increases to 4.96 when the variance increases to 0.15.

4. Conclusion

This paper provides a theory of endogenous formation of insurance groups in rural communities with heterogeneity on agents' risk aversion with asymmetric information and lack of contract enforceability. Insurance services are repeated over time, and members reneging on their commitment are permanently excluded from the group which gives rise to ad interim participation constraints. Agents vote on the level of income to be shared inside the insurance group, taking into account the fact that a higher sharing rule tightens participation constraints and therefore reduces the size of the group.

We show that in the long run, all group members agree on the same sharing rule, which maximizes their utility subject to the participation constraint. This equilibrium is constrained efficient, that is, given the participation constraint, no agent can be made better-off without reducing the utility of one agent. Interestingly, a mean preserving spread has a positive effect on both the level of the sharing rule and the size of the group, which unambiguously improves the quality of insurance and allows more agents to become members. Since a mean preserving spread increases the group size, three groups of agents emerge and are affected in different ways. First, the effect of a MPS has an ambiguous impact on the welfare of the agents who were already members prior to its introduction. Second, the "new members" who are the agents who entered the group thanks to the MPS. Interestingly, the MPS has a positive impact on their welfare. While the third group of agents who are still out of the insurance group is worse-off after the introduction of the MPS. Therefore, while the income risk increases, some agents' welfare may be improved. These results stress that one has to be careful about introducing risk-reducing policies without accounting for the adverse effect they might have on the stability of local communities.

5. Appendix

5.1. Proof and discussion of Lemma 1. Lemma 1 states that

$$\begin{aligned}\frac{\partial n^*}{\partial \alpha} &= \frac{\partial n_R}{\partial \alpha} / \left(1 - \frac{\partial n_R}{\partial n}\right) \\ &< 0 \Longleftrightarrow \frac{\partial n_R}{\partial n} < 1.\end{aligned}$$

To see why, it suffices to show that

$$\begin{aligned}\frac{\partial n_R}{\partial \alpha} &= -(N-1)\, F'(\Theta)\, \Theta_\alpha \\ &\leq 0\end{aligned}$$

since

$$\begin{aligned}\Theta_\alpha &\equiv \frac{\partial \Theta}{\partial \alpha} = \frac{1}{p\,(h-l)} \left[\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n}\right]^{-2} \left(\frac{\delta}{1-\delta} + \frac{1}{n}\right) \\ &> 0.\end{aligned}$$

The participation constraint will be respected by a lower number of agents if the share of income which members transfer to the pool increases (provided some agents in the community have a risk aversion level equal to Θ).

Let us now discuss the denominator. The participation constraint will be respected by a larger number of agents if the size of the insurance group increases:

$$\begin{aligned}\frac{\partial n_R}{\partial n} &= -(N-1)\, F'(\Theta)\, \Theta_n \\ &\geq 0,\end{aligned}$$

since

$$\begin{aligned}\Theta_n &\equiv \frac{\partial \Theta}{\partial n} = -\frac{\alpha}{n^2} \frac{1}{p\,(h-l)} \left[\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n}\right]^{-2} \\ &< 0.\end{aligned}$$

Let us finally discuss the relevance of the condition that $\frac{\partial n_R}{\partial n} < 1$. This condition is the least likely to hold is when α is large and n is small. Let us therefore compute $\frac{\partial n_R}{\partial n}$ for $\alpha = 1$ and $n = 2$. In this particular case, $\frac{\partial n_R}{\partial n}$ may be written as

$$\frac{\partial n_R}{\partial n} = \Omega \frac{(1-\delta)^2}{(3\delta-1)^2},$$

where

$$\Omega = \frac{(N-1)\, F'(\Theta)}{p\,(h-l)}.$$

If $F'(\Theta) = 0$, the condition is always trivially satisfied. If Θ takes one of the possible values in the distribution, posing $F'(\Theta) = 1/N$ which boils down to ignore distributional effects by imposing a discrete uniform distribution on risk aversion, one obtains that

$$\begin{aligned} \frac{\partial n_R}{\partial n} &= \frac{(N-1)}{Np\,(h-l)} \frac{(1-\delta)^2}{(3\delta-1)^2} \\ &< 1 \text{ if } \frac{(1-\delta)^2}{(3\delta-1)^2} < p\,(h-l)\,. \end{aligned}$$

This condition is satisfied when agents are sufficiently patient (δ close to 1) or if the probability of drawing a high income and the difference between high and low incomes are high. This means that agents will be willing to build a mutual insurance group if they are sufficiently patient.

5.2. Second order condition for α^*. The second order condition is always satisfied:

$$\begin{aligned} \frac{d^2 U_i(R)}{d\alpha^2} &= \frac{\partial U_i(R)}{\partial \Phi} \left(\begin{array}{c} \frac{\partial^2 \Phi}{\partial \alpha^2} + 2 \frac{\partial^2 \Phi}{\partial n \partial \alpha} \frac{\partial n^*}{\partial \alpha} + \frac{\partial^2 \Phi}{\partial n^2} \left(\frac{\partial n^*}{\partial \alpha} \right)^2 \\ + \frac{\partial \Phi}{\partial n} \left(\frac{\partial^2 n^*}{\partial \alpha^2} + \frac{\partial^2 n^*}{\partial \alpha \partial n} \frac{\partial n^*}{\partial \alpha} \right) \end{array} \right) \\ &< 0 \text{ since } \left\{ \begin{array}{l} \frac{\partial U_i(R)}{\partial \Phi} < 0 \\ \left(\begin{array}{c} \overset{(+)}{\frac{\partial^2 \Phi}{\partial \alpha^2}} + 2 \overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}} \overset{(-)}{\frac{\partial n^*}{\partial \alpha}} + \overset{(+)}{\frac{\partial^2 \Phi}{\partial n^2}} \overset{(+)}{\left(\frac{\partial n^*}{\partial \alpha} \right)^2} \\ + \overset{(-)}{\frac{\partial \Phi}{\partial n}} \left(\overset{(-)}{\frac{\partial^2 n^*}{\partial \alpha^2}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial n}} \overset{(-)}{\frac{\partial n^*}{\partial \alpha}} \right) \end{array} \right) > 0 \end{array} \right. . \end{aligned}$$

Indeed, the signs of the terms which compose the second term are:

$$\begin{aligned}
\frac{\partial^2\Phi}{\partial\alpha^2} &= \frac{2}{n}(n-1) \\
&> 0 \\
\frac{\partial^2\Phi}{\partial n\partial\alpha} &= -\frac{2}{n^2}(1-\alpha) \\
&< 0 \\
\frac{\partial\Phi}{\partial n} &= -\frac{1}{n^2}\alpha(2-\alpha) \\
&< 0, \\
\frac{\partial^2\Phi}{\partial n^2} &= \frac{2}{n^3}\alpha(2-\alpha) \\
&> 0,
\end{aligned}$$

and under the assumption that risk aversion follows a uniform distribution $U[a,b]$,

$$\begin{aligned}
\frac{\partial n^*}{\partial\alpha} &= \frac{\frac{\partial n_R}{\partial\alpha}}{1-\frac{\partial n_R}{\partial n}} \\
&< 0 \iff \frac{\partial n_R}{\partial n} < 1, \\
\frac{\partial^2 n^*}{\partial\alpha^2} &= \frac{\frac{\partial^2 n_R}{\partial\alpha^2}\left(1-\frac{\partial n_R}{\partial n}\right)+\frac{\partial n_R}{\partial\alpha}\frac{\partial^2 n_R}{\partial n\partial\alpha}}{\left(1-\frac{\partial n_R}{\partial n}\right)^2} \\
&< 0 \iff \frac{\partial n_R}{\partial n} < 1 \\
\frac{\partial^2 n^*}{\partial\alpha\partial n} &= \frac{\frac{\partial^2 n_R}{\partial n\partial\alpha}\left(1-\frac{\partial n_R}{\partial n}\right)+\frac{\partial n_R}{\partial\alpha}\frac{\partial^2 n_R}{\partial n^2}}{\left(1-\frac{\partial n_R}{\partial n}\right)^2} \\
&> 0
\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial n_R}{\partial\alpha} &= -\Omega\frac{1}{n}\frac{1}{p(h-l)(1-\delta)}\frac{\delta(n-1)+1}{\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)^2} \\
&< 0,
\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 n_R}{\partial \alpha^2} &= -2\Omega \frac{1}{p(h-l)} \frac{1}{n^2(1-\delta)^2} \frac{(\delta(n-1)+1)^2}{\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)^3} \\ &< 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 n_R}{\partial n^2} &= -2\frac{\alpha}{n^3}\Omega \frac{1}{p(h-l)} \left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3} \left(\frac{\delta}{1-\delta}(2-\alpha)\right) \\ &< 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2 n_R}{\partial \alpha \partial n} &= \Omega \frac{1}{p(h-l)(1-\delta)} \frac{(\alpha(1-\delta)+n(2+\alpha)\delta)}{\left(n\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)\right)^3} \\ &> 0.\end{aligned}$$

where

$$\Omega = \frac{N-1}{b-a}.$$

5.3. Preferred α of agents who don't respect the ad interim constraint for $\alpha = \alpha^*$. Two opposite forces are at work in the determination of these agents' preferred α. On the one hand, if they know they will have to leave the group for $\alpha = \alpha^*$, they might as well prefer a larger α and hope to benefit from a high insurance as long as they draw low incomes. On the other hand, they might prefer a level of α strictly lower than α^* in order to be able to satisfy the ad interim constraint and permanently benefit from the insurance group. Formally, choosing a lower α means to choose the level which makes their participation constraint binding, that is

$$\begin{aligned}a_j &= \Theta\left(\alpha_j^*\right) \equiv \left[p(h-l)\left(\frac{\delta}{1-\delta}(2-\alpha_j^*)-\frac{\alpha_j^*}{n}\right)\right]^{-1}, \\ \alpha_j^* &= \frac{1}{\frac{\delta}{1-\delta}+\frac{1}{n}}\left(2\frac{\delta}{1-\delta}-\frac{1}{a_j p(h-l)}\right).\end{aligned}$$

so that the utility of a respectful agent j writes

$$U_j(R) = \frac{\mu - a_j \Phi\left(\alpha_j^*, n_R\left(\alpha_j^*\right)\right)\sigma^2}{1-\delta}.$$

