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Co-optimization of energy and reserve with Incentives to Wind Generation

Yves Smeers, Sebastian Martin, *Member, IEEE,* and José A. Aguado, *Member, IEEE*

***Abstract*—Transitioning from fossil fuel dominated power systems to high penetrations of intermittent renewable generation is affecting classical electricity market designs. Here, a method is proposed to model and assess the absence of co-optimization of energy and reserve that prevails in the European system. Co-optimization is formulated through an optimization problem (COM), and its absence as an equilibrium problem (EQM) built on Karush-Kuhn-Tucker conditions of agents' optimization and market clearing equations. EQM cannot be reformulated as a single optimization problem. Market distortions are identified by comparing the complementarity conditions of both models. These are then discussed on system with Feed in Premium to wind generation. Parameters in the models allow to represent different market configurations regarding: available wind generation, Feed in Premium to wind, generators' risk aversion, and required reserve from wind generation.**

***Index Terms*—co-optimization, energy and reserve, complementarity conditions, market equilibrium**

NOTATION

Parameters are in upper case and variables in lower case.

A. Indices and sets

g, G	Dispatchable generators, $g \in G$.
w, WT	Wind turbines, $w \in WT$.
k, Ω	Index and set for scenarios, $k \in \Omega$.

B. Parameters

$\overline{A}/\underline{A}$	Upper/lower factor for required up. reserve (p.u.).
B	Lower bound factor for committed downward reserve respect to the committed upward reserve (p.u.).
$\overline{R}_g/\underline{R}_g$	Upward/Downward ramping slope respect to the generation capacity (p.u.).
C_g	Slope of gen. cost for dispatch. gen. g, (€/MWh).
FiP	Premium to the schedu. wind gen., (€/MWh).
M_y	Balancing reserve factor for wind turbines, (p.u).
M_x	Balancing reserve factor for dispatch. gen., (p.u.).
Pr^k	Probability of scenario k, (p.u.).
$\overline{X}_g/\underline{X}_g$	Rated / must-run power of dispat. gen. g, (MW).
$\overline{Y}_w^k/\underline{Y}_w^k$	Available /must-run gen. wind tur. w, scen. k, (MW).
$\overline{Y}_w^{max}$	Rated capacity of wind turbine w, (MW).
Γ_0	Constant of the inverse demand function, (€/MWh).
Θ	Confidence level for firm Conditional Value at Risk (CVaR), $\Theta \in (0, 1)$.
Λ	Parameter for ramping availability, $\Lambda \in [0, 1]$.
Φ_0	Slope of the inverse demand function (€/(MWh)^2).
Ψ	Level of risk aversion, $\Psi \in [0, 1]$, zero is risk neutral.

C. Variables

Primal variables

d	Energy sales of the firm in day-ahead, (MWh).
q^k	Correction cost for scenario k, (€).
$q^{k,-}/\zeta$	Auxiliary variables for $CVaR_\Theta$ calculation (€).
ru_g/rd_g	Committed upward/downward reserve capacity from dispatchable generator g, (MW).
s_g^k/u_g^k	Energy deployed from upward/downward reserve of dispatchable gen. g at scen. k, (MWh).
v_w^k	Production over the scheduled value for wind turbine w at scen. k, (MW).
x_g	Scheduled generation of dispatchable unit g, (MW).
y_w	Scheduled generation of wind turbine w, (MW).
z_w^k	Energy bought by wind turbine w at scen. k, (MWh).
ρ	Energy price in day-ahead (€/MWh).
ρ^*	Equilibrium energy price in day-ahead (€/MWh).

Dual variables, all in €/MWh except σ^k that can be interpreted as an scaled probability, $\Psi = \sum_{k\in\Omega} Pr^k \cdot \sigma^k$.

$\overline{\gamma}/\underline{\gamma}$	Upper bound committed upward/downward reserve.
$\overline{\delta}_g/\underline{\delta}_g$	Upper bound for upward/downward reserve that generator g can supply. Ramping constraint.
$\overline{\eta}_g^k/\underline{\eta}_g^k$	Upper bound for energy from committed upward / downward reserve, dispatch. gen. g, scen. k.
$\overline{\iota}_w^k/\underline{\iota}_w^k$	Upper/Lower bound for purchases of wind turbine w at scenario k.
$\overline{\kappa}/\underline{\kappa}$	Lower bound for committed up./down. reserve.
ν	Firm's power balance (energy price).
ξ_w^k	Upper bound for surplus sales wind tur. w at scen. k.
σ^k	$CVaR_\Theta$ constraint. Distorted probability.
τ_w^k	Constraint that relates the available wind surplus for wind turbine w and it use at scenario k.
$\overline{\phi}_g/\underline{\phi}_g$	Upper/Lower bound capacity of dispat. gen. g.
$\overline{\pi}^k/\underline{\pi}^k$	Balancing with surplus/shortage of wind at scen. k.
$\overline{\psi}_w$	Upper bound capacity constraint of wind turbine w.
$\underline{\psi}_w^k$	Lower bound capacity of wind tur. w at scen. k.

Y. Smeers is with the Center for Operations Research and Econometrics (CORE) at Université Catholique de Louvain (UCL), Belgium (e-mail: yves.smeers@uclouvain.be)

S. Martin and J. A. Aguado are with the Department of Electrical Engineering, University of Malaga, Malaga, Spain (e-mail: smartin@uma.es; jaguado@uma.es). Their work was supported in part by the Spanish Ministry of Economy and Competitiveness through the project ENE2016-80638-R, and in part by the University of Málaga (Campus de Excelencia Internacional Andalucía Tech).

I. INTRODUCTION

The relevance of co-optimization of energy and services pervades the restructuring of the power sector. It underpinned the integration of transmission with energy in the early days in the United States (US) and later led to the further integration of ancillary services. Restructuring in the European Union (EU) does not co-optimize energy and services. The power systems of the Member States (MS) are separated into energy cleared by Power Exchanges (PX) and transmission and reserves run by Transmission System Operators (TSOs). These are coordinated to some extent but in no way co-optimized. It is only when moving from the MS to the internal EU wide electricity market level that the need for some co-optimization of energy and transmission is recognized. This is achieved by a new agent, the "European Market Coupling Operator"; it deals with transmission but not with reserve.

The growing reliance on wind and solar generation in the energy transition and the need for coordinating energy and reserve suggests revisiting the role of co-optimization. Here we present a method for assessing the impact of the lack of co-optimization of energy and reserve and illustrate it on a system inspired by the Spanish market. This can be interpreted as a transposition to reserve of a question that was extensively debated in zonal vs. nodal transmission systems. The issue can be stated simply: in engineering terms co-optimization reduces costs; in economic terms an algorithm trades away arbitrages between energy and reserves. The case for co-optimization therefore appears overwhelming; but there may exist institutional obstacles. This is the case when energy and ancillary markets obey different rules in different geographical areas, as in EU, and are not subject to a federal (EU wide) authority. The objectives addressed in this paper are twofold: *(i)* to build a model for the electricity market that reflects the separation of energy and reserve, including relevant idiosyncratic features of European markets (FiP and reserve constraints), *(ii)* to define a method to compare the model with separation of energy and reserve to the model with co-optimization in terms of price formation, and allow the interpretation of price distortions in terms of arbitrage. This is used to identify and to explain the price distortions in the model with separation of price and reserve in comparison with co-optimization.

