MoM interaction evaluation intended for antennas printed on ultra-thin substrates

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***Abstract*—A new kernel is proposed for the accurate simulation of planar antennas printed on ultra-thin substrates by means of integral-equation methods. A scalar-potential representation is used and a number of spectral-domain asymptotic terms, in the form of a series of exponentials, are extracted and integrated into the spatial domain. The spatial-domain counterpart of the remainder is evaluated numerically via Hankel transforms to be simply convoluted numerically with basis and testing functions. So, a global Method of Moments (MoM) impedance matrix is obtained from the asymptotic and remaining parts, which allows for an efficient evaluation of the electromagnetic properties of antennas printed on very thin substrates. It is shown that the asymptotic parts can be estimated in closed form, which is validated against purely numerical or hybrid (numerical and analytical) solutions.**

***Index Terms*—Method of Moments, Printed antennas, Ultra thin substrate, Kummer extraction, Green's functions .**

I. INTRODUCTION

Printed electronics have a wide variety of applications such as high-frequency circuit elements or low profile antenna (i.e. with ultra-thin substrate). To study these structures, various numerical algorithms based on the method of moments (MoM) are used [1]–[3]. Considering the problem of thin substrates, the spatial Green's functions are often computed through the Sommerfeld integration, where the spatial Green's function is formulated as a Hankel transform of the closed-form spectral-domain Green's functions [4]–[9]. However, Green's function representation is not immediately in a suitable form to be employed in conjunction with the MoM method. That is mostly due to (a) the occurrence of surface wave poles [8], which influences the choice of the contour path of the Sommerfeld integral, (b) the moderately decaying character of the spectral-domain Green's functions, especially when the field location and source location are close (i.e., with ultra-thin substrates), such as in many of the VLSI and MMIC applications, and (c) the oscillatory profile of the spectral Green's functions [4].

To overcome this numerical challenge, a novel technique is reported in this work based on the scalar potentials with singularity subtractions [2], [10]. Our approach is focused essentially on improving the accuracy of the interaction between basis and test functions, used to build the MoM system of equations. Here, an extraction (Kummer) [11] method is adopted to build the total impedance matrix; namely, several asymptotic terms of the spectral Green's function, whose Hankel transform is known analytically, are extracted. These terms, which are singular or quasi-singular in the spatial domain, are integrated separately using semi-analytical and new fully analytical methods [12], [17] when convolutions with basis and testing functions are computed. The total MoM impedance matrix is the sum of the asymptotic and remaining parts [1], [2], [10]. The invertible MoM impedance matrix is then used to calculate the electromagnetic performance of circuits or antennas (input impedance, radiation pattern, efficiency, directivity, etc.).

This work is organized as follows. In Section II, the theoretical background about the scalar representation of Green's functions and their asymptotic forms is reminded [4]- [9]. Then, in Section III, the computation of the total impedance matrix based on the Kummer extraction is described. Using the method of [12], [13], semi-analytical and fully analytical methods are used to reduce the time of execution. In Section IV, we present and discuss the numerical results which validate the method. In the final Section, some conclusions are drawn.

II. NOTE ABOUT THE GREEN'S FUNCTIONS (SPECTRAL AND SPATIAL FORMS)

We consider a metallic structure printed on a grounded dielectric of thickness h and relative permittivity ϵ_r. As explained in [1], [2], [17], the MoM impedance matrix can be computed in spatial domain using scalar Green's functions as follows:

$$Z_{i,j} = \iint_{S_i} \iint_{S_j} \left[G_a(r-r')\bar{J}_t(r)\cdot\bar{J}_b(r') + G_v(r-r')\,\nabla\cdot\bar{J}_t(r)\,\nabla\cdot\bar{J}_b(r') \right] dSdS' \quad (1)$$

where, $\bar{J}_b$ and $\bar{J}_t$ are respectively the basis and testing functions used to discretize the printed metallic structure. The Green's functions (more precisely the "scalar and vector potentials") are known analytically in the spectral domain [4]–[9]:

$$\widetilde{G}_a = -\frac{1}{2\pi}\frac{1}{D_{\mathrm{TE}}} \quad (2)$$

$$\widetilde{G}_v = \frac{1}{\beta^2}\frac{1}{2\pi}\left(\frac{1}{D_{\mathrm{TE}}} - \frac{1}{D_{\mathrm{TM}}}\right) \quad (3)$$