On the other hand, starting from the assumption that the agent would not be able to satisfy the ad interim constraint, his preferred α maximizes U_j, where

$$\begin{aligned}
U_j &= pU_B + (1-p)\,u_R + (1-p)\,\delta\,(pU_B + (1-p)\,u_R) \\
&\quad + (1-p)^2\,\delta^2\,(pU_B + (1-p)\,u_R) + ... \\
&= (pU_B + (1-p)\,u_R) \sum_{t=1}^{+\infty} ((1-p)\,\delta)^{t-1} \\
&= \frac{1}{1-(1-p)\,\delta}\,(pU_B + (1-p)\,u_R)
\end{aligned}$$

Only u_R is affected by α, and the value which maximizes u_R is such that

$$\begin{aligned}
\frac{n-1}{n}\,(\mu - l) &= 2a_i \frac{n-1}{n^2} \alpha' \sigma^2 \\
\alpha' &= \frac{n\,(\mu - l)}{2a_i\sigma^2},
\end{aligned}$$

so that the maximal U_j is

$$\begin{aligned}
& \frac{1}{1-(1-p)\,\delta} \begin{pmatrix} p\left(h + \frac{\delta}{1-\delta}\,(\mu - a_i\sigma^2)\right) \\ +(1-p)\left(l + \frac{(\mu-l)^2(n-1)}{4\sigma^2 a_i}\right) \end{pmatrix} \\
\Leftrightarrow \; & \frac{1}{1-(1-p)\,\delta} \begin{pmatrix} \mu + p\left(\frac{\delta}{1-\delta}\,(\mu - a_i\sigma^2)\right) \\ +(1-p)\left(\frac{(\mu-l)^2(n-1)}{4\sigma^2 a_i}\right) \end{pmatrix}.
\end{aligned}$$

the agent whose risk aversion is lower than $\Theta\,(\alpha^*)$ prefers a lower α_j^* to a higher α_j' iff

$$\frac{\mu - a_j \Phi_j^* \sigma^2}{1-\delta} > \frac{1}{1-(1-p)\,\delta} \begin{pmatrix} \mu + p\left(\frac{\delta}{1-\delta}\,(\mu - a_i\sigma^2)\right) \\ +(1-p)\left(\frac{(\mu-l)^2(n-1)}{4\sigma^2 a_i}\right) \end{pmatrix}.$$

One can see that the LHS is divided by $(1-\delta)$, whereas the RHS is divided by $1-(1-p)\,\delta$. Therefore, if δ and p are sufficiently large, the LHS always dominates the RHS.

5.4. Comparative Statics. Let us assume for the comparative statics that the distribution of risk aversion is approximated by a continuous uniform distribution with support $[a; b]$. One can then rewrite n_R as

$$\begin{aligned} n_R &= 1 + (N-1)\left(1 - \frac{\Theta - a}{b - a}\right) \\ &= -\Omega\Theta + N + (N-1)\frac{a}{b-a}, \end{aligned}$$

where

$$\Omega = \frac{N-1}{b-a}.$$

The partial derivatives on n^* are:

$$\begin{aligned} \frac{\partial n^*}{\partial \alpha} &= \frac{\frac{\partial n_R}{\partial \alpha}}{1 - \frac{\partial n_R}{\partial n}} < 0, \\ \frac{\partial^2 n^*}{\partial \alpha^2} &= \frac{\frac{\partial^2 n_R}{\partial \alpha^2}\left(1 - \frac{\partial n_R}{\partial n}\right) + \frac{\partial n_R}{\partial \alpha}\frac{\partial^2 n_R}{\partial n \partial \alpha}}{\left(1 - \frac{\partial n_R}{\partial n}\right)^2} < 0, \\ \frac{\partial^2 n^*}{\partial \alpha \partial n} &= \frac{\frac{\partial^2 n_R}{\partial n \partial \alpha}\left(1 - \frac{\partial n_R}{\partial n}\right) + \frac{\partial n_R}{\partial \alpha}\frac{\partial^2 n_R}{\partial n^2}}{\left(1 - \frac{\partial n_R}{\partial n}\right)^2} > 0, \end{aligned}$$

with

$$\begin{aligned} \frac{\partial n_R}{\partial \alpha} &= -\Omega\Theta_\alpha < 0, \\ \frac{\partial^2 n_R}{\partial \alpha^2} &= -\Omega\Theta_{\alpha\alpha} < 0, \\ \frac{\partial^2 n_R}{\partial \alpha \partial n} &= -\Omega\Theta_{n\alpha} > 0, \\ \frac{\partial n_R}{\partial n} &= -\Omega\Theta_n > 0 \text{ but } < 1, \\ \frac{\partial^2 n_R}{\partial n^2} &= -\Omega\Theta_{nn} < 0. \end{aligned}$$

since

$$
\begin{aligned}
\Theta_n &\equiv \frac{\partial \Theta}{\partial n} = -\frac{\alpha}{n^2}\frac{1}{p(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-2} \\
&< 0, \\
\Theta_{nn} &\equiv \frac{\partial^2 \Theta}{\partial n^2} = 2\frac{\alpha}{n^3}\frac{\left(\frac{\delta}{1-\delta}(2-\alpha)\right)}{p(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3} \\
&> 0, \\
\Theta_\alpha &\equiv \frac{\partial \Theta}{\partial \alpha} = \frac{\left(\frac{\delta}{1-\delta}+\frac{1}{n}\right)}{p(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-2} \\
&> 0, \\
\Theta_{\alpha\alpha} &\equiv \frac{\partial^2 \Theta}{\partial \alpha^2} = 2\frac{\left(\frac{\delta}{1-\delta}+\frac{1}{n}\right)^2}{p(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3} \\
&> 0, \\
\Theta_{n\alpha} &\equiv \frac{\partial^2 \Theta}{\partial \alpha \partial n} = -\frac{1}{n^2}\frac{\left(\frac{\delta}{1-\delta}(2+\alpha)+\frac{\alpha}{n}\right)}{p(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3} \\
&< 0.
\end{aligned}
$$

We know that α^* is such that

$$\Psi(\alpha, n^*, \theta) \equiv \frac{\partial \Phi(\alpha, n^*)}{\partial \alpha} + \frac{\partial \Phi(\alpha, n^*)}{\partial n}\frac{\partial n^*}{\partial \alpha} = 0,$$

where

$$\frac{\partial n^*}{\partial \alpha} = \frac{\frac{\partial n_R}{\partial \alpha}}{1-\frac{\partial n_R}{\partial n}} < 0,$$

and

$$
\begin{aligned}
\frac{\partial \Phi}{\partial n} &= -\frac{\alpha(2-\alpha)}{n^2} < 0, \\
\frac{\partial \Phi}{\partial \alpha} &= -2\frac{n-1}{n}(1-\alpha) \leq 0, \\
\frac{\partial^2 \Phi}{\partial \alpha \partial n} &= -2\frac{1}{n^2}(1-\alpha) < 0, \\
\frac{\partial^2 \Phi}{\partial n^2} &= -2\frac{1}{n^3}\alpha(2-\alpha) < 0, \\
\frac{\partial^2 \Phi}{\partial \alpha^2} &= 2\frac{n-1}{n} > 0.
\end{aligned}
$$

We also know that

$$\begin{aligned}\frac{\partial\Psi}{\partial\alpha} &= \overset{(+)}{\frac{\partial^2\Phi}{\partial\alpha^2}} + 2\overset{(-)}{\frac{\partial^2\Phi}{\partial n\partial\alpha}}\overset{(-)}{\frac{\partial n^*}{\partial\alpha}} + \overset{(+)}{\frac{\partial^2\Phi}{\partial n^2}}\left(\overset{(+)}{\frac{\partial n^*}{\partial\alpha}}\right)^2 \\ &\quad + \overset{(-)}{\frac{\partial\Phi}{\partial n}}\left(\overset{(-)}{\frac{\partial^2 n^*}{\partial\alpha^2}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial\alpha\partial n}}\overset{(-)}{\frac{\partial n^*}{\partial\alpha}}\right) \\ &> 0.\end{aligned}$$

Therefore, the effect of any parameter θ on α^* is the following:

$$\frac{d\alpha^*}{d\theta} = -\frac{\frac{\partial\Psi}{\partial\theta}}{\frac{\partial\Psi}{\partial\alpha}},$$

so that

$$sign\left(\frac{\partial\alpha^*}{\partial\theta}\right) = sign\left(-\frac{\partial\Psi}{\partial\theta}\right),$$

where

$$\begin{aligned}\frac{\partial\Psi}{\partial\theta} &= \overset{(0)}{\frac{\partial^2\Phi}{\partial\alpha\partial\theta}} + \overset{(-)}{\frac{\partial^2\Phi}{\partial n\partial\alpha}}\overset{(?)}{\frac{\partial n^*}{\partial\theta}} + \left(\overset{(0)}{\frac{\partial^2\Phi}{\partial n\partial\theta}} + \overset{(+)}{\frac{\partial^2\Phi}{\partial n^2}}\overset{(?)}{\frac{\partial n^*}{\partial\theta}}\right)\overset{(-)}{\frac{\partial n^*}{\partial\alpha}} \\ &\quad + \overset{(-)}{\frac{\partial\Phi}{\partial n}}\left(\overset{(?)}{\frac{\partial^2 n^*}{\partial\alpha\partial\theta}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial\alpha\partial n}}\overset{(?)}{\frac{\partial n^*}{\partial\theta}}\right)\end{aligned}$$

with

$$\begin{aligned}\frac{\partial n^*}{\partial\theta} &= -\frac{-\frac{\partial n_R}{\partial\theta}}{\left(1-\frac{\partial n_R}{\partial n}\right)}, \\ \frac{\partial^2 n^*}{\partial\alpha\partial\theta} &= \frac{\frac{\partial^2 n_R}{\partial\alpha\partial\theta}\left(1-\frac{\partial n_R}{\partial n}\right) + \frac{\partial n_R}{\partial\alpha}\frac{\partial^2 n_R}{\partial n\partial\theta}}{\left(1-\frac{\partial n_R}{\partial n}\right)^2}.\end{aligned}$$

