

Phases and Amplitudes in Inclusive Ψ and Ψ' Decays

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Abstract

In inclusive decays of the Ψ (3097), electromagnetic and gluonic annihilation amplitudes add incoherently, namely they are 90° out of phase. We argue that this incoherence must persist in each exclusive decay channel. For inclusive Ψ' (3686) decays, we suggest the absence of a significant direct annihilation amplitude into three gluons and propose a new amplitude via QCD anomalies and the h_c (3526) off shell. Phenomenological implications for exclusive decay channels are pointed out.

1 Introduction

From a theoretical point of view, two generic classes of transition amplitudes have to be considered in order to describe Ψ (3097) and Ψ' (3686) decays : survival amplitudes and annihilation amplitudes.

By survival amplitudes (on shell) we mean transition amplitudes from the Ψ or Ψ' to a final state which contains a bound ($c\bar{c}$) pair. Of course, annihilation amplitudes correspond to transitions from Ψ or Ψ' to quarkless intermediate states.

In the context of QCD, survival and annihilation amplitudes are radically different : while the latter can be reasonably handled as perturbative processes, the former are intrinsically non perturbative. Because of the scale and axial anomalies, the largest survival amplitude is expected [1] to correspond to a transition from the Ψ' to the Ψ accompanied with gluons in a 0^{++} or 0^{-+} state which then hadronize. Needless to say, this agrees with the data. The dominant role of survival amplitudes is also illustrated by the large decay rates for radiative processes such as $\Psi \rightarrow \eta_c \gamma$ and $\Psi' \rightarrow \chi \gamma$.

Since the Ψ is the (next to) lowest state in the charmonium spectrum, the radiative decay $\Psi \rightarrow \eta_c \gamma$ exhausts the possible contributions from survival amplitudes. To leading order in QCD and QED, one is then left with perturbative annihilation amplitudes of the Ψ into three gluons or into one photon. One of the purposes of this note is to draw attention on the relative phase of these annihilation amplitudes. This will be done in the next paragraph. The main result is that we expect a *universal incoherence* of these amplitudes, that is to say they are 90° out of phase in every exclusive decay channel. This solves the “phase problem” in Ψ decays, at least in the approximation where bona fide final state interactions are neglected.

For the Ψ' , on the other hand, the overall phenomenological picture is much less clear. While there is no doubt that survival amplitudes account for the bulk of Ψ' decays, the impor-

tant question is to correctly identify the leading strong decay amplitude which is responsible for the remaining 20%. Usually this amplitude is assumed to be a direct annihilation of the Ψ' into three gluons. In § 3 we will argue first of all that there is overwhelming phenomenological evidence against this scenario. We then propose a new amplitude for the strong annihilation of the Ψ' into light hadrons namely an off shell survival amplitude where the $c\bar{c}$ pair has the quantum numbers of the not yet well-established h_c (3526) i.e. $J^{PC} = 1^{+-}$. More precisely, we suggest that the strong annihilation of the Ψ' into light hadrons is a two step process : in the first step the Ψ' goes, via anomalies, into two gluons in a 0^{++} or 0^{-+} state and an off shell h_c (3526); in the second step the off shell h_c annihilates into three gluons to produce light hadrons. In spirit, our model is somewhat akin to the Gell-Mann, Sharp and Wagner model [2] for the decay $\omega \rightarrow 3\pi$: here, the dominant intermediate state is an off-shell ρ accompanied by a π . There are many possible tests of the model we advocate, some of which will be mentioned at the end of §3.

Finally, to conclude this note, we briefly point out, in paragraph 4, some obvious but important phenomenological implications of our model for exclusive Ψ' decay modes. In particular, there is no direct strong annihilation amplitude for $\Psi' \rightarrow \rho\pi$ i.e. the “ $\rho\pi$ puzzle” [3] is solved in our model : the so-called 14% rule between Ψ' and Ψ branching ratios is expected to be valid only for purely electromagnetic annihilation processes.