We consider a market composed of energy and reserve, but not transmission, as in most of the literature on reserve. The analysis is restricted to a single jurisdiction because, among other reasons, there is today no pan-EU integration of reserves. The market is stylized from the Spanish design and simulated under two different regimes: Co-Optimization Model (COM) for energy and reserve, and separation of energy and reserves using an EQuilibrium Model (EQM). COM is the counterfactual, as it is unambiguously defined for a given representation of the physical system. In contrast, there exists many possible approaches for the coordination of separate energy and reserve. The model proposed here (EQM) simply splits the joint market clearing of energy and reserve of co-optimization into two separate markets respectively operated by the PX and the TSO as in the EU. In EQM the separate market clearing conditions cannot be derived from an optimization problem. Needless to say neither COM nor EQM are intended to provide a representative and meaningful model of the Spanish market.

Alternative coordination methods can resort to a mix of (administered or market driven) price and quantity regulations. This touches the domain of incomplete markets [1]. Adopting a mix of priced and non priced goods and services would dramatically complicate the formulation and introduce the possibility of a multiplicity of sets of solutions (quasi variational models) that we want to avoid in this first paper. As discussed later, COM and EQM both describe complete markets with the same set of goods and prices, and can be compared in those terms on a one to one basis. This comes from formulating them in complementarity form, with complementarity relations and variables in one to one correspondence.

Quantitative analysis of market designs are often conducted in terms of global welfare and its allocation among the different economic agents. We also discuss prices as these are signals to investors. It would be good news to find similar prices of energy and reserves in COM and EQM, as this would mean an absence of significant market distortion. The reality is that one can expect the results to depend on the structure of the generation system, which will evolve along the path to the energy transition. The pricing mechanisms in COM and EQM can also be investigated independently of numerical results, as done here. Co-optimization embeds an opportunity cost pricing mechanism, plants are charged differently depending on their implications on reserve. This is sometimes replaced by a policy of equal access or even priority of access for renewable in the absence of co-optimization. These are true market distortions.

A. Literature Survey

While the integration of energy and ancillary services is a well-established and thoroughly researched subject in the US, there is little corresponding analysis in Europe with essentially no explicit treatment of the impact of the separation of the PX and TSO. As is generally the case, the emphasis is put on cross border integration, in an organization that separates day ahead as a true spot market cleared by the PX and real time, seen as a correcting balancing mechanism different from energy. The computational integration of energy and reserve in a model is obviously always possible but it implicitly assumes an integration between PXs and TSOs that does not correspond to existing institutions.

As an example of this kind of study, the integration of balancing among different Nordic systems and Germany is addressed in [2]. The authors focus on the interaction between balancing and the clearing of the energy market, but keep energy and reserve markets separated. They conclude with the gain of integrating balancing, given a certain organization of energy. Other studies discuss design options for cross border balancing, as in [3] where again the emphasis is put on balancing as a product separated from energy.

The commitment and use of reserve in Europe is explored in [4], which quantifies its integration using a unit commitment model of the European market with representation of cross border trade. Energy is cleared first, followed by the procurement of reserve. This is done using a deterministic unit commitment with three different methods for coordinating cross border reserve procurement. Reserve is then activated in real time for different scenarios. The authors note the problem of allocating cross border capacity, how much to reserve and

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its dependence on the amount allocated to cross border energy. The study in [4] is extended in [5] considering that energy and reserve can be simultaneously co-optimized or cleared sequentially. The US Independent System Operators (ISOs) typify joint clearing while the EU remains with sequential clearing. The paper [5] also contains a nice survey of the literature that confirms that the main stream of the European literature remains with sequential clearing of energy and reserve clearing, [6] being an exception.

In [6] a stylized zonal model is used to study different organizations of the market. These again differ by the sequencing of reserve procurement and the clearing of the energy (reserve before energy, or conversely, or joint clearing) and the degree of coordination across zones in the management of reserve. Three organization that are effectively found in different European countries. They conclude to the advantage of joint clearing.

The restructuring of the electric market was driven by the implementation of a marginal cost pricing principle based on the merit order rule associated to fossil generation plants. Marginal cost pricing is the standard paradigm of market efficiency (the "neoclassical" model of microeconomics) with the consequence that inefficiencies can be studied as deviations with respect to this paradigm. We briefly survey the question in the computational based literature with the view of putting our methodology and its application to reserve in perspective. We refer to the standard paradigm of marginal cost pricing in static states of the world. Intermittent sources and their dynamics complicate both the merit order and the pricing that it implies, e.g. [7] and [8] for two different issues.

A significant part of the literature on market imperfections in electricity concentrates on market power. The Nash Cournot and Stackelberg paradigms are those most often transposed to computational models. This paper does not deal with market power and the interested reader can consult [9] for a thorough treatment of the subject. Non-convexities originating in operating constraints of power plants are an other source of deviations from marginal cost pricing. The subject generated an extensive literature: [10] is an early paper on the subject; [11] and [12] are illustrations of recent developments. These notions are effectively applied in both the US, e.g. the uplift and its minimization in convex hull pricing, and EU systems with the paradoxically rejected bids and the difficulties that they raise, [13]–[15]. More recently the emergence of ramping constraints introduced distortions of the standard merit order rule and created the need for an other type of uplift [8]. Here again we do not deal with these questions and refer the reader to the cited literature.

Our proposal concentrates on situations where marginal cost pricing does not apply because of regulation. These are thus true deviations with respect to the neoclassical model and its claims of market efficiency. Transmission, with its long lasting debate between zonal and nodal systems is the most extensively discussed example of that situation but in an unusual sense. The EU zonal system effectively applies marginal cost pricing but to a representation of the grid that can be a very poor approximation of the real one. We do not deal with transmission and refer the reader to [16] and the extensive literature therein.

A number of papers address the case where there is no attempt to apply marginal cost pricing. Average cost pricing of electricity is an early example of that policy. It prevailed in the "cost plus" regulated generation systems. The problem was formulated as an equilibrium model and solved by a sequences of optimization problems that were perturbations of marginal cost pricing models in [17]. A more recent example of this situation is the analysis of administered prices determined by some public authority presented in [18] and [19]. The problem is treated through complementarity models and applied to the Chinese gas market in [20] and electricity cooperation in the Gulf countries in [21]. Our proposal is directly inspired by that philosophy, but technicalities are different and the application is about reserve in a restructured electricity market.

One can note at the outset of this brief survey of the literature on reserve that some authors have already raised the issue of lack of integration between energy and services in the EU. Specifically [22] and [23] explicitly wonder why the EU does not co-optimize ancillary services and energy. Also [24] presents a computational model of this integration.

The literature on reserve is vast and growing in importance, in particular due to the penetration of renewable energy. It is mainly developed in an optimization context, which when applied to cost minimization implies marginal cost pricing. Models differ by the degree of details in the representation of the reserve, by the assessment of reserve requirements and by the sequencing of reserve commitment and operations in an uncertain environment. A simultaneous security-constrained market-clearing procedure for energy and reserves is presented in [25]. It is based on marginal cost pricing and provides a single price for all kinds of reserves at each node; that price is the nodal marginal cost of security. A method based on stochastic linear programming is presented in [26] with the focus on generator remuneration.

Two-stage stochastic programming is also used in [27] to determine the required reserve levels and also its cost in a system with high wind power penetration, also taking network constraints into account. A similar problem is solved in [28] using adaptive robust optimization with the objective of reducing the computational cost. Authors in [29] present a comprehensive model that simultaneously considers different space and time scales, several markets and relevant technical constraints; the model co-optimizes energy and reserves taking into account conventional and distributed energy resources. The problem of revenue sufficiency in the wholesale electricity market with increasing shares of renewable energies is studied in [30]. The comparison of the merit order rule emerging from sequential market-clearing of forward and balancing markets, and the market-clearing based on stochastic co-optimization is treated in [31].