The effect of any parameter θ on n^* is

$$\frac{dn^*}{d\theta} = \overset{(?)}{\frac{\partial n^*}{\partial\theta}} + \overset{(-)}{\frac{\partial n^*}{\partial\alpha}}\overset{(?)}{\frac{\partial\alpha^*}{\partial\theta}}.$$

5.4.1. *Proof of proposition 3.* We write h and l as functions of μ, σ and p:

$$\begin{aligned} h &= \mu + \sigma \sqrt{\frac{1-p}{p}}, \\ l &= \mu - \sigma \sqrt{\frac{p}{1-p}}, \\ p\,(h-l) &= \sigma p', \end{aligned}$$

where $p' = \frac{\sqrt{p}}{\sqrt{1-p}}$. The effect of σ on Ψ is

$$\begin{aligned} \frac{\partial \Psi}{\partial \sigma} &= \overset{(0)}{\frac{\partial^2 \Phi}{\partial \alpha \partial \sigma}} + \overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}} \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} + \left(\overset{(0)}{\frac{\partial^2 \Phi}{\partial n \partial \sigma}} + \overset{(+)}{\frac{\partial^2 \Phi}{\partial n^2}} \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} \right) \overset{(-)}{\frac{\partial n^*}{\partial \alpha}} \\ & \quad + \overset{(-)}{\frac{\partial \Phi}{\partial n}} \left(\overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial \sigma}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial n}} \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} \right) \\ &< 0 \end{aligned}$$

with

$$\begin{aligned} \frac{\partial n^*}{\partial \sigma} &= \frac{\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}}{\left(1 - \frac{\partial n_R}{\partial n}\right)} \\ &> 0, \\ \frac{\partial^2 n^*}{\partial \alpha \partial \sigma} &= \frac{\overset{(+)}{\frac{\partial^2 n_R}{\partial \alpha \partial \sigma}} \overset{(+)}{\left(1 - \frac{\partial n_R}{\partial n}\right)} + \overset{(-)}{\frac{\partial n_R}{\partial \alpha}} \overset{(-)}{\frac{\partial^2 n_R}{\partial n \partial \sigma}}}{\left(1 - \frac{\partial n_R}{\partial n}\right)^2} \\ &> 0, \end{aligned}$$

since

$$\begin{aligned} \frac{\partial^2 n_R}{\partial \alpha \partial \sigma} &= -\Omega \Theta_{\alpha\sigma} > 0, \\ \text{with } \Theta_{\alpha\sigma} &= -\sigma^{-1} \Theta_{\alpha} < 0, \end{aligned}$$

$$
\begin{aligned}
\frac{\partial n_R}{\partial \sigma} &= -\Omega \Theta_\sigma > 0, \\
\text{with } \Theta_\sigma &= -\sigma^{-1} \frac{1}{p\,(h-l)} \left[\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n} \right]^{-1} < 0,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial^2 n_R}{\partial n \partial \sigma} &= -\Omega \Theta_{n\sigma} < 0, \\
\text{with } \Theta_{n\sigma} &= \frac{\alpha}{n^2} \sigma^{-1} \frac{1}{p\,(h-l)} \left[\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n} \right]^{-2} > 0.
\end{aligned}
$$

It implies that

$$
\begin{aligned}
\frac{\partial \alpha^*}{\partial \sigma} &= -\frac{\frac{\partial \Psi}{\partial \sigma}}{\frac{\partial \Psi}{\partial \alpha}} \\
&> 0.
\end{aligned}
$$

The effect of σ on n^* is

$$
\begin{aligned}
\frac{dn^*}{d\sigma} &= \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} + \overset{(-)}{\frac{\partial n^*}{\partial \alpha}}\overset{(+)}{\frac{\partial \alpha^*}{\partial \sigma}} \\
&= \frac{1}{\frac{\partial \Psi}{\partial \alpha}}\left(1-\frac{\partial n_R}{\partial n}\right)^{-1}\left(\frac{\partial n_R}{\partial \sigma}\frac{\partial \Psi}{\partial \alpha} - \frac{\partial n_R}{\partial \alpha}\frac{\partial \Psi}{\partial \sigma}\right) \\
&= \frac{1}{\frac{\partial \Psi}{\partial \alpha}}\left(1-\frac{\partial n_R}{\partial n}\right)^{-1}\left(\begin{array}{c}
\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}\left(\begin{array}{c}
\overset{(+)}{\frac{\partial^2 \Phi}{\partial \alpha^2}} + 2\overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}}\overset{(-)}{\frac{\partial n^*}{\partial \alpha}} + \overset{(+)}{\frac{\partial^2 \Phi}{\partial n^2}}\overset{(+)}{\left(\frac{\partial n^*}{\partial \alpha}\right)^2} \\
+\overset{(-)}{\frac{\partial \Phi}{\partial n}}\left(\overset{(-)}{\frac{\partial^2 n^*}{\partial \alpha^2}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial n}}\overset{(-)}{\frac{\partial n^*}{\partial \alpha}}\right)
\end{array}\right) \\
-\frac{\partial n_R}{\partial \alpha}\left(\begin{array}{c}
\overset{(0)}{\frac{\partial^2 \Phi}{\partial \alpha \partial \sigma}} + \overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}}\overset{(+)}{\frac{\partial n^*}{\partial \sigma}} \\
+\left(\overset{(0)}{\frac{\partial^2 \Phi}{\partial n \partial \sigma}} + \overset{(+)}{\frac{\partial^2 \Phi}{\partial n^2}}\overset{(+)}{\frac{\partial n^*}{\partial \sigma}}\right)\overset{(-)}{\frac{\partial n^*}{\partial \alpha}} \\
+\overset{(-)}{\frac{\partial \Phi}{\partial n}}\left(\overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial \sigma}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial n}}\overset{(+)}{\frac{\partial n^*}{\partial \sigma}}\right)
\end{array}\right)
\end{array}\right) \\
&= \frac{1}{\frac{\partial \Psi}{\partial \alpha}}\left(1-\frac{\partial n_R}{\partial n}\right)^{-1}\left(\begin{array}{c}
\overset{(+)}{\overset{(+)}{\left(\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}\overset{(+)}{\frac{\partial^2 \Phi}{\partial \alpha^2}}\left(1-\frac{\partial n_R}{\partial n}\right)^2\right)}} \\
+\overset{(+)}{\left(\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}\overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}}\overset{(-)}{\frac{\partial n_R}{\partial \alpha}}\overset{(+)}{\left(1-\frac{\partial n_R}{\partial n}\right)}\right)} \\
+\overset{(-)}{\frac{\partial \Phi}{\partial n}}\left(1-\frac{\partial n_R}{\partial n}\right)\overset{(-)}{\left(\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}\overset{(-)}{\frac{\partial^2 n_R}{\partial \alpha^2}} - \frac{\partial n_R}{\partial \alpha}\overset{(+)}{\frac{\partial^2 n_R}{\partial \alpha \partial \sigma}}\right)} \\
+\overset{(-)}{\frac{\partial \Phi}{\partial n}}\frac{\partial n_R}{\partial \alpha}\overset{(+)}{\left(\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}\overset{(+)}{\frac{\partial^2 n_R}{\partial n \partial \alpha}} - \frac{\partial n_R}{\partial \alpha}\overset{(-)}{\frac{\partial^2 n_R}{\partial n \partial \sigma}}\right)}
\end{array}\right) \\
&> 0.
\end{aligned}
$$

Since

$$\begin{aligned}
\frac{\partial n_R}{\partial \sigma} &= \Omega\left(p'\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)\right)^{-1}\sigma^{-2} \\
&> 0 \\
\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}\overset{(-)}{\frac{\partial^2 n_R}{\partial \alpha^2}} - \overset{(-)}{\frac{\partial n_R}{\partial \alpha}}\overset{(+)}{\frac{\partial^2 n_R}{\partial \alpha \partial \sigma}} &= -\frac{1}{n^2(p')^2}\frac{\Omega^2}{\sigma^3}\frac{1}{(1-\delta)^2}\frac{((1-\delta)+\delta n)^2}{\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)^4} \\
&< 0. \\
\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}\overset{(+)}{\frac{\partial^2 n_R}{\partial n \partial \alpha}} - \overset{(-)}{\frac{\partial n_R}{\partial \alpha}}\overset{(-)}{\frac{\partial^2 n_R}{\partial n \partial \sigma}} &= 2\frac{1}{n^2(p')^2}\frac{\Omega^2}{\sigma^3}\frac{\delta}{(1-\delta)}\frac{1}{\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)^4} \\
&> 0.
\end{aligned}$$

5.4.2. *Proof of corollary 1.* The effect of a mean preserving spread of Y on the insured income risk is

$$\begin{aligned}
\frac{d\left(\Phi\sigma^2\right)}{d\sigma} &= \sigma^2\frac{d\Phi}{d\sigma}+2\sigma\Phi \\
&< 0 \Leftrightarrow -\frac{d\Phi}{d\sigma} > \frac{2\Phi}{\sigma},
\end{aligned}$$

where

$$\begin{aligned}
\frac{d\Phi}{d\sigma} &= \overset{(-)}{\frac{\partial \Phi}{\partial \alpha}}\overset{(+)}{\frac{\partial \alpha^*}{\partial \sigma}} + \overset{(-)}{\frac{\partial \Phi}{\partial n}}\left(\frac{\partial n^*}{\partial \sigma}+\overset{(+)}{\frac{\partial n^*}{\partial \alpha}}\frac{\partial \alpha^*}{\partial \sigma}\right) \\
&< 0.
\end{aligned}$$

