

Dynamics of the Wake Vortices of a Two-aircraft Formation, Hazard Assessment at Large Distances and Sensitivity Analysis

Ignace Ransquin*, Matthieu Duponcheel †, Philippe Chatelain‡
Institute of Mechanics, Materials and Civil Engineering
Université catholique de Louvain, 1348, Louvain-la-Neuve, Belgium

Extended formation flight is foreseen to be a viable technique for fuel consumption reduction in commercial aviation. It is based on a follower tracking the wake of a leader while maintaining a constant adequate separation. This leads however to different wake dynamics compared to those of the wake of an isolated aircraft. The uncertainty on the relative position between the follower and the wake of the leader further increases the analysis of the underlying vortex dynamics. Using Large Eddy Simulations, this paper presents a methodology to quantify the impact of that uncertainty on the propagation of the formation wake. Midway between a vortex method and grid-based CFD, the Vortex Particle-Mesh method combined the Immersed Lifting Lines is particularly well suited for the simulation of such long-lasting structures. The proposed methodology combines 3D Space Developing and 3D Time Developing simulations to capture the roll-up of the vortex sheets originating from the leader and the follower, their mutual interactions and their long term evolution in a turbulent environment. Simulations are performed for several formation flight configurations around a reference in order to produce a probabilistic representation for the wake propagation. Statistics on the wake vortices characteristics are deduced and are compared to the wake properties of an isolated aircraft.

Nomenclature

$\mathcal{R}$	=	aspect ratio
b	=	wing span
c	=	chord length
$C_{l,\alpha} = \frac{\partial C_l}{\partial \alpha}$	=	slope of the airfoil lift coefficient
$\Delta x, \Delta y$	=	leader's wake position relative to the follower's position
ϵ	=	eddy dissipation rate
f	=	probability density function
F_V	=	vertical Force
$\mathbf{l}$	=	sectional lift of the airfoil
M_{roll}	=	rolling Moment
m	=	aircraft mass
$RMC = \frac{M_{roll}}{\frac{1}{2}\rho V_\infty^2 S b}$	=	rolling Moment Coefficient
t	=	time
$\mathbf{u}$	=	local velocity
$VFC = \frac{F_V}{\frac{1}{2}\rho V_\infty^2 S}$	=	vertical Force Coefficient

*Ph.D. Candidate; ignace.ransquin@uclouvain.be.

†Research Engineer, Ph.D.; matthieu.duponcheel@uclouvain.be.

‡Professor; philippe.chatelain@uclouvain.be.

V_∞	=	norm of the upstream velocity
$\mathbf{x} = (x, y, z)$	=	spatial coordinate
$W = mg$	=	weight
w_i	=	weights
α	=	angle of attack
δ_{Ail}	=	aileron deflection
Γ_0	=	circulation distribution
θ	=	pitch angle
ρ	=	air density
$\omega = (\omega_x, \omega_y, \omega_z)$	=	local vorticity

I. Introduction

Formation flight is known to improve the efficiency of a two-aircraft formation (leader+follower), as proven by early investigations [1, 2]. Indeed, the follower benefits from substantial apparent drag reduction when flying in the upwash region of the leader's wake. This bio-inspired technique can therefore lead to significant fuel savings as shown either in a military context [3, 4] or very recently in commercial aviation [5, 6].

Whereas birds usually fly in close formation, civilian applications rather target the concept of extended formation flight for energy savings [7] that better addresses safety concerns. In that case, the follower is flying at several tens of wingspans downstream of the leader. The follower still benefits from a positive interaction as the leader's wake vortices are strong, long-lasting structures. However, simplified linear analysis [8] has shown that 50% of the benefit is lost if the follower cannot maintain a lateral and vertical relative position within 10% span of the leader, wings tip to tip. This has been also confirmed experimentally [9].

Because of that sensitivity to the relative position, an accurate tracking of the wake position is thus mandatory in order to perform efficient extended formation flight. Rather than following the leader itself, the follower needs to track its wake, which moves in its environment due to the wind or its self-induced velocity. Strategies based on the use of onboard Light Detection And Ranging (LIDAR) allow the follower to measure in real time the air flow velocities induced by the vortices created by the leader. Dedicated software determines the positions of the vortices that have to be followed [10]. Alternative vortex detection techniques rely on indirect measurements and on estimations of the wake vortex parameters. The first envisioned strategies rely on wing-distributed pressure measurements that are processed by an estimator to deduce the parameters of the wake [11, 12]. Those techniques rely on additional specific instrumentation; the use of conventionally available measurements has therefore been envisioned using the dynamics of the aircraft themselves feeding similar estimators [13, 14].

Another key research area relates the study of the evolution of the wake produced by a formation as it will govern the air traffic management decisions. The propagation of the wake of an isolated aircraft has long been studied and led to numerous studies describing and modeling their behavior depending on turbulence intensity, stratification or the ground proximity [15, 16]. While an isolated aircraft produces two counterrotating vortices that sink due to their self-induced velocity, a two-aircraft formation wake shows different behaviors with an important sensitivity to the relative position of the aircraft. In [17], Understrasser et al. model the wake of a two-aircraft formation as two pairs of rolled-up vortices. Using Large-Eddy Simulations (LES), they study the movement of each vortex individually and investigate the resulting far-field wake, and in particular the plume dimensions and the contrail formation.

Some crucial questions remain though. The aforementioned tracking strategies aims to maintain a constant relative position between the follower and the leader's wake. However, it is very likely that the relative distance will be noisy and uncertain due to the inaccuracy of the chosen tracking equipment or because of turbulence-induced perturbations of the leader's wake. The uncertainty on this relative position, associated

with the non-linear character of the interaction between vortices, leads to an uncertainty on the position of the wake vortices and this from their emission to the far-wake. In the present work, we introduce a methodology to estimate in a probabilistic sense the characteristics of a wake produced by two aircraft in formation flight whose relative position is uncertain. For that purpose, we employ LES in order to capture the roll-up of the vortex sheets originating from the leader and follower, their mutual interactions, and their evolution within ambient turbulence. The numerical tool is based on state-of-the-art Vortex Particle-Mesh (VPM) flow solver. We propose a calibrated combination of 3D Space Developing (SD) and 3D Time Developing (TD) VPM simulations to capture the full phenomenological chain of such wakes. The TD simulation is initiated from a location in the SD simulation where these roll-up 3D phenomena become negligible with respect to the large scales and long time evolution of the asymmetric wake emanating from the two-aircraft formation. To our knowledge, the combination of both SD and TD simulations has never been performed in the context of formation flight. Simulations are performed for several formation flight configurations around a reference, we then quantify the imprecision of the relative position on wake propagation and we study the consequences and constraints for the air traffic management, e.g., in terms of Rolling Moment Coefficient sensed in the wake of the formation.

In this paper, we first briefly review the VPM method and the associated modeling methods (Section II). We then present in III the methodology followed to perform either the SD and TD simulations and we describe the formation flight configurations studied. Opening the results section, we describe the sensitivity observed in the wake vortices evolution, and the wake vortex characteristics with respect to the relative positioning between the follower and the leader's wake (Section IV.A). We then present a probabilistic analysis of the wake evolution and describe and compare the global characteristics of the wake to those of a single aircraft wake (Section IV.B). We close the results section in presenting in Section IV.C the impact of formation flight on global wake characteristics.