Scarcity pricing is a particular example of the simultaneous marginal cost pricing of energy and reserve. It was initially presented in [32] and first implemented in the Electric Reliability Council of Texas (ERCOT). The method is now gaining traction in other ISOs, [33].

Models of electricity markets and market policies using complementarity problems appear in numerous publications.

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Examples are [34] for Nash-Cournot models, and [18] for a focus on policies. An in-depth analysis of energy markets using complementary models, including formulation issues and solution techniques, can be found in [9].

The European and US models embed renewable incentives. The effects of direct and indirect subsidies in environmental policies have also been extensively studied in the literature. We only refer to two illustrations of this type of work: [35] compares the contribution of capacity payments to feed-in-tariffs for the integration of renewable generation; [36] compares capacity and energy subsidies for promoting renewable generation, concluding that capacity subsidies are more effective in the long run while energy subsidies are more effective in the short run.

B. Contribution and Paper Organization

There is little discussion on the integration of energy and reserve in the EU. The reason is simple but fundamental: the dual allocation of transmission and ancillary services to the TSOs and of energy to the PXs prevent co-optimization. As discussed in the review of the literature, the EU market design does not recognize the notion of a real time energy market but only of a balancing mechanism. The review of the literature mentions an exception to that general trend with a recent paper [5] on the integration, of energy and reserve at the European level; but the study is conducted with an Unit Commitment (transposing the US approach), something which is institutionally impossible in the current EU market design. Given this background the main contributions of this work are:

1) To build an equilibrium model for the market with firms, Power Exchange and the TSO that reflects the separation of energy and reserve. Because of this latter requirement, this model cannot be an optimization problem. For the sake of realism it should also include some idiosyncratic features of European markets such as the FiP incentives and some particular constraints on reserves and real time settlement.
2) A method is proposed to compare two market models, one with separation of energy and reserves (EQM) and the other with co-optimization (COM), in terms of prices and price formation. The method relies on the comparison of the complementarity conditions of both models. It enables an interpretation of the price formation in terms of arbitrag, and the quantification of price distortion.

As most of the literature dealing with reserves in a wind generation environment, we rely on two stage stochastic models for the firms' operation. Reserves are committed in Day-Ahead (DA) and employed in Real Time (RT) to accommodate wind deviations with respect to DA forecast.

The paper is organized as follows. The general features of the power system models are described in Section II. COM is presented in mathematical form in Section III, and a brief discussion of the separation of energy and reserve in Section IV. EQM is presented in Section V. Computational issues related to solving the proposed models are discussed in Section VI. The method to compare two market models is described and applied to EQM and COM in Section VII to identify market distortions in EQM. Conclusions in Section VIII close the paper.

II. Models: General Features and Assumptions

This section presents the elements of the model. Variables are in lower case and parameters in upper case. The market consists of one generating firm[1] owning fossil and/or renewable plants; a TSO operates the reserve market and the consumers are represented by a demand curve. Because reserves respond to uncertainties, COM and EQM are formulated as two-stage models where the second stage describes an uncertain wind realization in RT. Agents are price takers. A Feed in Premium FiP to renewable generation is considered[2].

A. General Features Common to COM and EQM

The market is organized as an hourly two settlement system, Day-Ahead (DA) and Real Time (RT). The simulation time is 1 hour, making values in MW and MWh equivalent. Intra-day market is not considered[3].

Generation plants are either conventional or wind turbines. They participate in the DA market where conventional generators receive the DA price, and wind generators the sum of this price and a FiP. Generators also participate in the RT balancing mechanism. As discussed later (Subsection II-C), the pay-off from balancing operations is stochastic. The generation company is risk averse and assesses this random pay-off through an E-CVaR [37]. Both types of plants require balancing services but only conventional plants supply reserve for balancing. Cash flows are determined as follows:

- A wind turbine that under-delivers pays the upward balancing price, the DA energy price and the FiP on the deviation. The wind turbine thus does not receive the FiP for energy committed in DA but not generated in RT.
- A wind turbine that over-delivers receives the DA price, but not the FiP for its generation surplus. It pays the deviation at the downward balancing price.
- A conventional generator that ramps up receives the upward balancing price.
- A conventional generator that ramps down receives the downward balancing price but has to give up the DA energy price.

Consumers are aggregated in a single DA linear demand function, $\rho = \Gamma_0 - \Phi_0$ (Total demand in DA). They are not exposed to RT balancing risk but the TSO charges them the cost of the reserve capacity required for balancing.

B. Wind in the Market Design

Wind is modeled on two time scales respectively yearly and daily for variability and uncertainty. The year is decomposed in several representative wind days. Higher wind speed

[1] The single firm assumption raises several questions that are briefly discussed in the conclusion, with a full treatment left for further research.

[2] This point is introduced in this paper to illustrate the impact of some policy measures on the result.

[3] Modelling intra-day is beyond the scope of this paper, especially in view of the two quite different modes of intra-day, continuous trading and sequence of intra-day auctions, in place in Europe.

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implies higher wind generation, lower electricity prices and lower utilization of conventional power plants. This in turns determines revenue for both conventional and wind plants in the energy market and hence the demand for reserve services. A daily forecast of wind speed assigns the day in the set of representative wind days. This forecast is affected by an error that increases with the number of hours separating the forecast and generation times. Forecast errors must be compensated in RT, which has an impact on the required reserve capacity.

The market design adds a FiP for scheduled wind generation in DA, see Subsection II-A for more details. As FiP policies are continuously changing[4], a sensitivity analysis studies the effect of FiP from 0 to a high value.

C. TSO and Reserve Requirement

Reserves deal with uncertain events occurring at different time scales [40]; they are conventionally categorized as primary, secondary and tertiary reserves[5]. We focus the study on "flexibility reserves" or "operating reserves" [41], that provide ramping capability to compensate for the variability and forecast errors between commitment time and RT.

Ramping requirement depends on the difference of scheduled generation between consecutive hours [42] and hence cannot be directly represented in a single period model. Ramping constraints are considered through equations (3) and (4) here. The influence of other periods in the ramping capability is endogenous in multi-period models, but here it is considered through parameter $\Lambda \in [0,1]$. This parameter quantifies the amount of ramping capability used in scheduled generation (DA) in the periods connected to the current period. For instance, the capacity already committed (not available for flexibility) for the upward reserve is $\overline{R}_g{\cdot}\Lambda{\cdot}x_g$. That means the ramping capability (for flexibility) is minimum for $\Lambda = 1$, and maximum for $\Lambda = 0$. Parameter Λ is used for a sensitivity analysis to consider the different configurations that could happen in the multi-period case. This allows to concentrate the study on the effect of ramping availability, what would be more complex to observe in the multi-period case.

The TSO deals with deviations by procuring reserve resources (capacity committed) in DA and operating these resources (energy used) in RT in the balancing[6] mechanism to minimize ramp up and ramp down energy costs. This mechanism also gives upward and downward RT balancing prices. We adapt existing rules[7] to fit the information structure embedded in the model. The amount of required reserve is determined by the TSO and committed in DA on the basis of wind forecast and plant schedules.

