

Firm Heterogeneity, Capacity Utilization, and the Business Cycle*

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In a stochastic dynamic general equilibrium framework, we introduce the concept of capacity utilization (as opposed to capital utilization). We consider an economy where monopolistic firms use a putty-clay technology and decide on their productive capacity and technology under uncertainty. An idiosyncratic uncertainty about the exact position of the demand curve faced by each firm explains why some productive capacities remain idle and why individual capacity utilization rates differ across firms. A variable capacity utilization allows for a good description of some

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of the main stylized facts of the business cycle, propagates and magnifies aggregate technological shocks and generates endogenous persistence (i.e., the output growth rate displays positive serial correlation). *Journal of Economic Literature* Classification Numbers: E22, E32. © 1999 Academic Press

1. INTRODUCTION

Understanding the mechanisms generating economic fluctuations in response to technological shocks is a central theme in the real business cycle literature. In this perspective, several recent contributions have aimed at incorporating stronger propagation mechanisms into stylized business cycle models. The presence of idle resources that can be readily engaged in production activities has so reappeared as a potentially important issue in the understanding of business cycle fluctuations. In particular, several authors have introduced the possibility of variable capital utilization into standard real business cycle models. The present article investigates this issue while proposing explicit microfoundations of the existence of idle equipment in some firms.

The underutilization of some productive equipments in actual economies is rather well illustrated by the existing microeconomic evidence. For instance, Bresnahan and Ramey (1993) report that in the American automobile industry, the most usual way of adjusting production is to shut the plant down for a week. Similarly, the business surveys achieved in most Western European countries show consistently through time that an important proportion of firms run excess capacities.

Modeling the very idea of excess capacity or idleness of some production units is not a trivial undertaking. Most existing works in the business cycle literature do not tackle it explicitly but model instead the idea that physical capital is a production factor that can be used with a variable intensity. To this end, a capital utilization variable is introduced in the production function of a representative firm.¹ According to the reason why this utilization rate fluctuates through the cycle, those models can be divided into two subsets. A first set of works (a.o. Greenwood et al., 1988; Burnside and Eichenbaum, 1996; Licandro and Puch, 1995) assumes that capital utilization affects capital depreciation: a more intensive use of equipment implies a faster depreciation. This depreciation cost leads to an optimal utilization rate that is below one and varies with the state of the aggregate technology.

¹ In the case of a Cobb–Douglas technology, no concept of production *capacity* can actually be defined: indeed, the maximum level of output that a firm can produce with a given level of capital is *a priori* infinite. Even though this remark is quite incidental, it questions the relevance of a direct comparison between the capital utilization rate of such a model and the *capacity* utilization measure proposed by the Federal Reserve Bank.

A second strand of the literature (Kydland and Prescott, 1988; Bils and Cho, 1994) supposes that the utilization rate of capital increases when employees work a larger number of hours and/or more intensively. The optimal utilization level then reflects the trade-off between the output gain resulting from a longer or more intensive utilization and the utility loss induced by the marginal work effort corresponding to this increased utilization.

Both types of modeling have been rather successful. In both cases, the variability in capital utilization substantially magnifies and propagates the impact of technological shocks. These results sound promising but one may remain unsatisfied by the fact that in those models, the capital utilization variability relies on a mechanism without obvious microfoundations. A link between utilization and the number of worked hours—as assumed by the second set of works quoted above—seems quite plausible; but one may doubt that the cyclical behavior of utilization first follows from considerations of disutility of work and overtime payments. The depreciation in use assumption also appears disputable: capacity utilization fluctuations are unlikely to be guided by the concern of not depreciating equipments too much. As Burnside and Eichenbaum (1996) recognize, the “depreciation in use” assumption should thus only be viewed as a very crude approximation of the multiple means that firms use to regulate their production.

The choice of a highly stylized description of the underutilization phenomenon is a first natural step and possibly a useful shortcut. However, the first attempt to rationalize explicitly equipment idleness in a real business cycle framework (Cooley et al., 1995) has given rise to somewhat different conclusions about the importance of the underutilization phenomenon in propagating technological shocks. Cooley et al. (1995) assume that production takes place in different plants, which face idiosyncratic technological shocks. Operating a plant implies a fixed cost. Depending on the idiosyncratic shock (s)he observes, the plant manager must thus decide whether to operate the plant (and to bear the fixed cost) or to leave the plant idle. The macroeconomic equilibrium is characterized by a variable proportion of operating plants.² The authors conclude that, except for variations in factors shares, the cyclical properties of the model are close to the ones of a standard real business cycle economy. Moreover, introducing a variable capacity utilization does not increase the internal propagation mechanisms of the model: a more variable aggregate technological shock is even necessary for reproducing output variability.

² The Cooley et al. (1995) modeling of capacity idleness is theoretically close to the Hansen (1985) and Rogerson (1988) models of labor supply. In both cases, the existence of a fixed cost (to operate a plant or to go to work) introduces a nonconvexity in the agent's optimization program, which disappears at the aggregate level. See Hornstein and Prescott (1993) for a theoretical characterization of this type of environment.

In the face of these rather contrasted results, it seems to us quite worthwhile to investigate the cyclical implications of the capacity utilization phenomenon in a framework with explicit microfoundations. To this end, we construct a stochastic general equilibrium model that uses the description of the capacity utilization phenomenon proposed by Sneessens (1987) and Fagnart et al. (1997). These works rely on three basic intuitions: (i) the very concept of productive capacity suggests that in the short run, the possibilities of substitution between production factors are limited;³ (ii) given this technological rigidity, the presence of uncertainty at the time of capacity choices can explain why the installed productive equipments of the economy are usually underutilized in equilibrium; (iii) idiosyncratic uncertainty can explain why, as reported in business surveys, some firms produce at full capacity while others face demand shortages and run excess capacities. In purely descriptive terms, our model is thus qualitatively different from the one of Cooley et al. (1995) where idleness takes the form of a proportion of totally idle plants. In our case, there is a nondegenerated distribution of utilization rates across firms, which seems to have a higher descriptive realism.

The real business cycle exercise we perform allows us to show that the proportion of firms with excess capacities plays an important role in magnifying and propagating aggregate (technological) shocks, contrary to the conclusion of Cooley et al. (1995). Moreover, our model fits the main stylized facts of the business cycle and displays a positive serial correlation of output growth rate.

The rest of the paper is organized as follows: Section 2 presents and describes the model; Section 3 discusses the calibration; Section 4 presents the results and concludes.

2. THE MODEL

The general structure of our artificial economy with imperfect competition is rather standard. There are two productive sectors: a competitive sector produces a final good; a monopolistic sector produces intermediate goods. These intermediate goods are the only inputs necessary for producing the final good. Capital and labor are used in the production of the intermediate goods.

