A Schumpeterian vintage capital model: An attempt at synthesis

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Abstract

In this paper, we build up a general equilibrium model explicitly incorporating Schumpeterian growth *à la* Aghion and Howitt (1992) and a vintage capital structure in line with Solow (1960). In this set-up, we show that the investment rate is a fundamental determinant of the profitability of R&D. We characterize the balanced growth paths. We show that, although the model can generate multiple equilibria due to the presence of strategic complementarities, the unique stable equilibrium is indeed dominated by strategic substituabilities. At this equilibrium, the higher is the investment rate, the more resources are devoted to R&D, the faster the economy grows, the lower is the average age of capital and the higher is the rate of decline of the relative price of capital. Subsidizing both capital and research stimulates growth. The embodiment of technological progress does not necessarily affect negatively the efficiency of capital subsidy through the typical obsolescence costs because of the modernization effect of investment in such a context.

Keywords: Vintage capital, R&D, Creative destruction, Embodiment, Obsolescence, Modernization of capital.

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1 Introduction

The recent economic growth literature has been markedly influenced by the revival of vintage capital setups on one hand and by the development of research-based endogenous growth models on the other. Both ingredients are proving most useful in understanding and explaining the most interesting and puzzling stylized facts of the real economies.

The revival of vintage capital models is intimately linked to the renewed interest in the embodied component of technological progress. The contribution of Greenwood, Hercowitz and Krusell (1997) is fundamental in this respect: They show that embodied technical change accounts for 63% of US growth in the post-war period. Embodiment is indeed a key concept to understand the economic mechanisms at work since the first oil shock, as argued Greenwood and Yorukoglu (1997). Since then, the rate of decline of the relative price of capital has been higher on average (see also Gordon, 1990) and the accumulation of new equipment (mostly hi-tech products) has been boosted. Both features can be used to provide a highly appealing interpretation of the *productivity slowdown paradox* through a simple story of adoption costs and technology diffusion, as explained in Greenwood and Yorukoglu.

On the other hand, vintage capital structures turn out to be most useful to capture some of the most salient characteristics of investment activity and job allocation in real economies. Let us mention three of them. First at all, the investment at the plant level is fundamentally lumpy as reported in Doms and Dunne (1994). Because the vintage structure is the natural one to treat the investment-replacement-adoption decisions when technological progress is embodied in the new capital goods, it has been intensively used in the recent years to study the foundations of lumpy investment behaviour and its implications on business fluctuations (see Cooley *et al.*, 1997, and Boucekkine *et al.*, 1998 and 1999, for more details). Secondly, investment and job creation and destruction are found to be tightly linked in real economies. Davis *et al.* (1996) found that the employment associated with a plant is a decreasing function of its age: It increases during the first five years of the plant's lifetime and then it decreases steadily. Indeed the impressive magnitude and the qualitative characteristics of job creation and destruction at the plant level have been abundantly documented in the recent literature (see also Dunne *et al.*, 1989, and Davis and Haltiwanger, 1992), and all these contributions can be advocated to motivate a vintage approach. Finally, there is a substantial evidence of investment irreversibility at the plant level (see in particular Caballero *et al.*, 1995). The new plants incorporate the new capital inputs, which in turn embody the most recent technological advances, while the older plants only use the older generations of capital goods and the less efficient technologies. This induces an economic obsolescence mechanism and a continuous *creative destruction* process: While new plants are created with the up-to-date technology and capital inputs, the oldest are meanwhile eventually destroyed. It therefore clearly appears that vintage capital models are a natural setup for the Schumpeterian creative destruction to take place.

This Schumpeterian concept of creative destruction is also at the basis of one of the most influential research-based endogenous growth theories of the recent years, namely Aghion and Howitt's model (1992).[1] In this model, economic growth is generated thanks to a random sequence of innovations improving the quality ladders of the goods. The innovations result from intended R&D activities with random returns. Creative destruction arises as the better quality new goods cause the obsolescence of the older ones.[2] Caballero and Jaffé (1993) and Barro and Sala-i-Martin (1995), chapter 7, have provided multi-sectoral extensions of Aghion and Howitt's setting. More recently, Aghion and Howitt (1998), chapter 3, and Howitt and Aghion (1998) have amended Caballero and Jaffé's model so as to include capital accumulation. More precisely, Aghion and Howitt have combined within the same framework the basic characteristics of Solow's neoclassical growth model (1956) and those of their creative destruction model. As one can infer from reading the two contributions mentioned just above, the incorporation of the creative destruction story into the neoclassical growth model requires a substantial adaptation. We argue in this paper that a much simpler and more meaningful way to introduce capital accumulation into Aghion and Howitt's creative destruction model consists in combining the latter with another Solow's fundamental contribution, namely his vintage capital model (1960). Since vintage capital models are by construction consistent with creative destruction, combining them with Schumpeterian R&D specifications seem natural and even desirable.

There are some expected gains from this combination in terms of the explanatory power of the theory. In contrast to the basic creative destruction model, ours will reproduce the fundamental stylized facts linked to embodiment. Indeed, we will show that the rate of decline of the relative price of capital is negatively correlated with the growth rate of the economy along the balanced growth path. The predictions of our theory regarding the evolution of the investment to output ratio and job reallocation across plants are also completely consistent with the empirical studies mentioned above. Less immediate gains from our theory concern the potential multiplicity of long run equilibria. It is well known since Cooper and John (1988) that the presence of strategic complementarities favors the emergence of multiple equilibria. In our model, the strategic complementarity operates according to the following mechanism. A rise in R&D effort increases labor reallocation at the expense of the oldest plants, which rises the marginal return from investing and increases investment. One of the most salient characteristic of our model, as we will explain later, is that marginal return to R&D depends positively on the investment rate. Hence a drop in investment will stimulate R&D inducing a further upward shift in R&D effort. This is the way the strategic complementarity operates in our model and that is why it may yield multiple

[1]The first attempt at providing a Schumpeterian approach to endogenous growth theory is indeed due to Segerstrom *et al.* (1990).

[2]See also Aghion and Howitt (1994) for the implications of creative destruction in the labor market.

equilibria.

The occurrence of multiplicity in several endogenous growth models has been very often stressed in the recent literature (see the comprehensive survey of Evans *et al.*, 1998). Young (1993) obtained it by introducing strategic complementarities between the new and old technologies in Romer's model (1990). Redding (1996) also obtained multiple equilibria as the consequence of a strategic complementarity between R&D and investment in education. If we allow the marginal return to R&D to depend on the total number of researchers in Aghion and Howitt's model (1992), we would also generate multiple equilibria. Our model shows that the embodiment hypothesis (through our vintage structure) is a plausible source of multiplicity in a Schumpeterian creative destruction setup.

As mentioned above, one major characteristic of our model is that the investment rate appears as a determinant of marginal return to R&D. This fundamental property is due to our vintage specification and the underlying embodiment hypothesis. It is neither generated in the traditional models of vertical differentiation (as illustrated in Barro and Sala-i-Martin, 1995, chapter 7) since these models do not include capital accumulation, nor in the creative destruction setting based on the neoclassical growth model as built up by Aghion and Howitt (1998), chapter 3. That is to say capital accumulation is not sufficient to obtain our fundamental property. Instead, the vintage capital structure and especially the underlying embodiment hypothesis are absolutely required. In our model, the investment activity does not only matter for capital accumulation, it is also an important determinant of R&D profitability. If technological progress were disembodied, the demand for the new designs of the capital goods would depend on total capital stock. But since technological progress is embodied, the latter demand will be fully determined by investment. Hence, investment should be a fundamental determinant of R&D profitability under the embodiment hypothesis.

