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Domestic environmental policy and international cooperation for global commons

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ABSTRACT

The paper analyzes the strategic behavior of several countries engaged in capital accumulation, pollution mitigation, and environmental adaptation in the context of an environmental common good. Both cooperative and non-cooperative strategies are discussed. The non-cooperative strategy is a dynamic game in which each country makes its own environmental decision following the open-loop Nash equilibrium. The cooperative social planner problem assumes an international environmental agreement in force. The non-cooperative and cooperative solutions are compared in the symmetric case of two countries and extended to several identical countries. It is shown that the non-cooperative strategy in multi-country world leads to over-production, over-consumption, over-pollution, and over-adaptation.

1. Introduction

The economic literature recognizes that global environmental problems require a global solution, that is, cooperation among the involved countries. It is well established that the absence of cooperation leads to the under-provision of pollution mitigation compared to the level reasonable from a

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global standpoint. Countries interested in their own welfare tend to spend less on pollution mitigation. Because mitigating a global pollutant is a contribution to a public good, it cannot be globally optimal without some coordination among countries. However, countries' environmental policies are not restricted to mitigation. Starting from the late 90s, a new policy instrument has emerged in the policy debate, namely, adaptation. In short, adaptation consists of protecting a country from the adverse effects of pollution. When a country is unable to effectively control pollutions, for example, due to the lack of international cooperation in the case of a global pollutant, adaptation appears to be an appealing option for domestic policy. Instead of spending money in vein on mitigation, the country may be interested in spending on adaptation. The striking difference between mitigation and adaptation is that the former contributes to a public good while the latter is led by pure selfishness, to be understood here as the country's self-interest. The intuition thus suggests that the lack of cooperation should lead to too little mitigation and too much adaptation. To justify this intuition, we must understand a country' motive to invest in mitigation and adaptation in a multi-country setting, that is, understand how optimal domestic policies are shaped by international cooperation.

The emerging literature on the optimal policy mix between mitigation and adaptation usually considers adaptation as a spare wheel when mitigation fails. In this paper we question this statement. We will show that a much wider picture should be discussed when comparing domestic environmental policies under international cooperation and no-cooperation. This issue is of a major importance in designing effective environmental policies, and a key policy message of this paper is that international agreements and domestic policies should not be considered as separate issues.

During the last years, the literature devoted to the management of global commons used to start from the top-level design of an international agreement and then scale down to the nation level to find a domestic policy compatible with the international agreement, notably in terms of incentives (is the country willing to join the agreement?). Empirical evidence suggests that such an approach is ineffective. On one hand, the climate change multilateral negotiation process undergone under the United Nations Framework on Climate Change (UNFCCC) has failed to deliver concrete outcomes for many years. On the other hand, despite the slow pace of the UNFCCC process, several domestic initiatives (at the national or sub-national levels) have been recently documented. A remarkable example is the GLOBE International initiative. GLOBE International was originally founded in 1989 by legislators from the US Congress, European Parliament, Japanese Diet and the Russian State Duma with the mission to respond to urgent environmental challenges through the development and advancement of national legislation.[1] Fankhauser et al. (2014) have recently analyzed national and international factors that drive the adoption of legislation on climate change with a unique dataset of 419 national legislation pieces from 63 countries. They show that the passage to climate legislation is influenced by both domestic and international factors. This new perspective suggests that a positive policy context at the local level may eventually scale up at the global level and help an international agreement to come out. Their main conclusion warns against focusing too narrowly on international treaties as the sole solution to the climate problem, while domestic actions might be a possible route to unlock the stalemate in international negotiations. Indeed, strong interactions exist between the top and bottom decisions when it comes to managing global commons.

A recent interesting theoretic idea to bridge this top-down gap was proposed by Olstrom (2012) under the collective bi-disciplinary (economics and political sciences) research project initiated by Brousseau et al. (2012). Actually, a new strand of thinking is emerging that attempts to circumvent well-known drawbacks of the common top-down approach. A better understanding of how international agreements shape domestic policies is the key. The latest thread of analytic research shed an essential light on effective funding mechanisms and transfers to finance mitigation and adaptation activities on international scale, see, *e.g.*, Buob and Stephan (2013), Schenker and Stephan (2014), Pittel and Rübbelke (2015), Heuson et al. (2015), Buchholz et al. (2015), and the references therein.

[1] The main publication is the GLOBE Climate Legislation Study (fourth edition). The Study is produced in partnership with the Grantham Institute at the London School of Economics and serves to baseline existing climate laws and regulation, see http://globelegislators.org. It has been widely recognized for highlighting the growing trend of national climate change legislation.

In this paper, we focus a comprehensive analysis of the interplay between economic growth and environmental policy in a multi-country dynamic general equilibrium setting. We consider the main drivers of economic growth, such as investment in physical capital, investment in environmental adaptation, and spending in pollution mitigation. Our main outcomes can be summarized as follows.

We will first consider the simple case of a single country with the rest-of-the-world represented by a given exogenous external pollution stock. This setting will allow us to understand the potential effects due to asymmetry among countries. It turns out that the optimal adaptation policy depends on a combination of three factors, each of them having a strong economic rationale. First, the physical–technological potential for adaptation must be large enough, which is related to the country's geographical and physical characteristics. A country highly sensitive to global warming will probably have a wider technological potential than other countries. Second, the opportunity cost and incentive to invest in pollution abatement increase with the country's pollution level. These first two factors suggest that mitigation and adaptation may well be either substitutable or complementary policy instruments depending on the economy under analysis. Third, the wealth of the economy also plays a key role: a wealthier economy may have a stronger incentive to invest in adaptation than in mitigation, and it is optimal not to invest in adaptation if the economy is too poor. When discussing these factors, we use a synthetic indicator of *country's environmental harm and vulnerability* and prove it to be an appealing tool for policy development.

This one-country setting also allows us to analyze how the size of the economy influences its domestic policy. We show that the larger contribution of an economy to the global pollution stock leads to larger output, capital stock, and mitigation spending. Furthermore, a stronger environmental harmfulness reinforces this relationship. Understanding optimal adaptation decisions is more challenging. The optimal adaptation spending depends on both the harmfulness of the economy and its size. If the harmfulness is high, then optimal adaptation first increases and then decreases with the size of the economy. This is an important counter-intuitive result that would require to be empirically documented. It reveals that adaptation is not driven by selfishness only.

Then, we will consider a multi-country model. For the sake of clarity we will start with two countries before generalizing to $n > 2$ countries. Indeed, all the results displayed for two countries will hold for n countries. We restrict the analysis to symmetric countries in steady state. In such a setting, two polar scenarios can be considered, namely non-cooperative and cooperative. The former corresponds to the Nash equilibrium, the latter to the first-best Pareto solution. Comparing these two scenarios leads to a natural result, that the global pollution level is too high in the non-cooperative scenario with respect to the first best. Still, the reasons why it is so are not trivial. The first unexpected result (never mentioned in the literature to the best of our knowledge) is that the size of the economies significantly differs in the above two scenarios. The optimal size of the economy at steady state is always larger in the non-cooperative scenario than under cooperation. The lack of cooperation in the global pollution problem yields too fat economies, where not only pollution is too high, but also the output, capital and consumption are. Because countries cannot effectively control the global pollution level in the Nash scenario, they have no choice but to spend more on mitigation and adaptation, and because they need more income for that purpose, more production and capital accumulation are required. We also show that, expressed in terms of output, mitigation efforts are indeed too small in the Nash scenario (as it is well known in the literature), but that mitigation expenses are larger compared to what they could be under cooperation. An economy where people work hard to get income to be able to spend a lot in order to cope with pollution: is not it a wasteful economy? We also show that the degree of wastefulness increases with the degree of environmental harmfulness.

The optimal policy ratio between adaptation and mitigation displays an inversed U-shape with respect to the stage of development of the economy (represented by its total factor productivity), as in Bréchet et al. (2013). In other words, mitigation and adaptation are complementary policy instruments for poor countries but become substitutes at some higher stage of development. A new result is that cooperation moves this inversed U-shape to the left. As a consequence, the optimal adaptation becomes positive for smaller values of total productivity at Nash equilibrium than under cooperation, and the maximum of the optimal ratio between adaptation and mitigation is reached at a smaller total productivity value. This result has major implications for the current policy debate. It shows that the lack of cooperation does not necessarily lead to too much adaptation, but rather to too little adaptation in poor countries. It suggests that more should be spent in international adaptation funds to help developing countries in the absence of effective cooperation, which is a sensitive policy issue in the current international negotiation process.

The remainder of the paper is organized as follows. Section 2 provides a literature review. Section 3 introduces the model of a single country in a global environment, *i.e.*, with a given exogenous pollution flow coming from the rest of the world. This preliminary setting conveniently allows us to show how asymmetry among countries shapes domestic policies. We provide a steady-state analysis and obtain approximate formulas necessary for further analysis of a multi-country world. Section 4 investigates the model with several countries. We start with a competitive two-country world (Section 4.1), *i.e.*, a dynamic game in which each country makes its own environmental decision following the open-loop Nash equilibrium. A social planner problem, where decisions are taken on behalf of all countries, is described in Section 4.2. Those two cases are compared in Section 4.3. Finally, the simulation Section 4.4 illustrates and extends the obtained modeling outcomes to the case with positive capital deterioration. The generalization to $n > 2$ countries is presented in Section 4.4. Section 5 concludes.

2. Literature review

The current literature provides only partial insights to the question of the connection between domestic policies and international cooperation for a global common. It is mostly restricted to the applied integrated assessment modeling (IAM) framework (Bosello, 2008; Bosello et al., 2010; de Bruin et al., 2009a, 2009b). A more theoretic strand is represented by Tulkens and van Steenberghe (2009), Buob and Stephan (2011, 2013), Ebert and Welsch (2012), and Pittel and Rübbelke (2015). Let us review the main contributions of the aforementioned papers.

de Bruin et al. (2009a, 2009b) consider aggregate adaptation expenditures as an endogenous flow variable in the numerical DICE model. They obtain that adaptation and mitigation are complementary policy instruments, adaptation is stronger in the short run, mitigation stronger in the long run, and adaptation is better for low environmental damages. Bosello (2008) considers aggregate adaptation expenditures as an endogenous stock variable of the RICE model and argues that the optimal mitigation starts earlier, adaptation is postponed, and mitigation is better for low damages. The later work of Bosello et al. (2010) distinguishes between three adaptation categories and mitigation in the AD-WITCH optimal growth model of the 12-region world. All these papers focus on computational models rather than providing theoretical insights.

The first theoretic paper related to our research is Tulkens and van Steenberghe (2009). They extend the standard cost minimization model (called "$c + d$" for abatement costs and damage costs minimization) by including adaptation costs explicitly in the damage function. They characterize the optimal (cost minimizing) balance between policy options (abatement, adaptation, and suffering) in both static and dynamic settings. Tulkens and van Steenberghe show that, in the cooperative solution, the optimal adaptation level at any time t is such that its marginal cost is equal to the discounted value of future suffered damages that adaptation allows to avoid. This suggests how adaptation and mitigation ought, in their view, to complement each other.

