

Université catholique de Louvain
Louvain School of Engineering
Institute of
Information and Communication Technologies, Electronics
and Applied Mathematics (ICTEAM)



Simulation of coherent Doppler LIDAR signals and their analysis with the Cohen's class

Application to algorithms design for wake vortex
detection and characterization

Sébastien BROUSMICHE

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in engineering sciences*

PhD committee

Benoît MACQ	UCL/ICTM - Supervisor
Piotr SOBIESKI	UCL/ICTM - Supervisor
Grégoire WINCKELMANS	UCL/IMMC
Jean-Didier LEGAT	UCL/ICTM
Agnès DOLFI-BOUTEYRE	ONERA, France
Marc ACHEROY	RMA, Belgium
Christophe DE VLEESCHOUWER	UCL/ICTM - President

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Abstract

The problem of wind statistics measurements in the planetary boundary layer using a coherent Doppler LIDAR is addressed. More specifically, it focuses on the design of efficient algorithms in the time-frequency space dedicated to the detection and the characterization of atmospheric hazards, such as aircraft wake vortices or wind shears.

To support this study, a simulation program has been developed which combines the numerical simulation of LASER beam propagation in a turbulent medium with state-of-the-art Large Eddy Simulation (LES) of wake vortex in atmospheric turbulence or in ground effect. This tool constitutes a complete framework for optimizing the LASER source while developing and evaluating the performance of the whole estimation process.

Moreover, an analytical formulation of the coherent LIDAR Wigner-Ville spectrum has been derived along with its associated Cohen's class of time-frequency estimators. This theoretical basis leads to explicit equations of the wind statistics that are actually retrievable, depending on the measurement conditions and the LASER pulse parameters. It also yields to an adaptive spectral modeling algorithm which is used to detect the wake vortices and then estimate their wind speed probability density function as well as their extremum radial velocity profiles, related to the vortex maximum tangential velocity.

Another major achievement of this thesis is the development and the validation of a fast inversion method dedicated to wake vortex characterization, e.g. the estimation of its position and intensity. In this method, a Burnham-Hallock model, transformed by a new measurement model integrating the effects of the estimation process, is directly matched to the retrieved radial velocity map. Monte Carlo simulations have been carried out on both analytical models and LES to confirm its good performance.

Finally, the validation of the axial detection of vortices with a fiber-based LIDAR has been performed during a test campaign conducted by ONERA at Orly airport in April 2008 during the FIDELIO project. Wake vortex signatures have been successfully obtained with some of the algorithms developed in this thesis.

*“Experience is not what happens to a man;
it is what a man does with what happens to him”*

Aldous Huxley

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Symbols and notations

b	[m]	Aircraft wing span
b_0	[m]	Distance between vortex centers after the initial roll-up phase
c	[m.s ⁻¹]	Speed of light in vacuum, $c = (\mu_0\epsilon_0)^{-1/2}$
f_{IF}	[Hz]	Intermediate frequency (RF)
f_{D}	[Hz]	Aerosol particles induced Doppler frequency shift
$h(R)$	[-]	Normalized system gain, equivalent to SNR(R)
$i(t)$	[A]	Photodetector signal current, $i(t) = i_s(t) + i_{dc}(t) + i_h(t)$
$i_s(t)$	[A]	Direct detection signal current due to the backscattered field
$i_{dc}(t)$	[A]	Direct signal current due to the LO field
$i_h(t)$	[A]	Photodetector heterodyne current signal, bandwidth around f_{IF}
j	[-]	Standard imaginary unit with $j^2 = -1$
$j_{\text{T,BPLO}}$	[m ⁻²]	Irradiances of the normalized transmitted and BPLO fields
l_0	[m]	Smallest turbulent eddy scale, called <i>inner scale</i>
n	[-]	Real part of the medium refractive index, $n = (\epsilon/\epsilon_0)^{-1/2} \approx 1 + n_1$, $\langle n_1 \rangle = 0$
n_1	[-]	Random component of the refractive index
$p(t)$	[s ^{-1/2}]	Normalized LASER pulse profile
$r(t; R)$	[-]	Elementary signal from atmospheric slice dR at R
r_c	[m]	Radius of the maximum tangential velocity, $r_c \approx 3 - 5\% b_0$
r_p	[m]	Spatial extend of function $I(R; t)$
r_a	[m]	Spatial extend of function $I_a(R; t)$
$v_{\text{R}}(R)$	[m/s]	Radial component of the velocity field at range R
w	[m]	e^{-1} beam amplitude radius in free-space
w_e	[m]	Effective, long-term averaged, e^{-1} beam radius
w_p	[Hz]	e^{-1} radius of the pulse Fourier transform
w_{WS}	[Hz]	Spectral spreading induced by wind gradient
w_{V}	[Hz]	Spectral spreading induced by wind turbulence
u_{θ}	[m/s]	Tangential velocity of the wake vortex field
$\mathbf{r}$	[m]	Three-dimensional cartesian coordinate vector

$\mathbf{u}, \mathbf{v}$	[m]	Transverse coordinate vectors at the transmitter and receiver planes
$\mathbf{p}, \mathbf{q}$	[m]	Transverse coordinate vectors at the target planes
$\mathbf{w}$	[m]	Transverse coordinate vector at the detector plane
A_R	[m ²]	Receiver aperture area
B_w	[Hz]	Detector bandwidth
$C(\mathbf{p}, R, t)$	[-]	Coherent responsivity
C_n^2	[m ^{-2/3}]	Refractive-index structure constant
G^f, G	[m ⁻²]	Green's functions in free-space and in a turbulent medium
$H(R, t)$	[m ⁻¹]	System gain
$E(\mathbf{p}, R, t)$	[V.m ⁻¹]	Complex amplitude of the electric field
$F_p(\nu)$	[Hz ⁻¹]	Pulse power spectrum
$I(\mathbf{p}, R, t)$	[W.m ⁻²]	LASER beam irradiance
$I(R; t)$	[-]	Spatial weighting function at time t
$I_a(R; t)$	[-]	Spatial weighting function for the SPWVD
M^2	[-]	LASER beam quality factor, $M^2 \geq 1$
$K(R)$	[-]	One-way irradiance extinction at range R and wavelength λ
L_0	[m]	Largest turbulent scale, called <i>outer scale</i>
$\text{SNR}(R)$	[-]	Signal-to-noise ratio evolution with range
NEP	[W Hz ^{-1/2}]	Noise Equivalent Power of the detector intrinsic noises
R	[m]	Range along one line-of-sight
R_F	[m]	Size of the first Fresnel zone
Re	[-]	Reynolds number
S	[A/W]	Detector sensitivity
U_L	[J]	LASER pulse energy
V_0	[m/s]	Vortex descent speed
$\beta(\mathbf{p}, R)$	[m ⁻¹ sr ⁻¹]	Atmospheric backscattering coefficient
ε_0	[F.m ⁻¹]	Permittivity of free-space, $\varepsilon_0 = 8.854 \cdot 10^{-12} \text{ F.m}^{-1}$
ε	[F.m ⁻¹]	Permittivity of the medium
ϵ	[m ² .s ⁻³]	Eddy Dissipation rate, TEDR
$\eta_{Q,H,S}$		Quantum, heterodyne and system efficiency
κ	[rad/m]	Spatial frequency
λ	[m]	Wavelength, $\lambda = c/\nu_0$ (1.55 μm in this study)
μ_0	[H.m ⁻¹]	Magnetic permeability of free-space, $\mu_0 = 4\pi \cdot 10^{-7} \text{ H.m}^{-1}$
ν_0	[Hz]	Optical frequency of the LASER source
ϕ	[Hz.s ⁻¹]	Linear frequency chirp of the LASER pulse

ρ_0	[m]	Spatial coherence radius of an unbounded plane wave in Kolmogorov turbulence
σ_p	[s]	e^{-1} pulse duration
σ_I^2	[-]	Scintillation index
σ_1^2	[-]	Rytov variance at distance R , $\sigma_1^2(R) = 1.23 C_n^2 k^{7/6} R^{11/6}$
σ_{CR}	[m/s]	Cramer-Rao bound on the radial velocity estimate
τ_p	[s]	Pulse duration, $\tau_p = 2\sigma_p$
τ_{WS}	[s]	Decorrelation time induced by wind gradient
τ_V	[s]	Decorrelation time induced by wind turbulence
ν	[m ² .s ⁻¹]	Kinematic viscosity
Δt	[s]	FWHM pulse duration
Δr	[m]	Spatial extend of the pulse, $\Delta r = c\Delta t/2$
Δp	[m]	Spatial extend of the analyzing window
ΔR	[m]	Longitudinal range resolution, $\Delta R = \Delta p + \Delta r$
Γ_0	[m ² /s]	Total vortex circulation
$\Gamma(r)$	[m ² /s]	Vortex circulation profile
$\Omega(R)$	[-]	Telescope solid angle, $\Omega(R) = A_R/R^2$
$\Pi(t, \nu)$	[-]	Cohen's class smoothing function
Φ_n	[m ³]	Refractive index power spectrum
Φ'_V	[-]	Weighted velocity distribution
Φ''_V	[-]	Weighted velocity distribution, Cohen's class
$\mathcal{T}$	[-]	Dimensionless time for vortex analysis
$(\nabla \times)$		Curl operator
(∇^2)		Laplacian operator, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
$(\nabla_\perp^2)$		Transverse part of the Laplacian operator, $\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
$\otimes_t, \otimes_\nu$		Convolution operator in time or in frequency
$\otimes_{t,\nu}$		Convolution in the time-frequency domain
$\Re\{\}$		Real part of a complex quantity
$\mathcal{F}\{\}$		Forward Fourier transform
$\mathcal{F}^{-1}\{\}$		Backward Fourier transform
$\mathbf{F}\{\}$		2D Forward Fourier transform
$\mathbf{F}^{-1}\{\}$		2D Backward Fourier transform
$\mathcal{H}\{\}$		Hilbert transform
$P\{\}$		Probability
$\hat{\mathbf{e}}_r$		Unitary vector along the direction r
$\langle \cdot \rangle$		Ensemble average

$ \cdot $	L_1 norm, absolute value
$\ \cdot\ $	L_2 norm
$\hat{\cdot}$	Estimated quantity
$m_i(X)$	i -th order statistical moment of the quantity X
$M_X(\omega)$	Characteristic function of random variable X
$D_X(r)$	Structure function of random variable X
$\Phi_X(x)$	Probability density function of random variable X
$R_x(t_1, t_2)$	Covariance of a signal $x(t)$
$r_x(t_1, t_2)$	Correlogram of a signal $x(t)$
$W_x(t, \nu)$	Wigner-Ville distribution of a signal $x(t)$
$W_p(t, \nu)$	LASER pulse Wigner-Ville distribution
$S_x(t, \nu)$	Spectrogram of a signal $x(t)$
$SPW_x(t, \nu)$	Smoothed Pseudo Wigner-Ville distribution
AOM	Acousto-Optic Modulator
AWIATOR	<i>Aircraft Wing Advanced Technology Operation</i> , UE-FP5
BH	The Burnham-Hallock vortex model
BPLO	Back-Propagated Local Oscillator
CFD	Computational Fluid Dynamics
CLR	Coherent LASER Radar
DELICAT	<i>Demonstration of LIDAR based Clear Air Turbulence Detection</i> , an UE-FP7 project
EM	Expectation-Maximization algorithm
FFT	Fast Fourier Transform algorithm
FFTW	<i>Fast Fourier Transform in the West</i> Library
FWHM	Full Width at Half Maximum
FIDELIO	<i>Fiber laser DEvelopment for next generation LIdar Onboard detection system</i> , UE-FP6
GREENWAKE	an UE-FP7 Project
HIT	Homogeneous and Isotropic Turbulence
IGE	Wake vortex In Ground Effect
ICTEAM	Institute of Information and Communication Technologies, Electronics and Applied Mathematics at the UCL
IMMC	Institute of Mechanics, Materials and Civil Engineering at the UCL
IR	Infra-Red electromagnetic wave, $\sim 100 \mu\text{m} \rightarrow \sim 1 \text{ mm}$
I-WAKE	<i>Instrumentation systems for on-board wake-vortex and other hazards detection, warning and avoidance</i> , UE-FP5+FP6
IF	Intermediate Frequency, around 80 MHz

LES	Large Eddy Simulation
LASEF	<i>Laser A Source d'Emission Fibrée</i> , a Belgian RW-Réseaux Project
LASER	Light Amplification by Stimulated Emission of Radiation
LIDAR	Light Detection and Ranging
LO	Local Oscillator
LOS	Line-Of-Sight
MCMC	Markov Chain Monte-Carlo
MCMH	Markov Chain Metropolis-Hasting
MFLAME	<i>Future Laser Atmospheric Measurement Equipment</i> , UE Brite-EuRam III programme
MOPFA	Master Oscillator Power Fiber Amplifier
NEP	Noise Equivalent Power
NGE	Wake vortex Near Ground Effect
ONERA	The French Aeronautic and Space Research Center
OGE	Wake vortex Out of Ground Effect
PDF	Probability Density Function
PRF	Pulse Repetition Frequency
PSM	Phase-screen method
RADAR	Radio Detection and Ranging
RIN	Relative Intensity Noise
RLS	Recursive Least Square filtering
ROI	Spatial Region-Of-Interest
RV	Random Variable
SESAR	<i>Single European Sky ATM Research</i> Programme, www.sesarju.eu
SLC	Submarine Laser Communication
SNR	Signal-to-Noise Ratio
SPWVD	Smoothed Pseudo-Wigner-Ville Distribution
STFT	Short-Time Fourier Transform
TEDR	Turbulent Eddy Dissipation Rate
WAKE4D	a Wake vortex prediction platform software developed at IMMC, UCL
WV	Wake Vortex
WVD	Wigner-Ville Distribution

Introduction

REMOTE sensing of atmospheric hazards such as wake vortices, wind shear, and clear air turbulence has been a major concern for 20 years. A well-adapted system to detect them is the coherent Doppler LIDAR, acronym standing for *Light Detection And Ranging*. This high-resolution device offers complementary advantages to the classical RADAR, especially when operating in clear air conditions. It is therefore increasingly used for practical applications concerning, notably, safety in air traffic management (ATM).

Fiber-based LIDARS are known for their high flexibility and compactness despite their limited pulse energy. Hence, to reach optimal performance, it is often needed to precisely model the whole measurement process. This thesis mainly addresses the problem of estimating wind statistics and wake vortex intrinsic parameters with fiber-based coherent Doppler LIDARS. An integrated numerical simulation technique has therefore been developed and used as a complete framework for the design and the validation of advanced wake vortex detection and characterization algorithms.

In this chapter, the physics of wake vortices and LIDAR measurement are respectively described in Section 1.1 and 1.2. The performance constraints are described in Section 1.3 along with a discussion on how numerical simulation can help in optimizing the system design. The major contributions to the field are then presented in Section 1.4 and the structure of the present document is detailed in Section 1.5.

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1.1 Fundamentals of aircraft wake vortex

How could we better introduce the wake vortex phenomenon than by showing the two large counter-rotating swirling flows of air generated by an aircraft? In the picture below, we observe a Boeing 757 flying through clouds during its landing at the Gatwick airport. This picture is interesting in many ways since it captures some of the most important aircraft wake characteristics such as the decompression vapor over the wings as well as the generation of the vortices themselves that here mainly trails from the outer flaps. Wake vortices also come from the wing tips but cannot be seen here. One of the main reason for which



Figure 1.1: Illustration of the wake vortex phenomenon from a Boeing 757-2B7 flying through clouds. The aircraft is descending through 200 m about one minute from landing with full landing flap 30 selected to increase its lift coefficient and fly safely at lower speeds. In this configuration, the wake vortices are principally issued from the flaps. Courtesy of Steve Morris, from www.airliners.net.

they are studied is that these large turbulences are important enough for an aircraft, flying close to them, to be destabilized. Moreover, they are only visible under certain conditions of humidity and pressure, such as the condensation trails crossing the sky at high altitudes, or for high aerosol density, usually near the ground.

1.1.1 Wake vortex formation and decay

The lift force acting on an aircraft is due to the pressure difference between the lower side (pressure side) and the upper side (suction side) of its wing. As a result of this pressure difference, there is a spanwise flow at the edges of the wing from the pressure side to the suction side. This pressure difference forces

the suction side streamlines to converge toward the center of the wing and the pressure side streamlines to diverge from it. This spanwise flow combined with the free-stream velocity produces a swirling motion of the air trailing downstream of the wing. Just behind the trailing edge, a vortex sheet is shed which rolls-up rapidly within a few span lengths to form a pair of counter rotating vortices of equal strength but opposite signs.

The circulation, denoted Γ [m^2/s], of the velocity field $\mathbf{u}$ [m/s] for each vortex is a quantity of primary importance as it gives an image of the vortex strength. For a closed contour $\mathcal{C} = \partial\Omega$ enclosing a patch of vorticity defined by the field $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, it is written as

$$\Gamma = \int_{\Omega} \boldsymbol{\omega} \cdot \mathbf{n} \, dS = \oint_{\mathcal{C}} \mathbf{u} \cdot d\mathbf{l}. \quad (1.1)$$

The initial wake vortex intensity is noted Γ_0 [m^2/s] and is principally determined by the aircraft weight, speed, wingspan, configuration and angle of attack. The vortex strength decreases with time depending on various parameters such as the atmospheric stability, wind strength and direction or their distance to the ground. Hence, atmospheric turbulence induces a more rapid break up of the vortices. In general, they may persist for several minutes and stretch for many kilometers behind the aircraft.

More precisely, vortices are generated at the inner and outer flaps, at the horizontal stabilizer as well as at the wing tips. As illustrated in Fig. 1.2, they will recombine to produce two counter rotating vortices per wing, the first at its beginning, with a circulation Γ_2 , and the second at its tip, with a circulation Γ_1 . The two vortices from the inner flaps are separated by a distance b_2 and the ones from the tips by a distance b_1 . It takes a few wingspans behind the aircraft to produce the two main vortices separated by a distance b_0 and with a circulation Γ_0 . The momentum conservation allows one to bind the initial circulation of a vortex pair to the lift force L [N] itself, equal to the weight of the aircraft:

$$L = M g = \rho U b_0 \Gamma_0, \quad (1.2)$$

with

$$\Gamma_0 b_0 = \Gamma_1 b_1 - \Gamma_2 b_2 \quad \text{and} \quad \Gamma_0 = \Gamma_1 - \Gamma_2 \quad (1.3)$$

where U [m/s] is the flight speed, M [kg] is the mass of the aircraft, b_0 [m] is the distance between the vortices, g [m/s^2] is the standard gravity and ρ [kg/m^3] is the fluid density. The circulation of the resulting vortices is thus proportional to the mass of the aircraft and inversely proportional to its flight speed. Typical values for the circulation range in $\Gamma_0 \approx 400 - 700 \text{ m}^2/\text{s}$ for heavy aircraft. For an elliptical loading and a wingspan b , the distance b_0 is approximately equal to $\frac{\pi}{4}b$. The vortex spacing b_0 can be deduced from the lifting line theory developed by [Prandtl 1957]. Wake vortices generated at takeoff are very strong since the airplane evolves at low speed and is fully loaded.

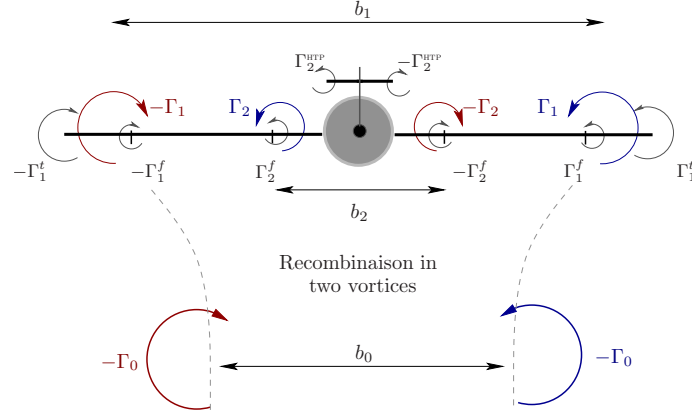


Figure 1.2: Illustration of the formation of a wake vortex pair from the vortices generated at the wing tips (superscript t), the flaps (superscript f) and the horizontal stabilizer (superscript HTP, for Horizontal Tail Plane). Two counter rotating vortices are created with absolute circulation Γ_0 . We have that $\Gamma_1 = \Gamma_1^f + \Gamma_1^t$ and $\Gamma_2 = \Gamma_2^f + \Gamma_2^{HTP}$

1.1.2 Why tracking wake vortices?

The effects of wake vortices on an aircraft passing near them depend on many factors and may vary in intensity from a slight rocking of its wings to a complete loss of control. In the general case, it will be subject to a rolling moment. More precisely, it will encounter a downward velocity if it flies between the two vortices and an upward velocity if it passes on one of the sides of the vortex pair. The potential to recover from the most severe effects will depend on the altitude and the manoeuvrability of the aircraft. The most dangerous situations therefore concern small aircraft flying into the wake of a larger one and evolving close to the ground, i.e. during takeoff or landing.

In order to reduce as much as possible the probability of wake vortex encounter (WVE), the air traffic control (ATC), applies a wake turbulence separation standard resumed in Table 1.1.2. It lists the separation distances depending on the Maximum Take Off Mass (MTOM) of the leading and following aircraft. Hence, a light aircraft is characterized by a MTOM of 7 tons or less, a heavy aircraft by a MTOM of at least 136 tons and a medium one lies between these two limits. These values are used for the approach phase of flight, while under RADAR control.

Instauring fixed time separations based on heuristics is a problem for two reasons. The first one is that, as all security margins, it is based on a worst-case scenario. These margins are therefore most of the time too long and decrease the

Leader aircraft max takeoff weight	Follower aircraft (metric tons)	Separation Nautical miles	Time delay [s]
Heavy	Heavy	4	106
Heavy	Medium	5	132
Heavy	Light	6	159
Medium	Light	5	132

Table 1.1: International Civil Aviation Organization (ICAO) aircraft separation distances, in nautical miles (1 NM = 1.852 km), to avoid wake vortex encounter during the approach phase. For all other combinations, the separation is 3 NM. Table reproduced from [Gerz 2001]. The time delay is given for an approach speed of 70 m/s. The aircraft are classified by their weight: light (≤ 7 T), medium (< 136 T) and heavy (≥ 136 T).

operation capacity of the air transport systems, e.g. the time between takeoff and landings. Second, as it is based on heuristics, it might not be a sufficient margin for some cases which can lead to WVE. These issues clearly point out the fact that airport security could be improved by adapting separations to the climate and aircraft conditions. It is necessary, for example, to take into account the possibility of wake displacement near the ground. Hence, a light crosswind may cause the vortices to drift. A 3 to 5 knot crosswind will tend to keep the upwind vortex in the runway area and may cause the downwind vortex to drift toward another runway. Moreover, as the two counter-rotating vortices induce a downward velocity to one another, the vortex pair comes closer to the ground and undergoes the so-called rebound phenomenon, when it reaches it. This tends to further increase the indetermination we have on the vortex position and strength. These considerations were emphasized by a number of research projects and have been the central issue of the most recent scientific breakthrough in the field. In particular, being able to detect and predict hazards, such as wake vortices or wind shear, is of the upmost importance for airport security. The WAKE4D software, developed by G. Winckelmans' team at UCL, allows one to predict the transport and decay of vortices, within a radius of a few tens of kilometers around an airport, by combining advanced simulation methods and meteorological measurements for the weather conditions. The use of RADAR and LIDAR systems to support, confirm and complement these predictions is nevertheless necessary for the whole system to be robust and reliable.

1.2 Wind measurement with a coherent Doppler LIDAR

A LIDAR is a detection and ranging system, such as the RADAR, and is used to measure the properties of a distant target. The measurement is done in two steps. The system first sends electromagnetic pulses in its direction. The target then scatters the emitted energy in all directions and a fraction of the scattered waves, directed back to the system receiver, is measured. The targets can be either solid objects or a diffuse quantity such as the aerosol particles or molecules present in the atmosphere. In contrary to the RADAR, the LIDAR is an optical remote sensing technology operating at shorter wavelengths in either the ultraviolet, visible or near-infrared spectrum. At these wavelengths and because of its exceptional spatial and temporal coherence, the LIDAR can be used in a wide range of applications going from the measurement of particular atmospheric components concentration, such as the CO_2 , to satellite communication and atmospheric turbulence analysis.

This thesis will exclusively focus on the coherent pulsed Doppler LIDAR used as a LASER anemometer and working in the Mie scattering regime. Thanks to the well-known *Doppler* effect, it measures the atmospheric velocity characteristics along a given direction of observation by estimating the mean velocity and the velocity dispersion of the atmospheric aerosol particles carried by the flow. The use of pulses allows to restrict the observation ranges at a given time to a fixed and known interval. Continuous systems, in comparison, must focalize the LASER beam to get a similar spatial selection effect. Moreover, in the *coherent*, also called heterodyne, LIDAR, the atmospheric information is retrieved from the backscattered wave by comparing it with a reference LASER beam at the surface of the photodetector as it will be detailed later. This further brings the signal spectrum from optical to radio frequencies where it can be more easily processed. An intrinsic limitation of this system is that it needs a sufficient concentration in aerosol particles and a good atmospheric transmission to reach acceptable performance. This particular condition can be found in the lower troposphere but, by no means, after rain or during dense fog. This incompatibility with high humidity conditions is what makes this system complementary to the RADAR since the latter requires scattering objects of the size of a rain drop, e.g. bugs, birds. New studies are currently undertaken, in particular by members of the newly started SESAR programme in which UCL is involved, to exploit this particularity in order to develop an all-weather wake vortex detection system.

This introduction on LIDAR wind measurement would not be complete without talking about direct, as opposed to coherent, UV LIDAR measurement. This technology is an important alternative to coherent systems for low aerosol concentration conditions and turbulence detection at short and medium ranges. This subject will be briefly presented in Section 1.2.5.

1.2.1 The coherent LIDAR principle

A general architecture for a coherent Doppler LIDAR is represented in Fig. 1.3. The heart of the system is a continuous LASER source producing an optical wave with a narrow linewidth, which is a determinant parameter to guarantee a high velocity resolution. This oscillator must also be stable both in intensity and frequency and produce a high-quality beam. This last property is essentially a measure of how tightly a LASER beam can be focused with a limited beam divergence. It is commonly quantified by the M^2 factor which must be as close as possible to unity. A M^2 larger than one traduces a beam with a non perfectly Gaussian transverse irradiance distribution.

Fiber-based LASER sources, operating at a wavelength of $1.55 \mu\text{m}$, are nowadays used as an alternative to classical solid-state sources because of their lower cost and higher compacity and flexibility. Although the pulse energy is rather low compared to the later, i.e. a few hundreds of microjoules instead of several millijoules, they produce a higher pulse repetition frequency (PRF) which allows to compensate this gap. Besides these technological reasons, LASER sources at $1.55 \mu\text{m}$ are most importantly chosen because they are eye-safe and because the atmospheric transmission at this wavelength is rather good, i.e. the molecular and aerosol absorption is limited. LASER source requirements for fiber-based coherent LIDAR are resumed in [Cariou 2006].

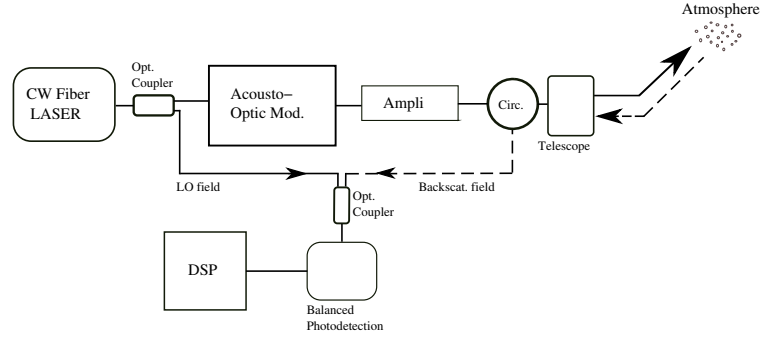


Figure 1.3: Classical coherent Doppler LIDAR architecture based on a fiber LASER source. An optical coupler first separates the continuous (CW) beam between the local oscillator (LO) beam used for the heterodyne detection and the one which will be transmitted. The backscattered beam is then mixed with the LO field and the signal produced is analyzed by the digital signal processor (DSP).

When passing through the acousto-optic modulator (AOM), the produced continuous (CW) wave is shifted around a given frequency and is modulated in amplitude to produce a LASER pulse with a global Gaussian shape. The

intermediate frequency is typically chosen around 70 MHz. The pulse duration may vary from 200 ns to 1 μ s depending on the needed spatial and velocity resolutions and some others optical constraints. This pulse is then amplified and emitted through the atmosphere using a telescope. The elevation and azimuth of the direction of observation are defined by the scanner position. The telescope configuration is usually monostatic which means that the energy is sent and received through the same aperture. This requires the use of a high-isolation free-space circulator. The monostatic configuration, in addition to be set up more easily than the bistatic one, is known to have a lower sensitivity to high spatial scales of the atmospheric turbulence.

As it propagates, the emitted beam is scattered by the aerosol particles. A portion of the backscattered energy is received at the telescope and is mixed with the local oscillator (LO) beam coming from the LASER source. A photodetector produces the heterodyne current which oscillates around the intermediate frequency with a deviation depending on the Doppler effect induced by the movement of the aerosol particles along the line-of-sight (LOS). The signals are then analyzed to retrieve the targetted information.

1.2.2 Wind statistics estimation

Each time a pulse is sent through the atmosphere, a backscattered wave is measured at the surface of the photodetector which, in turn, produces a current signal at its output. This narrowband signal contains some interesting information about the atmospheric state but is generally so contaminated by a multitude of detection noises that advanced estimation methods must be elaborated for this information to be extracted.

The difficulty to analyze a signal not only depends on the ratio between the power carried by the targetted information and the total power of all the involved noise sources, as defined by the signal-to noise ratio (SNR), but also on the correlation existing between the noise processes and the useful signal. We typically assume that two signals obtained from adjacent LOS carry highly correlated contents whereas the realizations of the noise processes are independent. This assumption is valid both for the additive photodetection noise generated by the detector and the multiplicative noise due to the atmospheric particle velocity dispersion and known as the speckle effect. The *pulse accumulation* technique is therefore always used and consists in the averaging of a number of successive signals either in the temporal domain through their correlograms or in the frequency domain until the noise power has been reduced to an acceptable level. This strategy is particularly suitable for fiber-based sources with lower pulse energy.

The uncertainty principle formulated by Heisenberg is also applicable. When translated to LIDAR measurement, it states that the instantaneous position

and velocity of the particles in an infinitesimal atmospheric volume cannot be simultaneously known with an arbitrary precision. Instead, these two quantities are linked together so that increasing the resolution of one of them will decrease the resolution in the other with the same factor. The tuning parameter, which enhances one quantity to the detriment of the other, is the LASER pulse duration. This problem will be covered in detail in Chapter 4. It has a major influence on the estimation of wind statistics as well as the wake vortex characterization problem.

1.2.3 Sensing wake vortices through appropriate scanning

As described in Section 1.1, wake vortices are localized atmospheric phenomena characterized by rapid fluctuations of the air velocity around each core. Their detection and characterization necessitates to optimize the LASER source parameters such as the PRF or the pulse duration and energy. It is also required to choose the scanning technique such that the vortices can be detected, at strategic locations of an airport for example, with the best sensitivity while reducing as much as possible the occurrence of false detections, either positive or negative. These wrong detections usually appear when the vortex signatures inside an estimated velocity map cannot be easily discriminated from the background fluctuations induced by both atmospheric turbulence and estimation errors. Hence, detecting a wake vortex whereas nothing is present in the observation volume is called a false positive detection. Inversely, which is obviously more problematic, a missed detection is called a false negative detection. An increase of the estimation error always leads to an increase of both error rates.

The reason why the scanning technique is so important is that only the radial component of the air velocity field, i.e. its projection onto the direction of propagation of the LASER, is retrievable. The scanning techniques are classically named depending on the way they cut the two turbulent vortices. We classically use a right-handed cartesian coordinate system where the x - y plane is horizontal and the x -axis is aligned with the flight path. The two following detection configurations, represented in Fig. 1.4, are the most frequently used:

- The *transverse detection* is a two-dimensional scanning configuration lying in the y - z plane. It is mostly used for ground-based detection and the LIDAR itself is positionned upstream from the runways for approach and downstream for takeoff. In this configuration, the measured radial velocity has its maximum dynamic which leads to a better LIDAR sensitivity and vortex detection capability.
- The *axial detection* has been introduced since the MFLAME project for onboard systems. They are therefore used for inflight wake vortices detection. The scanning is three-dimensional with a mean observation direction principally along the x -axis. Even if it has been demonstrated

by computational fluid dynamics (CFD) simulations of wake vortices that there actually exists an axial fluctuating velocity component, wake vortex detection and characterization in this configuration remain particularly sensitive to LIDAR performance.

Only these two configurations will be studied in the present work. The scanning pattern optimization with regards to the LIDAR PRF and pulse duration is therefore out of this thesis scope. Moreover, the axial configuration is presented here for vortex detection only in the context of the FIDELIO project. The wake vortex characterization problem has therefore exclusively been assessed for the transverse configuration.

1.2.4 Examples of coherent LIDAR systems

In this thesis, two LIDAR systems will be considered which have been developed and tested in the LASEF project from the Belgian Région Wallonne on the one hand and in the FIDELIO project, under the sixth framework programme of the European Commission, on the other hand. They are both based on a $1.55\text{ }\mu\text{m}$ LASER source and have a monostatic configuration. Their major difference is that the first one is a ground-based system aiming at detecting vortices with a transversal scanning whereas the second has been developed as an onboard system for axial detection.

a. The FIDELIO onboard system

The aim of this project was to demonstrate the feasibility of detecting wake vortices using a fiber-based LIDAR as an alternative to the solid-state LASER technology which do not meet the constraints of size, weight and reliability necessary for onboard implementations. The main challenges are related to the lower available energy of fibered sources, the weaker aerosol power return at high altitudes as well as the necessity to detect wake vortices in an almost axial direction. The complete system architecture is represented in Fig. 1.5. An innovative high-brightness pulsed $1.5\text{ }\mu\text{m}$ LASER source, developed by ONERA, has been used. It is based on a master oscillator power fiber amplifier (MOPFA) architecture with a large-core fiber. The achieved pulse energy is $120\text{ }\mu\text{J}$ with a pulse duration of 800 ns and a PRF of 12 kHz . A sinusoidal scanning pattern in the axial direction is performed by a 2-axis scanner lent by the Fraunhofer Institute. The measured signals along with the scanning data are recorded and the velocity estimates are computed in real-time with a platform of eight parallel TI-C6414 digital signal processors (DSP). The results are sent over a local network in order to be visualized by a graphical user interface (GUI) on several independent workstations.

Field tests have been carried out at Orly airport, France, in April 2008. The LIDAR position was about 800 m before the runway touchdown threshold,

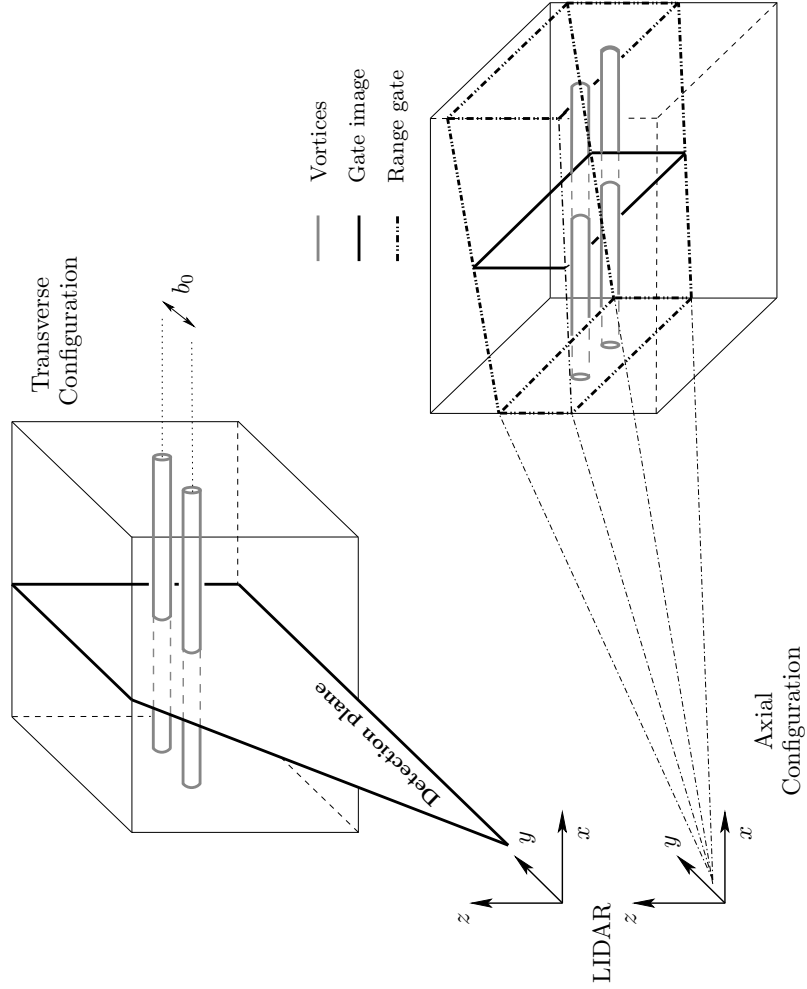


Figure 1.4: Wake vortex classical detection strategies: transverse configuration (top figure) and axial and axial configuration (bottom figure). The detection is performed for both cases in the y - z plane. In the axial case, the gate image is a representation of the velocity information in a volume called the range gate.

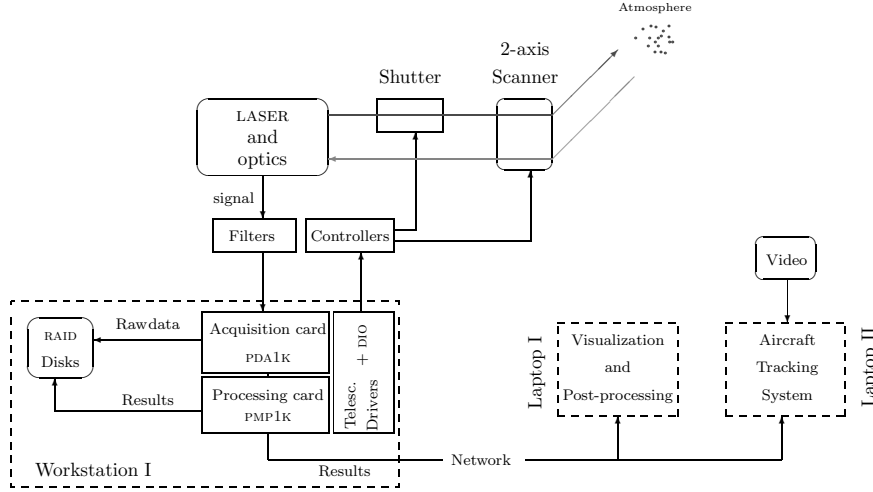


Figure 1.5: The complete FIDELIO system consisting on the LASER source and the beam focalization optics, the scanning mirrors as well as the acquisition and processing systems.

facing landing aircraft. During these tests, aircraft wake vortex signatures are successfully observed and acquired at a range of 1.2 km with an axial resolution of 75 m for the first time with fiber LASER sources. In order to validate the vortex detection, video recordings of the landing aircraft were made during the test campaign. A real-time aircraft detection, tracking, and geometric reconstruction algorithm was used in order to measure the distance between the aircraft and the sensor. Moreover, markers are written on the images whenever a strong-level return signal was received back from the LIDAR, corresponding to a reflection of the beam on the aircraft. For each aircraft, a few tens of scans are recorded. The acquisition was started when the aircraft was detected using the video tracking system. The acquired signals were transferred to the processing board through a buffer of multiple redundant array of independent disks (RAID). The results were sent to the GUI at the end of each scan during the flyback of the scanner. The global average wind velocity displayed in the main window was also validated with the atmospheric information coming from the airport tower.

b. The LASEF ground-based system

The LASER source, also based on a MOPFA architecture, has been developed by Multitel. It produces 350 ns pulses with an energy of 30 μJ and a PRF of 15 kHz. The spectral bandwidth is less than 8 kHz which provides a good

velocity resolution. A picture of the system is given in Fig. 1.6.

The Newton telescope is mounted on a scanning system which allows to scan in a transverse plane with an elevation angle up to 45 degrees. It has been developed by LambdaX under the specifications determined from the present work. This system has not yet been validated in situ but has shown promising LASER source properties.



Figure 1.6: The LASEF system with the LASER source and the telescope on the left.

1.2.5 The UV LIDAR technology for wind measurement

As a coherent Doppler LIDAR solely works in the Mie scattering regime, its performance rapidly deteriorates with a decrease of the aerosol concentration. This effect can be theoretically compensated by increasing the pulse energy and using, for example, a solid-state LASER source at $2\ \mu\text{m}$ instead of a fiber-based one. It is however not a good solution for onboard systems in civil aircrafts for which the constraints of compacity and power consumption are important.

The UV LIDAR technology is therefore preferably used for the measurement at high altitudes of clear air turbulence (not visible by the onboard RADAR), gusts or potentially wake vortices. The short-range detection, i.e. from 50 m to 300 m, has been validated in the frame of the AWIATOR project whereas its validation for medium range detection, i.e. from 10 km to 30 km, is the purpose of the ongoing DELICAT project.

An UV LIDAR is capable of measuring molecular scattering, i.e. it operates in the Rayleigh regime. It could also work in the Mie regime if the aerosol concentration is high enough. A coherent detection in that regime is not feasible due to the thermal broadening of the Rayleigh spectrum. A direct detection is performed instead. Hence, the wavelength shift induced by the wind is measured by a Fabry-Perot interferometer. Its function is to produce circular interferogrammes which are recorded by an image-intensified CCD

(charge coupled device) camera. The fluctuations of the circular pattern radius are directly proportional to the Doppler shifts corresponding to the change in relative speed of the airflow. The LASER source developed by EADS and Hovemere in the AWIATOR project has the following characteristics [Schmitt 2007]: it is a single-frequency, third-harmonics Nd:YAG operating at 355 nm with an average power of 3 W. It emits pulses of approximatively 8 ns width at a repetition rate of 18 kHz. The wind speed standard deviation is about 1 m/s and the longitudinal accuracy is around 15 m.

1.3 About performance constraints and simulation

Developping a state-of-the-art fiber-based coherent Doppler LIDAR stands at the crossroads of several scientific fields among which we find optics, atmospheric sciences, fluid mechanics, electrical engineering and applied mathematics. Consequently, optimizing the whole system for a given application, becomes a challenging process of integrating in a coherent way all the theories and techniques of each of these fields, while dealing with their intrinsic constraints and limitations.

A couple of important issues about LIDAR measurement have already been exposed. This analysis will be now completed by enumerating the major performance constraints, as well as the related design challenges, and explaining, when possible, how numerical simulation can help in making decisions about the various system parameters. For that purpose, two complementary points of view will be adopted here: a coherent detection principle view and a system view.

1.3.1 The coherent detection point-of-view

The first point-of-view consists in focusing on the coherent detection principle itself. In a coherent system, the power of the information-carrying part of the measured signal at the output of the photodetector is determined by the correlation between the backscattered and the LO fields. This means that any physical phenomenon, either atmospheric or optical, causing the propagating beam to lose its spatial coherence will alter the system efficiency.

The first requirements concern the LASER source itself which must therefore produce a perfectly Gaussian and stable beam. An M^2 of 1.3 is a typically reachable value. The polarization between the LO beam and the emitted beam, i.e. after having been modulated and amplified, must also match for the coherent mixing to be efficient.

The backscattered field is obviously not so spatially coherent. The main processes affecting its coherence are the speckle effect, caused by the uncorrelated response of the aerosol particles, and the refractive turbulence produced by local

fluctuations of the refractive index due to atmospheric turbulence. The latter has non negligible effects on the system efficiency, especially when operating at low altitudes and short wavelengths. Its study is therefore essential in determining optimal telescope parameters. By now, no analytical expression taking into account all the uppermentioned processes exist for the system efficiency. Some approximations clearly exists but must be considered with care when dealing with the double-passage problem, i.e. in monostatic configuration, through moderate-to-strong turbulence and with non-unitary M^2 beams propagating through complex optics. The field of the numerical simulation of LASER beam propagation in random media is nowadays dominated by the so-called phase-screen method. This technique decomposes the propagation path of an arbitrarily distributed LASER beam into successive transverse planes, each of them introducing either a structured or a random spatial perturbation of its phase depending, for example, if it passes through a lens or a turbulent slab of atmosphere. With this technique, complex optical systems can therefore be modeled and their performances assessed.

1.3.2 The measurement system point-of-view

This more global view draws the main interactions between different measurement subsystems identified as the most relevant for this study, i.e. the LASER source, the LIDAR system, the scanning system and the vortex parameter estimator. It is possible to highlight how the principal technological decisions, taken for one of them, influence the performance of the others. This section does not aim at being exhaustive but has the major interest to point the need for an integrated simulation of the measurement.

1. The LASER source. For fiber-based sources, a compromise must be found between the PRF, the pulse energy and its duration. It roughly depends on the maximum peak power admissible by fibers. The performance parameters are the pulse energy, the beam quality, the LO stability, the level of artefacts and noises, e.g. the relative intensity noise (RIN), and the pulse temporal profile.
2. The LIDAR system, including here the devices related to the coherent detection and the transmitted beam conditioning. The performance parameters which matter the most are the SNR dependency with range as well as the sensitivity to refractive turbulence.
3. The scanning device. It is characterized by a 3D pattern with a typically nonuniform resolution along the three axis. The performance parameters are the precision on the LOS direction, the time to perform a complete scan as well as the density of measurement points in a given spatial region-of-interest (ROI). They mainly depend on electro-mechanical factors that

will not be covered here.

4. The wake vortex estimator. It goes from the measured signal acquisition to the actual vortex detection and the estimation of its strength and position. The performance parameters are the bias and variance of the radial velocity field and the vortex parameter estimates as well as, for a system in real conditions, the rates of true and false detections.

Respect for LASER source specifications is a determinant factor influencing the overall performance in such a way that the most important efforts are generally concentrated on its optimization. Hence, a decrease of the pulse energy or a deterioration of the M^2 will - *ceteris paribus* - directly affect its SNR profile, the system visibility and eventually increase the estimation variances. Furthermore, estimation algorithms are particularly sensitive to any frequency fluctuations of the reference field as much as they are towards a change in the pulse profile symmetry. This last effect actually induces both an error on the radial velocity estimation for algorithms based on the spectral first-order moment as well as a range misallocation of the velocity estimates.

Moreover, though they usually operate at a low pulse energy, these sources provide a high PRF. The low SNR can therefore be theoretically compensated by pulse accumulation. This assertion may only be valid if the scanning pattern has been designed such that the adjacent LOS groups, of a size at least equal to the minimum requested accumulation level, contain measurement points with highly correlated velocity statistics. If not, information is simply lost due to excessive incoherent averaging. The constraints imposed on the scanner for the accumulation technique to be successful could be reformulated as the pattern allowing the highest and the most isotropic point density around each estimation position. Knowing that feasible scanning patterns are, for mechanical reasons, almost exclusively limited to a small subset of low-order Lissajous curves, it can be understood that optimizing it may not be such a trivial problem.

The telescope parameters, i.e. output beam size, lens size, solid angle and focalization distance, influence not only the SNR profile but also the sensitivity to turbulence. Hence, as soon as the geometrical or geographical constraints have been determined for the detection, these parameters are chosen such that the SNR profile meets certain properties of intensity and smoothness around the security region to cover. Unfortunately, a variation of the turbulence strength over the periods of the day or of the aerosol concentration after a rainfall, for example, will respectively change the SNR level and the system focalization. The telescope parameters have therefore to be optimized not for particular but for all weather conditions, which requires it to be done more statistically than deterministically.

Finally, the complex internal structures of the fluid flow to analyze have an important influence on the LIDAR backscattered signal properties, such

as decorrelation or time fading. It is therefore necessary to adjust the pulse characteristics and the signal analysis parameters in order to allow an optimal detection capability. Wake vortex detection and intensity estimation also require a good compromise between spatial resolution and velocity precision which also depends on the pulse duration.

The combination of all these constraints and observations leads us to the conclusion that the system design necessitates the parallel optimization of all the relevant parameters, with a deep understanding of the sensed atmospheric phenomena. Even if the experiences accumulated in previous projects in the field are necessary, this can only be done by an integrated numerical simulation of all these subsystems with the help of realistic fluid dynamics simulations. This thesis has been carried out in order to meet this precise objective.

1.4 Major contributions

The work performed in this thesis is intimately linked to the specifications of the LASEF and FIDELIO projects for which a prototype had to be developed and validated. Hence, it was mainly concentrated on the optimization of the LASER source parameters, the design of the LASEF telescope as well as the development of wake vortex detection and parameters estimation algorithms. The scanning pattern was a priori fixed in both projects.

The contributions are presented here as answers to a number of questions identified as being fundamental for wake vortex characterization. This therefore constitutes our general methodology which will be presented first.

1.4.1 General methodology

Due to the complexity of the LIDAR measurement, it is particularly difficult to design efficient signal processing and estimation algorithms without considering the following questions:

1. What are the characteristics of the physical phenomena to observe? How do they vary with space and time?
2. How do the system intrinsic parameters influence the estimation process? What are their optimum values with regards to some performance constraints?
3. How do the perturbing processes influence the measurement quality? Is it possible to attenuate their effects?
4. How to integrate all the information gathered from the previous questions to develop efficient estimation algorithms?

There exists two complementary ways to answer them: the numerical simulation approach and the analytical one for which we look for comprehensive equations. Both of them have been exploited in this thesis.

The LIDAR modeling part of this thesis has been realized in close collaboration with L. Bricteux from the Institute of Mechanics, Materials and Civil engineering (IMMC) at UCL. This work has therefore benefited from his realistic large eddy simulations (LES) of the wake vortex flows in different relevant conditions, i.e. in weakly turbulent atmosphere and in ground effect (IGE) with and without cross wind. Their analysis allowed one to answer the first set of questions. These simulations are detailed in his Ph.D. thesis [Bricteux 2008].

As explained in the previous section, the estimation process is at the very end of a complete measurement chain. It is tributary to the measured signal quality, mostly determined by the system intrinsic parameters. To have a LIDAR signal synthesis software at one's disposal is therefore necessary as a tool to study the influence of the system on the estimation process. Hence, algorithm validation can be performed by testing their performances for different system configurations and atmospheric conditions.

This tool must however take into account the most performance-limiting perturbing processes to be actually interesting. We have chosen to concentrate on four of them: the speckle effect, the detection noises, the refractive turbulence as well as the complex structures of the fluid flows. However, as mentioned in Section 1.3, no tractable equations are available for the backscattered beam statistics in the case of moderate-to-strong refractive turbulence. Besides their use for signal synthesis, they are also required for telescope parameters determination when measurements at low altitude are concerned. In particular, Monte Carlo simulations based on the PSM technique were needed to find a telescope design which attenuates the effect of refractive turbulence.

Equations for the LIDAR signal time-frequency distributions does not exist for general conditions but their use would be of great help for completely understanding the measurement process in the one hand, and for wake vortex characterization in the other hand. They have been developed as a complementary tool to numerical simulations. Furthermore, these analytical expressions integrate all the system parameters and can be directly used with CFD simulations.

1.4.2 LIDAR simulation: combining space and time models

An integrated LIDAR simulation software has been developed which combines the classical time-domain technique with the PSM used to compute the instantaneous measurement power and the LES of wake vortices to introduce the signal coherence loss induced by the fast variations of the radial velocity inside vortices. More specifically, it has been designed to allow the following operating modes:

1. The simulation of a coherent Doppler LIDAR system efficiency and SNR with given LASER and telescope parameters and refractive turbulence level. This mode is dedicated to performance analysis of a LIDAR depending on its characteristics and on the atmosphere properties (aerosol concentration, turbulence,...).
2. For a given LIDAR, generation of measured signals by scanning through either LES simulations or vortex algebraic models. This mode is typically used for the signal processing algorithms design and testing.

The LIDAR modeling and simulation are detailed in Chapter 3.

1.4.3 Wind field estimation: about retrievable statistics

Only approximations of the LIDAR periodogram exists for typical wind shear and wind turbulence conditions. One of our first motivation was thus to derive, from a general description of the wind statistics along any LOS, exact expressions of the LIDAR spectrum evolution with time.

This study led us to a complete formulation of the LIDAR signal Wigner-Ville distribution as well as its more general Cohen's class, for which the spectrogram and the Smoothed Pseudo Wigner-Ville distribution (SPWVD) are particular cases. These analytical developments have been realized from the knowledge of the general signal covariance depending on both the LIDAR and the atmospheric parameters. The effect of the signal analysis by a time-frequency observation window has also been taken into account.

This formulation has led to the equations of the retrievable wind distribution at each measurement time from the actual ones, defined at each range along a particular LOS. The spectrum can now be written as the convolution between this new statistical distribution and a spectral kernel expressed as a function of the LASER pulse spectrum and the frequency profile of the analyzing function. Expressions for the two first statistical moments computed from the Cohen's class for wind gradient and Gaussian wind turbulence have been obtained to illustrate these results.

This subject will be covered in Chapter 4.

1.4.4 The WV signal: a theory-supported spectral modeling

Knowing the exact LIDAR SPWVD is a major advantage for estimation algorithm development. This distribution has first been applied to wake vortex radial velocity profiles in order to principally analyze the effect of the pulse duration.

As the SPWVD can be expressed as a convolution product with a known kernel, it can be deconvolved to obtain the retrievable wind statistics. Such an algorithm has therefore been developed. It especially allows to estimate the wake vortex maximum velocity which is an important quantity for WV

characterization. It also permits to distinguish, for a given pulse duration range, the background wind distribution from the wake vortex one.

Finally, an adaptive algorithm has been developed which adapts the spectral model with time depending on a wake vortex detection algorithm. It has been applied to wake vortex detection in a transverse configuration.

The wake vortex signal analysis and related spectral modeling methods are presented in Chapter 5.

1.4.5 Wake vortex detection in an axial configuration

A major issue for onboard systems is the axial detection of wake vortices. At the outcome of the IWAKE project, in which this configuration was concerned, a heuristic bi-Gaussian spectral model has been developed. We will demonstrate that this model is actually a particular case of a more general one developed here. Its robustness and validity for wake vortex detection and characterization have been assessed here based on the theoretical results obtained for the average LIDAR Cohen's class as well as with LIDAR signal simulations achieved with LES simulation of wake vortex in weak atmospheric turbulence.

The signal processing for the FIDELIO project has been carried out in this thesis and the developed algorithms have resulted in successful wake vortex detection during the field tests at Orly airport.

All these observations and results are presented in Chapter 6.

1.4.6 WV characterization: a multimodal inversion technique

Another output of the various analytical developments is the design of a measurement model which can be directly used for wake vortex characterization. Hence, a method for evaluating the parameters, i.e. core positions and circulation, of wake vortices with a ground-based coherent Doppler LIDAR is proposed, assuming a scanning plane perpendicular to the runway. Its principle is to fit a Burnham-Hallock model, modified by the measurement model itself, on the velocity estimates given by the output of a LIDAR simulation module. This inversion technique has been used on the LES of wake vortices in weak turbulence or in ground effect. The robustness of this method has therefore been assessed, even when secondary vortices are present in the sensing volume.

This model-based estimation method is explained in Chapter 7.

1.5 Thesis outline

A complete view of the work performed during this thesis is represented in Fig. 1.7. It describes at the same time how the various contributions are interconnected to produce a complete wake vortex measurement software and

how the present document is structured. In this processing pipeline, only the *algorithms* box on the left corresponds to estimation techniques that are directly usable for actual measurements. The other boxes on its right are numerical simulations that allows their design and validation by providing realistic signals based on known CFD simulations, atmospheric characteristics and LIDAR parameters. The algorithm performance are evaluated by comparing the inputs and the outputs of this pipeline.

This thesis is divided into two parts respectively related to LIDAR simulation and wake vortex characterization. Reading it will make you travel from the optical simulations of the LASER beam propagation to the wake vortex parameters estimation in the transverse configuration. The purpose of the second chapter is to introduce the major aspects of LIDAR measurement in the boundary layer. Most of them are necessary to fully understand the numerical simulation results that constitute the vertebral column of Chapter 3 where the LASEF telescope design is presented. The second part begins with a theoretical study on retrievable wind statistics, in Chapter 4, which is particularized in Chapter 5 for the wake vortex signal. The two last chapters are respectively dedicated to wake vortex detection and characterization.

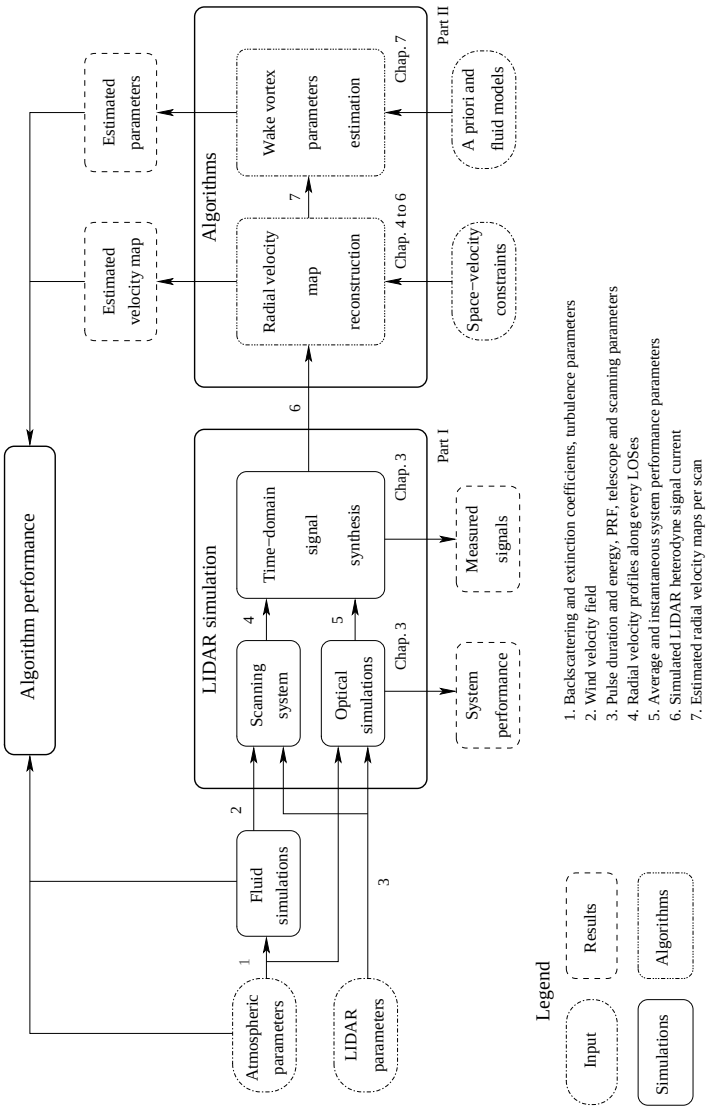


Figure 1.7: Complete pipeline of the contributions with reference to the thesis chapter.

Part I

Coherent LIDAR numerical simulation

The first part focuses on the development of a numerical simulation framework dedicated to the determination of the LIDAR performance parameters as well as the generation of realistic measurement signals. This LIDAR simulation allowed to develop and validate signal processing techniques for wind field and wake-vortex parameters estimation. These algorithms will be described in the second part of this thesis.

Chapter 2 exposes the fundamentals of coherent Doppler LIDAR sensing in the atmospheric boundary layer. The numerical simulation techniques are described into much details in Chapter 3.

LIDAR performance in the boundary layer

PROPAGATION of infrared waves at low altitudes in the troposphere is a complex phenomenon. The LASER beam is subject to molecular and aerosol extinction, scattering, refraction and scintillation. These effects are a manifestation of the meteorological state of the atmosphere and determine the performance of LIDAR systems.

This chapter contains the theoretical basics necessary to have a good understanding on how a LIDAR system works and how its performance is affected by the atmosphere when used for wind profile measurement in the boundary layer. The purpose is to introduce the major concepts that will be exposed in the following chapter on LIDAR performance numerical simulation and realistic measured LIDAR signal synthesis.

In Section 2.1, the influence of the refractive index fluctuation, called optical turbulence, will first be addressed. The propagation of a Gaussian LASER beam through this turbulence is explained in Section 2.2. Section 2.3 briefly presents the interaction of the LASER beam with atmospheric components. The principle of coherent detection is detailed in Section 2.4. Finally, this chapter will describe, in Section 2.5, the most frequently used performance parameters of a coherent Doppler LIDAR system.

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2.1 Optical turbulence in the atmosphere

Atmospheric turbulence has important effects on the propagation of electromagnetic waves as it produces variations in the refractive index. This turbulence is particularly important in the *boundary layer* which is the part of the troposphere where atmospheric dynamics are dominated by the interaction and heat exchange with the earth surface.

In clear air condition, characterized by long-range visibility and relatively low attenuation, the turbulence is the major phenomenon affecting LASER beam propagation. Large scale variations of the refractive index are primarily due to changes in the air density with altitude. In daytime, the ground is hotter than the air and the negative temperature gradients created tends to bend the light rays upwards producing the well-known mirage effect. Conversely, during nighttime, temperature gradients can be positive (the ground is cooler than air), bending the rays downwards. This is the looming phenomenon [Sasiela 1994]. Smaller scale effects commonly appear when temperature inversion occurs. This results in the mixing of the stratified air layers into finer structures thus producing smaller and smaller turbulent eddies until dissipation effects become important. The structure of the turbulence is strongly dependent on the altitude and surface conditions. The turbulence eddies typically have an order of magnitude ranging from millimeters to hundreds of meters.

In this section, the concept of optical turbulence is introduced. Although limited to the homogeneous and isotropic turbulence, this study is fundamental in understanding the effect of turbulence on ground-based LIDAR performance.

2.1.1 Description of the atmospheric turbulence

A viscous fluid is characterized by two distinct flow regimes: laminar and turbulent. Whether it falls in one state or the other depends on the velocity and dimensions of the flow. The laminar regime is characterized by smooth and constant fluid motion and the flow is dominated by viscous forces. On the other hand, the turbulent regime occurs when the flow is dominated by inertial forces. It then loses its uniform characteristics and undergoes irregular and random motions. Random subflows appear called turbulent eddies. Usually, atmospheric flows are in the turbulent regime. The Reynolds number Re is a measure of the flow stability and is expressed as the ratio between the inertial and the viscous forces. It is a nondimensional quantity defined by:

$$Re = v l / \nu, \quad (2.1)$$

where v [m/s] and l [m] are the characteristic velocity and length scale such as the average velocity and the altitude and ν is the kinematic viscosity [m²/s]. When exceeding a given value known as the critical Reynolds number, the fluid motion is completely turbulent and can be described in statistical terms.

The theory of local structure of turbulence of a fluid with a very large Reynolds number was first presented by Kolmogorov in 1941. In his theory, the small-scale structures of turbulence are statistically homogeneous, isotropic, independent of the large-scale structures and are uniquely determined by the average dissipation rate ϵ [$\text{m}^2.\text{s}^{-3}$] and the kinematic viscosity ν . The energy is introduced into the turbulence due to the variation of the average velocity, for instance due to wind shear or meteorological effects. The turbulent flow is composed of turbulent rotating fluid masses (eddies) with different size. The energy is transferred from the larger structures to the smaller ones. It thus forms a continuum from the outer scale L_0 , at which the energy is introduced to the turbulence, to the inner scale l_0 , at which energy is dissipated. This process is called the *Kolmogorov energy cascade*. The inner scale, also called the Kolmogorov scale, is defined by:

$$l_0 = (\nu^3/\epsilon)^{1/4}, \quad (2.2)$$

where ϵ is the dissipation rate which is the rate of conversion of turbulence into heat by molecular viscosity [Stull 1988]. This scale varies from 1 mm to 1 cm in the atmosphere. It inversely depends on ϵ which means that l_0 is smaller for stronger turbulence. Near the ground, the outer scale L_0 is approximatively equal to the altitude. The eddies of size higher than L_0 are considered anisotropic. The range between the outer and the inner scales is called the inertial subrange. Fig. 2.1 represents the turbulence energy spectrum with the wavenumber $\kappa = 1/l$.

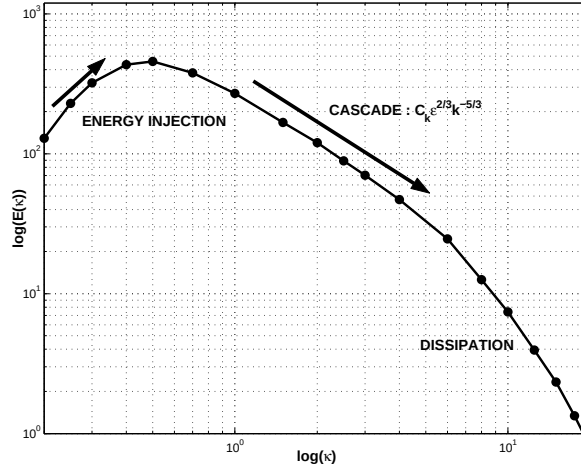


Figure 2.1: Experimental turbulence spectrum : Comte-Bellot and Corrsin experiment (J.F.M. 1971). Taken from [Bricteux 2008]

Only locally Homogeneous and Isotropic Turbulence (HIT) will be considered here. The assumption of statistical homogeneity (the counterpart of stationarity in time) allows one to treat the spatial fluctuations of the atmospheric quantities, such as the wind fluctuating velocity or the refractive index, in terms of covariance functions and power spectral density, using the Wiener-Khintchine theorem. However, homogeneity, which means that the mean value of the field of interest is constant, is not possible over large distances. The concept of local homogeneity is then used to describe a random field for which the difference between the value at two spatial points behaves like a statistically homogeneous field [Andrews 1998].

Hence, the structure function of the random velocity field is defined by the ensemble average of the difference between the radial velocity v_r at space location $\mathbf{r}_1$ and $\mathbf{r}_1 + r \hat{\mathbf{e}}_r$:

$$D_V(\mathbf{r}) = \left\langle [v_r(\mathbf{r}_1 + r \hat{\mathbf{e}}_r) - v_r(\mathbf{r}_1)]^2 \right\rangle \quad (2.3a)$$

$$= C_V^2 r^{2/3}, \quad l_0 \ll r \ll L_0 \quad (2.3b)$$

where $C_V^2 = 2 \epsilon^{2/3}$ is the velocity structure coefficient, the angle brackets indicate an ensemble average over the turbulence and $r = |\mathbf{r}|$ is the absolute distance. C_V^2 is a measure of the intensity of the turbulence. In the inertial subrange, $D_V(\mathbf{r})$ follows the universal 2/3 power law. The spatial power spectrum of the velocity exhibits a -11/3 power law, in the inertial subrange, given by:

$$\Phi_V(\kappa) = 0.033 C_V^2 \kappa^{-11/3}, \quad 1/L_0 \ll \kappa \ll 1/l_0 \quad (2.4)$$

with κ [rad/m], the one-dimensional spatial wavenumber.

When the *frozen atmosphere* Taylor's assumption is used [Andrews 1998], the temporal variations of any quantities, including the refractive index, at a given position $\mathbf{p}$ of the atmosphere are produced by the advection of these quantities by the mean wind speed flow and not by the changes in the quantities themselves. Hence, the spatial statistics can be converted to temporal statistics if the mean wind speed is known. Two time scales of the turbulence can be distinguished, one is associated with the advection and the other is the eddy turnover time. The first one is approximately equal to $L_0/v_\perp$ where $v_\perp$ is the mean velocity transverse to the observation path. The order of magnitude of this time scale is 1 s. The second time scale is slower and has an order of magnitude of 10 s.

2.1.2 Refractive index turbulence

The refractive index is one of the most important parameters governing the propagation of electromagnetic waves in the atmosphere. The refractive index is a complex number. The imaginary part is called the absorption index and

the real part is defined as the ratio between the wave velocity in vacuum and its velocity in the medium. No inhomogeneities in the absorption index have been considered in the present study. The term refractive index will then be used in the rest of the text to refer to the real part. The propagation of an optical wave through a randomly varying refractive index field will be discussed in Section 2.2.

For optical and infrared wavelengths, the Gladstone-Dale empirical law binds the fluctuations of the refractive index with fluctuations in the pressure P and the temperature T :

$$n(r) = 1 + \alpha_1 (1 + \alpha_2 \lambda^{-2}) \frac{P(r)}{T(r)}. \quad (2.5)$$

The coefficients are $\alpha_1 = 77.6 \cdot 10^{-6} \text{ K/mbar}$ and $\alpha_2 = 7.52 \cdot 10^{-3} \text{ m}^2$. This equation describes the behavior of the steady part of the refractive index. In addition to this steady part, there exists stochastic changes mainly due to random variations of the temperature field since local pressure fluctuations can be neglected.

The Kolmogorov theory has been applied to the potential temperature considered as a passive scalar leading to equations for the structure function and the temperature spectrum similar to Eq. 2.3b and Eq. 2.4 but with a new structure constant C_T^2 . The inner scale of the temperature field is given by $l_0 = 5.8 (D^3/\epsilon)^{1/4}$ where $D [\text{m}^2/\text{s}]$ is the thermal diffusivity in air. Fluctuations in the refractive index due to the temperature variations are called optical turbulence. The refractive index at position $\mathbf{r}$ and time t can be expressed by the following equation when separating its mean value n_0 to its fluctuating one n_1 :

$$n(\mathbf{r}, t) = n_0 + n_1(\mathbf{r}, t) \cong 1 + n_1(\mathbf{r}, t), \quad (2.6)$$

where $n_0 = \langle n(\mathbf{r}, t) \rangle \cong 1$ is the mean value given by Eq. 2.5 and $n_1(\mathbf{r}, t)$ is the random deviation from its mean value with $\langle n_1 \rangle = 0$. Moreover, according to the Taylor hypothesis, the time variation of the refractive index is given by the following equation:

$$n(\mathbf{r}, t + \tau) = n(\mathbf{r} - \mathbf{v}_\perp \tau, t), \quad (2.7)$$

where $\mathbf{v}_\perp$ is the mean wind velocity perpendicular to the mean propagation direction of the wave. However, this hypothesis fails when the parallel component of the wind becomes higher than its perpendicular one.

In the case of a locally homogeneous and isotropic turbulence, the structure function, in the inertial subrange, is given by:

$$D_n(\mathbf{r}) = \langle [n(\mathbf{r}_0) - n(\mathbf{r}_0 + r \hat{\mathbf{e}}_r)]^2 \rangle = C_n^2 r^{2/3}, \quad (2.8)$$

where $C_n^2 [\text{m}^{-2/3}]$ is the structure coefficient of the refractive index turbulence and the inner scale is $l_0 = 7.4 (\nu^3/\epsilon)^{1/4}$. The C_n^2 varies from 10^{-17} for *weak turbulence* conditions to 10^{-13} for *strong turbulence* conditions. Along an horizontal

paths, C_n^2 remains essentially constant. However, it decreases with altitude and therefore cannot be assumed constant over vertical or slant observation directions. One of the most widely used models is the Hufnagel-Valley model. It describes the evolution of the C_n^2 with altitude h [m]:

$$C_n^2(h) = 0.00594 (v/27)^2 (10^{-5} h)^{10} \exp(-h/1000) + 2.7 \cdot 10^{-16} \exp(-h/1500) + A \exp(-h/100), \quad (2.9)$$

where A is a nominal value of $C_n^2(0)$ at the ground level and v [m/s] is the wind-speed root mean square (RMS) [Andrews 1998]. Fig. 2.2 represents the evolution of C_n^2 for two values of the nominal ground turbulence levels. For altitude below 10 km, the windspeed v has very little effect so it has been fixed to 20 m/s. Other models exist such as the SLC Day and Night models [Belang 1993].