The expressions of D_{TE} and D_{TM} used to treat ultra-thin substrates are defined as [4]–[8]

$$D_{\text{TM}} = \frac{1}{\eta_0}\left(\frac{k_0}{k_z^{(0)}} + \frac{\epsilon_r k_0}{k_z^{(m)} \tanh(jk_z^{(m)}h)}\right) \tag{4}$$

$$D_{\text{TE}} = \frac{1}{\eta_0}\left(\frac{k_z^{(0)}}{k_0} + \frac{k_z^{(m)}}{k_0}\coth(jk_z^{(m)}h)\right) \tag{5}$$

with $k_z^{(m)} = -j\sqrt{\beta^2 - \epsilon_r k_0^2}$, $k_z^{(0)} = -j\sqrt{\beta^2 - k_0^2}$, β is the radial wavenumber in the plane (k_x, k_y), k_0 the free-space wavenumber and $\eta_0 = 376.7\Omega$ the free-space impedance.

The Green's function asymptotic expressions for large values of β are given by [4]–[8]

$$\widetilde{G}_v^{\text{as}} = \frac{1}{2\pi}\frac{j\eta_0}{\beta k_0}\frac{1}{(\epsilon_r+1)} \times \left[1 + \frac{2\epsilon_r}{(\epsilon_r+1)}\sum_{n=1}^{\infty}\left(\frac{\epsilon_r-1}{\epsilon_r+1}\right)^{(n-1)}(-1)^n e^{-2n\beta h}\right] \tag{6}$$

and

$$\widetilde{G}_a^{\text{as}} = \frac{1}{2\pi}\frac{k_0\eta_0}{j\beta}\frac{1}{2}(1 - e^{-2\beta h}) \tag{7}$$

Then, by applying a Hankel transformation, the following spatial or quasi-static forms of Green's functions are obtained [4], [8], [14], [15]:

$$G_v^{\text{as}}(\rho) = \frac{1}{2\pi}\frac{j\eta_0}{k_0}\frac{1}{(\epsilon_r+1)} \times \left[\frac{1}{\rho} + \frac{2\epsilon_r}{(\epsilon_r+1)}\sum_{n=1}^{\infty}\left(\frac{\epsilon_r-1}{\epsilon_r+1}\right)^{(n-1)}\frac{(-1)^n}{\sqrt{\rho^2+(2nh)^2}}\right] \tag{8}$$

$$G_a^{\text{as}}(\rho) = \frac{1}{2\pi}\frac{k_0\eta_0}{j}\frac{1}{2}\left[\frac{1}{\rho} - \frac{1}{\sqrt{\rho^2+(2h)^2}}\right] \tag{9}$$

with ρ the transverse coordinate of $r - r'$. The asymptotic parts of the Green's functions contain their singular and quasi-singular behaviour in the spatial domain. It is thus possible to decompose the Green's functions into a singular and a smooth parts [10], [11]:

$$G_{v,a} = (G_{v,a} - G_{v,a}^{\text{as}}) + G_{v,a}^{\text{as}} \tag{10}$$
$$= G_{v,a}^{\text{re}} + G_{v,a}^{\text{as}} \tag{11}$$

where $G_{v,a}^{\text{re}}$ is the "remainder" (re) part, which is a smooth function. In conformity with this representation of the spatial Green's functions, the total impedance matrix of the MoM method also comes in two parts: remainder and asymptotic.