The crucial role of investment in the growth process has been recently outlined in several empirical and theoretical contributions, often in connection with embodied technical change. For a sample of seven OECD countries, Wolff (1991) has found that catch-up in total factor productivity depends on capital accumulation, and that embodiment plays a central role in this respect. DeLong and Summers (1991) evidence according to which the countries with the higher growth rates are those who experienced higher investment rates in equipment and faster declines in the relative price of capita, is also consistent with our view of the role of investment. On the theoretical ground, the *modernization effects* of investment have been recently outlined by Bardhan and Priale (1999) using an endogenous growth vintage capital model. In this contribution, the considered vintage capital production function is Leontieff, technological progress builds on Arrowian learning-by-doing, and the investment activity is the channel through which technological advances are implemented. This particular role of investment is shown to take place through the average age (or the scrapping age) of capital goods: A drop in investment will decrease the average age of capital, which stimulates growth. Other recent endogenous growth models due to del Río (1999) and Hsieh (2000) also imply this particular role of investment. Both use the vintage capital production function of the Solow type as in our contribution while the research sector is modeled according to Romer (1987) and Romer (1990) respectively. While the modernization effects of investment are not deeply studied and just mentioned as side products in the two previous contributions, we propose here a complete description of the mechanisms at the basis of these effects and explain why they are most relevant in the appraisal of several related issues. In particular, the modernization effect of investment is shown to have another crucial transmission channel, namely the marginal return to research: A higher investment rate increases marginal return to R&D, thus stimulating further research effort and inducing more likely innovations.

Because of the strength of the modernization effects of investment in our model, our conclusions as to the efficiency of capital subsidy are different from those of Aghion and Howitt (1998), chapter 3. According to these authors, the magnitude of the (positive) effect of capital subsidy on growth is lower in the case of embodiment because of the obsolescence costs inherent to embodied technological change. In our model, the strong modernization effects of investment may well rule out this property. On the other hand, our analysis of the latter effects can be related to the well known "embodiment controversy". The view advocated by Phelps (1962) according to which embodiment and the associated modernization effects of investment are only important in the short run, is also shown to not hold in our research-based endogenous growth framework in the last section of this paper.

This paper is organized as follows. In the next section, we present our model in details and stress its main properties. Section 3 derives the balanced growth paths of the model and its comparative statics. Section 4 is devoted to the analysis of the efficiency of capital and research subsidies in connection with the study previously carried out by Aghion and Howitt (1998), chapter 3, and Howitt and Aghion (1998). Section 5 concludes.

2 The model

As announced in the introduction, our model can be seen as a mixture of Solow's vintage capital model (1960) and the Schumpeterian set-up *à la* Aghion-Howitt (1992). Solow's production function with vintage capital is introduced in the final goods sector, and the specification of the creative destruction process in the sector of intermediate capital goods is in line with Aghion and Howitt's story. Here are the salient features of our specifications.

Vintage capital in the final goods sector

The economy produces a unique final good which can be either consumed or invested.

As usual, we assume that the final goods sector is competitive. To introduce our vintage structure, we also assume that the final good is produced by a continuum of plants which differ by their age. More precisely, we hypothesize that the production at time t of a plant created at date z (therefore of age $t-z$) is given by:

$$y(z,t) = Bl(z,t)^{\alpha} \int_0^M \left(q^{\kappa_j(z)} k_j(z,z)\, e^{-\delta(t-z)} \right)^{1-\alpha} dj, \tag{1}$$

where $l(z,t)$ is the amount of labor used at time t to operate the plant constructed at time z, being B and α two positive constants with $0 < \alpha < 1$. All the plants are assumed to use a continuum of capital goods, $[0, M]$. Accordingly, $k_j(z,z)$ is the amount of the j-th capital good used at time z by the plant created at time z. Note that from (1), all capital goods depreciate over time at the same rate δ. However, for a given period z, capital goods are clearly differentiated by a quality index, $q^{\kappa_j(z)}$, where q is a positive constant greater than one, and $\kappa_j(z)$ is a positive real number. To unburden the presentation, we assume at this early stage that only the best quality of each capital good, namely $\kappa_j(z)$, is produced at each period, a property which will turn out to be true at equilibrium. Within this setting, the quality of the j-th capital good employed by a plant constructed at date z is equal to the one of the same capital good used by a plant built up one period before if no innovation occurs between $z-1$ and z. It is greater in the reverse case.

Summing up the outputs of all the (active) plants at time t, we get the aggregate production, $Y(t)$:

$$Y(t) = \int_{-\infty}^{t} Bl(z,t)^{\alpha} \int_0^M \left(q^{\kappa_j(z)} k_j(z,z)\, e^{-\delta(t-z)} \right)^{1-\alpha} dj dz \, . \tag{2}$$

The obtained vintage capital production function is similar to the specification discussed in details by Solow (1960). In particular, the characteristics of the adopted vintage technology imply that it is never optimal to close a plant whatever its age is.[3]

We now turn to the optimal behaviour of the firms. In our model, the representative firm has to choose the optimal amount of each type of capital in order to build up a new plant, and the optimal allocation of labor to the plants. This is done by maximizing discounted profits, given prices and the technological constraint (1). We assume that capital accumulation is subsidized at a rate β_k, so that the discounting factor of the firms is $e^{-\int_t^z r(s)ds + \beta_k(z-t)}$. The resulting first order conditions for an interior maximizer to exist are:

$$(1-\alpha)\, Bq^{\kappa_j(t)(1-\alpha)} k_j(t,t)^{-\alpha} \int_t^{\infty} e^{-\int_t^z r(s)ds - (\delta(1-\alpha)-\beta_k)(z-t)} l(t,z)^{\alpha}\, dz = p_{\kappa_j}(t) \ , \tag{3}$$

[3]Indeed, the specified production function is such that $\lim_{l\to 0} \frac{\partial y}{\partial l} = \infty$.

$$\alpha B l\left(z,t\right)^{\alpha-1}\int_0^M\left(q^{\kappa_j(z)}k_j\left(z,t\right)\right)^{1-\alpha}dj=w\left(t\right)\ , \qquad (4)$$

where $r\left(s\right)$ is the interest rate at date s, $w\left(t\right)$ is the wage at t, and $p_{\kappa_j}\left(t\right)$ is the price of the j-th capital good. We take the final good as the numéraire. Equation (3) determines the demand for the j-th capital good, it simply states that the discounted marginal return to the j-th capital good is equal to its price. Equation (4) establishes the way labor has to be optimally assigned to the plant built up at time z: The marginal return to labor at time t of a plant created at time z should be equal to the wage.

Now we turn to the definition of the aggregate variables, the major strength of the Solow-like vintage capital based models being precisely its tractability in this respect. To this end, as a characterization of the prices $p_{\kappa_j}\left(t\right)$ is required, we need an early information on research and the production of capital goods. Let us assume that the marginal cost of producing any capital good of any quality ladder is the same, here equal to η units of the final good. The researcher responsible for each quality improvement retains a monopoly right to produce the good at the obtained quality. This is the usual story in research-based growth models where innovations are typically stimulated through some form of market power accruing to the innovators. Here, we add a markedly Schumpeterian flavor to the model. We assume that prior innovators, those responsible for the previous quality advances for the same capital good, lose automatically their monopoly rentals when a further quality improvement occurs.[4] This feature generates a process of creative destruction, completely in line with the essence of Schumpeterian thought as previously figured out by Aghion and Howitt (1992).