2 Phases in inclusive and exclusive Ψ decays

In the conventional picture of inclusive hadronic Ψ decays, which we adopt, the amplitude for Ψ going into light hadrons is given by

$$A(\Psi \rightarrow \text{hadrons}) \equiv A_g^H + A_\gamma^H. \quad (1)$$

A_g^H is the hadronized (three gluon) QCD annihilation amplitude and, similarly, A_γ^H is the hadronized (one photon) QED annihilation amplitude. A priori, A_g^H and A_γ^H are complex numbers.

Experimentally [4]

$$\begin{aligned} Br(\Psi \rightarrow \text{hadrons}) &= 1 - Br(\Psi \rightarrow \eta_c \gamma, \ell^+ \ell^-) \\ &\approx 86.7\% \end{aligned} \quad (2)$$

but notwithstanding Eq.(1) one usually writes Eq.(2) in the form

$$Br(\Psi \rightarrow \text{hadrons}) = B(\Psi \rightarrow \text{gluons} \rightarrow \text{hadrons}) + B(\Psi \rightarrow \text{photon} \rightarrow \text{hadrons}). \quad (3)$$

Clearly this last equation holds only if there is no interference between A_g^H and A_γ^H or in other words, only if these amplitudes are 90° out of phase and thus add incoherently.

If φ is the relative phase angle between A_g^H and A_γ^H , the data per se, i.e. Eq.(2) together with

$$\begin{aligned} Br(\Psi \rightarrow \text{photon} \rightarrow \text{hadrons}) &\approx 3 \sum Q_i^2 \left(1 + \frac{\alpha_s}{\pi}\right) Br(\Psi \rightarrow \mu^+ \mu^-) \\ &\approx 13\% \end{aligned} \quad (4)$$

put only a mild constraint on its value

$$\varphi \lesssim 110^\circ. \quad (5)$$

There are, however, strong theoretical arguments in favor of $\varphi = \pi/2$. Indeed this value follows directly from the orthogonality of the three gluon and virtual photon states to leading order. In a symbolic but obvious notation, one has

$$A_g^H = \sum_h \langle h|3g\rangle \langle 3g|\Psi\rangle \quad (6)$$

and

$$A_\gamma^H = \sum_h \langle h|\gamma\rangle \langle \gamma|\Psi\rangle \quad (7)$$

Then, clearly,

$$A_g^{*H} A_\gamma^H = \langle \Psi|3g\rangle \langle 3g|(\sum_h |h\rangle \langle h|)|\gamma\rangle \langle \gamma|\Psi\rangle = 0 \quad (8)$$

is equivalent to

$$\langle 3g|\gamma\rangle = 0 \quad (9)$$

since $\sum_h |h\rangle \langle h| = \mathbf{1}$.

Incoherence between A_g^H and A_γ^H , or Eq.(3), has thus nothing to do with the hadronization process nor with the final states : it simply follows from the orthogonality relation, Eq.(9).

Incoherence or non interference at the inclusive level implies either non interference in every single exclusive channel or a conspiracy between channels. The latter possibility appears to be ruled out. Indeed consider all amplitudes as functions of m_Ψ (or m_c). Varying this parameter does not affect annihilation nor hadronization except in trivial ways i.e. it opens up or closes down, one at the time, a possible exclusive channel depending on its threshold. Thus, each of these channels must, by itself, exhibit non interference and hence we expect the latter property to hold channel by channel.

By this simple argument we thus expect *universal incoherence*, exclusive channel by exclusive channel, between the QCD and QED annihilation amplitudes of Ψ .

It has been known for quite a while that there is a very large phase angle of the order of $\frac{\pi}{2}$ between the electromagnetic and the gluonic decay amplitudes of the Ψ into two pseudoscalars

as well as into a pseudoscalar and a vector [5] or into a nucleon-antinucleon pair [5, 6]. This was recently rediscovered [7], at least in the mesonic channels, and interpreted as a large “final state interaction” phase [7, 8].