Two other analytic papers, Buob and Stephan (2011) and Ebert and Welsch (2012), use game-theoretic framework to analyze adaptation and mitigation in a multi-country setting. Buob and Stephan (2011) study a two-stage dynamic game of several identical regions with cooperative and non-cooperative behavior. They focus on the fundamental difference that adaptation benefits are private to a region while mitigation benefits are globally public. Some of their major qualitative conclusions are that poor countries should invest only in mitigation while rich countries should invest in both mitigation and adaptation. Thus, these countries provide public benefits of mitigation even in the case of non-cooperative behavior. Furthermore, the range of income for which the countries engage in adaptation is smaller under cooperation than under no-cooperation. Our analysis confirms and extends the results of Buob and Stephan (2011). Such a coincidence is surprising given the differences between the modeling approaches. Ebert and Welsch (2012) consider a two-country static game with endogenous production, pollution emissions, and adaptation expenditures, but they do not explicitly model mitigation expenditures. They come to a related conclusion that a larger productivity of emissions and larger adaptive capacity lead to greater emissions, in the home country and globally, in both cases of non-cooperative behavior and full cooperation. Moreover, they emphasize the controversial nature of this effect for policy making.

A more recent stream of game-analytic papers on mitigation and adaptation focuses on funding and transfers to finance mitigation and adaptation activities. Buob and Stephan (2013) consider a two-region two-stage dynamic game that includes mitigation in the first stage and adaptation in the second stage, and find that adaptation transfer can be beneficial to the donor country if adaptation and mitigation are complements. Schenker and Stephan (2014) use a dynamic multi-region CGE model to explore relationships between international trade, regional adaptation to climate change, and financial transfers for funding adaptation. They argue that adaptation funding can lead to welfare gains in both developing and developed countries. On the other side, Pittel and Rübbelke (2015) analyze the profitability of climate-related interregional transfers in a two-region static model and show that the induced welfare of both regions and related policy effects largely depend on the complementarity or substitutability between mitigation and adaption. Heuson et al. (2015) utilize a two-country two-stage game to demonstrate that many existing modes of mitigation and adaptation funding do not guarantee Pareto improvements for both countries. Buchholz et al. (2015) explore the so-called Bergstrom paradox, when unilateral matching of mitigation funding benefits the donor but harm the recipient country, and show how to avoid this transfer situation.

In this paper, we contribute to the literature by proposing a full-fledged multi-country dynamic model to address the optimal combination of domestic policy instruments, pollution mitigation, adaptation to the degraded state of the environment, and capital accumulation. To link those policy instruments to the country's endogenous economy, we exclude international trade and transfers from our analysis, realizing that it may seriously affect our qualitative results. Instead, we concentrate on the productive side of the economy and investigate how the optimal policy depends on the stage of development of a country and its position in the international area. The total factor productivity and the rate of time preference are among the main characteristics of a country's stage of development, and its power in international negotiations can be represented by the country's contribution to the global pollution flow. Also, despite the fact that there clearly exist big polluters and small polluters, there is no straight link between the pollution amount and the stage of development. A developed country may be a small polluter (*e.g.* Switzerland, the Netherlands, Singapore, Belgium) while a developing country may well be a big polluter (*e.g.* China, India, Brazil).[2] We will study how such heterogeneities affect domestic policies.

For pedagogical purposes, before building the multi-country model we will start with a simpler one-country model with an external pollution flow. It will produce some preliminary results that will be necessary for analyzing the multi-country model.

3. A single country in a global environment

Let us consider a country that is a part of the larger multi-country world and aggregate the rest of the world into a single exogenous zone. The country under study determines its optimal national policy by taking the rest of the world as given. Thus, it represents a Nash equilibrium constrained by exogenous players, *i.e.*, a game where one player (it could be generalized to n endogenous players) determines its optimal strategy while all other players have fixed strategies. As it is commonly assumed in the related literature, the only interaction between countries is the global climate. So, the rest of the world can be represented as an exogenous pollution flow. This flow complements the endogenous country's contribution to the global pollution, so that the sum of the two matches the world's pollution flow. Interestingly, such a simple modeling framework will allow us to analyze how optimal domestic policies are shaped by the size of the country under study. This is an innovative and convenient way

[2] Some countries contribution to world greenhouse gases emissions in 2006 (in million tons of CO_2 equivalent for all GHGs, and in percent): USA 7017.3 (24.5%); Russia Federation 2190.2 (7.6%); Belgium 136.9 (0.5%); Switzerland 53.2 (0.2%); *source*: UNFCCC, FCCC/SBI/2008/12. China 3649.8 (12.7%); Brazil 1477.1 (5.1%); India 1228.5 (4.2%); South Africa 361.2 (1.2%); Singapore 26.8 (0.1%); *source*: UNFCCC, FCCC/SBI/2005/18/Add.2.

of addressing asymmetry among countries in global pollution games: to what extent the optimal domestic policy would change with the contribution of the country to global pollution?[3]

3.1. Economic-environmental model of a country with external pollution stock

As in Bréchet et al. (2013), the endogenous country is described by the Solow–Swan type one-sector model. The programme of a benevolent social planner is to maximize the infinite-horizon utility of the infinitely lived representative household

$$\max \int_0^{\infty} e^{-\rho t} U[C(t), P(t), D(t)]\, dt \tag{1}$$

that depends positively on the consumption C, negatively on pollution P, and positively on adaptation D ($\rho > 0$ is the rate of time preference).

Assuming Cobb–Douglas technology, the country produces the aggregate final product $Y = AK^{\alpha}$ that is allocated across consumption C, investment I_K into physical capital K, investment I_D into environmental adaptation D, and pollution mitigation expenditures B:

$$Y(t) = AK^{\alpha}(t) = I_K(t) + I_D(t) + B(t) + C(t), \tag{2}$$

where $A > 0$ and $0 < \alpha < 1$ are parameters of the Cobb–Douglas production function.

The capital accumulation of productive physical capital K and adaptation capital D is described as

$$K'(t) = I_K(t) - \delta_K K(t), \quad K(0) = K_0, \tag{3}$$

$$D'(t) = I_D(t) - \delta_D D(t), \quad D(0) = D_0, \tag{4}$$

where $\delta_K \geq 0$ and $\delta_D \geq 0$ are deterioration rates of physical capital and adaptation capital.

Next, the interaction between the output Y, pollution emissions, and accumulated global pollution stock P is described by the following differential equation:

$$P'(t) = -\delta_P P(t) + \gamma \left[\frac{Y(t)}{B(t)} + Z(t) \right], \quad P(0) = P_0, \tag{5}$$

where an exogenous pollution flow $Z(t)$ represents the rest of the world, $\gamma > 0$ is the emission factor (the environmental dirtiness of the economy), and $\delta_P > 0$ is the natural pollution decay rate.[4] The pollution flow of the endogenous economy can be reduced by mitigation spending B, as in Smulders and Gradus (1996). Thus, the endogenous economy contributes only the part $\gamma Y/B$ to the worldwide pollution flow, while it suffers from the entire global pollution stock P.

In this setting, by varying the value Z, we can analyze how the optimal environmental policy of the country under study depends on its share in the global pollution flow. So, one of the key innovative features of this model is to endogenize both mitigation and adaptation as domestic policy measures in the presence of an external exogenous pollution stock. The externality typical to a public good appears here, as well as the asymmetry between adaptation and mitigation at the country level.

By (5), the economic activity Y increases the pollution stock P that negatively affects the utility (1), while the mitigation expenditure B allows to partially alleviate this adverse impact of the pollution stock. As in Bréchet et al. (2013), the utility function (1) is

$$U(C, P, D) = \ln C - \eta(D) \frac{P^{1+\mu}}{1+\mu}, \tag{6}$$

where the function $\eta(D)$ represents the *vulnerability* of the economy to pollution and the parameter $\mu > 0$ reflects the increasing marginal disutility of pollution. In our model, pollution P directly affects

[3] It must be stressed that "country size" and "country's contribution to climate change" are two different realities in an asymmetric world. Two countries with similar GDPs may well have very different pollution levels. Only the latter matters in our multi-country setting. For example, Canada and Spain had almost the same GDP level in 2011 (around 1400 billion US$, source: OECD) while GHGs emissions were twice larger in Canada (*source*: UNFCCC).

[4] For tractability reasons, we assume that γ is the same in the country under study and in the rest of the world.

the utility (6) rather than the production output Y, which is a common specification in climate models (Weitzman, 2010; Hritonenko and Yatsenko, 2013a, 2013b). In contrast, the simulation-based IAMs usually consider market damages of pollution as a decrease of the output Y but disregard related consumers' disutility (de Bruin et al., 2009b; Bosello et al., 2010).

The adaptation capital D reduces the environmental vulnerability of the economy $\eta(D)$ in (6), *i.e.*, the disutility of a given pollution stock level. It is natural to assume that vulnerability is maximal when no adaptation measures are taken, $\eta_{\max} = \eta(0)$, and that vulnerability gradually decreases to some minimum level when the adaptation efforts tend to infinity, $\eta_{\min} = \eta(\infty) > 0$. The potential gap $\eta_{\max} - \eta_{\min}$ represents the range of physical adaptation opportunities in the economy as it depends on the geographical or infrastructural characteristics of the country. In between this physical gap, the economic concept of efficiency must be introduced. In the remainder of the paper, the functional form of effective adaptation will be as follows:

$$\eta(D) = \eta_{\min} + (\eta_{\max} - \eta_{\min})e^{-aD}, \quad \eta_{\max} > \eta_{\min} > 0, \quad a > 0, \tag{7}$$

where the parameter a reflects the *marginal efficiency* of adaptation. The function (7) was earlier used in Bréchet et al. (2013) in one-country version of the model (1)–(7) and in Wang and McCarl (2013) as an improvement of the AD-DICE IAM model.

We are now equipped with the model of a country that is affected by the global climate change, for which it is only partially responsible, and chooses its optimal climate policy, which consists of emission mitigation and adaptation spending. To be able to extend this model to multi-country world, we will need the following auxiliary results on the steady state analysis of the model (1)–(7).

3.2. Steady-state analysis

The optimization problem (1)–(5) includes three control variables I_K, I_D, C, and four state variables K, D, B, P, related by four constraints-equalities (2)–(5). In order to keep the analytic complexity feasible for the forthcoming multi-country case, we restrict ourselves to the steady–state analysis assuming $Z(t) = \text{const} > 0$. We also assume that the capital depreciation is the same for physical and adaptation capital stocks ($\delta_K = \delta_D = \delta$). The following lemma extends the steady state structure obtained in Bréchet et al. (2013) for $Z = 0$.