Consistently with the properties above, we assume that for each capital good, all quality ladders are perfect substitutes as inputs in the production of the final good, and that $\left(1-\alpha\right)q>1$. The latter condition is particularly crucial in generating mark-up pricing by the innovator and the elimination of the position of previous innovators. It basically states that the quality differential between two successive innovations should be sufficiently large. In this case, the above mentioned creative destruction process does take place.[5] In particular, the researcher responsible for the last quality improvement retains a monopoly position and fixes a mark-up given demand, (3). The monopoly profit maximization implies that the mark-up should be constant and equal to

$$p_{\kappa_j}\left(t\right)=p_\kappa=\frac{\eta}{1-\alpha}.$$

Let us define gross investment at t as: $I\left(t\right)=\int_0^M\eta k_j\left(t,t\right)\ dj$. Once defined an aggregate quality index $Q\left(t\right)$,

[4] We assume implicitly here that the researchers who occupy an incumbent position do no research.

[5] More details on this standard condition and on the alternative cases can be found in Barro and Sala-i-Martin (1995), chapter 7. In the alternative case, $\left(1-\alpha\right)q\leq1$, the same results can be obtained if Bertrand-like competition is assumed instead.

$$Q(t) = \int_0^M q^{\kappa_j(t)\frac{1-\alpha}{\alpha}} di \ , \tag{5}$$

the production of the j-th capital good at t can be rewritten as follows using (3):

$$k_j(t,t) = \frac{I(t)}{\eta Q(t)} q^{\kappa_j(t)\frac{1-\alpha}{\alpha}} \ . \tag{6}$$

This equation simply establishes that the fraction of gross investment assigned to the j-th capital good depends mainly upon its relative efficiency. It is now possible to write aggregate counterparts for the production function and for the first order optimality conditions as well. Denote $L(t) = \int_{-\infty}^{t} l(z,t) \ dz$ total employment. Using the first order condition (4), one can find an expression $l(z,t)$. If this expression is used in the definition of total employment given just above, one can find:

$$w(t) = \alpha B \left(\frac{K(t)}{L(t)}\right)^{1-\alpha} \ , \tag{7}$$

where $K(t) = \int_{-\infty}^{t} \left(\int_0^M \left(q^{\kappa_j(z)} k_j(z,z) e^{-\delta(t-z)}\right)^{1-\alpha} dj\right)^{\frac{1}{1-\alpha}} dz$. $K(t)$ is the efficient aggregate capital stock. Equation (7) restates the optimality condition (4) into the standard rule: The wage should be equal to the marginal productivity of labor. It can be rewritten in terms of the detrended capital stock per capita, $k(t) = K(t)/L(t)Q(t)^{\frac{1}{1-\alpha}}$, as follows

$$\omega(t) = \alpha k(t)^{1-\alpha}$$

where $\omega(t)$ is the wage per efficiency unit, $\omega(t) = w(t)/Q(t)$. Using (6), we can rewrite the efficient capital stock as the weighted sum of past and current investments, being the weights equal to the productivity of each investment:

$$K(t) = \int_{-\infty}^{t} \frac{1}{\eta} Q(z)^{\frac{\alpha}{1-\alpha}} I(z) e^{-\delta(t-z)} dz \ . \tag{8}$$

If we differentiate (8), we obtain the accumulation law of efficient capital:

$$\overset{\bullet}{K}(t) = \frac{1}{\eta} Q(t)^{\frac{\alpha}{1-\alpha}} I(t) - \delta K(t) \ . \tag{9}$$

As in Solow (1960) and Greenwood, Hercowitz and Krusell (1997), the variation of the efficient capital stock is equal to efficient investment minus physical depreciation. There is, however, a crucial difference with respect to the previous contributions: The term $\frac{1}{\eta}Q(t)^{\frac{\alpha}{1-\alpha}}$, multiplying gross investment in the accumulation law, and as such interpretable as its marginal productivity or efficiency, is endogenous in our model.[6] Using the accumulation law of efficient capital and the definition of detrended capital stock per capita, we can derive the dynamics of detrended efficient capital stock per capita:

$$\dot{k}(t) = \frac{1}{\eta} i(t) - \left(\delta + \frac{1}{1-\alpha} g(t)\right) k(t) \ , \tag{10}$$

where $i(t) = I(t)/L(t)Q(t)$ is detrended investment per capita, and $g(t) = \dot{Q}(t)/Q(t)$ is the growth rate of the aggregate quality index.[7] Before moving to the aggregation of the main equations of the model given our definition of aggregate capital, let us define an average age of capital variable, a variable which turns out to be crucial in our interpretation of the main mechanisms at work in this model as announced in the introduction. Within our set-up, the most natural way to define the average age of capital, $m(t)$, is as follows:

$$m(t) = \frac{\int_{-\infty}^{t} \frac{1}{\eta} (t-z) \, Q(z)^{\frac{\alpha}{1-\alpha}} \, I(z) \, e^{-\delta (t-z)} \, dz}{K(t)}. \tag{11}$$

One can see that the integral appearing in the numerator is simply the “total age” of capital given the vintage formulation of aggregate efficient capital given in equation (8). As it should be in this kind of settings, all vintages (ie. past and current investments) are weighted by their efficiency level in both the numerator and the denominator of the fraction giving $m(t)$.

We now turn to the aggregate relations implied by our model so as to show that it is possible to reinterpret the vintage model of the final goods sector in much more standard terms. This is pretty clear for equation (7) as mentioned above. Indeed, using (4), (7) and the definition of the aggregate production function, (2), one obtains after some easy algebra:

$$Y(t) = BL(t)^{\alpha} K(t)^{1-\alpha} \ , \tag{12}$$

while detrended output per capita $y(t) = Y(t)/L(t)Q(t)$ is given by:

$$y(t) = Bk(t)^{1-\alpha} \ . \tag{13}$$

The first order condition (3) can be also rewritten in terms of the aggregate variables as:

$$(1-\alpha) B \left(\frac{K(t)}{L(t)}\right)^{-\alpha} = \mathbf{P}_K(t) \left(r(t) + \delta + \xi(t) - \beta_K\right) \ , \tag{14}$$

[6]This is a common characteristic with Krusell (1998).

[7]Equation (10) is obtained under the assumption $L(t) = L$ for all t, which is true at equilibrium since labor supply is assumed constant. We keep on using this property hereafter.

where $P_K(t) = p_\kappa\ Q(t)^{-\frac{\alpha}{1-\alpha}}$ is the quality-adjusted price of capital goods in terms of the final good, and $\xi(t) \equiv -\overset{\bullet}{P}_K(t)/P_K(t) = \frac{\alpha}{1-\alpha}g(t)$ is the obsolescence rate of capital. This relationship is absolutely crucial in assessing the behaviour of the relative price of capital during the growth process, as it will be clear in the next section. As one can see, equation (14) simply states that the marginal productivity of capital should be equal to its user cost.[8] Using the exact value of the monopoly price p_κ, we can rewrite the equation above in terms of detrended efficient capital stock per capita according to the following equation:

$$(1-\alpha) B\ k(t)^{-\alpha} = \frac{\eta}{1-\alpha}\left(r(t) + \delta + \xi(t) - \beta_K\right)\ . \tag{15}$$

We now turn to develop more comprehensively the research sector of the model.