5.4.3. *Proof of proposition 4.* First note that when the pre-transfer income variance is higher, more agents are part of the insurance group since $\frac{dn^*}{d\sigma} > 0$. To see an improvement of the welfare of these agents as the income variance increases, their welfare of being out of the agreement before the variance increase

$$U_i\left(B,\sigma'\right) = \frac{\mu - a_i\sigma'^2}{1-\delta},$$

has to be smaller than the welfare of being part of the group with a higher uninsured income variance

$$U_i(R,\sigma) = \frac{\mu - a_i \Phi \sigma^2}{1-\delta}$$

with

$$\begin{aligned} \sigma &= \sigma' + \Delta\sigma, \\ \Phi &= \Phi\left(\alpha^*(\sigma), n^*(\alpha^*(\sigma))\right). \end{aligned}$$

We have

$$\begin{aligned} \frac{\mu - a_i \sigma'^2}{1-\delta} &< \frac{\mu - a_i \Phi \sigma^2}{1-\delta} \\ &\Leftrightarrow \sigma'^2 > \Phi\sigma^2 \\ &\Leftrightarrow \Phi(\alpha^*, n^*) < \left(1 - \frac{\Delta\sigma}{\sigma}\right)^2. \end{aligned}$$

CHAPTER 3

The Interaction between Informal Insurance and Farm Households' Risk-taking Behaviors

In developing countries, the ability of farm households to cope with risk is a key determinant of their daily livelihood. This chapter aims at highlighting the relationship between self-insurance mechanisms (implemented before crop risks are realized) and mutual insurance, and the way this relationship is affected by household wealth. The evidence in this topic is that rich households both invest more in technology adoption and in mutual insurance than poor ones. We build a model in which agents, heterogeneously endowed in terms of buffer stocks, non-cooperatively decide on their level of subscription to mutual insurance and the riskiness of their production plans. Our results reproduce the stylised facts described above: households endowed with large buffer stocks fully subscribe to mutual insurance and take more risks in production. This behavior increases the risk of the income pool, which limits the participation and the risk-taking of poor households.

1. Introduction

In developing countries, the ability of farm households to cope with risk is a key determinant of their daily livelihood. Poverty, food security and insurance issues are intrinsically linked to one another. This chapter elaborates on these issues, highlighting the relationship between self- and mutual insurance, and the way this relationship is affected by household wealth. Indeed self- and mutual insurance are neither exclusive nor independent.

Self-insurance mechanisms are implemented before crop risks are realized. We can refer to them as ex-ante insurance. It covers income diversification, technology adoption and production choices or any individual means of mitigating risk ex ante. Kurosaki & Fafchamps (2002) provide empirical evidence that production choices belong to the set of risk-coping strategies in contexts of market imperfections. In line with our motivation, it has been empirically observed that household wealth does affect technology adoption, which tends to be low among the poorest households (Dercon & Christiaensen (2007)).

On the other hand, once crops, and potential shocks, are realized, consumption relies on the degree of ex-post insurance, which is mainly provided by households' involvement in mutual insurance mechanisms. Empirical evidence also suggests that poor households' consumption paths seem to be more affected by idiosyncratic shocks (Jalan & Ravallion (1999); Morduch (1995)). The general lack of access to ex post insurance in rural areas results from market failure and is well documented in the literature. To smooth consumption ex post, rural households can rely on informal insurance transfers. Due to moral hazard and imperfect commitment, full informal insurance has been rejected theoretically and empirically. On the one hand, from a theoretical perspective, the impact of limited commitment is widely acknowledged (Coate & Ravallion

(1993); Ligon et al. (2002); Murgai et al. (2002)) while on the other hand, the hypothesis of full income pooling is often empirically rejected (Townsend (1994); Jalan & Ravallion (1999); Hoogeveen (2002); Murgai et al. (2002); Morduch (1995)). Households may also have invested in assets with buffer purposes in the past and use them as another means of smoothing consumption. But it is naturally strongly dependent on household wealth.

One interpretation of the two above-mentioned stylized facts is that, considering the inability to smooth consumption ex post as given, poor farmers are reluctant to adopt high-return risky production strategies. However, we argue that a satisfactory explanation to this phenomenon can only be provided if one studies simultaneously both ex-ante and ex-post insurance decisions.

We present a model with agents heterogeneously endowed in terms of buffer stocks, who non-cooperatively decide on their risk-coping strategies, namely their level of subscription to the mutual insurance scheme and their production choices. Our theoretical contribution thus consists in considering both self- and mutual insurance as endogenous. The intuition behind our main result is the following: rich households tend to adopt risky production strategies that impact on the variance of the income pool. This gives an incentive to poorer farmers to partially withdraw from community level risk pooling. In addition, the model accounts for imperfect risk pooling without assuming neither information asymmetries nor imperfect commitment. Population heterogeneity is highlighted as a major determinant of incomplete community sharing.

Section 2 presents our assumptions regarding the risk environment and the way we model ex ante self-insurance strategies. In Section 3, the mutual insurance scheme is described. The Nash Equilibrium of the model is then computed in sections 4 and 5. Section 6 is aimed at providing a benchmark case in which population heterogeneity is removed. Section 7 concludes.

2. The Risk Environment and Self-Insurance Mechanisms

The community is characterized by a continuum of agents of size 1. Agents are denoted by $i \in [0,1] \equiv C$. Agent i's income Y_i is a random variable whose distribution is affected by its risk-taking behavior as well as by two types of shocks, namely zero-mean covariate shocks X and zero-mean idiosyncratic shocks Z.

$$Y_i = a + b\sigma_i + X_i + Z_i, \tag{2.1}$$

where $a \in R_+$ and $b \in R_+$ are constant for all i. Since X and Z have zero means, agent i's expected income is simply

$$E(Y_i, \sigma_i) = a + b\sigma_i. \tag{2.2}$$

The household's risk strategy $\sigma_i \in R_+$ is intended to capture the range of production choices that a farm household can take. Considering a given activity, the farmer can adopt technologies with different implications in terms of risk and return, choosing between traditional and improved seeds, for instance. From a broader point of view, the household may specialize in a given production strategy and gain from economies of scope or spread the allocation of its land and labour resources over a portfolio of activities in order to diversify income sources. We want to capture the idea that riskier strategies result in a twofold increase in the income mean and volatility. This is translated by the fact that while a unit increase in σ raises $E(Y)$by b, σ also affects positively the variance of shocks, as will be discussed after the description of covariate and idiosyncratic shocks.

On the one hand, the covariate shock , such as a drought or crop disease, is assumed perfectly positively correlated at the community level:

$$Corr(X_i, X_j) = 1;\ \forall i, j \in C\ . \tag{2.3}$$

On the other hand, the idiosyncratic shock Z, such as the death of a household member, is assumed independent across individuals as well as from X:

$$Z_i \perp Z_j;\ \forall i \neq j \in C\ , \tag{2.4}$$

$$X_i \perp Z_j;\ \forall i, j \in C\ . \tag{2.5}$$

What about the income variance? As previously mentioned, it is due to income shocks, and will be contingent on the agent's risk strategy. The larger σ, the larger variances of X and Z:

$$VAR(X_i; \sigma_i) = c\sigma_i^2, \tag{2.6}$$

$$VAR(Z_i; \sigma_i) = (1-c)\sigma_i^2, \tag{2.7}$$

where c is a constant that represents the fraction of total risk that is covariate, so that (2.5) implies that total income variance is simply

$$VAR(Y_i; \sigma_i) = VAR(X_i; \sigma_i) + VAR(Z_i; \sigma_i) = \sigma_i^2.$$

To summarize the information displayed up to now, agents face two types of risk (covariate and idiosyncratic) and can self-insure against them ex-ante by limiting their risk-taking behaviour σ (see (2.7)) but self-insurance also has the cost of decreasing expected income (see (2.2)).
Preferences are represented by a mean-variance utility function:

$$U_\rho(Y) = E(Y) - \rho VAR(Y),$$

where ρ is the agent's risk aversion coefficient as defined by the marginal disutility of variance, and $E(Y)$ and $VAR(Y)$ depend as previously mentioned on the risk strategy σ, but also on the household's implication in ex-post mutual insurance mechanisms, another choice variable of the household in our model[1]. The parameter ρ is inversely related to the household's ability to smooth consumption ex post through savings and dissavings, which can be interpreted as household's buffer stocks[2]. Indeed, higher buffer stocks allow to better mitigate the adverse effect of income variance. Therefore, in the developments below, we consider the heterogeneity in risk aversion as a reduced form of wealth inequalities. Precisely, C is a continuum of agents of size 1 who are heterogeneously endowed in terms of risk aversion: $\rho \sim F_\rho$.

The utility maximization problem is well-behaved, as benefits from risk-taking are linear, while its costs are convex. As an illustration, let us solve the optimization problem of an autarkic household, that is, a household that does not have the possibility to contract an informal insurance agreement with relatives or neighbors. In such a context, the optimal risk-taking strategy is:

$$\sigma_\rho \equiv \arg\max_{\sigma \in R_+} U_\rho(Y) = \frac{b}{2\rho}.$$

As expected, the optimal level of risk-taking is a decreasing function of risk aversion.

[1] Ex-post mutual insurance will be introduced in the next section. While ex-ante self-insurance enables the household to mitigate the impact of a shock of any type, ex-post mutual insurance only allows to trade idiosyncratic risks within the village. Hence, the distinction between the two types of shocks is needed.

[2] An important underlying assumption is that access to credit is constrained and cannot serve the purpose of consumption smoothing. Alternatively, one can consider that restrictions of access to credit are heterogeneously distributed in the population. Since, empirically, credit rationing is negatively correlated with wealth; again, wealthier households are better endowed in terms of protection against post-transfer residual shocks. Hence, interpretations go in the same direction.