Lemma 1 (Steady state). *The optimization problem (1)–(7) has the stationary state* ($\bar{K}, \bar{B}, \bar{C}, \bar{D}, \bar{P}$):

$$\bar{B} = A\bar{K}^{\alpha} - \frac{\bar{K}(\rho+\delta)}{\alpha}, \tag{8}$$

$$\bar{C} = \frac{\bar{K}(\rho+\delta)}{\alpha} - \delta(\bar{K}+\bar{D}), \qquad \bar{I}_K = \delta\bar{K}, \qquad \bar{I}_D = \delta\bar{D}, \tag{9}$$

$$\bar{P} = \frac{\gamma}{\delta_P}\left(\frac{A}{A - \bar{K}^{1-\alpha}(\rho+\delta)/\alpha} + Z\right), \tag{10}$$

where $\bar{K}$, $0 < \bar{K} < (\alpha A/(\rho+\delta))^{1/(1-\alpha)}$ *and* $\bar{D} \geq 0$ *are determined in the following way.*

First, the optimal capital stock $\bar{K} > 0$ *when the optimal adaptation is zero* ($\bar{D} = 0$) *is the unique solution of the following equation*:

$$\left(\frac{A}{A - \bar{K}^{1-\alpha}(\rho+\delta)/\alpha} + Z\right)^{-\mu} \frac{\left(A - \bar{K}^{1-\alpha}(\rho+\delta)/\alpha\right)^2}{\bar{K}^{1-\alpha}} = \frac{\gamma^{1+\mu}A^{1+\mu}\eta_{\max}}{(\delta_P+\rho)\delta_P^{\mu}}\left(\frac{\rho+\delta}{\alpha} - \delta\right), \tag{11}$$

provided the calculated value $\bar{K}$ *is small enough. Otherwise,* $\bar{K} > 0$ *and* $\bar{D} > 0$ *are determined by the following system of two nonlinear equations:*

$$\left(\frac{A}{A - \bar{K}^{1-\alpha}(\rho+\delta)/\alpha} + Z\right)^{-\mu} \frac{\left(A - \bar{K}^{1-\alpha}(\rho+\delta)/\alpha\right)^2}{\bar{K}^{-\alpha}}$$

$$= \frac{\gamma^{1+\mu}A}{(\delta_P+\rho)\delta_P^{\mu}}\left(\frac{\rho+\delta}{\alpha}\bar{K} - \delta\bar{K} - \delta\bar{D}\right)[\eta_{\min} + (\eta_{\max} - \eta_{\max})e^{-a\bar{D}}], \tag{12}$$

$$\left(\frac{\rho+\delta}{\alpha}\bar{K}-\delta\bar{K}-\delta\bar{D}\right)a(\eta_{\max}-\eta_{\max})e^{-a\bar{D}}\frac{\gamma^{\mu+1}}{\rho\delta_P^{\mu+1}(\mu+1)}=\left(\frac{A}{A-\bar{K}^{1-\alpha}(\rho+\delta)/\alpha}+Z\right)^{-\mu-1} \tag{13}$$

Proof. See Appendix A.

Analyzing positive solutions $\bar{D}$ and $\bar{K}$ to two nonlinear equations (12) and (13) is not straightforward. In particular, it turns out to be extremely difficult at $\delta > 0$ even in the case $Z = 0$ (Yatsenko et al., 2014). So, we split our forthcoming exposition into: (a) an analytic solution in the case $\delta = 0$ of zero capital deterioration; (b) a numeric clarification of outcomes obtained at $\delta > 0$ in more realistic cases with positive deterioration.

3.3. Optimal domestic policy: analytic solution at zero capital deterioration

At $\delta = 0$, Eq. (13) becomes

$$a(\eta_{\max}-\eta_{\min})e^{-a\bar{D}}\frac{\gamma^{\mu+1}\bar{K}}{\alpha\delta_P^{\mu+1}(\mu+1)}=\left(\frac{A}{A-\bar{K}^{1-\alpha}(\rho)/\alpha}+Z\right)^{-\mu-1} \tag{14}$$

and can be solved for D at a given $\bar{K}$ as:

$$D(\bar{K})=\frac{1}{a}\ln\left[\frac{a(\eta_{\max}-\eta_{\min})\gamma^{\mu+1}\bar{K}}{\alpha(\mu+1)\delta_P^{\mu+1}}\left(\frac{A}{A-\bar{K}^{1-\alpha}\rho/\alpha}+Z\right)^{\mu+1}\right]. \tag{15}$$

This leads us to the first proposition.

Proposition 1 (Optimal adaptation and capital stocks at steady-state). *Let $\delta = 0$. If the unique solution $\hat{K} > 0$ of the equation*

$$\frac{A\alpha}{\hat{K}^{1-\alpha}\rho}\left(1-\frac{\hat{K}^{1-\alpha}\rho}{A\alpha}\right)^{\mu+2}\left[1+Z\left(1-\frac{\hat{K}^{1-\alpha}\rho}{A\alpha}\right)\right]^{-\mu}=\frac{\eta_{\max}\gamma^{1+\mu}}{(\delta_P+\rho)\delta_P^{\mu}} \tag{16}$$

is such that $D(\hat{K}) \le 0$, then the optimal steady-state adaptation $\bar{D} = 0$ and the optimal capital $\bar{K} = \hat{K}$. Otherwise, the unique $\bar{K} > \hat{K}$ is found from the nonlinear equation:

$$\frac{A\alpha}{\bar{K}^{1-\alpha}\rho}\left(1-\frac{\bar{K}^{1-\alpha}\rho}{A\alpha}\right)^{\mu+2}=\frac{\eta_{\min}\gamma^{1+\mu}}{(\delta_P+\rho)\delta_P^{\mu}}\left[1+Z\left(1-\frac{\bar{K}^{1-\alpha}\rho}{A\alpha}\right)\right]^{\mu}$$
$$+\frac{\alpha(\mu+1)}{a\bar{K}(1+\rho/\delta_P)}\left(1-\frac{\bar{K}^{1-\alpha}\rho}{A\alpha}\right)^{\mu+1}\left[1+Z\left(1-\frac{\bar{K}^{1-\alpha}\rho}{A\alpha}\right)\right]^{-1}, \tag{17}$$

and the optimal adaptation $\bar{D} > 0$ is given by (15).

Proof. See Appendix A.

Eq. (15) shows that if the capital stock $\bar{K}$ is too small, then the optimal adaptation level should be negative (because of the logarithm), which is not possible. Thus, the optimal solution is no adaptation when the economy under analysis is very poor. As a consequence, the vulnerability of the economy is maximal $\eta(D) = \eta(0) = \eta_{\max}$ because the optimal adaptation is zero. This preliminary result shows that it is optimal to invest in adaptation only under some balance between three ingredients, each of which has an economic-environmental rationale. First, the wealth of the economy, represented by the level of its capital stock $\bar{K}$, must be considered. A wealthier economy has a stronger incentive to invest in adaptation. Second, the technological potential for adaptation $a(\eta_{\max}-\eta_{\min})$ must be wide enough. This potential naturally depends on each country's geographical and physical characteristics. A country located in an area highly sensitive to global warming (*e.g.*, near oceans or mountains) is expected to offer a wider technological potential than others. Third, what we may call the *environmental dirtiness* *ratio* γ/δ_P also plays a key role. It translates the fact that the actual detrimental impact of an economy on the environment depends not only on its polluting intensity γ but also on the natural decay rate of the pollution stock δ_P. So, the results gathered in Proposition 1 have important policy implications. These results will come out again later in the multi-country setting with new insights.

On the ground of Proposition 1 we can characterize the optimal climate policy of the country under study. Every economy must in fact be characterized in both dimensions, by its vulnerability to pollution and its contribution to pollution. So, reducing pollution is a benefit (avoided damages) as well as a cost (mitigation efforts). The optimal policy mix between the two depends on both physical characteristics and their economic counterparts and each country's strategic position depends on both dimensions. To proceed further we need a corner stone, and this one will prove to be a key indicator of both the vulnerability of the economy and its pressure on the environment. It is introduced in Bréchet et al. (2013) as the *κ-indicator of environmental harmfulness and vulnerability*

$$\kappa(\eta) = \frac{\eta\gamma^{1+\mu}}{(\delta_P + \rho)\delta_P^{\mu}}. \tag{18}$$

An early version of such an indicator that included the discount rate and costs of mitigation and adaptation, but not the environment self-cleaning capacity, earlier appeared in Buob and Stephan (2011). It is important to stress that our *κ-indicator* (18) is endogenous and depends on the country's adaptation efforts D (because $\eta = \eta(D)$).

The *κ-indicator* offers quite appealing economic intuitions. It combines both the pressure of human activity on the environment (the pollution intensity of production γ) and the pressure of the environment on welfare (the environmental vulnerability of the economy η).[5] An economy can be highly vulnerable to climate change because of the physical sensitivity to impacts (geography, infrastructure, location, etc.), subjective reasons (pollution disutility), and its polluting intensity, which will influence its mitigation cost (through the opportunity cost of mitigation). This synthetic indicator captures the whole picture. Even if the vulnerability of an economy is something given beforehand by country's physical characteristics, it can be changed by an adequate adaptation policy. Because D is related to other policy instruments B and K, the adaptation measures cannot be taken independently of other decisions.

Because the adaptation is bounded from below (no adaptation) and from above (full adaptation), the *κ-indicator* is also bounded. We can define the two boundaries $\kappa_{\min} < \kappa(\eta) \leq \kappa_{\max}$ as cases where vulnerability is minimal and maximal, *i.e.*:

$$\kappa_{\min} = \kappa(\eta_{\min}) \quad \text{and} \quad \kappa_{\max} = \kappa(\eta_{\max}). \tag{19}$$

The boundaries (19) are used in the proof of Proposition 1. In order to obtain approximate expressions for steady-state variables $\bar{K}, \bar{B}, \bar{C}, \bar{D}, \bar{P}$, we consider two extreme cases depending on whether the *κ-indicator* is large or small, defined as Case A and Case B below.

Case A reflects a (brown) economy that is both very polluting and heavily impacted by the global pollution. This boils down to assume a lower bound for κ given by:

$$1 \ll \kappa_{\min} \ll (1+Z)\left(\frac{A}{\rho}a^{1-\alpha}\right)^{1/\alpha}. \tag{20}$$

The interpretation of the expression (20) is rather straightforward. The left inequality (20) imposes that the lower limiting value $\kappa_{\min}$ in (19) is large enough, which means that pollution impact is high and the economy cannot overcome all adverse effects of pollution *even when all adaptation measures are implemented*. The second part of the inequality means that the ratio between the productivity level A and the discount factor ρ, and/or the adaptation efficiency a, and/or the external pollution are much larger than $\kappa_{\min}$. Bréchet et al. (2013) argue that Case A is rather realistic on the ground of

[5] In this sense, the *κ-indicator* represents a kind of theoretically reduced form of the DPSIR framework (Driving-Pressure-State-Impact-Response), showing that the interplay between the economy and climate goes both ways (every economy is polluted and is a polluter at the same time) and combines both physical and subjective dimensions.

current world aggregate data. If Case A holds, then the optimal steady state capital $\bar{K}$ is given by the approximate formula

$$\bar{K} \cong \left[\frac{\alpha A}{\kappa_{\min}\rho(1+Z)^{\mu}}\right]^{1/(1-\alpha)}. \tag{21}$$

Here and thereafter, the notation $f(\varepsilon)\cong g(\varepsilon)$ means that $f(\varepsilon)\equiv g(\varepsilon)[1+o(\varepsilon)]$ for some small parameter $0<\varepsilon\ll 1$ and $f(\varepsilon)\to g(\varepsilon)$ when $\varepsilon\to 0$. The proof of (21) is analogous to Bréchet et al. (2013).