[4]For instance, the 2014 Spanish Energy Reform [38], [39], still in force in 2021, changed the wind incentives from production to installed capacity, with the consequence that almost one third of the facilities lost their regulated remuneration. Operators try to recover at least part of their operating costs in the market, thus zero prices for wind generation vanished from the Spanish market since 2014.

[5]In EU terminology these correspond to: frequency containment reserves (FCR), frequency restoration reserves (FRR), and replacement reserves (RR). In order to simplify the discussion, we do not consider conventional reserves for dealing with contingencies.

[6]Recall that we simplify the presentation by not treating transmission.

[7]The Spanish Grid Code, in particular the Operation Procedure 7.2 [43], is considered here.

For illustrative and comparison purposes we assume that the TSO evaluates reserve needs using a criterion based on balancing reserve factors (MW of reserve/MW scheduled), M_x for conventional generators and M_y for wind generators. Balancing reserve factors have been used (with other name) in other studies to model reserve requirement, as for instance in [44]. The values of these parameters are calibrated on historical observations in the Spanish market. They correspond to the sum of the primary and secondary reserves. The real TSO procedure for determining reserve requirement for frequency control in the Spanish system is described in [45]. The requirement for the primary reserve is fixed annually and determined at the beginning of each year; the calculation is based on ENTSO-E recommendations. The formula used by the TSO is publicly available, but only part of the data necessary for its calibration is available. The other part of the reserve requirement is communicated by the TSO in DA or the intra-day markets. The model used by the TSO to determine this requirement is not published.

D. Physical Constraints

COM and EQM have identical physical constraints, which are presented in this section. The models differ in their economic structure as discussed in Sections III and IV.

Variables are non-negative except ζ and ν that are free.

1) Conventional generators' technical constraints:

$$x_g + ru_g \leq \overline{X}_g \quad \perp \ \overline{\phi}_g \geq 0, \quad \forall g \in G \tag{1}$$

$$x_g - rd_g \geq \underline{X}_g \quad \perp \ \underline{\phi}_g \geq 0, \quad \forall g \in G \tag{2}$$

$$ru_g \leq \overline{R}_g{\cdot}(\overline{X}_g - \Lambda \cdot x_g) \quad \perp \ \overline{\delta}_g \geq 0, \quad \forall g \in G \tag{3}$$

$$rd_g \leq \underline{R}_g{\cdot}(\overline{X}_g - \Lambda \cdot x_g) \quad \perp \ \underline{\delta}_g \geq 0, \quad \forall g \in G \tag{4}$$

where: (1)/(2) sets the upper/lower bounds on the capacity of dispatchable generators; (3)/(4) describes the upper bound for upward/downward reserves from dispatchable generators[8].

2) Wind turbines' constraints: Wind turbine w schedules a generation of y_w in DA and corrects that value in real time (in each scenario k) using z_w^k in case of wind shortage and v_w^k in case of wind surplus. Generation in real time is $y_w - z_w^k$ in case of wind shortage and $y_w + v_w^k$ in case of wind surplus. The upper bound for the scheduled wind y_w is the rated power $\overline{Y}^{max}$ (5), this condition is usually imposed by the System Operator. In the case wind generation in RT, $\overline{Y}_w^k$ (scen. k), is lower than scheduled generation, the wind turbine has to buy the energy $z_w^k \geq y_w - \overline{Y}_w^k$, (6) to fulfill its commitment. And it cannot trade more than its rated capacity by purchasing energy z_w^k from the conventional generators, (7). The minimum value of real production of wind turbine w, $y_w - z_w^k$, must be greater than its must-run capacity, (8). In case of wind surplus, $\overline{Y}_w^k > y_w$, the amount of additional wind generation that could be traded is $v_w^k \leq \overline{Y}_w^k - y_w$, we need to add the term z_w^k to accommodate the case of wind shortage $\overline{Y}_w^k < y_w$, resulting equation (9), to

[8]For a description of Λ see second paragraph in Subsection II-C.

avoid $v_w^k < 0$. Also the wind surplus is bounded by the available wind in RT, (10).

$$y_w - \overline{Y}_w^{\max} \le 0 \qquad \perp \overline{\psi}_w \ge 0,\ \forall w \tag{5}$$

$$y_w - z_w^k - \overline{Y}_w^k \le 0 \qquad \perp \underline{\iota}_w^k \ge 0,\ \forall k, w \tag{6}$$

$$z_w^k - \overline{Y}_w^{max} + \overline{Y}_w^k \le 0 \qquad \perp \overline{\iota}_w^k \ge 0,\ \forall k, w \tag{7}$$

$$\underline{Y}_w^k - y_w + z_w^k \le 0 \qquad \perp \underline{\psi}_w^k \ge 0,\ \forall k, w \tag{8}$$

$$y_w + v_w^k - z_w^k - \overline{Y}_w^k \le 0 \qquad \perp \tau_w^k \ge 0,\ \forall k, w \tag{9}$$

$$v_w^k - \overline{Y}_w^k \le 0 \qquad \perp \xi_w^k \ge 0,\ \forall k, w \tag{10}$$

Wind turbines lose the FiP for the scheduled energy that is not generated; this is quantified in the objective function by $z_w^k \cdot FiP$ in (20). That means that wind turbines are not selling and buying at the same time when $FiP > 0$ because that leads to a profit loss. In case $FiP = 0$ they gain nothing by selling and buying at the same time. Also the artificial increase in those variables is bounded by constraints (5)-(10), and it is in fact quite limited, such that it cannot be used to force the commitment and/or the deployment of more reserves. Thus the values of scheduled generation and profits are not affected. The value of v_w^k and z_w^k could be affected, but their net value $v_w^k - z_w^k$ is not altered.

3) Energy balance in DA. Scheduled generation must equal demand; ν sets the price for the consumer:

$$\sum_{g\in G} x_g + \sum_{w\in WT} y_w = d \quad \perp \nu, \tag{11}$$

4) TSO constraints for reserve capacity:

$$\frac{1}{\underline{A}} \sum_{g\in G} ru_g \ge M_x \cdot \sum_{g\in G} x_g + M_y \cdot \sum_{w\in WT} y_w \ \perp \overline{\kappa} \ge 0 \tag{12}$$

$$\frac{1}{\overline{A}} \sum_{g\in G} ru_g \le M_x \cdot \sum_{g\in G} x_g + M_y \cdot \sum_{w\in WT} y_w \ \perp \overline{\gamma} \ge 0 \tag{13}$$

$$\sum_{g\in G} rd_g \ge B \cdot \sum_{g\in G} ru_g \qquad \perp \underline{\kappa} \ge 0 \tag{14}$$

$$\sum_{g\in G} rd_g \le \sum_{g\in G} ru_g \qquad \perp \underline{\gamma} \ge 0 \tag{15}$$

These constraints reflect the TSO reserve rules in the Spanish grid code, [45]. Equation (12)/(13) sets lower/upper bound on upward reserve requirement, and (14)/(15) sets lower/upper bound on downward reserve requirement. Those rules in [45] require that committed upward reserve must be in the range between 90% and 110% of the estimated value for upward reserve, and that committed downward reserve must be in the range between 50% and 100% of the committed upward reserve. Instead of using numerical values for the ranges we use the parameters $\overline{A}$, $\underline{A}$ and B in the equations. The rule to estimate the required upward reserve is not explicitly stated in the TSO documents, but it must be a function of the scheduled generation x_g and y_w. That rule is here modeled using reserve balancing factors, M_x and M_y[9] that relate the scheduled generation, x_g, y_w, to the estimated upward reserve. The values for M_x and M_y can be estimated using a fit on historical data.