The main specificity of our modeling concerns the production technology of firms in the monopolistic sector. In order to obtain a simple concept of productive capacity, we assume that intermediate firms use a putty-clay

³ E.g., this may be the case simply because implementing those substitutions requires time.

technology: capital and labor are substitute *ex ante* (i.e., before investing) but complement *ex post* (i.e., once equipments are installed).⁴ This means that each firm makes a capacity choice when investing. Under monopolistic competition, this capacity choice problem is well defined: the optimal capacity depends indeed crucially on the firm's expectations regarding its future demand constraints.

When investing, a monopolistic firm is supposed to face uncertainty about both the aggregate state of the technology and the exact position of its demand curve. The latter source of uncertainty is purely idiosyncratic. It explains the presence of heterogeneity between firms in equilibrium, a positive proportion of firms running excess capacities. In order to keep our modeling tractable, we assume this idiosyncratic shock is not serially correlated. Therefore, its realization influences exclusively contemporary production and employment decisions, but not the investment decisions. This implies that monopolistic firms will always be *ex ante* identical and will differ *ex post* only. When investing, all the firms indeed have the same information regarding the future and thus make the same capacity choice. Once the idiosyncratic uncertainty is resolved, firms make different production and employment decisions.

In order to avoid a price aggregation problem in the monopolistic sector, we will furthermore assume that monopolistic prices are announced when firms know the state of the aggregate technology but have not observed yet the exact position of their demand curve. When setting prices, all the firms thus face the same uncertainty about their market share. This assumption allows us to extend the *ex ante* symmetry between monopolistic firms to price decisions (the *ex post* heterogeneity thus only bears on the decisions of production, employment and utilization). One should notice that this assumption on the price behavior of firms does not have any important effect on the way the economy responds to an aggregate technological shock. Prices are announced when the aggregate technology is known: they are thus perfectly flexible in this respect.

The model is closed by introducing an infinitely living consumer-worker.

2.1. Final Good Sector

There is a single final good produced by a representative firm. It is sold on a competitive market and can be used for consumption or investment. The production technology is represented by a constant returns-to-scale CES function defined over a continuum of intermediate goods (inputs). $\mathcal{Y}_t$ units of final good are produced with $y_t(j)$ units of each input j , $j \in [0, 1]$,

⁴ Our putty-clay assumption is very roughly modeled. We only assume that capital and blueprint employment are predetermined variables. See Section 2.2.1.

according to

$$\mathcal{Y}_t = \left[\int_0^1 y_t(j)^{(\theta-1)/\theta} v_t(j)^{1/\theta} dj \right]^{\theta/(\theta-1)}. \quad (1)$$

Each $v_t(j) (\geq 0)$ represents a productivity shock drawn from an i.i.d. distribution function $F(v)$, with unit mean and density function $f(v)$. F is defined over the support $[\underline{v}, \bar{v}]$ with $0 < \underline{v} < 1 < \bar{v}$.

The firm purchases inputs in the intermediate goods sector. The total supply of input j is limited to a quantity $q_t(j)$ (see Section 2.2.1 for details), equal to the productive capacity of the corresponding input supplying firm.

When maximizing its profit function, the final firm faces no uncertainty: it knows the input prices $\{p_t(j)\}_j$, the supply constraints $\{q_t(j)\}_j$ and the realizations of productivity shocks $\{v_t(j)\}_j$. The final good is assumed to be the numéraire of the economy and its price is normalized to 1. Since input firms are always *ex ante* identical [i.e., $p_t(j) = p_t$ and $q_t(j) = q_t$, $\forall j$], we can simplify our notations and write the optimization program of the final firm as follows:

$$\max_{\{y_t(j)\}_j} \mathcal{Y}_t - \int_0^1 p_t y_t(j) dj$$

subject to the supply constraints $y_t(j) \leq q_t$, $\forall j \in [0, 1]$, $\mathcal{Y}_t$ being defined by (1).

The presence of supply constraints in the above optimization program stems from the fact that input prices are set in advance. The demand for some inputs may thus be constrained by the productive capacity of the suppliers. In other words, and to use the terminology of Clower (1967), the notional demands for some inputs can be rationed; the final firm must then derive effective demands by taking the binding supply constraints into account.

The solution to the above problem can be described by the following system of equations: $\forall j \in [0, 1]$,

$$y_t(j) = \begin{cases} p_t^{-\theta} \mathcal{Y}_t v_t(j) & \text{if } v_t(j) \leq \tilde{v}_t \\ q_t & \text{if } v_t(j) \geq \tilde{v}_t, \end{cases} \quad (2)$$

$$\text{with } \tilde{v}_t = \frac{q_t}{p_t^{-\theta} \mathcal{Y}_t}, \quad (3)$$

$\mathcal{Y}_t$ being defined by (1). $\tilde{v}_t$ represents the critical value of the productivity shock $v_t(j)$ for which the unconstrained demand [i.e., $p_t^{-\theta} \mathcal{Y}_t v_t(j)$] equals the supply constraint q_t . The symmetry in capacities and prices implies that the critical value $\tilde{v}_t$ is the same for all j 's. We thus simplify our notations by omitting index j .

The law of large numbers implies that, for a proportion $F(\tilde{v}_t)$ of inputs, the realized value of the productivity shock is below $\tilde{v}_t$: an input for which $v_t \leq \tilde{v}_t$ is purchased in quantity $p_t^{-\theta} \mathcal{Y}_t v_t$. The other inputs [a proportion $1-F(\tilde{v}_t)$ of all inputs] are supply constrained and purchased in quantity q_t . From (1) and (2), the final output supply $\mathcal{Y}_t$ can thus be written as

$$\mathcal{Y}_t = \left\{ [p_t^{-\theta} \mathcal{Y}_t]^{(\theta-1)/\theta} \int_{\underline{v}}^{\tilde{v}_t} v dF(v) + (q_t)^{(\theta-1)/\theta} \int_{\tilde{v}_t}^{\bar{v}} v^{1/\theta} dF(v) \right\}^{\theta/(\theta-1)}. \quad (4)$$

This expression nests the standard case where no supply constraint is binding: when $F(\tilde{v}_t) = 1$ (or $\tilde{v}_t \rightarrow \bar{v}$), (4) implies indeed that the optimal production level $\mathcal{Y}_t$ is indeterminate; the relative price of inputs is then necessarily equal to 1. If some capacity constraints are binding, (4) determines $\mathcal{Y}_t$ as a function of the supply constraints q_t and the relative input price p_t (p_t then departing from 1 as we explain in Section 2.4).

2.2. Intermediate Inputs Sector

We now describe the monopolistic sector of the economy.