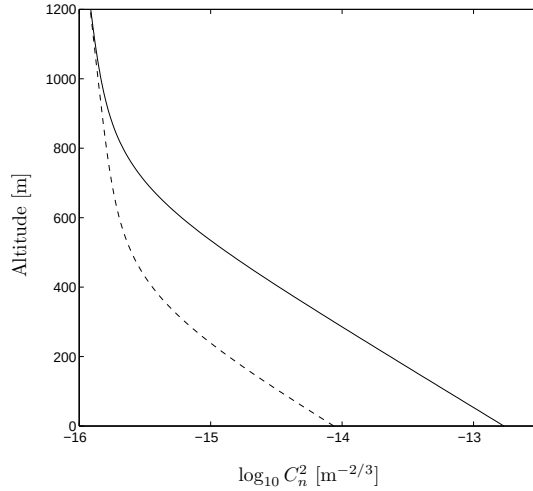


Figure 2.2: Hufnagel-Valley model of the C_n^2 up to 1200 meters for two values of the nominal turbulence level : $A=1.7 \cdot 10^{-13}$ (solid) and $A=8.4 \cdot 10^{-15}$ (dash). The RMS windspeed is 20 m/s.

For an isotropic and homogeneous turbulence, the covariance function of the refractive index $B_n(\mathbf{r}) = \langle [n_1(\mathbf{r}_0) n_1(\mathbf{r}_0 + \mathbf{r})]^2 \rangle$ is related to the power spectrum:

$$\Phi_n(\kappa) = \frac{1}{(2\pi)^3} \int_r B_n(r) e^{-i\kappa r} dr, \quad (2.10)$$

where κ [rad/m] is the wavenumber and r is absolute distance. Nevertheless, since the structure function is usually known, the inverse sine transform gives the desired relation between the structure function D_n and the turbulence

spectrum Φ_n [Sasiela 1994]:

$$\Phi_n(\kappa) = \frac{1}{(2\pi\kappa)^2} \int_0^\infty \frac{\sin(\kappa r)}{\kappa r} \frac{d}{dr} \left(r^2 \frac{dD_n(r)}{dr} \right) dr. \quad (2.11)$$

The well-known Kolmogorov spectrum for the refractive index fluctuations is given by:

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3}, \quad (2.12)$$

This model has a major drawback in that it is only valid in the inertial subrange and presents a singularity at $\kappa = 0$. As the distribution of the refractive-index power among the turbulent eddy cells is globally described by an inverse power law, the smallest cells have the weakest power and the largest cells the strongest. The Von Karman spectrum is also widely used because it integrates the effect of both the outer and the inner scales:

$$\Phi_n(\kappa) = 0.033 C_n^2 \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{11/6}}, \quad 0 \leq \kappa < \infty, \quad (2.13)$$

where $\kappa_m = 5.92/l_0$ and $\kappa_0 = 1/L_0$. In the inertial subrange, this equation is well approximated by the Kolmogorov model. Fig. 2.3 represents the two spectral models with $l_0=1$ cm and $L_0=10$ m. The Von Karman spectrum will be used for numerical simulation of LASER beam propagation.

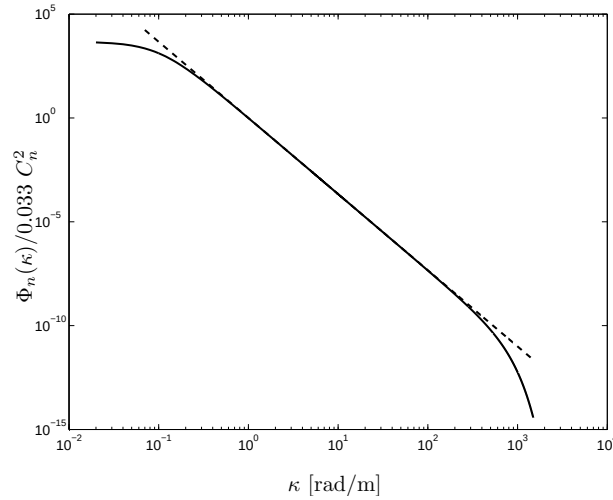


Figure 2.3: Spectral models of refractive index turbulence. Kolmogorov spectrum (dash) and Von Karman spectrum (solid) with $l_0=1$ cm and $L_0=10$ m.

2.2 Propagation in the atmospheric boundary layer

The propagation of electromagnetic waves through the turbulent atmosphere causes important distortions that may significantly reduce the performance of an optical system. In particular, a LASER beam propagating through the atmosphere experiences wandering (random motion of the beam centroid), additional spreading, loss of spatial coherence and scintillation. Moreover, the wave is affected differently by each scale of the refractive turbulence. Hence, scales smaller than the beam size generate irradiance fluctuations although larger ones cause beam wandering. Intuitively, it is clear that the loss of spatial coherence will be a major factor influencing the performance of monostatic systems based on coherent detection and beam wander for any bistatic systems. The knowledge of the various statistical moments of the beam irradiance is therefore mandatory for the design of efficient systems. They mainly depend on the initial LASER beam distribution and on the characteristics of the turbulence.

In this introductory section, the general wave equation for a LASER beam with a wavelength much smaller than the inner scale of turbulence will first be derived. The propagation of a Gaussian beam in free-space is then addressed. Finally, classical methods for evaluating the irradiance moments for both weak and strong fluctuation regimes are explained. It should be noted that no theoretical formulations are available for the moderate fluctuation regime for which the statistics are almost always obtained by numerical simulation with the phase-screen method.

2.2.1 The Maxwell's equations for a random medium

Analyzing the line-of-sight propagation of optical waves in the turbulent atmosphere requires solving Maxwell's equations with a stochastic permittivity ε [Goodman 1968]. In the absence of free charges, Maxwell's equations are given by:

$$\nabla \cdot \varepsilon \mathcal{E} = 0, \quad (2.14a)$$

$$\nabla \cdot \mu_0 \mathcal{H} = 0, \quad (2.14b)$$

$$\nabla \times \mathcal{E} = -\mu_0 \frac{\partial \mathcal{H}}{\partial t}, \quad (2.14c)$$

$$\nabla \times \mathcal{H} = \varepsilon \frac{\partial \mathcal{E}}{\partial t}, \quad (2.14d)$$

where c [m/s] is the speed of light and μ_0 [H m⁻¹] is the permeability. The quantities $\mathcal{E}$ and $\mathcal{H}$ are respectively the electric and the magnetic fields and have a time-harmonic variation with optical frequency ν_L [Hz]. The LASER source wavelength is thus $\lambda = c/\nu_L$ [m]. The electric field of a LASER beam, originating

at $z = 0$ and propagating along the positive z -axis, can be written as:

$$\mathcal{E}(\mathbf{p}, z, t) = E(\mathbf{p}, z, t) \exp(-2\pi i \nu_L t), \quad (2.15)$$

where $E(\mathbf{p}, z, t)$ is the complex amplitude of the field at the transverse coordinate $\mathbf{p} = \{x, y\}$ and z is the distance from the telescope transmitter. The complex amplitude may vary in time with a scale of variation much longer than the harmonic variation.

In the troposphere, it can be assumed that the fluctuations of the permittivity about its mean, noted $\langle \varepsilon \rangle$, are relatively small (typically of the order of 10^{-6}). If ε_1 is the permittivity perturbation, it is therefore written as $\varepsilon = \langle \varepsilon \rangle + \varepsilon_1$ with the condition that $\langle |\varepsilon_1| \rangle \ll \langle \varepsilon \rangle$. In general, the mean permittivity varies in space and time causing large scale refractive effects. Since the propagation distance of the wave is limited to a few kilometers in the present study, this effect won't be taken into account and therefore $\langle \varepsilon \rangle = 1$. For a small perturbation ε_1 , the previous Maxwell's equations can be rewritten as follows [Tatarskii 1971, Sasiela 1994]:

$$\nabla^2 E + k^2 E = -k^2 \varepsilon_1 E + \nabla \cdot (E \cdot \nabla \ln \varepsilon_1) - \frac{2ik}{c} \frac{\partial \varepsilon E}{\partial t} + \frac{\partial^2 \varepsilon E}{\partial t^2}, \quad (2.16)$$

where ∇^2 is the Laplacian operator. Further simplifications arise by considering that the wavelength λ is much smaller than the smallest scale of the turbulence, noted l_0 . With $\lambda \ll l_0$, the two last terms on the right of Eq. 2.16 are negligible compared to the two first ones. Moreover, the maximum scattering angle which is proportional to λ/l_0 is relatively small and so is the coupling between the polarization components of the electric field. The second term on the right of Eq. 2.16 is therefore removed and the electric field can be replaced by any of its polarization components [Fante 1985, Strohbehn 1978]. This remark is valid for any optical system even if the fields emitted by coherent LIDAR are usually linearly polarized. Taking into account the relation between the refractive index and the permittivity and noting that

$$\varepsilon = n^2(\mathbf{p}) \approx 1 + 2n_1(\mathbf{p}), \quad (2.17)$$

gives one the scalar Helmholtz equation:

$$\nabla^2 E + k^2 (1 + 2 n_1) E = 0. \quad (2.18)$$

A considerable number of studies have focused on the resolution of this scalar stochastic differential equation, particularly in the analytical formulation of the first four statistical moments of the field. The knowledge of these moments is of practical importance for the design of any optical system, including LIDAR. The first order moment at a distance z from the source, noted $\langle E(\mathbf{p}, z) \rangle$, determines the coherent portion of the field and the second order moment is

$$M(\mathbf{q}, \mathbf{p}, z) = \langle E(\mathbf{q}, z) E^*(\mathbf{p}, z) \rangle. \quad (2.19)$$

It is usually called the mutual coherence function of the field. It gives the mean irradiance in the particular case of $\mathbf{q} = \mathbf{p}$. The additional spreading due to turbulence as well as the spatial coherence length of the beam are also described by this moment. The fourth order moment,

$$\Gamma_4(\mathbf{q}, \mathbf{p}, \mathbf{u}, \mathbf{v}, z) = \langle E(\mathbf{q}, z) E^*(\mathbf{p}, z) E(\mathbf{u}, z) E^*(\mathbf{v}, z) \rangle, \quad (2.20)$$

determines the irradiance fluctuation and is used to evaluate the scintillation index. The importance of the irradiance fluctuations principally depends on the strength of the refractive index turbulence, described by C_n^2 and defined in the previous section, as well as the propagation distance of the optical wave. An important parameter for describing the effect of turbulence on a propagating wave is the scintillation index, noted σ_I^2 , which is the normalized variance of the irradiance:

$$\sigma_I^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1. \quad (2.21)$$

Theoretical studies are commonly classified according to their domain of validity. Hence, small perturbation theories [Ishimaru 1978], based on either the Born (additive perturbation) or the Rytov (multiplicative perturbation) approximations, are restricted to weak irradiance fluctuations although the asymptotic approximation has been introduced for strong fluctuations. When comparing the small perturbation methods with data obtained from a scintillometer, it has been observed that the Born approximation, in opposition to the Rytov one, only shows good results for extremely weak fluctuation regimes. Moreover, the extended Huygens-Fresnel principle is applicable under all conditions but is limited to the first and second moments for strong fluctuation regimes. This method yields relatively simple solutions for the fourth order moment although the equivalence with other methods has not been established [Andrews 1998]. In the present work, it has been necessary to introduce some basic concepts of the Rytov as well as the extended Huygens-Fresnel methods. Only numerical simulation methods give accurate results for moderate fluctuation. That is why the parabolic equation is used for the phase screen method, detailed in Section 3.1.

2.2.2 Propagation of a Gaussian beam in free-space

The propagation of a Gaussian LASER beam in free-space is described here in order to give some important properties that will be used later [Andrews 1998]. When the beam is not disturbed by turbulence, Eq. 2.18 becomes:

$$\nabla^2 E + k_0^2 E = 0. \quad (2.22)$$

The Huygens-Fresnel principle provides a general formulation of the complex amplitude of the beam at a propagation distance z from the source. The general

solution of Eq. 2.22 can be expressed in terms of the specific solution G^f through the convolution integral:

$$E(\mathbf{p}, z, t) = -2ik \int G^f(\mathbf{p} - \mathbf{u}, R) E(\mathbf{u}, 0, t - z/c) d\mathbf{u}, \quad (2.23)$$

where $\mathbf{u}$ and $\mathbf{p}$ are the transverse coordinate respectively in $z = 0$ and z , and $G^f(\mathbf{p}, \mathbf{u}; R) = G^f(\mathbf{p} - \mathbf{u}; R)$ is also known as the free-space Green's function. It is a spherical wave which gives the contribution at $\{\mathbf{p}, z\}$ of the field at $\{\mathbf{u}, 0\}$. Under the paraxial approximation, it can be expressed as:

$$G^f(\mathbf{p} - \mathbf{u}; z) = \frac{1}{4\pi z} \exp \left[ikz + \frac{ik}{2z} |\mathbf{p} - \mathbf{u}|^2 \right]. \quad (2.24)$$

Eq. 2.23 actually represents a two-dimensional spatial convolution in the transverse direction between the Green's function and the beam profile at $z=0$. The beam propagation can therefore be computed efficiently in the Fourier domain. The field of optics exploiting these properties is called Fourier optics. The plane and spherical wave approximations are not always adequate to characterize diverging or focusing waves. Thus, the Gaussian-beam wave model has been introduced, and is usually limited to its lowest order mode (TEM₀₀ for refractive turbulence propagation analysis. The presence of higher order modes degrades the performance of LIDAR systems. The quality of a LASER beam is therefore described by the parameter M^2 , which is the ratio between the energy in the TEM₀₀ mode and the total energy. Hence, a M^2 close to unity is usually a target for LIDAR designers.

The lowest Gaussian mode of the beam is given by the equation:

$$E_L(\mathbf{u}, 0, t) = g_L(t) e_L(\mathbf{u}, 0, t), \quad (2.25)$$

where $g_L(t)$ [V] is the temporal beam profile and $e_L(\mathbf{r}, 0, t)$ [m⁻¹] is the normalized electric field such that,

$$\int_{-\infty}^{\infty} |e_L(\mathbf{u}, 0, t)|^2 d\mathbf{u} = 1. \quad (2.26)$$

For a Gaussian beam, it is given by,

$$e_L(\mathbf{u}, 0, t) = \frac{1}{\sqrt{\pi} w_L} \exp \left(-\frac{\mathbf{u}^2}{w_L^2} - ik \frac{\mathbf{r}^2}{2F_L} \right), \quad (2.27)$$

with w_L [m], the e^{-1} beam amplitude radius and F_L [m], the radius of curvature which is positive for a focused beam. Collimated beams are obtained by imposing $F_L = \infty$. Inserting Eq. 2.24, 2.25 and 2.27 into the integral equation 2.23 provides an analytical expression for the beam at a distance z from the telescope:

$$E_L(\mathbf{p}, z, t) = \frac{g_L(t - z/c)}{\sqrt{\pi} w_L(z)} \exp \left(ikz - \frac{\mathbf{p}^2}{w^2(z)} - ik \frac{\mathbf{p}^2}{2F(z)} \right), \quad (2.28)$$

where $w(z)$ [m] is the spot size radius and $F(z)$ [m] the radius of curvature along the propagation path. The e^{-1} amplitude radius is given by:

$$w^2(z) = w_L^2 \left[\left(1 - \frac{z}{F_L} \right)^2 + \left(\frac{2z}{kw_L^2} \right)^2 \right]. \quad (2.29)$$

For a collimated beam, the divergence of the beam is inversely proportional to its initial radius w_L and is given by $(\lambda/\pi w_L)$. The irradiance or intensity of the optical wave is defined as the squared magnitude of the field:

$$I_L(\mathbf{p}, z, t) = \frac{c\epsilon_0}{2} |E_L(\mathbf{p}, z, t)|^2, \quad (2.30a)$$

$$= \frac{P_L(t)}{\pi w^2(z)} \exp \left(-\frac{2\mathbf{p}^2}{w^2(z)} \right), \quad (2.30b)$$

where $P_L(t)$ is the time-dependent power profile of the beam. Similarly as for the complex amplitude of the beam, the normalized irradiance is given by:

$$j_L(\mathbf{p}, z, t) = P_L(t)^{-1} I_L(\mathbf{p}, z, t), \quad (2.31)$$

It should be noted that in the literature the optical scalar field is also noted E but corresponds to a normalized electrical field with units $[\text{Wm}^{-2}]$ such that,

$$I_L(\mathbf{p}, z, t) = |E_L(\mathbf{p}, z, t)|^2. \quad (2.32)$$

2.2.3 The Rytov approximation for weak fluctuation

Restricted to weak fluctuation conditions, the Rytov perturbation expansion [Tatarskii 1971, Obukhov 1953] of the random scalar field $E(\mathbf{r}, z)$ propagating in a random medium is of the form:

$$E(\mathbf{p}, z) = E_0(\mathbf{p}, z) \exp(\psi_1(\mathbf{p}, z) + \psi_2(\mathbf{p}, z) + \dots), \quad (2.33)$$

where the i th complex phase perturbation due to turbulence, $\psi_i(\mathbf{p}, z)$, is the i -th Rytov approximation and $E_0(\mathbf{p}, z)$ is the unperturbed field. The phase perturbation can be decomposed into its log-amplitude χ and log-phase S such that $\psi = \chi + iS$.

If the Born's approximation is used, $\chi = \frac{1}{2} \ln(I/E_0^2)$ remains Gaussian distributed and the irradiance fluctuations follow the lognormal distribution:

$$p(I) = \frac{1}{2\sqrt{2\pi}I\sigma_\chi} \exp \left[-\frac{(\chi - \langle \chi \rangle)^2}{2\sigma_\chi^2} \right], \quad (2.34)$$

where σ_χ^2 is the variance of the log-amplitude, related to the scintillation index by:

$$\sigma_I^2 = \exp(4\sigma_\chi^2) - 1. \quad (2.35)$$

The Rytov theory is considered to be valid if the intensity fluctuations are less than 10% of the mean intensity. The mean field is given by

$$\langle E(\mathbf{p}, z) \rangle = E_0(\mathbf{p}, z) \exp(-a z), \quad (2.36)$$

with $a = 0.391 C_n^2 k^2 L_0^{5/3}$. For IR wavelengths, it tends to zero after a few meters. The second order moment, given by Eq. 2.19, leads to the equation of the mean irradiance at identical observation points $\mathbf{r} = \mathbf{p}$:

$$\langle I(\mathbf{p}, z) \rangle = \frac{w_L^2}{w_e^2(z)} \exp\left(-\frac{2\mathbf{p}^2}{w_e^2(z)}\right), \quad (2.37)$$

where $w_e(z)$ is the beam radius at a distance z including the natural divergence of the beam in free-space, $w(z)$, described by Eq. 2.29 and the additional spreading induced by the turbulence. $w_e(z)$ represents the long-term average of the e^{-1} radius of the beam and is given by:

$$w_e^2(z) = w^2(z) \left[1 + 1.33 \sigma_1^2 \Lambda^{5/6}(z)\right], \quad (2.38)$$

where

$$\Lambda(z) = \frac{2z}{k w^2(z)}, \quad (2.39)$$

and σ_1^2 is the Rytov variance which is the scintillation index of an unbounded plane wave propagating through a turbulence given by the Kolmogorov spectrum:

$$\sigma_1^2(z) = 1.23 C_n^2 k^{7/6} z^{11/6}. \quad (2.40)$$

This variance is only valid under the condition of weak irradiance fluctuations. It is however a good measure of the optical turbulence strength in moderate-to-strong fluctuation regimes. It increases either with C_n^2 or the propagation path z or both. The weak fluctuation regime is characterized by $\sigma_1^2 < 1$, the moderate regime by $\sigma_1^2 \approx 1$ and the strong regime by $\sigma_1^2 \gg 1$. The beam enters the saturation regime for $\sigma_1^2 \rightarrow \infty$. For a Gaussian-beam wave, it is necessary to take into account in these limits the diffractive property contained in the parameter Λ [Andrews 1998]:

$$\sigma_1^2 < 1 \quad \text{and} \quad \sigma_1^2 \Lambda^{5/6} < 1. \quad (2.41)$$

If one of these conditions fails, the beam is considered to be in the moderate-to-strong regime. It must be noted that the Rytov variance is proportional to $k^{7/6}$, which tells us that systems operating at smaller wavelengths are more sensitive to turbulence. This is the case, in particular, for fiber-based LASER sources working at $\lambda = 1.55 \mu\text{m}$ in comparison with LIDAR at $2 \mu\text{m}$ or $10 \mu\text{m}$.

The loss in spatial coherence is described by the evolution with range z of the modulus of the complex degree of coherence:

$$\text{DOC}(\mathbf{q}, \mathbf{p}, z) = \frac{|M(\mathbf{q}, \mathbf{p}, z)|}{[M(\mathbf{q}, \mathbf{q}, z)M(\mathbf{p}, \mathbf{p}, z)]^{1/2}} = \exp\left[-\frac{1}{2}D(\mathbf{q}, \mathbf{p}, z)\right], \quad (2.42)$$

where $D(\mathbf{q}, \mathbf{p}, z)$ is the structure function. The spatial coherence radius ρ_0 is defined as the e^{-1} value of the DOC function which corresponds to $D(\rho_0, z) = 2$. Again, in the case of an unbounded plane wave propagating in a Kolmogorov turbulence, the coherence radius is:

$$\rho_0(z) \cong \left[1.45 k^2 \int_0^z C_n^2(z') (1 - z'/z)^{5/3} dz' \right]^{-3/5}. \quad (2.43)$$

For a constant turbulence strength, the last equation becomes:

$$\rho_0(z) = (1.46 C_n^2 k^2 z)^{-3/5}. \quad (2.44)$$

It decreases with the propagation distance at a rate depending on the turbulence strength. For strong fluctuation conditions, it eventually becomes smaller than the inner scale of the turbulence. The coherence radius for a Gaussian beam in a turbulence described by a Von Karman spectrum is written analytically in [Andrews 1998].

Figure 2.4 gives the spatial coherence length of a plane wave propagating in a turbulent medium characterized by a Kolmogorov spectrum (no inner scale effect) and for different values of the refractive-index structure parameter C_n^2 . It can be observed that, as the intensity of the turbulence increases, the coherence length can be lower than the inner scale of the turbulence l_0 (in the order of 1 to 10 mm for atmospheric turbulence).

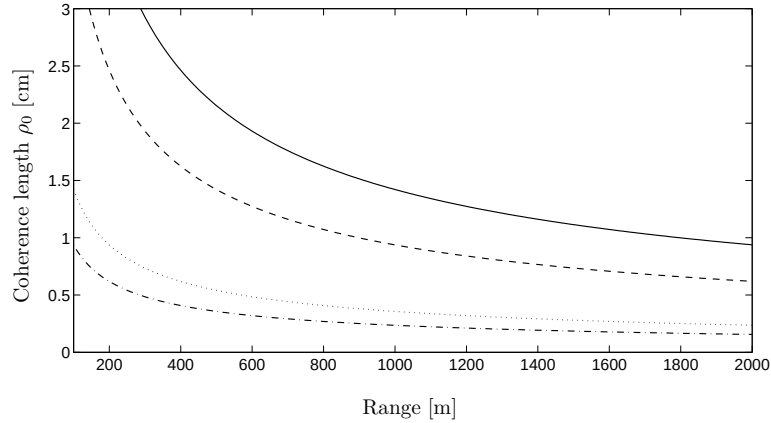


Figure 2.4: Spatial coherence length for a plane wave as a function of range at $C_n^2=5.10^{-14}$ (solid), $C_n^2=10^{-13}$ (dash), $C_n^2=5.10^{-13}$ (dot), $C_n^2=10^{-12}$ (dash-dot).

2.2.4 The strong fluctuation theories

The strong fluctuation regime is defined as soon as one of the conditions in Eq. 2.41 is no longer respected.

Within the extended Huygens-Fresnel theory, Eq. 2.23 giving the field at distance R can be rewritten using the Green's function for random media:

$$G(\mathbf{p}, \mathbf{u}; z) = \frac{1}{4\pi z} \exp \left[ikz + \frac{i}{2} \frac{k}{z} |\mathbf{p} - \mathbf{u}|^2 + \psi(\mathbf{p}, \mathbf{u}) \right], \quad (2.45)$$

where ψ is the phase perturbation. The distance z_0 is defined as the propagation distance at which the spatial coherence radius becomes smaller than the inner scale of turbulence [Andrews 1998] as:

$$z_0 = (C_n^2 k^2 l_0^{5/3})^{-1}. \quad (2.46)$$

Using the extended Huygens-Fresnel principle and under the condition $z \gg z_0$, the mean irradiance has an effective beam radius given by,

$$w_e^2(z) = w^2(z) [1 + 5 q(z) \Lambda/3], \quad (2.47)$$

where the parameter q is,

$$q(z) = \frac{R_F^2}{\rho_0^2} = 1.22(\sigma_1^2)^{6/5}, \quad (2.48)$$

and R_F is the Fresnel distance $R_F = (z/k)^{1/2}$. According to [Andrews 1998], Eq. 2.47 is sufficiently accurate to predict the effective spot size under nearly all atmospheric turbulence conditions.

2.2.5 The parabolic approximation of the wave equation

The first step in obtaining the parabolic approximation of the wave equation is to consider that as the wave propagates in the z direction, its phase progresses as ikz . Therefore, it comes that,

$$\mathcal{E}(\mathbf{p}, z, t) = E(\mathbf{p}, t) \exp(-2\pi i \nu_L t) \exp(ikz), \quad (2.49)$$

and using the paraxial approximation given by,

$$\left| \frac{\partial^2 E}{\partial z^2} \right| \ll \left| 2ik \frac{\partial E}{\partial z} \right|, \quad (2.50)$$

Eq. 2.18 becomes

$$\nabla_{\perp}^2 E + 2ik \frac{\partial E}{\partial z} + 2k^2 n_1 E = 0, \quad (2.51)$$

where $\nabla_{\perp}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the transverse part of the Laplacian operator. Equation 2.51 is known as the *parabolic or Fresnel approximation* of the wave equation [Ishimaru 1978]. An integral equation solution is obtained from Eq.2.51 which is useful in understanding the limitations of the parabolic equation [Tatarskii 1971]:

$$E(\mathbf{p}, z) = E(\mathbf{p}, 0) + \frac{k^2}{4\pi} \int_0^z \left[\iint_{-\infty}^{\infty} G(\mathbf{p}, \mathbf{u}; z - z') \varepsilon_1(\mathbf{u}, z') E(\mathbf{u}, z') d\mathbf{u} \right] dz', \quad (2.52)$$

where $G(\mathbf{p}, \mathbf{u}; z - z')$ is the Green's function in free-space. The solution of Eq. 2.51 is of the first order in z and its solution at a given distance z is therefore completely determined by the inhomogeneities located at $z' < z$. Hence, the reflection of waves due to the permittivity is neglected whereas these reflections are present in the complete wave equation.

The method used to establish the various statistical moments of the optical field from this parabolic equation is usually considered to be more fundamental [Andrews 1998] than other methods. However, no analytical formulations have been obtained by this method for moments higher than the second order. This equation is used as a basis for numerical simulation via the split-step method exposed in Chapter 3.

2.2.6 Elements of optical scintillation theory

The non-homogeneous character of the refractive index field has been described by means of geometrical optics as a collection of random positive and negative lenses with various sizes. With this analogy, each turbulent eddy with a given spatial scale l induces a focalization or a defocalization of the beam as a lens with diameter l . Small turbulent eddies compared to the beam size will induce local diffraction whereas large turbulence scales induce refraction, e.g. beam deviation. Moreover, as the beam size evolves with the propagation distance, the effect of the turbulent eddies will depend on their position in the propagation path. Hence, small scales close to the telescope won't have the same influence on the beam coherence than those far from it.

As detailed above, the Rytov variance increases exponentially with C_n^2 and z . However, the scintillation index which is a normalized variance, reaches a maximum above unity in the regime characterized by random focusing before tending to it in the so-called saturation regime. This behavior is explained in [Andrews 1999]. Hence, for an increasing path length or inhomogeneity strength, the irradiance first experiences the maximum focusing caused by large-scale inhomogeneities before being dominated by multiple scattering, thus reducing its spatial coherence. The fluctuations begin to saturate and the scintillation index reaches the unit value from above. The wave propagation in the saturation regime is described by the asymptotic theory which gives

convenient equations for the influence of refractive turbulence on LIDAR performance [Frehlich 1991].

Based on the weak-turbulence theory, Tatarskii predicted that the correlation length of the irradiance fluctuations in the tranverse plane is on the order of magnitude of the first Fresnel zone $R_F = \sqrt{z/k}$. However, for moderate-to-strong fluctuation regimes, the correlation length decreases with increasing value of the Rytov variance. Moreover, a large residual correlation tail emerges at large distances [Andrews 1999]. In strong fluctuation regimes, the correlation length is determined by the spatial coherence radius ρ_0 as well as the width of the residual tail, characterized by the scattering disk of size $L/k\rho_0$. The turbulent spatial scale that have a major influence on the LASER beam are the scales lower than ρ_0 for the diffractive effects and those higher than the scattering disk size for the refractive effect. The turbulent eddies with a scale between these two values have almost no influence on the scintillation process. Figure 2.5 shows the evolution of the characteristic coherence lengths for two

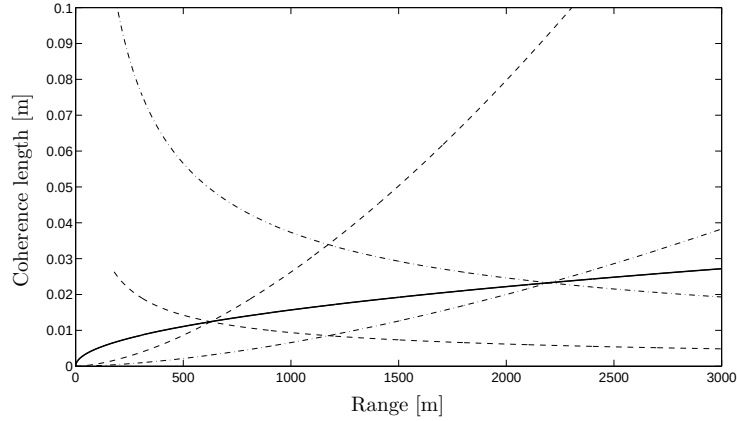


Figure 2.5: Evolution with range of the characteristic coherence lengths describing the scintillation phenomena for a $1.55 \mu\text{m}$ LASER beam for a moderate turbulence level $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ (dash-dot) and strong turbulence level $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash): length of the first Fresnel zone (solid, thick), coherence radius ρ_0 (decreasing curves) and the scattering disk size (increasing curves).

turbulence levels. At $C_n^2 = 10^{-13} \text{ m}^{-2/3}$, all the turbulence scales have an effect on the beam coherence before 600 m, in particular those with the same size than the first Fresnel zone. After this range, the eddies are divided into two classes, one with sizes lower than ρ_0 and the other with sizes higher than the scattering disk size. At 2 km, for example, the turbulent scales between 5 mm and 8 cm have no influence.

2.3 LASER beam interaction with atmospheric components

When propagating through the atmosphere, the LASER beam interacts with the atmospheric constituents, i.e. the aerosol particles and the molecules. This interaction induces both diffusion and absorption of the wave and results in a decrease of the energy in the forward direction. A fraction of the diffused energy is scattered backward towards the LIDAR telescope and is measured by the photodetector. This backscattered wave contains interesting information about the atmospheric particles typically estimated such as their concentration or velocity. This signal-to-noise ratio determining the quality of the system measurement is linearly influenced by the backscattering coefficient. The extinction of the wave in the forward direction characterized by the optical transmission is given by the energy loss caused by both the absorption and the diffusion over all directions. The total extinction coefficient $\alpha(z)$ [m^{-1}] at distance z from the transmitter is given by the following summation:

$$\alpha = \beta + \delta, \quad (2.53)$$

where β is the total diffusion from the molecules and aerosol particles and δ is the total absorption coefficient. The molecular extinction for infrared propagation is primarily a function of the absolute humidity. Thus LIDAR systems do not operate at high water vapor concentrations [Zeisse 2000]. In clear air conditions and for a wavelength of $1.55 \mu\text{m}$ wavelength, the molecular extinction is low providing a good optical transmission. Moreover, the backscattered energy is mainly dependent on the aerosol particle concentration which can be much lower for particular weather conditions such as during or after rain or dense fog.

The analytical solution of Maxwell's equations for scattering of electromagnetic waves by particles is given by Mie [van de Hulst 1981]. In this theory, the particles are assumed to be conducting spheres with typical diameter D and are homogeneous, i.e. characterized by a single refractive index n . They are also considered as embedded in an homogeneous medium. An important parameter x is introduced and is defined as the ratio between the particle diameter and the wavelength λ :

$$x = \frac{2\pi D}{\lambda}. \quad (2.54)$$

This parameter is used for the calculation of the various cross-sections, i.e. the absorption and the backscattering cross-sections. This theory is valid for any value of x . However, other solution exists such as the Rayleigh scattering, limited to the case $x \ll 1$, and the geometrical optic for $x \gg 1$. Given that the wavelengths considered by coherent Doppler LIDAR are around a few micrometers in length, the Rayleigh theory is used to explain the LASER interaction with the molecules and the Mie theory for the interaction with the aerosol particles.

The two following subsections focus on the calculation of the backscattering

coefficient β_π [$\text{m}^{-1}\text{sr}^{-1}$] for both molecules and aerosol particles as well as the Doppler effect induced by their movement.

2.3.1 Interaction with the molecules

The Rayleigh's theory predicts that the intensity of the light scattered by the molecules varies as a function of λ^{-4} . For standard pressure and temperature conditions, the backscattering coefficient of the molecules $\beta_{\pi,\text{mol}}$ is given by [Valla 2005]:

$$\beta_{\pi,\text{mol}}(\lambda) \approx \frac{1.272}{\lambda^4} 10^{-29} \text{m}^{-1}\text{sr}^{-1}. \quad (2.55)$$

For a wavelength of $1.55 \mu\text{m}$, one obtains $\beta_{\pi,\text{mol}}(\lambda) \approx 2.210^{-8} \text{m}^{-1}\text{sr}^{-1}$. The next section will show that this value cannot be neglected compared to the value obtained for aerosol particles.

Molecules, as well as aerosol particles, are subject to high velocity thermal random motion, known as Brownian motion, for which the velocity distribution is described by the isotropic Maxwell-Boltzmann density function $f(v)$ given by,

$$f(v) = 4\pi \left(\frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} v^2 \exp \left(-\frac{mv^2}{2k_B T} \right), \quad (2.56)$$

where m is the particle mass, T is the thermodynamic temperature and k_B is the Boltzmann constant. As the molecules have a much lower mass than the aerosol particles, they are characterized by a large range of velocities. Therefore, the LIDAR wave backscattered from air molecules possesses a very large spectral width. The spectral width is typically of a few hundreds of MHz and is thus much larger than the spectrum of a typical LASER pulse of a few hundreds of nanoseconds. Due to the low signal-to-noise ratio, the Rayleigh scattering is typically below the noise level.

2.3.2 Interaction with atmospheric aerosol particles

Aerosols are solid or liquid particles in suspension that can be classified into primary and secondary aerosols. Primary aerosols are mineral, dust, salt or soot while secondary aerosols results from chemical reactions. Their size ranges from 10 nm to more than 100 μm . This range of sizes represents scales from a gathering of a few molecules to the size where the particles no longer can be carried by the gas. The highest concentrations of aerosols are usually found in urban areas where it can be as high as 10^7 to 10^8cm^{-3} . Aerosols have a wide range of shapes and sizes and must therefore be described by their size distributions.

The scattering cross-section σ_s is defined as the portion of the incident wave energy scattered in all the directions. In the same way, σ_a is the portion of the

incident wave energy absorbed by the particle. By conservation of energy, the extinction cross-section, σ_e can be obtained by $\sigma_e = \sigma_s + \sigma_a$. The efficiency factors of the Mie theory are given by these cross-sections normalized by the geometrical cross-section of the spherical particle with a radius r . They are given by:

$$Q_e = \frac{\sigma_e}{\pi r^2}, \quad Q_a = \frac{\sigma_a}{\pi r^2} \quad \text{and} \quad Q_s = \frac{\sigma_s}{\pi r^2}. \quad (2.57)$$

Similarly, we define the aerosol backscattering coefficient $\beta_{\pi,a}$ which depends on the backscattering geometrical cross-section Q_b and on the aerosol size distribution,

$$\beta_{\pi,a} = \int_0^\infty \pi r^2 Q_b(\nu, r, n) \frac{dN(r)}{dr} dr, \quad (2.58)$$

where $N(r)$ is the total number of particles of size r per cm^3 and ν is the wave optical frequency. For the wavelength of interest and for an altitude of up to 1 km, a range of values for this coefficient are: $\beta_{\pi,a} \approx 10^{-7} - 10^{-6} \text{ m}^{-1}\text{sr}^{-1}$. Note that depending on the atmospheric condition, this coefficient can vary much more than this value. Moreover, as the aerosol particle concentration decreases with altitude, the measured backscattered energy from aerosols decreases similarly and molecular backscattering eventually becomes dominant. However, the total backscattering coefficient decreases and so does the signal-to-noise ratio. Thus LASER sources with higher energy are required at low altitude.

It is also interesting to quantify how well the aerosol particles follow the fluid flow to be measured. In other words, the question is: how much the Doppler frequency shift they induce on the backscattered wave is representative of the exact fluid radial velocity? The quantity that describes the time required for a particle to adjust or relax its velocity to new set of forces is called the relaxation time τ_r of the particle and is given by [Bricteux 2008]:

$$\tau_r = \frac{\rho_p d_p^2}{18 \mu}, \quad (2.59)$$

where ρ_d and d_p are the density and the diameter of the spherical particle and μ is the dynamic viscosity of the fluid. This is valid if the particle motion is in Stokes regime, i.e satisfying the condition

$$Re_p = \frac{\rho U_d d_p}{\mu} < 1, \quad (2.60)$$

where ρ is the fluid density. Considering an aerosol particle with diameter $d_p = 1 \mu\text{m}$ and density $\rho_p = 1 \text{ g/cm}^3$ leads to a relaxation time of $3 \cdot 10^{-8} \text{ s}$. This value, combined with a typical order of magnitude of particle speeds in atmospheric winds or vortex flows, $U_d \approx 10 \text{ m/s}$, gives one $Re_p = 0.66$. This means that aerosols are good tracers for the flows of interest. For $\lambda = 1.55 \mu\text{m}$, a radial velocity of 1 m/s produces a Doppler shift of 1.29 MHz .

2.4 Fundamental equations of the coherent Doppler LIDAR

In the previous sections, it has been described how a LASER beam propagates through a turbulent atmosphere and how it is backscattered toward the LIDAR receiver by the atmospheric components. This section details how a coherent Doppler LIDAR system works with a particular emphasis on the calculation of the LIDAR signal properties. It is mainly based on the work of Kavaya and Frehlich given in [Frehlich 1991].

2.4.1 Coherent detection

Figure 2.6 represents the detection principle of a coherent LIDAR. The telescope

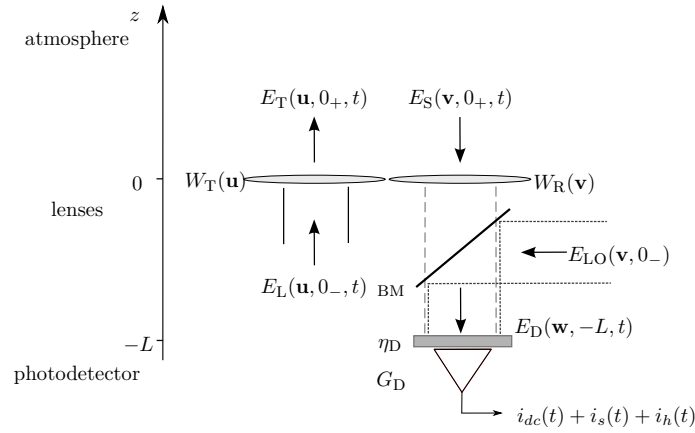


Figure 2.6: The coherent detection principle

configuration is assumed to be monostatic either monoaxial or biaxial, i.e. the emitted beam and the backpropagated beam from the atmosphere travel over essentially the same path. The transmitter and the receiver are modeled by a lens followed by a truncation window. The propagation distance z is defined such that $z = 0$ at the transmitter lens. The beam before the lens is at a distance $z = 0_-$ and the beam after the propagation through the lens at $z = 0_+$. The transverse coordinates at the emitter, target, receiver and detector plane are respectively denoted $\mathbf{u}$, $\mathbf{p}$, $\mathbf{v}$ and $\mathbf{w}$.

The continuous local oscillator (LO) that is used as a reference field for the coherent detection, i.e. optical mixing, has a power P_{LO} which is a fraction of the LASER source power. As it is a monochromatic wave, it is mainly described

by its space-function e_{LO} , defined in the receiver aperture by the transverse coordinate $\mathbf{v}$:

$$\mathcal{E}_{\text{LO}}(\mathbf{v}, 0_-) = \left(\frac{2P_{\text{LO}}}{\varepsilon_0 c} \right)^{1/2} e_{\text{LO}}(\mathbf{v}, 0_-) \exp(-j\omega_{\text{LO}}t), \quad (2.61)$$

where $\omega_{\text{LO}} = 2\pi\nu_{\text{LO}}$ is the angular frequency of the LO field, $\nu_{\text{LO}} = c/\lambda$ is the frequency of the source, c is the speed of light and ε_0 [Fm⁻¹] is the permittivity of free-space.

The wave field incident upon the transmitter aperture at $z = 0_-$, noted $\mathcal{E}_{\text{L}}(\mathbf{u}, 0_-, t)$, is an amplified and frequency shifted version of the LASER source field. It is also modulated in time in order to allow range-dependent analysis. It is given by

$$\mathcal{E}_{\text{L}}(\mathbf{u}, 0_-, t) = E_{\text{L}}(\mathbf{u}, 0_-, t) \exp(-j\omega_{\text{L}}t), \quad (2.62)$$

where $\omega_{\text{L}} = 2\pi\nu_{\text{L}}$ is the angular frequency, ν_{L} is the beam wave frequency. The frequency difference between the LO and emitted wave is called the intermediate frequency and is noted f_{IF} :

$$f_{\text{IF}} = \nu_{\text{L}} - \nu_{\text{LO}}. \quad (2.63)$$

As previously seen, the complex amplitude of the field can be written as the product of a time-dependent function $g_{\text{L}}(t)$ [V] and a space-dependent normalized complex amplitude $e_{\text{L}}(\mathbf{u}, z)$ [m⁻¹]:

$$E_{\text{L}}(\mathbf{u}, 0_-, t) = g_{\text{L}}(t) e_{\text{L}}(\mathbf{u}, 0_-), \quad (2.64)$$

with

$$g_{\text{L}}(t) = \left(\frac{2U_{\text{L}}}{\varepsilon_0 c} \right)^{1/2} p(t). \quad (2.65)$$

The quantity U_{L} [J] is the LASER pulse energy and $p(t)$ [s^{-1/2}] is the normalized complex amplitude of the pulse with

$$\int_{-\infty}^{\infty} |p(t)|^2 dt = 1. \quad (2.66)$$

The pulse profile is assumed to be Gaussian with 1/e intensity radius σ_{p} :

$$p(t) = (\sqrt{\pi}\sigma)^{-\frac{1}{2}} \exp\left(-\frac{t^2}{2\sigma_p^2}\right). \quad (2.67)$$

The pulse duration at 1/e is defined by $\tau_{\text{p}} = 2\sigma_{\text{p}}$ and its FWHM (full width at half maximum) duration is

$$\Delta t = 2\sqrt{\ln(2)}\sigma_{\text{p}}. \quad (2.68)$$

The transmitter emission has a frequency spread that is limited by the Fourier transform of the pulse duration. It is assumed that the pulse profile

varies slowly compared with the period of the optical field. The emitted pulse power [W] is given by

$$P_L(t) = \frac{\varepsilon_0 c}{2} \int_{-\infty}^{\infty} |\mathcal{E}_L(\mathbf{u}, z, t)|^2 d\mathbf{u} \quad (2.69a)$$

$$= U_L |p(t)|^2, \quad (2.69b)$$

where $|E|$ denotes the absolute value of the beam.

This beam then propagates through the transmitter aperture to produce the emitted beam, defined at $z = 0_+$ as

$$\mathcal{E}_T(\mathbf{u}, 0_+, t) = W_T(\mathbf{u}) E_L(\mathbf{u}, 0_-, t) \exp(-j\omega_L t), \quad (2.70)$$

where $W_T(\mathbf{u})$ is a space-dependent function taking into account the truncation by the transmitter aperture as well as the focalization or collimation of the beam. The fraction of the power P_L transmitted through this aperture is noted T_T . The transmitted power sent through the atmosphere is therefore,

$$P_T(t) = T_T P_L(t). \quad (2.71)$$

The emitted beam is scattered by the illuminated aerosol particles along the line-of-sight and a portion of the energy is sent back to the LIDAR receiver. The effective collecting area of the receiver, noted A_R [m²], is obtained by integrating the receiver transmittance over the receiver plane:

$$A_R = \int_{-\infty}^{\infty} |W_R(\mathbf{v})|^2 d\mathbf{v}. \quad (2.72)$$

The solid angle presented by the LIDAR receiver aperture at range z is given by $\Omega(z) = A_R z^{-2}$. This parameter represents the fact that the farther the position $\{\mathbf{p}, z\}$ of an atmospheric particle the smaller is the backscattered energy intercepted by the telescope.

The backscattered field after propagation through the receiver aperture is,

$$\mathcal{E}_s(\mathbf{v}, 0_-, t) = E_s(\mathbf{v}, 0_-, t) \exp(-j\omega t + j\theta), \quad (2.73)$$

where θ is the random phase of the backscattered field compared to the LO field, $\omega = \omega_L + \omega_D$ is the pulsation of the backscattered field and $\omega_D = 2\pi f_D$ is the pulsation difference induced by the atmospheric particles velocity. This field is then focalized on the surface of the photodetector at $z = -L$.

The total field illuminating the photodetector surface is given by the summation of the backscattered and the local oscillator fields:

$$\mathcal{E}_D(\mathbf{w}, -L, t) = \mathcal{E}_s(\mathbf{w}, -L, t) + \mathcal{E}_{LO}(\mathbf{w}, -L, t). \quad (2.74)$$

The irradiance [Wm^{-2}] of the resulting field on the detector plane is obtained from Eq. 2.61 and Eq. 2.73:

$$I_D(\mathbf{w}, t) = \frac{\varepsilon_0 c}{2} |E_{LO}(\mathbf{w}, t)|^2 + \frac{\varepsilon_0 c}{2} |E_s(\mathbf{w}, t)|^2 + \varepsilon_0 c E_s(\mathbf{w}, t) E_{LO}^*(\mathbf{w}, t) \exp(j\Delta\omega t + \theta_s), \quad (2.75)$$

where $\Delta\omega$ is the difference in pulsation between the backscattered and the LO fields:

$$\Delta\omega = \omega - \omega_{LO} = 2\pi f_{IF} + \omega_D. \quad (2.76)$$

The signal current [A] at the output of the photodetector is proportional to the irradiance at its surface:

$$i(t) = SG_D \int_D I_D(\mathbf{w}, t) d\mathbf{w}, \quad (2.77)$$

where $S = \eta_Q e / \hbar \nu$, G_D and η_Q are respectively the sensitivity [A/W], the gain and the quantum efficiency [e^- /photon] of the photodetector, e is the electronic charge [C/ e^-] and $\hbar$ [Js] is the Planck's constant. The quantum efficiency is supposed to be uniform over the surface of the detector. Combining Eq. 2.75 and Eq. 2.77 produces three terms corresponding respectively to the direct current i_{dc} from the LO field, the direct detection signal current i_s from the backscattered field and the heterodyne current i_h which has to be analyzed to estimate the aerosol particles velocity:

$$i(t) = i_{dc}(t) + i_s(t) + i_h(t). \quad (2.78)$$

The direct detection signal measured at the output of the detector is then

$$i_s(t) = \frac{\varepsilon_0 c}{2} S \int_D |E_s(\mathbf{w}, t)|^2 d\mathbf{w}. \quad (2.79)$$

The third term of Eq. 2.75 produces the intermediate-frequency current signal with equation

$$i_h(t) = G_D S \varepsilon_0 c \Re \left\{ \int_D E_s(\mathbf{w}, t) E_{LO}^*(\mathbf{w}, t) d\mathbf{w} \right\} e^{(j\omega_{IF} t + j\theta)}, \quad (2.80)$$

where $\Re$ indicates the real part and $\omega_{IF} = 2\pi f_{IF}$. As the heterodyne current is proportional to the backscattered field which is a random Gaussian variable, it is itself a centered random Gaussian variable. Its variance will be computed in Section 2.4.3 after introducing the backpropagated LO beam formulation.

2.4.2 The backscattered field

The total backscattered field is obtained by integrating the contribution of all the illuminated particles along the line-of-sight, i.e. over $\mathbf{p}$, z and the backscattering

cross-section of the particle σ_b [$\text{m}^2 \text{sr}^{-1}$]. As these contributions are known to be statistically independent, the backscattered irradiance is given by the summation of all the particles' backscattered wave irradiances. The atmospheric backscatter coefficient β [$\text{m}^{-1} \text{sr}^{-1}$] at a given location $(\mathbf{p}, z)$ is the number of particles per unit of volume per unit σ_b ,

$$\beta(\mathbf{p}, z) = \int_0^\infty \sigma_b N(\sigma_b; \mathbf{p}, z) d\sigma_b, \quad (2.81)$$

with $N(\sigma_b; \mathbf{p}, z)$, the density of particles per unit volume per unit σ_b . This quantity takes into account the backscattering by both the molecules, previously defined by $\beta_{\pi, \text{mol}}$, and the aerosols, defined by $\beta_{\pi, a}$. It is thus given by

$$\beta(\mathbf{p}, z) = \beta_{\pi, \text{mol}}(\mathbf{p}, z) + \beta_{\pi, a}(\mathbf{p}, z). \quad (2.82)$$

It is commonly assumed that the backscatter coefficient is uniform in the transverse direction over an area defined by the beam size, i.e. $\beta(\mathbf{p}, z) = \beta(z)$. The following equation gives ones the backscattered field,

$$\begin{aligned} E_s(\mathbf{v}, 0_+, t) = & \lambda \int_0^\infty K(z) \beta^{1/2}(z) \iint_{-\infty}^\infty E_T(\mathbf{u}, 0, t - 2z/c) \\ & \times G(\mathbf{p}; \mathbf{u}, z) G^*(\mathbf{v}; \mathbf{p}, z) \\ & \times \exp(j\theta(\mathbf{p}, z) - 4\pi j v_R(\mathbf{p}, z)t/\lambda) d\mathbf{u} d\mathbf{p} dz, \end{aligned} \quad (2.83)$$

where $G(\mathbf{p}; \mathbf{u}, z)$ is the Green function through a turbulent atmosphere with no extinction, $\theta(\mathbf{p}, z)$ is the random phase of the backscattered field, $v_R(\mathbf{p}, z)$ is the radial velocity of the particle defined as the velocity component parallel to the direction of propagation and $K(z)$ is the dimensionless one-way irradiance extinction at wavelength λ defined by

$$K(z) = \exp \left[- \int_0^z \alpha(z') dz' \right], \quad (2.84)$$

where $\alpha(z)$ [m^{-1}] is the linear extinction coefficient along the propagation path. Using the independence property between the response of two different particles, the mutual coherence function is obtain by [Frehlich 1991],

$$\begin{aligned} M_s(\mathbf{v}_1, \mathbf{v}_2, 0, t) = & \lambda^2 \int_0^\infty \beta(z) K^2(z) \iiint_{-\infty}^\infty M_T(\mathbf{u}_1, \mathbf{u}_2, 0_+, t - 2z/c) \\ & \times \langle G(\mathbf{p}; \mathbf{u}_1; z) G^*(\mathbf{p}; \mathbf{u}_2; z) \\ & \times G(\mathbf{v}_1; \mathbf{p}; z) G^*(\mathbf{v}_2; \mathbf{p}; z) \rangle d\mathbf{u}_1 d\mathbf{u}_2 d\mathbf{p} dz. \end{aligned} \quad (2.85)$$

The fourth moment of the Green's function appears in Eq. 2.85, but it has no analytical solution for moderate to strong fluctuation regimes.

2.4.3 The back-propagated LO field

For convenience in calculation and numerical simulation, the heterodyne mixing is usually viewed as taking place at the target plane. This is an application of the reciprocity theorem which allows to express the fields in the target plane in terms of the fields in the receiver plane. The LIDAR return is therefore given as the overlap integral of the transmitted beam and a backpropagated local oscillator beam, noted BPLO, at the target plane [Belmonte 2000a].

This formulation reduces the problem of calculating the backpropagation of the transmitted field, which is a random field, and its propagation inside the receiver. This reduces the complexity of simulating the interactions with the aerosol particles. Each particle generates a spherical wave with random phase and amplitude. The envelope of the backpropagated field is thus made of the combination of all the secondary waves from each of them. Using this formalism,

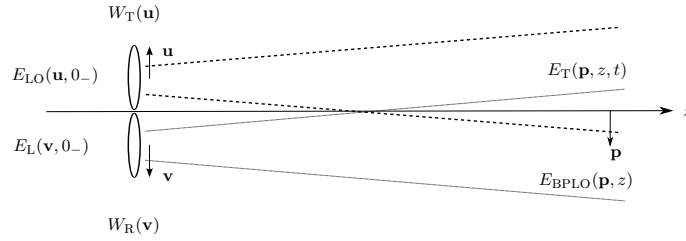


Figure 2.7: The back-propagated LO field principle.

it comes that $G(\mathbf{p}; \mathbf{v}, z) = G^*(\mathbf{v}; \mathbf{p}, z)$ and the intermediate-frequency current in Eq. 2.80 becomes,

$$i_h(t) = G_D S \varepsilon_0 c \Re \left\{ \int_D E_S(\mathbf{p}, z, t - z/c) E_{BPLO}^*(\mathbf{p}, z) d\mathbf{p} \right\} e^{(j\omega_{IF}t + j\theta)}, \quad (2.86)$$

where the equation of the BPLO field is given by backpropagating the LO field through the receiver aperture and then up to the target plane with the Green's function $G(\mathbf{p}; \mathbf{v}, z)$:

$$E_{BPLO}(\mathbf{p}, z) = \left(\frac{2P_{LO}}{\varepsilon_0 c} \right)^{\frac{1}{2}} K(z)^{\frac{1}{2}} \int_{-\infty}^{\infty} W_R(\mathbf{v}) e_{LO}^*(\mathbf{v}, 0_-) G(\mathbf{p}; \mathbf{v}, z) d\mathbf{v}. \quad (2.87)$$

The equation of the transmitted beam at the target plane after reflection by the aerosol particles is given by replacing E_T by its equation and inserting it in

Eq. 2.83:

$$E_s(\mathbf{p}, z, t) = \left(\frac{2U_L}{\varepsilon_0 c} \right)^{\frac{1}{2}} \lambda \int_0^\infty (\beta(z)K(z))^{\frac{1}{2}} \int_{-\infty}^\infty W_T(\mathbf{u}) e_L(\mathbf{u}, 0_-) G(\mathbf{p}; \mathbf{u}, z) \\ \times p(t - 2z/c) \exp(j\theta(\mathbf{p}, z) - 4\pi j v_R(\mathbf{p}, z)t/\lambda) d\mathbf{u} dz. \quad (2.88)$$

By inserting Eq. 2.87 and Eq. 2.88 in Eq. 2.86, the intermediate-frequency LIDAR current computed in the target plane then becomes,

$$i_h(t) = 2G_D S(U_L P_{LO})^{1/2} \Re \left\{ \int_0^\infty K(z) \beta(z)^{\frac{1}{2}} p(t - 2z/c) \right. \\ \left. \times \int_{-\infty}^\infty Q(\mathbf{p}, z) \exp(2\pi j(f_{IF} - 2v_R(\mathbf{p}, z)/\lambda)t + j\theta(\mathbf{p}, z)) d\mathbf{p} dz \right\}, \quad (2.89)$$

where the LIDAR system function $[m^{-1}]$ is defined by,

$$Q(\mathbf{p}, z) = \lambda \int \int_{-\infty}^\infty W_T(\mathbf{u}) W_R(\mathbf{v}) e_L(\mathbf{v}, 0) e_{LO}^*(\mathbf{v}, 0) \\ \times G(\mathbf{p}; \mathbf{v}, z) G(\mathbf{p}; \mathbf{u}, z) d\mathbf{u} d\mathbf{v}. \quad (2.90)$$

The variance of the heterodyne signal, $\langle i_h^2(t) \rangle$, gives the average heterodyne signal power, denoted $P(t)$. It can be written from Eq. 2.86 as:

$$P(t) = (G_D S \varepsilon_0 c)^2 \int_D \langle I_s(\mathbf{p}, z, t - z/c) I_{BPLO}(\mathbf{p}, z) \rangle d\mathbf{p}, \quad (2.91)$$

where the angle brackets denote the ensemble average over all the random phases of the backscattered fields from the aerosol particles as well as over the refractive turbulence. This expression will be used in Section 2.5 for the LIDAR performance equation as well as in the next chapter for numerical simulation. Using Eq. 2.89 also gives,

$$P(t) = 2(G_D S)^2 U_L P_{LO} \int_0^\infty K^2(z) \beta(z) |p(t - 2z/c)|^2 C(z, t) dz, \quad (2.92)$$

where the coherent responsivity is defined as [Frehlich 1991]

$$C(z, t) = \int_{-\infty}^\infty c(\mathbf{p}, z, t) d\mathbf{p}, \quad (2.93)$$

and the coherent responsivity density of the coherent Doppler LIDAR as,

$$c(\mathbf{p}, z, t) = \langle |Q(\mathbf{p}, z, t)|^2 \rangle \quad (2.94a)$$

$$= \lambda^2 \langle j_T(\mathbf{p}, z, t) j_{BPLO}(\mathbf{p}, z, t) \rangle, \quad (2.94b)$$

where $j_T(\mathbf{p}, z, t)$ and $j_{\text{BPLO}}(\mathbf{p}, z, t)$ are respectively the normalized irradiance of the transmitted and BPLO fields on the target plane. The system gain $H(z, t)$ is defined as,

$$H(z, t) = K^2(z)\beta(z)C(z, t). \quad (2.95)$$

It should be noted that for a bistatic configuration, when the refractive turbulence encountered by the emitted and the backscattered waves are independent, Eq. 2.94b becomes

$$c(\mathbf{p}, z, t) = \lambda^2 \langle j_T(\mathbf{p}, z, t) \rangle \langle j_{\text{BPLO}}(\mathbf{p}, z, t) \rangle. \quad (2.96)$$

The coherent responsivity in Eq. 2.94b and Eq. 2.96 is easily computed by using appropriate simulation techniques described in the next chapter. The overlap integral of the normalized irradiance of the two beams is estimated at the target plane. Ensemble averages are obtained by Monte Carlo simulation over a large number of propagations through turbulence.

2.4.4 The detection noises

Different noise sources are present in the measured signal. Some of them, like the thermal, the $1/f$ or the amplification noise, exist even when the photodetector is not illuminated. Others such as the quantum noise exist only due to the presence of the LO and the backscattered beams [Valla 2005].

The intrinsic detector noise sources are independent of each other and of the optical detected power. They are typically modeled by an equivalent noise source with an equivalent power NEP [$\text{W Hz}^{-1/2}$]. This noise is a zero-mean Gaussian noise corresponding to a noise current with power

$$\langle i_{\text{NEP}}^2 \rangle = S^2 \text{NEP}^2 B_w, \quad (2.97)$$

where B_w [Hz] is the detector bandwidth. This equivalent noise is not necessarily a white noise. The NEP value is computed as the sum of the different intrinsic noise powers.

The quantum noise, also called shot noise, is mainly generated by the LO field as the detected beam is relatively weak. As the number of photons in the LO field is large, this Poisson noise can be considered to be Gaussian and white. Its average power is given by,

$$\langle i_N^2 \rangle = 2eG_D^2 S P_{\text{LO}} B_w. \quad (2.98)$$

Another important noise comes from the irradiance fluctuations of the LO field and is called the RIN (Relative Intensity Noise) [Cariou 2006]. It can also be considered as Gaussian with a power spectral density decreasing with frequency

such that it is negligible around the signal intermediate frequency. It is given by

$$\langle i_{\text{RIN}}^2 \rangle = S^2 P_{\text{LO}}^2 \int_0^{B_w} 10^{\frac{\text{RIN}(f)}{10}} df, \quad (2.99)$$

where $\text{RIN}(f)$ gives its evolution with frequency f .

2.5 Performance of a coherent Doppler LIDAR

The study of LIDAR performance has largely been investigated using either analytical developments [Frehlich 1991, Frehlich 1993b] or numerical simulation techniques [Belmonte 2000a, Belmonte 2003, Frehlich 2000]. The first is somewhat limited to weak fluctuation due to refractive turbulence or the propagation of a untruncated Gaussian beam whereas numerical simulation methods, such as the phase-screen technique exposed in the next chapter, have a larger range of applications. The use of analytical expressions such as the one obtained by asymptotic approximations remains the preferred method for rapidly estimating the performance of a system under general conditions. The main performance parameters are defined in this section.

2.5.1 The Signal-to-Noise Ratio

The Signal-to-Noise Ratio, noted SNR, is defined as the ratio between the heterodyne signal power and the noise current power:

$$\text{SNR}(t) = \frac{\langle i_h^2(t) \rangle}{\langle i_N^2 \rangle}. \quad (2.100)$$

This equation implicitly considers that the detector is shot noise dominated, with a noise power given by Eq. 2.98. For narrow beams compared to the dimensions of the sensing volume in the z direction and typical atmospheric conditions, the functions $\beta(\mathbf{p}, z)$ and $N(\sigma_s; \mathbf{p}, R)$ behave like a delta function of $\mathbf{p}$ and $\beta(\mathbf{p}, z) = \beta(z)$. The target plane calculation of the SNR is obtain by inserting Eq. 2.92 and 2.98 in Eq. 2.100:

$$\text{SNR}(t) = \frac{\eta_Q U_L}{\hbar \nu B_w} \int_0^\infty K^2(z) |p(t - 2z/c)|^2 \beta(z) C(z) dz. \quad (2.101)$$

It depends on the backscattering coefficient of the aerosol particles in a volume defined by the spatial weighting function $|p(t - 2z/c)|^2$ around z . The influence of the telescope lenses, transmitted beam size and refractive turbulence are given by the variable $C(z)$. The spatial functions $\beta(z)$, $K(z)$ and $C(z)$ are also functions of altitude, as such the SNR does not have the same profile as a function of the elevation angle of the line-of-sight.

The SNR is usually represented as a functions of distance. This is given by calculating the SNR of an infinitesimal slice of atmosphere dz at a distance $z = ct/2$. It provides us:

$$\text{SNR}(z) = \frac{\eta_Q U_L}{\hbar \nu B_w} K^2(z) \beta(z) C(z). \quad (2.102)$$

A loss in SNR also exists with imperfect LASER beam ($M^2 > 1$). A simple equation has been derived in [Cariou 2006] which binds the SNR to the M^2 :

$$\text{SNR}(z)|_{M^2} = \frac{2}{1 + (M^2)^2} \text{SNR}(z)|_{M^2=1}. \quad (2.103)$$

A M^2 of $\sqrt{3}$ thus corresponds to a loss of 3 dB.

2.5.2 Heterodyne and system efficiency

The heterodyne efficiency is the measure of the loss in coherent power when the received backscattered field does not match perfectly the LO field. It is defined as a function of the detector output currents:

$$\eta_H(t) = \frac{\langle i_h^2(t) \rangle}{2 \langle i_{dc} \rangle \langle i_s(t) \rangle}. \quad (2.104)$$

It can be estimated by measuring the average coherent power $\langle i_h^2(t) \rangle$ and the ensemble average current of the detector which gives $\langle i_{dc} \rangle + \langle i_s(t) \rangle$. The average DC current is obtained when no backscattered field is present at the detector surface. In order to achieve high heterodyne efficiency, the pulsed transmitted and LO fields are both single mode radiations.

The system efficiency, also called antenna efficiency as it takes into account the receiver geometry, is defined by

$$\eta_s(z, t) = \frac{z^2 \lambda^2}{A_R} \int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, z, t) j_{\text{BPLO}}(\mathbf{p}, z) \rangle d\mathbf{p}. \quad (2.105)$$

This system performance parameter is particularly interesting for LIDAR design since it is independent of the transmitted power, the aerosol backscatter coefficient, the atmospheric extinction and the photodetector quantum efficiency [Frehlich 1993b]. This is the parameter to maximize to optimize the LIDAR performance. Given Eq. 2.94b and 2.93, it can also be written as [Frehlich 1991],

$$\eta_s(z, t) = \frac{C(z, t)}{\Omega(z)}, \quad (2.106)$$

where $\Omega(z) = A_R/z^2$ is the solid angle [sr] presented by the receiver. The system efficiency can therefore be interpreted as the effective portion of the aperture

collecting the heterodyne optical power. In [Belmonte 2003], this quantity is interpreted as the ratio between a coherent solid angle, given by the coherent responsivity and the effective solid angle. For a monostatic LIDAR modelled by Gaussian beams and a circular aperture, the system efficiency is bounded 40.1 %, i.e. $\eta_s \leq 0.401$ [Rye 1992]. The maximum is obtained for a truncation ratio $\sigma_L/R_p=0.8$ which corresponds to a heterodyne efficiency of 42 %. This result has also been obtained for a fiber LASER [Wang 1988, Winzer 1998, Valla 2005].

2.5.3 Heterodyne optical power gain

The heterodyne optical power from an infinitesimal atmospheric slice dz around distance z can be deduced from Eq. 2.92 and is defined by,

$$P(z, t) = U_L P_{LO} \lambda^2 K^2(z) \beta(z) \int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, z, t - z/c) j_{BPLO}(\mathbf{p}, z) \rangle d\mathbf{p}. \quad (2.107)$$

The heterodyne optical power gain is introduced in order to analyse the effect of refractive turbulence on the performance of a coherent LIDAR . It is given by the heterodyne optical power obtained in the presence of turbulence normalized by the case in free-space:

$$G_o(z, t) = \frac{P(z, t)}{P^f(z)} = \frac{\int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, z, t) j_{BPLO}(\mathbf{p}, z, t) \rangle d\mathbf{p}}{\int_{-\infty}^{\infty} j_T^f(\mathbf{p}, z) j_{BPLO}^f(\mathbf{p}, z) d\mathbf{p}}, \quad (2.108)$$

where the superscript f denotes the free-space condition. It should be noted that for free-space propagation the transmitted and LO beams are deterministic and therefore independent of time.

2.5.4 Normalized coherent power variance

An important measure of the LIDAR performance is given by the relative variance of the optical power which is due to the refractive turbulence. The normalized coherent power variance is defined by:

$$\sigma_c^2 = \frac{\left\langle \left[\int_{-\infty}^{\infty} j_T(\mathbf{p}, z, t) j_{BPLO}(\mathbf{p}, z, t) d\mathbf{p} \right]^2 \right\rangle}{\left[\int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, z, t) j_{BPLO}(\mathbf{p}, z, t) \rangle d\mathbf{p} \right]^2} - 1. \quad (2.109)$$

The fluctuation of the coherent power has almost the same effect as the speckle effect. The major difference is that they do not have the same time scale. Although the effect of speckle can be easily reduced by pulse accumulation, as it will be seen in the next part of this thesis, the effects of refractive turbulence are more difficult to average out due to their higher time scale.

2.5.5 Configuration gain

The configuration gain is the gain obtained when using a monostatic configuration instead of a bistatic one. It is given by the ratio between the heterodyne optical powers:

$$G_c(z, t) = \frac{P_m(z, t)}{P_b(z, t)} = \frac{\int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, z, t) j_{\text{BPLO}}(\mathbf{p}, z, t) \rangle d\mathbf{p}}{\int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, z, t) \rangle \langle j_{\text{BPLO}}(\mathbf{p}, z, t) \rangle d\mathbf{p}}. \quad (2.110)$$

The turbulence realizations for the two paths in the bistatic configuration are assumed to be independent.

LIDAR measurement simulation

IN the introductory chapter, the relevance of numerical simulation for designing efficient LIDAR systems has been addressed. Simulation programs have therefore been developed to contribute to the LASEF telescope design, determine the optimal parameters of the LASER source and assist in the development of wind velocity estimation algorithms.

The major interest of these simulations is that they integrate numerical methods from both the optical and the fluid dynamics fields. In the one hand, optical simulations of the beam propagation allows to consider the performance changes due to the beam quality, the telescope optics and the refractive turbulence, under general atmospheric conditions and arbitrary measurement configurations. The signal power fluctuations due to turbulence is also modeled which is of practical importance for the generation of realistic heterodyne LIDAR signals. On the other hand, the simulation of the fluid flows is needed to study, for example, the signal decorrelation due to the fast variation of the wind velocity.

In the first section of this chapter, the widely used phase-screen method for the simulation of electromagnetic wave propagation is described. The results exposed are principally dedicated to the development of the LASEF telescope. In Section 3.2, the Monte Carlo simulation of the LIDAR coherent responsivity is exposed. It is used in the last section to generate realizations of the signal power fluctuations due to turbulence. Section 3.3 will finally propose a complete LIDAR simulation method combining the feuilleté model of the LIDAR signal with the spatial simulations described in the previous sections. Some parts of the present chapter have been published in [Brousmiche 2007].

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3.1 Simulation of LIDAR performance parameters

The analysis of wave propagation in random media and, by extension the calculation of the LIDAR performance, requires the calculation of the fields statistics. This is particularly difficult to perform for moderate to strong fluctuation regimes for which no analytical solution exists for moments higher than the fourth one, except those obtained with asymptotic approximations.

This section presents a wave propagation numerical method known as the phase-screen method (PSM) widely used for analyzing the effect of a random medium on the statistics of a wave propagating through it. This technique has also been used here to simulate the propagation of the beam through the LIDAR telescope. The effect of beams with non-unitary M^2 has however not been taken into account.

3.1.1 The Markov assumption and the split-step method

The PSM is based on the parabolic theory of diffraction and represents the generalization of the Markov approximation. Hence, for this approach to be valid for stochastic radiations, the scales of the medium inhomogeneities as well as the scales of the field itself must greatly exceed the wavelength. Within this condition, the directivity diagram of the light scattered by the turbulent cells is therefore strongly oriented along the propagation direction, which means that the backscattering due to the permittivity fluctuation can be neglected [Kandidov 1996].

According to the Markov assumption, the wave fluctuations are independent of the local index of refraction variations. Mathematically, this is equivalent to the case for which the local refractive index fluctuations have a delta-correlation function along the z -axis [Tatarskii 1971, Belmonte 2000a]. The three-dimensional correlation function B_n between two spatial points with coordinates $(\mathbf{u}, z)$ and $(\mathbf{p}, z')$ is then given by:

$$B_n(\mathbf{u}, z; \mathbf{p}, z') = \langle n_1(\mathbf{u}, z) n_1(\mathbf{p}, z') \rangle \quad (3.1a)$$

$$= A_n(\mathbf{u} - \mathbf{p}, z) \delta(z - z'), \quad (3.1b)$$

where A_n is the two-dimensional transverse refractive index correlation function which can also be written as [Tatarskii 1971] :

$$A_n(\mathbf{p} - \mathbf{u}, z) = \int_{-\infty}^{\infty} B_n(\mathbf{u}, z; \mathbf{p}, z') dz'. \quad (3.2)$$

The local turbulent eddies therefore appear as flat disks oriented in the transverse direction. The approximation given by Eq. 3.1b is however only valid for propagation distances higher than the highest scale of turbulence L_0 . The conditions for the validity of the Markov approximation have been determined

for the averaged field, the mutual coherence function [Klyatskin 1971] as well as the fourth moment [Zavorotnyi 1978] and are equivalent to the small-angle approximation of the parabolic equation.

The simulation of wave propagation in a random media can be done by applying the split-step method on the parabolic equation (see Eq. 2.51) [Makaskill 1984, Tappert 1973]. Using the split-step method means that the terms dealing with field derivatives and those dealing with the medium refractive index are additively separable [Martin 1992]. Equation. 2.51 thus becomes:

$$\frac{\partial}{\partial z} E(\mathbf{p}, z, t) = i k \mathcal{S} \{E(\mathbf{p}, z, t)\}, \quad (3.3)$$

where $\mathcal{S}$ is an operator defined by :

$$\mathcal{S} = \left(\frac{1}{2 k^2} \nabla_{\perp}^2 \right) + (n^2 - 1) \quad (3.4a)$$

$$= \mathcal{D} + \mathcal{N}, \quad (3.4b)$$

where $\mathcal{D} = (2 k^2)^{-1} \nabla_{\perp}^2$ is the diffraction operator whereas $\mathcal{N} = (n^2 - 1) = 2n_1$ is the nonlinear operator taking into account the effect of the refractive index. The locality hypothesis [Belmonte 2000a] allows us to consider the solution of the wave at a range z to express the solution at a range $z + \delta z$ provided that δz is large compared with the largest atmospheric inhomogeneity but small enough such that the propagation through the layer δz can be considered to be in the small fluctuation regime. The solution for a small step δz in the forward direction is given by [Spivack 1989] :

$$E(\mathbf{p}, z + \delta z) \approx \exp \left[ik \int_z^{z+\delta z} \mathcal{D} dz \right] \exp \left[ik \int_z^{z+\delta z} \mathcal{N} dz \right] E(\mathbf{u}, z). \quad (3.5)$$

The two operators are applied on the input field $E(\mathbf{u}, z)$ and may therefore be solved sequentially. The second exponential term represents the phase perturbation due to refractive index inhomogeneities. The solution in free-space is just the solution of the last equation with $\mathcal{N}$ removed. Similarly, the solution of the $\mathcal{N}$ operator is just the solution of Eq. 3.5 with $\mathcal{D}$ removed.