5) Clearing of power in real time from committed reserves:

$$\sum_{g\in G} s_g^k = \sum_{w\in WT} z_w^k \qquad \perp \overline{\pi}^k,\ \ \forall k \tag{16}$$

$$\sum_{g\in G} u_g^k = \sum_{w\in WT} v_w^k \qquad \perp \underline{\pi}^k,\ \ \forall k \tag{17}$$

$$s_g^k - ru_g \le 0 \qquad \perp \overline{\eta}_g^k \ge 0,\ \ \forall k, g \tag{18}$$

$$u_g^k - rd_g \le 0, \qquad \perp \underline{\eta}_g^k \ge 0,\ \ \forall k, g \tag{19}$$

where: (16)/(17) is the balance equation for the power deployed from the upward/downward reserves: the power from reserves used by renewable generators in RT must be equal to the power for reserve provided from dispatchable generators in RT; and (18)/(19) sets the upper bound for the deployed power in RT from upward/downward reserves capacity committed in DA.

The cost of corrective actions in scenario k, recourse function (20), is the sum of three components: i) generation cost of energy from dispatchable generators $\sum_{g\in G} C_g \cdot (s_g^k - u_g^k)$, ii) Wind turbines loss of the FiP on the undelivered energy $\sum_{w\in WT} z_w^k \cdot FiP$, and iii) Cost of using reserves, that is a penalty for wind turbines and an income for dispatchable generators. Because of our assumption of a single firm, the value of this third term is zero (see discussion on this point in the conclusion). It is thus not present in (20).

$$q^k = \sum_{g\in G} C_g \cdot (s_g^k - u_g^k) + \sum_{w\in WT} z_w^k \cdot FiP, \qquad \forall k \in \Omega \tag{20}$$

III. Co-Optimization (COM): Complete Model

COM comprises the constraints (1)-(23), listed in Subsection II-D and below; the objective function (welfare) is (21):

$$\min_{\substack{d,x,y,z,rd,\\ ru,s,u,v,\zeta,q^-}} \Bigg\{ - \int_0^d (\Gamma_0 - \Phi_0 \cdot \omega) \cdot d\omega - \sum_{w\in WT} y_w \cdot FiP + \sum_{g\in G} C_g \cdot x_g + \text{E-CVaR}_\Theta(q^k, q^{k,-}) \Bigg\} \tag{21}$$

Where ω is the integration variable. The different terms of (21) are respectively the willingness to pay for electricity, the income from Feed in Premium to wind, the cost of conventional generation, and the E-CVaR described in Subsection III-A.

The decision variables in COM are the primal: $d, x_g, y_w, z_w^k, rd_g, ru_g, v_w^k, \zeta, q^{k,-}, s_g^k, u_g^k$, and the dual: $\overline{\phi}_g, \underline{\phi}_g, \overline{\psi}_w, \underline{\psi}_w^k, \overline{\delta}_g, \underline{\delta}_g, \overline{\iota}_w^k, \underline{\iota}_w^k, \tau_w^k, \xi_w^k, \sigma^k, \overline{\pi}^k, \underline{\pi}^k, \overline{\eta}_g^k, \underline{\eta}_g^k, \nu, \overline{\kappa}, \overline{\gamma}, \underline{\kappa}, \underline{\gamma}$.

A. Risk Aversion in EQM and COM: Definition of E-CVaR

The generator is exposed to the risk affecting its balancing transactions. This is formulated in an E-CVaR, (22), (23), which is presented here for COM but also applies to EQM. It is a convex combination of the expected value and the $CVaR_\Theta$ [37] of the recourse function q^k, (20). Parameter $\Psi \in [0, 1]$ characterizes the level of risk aversion of the generator, being

[9]See paragraph 4th in Subsection II-C for a description of these factors.

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risk neutral for $\Psi = 0$. Parameter $\Theta \in [0, 1]$ is the confidence level that determines the scenarios retained by the generator for CVaR, (22) is an auxiliary constraint for CVaR calculation.

$$\zeta + q^{k,-} \geq q^k, \quad (\sigma^k), \quad \forall k \in \Omega, \tag{22}$$

$$\text{E-CVaR}_\Theta(q^k, q^{k,-}) = (1-\Psi)\cdot\sum_k \text{Pr}^k q^k + \\ + \Psi\cdot\left(\zeta + \frac{1}{1-\Theta}\sum_{k\in\Omega}\text{Pr}^k\cdot q^{k,-}\right) \tag{23}$$

The dual variable σ^k scales the original probability into a risk neutral, or distorted, probability $Pr^k \cdot \sigma^k$ that represents the effect of risk aversion modeled in the CVaR. We have $\Psi = \sum_{k\in\Omega} Pr^k \cdot \sigma^k$ from Karush-Kuhn-Tucker (KKT) condition for ζ, and $\sigma^k \leq \frac{\Psi}{1-\Theta}$ from KKT condition for $q^{k,-}$.

B. Discussion: COM

By definition COM covers the sum of all costs and benefits incurred by the generator, the TSO and the consumer. A brief explanation justifies valuing the balancing cost inside the E-CVaR of the risk averse generator. Note first that the consumer is not exposed to risk and hence its willingness to pay needs not be included in the E-CVaR. The TSO operates balancing but does not incur its cost which is entirely passed to the generator: one should thus verify that including this cost in the E-CVaR is compatible with the risk free operation of the TSO. This is done by noting that the KKT conditions of the generator's problem computed with respect to the firm's and TSO's variables produce the optimality conditions of both the profit maximization of the firm and the clearing of the balancing market by the TSO. COM thus effectively expresses a simultaneous clearing of energy and reserve in DA, by a risk neutral TSO. Dual variables $\overline{\kappa}, \overline{\gamma}, \underline{\kappa}, \underline{\gamma}$ are then true values (prices) of the reserve in the mechanism that relates them to energy. The total value of the capacity committed by the TSO for reserve is then given by

$$\sum_{g\in G}(ru_g\cdot(\overline{\kappa}-\overline{\gamma}) + rd_g\cdot(\underline{\kappa}-\underline{\gamma})) \tag{24}$$

It is paid/charged by the TSO to the producer and recovered on the consumers as an access charge.

COM implements co-optimization both between energy and services but also between DA and RT through a stochastic program. This permits arbitraging between DA and RT operations but also between using resources for energy and reserve. Markets usually do not rely on pure stochastic optimization. But the stochastic model can be seen as a simplified representation of virtual trading implemented in the US markets to enable arbitrage between DA and RT markets. An elaboration of this statement goes beyond the scope of this paper[10]. It is however possible to illustrate a consequence of this arbitrage property: using the KKT conditions of COM one can show that the value of the DA capacity is equal to the risk adjusted revenue collected by the generator on the balancing market. The DA payment or charge to the generator for reserve capacity is then compensated by the incomes from balancing. This DA amount remains to be transferred to the consumer as a cost for a public good (the quality of the supply).

[10]In a nutshell, one can say that COM is a risk optimization problem that represents a financially complete and perfectly competitive market (or in other words statisfies the neoclassical paradigm of the well functioning market, see discussion in the conclusion). This is thus an ideal representation that offers an optimistic view of what can be achieved by the combination of co-optimization of energy and reserve, and virtual trading in US markets.