2.2.1. Individual Firm's Technology

We model the idea that input firms use a “putty-clay” technology in a very stylized way. Investments achieved during period $t-1$ become productive at the beginning of period t . When investing in $t-1$, a firm designs its future productive equipment by choosing simultaneously a quantity of capital goods k_t and a blueprint employment level b_t according to the following Cobb–Douglas technology:

$$q_t = A_t k_t^\alpha b_t^{1-\alpha} \quad \text{with } 0 < \alpha < 1.$$

Blueprint employment b_t represents the number of available work-stations in the firm. When all these work-stations are operated full time, the firm is at full capacity q_t . For the sequel, it is more convenient to model the investment decision as the choice of both k_t and the blueprint capital–labor ratio $x_t (= k_t/b_t)$. We thus rewrite the previous equation as follows:

$$q_t = A_t x_t^{\alpha-1} k_t, \quad (5)$$

where $A_t x_t^{\alpha-1}$ is the technical productivity of capital.

The firm has naturally the possibility of hiring less units of labor than b_t . Below b_t , the marginal productivity of labor on the installed equipments is constant and equal to $A_t x_t^\alpha$. If the firm hires $l_t \leq b_t$ units of labor, it thus produces $A_t x_t^\alpha l_t$ units of output.

During period t , the aggregate productivity parameter, A_t , is supposed to follow an autoregressive stochastic process:

$$\log A_t = \rho \log A_{t-1} + \epsilon_t \quad \text{where } 0 < \rho < 1. \quad (6)$$

ϵ_t is an i.i.d. normal innovation with zero mean and standard deviation σ_ϵ .

Finally, we assume that each firm has to bear a fixed cost of production equal to Φ units of final good. This assumption allows us to reduce the value of pure profits in the calibration of the model.

2.2.2. Decision Sequence

From a monopolistic firm's point of view, two exogenous variables exhibit a random behavior: the aggregate productivity parameter A_t and the idiosyncratic productivity shock v_t , which is perceived as a demand shock. The investment decisions k_t and x_t are made without knowing either A_t or v_t with certainty. After observing A_t , but under demand uncertainty, the firm takes its price decision p_t . Once all the uncertainty is resolved, the firm takes its production and employment decisions. We present now these decisions in a backward way.

Output and employment. The optimal production plan of a firm facing a demand shock v is summarized by (2). The firm adjusts instantaneously its labor demand l_t to the needs of the production plan, i.e.,

$$l_t = \frac{y_t}{A_t x_t^\alpha}. \quad (7)$$

Note that such a production/employment plan is necessarily the optimal one. Since monopolistic prices are set above the (constant) marginal cost of production (see below), it is profitable to produce as much as possible: if the firm has idle capacities, it is willing to serve all the demand that is forthcoming at the announced price. Hence, it produces as described in (2).

Price decision. After observing A_t , input firms can compute the equilibrium values of all the aggregate variables at date t (the remaining uncertainty is indeed purely idiosyncratic). The price decision of any firm at date t is thus the solution to the following static problem:

$$\max_{p_t} E_v \left[\left(p_t - \frac{w_t}{A_t x_t^\alpha} \right) y_t \right],$$

where y_t is given by (2); k_t and x_t are given. The expectation operator E_v represents firm's expectations when the only remaining uncertainty bears on v_t (i.e., conditionally on A_t and all past information). Hence, the wage rate w_t and the final output y_t can be perfectly foreseen. Only y_t remains uncertain. Its conditional expectation takes the following form [see (2)]:

$$E_v(y_t) = p_t^{-\theta} y_t \int_{\underline{v}}^{\tilde{v}_t} v dF(v) + q_t \int_{\tilde{v}_t}^{\bar{v}} dF(v), \quad (8)$$

where $\tilde{v}_t$ is given by (3).

The first order condition with respect to p_t can be written as

$$p_t = \left(1 - \frac{1}{\theta \pi(\tilde{v}_t)}\right)^{-1} \frac{w_t}{A_t x_t^\alpha}, \quad (9)$$

where $\pi(\tilde{v}_t)$ represents the elasticity of $E_v(y_t)$ with respect to $p_t^{-\theta} y_t$ (which is the expected demand in case of idle capacities), i.e.,

$$\pi(\tilde{v}_t) = \frac{p_t^{-\theta} y_t}{E_v(y_t)} \int_{\underline{v}}^{\tilde{v}_t} v \, dF(v). \quad (10)$$

From (8), the ratio $p_t^{-\theta} y_t / E_v(y_t)$ can be expressed as a function of $\tilde{v}_t$: the right-hand side of (10) thus only depends on $\tilde{v}_t$. Alternatively, the elasticity $\pi(\tilde{v}_t)$ can be interpreted as the (weighted) probability of excess capacity. The markup rate implied by the pricing rule (9) thus depends negatively on the probability of excess capacity $\pi(\tilde{v}_t)$: the smaller this probability, the greater the market power perceived by the firm.

Optimal capital and blueprint capital-labor ratio. During t , each input firm designs the production units it will use during $t + 1$, i.e., decides on the capital stock k_{t+1} and the blueprint capital-labor ratio x_{t+1} (See Section 2.2.1). The problem of the firm can be written recursively as

$$\begin{aligned} V(k_t, x_t) = & \max_{k_{t+1}, x_{t+1}} E_t[p_t y_t - w_t l_t - p_t(k_{t+1} - (1 - \delta)k_t + \Phi)] \\ & + E_t\left[\frac{1}{1 + r_{t+1}} V(k_{t+1}, x_{t+1})\right] \end{aligned}$$

subject to (8), (7) and (9), with k_t and x_t given.

The optimality condition for the capital stock is as follows:

$$\begin{aligned} 1 - E_t\left[\frac{1 - \delta}{1 + r_{t+1}}\right] \\ = E_t\left[\frac{1}{1 + r_{t+1}} \underbrace{\left(p_{t+1} - \frac{w_{t+1}}{A_{t+1} x_{t+1}^\alpha}\right) \frac{q_{t+1}}{k_{t+1}}}_{\mathcal{A}} [1 - F(\tilde{v}_{t+1})]\right]. \end{aligned} \quad (11)$$

The optimal capital stock is such that the expected marginal revenue of capital is equal to its expected user cost. The expected marginal revenue is the discounted increase in profits generated by an additional unit of capital, $\mathcal{A}$, weighted by the probability of operating this marginal unit, $[1 - F(\tilde{v}_{t+1})]$.