3. The Mutual Insurance Scheme

In addition to ex ante self-insurance, households also have the possibility to smooth consumption ex post by sharing risks within the community. In order to focus this discussion on the relation between self and mutual insurance, the analysis on the mutual insurance side is simplified by assuming that mutual insurance agreements are always enforceable.

For the sake of clarity, means of consumption smoothing can be chronologically listed. Incidentally, we define the structure of the game:

(1) Agents simultaneously and non-cooperatively decide on their risk coping strategies, namely their level of risk taking, $\sigma_\rho \in R_+$, and their level of subscription to mutual insurance, $\alpha_\rho \in [0,1]$.
(2) Income Y is realized.
(3) Insurance transfers take place.
(4) Households rely on buffer stocks to further smooth their consumption path.

We model risk sharing as the contribution to a virtual community income pool. The mutual insurance mechanism we present here has the property of being actuarially fair: the average transfer is zero, for each household and whatever the subscription level. In other words, no household systematically provides transfers to some other. With this assumption, we can focus on pure risk-sharing by distinguishing it from redistribution mechanisms.

The pool is redistributed according to the level of household's subscription, denoted by $\alpha \in [0,1]$. Zero corresponds to autarky and one to full subscription. More specifically, each household commits to provide a share of its income, noted

$$T_o = \alpha Y_\rho, \tag{3.1}$$

where stands T_o for transfer out. On the other hand, the household benefits from a transfer from the income pool that is a share of the pool. This transfer in is

$$T_I = s_\rho \int \alpha_\rho Y_\rho dF(\rho). \tag{3.2}$$

The income pool $\int \alpha_\rho Y_\rho dF(\rho)$ gathers the transfers out over the whole population and each household is then entitled to a share of the pool, noted s_ρ. In order for the insurance mechanism to be actuarially fair, this share must be such that the mean net transfer is nil:

$$\begin{aligned} E\left(T_I\right) &= s_\rho \int \alpha_\rho E\left(Y_\rho\right) dF(\rho) = \alpha_\rho \left(a + b\sigma_\rho\right) = T_o \\ &\Leftrightarrow s_\rho = \frac{\alpha_\rho \left(a + b\sigma_\rho\right)}{\int \alpha_\rho E\left(Y_\rho\right) dF(\rho)}. \end{aligned} \tag{3.3}$$

Re-writing the latter as

$$s_\rho = \frac{\alpha_\rho}{E_\rho\left(\alpha_\rho\right)} \frac{\left(a + b\sigma_\rho\right)}{\frac{E_\rho\left[\alpha_\rho\left(a+b\sigma_\rho\right)\right]}{E_\rho\left(\alpha_\rho\right)}},$$

we are able to highlight that the share of the income pool to which a type ρ household is entitled depends on two factors. On the one hand, it is an increasing function of household ρ's relative contribution to the income pool, $\frac{\alpha_\rho}{E_\rho(\alpha_\rho)}$. The higher the subscription to mutual insurance as compared to the others' subscriptions, the higher the transfer received. On the other hand, s_ρ depends on the ratio of the household's expected income, $(a + b\sigma_\rho)$, to the pool average $\frac{E_\rho[\alpha_\rho(a+b\sigma_\rho)]}{E_\rho(\alpha_\rho)}$.

Since utility is defined over the post-transfer income $R = Y - T_o + T_I$, it remains to compute its mean and variance. First, since the mutual insurance mechanism is actuarially fair,

$$E(R_\rho; \sigma_\rho) = E(Y_\rho, \sigma_\rho) = a + b\sigma_\rho.$$

Second, making use of the properties of both types of random variables in terms of variance and covariance, Appendix 1 provides the proof that the variance of R writes

$$(3.4)\quad VAR(R_\rho;\sigma_\rho,\alpha_\rho) = (1-c)\,(1-\alpha_\rho)^2\,\sigma_\rho^2 + c\left[\begin{array}{c}(1-\alpha_\rho)\,\sigma_\rho\\ +\alpha_\rho\frac{a+b\sigma_\rho}{a+b\theta}\theta\end{array}\right]^2,$$

where $\theta = \frac{E_\rho(\alpha_\rho\sigma_\rho)}{E_\rho(\alpha_\rho)}$ is the average risk taking behavior in the income pool.

The variance is composed of two terms. The first term is the idiosyncratic risk that keeps hurting the household. Notice that if the agent fully subscribes to the mutual insurance scheme, he can totally remove the impact of the idiosyncratic shock. The reason for this is twofold: on the one hand, these shocks are, by definition, independent between individuals, on the other hand, idiosyncratic risks are spread over a sufficiently large population. Hence, the Law of Large Numbers applies (see Appendix 1). The second term corresponds to the covariate fraction of the variance. It is a function of household 's own production choices but also on the risk taking behaviors of all the other participants to the insurance scheme. In that sense, any agent who subscribes to the risk pool exposes himself to the variance level induced by the others' production decisions. In the other direction, the more a household engages in risk sharing, the more it impacts on the risk composition of the income pool. This mechanism is a key to the model and gives rise to strategic interaction. In the following subsection, we compute the Nash Equilibrium of the game which is depicted above.

4. The Equilibrium Conditions

The first step consists in examining agents' strategies in terms of ex ante self insurance. The objective function of a type agent can be written as

$$(4.1)\qquad U_\rho\,(R_\rho) = E(R_\rho;\sigma_\rho) - \rho VAR(R_\rho;\sigma_\rho,\alpha_\rho).$$

The first order condition with respect to σ_ρ is as follows:

$$\begin{aligned}
\frac{b}{2\rho} &= (1-c)(1-\alpha_\rho)^2\sigma_\rho \\
&\quad +c\begin{pmatrix} \left[(1-\alpha_\rho)\sigma_\rho+\alpha_\rho\frac{a+b\sigma_\rho}{a+b\theta}\theta\right] \\ \left[(1-\alpha_\rho)+\alpha_\rho\frac{b\theta}{a+b\theta}\right] \end{pmatrix}
\end{aligned} \tag{4.2}$$

$$\Leftrightarrow \sigma^*_\rho(\alpha_\rho,\theta;\rho) = \frac{b(a+b\theta)^2-2\rho ac\alpha_\rho\theta\left((1-\alpha_\rho)a+b\theta\right)}{2\rho\begin{pmatrix}(1-c)(1-\alpha_\rho)^2(a+b\theta)^2 \\ +c\left((1-\alpha_\rho)a+b\theta\right)^2\end{pmatrix}}. \tag{4.3}$$

As in the autarkic situation, a marginal increase in individual risk taking results in a twofold increase of the income mean and volatility. As a function of its level of risk aversion, the household balances both effects. Equation (4.3) is a reaction function since it still depends on the strategy profile through θ, although the continuum of agents ensures that σ_ρ itself for a given ρ has a negligible impact on θ.

As a second step, regarding the desired subscription to the mutual insurance scheme, we work out the partial derivative of (4.1) with respect to α_ρ:

$$\frac{\partial U_\rho}{\partial\alpha_\rho} = -\rho\left(\frac{\partial VAR_I(R_\rho;\sigma_\rho,\alpha_\rho)}{\partial\alpha_\rho}+\frac{\partial VAR_C(R_\rho;\sigma_\rho,\alpha_\rho)}{\partial\alpha_\rho}\right), \tag{4.4}$$

where the subscripts I and C stand for idiosyncratic and covariate, respectively and where

$$\begin{aligned}
\frac{\partial VAR_I(R_\rho;\sigma_\rho,\alpha_\rho)}{\partial\alpha_\rho} &= -(1-c)(1-\alpha_\rho)^2\sigma_\rho \le 0 \\
\frac{\partial VAR_C(R_\rho;\sigma_\rho,\alpha_\rho)}{\partial\alpha_\rho} &= \begin{pmatrix} c\left[(1-\alpha_\rho)\sigma_\rho+\alpha_\rho\frac{a+b\sigma_\rho}{a+b\theta}\theta\right] \\ \left[\frac{a+b\sigma_\rho}{a+b\theta}\theta-\sigma_\rho\right]\end{pmatrix} \le 0
\end{aligned}$$

$$\Leftrightarrow \quad \sigma_\rho \ge \frac{E_\rho(\alpha_\rho\sigma_\rho)}{E_\rho(\alpha_\rho)} = \theta. \tag{4.5}$$

As already mentioned, community risk pooling is an effective means of absorbing the impact of idiosyncratic shocks on households' post-transfer income. However, the effect of income pooling on the extent of covariate risks is ambiguous and depends on the household's risk-taking behavior as compared to others. Making use of the above result, the population can be divided into two parts in the equilibrium strategy profile, agents whose risk taking behavior is below the pool weighted average and those whose risk level is above this same threshold.

For the latter category, the Nash equilibrium strategy is given by

$$\begin{aligned} \alpha_\rho &= 1 \\ \sigma_\rho^* &= \frac{(a+b\theta)^2}{2\rho c b\theta^2} - \frac{a}{b}. \end{aligned} \tag{4.6}$$

These agents take more risk than the pool and hence are willing to fully insure. Their exact level of risk-taking behavior is computed following (4.3) with $\alpha_\rho = 1$; it is still decreasing in ρ despite full insurance because c is non-zero.

For the former category, the equilibrium strategies are given by (4.3), in terms of risk-taking, and such that expression (4.4) is equal to zero. Indeed, agents who take less risk than the pool face a usual trade-off shown in the first-order condition rather than a corner solution. At their level of risk, a rise in the contribution to the pool decreases the exposure to idiosyncratic shocks, as is always the case, but increases their exposure to covariate risk because others are more prone to taking risks. This leads to an interior solution for α_ρ.

Together with the definition of $\theta = \frac{E_\rho(\alpha_\rho\sigma_\rho)}{E_\rho(\alpha_\rho)}$, this completes the set of equilibrium conditions.