IV. EQM (Separation of Energy and Reserve): Preliminaries

EQM is meant to represent a market where there is no co-optimization of energy and reserve. This corresponds to the EU organization where PXs clear energy and TSOs take care (by administrative or market means) of services, in this case reserves. The objective is to uncover a (computable) market interpretation of this dual process. Relations (12)-(15) play the key role in that endeavour. Their right-hand side is expressed in energy, x_g and y_w, while the left-hand sides is in terms of ramping capacities, ru_g and rd_g. The relations define joint (partially) interchangeable products. This is true irrespectively of the particular analytical form of the relation adopted in the paper. The TSO is in charge of procuring the reserve according to (12)-(15), taking the energy decisions of the generator but without any direct possibility of influencing them. The only possible influence is through the price of reserve.

We adapt a standard reasoning of supply and demand: the TSO needs to find prices for reserve such that the generator offers the reserve that its generation of energy requires. In contrast with COM, the energy market that determines this demand for reserve is of no concern to the TSO. This idea is implemented in (12)-(15) by considering $\overline{\kappa}$, $\underline{\kappa}$, $\overline{\gamma}$, $\underline{\gamma}$ as reserve capacity prices (incentives or penalties) for ramping up and ramping down to which generation reacts by both a supply (resulting from the optimization of operations) and demand (induced by energy production) of ru and rg. The TSO also clears the balancing market using these ramping capacities and redistributes the margin made by the plants to the generator. The generator receives the DA payment $\sum_{g\in G}(ru_g\cdot(\overline{\kappa}-\overline{\gamma})+rd_g\cdot(\underline{\kappa}-\underline{\gamma}))$ and the risk adjusted value of the RT payments $\sum_{g\in G} C_g\cdot(s_g^k - u_g^k)$ that it adds to the profit made on energy operation. The optimization of the overall profits of the generator in response to the reserve capacity prices $\overline{\kappa}$, $\underline{\kappa}$, $\overline{\gamma}$, $\underline{\gamma}$, determines the supply and demand function for reserve. The TSO pays $\sum_{g\in G}(ru_g\cdot(\overline{\kappa}-\overline{\gamma}) + rd_g\cdot(\underline{\kappa}-\underline{\gamma}))$ to the generator and recovers this amount from the consumer. The TSO and the consumers are thus risk free.

The difference between EQM and COM thus lies in the role and interpretation of $\overline{\kappa}, \overline{\gamma}, \underline{\kappa}, \underline{\gamma}$. They are parts of a general auction for energy and services in COM and the core of a particular auction for network services in EQM.

V. EQM: Complete Model

EQM can initially be structured in three blocks of relations:

1) A first block consists of the KKT conditions of the firm's profit maximization problem (1)-(11), (20), (22), (23), (25); it includes the DA revenue of the firm from reserve capacity ru_g and rd_g for given $\overline{\kappa}$, $\underline{\kappa}$ and $\overline{\gamma}$, $\underline{\gamma}$.

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2) The second block is made of the KKT conditions of the TSO's minimization of $\sum_{g\in G} C_g\cdot(s_g^k - u_g^k)$ subject to (16)-(19). With primal variables: s_g^k, u_g^k; and dual: $\overline{\eta}_g^k$, $\underline{\eta}_g^k$.
3) The third block contains the global constraints (12)-(15).

We briefly discuss each of these blocks and show that they can be reduced to two blocks.

A. Optimization Problem of the generator in EQM

The generator optimizes the E-CVaR of energy and reserve capacity in DA, and energy in RT subject to constraints (1)-(11), (20), (22), (23). This arbitrages between DA and RT. The objective function is stated as (25):

$$\min_{\substack{d,q,q^-,rd,ru\\ s,x,y,z,u,v,\zeta}} \Bigg\{ -(\Gamma_0 - \Phi_0\cdot d)\cdot d - \sum_{w\in WT} y_w\cdot FiP - \sum_{g\in G}(ru_g\cdot(\overline{\kappa}-\overline{\gamma}) + rd_g\cdot(\underline{\kappa}-\underline{\gamma})) + \sum_{g\in G} x_g\cdot C_g + \text{E-CVaR}_\Theta(q^k, q^{k,-}) \Bigg\} \tag{25}$$

The decision variables in EQM are the primal variables: $d, x_g, y_w, z_w^k, rd_g, ru_g, v_w^k, \zeta, q^{k,-}, s_g^k, u_g^k$; and the dual: $\overline{\phi}_g, \underline{\phi}_g, \overline{\psi}_w, \underline{\psi}_w^k, \overline{\delta}_g, \underline{\delta}_g, \overline{\iota}_w^k, \underline{\iota}_w^k, \tau_w^k, \xi_w^k, \sigma^k, \overline{\pi}^k, \underline{\pi}^k, \overline{\eta}_g^k, \underline{\eta}_g^k, \nu$.

The different terms in the firm's objective function are interpreted as follows:

- $(\Gamma_0 - \Phi_0\cdot d)\cdot d = \rho\cdot d$ is the profit from energy sales in DA. The firm is price taker and hence has no control on ρ. This implies $\frac{\partial}{\partial d}(\rho\cdot d) = \rho$ at agent's level ($\frac{\partial\rho}{\partial d} = 0$).
- $\sum_{w\in WT} y_w\cdot FiP$ is the profit accruing to wind turbines because of FiP on their scheduled generation.
- The profit accruing to dispatchable generators for supplying upward ru_g and downward rd_g reserve capacity, is equal to $\sum_{g\in G}(ru_g\cdot(\overline{\kappa}-\overline{\gamma}) + rd_g\cdot(\underline{\kappa}-\underline{\gamma}))$.
- $\sum_{g\in G} x_g\cdot C_g$ is the variable cost of dispatchable plants.
- E-CVaR$_\Theta$ (23) is the risk adjusted value of the revenues and costs incurred in balancing.

B. The TSO Problem and its Integration in the Firm Block

Using the same reasoning as in COM, see Subsection III-B, one sees that the equilibrium conditions of the TSO are obtained by inserting its equations in the generator problem. One indeed observes that $\sum_{g\in G} C_g\cdot(s_g^k - u_g^k)$, which is an exogenous part of q^k, (20), in the objective function of the firm, (25), (the s_g^k and u_g^k are TSO's variables), can produce the KKT conditions of the clearing of the balancing market if one extends the KKT derivatives of the generator's objective function to those with respect to TSO's variables. In other words, adding the TSO constraints, (16)-(19), to the firm's optimization problem, (1)-(11), (20), (22), (23), (25), and taking the KKT conditions of that problem with respect to both the firm and TSO variables (primal and dual) produces the TSO's and the firm's equilibrium conditions, without changing the behavior of the generator to which this cost will in any case be charged.

EQM can thus be seen as composed of only two blocks: One corresponds to the KKT conditions of the generator's problem that also include the TSO constraints. The second block expresses the balance of ramping services offered and demanded by the generator.

VI. Models Implementation and Solving

COM is a quadratic programming problem with linear constraints. As expected it is solved with no difficulty by standard off the shelf solvers such as CPLEX, CONOPT and MINOS under GAMS [46]. We crosschecked the model with two alternative formulations using the Karush-Kuhn-Tucker (KKT) conditions of COM, and solving it with PATH, also under GAMS. One formulation with handwritten KKT, and the other provided by the automatic generation of the dual by the Extended Mathematical Programming on GAMS. We got the same results with the three formulations.