The marginal condition on x_{t+1}^j is as follows:

$$\begin{aligned} E_t\left[\frac{1}{1 + r_{t+1}} \underbrace{\left(p_{t+1} - \frac{w_{t+1}}{A_{t+1} x_{t+1}^\alpha}\right) \frac{q_{t+1}}{x_{t+1}}}_{\mathcal{B}} \right. \\ \left. \times \left(\frac{\alpha(\theta - 1)}{\tilde{v}_{t+1}} \int_{\underline{v}}^{\tilde{v}_{t+1}} v \, dF(v) - \int_{\tilde{v}_{t+1}}^{\bar{v}} dF(v)\right)\right] = 0. \end{aligned} \quad (12)$$

It can be understood quite intuitively. When increasing the blueprint capital-labor ratio, the firm faces the following trade-off. On the one hand, it raises labor productivity, which has a favorable effect on its competitive position in case of excess capacities; this increases the expected profits by an amount $\mathcal{B}$ times the term in $\int_{\bar{v}}^{\bar{v}_{t+1}} v dF(v)$. On the other hand, by decreasing the blueprint employment level, the firm also reduces its maximum volume of sales; this decreases the expected profits by an amount $\mathcal{B}$ times $\int_{\bar{v}_{t+1}}^{\bar{v}} dF(v)$. The optimal x_{t+1}^j is such that the discounted marginal value of the former and latter effects are equal.

2.3. Households

This section is quite standard. We suppose identical and infinitely living households whose preferences are represented by a time separable utility function U_t defined over consumption and labor:

$$U_t = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} (\log(c_s) + v(1 - l_s)) \right]$$

where E_t represents the expectation operator given the information available in t . c_s and l_s are the consumption and labor supply during $s \geq t$. β is a constant subjective discount rate. Function v is strictly increasing, strictly concave and twice differentiable. In the sequel, v_{ll} represents the elasticity of $v'(\cdot)$.

The representative household enters into period t with a predetermined level of financial asset a_t . During the period, she receives a wage income, all firms' profits (represented below by Π_t), and the remuneration of her financial assets. She chooses how much to consume, to work and to save. The budget constraint of household in t is given by

$$a_{t+1} + c_t \leq (1 + r_t)a_t + w_t l_t + \Pi_t. \quad (13)$$

Given this period t budget constraint, the household problem can be written recursively as

$$V^H(a_t) = \max_{c_t, l_t, a_{t+1}} \{ \log(c_t) + v(1 - l_t) + \beta E_t[V^H(a_{t+1})] \}$$

The optimality conditions on l_t and c_t are quite standard, i.e., respectively,

$$w_t = v'(1 - l_t)c_t, \quad (14)$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1 + r_{t+1}}{c_{t+1}} \right]. \quad (15)$$

2.4. General Equilibrium and Stationary State

Given the initial productive equipments, k_0 and x_0 , the initial state of the aggregate technology A_0 and the stochastic process (6) for A_t , one can summarize the general equilibrium of the economy during any period $t \geq 0$ by a price vector $\{p_t, w_t, r_t\}$, a quantity vector $\{\mathcal{Y}_t, c_t, l_t, k_{t+1}, x_{t+1}\}$ and a proportion $F(\tilde{v}_t)$ of intermediate firms such that: (i) p_t , k_{t+1} and x_{t+1} satisfy (9), (11) and (12), with $\tilde{v}_t$ defined by (3); (ii) the final good market clears, i.e., $\mathcal{Y}_t = c_t + k_{t+1} - (1 - \delta)k_t + \Phi$ where the final output supply $\mathcal{Y}_t$ is given by (4) and c_t satisfies (15); (iii) the asset market clears, i.e., $a_{t+1} = k_{t+1}$; (iv) the labor supply (14) matches the aggregate labor demand equal to

$$l_t = \frac{p_t^{-\theta} \mathcal{Y}_t}{A_t x_t^\alpha} \int_{\underline{v}}^{\tilde{v}_t} v \, dF(v) + \frac{k_t}{x_t} \int_{\tilde{v}_t}^{\bar{v}} df(v). \quad (16)$$

By aggregating the individual capacity utilization rates (i.e., $p_t^{-\theta} \mathcal{Y}_t v / q_t$ if $v \leq \tilde{v}_t$ and 1 otherwise), one obtains an average utilization rate equal to $\mathcal{Y}_t / q_t$. Note finally that the aggregate value added in our economy is given by $\mathcal{Y}_t - \Phi$, which is denoted by Y_t in the sequel.

It is worth emphasizing the role played by the relative price p_t in the market share of an unconstrained input. As already mentioned, p_t departs from 1 when some supply constraints are binding. The equilibrium value p_t depends on $\tilde{v}_t$, i.e., on the proportion of firms with excess capacities. Indeed, a few manipulations of equation (4) allows us to express p_t as follows:

$$p_t = \left\{ \left[\int_{\underline{v}}^{\tilde{v}_t} v \, df(v) \right] + (\tilde{v}_t)^{(\theta-1)/\theta} \left[\int_{\tilde{v}_t}^{\bar{v}} v^{1/\theta} \, df(v) \right] \right\}^{1/(\theta-1)}. \quad (17)$$

The right-hand side of this expression is increasing in $\tilde{v}_t$ and bounded above by 1, with p_t equal to 1 only when $\tilde{v}_t \rightarrow \bar{v}$.⁵ The term $p_t^{-\theta}$ appearing in the demand function of a firm with excess capacities is thus larger than 1 and represents—at given $\mathcal{Y}_t$ —the positive spillover effects a firm with excess capacity benefits from. This term plays an important role in the model behavior as we will stress later.

In the absence of aggregate uncertainty, we can define a stationary equilibrium under idiosyncratic uncertainty. The stationary interest rate is given by the optimal consumption rule, i.e., $r = 1/\beta - 1$. The optimality condition (12) on x next determines the equilibrium value of $\tilde{v}$ as a function of

⁵ Intuitively, the presence of binding supply constraints raises the marginal cost of the final good at given input prices. Since, in equilibrium, the final good price equals the marginal cost, it is higher than the input price. The relative price of an input is thus smaller than 1.

the parameters θ , α and the form of the distribution function of the idiosyncratic shocks. By using (12) and the definition of $\pi(\tilde{v})$, one can show that the production of firms with excess capacity represents a proportion $\pi(\tilde{v})$ of total output given by

$$\pi(\tilde{v}) = \frac{1}{1 + (\theta - 1)\alpha} < 1. \quad (18)$$

This proportion is decreasing in both the degree of substitutability between goods and the capital coefficient α . Obviously enough, larger possibilities of substitution between intermediate goods reduce the problem of mismatch between input demands and supplies and thus lower the proportion of firms with excess capacity in equilibrium.⁶ The role played by parameter α is a bit less intuitive. If α was close to zero (almost no capital used in production), there would not exist any capacity constraint; as prices are set above the constant marginal cost of labor, all the firms would then feel demand constrained, i.e., π would tend to 1 as (18) shows. The higher α , the stronger the capacity constraint linked to a given capital choice and thus the larger the number of firms reaching this capacity limit in equilibrium [i.e., the lower $\pi(\tilde{v})$].