3.1.2 The phase screen method

By use of the Markov assumption, the continuous random medium can be decomposed into a series of statistically independent thin slabs. The effect of turbulence inside each slab is furthermore modelled by a single phase screen. Those screens are sufficiently thin to only introduce a spatially varying contribution to the wave phase. Amplitude fluctuations arise by diffraction over the slabs by application of the $\mathcal{D}$ operator. The phase screen method, resulting from this split-step method, lies in the class of pseudo-spectral numerical method since the diffraction is effectively computed in the Fourier domain.

a. Second order phase statistics

The slab of random medium is described in the interval $[z, z + \delta z]$ by the refractive index spatial spectrum, noted $\Phi_n(\boldsymbol{\kappa})$. To specify the statistics of each screen, the calculation of the correlation function as well as the spatial spectrum of the phase variations introduced by this slab on the beam is needed. The random phase increment θ introduced by one slab is given by

$$\theta(\mathbf{p}, z) = k \int_0^{\delta z} n_1(\mathbf{p}, z) dz. \quad (3.6)$$

The three-dimensional phase correlation function is found by using Eq. 3.1b:

$$B_\theta(\mathbf{p}, z; \mathbf{q}, z') = k^2 \int_0^{\delta z} \langle n_1(\mathbf{p}, z) n_1(\mathbf{q}, z') \rangle dz dz' \quad (3.7a)$$

$$= A_\theta(\mathbf{p} - \mathbf{q}, z) \delta(z - z'), \quad (3.7b)$$

where $A_\theta(\mathbf{p} - \mathbf{q}, z)$ is the two-dimensional transverse phase correlation function. To obtain Eq. 3.7b, the approximation that the interscreen distance is higher than the correlation length of the irregularities is used. This also means that the refractive-index fluctuations are aggregated along the propagation direction. The two-dimensional spatial phase spectrum can be written by:

$$\Phi_\theta(\kappa_x, \kappa_y) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} A_\theta(x, y) e^{-i(\kappa_x x + \kappa_y y)} dx dy. \quad (3.8)$$

Taking into account the definition of the three-dimensional refractive index spectrum,

$$\Phi_n(\kappa_x, \kappa_y, \kappa_z) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} B_n(x, y, z) e^{-i(\kappa_x x + \kappa_y y + \kappa_z z)} dx dy dz, \quad (3.9)$$

Finally, it comes that:

$$\Phi_\theta(\kappa_x, \kappa_y) = 2\pi k^2 \int_0^{\delta z} \Phi_n(\kappa_x, \kappa_y, \kappa_z = 0) dz \quad (3.10a)$$

$$= 2\pi k^2 \delta z \Phi_n(\kappa_x, \kappa_y, \kappa_z = 0). \quad (3.10b)$$

As explained in Section 2.2.6, the inner scale of turbulence, noted l_0 , has a great impact on the coherence of the wave. For that reason, the Von Karman spectrum is used instead of the Kolmogorov one. The generation of statistically independent phase screens $\theta(\mathbf{r}, z)$ with the appropriate second-order statistics given by the spatial spectrum given by Eq. 3.10b, can be done by a classical spectral factorization.

b. Formulation of the PSM

The sounding path is divided into layers of thickness δz . A random phase screen is simulated at each layer. Hence, the LASER beam undergoes phase distortion passing through the screen and is then diffracted within the layer. The algorithm for the propagation of the LASER beam $E_T(\mathbf{p}, z)$ through layer $[z, z + \delta z]$ is computed in four steps:

1. Perturbation of the beam $E(\mathbf{p}, z_-)$ by a phase screen $\theta(\mathbf{p}, z)$. The perturbed field $E_p(\mathbf{p}, z_+)$ is given by:

$$E_p(\mathbf{p}, z_+) = E(\mathbf{p}, z_-) \exp(i\theta(\mathbf{p}, z)), \quad (3.11)$$

where the notation z_- and z_+ respectively indicates the propagation distance before and after the phase-screen in z .

2. Fourier transform of $E_p(\mathbf{p}, z_+)$:

$$\tilde{E}_p(\boldsymbol{\kappa}, z_+) = \mathcal{F}\{E_p(\mathbf{p}, z_+)\}, \quad (3.12)$$

where $\boldsymbol{\kappa}$ is the two-dimensional spectral coordinate, and $\mathcal{F}$ is the two-dimensional Fourier transform operator.

3. Propagation in free-space of $E_p(\mathbf{p}, z_+)$ in a slab of length δz in the Fourier domain:

$$\tilde{E}(\boldsymbol{\kappa}, z + \delta z) = \exp(-i\pi \lambda \delta z \kappa^2) \tilde{E}_p(\boldsymbol{\kappa}, z_+). \quad (3.13)$$

4. Inverse Fourier transform of $\tilde{E}(\boldsymbol{\kappa}, z + \delta z)$ to get the beam field just at the input of the next phase screen:

$$E(\mathbf{p}, z + \delta z) = \mathcal{F}^{-1}\{\tilde{E}(\boldsymbol{\kappa}, z + \delta z)\}. \quad (3.14)$$

This procedure is repeated for all atmospheric layers up to the observation plane. The field $E(\mathbf{p}, z + \delta z)$ becomes the initial beam for the next iteration. The number of layers to use is noted N_p and depends on the intensity of the turbulence. Roughly speaking, the additional fluctuations of the beam irradiance produced by one layer must be small enough so that the propagation in this layer can be considered in the weak fluctuation regime.

The first, second and fourth moments of the wave fields are obtained by combining N_r realizations of the beam propagation through a random medium characterized by N_p uniformly separated phase screens [Spivack 1989]. Thus, the i^{th} realization gives a set of N_p screens $\{E^{(i)}(\mathbf{p}, j\delta z), j = 0 \dots N_p - 1\}$ with the transversal distribution of the optical wave at distance $j\delta z$ from the source. The Monte Carlo simulation involves the following procedures [Uscinski 1985] :

1. Simulation of the random fields $E^{(i)}(\mathbf{p}, 0)$ at the source and generation of the N_p phase screens of the random medium $\theta^{(i)}(\mathbf{p}, j \delta z)$ for all i .
2. Propagation of $E^{(i)}(\mathbf{p}, 0)$ through the N_p phase screens.
3. Statistical processing of the ensemble $\{E^{(i)}(\mathbf{r}, j \delta z), i = 0 \dots N_r - 1\}$ of wave fields obtained.

The first step is only necessary if a randomly distributed transverse beam profile is considered at the output of the LASER source. This is typically the case for a $M^2 > 1$, i.e. a not perfectly Gaussian beam. This case has not been examined in this thesis. Fig. 3.1 illustrates the propagation of a Gaussian beam through an homogeneous and isotropic turbulence (HIT). The effect of refractive turbulence, i.e. short-term beam spreading and spatial coherence loss, can be easily observed.

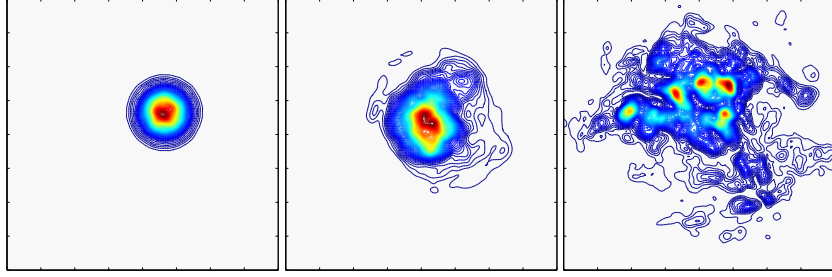


Figure 3.1: Example of beam irradiance after propagation through HIT: $C_n^2 = 10^{-15} \text{ m}^{-2/3}$, 100 m (left), $C_n^2 = 10^{-14} \text{ m}^{-2/3}$, 1900 m (middle) and $C_n^2 = 10^{-13} \text{ m}^{-2/3}$, 1900 m (right). The inner scale is $l_0=1 \text{ cm}$ and the outer scale is $L_0=1 \text{ m}$. The outer boxes corresponds to a surface of $30 \text{ cm} \times 30 \text{ cm}$. The initial beam radius is 3 cm.

c. Discrete algorithm and simulation constraints

The random phase screens as well as the transverse beam profiles are sampled on a square grid of size $N \times N$ with a uniform spatial resolution h . The physical size is therefore $\mathcal{L} = Nh$ [m]. The solution of the wave at step $i + 1$ corresponding to the propagation distance $z + \delta z$ is

$$E_{mn}^{i+1} = \mathbf{F}^{-1} \left\{ \exp(-i\pi\lambda\delta z\kappa^2) \mathbf{F} \left\{ E_{mn}^i \exp(i\theta_{mn}) \right\} \right\}, \quad (3.15)$$

where $\mathbf{F}$ and $\mathbf{F}^{-1}$ are respectively the direct and inverse two-dimensional FFT operators. The spectral resolution is

$$\Delta\kappa = 2\pi/Nh. \quad (3.16)$$

The random phase screen is represented as a set of real numbers $\{\theta_{mn}\}$ generated by spectral factorization, i.e. in the frequency domain from a set of complex random numbers

$$\tilde{\theta}_{pq} = A_{pq} + iB_{pq}. \quad (3.17)$$

The number A_{pq} and B_{pq} are independent zero-mean Gaussian random variables with variance [Coles 1995]:

$$\sigma_{pq}^2 = \langle |\tilde{\theta}_{pq}|^2 \rangle = \left(\frac{2\pi N}{h} \right)^2 \Phi_{\theta}(p\Delta\kappa, q\Delta\kappa). \quad (3.18)$$

The realization of the phase screen $\{\theta_{mn}\}$ is obtained by applying the inverse FFT on the resulting $\{\tilde{\theta}_{pq}\}$. Two independent phase screens are generated in the real and imaginary part of the FFT result. The fast Fourier transforms have been computed with the FFTW3 library described in [Frigo 2005].

The choice of the computational mesh depends on the important spatial scales of the random fields in the observation plane. Two conditions have to be verified for this algorithm to work effectively:

- As the numerical grid are used to compute both the irradiance profile and the phase perturbations, the resolution must be fine enough to capture all the scales of the irradiance field as well as the smallest scales of the turbulence, l_0 .
- The size $\mathcal{L}$ of the screens must be large enough to intersect the beam profile without truncation. The choice of $\mathcal{L}$ should depend on the natural diffraction of the beam and the additional spreading due to the turbulence.

The Rytov and the asymptotic theories described in Chapter 2 allows us to determine secure simulation conditions as they provide useful approximations for the long-term beam spreading and the transverse coherence length. Moreover, although the use of the FFT algorithm performs the numerical propagation of the field from screen to screen in an efficient way, it imposes the periodicity of the fields which is something that must be taken into account for the choice of the screen size.

Let's first discuss the choice of the grid resolution. According to the Shannon theory, the grid spacing must be two times smaller than the smallest sampled scale. The inner scale of turbulence, l_0 , is by definition the smallest scale of the phase perturbation. On the other hand, the smallest scale of the irradiance profile is given by the spatial coherence radius of the beam. It decreases depending on the turbulence strength and the propagation distance. Hence, for a given C_n^2 , there exists a distance from which the coherence radius becomes lower than the inner scale of turbulence. If the maximum propagation distance

is higher than this transition distance, the condition on h is given depending on ρ_0 otherwise it will be determined by l_0 :

$$h < \frac{1}{2} \min \{l_0, \rho_0\}. \quad (3.19)$$

The range at which the spatial coherence length of the beam is on the order of the inner scale is given in Fig. 3.2. The result for inner scale values is represented: 1 mm corresponding to a typical value near the ground and 1cm which is a value commonly selected for numerical simulation. Besides in the very weak fluctuation regime, this range varies with the turbulence level within the spatial region of interest. It is therefore necessary to choose the phase screen resolution depending on the spatial coherence length at the farthest sensing range, $\rho_0(z_{max})$, instead of the inner scale of turbulence. It can be shown that for long path-integrated turbulence, the Gaussian beam behaves like a plane wave regarding to ρ_0 . The equation of ρ_0 used has therefore been the one given for the propagation of a plane wave in a turbulence described by a Von Kàrmàn spectra given in [Andrews 1998].

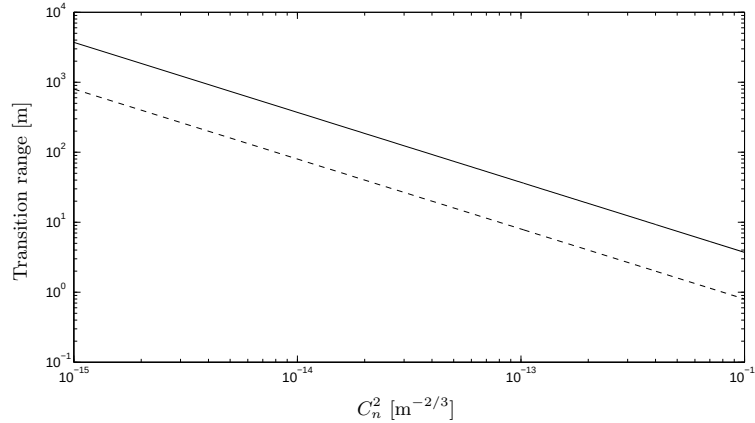


Figure 3.2: Transition range as a function of C_n^2 at which the transverse coherence length is on the order of the inner scale l_0 . The two curves represent its values for $l_0=1$ mm (solid) and $l_0=1$ cm (dash). The wavelength is $\lambda=1.55$ μm .

The next condition is related to the beam size. The long-term averaged beam must be included inside the screen. However, if spatial statistics have to be calculated, a security factor must be introduced in order for the screen to be large enough so that each point of interest is independent of any other point of the screen. This dependence is due to periodicity imposed by the FFT and leads

to the following condition:

$$\mathcal{L} > 4 m w_e(z_{\max}), \quad (3.20)$$

where m is the size of the buffer zone and $z_{\max}$ [m] is the maximum simulation range. The minimum screen size $N_{\min}$ is given by combining the conditions in Eq. 3.19 and Eq. 3.20:

$$N_{\min} = \frac{2\mathcal{L}}{\rho_0(z_{\max})} \quad (3.21a)$$

$$= 4 m \frac{w_e(z_{\max})}{\rho_0(z_{\max})}. \quad (3.21b)$$

The value of m is fixed to 3 in [Belmonte 2000a] and is comprised between 3 to 5 in [Kandidov 1996]. For the effective beam radius, Eq. 2.47 is used which have been computed with the extended Huygens-Fresnel principle. Fig. 3.3 shows the minimum phase screen dimension N needed depending on the maximum range and the C_n^2 .

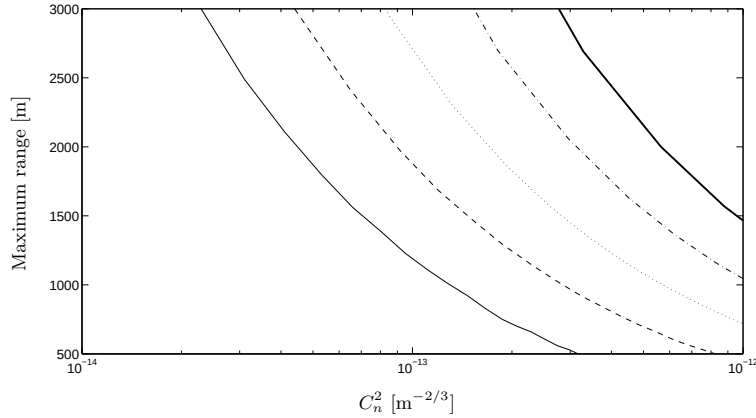


Figure 3.3: Minimum phase screen dimension $N_{\min}$ to use for simulation depending on the refractive-index structure constant C_n^2 and the maximum propagation range. The e^{-2} irradiance radius of the emitted beam is 3 cm. Different loci are represented for $N_{\min}=128$ (solid, thin), $N_{\min}=256$ (dash), $N_{\min}=512$ (dot), $N_{\min}=1024$ (dash-dot) and $N_{\min}=2048$ (solid, thick).

Large scales of the turbulence are less important for Gaussian beam waves since they are not sensitive to scales much higher than their physical size. More precisely, it has been found in [Martin 1988] that the effects of scales larger than ten times the Fresnel length R_F are very small. Hence, for the wavelengths

considered in this work, the outer scale L_0 used for simulation can be limited to a few meters. The condition on L_0 simply becomes $\mathcal{L} > L_0$.

Another important point concerns the propagation distance δz between adjacent phase-screens and therefore the number of screens to use depending on a given maximum observation range. The conditions on δz can be expressed in terms of the level of intensity fluctuations induced by the slab, described by the intensity variance $\sigma_I^2(\delta z)$. Hence, to ensure that the split-step method is accurate, the scattering must be weak over the interscreen propagation distance, i.e. $\sigma_I^2(\delta z) < 0.1$ [Martin 1992]. In this regime, the intensity variance is well approximated by the Rytov variance $\sigma_I^2(z)$ given by Eq. 2.40. Moreover, when an extended medium is modeled, it must be assumed that no intensity fluctuations are produced over δz and that the medium effects can be modeled as only a phase perturbation [Martin 1990]. The path has therefore to be sampled enough and an additional condition is required that the propagation over δz contributes less than 10 % of the total intensity variance at distance $z_{\max}$, i.e. $\sigma_I^2(\delta z) < 0.1 \sigma_I^2(z_{\max})$.

3.1.3 LIDAR performance parameters

In Section 2.4.3, it has been shown that the LIDAR return can be expressed as the overlap integral between the transmitted and the BPLO beams at the target plane. The main advantage of this formulation is that the complexity of computing the double-path propagation of a LASER beam with both refractive turbulence and aerosol speckle is removed. The simulation of the LIDAR performance parameters is thus realized by applying the PSM algorithm on both the emitted beam E_T and on the BPLO beam, E_{BPLO} . The SNR or the system efficiency described in Section 2.5 are thus obtained via the calculation of the coherent responsivity $C(z)$ given by

$$C(z, t) = \lambda^2 \int_{-\infty}^{\infty} \langle j_T(\mathbf{p}, z, t) j_{\text{BPLO}}(\mathbf{p}, z) \rangle d\mathbf{p}. \quad (3.22)$$

where $j_T(\mathbf{p}, z, t)$ and $j_{\text{BPLO}}(\mathbf{p}, z, t)$ are the normalized irradiances of the transmitted and the BPLO fields. The simulations can be made under general refractive turbulence conditions and taking into account beam truncation at the telescope aperture, beam-angle misalignment and arbitrary transmitter and receiver configurations.

The propagation through the transmitter with function W_T as well as the backpropagation through the receiver W_R , both inducing truncation effect on the beam, thus have to be simulated. The transmitter has the following function:

$$W_T(\mathbf{u}) = \begin{cases} \exp\left(-\frac{u^2}{\sigma_T^2} - ik\frac{u^2}{2F_T}\right) & \text{for } u \leq R_T \\ 0 & \text{for } u > R_T \end{cases}, \quad (3.23)$$

where σ_T [m] is the e^{-1} radius of the transmitter lens, F_T [m] its phase curvature and R_T [m] is the radius of the transmitter aperture. Similarly, the receiver is characterized by the parameters σ_R , F_R and R_R . This transfer function has been used in order to compare the PSM results with analytical expressions given by the asymptotic theories [Frehlich 1993b]. In particular, the real part of Eq. 3.23 must be removed for the diffraction induced by the lens to be modeled. In a monostatic configuration, the two LASER beams undergo the same phase distortion, i.e. are simulated using the same set of phase screens. However, the simulation of a bistatic configuration is obtained by using different realizations of the turbulence for the transmitted and the BPLO fields as the separation between the transmitting and the receiving apertures is large enough to ensure that both fields travel through statistically independent refractive turbulence.

The simulation of the performance parameters is carried out by performing the following steps N_r times:

1. Compute $E_T(\mathbf{u}, 0_+)$ by propagating $E_L(\mathbf{u}, 0_-)$ through $W_T(\mathbf{u})$.
2. Compute $E_{\text{BPLO}}^*(\mathbf{v}, 0_+)$ by backpropagating $E_{\text{LO}}^*(\mathbf{v}, 0_-)$ through $W_R^*(\mathbf{v})$.
3. Generate N_p phase-screens $\{\theta_{mn}^t\}$ along the transmission path as well as N_p phase-screens $\{\theta_{mn}^r\}$ along the reception path.
4. For each $z = i \delta z$ for $i = 1 \dots N_p - 1$
 - (a) Propagate $E_T(\mathbf{u}, (i-1) \delta z)$ through slice $[(i-1)\delta z, i\delta z]$
 - (b) Backpropagate $E_{\text{BPLO}}^*(\mathbf{v}, (i-1) \delta z)$ through slice $[(i-1)\delta z, i\delta z]$
 - (c) Computes the normalized irradiances $j_T(\mathbf{p}, i \delta z)$ and $j_{\text{BPLO}}(\mathbf{p}, i \delta z)$ as well as their spatial product.
5. Back to step (2) to generate other realizations.

The performance parameters are obtained by computing the two first statistical moments of the product $(j_T(\mathbf{p}, i \delta z) j_{\text{BPLO}}(\mathbf{p}, i \delta z))$ for all i and performing the integration along the 2D spatial coordinate $\mathbf{p}$. Statistics on $j_T(\mathbf{p}, i \delta z)$ such as beam wandering, short-term and long-term beam sizes and coherence length ρ_0 , as presented in Section 2.2, are also computed.

3.1.4 Results: design of the LASEF telescope

The results presented here have been obtained for the propagation of a Gaussian beam through 2 km of atmosphere using $N_p=20$ phase-screens. A minimum of $N_r=500$ realizations are computed in order to obtain statistics with good confidence. A Von Karman turbulence spectrum is used with an inner scale of $l_0=1$ cm and a outer scale of $L_0=1$ m for the phase-screen phase power spectral density. The screens have a resolution of $h=2$ mm and a size $N=1024$. The

Simulation parameters		
Wavelength	λ	1.55 μm
Atmos. turbulence	Spectral model	Von Karman
	l_0	10 mm
	L_0	1 m
	$C_n^2 [\text{m}^{-2/3}]$	From 10^{-15} to 10^{-12}
Telescope	Config.	monostatic, bistatic
	Beam radius σ_L	15, 30 and 60 mm
	Trunc. factor R_p/σ_L	1.67
PSM	N_p	20 screens
	Resolution h	2 mm
	Propa. distance δz	100 m
	Size	1024×1024
Monte Carlo	N_r realizations	500

Table 3.1: Simulation parameters associated with results presented in Section. 3.1.4. The LASER source parameters are not specified since the performance profiles with range z are normalized to their values at the telescope output ($z = 0$).

results presented here will be limited to a 1.55 μm LASER source. The truncation factor R_p/σ_L has been fixed to 1.67. When not specified, the e^{-2} irradiance beam radius is 3 cm at the output of the transmitter lens. The receiver aperture radius in this case is $R_p=5$ cm. The analysis below is mainly focused on typical diurnal conditions of moderate-to-strong turbulence, i.e. $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ and $C_n^2 = 10^{-12} \text{ m}^{-2/3}$. Results obtained at weaker turbulence levels are given for comparison purpose. Fig. 3.4 to Fig. 3.9 illustrates the influence of atmospheric turbulence on the Gaussian beam statistics, i.e. beam wandering, spreading and coherence loss. A number of simulations are also dedicated to the design of the LASEF telescope. The main results of this study are exposed here from Fig. 3.10 to Fig 3.17. The simulation parameters are given in Table. 3.1. All the simulations have been performed on a AMD Opteron 252/2.6 GHz with 4 Go of RAM. The computation time is about 8 hours for 500 realizations and 20 screens with the chosen resolution and size.

Fig. 3.4 represents the average beam wander, noted $\langle\beta(z)\rangle$, of two collimated beams with different e^{-2} irradiance radius in a strong turbulence. The beam wandering, which is caused by large scale inhomogeneities of the turbulence, is a major effect since it induces random displacement of the backscattered beam on the receiver plane. It can be observed that it is more important for smaller beam size having a larger natural spreading. The beam wandering standard deviation represented by the vertical segments also increases with range. Fig. 3.5 gives the long-term and the short-term irradiance beam radius. The long-term beam radius has been obtained by computing the mean irradiance of the field $\langle j_T(\mathbf{p}, z) \rangle$. By assuming that the ensemble average of the transversal intensity

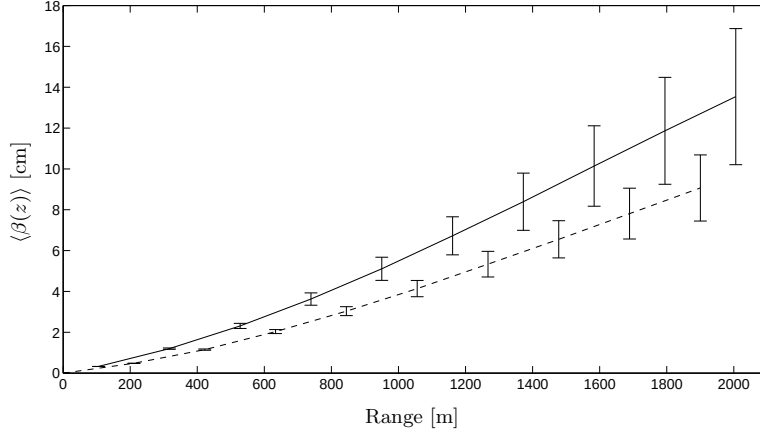


Figure 3.4: Wandering of two beams with irradiance radius of 1.5 cm (solid) and 6 cm (dash) in strong turbulence ($C_n^2 = 10^{-12} \text{ m}^{-2/3}$). The average value $\langle \beta(z) \rangle$ is given along with its standard deviation represented by the vertical segments.

profile is Gaussian, the second order spatial moment has been computed and compared with the results of the extended Huygens Fresnel principle effective beam radius given by Eq. 2.47. A good matching is observed between simulation and theory. The short-term irradiance radius is computed by the ensemble average value of the second-order spatial moment on each realization of the beam $j_T(\mathbf{p}, z)$. Fig. 3.6 and 3.7 illustrate the influence of the optical turbulence on a LASER beam focused at 500 m. Hence, increasing the turbulence level defocalizes the beam which radius profile tends to the collimated beam one. In Fig. 3.7 with $C_n^2 = 10^{-12} \text{ m}^{-2/3}$, it can also be observed that the collimated beam is partially focused around 200 m. This turbulence-induced phenomenon has an impact on the LIDAR performance as we will see later in this section. The spatial coherence radius ρ_0 of the beam is also computed and two cases are represented in Fig. 3.8 and Fig. 3.9. It is obtained by evaluating the modulus of the complex degree of coherence given by Eq. 2.42 and efficiently computed by the Fast Fourier Transform. The results are compared with both the theoretical plane wave case (see Eq. 2.44) and the Gaussian beam case given in [Andrews 1994]. For the very strong turbulence case given in Fig. 3.9, the coherent radius of the beam becomes lower than the inner scale of turbulence after nearly 800 m.

Let's focus now more specifically on the LIDAR performance parameters detailed in Section 2.5. Fig. 3.10 represents the effect of the LIDAR configuration on the sensitivity to refractive turbulence. It gives the evolution of the normalized SNR with range, given by Eq. 2.102 and normalized about its value at

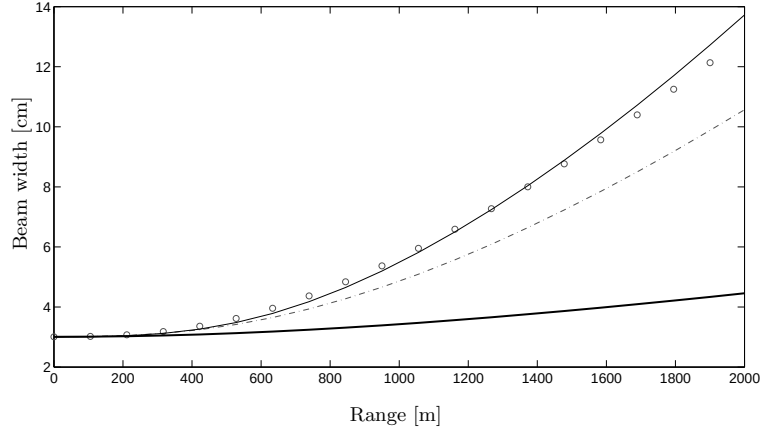


Figure 3.5: Theoretical and simulated long-term and short-term beam width as a function of range for a moderate turbulence level ($C_n^2 = 10^{-13} \text{ m}^{-2/3}$). The initial beam width is 3 cm and a wavelength of $1.55 \mu\text{m}$. Free-space (solid, thick), simulated long-term width (circles), simulated short-term width (dash-dot) and theoretical long-term width (solid, thin).

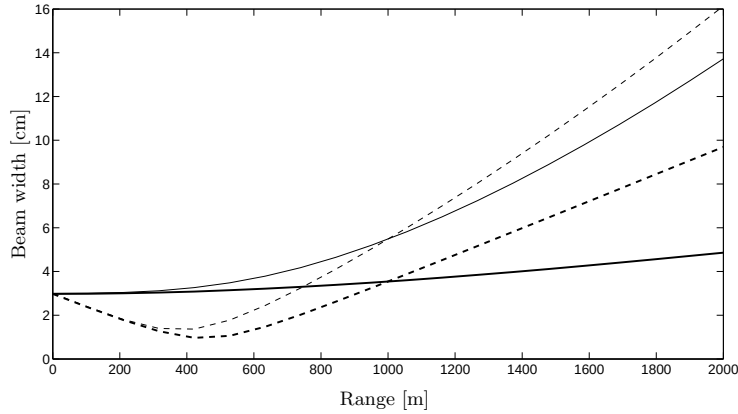


Figure 3.6: Simulated long-term beam irradiance radius as a function of range of a collimated and a focalized beam (at 500 m) for a moderate turbulence level ($C_n^2 = 10^{-13} \text{ m}^{-2/3}$). Collimated beam in free-space (solid, thick); Collimated beam in turbulence (solid, thin), Focused beam in free-space (dash, thick) and in turbulence (dash, thin).

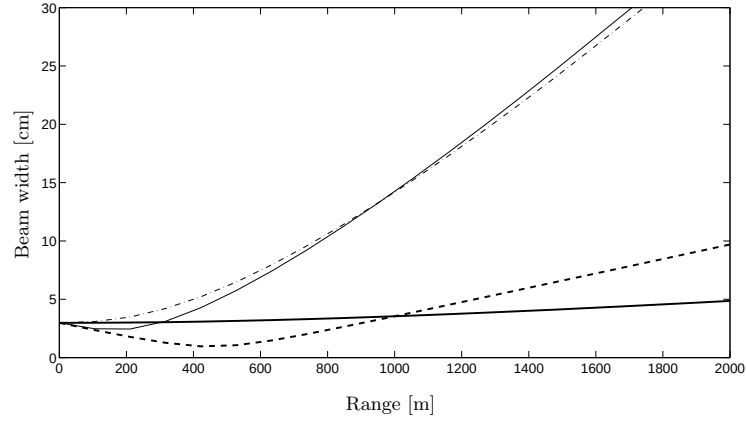


Figure 3.7: Same as Fig. 3.6 with strong turbulence level ($C_n^2 = 10^{-12} \text{ m}^{-2/3}$).

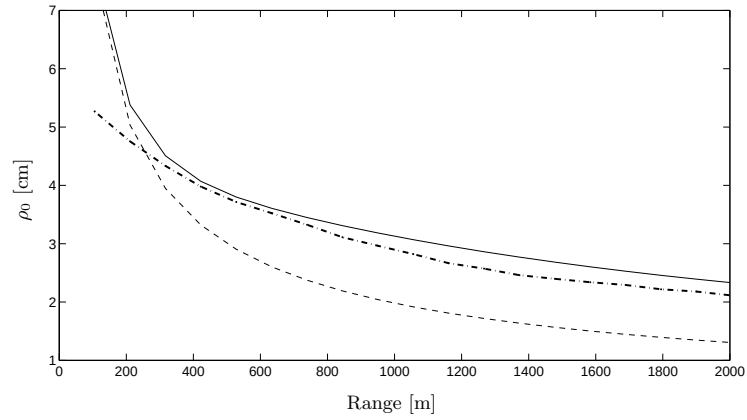


Figure 3.8: Theoretical and simulated transverse coherence radius as a function of range of a collimated beam for a moderate turbulence level ($C_n^2 = 10^{-13} \text{ m}^{-2/3}$ and an inner scale of 1 cm) : Simulated (dot-dash, thick); theoretical plane wave (dash), and theoretical Gaussian beam (solid).

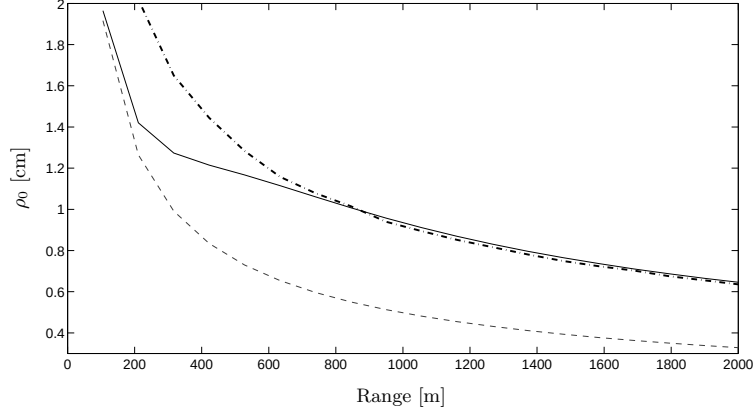


Figure 3.9: Similar to 3.8 with strong turbulence level ($C_n^2 = 10^{-12} \text{ m}^{-2/3}$).

$z = 0$. The evolution of the SNR in free-space is given for comparison. We first see that for both configurations, an increase of the turbulence level deteriorates the LIDAR minimum detection range by a faster decrease of the SNR with range. This can be easily interpreted by remembering that turbulence induces an additional spreading of the transmitted beam which in turn reduces the measured power for a fixed receiver solid angle. It can also be shown from asymptotic theory [Frehlich 1991] that, at long ranges, the SNR is inversely proportional to the square of the beam size. The variations of the SNR due to refractive index fluctuation, even in clear-air conditions with constant optical transmission and backscattering, may attain a few dB depending on the time of the day or even the telescope scanning elevation angle. A decrease of up to 10 dB also appears when choosing a bistatic configuration instead of a monostatic one. Let's define the LIDAR visibility as the maximum range at which the SNR is above a given threshold defined as a function of the maximum allowed estimation variance of the algorithm used. For a threshold of -5 dB, the visibility at $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ would be around 400 m for the bistatic case and around 1 km for the monostatic one. We also notice at short distances a strong focalization of the beam, which appears at high turbulence levels and with a monostatic configuration, at the expense of a faster decrease of the SNR at longer ranges. This well-known enhancement effect due to double-path propagation in the same turbulence (see Taylor assumption in Section 2.1.1) can be more easily quantified on the optical gain profiles given in Fig. 3.11 (top figure). It gives the heterodyne optical power gain comparatively to the free-space case only due to an increase of the refractive turbulence (see Eq. 2.108). The configuration gain, given by Eq. 2.110, is also represented. The coherent power variance, given by Eq. 2.109, is shown

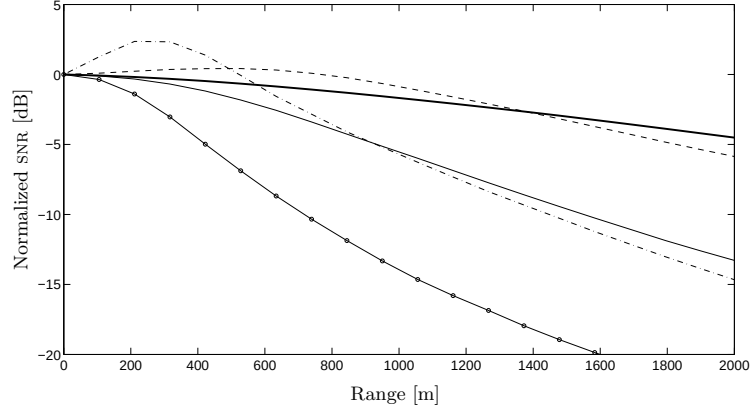


Figure 3.10: Evolution of the normalized SNR with range for monostatic and bistatic configuration and two level of turbulence. The LASER beam radius is 3 cm and the wavelength $\lambda=1.55 \mu\text{m}$: free-space (solid, tick); bistatic, $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (solid); monostatic, $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash); bistatic, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (solid-dot); monostatic, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash-dot).

in Fig. 3.12. This parameter is important since it gives the variability of the detected power and thus of the SNR between two successive shots. The coherent power variance is closely related to the aperture averaging principle which states that the higher the number of bright LASER spots in the receiver area, due to either the speckle effect and the scintillation, the smaller the power variance. For strong-turbulence conditions and large propagation paths, the LASER beam size at the target no longer depends on the transmitter aperture. The beam average depends on the turbulence beam spreading and the effective transmitter aperture is the coherent diameter of the beam [Belmonte 2000b, Andrews 2000]. As the coherent diameter is small, the beam-averaging effect is important and the heterodyne variance is reduced. A contrario, for shorter propagation paths, the partial focusing of the beam due to the refractive turbulence produces larger spots and reduces the averaging. This explains why in Fig. 3.12 the variance increases faster at shorter ranges for higher turbulence levels and decreases more rapidly at longer path distances. We also observe that the variance is more important for a bistatic configuration. Fig. 3.10 to Fig. 3.12 clearly show the importance of monostatic configurations to reduce the sensitivity to refractive turbulence.

The next analysis concerns the choose of the transmitted beam characteristics, i.e. size and focalization, for a given truncation factor. In [Belmonte 2000a], it has been shown that, at short wavelengths (around $1 \mu\text{m}$), small apertures and

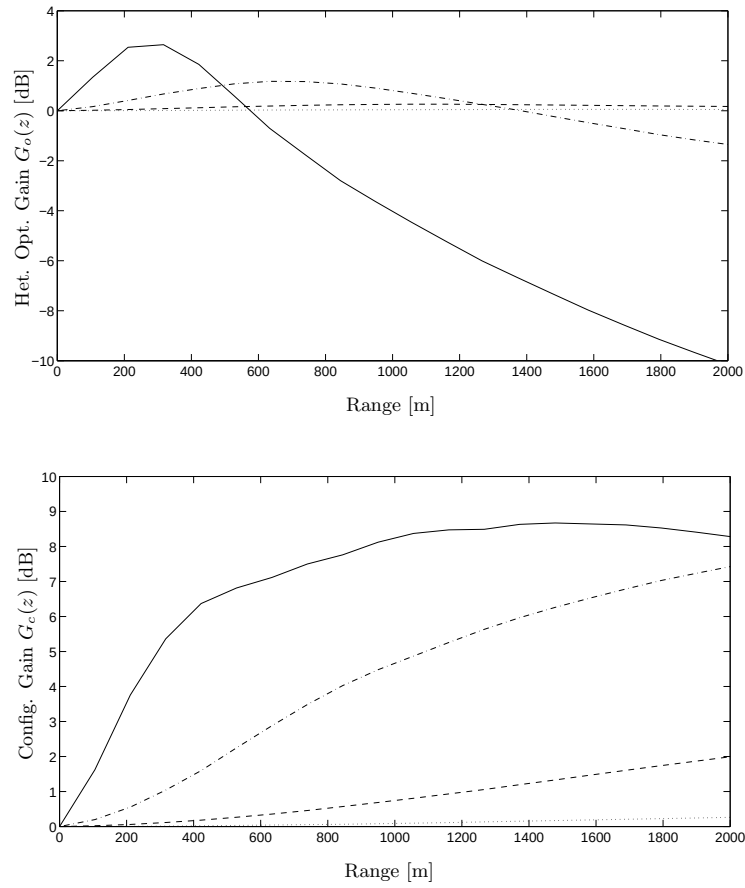


Figure 3.11: Optical gain (top figure) and configuration gain (bottom figure) of a coherent LIDAR in monostatic configuration for a different level of turbulence: $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ (dot), $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ (dash), $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash-dot), $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (solid).

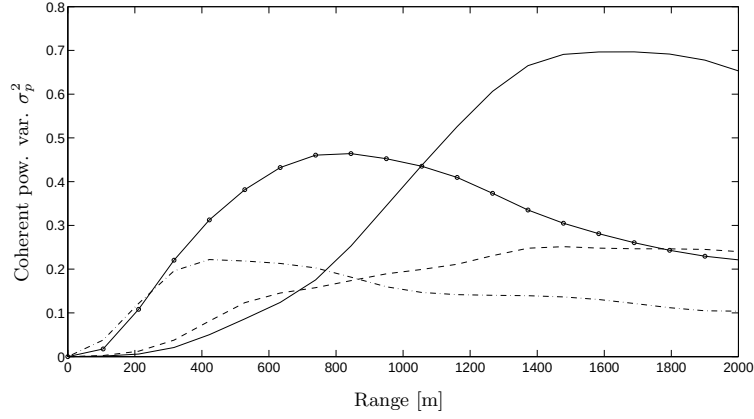


Figure 3.12: Evolution of the normalized coherent power variance with range for monostatic and bistatic configuration and two levels of turbulence. Legend similar than Fig. 3.10.

thus small output beams can exceed the performance of larger telescope with a focal length adapted to the range of interest. This consideration is of the most practical importance for ground-based LIDAR systems and point the fact that focalizing the transmitted beam is not necessarily a good option. Moreover, in the next figures, we will show that due to the scintillation and the variation of the turbulence level with time, the maximum performance range may vary significantly. Fig. 3.13 gives the SNR with range for different focalization of a beam propagating in weak to strong turbulence. The main message driven by this figure is that whatever the focalization of the beam at weak turbulence, the range at which the best SNR is obtained tends to be the same at strong turbulence, around 200 m in this example. Moreover, we observe at $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ that the collimated beam have a sensibly better performance than the two focalized cases. It is also worth remembering that choosing a focalized transmitted beam is always a trade-off between enhancing the performance at a given short range and the faster decrease of the SNR at longer ranges. This is particularly visible at the $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ case of Fig. 3.13. The corresponding heterodyne optical power profiles are represented in Fig. 3.14. We observe that this gain is upper bounded by the collimated beam case for ranges up to 1600 m for $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ and 1300 m for $C_n^2 = 10^{-12} \text{ m}^{-2/3}$. The influence of moderate turbulence level on the coherent power variance is given in Fig. 3.15. The variance is less important for a range interval which tends to be smaller for an increasing turbulence level. This variance reduction with decreasing focalization distance is however not significant enough to provide a

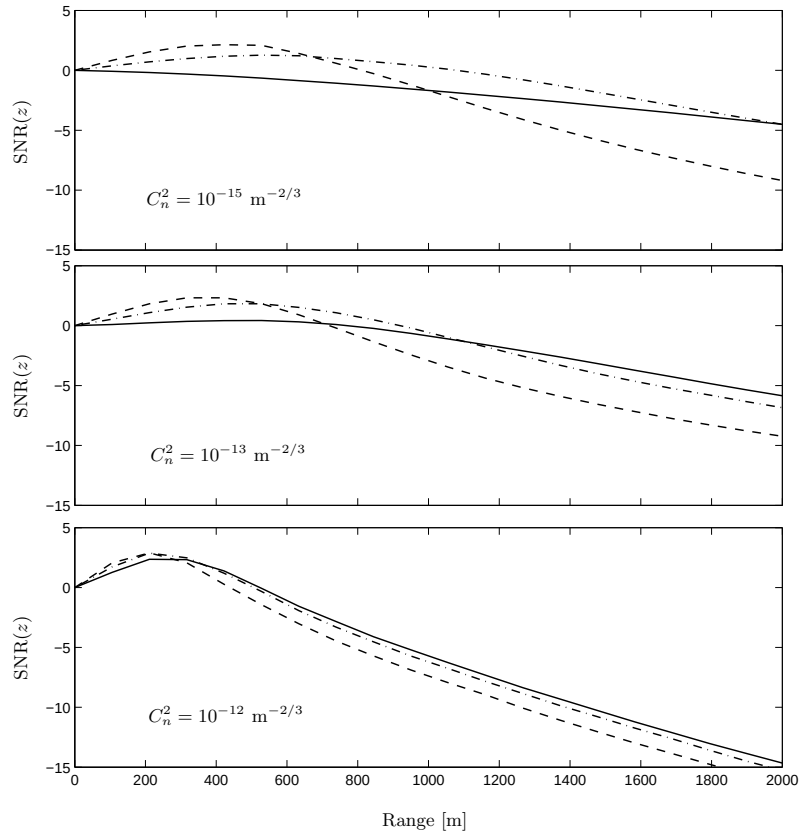


Figure 3.13: Influence of the refractive turbulence on the signal-to-noise ratio for a monostatic configuration and focalized beams at $F_T = \infty$ (solid), $F_T=500$ m (dash) and $F_T=1000$ m (dash-dot). The output beam radius is 3 cm.

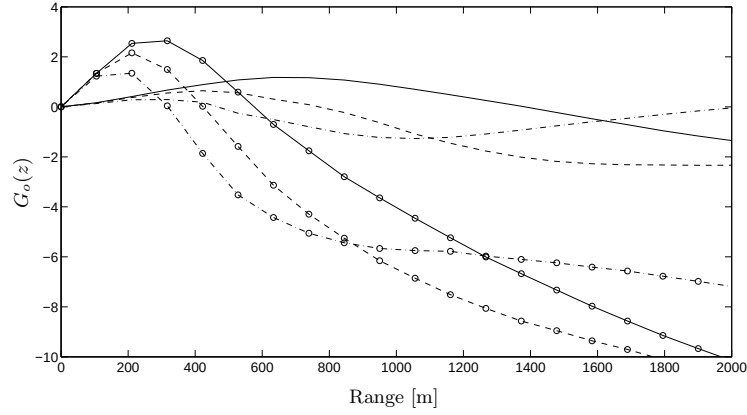


Figure 3.14: Heterodyne optical power gain for a monostatic configuration and focalized beams at $F_T = \infty$, $F_T=500$ m and $F_T=1000$ m. The output beam radius is 3 cm. Collimated, $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (solid); $F_T=500$ m, $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash-dot); $F_T=1000$ m, $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash); Collimated, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (solid, $\circ$); $F_T=500$ m, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash-dot, $\circ$); $F_T=1000$ m, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash, $\circ$).

practical interest. The gain of the collimated case in this range interval also tends to decrease with the turbulence. The system efficiency (see Eq. 2.105) is given in Fig. 3.16. It can be seen that, when the emitted beam is focalized, the system efficiency increases faster than for a collimated beam but saturates also faster to a lower value. However at $C_n^2 = 10^{-12} \text{ m}^{-2/3}$, the system efficiency has nearly the same profile with range in both case. We also observe that the system efficiency can be higher than its value in the absence of refractive turbulence. The observations exposed before about the possible necessity to focalize the beam leads us to the conclusion that, for a ground-based LIDAR essentially operating at a moderate-to-strong turbulence level, the collimated case was a rather good choice.

The remaining question about the design of the telescope concerns the beam size. We already know from Section 2.2.2 that the smaller the beam size, the more important its divergence will be. In terms of LIDAR performance, a decrease of the output beam radius provides a better SNR at short distances but also a more rapidly decreasing one at larger distances. A nearly constant SNR profile with range would also be needed when the phenomenon to analyze is not comprised in a determined range interval. Fig. 3.17 gives the influence of refractive turbulence on the SNR for beam radii of 1.5 cm, 3 cm and 6 cm and a constant truncation factor. The SNR profiles for the 1.5 cm and 6 cm

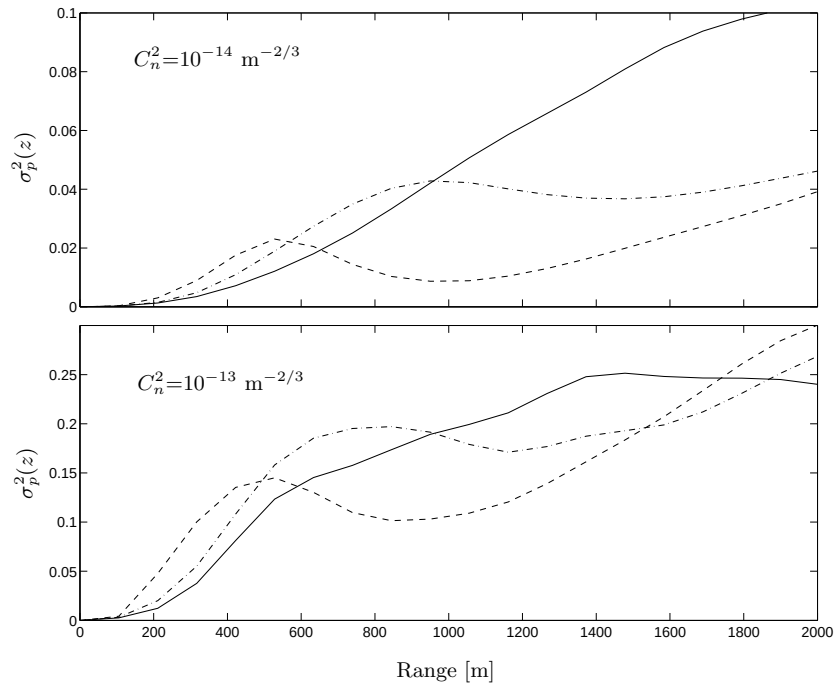


Figure 3.15: Influence of the refractive turbulence on the normalized coherent power variance for focalized beams at $F_T = \infty$ (solid), $F_T=500$ m (dash) and $F_T=1000$ m (dash-dot). The turbulence levels are $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ for the top figure and $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ for the bottom figure. The output beam radius is 3 cm.

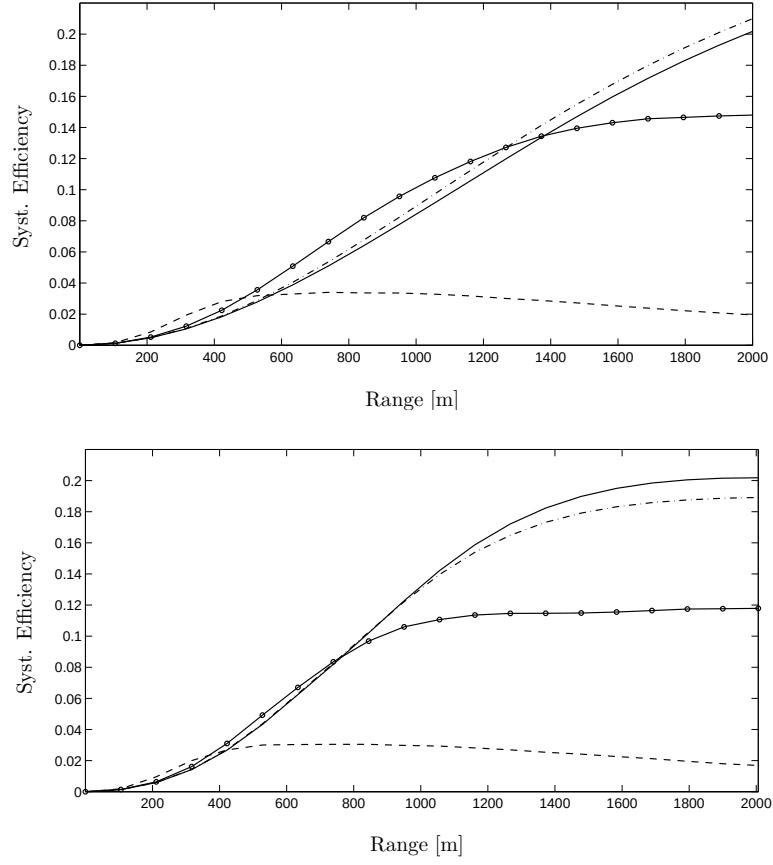


Figure 3.16: System efficiency for a monostatic configuration and an output beam radius of 3 cm. The top figure is for a collimated beam and the bottom one for a focalized beam with $F_T=1000 \text{ m}$. Free-space (solid), $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ (dash-dot), $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (solid, o), $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash).

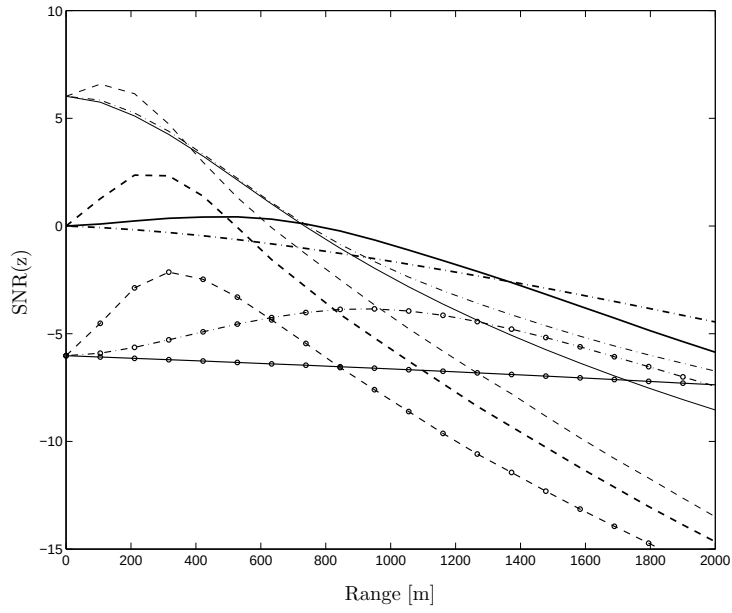


Figure 3.17: Influence of the refractive turbulence on the signal-to-noise ratio for a monostatic configuration, and different output collimated beam radius: $\sigma_T=3$ cm (middle three), $\sigma_T/2$ (top three) and $2\sigma_T$ (bottom three). The SNR profiles are normalized about those obtained at σ_T ; $\sigma_T/2$, $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ (solid, thin); $\sigma_T/2$, $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash-dot, thin); $\sigma_T/2$, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash, thin); σ_T , $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ (solid, thick); σ_T , $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash-dot, thick); σ_T , $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash, thick); $2\sigma_T$, $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ (solid, $\circ$); $2\sigma_T$, $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (dash-dot, $\circ$); $2\sigma_T$, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash, $\circ$).

cases are normalized to their equivalent value at 3 cm. We first observe that the sensitivity to turbulence is reduced with decreasing beam size. The decrease rate also tends to be independent on the beam size for strong turbulence. An e^{-2} irradiance beam radius of 3 cm has been chosen as a trade-off between SNR decrease rate at large propagation distance and sensitivity to refractive turbulence.

In conclusion, the LASEF telescope has been chosen to be monostatic with a Newton configuration. The emitted beam is collimated and has a e^{-1} radius of 30 mm. The aperture radius is 50 mm. The Newton configuration was necessary due to the unavailability at the time of the development of an high-isolation free-space isolator able to work at the targetted pulse energy. Unfortunately, the deflection mirror introduces an unnegeable detected power loss.

3.2 Monte Carlo simulation of the coherent responsivity

Fluctuations of the received signal power due to atmospheric turbulence have almost the same effect as those resulting from speckle. Both phenomena have a major impact on the degradation of the various estimates obtained from the coherent signal such as the wind velocity and dispersion. However, the time scale of the power fluctuations due to turbulence is order of magnitude larger than that induced by speckle. They are therefore more difficult to reduce by pulse accumulation even when using systems with a high pulse repetition frequency. Consequently, turbulence-induced fluctuations have to be simulated in order to provide realistic LIDAR signal simulations.

As we have seen in the previous section, the PSM allows us to estimate the LIDAR performance statistics as well as the instantaneous signal power over a few hundred beam propagation simulations [Banakh 2000]. It also provides the evolution of the coherent power variance with range denoted σ_C^2 . The major limitation of this method is the important computational cost needed to generate realizations of the signal power for a whole scan of a few thousand of line-of-sights. This is particularly true for small beam sizes, high turbulence level and long propagation paths which generally requires a finer grid resolution and a larger screen size. For those reasons, LIDAR signal simulations typically use different estimators of the mean signal power obtained either numerically with the PSM on a limited number of propagation simulations or theoretically by asymptotic methods for weak and strong fluctuations of the LASER beam intensity [Frehlich 1991]. Since the signal statistics and LIDAR performance parameters directly depend on the coherent responsivity, we will describe, in this section, a method to generate an infinite number of realizations of this parameter by means of the random-walk Metropolis-Hasting Monte Carlo method (MCMH).

3.2.1 Simulation principle

The coherent responsivity is an ensemble averaged quantity, noted $C(z, t)$ and defined by Eq. 2.93, which determines the effect of the atmospheric refractive turbulence as well as the beam characteristics on the system efficiency and on the average heterodyne power. It can be considered as the ensemble average of a random variable $X(z, t)$ such that:

$$C(z, t) = \langle X(z, t) \rangle, \quad (3.24)$$

and

$$X(z, t) = \lambda^2 \int_{-\infty}^{\infty} j_T(\mathbf{p}, z, t) j_{\text{BPLO}}(\mathbf{p}, z) \, d\mathbf{p}. \quad (3.25)$$

The instantaneous heterodyne signal power is thus obtained by replacing $C(z, t)$ by $X(z, t)$ in Eq. 2.92:

$$P(t) = 2(G_D S)^2 U_L P_{\text{LO}} \int_0^{\infty} K^2(z) \beta(z) |p(t - 2z/c)|^2 X(z, t) \, dz. \quad (3.26)$$

A direct simulation of this process would be to draw samples from a N_p -dimensional joint probability density function estimated from the N_r PSM realizations. However, the number of PSM simulations necessary to have a good confidence on this complex density would be too important. This problem is known as the Hughes effect or also the curse of dimensionality. We have thus limited the complexity of the problem to the estimation of the joint densities between each pair of adjacent screens. Doing this, we assume that the stochastic process $X(z, t)$ with probability density function $T_X[x(z)]$ is a Markovian process satisfying

$$T_X[x(z_n)|x(z_0), x(z_2), \dots, x(z_{n-1})] = T_X[x(z_n)|x(z_{n-1})]. \quad (3.27)$$

The Markov assumption is true for $X(z, t)$ under the same conditions as the ones imposed on the split-step method to be valid.

We have also seen in the previous section that the intensity fluctuations of the beam induced by the turbulence in an atmospheric slab of length δz should be considered in the weak scattering regime and that the condition on the irradiance variance $\sigma_I^2(\delta z) < 0.1$ has to be satisfied. It therefore determines an upper bound on δz for the split-step method to be accurate, i.e. for the statistics of the beam irradiance in z to only depend on the statistics of the beam irradiance at $z - \delta z$ as well as to those of the turbulence. In other words, the condition on $\sigma_I^2(\delta z)$ means that the beam irradiance fluctuations on consecutive screens must be kept small, imposing a minimum correlation between them. Fig. 3.18 represents the correlation coefficient between a number of screens and those positionned at shorter distances. We observe that the correlation coefficient decreases more rapidly for higher turbulence levels.

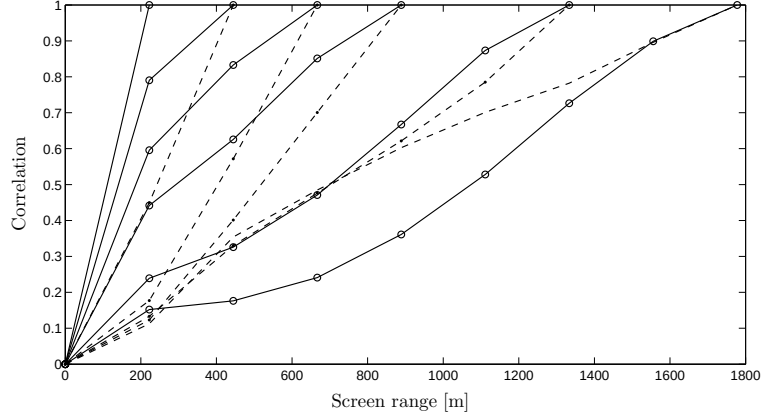


Figure 3.18: Correlation coefficient between variables $X(z, t)$ on phase-screens positionned at distances $\{1, 2, 3, 5\} \times \delta z$ with the screens at shorter distances for two levels of turbulence: $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ (solid) and $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash). The unit-correlation points corresponds to the screens for which we want analyze the contribution of the preceding ones.

As $X(z, t)$ is sampled in space at the PSM level, we consider the discrete-space and continuous-state process $X_n = X(n \delta z, t)$ for $n = 0 \dots N_p - 1$ where the variable X_0 at the output of the transmitter lens is considered to be deterministic. In the present case, the sequence $\{X_0, X_1, \dots, X_{N_p-1}\}$ is a non-homogeneous Markov chain described by its transition densities for which we have the following inequalities

$$T_X[x_{i+1}|x_i] \neq T_X[x_i|x_{i-1}], \quad \forall i \in \{0, N_p - 1\} \quad (3.28)$$

where $T_X[x_{i+1}|x_i]$ is the transition probability density from X_i to X_{i+1} . Note that the first density $T_X[x_1|x_0]$ is actually a 1-D density since X_0 is deterministic.

The main idea of this simulation is thus to generate a realization of the sequence $\{X_n\}$ per line-of-sight from the transition probabilities $T_X[x_n|x_{n-1}]$ estimated from the PSM Monte-Carlo simulations detailed in Section 3.1.2 and then to interpolate it in space in order to get a realization of coherent responsivity for all z . The generation of the sequences is realized by the Metropolis-Hasting algorithm [Metropolis 1953, Hastings 1970].

3.2.2 Estimation of the inter-screens joint densities

By the Bayes' theorem, the conditional probability densities $T_X[x_n|x_{n-1}]$ can be obtained from the joint density $T_X[x_n, x_{n-1}]$ which is estimated by the sequences $\{X_n^r\}$ produced by N_r PSM simulations. Two estimations techniques

have been tested in this thesis: a kernel-based algorithm known as the balloon estimator [Silverman 1986, Parzen 1962, Scott 2005] as well as a EM algorithm (Expectation-Maximization) [Dempster 1977]. Fig. 3.19 gives an illustration of the estimation technique.

The basic form of the balloon estimator, noted $\hat{f}_{\mathbf{x}}(\mathbf{x})$, is given by:

$$\hat{f}_{\mathbf{x}}(\mathbf{x}) = \frac{1}{N_r |H(\mathbf{x})|^{1/2}} \sum_{i=1}^{N_r} K \left(H(\mathbf{x})^{-1/2} (\mathbf{x} - \mathbf{x}_i) \right), \quad (3.29)$$

where K is the kernel function and $H(\mathbf{x})$ is a positive-definite smoothing matrix associated with the estimation point $\mathbf{x}$. A kernel estimator is thus an equal

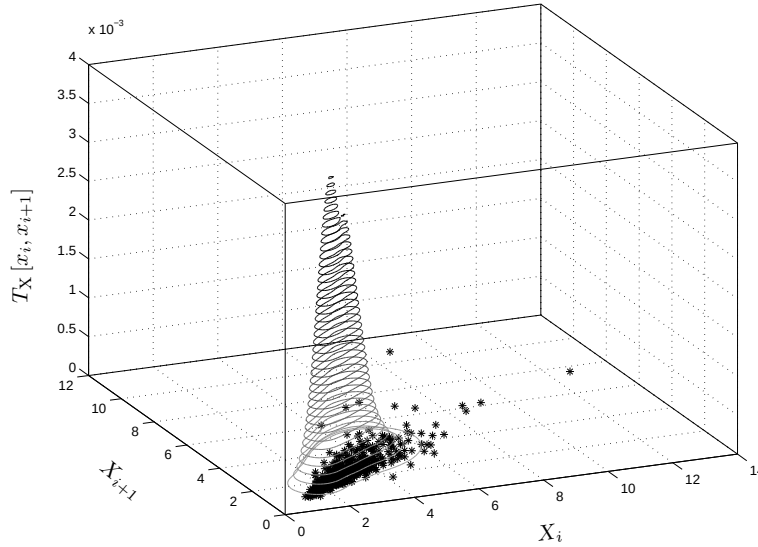


Figure 3.19: Example of the estimation of a joint probability density function $T_{\mathbf{x}}[x_i, x_{i+1}]$ using the balloon estimator. The values of the coherent responsivity samples X_i are given with normalizing factor 10^{-10} . The $\star$ symbols represent the PSM realizations.

mixture of N_r kernels, centered at the N_r data points. We use the k -nearest neighbor (k -NN) estimator [Loftsgaarden 1965]. It is obtained by choosing a kernel which is a uniform density on the unit sphere with $H(\mathbf{x}) = h_k(\mathbf{x}) I_d$ and letting $h_k(\mathbf{x})$, called the bandwidth, be the distance from $\mathbf{x}$ to the k -th nearest neighbor data point. This estimator thus tries to incorporate larger bandwidths in the tails of the density. In practice, we have observed that the density in the sparse regions tends to be overestimated. This consequently induces the generation of sequences with statistically more important gradients between

screen positions which is not physically consistent. Moreover, as the kernels are isotropic, narrow densities characterizing high correlation between successive X_n variables, cannot be modelled precisely.

To overcome these problems, an EM algorithm using Gaussian clustering has been used instead. The joint density is now modelled as a mixture of $N_e < N_r$ distributions for which the maximum likelihood of their parameters are estimated recursively:

$$\hat{f}_x(\mathbf{x}) = \sum_{i=1}^{N_e} \alpha_i p(\mathbf{x}|\omega_i), \quad (3.30)$$

where the normal densities are given by, for $i=1 \dots N_p - 1$,

$$\begin{aligned} p(\mathbf{x}|\omega_i) &= p(\mathbf{x}|\boldsymbol{\mu}_i, \Sigma_i) \\ &= |2\pi\Sigma_i|^{-1/2} \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_i)^T \Sigma_i^{-1}(\mathbf{x} - \boldsymbol{\mu}_i)\right), \end{aligned} \quad (3.31)$$

with $\sum \alpha_i = 1$ and $\alpha_i \geq 0$. The parameters to estimate are the mixing weights α_i , the mean vectors $\boldsymbol{\mu}_i$ as well as the covariance matrices Σ_i . The recursive algorithm is described in [Cwik 1996]. The number of distributions to use mainly depends on the complexity of the joint density but can be adjusted to avoid overestimation. The initialization is done by selecting the N_e centroids randomly inside the PSM simulation data set and computing the initial covariance matrices associated to each densities $p(\mathbf{x}|\omega_i)$ with a k -NN estimator.

3.2.3 The Metropolis-Hasting algorithm

Once each joint density $T_x[x_n, x_{n-1}]$ has been estimated, sequences $\{X_n\}$ are generated by the following steps executed N_m times:

1. Computes X_0 which is a system-dependent deterministic value.
2. Generate X_1 with density $T_x[x_1|x_0] = T_x[x_1]$.
3. For $i = 2 \dots N_p$,
 - (a) Knowing X_{i-1} , computes the density $T_x[x_i|x_{i-1}]$ as the slice of the joint density $T_x[x_i, x_{i-1}]$ at $x_{i-1} = X_{i-1}$.
 - (b) Draw X_i with conditional density $T_x[x_i|x_{i-1}]$.
4. Go back to step 2 to generate the following realization $\{X_n\}$.

The random-walk Metropolis-Hasting algorithm is used to generate the realizations of the random variables X_i from the conditional densities. It is a Markov chain Monte Carlo method described in much details in [Chib 1995].

A practical advantage of the MCMH algorithm is that it permits to use the same PSM statistics defining a given LIDAR for fixed refractive turbulence conditions with different CFD simulations. Moreover, the detector parameters (bandwidth, noises), pulse characteristics (energy, duration) as well as the $K(z)$ and $\beta(z)$ profiles can also be modified.

3.2.4 Results and discussion

The total extinction coefficient as well as the aerosol backscatter coefficient have been assigned to typical values in clear-air conditions at low altitudes and at $\lambda=1.55 \mu\text{m}$: $\alpha=7 \cdot 10^{-5} \text{ m}^{-1}$ and $\beta=10^{-7} \text{ m}^{-1}\text{sr}^{-1}$. The PSM parameters value are the same than those in Section 3.1.4. The maximum range is 2 km. The number of MCMH realizations is $N_m=1200$.

The results of the MCMH algorithm are analyzed by comparing the marginal densities $T_X[x]$ of the data obtained by the PSM and those computed via the MCMH algorithm at every distance where a phase-screen is positionned. Practically, this is performed by estimating the error either directly on $T_X[x]$ or on its first four statistical moments. Fig. 3.20 demonstrates the good matching between the statistics of the PSM and MCMH simulations. Fig. 3.21 enables us to estimate visually the interval of power variations as well as their typical longitudinal scales.

The importance of considering the fluctuations of the heterodyne power induced by the refractive turbulence for each line-of-sight mainly depends on three factors: the irradiance fluctuation regime, the pulse duration and the accumulation level defined as the number of successive signals necessary to produce a single radial velocity profile estimate. Depending on these factors, it could be more appropriate to use the average coherent responsivity $C(z, t)$ computed with the PSM simulations instead of the realizations $X(z, t)$ computed either directly by the PSM or the MCMH simulations. In the case of the weak fluctuation regimes, the normalized coherent power variance is small and so is the variations of the heterodyne power. Moreover, we have seen in Eq. 3.26 that $P(t)$ has a time scale defined by the spatial filtering of the irradiance extinction $K(z)$, the backscatter coefficient $\beta(z)$ and the coherent responsivity $X(z, t)$ by a weighting function given by the square modulus of the LASER pulse profile in space. The longitudinal fluctuations of $X(z, t)$ will therefore have a smaller effect on the heterodyne signal if the spatial extend of the pulse is large compared to the spatial scales of $X(z, t)$. Nevertheless, for pulse durations of a few hundred of nanoseconds and typical refractive index levels, this filtering is not important. Similarly, for very low SNR, the pulse accumulation can be large and the accumulated value of $X(z, t)$ tends to $C(z, t)$ which reduce the necessity of the MCMH simulation. Furthermore, if the PSM realizations can be saved and for a small number of line-of-sight, a simple permutation between the sequences can be used to generate different measurement scans. In conclusion,

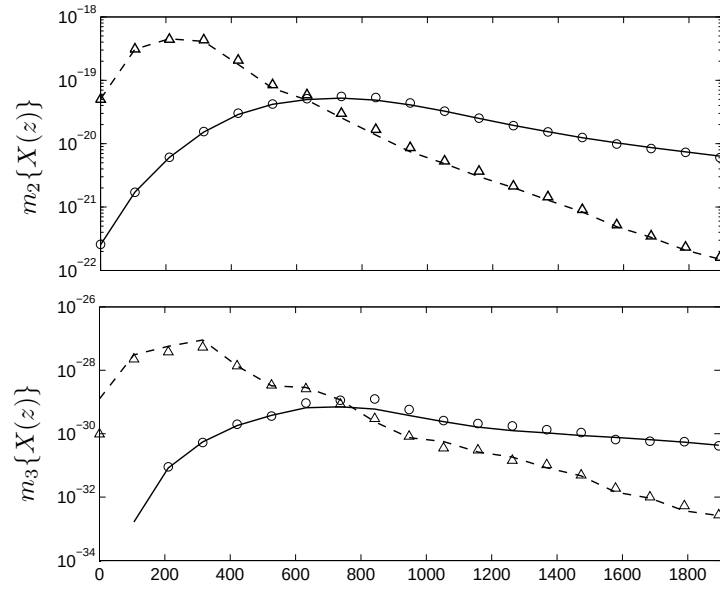


Figure 3.20: Second and third statistical moments of the marginal densities $T_x[x]$ with range for two different measurement conditions: $F_T=1000$ m, $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ (solid for PSM and $\circ$ for MCMH) and collimated, $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ (dash for PSM and $\triangle$ for MCMH).