II. Numerical method

A. Flow Solver

In this work, we use a state-of-the-art Vortex-Particle Mesh (VPM) method to simulate the wake flow behind the formation. It combines the advantages of a particle method and of a mesh-based approach, which makes it an hybrid approach among the many variants of vortex methods [18]. The method is based on the vorticity-velocity formulation ($\omega - \mathbf{u}$) of the Navier-Stokes equations for incompressible flows ($\nabla \cdot \mathbf{u} = 0$)

$$\frac{D\omega}{Dt} = (\nabla \mathbf{u}) \cdot \omega + \nu \nabla^2 \omega + \nabla \cdot \mathbf{T}^M. \quad (1)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ represents the Lagrangian derivative, ν is the dynamic viscosity and $\mathbf{T}^M$ is the sub-grid scales (SGS) model. The velocity field can be decomposed as $\mathbf{u} = \mathbf{U}_\infty + \mathbf{u}_\omega$ in which one recovers $\mathbf{u}_\omega$ from the vorticity through the resolution of the Poisson equation

$$\nabla^2 \mathbf{u}_\omega = -\nabla \times \omega. \quad (2)$$

Similarly to classical vortex particle methods, the advection term is handled in a Lagrangian way. The flow is discretized using a set of particles characterized by a position $\mathbf{x}_p$, a volume V_p and a strength $\alpha_p = \int_{V_p} \omega d\mathbf{x}$. Particles are advected by the flow field and their strength is modified to account for the vortex stretching and

diffusion

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}(\mathbf{x}_p) \quad (3)$$

$$\frac{d\boldsymbol{\alpha}_p}{dt} = \int_{V_p} \left((\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} \right) d\mathbf{x}. \quad (4)$$

The previous equations are here solved using a third order Runge-Kutta integration scheme. Since particles are used solely for the advection treatment, they enable to avoid dispersion errors and waive classical time stability constraints (i.e. CFL Number). In the present hybrid VPM method, all the spatial differential operations are computed on an underlying mesh, thus in an Eulerian manner, which offers several advantages. First, the velocity field, Eq. (2), is solved using a Fourier-based fast Poisson Solver which allows the combination of unbounded directions and inlet/outlet boundaries. The right-hand side of Eq. (4) uses fourth-order finite differences, and the turbulence SGS model $(\nabla \cdot \mathbf{T}^M)$ also uses the grid and is implemented as in [19]. Finally, the mesh is exploited to remedy the loss of accuracy due to Lagrangian distortion through remeshing. This consists in the periodic regularization onto a grid of the particle set via high order interpolation schemes to recover information back and forth between the particles and the mesh. Reprojection of the vorticity field is also periodically applied to maintain the solenoidal property of the vorticity field which is otherwise not directly enforced by the solver. We refer to the reviews by Cottet and Komoutsakos [20] and Winckelmans [21] for an in-depth description of the application of VPM methods and the theory behind. Additionally, a description of the present implementation on massively parallel architecture has been provided by Chatelain et al [22]. This VPM method is used in various applications, ranging from aircraft wakes [23] and helicopter wakes [24] to vertical axis wind turbines or horizontal axis wind turbine wakes [25, 26].

B. Wing model using Immersed Lifting Lines

The generation of vorticity along the lifting surfaces of an aircraft is accounted for through an Immersed Lifting Lines (ILL) method [26]. Like the Actuator Line Method [27], the ILL takes care of the generation of the vorticity in the flow, instead of applying a force in the volume. As we consider slender lifting devices, the flow around the airfoil can be treated as essentially two-dimensional. The wing is thus approximated as a bound vortex concentrated along the quarter chord line. Based on the instantaneous velocity and angle of attack of every wing segment, the lift and the drag produced by the airfoil are retrieved from aerodynamic polars. Then, the circulation around the local 2-D airfoil is recovered from the Kutta-Joukowski theorem

$$l = \rho \Gamma \times V_{rel},$$

where l is the lift per unit span, V_{rel} is the relative velocity, and Γ is the circulation vector. The bound vortex is thus discretized along the wing span as a line of vortex particles, thickened through a spatial mollification. Required for numerical purposes, this mollification corresponds to a Gaussian spreading (of mollification factor σ) that covers a few mesh sizes h in the direction of l . The vorticity shed in the flow as new particles (mollified as well) is deduced from the temporal and spatial variation of the bound vorticity [28]. These new particles are then merged with the existing vorticity in the flow. The ILL method thus preserves the Lagrangian character of the overall numerical method unlike the ALM.

C. Aircraft Model

While the simulation of the wake of a single aircraft does not necessarily require to take into account the vehicle dynamics, the simulation of the wake of a formation requires to be able to trim the dynamics of the follower. Indeed, in a formation flight context, the follower aircraft benefits from the upwash generated by the leader's wake, which ultimately results in excess lift if we were to the follower at the same pitch angle

as the leader's. Moreover, the follower undergoes a rolling moment, which has to be compensated by an action of the ailerons. Those control actions resulting from formation flight ultimately impact the complete 6 degrees-of-freedom dynamics of the aircraft, as clearly described in [14]. For the sake of simplicity though, we adopt an aircraft configuration brought to its essential components. In this study, we represent the aircraft as a single wing whose pitch angle θ can be modified and which is equipped with outboard ailerons, as depicted in Fig. 1. The ailerons deflection is modeled as a local modification of the polar in order to model the corresponding change in aerodynamic loads. The controller presented in the Fig. 2 uses a Proportional-Integral-Derivative (PID) feedback to adapt the pitch angle θ and the deflection of the ailerons δ_{Ail} in order to compensate exactly the weight of the plane by its lift (i.e. to cancel the resulting vertical force F_Z), and to cancel the rolling moment M_L . For each simulation presented in this work, the inertial position of the aircraft is fixed over time, as well as the roll and yaw angles of the aircraft which are set to 0. In spite of these simplifications, the force and moment balances ensure that the vortices shed by the follower are captured accurately.

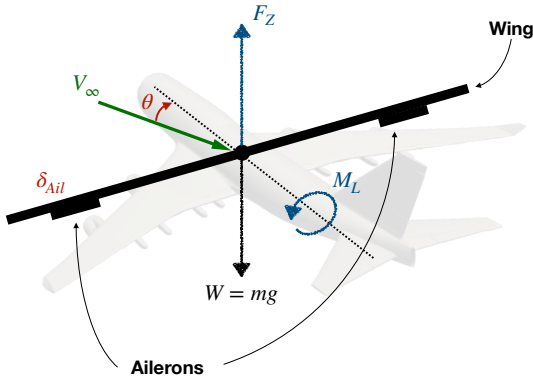


Figure 1 Aerodynamic model of the aircraft represented as a single ILL equipped with ailerons, the deflection of which, δ_{Ail} , is used to compensate the rolling moment M_L ; the pitch angle can be modified to compensate the weight W with the appropriate lift F_Z .

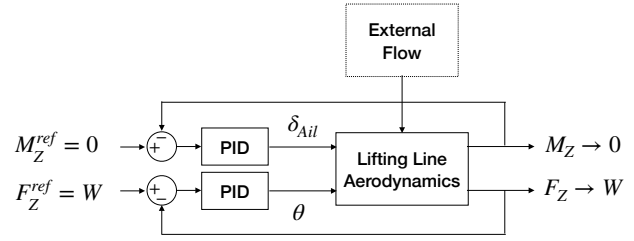


Figure 2 Block diagram of the PID controllers designed to adapt the ailerons deflection δ_{Ail} and the pitch angle θ of the aircraft in level flight in order to trim the aircraft under the influence of external flows (i.e., turbulence or wake vortices).

III. Simulations Setup

In this work, we use Space Developing (SD) simulations to accurately reproduce the 3D interactions phenomena occurring during the roll-up of the vortex sheets emanating from the leader and the follower. From these simulations, we generate an initial condition for a Time Developing (TD) simulation, which then allows to study the large scale and long time evolution of the asymmetric wake resulting from the formation. We study a formation flight scenario with two identical aircraft with geometrical properties similar to an A350 or B777: the ILLs used in the simulation have a wingspan $b = 64m$, with an elliptic chord distribution and an aspect ratio $\mathcal{A}R = 9.75$. Each aircraft has a mass $m_L = m_F = 250T$ and is flying at $V_\infty = 250m/s$ at FL330. As we consider steady flight configurations, we deduce the circulation of the wake vortices of the leader $\Gamma_{0,L}$ using

$$m_L g = \rho V_\infty \Gamma_{0,L} b_{0,L} \quad (5)$$

where g is the gravity constant, ρ the ambient air density and $b_{0,L} = b(\frac{\pi}{4})$ the separation distance between the two vortices of the leader. We obtain $\Gamma_{0,L} \simeq 485m^2/s$. We then compute the characteristic quantities

$$V_0 = \frac{\Gamma_0}{2\pi b_0} \simeq 1.55m/s \quad (6)$$

$$t_0 = \frac{b_0}{V_0} \simeq 32.6s, \quad (7)$$

where V_0 is the vertical self-induced velocity of the wake, and t_0 is the characteristic time. Under those conditions, the simulation of the wake propagation over one characteristic time t_0 requires to simulate the wake over a distance $z_0 = V_\infty \cdot t_0 = 8150m \simeq 130b$. The entailed simulation costs are prohibitive if we wish to represent all the desired phenomena with enough accuracy, which justifies the combined use of SD and TD simulations.