Note, finally, that the variance of distribution function $F(v)$ affects the equilibrium capacity utilization rate: by increasing the dispersion in the distribution of input demands, a higher variance of v implies a lower average capacity utilization rate at any capacity level.

3. CALIBRATION

The model is solved numerically by log-linear approximation around its stationary state (King et al., 1987). The calibration relies mainly on Cooley et al. (1995). The utility of leisure $v(\cdot)$ is assumed to be linear (as in Cooley et al., 1995), and corresponds to the Hansen (1985) indivisible labor model. To determine the other parameters, we impose restrictions on our artificial economy in order to obtain a nonstochastic stationary state consistent with a list of standard growth facts. This list is displayed in Table I. It includes the facts that in US post-Korean war data, the average quarterly capital-output ratio is equal to 13.28, the share of investment in value added is 0.25, the capital share in value added is 0.4, the ratio between the average working time and the total available time is 0.31.

At given capital-output ratio, the restriction on the investment share determines the value of parameter δ . The restrictions on the capital-output

⁶ In the limit case where inputs would be perfect substitutes ($\theta \rightarrow \infty$), no firm would feel demand constrained in equilibrium, i.e., π would tend to 0 as (18) shows.

TABLE I
Steady State Ratios

monopolistic profits/ Y	investment/ Y	rk/Y	l
0	0.25	0.4	0.31
capacity utilization	k/Y	markup ratio	
82.8%	13.2	0.7	

TABLE II
Calibration

α	β	δ	θ	σ_v	Φ	v_{LL}	ρ
0.3495	0.9946	0.0188	8.7364	0.8959	0.1057	0	0.95

ratio, the labor share in value-added and the absence of pure profits in the monopolistic sector (as documented in Rotemberg and Woodford, 1995) determine the values of α , β and Φ . We assume that the distribution function $F(v)$ is log normal and choose its variance σ_v and the parameter θ in order to reproduce the average capacity utilization rate measured by the Federal Reserve Board of Governors (82.8%) and to obtain a markup level consistent with the empirical studies for the US industry (Hall, 1988; Hall, 1990, Morrison, 1990). This gives us a proportion of demand constrained firms consistent with the estimate of Burda (1990) on US data. The stationary proportion of demand constrained firms is equal to 27% and the markup ratio is 0.7.

The last point concerns the calibration of aggregate uncertainty. As in Cooley et al. (1995), ρ is set to 0.95. As the approximated model and Hodrick–Prescott filter are linear, the absolute level of the variance of the technological innovation does not matter for the relative moments, and we do not need to calibrate it for the experiments that we perform with the model.⁷

⁷ An alternative strategy would have been to set ρ and the innovation s.d. σ_ε using an auxiliary model, as in Cooley et al. (1995). More precisely, the idea is to estimate on the simulated data a *naïve* Solow residual $SR_t = \Delta \log(Y_t) - \omega \Delta \log(K_t) - (1 - \omega) \Delta \log(L_t)$, where ω is the average share of capital income in total income. ρ and σ_ε can then be set in a such way that the serial correlation and innovation of the AR(1) process estimated on the artificial SR data are equal to the ones of the AR(1) estimated on the actual data ($\rho = 0.95$ and $\sigma_\varepsilon = 0.715\%$ according to Hansen, 1985). With such a strategy, we obtain $\rho = 0.994$ and $\sigma_\varepsilon = 0.65\%$, and the results are very similar to those reported with $\rho = 0.95$ (see below in the Sec. 4.2.4).

4. RESULTS

4.1. Impulse Responses to an Aggregate Technological Shock

We start this section by analyzing the impulse responses to an aggregate technological shock (Figures 1 and 2). These IRF show that the presence of idle capacities propagates and magnifies technological shocks.

4.1.1. *Magnification*

Since the technological shock bears on the installed capital stock, it automatically increases firms productive capacity. Consequently, a rise in capacity utilization will only occur if the output increase is larger than the productivity one. This is what happens in our artificial economy. The economic mechanism can be explained as follows. Because of consumption smoothing and logarithmic preferences, real wages follow only partially the initial productivity increase. Therefore, unit labor costs fall and push real input prices downwards. The resulting increase in input demands induces a more extensive use of productive capacities in all the firms with idle equipments. Naturally, the decrease in input prices due to the reduction in unit labor costs is partially offset by the increase in markups induced by the higher capacity utilization. In response to a positive technological shock, real input prices thus exhibit a countercyclical behavior whereas capacity utilization and markups are procyclical. This procyclical behavior of markups tends to dampen the output response. But this markup increase cannot overcome the magnification effect linked to the fall in real input prices since it is only a “second-round” effect in the propagation of the shock.

4.1.2. *Persistence*

Since real wages follow only partially the productivity increase, firms invest initially in a less capital intensive technology (x decreases), i.e., make investment choices more favorable to employment. During the following period, employment increases accordingly and the one period ahead response of output is greater than the instantaneous one. The resulting hump-shape response of output will account for the positive serial correlation of output growth (see below).

In summary, the above analysis suggests that the presence of idle capacities magnifies the effects of technological shocks. We should emphasize that we have not introduced technological shocks in the most favorable way for our model: the shock increases the productivity of all the installed equipments and thus allows firms to increase their production without requiring necessarily a more *extensive* use of existing capacities. A technological shock only embodied in the newly installed equipments, as in Greenwood