EQM is a linear complementarity problem that modifies COM's KKT conditions. This problem misses the perfect arbitraging between reserve and energy of COM. An interesting question is whether this is reflected in solving capabilities. And indeed a direct application of PATH (under GAMS) on EQM failed in some cases while PATH never failed on the corresponding COM problems. We accordingly used an iterative approach that interacts PATH with a linear sub-problem calculating the CVaR to update the risk neutral probabilities at each iteration. The iterative calculation of the CVaR follows the algorithm described in [47]. This worked in all studied cases, but it is at this stage a heuristic.

COM is a convex problem that, except for degeneracy, always has an unique solution. Scalability is not an issue as the (very high) capabilities of LP commercial codes set the limits. The situation is different for EQM that is not amenable to optimization. One needs to verify the existence of an equilibrium and the possibility of multiple solutions; one must also examine scalability. One can prove the existence of a solution by a direct application of section 2 of [48]: the two functions f (starting point in the homotopy) and g (end point in the homotopy) in the paper are the set of complementarity conditions of COM and EQM respectively with and without the "additional terms" given in Table I. The theorem is stated for fixed point problems but, as stated in [48], is valid for both, fixed point and zero of equations.

A final question is whether multiplicity of solution can occur in practice, and this is indeed the case. The analysis and interpretation of multiplicity of solutions goes beyond this paper.

VII. Model formulation and comparative analysis of COM and EQM

It is standard to resort to complementarity models to study restructured power markets, [9]. We follow suit and compare the complementarity conditions of both EQM and COM to characterize the differences between the two models and their interpretation in terms of pricing rules. These conditions can be partitioned into four blocks respectively describing:

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TABLE I
COMPARISON OF KKT CONDITIONS FOR COM AND EQM

	Var.	Common terms in COM and EQM	Additional terms for COM	
1	x_g	$C_g + \overline{\phi}_g - \underline{\phi}_g + \overline{\delta}_g \cdot \overline{R}_g \cdot \Lambda + \underline{\delta}_g \cdot \underline{R}_g \cdot \Lambda - \nu$	$+\overline{\kappa} \cdot \underline{A} \cdot M_x - \overline{\gamma} \cdot \overline{A} \cdot M_x$	≥ 0
2	y_w	$-FiP + \overline{\psi}_w + \sum_{k\in\Omega} Pr^k \cdot (-\underline{\psi}_w^k + \underline{\iota}_w^k + \tau_w^k) - \nu$	$+\overline{\kappa} \cdot \underline{A} \cdot M_y - \overline{\gamma} \cdot \overline{A} \cdot M_y$	≥ 0
3	rd_g	$-\underline{\kappa} + \underline{\phi}_g + \underline{\delta}_g + \underline{\gamma} - \sum_{k\in\Omega} Pr^k \cdot \underline{\eta}_g^k$		≥ 0
4	ru_g	$-\overline{\kappa} + \overline{\phi}_g + \overline{\delta}_g + \overline{\gamma} - \sum_{k\in\Omega} Pr^k \cdot \overline{\eta}_g^k$	$+\underline{\kappa} \cdot B - \underline{\gamma}$	≥ 0
5	s_g^k	$C_g \cdot (1 - \Psi + \sigma^k) - \overline{\pi}^k + \overline{\eta}_g^k$		≥ 0
6	u_g^k	$-C_g \cdot (1 - \Psi + \sigma^k) - \underline{\pi}^k + \underline{\eta}_g^k$		≥ 0
7	z_w^k	$FiP \cdot (1 - \Psi + \sigma^k) + \underline{\psi}_w^k + \overline{\iota}_w^k - \underline{\iota}_w^k - \tau_w^k + \overline{\pi}^k$		≥ 0
8	v_w^k	$\xi_w^k + \tau_w^k + \underline{\pi}^k$		≥ 0
9	d	$-(\Gamma_0 - \Phi_0 \cdot d) + \nu$		≥ 0
10	ζ	$\Psi - \sum_{k\in\Omega} Pr^k \cdot \sigma^k$		$= 0$
11	$q^{k,-}$	$\frac{\Psi}{1-\Theta} - \sigma^k$		≥ 0

1) The economics of dispatchable, x_g, and wind y_w generation.
2) The procurement of upward ru_g and downward rd_g reserve capacity.
3) The RT valuation of reserve capacity by dispatchable generation.
4) The RT valuation of balancing energy used by wind turbines from reserve.

The comparison between EQM and COM suggests the following remarks relative to EQM:

1) Remuneration collected on the energy market does not include the opportunity cost of the implied demand for reserve.
2) Even though RT is not seen as a market in the EU, it is considered as such in EQM in order to preserve the comparability of the models. In EQM this permits the arbitrage between DA and RT that is achieved in COM by construction (through virtual trading).

To facilitate the identification and interpretation of probabilities adjusted by a risk premium, constraints involving k (scenario) indexed variables can be multiplied by the scenario probability Pr^k. This modifies neither the equation nor the values of primal variables but scales the dual variables by the scenario probability.

Table I lists the complementarity conditions that are common to both models and the add ups in COM that result from the co-optimization of energy and reserve. The add ups express the different pricing of reserves. Recall from preceding discussion that the variables referring to scenario k are already expressed in the risk neutral probabilities defined by the probability measure $(1 - \Psi + \sigma^k)$. Conversely, deterministic values such as C_g must be multiplied by $(1 - \Psi + \sigma^k)$ to be converted to the risk neutral world. Differences of complementarity conditions in EQM and COM are discussed in what follows.

A. The Economics of Dispatchable x_g and Wind y_w Generation

The margin made by the scheduled generation x_g from conventional generator g (row 1 in Table I) is what is left from the electricity price ν after paying the fuel cost C_g, the remuneration of capacity services ($\overline{\phi}_g$ - $\underline{\phi}_g$), and the flexibility capabilities ($\overline{\delta}_g \cdot \overline{R}_g \cdot \Lambda + \underline{\delta}_g \cdot \underline{R}_g \cdot \Lambda$). Similarly the remuneration of the scheduled generation y_w of wind turbine w, row 2 in Table I, boils down to what is left from the electricity price ν and FiP after paying for capacity $(\overline{\psi}_w - \sum_{k\in\Omega} Pr^k \cdot \underline{\psi}_w^k)$ and flexibility capabilities, $(\sum_{k\in\Omega} Pr^k \cdot (\underline{\iota}_w^k + \tau_w^k))$.

Rows 1 and 2 in Table I, show that COM corrects the remunerations obtained in EQM by also charging the opportunity cost of the marginal reserve required by the operations of conventional and renewable plants: $(\overline{\kappa} \cdot \underline{A} \cdot M_x - \overline{\gamma} \cdot \overline{A} \cdot M_x)$ for x_g and $(\overline{\kappa} \cdot \underline{A} \cdot M_y - \overline{\gamma} \cdot \overline{A} \cdot M_y)$ for y_w. This cost applies directly to the plants in COM but does not appear in EQM, which does not differentiate the access charge of technologies by their requirement for reserve. This equal access to reserve policy is a market distortion, [49].

B. Upward ru_g and Downward rd_g Reserve Capacities

The terms $(-\underline{\kappa}+\underline{\gamma})$ for rd_g in row 3 of Table I, and $(-\overline{\kappa}+\overline{\gamma})$ for ru_g in row 4 of Table I value the reserve capacities in DA. They have the same analytical expressions in COM and EQM but do not carry the same economic interpretation and do not take the same values. Prices $\overline{\kappa}$, $\underline{\kappa}$, $\overline{\gamma}$ and $\underline{\gamma}$ clear the reserve market in EQM without embedding the opportunity cost of energy development on reserve; in contrast they effectively embed that cost in COM. Interestingly $(\underline{\phi}_g + \underline{\delta}_g)$ for rd_g and $(\overline{\phi}_g + \overline{\delta}_g)$ for ru_g value the opportunity cost of reserve on generation in the same way in both models. In other words, the opportunity cost of energy on reserve is missing in EQM while the opportunity cost of reserve on energy is present in both models .