The arguments presented above prove that the phase angle of $\frac{\pi}{2}$ is *independent* of the final states but originates from non interacting or orthogonal intermediate states.

To conclude this section let us add a comment and a caveat. The comment is that, despite large errors in the branching ratios, the data on $\Psi \rightarrow \text{tensor meson} + \text{vector meson}$ also appear compatible with non interference thus strenghtening our conclusion. The caveat is that in extracting the relative phase between A_g and A_γ from exclusive channels one should not a priori neglect the genuine final state interaction phases namely the eigenphases of the hadronic S matrix. For example in the decay $\Psi \rightarrow p\bar{p}$, a relative phase between the isospin amplitudes $A_{g,\gamma}(I=0)$ and $A_\gamma(I=1)$ is in principle present.

3 A new amplitude for hadronic Ψ' decays

As mentioned in the introduction, survival amplitudes are responsible for the bulk of Ψ' decays. However, the Ψ' does also decay into light hadrons [4]

$$\begin{aligned} Br(\Psi' \rightarrow \text{hadrons}) &= 1 - Br(\Psi' \rightarrow \Psi\pi\pi, \Psi\eta, \chi\gamma, \eta_c\gamma, \ell^+\ell^-) \\ &\cong 20\%. \end{aligned} \tag{10}$$

The standard description of these decays is in terms of the QCD and QED perturbative annihilation amplitudes $A(\Psi' \rightarrow 3g)$ and $A(\Psi' \rightarrow \gamma)$. In our opinion, this picture is essentially incorrect.

Indeed, let us for a moment concentrate on the three gluon intermediate state. This is the same intermediate state as in $\Psi \rightarrow 3g$ except that the gluons are slightly more energetic. Of course this will open up new hadronic channels but for exclusive channels common to

Ψ and Ψ' there can not be significant differences in relative abundances if the three gluon intermediate state makes any physical sense. The data and in particular the recent BES data blatantly contradict this theoretical expectation. The pattern of observed channels in Ψ' decays is totally different from the one in Ψ decays : no $(\rho\pi)$ neither a (ρa_2) channel in Ψ' decays while these channels are among the dominant ones in Ψ decays. On the other hand, the $b_1\pi$ channel appears dominant in Ψ' decays while in Ψ decays it is one of many with branching ratios of a few times 10^{-3} . Further evidence comes from the comparison of the “ $K^* - \bar{K}$ ” and “ $K_1 - \bar{K}$ ” type channels which exhibit different sensitivity to flavor $SU(2)$ and $SU(3)$ symmetry breakings, respectively. These completely different ordering patterns in Ψ and Ψ' decays into two mesons are *incompatible* with hadronization of *identical* (in quantum numbers) intermediate states. We believe that a natural explanation of these facts is simply that the Ψ' does not significantly annihilate into three gluons i.e.

$$Br(\Psi' \rightarrow 3g) \ll 20\%. \quad (11)$$

Actually this phenomenological conclusion can also be motivated by analogy with the positronium data. There, the decay width of the 2^3S_1 state into three photons is a factor eight smaller than for the 1^3S_1 ! From a more theoretical point of view, if both the physical Ψ and Ψ' did dominantly annihilate into three gluons, they would mix and could thus not be the physical eigenstates of the effective strong $(c\bar{c})$ hamiltonian which they are. In a non relativistic model, for example, the Ψ' is simply a radial excitation of the Ψ . This is a well defined picture in which Ψ and Ψ' are orthogonal states. If the annihilation into three gluons could be treated as a “perturbation” to the non relativistic potential, then clearly the unperturbed states would mix and rearrange themselves into *orthogonal* physical states i.e. the analog of Eq.(11) would obviously be true. From that point of view, it is also illuminating that the asymptotic behavior of the $(e^+e^- \rightarrow \text{light quarks})$ cross-section requires an infinite

tower of vector meson radial excitations... with vanishingly small hadronic decay widths.