In addition, it can be shown that a strategy profile symmetric in terms of risk-taking behaviors cannot be an equilibrium.

PROOF. Suppose an initial situation in which $\sigma_\rho = \widetilde{\sigma}, \forall \rho$, where $\widetilde{\sigma}$ denotes an arbitrary amount in R_+. In such an event, $\theta = \frac{E_\rho(\alpha_\rho \sigma_\rho)}{E_\rho(\alpha_\rho)} = \widetilde{\sigma}$ for any vector of α_ρ. It follows that condition (4.5) holds with equality for all the players. Hence, contribution to the community income pool does not affect the covariate fraction of any household's income variance. The whole population has an incentive to fully subscribe and hence choose $\alpha_\rho = 1$. As a final step, we prove that the latter is not compatible with an homogeneous choice of α_ρ. Indeed, as is evident from (4.6),

$$\frac{\partial \sigma_\rho^* (1, \theta; \rho)}{\partial \rho} < 0.$$

Hence, a strict heterogeneity in term of risk aversion results in strictly differentiated production choices in the population. □

Up to this point, the equilibrium is only characterized in terms of behaviors. The section to follow is aimed at establishing the relationship between the equilibrium strategies and the exogenous characteristics of the households, such as their wealth or endowment in terms buffer stocks embodied by risk aversion in the model.
However, two preliminary results are worth discussing. First, the extent of risk taking is positively related to participation in risk pooling mechanisms. Put in another way, ex ante self insurance and ex post mutual insurance are substitutes rather than complements. A configuration in which risk-averse agents are simultaneously deeply involved in community risk sharing and opt for stable income sources should not be observed. The second result, discussed in section 6, is a pattern of incomplete risk sharing. Indeed, all players whose equilibrium level of risk taking is below the pool average opt for partial subscription to mutual insurance. The model therefore accounts for imperfect risk pooling without having recourse to neither moral hazard nor imperfect commitment. As

argued in section 6, incomplete risk pooling is the outcome of population heterogeneity.
In the following section, the distribution of types is specified in order to draw a prediction regarding the link between individual risk-taking, subscription to mutual insurance and risk aversion.

5. Numerical Examples

A positive relationship between risk-taking and subscription to mutual insurance emerges as the outcome of the above equilibrium conditions. This prediction remains to be confronted with the distribution of types. Intuition suggests that, due to a higher ability to rely on buffer stocks to absorb residual fluctuations, rich households should be those that engage in relatively risky production practices while being relatively more involved in risk pooling mechanisms. The converse should be true for poor households. In this subsection, the distribution of types in the population is specified and numerical examples are presented in order to test for this pattern of behaviors. More specifically, let us suppose that risk aversion is uniformly distributed: $\rho \sim U\left[0.05; 10\right]$. If strictly risk neutral agents had to be included in the population, their level of risk taking would tend to infinity and would prevent the model from converging. In addition, assume the following regarding the exogenous parameters: $a = 0.1; b = 1$.
Figures 1a and 1b present the equilibrium strategy profile in terms of α and σ, respectively, for a situation where 30% of total production risk is covariate, $c = 0.3$. Numerical computations confirm that subscription to mutual insurance (figure 1a) and the level of risk taking are both a decreasing function of risk aversion. Although the shape of the former may slightly vary depending on the hypothesized parameter values, the relationship remains downward sloping for any $(a, b) \in R_+^2$, $c \in [0, 1]$. The negative slope of the former

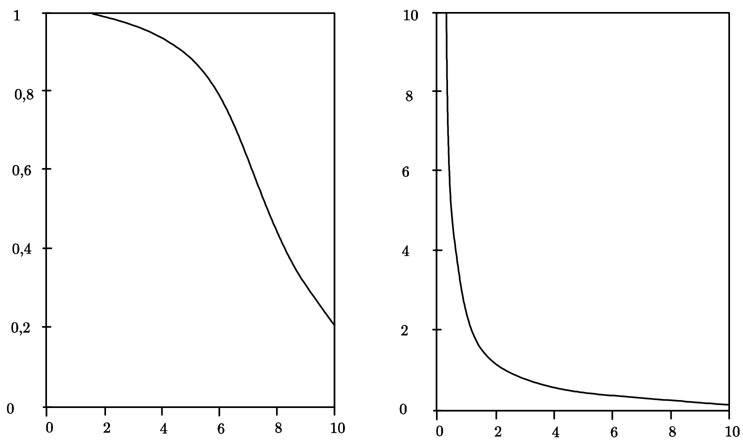

FIGURE 1. (a) Subscription to Mutual Insurance α as a Function of Risk Aversion ρ and (b) Risk-Taking σ as a Function of Risk Aversion ρ; under $a = 0.1$; $b = 1$ and $c = 0.3$

relationship is also robust to arbitrary changes in the parameter values.

Table 1 describes the equilibrium profile under different assumptions regarding the fraction of covariate shocks in total production risk. As in figures 1a and 1b, $a = 0.1; b = 1$.

c	$E_\rho(\alpha_\rho)$	$E_\rho(\sigma_\rho)$	θ
0.1	0.99	3.03	3.06
0.2	0.94	1.55	1.63
0.3	0.77	1.03	1.29
0.4	0.51	0.74	1.33
0.5	0.34	0.57	1.46
0.6	0.23	0.47	1.65
0.7	0.15	0.40	1.95
0.8	0.09	0.35	2.48
0.9	0.04	0.31	3.68

TABLE 1. Equilibrium profiles under different values of c with $a = 0.1; b = 1$.

The second column of table 1 reports the equilibrium average subscription to mutual insurance. It can be noted that this amount tends to decrease when c gets larger. This result is conform to expectations. Indeed, risk sharing at the community level is an effective means of absorbing orthogonal shocks. It follows that the larger is the fraction of those types of risk, that is the lower is c, the higher is the benefit from collective insurance.
Average risk taking is reported in the third column of table 1. Even if a change in the risk composition does not impact on the extent of total risk, self insurance behaviors appear to depend on c. The reason for this negative relationship is twofold. On the one hand, lower levels of risk taking result from the reduction in the extent of equilibrium risk sharing. Indeed, faced with a lack of protection against idiosyncratic shocks, households rely on ex ante self insurance to cope with income fluctuations. On the other hand, since the partial derivative of (4.6) with respect to c is negative, the share of the population that remains at a corner solution ($\alpha = 1$) tends to opt for more self insurance as c increases. This is due to the fact that the fraction of total risk that is absorbed through community risk pooling is reduced.
As fourth and last observation, you may notice that θ, and hence the average risk taking in the income pool, is non-monotonic in c. The intuition is as follows: as discussed above, a higher c tends to result in a lower σ for the entire population. However, we know that the composition of the income pool is biased towards relatively neutral households. Moreover, as a change in c affects the tradeoff between idiosyncratic and covariate risks for more risk averse households, the reduction in participation in mutual insurance is

primarily due to the withdrawal of more averse agents. Hence, average risk taking in the income pool increases as the outcome of a change of its composition.

6. The Case of Homogeneous Preferences

The purpose of this section is twofold. On the one hand, it is aimed at providing a benchmark case in which population heterogeneity is removed. This enables us to show that the latter is precisely the source of incomplete risk pooling in our framework. On the other hand, this simplifying assumption acts as a plausibility check of the model. Indeed, the homogeneity assumption allows to illustrate the argument of moral hazard in risk-taking (see, for example, Miller & Paulson (2007), for an empirical illustration). In a non-cooperative framework, if one considers community insurance as given, one should expect excessive risk-taking as compared to a second best[3] situation. It happens that the model has the desirable property of replicating this result.

Consequently, let us suppose that risk aversion is identical to all player . As shown in section 4, a strategy profile symmetric in terms of risk-taking cannot be an equilibrium under strict heterogeneity. As a direct consequence of the homogeneity assumption, the argument of section 4 does not apply. Hence, let us consider this possibility. As already argued, if, at equilibrium, the whole population chooses the same level of σ, then, at the margin, an additional subscription to the income pool does not entail any cost in terms of covariate variance. In other words, condition (4.5) holds with equality. It follows that the whole population ends up at a corner solution ($\alpha = 1$) and fully participates in the mutual insurance scheme. Under homogeneous preference, there is therefore complete risk sharing.

[3]The first best situation is the existence of a competitive insurance market.

Let us briefly elaborate on this result. Notice, first, that such an outcome is not consistent with empirical observations since full income pooling is generally rejected (Townsend (1994); Jalan & Ravallion (1999); Hoogeveen (2002); Murgai et al. (2002); Morduch (1995)). The reason is that our model rules out the usual theoretical explanations for incomplete risk pooling. Indeed, in the literature on informal insurance, two sets of arguments are put forward as rationale for imperfect risk sharing. On the one hand, if damage avoidance entails effort or expenses and if information is imperfect, the insurance agreement may suffer from moral hazard[4]. On the other hand, commitment issues may restrict the scope for risk pooling mechanisms (Coate & Ravallion (1993)). Second, population heterogeneity appears to be the source of a differentiated pattern of subscription to mutual insurance and hence of incomplete risk sharing. We are therefore able to highlight that population heterogeneity acts as a source of incomplete risk pooling. Indeed, partial ex post insurance is voluntarily chosen by some agents in order reduce their exposure to externalities.

Let us complete the resolution of the benchmark case. Incorporating the facts that $\alpha = 1$ for all the players and that the strategy profile is symmetric, the first order condition with respect to σ (4.2) can be rewritten as

$$\frac{VAR(Y_i)}{E(Y_i)} = \frac{\sigma_i^2}{a + b\sigma_i} = \frac{1}{2c\rho}, \ \ \forall i \in C \ . \tag{6.1}$$

Given that this ratio is an increasing function of σ, the following can be highlighted: on the one hand, as intuition suggests, risk-taking is a decreasing function of risk aversion. On the other hand,

[4]In our model, technically speaking, partial insurance results from the inability to contract on individual risk-taking. Hence, the only difference with classical moral hazard is that we do not assume asymmetry of information, but directly assume that risk-taking is not contractible. It can be argued that enforceability issues would naturally arise.

the higher is the covariate fraction of total risks, the lower is σ. The reason for this has already been mentioned in the preceding section: community risk pooling only allows to absorb idiosyncratic shocks. As a result, the residual risk increases with c.