The Schumpeterian research sector

As mentioned above, we adopt a comprehensive (typical) Schumpeterian stochastic modeling in the research sector. Recall from the previous sub-section that monopoly profit maximization by an innovator-producer of a capital good at the leading quality yields a mark-up $p_{\kappa_j}(t) = p = \frac{\eta}{1-\alpha}$. The resulting flow of profits is $\pi_i(t) = \frac{\alpha\,\eta}{1-\alpha}\ k_j(t,t)$. Using equation (6), this could be rewritten as:

$$\pi_j(t) = \frac{\alpha}{1-\alpha}\,\frac{I(t)}{Q(t)}\,q^{\kappa_j(t)\frac{1-\alpha}{\alpha}}\ . \tag{16}$$

How does the creative destruction process take place in the research sector? The story adopted here builds on the stochastic model of Aghion and Howitt (1992). A successful innovator increases the quality of the j-th capital good at t from grade $q^{\kappa_j(t)}$ to $q^{\kappa_j(t)+\tau}$ at time $t+dt$, where τ is a strictly positive constant. The innovation is assumed to come out according to a Poisson probability scheme. Denote by $\gamma_{\kappa j}(t)$, the Poisson probability for such an innovation to occur. We set: $\gamma_{\kappa_j}(t) = n_{\kappa_j}(t)\,\phi(\kappa_j(t))$, where $n_{\kappa_j}(t)$ is the flow of total resources (in terms of the final good) devoted to research in quality improvements of the j-th capital good at date t when the leading quality is $q^{\kappa j(t)}$.[9] We also assume that the Poisson probability $\gamma_{\kappa j}(t)$ is a decreasing function of the research task, here captured by the quality index $\kappa_j(t)$. Hence, $\phi'(\kappa_j(t)) < 0$. More concretely, we set $\phi(\kappa_j(t)) = \lambda q^{-(\kappa_j(t)+\tau)\frac{1-\alpha}{\alpha}}$, where λ is a positive parameter.

[8]To obtain (14), an expression for $l(z,t)$ is first derived thanks to (4), and we use it in (3). Then (7) is used to eliminate the wage variable from the modified (3). After some further simple algebra on the latter involving (6), we get our fundamental relation by differentiation with respect to time.

[9]Since the final good is the unique input of the R&D activites, we are implicitly assuming that these activities use capital and labor according to the same technology as in the production of the final good. This hypothesis is quite standard in the literature, see Romer (1987), Rivera-Batiz and Romer (1991), Barro and Sala-i-Martin (1995, Chapters 6, 7), Howitt and Aghion (1998) and Evans *et al.* (1998).

The remaining term $q^{-(\kappa_i(t)+\tau)\frac{1-\alpha}{\alpha}}$ represents the negative effect of the complexity of the research task on the Poisson probability for an innovation to occur.[10] This choice is consistent, as we will see later, with an equilibrium Poisson probability ultimately independent of the complexity of the research task.

Indeed, a successful innovator is allowed to set a mark-up pricing when selling the demanded amount of the j-th capital good as formulated in (6). Given the flow of profits' expression (16), and the Poisson probability specifications above, the value of an innovation improving the quality ladder of the j-th capital good at t is:

$$V_{\kappa_j}(t) = \frac{\alpha}{(1-\alpha)} \int_t^{\infty} \frac{I(z)}{Q(z)} q^{(\kappa_j(t)+\tau)\frac{1-\alpha}{\alpha}}\, e^{-\int_t^z r(s)ds}\, e^{-\int_t^z \gamma_{\kappa_j}(s)ds}\, dz\ , \tag{17}$$

where the two exponential terms in the integrand represent respectively the discount factor and the probability of the quality grade $\kappa_j(t)+\tau$ still leading at time $z > t$. We assume that the research sector is subsidized at a rate β_R. The arbitrage condition for the equilibrium level of resources spent in R&D activities should stipulate that the marginal cost of research is equal to or greater than the expected present value of profit with equality if the amount of resources devoted to R&D is strictly positive, that is:

$$1-\beta_R \geq \lambda q^{-(\kappa_j(t)+\tau)\frac{1-\alpha}{\alpha}} V_{\kappa_j}(t) \equiv \widetilde{V}_{\kappa_j}(t)\ , \tag{18}$$

with equality if $n_{\kappa_j}(t) > 0$. Assuming that resources devoted to R&D are never nil, the arbitrage condition implies $\overset{\bullet}{V}_{\kappa_j}(t) = 0$, which yields by differentiation of (17)

$$V_{\kappa_j}(t) = \frac{\alpha}{1-\alpha} \frac{I(t)}{Q(t)} \frac{q^{(\kappa_j(t)+\tau)\frac{1-\alpha}{\alpha}}}{r(t)+\gamma_{\kappa_j}(t)}\ . \tag{19}$$

From (19)-(18), it turns out that the Poisson probability, $\gamma_{\kappa_j}(t)$, does not depend on the complexity of the research task, that is $\gamma_{\kappa_j}(t) = \gamma(t)\ \forall \kappa_j(t)$. This means that quality improvements can occur for all types of capital goods with the same probability and whatever is the reached quality grade. This property of the model is entirely due to the specification of function $\phi(\kappa_j(t))$ as outlined in Barro and Sala-i-Martin (1995). In principle, the Poisson probability $\gamma_{\kappa_j}(t)$ is affected by $\kappa_j(t)$ in two opposite ways. First, the monopoly profits accruing to an innovator increase with $\kappa_j(t)$ since the amount of demanded capital good rises with the quality of the capital good as it transpires (6). Secondly, by assumption, the probability of innovating decreases with the difficulty of the task, measured by $\kappa_j(t)$. When the specification $\phi(\kappa_j(t)) = q^{(\kappa_j(t)+\tau)\frac{1-\alpha}{\alpha}}$ is adopted, the two effects exactly offset.

Assuming that resources devoted to R&D are never nil, and using (19) and (13), the arbitrage condition can be rewritten as

[10] The same specification is considered in Barro y Sala-i-Martin (1995).

$$1-\beta_R=\frac{\lambda B\alpha}{(1-\alpha)}\frac{s\,(t)\,k\,(t)^{1-\alpha}\,L\,(t)}{r\,(t)+\gamma\,(t)}\ , \tag{20}$$

where$s\,(t)$ is the investment-output ratio:

$$s\,(t)\equiv\frac{I\,(t)}{Y\,(t)}=\frac{i\,(t)}{y\,(t)}\ .$$

It is worthwhile pointing out that the marginal return to innovation does depend on the investment to output ratio in our model. This is probably its most salient property. Such a property does not arise in the basic research-based growth models, both deterministic (Romer, 1990) or stochastic (Aghion and Howitt, 1992). In the latter setting, capital goods are not durable and hence there is no capital accumulation. However, incorporating capital in the creative destruction model as in Aghion and Howitt (1998), chapter 3, is not sufficient to get the investment rate as a determinant of the profitability of R&D, as we explain in the introduction section. Indeed, this property is due to our vintage structure and to the underlying embodiment hypothesis. If technological progress were disembodied, the demand for the new designs of the capital goods would depend on total capital stock. In contrast, when technological progress is embodied, or in other words when technological progress is investment-specific, investment becomes the key variable of the profitability of the innovators' activities.

Finally, the impact of the successive innovations on the aggregate quality index is derived in the usual way by applying the law of large numbers (see Aghion and Howitt, 1992, and Barro and Sala-i-Martin, 1995, chapter 7). In case of a quality improvement of the j-th capital good, the proportionate change in the quality grade is quality is $\widetilde{q}=\left(q^{\frac{1-\alpha}{\alpha}\tau}-1\right)$. As established just above, the equilibrium Poisson probability of innovating is equal for all capital goods, which implies that the expected proportionate change in aggregate quality $Q(t)$ is $\gamma(t)\ \widetilde{q}$. By the Law of Large Numbers, one can set the average growth rate of $Q(t)$ (measured over any finite time interval) equal to the latter discrete proportionate change:

$$g\,(t)\equiv\frac{\dot{Q}\,(t)}{Q\,(t)}=\gamma\,(t)\,\widetilde{q}. \tag{21}$$

Closing up the model

The specification of the economy is completed by a standard modeling of consumers' behaviour. We consider an infinitely lived representative household endowed with L units of labor. The household maximizes its intertemporal utility $U=\int_0^\infty\frac{C(t)^{1-\sigma}}{1-\sigma}e^{-\rho t}dt$, where $\rho > 0$ is the time preference rate and $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution. As usual, the optimality condition is

$$\frac{\overset{\bullet}{C}(t)}{C(t)} = \frac{1}{\sigma}(r(t) - \rho) \ . \tag{22}$$

From the other hand, we assume that labor supply is inelastic and constant overtime, so that the clearing condition in the labor market writes simply

$$L(t) = L \tag{23}$$

Using (21) and (23), the Euler equation (22) can be rewritten as

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}(r(t) - \rho) - g(t)\,, \tag{24}$$

where $c(t) = C(t)/LQ(t)$ is detrended consumption per capita.