Fig. 3 describes the setups of the SD and TD simulation used to compute the formation flight scenarios.

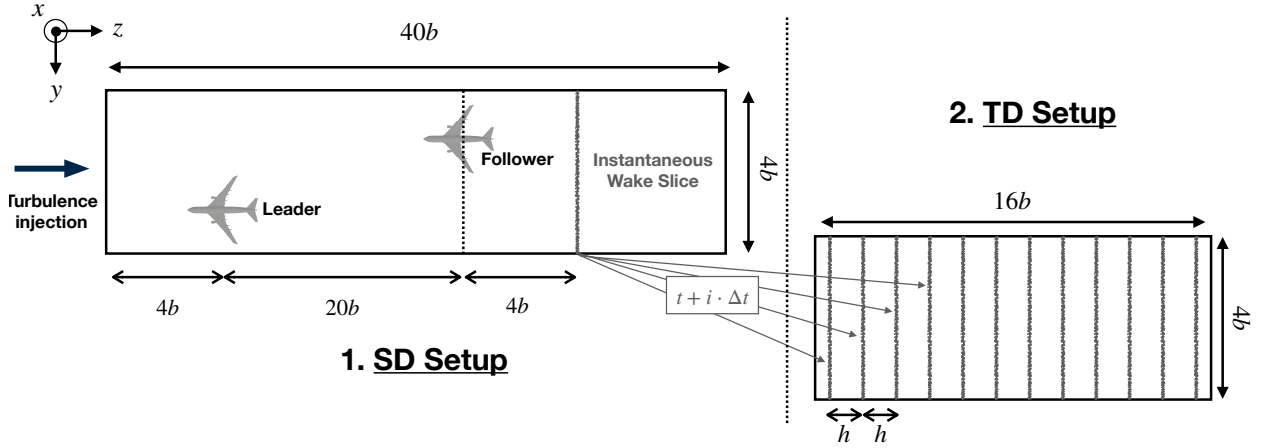


Figure 3 Description of the SD and TD simulation setups.

For the SD simulations, the follower is located twenty spans behind the leader in the streamwise direction. This distance ensures the roll-up of the leader's wake without increasing disproportionately the computational domain size, here at $4b \times 4b \times 40b$. Synthetic turbulence, reproducing the spectrum of Homogenous Isotropic Turbulence (HIT), and generated using the Mann algorithm [29], is smoothly injected at the inlet of the computational domain to reproduce as closely as possible actual flight and wake conditions. This affects the wake generation mechanism itself and seeds the flow with perturbations that may trigger instabilities. The turbulence is characterized by an integral length scale $L = 530m$ and a dimensionless Eddy Dissipation Rate (EDR) $\epsilon^* = \frac{\epsilon b_0}{V_0^3} \simeq 2 \cdot 10^{-4}$, corresponding to weak turbulence [30], which is a reasonable assumption for a formation flight application. According to the flow perceived (i.e., turbulence and wake vortices), the leader and follower adapt their pitch and aileron deflections in order to achieve vertical and roll axis equilibrium as described in Section II.C.

We observe that beyond two to three spans b behind the follower, the 3D interaction phenomena are no longer governing the flow which becomes mainly 2D. It is therefore possible to generate an initial condition for a TD simulation by using the instantaneous vorticity fields observed on the same 2D slice. As depicted in Fig. 3, we select the slice located 4 spans behind the follower and we copy into a computational domain of size $4b \times 4b \times 16b$ the 2D vorticity field computed by the SD simulation every time step such that $\Delta t_{S-D} = \frac{h}{V_\infty}$, where h is grid resolution of the mesh used in the TD simulation. The temporal sampling of the slice observed from the SD simulation associated with the spatial discretization of a TD simulation guarantees to capture

the the instability modes up to the theoretical fastest-growing Crow mode [31] of the equivalent pair. This explain the choice for a $16b$ -long TD computational domain.

A summary of the aircraft characteristics and the simulations setup is provided in Tables 1 and 2.

Table 1 Geometrical parameters of the aircraft

Wing	
Span	$b = 64\text{m}$
Aspect ratio	$\mathcal{R} = 9.75$
Chord distribution	Elliptic
Sectional Lift Slope	$a_0 = 2\pi$
mass	$m = 250000\text{kg}$
Speed	$U_\infty = 250\text{m/s}$
Air density	$\rho = 0.4\text{kg/m}^3$

Table 2 Numerical parameters used in the VPM simulations.

Space developing	
Domain size ($L_x \times L_y \times L_z$)	$4b \times 4b \times 40b$
Grid resolution	$h/b = 1/64$
Turbulence integral length scale	$L = 530m$
Eddy Dissipation Rate (EDR)	$\epsilon^* = \frac{\epsilon b_0}{V_0^3} \approx 0.0002$
Time developing	
Domain size ($L_x \times L_y \times L_z$)	$4b \times 4b \times 16b$
Grid resolution	$h/b = 1/64$

Fig. 4 presents all the relative position used in this work. We chose the relative position $(\Delta x, \Delta y) = (0b, 1.15b)$ as the reference position, considering it as optimal in terms of energy savings, while guaranteeing a security margin with respect to the position of the leader's vortices. The other simulated relative positions represent the uncertainty of the reference.

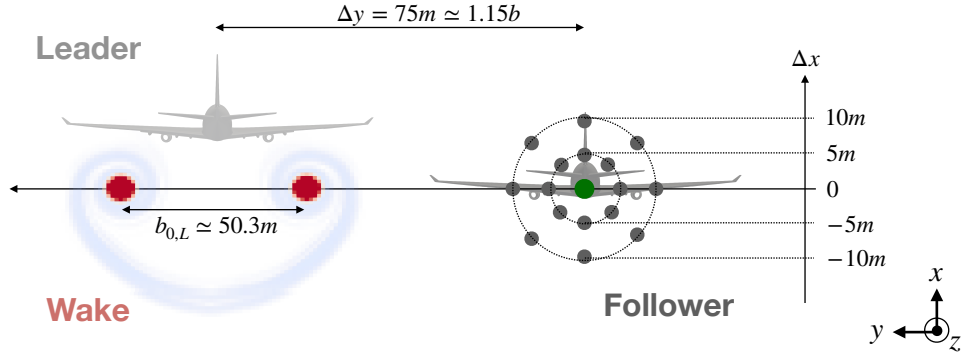


Figure 4 Sketch of the positions of the follower used for the simulations: the green dot represents the reference position of the follower while the grey dots indicates positions that the follower might reach when tracking the leader's wake.

IV. Results

The results section starts with a presentation of the reference configuration simulation. A 3D visualization of the vorticity produced by the two-aircraft formation is shown in Fig. 5. The vortex sheet emanating from the leader rolls up into the Leader Outboard Vortex (LOV) and Leader Inboard Vortex (LIV) before reaching the follower. Downstream of the follower, those vortices first interact with the follower's vortex sheet, which leads to complex 3D phenomena impacting the roll-up of that same sheet into the Follower Inboard Vortex (FIV) and the Follower Outboard Vortex (FOV). Those interactions play a major role in the propagation of the four wake vortices and are accurately modeled thanks to the SD simulation.