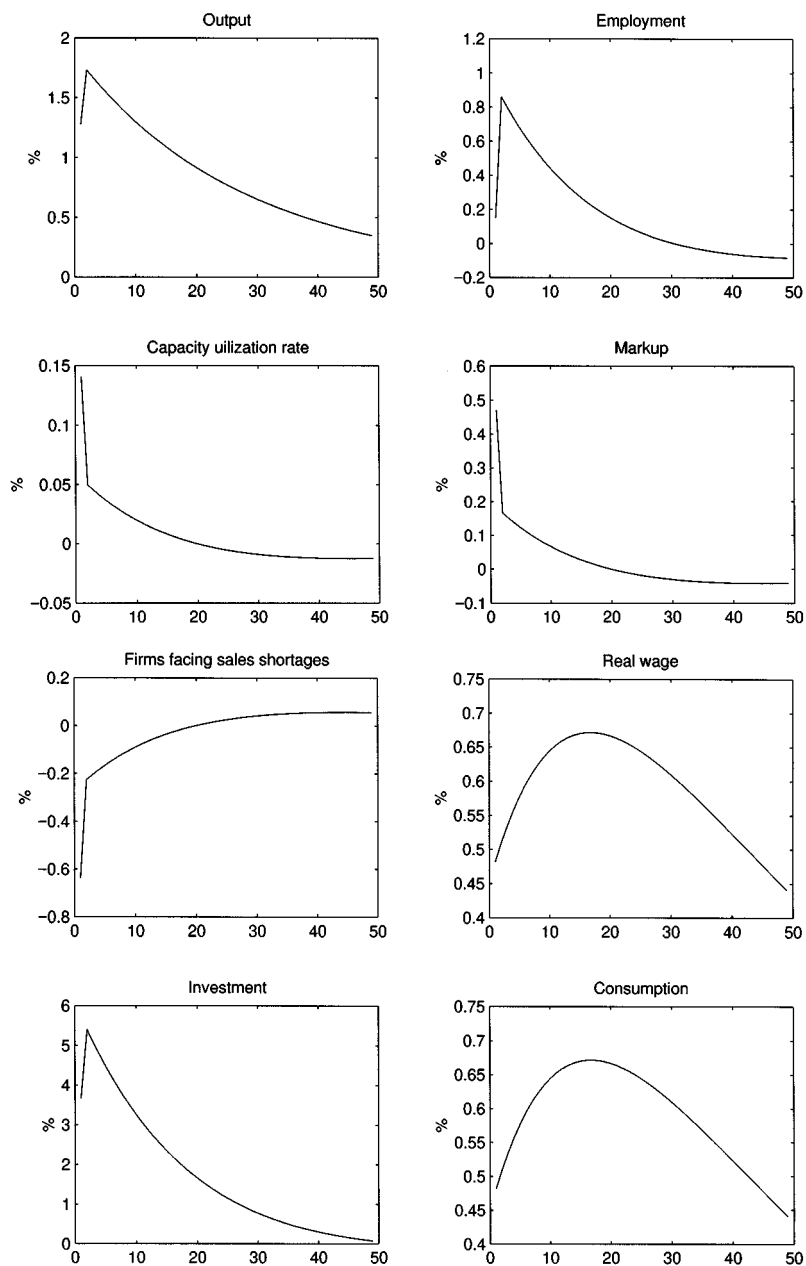


FIG. 1. Impulse response to a technological shock (i).

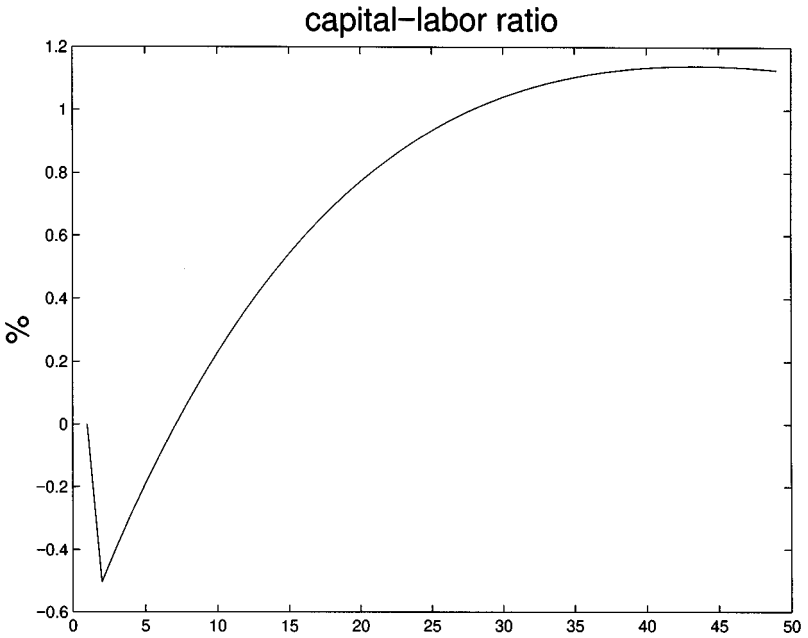


FIGURE 2

et al. (1988), would have implied more variation in the capacity utilization margin.

4.2. Simulation Results

All the simulated series have been detrended by using the Hodrick–Prescott filter. The results are obtained from 100 simulations of 150 points each.

4.2.1. *The Artificial Business Cycle*

The artificial business cycle displayed by the model is given in Table III. The “data” columns correspond to quarterly US time series from 1954:1 to 1991:3. Output is gross national product, consumption corresponds to purchases of nondurable and services, and investment (I in Table III) is fixed investment. All these variables are measured in 1982 dollars. Hours represent total weekly hours in all industries based on the current population survey.

The relative variations of investment and consumption are well reproduced. Hours are slightly more variable than productivity. As in the US business cycle, the variability of the labor share is about one-third of the

TABLE III
Cyclical Properties of Actual and Artificial Economies

Series	$sd/sd(Y)$		$cor(\cdot, Y)$	
	Data	Model	Data	Model
c	0.497	0.31	0.95	0.955
I	3.09	3.14	0.99	0.996
l	0.867	0.50	0.93	0.907
Y/l	0.509	0.487	0.95	0.902
wl/Y	0.312	0.308	-0.32	-0.82

one of output and is countercyclical (but substantially more in the model than in the data).

4.2.2. Amplification Measures

As in Burnside and Eichenbaum (1996), we measure the part of output variation that is explained by the internal propagation mechanisms of the model (denoted y_t) and the part that is directly related to exogenous movements in A_t . We thus represent the log of output as

$$\log(Y_t) = \log(A_t) + y_t.$$

Without any internal propagation mechanisms, output Y_t^* would evolve according to

$$\log(Y_t^*) = \log(A_t) + y,$$

where y is the steady state level of y_t . The measure of amplification proposed by Burnside and Eichenbaum (1996) is therefore the ratio between the standard deviation of Y and the standard deviation of Y^* (both series being detrended with an HP filter). Table IV compares the measure so obtained with the ones corresponding to the models of Burnside and Eichenbaum (1996) (BE) and Cooley et al. (1995) (CHP). The amplification measure obtained for our artificial economy is 48% and is thus as high as in the Burnside and Eichenbaum (1996) model with variable capital utilization (47%). In order to fix the ideas, let us recall that in the Burnside et al. (1993) model of labor hoarding, this measure is not higher than 1.01. In the Cooley et al. (1995) model with variable capacity utilization, the standard deviation of output is equal to 1.69 or to 1.38 in the function of the version of the model (variable or fixed number of plants in the economy), with a technological shock process given by

$$A_t = 0.95A_{t-1} + \epsilon_t$$

TABLE IV
Amplification Measure

Model	BE (94)	CHP (95) I	CHP (95) II	Our model
σ_Y/σ_{Y^*}	1.47	1.25	1.02	1.48

with $\sigma_\epsilon = 1.06$. By the mean of simulations, we computed the HP cyclical component of A , the standard deviation of which turned out to be 1.35%. Accordingly, the implicit propagation measures of the model are, respectively, 1.25 and 1.02. Our model thus exhibits stronger internal amplification mechanisms than the one of Cooley et al. (1995).