Whatever the merit of these arguments, if Eq.(11) is phenomenologically correct, the obvious question is then how does the Ψ' eventually annihilate into light hadrons?

The answer is almost self evident if one goes back to the dominant transitions observed in the $c\bar{c}$ system, namely the transition from the $\Psi'(1^{--})$ to the $\Psi(1^{--})$ via the scale or axial anomaly, i.e. gluons in a 0^{++} or 0^{-+} state [1] :

$$\Psi' \rightarrow \Psi(1^{--}) + (0^{++} \text{ or } 0^{-+}). \quad (12)$$

Exactly the same mechanism allows for one and only one other transition,

$$\Psi' \rightarrow h_c(1^{+-}) + (0^{++} \text{ or } 0^{-+}). \quad (13)$$

While the Ψ in Eq.(12) is on shell when the two gluons hadronize into $\pi\pi$ or η , the h_c in Eq.(13) cannot be on shell and, as such, it has only one way to go namely annihilate into three gluons.

Thus the new amplitude we propose as the dominant mechanism for (light) hadronic decays of the Ψ' corresponds to the two step process

$$\Psi' \rightarrow h_c(1^{+-}) + (0^{++} \text{ or } 0^{-+}) \rightarrow 3g(1^{+-}) + 2g(0^{++} \text{ or } 0^{-+}). \quad (14)$$

Specifically, our new annihilation amplitude is, in some sense, when the charmed quarks have finally disappeared, a *particular* five gluon configuration albeit not a perturbative one : two of the gluons (the non perturbative ones) carry dominantly the quantum numbers 0^{++} or 0^{-+} while the three others (perturbative gluons) carry the quantum numbers of the h_c namely 1^{+-} . To leading order, hadronization should preserve these quantum numbers.

Many possible tests of our model come to mind. Perhaps the simplest ones are in the analysis of inclusive spectra $\Psi' \rightarrow \pi^+\pi^-X$, $\Psi' \rightarrow \eta\tilde{X}$, $\Psi' \rightarrow \eta'\tilde{\tilde{X}}$: the states $X, \tilde{X}, \tilde{\tilde{X}}$ should be dominantly in a 1^{+-} configuration.

As far as exclusive channels are concerned, we expect $(\pi\pi)h_1(1170)$ or $\eta(\eta')h_1(1170)$ to be important decay modes of the Ψ' but, of course, final state interactions will lead to other configurations as well : for example $SU(3)$ elastic final state interactions do allow the transition $\eta h_1(1170) \rightarrow \pi b_1(1235)$ and the importance of the latter channel in the BES data [9] is perhaps a hint that we may not be on the wrong track.

4 Summary and Conclusions

The main points of this note are the following

- in Ψ decays into light hadrons, the three gluon annihilation amplitude and the QED amplitude add incoherently in all channels ;
- in Ψ' decays into light hadrons, the dominant QCD annihilation amplitude is not into three gluons but via a two step process into a specific configuration of five gluons.

To conclude let us simply point out some obvious phenomenological consequences of all our qualitative arguments. Our model for Ψ' hadronic decays predicts a sizeable $\Psi' \rightarrow (\pi^+\pi^-$ or $\eta)X(1^{+-})$ branching ratio. At the exclusive level, it implies that :

- a) to leading order there is no strong decay amplitude for the processes $\Psi' \rightarrow \rho\pi$ and $\Psi' \rightarrow K^*\overline{K}$;
- b) the well-known 14% rule relating Ψ' and Ψ branching ratios should hold for hadronic processes like $\Psi^{(\prime)} \rightarrow \omega\pi^0$ which take place via the QED amplitude only [10].

Finally we note that our model can easily be extended to the Υ system.

Acknowledgements

We have enjoyed many conversations with J. Pestieau and C. Smith.

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