Case B describes a (green) economy with a lightly polluted environment, which boils down to assume:

$$\kappa_{\max} \ll 1. \tag{22}$$

In this case, the upper bound of $\kappa_{\max}$ is small, which means that the vulnerability is low even without any adaptation. For Case B's countries, the optimal steady state capital $\bar{K}$ is determined by the approximate formula:

$$\bar{K} \cong \left[\frac{\alpha A}{\rho}(1-\kappa_{\max})^{1/(\mu+2)}\right]^{1/(1-\alpha)}. \tag{23}$$

By considering Cases A and B, we are able to find the optimal steady-steady capital stock $\bar{K}$ from (21) or (23) and all other steady-state variables $\bar{B}$, $\bar{C}$, $\bar{D}$, $\bar{P}$, which are explicitly determined by (8)–(10), (15) and Proposition 1. So, the formulas (21) and (23) allow us to describe the optimal domestic climate policy of a country under study and to understand how it is shaped by its contribution to the global pollution flow.

In the case $Z=0$ (a world economy), mitigation and adaptation turn out to be substitutable policy instruments and the optimal policy mix between adaptation and mitigation is shaped by country's wealth, in particular by its total factor productivity A. The optimal ratio D/B is bell-shaped: it is first increasing in A, and then decreasing (see Bréchet et al., 2013 for further details). An analysis of Eqs. (16) and (17) demonstrates that all these results hold for any $Z>0$.

A new issue compared to Bréchet et al. (2013) is the influence of Z on the domestic policy. The analysis of formulas (21) and (23) demonstrates how the optimal size of the economy, adaptation, and mitigation depend on the external pollution stock Z and the country's relative size (in terms of its contribution to the global pollution).

Proposition 2 (Optimal domestic policy *wrt* country's contribution to pollution).

(i) *The smaller the country's share in global pollution stock (larger Z), the smaller its mitigation spending $\bar{B}$, capital stock $\bar{K}$, output $\bar{Y}$, and consumption $\bar{C}$ levels.*
(ii) *The influence of Z on the domestic policy is weak at small κ-indicator values ($\kappa_{max}\ll 1$). If $\kappa_{\min}\gg 1$, then capital $\bar{K}$ decreases in Z following (21), the consumption $\bar{C}$ is proportional to $\bar{K}$, while the output and mitigation expense decrease as $\bar{K}^{\alpha}$.*
(iii) *At $\kappa_{max}\ll 1$, the optimal adaptation $\bar{D}$ starts earlier and increases with Z. In the case of $\kappa_{\min}\gg 1$, the optimal $\bar{D}$ decreases with Z when $\mu<(1-\alpha)/\alpha$, and increases otherwise.*

Proof. See Appendix A.

The larger the country's contribution to global warming, the better the country can control the global pollution stock by mitigating its own pollution and, thus, preserve its own welfare. So, there is a strong incentive to spend money on mitigation rather than on adaptation. Then, more income, capital, and output are required to mitigate more. As a result, the size of the economy becomes larger. Naturally, this dependence is stronger when the pollution vulnerability is essential.

As highlighted in the item (iii) of Proposition 2, the impact of Z on the optimal adaptation level $\bar{D}$ is less straightforward. It goes through two channels. Indeed, the optimal $\bar{D}$ is smaller (or even zero) if the economy is too poor ($\bar{K}$ very small). But on the other hand, Z directly increases pollution, and correspondingly the adaptation efforts by (15). The final effect depends on the κ-indicator and parameters μ and α. In particular, it is positive for a small κ-indicator. Then, $\bar{K}$ does not depend on Z by

(23), therefore, increasing Z directly increases adaptation. If the κ-indicator is large, both situations are possible depending on the pollution disutility power μ in (6). If μ is small (*i.e.* the disutility of pollution in (6) is almost linear), then the optimal adaptation level D decreases as the country's contribution to pollution decreases (larger Z). But if the pollution disutility $\mu > (1-\alpha)/\alpha$, then the optimal adaptation level D increases with Z. The above condition on μ roughly corresponds to $\mu > 0.6$ in the current world economy, assuming the share of labor in output $\alpha = 2/3$. In other words, an increased relevance of pollution in society's objectives leads to an increase of adaptation in absolute units even when the economy becomes weaker and all other economic characteristics become smaller (at a larger Z).

The impact of the country's size on the optimal ratio D/B is stronger as μ is larger. When μ is small, then Z does not affect B and K by (21), while it still impacts the optimal adaptation level D. The reaction of optimal adaptation is, thus, much more complex. This will explain some of our results in the multi-country setting. We shall notice that, if the country size is small, then its welfare (1) may become negative because of climate change adverse effects (the pollution P is large while both $\bar{K}$ and $\bar{C}$ remain small for large Z).

4. The multi-country case

Let us now explore the case of a multi-country world. For the sake of readability, we will first consider only two countries, and the generalization to n countries will be discussed in the last subsection. We are interested in comparing the optimal domestic climate policies in two scenarios: when each country maximizes its own welfare, and when both countries coordinate their policies. The former is known as a non-cooperative Nash equilibrium and the latter as a cooperative solution. Because the pollution stock is common to both countries, it is clear that the Nash equilibrium is sub-optimal. The cooperative solution, on the other side, leads the world to the first best situation. Our purpose is not to analyze game-theoretical issues related to cooperation (Breton et al., 2006; Fanokoa et al., 2011; Legras and Zaccour, 2011), but rather to scrutinize how domestic policies are shaped by cooperation. The usual result in the literature is that non-cooperation leads to too low emission mitigation compared to what is socially optimal. We will see in this section that much more results can emerge from comparing Nash equilibrium and cooperation. Actually, a general equilibrium standpoint allows showing that countries may look completely different in these two situations. This is not just a question of mitigation spending, as usually discussed in the literature, but also of adaptation, capital stock, and consumption.

We start by providing analytical solutions to the two scenarios (non-cooperation, then cooperation, Sections 4.1 and 4.2) and their comparison (Section 4.3) at zero depreciation rates. Then, in Section 4.4 numerical simulations will be proposed at positive depreciation rates.

4.1. The non-cooperative scenario

Let us consider an economic-environmental system (the world) that consists of two countries that share the same pollution stock: Country 1 and Country 2. The external emission flow Z now becomes endogenous and results from the economic activity of Country 2:

$$Z(t) = \frac{Y_2(t)}{B_2(t)}. \tag{24}$$

We assume that countries do not cooperate. Each one decides about its own policy by maximizing its utility function, taking the policy of the other country as given. The corresponding optimization problem is as follows:

$$\max_{I_{K_i}, I_{D_i}, C_i} \int_0^\infty e^{-\rho t} \left[\ln\, C_i(t) - \eta_i(D_i(t)) \frac{P(t)^{1+\mu}}{1+\mu} \right]\, dt, \tag{25}$$

under the constraints

$$I_{K_i}(t) \ge 0, \quad I_{D_i}(t) \ge 0, \qquad C_i(t) \ge 0,$$

$$Y_i(t) = A_i K_i^{\alpha i}(t) = I_{K_i}(t) + I_{D_i}(t) + B_i(t) + C_i(t), \tag{26}$$

$$K_i'(t) = I_{K_i}(t) - \delta_{K_i} K_i(t), \quad K_i(0) = K_{0i}, \tag{27}$$

$$D_i'(t) = I_{D_i}(t) - \delta_{D_i} D_i(t), \quad D_i(0) = D_{0i}, \quad i = 1, 2, \tag{28}$$

$$P'(t) = -\delta_P P(t) + \gamma \frac{A_1 K_1^{\alpha_1}(t)}{B_1(t)} + \gamma \frac{A_2 K_2^{\alpha_2}(t)}{B_2(t)}, \quad P(0) = P_0. \tag{29}$$

It is the dynamic continuous game of two players (Countries 1 and 2) with the pay-off functions (25), $i = 1, 2$. Each player solves the optimization problem (25)–(29) with its own endogenous variables: consumption C_i, investments in physical capital and in adaptation I_{K_i}, I_{D_i}, and mitigation B_i, $i = 1, 2$. The externality is the global pollution stock P.

As the solution strategy for this game we use the classic *open-loop Nash equilibrium* (OLNE), a pre-commitment strategy. It assumes that each player maximizes its own payoff (25) by taking the other player's OLNE decision as given.

Remark. Another possible strategy for dynamic games is the *feedback* or *Markov-perfect Nash equilibrium strategy* (Long, 2010), when the decision depends on a current state. There are several important reasons for choosing the OLNE strategy over the feedback Nash equilibrium strategy in our model. First of all, the players are countries and as such are *a priori* less manipulative and more committed to planned actions as compared to, say, separate firms or private agents. Second, the anticipatory nature of the majority of adaptation projects requires careful preliminary design and multi-year investments, which matches well the OLNE assumption that each player knows the other player's optimal strategy (which is the same as their own). For the same reason, we have chosen our adaptation control D as the stock of adaptation capital, which takes years to build. In the case of the feedback Nash equilibrium strategy, each player can instantaneously change its optimal decision if the measured value of the state variable (in our case, pollution) deviates from its anticipatory optimal Nash value. One more reason is *a posteriori*. Proposition 3 below indicates a large overproduction in the non-cooperative case under the assumption (20). On the other hand, it is well known (Long, 2010, pp. 10–12) that the feedback strategy leads to larger pollution emission and overproduction in pollution games than OLNE does. It gives us a sensible motive to stay with OLNE in our adaptation–mitigation policy analysis. A technical advantage of OLNE is that it is easier to analyze (Long, 2010), which is imperative in such a complicated dynamic game as (25)–(29).

Let $\delta_{K_i} = \delta_{D_i} = 0$. Similar to Lemma 1, possible steady states $\bar{K}_i, \bar{B}_i, \bar{C}_i, \bar{D}_i, \bar{P}$ for each player $i = 1, 2$ should satisfy Eqs. (8)–(13) at $Z = \bar{Y}_{3-i}/\bar{B}_{3-i}$ or the following system of nine equations

$$\bar{B}_i = A_i \bar{K}_i^{\alpha_i} - \frac{\bar{K}_i \rho}{\alpha_i}, \tag{30}$$

$$\bar{C}_i = \frac{\bar{K}_i \rho}{\alpha_i}, \tag{31}$$

$$\bar{D}_i = \max\left\{0, \frac{1}{a_i} \ln \frac{a_i(\eta_{\max_i} - \eta_{\min_i}) \bar{K}_i \bar{P}^{\mu+1}}{\alpha_i(\mu+1)}\right\}, \quad i = 1, 2, \tag{32}$$

$$\bar{P} = \frac{\gamma}{\delta_P}\left(\frac{A_1}{A_1 - \bar{K}_1^{1-\alpha_1}\rho/\alpha_1} + \frac{A_2}{A_2 - \bar{K}_2^{1-\alpha_2}\rho/\alpha_2}\right), \tag{33}$$

$$\frac{\gamma \bar{P}^{\mu}}{(\delta_P + \rho)}[\eta_{\min_i} + (\eta_{\max_i} - \eta_{\min_i})e^{-a_i \bar{D}_i}] = \frac{[\alpha_i A_i - \bar{K}_i^{1-\alpha_i}\rho]^2}{\rho \alpha_i A_i \bar{K}_i^{1+\alpha_i}}, \quad i = 1, 2, \tag{34}$$

with respect to $\bar{K}_1, \bar{K}_2, \bar{B}_1, \bar{B}_2, \bar{C}_1, \bar{C}_2, \bar{D}_1, \bar{D}_2$ and $\bar{P}$.