The clearing condition in the market of the final good:

$$C(t) + I(t) + R(t) = Y(t) \ , \tag{25}$$

where $R(t)$ is the total amount of the final good devoted to research, $R(t) = \int_0^M n_{\kappa_j}(t)\,dj$. Since the equilibrium Poisson probability is the same for all capital goods, $\gamma(t) = n_{\kappa_j}(t)\,\phi(\kappa_j(t))$, we get after some trivial algebra:

$$R(t) = \frac{\gamma(t)}{\lambda}(\tilde{q} + 1)\,Q(t) \ . \tag{26}$$

Using (13), (23) and (26), the clearing condition in the final good market can be rewritten, as:

$$c(t) + i(t) + \frac{\gamma(t)}{\lambda L}(\tilde{q} + 1) = Bk(t)^{1-\alpha} \ . \tag{27}$$

We now turn to characterize the balanced growth equilibria of our model.

3 Characterizing balanced growth equilibria

We define a balanced growth path (BGP) as an equilibrium path along which the probability of successful innovation is constant, that is $\gamma(t) = \gamma$, while the growth rates of consumption, investment, resources devoted to R&D and output are kept also constant, equal to the rate of endogenous technological progress $\frac{\dot{Q}}{Q} = g = \gamma\widetilde{q}$. For their crucial economic relevance, we will focus more on the steady-states values of capital intensity (k), the Poisson arrival rate (γ), the rate of interest (r), the growth rate (g), the obsolescence rate (ξ), the saving rate (s), and the average age of capital (m). These seven variables are simultaneously determined by the system:

$$k^{-\alpha} = \frac{\eta}{(1-\alpha)^2 B}(r + \xi + \delta - \beta_K) \tag{K}$$

$$1 - \beta_R = \frac{\lambda B\ L\ \alpha}{(1-\alpha)} \frac{sk^{1-\alpha}}{(r+\gamma)} \tag{G}$$

$$r = \sigma g + \rho \tag{R}$$

$$\gamma = \widetilde{q}^{-1} g \tag{A}$$

$$\xi = \frac{\alpha}{1-\alpha} g \tag{O}$$

$$m = \left(\frac{g}{1-\alpha} + \delta\right)^{-1} \tag{M}$$

$$s = \frac{\eta}{B\ m} k^{\alpha} \tag{S}$$

Equation (K) is obtained from (15) and (24), it states the equality between the user cost of capital and its marginal productivity. Equation (G) is the arbitrage condition in the R&D sector (20) along the BGP. Equation (R) is the familiar Fisher equation showing up how the interest rate depends on the growth rate of consumption. Equation (A) describes the relationship between the rate of technical progress γ and the growth rate g in the BGP. Equation (O) states that in the long run the obsolescence rate of capital is proportional to the growth rate. This relationship simply comes from the fact that the obsolescence rate is the rate of decline of the relative price of capital $P_K(t)$, which is equal by definition to $\frac{\eta}{1-\alpha}\ Q(t)^{-\frac{\alpha}{1-\alpha}}$. Equation (M), corresponding to equation (11) in the BGP, implies that the average age of capital is a decreasing function of the growth rate in the long run. Finally, equation (S) expressing the steady state saving rate in terms of the long run values of the average age of capital and capital intensity is obtained from equations (10) and (13) using (M).

The most salient feature of our model is the crucial role of investment in the growth process. As mentioned before, our model predicts that the investment rate is a crucial determinant of the research effort as captured by the total amount of resources devoted to R&D. This transpires from the fundamental equation (G). Hence, a rise in the investment rate stimulates the innovation, which increases the growth rate, and lowers the average age of capital by equation (M). We will refer to this property as *the modernization effect* of investment. In our model, technological progress is embodied, and investment is the unique channel through which technological advances are incorporated into the productive activities. The investment activity is not merely for capital accumulation in the traditional sense, it is also the unique way to enforce technological improvements and to modernize the productive economy. Bardhan and Priale (1999), del Río (1999) and Hsieh (2000) have already mentioned this role of investment when technological progress is embodied, as explained in the introduction section. In this paper, we first give a comprehensive and fine description of the mechanisms at the basis of the above mentioned modernization effects. A higher investment rate s rises the profitability of R&D, thus stimulating research efforts and inducing more likely quality improvements, which ultimately increases the growth rate and lowers the average age of capital.

Our model turns out to be consistent with many recent views and empirical contributions on the role of investment and embodied technological change in the growth process. On the empirical side, our story is for example consistent with DeLong and Summers (1991) evidence according to which the countries with the higher growth rates are those who experienced higher investment rates in equipment and faster declines in the relative price of capital. Recently, McGrattan (1998) revisited the arguments put forward by Jones (1995) to question the tight correlation between investment and growth rates as implied by the AK models. In particular, she showed that if longer time series and more countries are considered in the data, this tight correlation is definitely preserved. For a sample of seven OECD countries, Wolff (1991) found that catch-up in total factor productivity is highly correlated with capital accumulation, and that embodiment plays a central role in this relationship.

As expected, our model is also able to reproduce the main empirical facts linked to embodiment, as pointed out previously by Greenwood, Hercowitz and Krusell (1997) and Greenwood and Yorukoglu (1997). The rate of decline of the relative price of capital and the growth rate of the ratio efficient capital to output are equal and more importantly, proportional to the rate of technological progress g. Indeed, by equation (10), the efficient capital per capita increases at a rate $\frac{g}{1-\alpha}$. As usual, due to embodiment, the growth rate of the capital stock, measured in efficiency units, is greater than the growth rate of output. The ratio efficient capital to output should consequently grow at a rate $\frac{\alpha}{1-\alpha}\, g$ in the BGP, which is incidentally the rate of decline of the relative price of capital given by equation (O).

Last but not least, and as announced in the introduction, our vintage structure does also allow us to bring out several lessons as to optimal allocation of labor resources in the long run. Regarding the distribution of labor per plant, our model is very satisfactory. Indeed, along a BGP, the evolution of labor over time and its distribution across the plants is given by: $l(z,t) = \frac{L}{m}\, e^{-(\frac{g}{1-\alpha}+\delta)\,(t-z)}$. This equation is obtained by trivially combining equations (4) to (7) and (M) in the BGP. Recall that $l(z,t)$ is the amount of labor devoted to operate at t a plant constructed at $z < t$. In the BGP, as the plants get older, a lower amount of labour is assigned to them. Labor reallocation (namely creation plus desctruction), being equal to $2\frac{L}{m}$, is also found to increase when the growth rate g rises *via* the average age of capital variable. In this sense, our model does reproduce the most important stylized facts related to job creation and destruction as reported in Dunne *et al.* (1989), Davis and Haltiwanger (1992), and Davis *et al.* (1996).

To summarize the outcomes of this first long run analysis, our model implies that the higher is the investment rate, the more resources are devoted to R&D, the faster the economy will grow, the lower will be the average age of capital, the higher will be the rate of decline of the relative price of capital and the growth rate of the ratio efficient capital to output, and the more labor resources will be assigned to the newly created plants. We now turn briefly to more technical considerations.