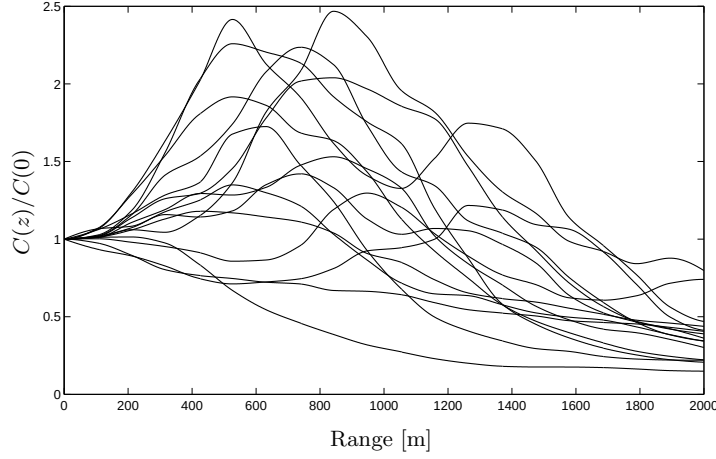


Figure 3.21: Example of realizations of the normalized coherent responsivity profile $C(z)$ for collimated beam at $C_n^2 = 10^{-12} \text{ m}^{-2/3}$.

the MCMH simulations has its major interest for diurnal conditions when the σ_c^2 is high and with low accumulation levels.

Concerning the computation time, as soon as the PSM realizations are available, the complete MCMH simulation including the PDF estimation step only takes a few tens of seconds for a complete scan of a few thousands line-of-sights on a workstation with average performance.

3.3 An integrated LIDAR signal simulation

One of the purpose of the LIDAR simulation program developed in this thesis is the generation of realistic measured signals for the design and the validation of efficient air velocity estimators. The main originality of the proposed simulation method is the integration of both optical and fluid dynamics numerical methods to take into account the coherent Doppler signal coherence loss due to the refractive turbulence, the speckle effect as well as the fine structures of the wake vortex velocity field. The expressions of the coherent Doppler LIDAR signal and its variance have been detailed in Section 2.4 and are respectively given by Eq. 2.89 and Eq. 2.92. We will now describe how the different physical quantities involved are effectively simulated.

3.3.1 Classical time-domain signal synthesis

The numerical simulation of the LIDAR signal is performed by discretizing Eq. 2.89 by means of the Feuilleté model presented in [Salamitou 1995] where the scattering volume is divided into slices of depth $\Delta z \neq \delta z$ in which the atmospheric properties are considered homogeneous. The depth must satisfy the following constraints:

$$\lambda \ll \Delta z \ll \frac{c}{\nu_1}, \quad (3.32)$$

where ν_1 [Hz] is the LASER linewidth. The lower bound is dictated by the use of the Huygens-Fresnel integral formulation on which this model is based. The upper bound is related to the spectral sharpness of the LASER and translates the fact that the slice depth should be small compared to the coherent length. Another constraint should be added to take into account the physical aspect of the flow under investigation. A vortex with a core size of $r_c=2.5$ m, for instance, will be well discretized if $\Delta z \leq r_c/3 = 0.8$ m. For convenience in calculation, the depth is sometimes determined by the sampling frequency F_s by

$$\Delta z = \frac{c}{2 F_s}, \quad (3.33)$$

which is the distance the pulse propagates during the sampling period. For a pulse duration τ_p defined at e^{-1} of its maximum value, the number of slices M contributing to the signal at time t is given by

$$M = \frac{c \tau_p}{2 \Delta z}. \quad (3.34)$$

Due to the large number of aerosol particles in each slice, the total backscattered field from this slice is the superposition of many scattered fields with random phase. The response of an atmospheric slice at distance z can thus be modelled by a complex Gaussian random variable α_z . Moreover, the response between two slices are statistically independent so that we can write:

$$\langle \alpha_z \alpha_{z'}^* \rangle = \delta(z - z'), \quad (3.35)$$

where δ is the Dirac distribution. Taking into account the homogeneity assumption inside each slice, the equation of the heterodyne current measured at the output of the photodetector in Eq. 2.89 can be rewritten as:

$$i_h(t) = 2G_D S(U_L P_{Lo})^{1/2} \Re \left\{ \int_0^\infty p(t - 2z/c) \alpha_z H(z)^{\frac{1}{2}} \times \exp(2\pi j(f_{IF} - 2v_R(z)/\lambda)t) dz \right\}, \quad (3.36)$$

where the quantity $H(z)$ is the average system gain over the turbulence given by Eq. 2.95. Simulations with the previous equations will provide measured

signals where the only existing random processes are the multiplicative noise α_z related to the speckle effect and the additive detection noise $n(t)$. Analytical expressions for $C(z)$ from the asymptotic theory are usually used and can be found in [Frehlich 1991, Frehlich 1993b]. The discrete form of the heterodyne signal at the output of the photodetector is finally given by

$$s(kT_s) = 2G_D S(U_L P_{Lo})^{1/2} \Delta z \Re \left\{ \sum_{l=-L_c}^{L_c} \alpha_l p(kT_s - lT_l) \right. \\ \left. \times H^{\frac{1}{2}}(l\Delta z) \exp(-4\pi j v_R(l\Delta z) kT_s/\lambda) \right\} + n(t), \quad (3.37)$$

where $n(t)$ is the detection noise, $T_l = 2\Delta z/c$, T_s [s] is the sampling interval and L_c is the summation length chosen to include most of the illuminated aerosol targets. The amplification noise and the shot noise are assumed to be independent Gaussian additive noises. For convenience in the simulation, they are assimilated to a unique Gaussian noise with variance σ_n^2 given by the following summation:

$$\sigma_n^2 = \langle i_{\text{NEP}}^2 \rangle + \langle i_{\text{N}}^2 \rangle, \quad (3.38)$$

where $\langle i_{\text{NEP}}^2 \rangle$ and $\langle i_{\text{N}}^2 \rangle$ are respectively the amplification noise and the shot noise variances given by Eq.2.97 and Eq.2.98. The RIN noise is not modeled since it appears at low frequencies compared to the intermediate frequency and can therefore be easily filtered.

3.3.2 Combining space and time models

The radial velocity profiles $v_R(z)$ in Eq. 2.95 are obtained by scanning through fluid dynamics simulations of wake vortices and atmospheric turbulence. In this thesis, simulations of only two-dimensional scans are considered although the simulation technique is obviously valid for any scanning pattern. As the fluid flow velocity fields issued from the CFD simulations are computed on a cartesian grid, it is necessary to perform a projection and an interpolation of the velocity field on the line-of-sights in order to obtain the radial velocities. The interpolation scheme selected in this work to interpolate information from a fluid simulation grid to the LIDAR measurement points is the M'_4 scheme [Winckelmans 2004].

To take into account the fluctuations of the instantaneous signal power due to refractive turbulence, the term $C(z)$ in Eq. 2.95 of the system gain $H(z)$ has been replaced by the realizations of $X(z)$, obtained from the MCMH algorithm exposed in the previous section. Hence, a single sequence is used for each line-of-sight. However, assigning different sequences $\{X_n\}$ for successive measured signals (or adjacent line-of-sights) is equivalent to impose that the turbulence along the propagation paths along which they have been obtained is independent and thus that the power fluctuations due to turbulence are

uncorrelated. To induce a partial correlation in the transverse direction, a curvilinear Gaussian filtering is performed over every line-of-sights for a fixed distance z with a spatial correlation length given by a fraction of the beam long-term size at the same distance.

Moreover, it is important to point out that both the variables $H(z)$ and $v_r(z)$ take into account the effect of the turbulence although they do not consider exactly the same spatial scales. On the one hand, the spatial scales simulated by the LES depend on the grid resolution and goes to typically less than the meter, e.g. 0.8 m for the wake-vortex simulations, to a few tens of meters. On the other hand, it has been shown in Section 2.2.6 that, for moderate-to-strong fluctuation regimes, the scales of turbulence, that are important to model the scintillation effect, are those lower than the transverse coherent radius and higher than the scattering disk size (see Fig. 2.5). Furthermore, for a monostatic configuration, the larger scales have no influence on the beam statistics. The turbulent scales considered for the signal instantaneous power fluctuations and those taken into account by the LES are therefore more likely to be distinct under these conditions. In contrary, for weak fluctuation regimes, the important scales have a size of the order of magnitude of the first Fresnel zone and a recovery can exist between the two simulations. The following general assumption has thus been introduced:

Assumption 3.1 *The spatial scales of the atmospheric turbulence lower than the beam size are assumed to only have an effect on the beam spatial coherence and therefore on the instantaneous signal power represented by $H(z)$. On the other hands, the spatial scales higher than the beam size are assumed to only have an effect on the temporal correlation length of the coherent signal.*

In other words, the LIDAR optical performance parameters are not altered locally by the presence of a wake vortex. This assumption allows us to consider an intensity of the turbulence, given by the C_n^2 , which is not dependent on the CFD simulations, and thus to analyze different LIDAR performance situations with the same CFD database.

Another point that must be taken into account concerns the possible variation in particle density inside the wake vortex. The following assumption states that, as a results of the speckle effect itself inducing signal envelope fadings, the variations of the backscatter coefficient inside the wake vortex region are not considered since they are not estimable.

Assumption 3.2 *The aerosol particle density does not vary inside the wake vortex although a kind of stratification of their concentration can normally appear. The same population of aerosol particles are therefore present everywhere with the same density and radius distribution. In this case the aerosol backscatter coefficient is not supposed to vary in the wake vortex region.*

3.3.3 Results and discussion

Fig. 3.22 gives the evolution of the signal-to-noise ratio obtained from simulated signals with different turbulence levels by computing the signal and the noise variance on a complete scan of more than 5000 signals.

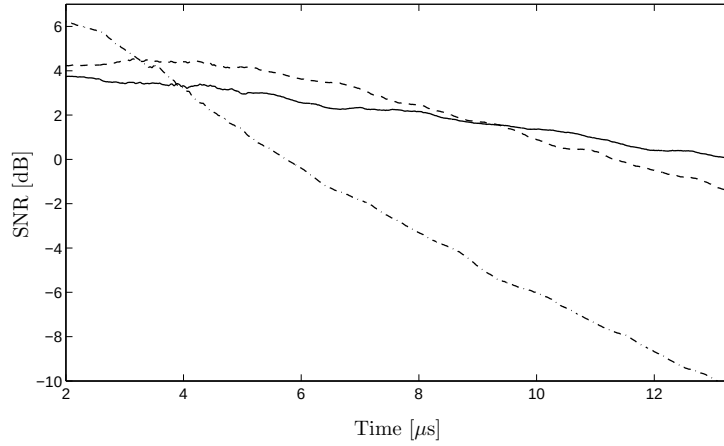


Figure 3.22: Evolution of the SNR with time computed from simulated signals for $C_n^2 = 10^{-15}$ (solid), $C_n^2 = 10^{-13}$ (dash) and $C_n^2 = 10^{-12}$ (dash-dot).

Fig. 3.23 illustrates the time decorrelation of the heterodyne signal due to the speckle effect as well as the fast variations of the radial velocity profile inside the wake vortex spatial region. The importance of this decorrelation can be easily apprehended by observing the time interval between two successive fades in the signal envelope. This interval is proportional to the pulse duration Δt when only speckle is present. It decreases substantially inside the wake vortex region before 6 μs . The effect of the LIDAR parameters on the signal statistics will be detailed in Chapter 4.

The computation time for the generation of one line-of-sight essentially depends on the sampling frequency, the maximum range and the pulse duration. It typically takes less than one second for a propagation distance of 2 km, a sampling frequency of 250 MHz and a pulse duration of more than 400 ns.

3.4 Conclusions

The main originality of the LIDAR simulator developed in this thesis resides in the integration of different levels of numerical simulation in order to give a complete

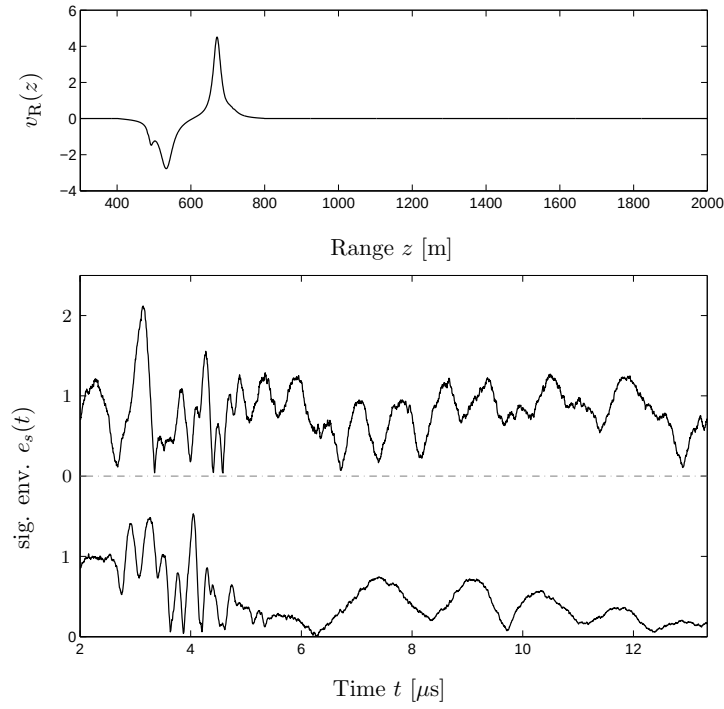


Figure 3.23: Illustration of the signal decorrelation due to speckle and wake vortex in ground effect (IGE). The top figure gives the radial velocity profile $v_R(z)$ used to generate the heterodyne signal. The bottom figure gives two realizations of the signal envelope $e_s(t)$ for $\Delta t=200$ ns (upper envelope) and $\Delta t=400$ ns (lower envelope). The relation between t and z is $z = ct/2$.

vision of the LIDAR design problem. Indeed, in comparison to existing simulators, it combines simulation techniques dedicated to fluid dynamics, optical propagation through the turbulent atmosphere and time-domain LIDAR signal synthesis. More specifically, this simulator has been designed to support the LIDAR development in two complementary ways.

The first way is the simulation of the coherent Doppler LIDAR performance depending on the LASER source, the telescope configuration and the atmospheric state. It reposes on the well-known phase-screen method (PSM) which allows one to simulate the propagation of an arbitrarily distributed LASER beam in a turbulent media as well as complex optical systems. This technique has notably been applied to the design of the telescope in the LASEF project. A number of simulations have thus been performed in order to determine its optimal parameters. This led to the development of a monostatic system based on a Newton telescope emitting a collimated Gaussian beam with a 3 cm e^{-1} irradiance radius. The receiver lens area has an aperture with a 10 cm diameter.

The second purpose of the LIDAR simulator is the generation of measured signals by scanning through either analytical models or LES databases of wake vortices in ground effect or in weak atmospheric turbulence. It also simulates the instantaneous fluctuations of the signal heterodyne power caused by atmospheric turbulence, in addition to those produced by the speckle effect. A Monte-Carlo Metropolis Hasting simulation is therefore performed from the statistics of the LIDAR coherent responsivity obtained by the PSM and thereby provides enough signal power realizations for a complete scanning containing thousands of line-of-sights. This operating mode has been extensively used for the design and the validation of advanced signal processing and estimation algorithms.

The simulation of the LIDAR measurement highlights the problem of considering the atmospheric turbulence in both the fluid and the optical propagation simulations. In this work, based on the optical scintillation theory and for the monostatic configuration, it has been assumed that the overlap between the turbulent scales simulated by these two simulations was sufficiently small such that they do not participate to the same performance-limiting processes. Hence, scales of turbulence on the order of magnitude or smaller than either the length of the first Fresnel zone or the beam coherence radius, depending on the fluctuation regime, only generate optical perturbations, i.e. optical scintillation, heterodyne power fluctuation and system gain deterioration with range. Fluid simulations are however supposed to only induce a decrease of the signal correlation length due to both the turbulence and the fast varying velocity fields. This assumption allows to consider different level of atmospheric turbulence with the same LES simulations. More advanced studies should nevertheless be done to integrate further these two simulation modes in order, for example, to generate an optical turbulence for which the statistics, e.g. structure functions or turbulence spectrum, are consistent with those of the fluid simulations.

Part II

Aircraft wake vortex characterization

In the first part of this thesis have been exposed the development of LIDAR numerical simulation techniques to evaluate the optical performances of the system when used in the boundary layer of the atmosphere. In chapter 3, it has also been described how to simulate the Doppler LIDAR signal measured at the output of the photodetector with given LIDAR and atmospheric parameters.

The following part describes how practically these systems can be used for air velocity measurement and wake-vortex parameters estimation. In Chapter 4, the theoretical results obtained on the estimation of the air velocity statistics using the Cohen's class of time-frequency distributions are exposed. These results are then applied, in Chapter 4, to the analysis of the wake vortex signal as well as on the development of appropriate estimation algorithms. Wake vortices detection in both transversal and axial configuration will be exposed in Chapter 5 and the results of the Orly field tests will be presented. We will finally finish this part, in Chapter 7, with the description of algorithms dedicated to the estimation of wake vortex parameters such as their position and intensity.

On wind velocity estimation

ONE of the most relevant question to consider when estimating the wind velocity with a coherent Doppler LIDAR is how close can we estimate its exact statistics. In this analysis, both the LIDAR and processing parameters must be taken into account and the derivation of the signal statistics in both the time and frequency domains is required.

In this thesis, we have obtained the expressions for the most important members of the Cohen's class of time-frequency distributions for general wind radial velocity statistics. From this analysis, a complete formulation of what is actually estimable is presented. The particularization of these results for the wake vortex signal will be detailed in the next chapter.

The statistics of the Doppler LIDAR signal are first given in Section 4.1 with a particular interest to its covariance. In Section 4.3, we develop the equation of its Wigner-Ville distribution in order to analyze the effect of LIDAR parameters on the estimation of the radial velocity statistical distribution and to provide a complete description of the evolution of the signal spectrum with time. We will then focus, in Section 4.4, on two time-frequency energy distributions that are members of the Cohen's class: the spectrogram and the smoothed Pseudo Wigner-Ville distribution. We conclude this chapter in Section 4.5 with the results of a wind statistics estimator based on the SPWVD. The theoretical concepts about the time-frequency distributions presented here use the formalism found in [Flandrin 1993]. The reader will find in Appendix B a synthesis of the concepts exposed here. The proof of the theorems and their corollaries appearing in this chapter are also given in Appendix C.

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4.1 Statistical properties of the LIDAR signal

In this section, we focus on the derivation of the signal covariance which will serve as the basic material for subsequent sections dedicated to time-frequency representations of the LIDAR signal.

4.1.1 The analytic LIDAR signal

At a given time t , the coherent Doppler LIDAR signal is obtained by the contribution of many random phase wave fields backscattered by all the aerosol particles when illuminated by the LASER beam along the line-of-sight. As the total amount of particles is huge, the LIDAR signals are well approximated by zero-mean Gaussian random processes as stated by the central limit theorem. The atmospheric properties can also be regarded as homogeneous in the transverse direction at a given range from the telescope. In this chapter, we assume that the noise is dominated by the shot-noise. Since the photodetector operates at a high regimes the shot noise which is a Poisson process is assumed to be Gaussian. These assumptions are classically used in LIDAR signal analysis. In addition, for this theoretical study, we suppose that there is no fluctuation of the instantaneous signal power due to refractive turbulence and that the signal envelope is mainly determined by the average system gain $H(z)$, given by Eq. 2.95, and the fluctuations of the radial velocity.

Assumption 4.1 *The fluctuations of the signal instantaneous power are only due to the speckle effect and the fluctuations of the Doppler frequency. For a given pulse duration, the signal envelope is thus determined by the average system gain $H(z)$.*

The measured signal at the output of the photodetector is real but is usually converted to complex data in order to be processed by advanced velocity estimation algorithms. Hence, for real measured data $s(t)$ obtained from one LASER shot, we associate a complex valued analytic signal $x(t)$ defined as

$$x(t) = s(t) + j\mathcal{H}\{s(t)\}, \quad (4.1)$$

where $\mathcal{H}\{\}$ is the Hilbert transform of $s(t)$ and $j^2 = -1$. This leads to the following relationship in the frequency domain:

$$X(\nu) = \begin{cases} 2S(\nu) & \text{for } \nu \geq 0 \\ 0 & \text{for } \nu < 0 \end{cases}, \quad (4.2)$$

where ν is the frequency variable and $S(\nu)$ and $X(\nu)$ are respectively the Fourier transform of $s(t)$ and $x(t)$. Using analytical signals instead of real ones validates the approximation of $x(t)$ as a modulated envelope $m(t)$:

$$x(t) = m(t) e^{-j\omega(t)}, \quad (4.3)$$

where ω is the time-dependent pulsation. It also allows one to reduce the level of interference in the Wigner-Ville distribution as we will see in Section 4.3.

The heterodyne current is independent of the additive uncorrelated photodetection noise $\tilde{n}(t)$ with the following properties:

$$\langle \tilde{n}(t) \rangle = 0, \quad \langle \tilde{n}(t) \tilde{n}^*(t') \rangle = N \delta(t - t'). \quad (4.4)$$

The average noise power is $N = \langle |\tilde{n}|^2 \rangle$. Moreover, the signal $s(t)$ is normalized by N such that its average power is directly related to the signal-to-noise ratio, noted SNR, by

$$\langle |x(t)|^2 \rangle - 1 = \int_0^{+\infty} h(z) |p(t - 2z/c)|^2 dz \quad (4.5a)$$

$$= \text{SNR}(t), \quad (4.5b)$$

Considering all the previous assumptions, one realization of the analytical LIDAR signal is obtained by the following spatial integration:

$$x(t) = \int_0^{+\infty} h(z)^{\frac{1}{2}} \alpha_z p(t - 2z/c) e^{-2\pi j(f_{\text{IF}} - f_D(z))t} dz + n(t), \quad (4.6)$$

where $p(t)$ [$s^{-\frac{1}{2}}$] is the normalized complex amplitude of the LASER pulse, $n(t)$ is a unit variance Gaussian noise, f_{IF} [Hz] is the intermediate frequency, $f_D(z)$ [Hz] is the Doppler shift induced by particles at a distance z from the LIDAR, α_z is the statistically independent zero-mean Gaussian random variable with

$$\langle \alpha_z \rangle = 0, \quad \langle \alpha_z \alpha_{z'}^* \rangle = \delta(z - z'), \quad (4.7)$$

where δ is the Dirac distribution such that $\int \delta(t) dt = 1$. In accordance with Assumption 4.1, the quantity $h(z)$ occurring in 4.6 is a system dependent factor defined by

$$h(z) = \left(\frac{\eta_Q U_L}{\hbar \nu_0 B_w} \right) H(z), \quad (4.8)$$

where $H(z)$ is the ensemble average of the system gain over the refractive turbulence varying slowly compared to the spatial extend of the pulse (see Eq. 2.95), η_Q [electrons/photon] is the detector quantum efficiency, U_L [J] is the pulse energy, $\hbar$ [Js] is the Planck constant, ν_0 [Hz] is the LASER frequency and B_w [Hz] is the detector bandwidth. Note that $H(z)$ is exactly the range-dependent SNR given at Eq. 2.102. When the temporal pulse profile is assumed to be Gaussian, it is described by the following equation:

$$p(t) = (\sqrt{\pi} \sigma_p)^{-\frac{1}{2}} \exp \left(-\frac{t^2}{2\sigma_p^2} + \pi j \phi t^2 \right), \quad (4.9)$$

where σ_p [s] is the e^{-1} intensity pulse radius and ϕ [Hz s $^{-1}$] is a linear frequency chirp. It is normalized such that

$$\int_{-\infty}^{+\infty} |p(t)|^2 dt = 1. \quad (4.10)$$

A measure of the range resolution of the pulse is given by $\sigma_z = c\sigma_p/2$ [m]. Hence, for a pulse duration of 200 ns, defined by its full width at e^{-1} of its maximum, the range resolution is equal to 30 m. As it will be used later, we also provide the spectrum of the pulse with no chirp given by the Fourier transform of $p(t)$:

$$P(\nu) = (\sqrt{\pi}w_p)^{-\frac{1}{2}} \exp\left(-\frac{\nu^2}{2w_p^2}\right), \quad (4.11)$$

where $w_p = (2\pi\sigma_p)^{-1}$ [Hz] is its frequency standard deviation. The pulse power spectrum is given by $F_p(\nu) = |P(\nu)|^2$.

The Doppler shift $f_D(z)$ [Hz] in Eq. 4.6 depends on the radial velocity $v_R(z)$ [m/s] of the aerosol particles at the distance z and is defined as the velocity component parallel to the direction of propagation of the beam. It is therefore given by:

$$f_D(z) = -2\lambda^{-1}v_R(z), \quad (4.12)$$

where λ [m] is the LASER wavelength. For instance, a radial velocity of 1 m/s corresponds to a Doppler shift of 1.29 MHz at a wavelength of 1.55 μ m. We will use the following assumption for the random radial velocity:

Assumption 4.2 *The particles illuminated by the LASER in an infinitesimal slice dz around the distance z from the transmitter have a radial velocity probability density function given by $\Phi_v(v; z)$. The frozen atmosphere (Taylor) hypothesis is used such that this quantity is independent of time at least for the time scale of the LIDAR measurement.*

Due to the independent contributions from different atmospheric slices, the signal envelope is modulated in time with a temporal scale depending on the time required for the pulse to travel a distance $c\sigma_p/2$ after which it illuminates a new collection of independent atmospheric scatterers.

In order to simplify the calculation of the signal covariance and its different time-frequency distributions appearing in this chapter, the LIDAR signal $x(t)$ is decomposed into a sum of randomly weighted complex waveforms. They are defined as follows:

Definition 4.1 *An elementary signal $r(t; z)$ [s $^{-1/2}$] is a function of time representing the contribution of the atmospheric slice at distance z to the LIDAR signal $x(t)$ at time t :*

$$r(t; z) = p(t - 2z/c) e^{-2\pi j f(z)t}. \quad (4.13)$$

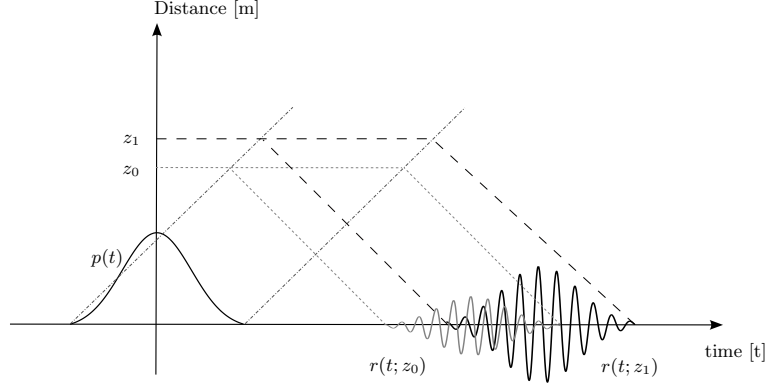


Figure 4.1: Illustration of the generation of a LIDAR signal $x_2(t)$ given by the contributions of two atmospheric slabs at respective ranges z_0 and z_1 . The temporal pulse profile is denoted $p(t)$ and the functions $r(t; z_i)$ for $i \in \{1, 2\}$ are the temporal waveforms generated by the two slabs.

This random signal is given by the modulation of the pulse envelope with a random frequency

$$f(z) = f_{\text{IF}} - 2\lambda^{-1}v_{\text{R}}(z). \quad (4.14)$$

Mathematically, the waveform $r(t; z)$ is obtained by applying a translation in time and a modulation in frequency on the pulse profile $p(t)$. The variables $v_{\text{R}}(z)$ and $f(z)$ will be used equivalently through the text as they describe the same process. If the pulse profile is symmetric about its maximum value, the waveform generated by a particle at distance z will have its stronger effect at a time $t = 2z/c$ in the measured signal. We also observe that the frequency component $f(z)$ is present in the signal in a time interval of length $2\sigma_p$ around t . For a Gaussian pulse profile and a constant $f(z)$, the waveforms $r(t; z)$ are called *Gabor atoms* [Torrésani 1995]. Using this decomposition, the equation of the analytical LIDAR signal (see Eq. 4.6) then becomes:

$$x(t) = \int_0^{+\infty} h(z)^{\frac{1}{2}} \alpha_z r(t; z) dz + n(t). \quad (4.15)$$

In the previous equations, the random variables α_z and $f(z)$ have deliberately been separated and we further introduce the following assumption that the power backscattered by an aerosol particle is independent on its velocity:

Assumption 4.3 *The random variables α_z and $f(z)$ are independent. In other words, the Doppler shift induced by a aerosol particle is independent on the quantity of energy it scatters back to the telescope.*

Instead of defining the contribution of one atmospheric slice to the signal, we can also define the portion of the atmosphere which contributes to the realization of one signal sample at time t . This is given by defining the following weighting function:

Definition 4.2 *The spatial weighting function is a function of z determining the contribution of the particles in atmospheric slice dz around z to the signal $x(t)$ at time μ . It is given by the following normalization*

$$I(z; \mu) = \frac{c(z; \mu)}{\int_0^\infty c(z; \mu) dz}, \quad (4.16)$$

of a function

$$c(z; \mu) = h(z) |p(\mu - 2z/c)|^2. \quad (4.17)$$

The position of the pulse at time t is $\rho = c\mu/2$.

We also have that $\int I(z; \mu) dz = 1$. The smaller the distance $|z - \rho|$ the higher is the contribution of z at time μ . Note that if $h(z)$ is approximatively constant over the spatial domain defined by pulse at time μ , we have that $I(z; \mu) = |p(\mu - 2z/c)|^2$.

In this chapter, we define the Doppler LIDAR signal $x_N(t)$ obtained using the atmospheric model similar to the one presented in Chapter 3 for which the line-of-sight is decomposed into N slices with depth Δz :

$$x_N(t) = \Delta z \sum_{i=0}^{N-1} h(z_i)^{\frac{1}{2}} \alpha_i r_i(t; z_i) + n(t). \quad (4.18)$$

In this model, each slice at distance z_i contains a huge amount of aerosol particles. As illustrated in Fig. 4.1, a LIDAR signal $x_2(t)$ that would have been produced by two slices of aerosol particles at respective distance z_0 and z_1 from the LIDAR telescope is given by

$$x_2(t) = x_0(t) + x_1(t) \quad (4.19a)$$

$$= h(z_0)^{\frac{1}{2}} \alpha_0 r(t; z_0) + h(z_1)^{\frac{1}{2}} \alpha_1 r(t; z_1) + n(t). \quad (4.19b)$$

4.1.2 The estimation principle

A line-of-sight is defined by the position, i.e. azimuth and elevation, of the telescope at the time a LASER pulse is sent through the atmosphere. A Doppler LIDAR signal $x(t)$ is measured for each of them. The definition of a LIDAR signal processing algorithm is given by:

Definition 4.3 *A wind velocity estimator is an algorithm applied on a number of measured LIDAR signals in order to estimate the moments of the probability density function $\Phi_v(v; z)$ of the radial velocity $v_R(z)$ for all z along a given direction of observation.*

The statistical moments to estimate are mainly the two first one giving the mean radial velocity and its dispersion. We will see in Section 4.3 that, at the exception of some trivial cases, only approximations of the statistical moments of $\Phi_V(v; z)$ can be retrieved as a result of the finite pulse duration. The wind statistics are actually smoothed by the pulse during the measurement process. The atmospheric turbulence parameters such as the turbulent eddy dissipation rate (TEDR) as well as the integral scale of turbulence can be estimated from the second order moment as in [Frehlich 1999, Smalikho 2005].

The Doppler LIDAR signal $x(t)$ is sampled with a period T_s and each sample at time $t_k = kT_s$ corresponds to a distance $z_k = ct_k/2$. The spatial extend of the pulse, noted Δr , gives the dimension of the aerosol region illuminated at a given time and is defined by its spatial full width at half-maximum (FWHM) :

$$\Delta r = c\Delta t/2, \quad (4.20)$$

where Δt is temporal FWHM of the pulse given by

$$\Delta t = 2(\ln 2)^{1/2} \sigma_p. \quad (4.21)$$

The characteristics of the radial velocity profile along a given LOS are classically estimated on a temporal sliding window $g(t)$ of length $T = MT_s$ where M is the number of complex data points per estimate. We therefore define the range gate as the distance $\Delta p = cT/2$ that the pulse moves during the estimation interval T . The actual range resolution depending on the pulse duration and the number of samples needed to perform an estimation is

$$\Delta R = \Delta r + \Delta p. \quad (4.22)$$

Let's consider the following example with values taken from [Kopp 2004]. The LASER source produces a pulse with length $\Delta t=400$ ns. This means that the pulse has a spatial extend of $\Delta r=60$ m. If the data are sampled at 500 MHz and that a velocity estimate is produced from $M = 256$ successive samples, the actual range resolution is $\Delta R=137$ m.

Due to the speckle effect and the low SNR, information from adjacent line-of-sight, i.e. obtained from successive emitted pulses, must be accumulated. The accumulation technique is usually performed either on the signal correlogram or on estimates of the spectral density of the signal. In this case, the estimation of the wind statistics are retrieved along a direction which does not necessarily correspond to a line-of-sight. This direction will be defined depending on the spatial accumulation strategy used. In this chapter, we will only discuss the estimation along a single LOS.

4.2 The LIDAR signal covariance

As the LIDAR signal is well represented by a Gaussian random process, it is completely described by its two first order statistical moments. The signal correlation time due to the random wind field cannot be found for general atmospheric conditions but will be given later in this section for wind gradients and Gaussian wind turbulence. The equation of its covariance is developed here and will be used in Section 4.3 to determine the equation of the average Wigner-Ville distribution of the LIDAR signal. The ensemble averages are computed over the random phases of the backscattered signal from shot to shot. The ensemble average over the refractive turbulence is assumed to have already been computed (see Assumption 4.1 and Eq. 4.8).

The signal covariance is generally defined as $R_x(t_1, t_2) = \langle x(t_1)x^*(t_2) \rangle$. Here we use the time variables $t = (t_1 + t_2)/2$ and $\tau = t_2 - t_1$ which gives:

$$R_x(t, \tau) \triangleq \langle x(t + \tau/2) x^*(t - \tau/2) \rangle \quad (4.23a)$$

$$= \iint_0^{+\infty} h(z)^{\frac{1}{2}} h(z')^{\frac{1}{2}} \langle \alpha_z \alpha_{z'}^* \rangle R_r(t, \tau; z) dz dz' + \delta(\tau), \quad (4.23b)$$

where $R_r(t, \tau; z)$ is the covariance of an elementary waveform $r(t; z)$. Since the LIDAR return from two different atmospheric volumes are independent (see Eq. 4.7), we find the following relation between the covariances of the signal $x(t)$ and the waveforms $r(t; z)$

$$R_x(t, \tau) = \int_0^{+\infty} h(z) R_r(t, \tau; z) dz + \delta(\tau). \quad (4.24)$$

As the covariance is dependent on time t , the Doppler LIDAR signal is considered to be non-stationary. Using Eq. 4.13 for the $r(t; z)$, we find for their covariance $R_r(t, \tau; z)$:

$$R_r(t + 2z/c, \tau; z) = p(t + \tau/2) p^*(t - \tau/2) \langle e^{-2\pi j f(z) \tau} \rangle, \quad (4.25)$$

where the brackets denotes the ensemble average over the random wind field. The third term of the product is known as the characteristic function of the random process $f(z)$ defined by:

$$M_F(\tau; z) \triangleq \langle e^{-2\pi j f(z) \tau} \rangle \quad (4.26a)$$

$$= \int_{-\infty}^{\infty} \Phi_F(f; z) e^{-2\pi j f(z) \tau} df, \quad (4.26b)$$

where $\Phi_F(f; z)$ is the probability density function of $f(z)$. The dual formula of Eq. 4.26b is

$$\Phi_F(f; z) = \int_{-\infty}^{\infty} M_F(\tau; z) e^{2\pi j f(z) \tau} d\tau. \quad (4.27)$$

The statistics of $f(z)$ are determined by the characteristics of the wind turbulence. Since $v_R(z) = \lambda(f_{IF} - f(z))/2$, the relations between the statistics of $f(z)$ and $v_R(z)$ are:

$$\Phi_V(v; z) = \frac{2}{\lambda} \Phi_F(f_{IF} - 2\lambda^{-1}v(z); z) \quad (4.28a)$$

$$\Phi_F(f; z) = \frac{\lambda}{2} \Phi_V\left(\frac{\lambda}{2}(f_{IF} - f); z\right). \quad (4.28b)$$

When the radial velocity is assumed to be a Gaussian random variable with mean $\langle v_R(z) \rangle$ and variance σ_V^2 , its characteristic function becomes:

$$M_F(\tau; z) = \exp(-2\pi j \langle v_R(z) \rangle \tau) \exp(-2\pi^2 \sigma_V^2 \tau^2). \quad (4.29)$$

The performance of a Doppler LIDAR typically lies into two physical regimes when the statistics of the signal are determined either by the pulse or by the atmospheric parameters [Frehlich 1997]. In the pulse-dominated regime, the statistical properties of the signal are determined exclusively by the pulse profile. It typically happens when the length of the sensed volume defined by the pulse duration is small compared to the spatial correlation length of the wind field and the scale of variation of the LIDAR system factor $h(z)$. In the atmospheric regime, when the pulse duration is large, the signal covariance is essentially determined by the statistics of the wind field and the fluctuations of the aerosol backscatter coefficient.

4.2.1 Signal covariance with Gaussian pulse

Results obtained in this chapter will be systematically particularized for a Gaussian pulse with equation given by Eq. 4.9. Under this assumption, the covariance of $r(t; z)$ in Eq. 4.25 can be decomposed as

$$R_r(t, \tau; z) = |p(t - 2z/c)|^2 \gamma_r(\tau; z), \quad (4.30)$$

with

$$\gamma_r(\tau; z) = \exp(-\tau^2/\tau_p^2) M_F(\tau; z), \quad (4.31)$$

is the decorrelation function which takes into account the time decorrelation of the LIDAR signal due to the pulse profile as well as the time decorrelation due to the random wind field. The signal correlation time τ_p that is due to the pulse is defined as the $1/e$ full width of the signal covariance and is therefore given by $\tau_p = 2\sigma_p$. A signal with a covariance satisfying this separability property between its behavior in t and τ as in Eq. 4.30, is said to be locally stationary. The covariance then appears as the modulation in time by the function $|p(t)|^2$ of a stationary correlation function $\gamma_r(\tau; z)$ depending on particle velocity. Inserting Eq. 4.30 in Eq. 4.24 gives the signal covariance for a Gaussian pulse:

$$R_x(t, \tau) = \int_0^{+\infty} c(z; t) \gamma_r(\tau; z) dz + \delta(\tau), \quad (4.32)$$

where $c(z; t)$ has been introduced in Definition 4.2.

The well known Zrnić covariance model used in RADAR signal processing is obtained by assuming, in addition to the Gaussian pulse condition, that the system parameter $h(z)$ and the wind velocity have a spatial scale of variation much larger than the length of the volume sensed by the pulse. Under these assumptions the signal is stationary and the statistics are described by the transmitted pulse (pulse-dominated regime) [Churnside 1983, Frehlich 1993b]. Using Eq. 4.5b defining the SNR, the covariance of the signal at time μ becomes

$$R_x(\tau) = \text{SNR}(\mu) \gamma_r(\tau; \rho) + \delta(\tau), \quad (4.33)$$

where $\rho = c\mu/2$ is the spatial position of the pulse along the line-of-sight at time μ . The function $\gamma_r(\tau; \rho)$ is typically written in this model by

$$\gamma_r(\tau; \rho) = \gamma_n(\tau) \exp(4\pi j v_R(\rho) \lambda^{-1} \tau), \quad (4.34)$$

where $\gamma_n(\tau)$ is the normalized covariance of the signal defined by

$$\gamma_n(\tau) = \exp(-2\pi^2 w^2 \tau^2) \quad (4.35)$$

and w is the signal spectral width which is, in general, different than w_p . As soon as the temporal scale of variation of the velocity fluctuations becomes smaller than the size of the sensed volume, the LIDAR signal becomes non-stationary and this model is not valid anymore. It is nevertheless classically used as a basic model for most of the time-domain and frequency-domain maximum likelihood estimators of the mean radial velocity.

4.2.2 Decorrelation by wind gradient

The particularization of the Doppler LIDAR signal covariance in the case of a wind gradient profile is detailed in [Frehlich 1994]. It is exposed in this section in order to be used later for the calculation of its Wigner-Ville distribution. We will assume here that the beam is narrow and that the system factor $h(z)$ is constant over the sensing volume of the pulse.

The wind gradient profile is obtained by approximating the wind field along the line-of-sight by the first two terms of the Taylor expansion of $v_R(z)$ in z . The radial velocity is defined by its value at a distance ρ , noted $v_R(\rho)$, and the gradient along the z direction given by

$$g(\rho) = \left. \frac{dv_R(z)}{dz} \right|_{z=\rho}. \quad (4.36)$$

The radial velocity profile is thus given by:

$$v_R(z) = v_R(\rho) + g(\rho) (z - \rho). \quad (4.37)$$

Under these assumptions, the signal covariance at time μ becomes:

$$R_x(\mu, \tau) = h(\rho) \frac{c}{4} \exp \left(-2\pi j [f_{\text{IF}} - 2\lambda^{-1}v_R(\rho)] \tau - \frac{\tau^2}{\tau_T^2} \right) + \delta(\tau), \quad (4.38)$$

with $\mu = 2\rho/c$, the time corresponding to the distance ρ and τ_T [s], the total correlation time of the signal given by

$$\frac{1}{\tau_T^2} = \frac{1}{\tau_p^2} + \frac{1}{\tau_{\text{WS}}^2}. \quad (4.39)$$

The quantity τ_{WS} [s] is the correlation time due to wind gradient at a given time t . It is given by

$$\tau_{\text{WS}}(t) = \frac{\lambda}{2\pi r_p g(ct/2)}, \quad (4.40)$$

where $r_p = c\sigma_p/2$ is the range resolution of the pulse and $r_p g(\rho)$ is the radial velocity variation over this distance. The additional spectral spreading due to wind gradient is $w_{\text{WS}} = (\pi\tau_{\text{WS}})^{-1}$.

From Eq. 4.39, we see that the wind gradient has an important decorrelation effect on the signal if the condition $\tau_{\text{WS}} \ll \tau_p$ is satisfied, i.e. $\tau_T \approx \tau_{\text{WS}}$, which defines the atmospheric regime ($w_{\text{WS}} \gg w_p$). For a wind gradient of 0.01 s^{-1} which corresponds to a variation of the radial velocity of 1 m/s over 100 m, and for $\lambda = 1.55 \text{ }\mu\text{m}$, we have that the decorrelation effect becomes important for pulse duration higher than 287 ns. Similarly, we can write a condition on $g(z)$ for a given pulse duration:

$$g(\rho) > \frac{1}{4\pi r_p \sigma_p}. \quad (4.41)$$

For the same wavelength, the signal obtained with a pulse duration of 400 ns will be affected by a wind gradient higher than 0.02 s^{-1} .

4.2.3 Decorrelation by a Gaussian turbulence

When $v_R(z)$ is a stationnary Gaussian random variable with mean $\langle v_R(z) \rangle$ and variance σ_v^2 , the probability density function of $f(z)$ is given by

$$\Phi_F(f; z) = \frac{1}{\sqrt{2\pi}\sigma_F(z)} \exp \left(-\frac{(f - \langle f(z) \rangle)^2}{2\sigma_F^2(z)} \right), \quad (4.42)$$

where $\bar{f}(z)$ is the mean value and $\sigma_F^2(z) = 4\sigma_v^2(z)/\lambda^2$ is the variance corresponding to a velocity dispersion σ_v [m/s]. The characteristic function of $f(z)$ is [Papoulis 1991]:

$$M_F(\tau; z) = \exp \left(-2\pi j \langle f(z) \rangle \tau - \frac{\tau^2}{\tau_F^2} \right), \quad (4.43)$$

where $\tau_F = (\pi\sigma_F(z))^{-1}$ is the correlation time due to the wind turbulence.

By combining Eq. 4.31, 4.32 and 4.43, we find the following equation for the signal covariance, which is equivalent to the one given by Churnside and Yura in [Churnside 1983]:

$$R_x(t, \tau) = \int_0^{+\infty} h(z) |p(t - 2z/c)|^2 \exp\left(-2\pi j \langle f(z) \rangle \tau - \frac{\tau^2}{\tau_R^2}\right) dz + \delta(\tau), \quad (4.44)$$

where we have defined the total correlation time τ_R [s] by

$$\frac{1}{\tau_R^2(z)} = \frac{1}{\tau_p^2} + \left(\frac{2\pi\sigma_v(z)}{\lambda}\right)^2. \quad (4.45)$$

If we assume that the radial velocity is approximatively constant over the sensed volume, we find that

$$R_x(t, \tau) = h(\rho) \exp\left(-2\pi j [f_{IF} - 2\lambda^{-1} \langle v_R(\rho) \rangle] \tau - \frac{\tau^2}{\tau_R^2}\right) + \delta(\tau), \quad (4.46)$$

where $\rho = ct/2$. The decorrelation time τ_R is now the exact correlation of the signal at a given time t . Again, the condition for the wind randomness to induce important decorrelation of the signal is given by:

$$\sigma_v(z) > \frac{\lambda}{4\pi\sigma_p}. \quad (4.47)$$

The smaller the pulse duration, the bigger the velocity variance at distance z while preserving the correlation time τ_p .

This case is also useful to describe the effect of the accumulation process on adjacent line-of-sights, performed prior to the radial velocity estimation, for which the radial velocity profiles may vary significantly in the transverse direction, e.g. inside a wake-vortex. If these fluctuations are well described by their first two statistical moments, the equations above can be used by replacing σ_v by a modified standard deviation

$$\sigma_v'^2 = \sigma_v^2 + \sigma_{v,T}^2, \quad (4.48)$$

which includes the random fluctuations inside the atmospheric slices in z , σ_v , as well as the inter-LOS fluctuation variance $\sigma_{v,T}^2$. Hence in spatial regions where the velocity changes rapidly as, for example, inside wake-vortices and for high accumulation level, the spectral estimates will be broadened consequently. Furthermore, under the same assumption of Gaussianity, we may introduce in this variance, the fluctuations of the LASER frequency.

4.3 Wigner-Ville Distribution of the LIDAR signal

In this section, the time-frequency properties of the Doppler LIDAR signal are calculated through the formulation of its Wigner-Ville distribution (WVD). We demonstrate that the signal WVD can be expressed as the convolution in frequency between the pulse power spectrum and a weighted radial velocity distribution defined for all time instead of space. This formulation allows to clearly exhibit the limitations of the estimation principle as exposed in Definition 4.3 and provides an analysis on what information can be inferred on the wind statistics from the analysis of the signal WVD.

As the LIDAR signal is random by nature, we will distinguish the ideal case where the signal covariance is known exactly and the one in which it has to be estimated by pulse accumulation. We will then talk of the *average Wigner-Ville distribution* for the first case and of the *sample Wigner-Ville distribution* for the second one. In the signal processing literature, when stochastic processes are analyzed, one also refer to respectively the Wigner spectrum and the stochastic Wigner-Ville distribution.

The analytical results presented here have been obtained by applying the properties of the Wigner-Ville distribution on the LIDAR signal given by Eq. 4.6 and characterized by its covariance in Eq. 4.24. The main properties of the Wigner-Ville distribution are detailed in Appendix A.

4.3.1 The pulse WVD

The Wigner-Ville distribution of the pulse determines the resolution of the average signal WVD and thus governs the precision with which the wind velocity statistics can be retrieved. The localization property in Eq. A.16 can be applied to the pulse profile given by Eq. 4.9 to show the effect of the linear frequency chirp on its WVD. In this case, the pulse WVD is an elliptical Gaussian function in the time-frequency plane with equation:

$$W_p(t, \nu) = \frac{1}{\pi \sigma_p w_p} \exp \left(-\frac{t^2}{\sigma_p^2} - \frac{(\nu - 2\pi\phi t)^2}{w_p^2} \right), \quad (4.49)$$

where σ_p^2 and w_p^2 are respectively its variances in time and frequency. The product time-bandwidth is constant and reaches the Heisenberg uncertainty principle lower bound, i.e. $2\pi\sigma_p w_p = 1$. The Gaussian pulse time-frequency support is therefore the most compact that is why this profile is usually targetted. The presence of a chirp would provide a way to adapt the resolution in time and frequency and act as a rotation factor on its WVD. In this thesis, no chirp will be considered, i.e. $\phi = 0$, and Eq. 4.49 is therefore written as

$$W_p(t, \nu) = |p(t)|^2 F_p(\nu). \quad (4.50)$$

This expression can be found by replacing $x(t)$ by $p(t)$ in Eq. A.13. We will see in Section 4.4.2 that, when using a time-frequency signal analyzing technique lying in the Cohen's class, the chirp can be effectively replaced by an appropriate transformation on the observation window. Note also that from Eq. A.12, we have,

$$\iint W_p(t, \nu) \, d\nu dt = \int |p(t)|^2 dt = 1. \quad (4.51)$$

4.3.2 The average LIDAR signal WVD

We derive here the WVD of the response signal $r(t; z)$ of an isolated atmospheric slice dz at a distance z sensed with a pulse $p(t)$ and describe how it is influenced by the radial velocity statistics of the particles in this slice. The average WVD of the LIDAR signal is then established by considering the response of all the particles along the line-of-sight. A second formulation of this distribution is also presented which gives a better understanding on what radial velocity distribution can be estimated from the measured signal. The following notation will be adopted when the frequency profile of the distribution is analyzed at a given time instead of the global time-frequency evolution of the signal. This convention will also be used for the more general Cohen's class.

Notation 4.1 *The average WVD analyzed at a time μ is noted $W_{x,\mu}(\nu)$. At this particular time, the signal spectrum is determined by the contribution of atmospheric volumes defined by the spatial weighting window $I(z; \mu)$ centered around distance $\rho = c\mu/2$.*

The first theorem shows that the WVD of an elementary waveform $r(t; z)$ depends both on the pulse profile and on the probability density function $\Phi_F(f; z)$ of the atmospheric particles at distance z . The proof of the Theorems and Corollaries in this chapter are given in Appendix C.

Theorem 4.1 *The Wigner-Ville distribution of an elementary waveform $r(t; z)$, noted $W_r(t, \nu; z)$, is obtained by the convolution in frequency between the pulse WVD and the probability density function of $f(z)$, noted $\Phi_F(f; z)$:*

$$W_r(t, \nu; z) = \int_{-\infty}^{\infty} W_p(t - 2z/c, \nu - \xi) \, \Phi_F(\xi; z) \, d\xi. \quad (4.52)$$

We observe that when the radial velocity dispersion of the particles inside an atmospheric slice dz at distance z increases, the average WVD of the elementary waveforms broadens along the frequency direction. In contrary, when the particles all have the same radial velocity, the WVD becomes

$$W_r(t, \nu; z) = W_p(t - 2z/c, \nu - f(z)), \quad (4.53)$$

It is also obtained by replacing $\Phi_F(\xi; z)$ by $\delta(\xi - f(z))$ in Eq. 4.52 and using the property of the Dirac function given in Eq. A.2. The pulse WVD has simply been translated around the time-frequency coordinate $\{t - 2z/c, \nu - f(z)\}$. For a Gaussian pulse, the pulse WVD is separable in time and frequency and so is $W_r(t, \nu)$.

Corollary 4.1 *For a Gaussian pulse profile, the Wigner-Ville distribution of the waveforms $r(t; z)$ is separable in time and frequency:*

$$W_r(t, \nu; z) = |p(t - 2z/c)|^2 \Gamma_r(\nu; z), \quad (4.54)$$

where $\Gamma_r(\nu; z)$ is given by

$$\Gamma_r(\nu; z) = \int_{-\infty}^{+\infty} F_p(\nu - f) \Phi_F(f; z) df, \quad (4.55)$$

which is the Fourier transform of the decorrelation function $\gamma_r(\tau; z)$.

As $\gamma_r(\tau; z)$, given by Eq. 4.31, is the product of two functions of τ , the multiplication property of the Fourier transform actually leads to a convolution in the spectral domain between the two functions, $F_p(\nu)$ and $\Phi_F(\nu; z)$. We therefore observe that analyzing the WVD of an elementary waveform at a given time $t = 2z/c$ provides us with a filtered version of the frequency probability density function. The filtering kernel is simply the Gaussian function $F_p(\nu)$. As $F_p(\nu)$ is Gaussian, the mean value of $f(z)$ is estimated directly from $W_r(t, \nu)$ whereas the second order moment is affected by the pulse spectral width w_p .

The average Wigner-Ville distribution is the ensemble average WVD of the LIDAR signal over the random parameters α_z and $v_R(z)$ under the independency Assumption 4.3. This distribution is given by the weighted sum of an infinite number of elementary WVD as given by the following theorem.

Theorem 4.2 *The average Wigner-Ville distribution of the LIDAR signal is the sum of the WVD of all the contributions $r(t; z)$ weighted by the system dependent factor $h(z)$:*

$$W_x(t, \nu) = \int_0^\infty h(z) W_r(t, \nu; z) dz + 1, \quad (4.56)$$

where $W_r(t, \nu; z)$ is given by Eq. 4.52 of Theorem 4.1.

In other words, the WVD of the signal $x(t)$ is the superposition of the pulse WVD centered around all the time-frequency coordinates $\{t - 2z/c, \nu + \bar{f}(z)\}$, where $\bar{f}(z)$ is the mean Doppler shift induced by particles in z , and with a broadening depending on the higher moments of Φ_F .

For a deterministic radial velocity profile, Eq. 4.56 can be interpreted as the 2-D convolution of an ideal time-frequency trajectory convolved with the

pulse WVD. From this point-of-view, we understand that the time-frequency resolution at which the radial velocity profile will be analyzed strongly depends on the characteristic of the pulse profile, especially where the velocity profile vary with a spatial scale lower than the support of $I(z; \mu)$. Hence, by reducing the pulse duration, the time or spatial resolution increases at the expense of a poorer frequency or velocity resolution. The Gaussian pulse case can be deduced from Corollary 4.1.

Corollary 4.2 *For a Gaussian pulse profile, the LIDAR average WVD, analyzed at a given time μ , is an infinite sum of spectral functions $\Gamma_r(\nu; z)$ weighted by $I(z; \mu)$:*

$$W'_{x,\mu}(\nu) = \int_0^{+\infty} I(z; \mu) \Gamma_r(\nu; z) dz + 1. \quad (4.57)$$

For a deterministic wind velocity profile, the WVD of the signal $x(t)$ analyzed at a given time μ is thus given by the infinite sum of Gaussian functions $\Gamma_r(\nu)$ with central frequency $f(z)$ and a weighting factor equal to $I(z; \mu)$. Their variances depends on the pulse duration as well as on radial velocity fluctuations at distance z (see Eq. 4.55). In Eq. 4.57, the prime symbol of $W'_{x,\mu}(\nu)$ is used to indicate the normalization induced by replacing the production $h(z) |p(t - 2z/c)|^2$ (see Eq. 4.56 and 4.54) by $I(z; \mu)$. This operation is necessary for $W'_{x,\mu}(\nu)$ to be interpreted as an actual statistical distribution.

Fig. 4.2 and Fig. 4.3 represents the average WVD of a LIDAR signal $x_3(t)$ obtained by the contribution of three atmospheric slices at distances 240, 360 and 420 m from the telescope for pulse durations of 100ns and 400ns. The particles in these slices has a mean radial velocity of respectively -3 m/s, 3 m/s and 0 m/s. We consider here a constant $h(z)$ and an intermediate frequency of 80 MHz. The marginal densities $|x_3(t)|^2$ and $|X_3(\nu)|^2$ are also represented. We observe that the three $W_r(t, \nu)$ atoms are positionned in the time-frequency space relatively to the characteristics position/velocity of their atmospheric slice with a shape corresponding to $W_p(t, \nu)$. By analyzing the marginal densities in time and frequency, we clearly see that increasing the pulse duration increase the frequency (or velocity) resolution at the expense of a decrease in time (or space) resolution.

It comes from Theorem 4.1 that, due to the measurement process, the probability density functions $\Phi_F(f; z)$ for all z along the LOS are not retrievable from the signal WVD and, by extension, from any time-frequency representation. In order to clarify what can actually be estimated, we will now express the previous equations in a different way by introducing an estimable radial velocity density probability, noted $\Phi'_F(f; \mu)$, defined for each time μ and corresponding to a mean range $\rho = c\mu/2$. The following equations are only valid for a Gaussian pulse profile since it requires the separability property of W_p given by Eq. 4.50.

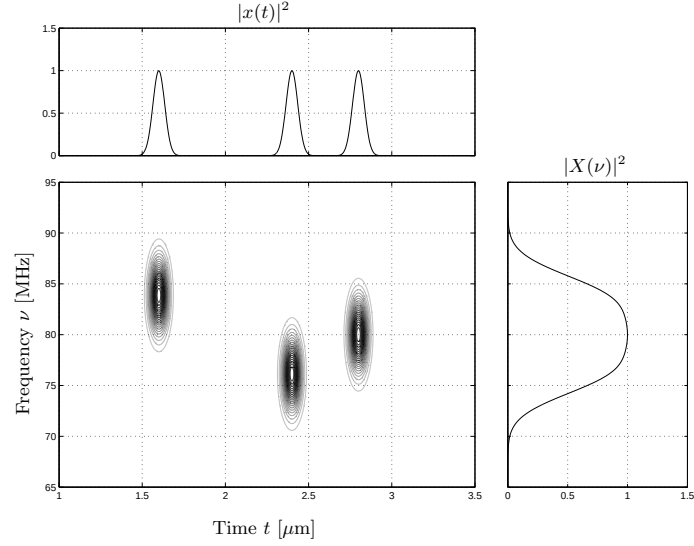


Figure 4.2: Average WVD of a LIDAR signal $x_3(t)$ generated by the contribution of three particles with following position/radial velocity $\{z_i, v_R(z_i)\}$ parameters: (1) 240m, -3 m/s; (2) 360m, 3 m/s; (3) 420m, 0 m/s. The pulse duration is $\tau_p=100$ ns and the intermediate frequency is 80 MHz.

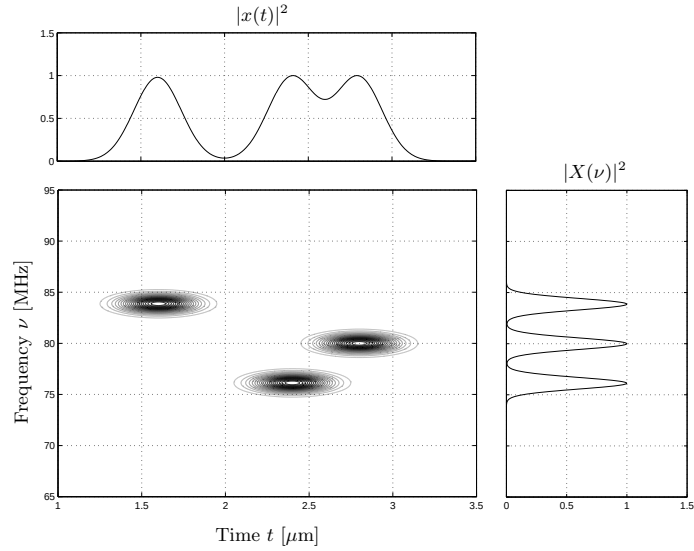


Figure 4.3: Similar to Fig. 4.2 with a pulse duration of $\tau_p=400$ ns.

Theorem 4.3 *The Wigner-Ville distribution of the LIDAR signal at a time μ can be written as the convolution between the pulse power spectrum and a weighted frequency distribution:*

$$W'_{x,\mu}(\nu) = \int_{-\infty}^{\infty} F_p(\nu - f) \Phi'_F(f; \mu) df + 1, \quad (4.58)$$

where we define the following quantity

$$\Phi'_F(f; \mu) = \int_0^{\infty} I(z; \mu) \Phi_F(f; z) dz, \quad (4.59)$$

which can be interpreted as the weighted average distribution of the signal frequency at time μ , noted $f(\mu)$, taking into account the contribution of the atmospheric slices at distances z around $\rho = c\mu/2$.

This point will be detailed in terms of the evaluation of the statistical moments of Φ_V in Section 4.3.3. The signal WVD can also be interpreted as a function of the mean range ρ and the radial velocity as in the following corollary.

Corollary 4.3 *The Wigner-Ville distribution can be written as a space-velocity distribution:*

$$W'_x(\rho, v) = \int_{-\infty}^{\infty} F_p(v - v') \Phi'_V(v'; z) dv' + 1, \quad (4.60)$$

where $\rho = c\mu/2$ and

$$\Phi'_V(v; \mu) = \int_0^{\infty} I(z; \mu) \Phi_V(v; z) dz. \quad (4.61)$$

We thus complement the definition 4.3 with the following statements.

Proposition 4.1 *Due to the finite pulse duration, we have only access to a smoothed radial velocity distribution $\Phi'_V(v; \mu)$ defined at each time μ and given by Eq. 4.61 instead of the exact one $\Phi_V(v; z)$ defined at each distance z .*

In the particular case of a determinist radial velocity profile, i.e. $\Phi_V(v; z) = \delta(v - v_R(z))$, the quantity $\Phi'_V(v; \mu)$ has to be interpreted as a weighted velocity distribution instead of a weighted probability density function. From Theorem 4.3, the two regimes of Frehlich exposed in Section 4.2 can also be restated.

Definition 4.4 *The pulse-dominated regime is defined as the regime in which the frequency extend of the pulse power spectrum is large compared to the second-order moment of the weighted frequency probability density function $\Phi'_F(f; \mu)$. In opposition, the atmospheric regime is characterized by a second order moment of $\Phi'_F(f; \mu)$ large compared to the spreading of the pulse power spectrum.*

It should be noted that the LIDAR regime is now defined for the mean spatial position $\rho = c\mu/2$.

About interference terms: It is interesting to point that no interference terms are present in the average WVD although it is an intrinsic property of the Wigner-Ville distribution of a signal given by a sum of elementary components. This is due to the fact that the signal components are here weighted by statistically independent random variables α_z which has the effect of annihilating all the cross-WVD terms. This can be easily seen in the case of a two particles system described by Eq. 4.19b. The quadratic superposition produces an interference term given by the cross-WVD of the waveforms $r(t; z_0)$ and $r(t; z_1)$, noted $W_{r_0, r_1}(t, \nu)$:

$$W_{x_2}(t, \nu) = W_{r_0}(t, \nu) + W_{r_1}(t, \nu) + 2\Re\{W_{r_0, r_1}(t, \nu)\}, \quad (4.62)$$

where, for $z_0 \neq z_1$ and thanks to Eq. 4.7, we have:

$$W_{r_0, r_1}(t, \nu) = \int_{-\infty}^{\infty} h(z_0)^{\frac{1}{2}} h(z_1)^{\frac{1}{2}} \langle \alpha_0 \alpha_1^* \rangle \langle r(t, z_0) r^*(t, z_1) \rangle e^{-2\pi j \nu \tau} d\tau \quad (4.63a)$$

$$= 0. \quad (4.63b)$$

We will see in Section 4.3.5 that the interference terms between the LIDAR signal components actually exists in the sample WVD but are significantly reduced by an appropriate accumulation method either on the WVD itself or on the signal covariance.

4.3.3 Estimation of the radial velocity distribution

As pointed by Proposition 4.1, analyzing the statistical moments of the Doppler LIDAR signal WVD gives in some extends those of the statistical moments of $v_r(z)$. The precision depends mainly on the pulse duration and on the system parameter $h(z)$. Particularly, the computation of the instantaneous frequency from the average Wigner-Ville transform gives us fundamental information about the influence of pulse duration as well as atmospheric parameters on the estimation of the radial velocity profile. The second order moments gives the signal time decorrelation, and therefore the spectral width as well as an approximation of the velocity dispersion. The two first moments are computed on the normalized WVD given by Eq. 4.58.

Proposition 4.2 *The statistical moments computed from the average Wigner-Ville distribution at time μ gives the equivalent moments of a smoothed version of the weighted probability density function $\Phi'_v(v; \mu)$ instead that those of the exact one $\Phi_v(v; \rho)$ assigned at distance $\rho = c\mu/2$.*

a. The mean radial velocity

For a LIDAR signal described by Eq. 4.6, the instantaneous frequency takes into account the local characteristics of all the signal components $r(t; z)$ as well as the phase relation existing between them. Consequently, the instantaneous frequency is better defined as a median frequency modulated in amplitude. The first order moment of the signal WVD at a given time μ can be written as follows:

$$m_{1,x}(\mu) = \int_{-\infty}^{\infty} \nu W'_{x,\mu}(\nu) d\nu = \int_0^{\infty} I(z; \mu) \bar{f}(z) dz, \quad (4.64)$$

where $\bar{f}(z)$ is the first order moment of the frequency distribution $\Phi_F(f; z)$ and $I(z; \mu)$ is the spatial weighting function given by Eq. 4.16. Note that this result has been obtained by assuming a pulse profile which is symmetric about its maximum. We therefore have that that we have

$$m_{1,x}(\mu) = \bar{f}'(\mu), \quad (4.65)$$

which is the first order moment of $\Phi'_F(f; \mu)$. We thus observe that, if the spatial scale of variation of v_R is smaller than the spatial pulse extend, it is not possible to extract the exact $\bar{f}(z)$ from the signal for all z since we can only estimate a filtered estimate given by Eq. 4.64. Moreover, we see that the way the pulse spectrum filters the weighted frequency distribution has no influence on the estimation of the instantaneous frequency in the Gaussian case because $P(\nu)$ is symmetric about its maximum.

By using Eq. 4.28a, giving the relation between the velocity and frequency distributions, we find that the best radial velocity estimate at time μ , noted $\hat{v}_R(\mu)$, is written as follows:

$$\hat{v}_R(\mu) = \int_0^{\infty} I(z; \mu) \bar{v}_R(z) dz, \quad (4.66)$$

where $\bar{v}_R(z)$ is the mean radial velocity, i.e. the first order moment of $\Phi_V(v; z)$, at a distance z from the telescope. This velocity represents the mean radial velocity of particles in a range gate centered around range $\rho = c\mu/2$ with a spatial extend defined by $I(z; \mu)$. This value must be assigned to a distance corresponding to the centroïd of $I(z; \mu)$ which is ρ for a Gaussian pulse profile. This convolution integral (see also Definition 4.2) tells us that the spatial scales of the radial velocity field smaller than $\tau_z = c\tau_p/2$ cannot be measured with the first-order moment. Since it has been obtained from the average WVD, it is the best radial velocity that would be retrieved if we could estimate it from only one signal sample in absence of speckle and additive noise. This equation will be used later as a reference for the comparison of the estimated radial velocity profile obtained by different estimation algorithms.

With Eq. 4.66, we have proven that the approximation for the mean Doppler velocity proposed by Frehlich in [Frehlich 1997] is true when $h(z)$ is constant over the sensing volume. In the next sections, we will extend this results to take into account the effect of signal processing and analysis.

b. The velocity second-order moment

The second order moment of the signal WVD estimated at time μ is given by

$$m_{2,x}(\mu) \equiv \int_{-\infty}^{\infty} (\nu - m_{1,x}(\mu))^2 W'_{x,\mu}(\nu) d\nu. \quad (4.67)$$

For a Gaussian pulse with spectral variance w_p^2 , the convolution integral in Eq. 4.58 directly gives that

$$m_{2,x}(\mu) = \frac{w_p^2}{2} + \sigma_f^2(\mu), \quad (4.68)$$

where $\sigma_f^2(\mu)$ is the second order moment of the weighted distribution $\Phi'_f(f; \mu)$. This is the spectral broadening due to the inhomogeneity of the wind field inside the sensing volume. The error in estimating the radial velocity dispersion naturally increases with the ratio $w_p/\sigma_f(\mu)$. Hence, for the case of a constant $\Phi_v(v; z)$ with z , a higher pulse duration is preferable.

This means that, knowing the pulse spectral width, it is possible to estimate the radial velocity dispersion at distance ρ on a range interval defined by $I(z; \mu)$ by computing $m_{2,x}(\mu)$ from the signal WVD as well as, thanks to Eq. 4.61

$$\hat{\sigma}_v^2(\rho) = \frac{\lambda^2}{4} \left(m_{2,x}(\mu) - \frac{w_p^2}{2} \right). \quad (4.69)$$

The only trivial case for which the estimated velocity dispersion is equal to the exact one at distance z is when $\Phi_v(v; z)$ is constant over the sensing volume. In any other case, the estimated velocity dispersion at ρ is also dependent on the radial velocity gradient at this distance. The correlation time $\tau(\mu)$ at time μ of the signal is computed from the second order moment of the WVD and the following equation

$$\tau(\mu) = \frac{1}{\pi \sqrt{m_{2,x}(\mu)}}. \quad (4.70)$$

Note that this equation is valid if $W_{x,\mu}(\nu)$ is Gaussian, i.e. in the pulse-dominated regime. Nevertheless, this gives a good approximation in the general case.

4.3.4 Analysis of particular wind fields

The average Wigner-Ville for a LIDAR signal is now derived for the wind gradient and wind turbulence cases developed in Section 4.2.

a. The wind gradient profile

For a covariance given by Eq. 4.38, the Wigner-Ville distribution can be written as

$$W_x(t, \nu) = \frac{c}{2} \frac{h(ct/2)}{\sqrt{2\pi}w_T} \exp\left(-\frac{[\nu - f_{IF} + 2\lambda^{-1}v_R(ct/2)]^2}{2w_T^2}\right) + 1, \quad (4.71)$$

where $v_R(z)$ is the radial velocity profile given by the approximation in Eq. 4.37 and w_T^2 is the frequency second-order moment, related to the correlation time τ_T in Eq. 4.39, and given by

$$w_T^2 = (\pi\tau_T)^{-2} \quad (4.72a)$$

$$= \frac{w_p^2}{2} + w_{ws}^2, \quad (4.72b)$$

where w_{ws} [Hz] is the wind gradient induced spectral broadening given by

$$w_{ws} = g(\rho) \frac{c\sigma_p}{\lambda}. \quad (4.73)$$

We observe that, when the pulse profile is Gaussian, the first-order moment of the WVD directly provides the radial velocity at the distance $z = ct/2$. This is a results of the perfect localization property of the Wigner-Ville distribution given by Eq. A.16. Moreover, the presence of the wind gradient induces an additional broadening of the signal spectrum. Fig. 4.4 illustrates the influence of wind gradient on signal decorrelation time and spectral width for different radial velocity gradients. The upper figure shows the limit on the pulse duration after which the wind gradient effect becomes important, i.e. when $\tau_{ws} < \tau_p$. The undecorrelated signal is represented by the solid line. The total spectral width w_T is represented at the bottom figure.

b. Gaussian random velocity field

The corollaries 4.1 and 4.2 can be used directly to find the WVD in this case. The quantity $\Gamma_r(\nu; z)$ is defined in Eq. 4.55 as the convolution between two Gaussian functions F_p and Φ_F with respective variances $w_p^2/2$ and σ_F^2 and is therefore itself a Gaussian function with variance

$$w_R^2 = \frac{w_p^2}{2} + \left(\frac{2\sigma_v}{\lambda}\right)^2. \quad (4.74)$$

The effect of wind turbulence becomes important when the decorrelation of the LIDAR signal due to turbulence is higher than the decorrelation due to the pulse, i.e. $w_p < w_F$. For a radial velocity and a system gain approximatively constant

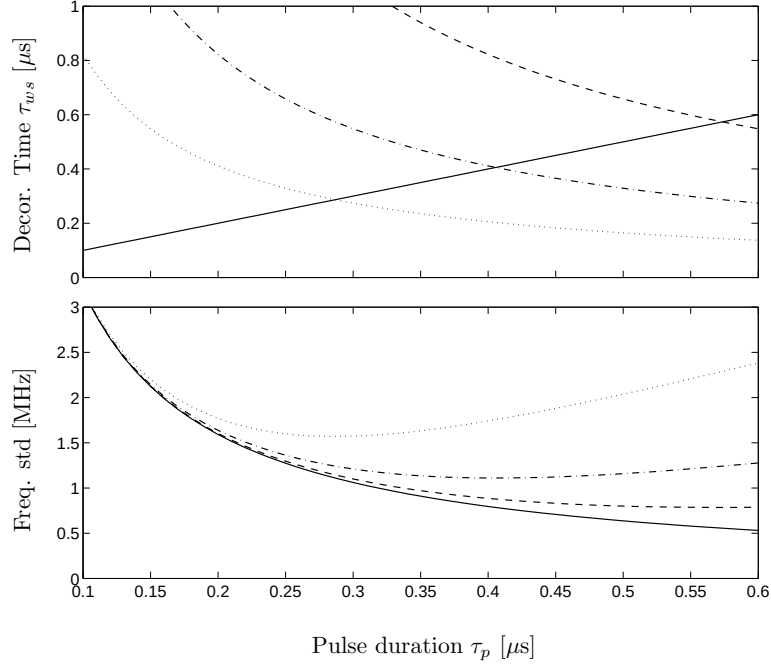


Figure 4.4: Decorrelation time τ_{ws} and spectral broadening due to wind gradient as a function of the pulse duration $\tau_p = 2\sigma_p$ for different wind gradient value $g(\rho)$: no wind gradient (solid); $g(\rho)=0.01 \text{ s}^{-1}$ (dash), $g(\rho)=0.02 \text{ s}^{-1}$ (dash-dot) and $g(\rho)=0.04 \text{ s}^{-1}$ (dot-dot).

over the sensed volume, the WVD is then

$$W_x(t, \nu) = \frac{h(ct/2)}{\sqrt{2\pi}w_R} \exp\left(-\frac{[\nu - f_{IF} + 2\lambda^{-1}v_R(ct/2)]^2}{2w_R^2}\right). \quad (4.75)$$

Again, the first-order moment is exactly the mean radial velocity.