The existence of the Nash equilibrium for continuous dynamic games is not guaranteed. In our game (25)–(29), it depends on the solvability of the nonlinear system (30)–(34). Solving the game with asymmetric players is not feasible. So, we will consider the symmetric case (*two identical countries*). Let:

$$A_1 = A_2, \qquad \alpha_1 = \alpha_2, \qquad \eta_{\min_1} = \eta_{\min_2}, \quad a_1 = a_2, \quad b_1 = b_2. \tag{35}$$

At (35), if the OLNE equilibrium state exists, it is the same for both players: $\bar{K}_1 = \bar{K}_2 = \bar{K}_N$, $\bar{B}_1 = \bar{B}_2 = \bar{B}_N$, $\bar{C}_1 = \bar{C}_2 = \bar{C}_N$, $\bar{D}_1 = \bar{D}_2 = \bar{D}_N$, where the subscript N stands for Nash. Then, Eqs. (30)–(34) for the unknown $\bar{K}_N, \bar{B}_N, \bar{C}_N, \bar{D}_N$ and $\bar{P}_N$ lead to

$$\bar{B}_N = A\bar{K}_N^{\alpha} - \frac{\bar{K}_N \rho}{\alpha}, \tag{36}$$

$$\bar{C}_N = \frac{\bar{K}_N \rho}{\alpha}, \tag{37}$$

$$\bar{P}_N = \frac{2\gamma}{\delta_P}\left(\frac{A}{A - \bar{K}_N^{1-\alpha}\rho/\alpha}\right), \tag{38}$$

$$\bar{D}_N = \max\left\{0, \frac{1}{a}\ln\frac{a(\eta_{\max} - \eta_{\min})\bar{K}_N \bar{P}_N^{\mu+1}}{\alpha(\mu+1)}\right\}, \tag{39}$$

where $\bar{K}_N > 0$ is found from the nonlinear equation (16) at $\bar{D}_N = 0$ or (17) at $\bar{D}_N > 0$, provided $Z = A\bar{K}_N^{\alpha}/\bar{B}_N$.

As in Section 3, if condition (20) holds, then the unique optimal $\bar{K}_N$ is determined as:

$$\bar{K}_N \cong \left[\frac{\alpha A(\rho + \delta_P)\delta_P^{\mu}}{\eta_{\min}\gamma^{\mu+1}2^{\mu}}\right]^{1/(1-\alpha)} = \frac{\bar{K}}{2^{\mu/(1-\alpha)}}. \tag{40}$$

If (22) holds: $\kappa_{\min} \ll 1$, then the optimal $\bar{K}_N \cong \bar{K}$ is determined by the formula (23).

The set of the same strategies $\bar{K}_N, \bar{B}_N, \bar{C}_N, \bar{D}_N$ for both players constitutes the *open-loop Nash equilibrium strategy* in the dynamic game (25)–(29). Using (7) and (18), the corresponding optimal vulnerability function and κ-indicator are calculated as

$$\bar{\eta}_N = \eta(\bar{D}_N), \qquad \kappa_N = \kappa(\bar{\eta}_N). \tag{41}$$

It is clear that the Nash equilibrium does not yield the best cumulative payoff for the players. In many situations, the players might improve their own payoffs, as well as global welfare, by following some cooperative strategies. The next section considers that scenario.

4.2. The cooperative scenario

In this section we analyze the case of a full international cooperation in our two-country world. It means that the countries coordinate the policy that maximizes their joint welfare. We assume that a benevolent social planner represents a multinational governmental body with complete control on the environmental policies of Countries 1 and 2. The corresponding optimization problem is as follows:

$$\max_{I_{K_1}, I_{D_1}, C_1, I_{K_2}, I_{D_2}, C_2} \int_0^{\infty} e^{-\rho t}\left[\ln C_1(t) + \ln C_2(t) - \eta(D_1(t))\frac{P(t)^{1+\mu}}{1+\mu} - \eta(D_2(t))\frac{P(t)^{1+\mu}}{1+\mu}\right] dt,$$
$$I_{K_i}(t) \geq 0, \quad I_{D_i}(t) \geq 0, \quad C_i(t) \geq 0, \tag{42}$$

under the constraints (26)–(29).

This problem is an extension of the optimization problem (1)–(6) with a double set of endogenous variables: the consumptions C_1 and C_2, the investments $I_{K_1}, I_{K_2}, I_{D_1}, I_{D_2}$, and the mitigation expenses B_1 and B_2 of Countries 1 and 2 respectively. The objective function describes the joint cumulative payoff of both countries. In such a case, the externality associated to the global pollution stock P is fully internalized and the outcome is the first best. The optimization problem includes six decision variables I_{K_1}, I_{K_2}, I_{D_1}, I_{D_2}, C_1, C_2, seven state variables K_1, K_2, D_1, D_2, B_1, B_2, P, and nine constraints-equalities (26)–(29). Despite its double size, it can be treated analogously to the problem (1)–(6).

Assuming $\delta_{K_i} = \delta_{D_i} = \delta$ and providing the comparative static analysis, we obtain the following interior first order conditions for nine steady-state variables $\bar{K}_1, \bar{K}_2, \bar{B}_1, \bar{B}_2, \bar{C}_1, \bar{C}_2, \bar{D}_1, \bar{D}_2, \bar{P}$:

$$\alpha_i A_i \bar{K}_i^{\alpha_i} - \alpha_i \bar{B}_i = \bar{K}_i(\delta + \rho), \tag{43}$$

$$A_i \bar{K}_i^{\alpha_i} = \delta \bar{K}_i + \delta \bar{D}_i + \bar{B}_i + \bar{C}_i, \tag{44}$$

$$-\eta_i'(\bar{D}_i)\frac{\bar{P}^{\mu+1}}{(1+\mu)} = \frac{(\delta+\rho)}{\bar{C}_i}, \quad i = 1, 2, \tag{45}$$

$$\bar{P}^{\mu}[\eta_1(\bar{D}_1) + \eta_2(\bar{D}_2)] = \frac{(\delta_P + \rho)\bar{B}_1^2}{\gamma A_1 \bar{K}_1^{\alpha_1} \bar{C}_1} = \frac{(\delta_P + \rho)\bar{B}_2^2}{\gamma A_2 \bar{K}_2^{\alpha_2} \bar{C}_2}, \tag{46}$$

$$\delta_P \bar{P} = \gamma \left(\frac{A_1 \bar{K}_1^{\alpha_1}}{\bar{B}_1} + \frac{A_2 \bar{K}_2^{\alpha_2}}{\bar{B}_2} \right). \tag{47}$$

An analysis of the system (43)–(47) indicates the way of its sequential solution, which is the extension of the technique used in Section 2.

Assuming $\delta = 0$, the steady state $\bar{K}_i, \bar{B}_i, \bar{C}_i, \bar{D}_i$, $i = 1, 2$, $\bar{P}$ satisfies the system of nine nonlinear equations (30)–(33) and

$$\frac{\gamma \bar{P}^{\mu}}{(\delta_P + \rho)} \sum_{i=1}^{2} [\eta_{\min_i} + (\eta_{\max_i} - \eta_{\min_i}) e^{-a_i \bar{D}_i}] = \frac{[\alpha_i A_i - \bar{K}_i^{1-\alpha_i} \rho]^2}{\rho \alpha_i A_i \bar{K}_i^{1+\alpha_i}}, \quad i = 1, 2. \tag{48}$$

This system differs from the Nash equilibrium case only by Eq. (48). Substituting the expressions of $\bar{D}_1, \bar{D}_2$, and $\bar{P}$ *via* $\bar{K}_1$ and $\bar{K}_2$ into (48), we obtain two nonlinear equations with respect to $\bar{K}_1$ and $\bar{K}_2$. This system of two equations is analogue to Eq. (14) for $\bar{K}$ in the one country case. Its solution is simple in the case (35) of symmetric countries. Then, by symmetry considerations, the steady state satisfies conditions $\bar{K}_1 = \bar{K}_2 = \bar{K}_{CO}, \bar{B}_1 = \bar{B}_2 = \bar{B}_{CO}, \bar{C}_1 = \bar{C}_2 = \bar{C}_{CO}, \bar{D}_1 = \bar{D}_2 = \bar{D}_{CO}$, where the subscript CO stands for "Cooperation". Substituting the last expressions into the system (40)–(43), we obtain the same Eqs. (8)–(13) for the unknown $\bar{K}_{CO}, \bar{B}_{CO}, \bar{C}_{CO}, \bar{D}_{CO}, \bar{P}_{CO}$, in which $\delta = 0$, $Z = 0$, and the coefficient γ is replaced by 2γ.

As in Section 2, if the condition (20) holds, then the unique optimal $\bar{K}_{CO}$ is found as

$$\bar{K}_{CO} \cong \left[\frac{\alpha A(\rho + \delta_P)\delta_P^{\mu}}{\rho \eta_{\min}(2\gamma)^{\mu+1}} \right]^{1/(1-\alpha)} = \frac{\bar{K}}{2^{(\mu+1)/(1-\alpha)}} = \frac{\bar{K}_N}{2^{1/(1-\alpha)}}. \tag{49}$$

If (22) holds, $\kappa_{\min} \ll 1$, then the optimal $\bar{K}_N$ is determined by the approximate formula

$$\bar{K}_{CO} \cong \left[\frac{\alpha A}{\rho} (1 - 2^{(\mu+1)/(\mu+2)} \kappa_{\max}^{1/(\mu+2)}) \right]^{1/(1-\alpha)} < \bar{K}_N. \tag{50}$$

After finding $\bar{K}_{CO}$, the corresponding steady-state levels $\bar{B}_{CO}, \bar{C}_{CO}, \bar{D}_{CO}, \bar{P}_{CO}, \bar{\eta}_{CO}$ and κ_{CO} are also known, in particular,

$$\bar{\eta}_{CO} = \eta(\bar{D}_{CO}), \qquad \kappa_{CO} = \kappa(\bar{\eta}_{CO}). \tag{51}$$

We can now compare the optimal domestic policy mix in the non-cooperative and cooperative scenarios.