4.2.3. Persistence

Cogley and Nason (1993) have shown that many real business cycle models imply that the growth rate of output is close to a white noise. This is in sharp contrast with the actual US growth rate which displays a positive persistence. According to these authors, the weakness of the internal propagation mechanisms of standard real business cycle models accounts for this discrepancy. We have computed the implied serial correlation of output growth for our economy and present the results in Table V.

The model exhibits a positive serial correlation of order 1 in output growth, which, however, remains lower than in US data (0.4) and in Burnside and Eichenbaum’s (1996) depreciation in use model (0.4). The main source of persistence is the one period of time needed to achieve factor substitution (adjusting x). This mechanism is qualitatively comparable to the “labor hoarding” assumption in Burnside et al. (1993) and Burnside and Eichenbaum (1996). The capital–labor ratio is changed one period after the realization of the shock, which implies that output growth has the same sign during two periods as it appears clearly in the output impulse-response function.

On the basis of this analysis, it seems to us worthwhile to consider more sophisticated specifications of the factor substitution process (for example, by assuming that this process takes more than one quarter) as they could produce more persistence and contribute to a serial correlation of the output growth beyond the first order.

TABLE V
Serial Correlation of Output Growth

Order 1	Order 2	Order 3	Order 4
0.24	−0.03	−0.02	−0.02

4.2.4. *Robustness to Alternative Calibrations*

The simulation of our model relies on the calibration of two parameters that are not standard in the literature: the idiosyncratic uncertainty (σ_v) and the elasticity of substitution between intermediate goods (θ). These two parameters are key determinants of the stationary capacity utilization level. Figure 3 displays the results of a sensitivity analysis of the magnification and persistence effects with respect to σ_v and θ .

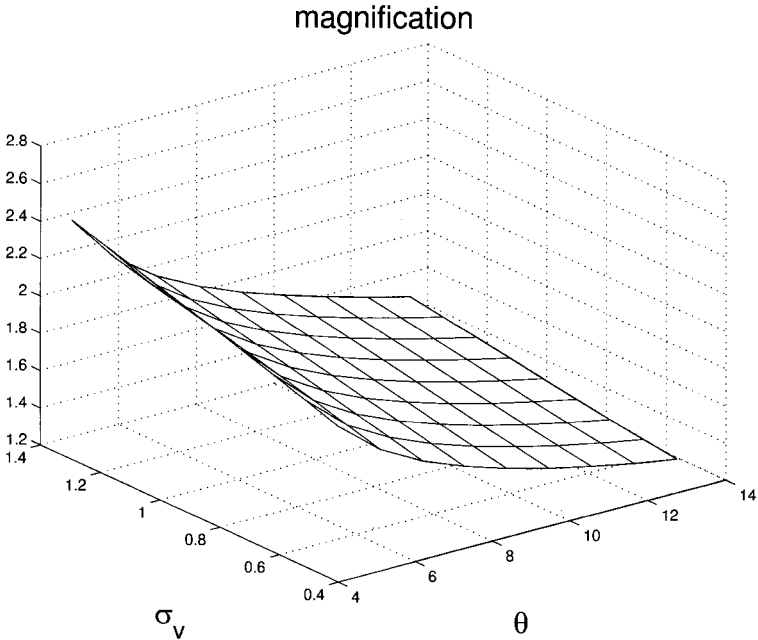
As explained at the end of Section 2.4, capacity utilization is decreasing in σ_v and increasing in θ .⁸ Therefore, when σ_v is high and θ is low, the aggregate utilization rate is low: the economy can respond more strongly and more persistently to technological shocks, as shown on Fig. 3. This figure also shows that our calibration is rather conservative in this respect: the magnification measure can reach 2.4 and the order-one serial correlation of output growth 0.4 for values of θ and σ_v that are, respectively, 50% lower and greater than in our benchmark calibration.

The persistence of the aggregate shock (ρ) and the capital coefficient (α) is a second set of particularly important parameters. A sensitivity analysis with respect to these two parameters is presented in Fig. 4. The effect of these two parameters on magnification and persistence is fairly standard. The higher the weight of capital (α) in production, the less volatile the economy: accordingly, the lower are the magnification and persistence effects. Similarly, highly persistent shocks (high ρ) induce important wealth effects and thus make the economy less volatile. Again, one should notice that our benchmark calibration is rather conservative.

5. CONCLUDING COMMENTS

In this paper, we have introduced idiosyncratic (demand) uncertainty and a richer modeling of the production sector (firms heterogeneity and absence of an aggregate production function) within a business cycle model with monopolistic competition. In equilibrium, a variable proportion of firms face demand shortages and have idle capacities; others are at full capacity and unable to serve any extra demand. A real business cycle exercise has shown that the capacity utilization variability is a mechanism that magnifies and propagates technological shocks. In a setup accounting for the phenomenon of capacity idleness and heterogeneity, we have thus obtained quantitative results that are similar to the ones of the representative firm “depreciation in use” model of Burnside and Eichenbaum (1996).

⁸ The higher σ_v , the lower the utilization rate in the firms experiencing the worst individual shocks. The higher θ , the easier the substitution between the supply constrained inputs and the others, which increases the demand for the unconstrained inputs.