4.3.5 The sample WVD

In the previous sections, we have assumed that the signal covariance is known exactly which gives a direct knowledge of the average WVD. Practically, this covariance must be estimated from a number of sample signals measured from different shots. Interference terms then appears when calculating the WVD since each atmospheric slice responds to the pulse with an unknown but determinist α_z value.

We first describe the nature of the interference terms between the response of N atmospheric slices as described by the signal model $x_N(t)$ introduced in Section 4.1.1. The accumulation techniques, that have been implemented in this thesis, will then be considered.

Notation 4.2 *The sample Wigner-Ville distribution of sample signal $x(t)$ is noted $W_x^1(t, \nu)$ where the superscript ¹ denotes a WVD obtained from a single realization. Similarly, the sample WVD obtained from the accumulation of M successive shots is denoted $W_x^M(t, \nu)$.*

The quantity $W_x^M(t, \nu)$ will be called the accumulated WVD of the LIDAR signal.

a. Interference terms

For a Doppler LIDAR signal $x(t)$ measured for one single shot, the Wigner-Ville distribution definition becomes:

$$W_x^1(t, \nu) \triangleq \int_{-\infty}^{+\infty} x(t + \tau/2) x^*(t - \tau/2) e^{-2\pi j \nu \tau} d\tau. \quad (4.76)$$

If we analyze the WVD of a signal with Eq. 4.18 which is given by the contribution of N atmospheric slices, we obtain:

$$W_x^1(t, \nu) = (\Delta z)^2 \sum_{n=1}^N h(z_n) (\alpha_n \alpha_n^*) W_{r_n}^1(t, \nu; z_n) + I_x^1(t, \nu) + n(t)n^*(t), \quad (4.77)$$

where $W_{r_n}^1(t, \nu; z_n)$ is the sample WVD of a given waveform $r_n(t; z_n)$ given by

$$W_{r_n}^1(t, \nu; z_n) = W_p(t - 2z_n/c, \nu - f(z_n)) \quad (4.78)$$

and $I_x^1(t, \nu)$ is the sum of all the interference terms between every couple of waveforms $\{r_n, r_k\}$:

$$I_x^1(t, \nu) = \Delta z^2 \sum_{n=1}^N \sum_{k=n+1}^N \Re \left\{ h^{\frac{1}{2}}(z_n) h^{\frac{1}{2}}(z_k) (\alpha_n \alpha_k^*) W_{r_n, r_k}^1(t, \nu; z_n, z_k) \right\}, \quad (4.79)$$

where $W_{r_n, r_k}^1(t, \nu; z_n, z_k)$ is the sample cross-WVD between $r_n(t; z_n)$ and $r_k(t; z_k)$. It can be shown, if we refer to the geometrical interpretation of the interference terms construction in [Flandrin 1993], that an interference term between two atmospheric slices is positioned around the coordinate $\{t - (z_n + z_k)/c, (f(z_n) + f(z_k))/2\}$, i.e. in the middle position on a straight line connecting atoms W_{r_n} and W_{r_k} . Moreover, one interference has a time-frequency shape which corresponds to a modulated WVD similar to W_p in the perpendicular direction

of this same line. The modulation frequency increases with the time-frequency distance between the two atoms.

Fig. 4.5 and Fig. 4.6 gives the sample WVD of a signal $x_3(t)$ from three atmospheric slices with same characteristics as in Section 4.3.4. Those figures must be interpreted in conjunction with Fig. 4.2 and Fig. 4.3 giving the corresponding average WVD. We observe the apparition of the interference terms. The amplitude of the W_r atoms is now a random variable. The marginal densities are given with their average value given from the average WVD.

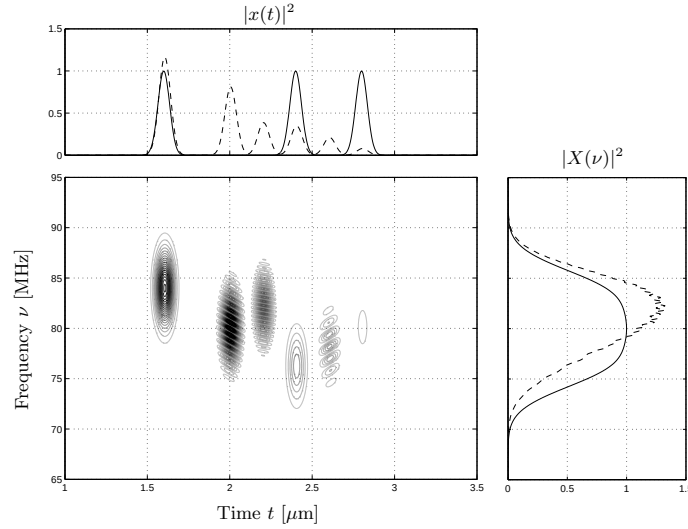


Figure 4.5: Sample WVD of a LIDAR signal realization $x_3(t)$ generated by the return of three particles with following position/radial velocity $\{z_i, v_{R(z_i)}\}$ parameters: (1) 240m, -3 m/s; (2) 360m, 3 m/s; (3) 420m, 0 m/s. The pulse duration is $\tau_p=100$ ns and the intermediate frequency is 80 MHz; Marginal densities from the sample signal (dash) and from the average WVD (solid).

b. Accumulation on WVD

Reducing the interference terms is necessary if we want to use this distribution in practice. This can be done, for example, by averaging either the correlograms or the sample WVDs estimated from successive sample signals.

In the first method, the accumulated WVD computed on M successive signals can be written by the following summation:

$$\widehat{W}_x^M(t, \nu) = \frac{1}{M} \sum_{i=1}^M W_x^i(t; \nu). \quad (4.80)$$

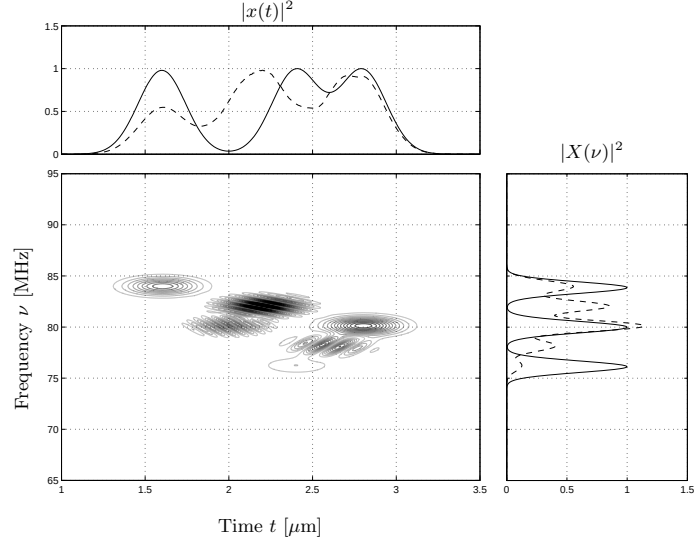


Figure 4.6: Similar to Fig. 4.5 with a pulse duration of $\tau_p=400$ ns.

The second method requires first to accumulate the M correlograms producing the estimate $\hat{r}_x$ of R_x given by

$$\hat{r}_x(t, \tau) = \frac{1}{M} \sum_{i=1}^M x(t + \tau/2) x^*(t - \tau/2), \quad (4.81)$$

and then to estimate the mean WVD of the signals by

$$\widehat{W}_x^M(t, \nu) = \int_{-\infty}^{\infty} \hat{r}_x(t, \tau) e^{-2\pi j \nu \tau} d\tau. \quad (4.82)$$

In both cases, it can be shown that when the level of accumulation M increases, the level of the interference terms tends to zero while the accumulated distribution tends to the average WVD. Hence we will have that, for two slices at z_n and z_k and in accordance with Eq. 4.7:

$$\lim_{M \rightarrow \infty} \sum_{i=1}^M \alpha_{n,i} \alpha_{n,i}^* = 1 \quad (4.83a)$$

$$\lim_{M \rightarrow \infty} \sum_{i=1}^M \alpha_{n,i} \alpha_{k,i}^* = 0, \quad (4.83b)$$

and therefore

$$\lim_{M \rightarrow \infty} I_x^M(t, \nu) = 0 \quad (4.84a)$$

$$\lim_{M \rightarrow \infty} \widehat{W}_x^M(t, \nu) = W_x(t, \nu), \quad (4.84b)$$

as first introduced in Section 4.3.2 for three particles.

As the aerosol particles move over distances much larger than the LIDAR wavelength between successive shots, the signal phase is random and successive LIDAR signals are uncorrelated. That is why the accumulation process allows one to reduce the amplitude fluctuations due to the speckle effect. We see here that the interference terms are also uncorrelated and are reduced the same way. The number of sample signals to accumulate before convergence is however relatively large and the trade-off between interference and speckle reduction and spatial resolution perpendicularly to the mean line-of-sight direction. We will see in the next section that a filtering of the accumulated WVD is possible directly in the time-frequency plane by the use of other representation of the Cohen's class such as the well-known spectrogram. Fig. 4.7 shows the decrease of the normalized MSE between the sample WVD and the average WVD. All the signal realization have all been computed with the same radial velocity profile.

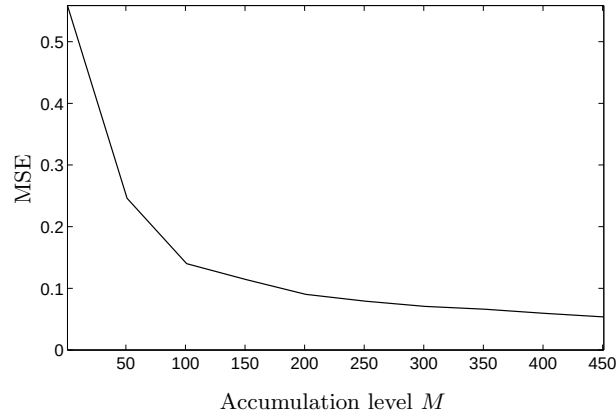


Figure 4.7: Evolution of the MSE between the accumulated WVD and the average WVD for a classical LIDAR signal with a pulse duration of 400 ns.

4.4 The LIDAR signal through the Cohen's lens

We have seen that, although it gives interesting information about the evolution in time of the spectral distribution of the signal, the Wigner-Ville distribution is not usable as it is for velocity characteristic estimation due to the presence of interference terms. In this section, we will focus on a particular class of time-frequency representations called the Cohen's class which gives a way to

reduce them by the choose of an appropriate time-frequency observation window. The Cohen's class is a class of quadratic time-frequency energy distribution which is covariant by translations in time and frequency. The spectrogram, which is the mostly used algorithm in nowadays LIDAR signal processing, as well as the Wigner-Ville distribution are particular cases of this class.

In this section, the Gaussian pulse assumption is used. The Wigner-Ville distribution of the LIDAR signal is thus given by Eq. 4.57. A distinction between the average and the sample Cohen's class of the signal is still possible as for the Wigner-Ville distribution. The properties of the Cohen's class are exposed in Appendix A.

4.4.1 Application to the Doppler LIDAR signal

The Cohen's class C_x of an analytical signal $x(t)$ is defined by a 2-D convolution in the time-frequency domain:

$$C_x(t, \nu; \Pi) \triangleq \iint_{-\infty}^{\infty} \Pi(s - t, \xi - \nu) W_x(s, \xi) ds d\xi, \quad (4.85)$$

where $W_x(s, \nu)$ is the Fourier transform of the signal and $\Pi(t, \nu)$ is a smoothing function equal to the two-dimensional Fourier transform of a function $F(\xi, \tau)$ called the parameterization function:

$$\Pi(t, \nu) = \iint_{-\infty}^{\infty} F(\xi, \tau) \exp(-j2\pi(\nu\tau + \xi t)) d\tau d\xi. \quad (4.86)$$

We will now derive the equations of the average Cohen's class of the LIDAR signal $x(t)$ in such a way that it can be analyzed as the average Wigner-Ville distribution of a LIDAR signal $x'(t)$ which would have been obtained by a LASER pulse completely defined in the time-frequency domain and taking into account both the actual pulse and the analyzing function $\Pi(t, \nu)$.

Theorem 4.4 *The average Cohen's class, denoted $C_x(t, \nu; \Pi)$, of a Doppler LIDAR signal $x(t)$ obtained with a pulse profile $p(t)$ and analyzed by a function $\Pi(t, \nu)$ is the average Wigner-Ville distribution of an analytical LIDAR signal $x'(t)$ that would have been obtained if the atmospheric volume had been sensed with a pulse profile $k(t)$ having the following Wigner-Ville distribution:*

$$W_k(t, \nu; \Pi) = \iint_{-\infty}^{\infty} W_p(s - t, \xi - \nu) \Pi(s, \xi) ds d\xi, \quad (4.87)$$

where $W_p(t, \nu)$ is the WVD of $p(t)$.

This theorem is of particular importance since it allows one to transpose the results of the previous section for the most important time-frequency representations used for LIDAR signal processing.

Theorem 4.5 *The average Cohen's class of a Doppler LIDAR signal is a weighted sum of elementary Wigner-Ville distributions, denoted $W_c(t, \nu; \Pi, z)$, giving the contribution of each atmospheric slice z :*

$$C_x(t, \nu; \Pi) = \int_0^{+\infty} h(z) W_c(t, \nu; \Pi, z) dz + 1, \quad (4.88)$$

where $W_c(t, \nu; \Pi, z)$ is the WVD of an elementary waveform $c(t; z, \Pi)$ and is defined by the following convolution integral:

$$W_c(t, \nu; \Pi, z) = \int_{-\infty}^{\infty} W_k(t - 2z/c, \nu - \xi; \Pi) \Phi_F(\xi; z) d\xi, \quad (4.89)$$

which is the contribution of the atmospheric slice z to the signal time-frequency distribution.

This corollary is the exact transposition of the Theorem 4.2 to the Cohen's class of a LIDAR signal for which the Wigner-Ville distribution $W_r(t, \nu; z)$ of the elementary waveforms $r(t; z)$ is replaced by the distribution $W_c(t, \nu; \Pi, z)$ of hypothetical elementary waveforms $c(t; z, \Pi)$ which are now interpreted as the response of the atmospheric slice z to a LASER pulse $k(t)$. The time-frequency resolution now depends on $W_k(t, \nu; \Pi)$ instead of $W_p(t, \nu)$. More precisely, the estimation of an ideal time-frequency trajectory $\{t - 2z/c, \nu - f(z)\}$ will be only possible on a time-frequency representation for which this trajectory had been smoothed by the kernel $W_k(t, \nu; \Pi)$.

4.4.2 The equivalent Doppler LIDAR signal principle

In accordance with Theorem 4.4 and using a decomposition equivalent to the one of Eq. 4.15, the equation of the analytical LIDAR signal $x'(t)$ then becomes:

$$x'(t) = \int_0^{+\infty} h(z)^{\frac{1}{2}} \alpha_z c(t; z, \Pi) dz + n'(t). \quad (4.90)$$

where the noise $n'(t)$ is a filtered version of the initial detection noise $n(t)$. For a Gaussian noise, the average WVD of the noise $n(t)$ is constant and so is the one of $n'(t)$. However, although the sample WVD of $n(t)$ is δ -correlated, the correlation length in time and frequency of $n'(t)$ is fixed by $\Pi(t, \nu)$, instead of W_k , and in some extent to the accumulation level M . This is an important point to keep in mind especially when developing a spectral modeling technique since it determines the quality of the separation between the signal and noise modes. In other words, the more Π and W_k are different depending on some time-frequency criteria, the more the noise distribution will be easily removed from the signal Cohen's class.

Thanks to the statistical independence between the response of two different atmospheric slices, we can write, as an extension to Eq. 4.24, that the covariance of $x'(t)$ is

$$R_{x'}(t, \tau) = \int_0^{+\infty} h(z) R_c(t, \tau; z, \Pi) dz + R_{n'}(t, \tau), \quad (4.91)$$

where R_c and $R_{n'}$ are the correlation of respectively $c(t; z, \Pi)$ and $n'(t)$ which is still a white noise in this case.

From Eq. 4.87, we easily see that the same LIDAR Cohen's class is obtained by interchanging the function of W_p and Π , i.e. by sending a pulse Π and analyzing the signal with an observation window W_p . The only validity constraint is that Π must be the Wigner spectrum of a realizable pulse profile. In particular, it appears that applying a rotation factor on Π in the time-frequency plane may lead to the same results as if a chirp had been applied on the pulse profile itself.

4.4.3 The smoothed pseudo Wigner-Ville distribution

We will here restraint our study to a smoothing function which is a Gaussian kernel separable in time and frequency in order to study further the signal statistics.

Assumption 4.4 *The smoothing kernels $\Pi(t, \nu)$ are separable in time and frequency, i.e.*

$$\Pi(t, \nu) = g(t) U(-\nu), \quad (4.92)$$

where $U(\nu)$ is the Fourier transform of a temporal smoothing window $u(t)$. Both $g(t)$ and $u(t)$ are Gaussian functions with respective time (frequency) variances σ_g^2 (w_g^2) and σ_u^2 (w_u^2).

Hence, we have that

$$\Pi(t, \nu) = \frac{1}{2\pi\sigma_g w_u} \exp\left(-\frac{t^2}{2\sigma_g^2} - \frac{\nu^2}{2w_u^2}\right), \quad (4.93)$$

Under assumption 4.4, the Cohen's class becomes the Smoothed-Pseudo Wigner-Ville distribution. The spectrogram is obtained with $g(t) = u(t)$ or equivalently $U(\nu) = G(\nu)$. Depending on $g(t)$ and $u(t)$, it is possible to have a progressive and independent control of the time and frequency resolutions as it allows a passage from the spectrogram to the WVD. The WVD of the theoretical pulse profile $k(t)$ can be now expressed as the product of two convolution integrals in time and frequency:

$$W_k(t, \nu; \Pi) = a(t) B(-\nu), \quad (4.94)$$

where

$$a(t) = \int_{-\infty}^{\infty} g(s-t) |p(t)|^2 ds \quad (4.95a)$$

$$B(-\nu) = \int_{-\infty}^{\infty} U(\xi - \nu) F_p(\nu) d\xi, \quad (4.95b)$$

Eq. 4.50 has been used for the pulse WVD. Functions $a(t)$ and $B(\nu)$ have time (frequency) standard deviations noted respectively $\sigma_a(w_a)$ and $\sigma_b(w_b)$. The two important quantities σ_a^2 and w_b^2 provide the global resolution in time and frequency which are determined by both the measurement process ($p(t)$) and the signal analysis one ($\Pi(t, \nu)$). They are given by

$$\sigma_a^2 = \sigma_p^2/2 + \sigma_g^2, \quad (4.96a)$$

$$w_b^2 = w_p^2/2 + w_u^2. \quad (4.96b)$$

For $W_k(t, \nu; \Pi)$ to be the WVD of a realizable Gaussian pulse $k(t)$, $\Pi(t, \nu)$ must be a 2-D Gaussian function such that the condition $2\pi\sigma_k w_k = 1$ is satisfied, where $\sigma_k^2 = \sigma_a^2$ and $w_k^2 = w_b^2$. The corollary 4.1 can then be translated for the function $c(r; z, \Pi)$:

$$W_c(t, \nu; \Pi, z) = a(t - 2z/c) \Gamma_c(\nu; z), \quad (4.97)$$

with

$$\Gamma_c(\nu; z) = \int_{-\infty}^{\infty} B(\nu - f) \Phi_F(f; z) df. \quad (4.98)$$

With the new signal model in Eq. 4.90, the atmosphere is now sensed with a longer pulse $c(t; z, \Pi)$ and the Definition 4.2 giving the portion of the atmosphere which contributes to the realization of one signal sample at time t must be reviewed. In the particular case of a separable $\Pi(t, \nu)$ function, the weighting spatial function is determined by $a(t)$.

Definition 4.5 *The weighting spatial function determining the contribution of the particles in atmospheric slice z to the distribution $\text{SPW}_{x,\mu}(\nu)$ at a given time μ is defined by*

$$I_a(z; \mu) = \frac{c_a(z; \mu)}{\int_0^\infty c_a(z; \mu) dz}, \quad (4.99)$$

with $c_a(z; \mu) = h(z) a(\mu - 2z/c)$.

We also have that $\int I_a(z; \mu) dz = 1$. The covariance of $c(t; z, \Pi)$ is obtained by using the definition of the Wigner-Ville distribution and applying the inverse Fourier transform along ν on Eq. 4.97:

$$R_c(t, \tau; z, \Pi) = a(t - 2z/c) \gamma_c(\tau; z, \Pi), \quad (4.100)$$

where γ_c is the decorrelation function which, as γ_r in Eq. 4.31, gives the additional decorrelation of the signal due to the particle velocity randomness in slice z . It is given by the inverse Fourier transform of Eq. 4.98, i.e.

$$\gamma_c(\tau; z, \Pi) = b(\tau) M_F(f; z). \quad (4.101)$$

We finally obtain the average SPWD of the Doppler LIDAR signal

$$\text{SPW}_{x,\mu}(\nu) = \int_0^\infty c_a(z; \mu) \Gamma_c(\nu; z) dz + 1, \quad (4.102)$$

which is similar to Eq. 4.57 for the Wigner-Ville distribution for the Gaussian pulse case. Again, a normalized SPWVD, noted $\text{SPW}'_{x,\mu}(\nu)$, is used by replacing $c_a(z; \mu)$ by $I_a(z; \mu)$ in Eq. 4.102. It is now possible to extend the Theorem 4.3 for the SPWVD.

Theorem 4.6 *The Smoothed Pseudo-Wigner-Ville distribution of the LIDAR signal at time μ is obtained by the following convolution*

$$\text{SPW}'_{x,\mu}(\nu) = \int_{-\infty}^\infty B(\nu - f) \Phi_F''(f; \mu) df + 1, \quad (4.103)$$

where we define the following quantity

$$\Phi_F''(f; \mu) = \int_0^\infty I_a(z; \mu) \Phi_F(f; z) dz. \quad (4.104)$$

The weighted average distribution of $f(z)$, Φ_F'' is now obtained by considering atmospheric slices defined under a spatial weighting function given by the convolution of the pulse profile and the observation window $g(t)$ (see Eq. 4.95a). The spatial resolution is therefore smaller than for the $W_{x,\mu}(\nu)$. Moreover, Eq. 4.103 tells us that the resolution in frequency is also smaller due to the convolution in Eq. 4.95b. The following theorem is the extension of the Theorem 4.3 for the SPWVD.

Theorem 4.7 *The Smoothed Pseudo-Wigner-Ville distribution of the LIDAR signal can be written as a space-velocity distribution:*

$$\text{SPW}'_x(\rho, v') = \int_{-\infty}^\infty B(v' - v) \Phi_V''(v; z) dv + 1, \quad (4.105)$$

where $\rho = c\mu/2$ and

$$\Phi_V''(v; \mu) = \int_0^\infty I_a(z; \mu) \Phi_V(v; z) dz. \quad (4.106)$$

4.4.4 The spectrogram

We will now derive the expression of the spectrogram since it is one of the mostly used time-frequency distribution in the field. It is actually a particular case of the previous paragraph. For a temporal observation window $g(t)$, the smoothing kernel is simply given by

$$\Pi(t, \nu) = g(t)G(-\nu), \quad (4.107)$$

which is a 2D Gaussian kernel verifying the condition $2\pi\sigma_g w_g = 1$. Theorem 4.4 thus becomes

Theorem 4.8 *The Spectrogram of a LIDAR signal $x(t)$, noted $S_x(\nu)$, obtained by a Gaussian pulse profile $p(t)$ and analyzed with a Gaussian window $g(t)$ is the average Wigner-Ville distribution of an hypothetical LIDAR signal $x'(t)$ obtained by sensing the same volume with a realizable pulse $b(t)$ given by*

$$b(t) = \int_{-\infty}^{\infty} g(s-t) |p(t)|^2 ds. \quad (4.108)$$

The duration of $b(t)$ defined at e^{-1} is $\tau_b = 2\sigma_a$ and its FWHM duration is $\Delta t = (\ln 2)^{1/2} \tau_b$. Hence, increasing the duration of the observation window has the same effect on the spatial-velocity resolution as increasing the pulse duration. Moreover, all the equations obtained for the average WVD are therefore directly usable for the spectrogram. This allows one to use these equations without having to set independently the pulse duration and the observation window length in our study. Note that $b(t)$ is a realizable LASER pulse and so is $x'(t)$.

4.4.5 Estimation of the radial velocity distribution

We will now address the problem of estimating the radial velocity distribution characteristics from the SPWVD. If the exact signal covariance is known, the only factors influencing the resolution at which these characteristics are estimated are those of the pulse characteristics as well as the smoothing function $\Pi(t, \nu)$. Proposition 4.2 can be naturally extended for the SPWVD.

Proposition 4.3 *The statistical moments computed from the Smoothed Pseudo Wigner-Ville distribution at time μ gives the equivalent moments of a smoothed version of the weighted probability density function, noted $\Phi'_V(v; \mu)$, instead of those of the exact one $\Phi_V(v; \rho)$ assigned at distance $\rho = c\mu/2$.*

a. The mean radial velocity

By proceeding as in Section 4.3.3, we find that the instantaneous frequency of a smoothed-pseudo Wigner-Ville transform of the LIDAR signal is given by

$$m_{1,x}(t) = \int_0^\infty I_a(z; \mu) \bar{f}(z) dz. \quad (4.109)$$

We see that the use of a frequency filtering function $U(\nu)$ has no influence on the estimation of the instantaneous frequency unless it becomes non-symmetrical. The radial velocity estimate at time μ is now given by

$$\hat{v}_R(\mu) = \int_0^\infty I_a(z; \mu) \bar{v}_R(z) dz. \quad (4.110)$$

It must be noted that the radial velocity smoothing is now performed over a distance $r_a = \sigma_a c/2$ instead of $r_p = c\sigma_p/2$.

b. The radial velocity dispersion

For a Gaussian pulse with spectral variance w_p^2 computed on $F_p(\nu)$, the convolution integral in Eq. 4.103 directly gives the second-order moment at time μ of $\text{SPW}_{x,\mu}(\nu)$, noted $m_{2,x}(\mu)$:

$$m_{2,x}(\mu) = \frac{w_p^2}{2} + w_u^2 + \sigma_f^2(\mu), \quad (4.111)$$

where $\sigma_f^2(\mu)$ is the second order moment of the weighted distribution $\Phi_f''(f; \mu)$. This means that, knowing the pulse spectral width, it is possible to estimate the radial velocity dispersion on a range interval defined by $I_a(z; \mu)$ by computing $m_{2,x}(\mu)$ from the signal SPWVD as well as, thanks to Eq. 4.106

$$\hat{\sigma}_v^2(\mu) = \frac{\lambda^2}{4} (m_{2,x}(\mu) - w_b^2). \quad (4.112)$$

4.4.6 Analysis of particular wind fields

We will now continue our analysis of the wind gradient and wind turbulence cases by considering the effect of the signal processing in addition to the measurement process on the estimates.

a. The wind gradient profile

The SPWVD has been computed from the corollary 4.4 and with assumption 4.4 for the radial velocity profile at Eq. 4.37. It is given by:

$$\text{SPW}_x(t, \nu) = \frac{c}{2} \frac{h(ct/2)}{\sqrt{2\pi}w_{T'}} \exp\left(-\frac{[\nu - f_{\text{IF}} + 2\lambda^{-1}v_{\text{R}}(ct/2)]^2}{2w_{T'}^2}\right) + 1, \quad (4.113)$$

with $w_{T'}$, its second-order moment, given by

$$w_{T'}^2 = \frac{w_p^2}{2} + w_u^2 + w_s^2, \quad (4.114)$$

where

$$w_s = \frac{c}{\lambda} \frac{\sigma_a g(\rho)}{\lambda}, \quad (4.115)$$

is the wind gradient induced spectral broadening. We observe that, as pointed by Theorem 4.4 and in comparison with the results of the wind gradient WVD in Section 4.3.4, the atmospheric is now sensed as if we had used a pulse profile with a duration $2\sigma_a$ and a spectral width w_b . For the spectrogram with a Gaussian observation window $g(t)$ we must replace w_u by $w_g = (2\pi\sigma_g)^{-1}$ in Eq. 4.114.

b. The Gaussian random velocity field

With corollary 4.4 and Eq. 4.97 and 4.98, we find that $\Gamma_c(\nu; z)$ is a Gaussian density with variance

$$w_c^2 = \frac{w_p^2}{2} + w_u^2 + \left(\frac{2\sigma_v}{\lambda} \right)^2, \quad (4.116)$$

and so is the SPWVD when the system gain and Φ_v are constant over the $I_a(z; t)$ support. Under these assumptions, it can be easily shown, in contrary to the wind gradient case, that the time smoothing has no influence on the estimation of the second-order moment.

4.4.7 The sample SPWVD

An optimized implementation of the sample SPWVD can be obtained by exploiting the properties of the Cohen's class. The reader is advised to read Section A.2.2 for more details. The distributions are actually computed in the $\{\xi, \tau\}$ domain and projected back to the $\{t, \nu\}$ domain by using the two-dimensional Fourier transform. The accumulation itself is realized as in Section 4.3.5 for the WVD.

For a given accumulation level M , the sample SPWVD is obtained by first estimating the signal covariance $\hat{r}_x(t, \tau)$ with Eq. 4.81 and computing its Fourier transform along t to obtain an estimate of the ambiguity function $\hat{A}_x^M(\xi, \tau)$ which is the two-dimensional Fourier transform of the Wigner-Ville distribution $\hat{W}_x^M(t, \nu)$. The parameterization function $F(\xi, \tau)$ is then computed depending on the parameters $\{\sigma_g, w_u\}$ of the associated smoothing function $\Pi(t, \nu)$. The two-dimensional Fourier transform of the product $\hat{A}_x^M F$ finally gives the sample SPWVD. The average SPWVD is usually for comparison and is directly computed from Theorem 4.5.

4.5 Numerical results and discussion

To illustrate these results, we will now consider a radial velocity profile composed of a first deterministic component with a quadratic variation with range and a second component which is a Gaussian random variable with a range-dependent variance σ_v^2 . The objective is to estimate both the mean radial velocity and the standard deviation of the Gaussian turbulence. These estimates are respectively denoted $\hat{v}_R$ and $\hat{\sigma}_v$. The velocity profile is thus modeled by

$$v_R(z) = \langle v_R(z) \rangle + v'_R(z), \quad (4.117)$$

where $\langle v_R(z) \rangle$ is the deterministic component given by

$$\langle v_R(z) \rangle = \langle v_R(0) \rangle + \alpha z^2. \quad (4.118)$$

where $\alpha [\text{m}^{-1} \text{s}^{-1}]$ is a real constant and $v'_R(z)$ is the random component, given by $v'_R(z) = v_R(z) - \langle v_R(z) \rangle$. The deterministic component can be approximated at any distance ρ by

$$\langle v_R(z) \rangle \approx \langle v_R(\rho) \rangle + g(\rho) (z - \rho), \quad (4.119)$$

where

$$g(\rho) = \left. \frac{dv_R(z)}{dz} \right|_{z=\rho}. \quad (4.120)$$

The Monte Carlo simulation is performed by first generating 5000 signals using Eq. 4.6 and Eq. 4.12 with different realizations of the speckle and $v_R(z)$ given by Eq. 4.117. The pulse has a Gaussian profile with a duration fixed to $\tau=480$ ns ($\Delta t=400$ ns). The system gain $h(z)$ is constant with a value corresponding to a SNR of -6 dB. The sampling frequency is 200 MHz and the intermediate frequency 30 MHz. The accumulation level varies from 10 to 50 signals. For each test, 400 signals are selected randomly among the 5000 available. We choose a mean radial velocity increasing quadratically from 5 m/s at the ground level to 15 m/s at a range of 3 km. The standard deviation of v'_R decreases linearly from 4 m/s to 0 m/s over the same distance. These values have been chosen to reduce as much as possible the computational time by maximizing the time-frequency occupancy of the signals.

Since the average SPWVD at any time has a Gaussian shape with a constant noise level outside the signal bandwidth, the spectral moments of the sample SPWVD as well as the signal and noise power can be computed by the classical spectral-based ML estimators (see Appendix E). The first-order moment gives the estimate of the mean radial velocity $\langle v_R(z) \rangle$, noted $\hat{v}_R$. It is obvious, from Section 4.4.6, that the estimate of the second-order moment at time μ of the average SPWVD, noted $\hat{m}_{2,x}(\mu)$, is given by

$$\hat{m}_{2,x}(\mu) = \frac{w_p^2}{2} + w_u^2 + \frac{c \sigma_a \hat{g}(\rho)}{\lambda} + \left(\frac{2\hat{\sigma}_v}{\lambda} \right)^2, \quad (4.121)$$

where $\hat{g}(\rho)$ and $\hat{\sigma}_v$ are respectively the targetted estimates of the wind gradient $g(\rho)$ and the standard deviation σ_v of the turbulence at mean range $\rho = c\mu/2$. The other parameters are known. The wind gradient component is estimated by derivating $\hat{v}_R$ and $\hat{\sigma}_v$ is obtained by inserting $\hat{g}(\rho)$ in Eq. 4.121.

Fig. 4.8 gives the sample SPWVD obtained for $M=50$ and different smoothing functions. The quadratic evolution of the mean radial velocity with range as well as the decrease in Gaussian turbulence can be observed in all of them. We see that an increase of the turbulence induces a broadening of the SPWVD which, in turn, becomes more sensitive to noise. The estimation variances will be therefore analyzed depending on σ_v and the accumulation level M as the SNR is fixed in this study. We must point that only the two distributions lying on

the anti-diagonal of this figure, with parameters $\{\sigma_p/2, 2w_p\}$ and $\{2\sigma_p, w_p/2\}$, are spectrograms. The two others cannot therefore be obtained from a simple temporal observation window.

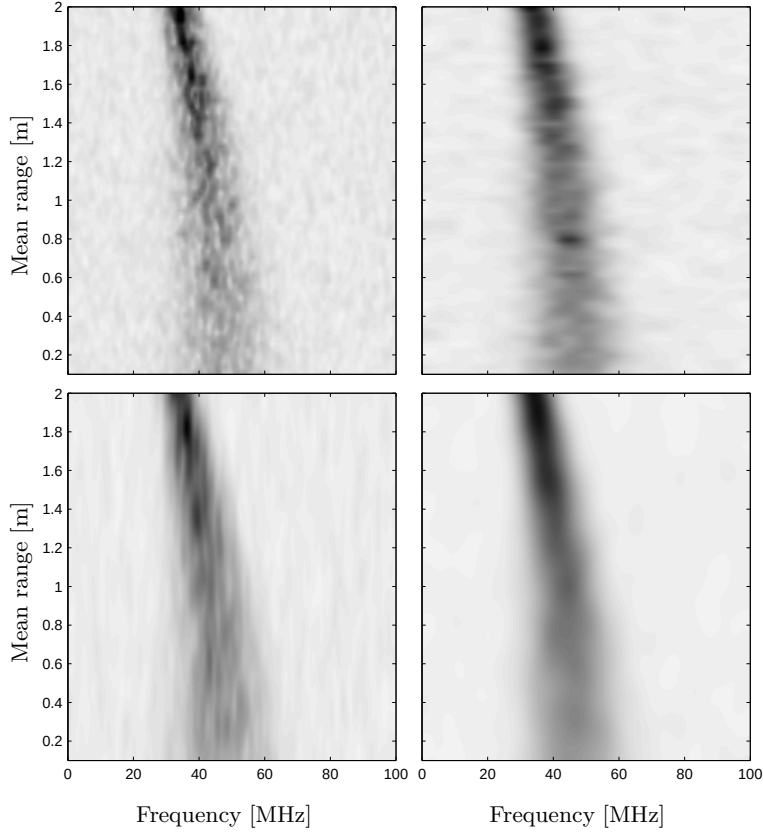


Figure 4.8: Influence of the smoothing function $\Pi(t, \nu)$ with parameters $\{\sigma_g, w_u\}$ on the SPWVD for a quadratic radial velocity field and a Gaussian turbulence decreasing with range. The pulse duration is $2\sigma_p=480$ ns ($\Delta t=400$ ns). $\sigma_g=\sigma_p/2$, $w_u=w_p/2$ (top,left); $\sigma_g=\sigma_p/2$, $w_u=2w_p$ (top,right); $\sigma_g=2\sigma_p$, $w_u=w_p/2$ (dash); $\sigma_g=2\sigma_p$, $w_u=2w_p$ (dot). The accumulation level is 50.

Fig. 4.9 and 4.10 respectively gives the influence of σ_v on the estimates $\hat{v}_R$ and $\hat{\sigma}_v$. The accumulation is there fixed to 10 signals per estimates. The upper and lower bounds are respectively determined by the SPWVD for which

the smoothing functions has the parameters $\{\sigma_p/2, w_p/2\}$ and $\{2\sigma_p, 2w_p\}$. The same is observable in Fig. 4.11 which gives the influence of the accumulation level. For more complex signals, with an instantaneous frequency varying more rapidly with range, such as the wake vortex signal studied in the next chapter, the time-frequency support of $\Pi(t, \nu)$ cannot be arbitrarily widen and a compromise must therefore be chosen between the conservation of either the temporal evolution of the mean frequency or the frequency contents.

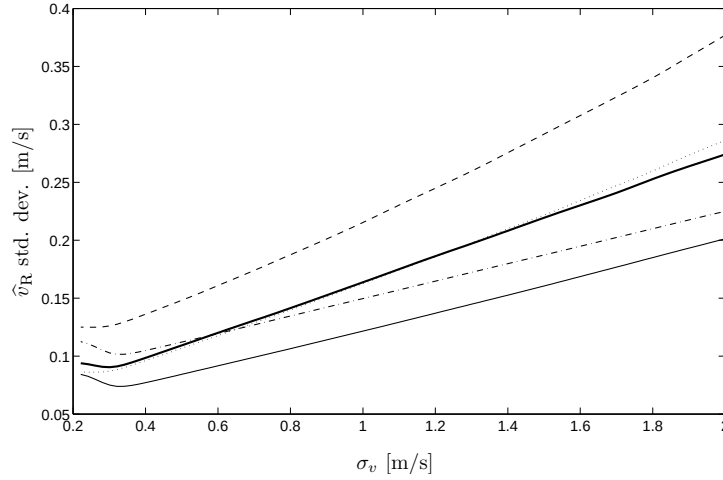


Figure 4.9: Influence of a Gaussian wind turbulence with a standard deviation σ_v on the SPWVD first order moment estimates for a quadratic wind profile, a pulse duration σ_p and various $\Pi(t, \nu)$: $\sigma_g=\sigma_p, w_u=w_p$ (solid,thick); $\sigma_g=2\sigma_p, w_u=2w_p$ (solid,thin); $\sigma_g=\sigma_p/2, w_u=w_p/2$ (dash); $\sigma_g=\sigma_p/2, w_u=2w_p$ (dot); $\sigma_g=2\sigma_p, w_u=w_p/2$ (dash-dot). The accumulation level is 10.

4.6 Conclusions

Even if they are necessary at different levels of the system design, numerical simulations of the LIDAR measurement are not always appropriate for analyzing the influence of the system parameters with given atmospheric conditions. The major limiting factor is the computation time, i.e. hours for LIDAR simulations and weeks for realistic fluid dynamics ones. Complementary but yet powerful tools for wind statistics and wake vortex circulation estimation are the LIDAR time-frequency statistical distributions which extend the classical concept of signal spectrum. They can be used for direct processing when little knowledge about the signal physics is available but also, in contrary, for signal modeling and advanced estimation algorithm design.

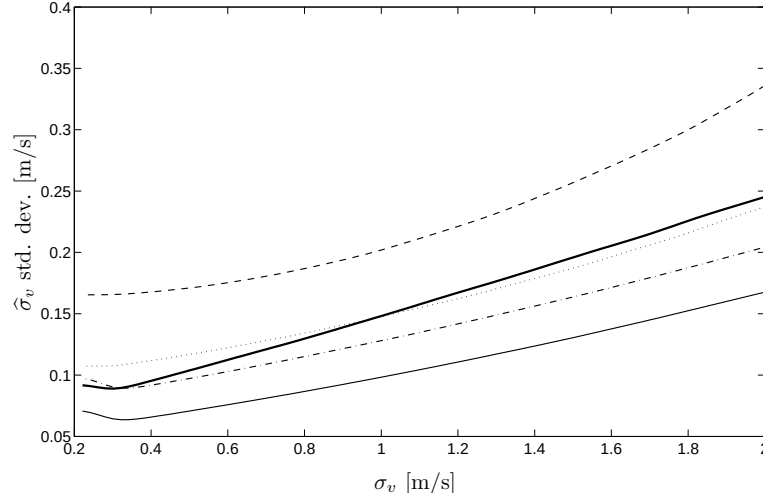


Figure 4.10: Influence of a Gaussian wind turbulence with a standard deviation σ_v on the estimate of σ_v itself, noted $\hat{\sigma}_v$, for a quadratic wind profile, a pulse duration σ_p and various $\Pi(t, \nu)$. The legend is similar than Fig. 4.9.

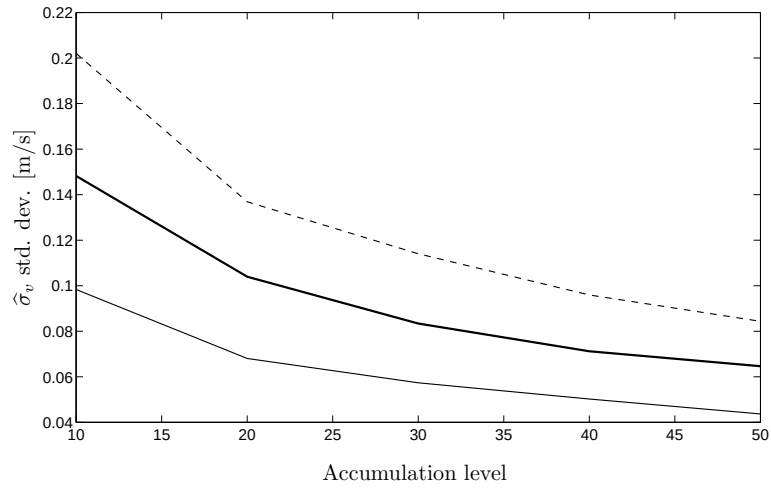


Figure 4.11: Influence of the accumulation level on the σ_v estimate for a Gaussian turbulence of $\sigma_v=0.1$ m/s and different $\Pi(t, \nu)$ parameters: $\sigma_g=\sigma_p$, $w_u=w_p$ (solid,thick); $\sigma_g=2\sigma_p$, $w_u=2w_p$ (solid,thin); $\sigma_g=\sigma_p/2$, $w_u=w_p/2$ (dash).

Based on already existing analytical expressions of the LIDAR signal covariance, a complete formulation of the LIDAR signal Cohen's class has been derived in this thesis. Thanks to this formulation, it is now possible to immediately obtain the time fluctuations of the signal spectrum for a given velocity profile and any LASER source parameters. This formulation has been applied to the general case of wind statistics estimation and has been particularized to the spectrogram and the Smoothed Pseudo Wigner-Ville distribution (SPWVD) of LIDAR signals measured for wind gradient and Gaussian wind turbulence.

Besides that, it also provides us with a clear understanding on what is actually estimable and with which precision. Hence, it has been demonstrated that the Wigner-Ville distribution (WVD) of the LIDAR signal, observed at a given observation time, is obtained by the convolution in frequency between the pulse WVD and a weighted Doppler frequency distribution which depends on the exact velocity distribution and on the LIDAR and atmospheric parameters. This analysis has been carried out a step further by introducing a time-frequency observation window and therefore considering the effect of the signal processing itself on the quality of the estimation.

Concerning the estimation of the mean radial velocity and the velocity dispersion, expressions for the first and second-order statistical moments of the average LIDAR SPWVD have been derived under the Gaussian pulse assumption. They confirm that the retrieved radial velocity profile is given by the spatial convolution of the exact radial velocity with a filtering kernel which is a function of the spatial pulse profile, the analyzing window and the LIDAR system gain. Similar results have been obtained for the second order moment. It gives us a way to estimate the velocity dispersion for particular conditions in which the velocity is assumed to be statistically homogeneous in the volume analyzed at a given observation time.

Having derived the LIDAR equations for the general Cohen's class allows us to extend the existing estimation algorithms to a wider set of methods performing directly the signal analysis in the time-frequency plane. Nevertheless, we have limited our study to separable in time and frequency Gaussian analyzing windows where more sophisticated ones could be used. This constraint have been imposed by the necessity to conserve the physical interpretation of the derived equations, which is not possible for more complex windows such as the one of Choi-Williams or Rihaczek having the interesting property to reject the interference terms outside the signal location. This study must be extended to the estimation of an atmospheric wind turbulence described by a more general structure function. It would notably allow to develop estimation algorithms of the turbulent eddy dissipation rate (TEDR) based on the estimation of the spectral width. The background wind turbulence is indeed an important factor to evaluate the wake vortex dwell time.

Analysis of the wake vortex signal

CHAPTER 4 was dedicated to the general problem of estimating the wind field statistics from the Doppler LIDAR signal. We will now more specifically analyze the wake vortex signal by means of the previously formalized time-frequency distributions. This study has led to the development of an adaptive algorithm for the estimation of the Doppler frequency distribution as well as the maximum radial velocity which is sometimes used for the evaluation of the vortex strength.

The wake vortex signal is first analyzed through the Cohen's lens and the results of this study are presented in Section 5.1. The maximum velocity estimation problem is exposed in Section 5.2 and the adaptive algorithm is then exposed in Section 5.3.

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5.1 Analysis of the wake vortex signal

In this section, the previous average time-frequency distributions are exploited for the analysis of LIDAR signals with typical aircraft wake vortex velocity profiles. We will principally focus on model-based radial velocity estimators which provides more precise results than the first-order spectral moment, especially when considering noisy accumulated distributions. The following terminology will be used:

Definition 5.1 *The coherent Doppler LIDAR signal measured when a pair of wake vortices are present in the sensing volume is called the wake vortex signal. Moreover, the part of the atmosphere in which the influence of the vortices is negligible will be called the background region.*

A wake vortex signal obviously contains information from both the wake vortex and the background regions, in time intervals which overlaps depending on the pulse duration. It must be noted that, as we will now deal with fluid simulations, the coordinate system defined in Section 1.2.3 will be used. The range along a given line-of-sight will be noted R instead of z as used in the first chapters.

Notation 5.1 *When dealing with wake vortex signals, the quantities related to the nearest and farthest cores are noted with the subscripts 1 and 2 respectively, in particular for the parameters couple $\{t_i, z_i\}$ giving their positions in space and measurement time.*

Fig. 5.1 represents a contour plot of the radial velocity of a wake vortex pair in ground effect (IGE) and after rebound obtained in the transverse direction. This map has been obtained by scanning through a CFD simulation at a Reynolds number of 20000 and with initial parameters of $\Gamma_0=400 \text{ m}^2/\text{s}$ for the circulation and $b_0=50 \text{ m}$ for initial spatial core separation. Three typical line-of-sights are also shown and the ideal radial velocity profiles corresponding to these directions of observation are depicted in Fig. 5.2. The two vortex core centers are positionned respectively around the ranges $R_1=530 \text{ m}$ and $R_2=670 \text{ m}$ which gives a kernel separation of about 140 m. We first observe that the spatial scales of the velocity are of the same order than typical values of the spatial pulse extend, i.e. a few tens of meters. The performance of the velocity estimation algorithms will therefore be strongly sensitive to signal decorrelation.

5.1.1 Analysis of its average WVD

Let's begin this study by describing a short example illustrated by results exposed in Fig. 5.3 and Fig. 5.4. The average Wigner-Ville distribution has been obtained by applying Theorem 4.2 and Eq. 4.53 on a particular profile extracted from a CFD database and given by the solid curve of Fig. 5.2. The

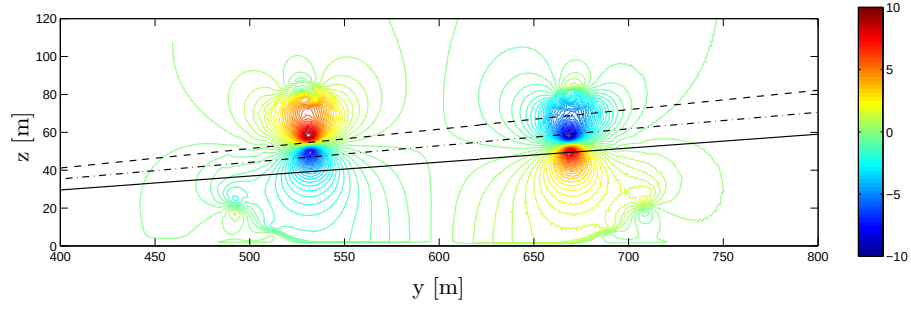


Figure 5.1: Example of a radial velocity $v_r(R)$ [m/s] map in contour plot of a wake vortex pair in ground effect obtained by numerical simulation. The three lines are the selected LOSes represented in Fig. 5.2.

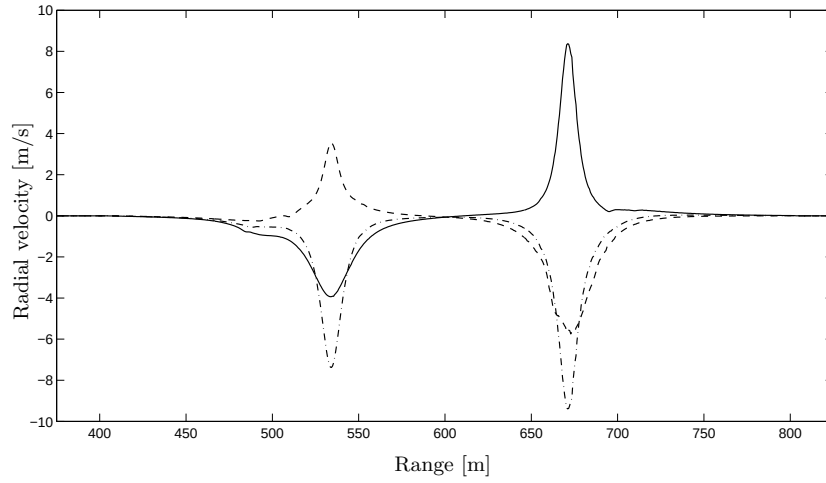


Figure 5.2: Examples of radial velocity profiles for three LOS selected obtained by scanning through the wake vortex IGE simulation for which a plane is represented in Fig. 5.1.

Gaussian pulse has a duration $\tau_p = 2\sigma_p = 200$ ns. The size of the sensing volume is therefore $r_p = c\sigma_p/2 = 30$ m ($\Delta r = 25$ m). The WVD is given at the center of Fig. 5.3 where the transformation

$$W_x(t, v) = W_x(t, f_{IF} - 2\lambda^{-1}v) \quad (5.1)$$

has been applied in order for the WVD to be interpreted as a time-velocity distribution. The radial velocity profile presents two important fluctuations around ranges $R_1 = 530$ m and $R_2 = 670$ m corresponding to the position of the two vortex cores and which induces changes in the WVD around times $t_1 = 2R_1/c = 3.5$ μ s and $t_2 = 4.5$ μ s. The right figure in Fig. 5.3 represents three cuts of the WVD at times $t_0 = 2.5$ μ s, t_1 and t_2 . They are denoted $W_{x,t_i}(v)$ with $i \in \{0, 1, 2\}$ (see Notations 4.1 and 5.1). The bottom and top figures give respectively the evolution of the $W_x(t, v)$ maximum with time as well as the signal correlation time computed from the WVD second-order moment by using Eq. 4.70. The top solid curves of Fig. 5.4 gives the exact radial velocity profile as a function of time t from particles at range $R = ct/2$ along with the estimated radial velocity obtained by computing the first-order moment of $W_{x,t}(v)$. The bottom figure is the estimated radial velocity variance $\hat{\sigma}_v^2(t)$ of particles inside the sensing volume around the same range R and obtained from Eq. 4.69.

We now analyze the average WVD under the light of Theorem 4.3. Before the LASER pulse attains the distance $(R_1 - r_p/2)$, all particles have the same radial velocity and we have thus that $\Phi_v(v) = \Phi'_v(v) = \delta(0)$ and $W_{x,t}(v) = F_p(v)$ is therefore a Gaussian function centered around $v = 0$ m/s with a variance $m_{2,x}(t) = (\lambda w_p/2)^2$. The estimated velocity standard deviation is naturally $\hat{\sigma}_v^2(t) = 0$ m/s. Since there is no important variations of the radial velocity profile, the signal correlation time is equal to $\tau_p = 200$ ns. The pulse then entered into the spatial region around distance R_1 when the nearest vortex core is present. The velocity distribution $\Phi'_v(v, t)$ spreads towards negative values of v (see the evolution of $\hat{\sigma}_v^2$ in Fig. 5.4) and so is $W_x(t, v)$ as described the convolution in Eq. 4.60. This spreading induces a decrease of the WVD maximum as well as of the correlation time of the signal. The same phenomenon is observed around $t = 4.5$ μ s when the pulse is inside the farthest vortex core at range R_2 . The radial velocity is now spread towards positive value of v . As the variation of velocity is stronger and narrower around R_2 than it is around R_1 , the effects on the signal will be more important as depicted by the evolution of the signal correlation time which decreases up to 50 ns. The velocity distribution is also richer and $W_{x,t_2}(v)$ becomes multimodal in the sense that it can be approximated by a finite sum of clearly separated Gaussian kernels. The two following propositions are given to stress the previous observations.

Proposition 5.1 *When analyzing the wake vortex signal, the first-order moment of WVD leads to an estimate of the radial velocity profile which is a smoothed version of the exact radial velocity profile.*

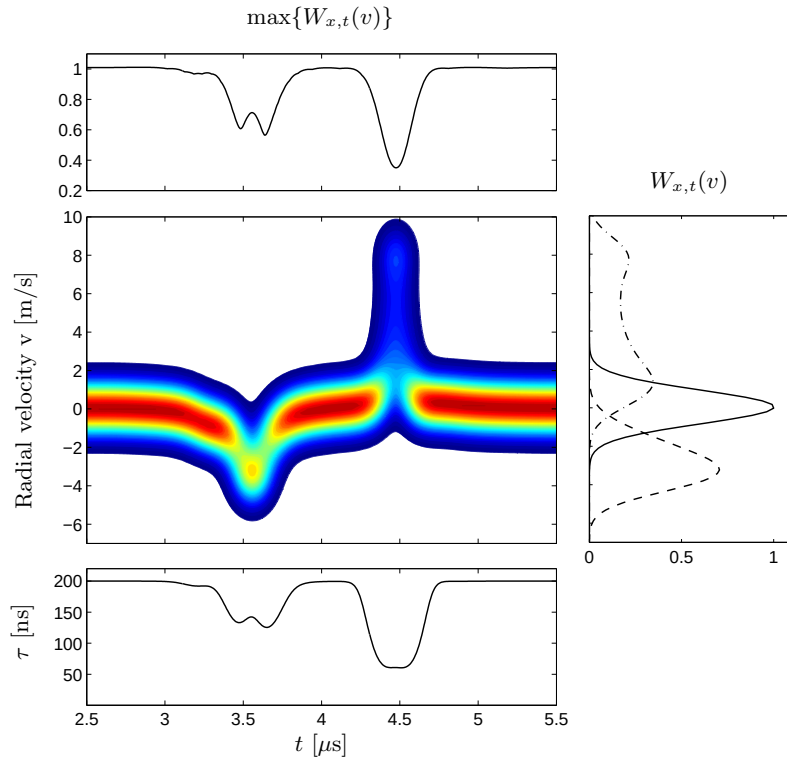


Figure 5.3: Average Wigner-Ville distribution $W_x(t, v)$ of a wake vortex system obtained with a 200 ns pulse duration. The central figure gives the time-velocity distribution in contour plot. The upper figure is an image of temporal signal energy given by $\max\{W_{x,t}(v)\}$. The correlation time is given in the lower figure. The right figure gives three $W_{x,t}(v)$ profiles: outside vortex at $t=2.5 \mu\text{s}$ (solid); at $t=3.5 \mu\text{s}$ (dash; center of the nearest vortex core) and at $t=4.5 \mu\text{s}$ (dash-dot; center of the farthest vortex core).

This proposition, which is a direct translation of Eq. 4.66, is of particular importance since the estimation error that is made on the radial velocity profile may induce an estimation error on the wake vortex circulation and, in an indirect way, on its other parameters, i.e. core size and position. This point is exposed in much details in Chapter 7 where a model-based technique to reduce these errors is proposed. Fig. 5.5 gives the estimated radial velocity and its dispersion for a more complex profile obtained from a CFD of wake vortex in ground effect with cross wind. It can be seen, especially on the farthest vortex core that some of the characteristic spatial scales are definitely lost during the sensing process for the $\tau_p=300$ ns case. Another observation that can be made is that, when the pulse duration increases, the estimated profiles tends to be modelled by a weighted sum of $I(R; \mu)$ functions shifted in space, where $I(R; \mu)$ is the normalized spatial profile of the pulse given at Definition 4.2.

Proposition 5.2 *When analyzing the wake vortex signal, the fast variations of the radial velocity profile induces a decorrelation of the LIDAR signal which can be observed by a decrease of the SNR in conjunction with an increase of the spectral width. Moreover, in the atmospheric regime (see Definition 4.4), the WVD becomes multimodal since the pulse power spectrum $F_p(f)$ is small compared to $\Phi'_f(f; \mu)$ (Theorem 4.3).*

It is thus possible to detect wake vortices only by analyzing the reconstructed map of either the computed correlation time or the second-order moment of the WVD. Fig. 5.6 illustrates this for two pulse durations of 100 ns and 300 ns. The relation between the correlation time τ and the second-order moment of the WVD is given by Eq. 4.70. We clearly observe a decrease of the correlation time around time instants t_1 and t_2 to a value independent to the pulse duration (respectively around 70 ns and 60 ns). The variations of the second-order statistics of the signal is however larger for longer pulse duration which increases the efficiency of any wake vortex detection algorithm based on these parameters.

Important remark 5.1 *When sensing phenomena such as wake vortices or wind shear, care must be taken not to interpret directly spatial variations of the average signal SNR as only a modification of other atmospheric parameters such as the aerosol particles density. The analysis of the signal decorrelation time gives a way to reduce this ambiguity.*

The Cramer-Rao Bound for the spectral maximum likelihood of the radial velocity with accumulation has been computed in [Rye 1993] for a Gaussian power spectrum with equation

$$S_x(f) = \frac{\delta}{\sqrt{2\pi}w} \exp(-(f - \bar{f})^2/(2w^2)) + N, \quad (5.2)$$

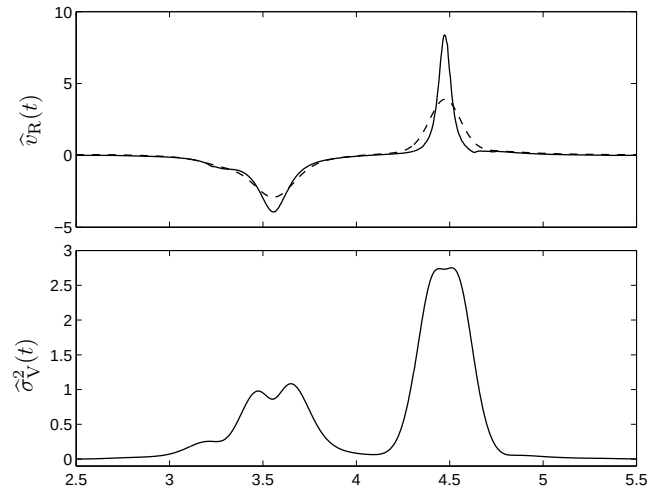


Figure 5.4: Estimation of the two first-order moments of the average Wigner-Ville distribution of a wake vortex pair obtained with a 200 ns pulse duration. The upper figure gives the exact radial velocity profile (solid) as well as the estimated profile $\hat{v}_R$ (dash) at each time t . The lower figure represents the second-order moment $\hat{\sigma}_V^2(t)$.

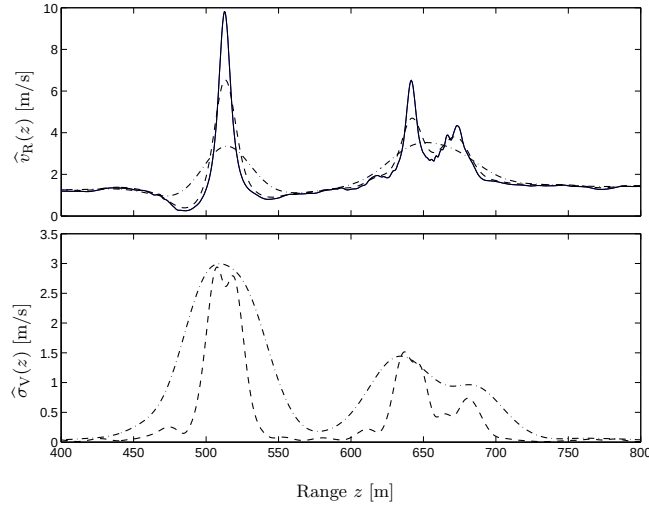


Figure 5.5: Influence of pulse duration on the average WVD in the case of a wake vortex pair in ground effect with cross-wind; The top figure gives estimated radial velocity profile: exact from CFD (solid), $\tau_p = 100$ ns (dash), $\tau_p = 300$ ns (dash-dot). The bottom figure gives the associated estimated velocity dispersion.

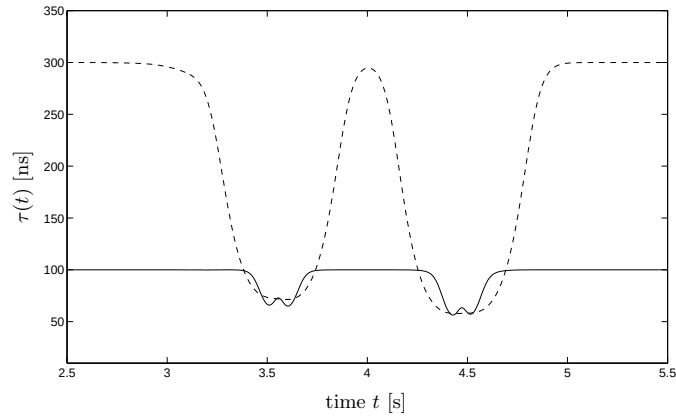


Figure 5.6: Influence of pulse duration on the signal correlation time; $\tau_p = 100$ ns (solid) and $\tau_p = 300$ ns (dash). The time instants $t_1=3.5 \mu\text{s}$ and $t_2=4.5 \mu\text{s}$ corresponds to twice the time after which the pulse encountered the first and the second vortex cores.

where $\bar{f}$ is the mean frequency, w is the second spectral moment, N is the unit noise level, δ is the wideband SNR, i.e. the ratio between the total signal power and the total noise power over the entire spectral bandwidth. The lower bound on the estimation variance is given by the following equation for low SNR:

$$\sigma_{\text{CR}}^2 = \frac{4\sqrt{\pi}w^3}{nM\delta^2}, \quad (5.3)$$

where n is the accumulation level and M is the number of spectral channel. Although not directly applicable in the wake vortex regime, this lower bound, combined with the observations already made about the wake vortex signal, tells us that the estimation variance drastically increases inside the wake vortex due to the spreading of the frequency distribution $\Phi_f''(f; \mu)$. Hence, we have a decrease of δ^2 with an increase of w^3 which can only be reduced by an increase of the accumulation level M , at least for high PRF.

5.1.2 Estimation of the radial velocity distribution

We have just seen that the estimation of the radial velocity profile from the computation of the first-order moment of one of the time-frequency distributions exposed in Section 4.4, only gives a smoothed version of the exact velocity profile. We will now discuss the use of spectral modeling techniques to improve the radial velocity estimates. The Theorem 4.3 is used as a reference to the average WVD and the Theorem 4.6 for the average SPWVD.

Fig. 5.7 represents a mesh of the average WVD of a LIDAR signal obtained with a 200 ns pulse duration and a radial velocity profile given by the dash-dot curve of Fig. 5.2. As it is shown in Fig. 5.3, unless for very short pulses, the classical assumption according to which the signal spectrum is Gaussian for a Gaussian pulse is not valid for a wake vortex signal. Moreover, if we analyze the distribution at R_1 and R_2 , we notice as well that the exact radial velocities which are respectively of about 7 and 9 m/s have to be retrieved at the extremity of the distribution. We therefore have to look at advanced techniques able to produce more precise estimates than the first-order moment.

The mostly used spectral model is the sum of Gaussian with a constant photodetector noise spectrum model. The case of a single Gaussian is typically the one targetted by classical spectral-based maximum likelihood estimators based on the Zrnić model as exposed in Appendix E. Note that the presence of a RIN noise which appears at low frequencies, i.e. far from the signal bandwidth, won't be taken into account since it can be removed by well-known filtering techniques either in the time or in frequency domain. The number of kernels to use depends on the pulse duration as well as on the radial velocity profile distribution. The kernel shape is determined by $B(\nu)$ defined in Eq. 4.95b ($F_p(\nu)$ for the WVD) which leads to the following proposition.

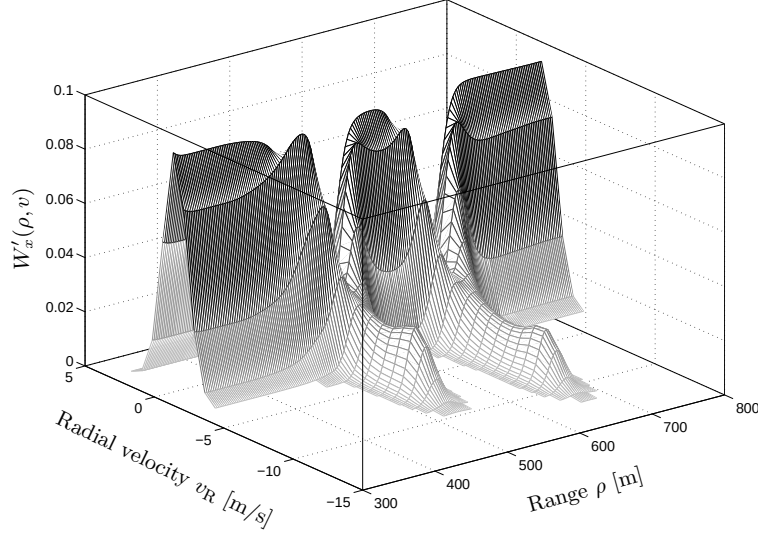


Figure 5.7: Representation of the average WVD of a wake vortex signal obtained with a pulse duration $\tau_p=200$ ns with a radial velocity profile given by the third profile of Fig. 5.2 (dash-dot line).

Proposition 5.3 *The main objective of the LIDAR spectrum modeling is to estimate the weighted radial velocity distribution $\Phi''_v(v; t)$ (or $\Phi'_v(v; t)$ for the WVD). The use of a kernel-based approach with kernels lying in the family of scaled and shifted versions of $B(\nu)$ ($F_p(\nu)$) has therefore to be interpreted as a spectral deconvolution technique.*

The following assumption is used for the development given below.

Assumption 5.1 *The aerosol particles outside the spatial region of the wake vortex core, called the background region, are characterized by a mean radial velocity $\bar{v}_{R,b}$ corresponding to a frequency denoted $\bar{f}_b$ and a negligible dispersion compared to the support of $B(\nu)$ in the velocity space.*

Most of the time, for fluid simulations without crosswind, we consider $\bar{f}_b = f_{IF}$ corresponding to $\bar{v}_{R,b} = 0$. It is worth remembering that the spatial extend of the distribution $\Phi''_F(f; t)$ depends either on the pulse profile and on the observation window noted $g(t)$.

To illustrate this modeling process, Fig. 5.8 gives the various WVDs, $W_{x,t_1}(v)$, obtained for different pulse durations and analyzed at a time corresponding to the time response of the first vortex core. The vertical line gives the value of the exact radial velocity. The following observations can be made which have to be

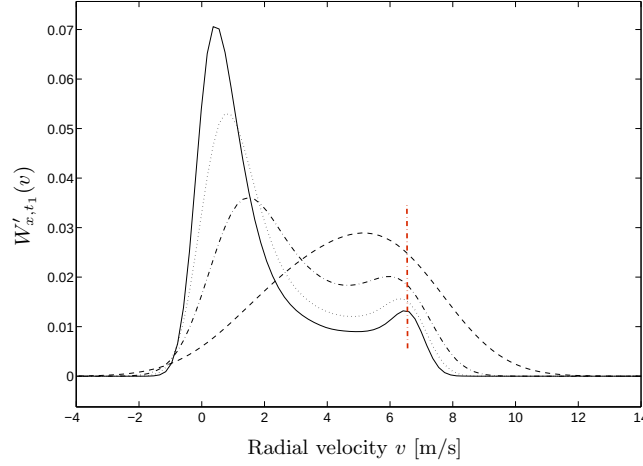


Figure 5.8: Influence of the pulse duration on $W'_{x,t_1}(v)$ inside a wake vortex core; $\tau_p = 100$ ns (dash), $\tau_p = 200$ ns (dot-dash), $\tau_p = 300$ ns (dot) and $\tau_p = 400$ ns (solid). The vertical line corresponds to the exact value of the radial velocity $v_R(z_1)$ at range z_1 .

related to the two LIDAR regimes of Definition 4.4. Since we essentially analyze the effect of fast varying deterministic velocity profiles, the atmospheric regime defined previously should preferably be called in this section the *wake vortex regime*. Although it is a subclass of the atmospheric regime, this distinction is used to stress the fact that multimodal distributions are more likely to appear as the results of this kind of velocity profiles, for long pulse durations. For a Gaussian pulse power spectrum, a faithful indicator of this regime is its skewness, i.e. the third-order moment. Let's analyze these two cases.

In the *pulse-dominated regime*, represented by the dash line of Fig. 5.8, the pulse profile is small compared to the spatial scale of variation of the wake vortex radial velocity profile. The pulse power spectrum is therefore large compared to the weighted frequency distribution $\Phi''_F(f; t)$ of which details are suppressed due to the convolution integral in Eq. 4.103. Moreover, particles in the background region does not contribute to the frequency distribution and no significant peak is observed around $\bar{v}_{R,b}$. The spectrum can therefore be modelled by a small number of kernels and the first-order moment of the distribution gives a good approximation of the exact radial velocity value.

In the *wake vortex regime*, mostly represented by the solid line of Fig. 5.8, the frequency distribution is richer due to the larger volume illuminated by the LASER pulse. A peak is also present around $\bar{v}_{R,b}$ due to the stronger contribution

of the background region in $\Phi_F''(f; t)$. It can be seen from this figure that its strength increases with increasing τ_p and its position tends to $\bar{v}_{r,b}$. The first order moment is a poor estimation of the exact radial velocity distribution at distance ρ since it is attracted to the value $\bar{v}_{r,b}$ outside the WV region and the distribution must be modeled by a large number of spectral kernels, since the exact value of the velocity is present in the queue of the distribution. Note that the frequency corresponding to the mean spectral peak will only give an estimation of the radial velocity of particles in the background region. It is thus quite obvious that the velocity corresponding to the maximum of the distribution is not a good estimate at all in this regime.