Existence and uniqueness of the balanced growth paths

Does the system (K)-(S) yield positive solutions for the targeted variables? Are the solutions unique? Do the positive solutions of this system imply positive values for the other variables of the system, namely consumption and investment? The following proposition gives simple sufficient conditions under which unique positive solutions are ensured. To unburden the presentation and without any loss in the scope of our finding, we assume at the moment that the subsidy rates β_K and β_R are nil. The effects of these policy instruments on growth is reported in the next section.[11]

Proposition 1 *If $\rho > (1-\sigma)\, g$, where g solves the system (K)-(S), and $L > \frac{(\delta+\rho)^{\frac{1}{\alpha}}\rho}{\lambda\delta\Gamma}$, where $\Gamma = \left(\frac{(1-\alpha)^2}{\eta}\right)^{\frac{1-\alpha}{\alpha}} B^{\frac{1}{\alpha}} \frac{\alpha}{1-\alpha}$, there exists a unique solution to the system (K)-(S) with $g > 0$.*

Proof: The condition $\rho > (1-\sigma)\, g$ is needed to get utility bounded along the BGP for any $\sigma > 0$. Moreover, condition $\rho > (1-\sigma)\, g$ is sufficient to guarantee that consumption and investment are positive in the BGP. Let us prove it. From (K), (R), (A) and (O), it follows that

$$k = \left(\frac{(1-\alpha)^2 \frac{B}{\eta}}{\left(\sigma + \frac{\alpha}{1-\alpha}\right) g + \delta + \rho} \right)^{\frac{1}{\alpha}} , \tag{28}$$

k is strictly positive for all $g \geq 0$. From (27) and (A), we obtain

$$c = Bk^{1-\alpha} - i - \frac{\widetilde{q}+1}{\widetilde{q}\lambda L} g \; . \tag{29}$$

[11] Since the proof is simple and informative, we include in the main text.

Using equations (S) and (10), and by definition of the saving rate s, we get

$$k = \frac{Bm}{\eta} i \ . \tag{30}$$

Using the equation just above, we can rewrite (29) as follows

$$c = \left(\frac{Bm}{\eta} k^{-\alpha} - 1 \right) i - \frac{\widetilde{q} + 1}{\widetilde{q} \lambda L} g \ . \tag{31}$$

On the other hand, the relations (G), (R) and (A), together with $sBk^{1-\alpha} = i$, imply

$$i = \frac{(1-\alpha)}{\alpha \lambda L} \left(\left(\sigma + \frac{1}{\widetilde{q}} \right) g + \rho \right) \ , \tag{32}$$

which implies that investment is positive in the BGP if the growth rate g is positive. Finally, substituting (28)and (32) into (31) yields the following fundamental relation

$$c = \frac{\widetilde{q} + 1}{\widetilde{q} \lambda L} g \left[\left(m \frac{\left(\sigma + \frac{\alpha}{1-\alpha} \right) g + \delta + \rho}{(1-\alpha)} - (1-\alpha) \right) \frac{(\widetilde{q}\sigma + 1) g + \widetilde{q}\rho}{\alpha (\widetilde{q} + 1) g} - 1 \right]$$

If $r = \sigma g + \rho > g$ then, using (M), and for all $g \geq 0$, we get

$$\begin{aligned} c \ &> \ \frac{(\widetilde{q} + 1) g}{\widetilde{q} \lambda L} \left[\left(\frac{\frac{\alpha}{1-\alpha} g}{g + \delta (1-\alpha)} + \alpha \right) \frac{(\widetilde{q}\sigma + 1) g + \widetilde{q}\rho}{\alpha (\widetilde{q} + 1) g} - 1 \right] > \\ &> \ \frac{g}{\lambda L} \frac{\frac{1}{1-\alpha} g}{g + \delta (1-\alpha)} > 0 \ . \end{aligned}$$

So under the conditions of Proposition 1, consumption is positive in the BGP as long as $g > 0$. To study the positivity of g, some tedious algebra is needed. Indeed after successive substitutions involving equations (K), (G), (R), (A), (O), (S) and (M), we can write g as an implicit function of the sole parameters of the problem:

$$1 = \frac{\Gamma \lambda L \left(\frac{1}{1-\alpha} g + \delta \right)}{\left[\left(\sigma + \widetilde{q}^{-1} \right) g + \rho \right] \left[\left(\sigma + \frac{\alpha}{(1-\alpha)} \right) g + \delta + \rho \right]^{\frac{1}{\alpha}}} \equiv \widetilde{V}(g) \tag{33}$$

where $\Gamma = \left(\frac{(1-\alpha)^2}{\eta} \right)^{\frac{1-\alpha}{\alpha}} B^{\frac{1}{\alpha}} \frac{\alpha}{1-\alpha}$. Equation (33) is obviously equation (0) expressed in terms of the unique endogenous variable g. In particular, the function $\widetilde{V}(g)$ represents the return to innovation as a function of g. It is easy to check that function $\widetilde{V}(g)$ have the following properties: (i) $\widetilde{V}(0) = \frac{\Gamma\lambda L\delta}{(\rho+\delta)^{\frac{1}{\alpha}}\rho}$, (ii) the limit of $\widetilde{V}$ is zero when g tends to infinity, (iii) $\widetilde{V}$ is continuous and strictly increasing in L and (iv) there is at most one $g > 0$ such that $\widetilde{V}'(g) = 0$. From properties (i)-(iv) follows that for all $L > \frac{(\delta+\rho)^{\frac{1}{\alpha}}\rho}{\lambda\delta\Gamma}$ there is only one strictly positive solution to (33). □

Proposition 1 states, among other things, that the labor resources of the economy should be sufficiently big for a BGP with positive growth rate to be sustainable. This is one of the most crucial properties of this model. Though this kind of conditions is very often required in endogenous growth models (even in the simplest ones, see Romer, 1986), it is absolutely needed in our framework to additionally rule out multiplicity. Indeed, our model presents strategic complementarities and substituabilities between R&D activities, and it is well known, since Cooper and John (1988) that multiple equilibria can be generated in the presence of strategic complementarities. In our model, a simple way to identify these features consists in studying the slope of $\widetilde{V}(g)$. If this slope is positive (Resp. negative), an increase in the growth rate g rises (Resp. reduces) the return to innovation, which corresponds to strategic complementarities (Resp. substituabilities). The strategic complementarity operating in our model comes from a characteristic mentioned just above: An increase in the resources devoted to R&D involves a job reassignment at the expense of the old plants. This implies a rise in the marginal return to investment given the production function in the final goods sector, an increase in investment, and finally a further increase in the resources devoted to R&D since the drop in investment shifts upward the marginal return to innovation by (G). The whole mechanism is close to *the crowding out effect* described by Aghion and Howitt (1998). However as one can see, it fundamentally comes from the the modernization role of investment in our setting: For the increase in investment to yield an increment in the research effort, the former should be explicitly or implicitly a determinant of R&D profitability, which is ensured in our model thanks to the modernization mechanism as described above.

In addition to that, our model generates two strategic substituabilities. The first one recovers an *obsolescence effect*. More resources devoted to R&D rises the discounting rate of the firms operating in the final goods sector, say $\widetilde{r}$ *via* an increase in economic obsolescence. Indeed, $\widetilde{r} = r + \xi + \delta = r + \frac{\alpha}{1-\alpha}g + \delta$, where $\xi = \frac{\alpha}{1-\alpha}g$, by equation (O), is the obsolescence rate. Therefore, the marginal return to investment diminishes, and so will do successively investment and the marginal return to innovation, which will typically divert resources from the research sector. The second strategic substituability comes from the well known *business stealing effect*. It is inherent to Schumpeterian models *'a la* Aghion and Howitt (1992), and as such it cannot be found in deterministic frameworks like Krusell's model (1998) where the concept of planned obsolescence is more adapted. In our model, it works as follows: By construction of Poisson arrival rates, a rise in resources devoted to R&D reduces the expected lifetime of the monopoly power accruing to an innovator, and so it decreases the marginal return to innovation, which discourages in turn research effort.