Equation (6.1) describes equilibrium risk-taking in a non-cooperative setting when preferences are homogeneous. Let us compare it with a second-best measure of individual risk-taking. First notice that risk pooling reduces the impact of idiosyncratic shocks without entailing any social cost. Hence, in the second best situation, $\alpha^* = 1$. Therefore, if a social planner had to select a level of individual risk-taking, he would maximize

$$\int_{i \in C} U_i di = a + b\sigma - c\rho\sigma^2.$$

The first order condition with respect to σ gives $\sigma^* = \frac{b}{2c\rho}$ as solution. In order to compare the latter amount with the non-cooperative solution, we simply compute the corresponding ratio:

$$\begin{aligned} \frac{VAR\left(Y_i; \sigma^*\right)}{E(Y_i; \sigma^*)} &= \frac{b^2}{2c\rho\left[2ac\rho + b^2\right]} < \frac{1}{2c\rho} = \frac{VAR\left(Y_i; \sigma^{NC}\right)}{E(Y_i; \sigma^{NC})} \\ &\Leftrightarrow \sigma^* < \sigma^{NC}, \end{aligned}$$

where the superscript NC stands for non-cooperative. We can conclude that, as a result of the protection offered by mutual insurance, the risk level taken by non-cooperative players is larger than the optimal level. This is expected as risk-taking behaviors generates externalities through the income pool. Therefore, our model replicates the result of moral hazard in risk-taking.

Excessive risk-taking is hardly a relevant problem in the context of a rural third-world community. Our model stresses that some agents will take excessive risks – actually all agents would do so if preferences were homogenous. It is therefore clear that reducing ex ante inequality towards risk exposure must be accompanied by risk-limiting norms. Actually, even when preferences are heterogenous, curbing the risk taken by those having riskier behaviors than the average is a way to incite others to participate more in the insurance scheme and to take more risk on their own. In short, bounding the maximum risk taken may be a way to promote the average risk (if need be). This is especially relevant and possible in a context where participation constraints and group formation are not an issue, i.e. insurance within a clan or a lineage.

7. Conclusion

This paper has been motivated by a twofold empirical observation: on the one hand, poorer farm households are reluctant to adopt modern technologies characterized by high risk and return levels. On the other hand, poor farmers tend to suffer from a higher exposure to idiosyncratic shocks. Unequal endowments in terms of buffer stocks only partially account for the latter observation since it appears that, somewhat paradoxically, poor households are less involved in informal insurance schemes. As shown in this chapter, a game in which the players non-cooperatively decide on self- and mutual insurance strategies can replicate such an outcome. As a by-product, our model is a first attempt to deal with ex ante and ex post risk-coping strategies in a unified framework. The following results can be highlighted.

First, as we have shown, a pattern of differentiated production choices emerges as the outcome of population heterogeneity. At equilibrium, risk-takers have an incentive to fully participate in mutual insurance. Indeed, by subscribing to the community income pool, they free ride on the self insurance costs incurred by others, thereby reducing the covariate fraction of their income variance. On the contrary, players who take relatively cautious production decisions are faced with a trade-off since participation in mutual

insurance, while reducing the extent of idiosyncratic shocks, entails an increase in the covariate fraction of their income variance. As a consequence, they partially withdraw from risk sharing mechanisms on a voluntarily basis.
Second, risk-taking can be negatively linked to risk aversion, or to the ability to rely on wealth to smooth consumption. It results that the composition of the community income pool tends to be biased towards relatively neutral players.
Finally, from a more theoretical perspective, our framework offers a third way of rationalizing incomplete risk pooling. Indeed, while classical moral hazard and imperfect commitment are ruled out, the model highlights population heterogeneity as a potential source of externalities. Partial subscription to mutual insurance follows.
To conclude this chapter, let us observe that our theoretical prediction is pessimistic with respect to poverty traps issues. Indeed, provided poverty is directly linked to the ability to cope with risk ex post, the model predicts that the lower is this ability, the less the household is likely to be involved in risk pooling. As a consequence, ex post means of insurance are weak on the aggregate and poor household have to rely on ex ante self-insurance mechanisms. Hence, their production decisions are highly affected by risk-coping considerations that make them reluctant to adopt the more profitable technologies..

8. Appendix: Computation of the Variance of Income after Transfers

Recall that $R \equiv Y - T_o + T_I$. From (3.1) and (3.2), income after transfers of household i can be re-written as

$$R_i = (1-\alpha_i) Y_i + s_i P,$$

where $P = \int \alpha_\rho Y_\rho dF(\rho)$ is the income pool.
By definition and since s_ρ is a constant,

$$\begin{aligned} VAR(R_i) &= (1-\alpha_i)^2 VAR(Y_i) + s_i^2 VAR(P) \\ &\quad + 2(1-\alpha_i) s_i COV(Y_i, P). \end{aligned} \tag{8.1}$$

Let us compute each of the three terms on the right hand side of (8.1).

8.1. The First Term of (8.1). By (2.7), the first term simply writes

$$(1-\alpha_i)^2 VAR(Y_i) = (1-\alpha_i)^2 \sigma^2. \tag{8.2}$$

8.2. The Second Term of (8.1). With respect to the second term, by the definition of the variance and making use of (2.1), the fact that X and Z are assumed to be zero mean random variables implies

$$VAR(P) = E\left[(P - E(P))^2\right] = E\left[\left(\int \alpha_\rho (X_\rho + Z_\rho) dF(\rho)\right)^2\right].$$

Expanding the right hand side gives

$$\begin{aligned} VAR(P) &= VAR\left(\int \alpha_\rho X_\rho dF(\rho)\right) + VAR\left(\int \alpha_\rho Z_\rho dF(\rho)\right) \\ &\quad + 2COV\left(\int \alpha_\rho X_\rho dF(\rho), \int \alpha_\rho Z_\rho dF(\rho)\right). \end{aligned} \tag{8.3}$$

In order to compute (8.3), the following preliminary result is needed:

Lemma 2. *Suppose a vector of n random variables. If these variables are perfectly correlated, then the variance of any linear combination of these variables equals the squared linear combination of their standard deviations. In maths,*

$$Corr(Xi, Xj) = 1, \forall i, j \in (1, 2, ...n)$$

$$\Rightarrow VAR\left(\sum_{i=1}^{n} \alpha_i X_i\right) = \left(\sum_{i=1}^{n} \alpha_i stdev\left(X_i\right)\right)^2, \forall\left(\alpha_1, \alpha_2, ...\alpha_n\right) \in R^n.$$

Proof. First note that, as a consequence of perfect correlation,

$$COV(\alpha_i Xi, \alpha_j Xj) = \sqrt{VAR\left(\alpha_i Xi\right) VAR\left(\alpha_j Xj\right)}$$

Then, making use of this result, we obtain what follows:

$$\begin{aligned} VAR\left(\sum_{i=1}^{n} \alpha_i X_i\right) &= VAR\left(\alpha_n X_n\right) + VAR\left(\sum_{i=1}^{n-1} \alpha_i X_i\right) \\ &\quad +2COV\left(\alpha_n X_n, \sum_{i=1}^{n-1} \alpha_i X_i\right) \\ &= VAR\left(\alpha_n X_n\right) + VAR\left(\sum_{i=1}^{n-1} \alpha_i X_i\right) \\ &\quad +2\sqrt{VAR\left(\alpha_n X_n\right) VAR\left(\sum_{i=1}^{n-1} \alpha_i X_i\right)} \\ &= \left(stdev\left(\alpha_n X_n\right) + stdev\left(\sum_{i=1}^{n-1} \alpha_i X_i\right)\right)^2, \end{aligned}$$

Incidentally,

$$VAR\left(\sum_{i=1}^{n-1} \alpha_i X_i\right) = \left(stdev\left(\alpha_{n-1} X_{n-1}\right) + stdev\left(\sum_{i=1}^{n-2} \alpha_i X_i\right)\right)^2$$

$$\Leftrightarrow stdev\left(\sum_{i=1}^{n-1} \alpha_i X_i\right) = stdev\left(\alpha_{n-1} X_{n-1}\right) + stdev\left(\sum_{i=1}^{n-2} \alpha_i X_i\right).$$

Applying recursively, we end up with

$$VAR\left(\sum_{i=1}^{n}\alpha_i X_i\right)=\left(\sum_{i=1}^{n}\alpha_i stdev\left(X_i\right)\right)^2.$$

By (2.3), lemma 1 applies to the first term on the right hand side of (8.3):

$$\begin{aligned} VAR\left(\int\alpha_\rho X_\rho dF(\rho)\right) &= \left(\int\alpha_\rho stdev\left(X_\rho\right)dF\left(\rho\right)\right)^2 \\ &= \left(\int\alpha_\rho\sqrt{c}\sigma_\rho dF\left(\rho\right)\right)^2 \\ &= c\left[E\left(\alpha_\rho\sigma_\rho\right)\right]^2. \end{aligned}$$

where use has been made of (2.6). Second, since the Law of Large Numbers applies to the case of *i.n.i.d.* (independent but not identically distributed) random variables, the second term on the right hand side of (8.3) vanishes: On the one hand, note that we are indeed in a case of *i.n.i.d.* variables given that the variables are independent to one another (2.4) and taking into account that the variance and hence the distribution of $\alpha_\rho Z_\rho$ are allowed to differ between individuals. Indeed,

$$VAR\left(\alpha_\rho Z_\rho\right)=\alpha_\rho^2\left(1-c\right)^2\sigma_\rho^2,$$

and both α_ρ and Z_ρ belong to the players' strategy space. They may therefore diverge from one individual to the other. On the other hand, the fact that players belong to a continuum allows to consider $\int\alpha_\rho Z_\rho dF(\rho)$ as the average of a sample whose size tends to infinity. This last element completes the proof. □