Since Theorem 4.6 states that, for a Gaussian pulse profile and under Assumption 4.4, the SPWVD can be expressed as the convolution of an unknown distribution Φ_F'' with a Gaussian kernel $B(\nu)$, it is natural to select a model which is composed of a mixture of weighted and shifted versions of $B(\nu)$. The general approximation $\hat{S}_{x,\mu}^K(\nu)$ of the $\text{SPW}'_{x,\mu}(\nu)$ obtained with K kernels is

$$\hat{S}_{x,\mu}^K(f) = \frac{1}{K} \sum_{i=0}^{K-1} y_i G^i \left(\frac{f - \bar{f}_i}{\sigma_i} \right) + N(f), \quad (5.4)$$

where K is the number of kernels, $G^i(f)$ is the i^{th} kernel defined by a frequency position $\bar{f}_i$ and a standard deviation σ_i , y_i is the weighting factor and $N(f)$ is the noise power spectrum. It can be proved that, for Gaussian kernels,

$$\lim_{K \rightarrow \infty} \| \hat{S}_{x,\mu}^K(f) - \text{SPW}'_{x,\mu}(f) \| < \epsilon, \quad (5.5)$$

where ϵ is an arbitrarily small real number and $\| \cdot \|$ is the mean square error. Let's now assume that $\phi_F(f; \mu)$ is the probability density function of a discrete random variable f given by [Papoulis 1991]

$$\phi_F(f; \mu) = \sum_{i \in \mathcal{S}} p_i \delta(f - f_i), \quad (5.6)$$

where $p_i = P\{f = f_i\}$ and $\mathcal{S}$ is the frequency support of $\text{SPW}'_{x,\mu}(\nu)$. If ϕ_F is obtained by sampling Φ_F'' with a period Δf , such that

$$\lim_{\Delta f \rightarrow 0} \phi_F(f; \mu) = \Phi_F''(f; \mu), \quad (5.7)$$

Eq. 4.103 of Theorem 4.6 simply becomes

$$\text{SPW}'_{x,\mu}(f) = \int_{-\infty}^{\infty} B(f - f') \phi_F(f'; \mu) df' + 1 \quad (5.8a)$$

$$= \sum_{i \in \mathcal{S}} p_i B(f - f_i) + 1. \quad (5.8b)$$

An approximation of $\Phi_F''(f; \mu)$ can thus be theoretically obtained by the following algorithm for which the kernels have all the same width $\sigma_i = w_b$:

1. Sample the interval domain $\mathcal{S}$ to select K equidistant frequencies f_i with a sampling interval Δf .
2. Estimate the noise spectrum $N(f)$ either by analyzing the SPWVD for distant range gates or by approximating it by a constant value, obtained, for example, with the median value of $\text{SPW}'_{x,\mu}(f)$.
3. Initialize values y_i and create the model:

$$\hat{S}_{x,\mu}^K(f) = \frac{1}{K} \sum_{i=0}^{K-1} y_i B(f - f_i) + N(f), \quad (5.9)$$

4. Computes the values of y_i minimizing the cost function

$$c(y) = \| \hat{S}_{x,\mu}^K(f) - \text{SPW}'_{x,\mu}(f) \| . \quad (5.10)$$

5. The results of the model parameters estimation leads to the values of p_i for each f_i and thus to $\phi_f(f; \mu)$ approximating Φ_f'' .

This algorithm suffers from the intrinsic properties of Gaussian functions, e.g a Gaussian can be expressed as the sum of two other Gaussian. It is therefore difficult to tune in practice. Nevertheless, it has been used in this thesis in order to validate the equations of the main theorems of this chapter by comparing the resulting frequency distributions (and its various statistical moments) with the one computed directly from the radial velocity pulse profile.

Different kernel-based spectral modeling techniques from the literature are particular cases of Eq. 5.4 and have been tested. However, they are not of practical interest for wake vortex characterization.

- A.** $K=1$ with unknown parameters $\{\bar{f}_1, \sigma_1\}$. This case corresponds to the Periodogram ML estimator described in [Frehlich 1994] and partially presented in Appendix E. This is the simplest model and is limited to very short pulses (pulse-dominated regime) or for radial velocity estimation in the background region. In this case, the physical meaning of the parameters are directly related to the statistical moments of $\Phi_f'(f; \mu)$.
- B.** $K=2$ with unknown parameters $\{y_i, \bar{f}_i, \sigma_i\}$. It is an heuristic model suggested by [Douxchamps 2008]. The parameters have no physical meaning and this model is only valid for particular values of the pulse duration. Nevertheless, the kernel associated with the lower radial velocity is more likely to describe the background region and the second one, the vortex region. In the wake vortex regime, this model has to be used with great care after having to validate it on characteristic spectra. If used as a background separation algorithm, the following conditions must be satisfied: $y_1 > y_2$, $|f_{\text{IF}} - f_1| < |f_{\text{IF}} - f_2|$ and $\sigma_1 < \sigma_2$. Examples of good

conditions are given by cases $\tau_p=100$ ns and 200 ns illustrated in Fig. 5.8. This algorithm has been tested for axial WV detection and the results are presented in the next chapter.

- C. K kernels with unknown parameter y_i . The position $\bar{f}_i$ and standard deviation $\sigma_i = w_b$ are fixed. This corresponds to the general model described previously for the estimation of Φ''_F . It is used as a deconvolution algorithm and the number of kernels is relatively high.

5.2 Maximum radial velocity estimation

We will describe here an estimation method for the extremum of the radial velocity profile. It is based on the observation made in [Kopp 2004] that when the spectrum becomes multimodal, the central frequency corresponding to the farthest mode is a good estimation of the exact maximum radial velocity of the vortex core. This can be observed for the $\tau_p=400$ ns case of Fig. 5.8 where the smallest frequency bump is centered around the exact velocity value. We propose here to analyze this further and define the advantages and limits of this technique by using the equations of the average WVD developed in this thesis. The average WVDs are computed by direct application of the Theorem 4.3. For comparison purpose with the precited paper, we will use Δt instead of $\tau_p = 2\sigma_p$, which are related by Eq. 4.21.

Fig. 5.9 represents the WVDs at time t_2 for two pulses with respective Δt of 400 ns and 800 ns. The distributions clearly exhibit two modes indicated by labels L_b for the background region and L_w for the wake vortex region. We observed that when the pulse duration increases, the power of P_b increases at the expense of a decrease of P_w which expresses the fact that the contribution of the background region in the sensing volume of the pulse becomes more important than the contribution of the wake vortex region in the same volume. In this example, as the background radial velocity is $\bar{v}_{r,b} = 0$ m/s, the position of the peak P_b tends to 0. The radial velocity profile used for this simulation is shown in Fig 5.10 by the tick curve. The first-order moment of $W_{x,t_2}(v)$ is also represented for the two Δt . The horizontal lines near the $v_r(z)$ peaks are the estimates of the radial velocity profile extrema. These estimates have been obtained by the following algorithm:

1. Selection of the LOS to analyze in the CFD database of the $v_r(z)$ profile.
2. Computation of $W'_x(\rho, v)$ and its moments $\bar{v}_r(\rho)$ and $\hat{\sigma}_v^2(\rho)$.
3. Estimation of the velocity v_c corresponding to the nearest minimum of $W'_{x,\rho}(v)$ from $\bar{v}_r(\rho)$. This velocity separates the two modes of the distribution.

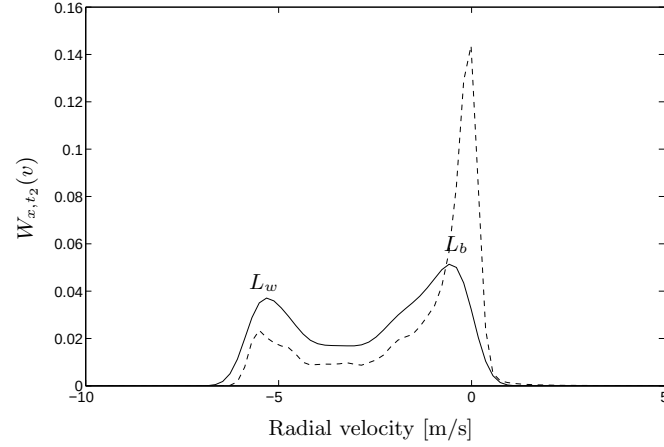


Figure 5.9: Average wVD computed at time t_2 , noted $W_{x,t_2}(v)$, as a function of the radial velocity v for two pulse durations Δt (FWHM): $\Delta t=400$ ns (solid) and $\Delta t=800$ ns (dash). The two maxima of the distribution are noted P_1 for the mode related to the background region and P_2 for the wake vortex region.

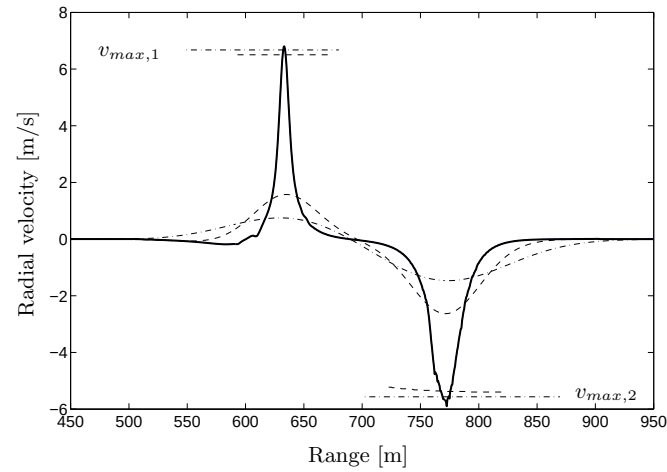


Figure 5.10: Evolution with range ρ of the exact radial velocity profile (solid, tick), the first-order moment of the distributions $W_{x,t_2}(v)$ given in Fig. 5.9 and its extremum values v_1 and v_2 estimated for $\Delta t=400$ ns (dash) and $\Delta t=800$ ns (dash-dot).

4. Modeling of the two parts of the distribution for $v < v_c$ and $v \geq v_c$ and computation of the maximum positions v_i and value $P_i = W'_{x,\rho}(v_i)$ for $i = \{1, 2\}$.

The actual modeling of the two parts of the distribution will depends on its quality, i.e. smoothness and SNR. A more robust technique to separate the wake vortex velocity distribution from the background distribution is presented in the next paragraph. It can be seen that the range interval on which the velocity maximum can be estimated increases with Δt and that its value is constant over this interval. Fig. 5.11 illustrates the ratio P_w/P_b which gives a reliable image of the estimation uncertainty on the parameters $\{P_w, v_w\}$ when the pulse duration increases. In other words, the smaller the ratio the more difficult it is to perform a good spectral modeling to retrieve the information on P_w . We also observe that this ratio presents a maximum at R_1 and R_2 which allows one to correctly assign the maximum values to the appropriate range. Fig. 5.12 represents the

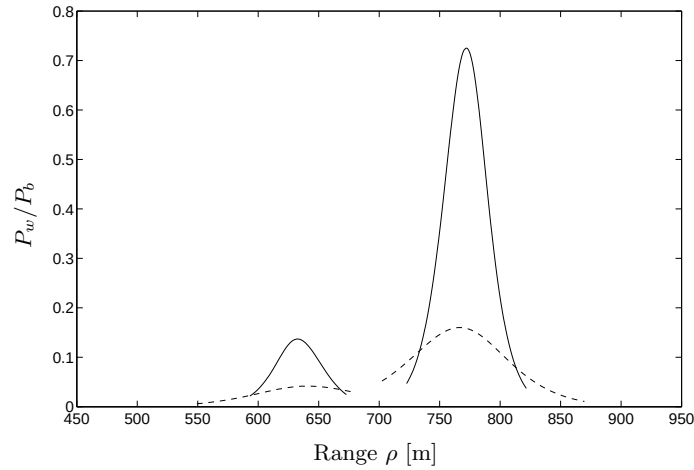


Figure 5.11: Evolution with range of the Ratio P_w/P_b computed on the distributions $W_{x,t_2}(v)$ given in Fig. 5.9 and for two pulse durations (FWHM): $\Delta t=400$ ns (solid) and $\Delta t=800$ ns (dash).

evolution of the intensity P_w and P_b with a pulse duration Δt varying from 300 to 800 ns. The intensities have been normalized by the maximal value of the spectral peak in the background region. The top figure gives the intensities for the two cores and the bottom figure, the ratio P_w/P_b . We observe that when the pulse duration increases it becomes more difficult to retrieve the wake vortex mode from the signal whereas the background mode tends to be dominant.

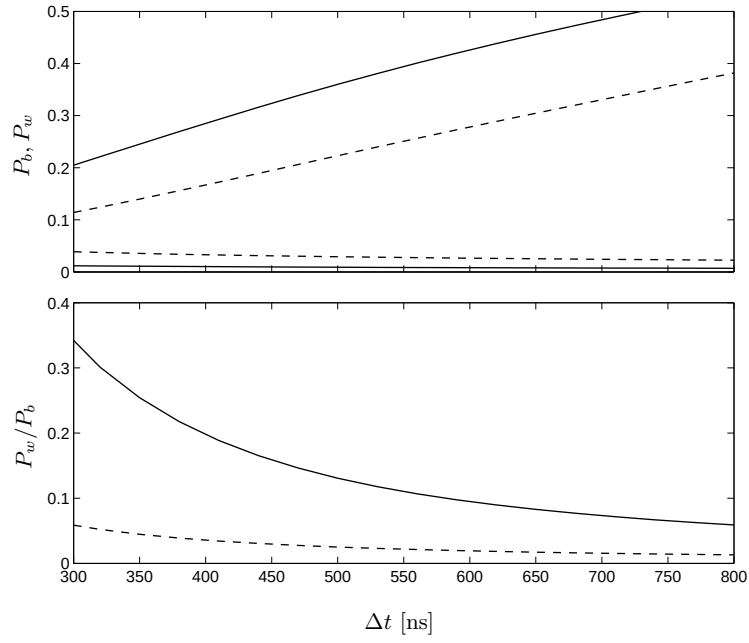


Figure 5.12: Evolution of the intensity of P_w and P_b as well as the ratio P_w/P_b with Δt . The top figure gives the evolution of P_w (lower curve) and P_b (highest curve) at z_1 (solid) and z_2 (dash). The bottom figure gives the ratio P_w/P_b at z_1 (dash) and z_2 (solid).

5.3 An adaptive algorithm based on Cohen's class

In this section, we will describe one of the algorithms developed in this thesis to estimate the radial velocity profile characteristics. It integrates some of the results and observations exposed previously. Before describing it, the following assumptions must be introduced in accordance with the observations made in the last section:

Assumption 5.2 *The accumulation level M is chosen sufficiently high such that the fluctuations outside the signal bandwidth due to measurement noises are low compared to the amplitude of the weakest spectral mode of the wake vortex signal.*

This condition is quite strong but is necessary for more efficient algorithms to be used, especially when estimating the radial velocity profile extrema. When not satisfied, the best estimate for the radial velocity is the first-order moment of the main spectral peak which, as we have seen, tends to estimate the background velocity for high pulse durations. This estimate can be obtained in this case by using a well-known spectral modeling technique based typically on one or two Gaussian kernels (see models A and B of Section 5.2). The algorithm described here is, as a result, only valid when Assumption 5.2 is satisfied.

The main purpose of our algorithm is to estimate the distribution $\Phi_v''(v; \mu)$ for each time μ as well as its first two order moments $\hat{v}_R(\mu)$ and $\hat{\sigma}_R(\mu)$. The extremum of the radial velocity for each vortex core, noted $\hat{v}_{\max,1}$ and $\hat{v}_{\max,2}$, are then computed from the estimated velocity distribution. The main characteristic of this algorithm is to detect the presence of a wake vortex along the line-of-sight and to adjust the spectral model as well as the time-frequency resolution depending on the position of the observation window, i.e. inside or outside the wake vortex region. It is based on the following observations:

1. The signal regime outside the wake vortex region can be considered to be pulse-dominated. As a result, the observation window can be large in time in order to reduce noise level and provides a high frequency resolution. The spectrum is typically Gaussian and the spectral modeling thus uses only one kernel. The range resolution can be low in this region since the scales of variation of the wind profile are large.
2. Inside the wake vortex region, the correlation time of the signal decreases and the observation window $g(t)$ must therefore be adapted since the signal becomes highly non-stationary. The spectrum is complex and more kernels are needed for the modeling.

5.3.1 Algorithm description

Fig. 5.13 details the structure of the algorithm. The accumulated correlogram,

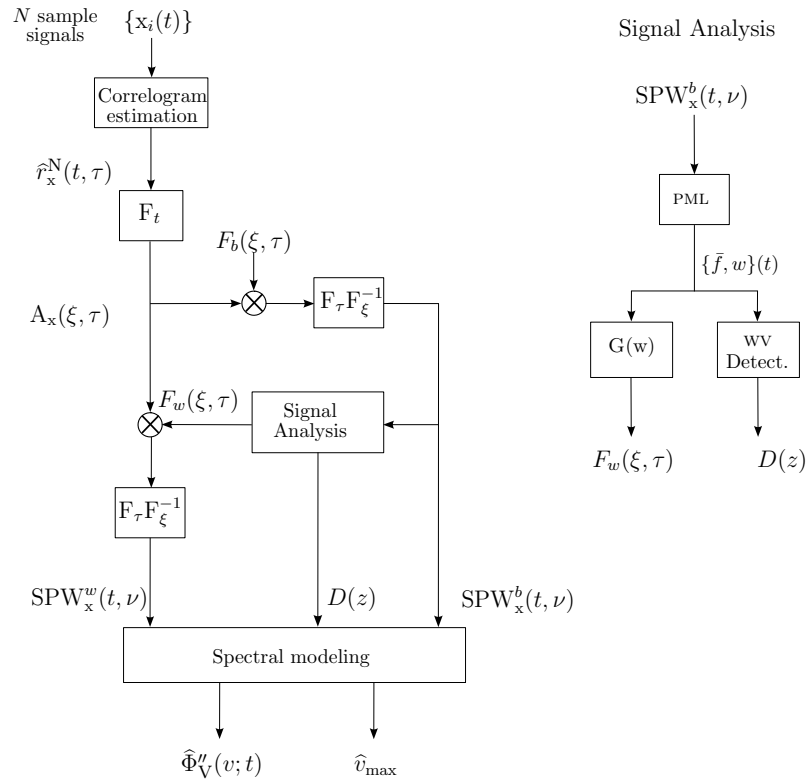


Figure 5.13: Description of the algorithm. The right scheme describes the different steps to determine the velocity distribution as well as the profile velocity extrema. The left scheme detailed the signal analysis process.

noted $\hat{r}_x^N(t, \tau)$ is first computed using Eq. 4.81 from N realizations of the LIDAR signal measured on successive line-of-sights for which the assumption is made that the radial velocity profile is approximatively the same. The ambiguity function $A_x(\xi, \tau)$ is then estimated by applying the Fourier transform on this correlogram along the variable t . A two-dimensional Fourier transform on $A_x(\xi, \tau)$ or a Fourier transform of $\hat{r}_x^N(t, \tau)$ along τ would directly give us the accumulated Wigner-Ville distribution of the signal. As we have seen before, this distribution is not usable directly since it still contains interferences terms that have to be removed by an appropriate smoothing. This filtering is performed in the transformed domain $\{\xi, \tau\}$ by multiplying the ambiguity function by a parameterization function $F(\xi, \tau)$ as given by Eq. A.23. The Smoothed Pseudo Wigner-Ville $\text{SPW}_x(t, \nu)$ is obtained by a two-dimensional Fourier transform on the product $A_x(\xi, \tau)F(\xi, \tau)$. Note that functions F are obtained from $\Pi(t, \nu) = g(t)U(-\nu)$ which is a 2D Gaussian function and are thus defined in this plane by the parameters σ_g and w_u .

We define two parameterization functions F_b and F_w which are chosen to enhance the characteristics of respectively the background and the wake vortex regions. The function F_b produces the distribution $\text{SPW}_x^b(t, \nu)$. It is used to analyze the LIDAR signal and to detect the presence of a wake vortex by computing the second-order spectral moment and the signal correlation time. If a significant decrease in the correlation time is observed a binary function $D(t)$ is set which defines the time interval where a wake vortex profile should be retrieved. It is thus given by

$$D(t) = \begin{cases} 1 & \text{for } \tau \leq \tau_{\min} \\ 0 & \text{otherwise} \end{cases}, \quad (5.11)$$

where $\tau_{\min}$ is the correlation time under which a wake vortex is detected. The function $F_b(\xi, \tau)$ is determined such that $\Pi(t, \nu)$ is given by the Wigner-Ville distribution of the pulse $W_p(t, \nu)$. We thus have that $F_b(\xi, \tau) = A_p(\xi, \tau)$ and $\sigma_g = \sigma_b$. Under this condition the number of independent samples is $M_I = 1$ and the signal envelope fluctuations due to the speckle effect are well attenuated. The parameter σ_g of Π_w is determined depending on the minimum correlation length of the signal whereas w_u is kept equal to w_p . The distributions are then estimated and a spectral modeling is performed on this two distributions in order to estimate the velocity distribution Φ_v'' and the radial velocity profile extrema $v_{\max}$ for all the distances $\rho = ct/2$.

a. Spectral enhancement

Each time a time-frequency distribution is estimated the fluctuations of the noise outside the signal bandwidth are removed for the subsequent spectral modeling step to work efficiently. The measurement noise is assumed shot-noise dominated and therefore white. The average noise spectrum $N(f)$ is therefore

constant with f . Due to the finite accumulation level, the estimated noise spectrum is a fluctuating function with mean value N_0 and standard deviation σ_n . The median value of the time-frequency plane is used as an estimation of N_0 and the standard deviation is computed outside the signal spectrum. The noise is removed from the time-frequency distribution for all μ by the following operation:

$$S'_{x,\mu}(f) = \begin{cases} S_{x,\mu}(f) & |S_{x,\mu}(f) - N_0| \leq \beta \sigma_n \\ 0 & \text{otherwise} \end{cases}, \quad (5.12)$$

where $\beta > 3$. With this techniques spectral modes with $y_i < \beta \sigma_n$ are removed. The main constraint is to preserve the lowest modes of the velocity distribution. The condition on M of Assumption 5.2 can then be written as

$$M \text{ s.t. } \min\{y_i\} > \beta \sigma_n. \quad (5.13)$$

If RIN noise is present at low frequencies is must first be removed by an appropriate low-pass filtering. Fig. 5.14 gives an example of spectral enhancement applied to the distribution $SPW_{x,t_1}(\nu)$ at 10 dB and an accumulation level of 100 signals.

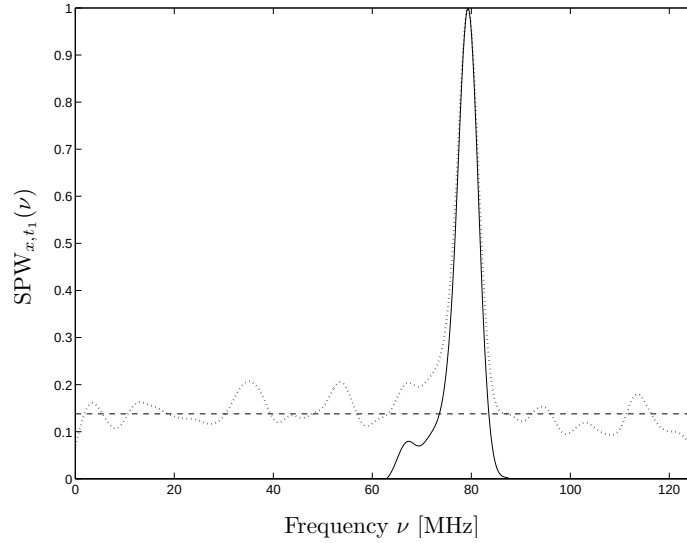


Figure 5.14: Illustration of the spectral enhancement algorithm for a pulse with duration $\Delta t=300$ ns, a SNR before accumulation of 10 dB and an accumulation level of 100 signals: accumulated distribution at t_1 $SPW_{x,t_1}(\nu)$ (dot), its median value (dash), the enhanced distribution (solid) and the theoretical average distribution (dash-dot). The intermediate frequency is 80 MHz.

b. Spectral modeling

The wake vortex detection is performed on the $\text{SPW}_x^b(t, \nu)$. The spectral parameters are estimated using the classical spectral maximum likelihood with unknown variables $\{y_b, \bar{f}_b, \sigma_b\}$. The value $\bar{f}_b$ directly gives the radial velocity profile outside the wake vortex region.

A more complex spectral modeling is performed on $\text{SPW}_x^w(t, \nu)$. This is an adaptive model for which the number of kernels is computed automatically at each time μ by the nearest integer value of the following expression:

$$K(\mu) = \max \left\{ n \left(\frac{m_{2,x}(\mu)}{w_b^2} \right)^{\frac{1}{2}}, K_{\min} \right\}, \quad (5.14)$$

where $m_{2,x}(\mu)$ is the second order moment of $\text{SPW}_{x,\mu}^w(\nu)$ at time μ , w_b^2 is the variance of the smoothing function $B(\nu)$ (see Eq. 4.105), $n \geq 1$ is a parameter determining the sensitivity of the model and $K_{\min}$ is a minimum imposed value. It depends on the spectral width as well as on the accumulation level since the presence of speckle generated more complex spectra. The optimization is thus performed for increasing time with appropriate constraints on each parameters. The parameter initialization for the optimization at time μ is done by using the final value of the parameters obtained at $\mu - d\mu$ where $d\mu$ is the processing time step. If the number of kernels has increased, i.e. $K(\mu) > K(\mu - d\mu)$, new kernels are added with random frequency position in the signal bandwidth. On the other hand, if the number of kernels decreases, only those with the highest energy from the last iteration are conserved. The parameter K is filtered recursively, by an RLS filter, to remove the erratic variations of $m_{2,x}(\mu)$. Fig. 5.15 illustrates the computation of the number of spectral kernels with range for a wake vortex profile with $R_1=700$ m and $R_2=200$ m.

c. Spectral modes separation algorithm

The kernels are then separated into two classes depending on their frequency position $\bar{f}_i$ and amplitude y_i to form a background spectrum $S_b(f; \mu)$ and a wake vortex spectrum $S_w(f; \mu)$. This classification is performed by using an average background spectrum $S_c(f; \mu)$ which is updated at each time μ by a classical recursive least-square filter:

$$S_c(\bar{f}_i; \mu) = \lambda_c S_c(f; \mu - d\mu) + \text{SPW}_{x,\mu}^w(f), \quad (5.15)$$

where λ_c is called the forgetting factor. A kernel i with position $\bar{f}_i$ is contained in the wake vortex class if the following condition is satisfied:

$$S_c(\bar{f}_i; \mu) \leq \beta_c \max\{S_c(f; \mu)\}, \quad (5.16)$$

where β_c is used as a threshold. A value of $\beta_c = e^{-1}$ has shown to produce a good separation between the two classes.

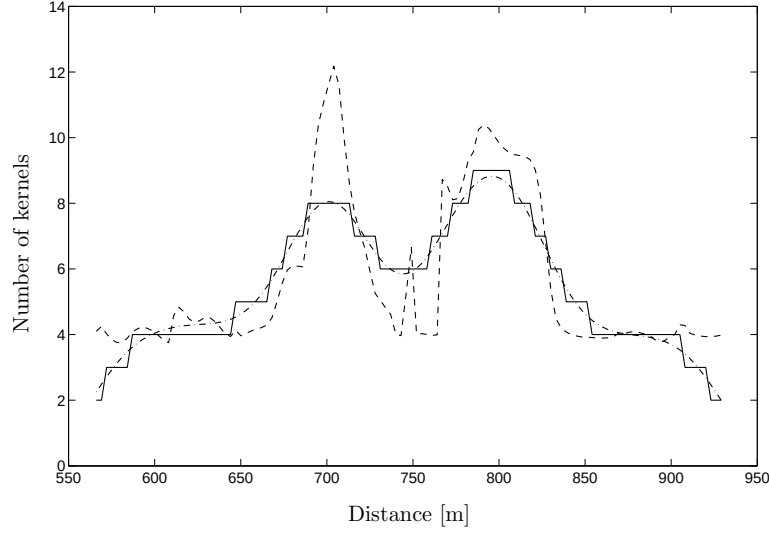


Figure 5.15: Computation of the number of kernels at each time $K(\mu)$ with n : before the filtering (dash), after the filtering (dash-dot) and after the quantization (solid).

d. Computed radial velocity estimates

A number of characteristic radial velocity estimates are computed presenting different sensitivity to noise:

1. The first-order moment of $\text{SPW}_{x,\mu}^w(f)$ noted $\hat{v}_R$. This is the estimates governed by Eq. 4.109. The main interest of this estimate is that it is predictable, i.e. we know that this is a smoothed estimate obtained with the filtering kernel $I_a(z; \mu)$.
2. The two farthest modes of $S_w(f; \mu)$ from each sides of the first-order moment $\text{SPW}_{x,\mu}^w(f)$. The mode with the highest intensity is used as an estimate of the extremum velocity for the nearest vortex core and is denoted $\hat{v}_{\max}$. Their intensity is usually closer to the noise level and these estimates highly depend on the quality of the spectral enhancement method which is in turn improved by an increase of the accumulation level. This estimate only has a meaning at the core position R_1 and R_2 but retrieving the locus of $\hat{v}_{\max}$ also provides us a useful information about the vortex positions.
3. The first-order moment of $S_w(f; \mu)$ noted $\hat{v}_w$. It provides a better estimate of the velocity profile extrema when the previous estimation is impossible

due to noise fluctuations.

4. The first-order moment of $S_b(f; \mu)$ denoted $\hat{v}_b$ which gives the radial velocity of the background region. This estimate has no practical interest other than providing a evaluation of the algorithm performance.

5.3.2 Results and discussion

This estimation algorithm has been tested on simulated data using the LIDAR simulation program detailed in Chapter 3 and with two analytical wake vortex velocity profiles obtained from a Burnham-Hallock model. The estimation statistics has been computed by Monte-Carlo simulation with $N=200$ realizations per test case and a fixed accumulation level M . For each test, M signals $x_i(t)$ are randomly chosen from a set of 10 000 signals and are processed by the algorithm represented in Fig. 5.13. The results below gives the average velocity estimates as well as their standard deviation for a SNR before accumulation of 10 dB. The range resolution, defined by the time interval between two estimates, has been fixed to 24 m outside the wake vortex region and 3 m inside.

Fig. 5.16 illustrates the separation algorithm. All the previous estimates have been retrieved. The velocity distribution $\Phi_V''(v; \mu)$ is then obtained from $\text{SPW}_{x,\mu}^w(f)$ by the use of the deconvolution algorithm described in Section 5.1.2.

Fig. 5.17 gives the results obtained for a 300 ns (FWHM) pulse and an accumulation level of 100 signals. We first observe that all the estimates have a spatial scale of variation governed by the pulse duration and the smoothing function Π_w (see Eq. 4.96b and Theorem 4.7). This is in particular true for the first-order moment $\hat{v}_R$ which only provides a filtered radial velocity profile estimate which has a variance increasing with the pulse duration. It can also be seen that the estimation variance is higher for the estimates $\hat{v}_{\max}$ and $\hat{v}_w$ than the background velocity estimate $\hat{v}_b$ and the first order moment $\hat{v}_R$. This is due to the fact that they have been retrieved from the wake vortex spectrum which is more sensitive to the detection noise and the amplitude fluctuations caused by the speckle effect.

Fig. 5.18 represents the average estimates with a higher pulse duration of $\Delta t=400$ ns and for two velocity profiles. A lower accumulation level of 25 signals has also been chosen. The associated standard deviations are given in Fig. 5.19. The estimation bias of the extremum velocity profile around distances R_1 and R_2 is on the order of a few meters per second and is higher for the velocity profile, at the vortex centers, which are narrow compared to the pulse duration. This is an important observation that should be taken into account for any wake vortex circulation estimators based on the velocity profile maxima as in [Kopp 2004]. Moreover, the estimator standard deviation is also of the same order and also increases with narrower velocity profile (see top figure of

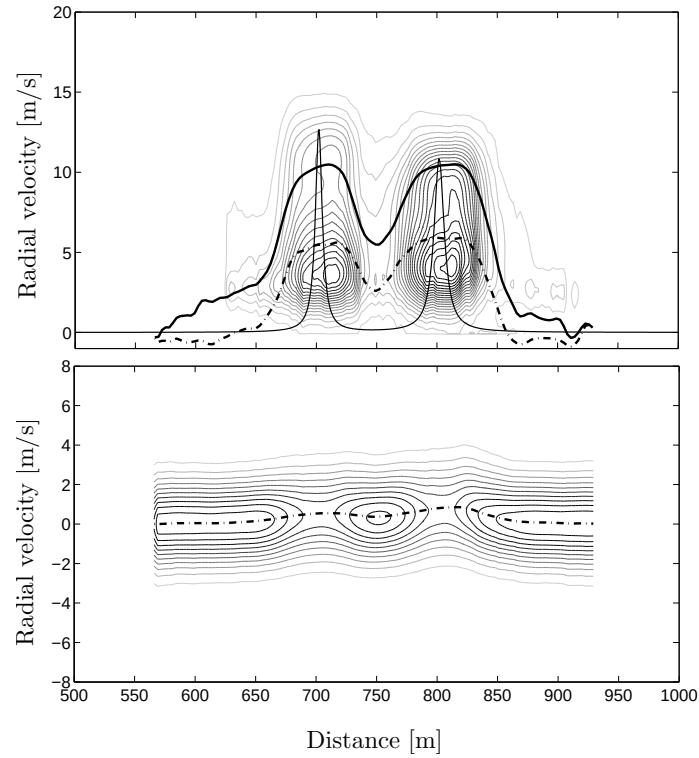


Figure 5.16: Illustration of the spectral estimation technique for a pulse duration of $\Delta t=300\text{ns}$. The averaging is computed on 10 000 signals and an accumulation level of 100. The top figure gives the wake vortex distribution $S_w(\rho, v)$ with the exact radial velocity profile v_R (solid, thin) as well as the averaged estimates $\hat{v}_{\max}$ (solid, thick) and $\hat{v}_w$ (dash-dot, tick). The bottom figure gives the accumulated background distribution $S_b(\rho, v)$ along with the estimate $\hat{v}_b$.

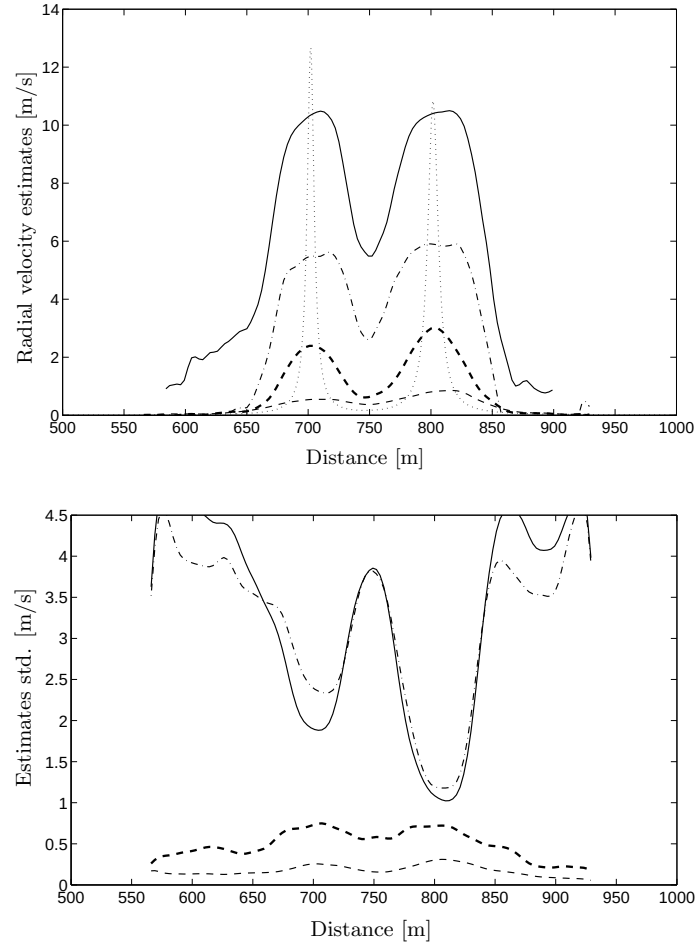


Figure 5.17: Average estimates (top figure) and their standard deviation (bottom figure) as a function of range for a typical velocity profile and a pulse duration (FWHM) of 300 ns. The accumulation level has been fixed to $M=100$; ideal radial velocity (dot), first-order moment $\hat{v}_R$ (dash,tick), velocity extrema $\hat{v}_{\max}$ (solid), background velocity $\hat{v}_b$ (dash,thin) and first-order moment of the wake vortex distribution $\hat{v}_w$ (dash-dot).

Fig. 5.19) and is naturally more important than for the first case in Fig. 5.17 with $N=100$.

The computational time of the algorithm presented in this section mainly depends on the accumulation level, the time-frequency resolution, the spatial sampling in the background and the wake-vortex regions and the number of spectral kernels. The computation on one line-of-sight may take several minutes on an Intel Centrino Core 2 Duo T9300/2.5GHz with 2 Go of RAM.

5.4 Conclusions

The theoretical foundation, exposed above, has been proven to be an interesting source of development for wake vortex spectral modeling. The average LIDAR spectrogram and SPWVD have shown to be obtained by the space-velocity convolution between the actual radial velocity profile along a given line-of-sight and a space-velocity function depending on both the pulse WVD and the analyzing window (see Theorem 4.7). Not only this knowledge tells us how to deconvolve the time-frequency plane, i.e. with which kernel, but it also informs us on what information is the most likely to disappear as a result of this smoothing. This latter issue is actually an extension of the concepts of atmospheric and pulse-dominated regimes proposed by Frehlich in [Frehlich 1997]. The pulse-dominated regime has therefore been introduced to traduce the oversmoothing of the wind statistics by the pulse power spectrum in such a way that its statistical moments become very difficult to estimate. The same phenomenon appears when sensing wake vortices. In this case, the weighted velocity distribution, defined in Theorem 4.3, is multimodal and contains, for example, a clearly distinct mode at the location of the maximum radial velocity, related to the maximum tangential velocity of the vortex field. If the pulse WVD, the time-frequency analyzing function or both are too large compared to the fluctuations of the mean radial velocity, this modality is lost. To characterize this effect, we have therefore introduced the wake-vortex regime.

Based on this analysis, an adaptive estimation algorithm has been developed which uses a spectral model with a range-varying number of kernels. The model order depends on the regime and therefore introduce more kernels in the atmospheric or wake vortex regimes than it does in the pulse-dominated one. One of the estimates extracted from this model is the maximum radial velocity which can be used for vortex strength estimation. This adaptive processing thus allows to better describe the radial velocity distribution, in particular when it is the less altered.

The average LIDAR Cohen's class given by Theorem 4.5 suggests to exploit the time-frequency correlation imposed by the pulse duration and the analyzing window, and therefore to model the wake vortex signal directly in the time-frequency domain. This two-dimensional modeling is more likely to provide

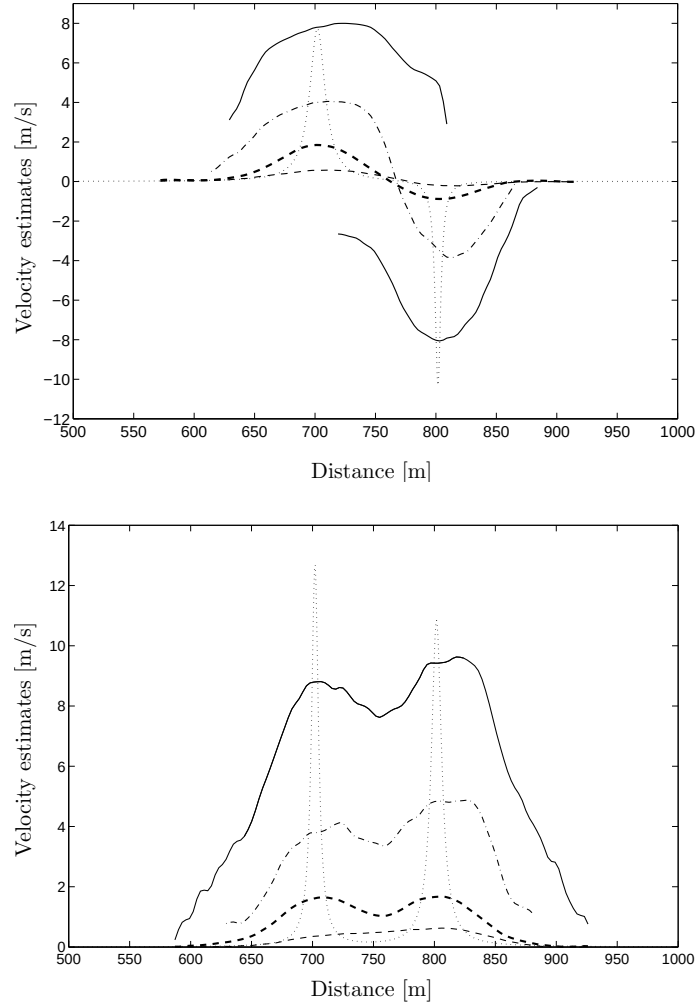


Figure 5.18: Mean radial velocity estimates with range for two typical profiles obtained with a pulse duration (FWHM) of 400 ns, a SNR of 10 dB and an accumulation level of 25 signals: ideal radial velocity (dot), first-order moment $\hat{v}_R$ (dash,tick), velocity extrema $\hat{v}_{\max}$ (solid), background velocity $\hat{v}_b$ (dash,thin) and first-order moment of the wake vortex distribution $\hat{v}_w$ (dash-dot).

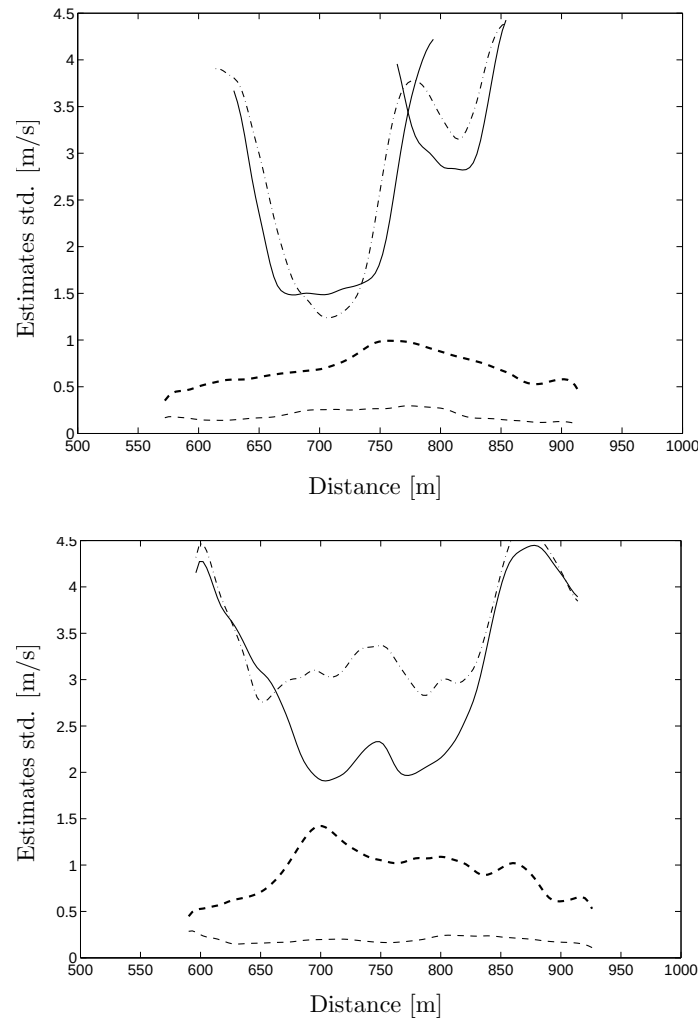


Figure 5.19: Standard deviation with range. The simulation parameters are similar to those used for Fig. 5.18.

better estimates for high pulse accumulation regimes than the classical spectral model defined at each time instant. Nevertheless, its robustness to speckle and noise should be assessed. Moreover, the robustness of the estimation algorithm of the radial velocity maximum to measurement conditions must also be performed in order to provide an evaluation of their estimation variance, a quantity necessary for the validation of the wake-vortex circulation estimator based on it.

Wake vortex detection strategies

UNTIL now, we have considered the problem of estimating the velocity statistics along an isolated line-of-sight from signal realizations which were supposed to contain the same mean radial velocity profile. This was performed by estimating, through an accumulation process, either the covariance or one of the time-frequency distributions of the Cohen's class, e.g the spectrogram or the smoothed pseudo Wigner-Ville distribution.

In this chapter, we will describe how to reconstruct the radial velocity statistics on a spatial map representing the sensed volume and mainly defined by the scanning pattern and the pulse repetition frequency (PRF). The accumulation, in this case, is therefore realized on signals coming from different line-of-sights based on correlated, but not identical, velocity profiles. The reconstructed maps are eventually used for wake vortex detection and characterization. This is the purpose of the next chapter.

In the first section, we expose two reconstruction algorithms that can be used for transverse wake vortex detection. The evolution of the mean radial velocity error in the vicinity of the vortex core is also analyzed. Section 6.3 focuses on the axial configuration and exposes the field test results obtained with the FIDELIO system. A number of simulation results are also presented to discuss the use of the bi-Gaussian model for axial detection.

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6.1 Introduction

A spatial reconstruction algorithm is based on a particular accumulation technique which spatially aggregates the time-frequency information obtained from a number of adjacent line-of-sights. This process is necessary to produce more reliable estimates of the velocity statistics by reducing the effects of both the speckle and the photodetection noise.

The spatial resolution of the reconstructed maps mainly depends on the scanning angular resolutions and the pulse duration. It is also determined by the size of the spatial region-of-interest (ROI) used for the accumulation and defined around each spatial point where we want to estimate the velocity statistics. Moreover, producing velocity statistics with a small estimation variance usually necessitates to accumulate on large spatial regions which subsequently decreases the spatial resolution. A trade-off has therefore to be found between the spatial resolution and the estimation variance of the reconstructed maps. This is therefore especially important when sensing wake vortices, where large radial velocity fluctuation may be observed. The problem of pulse duration has been already exposed in the previous chapters.

6.1.1 Reconstruction principle and formalism

The same quantities as those introduced in Section 4.1.2 for the estimation principle, are adopted here. Hence, the spatial extend of the pulse is $\Delta r = c\Delta t/2$ where Δt is the FWHM pulse duration. The signals are analyzed with an observation window with a length T [s] in time which defines the range gate $\Delta p = cT/2$. Note that the time t appearing here is the relative time defined for each LOS such that $t=0$ is when the maximum of the pulse power profile is at range $R=0$, corresponding to the transmitter lens position.

We distinguish the processing map, denoted $\mathcal{K}$, to the results representation map, denoted $\mathcal{P}$. The first one contains the *raw* time-frequency information I_k at each spatial point $\mathbf{k}$ obtained from each individual LOS. The second one is the result of the accumulation process and is defined by the set of spatial points $\mathbf{p}$. The range interval ΔR_k between two adjacent points $\mathbf{k}$ along a given LOS is determined by the time interval T_k between two successive positions of the analyzing window so that we have $\Delta R_k = cT_k/2$. For a time sampling frequency F_s [Hz], its minimum value is given by $c/2F_s$ corresponding to the pulse propagation distance during the sampling period $T_s = 1/F_s$. The resolution in $\mathcal{P}$ is typically lower than that of $\mathcal{K}$ and is determined by a classical compromise between the minimum correlation needed for the subsequent processing between two adjacent points $\mathbf{p}$ and the computational cost. Note that $\mathcal{P}$ is not necessarily a subset of $\mathcal{K}$, i.e. the points $\mathbf{p}$ are not chosen among the $\mathbf{k}$ points. We further define the accumulation set $\mathcal{C}$, associated to a given point $\mathbf{p}$, as the set of points $\mathbf{k}$ lying in a defined close spatial region around $\mathbf{p}$.

In the following accumulation methods, each measured signal is first converted to an analytical signal $x_i(t)$ using the Hilbert transform where the subscript i corresponds to the i^{th} LOS of the current scan. These signals are then processed to estimate the velocity statistics from the spectrum $S_{x,p}(\nu)$ at each point $\mathbf{p}$ obtained by the following steps:

- I. Computation of the information vectors I_k , which are either the correlogram $r_{x,i}(t, \tau)$ or the time-frequency distribution $C_{x,i}(t, \nu)$, computed for all points $\mathbf{k}$ of the LOS i at instants $t_k = 2R_k/c$.
- II. Accumulation of all the I_k from the $\mathbf{k}$ -points lying in the predefined accumulation region $\mathcal{C}$ around each point $\mathbf{p}$ of the representation map $\mathcal{P}$ to retrieve the vectors I_p .
- III. For all $\mathbf{p}$, estimation of $S_{x,p}(\nu)$ from the corresponding I_p .

Once the spectra $S_{x,p}(\nu)$ has been estimated for each grid points of the map $\mathcal{P}$, a number of estimates can be represented such as the statistical moments of the weighted radial velocity distribution Φ_V'' (see Section 4.4.5), the maximum radial velocity or the various estimates from the background and wake-vortex distributions produced by the algorithm presented in Section 5.3. We will here concentrate on the two first-order statistical moments of Φ_V'' . The first one is a filtered version of the exact radial velocity and is important for wake-vortex circulation estimation whereas the second-order moment has shown in the previous chapter to be a good parameter for wake-vortex detection.

6.1.2 Basic assumptions

We will suppose that the scanner has a perfectly constant angular velocity, noted ω [rad/s], and that the exact azimuth and elevation angles of the scanner can be measured each time a LASER pulse is sent. The effects due to the scanning on the LASER beam propagation, e.g. misalignment due to the elapsed time between the pulse emission and the corresponding signal measurement, are neglected so that the reconstruction process can be modelled exclusively by mechanical and geometrical quantities.

6.2 Transverse detection algorithms

We will first focus on wake vortex detection with a ground-based system scanning in a plane perpendicular to the runways. Two reconstruction algorithms are proposed and are based on either a sectorial or a kernel-based spatial accumulation.

6.2.1 The reconstruction techniques

For the transverse detection, the angular resolution $d\theta$ is simply given by $d\theta = \omega \text{ PRF}^{-1}$. The processing map is defined by the elevation angle θ_k and the range R_k from the telescope. The range resolution ΔR_k has been already defined. The elevation resolution $\Delta\theta_k$ is simply given by $d\theta$ when all measured signals are analyzed. This may not be the case when the PRF is too high in comparison to the acquisition rate or the real-time processing duration. Similarly, the points $\mathbf{p}$ of $\mathcal{P}$ has polar coordinates $\{R_p, \theta_p\}$ and resolutions $\{\Delta R_p, \Delta\theta_p\}$.

Let's now discuss the two reconstruction methods. Table 6.1 gives their respective range and angular resolutions in $\mathcal{P}$ and Fig. 6.1 illustrates the spatial accumulation principle.

Method	ΔR	$\Delta l_{\perp}$	$\Delta\theta$
Angular	Δp	$\approx 2 \sigma_{\theta} R$	$2 \sigma_{\theta}$
Kernel-based	$2 \rho(R) + \Delta p$	$2 \rho(R)$	$\approx 2 \rho(R)/R$

Table 6.1: Longitudinal range resolution ΔR , transversal range resolution $\Delta l_{\perp}$ and angular resolution $\Delta\theta$ for an accumulation level M . The observation window of duration T and the corresponding range gate is Δp . The kernel radius at point $\mathbf{p}$ is $\rho(R_p)$. The angular weighting function for the angular accumulation is a Gaussian function with standard deviation σ_{θ} [rad].

6.2.2 Angular accumulation

The complete algorithm is given in Table 6.2. The accumulation for each point $\mathbf{p}$ is performed on M points $\mathbf{k}$ from the adjacent LOS for which $R_k = R_p$ and $|\theta_k - \theta_p| \leq Md\theta/2$. As illustrated by the left figure of Fig. 6.1, it gives a set $\mathcal{C}$ with a sectorial geometry having an arc of length at R_p given by $R_p \Delta\theta$ and a depth Δp .

We introduce an angular weighting function $A(|\theta - \theta_p|)$ which gives a more important consideration to spatially closer $\mathbf{k}$ -points to the current grid point $\mathbf{p}$. A Gaussian function with a standard deviation σ_{θ} [rad] has been chosen here. An equal longitudinal and transverse resolution is obtained by choosing $\sigma_{\theta}(R) \approx 2R\Delta p^{-1}$. It should also be possible to define a range-dependent angular resolution such that the transversal resolution would stay constant with range. The problem in this case would be that the quality of the information I_p would decrease with range due to an increase of the average distance between the $\mathbf{k}$ points in $\mathcal{C}$. We also have to take into account the fact that the quality of the I_k already decreases due to the range evolution of the SNR. The non-

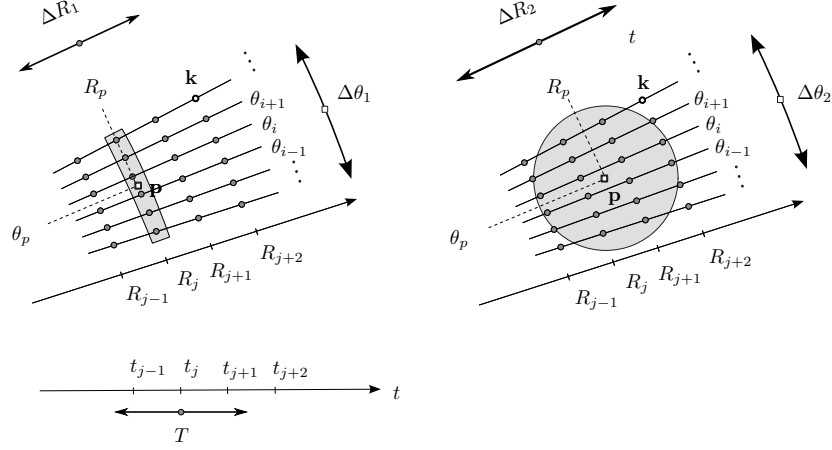


Figure 6.1: Principle of the angular (left) and the kernel-based accumulation (right). The points $\mathbf{k}$ in the processing map $\mathcal{K}$ are represented by the circles and the LOS by the lines. The dark spatial regions represents the accumulation set $\mathcal{C}$ of the points $\mathbf{p}$ represented by a square. The range and angular resolution are respectively denoted ΔR and $\Delta \theta$. The observation window length is T .

-
- I. For each LOS i with elevation θ_i ,
 $\forall \mathbf{k} \in \mathcal{K} \text{ s.t. } \theta_k = \theta_i$,
 Estimates I_k from signal $s_i(t_k)$ where $t_k = 2R_k/c$
 - II. Define the angular weighting function $A(|\theta - \theta_p|)$
 $\forall \mathbf{p} \in \mathcal{P}$,
 Define $\mathcal{C}$ as a subset of $\mathcal{K}$ s.t. $R_k = R_p$ and $|\theta_k - \theta_p| \leq Md\theta/2$
 $I_p \leftarrow 0$;
 $\forall \mathbf{k} \in \mathcal{C}$
 $I_p \leftarrow (I_p + A(|\theta_k - \theta_p|) I_k)$;
 - III. $\forall \mathbf{p} \in \mathcal{P}$,
 Computes $S_{x,p}(\nu)$ from I_p
-

Table 6.2: Angular accumulation algorithm.

uniform spatial resolution is an important issue for the wake vortex detection and characterization algorithms because it will result in a range-dependent estimation variance, on the vortex strength for instance.

This accumulation strategy has the major advantage to be easily implemented in real-time because the computation of the vectors I_p can be realized recursively as new signals are acquired. All the estimates such as the air velocity statistical moments obtained from the $S_{x,p}(\nu)$ for $\mathbf{p}$ points with the same elevation θ_p are computed concurrently and are therefore produced with a constant rate.

6.2.3 Kernel-based spatial accumulation

In the kernel-based accumulation method, the objective is to find a spatial region-of-interest which provides the highest accumulation level while preserving a given spatial resolution. We will here consider the case of a circular region $\mathcal{C}$ as illustrated by the right figure of Fig. 6.1.

The algorithm is given at Table 6.3. We use a Gaussian spatial weighting function $W(\|\mathbf{k} - \mathbf{p}\|)$ with a range-dependent radius $\rho(R)$. The difference

-
- | | |
|------|---|
| I. | For each LOS i with elevation θ_i ,
$\forall \mathbf{k} \in \mathcal{K}$ s.t. $\theta_k = \theta_i$,
Estimates I_k from signal $s_i(t_k)$ where $t_k = 2R_k/c$ |
| II. | Define the spatial weighting function $W(\ \mathbf{k} - \mathbf{p}\)$
$\forall \mathbf{p} \in \mathcal{P}$,
Define $\mathcal{C}$ as a subset of $\mathcal{K}$ s.t. $\ \mathbf{k} - \mathbf{p}\ \leq \alpha_r \rho(R_p)$
$n \leftarrow 0$;
$I_p \leftarrow 0$;
$\forall \mathbf{k} \in \mathcal{C}$
$I_p \leftarrow (I_p + W(n) I_k)$;
$n \leftarrow n + 1$; |
| III. | $\forall \mathbf{p} \in \mathcal{P}$,
Computes $S_{x,p}(\nu)$ from I_p |
-

Table 6.3: Kernel-based accumulation algorithm for a circular region at point $\mathbf{p}$ of radius $\rho(R)$. The parameter α_r is set to 3.

in resolution between the longitudinal and the transversal directions is given by Δp . As a result, the actual spatial filtering introduced by the combination of both the measurement and the accumulation process is done by an elliptic kernel. Fig. 6.2 represents the evolution of the accumulation level with range

for different kernel radius. It decreases with range as $1/R^2$. We should note that accumulating M signal consecutive segments I_k along the same LOS or for a given range from adjacent LOSes does not lead to the same estimation quality. Indeed, the manifestations of the speckle effect are correlated in the first case whereas they are independent in the second case.

When the accumulation level is high, computing the subset $\mathcal{C}$ may be time-consuming and may compromise its real-time implementation. What has been done in practice is to compute two reconstruction tables. A first one that assigns for every point $\mathbf{p}$, the corresponding points $\mathbf{k}$, and a second one which does the opposite, i.e. tells to which points $\mathbf{p}$ a point $\mathbf{k}$ belongs to. This last table greatly helps when efficient processing has to be done since the I_k does not have to be stored in memory.

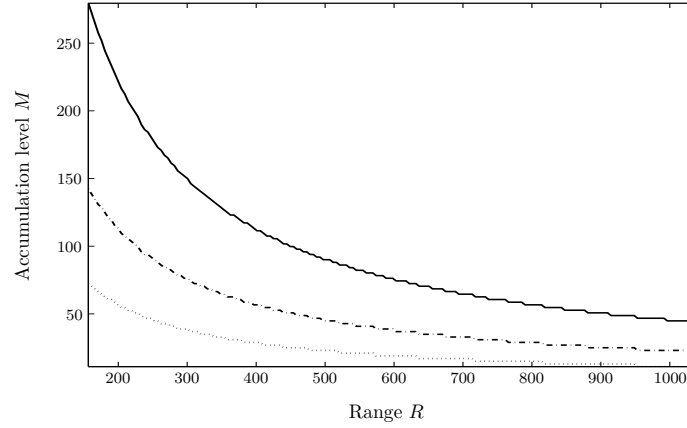


Figure 6.2: Evolution of the accumulation level with range for different, constant with range, circular kernel radius: $\rho=1$ m (dot), $\rho=2$ m (dash-dot) and $\rho=4$ m (solid).

6.2.4 Results and discussion

Only results obtained with the kernel-based method will be exposed here. They have been published in [Brousmiche 2009b]. The vectors I_k and I_p are either the signal correlogram or the spectrogram. The first-order moment is computed for the spectrogram case and provides an estimate of the mean radial velocity, noted $\hat{v}_R$. However, when the correlogram is used, the spectrum $S_{x,p}(\nu)$ is computed with a parametric estimation technique in which the signals are supposed to have been generated by an auto-regressive (AR) process. This method is exposed in Section E.2. The mean radial velocity is estimated in this case via the computation of the dominant pole angle. The estimation error $\sigma_v^2(\mathbf{p})$ is computed in both cases by the mean square error between the

estimated radial velocity $\tilde{v}_r(\mathbf{p})$ and the exact value of the first-order moment $\hat{v}_r(\mathbf{p})$ derived in the previous chapter (see Eq. 4.110):

$$\sigma_v^2(\mathbf{p}) = |\tilde{v}_r(\mathbf{p}) - \hat{v}_r(\mathbf{p})|^2. \quad (6.1)$$

We are interested here in comparing the performance of the two methods in the vicinity of a wake vortex core. The LES simulation of wake vortices in ground effect with no wind has been used at a dimensionless time of $\mathcal{T} = 2.5$ defined by

$$\mathcal{T} = \frac{V_0}{b_0} t, \quad (6.2)$$

where t is the physical time, b_0 [m] is the distance between the pair of vortices and V_0 [m/s] is the descent velocity OGE. This value of $\mathcal{T}$ corresponds to a relative spacing of $3 b_0$, after rebound. The initial vortex parameters are $\Gamma_0=400$ m²/s and $b_0=50$ m. In order to have the same spatial resolution at each point $\mathbf{p}$ a constant radius ρ with distance R has been chosen. The vortex pair is situated around 400 m from the LIDAR. At this distance, a radius ρ of 1, 2 and 4 m corresponds to an accumulation level per velocity estimate of respectively 20, 60 and 120 k-points. Using a fixed radius implies that the variance of the spectral density estimates increases with range due to the decreasing accumulation level and tends to make worse the effect of the range-decreasing SNR.

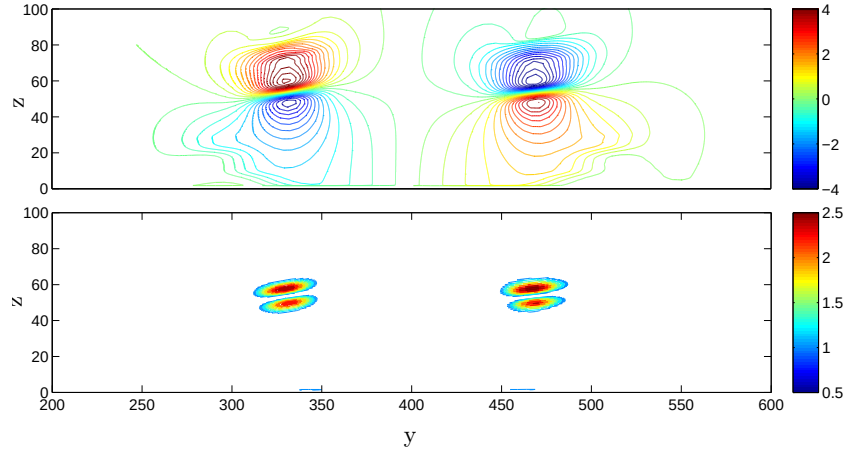


Figure 6.3: Influence of the accumulation process on the estimation error: estimated radial velocity map $\tilde{v}_r$ (top) and standard deviation σ_v [m/s] (bottom).

Fig. 6.3 gives the spatial variation of the velocity error by comparing the estimates with the exact radial velocity field. We observe that the estimation error increases inside the vortex where the velocity gradients along the propagation

direction are the most important. High velocity gradients decrease the temporal coherence of the signal and consequently induce a broadening of the signal spectrum with a decrease of its maximum value. Moreover, the accumulation process as well as the pulse smoothing effect tend to promote narrow spectra computed outside the vortex. This effect, observed in the previous chapter, must be taken into account since it distorts the retrieved vortex shape and may increase the error made on the circulation if it is not taken into account.

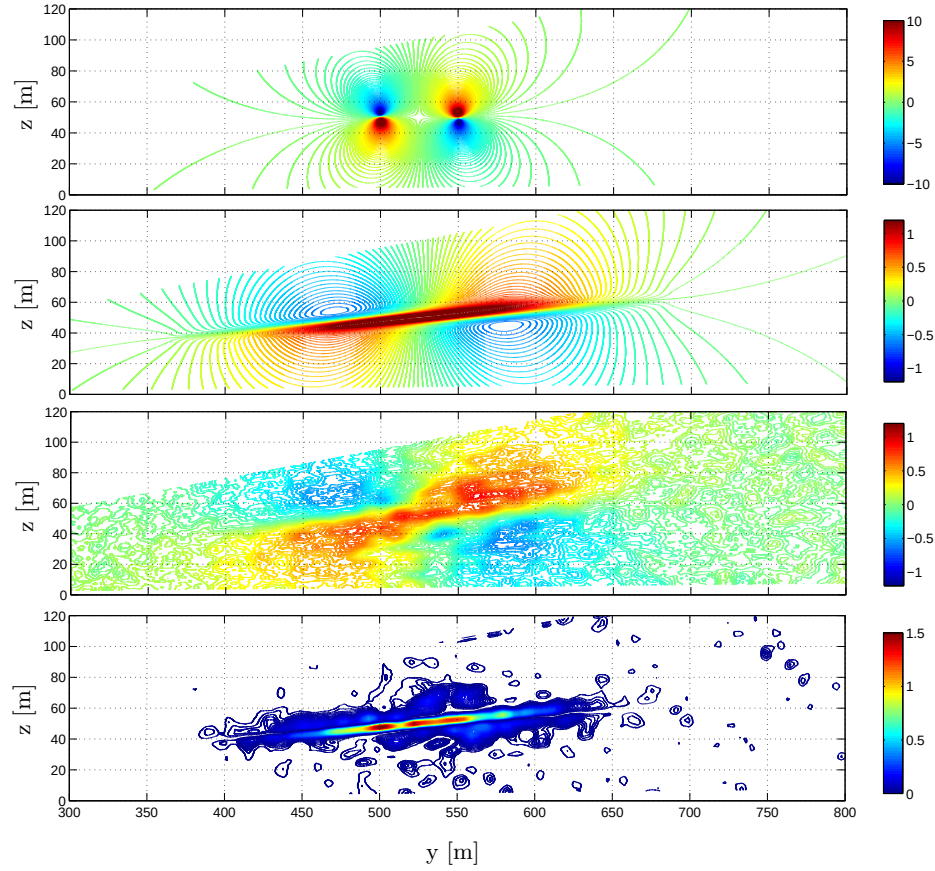


Figure 6.4: Example of estimated radial velocity map for a LES of a pair of wake vortices in weak turbulence: exact radial velocity map [m/s] (top), $\hat{v}_R(\mathbf{p})$ [m/s] (second), $\tilde{v}_R(\mathbf{p})$ [m/s] (third) and $\sigma_v^2(\mathbf{p})$ [m/s] (bottom). A kernel-based accumulation is used with a kernel radius of $\rho=2$ m. The pulse duration is 480 ns ($\Delta t=400$ ns). The initial parameters of the wake vortex pair are $\Gamma_0=400$ m²/s and $b_0=50$ m.

The processing results obtained for a wake vortex pair in weak atmospheric

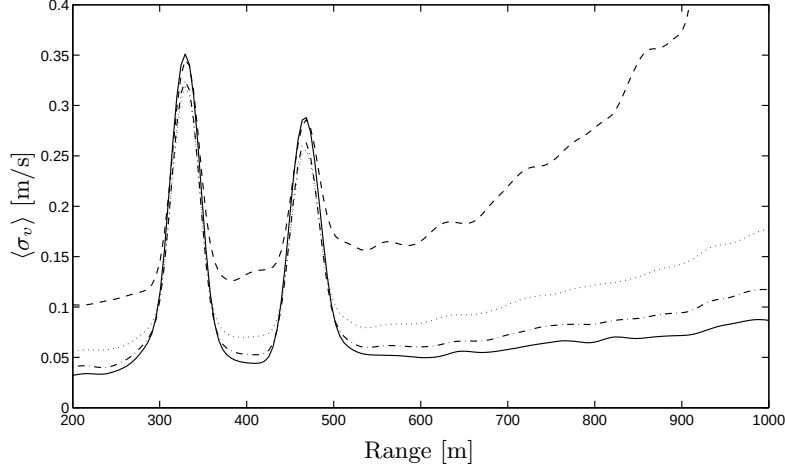


Figure 6.5: Influence of the pulse energy on σ_v . $E_t = 50 \mu J$ (dash); $E_t = 150 \mu J$ (dot-dot); $E_t = 300 \mu J$ (dash-dot) and $E_t = 600 \mu J$ (solid) obtained with the correlogram accumulation method ($\rho = 2$ m). The pulse duration is 200 ns. The refractive index is $C_n^2 = 10^{-13} \text{ m}^{-2/3}$.

turbulence with initial parameters $\Gamma_0 = 400 \text{ m}^2/\text{s}$ and $b_0 = 50 \text{ m}$ is given in Fig. 6.4. The signal segments have a fixed length of 64 samples which corresponds to a spatial resolution of 38.4 m. The pulse duration is $\tau = 480 \text{ ns}$ and the radius of the circular kernel is $\rho = 2 \text{ m}$. This case is important since it shows the filtering induced by a pulse which has a spatial extend larger than the distance between the vortices. Fig. 6.5. shows the standard deviation of the radial velocity estimates averaged over all the line-of sight, noted $\langle \sigma_v \rangle$, for different values of the pulse energy U_L . It has been obtained in the early case with a pulse duration of 200 ns and strong turbulence $C_n^2 = 10^{-13} \text{ m}^{-2/3}$. The algorithm used is the correlogram-accumulation method. We observe that reducing the pulse energy has no major influence on the estimation variance inside the vortices whereas it monotonically increases with range. It must be noted that, as pointed by Eq. 2.102, for the same telescope configuration and beam sizes, the SNR is identical provided that we keep the ratio $\beta U_L / B_w$ constant. Hence, decreasing the pulse energy by a factor of 2, for example, gives the same estimation variance than decreasing by the same factor the aerosol backscatter coefficient. The comparison between the two accumulation strategies are given in Fig. 6.6. The standard deviation of the velocity estimate are given after having removed the averaged evolution due to SNR variation. The correlogram accumulation gives better results inside the vortices at 200 ns and 400 ns. The error profiles are

both narrower (spatial resolution) and smaller (velocity resolution).

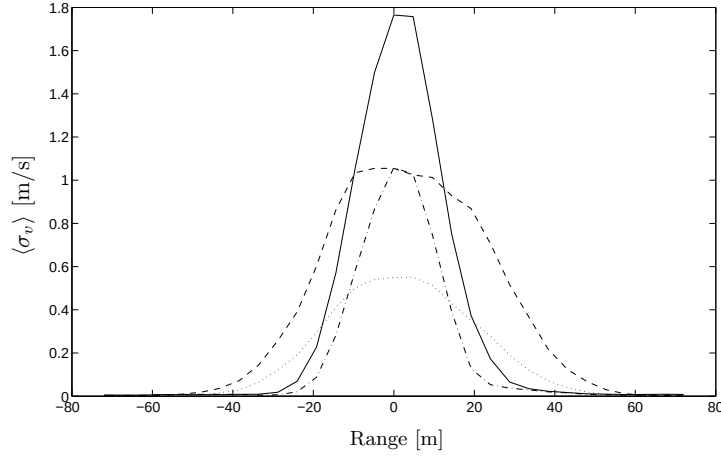


Figure 6.6: Comparison of the correlogram (CA) and spectrogram (PA) accumulation methods: PA, 200ns (solid); CA, 200ns (dash-dot); PA, 400ns (dash); CA, 400ns (dot). The refractive index is $C_n^2 = 10^{-15} \text{ m}^{-2/3}$. The range is given here such that $R = 0 \text{ m}$ at the vortex center.

6.3 An axial detection algorithm

6.3.1 Description of the LIDAR simulations

The detection principle is shown in Fig. 6.7. The LIDAR field of view has a 3° angle of variation in elevation and 12° in azimuth. The scanner mean elevation is pointed toward the 3° glideslope. The detection ranges are comprised between a minimum of 800 m and an expected maximum of 2.4 km. At these two distances, the observation regions in the $\{y, z\}$ plane are bounded by a rectangle with respective size $168 \times 42 \text{ m}$ and $500 \times 125 \text{ m}$. The scanning limits are represented in the figure by the pairs of lines originating from the LIDAR on the left. In this project, the LIDAR simulations have been carried out by ONERA with the fluid simulation databases provided by immc and described in [Bricteux 2008].