III. Calculation of the Total Impedance Matrix Based on the Extraction Procedure

Given the previously defined Green's functions, the elements of the MoM matrix Z can be computed following (1). An element of the impedance matrix Z for basis function $\bar{J}_b$ and testing function $\bar{J}_t$ can be written as

$$\begin{aligned} Z_{i,j} = &\iint_{S_i}\iint_{S_j}[G_a^{\text{re}}(r-r')\bar{J}_t(r)\cdot\bar{J}_b(r') \\ &+ G_v^{\text{re}}(r-r')\nabla\cdot\bar{J}_t(r)\,\nabla\cdot\bar{J}_b(r')]\,dSdS' \\ &+ \iint_{S_i}\iint_{S_j}[G_a^{\text{as}}(r-r')\bar{J}_t(r)\cdot\bar{J}_b(r') \\ &+ G_v^{\text{as}}(r-r')\nabla\cdot\bar{J}_t(r)\,\nabla\cdot\bar{J}_b(r')]\,dSdS' \end{aligned} \tag{12}$$

These two integrals are computed using different approaches. The first one deals with the non-singular G_a^{re} and G_v^{re} functions: G_a^{re} and G_a^{re} are estimated using a numerical Hankel transform and tabulated. Their values are then interpolated to compute the integral numerically. For the second integral, the Green's functions are first written as a series of functions of type $1/\sqrt{\rho^2+(2nh)^2}$ using (8) and (9).

Let us decompose the Z_{ij} matrix into two parts denoted as Z_{ij}^{re} and Z_{ij}^{as}, according to the extraction procedure:

$$Z_{ij} = Z_{ij}^{\text{re}} + Z_{ij}^{\text{as}} \tag{13}$$

where Z_{ij}^{as} can be written as

$$Z_{ij}^{\text{as}} = Z_{ij}^{\text{as},a} + Z_{ij}^{\text{as},v} \tag{14}$$

Now, let us write

$$Z_{ij}^{\text{as},a} = \sum_{n=0}^{1}\alpha_n I_n(i,j) \tag{15}$$

with

$$I_n(i,j) = \iint_S \bar{J}_t(r)\cdot\iint_{S'}\frac{\bar{J}_b(r')}{\sqrt{\rho^2+(2nh)^2}}\,dS'dS \tag{16}$$

and

$$Z_{ij}^{\text{as},v} = \sum_{n=0}^{N}\beta_n I'_n(i,j) \tag{17}$$

with

$$I'_n(i,j) = \iint_S \nabla\cdot\bar{J}_t(r)\iint_{S'}\frac{\nabla\cdot\bar{J}_b(r')}{\sqrt{\rho^2+(2nh)^2}}\,dS'dS \tag{18}$$

where α_n and β_n are coefficients that can readily be obtained from the asymptotic spatial expressions of $G_{a,v}$ in (8) and (9).

The accurate evaluation of integrals (16) and (18) may be challenging for low values of n when the distance ρ between the basis and testing functions is small. Gaussian integration techniques may lead to poor results given the singular or nearly-singular behaviour of the integrand. In [13], an analytical solution to the integration over the basis function is proposed. This solution is smoother than the original integrand and can thus be evaluated numerically over the testing function. In [12], a fully analytical solution to (16) and (18) was proposed. The solution involves a large number of different terms, so that its evaluation may require special precautions in order to be competitive with semi-analytical or fully numerical evaluation [16].

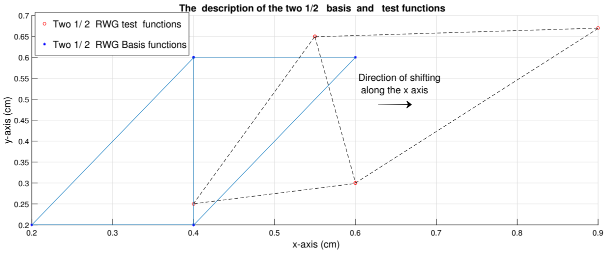


Fig. 1. RWG basis and testing functions used in this study.