4.3. Comparing cooperative and non-cooperative scenarios in terms of domestic policies

The comparison between the two scenarios is carried out by comparing approximate formulas (36)–(40) (in Nash) and (49) (in cooperation). This will confirm a couple of expected results, but also yield many unexpected ones. A naturally expected result is that pollution is too high in the non-cooperative scenario compared to the first best. Interestingly, the reasons for such relationship are not trivial at all. Namely, the first important result is that the size of economies sharply differs between two scenarios. This is summarized in the following proposition.

Proposition 3 (Size of the economy). *The optimal steady state capital level is always larger in the non-cooperative case than under cooperation: $\bar{K}_N > \bar{K}_{CO}$. If the κ-indicator is small ($\kappa_{\max} \ll 1$), then the difference is small ($\bar{K}_N \cong \bar{K}_{CO}$); if the κ-indicator is large, then the difference is large. Formally, if $\delta = 0$ and the condition (20) holds, then:*

$$\bar{K}_N \cong 2^{1/(1-\alpha)}\bar{K}_{CO}, \qquad \bar{Y}_N \cong 2^{\alpha/(1-\alpha)}\bar{K}_{CO}. \tag{52}$$

Proposition 3 states that the size of the economy is always too large in Nash equilibrium. This result will drive many other ones, as we will see later in this section. In other words, the lack of cooperation does not only lead to too much pollution but also (and it may be more important) to over-accumulation of capital. Actually, the whole economy is much too fat in Nash. And it is even worse when the economy environmental harmfulness is high. In fact, the interpretation of Proposition 3 is rather intuitive. If the environmental self-cleaning capability δ_P is strong and the emission impact factor γ and environmental vulnerability η are negligible, then the pollution hardly affects the optimal policy. Indeed, in the case of a small $\kappa_{\max}$ (*i.e.*, $\eta\gamma^{\mu+1} \ll \rho\delta_P^{\mu}$), the Nash equilibrium and cooperative optimal strategies are similar and do not depend on environmental parameters. The gap between $\bar{K}_N$ and $\bar{K}_{CO}$ monotonically increases from zero to the maximal factor $2^{1/(1-\alpha)}$ as the environmental harmfulness increases. It is optimal to shrink the environmentally vulnerable economy, which is intuitive, but the shrink is not strong enough under Nash equilibrium. This occurs because countries are not able to effectively control the global pollution level when international cooperation fails.

In the case of large emission impact γ, environmental vulnerability η, economic and adaptation efficiencies A and a, and negligible environmental self-cleaning δ_P, the Nash value (50) of capital is up to $2^{1/(1-\alpha)}$ times larger than the optimal capital amount (49) in the cooperative optimization problem. Then, the Nash output $\bar{Y}$ is $2^{\alpha/(1-\alpha)}$ times larger than the optimal one in the cooperative case. The consumption and adaptation levels are also larger in Nash (than in the cooperative scenario), which seems to be good for welfare. However, the pollution level is so high that it overpowers these two components. By the end, the resulting welfare is smaller.

Let us now consider the climate policies and concentrate on mitigation first. In both scenarios, the optimal mitigation spending is determined by the same formula (30) that depends on the capital level only. So it can formally be analyzed separately from the adaptation decision. This does not mean that adaptation and mitigation decisions are separate (we will see it below), but it suggests that the policy-maker should try first to control the pollution flow (with mitigation spending) and, then, to alleviate adverse impacts with adaptation. Mitigation $\bar{B}_N$ in Nash equilibrium is larger than the optimal mitigation in cooperation $\bar{B}_{CO}$. Countries spend too much on mitigation in order to keep the pollution flow under control. This is in sharp contrast with the current literature: in Nash, mitigation efforts are too large in dollars because the economy is too fat. In a certain sense, the lack of cooperation on the environmental common draws to wasteful economies: too much production is required to finance too much mitigation. It can be noted that, due to our assumptions about the pollution motion law (6), the pollution flow Y/B cannot decrease down to zero, which is realistic for greenhouse gases.

What is the influence of country's characteristics (the productivity parameter A and impatience rate ρ) on the optimal domestic policy? Such an analysis remains qualitatively similar to Section 2 in both non-cooperative and cooperative scenarios. The optimal policy mix between adaptation and mitigation $\bar{D}/\bar{B}$ is initially zero for very small A (no adaptation is optimal). Then it grows with A and decreases later producing an inverted U-shape. When adaptation is positive, the size of the economy is larger, the pollution stock is larger, and mitigation efforts are smaller compared to the case with no adaptation. So, the policy mix displays the same qualitative properties in both scenarios.

We know from Eq. (15) that adaptation is positive starting with some minimal capital threshold size. On the other side, the capital stock is larger in Nash than in cooperation. This means that adaptation will start earlier, *i.e.*, for poorer economies, in the absence of cooperation. It is an important result as it suggests that a large pollution stock is not the only reason why poor countries have to start adaptation. And a key policy implication is related to financial transfers among countries (adaptation funds): too much money is needed in developing countries for adaptation in absence of cooperation.

Another result of interest is the following. In Nash, the optimal policy mix $\bar{D}/\bar{B}$ will reach its maximum for weaker economies (compared to the cooperative case). This will be illustrated in Section 4.4.

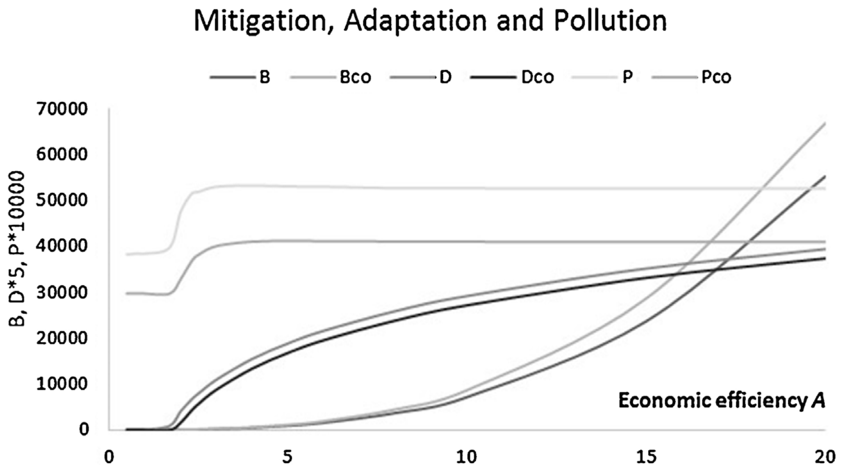

Fig. 1. The optimal adaptation $\bar{D}$, abatement $\bar{B}$, and pollution level $\bar{P}$ for various values of the country productivity A in the competitive OLNE case and the cooperative case.

This shows that a definitive difference between cooperation and non-cooperation does not only lie in mitigation efforts, but also on the whole domestic policy mix. All these results are summarized in the following statement.

Proposition 4. *In both cooperative and non-cooperative scenarios, optimal adaptation is zero when the total factor productivity A is too small; then $\bar{D}/\bar{B}$ increases with A and decreases later. In the absence of cooperation, the adaptation becomes positive at a smaller A and the maximum of $\bar{D}/\bar{B}$ occurs at smaller values of A than under cooperation.*

4.4. Numerical simulations with positive capital deterioration

To analyze the optimal sustainable dynamics in our model with positive capital deterioration $\delta > 0$, we solve the optimization problems (25) and (42) numerically for different ranges of the model parameters. The simulation was done using Excel Solver. It confirms and clarifies the above theoretical outcomes, including Propositions 3 and 4.

The benchmark simulation scenario uses the following model parameters: $\rho = 0.01$, $\alpha = 0.66$, $\delta_P = 0.002$, $\gamma = 0.1$, $\mu = 0.1$, $\underline{\eta} = 0.1$, $\bar{\eta} = 0.2$, $a = 0.001$, $\delta = 0.05$, and $A = 20$. The dimensional analysis of all model variables is discussed in Bréchet et al. (2013). This dataset is based on Bréchet et al. (2013) and Yatsenko et al. (2014), but describes a more favorable environmental situation. Namely, we take the same values of ρ, α, γ, a, but faster decaying pollution δ_P, less damaging pollution disutility μ, and wider range $[\underline{\eta}, \bar{\eta}]$, of environmental vulnerability regulation. To determine sustainable optimal policies at different levels of economic development and environmental vulnerability, we provide simulations for different ranges of A, γ, and ρ.

4.4.1. Case of increasing economic efficiency

The first simulation run is for different values of the economic efficiency A from 1 to 25 (the rest of parameters are from the basic scenario). The results are provided in Figs. 1–4. They compare the competitive Nash-equilibrium policy of a separate country to the cooperative case of two countries. The main insights are the following.

As seen in Fig. 1, the optimal steady-state levels of mitigation B, adaptation D, and pollution P in cooperation remain below the ones in Nash equilibrium for any productivity A. The exponential dependence of the optimal output Y, consumption C, and capital K on A is similar to B and is not shown. The optimality of zero-adaptation policy for low productivity levels predicted by Proposition 1 is clearly visible in Figs. 1 and 2. Namely, a sharp increase in the optimal adaptation level $\bar{D}$ from

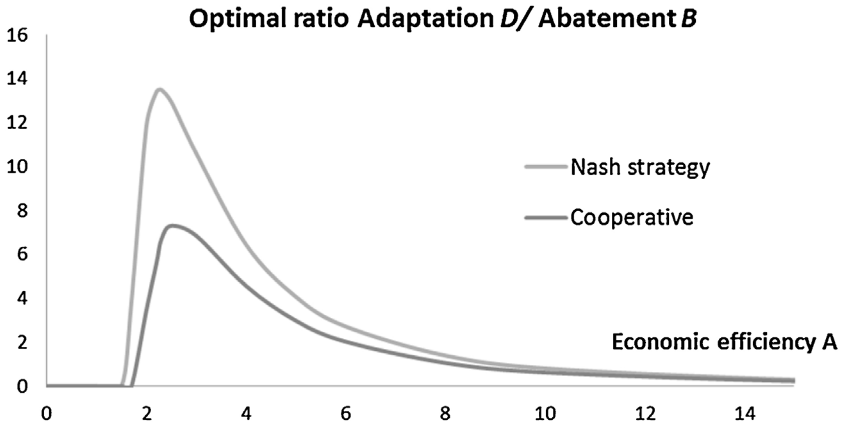

Fig. 2. The adaptation–abatement ratio D/B for various values of the country productivity A.

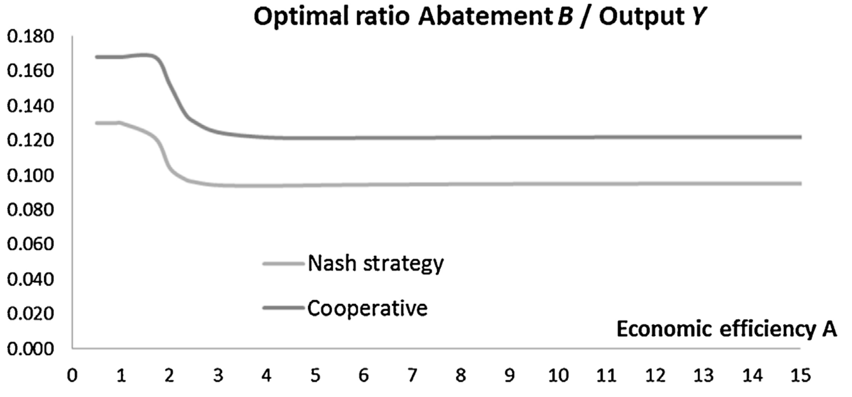

Fig. 3. The abatement–output ratio B/Y for various values of the country productivity A.