The LES simulations consists of a wake vortices pair in a weak atmospheric turbulence. They have mainly been obtained in three steps. The first one is the generation of homogeneous and isotropic turbulence (HIT) itself by using a parallel three-dimensional pseudo-spectral code in LES mode with a grid of 256^3 . The simulation of the wake vortex system requires then to choose a distribution

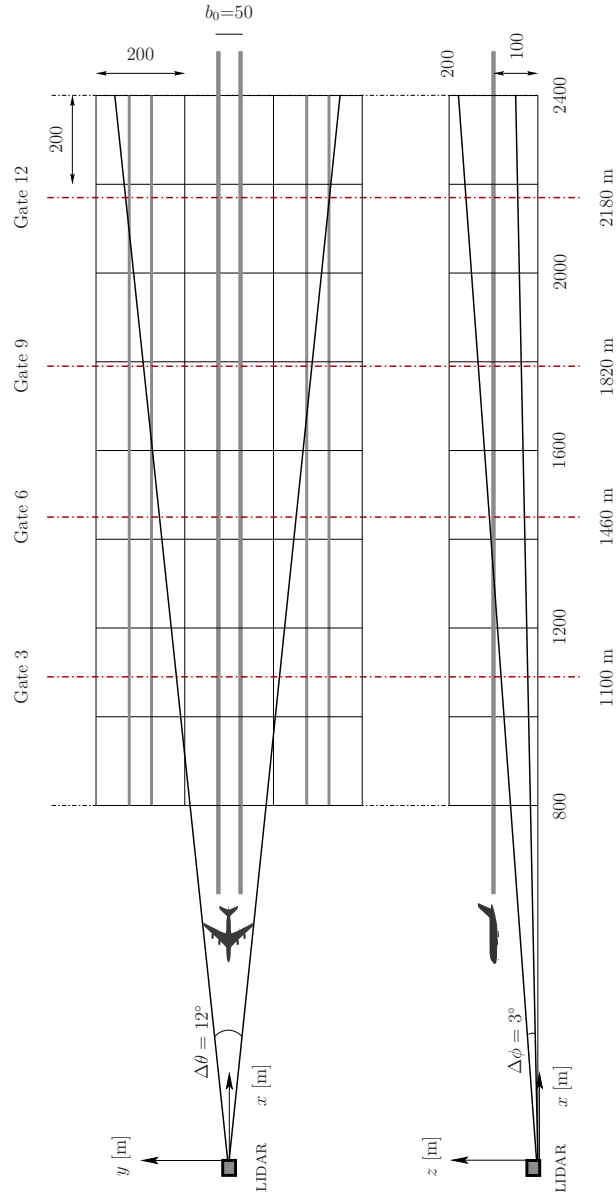


Figure 6.7: Axial detection configuration and simulation setup. The scanning is sinusoidal with an azimuth variation of $\Delta\theta=12^\circ$ and $\Delta\phi=3^\circ$ in elevation. The grid represents the reproduction along x and y of the LES volume with size $200 \times 200 \times 200$ m.

function to initialize the wake flow. A Burnham-Hallock model is used for this purpose. The evolution of the whole fluid flow is finally computed. Only the fields at time $t=20$ s (early case) and $t=120$ s (late case) have been exploited in the present study. The initial physical parameters are given in Table 6.4. As

Description	Symbol	Value
Vortex circulation	Γ_0	400 m ² /s
Vortex spacing	b_0	50 m
Core radius	r_c	0.05 b_0
Atmospheric EDR	ϵ	10^{-5} m ² /s ³
Periodicity length	L_x, L_y, L_z	4 b_0
Number of Fourier modes	N_x, N_y, N_z	256

Table 6.4: Parameters of the LES simulations of wake vortex in weak atmospheric turbulence. The wake vortex parameters b_0 and Γ_0 are given before evolving in HIT.

the 200³ m³ simulated volume is periodic along the three directions, multiple boxes can be concatenated to cover the whole detection volume without any discontinuity. The simulation volume is thus composed of 3×8 boxes. The resulting periodization of the wake vortex can be seen in Fig. 6.7. The relation $L = 4 b_0$ has been chosen such that the influence of the vortex periodic images are minimized. The vortex replica are visible in the simulations from a range of approximatively 1600 m.

The LIDAR simulations are based on the *feuilletté* model [Salamitou 1995]. It includes effects of LASER beam propagation in the instrument and the atmosphere, diffraction, defocus, refractive turbulence, Mie scattering on aerosols, LASER speckle and receiver noise. The LASER beam shape, which is a critical aspect of large-core fiber amplifier, is introduced by its M² beam quality factor affecting the Gaussian beam divergence [Dolfi-Bouteyre 2009].

6.3.2 The reconstruction technique

We first have to introduce the simple concept of a range gate. It is the spatial volume that we actually select if, for every LOS of a scan, we analyse a signal segment of duration T around a given instant t . This volume has a mean range $R = ct/2$ and a depth $\Delta R = cT/2$. The gate image is the projection of the velocity information contained in those segments on a plane normal to the mean direction of scan and positionned at a range R along this direction. We must point that this kind of representation, although it is a rather convenient way to map the radial velocity, is not exact geometrically since the surface considered is actually a spherical rectangle.

We follow here the same formalism as the one introduced in Section 6.2 with the difference that the processing map $\mathcal{K}$ and the representation map $\mathcal{P}$ now describe three-dimensional spatial regions. Each point $\mathbf{k}$ is represented by the position $t_k = 2R_k/c$ of a sliding window with duration T on a measured signal obtained from a given LOS with an azimuth angle θ_k and an elevation angle ϕ_k . The interval T_k between two $\mathbf{k}$ -points on a same LOS corresponds to a distance $R_k = cT_k/2$. For the map $\mathcal{P}$, two successive gate images are separated by a distance ΔR_p and are represented by a uniform grid in the $\xi \equiv \{\theta, \phi\}$ space with coordinate $\{\theta_p, \phi_p\}$ and resolution $d\theta = d\phi$. Each point $\mathbf{p}$ is associated to a circular accumulation region $\mathcal{C}$, also defined in the space ξ , with a radius ϱ [rad]. A point $\mathbf{k}$ is therefore included in the ROI of a point $\mathbf{p}$ if the distance $\|\mathbf{k} - \mathbf{p}\|_\xi$ in ξ , defined by

$$\|\mathbf{k} - \mathbf{p}\|_\xi \equiv \sqrt{(\theta_p - \theta_k)^2 + (\phi_p - \phi_k)^2}, \quad (6.3)$$

is less or equal than ϱ . In $\{x, z\}$ and for a given point $\mathbf{p}$, the gate images have a resolution $\Delta x = \Delta z = R_p d\theta$ and a spatial ROI with radius $\rho(R_p) = R_p \varrho$. We also define a weighting function $A(\|\mathbf{k} - \mathbf{p}\|_\xi)$ in ξ which is a 1-D Gaussian function with variance σ_ϱ [rad]. The ratio σ_ϱ/ϱ is typically chosen equal to 1/3. The algorithm is detailed in Table 6.5 and the accumulation is illustrated in Fig. 6.8.

-
- | | |
|------|--|
| I. | For each LOS i with direction $\{\theta_i, \phi_i\}$,
$\forall \mathbf{k} \in \mathcal{K}$ s.t. $\theta_k = \theta_i$ and $\phi_k = \phi_i$,
Estimates I_k from signal $s_i(t_k)$ where $t_k = 2R_k/c$ |
| II. | Define the angular weighting function $A(\ \mathbf{k} - \mathbf{p}\ _\xi)$
$\forall \mathbf{p} \in \mathcal{P}$,
Define $\mathcal{C}$ as a subset of $\mathcal{K}$ s.t. $R_k = R_p$ and $\ \mathbf{k} - \mathbf{p}\ _\xi \leq \varrho$
$I_p \leftarrow 0$;
$\forall \mathbf{k} \in \mathcal{C}$
$I_p \leftarrow (I_p + A(\ \mathbf{k} - \mathbf{p}\ _\xi) I_k)$; |
| III. | $\forall \mathbf{p} \in \mathcal{P}$,
Computes $S_{x,p}(\nu)$ from I_p |
-

Table 6.5: Axial accumulation algorithm for a circular region $\mathcal{C}$ at point $\mathbf{p}$ of radius $\rho(R_p)$.

As the accumulation process is defined independently to the range gate position, the reconstruction can be computed more efficiently by creating reconstruction tables as discussed previously for the transverse case. Moreover, due

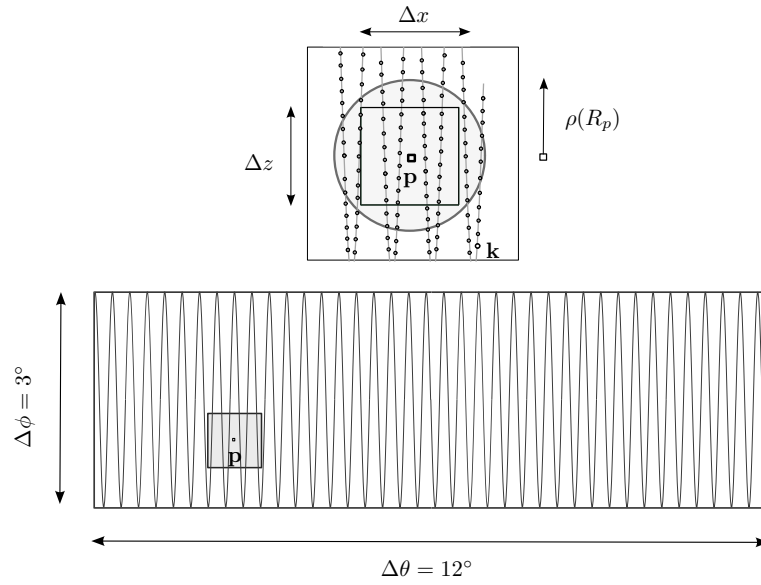


Figure 6.8: Principle of the accumulation technique in axial detection: scanning pattern (bottom) and accumulation and pixel regions (top). The points $\mathbf{k}$ in the processing map $\mathcal{K}$ are represented by the circles which also corresponds to the LOS intersection with the gate plane. The dark circular region in the top figure represents the accumulation set $\mathcal{C}$ of the point $\mathbf{p}$ represented by a square.

to the sinusoidal scanning pattern and the fixed circular ROI, the accumulation level M is non-uniform inside the gate image, i.e. it depends on ϕ_p and θ_p but not on R_p . It tends to be higher at the image boundaries. We typically impose a maximum accumulation level M_m which has also the advantage, for real-time processing, to bring the computational time under control. This constraint has a real interest if the reconstruction tables are constructed such that the points $\mathbf{k}$ for a given point $\mathbf{p}$ are sorted with increasing distance in order to consider only the M_m nearest ones.

6.3.3 Results on simulated data and discussion

Since Chapter 4, we know that increasing the pulse duration τ results in a filtering of the wind statistics. As the radial velocity dynamic in axial configuration is already very low, it may result in a decrease of the contrast between the wake vortex region and the atmospheric turbulence and eventually compromise the detection capability of the system as first exposed in the introductory chapter. The pulse duration must however be kept large enough to guarantee a good velocity resolution. We have also to keep in mind that, using LIDAR simulations for which the receiver is adapted, may lead to unappropriated conclusions since the detection noise power is higher for shorter pulse durations.

The results presented here have been obtained with a PRF of 4 kHz and a pulse duration τ of either 400 ns or 800 ns. The pulse energy is 1 mJ. In order to optimize the computational time, the sampling frequency has been fixed to 50 MHz with an intermediate frequency of 10 MHz. For the radial velocity map reconstruction, an angular resolution of $d\theta=3.8$ mrad has been chosen corresponding to 55×14 gate images. The accumulation is performed on the spectrogram on which a bi-Gaussian model is fitted. Range gates of $\Delta R_p=120$ m are used.

A point that had to be clarify is the fact that, when using a bi-Gaussian spectral model, the information carried by the larger one is related to the wake vortex [Douxchamps 2008]. The study made on the wake vortex signal and exposed in the previous chapter both supports and refutes this postulate. It is supported by the fact that inside the wake vortex the statistical distribution of the radial velocity may becomes multimodal and, depending on the pulse duration, the resulting spectrogram may conserve this characteristic. However, imposing two spectral kernels for very long pulse durations can result in particularly uncertain results since the wake vortex information tends to disappear under the noise spectrum at low accumulation levels. In contrary, for very short pulses, the spectral smoothing is such that both kernels contain vortex information.

Fig. 6.9 illustrates the wake vortex detection by a bi-Gaussian spectral model in the late case ($t=120$ s). The results for the mean radial velocity and

the velocity dispersion computed from the second Gaussian width are given for the four range gates selected in Fig. 6.7. We observe that the detection is rather good in this case on the dispersion images. In contrary, it becomes more and more difficult as the range increases to discern the signature of the central vortex pair in the mean radial velocity one. This can be explained by the very low velocity dynamic, i.e. ± 0.4 m/s, and the decrease of the vortex spatial sampling as range increases. The vortex replica are better detected since the velocity projection is larger when the azimuth angle has its maximum values. The axial configuration tends therefore to be less efficient at the mean observation direction, i.e. at the center of the gate images. This constatation also suggests to install the LIDAR system with a slight azimuthal angle with respect to the glide slope. However, the drawback of an off-axis position is that it potentially reduces the intersection between the scanning field-of-view and the aircraft trajectory. It is finally interesting to remark, in particular in the first two gates of this figure, the presence of the Rankine oval which is the spatial zone inside which the vortex pair turbulence is confined.

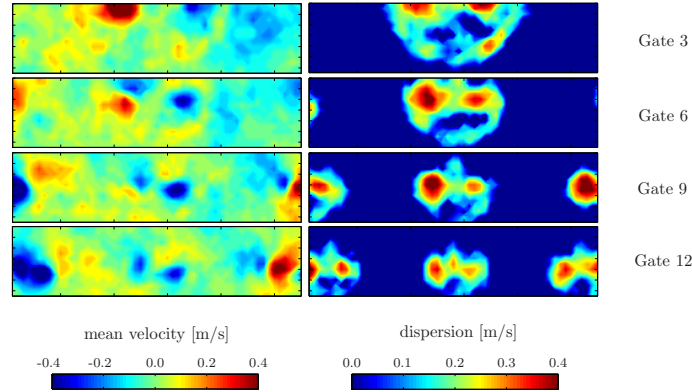


Figure 6.9: Wake vortex detection at a pulse energy of 1mJ using a bi-Gaussian spectral model. The pulse duration is 800 ns, the PRF is 4 kHz and the ROI radius is $\varrho = d\theta = 3.8$ mrad. The four pairs of images are given for the range gates which center is represented on Fig. 6.7. The vortex time is $t=120$ s.

We have observed that the optimization process is particularly sensitive to parameters initialization as well as on constraints imposed on them, especially if we want to distinguish the vortex Gaussian from the atmospheric turbulence. In conclusion, even if good results have been obtained for good SNR conditions, in any case the Gaussian parameters in such a model can be used directly to characterize wake vortices. In the best case, this algorithm can serve for wake vortex detection.

6.3.4 Only field tests results

For various technical reasons, the LASER source at 1 mJ was not available for the field tests. The LIDAR system, presented at Section 1.2.4 and developed by ONERA, has been used instead with a lower pulse energy of $120 \mu\text{J}$. It was therefore necessary to perform additional simulations with the new source parameters in order to evaluate the eventual gap in performance. Fig. 6.10 allows us to expose some of the observations made. The SNR is nearly eight times lower and the noise fluctuations on the spectrogram are high enough for the bi-Gaussian model to become unusable, even with an increase of the accumulation level. A classical MLE estimator of the mean radial velocity and velocity dispersion has therefore been tested instead. The estimation variance in these parameters is however important in comparison to the velocity dynamic. The vortex signature are therefore more difficult to detect in the gate images and the velocity dispersion appears relatively noisy. More generally, the system visibility in clear conditions has decreased around 1.5 km in comparison to the 2.4 km expected.

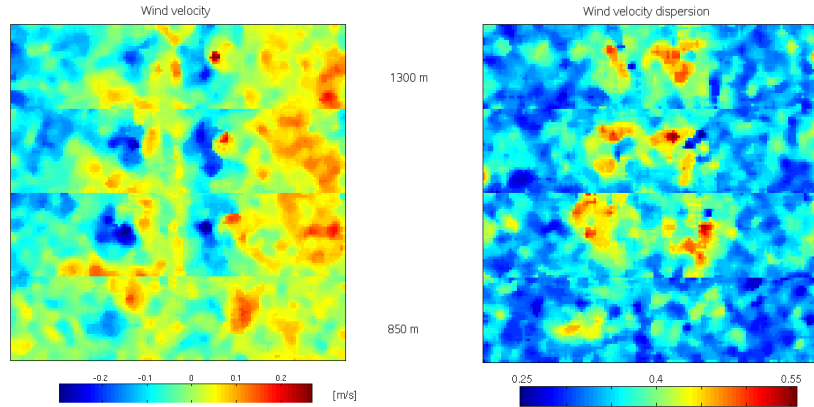


Figure 6.10: Processing of the simulated data with the parameters of the field test (pulse energy of $120 \mu\text{J}$). The range gates have a depth of 75 m and the gate images represented corresponds to the gate numbers 1, 3, 5 and 7.

Let's describe now the results obtained during the field tests. The information obtained for each range gate image is updated every scan. Up to 16 images are computed per scan corresponding to 16 range gates of 75 m when they do not overlap. The image resolution has been fixed to 128×32 pixels, giving an angular resolution of $d\theta = 1.6 \text{ mrad}$. The averaged spectrograms are first processed to isolate the main spectral peak corresponding to the heterodyne signal. In its simplest form, the one implemented in real time, the algorithm only extracts

the central peak position (mean velocity), the peak width (velocity dispersion), and its maximum (related to the local SNR). During postprocessing, the wind velocity dispersion is computed by fitting the logarithm of the spectral peak upper part with a second-order polynomial function. The higher polynomial coefficient is inversely proportional to the peak variance. This last algorithm was necessary due to the presence of an artefact near the intermediate frequency.

Fig. 6.11 shows a set of velocity dispersion images computed after the passing of two successive aircraft. Each box represents the $12^\circ \times 3^\circ$ field-of-view image acquired at different ranges (vertical axis) and different times (horizontal axis). We can clearly see the vortex signatures in scans 0 and 1 for the first plane, and in scans 4 and 5 for the second one. The elapsed time between two scans is 6 seconds. The disappearing of turbulence is mainly due to the cross-wind, moving them outside the spatial region of scan. The distance of the vortex signature to the LIDAR corresponds to the distance between the aircraft and the sensor measured by the video detection and tracking system.

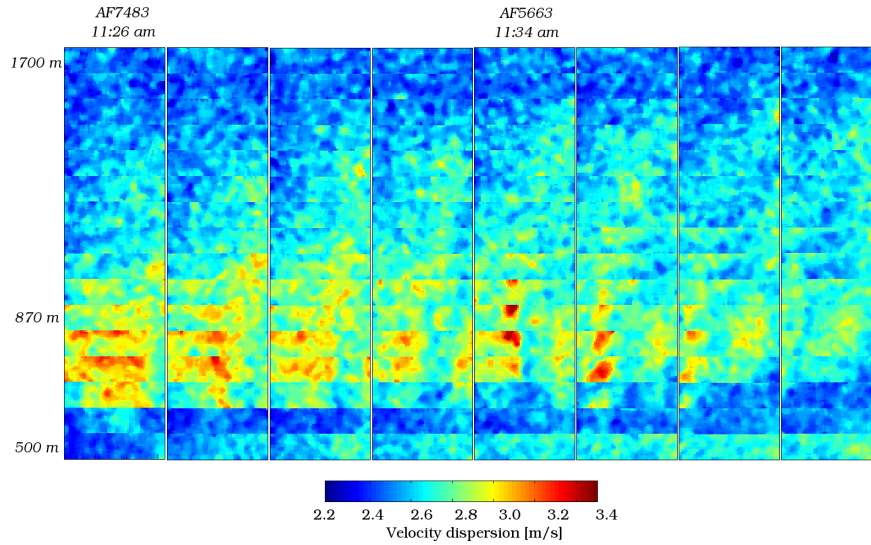


Figure 6.11: Velocity dispersion images obtained for eight successive scans of 16 range gates of 75 m. Two aircrafts crossed the LIDAR field-of-view (FOV) during the scan 1 (aircraft label AF7483) and the scan 5 (label AF5663). The vortex signatures are visible in every scans around 870 m. The wind direction is such that the vortices are carried outside of the FOV to the left of the gate images. They are therefore no more present in the two last scans.

Figs. 6.12 and 6.13 show 3-D representations of the wake vortex detected on two consecutive scans, a few seconds after a B747 landing. Each rectangle

corresponds to a range gate. The x -axis is the 12° horizontal field of view, z -axis the 3° vertical field of view, and y -axis the distance of the range gates. The detection was possible on wind velocity that shows strong discontinuities. The blue regions (gates 7 to 10 around 1.2 km) are again consistent with the aircraft location. We can see the wake vortex decay on the two consecutive scans. This range detection is in accordance with simulated results.

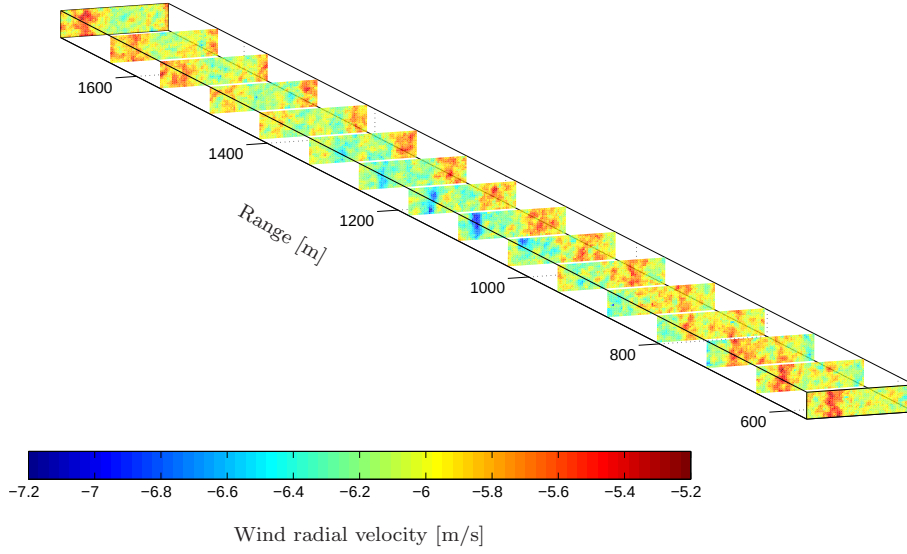


Figure 6.12: 3-D view of mean velocity images obtained during the landing of a B747. The LIDAR is positionned in bottom right angle of the image looking through the left. Vortex signatures are visible for ranges between 1000 m and 1300 m. The mean radial velocity is -6.2 m/s.

6.4 Conclusions

Pulse accumulation is a necessary signal processing technique to obtain more reliable estimates of the wind statistics by exploiting the existing correlation of the instantaneous frequency between adjacent line-of-sights while reducing the effects of independent processes such as the speckle or the photodetection noise. However, this aggregation must be carried out by taking into account a number of spatial constraints on the velocity maps to reconstruct. Hence, an increase of the accumulation level always leads to a decrease of the estimation variance at the expense of the spatial resolution and inversely. Reconstruction methods have been formalized in this thesis and tested for both the transverse and the

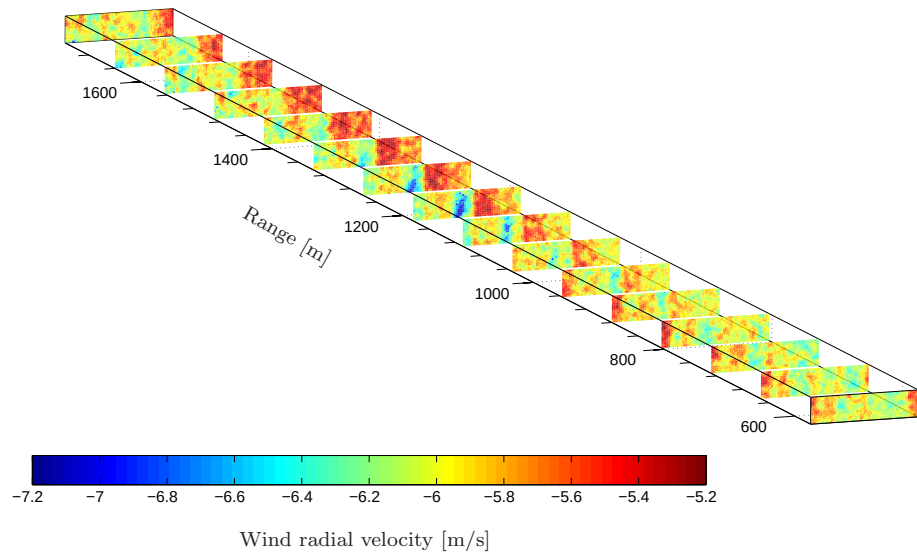


Figure 6.13: 3-D view of mean velocity images obtained during the landing of a B747 (second scan). The LIDAR is positioned in bottom right angle of the image looking through the left. Vortex signatures are visible for ranges between 1000 m and 1300 m. The mean radial velocity is -6.2 m/s.

axial wake vortex detection cases.

For the transverse detection case, a kernel-based accumulation is proposed in order to better define the trade-off between the estimation variance and the spatial resolution. This analysis has pointed out the fact that, as a result of the angular scanning and the decreasing signal-to-noise ratio, the kernel size should grow with range if a homogeneous estimation variance is needed. The resulting decrease of the spatial resolution with range is found to have an impact on the subsequent wake vortex characterization algorithms. The exact influence of this effect on the estimation variance of the vortex circulation must however be studied further.

In the framework of the FIDELIO project, the main efforts were directed toward the validation of wake vortex detection with a fiber-based LASER source and an axial configuration. In this configuration, the problems related to the velocity statistics smoothing or the estimation variance reduction, while keeping an acceptable spatial resolution, reaches their paroxysm due to the low radial velocity dynamics. Hence, although it has been proven by simulation in other studies that an axial velocity component actually exists, the estimation noise combined with the effects of atmospheric turbulence prevents us, at low pulse energy, to detect discriminant vortex signatures inside the reconstructed maps. High pulse repetition frequencies are therefore necessary. Despite of a pulse energy limited to $120 \mu\text{J}$, vortex signatures have clearly been detected with the ONERA source during the Orly campaign thanks to a frequency of 8 kHz and a good beam quality. Even if works have still to be done to validate it completely, especially at high altitudes, these tests have permitted us to position the fiber LASER sources as concurrents to classical solid-state sources.

The FIDELIO project was also a good occasion to test the bi-Gaussians spectral model developed earlier in the framework of the IWAKE project. The results obtained in this project with this model have shown that the larger Gaussian would carry more discriminant features about the wake vortex itself than a classical single-kernel model. This model turns out to be a particular case of the more general spectral model developed in this thesis. It has furthermore been observed by processing simulated signals, provided by ONERA, that it is only valid for specific conditions binding the pulse duration to the radial velocity distribution in the sensed volume. The statistics obtained from this second Gaussian cannot be related to any radial velocity statistics and it is therefore not adapted for wake vortex characterization. However, in its validity range, simulations have confirmed its good wake vortex detection capabilities.

Wake vortex parameters estimation

A NUMBER of projects in the past ten years have shown that the coherent Doppler LIDAR is an optimal tool for detecting and tracking wake vortices in clear air conditions [Hannon 1994, Harris 2002, Besson 2009]. However, the estimation of their circulation is still a challenging problem since the algorithms sensitivity to measurement conditions cannot be easily assessed on real data.

We have demonstrated in Chapter 4 that only a space-averaged radial velocity distribution can be retrieved with a pulsed Doppler LIDAR. This effect is of major importance since it deteriorates the measured velocity intensity and gradients along the observation path, which may eventually lead to an underestimation of the maximal tangential velocity and the exact vortex strength. A model-based approach for evaluating the wake vortex parameters has thus been developed to compensate this effect. It consists of a measurement model exploiting the properties of the LIDAR average Smoothed Pseudo Wigner-Ville distribution and takes into account both the LIDAR parameters and the estimation process itself. It has been validated for transverse detection on realizations of the Burnham-Hallock model as well as on LES simulations of wake vortex in ground effect .

In Section 7.1, we introduce the wake vortex characterization problem and explain why the use of a priori models is mandatory. We then present and discuss, in Section 7.2, an algorithm based on a property of the Burnham-Hallock model binding the radial and the tangential velocity which computes the circulation profile directly from the estimated radial velocity field. We will finally describe our wake vortex characterization algorithm in Section 7.3.

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7.1 Introduction

The reasons why an estimator based on a vortex model has been developed are exposed in this section. Both the wake vortex and the measurement models are then detailed.

7.1.1 About the need of model-based estimators

The evaluation of the vortex circulation, noted Γ [m²/s], requires to integrate the wind field $\mathbf{u}$ [m/s] over a closed contour $\mathcal{C}$ around the vortex center, i.e. by calculating

$$\Gamma = \oint_{\mathcal{C}} \mathbf{u} \cdot d\mathbf{l}. \quad (7.1)$$

However, as we have seen in Chapter 4, the conjunction of both the measurement and the signal analysis processes prevents us from estimating the exact components of this field. Hence, computing the vortex strength by replacing $\mathbf{u}$ in Eq. 7.1 by the estimated radial velocity field would induce an underestimation of its value which deteriorates further with increasing pulse duration or analyzing window length. More precisely, Proposition 4.3 tells us that only a smoothed version of the radial velocity distribution is retrievable. In conclusion, even if we have shown that it is possible to detect vortices with very little information about their spatial structure and velocity distribution, characterizing them is a more challenging process.

It is therefore necessary to introduce enough a priori knowledge to create a more faithful representation of the wake vortex velocity field, i.e. by reconstructing the unknown velocity components. Among the variety of existing wake vortex models, the Burnham-Hallock one is the mostly used for this purpose. It is a low order algebraic model having the main advantage to be set up easily. Nevertheless, it is quite obvious that the choice of a vortex model will directly restrict the estimation algorithm on which it is based, to its intrinsic validity limits. In particular, this model will not be complex enough to describe with precision a wake vortex flow in ground effect with wind. Consequently, we have to introduce the following assumption.

Assumption 7.1 *The fluid flow to be analyzed behaves like a Burnham-Hallock model for which the unknown parameters are the central position in the polar coordinate system $\{R_c, \beta_c\}$, the radius r_c as well as the circulation Γ of both vortices.*

For a pair of vortices, the subscripts 1 and 2 will be used to distinguish respectively the parameters of the nearest and the farthest vortices from the LIDAR telescope. This assumption will be exploited differently for the two estimation techniques presented here. In Section 7.2, we use a well-known

relation existing between the radial velocity and the tangential velocity whereas in Section 7.3, this model is used almost directly to match the reconstructed radial velocity map.

7.1.2 The Burnham-Hallock model

The Burnham-Hallock model is a low order algebraic model in which the induced velocity field due to one vortex has the form:

$$\mathbf{u} = u_\theta(r) \hat{\mathbf{e}}_\theta, \quad (7.2)$$

where $\{r, \theta\}$ is the chosen polar coordinate system with $r=0$ at the vortex center, $\hat{\mathbf{e}}_\theta$ is the tangential unitary vector and $u_\theta(r)$ [m/s] is the tangential velocity. For this axisymmetric vortex, it is natural to choose a circular integration contour $\mathcal{C}$ in Eq. 7.1, and it follows that

$$\Gamma(r) = 2\pi r u_\theta(r). \quad (7.3)$$

The vorticity, noted $\omega(r)$, as well as the circulation and the tangential distribution are given by the following relations:

$$\omega(r) = \frac{\Gamma_0}{\pi} \frac{r^2}{(r^2 + r_c^2)}, \quad (7.4a)$$

$$\Gamma(r) = \Gamma_0 \frac{r^2}{(r^2 + r_c^2)}, \quad (7.4b)$$

$$u_\theta(r) = \frac{\Gamma(r)}{2\pi r}. \quad (7.4c)$$

The model parameters are the total circulation Γ_0 [m²/s] of the vortex and the radius r_c [m] of the maximum induced tangential velocity. This last quantity is an indicator of the vortex core size and typically amounts, after the initial roll-up phase, to 3 – 5% of the wing span b [m]. A typical velocity profile for a pair of vortices is illustrated in Fig. 7.1. The wake vortex time evolution is usually defined by the dimensionless time parameter

$$\mathcal{T} = \frac{V_0}{b_0} t, \quad (7.5)$$

where t is the physical time, b_0 [m] is the distance between the pair of vortices and V_0 [m/s] is the descent velocity OGE given by

$$V_0 = \frac{\Gamma_0}{2\pi b_0}. \quad (7.6)$$

We see from the last equation that the total circulation could be theoretically measured by tracking the vortex from successive scans and computing the descent

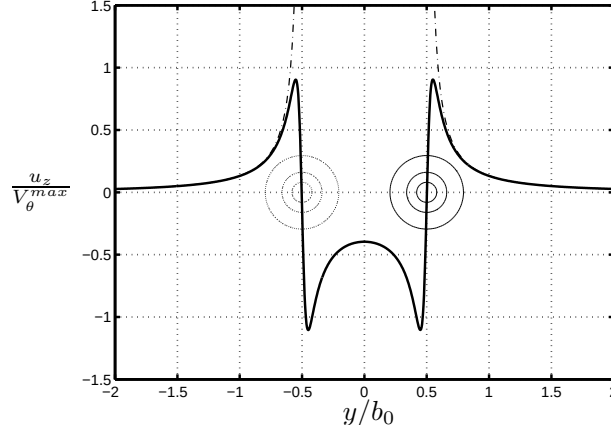


Figure 7.1: Vertical velocity profile in the far wake of an aircraft. The dashed-dotted lines represent the potential velocity profile $V_\theta = \frac{\Gamma_0}{2\pi r}$. The curve is normalized by $V_\theta^{max} = \frac{\Gamma_0}{4\pi r_c}$ in the case of a low order algebraic model.

speed from its position. We haven't explored this possibility in depth. Although it gave good results on very simple cases, it was sufficient to observe the behavior of wake vortices in ground effect, experiencing the rebound phenomenon, or with cross-wind, to understand its limitation.

7.1.3 The measurement and estimation models

The actual radial velocity in the transverse plane at a distance R along a LOS with elevation ϕ is denoted $v_R(R, \phi)$. For an estimates $\hat{v}_R(\mu, \phi)$ of this velocity, given by the first order moment of the LIDAR SPWVD at time $\mu = 2\rho/c$, where ρ is the mean estimation range, it has been shown that

$$\hat{v}_R(\rho, \phi) = \int_0^\infty I_a(R; \mu) v_R(R, \phi) dR, \quad (7.7)$$

where $I_a(R; \mu)$ is the spatial weighting function first introduced in Definition 4.5. This function takes into account the main measurement parameters such as the pulse profile, the evolution of the system gain or also the analyzing time-frequency window, $\Pi(t, \nu)$ (see Chapter 4), used for signal processing. Since these parameters are either known or estimable with a good precision, this equation informs us on how the exact velocity field is actually transformed by the measurement. This equation will be called the *measurement model*.

The radial velocity field obtained at the output of the LIDAR signal processing step also includes an additive velocity estimation error induced by all the

perturbating processes inherent to the LIDAR measurement, e.g. the speckle effect and the detection noises, on the accuracy of the signal processing. This noise, noted v_e [m/s], is supposed to be a zero-mean Gaussian random variable with variance σ_e . The succession of the measurement model and the measurement noise is called the *estimation model*.

Both models are defined in the velocity space and are decomposed as in Fig. 7.2. The measurement model takes the LIDAR and processing parameters $h(R)$, τ and Π defined in Chapter 4.

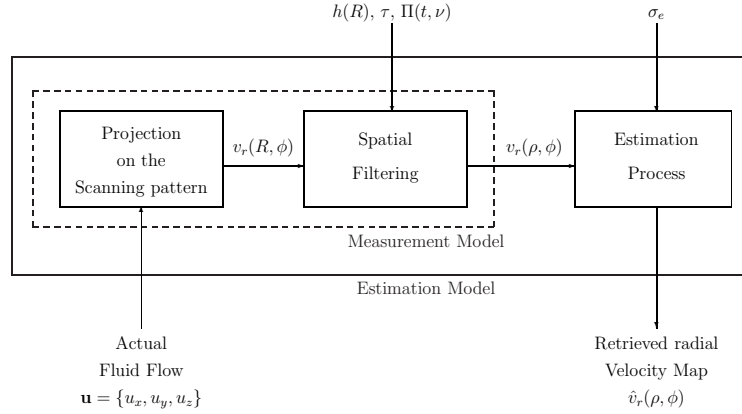


Figure 7.2: Measurement and estimation models defined in the velocity space. The exact wind field is given by $\mathbf{u}$

We consider here a two-dimensional scanning pattern in the $\{y, z\}$ transverse plane and define the LIDAR polar coordinate system $\{R, \phi\}$ where R is the range and ϕ is the elevation angle.

7.2 Circulation estimation from the radial velocity field

In this section, we will present an algorithm for estimating the circulation profile $\Gamma(r)$ directly from the exact radial velocity maps $v_r(R, \phi)$ where R is the range along a given LOS with elevation ϕ . It is based on a particular property of the Burnham-Hallock (BH) models according to which, for a single vortex and at given locations, the radial velocity equals the tangential one, u_θ . Assumption 7.1 is thus explicitly used. It could also be used on the retrieved field $\hat{v}_r(\rho, \phi)$ but, as we will see here, the estimation bias introduced increases exponentially with the spatial extend of the LASER pulse. This section is also aimed at demonstrating the limits of direct methods

7.2.1 Principle

The tangential velocity $u_\theta(r)$ found at one point of a given streamline is the same everywhere on the same curve defined by a constant value of r when θ goes from 0 to 2π . There exists two line-of-sights which are tangent to any circle $\mathcal{C}$ of radius r centered around the vortex center. It is also known that, at the two intersections between these LOS and the contour $\mathcal{C}$, the absolute value of the radial velocity, noted $|v_r(R, \phi)|$, reaches its maximum and is strictly equal to the tangential velocity at those particular points. Hence, the radial velocity field, induced by a single vortex, is given by

$$v_r(R, \phi) = u_\theta(r) \sin(\theta - \phi), \quad (7.8)$$

with the distance r between the vortex center to the point $\{R, \phi\}$ is:

$$r^2 = R^2 + R_c^2 - 2 R R_c \cos(\theta - \phi). \quad (7.9)$$

The radial velocity is therefore equal to the tangential velocity when $\theta = \phi \pm \frac{\pi}{2}$. We define the locus ℓ as the ensemble of positions $\{R, \phi\}$ for which $v_r(R, \phi) = u_\theta(r)$. The radial velocity profile obtained on ℓ is noted $e_r(\ell)$. We finally deduce two estimates of the circulation profile by starting from the vortex center, following ℓ along the two possible directions and using Eq. 7.3. The first one, noted $\Gamma_{\max}(r)$, is given by following the positive values of $v_r(R, \phi)$ and the second, noted $\Gamma_{\min}(r)$, by considering its negative values.

7.2.2 Algorithm description

This estimation principle is obviously easier to implement when considering an ideal pair of BH vortices. In practice, we have to deal with the random fluctuations of the velocity field due to atmospheric turbulence as well as the estimation errors. The following algorithm has been developed to consider those cases. It can be decomposed in five steps from the vortex detection to the $\Gamma(r)$ estimation:

1. Extraction of the spatial sector containing the two vortices by keeping the range interval defined by the conditions

$$\langle |v_r(R, \phi)| \rangle_\phi \geq \beta_r \max\{|v_r(R, \phi)|\}, \quad (7.10)$$

where $\langle \rangle_\phi$ is the ensemble average over all the elevation angles and β_r is a pre-defined threshold. This operation is possible on a radial velocity field in which the background fluctuations have been removed, e.g. by spatial filtering.

2. Spatial filtering and interpolation of the selected spatial region in order to respectively reduce the estimation variance, remove eventual outliers and

obtain a sufficient resolution on both vortices, especially for the farthest one.

3. Computation of the optimal separation range between the two vortices from the averaged profile obtained at step (1) and disjunction of the two cores. The following steps are applied independently on each of them.
4. Estimation of the exact locus ℓ of the radial velocity extrema $e_r(\ell)$. The estimated locus, noted $\hat{\ell}$, as well as the associated $\hat{e}_r$ profile are generally noisy curves. An adequate filtering must therefore be performed. Since it is physically consistent to assume that the locus ℓ is a smooth function, $\hat{\ell}$ is filtered using a succession of a median and a RLS (Recursive Least Square) filter. The first one is used for outlier rejection although the second one serves for the actual smoothing. The extrema profile $\hat{e}_r(\hat{\ell})$ is recomputed consequently along the new trajectory. The vortex center is then determined by the position of the minimum value of $|\hat{e}_r(\hat{\ell})|$ lying between its minimum and maximum.
5. Computation of the circulation profiles $\Gamma_{\max}(r)$ and $\Gamma_{\min}(r)$ by following the locus $\hat{\ell}$ from the vortex center to the two possible directions. An estimation of $\Gamma(r)$ is given by averaging those loci, which are two different estimates of the $u_\theta(r)$ profile, and then by using Eq. 7.3.

7.2.3 Results and discussion

This algorithm has been first tested on an ideal BH model with a total circulation of $\Gamma_0=400 \text{ m}^2/\text{s}$, a distance between vortices of $b_0=50 \text{ m}$ and a radius of $r_c = 0.05 \cdot b_0=2.5 \text{ m}$. The spatial filtering of the radial velocity imposed by the LIDAR measurement has therefore not be taken into account. We consider a Gaussian estimation error on the velocity field with a standard deviation of 0.1 m/s . Fig. 7.3 illustrates the estimation principle by representing the radial velocity field along with the estimated and the actual loci for both vortices. Fig. 7.4 gives the circulation evolution with r of the model as well as the estimated $\Gamma_{\max}(r)$ and $\Gamma_{\min}(r)$ with and without estimation noise. We observe that, whereas the matching is good up to about $3 r_c$, the estimated circulation tends to about 90% of Γ_0 in average. This effect can partially be explained by the limited transverse resolution used for this simulation. The fluctuations due to the 0.1 m/s noise are less than 0.4 about their mean values.

We have then considered the complete measurement model with a pulse duration $\tau_p = 2\sigma_p$ and an analyzing function $\Pi(t, \nu) = \delta(t)\delta(\nu)$, i.e. as if the WVD were used. We have analyzed the effect of the pulse duration, with spatial extend $r_p = c\sigma_p/2$, on the estimation of the maximum tangential velocity $V_\theta^{\max}$

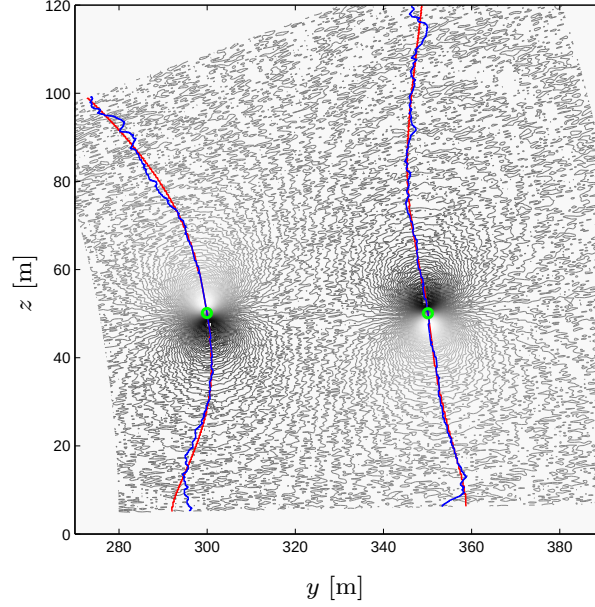


Figure 7.3: Radial velocity map of a BH vortex pair with a Gaussian estimation error with a 0.1 m/s std. dev. The exact and estimated extremum loci are respectively represented by the red and blue curves. The vortex centers are given by the green circles.

[m/s] related to the total circulation by

$$V_{\theta}^{\max} = \frac{\Gamma_0}{4\pi r_c}. \quad (7.11)$$

The results are given in Fig. 7.5. A typical value for the ratio r_p/b_0 is 1.2 obtained for a pulse duration of $\tau_p=400$ ns and $b_0=50$ m. Under the assumption that r_c is known or estimable, the conclusion of this test is that, with increasing values of the ratio r_p/b_0 , the estimation of the total circulation of the wake vortex monotonically decreases. This simulation confirms that no valid estimation on the circulation can be made from the radial velocity estimates given by the first moment of the LIDAR signal spectrum.

Moreover, as we have seen in Section 5.2, the most distant spectral mode of the LIDAR spectrum or by extension, of its Smoothed Pseudo Wigner-Ville distribution (SPWVD), would lead to a better estimate of the maximum radial velocity $V_{\theta}^{\max}$ and eventually to a more precise circulation. This method is nevertheless sensitive to the pulse duration and may only be efficient for specific conditions, as pointed out in the related section. However, in comparison to the

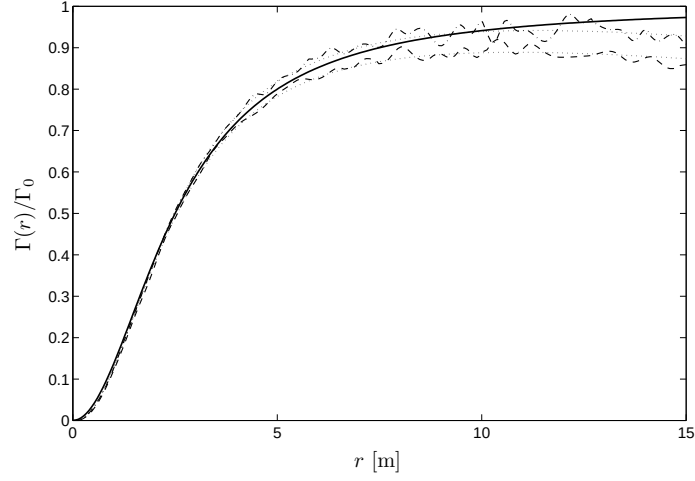


Figure 7.4: Estimation of the circulation profiles $\Gamma(r)$ for the nearest vortex: theoretical profile (solid), $\Gamma_{\max}(r)$ estimated from the positive part of $v_r(\rho, \phi)$ (dash-dot) and $\Gamma_{\min}(r)$ from its negative (dash) part and the estimates for both profiles with no estimation noise (dot).

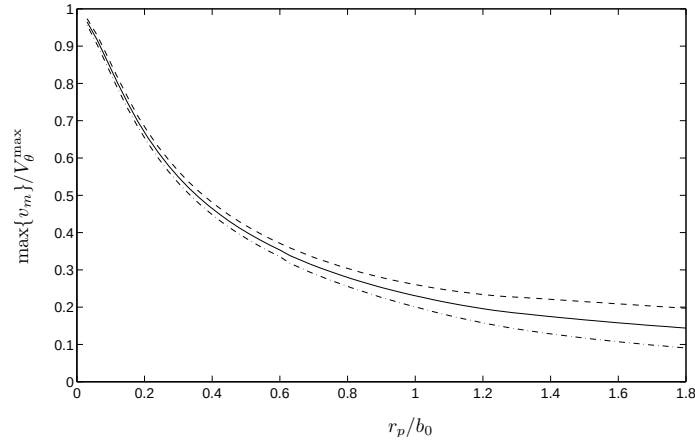


Figure 7.5: Influence of the pulse duration on the estimation of $V_{\theta}^{\max}$. The evolution is given depending on the ratio between the spatial pulse extend r_p and the distance b_0 .

method in [Kopp 2004], estimating the maxima with the adaptive model-based technique presented in Section 5.3 allows to decrease the sensitivity of the algorithm to noise.

7.3 A Model-based estimation method

From the analytical results and observations exposed in the previous chapters, we have developed an estimation technique based on a modified Burnham-Hallock model depending on a complete LIDAR measurement model. This is an inversion method which tends to reduce the effects of both the LIDAR measurement and the signal processing on the wake vortex estimates. It has been validated on realizations of a BH model and tested on LES simulations of wake vortex in ground effect. Parts of the present section have been published in [Brousmiche 2009a].

7.3.1 The inversion principle

Under Assumption 7.1, the inversion method represented in Fig. 7.6 has been elaborated. Its principle is to compute the parameters of an ideal Burnham-

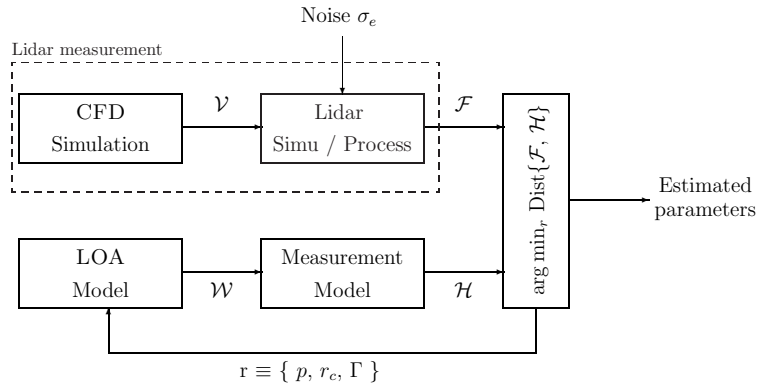


Figure 7.6: Optimization scheme for the direct simulation method.

Hallock model $\mathcal{W}$ which minimizes a given distance between the radial velocity field $\mathcal{H}$ obtained at the output of the measurement model, defined previously, and the estimated field $\mathcal{F}$. The parameters obtained after convergence of the algorithm are supposed to be closer to those of the exact field $\mathcal{V}$ than the ones that should have been obtained directly from $\mathcal{F}$.

In the proposed method, the estimate of the radial velocity is obtained by computing the first-order moment of the LIDAR SPWVD. Note also that the

computation of $\mathcal{H}$ and $\mathcal{F}$ necessitates the projection of the velocity field $\mathcal{V}$ and $\mathcal{W}$ onto the scanning pattern to get the radial velocity components.

7.3.2 Parameters estimation

This estimation method is actually a constrained non-linear optimization problem with four parameters per vortex, i.e. center coordinates, radius and circulation. In order to reduce the complexity to a 4-D problem and increases its robustness, this optimization is performed in three distinctive steps:

1. Vortex core separation and initialization of their position by analyzing the averaged radial velocity profiles in $\mathcal{F}$ along the two directions y and z . This step corresponds to the steps (1) to (3) of the algorithm described in Section 7.2.2.
2. 4-D optimization on each vortex independently, i.e. on parameters $\{y_c, z_c, r_c, \Gamma\}$, to refine the estimation of the vortex position. When the initialization of the position is considered accurate enough, this step is omitted.
3. Concurrent optimization on the parameters r_c and Γ_0 for both vortices. This step is necessary due to the existing interaction between vortices. It is however dependent on the quality of the vortex position estimates.

It has been observed that the convergence depends on the ratio between the spatial extend of the function $I_a(R; \rho)$ (see Eq. 7.7) and the distance b_0 between them. It possibly becomes unstable for too high values of it. This remark is comprehensible seeing that an increase of the pulse duration and of the duration of the observation window or both tends to join together the two vortices and decreases at the same time the precision on their position.

The cost function is the mean square error (MSE) between the $\mathcal{F}$ and $\mathcal{H}$ radial velocity fields and its gradient at each iteration is computed by the use of a 4-dimensional Sobel operator given in Appendix A. The mutual information between both maps has also been tested with poorer results. Moreover, in order to limit the computational time, the Burnham-Hallock model map is not necessarily recomputed at each iteration of the step 2. Instead, a scaled and sheared version of an initial radial velocity map is computed.

The convergence is also improved by multiplying the cost function by a spatial weighting function centered around the two vortices given by

$$W(r) = \exp(-r^2/2\sigma_k^2), \quad (7.12)$$

where r is the distance to the vortex center and σ_k [m] is the parameter controlling the size of the region-of-interest. It helps, in particular, to reduce the influence of radial velocity estimation errors outside the wake-vortex region.

An illustration of the optimization process is given in Fig. 7.7. It gives the results of the different velocity fields after convergence of the algorithm for a LES of a wake vortex pair in weak atmospheric turbulence. This fluid simulation is described in [Bricteux 2008]. The field $\mathcal{F}$ obtained at the output of the radial velocity estimator for a pulse duration of 400 ns is affected by a Gaussian noise with a standard deviation of 2 m/s. The difference between the actual field $\mathcal{V}$ from the LES and the optimal solution $\mathcal{W}$ is also represented. We observe on this velocity map the contributions of the atmospheric turbulence as well as the turbulence created by the wake vortex system itself and localized in the Rankine oval. It also contains the optimization error which is mainly concentrated inside the vortex cores, i.e. for $r < r_c$, when no error on the vortex centers is made.

7.3.3 Results and discussion

The first set of results are given for an ideal Burnham-Hallock model with $\Gamma_0=400 \text{ m}^2/\text{s}$, $b_0=50 \text{ m}$ and $r_c=2.5 \text{ m}$ (5% b_0). It takes into account the estimation error on the radial velocity at the output of an ideal signal processing estimator with an additional zero-mean Gaussian random variable with standard deviation σ_e [m/s]. The radial velocity map $\mathcal{F}$ has a resolution of $\Delta R=3.75 \text{ m}$ in range and $\Delta\theta=0.1^\circ$ in elevation. These values have been chosen such that the vortex cores are sampled enough. Fig. 7.8 and Fig. 7.9 respectively gives the standard deviation on the vortex parameters Γ and r_c obtained by a Monte Carlo simulation over 200 realizations and for an increasing pulse duration τ . The results are represented along with their polynomial regression. We first observe that the estimation error increases linearly with the velocity error, at least in this interval of σ_e . It also further increases with the pulse duration as the variation of the smoothed radial velocity decreases compared to the noise standard deviation. The estimation bias on the circulation is below $1 \text{ m}^2/\text{s}$ for all pulse durations provided that the standard deviation of the radial velocity estimation noise is kept lower than $\sigma_e=1.5 \text{ m/s}$. For stronger estimation noises, the bias increases exponentially for pulse durations higher than 400 ns. It is however limited to $3 \text{ m}^2/\text{s}$ below $\sigma_e=3 \text{ m/s}$ for shorter pulses. The same results for the estimation bias and variance have been obtained with a core radius of $r_c = 3\% b_0$.

A second set of simulations have been performed on a LES of a wake vortex pair in weak atmospheric turbulence. It is the same fluid simulation that has been used in Section 6.3 to assess the axial detection problem. The initial parameters of this vortex pair, listed in Table 6.4, are $\Gamma_0=400 \text{ m}^2/\text{s}$, $b_0=50 \text{ m}$ and $r_c=2.5 \text{ m}$ (5% b_0). The exact values of the vortex parameters at the time of the simulation, computed directly on the LES, are $\Gamma=379 \text{ m}^2/\text{s}$ and $r_c=2.4 \text{ m}$ for the nearest vortex and $\Gamma=385 \text{ m}^2/\text{s}$ and $r_c=2.5 \text{ m}$ for the farthest one. The velocity maps have the same resolutions in range and elevation than for the first set of simulations. The map size is 150 m in range and 30° in elevation,

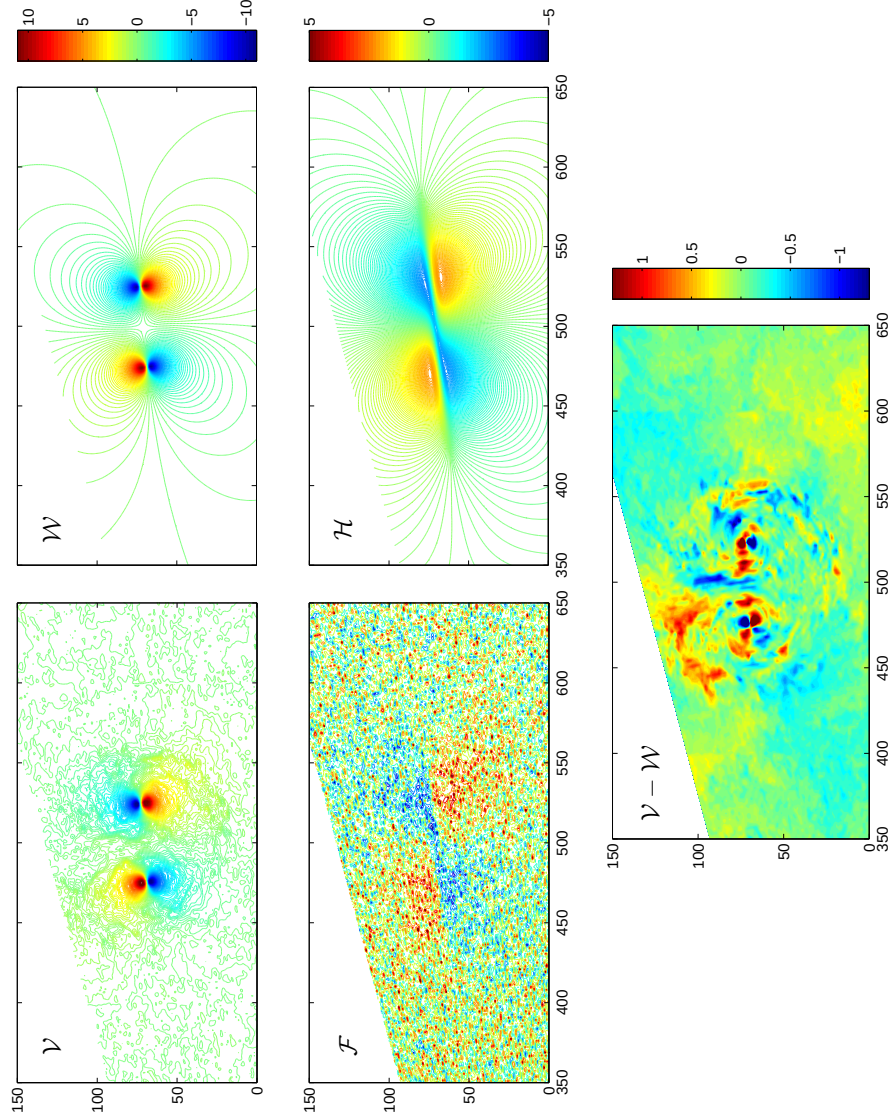


Figure 7.7: Illustration of the model-based inversion technique on a LES of a vortex pair in weak atmospheric turbulence with initial parameters $\Gamma_0=400 \text{ m}^2/\text{s}$, $b_0=50 \text{ m}$, $r_c=5\% b_0$ and $\epsilon=10^{-5} \text{ m}^2/\text{s}^3$. The pulse duration is $\tau_p=400 \text{ ns}$ producing a ratio r_p/b_0 of 1.2. The estimation noise std on the radial velocity is 2 m/s. The fields $\mathcal{V}$, $\mathcal{F}$ and $\mathcal{W}$ are respectively the actual, measured and estimated radial velocity fields. The field $\mathcal{H}$ is the output of the measurement model after convergence of the algorithm.

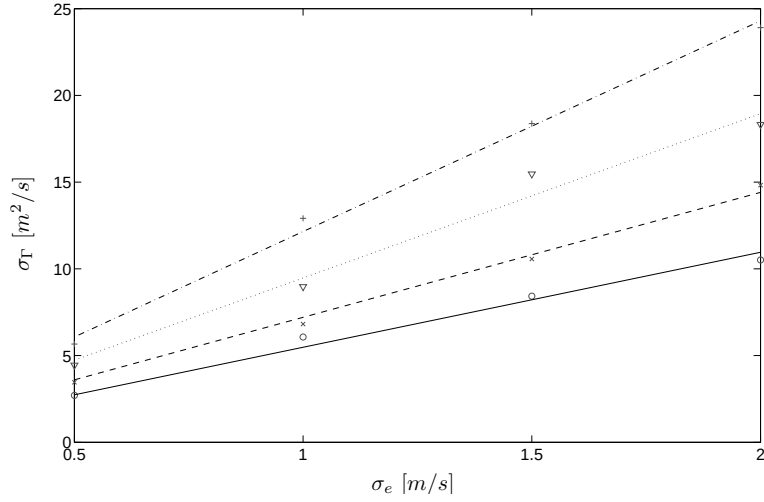


Figure 7.8: Influence of the radial velocity estimation error, described by its standard deviation σ_e , on the circulation estimation standard deviation σ_Γ for different pulse durations τ : 200 ns (solid), 400 ns (dash), 600 ns (dot), 800 ns (dash-dot).

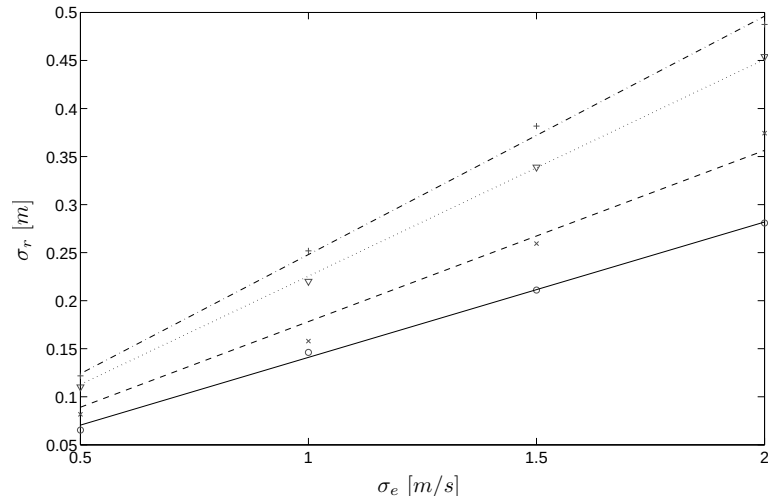


Figure 7.9: Influence of the radial velocity estimation error, described by its standard deviation σ_e , on the vortex radius r_c estimation standard deviation σ_r for different pulse durations τ : 200 ns (solid), 400 ns (dash), 600 ns (dot), 800 ns (dash-dot).

		Circulation Γ [m ² /s]	Core radius r_c [m]
200 ns	B	-1.3 / -1.3	0.10 / -0.09
	SD	1.8 / 1.9	0.05 / 0.06
400 ns	B	-2.9 / -4.3	0.24 / -0.15
	SD	2.5 / 2.7	0.06 / 0.07
600 ns	B	-11.2 / -14.2	0.32 / -0.22
	SD	3.4 / 3.5	0.08 / 0.10

Table 7.1: Influence of the pulse duration Δt on the estimation bias and std. deviation for the case of a LES simulation with weak atmospheric turbulence. The estimation noise has a std. deviation of $\sigma_e=0.5$ m/s. Two values are given for the Nearest and the Farthest vortex (N/F).

corresponding to 40 range gates and 300 LOS. The estimation bias and standard deviation of the vortex circulation and core radius have been computed on 500 realizations of the measured radial velocity map. Fig. 7.10 represents the results obtained for a pulse duration of $\Delta t=400$ ns. The probability density functions (PDF) of the estimates of both parameters are given for each vortex. A Gaussian PDF has also been fitted on them. The influence of the pulse duration on these statistics is synthesized in Table 7.1 for a noise of $\sigma_e=0.5$ m/s and in Table 7.2 for a noise of $\sigma_e=1$ m/s. It clearly appears that the bias on the circulation increases exponentially with Δt whereas the standard deviation increases more slowly. The vortex position is also an important factor since a decrease of the grid resolution at the vortex core center, e.g. for the farthest one, necessarily induces an increase of the estimation error. This constatation must be kept in mind when analyzing the wake vortex evolution with time if they happen to move in the detection plane. An important remark is that, when increasing the noise level from 0.5 to 1 m/s, only the standard deviation increases and not the bias. This results suggests that the bias is more sensitive to the grid resolution. With the chosen velocity map resolution, one estimation of the vortex parameters is provided in about one minute with a laptop based on an Intel Centrino Core 2 Duo T9300/2.5GHz with 2 Go of RAM. The computation time decreases with decreasing angular resolution and is about 25 seconds for $\Delta\theta=0.5^\circ$. However, both the estimation bias and variance increase with decreasing angular resolution, and inversely. For the $\sigma_e=1$ m/s case, they respectively reach -2.8 m²/s and 11.1 m²/s for the nearest vortex at 0.5° with a pulse duration of $\Delta t=400$ ns and the same noise variance.

A number of simulations have also been carried out to analyze the influence of pulse duration on the estimation of the vortex strength in ground effect, i.e. when secondary vortices are present. Fig. 7.11 gives two examples of radial velocity fields simulated in this case at dimensionless times $\mathcal{T} = 2.5$ (early case)

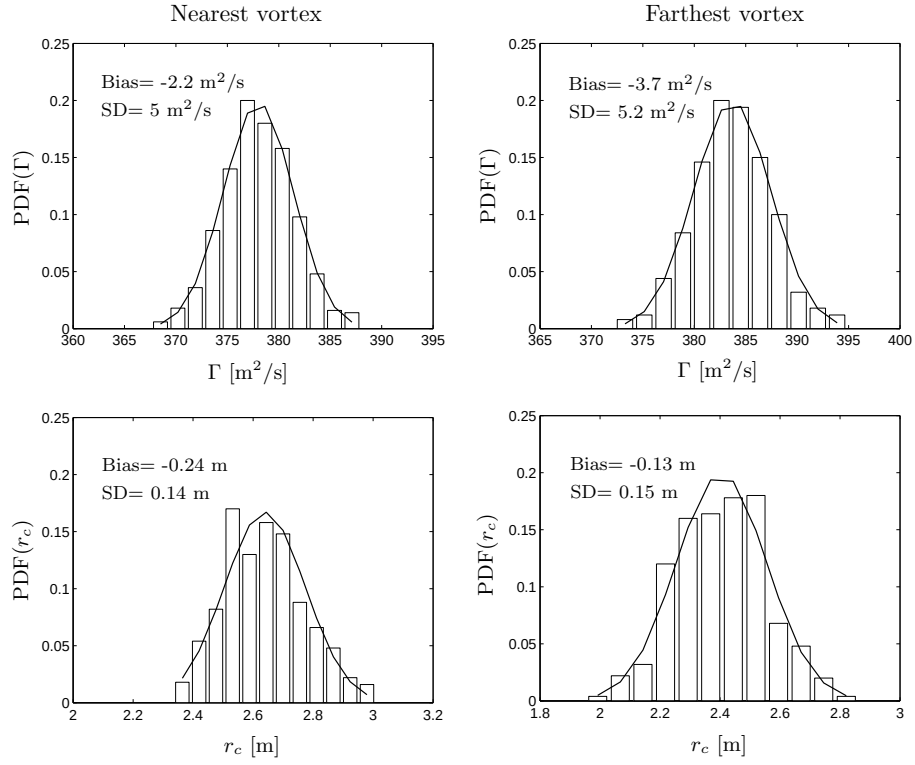


Figure 7.10: Computed (bar plot) and fitted (solid) probability density function of the vortex estimates (Γ and r_c) for a LES simulation of a pair of wake vortices in weak atmospheric turbulence. The pulse duration Δt is 400 ns and the estimation noise std. on the radial velocity is 1 m/s. The exact values of the LES are $\Gamma=379 \text{ m}^2/\text{s}$ and $r_c=2.4 \text{ m}$ for the nearest vortex and $\Gamma=385 \text{ m}^2/\text{s}$ and $r_c=2.5 \text{ m}$ for the farthest one. The velocity map resolution is $\Delta R=3.75 \text{ m}$ in range and $\Delta \theta=0.1^\circ$ in elevation.

		Circulation Γ [m ² /s]	Core radius r_c [m]
200 ns	B	-1.3 / -1.3	0.10 / -0.09
	SD	3.4 / 3.8	0.09 / 0.10
400 ns	B	-2.2 / -3.7	0.24 / -0.13
	SD	5.0 / 5.2	0.14 / 0.15
600 ns	B	-10.5 / -13.6	0.33 / -0.21
	SD	6.5 / 6.6	0.17 / 0.19

Table 7.2: Influence of the pulse duration Δt on the estimation bias and std. deviation for the case of a LES simulation with weak atmospheric turbulence. The estimation noise has a std. deviation of $\sigma_e=1$ m/s. Two values are given for the Nearest and the Farthest vortex (N/F).

and $\mathcal{T} = 5$ (late case). The secondary vortex is rotating in the opposite direction compared to the primary one, i.e. its circulation thus has the opposite sign. A typical radial velocity error map given by the standard deviation between the $\mathcal{W}$ and the $\mathcal{V}$ fields at $\mathcal{T} = 2.5$ is given in Fig. 7.12. As it has been computed without estimation noise, it gives a good image of the difference between the BH model and the actual radial velocity map when secondary vortices exist. The evolution of $\Gamma(r)$ for the two vortices at these two times and for a given transverse plane are compared to the one of a BH model with the same initial parameters in Fig. 7.13. Moreover, we can observe in Fig. 7.14 the fluctuations of $\Gamma(r)$ due to the presence of these secondary vortices.

Fig. 7.15 and Fig. 7.16 shows the vortex circulation obtained for a pulse duration varying from 100 ns to 1 ms and referred respectively to the early and the late cases. Three estimates are shown: the circulations Γ_1 and Γ_2 of the left (nearest one) and the right vortices as well as the mean value of both. For comparison purpose are also given the exact circulation Γ_0 computed on the CFD simulation and the ideal Γ_{5-15} computed on the unfiltered radial velocity map and defined by

$$\Gamma_{5-15} = \frac{6}{b} \int_{\frac{b}{12}}^{\frac{b}{4}} \Gamma(r) dr \quad (7.13)$$

where b [m] is the aircraft wingspan. The Γ_{5-15} at dimensionless time $\mathcal{T} = 2.5$ and $\mathcal{T} = 5$ are respectively equal to 90 % and 60 % of Γ_0 . This result can be found in [Bricteux 2008].

We first observe that a difference exists between the estimates Γ_1 and Γ_2 equal to about 10 % of the exact circulation value Γ . Normally, such a variation should not be observed when no wind is present. As we have just seen, this difference is related to the non-uniform spatial resolution imposed by the angular scanning. A second point, which has not been observed at all with the tests

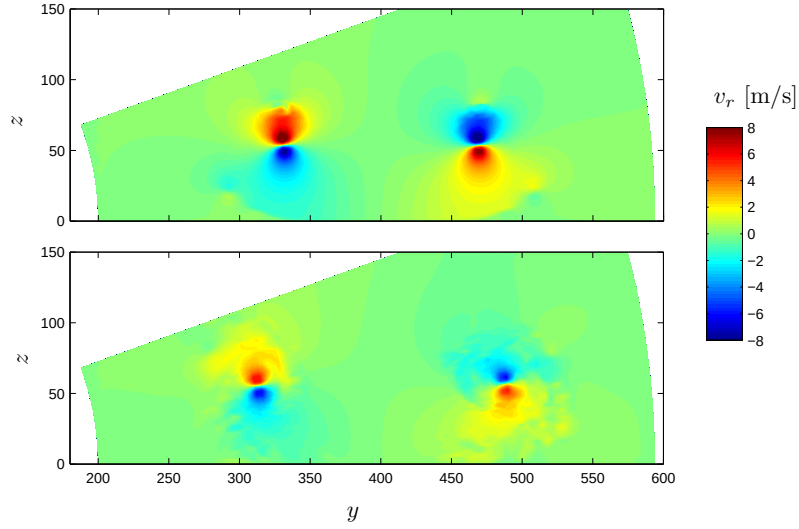


Figure 7.11: Examples of radial velocity maps extracted from CFD simulation of wake vortex IGE without cross-wind at dimensionless time $\mathcal{T} = 2.5$ (top figure) and $\mathcal{T} = 5$ bottom figure. The colormap goes from -8 m/s (blue) to 8 m/s (red).

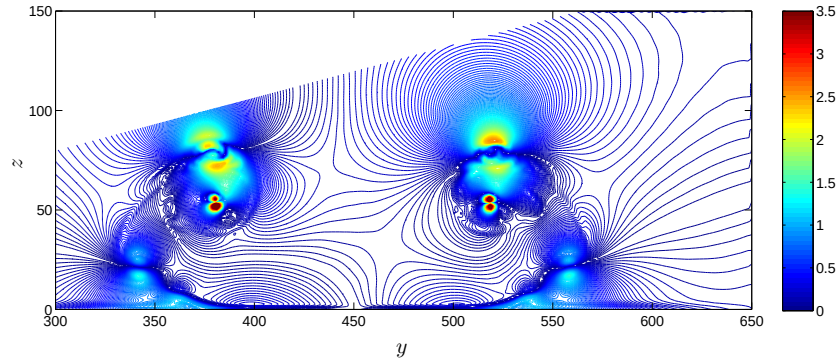


Figure 7.12: Typical radial velocity error map given by the standard deviation in [m/s] between the $\mathcal{W}$ and the $\mathcal{V}$ fields. It has been obtained from a LES simulation of wake vortex IGE at dimensionless time $\mathcal{T} = 2.5$. Secondary vortices are visible for each vortex, the first one just above the ground and the other one already gravitating around it.