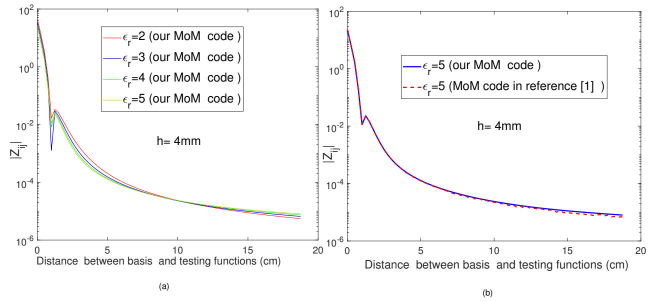


Fig. 2. $|Z_{ij}|$ versus the distance of interaction between basis and testing functions with different permittivities ϵ_r. Frequency $f = 0.868\,\mathrm{GHz}$ and height substrate $h = 4\,\mathrm{mm}$.

In the next section, I_n and I'_n are evaluated using fully-numerical, semi-analytical and fully analytical techniques. The computation times and results of each method are compared.

IV. Validation and Numerical results

In order to validate the method described in the previous section, we studied the interaction between two RWG functions. The basis and testing functions used in this study are described in Fig. 1. We first considered the variation of $|Z_{ij}|$ versus the distance between the basis and testing functions for a thick substrate ($h = 4$ mm) at a frequency $f = 0.868$ GHz and several relative permittivities of the dielectric substrate. The results are displayed in Fig. 2(a). A specific validation with $\epsilon_r = 5$ is shown in Fig. 2(b) using the MoM code of [1], which is devoted to the simulation of thick substrates (h larger than free-space wavelength divided by 100), and the MoM code developed based on the formulation presented here. The matching between both methods is very good. Note that, in [1] only one singularity is removed. It corresponds to the Green's function associated with the infinitesimal dipole in an average homogeneous medium. In our work, besides that, an arbitrary number of "images" are extracted.

In Fig. 3, we study the convergence of $|Z_{ij}|$ with the number of terms in the series expansion (8). The cases of a thick substrate ($h = 4\,\mathrm{mm}$) and a thin substrate ($h = 0.125\,\mathrm{mm}$) are displayed in Fig. 3. For the thick substrate, 5 asymptotic terms are required to reach convergence, while 11 terms are required for the thin substrate. As expected, if the substrate is ultra-thin, the number of extractions required to reach convergence is larger. As another validation, a very good agreement is

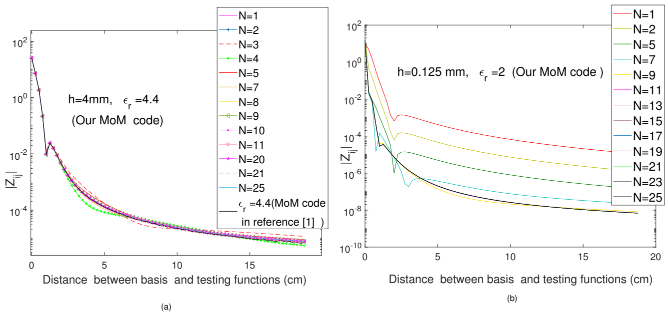


Fig. 3. Convergence of $|Z_{ij}|$ versus the number of extracted terms for (a) a thick substrate [$\epsilon_r = 4.4$ and $h = 4\,\mathrm{mm}$] and (b) an ultra thin substrate [$\epsilon_r = 2$ and $h = 0.125\,\mathrm{mm}$] at $f = 0.868\,\mathrm{GHz}$.

obtained between the converged results in Fig. 3(a) and the results obtained from the MoM code of [1].

Last, we concentrate on the evaluation of the asymptotic term Z_{ij}^{as} and more particularly to the evaluation the integrals I_n and I'_n involved in the asymptotic term (cf. Equations (15) to (18)).

We evaluate the asymptotic behaviour as a separate part: the asymptotic Z_{ij}^{as} is mainly calculated based on the integrations over basis and testing functions and functions of the type $1/\sqrt{\rho^2 + (2nh)^2}$ [for more details, see the asymptotic spatial forms of Green's functions G_a^{as} and G_v^{as} in (16) and (18)].