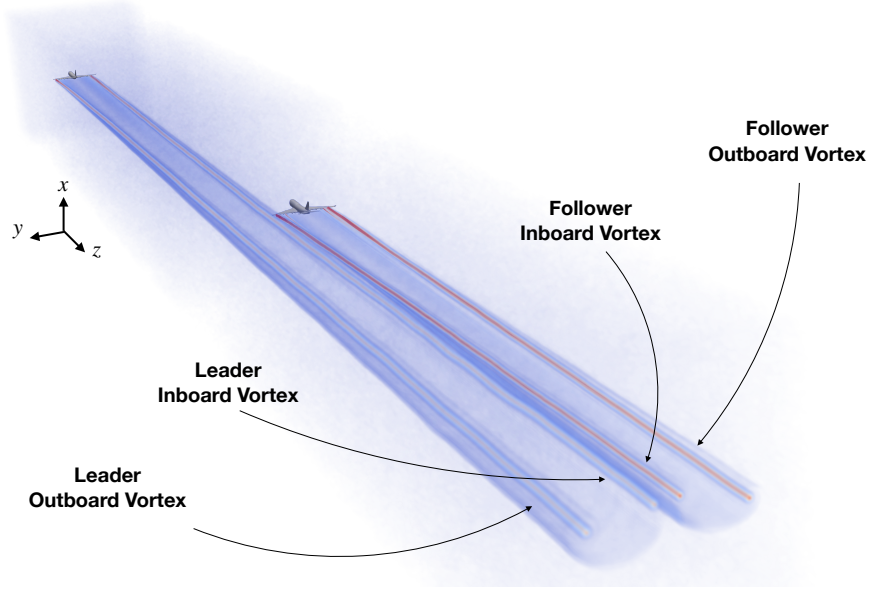


Figure 5 Volume rendering of the vorticity in the wake of the reference two-aircraft formation configuration computed by the SD simulation.

Following the methodology described in Section III, the initial vorticity field $t/t_0 = 0$ shown in Fig. 6a is deduced from successive instantaneous vorticity fields observed at one defined slice downstream of the follower. This initial condition is propagated over time to study the long term behavior of the wake, as observed in Fig. 6. Initially, the vortices of the formation are mainly straight, with a weak oscillation of the LOV and LIV betraying the impact of the ambient turbulence during the SD simulation. The LIV and LOV are almost totally rolled up and turbulent, unlike the FIV and FOV, between which we still observe a part of the vortex sheet of the follower. The LIV and FIV then move up and towards the left-hand side of the computational domain. This movement is explained by the self-induced velocity of the vortices, and by the asymmetry in terms of vortex roll-up and intensity that leads to the leftward displacement. Because of the small distance between the LIV and FIV, a Crow-type oscillation appears. However, as they have slightly different circulation (due to "younger" FIV), the reconnection events do not lead to typical vortex rings. One rather observes the generation of secondary structures, akin to the omega loops observed in [32]. These structures undergo stretching and instabilities of their own, leading to the dissipation of the inboard vortex pair. In the reference configuration, the LOV and FOV remain far from each other and from the inboard pair, which results in weak interactions, and their persistence over time.

In the following subsections, we first investigate the sensitivity of the vortex positions to the relative initial position based on the average axial vorticity field. We then investigate the characteristics of the instantaneous field in order to provide a statistical picture of the propagation of the wake. Finally, we close this result section by comparing the operational characteristics of the wake of a formation with those of the wake of an isolated aircraft.

A. Relative Positioning Sensitivity

In this section, we are interested in the longitudinally averaged vorticity field

$$\bar{\omega}_z(x, y, t) = \frac{1}{L_z} \int_0^{L_z} \omega_z(x, y, z, t) dz, \quad (8)$$

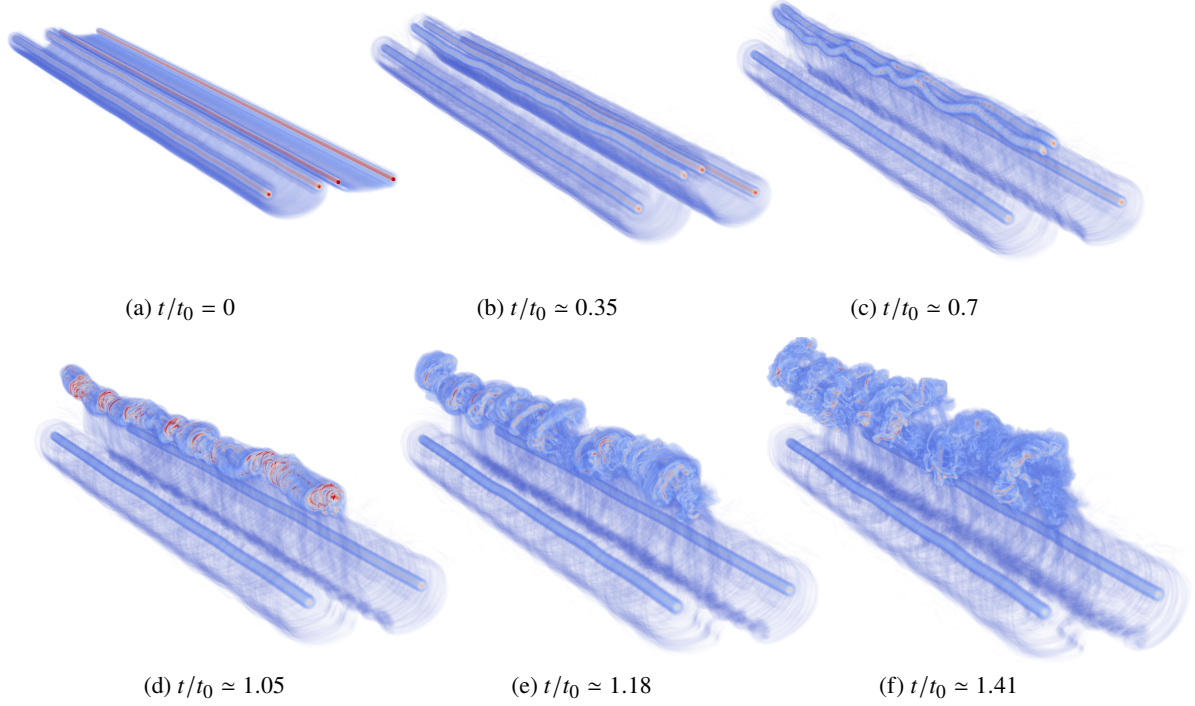


Figure 6 Volume rendering of the vorticity at six different wake ages for the reference formation configuration. The initial condition $t/t_0 = 0$ of this TD simulation is extracted from the SD simulation presented in Fig. 5.

where $L_z = 16b$ is the longitudinal length of the TD computational domain. By averaging the vorticity along the z axis, we remove the secondary vortical structures observed in Fig. 6, and we study the mean evolution of the wake over time. In particular, this enables to determine the trajectory of the vortices for the different configurations. In order to measure the mean displacement of the vortices, we compute the position of the vortex centroid using

$$\langle \bar{\omega}_z \rangle_k = \int_{\Omega} \frac{1}{\pi R^2} \exp \left(-\frac{((x - x_{c,k}(t))^2 + (y - y_{c,k}(t))^2)}{R^2} \right) \bar{\omega}_z(x, y, t) d\Omega, \quad (9)$$

$$x_{c,k}(t) = \frac{1}{\langle \bar{\omega}_z \rangle_k} \int_{\Omega} x \frac{1}{\pi R^2} \exp \left(-\frac{((x - x_{c,k}(t))^2 + (y - y_{c,k}(t))^2)}{R^2} \right) \bar{\omega}_z(x, y, t) d\Omega, \quad (10)$$

$$y_{c,k}(t) = \frac{1}{\langle \bar{\omega}_z \rangle_k} \int_{\Omega} y \frac{1}{\pi R^2} \exp \left(-\frac{((x - x_{c,k}(t))^2 + (y - y_{c,k}(t))^2)}{R^2} \right) \bar{\omega}_z(x, y, t) d\Omega \quad (11)$$

where Ω is the area covered by the slice (i.e., the $4b$ square), $R = b/32$ is a smoothing parameter and $\langle \bar{\omega}_z \rangle_k$ is the Gaussian-smoothed average vorticity around the vortex centroid in $(x_{c,k}(t), y_{c,k}(t))$. This is thus an iterative procedure performed for each one the LOV, LIV, FIV and FOV.

Fig. 7 shows the average vorticity fields with the vortex trajectories at different ages of the wake and for five formation flight configurations, including the reference i.e., $(\Delta x, \Delta y) = (0m, 75m)$. We observe that a change in the relative position completely modifies the wake behavior over time, and in particular the vortex trajectories.