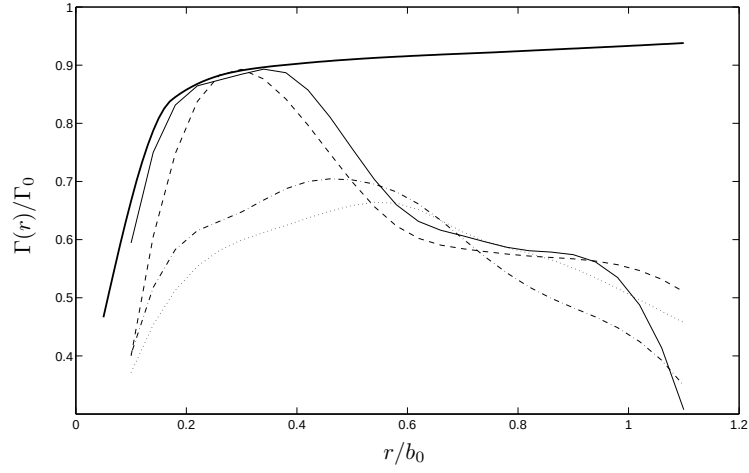


Figure 7.13: Evolution of the $\Gamma(r)$ with radius r in the early and late case for both vortices compared to the Burnham-Hallock model: model (solid, thick); early, left vortex (solid, thin); early, right vortex (dash); late, left vortex (dash-dot) and late, right vortex (dot).

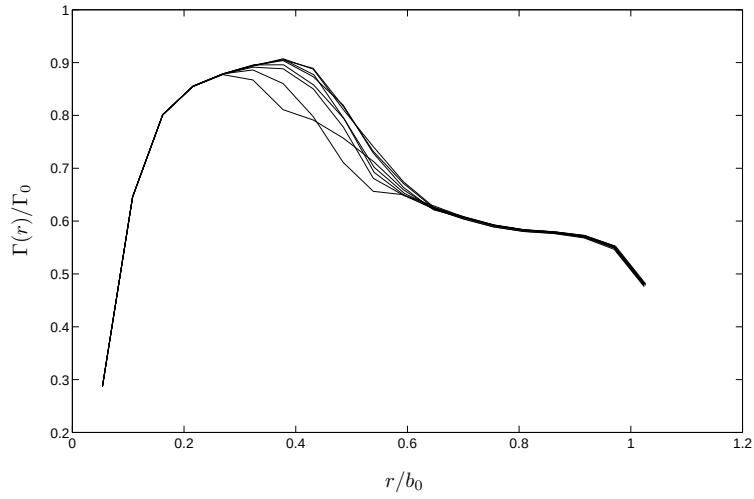


Figure 7.14: Effect of the secondary vortices on the $\Gamma(r)$ profiles in the early case.

on the BH model, concerns the increase of the estimation bias when the pulse duration increases. The relative circulation difference at $\mathcal{T} = 2.5$ is about 9% at 400 ns and 12% at 1 ms. In the late case, it becomes 26% at 400 ns and 29% at 1 ms. Two reasons are proposed to explain this phenomenon. It is first due to the presence of the secondary vortices, with opposite circulation, which induces an underestimation of the primary vortex circulation on which the model is positioned. In this case, it is not considered as being an estimation error. The bias is naturally higher for larger pulses. The second reason is related to the fact that the more important is the radial velocity smoothing the higher is the error on the vortex positions. This directly induces an error on the primary vortex strength due to the algorithm splitting (see Section 7.3.2).

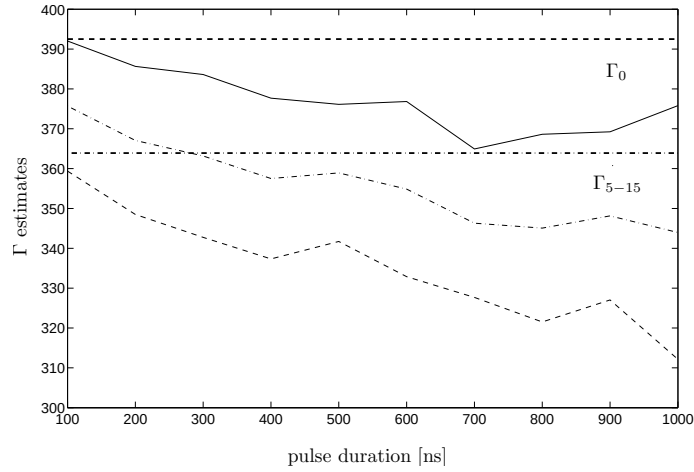


Figure 7.15: Wake vortices circulation estimates (left and right vortices and mean value) obtained for different pulse duration from 100ns to 1ms in the early case (at dimensionless time $\mathcal{T}=2.5$). The values of Γ_0 and Γ_{5-15} (computed on the CFD database) are given for comparison.

Moreover, when the vortex age increases, their spatial coherence decreases and the intensity of the secondary vortices becomes more important compared to the intensity of the primary one. The destructive interaction between them also induces a loss in its axial symmetry. Therefore, the use of a Burnham-Hallock model tends to be less physically relevant. It can also be seen that the estimates obtained at $\mathcal{T} = 5$ are more accurate than the Γ_{5-15} . One of the reasons is that the method proposed here is less sensitive to spatial coherence loss as it uses a model-based estimation which tends to reduce the estimation variance over the whole vortex region.

Fig. 7.17 illustrates the influence of the size of spatial weighting function,

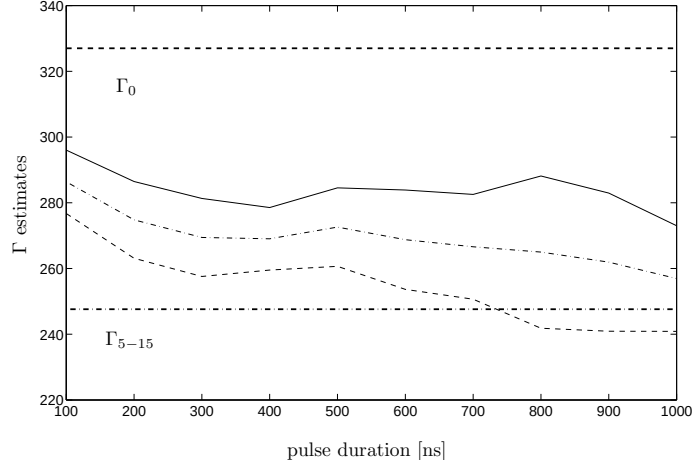


Figure 7.16: Wake vortices circulation estimates (left and right vortices and mean value) obtained for different pulse duration from 100ns to 1ms in the late case (at dimensionless time $\mathcal{T}=5$). The values of Γ_0 and Γ_{5-15} (computed on the CFD database) are given for comparison.

$W(r)$, given at Eq. 7.12, on the estimation of the nearest primary vortex circulation at $\mathcal{T}=2.5$. The estimation bias decreases by reducing the size of $W(r)$. Although, the bias tends to zero, the algorithm is more likely to be unstable for small σ_k .

A maximum likelihood estimation of the vortex parameters has also been proposed in [Frehlich 2005]. This vortex tracking algorithm directly works on the average spectrum obtained from adjacent line-of-sights and the radial velocity profile used for the optimization is given by a Burnham-Hallock model. The vortex core radius is a priori known. A comparison of the performance of this algorithm and the one proposed in this thesis is difficult because the estimation bias and variances are not given depending on the same quantity, i.e. the ratio accumulation level versus SNR instead of the radial velocity estimation noise. Nevertheless, the algorithm proposed here has the major advantage to be much faster thanks to the measurement model which allows to work directly on the estimated radial velocity maps where the other one has to use the spectral estimate for each point of the measurement grid.

7.4 Conclusions

Whereas recent projects have clearly confirmed the ability of fiber-based LIDAR systems to track and detect wake vortices, the measure of their intrinsic

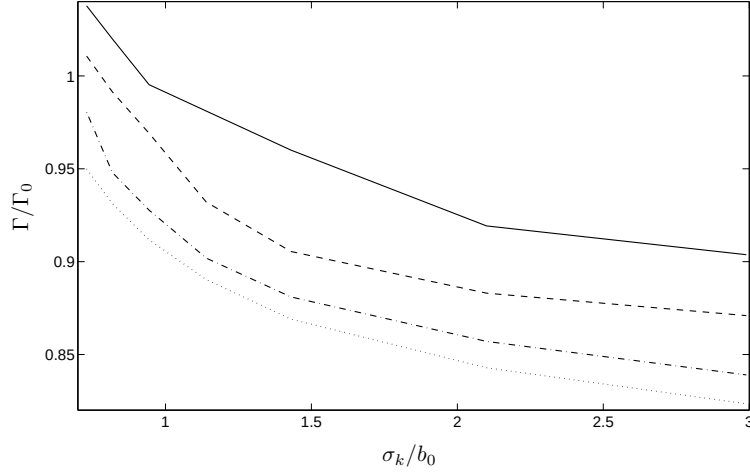


Figure 7.17: Influence of the kernel size σ_k on the estimation of the primary vortex circulation for different pulse durations in the early case : 200 ns (solid), 400 ns (dash), 800 ns (dash-dot), 1 μ s (dot).

parameters, such as their circulation, is still a challenging problem at the low pulse energy imposed by those devices. The main limitation is related to the infeasibility for monostatic LIDAR systems to measure all the components of the wind field. A combinaison of two LIDAR could be used but yields in an increase of the system complexity and set up difficulties.

A priori information is needed and the use of vortex models, such as the Burnham-Hallock one, greatly helps in reducing the indetermination on the vortex strength. However, these models are not applicable directly due to the multiple phenomena affecting the vortex distribution on the radial velocity maps. Thanks to the theoretical studies made in this thesis on the LIDAR time-frequency distributions, a complete measurement model has been developed based on the derivation of the SPWVD first order moment. This model takes into account the effects of the range-dependent SNR as well as the filtering induced by both the LASER pulse and the time-frequency analyzing windows. Moreover, as it is defined in the velocity space, it becomes a powerfull tool to predict what would be the measured velocity map for any input wind fields, measurement parameters and scanning configuration.

In this thesis, both the Burnham-Hallock and the measurement models have been integrated in a wake vortex characterization algorithm. This estimation method takes the form of an inversion problem in which the retrieved radial velocity maps are compared with the vortex model after its transformation by

the measurement one. The vortex model parameters are thus optimized until the output of the measurement model perfectly matches with the measured velocity map. This algorithm has been validated on realizations of a Burnham-Hallock model with a Gaussian estimation error and the effect of the pulse duration has been assessed. The same study has been done on a LES of a vortex pair in weak atmospheric turbulence. Furthermore, the influence of secondary vortices in the case of wake vortex in ground effect has also been analyzed thanks to the use of realistic LES simulations.

The measurement model could be exploited systematically to analyze to influence, or even optimize, the scanning configuration. It could for example be used to further study the performance of an onboard system with axial detection. The algorithm presented above for estimation of the maximum radial velocity is also a promising tool for the estimation of the maximum tangential velocity as well as the total circulation. This method must still be validated and compared to the model-based estimation algorithms.

Conclusions

THE characterization of wake vortices by means of remote sensing devices such as the coherent Doppler LIDAR is a complex problem which cannot be solved by restraining the analysis to only LIDAR or fluid dynamics considerations. Fortunately, this work has benefited from the interaction with a number of partners from different science and engineering disciplines, in the framework of various wake vortex projects. As a result, it has been possible to address the subject through different aspects, such as LIDAR performance simulation and design or the development of estimation algorithms. Furthermore, our collaboration with the G. Winckelmans' team at the UCL brought us their expertise in fluid dynamics and provides us with realistic large eddy simulations (LES) databases of wake vortices in ground effect, with or without cross wind, or in weak turbulence. The participation to test campaigns, particularly at the Orly airport with ONERA, has also been an opportunity for us to keep this study as close as possible to practical implementation problems.

Now is the time to conclude this thesis by reviewing its main contributions to the field and exposing the more relevant future works.

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8.1 Towards an integrated LIDAR simulation

The main originality of the LIDAR simulator developed in this thesis resides in the integration of different levels of numerical simulation in order to give a complete vision of the LIDAR design problem. Indeed, it combines simulation techniques dedicated to fluid dynamics, optical propagation through the turbulent atmosphere and time-domain LIDAR signal synthesis. More specifically, this simulator has been designed to support the LIDAR development in two complementary ways.

The first way is the simulation of the coherent Doppler LIDAR performance depending on the LASER source, the telescope configuration and the atmospheric state. It reposes on the well-known phase-screen method (PSM) which allows one to simulate the propagation of an arbitrarily distributed LASER beam in a turbulent media as well as complex optical systems. This technique has notably been applied to the design of the telescope in the LASEF project. A number of simulations have thus been performed in order to determine its optimal parameters. This led to the development of a monostatic system based on a Newton telescope emitting a collimated Gaussian beam with a 3 cm e^{-1} irradiance radius. The receiver lens area has an aperture with a 10 cm diameter.

The second purpose of the LIDAR simulator is the generation of measured signals by scanning through either analytical models or LES databases of wake vortices in ground effect or in weak atmospheric turbulence. It also simulates the instantaneous fluctuations of the signal heterodyne power caused by atmospheric turbulence, in addition to those produced by the speckle effect. A Monte-Carlo Metropolis Hasting simulation is therefore performed from the statistics of the LIDAR coherent responsivity obtained by the PSM and thereby provides enough signal power realizations for a complete scanning containing thousands of line-of-sights. This operating mode has been extensively used for the design and the validation of advanced signal processing and estimation algorithms.

The simulation of the LIDAR measurement highlights the problem of considering the atmospheric turbulence in both the fluid and the optical propagation simulations. In this work, based on the optical scintillation theory and for the monostatic configuration, it has been assumed that the overlap between the turbulent scales simulated by these two simulations was sufficiently small such that they do not participate to the same performance-limiting processes. Hence, scales of turbulence on the order of magnitude or smaller than either the length of the first Fresnel zone or the beam coherence radius, depending on the fluctuation regime, only generate optical perturbations, i.e. optical scintillation, heterodyne power fluctuation and system gain deterioration with range. Fluid simulations are however supposed to only induce a decrease of the signal correlation length due to both the turbulence and the fast varying velocity fields. This assumption allows to consider different level of atmospheric turbulence

with the same LES simulations. More advanced studies should nevertheless be done to integrate further these two simulation modes in order, for example, to generate an optical turbulence for which the statistics, e.g. structure functions or turbulence spectrum, are consistent with those of the fluid simulations.

8.2 The LIDAR signal Cohen's class

Even if they are necessary at different levels of the system design, numerical simulations of the LIDAR measurement are not always appropriate for analyzing the influence of the system parameters with given atmospheric conditions. The major limiting factor is the computation time, i.e. hours for LIDAR simulations and weeks for realistic fluid dynamics ones. Complementary but yet powerful tools for wind statistics and wake vortex circulation estimation are the LIDAR time-frequency statistical distributions which extend the classical concept of signal spectrum. They can be used for direct processing when little knowledge about the signal physics is available but also, in contrary, for signal modeling and advanced estimation algorithm design.

Based on already existing analytical expressions of the LIDAR signal covariance, a complete formulation of the LIDAR signal Cohen's class has been derived in this thesis. Thanks to this formulation, it is now possible to immediately obtain the time fluctuations of the signal spectrum for a given velocity profile and any LASER source parameters. This formulation has been applied to the general case of wind statistics estimation and has been particularized to the spectrogram and the Smoothed Pseudo Wigner-Ville distribution (SPWVD) of LIDAR signals measured for wind gradient and Gaussian wind turbulence.

Besides that, it also provides us with a clear understanding on what is actually estimable and with which precision. Hence, it has been demonstrated that the Wigner-Ville distribution (WVD) of the LIDAR signal, observed at a given observation time, is obtained by the convolution in frequency between the pulse WVD and a weighted Doppler frequency distribution which depends on the exact velocity distribution and on the LIDAR and atmospheric parameters. This analysis has been carried out a step further by introducing a time-frequency observation window and therefore considering the effect of the signal processing itself on the quality of the estimation.

Concerning the estimation of the mean radial velocity and the velocity dispersion, expressions for the first and second-order statistical moments of the average LIDAR SPWVD have been derived under the Gaussian pulse assumption. They confirm that the retrieved radial velocity profile is given by the spatial convolution of the exact radial velocity with a filtering kernel which is a function of the spatial pulse profile, the analyzing window and the LIDAR system gain. Similar results have been obtained for the second order moment. It gives us a way to estimate the velocity dispersion for particular conditions in which the

velocity is assumed to be statistically homogeneous in the volume analyzed at a given observation time.

Having derived the LIDAR equations for the general Cohen's class allows us to extend the existing estimation algorithms to a wider set of methods performing directly the signal analysis in the time-frequency plane. Nevertheless, we have limited our study to separable in time and frequency Gaussian analyzing windows where more sophisticated ones could be used. This constraint have been imposed by the necessity to conserve the physical interpretation of the derived equations, which is not possible for more complex windows such as the one of Choi-Williams or Rihaczek having the interesting property to reject the interference terms outside the signal location. This study must be extended to the estimation of an atmospheric wind turbulence described by a more general structure function. It would notably allow to develop estimation algorithms of the turbulent eddy dissipation rate (TEDR) based on the estimation of the spectral width. The background wind turbulence is indeed an important factor to evaluate the wake vortex dwell time.

8.3 Analyzing the wake vortex signal

The theoretical foundation, exposed above, has been proven to be an interesting source of development for wake vortex spectral modeling. The average LIDAR spectrogram and SPWVD have shown to be obtained by the space-velocity convolution between the actual radial velocity profile along a given line-of-sight and a space-velocity function depending on both the pulse WVD and the analyzing window (see Theorem 4.7). Not only this knowledge tells us how to deconvolve the time-frequency plane, i.e. with which kernel, but it also informs us on what information is the most likely to disappear as a result of this smoothing. This latter issue is actually an extension of the concepts of atmospheric and pulse-dominated regimes proposed by Frehlich in [Frehlich 1997]. The pulse-dominated regime has therefore been introduced to translate the oversmoothing of the wind statistics by the pulse power spectrum in such a way that its statistical moments become very difficult to estimate. The same phenomenon appears when sensing wake vortices. In this case, the weighted velocity distribution, defined in Theorem 4.3, is multimodal and contains, for example, a clearly distinct mode at the location of the maximum radial velocity, related to the maximum tangential velocity of the vortex field. If the pulse WVD, the time-frequency analyzing function or both are too large compared to the fluctuations of the mean radial velocity, this modality is lost. To characterize this effect, we have therefore introduced the wake-vortex regime.

Based on this analysis, an adaptive estimation algorithm has been developed which uses a spectral model with a range-varying number of kernels. The model order depends on the regime and therefore introduce more kernels in the

atmospheric or wake vortex regimes than it does in the pulse-dominated one. One of the estimates extracted from this model is the maximum radial velocity which can be used for vortex strength estimation. This adaptive processing thus allows to better describe the radial velocity distribution, in particular when it is the less altered.

The average LIDAR Cohen's class given by Theorem 4.5 suggests to exploit the time-frequency correlation imposed by the pulse duration and the analyzing window, and therefore to model the wake vortex signal directly in the time-frequency domain. This two-dimensional modeling is more likely to provide better estimates for high pulse accumulation regimes than the classical spectral model defined at each time instant. Nevertheless, its robustness to speckle and noise should be assessed. Moreover, the robustness of the estimation algorithm of the radial velocity maximum to measurement conditions must also be performed in order to provide an evaluation of their estimation variance, a quantity necessary for the validation of the wake-vortex circulation estimator based on it.

8.4 Detection of wake vortex

Pulse accumulation is a necessary signal processing technique to obtain more reliable estimates of the wind statistics by exploiting the existing correlation of the instantaneous frequency between adjacent line-of-sights while reducing the effects of independent processes such as the speckle or the photodetection noise. However, this aggregation must be carried out by taking into account a number of spatial constraints on the velocity maps to reconstruct. Hence, an increase of the accumulation level always leads to a decrease of the estimation variance at the expense of the spatial resolution and inversely. Reconstruction methods have been formalized in this thesis and tested for both the transverse and the axial wake vortex detection cases.

For the transverse detection case, a kernel-based accumulation is proposed in order to better define the trade-off between the estimation variance and the spatial resolution. This analysis has pointed out the fact that, as a result of the angular scanning and the decreasing signal-to-noise ratio, the kernel size should grow with range if a homogeneous estimation variance is needed. The resulting decrease of the spatial resolution with range is found to have an impact on the subsequent wake vortex characterization algorithms. The exact influence of this effect on the estimation variance of the vortex circulation must however be studied further.

In the framework of the FIDELIO project, the main efforts were directed toward the validation of wake vortex detection with a fiber-based LASER source and an axial configuration. In this configuration, the problems related to the velocity statistics smoothing or the estimation variance reduction, while keeping

an acceptable spatial resolution, reaches their paroxysm due to the low radial velocity dynamics. Hence, although it has been proven by simulation in other studies that an axial velocity component actually exists, the estimation noise combined with the effects of atmospheric turbulence prevents us, at low pulse energy, to detect discriminant vortex signatures inside the reconstructed maps. High pulse repetition frequencies are therefore necessary. Despite of a pulse energy limited to $120\ \mu\text{J}$, vortex signatures have clearly been detected with the ONERA source during the Orly campaign thanks to a frequency of 8 kHz and a good beam quality. Even if works have still to be done to validate it completely, especially at high altitudes, these tests have permitted us to position the fiber LASER sources as concurrents to classical solid-state sources.

The FIDELIO project was also a good occasion to test the bi-Gaussians spectral model developed earlier in the framework of the IWAKE project. The results obtained in this project with this model have shown that the larger Gaussian would carry more discriminant features about the wake vortex itself than a classical single-kernel model. This model turns out to be a particular case of the more general spectral model developed in this thesis. It has furthermore been observed by processing simulated signals, provided by ONERA, that it is only valid for specific conditions binding the pulse duration to the radial velocity distribution in the sensed volume. The statistics obtained from this second Gaussian cannot be related to any radial velocity statistics and it is therefore not adapted for wake vortex characterization. However, in its validity range, simulations have confirmed its good wake vortex detection capabilities.

8.5 Wake vortex characterization

Whereas recent projects have clearly confirmed the ability of fiber-based LIDAR systems to track and detect wake vortices, the measure of their intrinsic parameters, such as their circulation, is still a challenging problem at the low pulse energy imposed by those devices. The main limitation is related to the infeasibility for monostatic LIDAR systems to measure all the components of the wind field. A combinaison of two LIDAR could be used but yields in an increase of the system complexity and set up difficulties.

A priori information is needed and the use of vortex models, such as the Burnham-Hallock one, greatly helps in reducing the indetermination on the vortex strength. However, these models are not applicable directly due to the multiple phenomena affecting the vortex distribution on the radial velocity maps. Thanks to the theoretical studies made in this thesis on the LIDAR time-frequency distributions, a complete measurement model has been developed based on the derivation of the SPWVD first order moment. This model takes into account the effects of the range-dependent SNR as well as the filtering induced by both the LASER pulse and the time-frequency analyzing windows. Moreover,

Algorithms	Constraint on τ	Results
Estimation of V_{θ}^{max}	low τ	Fig. 5.12
Adaptive algorithm	low τ	Fig. 5.17 vs. 5.19
Wake vortex detection	high τ	Fig. 5.6
Wake vortex characterization	low τ	Fig. 7.8 and Table. 7.1

Table 8.1: Constraint on the pulse duration τ for the different algorithms developed in this thesis. A short pulse is typically defined by $\Delta t < 400$ ns.

as it is defined in the velocity space, it becomes a powerful tool to predict what would be the measured velocity map for any input wind fields, measurement parameters and scanning configuration.

In this thesis, both the Burnham-Hallock and the measurement models have been integrated in a wake vortex characterization algorithm. This estimation method takes the form of an inversion problem in which the retrieved radial velocity maps are compared with the vortex model after its transformation by the measurement one. The vortex model parameters are thus optimized until the output of the measurement model perfectly matches with the measured velocity map. This algorithm has been validated on realizations of a Burnham-Hallock model with a Gaussian estimation error and the effect of the pulse duration has been assessed. The same study has been done on a LES of a vortex pair in weak atmospheric turbulence. Furthermore, the influence of secondary vortices in the case of wake vortex in ground effect has also been analyzed thanks to the use of realistic LES simulations.

The measurement model could be exploited systematically to analyze the influence, or even optimize, the scanning configuration. It could for example be used to further study the performance of an onboard system with axial detection. The algorithm presented above for estimation of the maximum radial velocity is also a promising tool for the estimation of the maximum tangential velocity as well as the total circulation. This method must still be validated and compared to the model-based estimation algorithms.

Table 8.1 gives a short synthesis on what pulse duration to use depending on the estimation algorithm, with a reference on the main figures or tables describing the results. The wake vortex detection is the only algorithm for which a high pulse duration is requested since it is based on the signal correlation time. In this case, we enter in the atmospheric regime where the spectral width varies depending on the wind gradients and turbulence.

Appendices

Basic concepts and definitions

THE contents of this appendix are presented to clarify some points of terminology. It presents a number of basic concepts and definitions used in this thesis. In particular, the Cohen's class of time-frequency distributions is exposed since its understanding is required to exploit practically the results issued from Chapter 4. Algorithms and estimators which are usually used for LIDAR signal processing are presented in Appendix E.

A.1 Operators and distributions

A.1.1 The Dirac distribution

The Dirac distribution, noted $\delta(x)$, has the following properties:

$$\int \delta(x) \, dx = 1 \quad (\text{A.1})$$

and

$$\int f(x) \delta(x - x_0) \, dx = f(x - x_0). \quad (\text{A.2})$$

A.1.2 The Fourier transform

The Fourier transform $S(\nu)$ of a complex one-dimensional signal $s(t)$ is given by

$$S(\nu) = \int_{-\infty}^{\infty} s(t) e^{-2\pi j \nu t} dt, \quad (\text{A.3})$$

where ν is the frequency variable. Similarly, the inverse Fourier transform is

$$s(t) = \int_{-\infty}^{\infty} S(\nu) e^{2\pi j \nu t} d\nu \quad (\text{A.4})$$

Practically, the FFTW [Frigo 2005] implementation is used both for 1-D signals and 2-D ones, e.g. for the beam propagation simulations (phase-screen method).

A.1.3 The Hilbert transform

Using analytical signals instead of real ones validates the approximation of $x(t)$ as a modulated envelope $m(t)$:

$$x(t) = m(t)e^{-j\omega(t)}, \quad (\text{A.5})$$

where ω is the time-dependent pulsation.

The measured signal at the output of the photodetector is real but is usually converted to complex data in order to be processed by advanced velocity estimation algorithms. Hence, for real measured data $s(t)$ obtained from one LASER shot, we associate a complex valued analytic signal $x(t)$ defined as

$$x(t) = s(t) + j\mathcal{H}\{s(t)\}, \quad (\text{A.6})$$

where $\mathcal{H}\{\}$ is the Hilbert transform of $s(t)$ and $j^2 = -1$. This leads to the following relationship in the frequency domain:

$$X(\nu) = \begin{cases} 2S(\nu) & \text{for } \nu \geq 0 \\ 0 & \text{for } \nu < 0 \end{cases}, \quad (\text{A.7})$$

where ν is the frequency variable and $S(\nu)$ and $X(\nu)$ are respectively the Fourier transform of $s(t)$ and $x(t)$.

A.1.4 The L_1 median filter

The L_1 median filter is mainly used in this work to remove outliers, i.e. an estimate value that is markedly distant from the others, either on 1D or 2D signals. The description below is partially taken from [Barni 1993] where a fast algorithm is proposed.

Given N vectors $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ in $\mathbb{R}^M$, representing N points inside the current filter window, the output $\mathbf{x}_{\text{VM}}$ of the L_1 metric vector median filter is defined by

$$\mathbf{x}_{\text{VM}} \in \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}, \quad (\text{A.8})$$

such that

$$\sum_{i=1}^N |\mathbf{x}_{\text{VM}} - \mathbf{x}_i| \leq \sum_{i=1}^N |\mathbf{x}_j - \mathbf{x}_i| \quad j = 1, 2, \dots, N \quad (\text{A.9})$$

where $|\cdot|$ is the L_1 norm and the subscript VM denotes the vector median operator. In the case where, in the contrary to condition A.8, the output is not constrained to be one of the points inside the filter window, the output is calculated by looking for the vector $\mathbf{x}_{\text{VM}}$ which minimizes the function $f(\mathbf{x}) : \mathbb{R}^N \rightarrow \mathbb{R}$ defined as

$$f(\mathbf{x}) = \sum_{i=1}^N |\mathbf{x} - \mathbf{x}_i| = \sum_{j=1}^M \sum_{i=1}^N |\mathbf{x}(j) - \mathbf{x}_i(j)|, \quad (\text{A.10})$$

where $\mathbf{x}(j)$ denotes the j -th component of vector $\mathbf{x}$.

A.1.5 The 4-D Sobel operator

The Sobel operator is a discrete differentiation operator mostly used to compute an approximation of the gradient of a multidimensional signal. In this thesis, it is used to estimate the gradient of the cost function in the constrained 4-D optimization problem exposed in Chapter 7 for wake vortex characterization.

The Matlab code for the 4-dimensional Sobel filter is given by:

```
Si=zeros(3,3,3,3);
Si(1,1,:,:)=[-1 -2 -1; -2 -4 -2; -1 -2 -1];
Si(1,2,:,:)=2*[-1 -2 -1; -2 -4 -2; -1 -2 -1];
Si(1,3,:,:)=[-1 -2 -1; -2 -4 -2; -1 -2 -1];
Si(2,:,:,:)=zeros(3,3,3);
Si(3,1,:,:)= [1 2 1; 2 4 2; 1 2 1];
Si(3,2,:,:)=2*[1 2 1; 2 4 2; 1 2 1];
Si(3,3,:,:)= [1 2 1; 2 4 2; 1 2 1];
Si=Si/64;
Sj=permute(Si,[4 1 2 3]);
Sk=permute(Si,[3 4 1 2]);
Sl=permute(Si,[2 3 4 1]);
```

A.2 Time-Frequency distributions

A.2.1 The Wigner-Ville distribution

The Wigner-Ville distribution of a non-stationary random signal $x(t)$, noted $W_x(t, \nu)$, is a joint time and frequency energy density defined by

$$W_x(t, \nu) \triangleq \int_{-\infty}^{+\infty} R_x(t, \tau) e^{-2\pi j \nu \tau} d\tau, \quad (\text{A.11})$$

where $R_x(t, \tau) = \langle x(t + \tau/2)x^*(t - \tau/2) \rangle$ is the signal covariance, given by Eq. 4.24 and 4.25 for the LIDAR signal. This distribution is also known as the Wigner spectrum in the signal processing literature. It is covariant by translations in time and frequency which means that if the signal is delayed and modulated, its WVD will be translated of the same quantities in the time-frequency plane. It also satisfies the marginal properties:

$$\int_{-\infty}^{\infty} W_x(t, \nu) d\nu = \langle |x(t)|^2 \rangle, \quad (\text{A.12a})$$

$$\int_{-\infty}^{\infty} W_x(t, \nu) dt = \langle |X(\nu)|^2 \rangle, \quad (\text{A.12b})$$

where $X(\nu)$ is the Fourier transform of $x(t)$. The quantities $\langle |x(t)|^2 \rangle$ and $\langle |X(\nu)|^2 \rangle$ are the marginal energy densities respectively in time and frequency.

Two other important properties are the compatibility with filtering and modulation. The coefficients of the WVD are not necessarily positive. For a determinist analytical signal, Eq. A.11 simply becomes

$$W_x(t, \nu) \triangleq \int_{-\infty}^{+\infty} x(t + \tau/2) x^*(t - \tau/2) e^{-2\pi j \nu \tau} d\tau. \quad (\text{A.13})$$

If $x(t)$ is a sample signal of a stochastic process, this integral is also known as the stochastic Wigner-Ville distribution. In Chapter 4, we refer to the sample Wigner-Ville distribution.

The Wigner-Ville distribution also gives the best time-frequency resolution but has the major drawback of being quadratic. In other words, it is not easily interpretable due to the presence of interference terms between the signal components represented by the waveforms $r(t; z)$ defined in Section 4.1.1. The WVD is usually filtered to be usable in practice which introduces a trade-off between the quantity of interference left and the number of good properties. The class of time-frequency distributions covariant by translations in time and frequency obtained by the smoothing of the WVD is known as the Cohen's class and will be detailed in the next section.

The instantaneous frequency $\nu_x(t)$ of the analytical LIDAR signal is defined at time t by the following expression

$$\nu_x(t) = \frac{1}{2\pi} \frac{d}{dt} \arg\{x(t)\}. \quad (\text{A.14})$$

This quantity gives a description of the signal limited to mono-component signals for which the signal energy is supposed to exits around a given frequency. Under certain conditions, the instantaneous frequency $\nu_x(t)$ can be recover from its Wigner-Ville distribution as its first order moment in frequency, noted $m_{1,x}(t)$:

$$m_{1,x}(t) = \nu_x(t) = \frac{\int_{-\infty}^{\infty} \nu W_x(t, \nu) d\nu}{\int_{-\infty}^{\infty} W_x(t, \nu) d\nu}. \quad (\text{A.15})$$

This property will be used in Section 4.3.3 to estimate the mean radial velocity. The Wigner-Ville distribution is perfectly localized on linear chirp signals. Thus, for a signal $x(t)$ with envelope $g(t)$ and instantaneous frequency $\nu_x(t) = \nu_0 + 2\beta t$, we can write that

$$W_x(t, \nu) = W_g(t, \nu - (\nu_0 + \beta t)). \quad (\text{A.16})$$

A function of particular interest in RADAR signal processing is the narrow-band ambiguity function, noted $A_x(\xi, \tau)$. It gives a measure of the time-frequency correlation of the signal and is defined as

$$A_x(\xi, \tau) = \int_{-\infty}^{\infty} R_x(s, \tau) e^{-2\pi j \xi s} ds, \quad (\text{A.17})$$

where the variables τ and ξ are known respectively as the delay and Doppler coordinates. The elements of the ambiguity function that corresponds to the signal components are mainly located around the origin $A_x(0, 0)$ although the interference terms between signal components appears at distance from the origin proportional to the time-frequency distance between the involved components [Flandrin 1993]. There exists a strong relation between the ambiguity function and the Wigner-Ville distribution as they are dual in the sense of the two-dimensional Fourier transform:

$$A_x(\xi, \tau) = \iint_{-\infty}^{\infty} W_x(t, \nu) e^{-2\pi j(\nu\tau - \xi t)} dt d\nu. \quad (\text{A.18})$$

A.2.2 The Cohen's class

The Cohen's class C_x of an analytical signal $x(t)$ is defined by a 2-D convolution in the time-frequency domain:

$$C_x(t, \nu; \Pi) \triangleq \iint_{-\infty}^{\infty} \Pi(s - t, \xi - \nu) W_x(s, \xi) ds d\xi \quad (\text{A.19})$$

where $W_x(s, \nu)$ is the Fourier transform of the signal and $\Pi(t, \nu)$ is a smoothing function equal to the two-dimensional Fourier transform of a function $F(\xi, \tau)$ called the parameterization function:

$$\Pi(t, \nu) = \iint_{-\infty}^{\infty} F(\xi, \tau) \exp(-j2\pi(\nu\tau + \xi t)) d\tau d\xi \quad (\text{A.20})$$

The major properties of the Cohen's class are conserved if we impose the following normalization:

$$\iint_{-\infty}^{\infty} \Pi(t, \nu) dt d\nu = 1 \quad (\text{A.21})$$

From Eq. A.19, we see that the Wigner-Ville distribution is the element of the Cohen's class for which the function Π is a double Dirac

$$\Pi(t, \nu) = \delta(t) \delta(\nu) \quad (\text{A.22})$$

One of the main properties of the Cohen's class is to convert the classical compromise between time and frequency resolutions to a trade-off between the joint time-frequency resolution and the level of interference terms [Flandrin 1993]. We will principally focus on separable Π functions.

The dual formula of Eq. A.19, using the ambiguity function $A_x(\xi, \tau)$, is given by:

$$C_x(t, \nu; \Pi) = \iint_{-\infty}^{\infty} F(\xi, \tau) A_x(\xi, \tau) e^{-2\pi j(\nu\tau - \xi t)} d\xi d\tau \quad (\text{A.23})$$

This equation allows us to interpret the parameterization function $F(\xi, \tau)$ as a weighting function that rejects the interference terms which are not present in the region of the ambiguity function near its center $A_x(0, 0)$. This equation presents a strong interest for adaptive algorithms since different distributions can be obtained for one computation of the ambiguity function simply by changing F . The Wigner-Ville distribution corresponds to $F(\xi, \tau) = 1, \quad \forall \xi, \tau$.

The spectrogram of $x(t)$ is defined as the squared modulus of the Short-Time Fourier Transform. Due to its simplicity of computation, it is one of the most used time-frequency representation for velocity estimation. The relation between the spectrogram noted $S_x(t, \nu)$, using a temporal analysing window $g(t)$ and the Wigner-Ville distribution is given by the Moyal's formula:

$$S_x(t, \nu) \triangleq \left| \int_{-\infty}^{\infty} x(s) g^*(s-t) e^{-2\pi j \nu s} ds \right|^2 \quad (\text{A.24})$$

$$= \iint_{-\infty}^{\infty} W_g(s-t, \xi-\nu) W_x(s, \xi) ds d\xi \quad (\text{A.25})$$

where $W_g(t, \nu)$ is the WVD of the observation window. In other words, the spectrogram is obtained by 2D smoothing in the time-frequency domain the WVD of $x(t)$ with the WVD of the window $g(t)$.

The spectrogram evaluated at a given μ is usually called periodogram and will be noted $S_{x,\mu}(\nu)$.

The well-known trade-off between time and frequency resolutions of the LIDAR signal spectrogram can be expressed as a conjunction of the fixed LASER pulse duration and the length of the analyzing observation window.

The admissibility conditions for the estimation of the instantaneous frequency as the first order moment of the Cohen's class are expressed in terms of conditions on the parameterization function. The first two conditions concern the existence of the marginal properties:

$$F(0, \tau) = 1 \quad \text{and} \quad F(\xi, 0) = 1. \quad (\text{A.26})$$

It is also necessary to satisfy that:

$$\frac{\delta F}{\delta \tau}(\xi, 0) = 0. \quad (\text{A.27})$$

This last condition is verified for the Gaussian parameterization function.

Wind estimable statistics: a synthesis

IN this appendix are synthesized the main concepts and results obtained in Chapter 4 dealing with estimable wind statistics. It uses the material given in Appendix A about time-frequency representation.

The analytical LIDAR signal

The classical model of the signal:

$$x(t) = \int_0^{+\infty} h(z)^{\frac{1}{2}} \alpha_z r(t; z) dz + n(t)$$

where

- $h(z)$: ensemble averaged system gain over refractive turbulence.
- α_z : multiplicative Gaussian noise with $\langle \alpha_z \rangle = 0$ and $\langle \alpha_z \alpha_{z'}^* \rangle = \delta(z - z')$
- $n(t)$: additive Gaussian noise (shot noise and NEP, no RIN)
- $r(t; z)$: atmospheric slice z at time t for a pulse profile $p(t)$ and at wavelength λ

$$r(t; z) = p(t - 2z/c) e^{-2\pi j f(z)t} \quad \text{and} \quad f(z) = f_{\text{IF}} - 2\lambda^{-1} v_{\text{R}}(z).$$

Remarks:

- . $v_{\text{R}}(z)$ is the **radial** aerosol particle velocity with PDF $\Phi_{\text{V}}(v; z)$
- . $f(z)$ with PDF $\Phi_{\text{F}}(f; z)$
- . Random variables (RV) α_z and $f(z)$ stat. independent

This model corresponds to the *Feuilleté model* of Salamiou or the *Fading Dispersive Model* of Van Trees

Ideal LIDAR signal processing algorithm

If $v_r(z)$ is a random variable with PDF $\Phi_v(v; z)$,

$\Rightarrow$ The (ideal) estimator gives the statistical moments of $\Phi_v(v; z)$, $\forall z$!

The LIDAR Wigner spectrum ($\equiv$ average WVD)

$\Rightarrow$ Perfect localization: first order moment = instantaneous frequency

$$W_x(t, \nu) \triangleq \int_{-\infty}^{+\infty} R_x(t, \tau) e^{-2\pi j \nu \tau} d\tau$$

where $R_x(t, \tau)$ is the signal covariance.

$\Rightarrow$ Gives the **best** Time-Frequency resolution

Estimation of $\Phi_v(v; z)$ from the Wigner spectrum

If $F_p(v)$ is the pulse power spectrum with spectral width w_p ,

the Wigner spectrum at time μ is

$$W'_{x,\mu}(v) = \int_{-\infty}^{\infty} F_p(v - f) \Phi'_F(f; \mu) df + 1,$$

where

$$\Phi'_F(f; \mu) = \int_0^{\infty} I(z; \mu) \Phi_F(f; z) dz.$$

is the **estimable** radial velocity distribution depending on the spatial weighting funct. $I(z; \mu)$ (see Eq. 4.16)

◦ Two **LIDAR regimes**:

▷ Atmospheric regime: $\Phi_F(f; z)$ larger than $F_p(f)$

$\Rightarrow$ sig. stat. defined by $\Phi_v(v; z)$

▷ Pulse-dominated regime : $\Phi_F(f; z)$ shorter than $F_p(f)$

$\Rightarrow$ sig. stat. defined by $F_p(f)$

◦ The **estimable statistical moments** are:

$$\hat{v}_r(\mu) = \int_0^{\infty} I(z; \mu) \bar{v}_r(z) dz \neq \bar{v}_r(z) \quad \Rightarrow \text{pulse filtering}$$

$$\text{and} \quad \hat{\sigma}_v^2(\rho) = \frac{\lambda^2}{4} \left(m_{2,x}(\mu) - \frac{w_p^2}{2} \right)$$

Estimation of the Wigner spectrum by pulse accumulation

The Wigner-Ville distribution contains interference terms!
But tends by ens. averaging to the Wigner spectrum

(Low-level) speckle and interference terms (high-level speckle)
 are thus reduced by pulse accumulation:

on the correlograms to get an estimate of the covariance $R_x(t, \tau)$

or on the sample WVD $\equiv$ (stochastic) WVD

$\Rightarrow$ Does not converge fast enough ! (see Fig. 4.7)

$\Rightarrow$ The Cohen's class gives a way to go further

A class of Wigner spectrum estimators

The Cohen's class of time-frequency distributions is

$$C_x(t, \nu; \Pi) \triangleq \Pi(t, \nu) \otimes_{t, \nu} W_x(t, \nu)$$

where $\Pi(t, \nu)$ is a smoothing function.

$\Rightarrow$ **Doubly biased estimator** in time and frequency !

$\Rightarrow$ A combinaison of ensemble averaging and TF filtering is needed

o Particular cases:

- The **Wigner-Ville distribution**: $\Pi(t, \nu) = \delta(t) \delta(\nu)$
 $\Rightarrow$ Unreadable time-frequency plane!

- The **spectrogram** with observation window $u(t)$: $\Pi(t, \nu) = W_u(t, \nu)$
 $\Rightarrow$ Limited by the Heisenberg principle

★ The **SPWVD** with separable kernel : $\Pi(t, \nu) = g(t) U(-\nu)$
 $\Rightarrow$ Arbitrary resolution in time and frequency possible !

The SPWVD as a Wigner spectrum estimators

With $a(t) = g(t) \otimes_t |p(t)|^2$ and $B(\nu) = U(\nu) \otimes_\nu F_p(\nu)$,
the SPWVD at time μ is:

$$\text{SPW}'_{x,\mu}(\nu) = \int_{-\infty}^{\infty} B(\nu - f) \Phi_F''(f; \mu) \, df + 1$$

where

$$\Phi_F''(f; \mu) = \int_0^\infty I_a(z; \mu) \Phi_F(f; z) \, dz$$

is the **new estimable radial velocity distribution**,

depending on the spatial weighting funct. (see Eq. 4.99)

$$I_a(z; \mu) \approx I(z; \mu) \otimes a(2z/c)$$

◦ Two new **LIDAR regimes**:

▷ Atmospheric regime: $\Phi_F''(f; z)$ larger than $B(f)$

⇒ sig. stat. defined by $\Phi_V''(v; z)$

▷ Pulse-dominated regime : $\Phi_F''(f; z)$ shorter than $B(f)$

⇒ sig. stat. defined by $B(f)$

◦ The **estimable statistical moments** become:

$$\hat{v}_r(\mu) = \int_0^\infty I_a(z; \mu) \bar{v}_r(z) \, dz \quad \Rightarrow \text{No effect of } U(\nu)$$

$$\text{and} \quad \hat{\sigma}_V^2(\rho) = \frac{\lambda^2}{4} (m_{2,x}(\mu) - w_b^2)$$

**The SPWVD's equation suggests to use a
kernel-based spectral modeling
to estimate $\Phi_F''(f; z)$**

On the use of spectral models: deconvolution

Approximation of $\text{SPW}'_{x,\mu}(\nu)$ with K kernels $G(f)$:

$$\hat{S}_{x,\mu}^K(f) = \frac{1}{K} \sum_{i=0}^{K-1} y_i G^i(f - \bar{f}_i) + N(f) \approx \text{SPW}'_{x,\mu}(\nu)$$

where

- $G^i(f)$ is the i^{th} kernel at $\bar{f}_i$
 - $\Rightarrow$ Choose $G(f) = B(f)$ with equidistant $\bar{f}_i \dots$
 - y_i are the weighting factors
 - $\Rightarrow$ To find a (discrete) approximation of $\Phi''_F(f; \mu)$
 - $N(f)$ is the noise power spectrum
 - $\Rightarrow$ Estimable at longer ranges where the SNR is very low
- $\Rightarrow$ **Gives an estimation of $\Phi''_F(f; \mu)$!**

The equivalent Doppler LIDAR signal principle

Given a signal $x(t)$ obtained with a pulse $p(t)$ and analyzed with a window $\Pi(t, \nu)$, the equivalent signal $x'(t)$ is:

$$x'(t) = \int_0^{+\infty} h(z)^{\frac{1}{2}} \alpha_z c(t; z, \Pi) dz + n'(t)$$

where

- $h(z)$: ensemble averaged system gain over refractive turbulence.
- α_z : multiplicative Gaussian noise with $\langle \alpha_z \rangle = 0$ and $\langle \alpha_z \alpha_{z'}^* \rangle = \delta(z - z')$
- $n'(t)$: additive noise, filtered version of $n(t)$ such that

$$W_{n'}(t, \nu; \Pi) = W_n(t, \nu) \otimes_{t, \nu} \Pi(t, \nu)$$

- $c(t; z, \Pi)$: response of the atmospheric slice z at time t of a pulse $k(t)$ with Wigner spectrum

$$W_k(t, \nu; \Pi) = W_p(t, \nu) \otimes_{t, \nu} \Pi(t, \nu)$$

Remarks:

- . The functions of $W_p(t, \nu)$ and $\Pi(t, \nu)$ are interchangeable
 - $\Rightarrow$ A chirp on $p(t)$ could rather be applied on $\Pi(t, \nu)$.

Proofs of theorems

SIGNAL analysis tools have been exposed, notably in Chapters 4, in the form of theorems and corollaries. Their proof is given here. The first section is related to the LIDAR signal Wigner-Ville distribution and the last one on the average LIDAR Cohen's class.

C.1 The LIDAR Wigner-Ville distribution

Theorem C.1 (4.1) *The Wigner-Ville distribution of an elementary waveform $r(t; z)$, noted $W_r(t, \nu; z)$, is obtained by the convolution in frequency between the pulse WVD and the probability density function of $f(z)$, noted $\Phi_F(f; z)$:*

$$W_r(t, \nu; z) = \int_{-\infty}^{\infty} W_p(t - 2z/c, \nu - \xi) \Phi_F(\xi; z) d\xi.$$

Proof : We first insert the covariance of $r(t; z)$ given by Eq. 4.25 in the definition of the WVD in Eq. A.11 leading to

$$W_r(t, \nu; z) = \int_{-\infty}^{+\infty} p(t - \tau/2) p^*(t + \tau/2) M_F(\tau; z) e^{-2\pi j \nu \tau} d\tau.$$

The final equation is then found by using Eq. 4.26b and remembering that the WVD is compatible with modulation. ■

Corollary C.1 (4.1) *For a Gaussian pulse profile, the Wigner-Ville distribution of the waveforms $r(t; z)$ is separable in time and frequency:*

$$W_r(t, \nu; z) = |p(t - 2z/c)|^2 \Gamma_r(\nu; z),$$

where $\Gamma_r(\nu; z)$ is given by

$$\Gamma_r(\nu; z) = \int_{-\infty}^{+\infty} F_p(\nu - f) \Phi_F(f; z) df,$$

which is the Fourier transform of the decorrelation function $\gamma_r(\tau; z)$.

Proof : By replacing in Eq. 4.52, the pulse WVD by its equation in Eq. 4.50 and separating time-dependent and frequency-dependent terms, we easily find Eq. 4.54. ■

Theorem C.2 (4.2) *The average Wigner-Ville distribution of the LIDAR signal is the sum of the WVD of all the contributions $r(t; z)$ weighted by the system dependent factor $h(z)$:*

$$W_x(t, \nu) = \int_0^\infty h(z) W_r(t, \nu; z) dz + 1,$$

where $W_r(t, \nu; z)$ is given by Eq. 4.52 of Theorem 4.1.

Proof : This result is obtained by replacing the signal covariance in Eq. A.11 by its equation given by Eq. 4.24 and switching the integrals over z and τ :

$$W_x(t, \nu) = \int_0^\infty h(z) \int_{-\infty}^{+\infty} R_r(t, \tau; z) e^{-2\pi j \nu \tau} d\tau dz + 1,$$

where $R_r(t, \tau; z)$ is given by Eq. 4.25. ■

Corollary C.2 (4.2) *For a Gaussian pulse profile, the LIDAR average WVD, analyzed at a given time μ , is an infinite sum of spectral functions $\Gamma_r(\nu; z)$ weighted by $I(z; \mu)$:*

$$W'_{x,\mu}(\nu) = \int_0^{+\infty} I(z; \mu) \Gamma_r(\nu; z) dz + 1$$

Proof : This results is obtained by replacing Eq.4.54 in Eq. 4.56 and using $c(z; \mu) = h(z)|p(t - 2z/c|^2$. The normalized WVD $W'_{x,\mu}(\nu)$ is obtained by replacing $c(z; \mu)$ with $I(z; \mu)$. ■

Theorem C.3 (4.3) *The Wigner-Ville distribution of the LIDAR signal at a time μ can be written as the convolution between the pulse power spectrum and a weighted frequency distribution:*

$$W'_{x,\mu}(\nu) = \int_{-\infty}^{\infty} F_p(\nu - f) \Phi'_F(f; \mu) df + 1$$

where we define the following quantity

$$\Phi'_F(f; \mu) = \int_0^\infty I(z; \mu) \Phi_F(f; z) dz$$

which can be interpreted as the weighted average distribution of the signal frequency at time μ , noted $f(\mu)$, taking into account the contribution of the atmospheric slices at distances z around $\rho = c\mu/2$.

Proof : Combining Eq. 4.54 and Eq. 4.55 with Eq. 4.16 gives the following normalized average WVD

$$\begin{aligned} W'_{x,\mu}(\nu) &= \int_0^\infty I(z; \mu) \int_{-\infty}^{\infty} F_p(\nu - f) \Phi_F(f; z) df dz + 1 \\ &= \int_{-\infty}^{\infty} F_p(\nu - f) \Phi'_F(f; \mu) df + 1 \quad \blacksquare \end{aligned}$$

Corollary C.3 (4.3) *The Wigner-Ville distribution can be written as a space-velocity distribution:*

$$W'_x(\rho, v) = \int_{-\infty}^{\infty} F_p(v - v') \Phi'_v(v'; z) dv' + 1,$$

where $\rho = c\mu/2$ and

$$\Phi'_v(v; \mu) = \int_0^{\infty} I(z; \mu) \Phi_v(v; z) dz.$$

Proof : The weighted distribution of the radial velocity is obtained by

$$\begin{aligned} \Phi'_v(v; \mu) &= \frac{2}{\lambda} \Phi'_F(f_{IF} - 2\lambda^{-1}v; z) + 1 \\ &= \int_0^{\infty} I(z; \mu) \Phi_v(v; z) dz + 1 \end{aligned}$$

Moreover, if we further project $F_p(\nu)$ in the velocity space using Eq. 4.11 to get

$$F_p(v) = (\sqrt{\pi}w_v)^{-1} \exp(-v^2/w_v^2)$$

where $w_v = 2w_p/\lambda$ [m/s] is the velocity standard deviation of $F_p(v)$, Eq. 4.58 leads to Eq. 4.60. ■

C.2 The LIDAR Cohen's class

Theorem C.4 (4.4) *The average Cohen's class, denoted $C_x(t, \nu; \Pi)$, of a Doppler LIDAR signal obtained with a pulse profile $p(t)$ and analyzed by a function $\Pi(t, \nu)$ is the average Wigner-Ville distribution of an analytical LIDAR signal that would have been obtained if the atmospheric volume had been sensed with a pulse profile $k(t)$ having the following Wigner-Ville distribution:*

$$W_k(t, \nu; \Pi) = \iint_{-\infty}^{\infty} W_p(s - t, \xi - \nu) \Pi(s, \xi) ds d\xi,$$

where $W_p(t, \nu)$ is the WVD of $p(t)$.

Corollary C.4 (4.5) *The average Cohen's class of a Doppler LIDAR signal is a weighted sum of elementary Wigner-Ville distributions, denoted $W_c(t, \nu; \Pi, z)$, giving the contribution to each atmospheric slice z :*

$$C_x(t, \nu; \Pi) = \int_0^{+\infty} h(z) W_c(t, \nu; \Pi, z) dz + 1$$

where $W_c(t, \nu; \Pi, z)$ is the WVD of an elementary waveform $c(t; z, \Pi)$ and is defined by the following convolution:

$$W_c(t, \nu; \Pi, z) = \int_{-\infty}^{\infty} W_k(t - 2z/c, \nu - \xi; \Pi) \Phi_F(\xi; z) d\xi$$

Proof : The proof of the previous statements (Theorem 4.4 and Corollary 4.5) is relatively straight forward. It is obtained by inserting Eq. 4.52 and Eq. 4.56 in Eq. A.19 and switching the integrals to introduce the Wigner-Ville distributions $W_c(t, \nu; \Pi, z)$ and then $W_k(t, \nu; \Pi)$. ■

The Lidarsim library

LIDARSIM is a simulation program based on a collection of libraries initially developed as a tool for coherent pulsed Doppler LIDAR design. It is principally suited for the optimization of systems used to detect wind shears or wake vortices. It also provides a complete framework for signal processing algorithms development, e.g. radial velocity profile and wake vortex parameters estimation. This work is the result of the collaboration of two institutes of the Université catholique de Louvain, UCL : the IMMC for the fluid dynamics numerical simulations and icTEAM for the aspects related to optical propagation and signal processing.

D.1 Introduction

The need for such a simulation is primarily related to the increasing use of fiber-based LASER sources for environmental and security applications. Though they usually operate at low pulse energy (less than the millijoule), they provide a high pulse repetition rate. The low signal-to-noise ratio obtained with those sources compared to classical ones is therefore commonly compensated by pulse accumulation at the signal processing level. When operating at low altitude, the effect of refractive turbulence has to be taken into account due to its unnegligible effects on the system efficiency. Inversely, at higher altitudes, the signal-to-noise ratio is reduced by the decrease in aerosol particles density. The complex internal structures of the fluid flow to analyze have in return an important influence on the LIDAR backscattered signal properties, such as decorrelation or time fading, and it is therefore necessary to adjust the pulse characteristics as well as the scanning configuration in order to attenuate their effects. The combination of these observations leads us to the conclusion that the design of a LIDAR system for a given application necessitates the parallel optimization of the LIDAR (LASER source and telescope) and the algorithms used to analyse the signals, with a deep understanding of the atmospheric phenomena.

Lidarsim integrates different levels of numerical simulation in order to give a complete vision of the design problem. Hence, it combines simulations techniques dedicated to fluid dynamic processes, optical propagation through turbulent

atmosphere and time-domain LIDAR signal synthesis. More specifically, it has been designed to allow the following operating modes :

1. The simulation of a coherent Doppler LIDAR system efficiency and SNR with given LASER and telescope parameters and refractive turbulence level. This mode is dedicated to the analysis of the performance of a LIDAR depending on its characteristics and on the atmosphere properties (aerosol concentration, turbulence,...).
2. For a given LIDAR, generation of measured signals by scanning through either CFD (Computational Fluid Dynamics) simulations or synthetical fluid flows. This mode is typically used for the design of signal processing algorithms or for the performance evaluation of an existing processing algorithm on different LIDAR systems.

These operating modes are explained in more details in the next section. illustrate the different functionalities.

D.2 Simulations description

Lidarsim is composed of interconnected but separately usable numerical simulation libraries. The main characteristics of the whole simulation are the followings :

1. It is designed to be connected with the CFD simulations. The use of synthetic fluid flows based on classical models such as Burnham-Hallock is also possible. The LIDAR simulation program scans inside the CFD volume in order to retrieve the radial velocity profile for each line-of-sight. The scanning pattern is for the moment limited to a two-dimensional slice of atmosphere defined inside the three-dimensional fluid flow database.
2. It uses optical numerical simulation techniques in order to take into account LASER propagation inside the telescope such as focalization of the beam and truncature effects. The type of telescope is selectable and the characteristics of the output LASER beam can be defined.
3. The propagation of the LASER beam through a turbulent atmosphere is simulated in order to compute the average system efficiency as well as the signal-to-noise ratio with given output beam characteristic and atmospheric turbulence parameters. The instantaneous signal power is generated for each line-of-sight by use of appropriate Monte Carlo simulations.
4. A measured signal is generated for each line-of-sight. The effect of the refractive turbulence on the signal is recomputed for each line-of-sight

in order to take into account the variation of the SNR with time. The parameters of the LIDAR (wavelength, pulse energy and duration, pulse repetition frequency,...), the receiver, the scanner and the telescope can be defined separately.

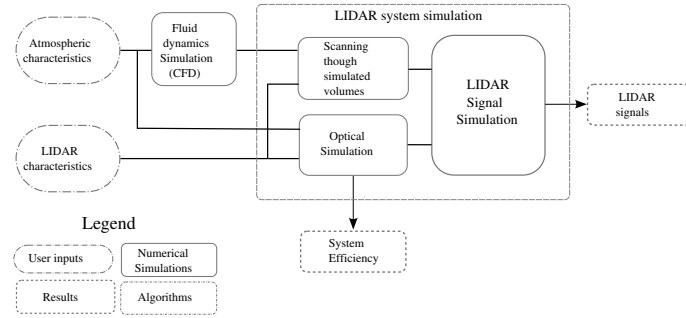


Figure D.1: Global architecture of the Lidarsim library.

D.3 Library architecture

Lidarsim is a C++ library composed of four interconnected numerical simulation modules parameterized by two input files listing the LIDAR (LASER source, telescope and receiver) parameters and the atmospheric parameters :

1. The optical simulation: it computes the performance of a given lidar for given atmospheric conditions. The main LIDAR parameters are : wavelength, pulse duration and energy, telescope geometry, receiver bandwidth, photodetection noise power, detector sensitivity. The main atmospheric parameters are: aerosol particles density, molecular attenuation coefficient, refractive turbulence level and characteristic scales.
2. The fluid dynamics simulations: The whole simulator has been tested on different fluid databases: wake vortex in ground effect with or without cross wind, wind alone.
3. The scanning simulation: The line-of-sight of the complete scanning pattern are defined and the radial wind velocity profiles are retrieved.
4. The LIDAR signal simulation: this is a central module connecting the information given by the optical and scanning simulation in order to generate realistic measured signals.

The figure D.1 represents the architecture of the simulation program.

D.4 Some results

Fig. D.2 to D.3 illustrates some of the output of the simulation program.

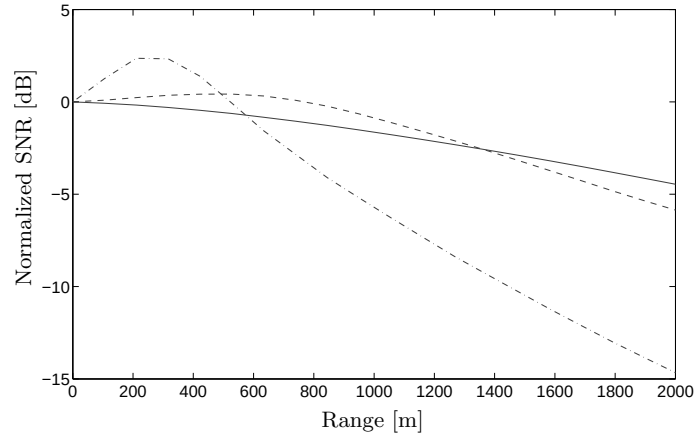


Figure D.2: Normalized signal-to-noise ratio of a $1.55\mu\text{m}$ coherent lidar in monostatic configuration.

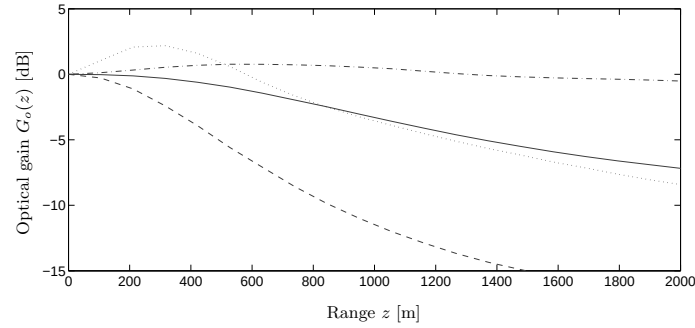


Figure D.3: Optical gain of a $2\mu\text{m}$ coherent lidar in monostatic and bistatic configurations for a moderate ($C_n^2 = 10^{-13} \text{ m}^{-2/3}$) and strong ($C_n^2 = 10^{-12} \text{ m}^{-2/3}$) turbulence levels: bistatic, moderate turb. (solid); bistatic, strong turb. (dash); monostatic, moderate turb. (dot-dash); monostatic, strong turb. (dot). The output beam is collimated with a $1/e^2$ irradiance radius of 30 mm.

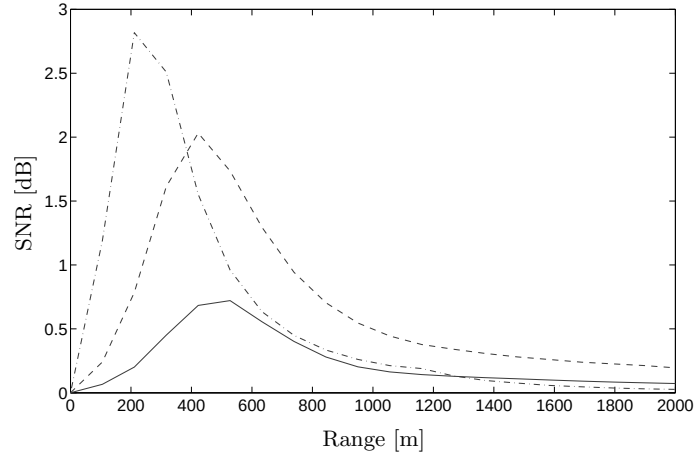


Figure D.4: Standard deviation of the signal-to-noise ratio of a $1.55\mu\text{m}$ coherent lidar in monostatic configuration with a 30 mm output beam focalized at 500 m : weak turb. ($C_n^2 = 10^{-15} \text{ m}^{-2/3}$, solid); moderate turb. ($C_n^2 = 10^{-13} \text{ m}^{-2/3}$, dash); strong turb. ($C_n^2 = 10^{-12} \text{ m}^{-2/3}$, dot-dash);

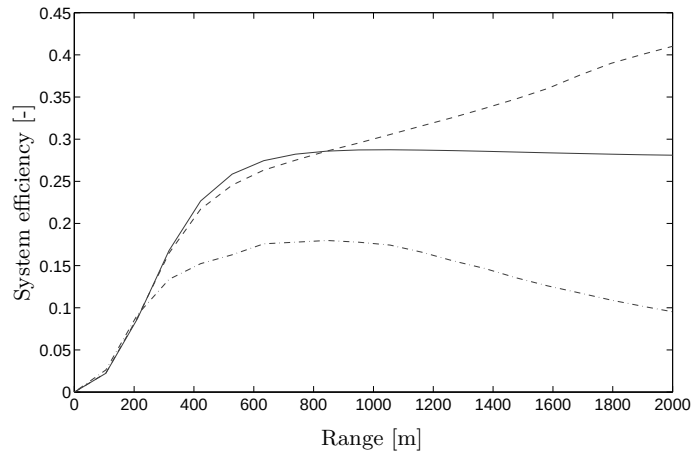


Figure D.5: System efficiency of a $1.55 \mu\text{m}$ coherent lidar in monostatic configuration focalized at 1 km for different levels of refractive turbulence : weak turb. ($C_n^2 = 10^{-15} \text{ m}^{-2/3}$, solid); moderate turb. ($C_n^2 = 10^{-13} \text{ m}^{-2/3}$, dash); strong turb. ($C_n^2 = 10^{-12} \text{ m}^{-2/3}$, dot-dash); The output beam has a $1/e^2$ irradiance radius of 15 mm.

Classical velocity profile estimation techniques

THE algorithms described in this section are commonly used in today's LIDAR signal processing. They are presented here in order to be used later as a reference to analyze the performance of the new algorithm proposed in this thesis.

E.1 Maximum likelihood estimation

The atmospheric signal parameters $\Theta = \{v, S, w_v\}$ to estimate are the mean radial velocity over the sensing volume v , the average signal power S as well as the radial velocity standard deviation w_v . The noise power N is directly estimated from signal samples at the end of the recording associated to farthest propagation distance where the SNR is extremely low. The signal data are represented by:

$$x_k = s_k \exp(4\pi j v T_s k / \lambda) + n_k, \quad (\text{E.1})$$

where the first term is the signal from the atmospheric return and n_k is the uncorrelated noise. The covariance matrix $R_{kl}(\Theta)$ is given by the Zrnić model:

$$R_{kl}(\Theta) = S \exp(4\pi j v (k - l) T_s / \lambda - 2\pi^2 w^2 T_s^2 (k - l)^2), \quad (\text{E.2})$$

where $w = 2w_v / \lambda$ is the spectral width. The vector of M consecutive data sample is noted $\mathbf{x}$ and the corresponding covariance matrix $\mathbf{R}(\Theta)$. In the limit of large number of data samples per velocity estimate the ML estimator is unbiased and approaches the Cramer-Rao Bound (CRB). It is therefore an asymptotically efficient estimator [Van Trees 2001a]. The ML estimator $\hat{\Theta}$ is the value of Θ that maximizes the log-likelihood function $L(\mathbf{x}, \Theta)$. For a zero-mean circular Gaussian random process, we have:

$$L(\Theta, \mathbf{x}) = -\mathbf{x}^{*T} \mathbf{R}^{-1}(\Theta) \mathbf{x} - \ln[|\mathbf{R}(\Theta)|] - M \ln \pi \quad (\text{E.3})$$

where $*$ denotes the complex conjugates, T the transpose and $|\cdot|$ the determinant. The ML estimator is numerically intensive if all the signal parameters are all unknown. If many pulses are transmitted and if the atmosphere is stationary over the total observation time, the SNR can be reliably estimated by other

techniques. Moreover, in the pulse-dominated regime, the spectral width w is determined by the pulse profile and therefore known a priori [Frehlich 1994]. Nevertheless, in the scope of this thesis, both the mean radial velocity and the spectral width must be estimated.

E.1.1 The ML estimator of the mean frequency

This is the simplest ML estimator for which we assume that the data are stationary with statistics determined by the pulse profile. An unbiased estimate of S is obtained by subtracting the computed noise power N to the estimated total power of the signal data:

$$S = M^{-1} \sum_{k=0}^{M-1} |x_k|^2 - N. \quad (\text{E.4})$$

For the covariance model of Zrnić, the ML estimator for $f = f_{\text{IF}} - 2\lambda^{-1}v$ is the value $\hat{f}$ that maximizes the log-likelihood function:

$$L(f, \mathbf{x}) = - \sum_{m=0}^{M-1} d_m \cos(2\pi T_s m \hat{f}), \quad (\text{E.5})$$

where

$$d_m = \sum_{k=0}^{M-1} x_k^* x_{k+m} D_{k,k+m}(S, w), \quad (\text{E.6})$$

where $D(\Theta) = R^{-1}(\Theta)$. For pulse accumulation, the accumulated covariance array d_m from different signals are accumulated before evaluating the maximum. The maximum is determined by using the FFT algorithm with zero padding and interpolating the maximum with a parabolic function to produce the velocity estimates. The use of a parabolic interpolation function is chosen since it is the best approximation of a Gaussian function around its maximum.

E.1.2 The periodogram ML estimator

When the number of data sample is large, the periodogram coefficients are mutually uncorrelated and the ML estimator of the mean frequency is the value maximizing [Frehlich 1994]:

$$L(v, S, w) = - \sum_{m=0}^{M-1} \frac{\hat{P}(m)}{P(m; v, S, w)}, \quad (\text{E.7})$$

where $P(m)$ is the spectrum model as described in [Levin 1965, Rye 1993] and $\hat{P}(m)$ is the accumulated periodogram. The performance of the ML estimator compared to the PML estimator are given in [Frehlich 1993a] where it is said

that for uncorrelated frequency bins, the approximate CRB are equal to the exact one. In this work, the periodogram-ML estimator has been computed by using a constrained non-linear optimization over either both S and v or on the three signal parameters.

The asymptotic bounds σ_{CR}^2 on the mean frequency for the PML with spectral accumulation have been determined in [Rye 1993] in the cases of very low SNR and for high SNR for a purely Gaussian spectral model:

$$\sigma_{\text{CR}}^2 = \begin{cases} \frac{4\pi^{1/2}w^3}{nM\text{SNR}^2} & \text{for SNR} \ll 1 \\ \frac{12w^4}{nM} & \text{for SNR} \gg 1 \end{cases}, \quad (\text{E.8})$$

where n is the number of accumulated periodograms. The influence of M and n is quite obvious. We also observe that the CRB is degraded with SNR^{-2} and w^3 for weak signal regime.

E.2 Estimation based on AR spectral model

A parametric spectral estimation method is also possible usually leading to better velocity estimates at moderate noise level. The signal sample $x[n]$ at time nT_s at a given position of the window can be modeled by a p -order autoregressive model, noted $\text{AR}(p)$. The difference equation describing this model is

$$x[n] = -\sum_{k=1}^p a_k x[n-k] + w[n], \quad (\text{E.9})$$

where $\{a_k\}$ are the parameters of the model and $w[n]$ is the input sequence of the model, i.e. a white noise with zero mean and variance σ^2 . The corresponding power spectrum estimate can then be expressed in term of the AR coefficients

$$\hat{P}_s^{\text{AR}}(k\Delta f) = \frac{\sigma^2}{|A(k)|^2} \quad (\text{E.10})$$

$$= \frac{\sigma^2}{|1 + \sum_{k=1}^p a_k e^{-j2\pi kf}|^2}. \quad (\text{E.11})$$

In this method, we first estimate of the signal covariance by accumulation of correlograms computed on successive line-of-sight on temporal windows located at the same relative time in the signal. This estimate is used to solve the AR model parameters using the recursive Levinson-Durbin algorithm. The model parameters estimation technique is described in [Proakis 2006]. This algorithm have been tested for wind velocity profile estimation in the boundary layer in [Brousmiche 2007].

The spectrum of an $\text{AR}(p)$ process has at most p poles, i.e. frequencies minimizing $A(k)$. Those models are good candidates to represent sharp spectral peaks but not spectral valleys since they have no zeros. However, if p is chosen large enough, AR techniques can approximate adequately any kind of spectral shapes.

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