So unsurprisingly, our model is able to generate multiple equilibria under certain conditions. However, we prefer to focus on the uniqueness case described by Proposition 1 for the following reason. Indeed, the BGP dominated by strategic complementarities turns out to arise only for sufficiently low values of labor resources L, and more importantly, to be instable (in contrast to Young, 1993, for example).[12] Moreover, the latter BGP arises for definitely unrealistic values of the parameters.[13] Since the multiplicity of equilibria generated in our model is indeed fictitious, we abstract away from the related mathematical developments. Nonetheless, the idea that our model presents different sources of strategic complementarities and substituabilities is appealing, and we still use it in the next comparative statics exercises.

Some comparative statics

Indeed, we perform some comparative statics exercises to complete the characterization of the BGP. Straightforwardly, we get the following results:

Proposition 2 *Under the assumptions of Proposition 1, the steady state growth rate g depends positively on λ, $\widetilde{q}$, L, and on B. On the contrary, g is negatively correlated with σ, ρ, δ, and with η.*

Proof: The proof is trivial. An increase in λ, L, B and $\widetilde{q}$ rises $\widetilde{V}(g)$ for all $g \geq 0$. Since the equilibrium growth rate is determined by $\widetilde{V}(g) = 1$ thanks to equation (33), the first part of the proposition follows immediately. The same argument applies for the second part since it can be trivially checked that an increase in σ, ρ, δ and η reduces $\widetilde{V}(g)$ for all $g \geq 0$.□

The comparative statics yield some interesting results. Note that an increase in λ rises the return to innovation by equation (18), and as such it should stimulates both innovation and growth. The same argument applies to the size of innovations parameter $\widetilde{q}$ and to the marginal cost of capital goods η (in the reverse direction of course). An increase in $\widetilde{q}$ lowers the magnitude of the business stealing effect, which is good for growth. In contrast, an increase in η is bad for the return to investment, which tends to discourage investment, thus affecting negatively the growth rate.

As usual in endogenous growth models, there is a scale effect: The greater is the size of the labor force, the larger will be output, and the more resources will be devoted to

[12] One can check that if $\widetilde{V}'(0) > 0$, then there exists a value $L^0 > 0$ such that if $L^0 < L < \frac{(\delta+\rho)^{\frac{1}{\alpha}}\rho}{\lambda\delta\Gamma}$, three BGP arise, two of them with strictly positive growth rates g. Only one of the two relevant equilibria, namely the one dominated by strategic substituabilities, turns out to be saddlepoint stable. All the details of the characterization of BGP's existence and stability for any value of L can be found in del Río (1999).

[13] For multiplicity to hold, we need $\widetilde{V}'(0) > 0$, which implies $\Lambda < 1$, where $\Lambda = \frac{1-\alpha}{\rho}\delta\left(\sigma + \widetilde{q}^{-1} + \frac{\sigma+\frac{\alpha}{1-\alpha}}{\alpha(\rho+\delta)}\right)$. Set $\rho = 0.02$, $\delta = 0.05$, $\alpha = 0.6$ and $\sigma = 2$, then $\Lambda \geq 85$ $\forall \widetilde{q} \geq 0$! The condition $\Lambda < 1$ is impossible to obtain with acceptable parameters' values.

research. The same mechanism is present in the lab-equipment model of Rivera-Batiz and Romer (1991) for example. A similar scale effect is generated by the production function scale parameter B. The negative effect on growth of an increase in the impatience rate ρ or in the preference parameter σ is abundantly documented in the literature. There are however two main departures with respect to Krusell (1998). He finds that the effect on growth of a rise in the physical depreciation rate, δ, is ambiguous. Indeed, in his deterministic framework, an increase in δ also "rises the extent to which planned obsolescence hampers technological development" (Krusell, 1998, page 138), thus increasing the growth rate. Since our research sector is Schumpeterian *à la* Aghion-Howitt, this effect disappears, and we only have the usual negative effect of depreciation on capital accumulation.

Another difference with respect to Krusell resides in the role of the number of capital goods M. Krusell finds that an increase in M should lower the growth rate through the following channel: Since his research sector uses labor, more varieties of capital goods means less labor available for each capital type, which implies in a symmetric equilibrium a lower growth rate. The same result is obtained by Young (1998). In our lab-equipment type modeling of the research sector, this property does not arise. An increase in M will neither increase nor lower the long run growth rate. Since the amount of resources devoted to research can steadily increase in our model, there is no immediate negative effect inherent to a drop in the number of capital goods. Indeed, an increase in M requires more resources in the research sector exactly as in Krusell and Young. But the required increment is sustainable in our lab-equipment type model in that it does not necessarily imply a reduction in the growth rate. And the rise in M will increase technological progress $Q(t)$. In the BGP of our model, the cost involved by the presence of more varieties (more resources to operate the research sector) is sustainable and it is exactly offset by the subsequent increment in technological progress. This explains why the long run growth rate does not depend on M in our model, in contrast to Krusell and Young.

We now turn to study the policy implications of our model in the BGP.

4 On the impact of subsidies and embodiment

In this section, we relax the assumption $\beta_K = \beta_R = 0$. We first present some insight into how the long run analysis is affected by this change. Then, we study more closely the effect of technological progress embodiment on the efficiency of capital subsidy, in connection with the discussion of Howitt and Aghion (1998).

4.1 The long run effects of capital and research subsidies

In such a case, the fundamental relation (33), equalizing the marginal cost and revenue of an innovation in the long run, is modified according to

$$1-\beta_R=\frac{\Gamma\lambda L\left(\frac{1}{1-\alpha}g+\delta\right)}{\left[\left(\sigma+\widetilde{q}^{-1}\right)g+\rho\right]\left[\left(\sigma+\frac{\alpha}{(1-\alpha)}\right)g+\delta+\rho-\beta_K\right]^{\frac{1}{\alpha}}}\equiv\widetilde{V}\left(g\right).$$

The conditions on the parameters for a BGP to exist are trivial extensions of the ones established in the benchmark case $\beta_K=\beta_R=0$. A sufficient condition for a unique BGP to exist is $\widetilde{V}(0)>1-\beta_R$, which implies as before a lower bound for labor supply L. Again, for sufficiently high labor endowment L, only one BGP arises, and this BGP is dominated by strategic substituabilities. The effects of capital and research subsidy on growth turn out to be clear as reflects the following proposition.

Proposition 3 *If the parameters of the model check the condition $\widetilde{V}(0)>1-\beta_R$, the unique positive steady state growth rate g increases when either β_K or β_R increase.*

The proof is trivial and follows exactly the same arguments as in the proof of Proposition 2. Both subsidies stimulate growth in the long run. A rise in capital subsidy increases the marginal return to investment, and the main mechanism behind this is straightforward. An increase in β_k reduces the discount rate of the firms in the final goods sector since in this case $\widetilde{r}=r+\xi+\delta-\beta_K=r+\frac{\alpha}{1-\alpha}g+\delta-\beta_K$. This stimulates investment and research following the indirect modernization effect of investment. Indeed, an increase in β_K tends to shift upward the marginal return to innovation, $\widetilde{V}(g)$, while the marginal cost of innovation is kept constant. On the other hand, an increase in research subsidy β_R has the same expansive effects through a direct reduction in the marginal cost of research.