order 1 serial correl of $\Delta \log(Y)$

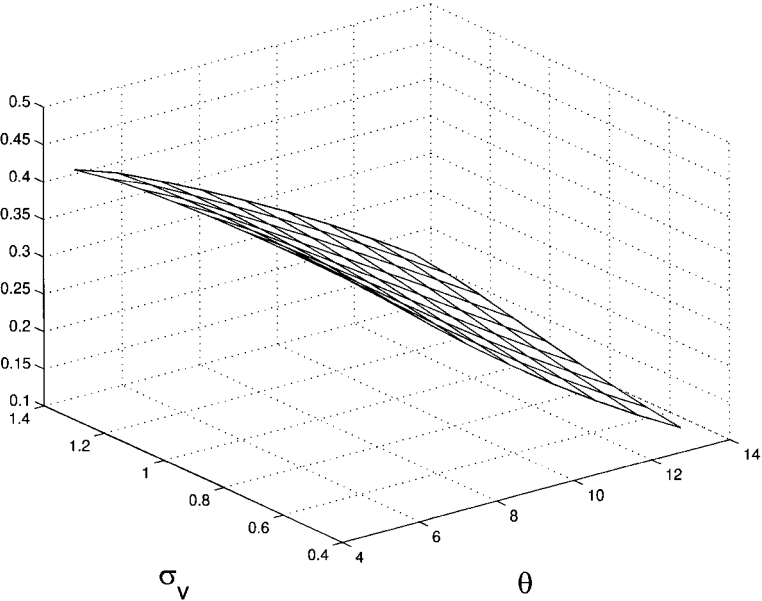


FIGURE 3

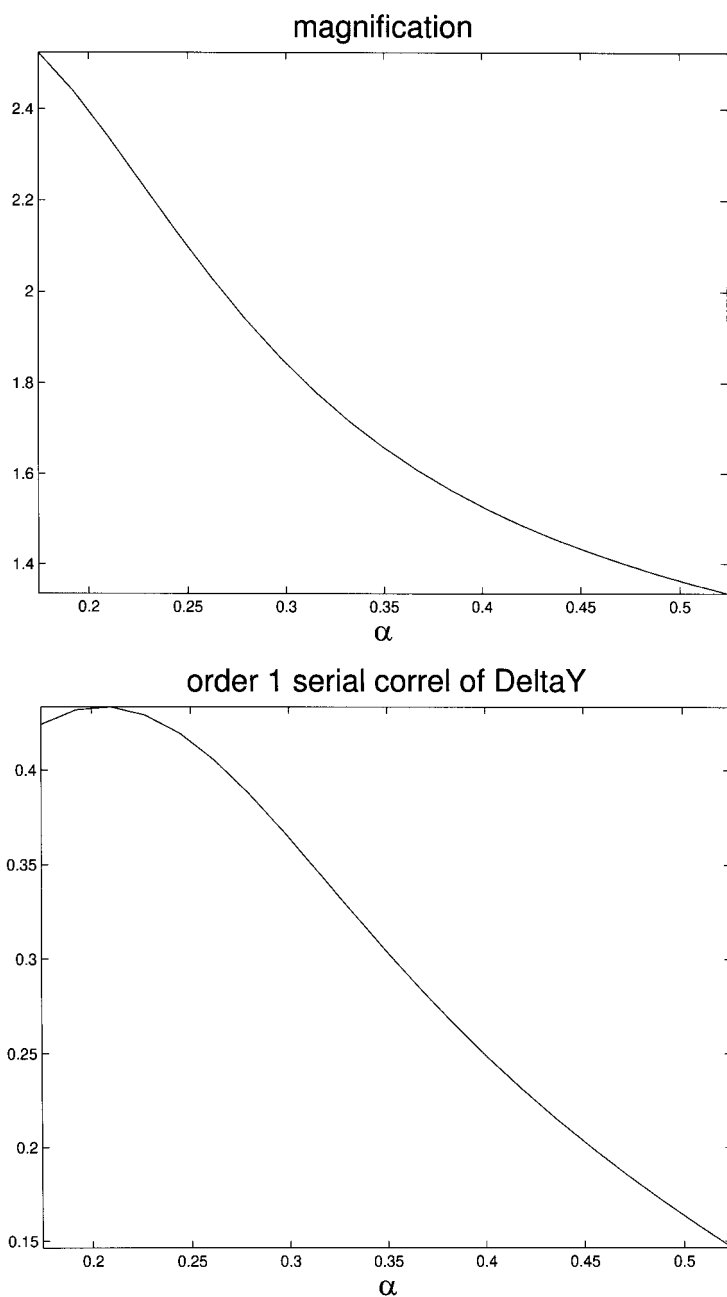


FIGURE 4 A

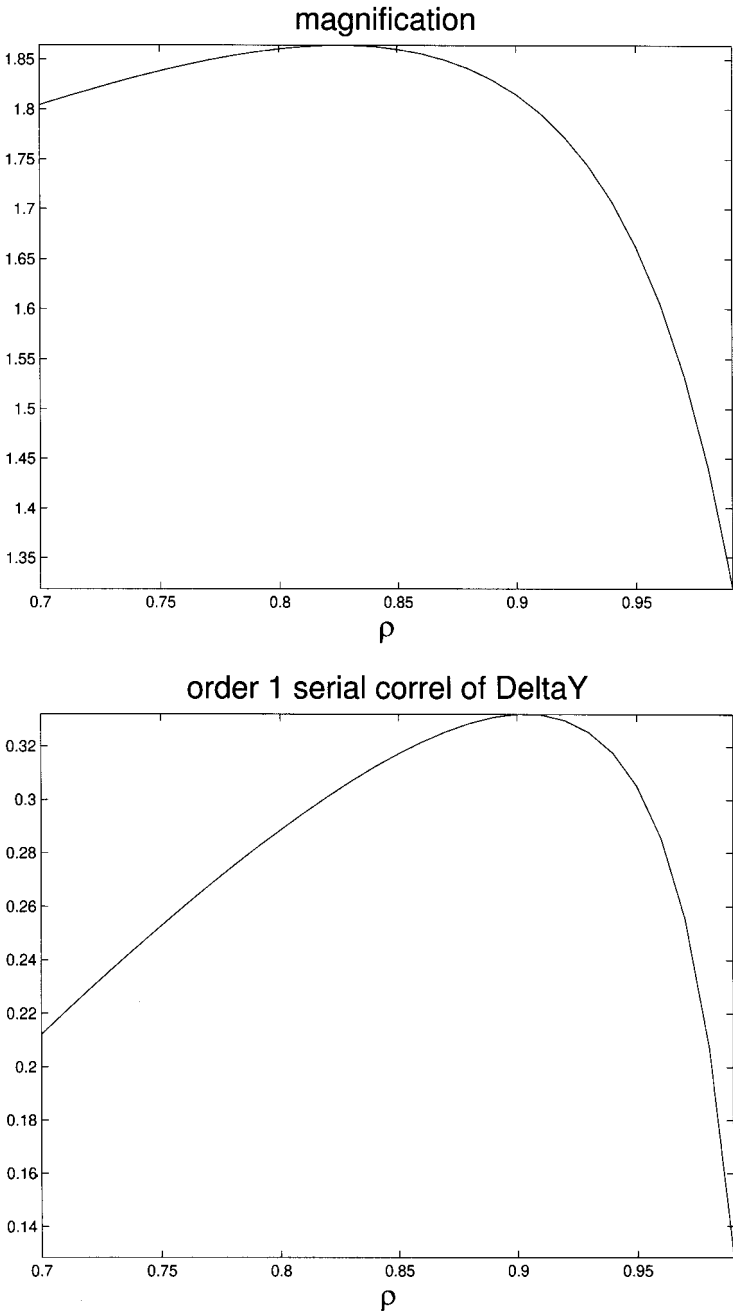


FIGURE 4 B

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