The additional term $(\underline{\kappa} \cdot B - \underline{\gamma})$ in COM for ru_g prices the constraint relating ru_g and rd_g quantities in DA. The constraint also exists in EQM but it is not priced because EQM overlooks the opportunity cost of energy on reserve that is considered in COM. The terms $(\sum_{k\in\Omega} Pr^k \cdot \underline{\eta}_g^k)$ for rd_g, and $(\sum_{k\in\Omega} Pr^k \cdot \overline{\eta}_g^k)$ for ru_g remunerate the plants for their participation to the balancing market. They are present in both models and stand for the risk adjusted value of reserve in RT. In both models we have assumed the same role of RT and hence the same arbitrage of reserve between DA and RT. This assumption is a bit optimistic for COM, as it would correspond to the combination of virtual trading of energy between DA and RT and the co-optimization of reserve and energy in the US system, which we model through a stochastic program using a coherent risk function. It is certainly an optimistic view of a COM EU where balancing is not seen as an energy market, with the consequence that there is no mechanism to implement this arbitrage between DA and RT. One could argue that intra-day will be a good substitute if it closes near RT. But intra-day is today conceived, at EU level, as an "order book" system and hence would be quite unsuited to achieve this arbitrage.

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VIII. Conclusion

The proposed methodology can be used to identify and quantify price distortions in electricity markets with separation of energy and reserve (compared to co-optimization). And to interpret those distortions in terms of arbitrage. Using a sensitivity analysis on the main parameters in the models it can identified situations where flexibility, in terms of reserve, has / does not have value. This is relevant in market conditions where energy prices are insufficient for remunerating plants that are necessary for the functioning of the system. In those situations reserve provided by the generators could constitute a valuable source of revenue. Using the proposed methodology one can quantify whether co-optimization of energy and reserve could modify the value of the reserves. Three different situations can be identified: *i)* Reserve is available as an abundant byproduct of generation and hence has "no value"; *ii)* Reserve is excessive, it hits the upper value of the TSO's interval, and has a negative value. Increasing the upper bound to a level where it is no longer binding will extend the range where it has "no value"; and *iii)* The case of most interest is when reserve is effectively a scarce resource; it hits the lower value of the TSO's interval and should be remunerated.

We find that there is no pricing power in reserve when generation capacity is ample compared to energy demand, and fossil plants are unable to cover their fixed and variable operating costs on the energy market. In this case market design, whether COM or EQM, is irrelevant. But a high demand for reserve due to wind combined with a reduction of economically redundant generation capacity can restore plant profitability. This reveals the value of reserve and the relevance of co-optimization. The revenue accruing from reserve can significantly contribute to the viability of flexible plants in COM and EQM, but because of a more efficiently use of reserves COM decreases the price to the consumer and increases demand and welfare. These advantages are fragile regarding to an increase of reserve capability. Increasing the reserve potential of existing plant could quickly decrease their value both in COM and EQM.

As supplementary content on a file in arXiv [50] (arXiv:2107.09636v1 [q-fin.GN]) we provide a case study inspired by the Spanish electricity market with a sensitivity analysis on wind condition μ, Feed in Premium FiP, generators' risk aversion Ψ, and reserve requirement M_y.

The whole discussion is conducted on the basis of two standard but important assumptions: i) There is a single firm and, ii) all agents (that is the firm, the consumer and the system operator) are price takers. We briefly discuss what could happen by removing these assumptions.

One can first note that the single firm assumption is implicit in all dispatch models. Whatever the number of firms on the market, the dispatch does not keep track of equipment ownership. The same would be true of a two-stage stochastic dispatch. The same is thus also true of the KKT conditions of that model. The model is thus agnostic to the single firm assumption up to here. Things change if we consider several firms with their own risk aversions. This happens for instance with different stochastic discount rates whether exogenous or derived from a risk function as in our models. The equilibrium problem is then no longer equivalent to an optimization problem. Calling upon a result from [51], one can construct an equivalent single firm problem if one assumes that the risk market is "complete" that is that there exists a financial market capable of trading all the risks occurring in the RT market. The completeness assumption is a standard neoclassical assumption (it is part of the perfect market when risk is present) and this corresponds to what we did. This applies to both COM and EQM.

But this does not close the subject. Besides the separation of energy and services which is common to all European systems, power markets generally differ by some idiosyncratic features that add complications and market imperfections. Specifically, Section II explains that the DA ahead price intervenes in the settlement of balancing in RT. This introduces an externality between DA and RT that hinders the application of [51] and hence invalidates the assumption of a single representative firm. This type of interaction between DA and RT appears in different forms in European markets and has important consequences. It makes it impossible to construct a co-optimization market, as the COM is replaced by a fixed-point problem. As a result of the need to explicitly represent different firms, EQM becomes much more complex, this will be treated (where it only in terms of formulation) in another paper.

We suppose price taking agents, which means that we neglect market power. This is the standard neoclassical assumption in well functioning markets. It is a standard assumption in many, but by far not all, energy models. Market power is indeed a favorite subject of energy modelers but most often working on the sole energy market. We are not aware of a literature discussing the economic issues raised by market power in an energy/reserve context; [52], which is a relatively recent paper coming from a group with extensive knowledge of the literature does not cite many papers on market design in energy and reserve; this suggests that those dealing with market power in energy/reserve market are still less numerous.

A key question is to first define how market power is exercised as again, and in contrast with the neoclassical paradigm, the notion of market power is ambiguous. A natural exercise of market power in COM is to place this model as the second stage of a bidding model where generators bid the characteristics of their plants where they exercise market power. The problem would then be an MPEC with the second stage being an optimization problem, something that is now commonly found in the literature (e.g. bidding in upper stage subject to second stage representing a market clearing). There would not be much change in EQM, except for the fact that the lower stage model would be a more complex equilibrium problem, especially if one also wants to consider multiple firm with DA price intervening in RT. Illustrations of these MPEC problems are [53] and [54], in one of them the second stage is a COM type (optimization) problem. Except for noting that market power will further degrade efficiency, commenting how our numerical results would be affected would be pure conjectures.

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Yves Smeers received the M.S. and Ph.D. degrees from Carnegie Mellon University, Pittsburgh, PA, in 1971 and 1972, respectively.

He is currently a Professor Emeritus and a member of the Center for Operations Research and Econometrics, at Université Catholique de Louvain, Louvain-la-Neuve, Belgium.

Sebastian Martin (S'08, M'14) was born in Málaga, Spain, in 1980. He graduated from University of Granada, Spain, as Civil Engineer in 2003. He received an Industrial Engineer and a PhD degrees from University of Málaga, Spain, in 2007 and 2014 respectively.

He is currently a teaching assistant at the University of Málaga. His research interests include: optimization theory, methodologies for teaching, and economics of electric energy systems.

José Antonio Aguado (M'01) received the Ingeniero Eléctrico and Ph.D. degrees from the University of Málaga, Málaga, Spain, in 1997 and 2001, respectively.

Currently, he is full Professor and Head of the Department of Electrical Engineering at the University of Málaga. His research interests include operation and planning of smart grids and numerical optimization techniques.