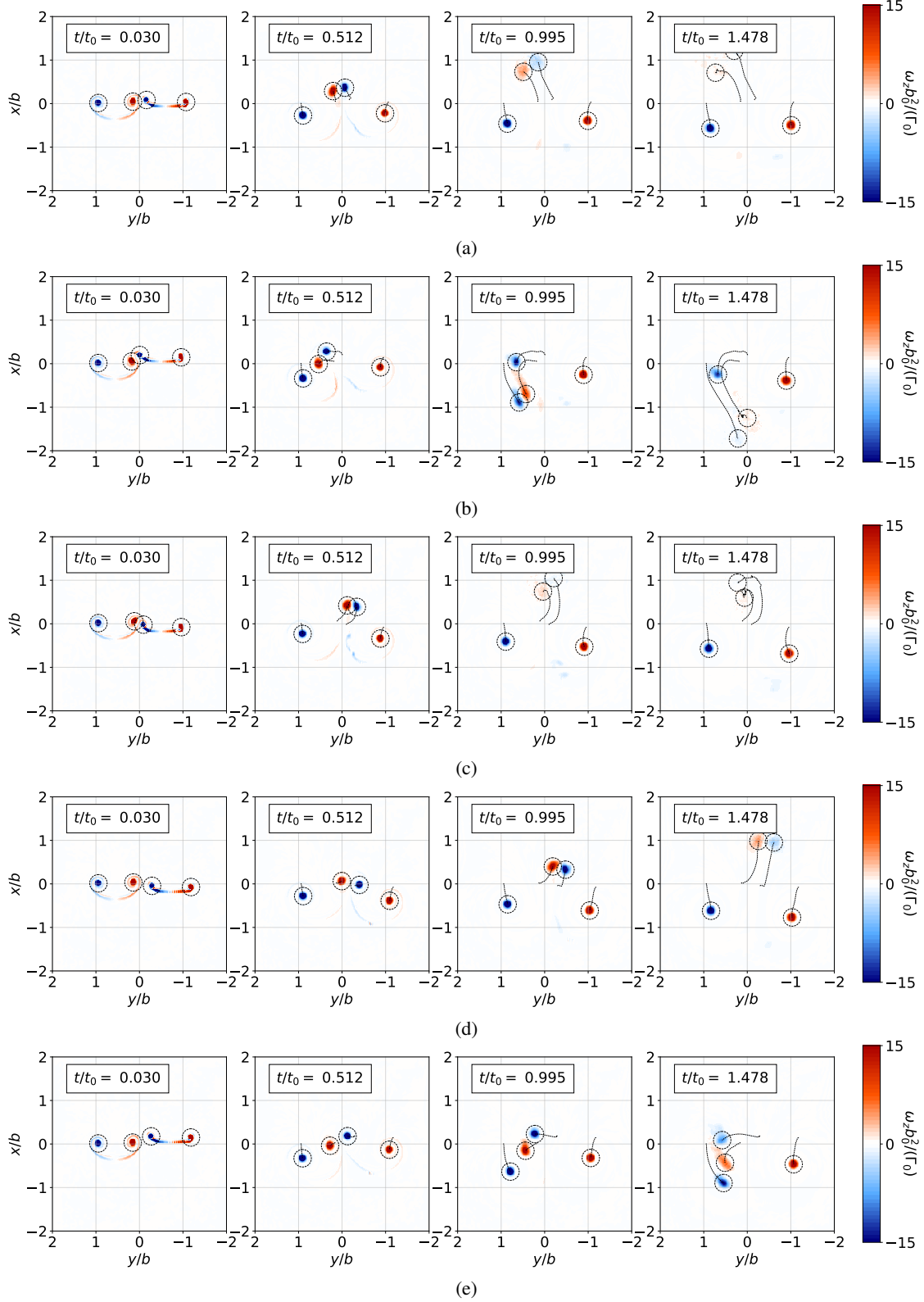


Figure 7 Streamwise averaged vorticity field $\bar{\omega}_z$ at different ages of the wake behind the two-aircraft formation for different initial configurations. The dashed circle indicates the instantaneous position of each vortex, while the dashed line is the trajectory. (a) $(\Delta x, \Delta y) = (0m, 75m)$, (b) $(\Delta x, \Delta y) = (-7.1m, 67.9m)$, (c) $(\Delta x, \Delta y) = (7.1m, 67.9m)$, (d) $(\Delta x, \Delta y) = (7.1, 82.1m)$, (e) $(\Delta x, \Delta y) = (-7.1, 82.1m)$.

B. Probabilistic Wake Propagation

In this section, we perform a statistical analysis of the wake propagation around the reference two-aircraft formation configuration. For this purpose, we associate weights $\bar{w}_i$ to each one of the 17 configurations presented in Fig. 4. They correspond to the probability for the formation to be in the configuration i during a formation flight tracking procedure. These weights are computed using the Gaussian distribution

$$w_i = \frac{1}{\pi\sigma_t^2} \exp\left(-\frac{((\Delta x_i - \Delta x_{ref})^2 + (\Delta y_i - \Delta y_{ref})^2)}{\sigma_t^2}\right) \quad (12)$$

$$\bar{w}_i = \frac{w_i}{\sum_j^N w_j} \quad (13)$$

where $(\Delta x_{ref}, \Delta y_{ref})$ and $(\Delta x_i, \Delta y_i)$ are the relative positions used for the reference configuration and the configuration i , $\sigma_t = 10m$ is the tracking variance and $N = 17$ is the total number of configurations. We then select 32 slices distributed in the TD computational domain along the streamwise direction (i.e., z axis) for which we compute the instantaneous position and circulation associated with each of the four vortices over time. By gathering these statistics for each of the 17 configurations described, we can determine a statistical model of the wake propagation and its intensity over time.

Fig. 8 presents the Probabilistic Density Function (PDF) $f_{V_k}(x, y, t)$ of the position of each one of the V_k vortices (i.e., LOV, LIV, FIV and FOV) for four different times (i.e., ages of the vortices). The PDF $f_{V_k}(x, y, t)$ is computed in assuming that each one of the 32 slices extracted from the TD simulation follows a 2D Gaussian distribution which gives

$$f_{V_k}(x, y, t) = \sum_{i=1}^N \bar{w}_i \sum_{j=1}^{32} \frac{1}{\pi\sigma_v^2} \exp\left(-\frac{((x - x_{V_k}^j(t))^2 + (y - y_{V_k}^j(t))^2)}{\sigma_v^2}\right) \quad (14)$$

where $[x_{V_k}^j(t), y_{V_k}^j(t)]$ is the position of the vortex V_k in the slice j at time t , and σ_v is a mollification parameter associated to the position of the vortex chosen similar to the discretization of the tracking Gaussian in Eqs. (15). The positions $[x_{V_k}^j(t), y_{V_k}^j(t)]$ are computed using the same methodology as the one described in Section IV.A in considering the 32 instantaneous slices this time.

We first observe the characteristics of the FOV (i.e., right most column in Fig. 8), the distribution of which remains very concentrated over time. This is due to the absence of interactions with other vortices during the period considered. It only undergoes a downward velocity induced mainly by the LOV. The FIV and LIV forming the inboard pair present much wider distributions which can be explained by their strong mutual interactions in all of the flight configurations studied. Also, these two vortices potentially interact with the LOV, which stretches their distribution even more. Finally, we observe that the LOV presents a distribution as concentrated as the FOV. Nevertheless we observe the appearance of secondary clusters, explained by possible interactions with the inboard pair.