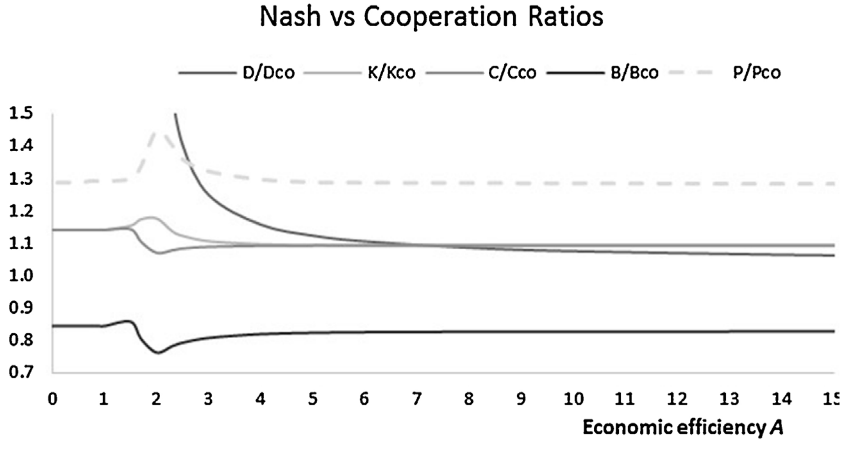

Fig. 4. The optimal ratios of Y, C, K, B, D, and P between cooperative and competitive cases for various values of A.

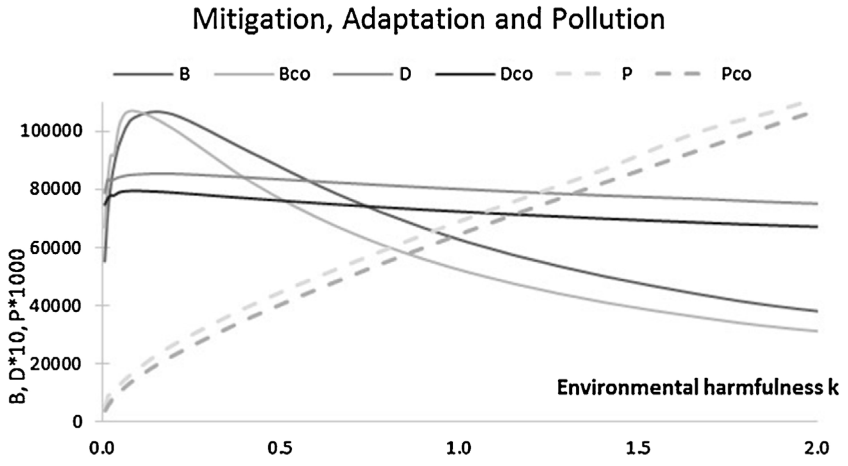

Fig. 5. The optimal adaptation $\bar{D}$, abatement $\bar{B}$, and pollution level $\bar{P}$ for various values of the environmental harmfulness indicator κ in two cases: (a) Nash equilibrium and (b) cooperation.

zero occurs when the economic efficiency A reaches the threshold value ≈1.7 in competitive case and ≈2 in the presence of cooperation.

In Fig. 1, both adaptation D and abatement B increase in absolute units, but B increases faster. The resulting *inverted-U pattern* of the optimal ratio D/B between adaptation and abatement is shown in Fig. 2. Simulations show that the optimal ratio D/B reaches its maximum at the smaller value $A \approx 2.3$ of economic efficiency in the competitive Nash case than at $A \approx 2.5$ in cooperation. This fact was predicted in Proposition 4. The maximal ratio D/B is twice larger in the competitive Nash case than in cooperation, but this difference disappears with further increase of A (see also Fig. 4).

Fig. 3 shows that the part of abatement spending B in the output Y is smaller in the presence of positive adaptation D that starts with the productivity $A > 1.7$–2. Therefore, these two policy instruments are partially substitutable. The abatement-output ratio B/Y is 30–35% larger in the range $0 < A < 1.7$ without adaptation than in the range $A > 2$ with adaptation.

Fig. 4 highlights that the optimal ratios of all major variables Y, C, K, B, D, and P between cooperative and competitive cases remain practically the same for different values of productivity A when $A < 1.7$ and $A > 3$. Those ratios vary only in the range $1.7 < A < 3$ of the policy change where the adaptation becomes a viable policy instrument. It means that the presence or the absence of adaptation matters more than the productivity level itself on the optimal policy mix. In particular, an increase in the pollution level P due to the absence of cooperation has a stronger effect (30%) than the increase in the size of the economy (both capital K and consumption C differ by less than 15% from their levels with cooperation).

4.4.2. *Cases of increasing environmental harmfulness*

The next simulation run analyzes how an increase in the environmental harmfulness affects the country's optimal domestic policy. Figs. 5 and 6 compare optimal sustainable levels of the country's economy in cases with and without cooperation for different values of the pollution emission factor γ in the range $0.005 < \gamma < 0.1$. By formula (18), the corresponding κ-indicator of environmental harmfulness varies from 0.007 to 2.5.

As displayed in Fig. 5, the steady-state pollution P increases, while both optimal mitigation B and adaptation D decrease when the environmental harmfulness κ increases. The optimal output Y, consumption C, and capital K also decrease exponentially to zero (not shown in the figure). As a result, the sustainable *welfare level* W (described by the utilities (25) and (42)) decreases and becomes negative at $\kappa > 1$ in the competitive Nash equilibrium, and at $\kappa > 1.5$ in cooperation.

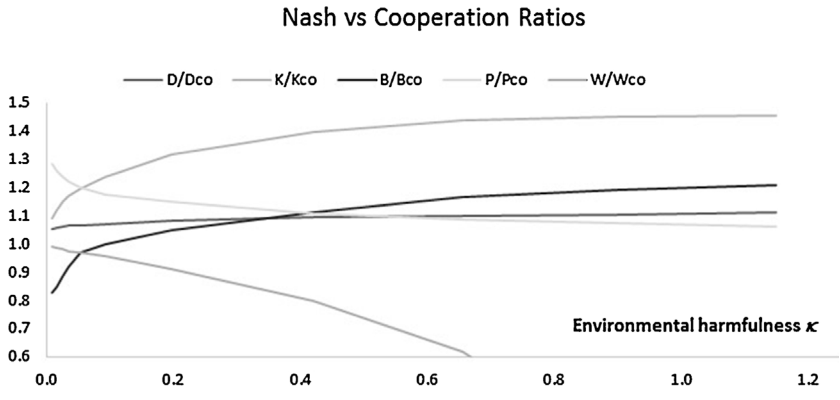

Fig. 6. The optimal ratios of D, K, B, P, and W between cooperative and competitive cases for various values of the environmental harmfulness indicator κ.

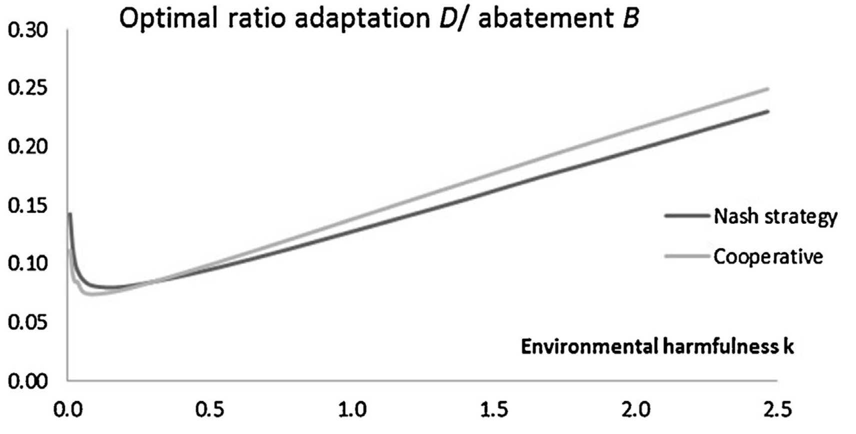

Fig. 7. The adaptation/abatement ratio D/B for cases for various values of κ.

Fig. 6 shows the optimal ratios of all economic variables Y, C, K, B, and D between cooperative and competitive cases when γ and κ increase. In particular, the output Y is larger by 30% and the capital K and consumption C are larger by 50% and more from their levels with cooperation at $\kappa > 0.9$. The observation confirms and clarifies Proposition 3. Still, the simulated increase of the economy size is smaller than predicted by the formula (52) for the case that combines $\kappa \gg 1$, $\delta = 0$, and the condition (20). It confirms that such case is an upper bound and that the real increase of the capital K would be smaller than $2^{\alpha/(1-\alpha)}$. Fig. 6 shows that, with increasing environmental harmfulness, the increase of pollution P due to the absence of cooperation becomes less essential compared to an increase of the economy size.

As before, the optimal steady-state levels of output Y, consumption C, capital K, adaptation D, and pollution P in cooperation remain below the ones in Nash equilibrium for any pollution emission factor γ (the mitigation B is smaller in Nash for small γ).

In Fig. 5, both adaptation D and abatement B decrease in absolute units and B decreases faster. The resulting *U-pattern* of the optimal ratio D/B at increasing environmental harmfulness is shown in Fig. 7 and is the inverse of Fig. 2 in a certain sense. It simply demonstrates that the role of adaptation becomes more important than of mitigation when the environmental harmfulness increases at the same level of economic efficiency.

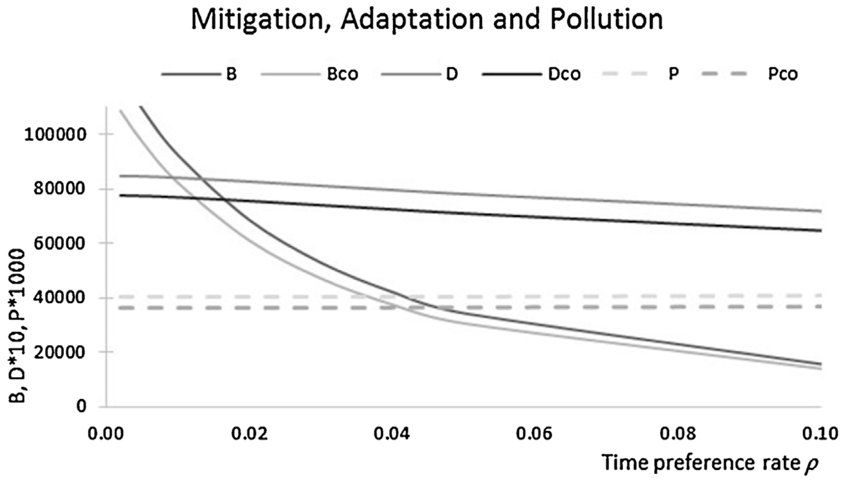

Fig. 8. The optimal adaptation $\bar{D}$, abatement $\bar{B}$, and pollution level $\bar{P}$ for various time preference rates ρ in the competitive OLNE case and the cooperative case.