Therefore, subsidizing both capital or research has the same qualitative effects in the long run. As pointed out by Howitt and Aghion (1998), the traditional dichotomy between the accumulation of capital as a short run determinant of the growth process and R&D as the exclusive determinant of long run growth *via* technological advances does not make sense neither theoretically nor empirically. It is not relevant on the theoretical ground because extremely stringent conditions are required to generate such a property. Even if we abstract away from the embodiment hypothesis and the Arrowian learning by doing, capital accumulation is important for long run growth because capital is required for producing ideas on one hand, and for implementing and exploiting them on the other. Unless we hypothesize some extreme production functions in the research sector (as in Romer, 1990), the previous dichotomy cannot be generated.

Finally, note that our results are obtained on the BGP dominated by strategic substituabilities. The results would be inverted if a BGP dominated by strategic complementarities were to be considered. The reason for that has been already clearly identified by Young (1993): "...If one raises the return to an endogenous activity in a situation in which the return to economic actors is locally decreasing in their effort level, then a return to equilibrium will require an increase in their level of activity. However, if the return to the economic actors is locally increasing in their effort level, as is the case near the complementarity steady state, then a return to equilibrium requires a paradoxical reduction in their level of activity". In our model the BGP dominated by strategic complementarities does arise for unrealistic parameters' values and turns out to be instable. Therefore, the unique sustainable policy assessment lesson of our model is that both capital and research subsidy are growth improving.

4.2 Embodiment and the efficiency of capital subsidy

Aghion and Howitt (1998), chapter 3, argue that an increase in the capital subsidy has a lower (positive) effect on growth if technological progress is embodied. Precisely, the two authors compare a situation in which the obsolescence rate is zero due to a putty-putty specification (the non-embodiment case) with a situation in which a putty-clay technology yields a non-zero obsolescence rate equal to the innovations obsolescence rate, which is the Poisson probability $\gamma = \tilde{q}^{-1}g$ in our model (the embodiment case). In both cases, a rise in capital subsidy stimulates growth. However, in the embodiment case, the subsequent drop in the growth rate induces an obsolescence effect, which affects negatively capital accumulation. This obsolescence effect specific to the embodiment case implies that capital subsidies are more efficient in terms of growth promotion in the non-embodiment configuration.

We argue here that Aghion and Howitt's story is incomplete since it does not take into account the modernization effect of investment when technological progress is embodied. In our model, the embodiment assumption implies that the marginal return to innovation is positively correlated with the investment rate, and negatively correlated with the average age of capital. This is a characteristic of our truly vintage structure, which captures entirely the embodiment assumption in contrast to the putty-clay technology adopted by Aghion and Howitt. If the modernization effect is taken into account, then we have an additional ingredient with respect to the latter authors. The rise in investment due to the capital subsidy yields indeed a decline in the average age of capital, which again stimulates research and growth. This additional third effect may well rule out the main lesson from Aghion and Howitt's model, namely the negative effect of embodiment on the efficiency of capital subsidy.

Let us give a very simple insight into this issue.[14]Let us consider the two following

[14]The treatment given below is just for an illustrative purpose. A much more rigorous appraisal of this question requires the accurate comparison of the exact effects of capital subsidy in our model and in

equations characterizing the BGP of the model:

$$k^{-\alpha} = \frac{(1-\alpha)^2 B}{\eta} \left(\sigma g + \xi + \delta + \rho - \beta_K\right) \tag{K'}$$

$$1 - \beta_R = \frac{\lambda L \alpha \, \eta}{1-\alpha} \frac{k}{m\left(\sigma g + \rho + \tilde{q}^{-1} g\right)} \tag{G'}$$

The equation (K') and (G') are simply obtained by rewriting equations (K) and (G) using the other equations of the system (K)-(S), once the assumption $\beta_K = \beta_R = 0$ is relaxed. Recall that ξ and m represent the obsolescence cost and the average age of capital respectively. A higher growth rate g supposes an increase in ξ but a decline in m through the modernization effect of investment. How do these effects show up? In Figure 1, the system (K')-(G') is represented. The two curves intersect in a point O which coordinates are the long run values for the growth rate and capital intensity. For fixed ξ and m, if we rise the capital subsidy β_K, the curve (K') will shift upward, and we get a new equilibrium D with a higher growth rate. If technological progress were disembodied, the story would stop here. Under embodiment, ξ will rise and this obsolescence effect will imply a further downward shift in the curve (K'), with a new equilibrium point B. At B, the growth rate is lower than in D, which reflects exactly the argument of Aghion and Howitt. Now, if we account for the modernization effect, an increase in g will lower the average age of capital m, which shifts the curve (G') upward, and we get a final equilibrium point I. At this point, the equilibrium point could be perfectly greater than at point O.

Our analysis traces back indeed to one of the most famous controversies in the sixties, *the embodiment controversy* (see Hercowitz, 1998, for a modern reincarnation of this debate). Those who supported the embodiment assumption argued that investment is the right channel through which innovations are implemented, and the capital stock is modernized, so that the investment rate should be a crucial determinant of long run growth. In a truly devastating paper, Phelps (1962) radically questioned this view. Within an exogenous growth framework with both embodied and neutral technological progress, he showed that while investment and its modernization effect are important for the pace of the growth process in the short run, they are not so in the long run: The distribution per vintage of the capital stock is ultimately independent of the total amount of capital and of the investment rate.[15] Our model makes clear that this issue is far from simple in a research-based endogenous growth setting. In our vintage

a kind of "disembodied technological progress" counterpart. We think that our treatment is enough meaningful to make the point in economics terms without any need to any heavy complementary analytical developments.

[15]Denison (1964) denied any empirical relevance to the distinction between embodied and neutral technological progress. According to this contribution, embodiment is important if the modernization effect is. Through some simple (and questionable) accounting exercises, Denison found that this modernization effect is of a negligible magnitude. So he concluded for "the unimportance of the embodied question".

capital model, the growth rate is not independent of the average age of capital. An increase in investment stimulates research and growth, which lowers the average age of capital and induces a further expansion in investment and growth. In our context, the modernization effect is therefore a fundamental determinant of long run growth.

5 Conclusion

In this paper, we have developed a general equilibrium model which incorporates both the creative destruction setup due to Aghion and Howitt (1992) and the vintage capital production function due to Solow (1960). In our view, this is the most natural way to introduce capital accumulation into the canonical creative destruction model. As we have shown along this paper, the previous attempt by Aghion and Howitt (1998) to do so is certainly not the most complete. It fails to reproduce some fundamental stylized facts of the real economies related to embodied technological progress, and it implies some questionable policy implications precisely because its appraisal of the importance of embodiment is not comprehensive enough.

The most salient characteristic of our endogenous growth research-based model is that the profitability of R&D depends on the investment rate. This property is intimately related to the embodied nature of technological progress as we have shown. In such a case, the investment variable fully determines the demand for the new designs of capital goods, and thus the (expected) profits of the innovators. This generates a clear modernization effect of investment, an increase in investment stimulates the research efforts and the probability of successful innovations, which tend to accelerate the rate of technological progress and growth. This modernization effect can lead to rule out one of the main policy lessons brought out by Aghion and Howitt (1998), chapter 3: The magnitude of the effect on growth of a capital subsidy need not be lower under embodiment. Clearly, embodied technological progress generates obsolescence costs. But the modernization effect of investment inherent to embodiment may well counterbalance these costs.

Our view of investment is, as we have pointed out along this paper, completely consistent with the recent empirical evidence on the role of investment in explaining the divergence or convergence of growth patterns. Especially, the empirical contributions of DeLong and Summers (1991) and Wolff (1991) have explicitly or implicitly pointed at the embodied nature of technological progress as the main reason behind their findings. Our paper presents a complete framework in which the modernization effects of investment are thoroughly identified and tightly related to the embodied nature of technological progress. In comparison with the previous attempts to merge the Schumpeterian creative destruction and capital accumulation, we think it definitely more relevant and useful, despite its simplicity.

6 References

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