Fig. 9 presents the evolution of the average circulation $\bar{\Gamma}_{0,V_k}(t)$ for each one the vortices as well as their standard deviation $\sigma_{V_k}(t)$. Means and standard deviations are computed using a weighted approach,

$$\bar{\Gamma}_{0,V_k}(t) = \sum_{i=1}^N \bar{w}_i \sum_{j=1}^{32} \frac{1}{32} \cdot \Gamma_{0,V_k}^j(t) \quad (15)$$

$$\sigma_{V_k}^2(t) = \sum_{i=1}^N \bar{w}_i \sum_{j=1}^{32} \frac{1}{32} \cdot \left(\Gamma_{0,V_k}^j(t) - \bar{\Gamma}_{0,V_k}(t)\right)^2 \quad (16)$$

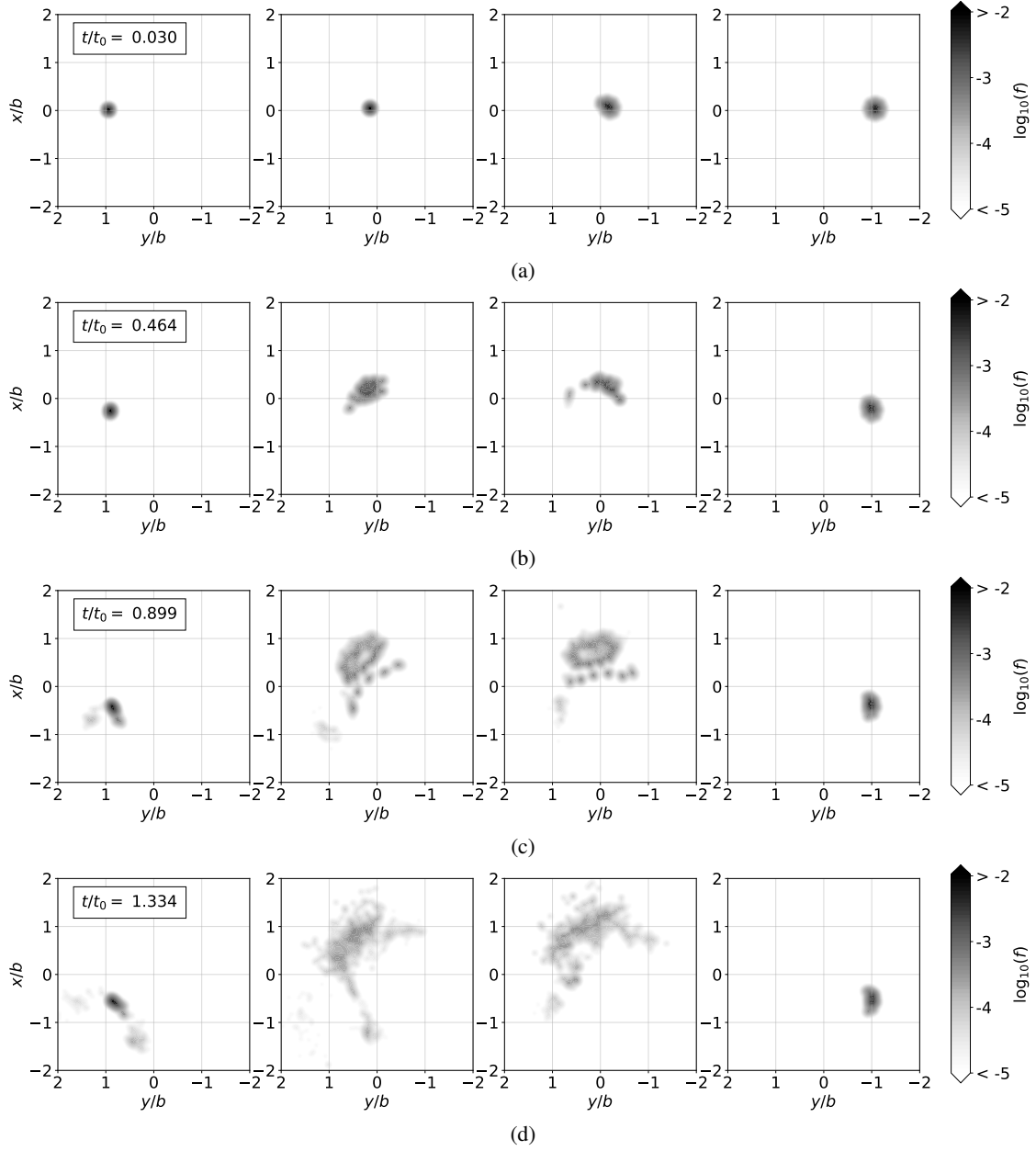


Figure 8 Probability Density Function $f_{V_k}(x, y, t)$ at four different time of the position of the four wake vortices (i.e., LOV, LIV, FIV and FOV from left to right).

where $\Gamma_{0, V_k}^j(t)$ is the maximum circulation of the vortex V_k on the slice j at time t .

As expected, we observe that the circulation of the FOV is constant over time showing once again the absence of interactions with the other vortices. The inboard vortices are the most impacted and their circulation decreases rapidly. However, the important standard deviation reminds that the instantaneous vortices circulation can still be intense despite their progressive dissipation. Finally, the LOV also show a decreasing behavior, which is substantially slower than the inboard vortices, as its interactions with the latter occurs later and with a lower probability.

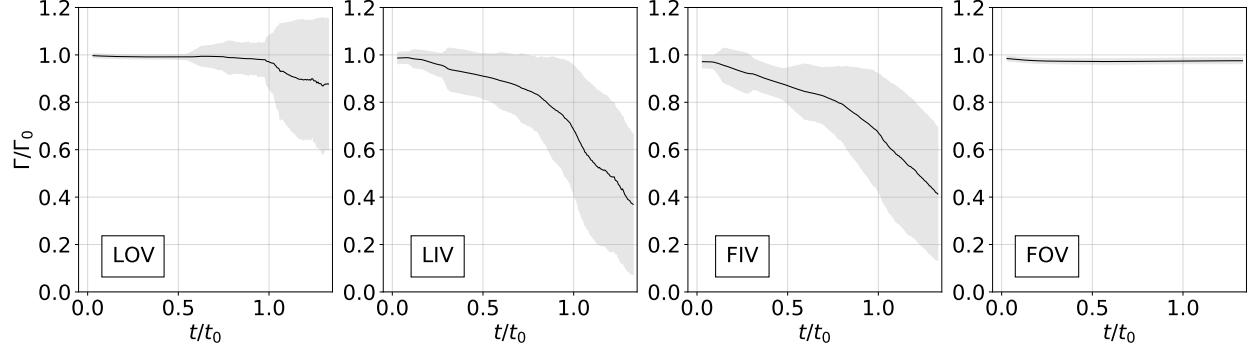


Figure 9 Evolution of the mean and standard deviation of the circulation of the four wake vortices (i.e., LOV, LIV, FIV and FOV from left to right) over time.

C. Operational Analysis

The propagation of the wake of an isolated aircraft has long been studied and led to numerous studies, among others in an operational and air traffic management context. Specifically, the hazard posed to a following aircraft has been studied and quantified through the rolling moment induced by the vortices on this hypothetical follower. In the context of formation flight, this induced rolling moment is of course controlled by an action on the aircraft dynamics (e.g., aileron deflections), the follower willing to stay in the vicinity of the leader's vortices. However, as shown in Section IV.A, the wake of a two-aircraft formation exhibits complex behaviors, which would therefore lead to a different rolling moment induced on a third aircraft.

We denote the vertical velocity field of an existing wake encountered by an aircraft at a location y' along the wingspan of that aircraft by $u_f(y')$. We assume that this velocity is small compared to the flight velocity V_∞ of the aircraft. This allows to consider the effect of the wake as a small perturbation on the wing loading of the aircraft. The induced rolling moment on the aircraft is then obtained as

$$M_{roll}(x, y, t) = \frac{1}{2} \rho V_\infty^2 \int_{-b/2}^{b/2} C_{l,\alpha}(y') c(y') \frac{u_f(y')}{V_\infty} y' dy' \quad (17)$$

where $C_{l,\alpha}(y') = \frac{\partial C_l}{\partial \alpha}$ is the lift slope coefficient of the aerodynamic profile at location y' , $c(y')$ is the chord at that profile. The Rolling Moment Coefficient (RMC) is then defined as

$$RMC = \frac{M_{roll}}{\frac{1}{2} \rho V_\infty^2 S b} \quad (18)$$

where S is the surface of the wing of the aircraft.