Case of increasing time preference rate. Interestingly, there exists another way of increasing environmental harmfulness: it is by decreasing the rate of time preference ρ in (1), (25) and (42). The corresponding simulation is shown in Fig. 8 for different rates ρ lying in the range $0.002 < \rho < 0.1$. By Eq. (18), the corresponding κ-indicator of environmental harmfulness varies between 0.05 and 1.3. Then, the pollution level remains the same, but its perception for future generations decreases when ρ increases. As result, both mitigation and adaptation efforts decrease (in both cases with and without cooperation). The mitigation B decreases faster than adaptation D. The optimal output Y, consumption C, and capital K have the same decreasing exponential dependence on ρ as mitigation B shown in Fig. 8. As before, the optimal steady-state levels of mitigation B, adaptation D, and pollution P in cooperation are lower than the levels at Nash equilibrium for any rate ρ. Finally, the optimal ratios of all major variables Y, C, K, B, D, and P between cooperative and competitive cases are the same for any value of ρ. The increase of pollution P due to the absence of cooperation is less severe compared to the increase of the economy size (both capital K and consumption C are larger by 40% compared to their levels with cooperation).

4.5. Generalization to n countries

Until now we restricted our analysis to a two-country case for the sake of readability. Generalizing our results to n symmetric countries is straightforward. Proposition 2 holds replacing 2 by n. In particular, the Nash equilibrium output $\bar{Y}$ may be $n^{\alpha/(1-\alpha)}$ times larger than the optimal output in the case of full cooperation for a brown economy. The Nash equilibrium capital $\bar{K}$ will be $n^{1/(1-\alpha)}$ times larger than the capital in cooperative case. In summary, the Nash strategy in a multi-country world under study involves overproduction, which leads to overconsumption, over-pollution and over-adaptation. The primary problem comes from overproduction that appears because the country controls only its own pollution emission (so, a decrease in production causes much smaller decrease of pollution emissions than in the cooperative scenario). The corresponding adaptation investment depends on both production and pollution, so over-adaptation is the result of overproduction. All the results proved for the two-country model hold for n symmetric countries.

5. Conclusion and further research directions

In this paper we develop a multi-country dynamic general equilibrium model to understand how international cooperation shapes domestic policies. Our model is built upon the classic Solow–Swan framework and captures essential nonlinearities in production and pollution dynamics. Its contribution to the literature includes several insights. First, the optimal mitigation effort is always positive for all involved countries.[6] Second, the optimal ratio between adaptation and mitigation is zero for poor countries, maximal for middle-income countries, and it decreases but remains positive as the country becomes richer. Buob and Stephan (2011) agree that the adaptation is zero for poor countries, but they obtain a polar result that the richest regions should invest in adaptation only. Third, the non-cooperative (Nash) strategy in the multi-country world leads to overproduction, overconsumption, over-pollution and over-adaptation in comparison to cooperative case. This result could not be displayed by Buob and Stephan (2011) because their production is exogenous.

We shall notice that our analysis is limited to the symmetrical case (identical countries). Extending our model to a multi-country model with asymmetry among countries would be highly desirable to address strategic behavioral differences between rich and poor countries, but it raises additional mathematical challenges and we leave it for further research. Nevertheless, our modeling approach with an exogenous pollution flow proves to be an appealing simple way of introducing heterogeneity in terms of country size.

Several ideas for our model extension can be borrowed from the environmental games, *e.g.*, Fanokoa et al. (2011) or Eyland and Zaccour (2014). We are planning to consider a two-player asymmetric pollution-control differential game where one player is non-vulnerable to pollution. It may simplify analytics and lead to interesting insights. Another potential idea is to use a border tax instead of a cooperative international agreement (Eyland and Zaccour, 2014). In terms of Breton et al. (2006), our model describes an "autarky" adaptation. Its possible extension to access merits of cooperation is in considering the "joint implementation" when players can invest into adaptation at home and abroad (where adaptation costs are lower). Multiple interacting pollutants with synergetic effects can be considered as in Legras and Zaccour (2011).

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Appendix A.

Proof of Lemma 1. It is analogous to Yatsenko et al. (2014) for the case $Z=0$. The current-value Hamiltonian for the problem (1)–(7) is

$$H = e^{-\rho t}\left(\ln C - \eta(D)\frac{P^{1+\mu}}{1+\mu}\right) + \lambda_1(AK^{\alpha} - I_K - I_D - B - C) + \lambda_2\left(-\delta_P P + \frac{\gamma AK^{\alpha}}{B} + \gamma Z\right)$$
$$+ \lambda_3(I_K - \delta K) + \lambda_4(I_D - \delta D) + \mu_1 I_K + \mu_3 I_D + \mu_2 C, \tag{A1}$$

where the dual variables λ_1, λ_2, λ_3, λ_4 are associated with equalities (2)–(5) and μ_1, μ_2, μ_3 reflect the irreversibility constraints. Deriving the first order conditions for I_K, I_D, C, K, B, P, D from (A1) and excluding the dual variables, we obtain the following nonlinear system for possible *interior* optimal trajectories K, B, C, D and P:

$$AK^{\alpha} = \delta K + K' + \delta D + D' + B + C, \tag{A2}$$

[6] In contrary, in Buob and Stephan (2011), regions can invest in mitigation or in adaptation only (or neither in adaptation nor mitigation), and the regions completely switch from mitigation to adaptation as the number of regions becomes large, which does not happen in our model.

$$\alpha AK^{\alpha-1} - \alpha\frac{B}{K} - \frac{C'}{C} = \delta + \rho, \tag{A3}$$

$$P' + \delta_P P = \gamma\left(\frac{AK^\alpha}{B} + Z\right), \tag{A4}$$

$$\eta(D)P^\mu = \frac{B}{\gamma AK^\alpha C}\left(B(\delta_P + \rho) + \alpha\frac{B}{K}K' + \frac{B}{C}C' - 2B'\right), \tag{A5}$$

$$-\eta'(D)\frac{P^{\mu+1}}{\mu+1} = \frac{1}{C}\left[\rho + \delta + \frac{C'}{C}\right]. \tag{A6}$$

Next, we analyze possible steady states of the problem under study. By (A2)–(A6), any positive constant (stationary) state $(\bar{K}, \bar{B}, \bar{C}, \bar{D}, \bar{P})$ of the problem (1)–(6), if it exists, should satisfy the following system of nonlinear equations:

$$A\bar{K}^\alpha = \bar{B} + \bar{C} + \delta\bar{K} + \delta\bar{D}, \tag{A7}$$

$$\alpha A\bar{K}^{\alpha-1} - \alpha\frac{\bar{B}}{\bar{K}} = \delta + \rho, \tag{A8}$$

$$\delta_P\bar{P} = \gamma\left(\frac{A\bar{K}^\alpha}{\bar{B}} + Z\right), \tag{A9}$$

$$[\underline{\eta} + (\bar{\eta} - \underline{\eta})e^{-a\bar{D}}]\bar{P}^\mu = \frac{\bar{B}^2}{\gamma A\bar{K}^\alpha\bar{C}}(\delta_P + \rho), \tag{A10}$$

$$\frac{\bar{P}^{\mu+1}}{\mu+1}a(\bar{\eta} - \underline{\eta})e^{-a\bar{D}} = \frac{\rho+\delta}{\bar{C}}. \tag{A11}$$

The rest of the proof follows (Yatsenko et al., 2014). Lemma is proved. □

Proof of Proposition 1. It extends the analysis of Bréchet et al. (2013) for $Z = 0$ and includes similar steps. Namely, substituting (15) into (14), we get Eq. (17). Next, introducing the dimensionless unknown variable $x = (\rho/\alpha A)\bar{K}^{1-\alpha}$, $0 < x < 1$, and the parameters

$$\kappa_{\min} = \frac{\eta_{\min}\gamma^{1+\mu}}{(\delta_P + \rho)\delta_P^\mu}, \qquad \kappa_{\max} = \frac{\eta_{\max}\gamma^{1+\mu}}{(\delta_P + \rho)\delta_P^\mu}, \tag{A12}$$

we obtain from (17) the equation

$$\frac{(1-x)^{2+\mu}}{x} = \kappa_{\min}[1 + Z(1-x)]^\mu + \frac{\alpha\delta_P(\mu+1)}{a(\delta_P+\rho)(\alpha A/\rho)^{1/(1-\alpha)}}\frac{(1-x)^{1+\mu}}{x^{1/(1-\alpha)}[1+Z(1-x)]^\mu} \tag{A13}$$

The left-hand side $F(x) = ((1-x)^{2+\mu})/x$ of Eq. (A13) strictly decreases from ∞ at $x = 0$ to $F(1) = 0$ and is the same as in the case $Z = 0$. The right-hand function

$$G(x) = \kappa_{\min}[1 + Z(1-x)]^\mu + \frac{\alpha\delta_P(\mu+1)}{a(\delta_P+\rho)(\alpha A/\rho)^{1/(1-\alpha)}}\frac{(1-x)^{1+\mu}}{x^{1/(1-\alpha)}[1+Z(1-x)]^\mu}$$

strictly decreases from ∞ at $x = 0$ to $\kappa_{\min}$ at $x = 1$ (exactly as at $Z = 0$).

Similar to Bréchet et al. (2013), now we can prove that, if $D((\alpha A\hat{x}/\rho)^{1/(1-\alpha)}) > 0$, where $\hat{x}$ is the solution of the equation

$$\frac{(1-\hat{x})^{2+\mu}}{\hat{x}} = \kappa_{\min}[1 + Z(1-\hat{x})]^{1+\mu}, \tag{A14}$$

then the functions $F(x)$ and $G(x)$ intersect at a unique point x^*, $\hat{x} < x^* < 1$, and, therefore, Eq. (A13) has a unique solution. The proposition is proven. □

Proof of Proposition 2. The case of a small κ-indicator ($\kappa_{\max} \ll 1$) is obvious. Let us focus on the case (20) when the κ-indicator is high. Then, the dimensionless unknown $x = (\rho/\alpha A)\bar{K}^{1-\alpha}$, introduced in the proof of Proposition 1 is small: $0 < x \ll 1$. Correspondingly, by (8) and (9),

$$\bar{C} = \frac{\bar{K}\rho}{\alpha}, \qquad \bar{B} = A\bar{K}^{\alpha}\left(1 - \frac{\bar{K}^{1-\alpha}\rho}{\alpha A}\right) \approx A\bar{K}^{\alpha}.$$

Finally, $\bar{D}(\bar{K})$ is defined by (15). Assuming (20) and substituting (21) into (15), we obtain that $\bar{D}(Z) \sim \ln\{[(1+Z)^{-\mu}]^{1/(1-\alpha)}(1+Z)^{\mu+1}\}$. So, $\bar{D}$ increases in Z when $\mu < (1-\alpha)/\alpha$. This completes the proposition proof. □

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