Finally, the third term on the right hand side of (8.3) is also 0 by (2.5). It follows that

$$s_i^2 VAR\left(P\right)=cs_i^2\left[E_\rho\left(\alpha_\rho\sigma_\rho\right)\right]^2. \tag{8.4}$$

8.3. The Third Term of (8.1). By definition,

$$COV\left(Y_i, P\right) = E\left(Y_i P\right) - E\left(Y_i\right) E\left(P\right),$$

where

$$E\left(Y_i P\right) = E\left(Y_i \int \alpha_\rho Y_\rho dF(\rho)\right).$$

Making use of (2.1) and distributing the expectation operator, we obtain

$$\begin{aligned} E\left(Y_i P\right) &= \int \alpha_\rho \left(a + b\sigma_i\right)\left(a + b\sigma_\rho + E\left(X_\rho\right) + E\left(Z_\rho\right)\right) dF\left(\rho\right) \\ &\quad + E\left(X_i\right) \int \alpha_\rho \left(a + b\sigma_\rho\right) dF\left(\rho\right) + \int \alpha_\rho E\left(X_i X_\rho\right) dF\left(\rho\right) \\ &\quad + \int \alpha_\rho E\left(X_i Z_\rho\right) dF\left(\rho\right) + E\left(Z_i\right) \int \alpha_\rho \left(a + b\sigma_\rho\right) dF\left(\rho\right) \\ &\quad + \int \alpha_\rho E\left(Z_i X_\rho\right) dF\left(\rho\right) + \int \alpha_\rho E\left(Z_i Z_\rho\right) dF\left(\rho\right). \end{aligned}$$

Given that X and Z have zero means, we are left with

$$\begin{aligned} E\left(Y_i P\right) &= \left(a + b\sigma_i\right) \int \alpha_\rho \left(a + b\sigma_\rho\right) dF\left(\rho\right) \\ &\quad + \int \alpha_\rho E\left(X_i X_\rho\right) dF\left(\rho\right) + \int \alpha_\rho E\left(X_i Z_\rho\right) dF\left(\rho\right) \\ &\quad + \int \alpha_\rho E\left(Z_i X_\rho\right) dF\left(\rho\right) + \int \alpha_\rho E\left(Z_i Z_\rho\right) dF\left(\rho\right). \end{aligned} \tag{8.5}$$

By the same argument, the mean cross products in (8.5) are covariances. Since the variables are perfectly correlated,

$$\frac{E\left(X_i X_\rho\right)}{\sqrt{VAR\left(X_i\right) VAR\left(X_\rho\right)}} = 1 \Leftrightarrow E\left(X_i X_\rho\right) = c\sigma_i \sigma_\rho.$$

It follows from the independence assumption (2.5) that the third and fourth terms are zero. Finally, the last term on the right hand side of (8.5) is also zero given that, on the one hand, the idiosyncratic shocks are independent between individuals (2.4) and that, on the other hand, the weight of individual i tends to zero in the integral. In other words, individual i is atomistic in the continuum. As a consequence of the above arguments,

$$E\left(Y_i P\right) = \left(a + b\sigma_i\right)\left[aE_\rho\left(\alpha_\rho\right) + bE_\rho\left(\alpha_\rho\sigma_\rho\right)\right] + c\sigma_i E_\rho\left(\alpha_\rho\sigma_\rho\right).$$

In addition,

$$E\left(Y_i\right)E\left(P\right) = \left(a + b\sigma_i\right)\left[aE_\rho\left(\alpha_\rho\right) + bE_\rho\left(\alpha_\rho\sigma_\rho\right)\right],$$

hence,

$$COV\left(Y_i, P\right) = c\sigma_i E_\rho\left(\alpha_\rho\sigma_\rho\right).$$

The third and final term of (8.1) is therefore

$$2\left(1-\alpha_i\right)s_i COV\left(Y_i, P\right) = 2c\left(1-\alpha_i\right)s_i\sigma_i E_\rho\left(\alpha_\rho\sigma_\rho\right). \tag{8.6}$$

As a final step, we are now able to sum the three terms (8.2), (8.4) and (8.6) in order to find the variance of R_i:

$$\begin{aligned} VAR\left(R_i\right) &= \left(1-\alpha_i\right)^2\sigma_i^2 + cs_i^2\left[E_\rho\left(\alpha_\rho\sigma_\rho\right)\right]^2 + 2c\left(1-\alpha_i\right)s_i\sigma_i E_\rho\left(\alpha_\rho\sigma_\rho\right) \\ &= \left(1-c\right)\left(1-\alpha_i\right)^2\sigma_i^2 + c\left[\left(1-\alpha_i\right)\sigma_i + s_i\left(E_\rho\left(\alpha_\rho\sigma_\rho\right)\right)\right]^2. \end{aligned}$$

Plugging (3.3) in the latter expression, one can easily find equation (3.4).

Bibliography

Barro, R. J. (1991). Economic growth in a cross section of countries. *The Quarterly Journal of Economics*, 106(2), 407–43.

Barro, R. J. (1998). *Determinants of Economic Growth: A Cross-Country Empirical Study*. MIT Press Books. The MIT Press.

Bold, T. (2009). Implications of endogenous group formation for efficient risk-sharing. *Economic Journal*, 119(536), 562–591.

Brams, S. & Merrill, S. (1994). Would ross perot have won the 1992 presidential election under approval voting? *PS: Political Science and Politics*, 29, 39–44.

Coate, S. & Ravallion, M. (1993). Reciprocity without commitment, characterization and performance of informal insurance arrangements. *Journal of Development Economics,*, 40, 1–24.

Copeland, A. H. (1951). A reasonnable social choice welfare function. *Seminar on Application of Mathematics to Social Sciences, University of Michigan*.

Deaton, A. (1992). Household saving in ldcs: Credit markets, insurance and welfare. *Scandinavian Journal of Economics*, 94(2), 253–73.

Dercon, S. & Christiaensen, L. (2007). Consumption risk, technology adoption, and poverty traps : evidence from ethiopia. *The World Bank Policy Research Working Paper Series*, (4257).

Dutta, B., Jackson, M. O., & Le Breton, M. (2001). Strategic candidacy and voting procedures. *Econometrica*, 69(4), 1013–37.

Ehlers, L. & Weymark, J. A. (2003). Candidate stability and nonbinary social choice. *Economic Theory*, 22(2), 233–243.

Eraslan, H. & McLennan, A. (2004). Strategic candidacy for multivalued voting procedures. *Journal of Economic Theory*, 117(1), 29–54.

Genicot, G. & Ray, D. (2003). Group formation in risk–sharing arrangements. *Review of Economic Studies*, 70(1), 87–113.

Gertler, P. & Gruber, J. (2002). Insuring consumption against illness. *American Economic Review*, 92, 51–70.

Gibbard, A. (1973). Manipulation of voting schemes: a general result. *Econometrica*, 41, 587–601.

Grimard, F. (1997). Household consumption smoothing through ethnicities: Evidence from côte d'ivoire. *Journal of Development Economics*, 53, 391–422.

Hoogeveen, H. (2002). Evidence on informal insurance in rural zimabwe. *Journal of African Economies*, 11(2), 249–278.

Jalan, J. & Ravallion, M. (1999). Are the poor less well insured? evidence on vulnerability to income risk in rural china. *Journal of Development Economics*, 58, 61–81.

Kocherlakota, N. (1996). Implications of efficient risk sharing without commitment. *Review of Economic Studies*, 63, 595–609.

Kramer, G. H. (1977). A dynamical model of political equilibrium. *Journal of Economic Theory*, 16(2), 310–334.

Kurosaki, T. & Fafchamps, M. (2002). Insurance market efficiency and crop choices in pakistan. *Journal of Development Economics*, 67(2), 419–453.

Ligon, E., Thomas, J., & Worrall, T. S. (2002). Informal insurance arrangements with limited commitment: Theory and evidence from village economies. *The Review of Economic Studies*, 69, 209–244.

Maskin, E. (1985). *The theory of implementation in Nash equilibrium: a survey. In: Social Goals and Social Organizations: Essays in memory of Elisha Pazner*. Cambridge Univ. Press, Cambridge.

May, K. (1952). A set of independent necessary and sufficient conditions for simple majority decision. *Econometrica*, 20, 680–684.

Miller, D. L. & Paulson, A. (2007). Risk taking and the quality of informal insurance: gambling and remittances in thailand. *Federal Reserve Bank of Chicago Working Paper Serie*, WP-07-01.

Morduch, J. (1995). Income smoothing and consumption smoothing. *Journal of Economic Perspectives*, 9(3), 103–14.

Murgai, R., Winters, P., Sadoulet, E., & de Janvry, A. (2002). Localised and incomplete mutual insurance. *Journal of Development Economics*, 67, 245–274.

North, D. C. (1994). Economic performance through time. *American Economic Review*, 84(3), 359–68.

Rodríguez-Álvarez, C. (2006). Candidate stability and voting correspondences. *Social Choice and Welfare*, 27(3), 545–570.

Satterthwaite, M. A. (1975). Strategy-proofness and arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2), 187–217.

Simpson, P. B. (1969). On defining areas of voter choice: Professor tullock on stable voting. *The Quarterly Journal of Economics*, 83(3), 478–90.

Townsend, R. M. (1994). Risk and insurance in village india. *Econometrica*, 62, 539–591.

Udry, C. (1994). Risk and insurance in a rural credit market: An empirical investigation in northern nigeria. *Review of Economic Studies*, 61(3), 495–526.

Young, H. & Levenglick, A. (1978). A consistent extension of condorcet's election principle. *SIAM Journal of Applied Mathematics*, 35, 285–300.

Young, H. P. (1975). Social choice scoring functions. *SIAM Journal of Applied Mathematics*, 28, 824–838.