The RMC is computed for every possible location in the wake of the sampled formation configurations described in Section III, and we determine the maximum of the RMC at every age of the formation wake. Following a approach as similar as the one described in Section IV.B, we determine the means and standard deviations of the maximum RMC for the flight configurations, and we compare it the maximum RMC experienced in the wake of a single aircraft in Fig. 10a.

We observe that the maximum RMC experienced in the wake of a formation can be nearly 50% higher than the one experienced in the wake of an isolated aircraft. This maximum is observed near the inboard vortex pair of the formation (i.e., LIV and FIV, solid line with circle ($\circ$) in Fig. 10a), which also explains its rapid decay, as well as a important standard deviation. Once this inboard pair has dissipated, the maximum RMC is found near the FOV (i.e., solid line with square ($\square$) in Fig. 10a). As the FOV interacts little with other vortices in the formation, the maximum RMC becomes constant over time with a small standard deviation.

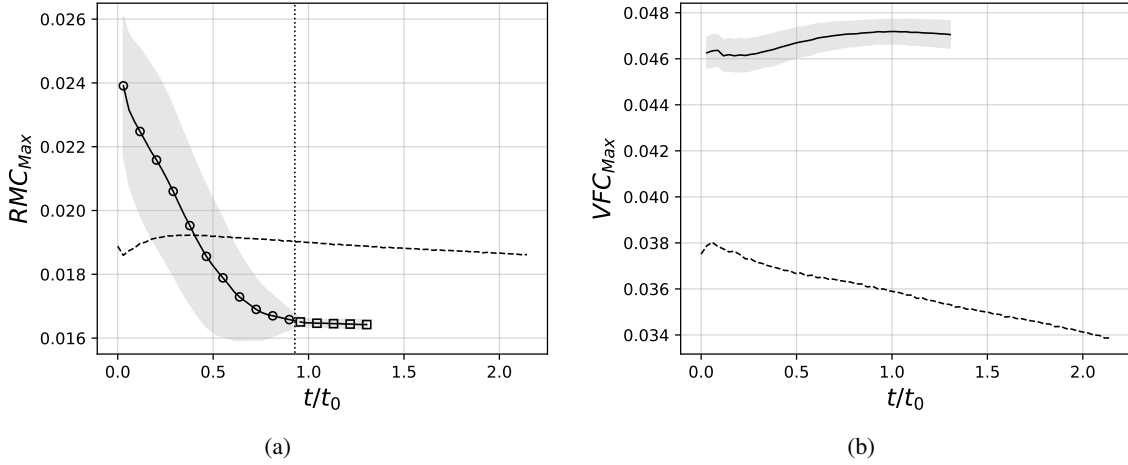


Figure 10 Rolling Moment Coefficient (a) and Vertical Force Coefficient (b) experienced in the wake of the reference two-aircraft formation (solid line) and in the wake of an isolated aircraft (dashed line). The grey area represents the 1-standard deviation envelope.

The maximum RMC is then weaker than the one observed behind an isolated wake, which makes it *a priori* less hazardous.

In the context of formation flight, we get interested in a second wake global quantity that we name the Vertical Force Coefficient (VFC) that characterizes the increase in vertical force experienced by an aircraft flying in a wake because of the upwash $u_f(y')$. An important VFC characterizes the "free lift" induced by the wake. We compute the vertical force F_x and the resulting VFC using the equations

$$F_x(x, y, t) = \frac{1}{2} \rho V_\infty^2 \int_{-b/2}^{b/2} C_{l,\alpha}(y') c(y') \frac{u_f(y')}{V_\infty} dy' \quad (19)$$

$$VFC = \frac{F_x}{\frac{1}{2} \rho V_\infty^2 S}. \quad (20)$$

Again, we compare the maximum VFC computed in the wake of the two-aircraft formation and in the wake of an isolated aircraft in Fig. 10b. We observe this time that the maximum VFC is at least 25% greater in the wake of a formation. Furthermore, we measured this maximum in the outboard vicinity of the FOV. There would therefore be of great interest to extend the formation in adding a third aircraft that would benefit from an additional vertical force than the second one in the formation.

V. Conclusion

In this paper, we presented a methodology used to quantify in a probabilistic sense the characteristics of the wake produced by two aircraft in formation flight whose relative position is uncertain. We employed Large Eddy Simulations performed using the Vortex Particle-Mesh method with Immersed Lifting Lines to simulate several formation flight configurations around a reference. The combination of Space Developing and Time Developing simulations enabled to accurately reproduce the interactions phenomena occurring during the roll-up of the vortex sheets emanating from the aircraft, as well as the large scale and long time evolution of the asymmetric wake resulting from each one of the configurations. In particular, we managed to accurately model the intrinsic asymmetry of the wake of a formation coming from the age difference between the leader's vortices and the follower's vortex sheet. The simplified aircraft dynamics that were taken into account in the simulations also participate to the wake asymmetry. Indeed, controllers adapt the ailerons

deflections to compensate the induced rolling moment of the follower which in return locally modifies the vorticity produced by the Immersed Lifting Lines.

The high fidelity of the simulations allowed to highlight the sensitivity of the propagation of the wake vortices to the relative position between the follower and the leader's wake. We described the trajectories of the vortices for the different configurations using time average vorticity fields. We then combined the results produced for all the formation configurations studied, and we deduced a statistical model of the wake propagation of the formations, thanks to the use of instantaneous vorticity fields. We showed that the outboard vortex of the follower had a very concentrated statistical distribution both in terms of position and intensity. The latter was therefore very little impacted by the rest of the wake. The inner vortices showed a very large variability in terms of position and intensity which was explained by their very strong mutual interaction in the form of Crow-type oscillations. Due to the asymmetry of the wake, we observed that the inner pair tended to move towards the leader's outboard vortex, which increased the probability of interactions between that pair and the leader outboard vortex. The latter thus showed a concentrated position distribution associated to a wide distribution in terms of vortex intensity.

The aforementioned probabilistic analysis led us to observe the impact of a formation wake in an operational context, and to compare it to the wake of an isolated aircraft. By computing a Rolling Moment Coefficient in the wake of both the formations and the isolated aircraft, we observed that the wake of a formation was less constraining in terms of Rolling Moment Coefficient intensity. However, it did not decrease as fast as the one computed in the wake of an isolated aircraft. We also defined a Vertical Force Coefficient that characterized the increase in vertical force experienced by an aircraft flying in a wake because of upwash winds. That Vertical Force Coefficient was shown more intense in the wake of a formation, compared to the wake of an isolated aircraft. This observation would justify the deployment of larger aircraft formations, as a third aircraft would enjoy an even greater benefit.

The simulations carried out in this paper allowed us to highlight the complexity and diversity of the dynamics of vortices in the wake of a two-aircraft formation. They allowed to evaluate the medium-term behaviors after the formations, and only need to be pursued on even longer propagation periods to bring a real light on the impact of the formation flight on the air traffic management. The present probabilistic approach also lays the foundations for the design of an operational model of the propagation of aircraft formation wake.

Acknowledgments

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement no. 725627).

The development work benefited from the computational resources provided by the supercomputing facilities of the Université catholique de Louvain (CISM/UCL) and the Consortium des Équipements de Calcul Intensif (CÉCI) en Fédération Wallonie Bruxelles (FWB) funded by the Fond de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under Convention No. 2.5020.11.

References

- [1] Beukenberg, M., and Hummel, D., "Aerodynamics, performance and control of airplanes in formation flight," *ICAS, Congress, 17 th, Stockholm, Sweden, Proceedings.*, 1990, pp. 1777–1794.
- [2] Hummel, D., "The use of aircraft wakes to achieve power reductions in formation flight," *The Characterization & Modification of Wakes from Lifting Vehicles in Fluid*, Vol. AGARD-CP-584, NATO, 1996, pp. 1–13.
- [3] Vachon, M., Ray, R., Walsh, K., and Ennix, K., "F/A-18 aircraft performance benefits measured during the autonomous formation flight project," Tech. Rep. TM-2003-210734, NASA, 2002. doi:10.2514/6.2002-4491.
- [4] Bieniawski, S. R., Rosenzweig, S., and Blake, W. B., "Summary of Flight Testing and Results for the Formation

- Flight for Aerodynamic Benefit Program,” *52nd Aerospace Sciences Meeting*, AIAA, 2014, pp. 1–26. doi:10.2514/6.2014-1457.
- [5] Flanzer, T. C., Bieniawski, S. R., and Brown, J. A., “Advances in Cooperative Trajectories for Commercial Applications,” *AIAA Scitech 2020 Forum*, 2020. doi:10.2514/6.2020-1257, URL <https://arc.aiaa.org/doi/abs/10.2514/6.2020-1257>.
 - [6] Airbus, “Airbus and its partners demonstrate how sharing the skies can save airlines fuel and reduce CO2 emissions,” , November 2021. URL <https://www.airbus.com/en/newsroom/press-releases/2021-11-airbus-and-its-partners-demonstrate-how-sharing-the-skies-can-save>.
 - [7] Ning, A., Flanzer, T. C., and Kroo, I. M., “Aerodynamic performance of extended formation flight,” *Journal of Aircraft*, Vol. 48, No. 3, 2011, pp. 855–865. doi:10.2514/1.C031046.
 - [8] Blake, W., and Multhopp, D., “Design, performance and modeling considerations for close formation flight,” *CLJ*, Vol. 150, 1998, p. 2.
 - [9] Hanson, C. E., Pahle, J., Reynolds, J. R., and Andrade, S., “Experimental Measurements of Fuel Savings During Aircraft Wake Surfing,” *2018 Atmospheric Flight Mechanics Conference*, 2018, p. 3560. doi:10.2514/6.2018-3560.
 - [10] Michel, D. T., Dolfi-Bouteyre, A., Goular, D., Augère, B., Planchat, C., Fleury, D., Lombard, L., Valla, M., and Besson, C., “Onboard wake vortex localization with a coherent 1.5 μm Doppler LIDAR for aircraft in formation flight configuration,” *Optics Express*, Vol. 28, No. 10, 2020, pp. 14374–14385. doi:10.1364/OE.377049, URL <http://www.opticsexpress.org/abstract.cfm?URI=oe-28-10-14374>.
 - [11] Hemati, M. S., Eldredge, J. D., and Speyer, J. L., “Wake Sensing for Aircraft Formation Flight,” *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 2, 2014, pp. 513–524. doi:10.2514/1.61114.
 - [12] DeVries, L., and Paley, D. A., “Wake Sensing and Estimation for Control of Autonomous Aircraft in Formation Flight,” *Journal of Guidance, Control, and Dynamics*, 2015, pp. 1–10. doi:10.2514/1.G001138.
 - [13] Caprace, D.-G., Winckelmans, G., Chatelain, P., and Eldredge, J., “Wake Vortex Detection and Tracking for Aircraft Formation Flight,” *AIAA Aviation 2019 Forum*, edited by AIAA, 2019. doi:10.2514/6.2019-3329, URL <https://arc.aiaa.org/doi/abs/10.2514/6.2019-3329>.
 - [14] Ransquin, I., Caprace, D.-G., Duponcheel, M., and Chatelain, P., “Wake Vortex Detection and Tracking for Aircraft Formation Flight,” *Journal of Guidance, Control, and Dynamics*, Vol. 0, No. 0, 2021.
 - [15] Holzäpfel, F., “Probabilistic Two-Phase Wake Vortex Decay and Transport Model,” *Journal of Aircraft*, Vol. 40, No. 2, 2003, pp. 323–331. doi:10.2514/2.3096, URL <https://doi.org/10.2514/2.3096>.
 - [16] De Visscher, I., Bricteux, L., and Winckelmans, G., “Aircraft vortices in stably stratified and weakly turbulent atmospheres: simulation and modeling,” *AIAA Journal*, Vol. 51, No. 3, 2013, pp. 551–566. doi:10.2514/1.J051742.
 - [17] Unterstrasser, S., and Stephan, A., “Far field wake vortex evolution of two aircraft formation flight and implications on young contrails,” *The Aeronautical Journal*, Vol. 124, No. 1275, 2020, pp. 667–702. doi:10.1017/aer.2020.3.
 - [18] Mimeau, C., and Mortazavi, I., “A Review of Vortex Methods and Their Applications: From Creation to Recent Advances,” *Fluids*, Vol. 6, No. 2, 2021. doi:10.3390/fluids6020068, URL <https://www.mdpi.com/2311-5521/6/2/68>.
 - [19] Cocle, R., Winckelmans, G., and Daeninck, G., “Combining the vortex-in-cell and parallel fast multipole methods for efficient domain decomposition simulations,” *Journal of Computational Physics*, Vol. 227, No. 21, 2008, pp. 9091–9120. doi:10.1016/j.jcp.2007.10.010.
 - [20] Cottet, G.-H., and Koumoutsakos, P., *Vortex Methods, Theory and Practice*, Cambridge University Press, 2000.
 - [21] Winckelmans, G. S., *Encyclopedia of Computational Mechanics*, John Wiley & Sons, Ltd, 2004, Chap. Vortex Methods, pp. 129–153. doi:10.1002/0470091355.ecm055.

- [22] Chatelain, P., Curioni, A., Bergdorf, M., Rossinelli, D., Andreoni, W., and Koumoutsakos, P., “Billion vortex particle Direct Numerical Simulations of aircraft wakes,” *Computer Methods in Applied Mechanics and Engineering*, Vol. 197, No. 13, 2008, pp. 1296–1304. doi:10.1016/j.cma.2007.11.016.
- [23] Caprace, D.-G., Winckelmans, G., and Chatelain, P., “An immersed lifting and dragging line model for the vortex particle-mesh method,” *Theoretical and Computational Fluid Dynamics*, Vol. 34, 2020, pp. 21–48. doi:10.1007/s00162-019-00510-1.
- [24] Caprace, D.-G., Winckelmans, G., and Chatelain, P., “Assessment of the Vortex Particle-Mesh Method for Efficient LES of Hovering Rotors and their Wakes,” *AIAA Scitech 2021 Forum*, 2021. doi:10.2514/6.2021-0738.
- [25] Balty, P., Caprace, D.-G., Waucquez, J., Coquelet, M., and Chatelain, P., “Multiphysics simulations of the dynamic and wakes of a floating Vertical Axis Wind Turbine,” *TORQUE*, Vol. 1618, IOP Publishing, Delft, 2020, p. 062053. doi:10.1088/1742-6596/1618/6/062053.
- [26] Chatelain, P., Backaert, S., Winckelmans, G., and Kern, S., “Large eddy simulation of wind turbine wakes,” *Flow, Turbulence and Combustion*, Vol. 91, No. 3, 2013, pp. 587–605. doi:10.1007/s10494-013-9474-8.
- [27] Sørensen, J. N., Mikkelsen, R. F., Henningson, D. S., Ivanell, S., Sarmast, S., and Andersen, S. J., “Simulation of wind turbine wakes using the actuator line technique,” *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 373, No. 2035, 2015. doi:10.1098/rsta.2014.0071.
- [28] Caprace, D.-G., Chatelain, P., and Winckelmans, G., “Lifting Line with Various Mollifications: Theory and Application to an Elliptical Wing,” *AIAA Journal*, Vol. 57, No. 1, 2019, pp. 17–28. doi:10.2514/1.J057487.
- [29] Mann, J., “Wind field simulation,” *Probabilistic Engineering Mechanics*, Vol. 13, No. 4, 1998, pp. 269–282. doi:10.1016/S0266-8920(97)00036-2.
- [30] De Visscher, I., “Interaction of wake vortices with the atmosphere and the ground : advanced numerical simulation and operational modeling,” Ph.D. thesis, Université catholique de Louvain, 2012.
- [31] Crow, S. C., “Stability theory for a pair of trailing vortices,” *AIAA Journal*, Vol. 8, No. 12, 1970, pp. 2172–2179. doi:10.2514/3.6083.
- [32] Ortega, J. M., Bristol, R. L., and Savas, Ö., “Experimental study of the instability of unequal-strength counter-rotating vortex pairs,” *Journal of Fluid Mechanics*, Vol. 474, 2003, pp. 35–84.