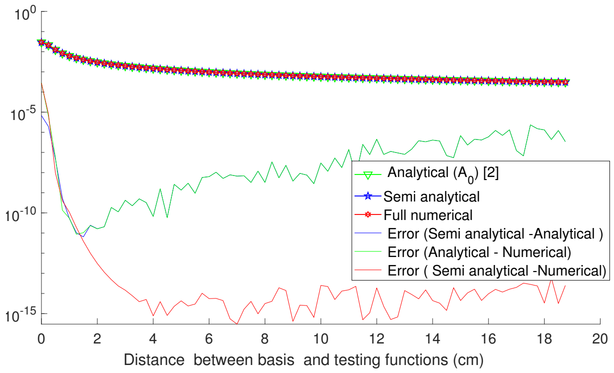
The interaction integrals I_n and I'_n are computed using the fully numerical, semi-analytical, and fully analytical techniques. The value of the integral and the absolute errors between the different methods are depicted in Figs. 4(a) and (b). For small lateral distances, the absolute errors between analytical-numerical and hybrid-numerical are the highest, indicating that the numerical integration is inaccurate. However, as the distance increases, the numerical and semi-analytical results converge to the same values, while the error of the analytical method increases, as described in [12]. From the behaviour of the different methods, it can be inferred that the fully analytical method is more accurate than the semi-analytical method for small distances ρ, which was confirmed using additional tests (not shown).

The average time required by each technique to compute a single integral is given in table I. The analytical integration represents the best suitable solution that permits reducing the computation time of the asymptotic part of the interaction [12].

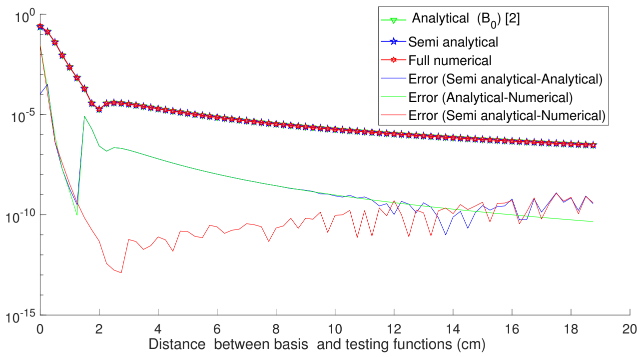
The analytical approach provides a precise and rapid integration near the origin, as indicated in [12].

TABLE I
Average elapsed time consumption (in milliseconds) during the integration operation over (1 basis function, 1 testing function)

	I_n	I'_n
Hybrid (semi-analytic) integration	0.805	0.748
Analytic integration	0.230	0.230
Numerical integration	0.847	0.891



(a)



(b)

Fig. 4. (a) $|I_n|$ and (b) $|I'_n|$ versus the distance of interaction between the basis function and testing function. 'continuous green line with downward-pointing triangle': Analytical integration (Approach in reference [2]) for A_0 and B_0, 'continuous blue line with an asterisk': Semi-analytical integration, 'continuous red line with hexagram': Full numerical integration, 'continuous blue line': Error between semi-analytical and analytical integrations, 'continuous red line': Error between semi-analytical and numerical integrations and 'continuous green line': Error between analytical and numerical integrations. $f = 0.868\,\text{GHz}$, $\epsilon_r = 2$, n=1 (index term of the series development), and $h = 0.125\,\text{mm}$.

Applying full numerical integration presents an inconvenience to calculate the singularity near the origin ($n = 0$). Only the two other methods (analytical and hybrid) are truly adequate to implement the MoM method. As given in Table I, in terms of execution time, the analytical integration is the most suitable solution to realize the MoM code without any problem at the origin (when $n = 0$). Note that the evaluation time depends on the operating environment (both hardware and the operating system), as given in [12].

V. Conclusion

This paper has described implementation for the Z matrix of the Method of Moments obtained by carrying out the extraction procedure (Kummer procedure), following a Green's functions decomposition comprising asymptotic and remainder parts, particularly well suited for the analysis of circuits or antennas on ultra-thin substrates. The integration over basis and testing functions for the asymptotic part is evaluated by means of different methods. The more desirable solution presented here consists of using the analytical integration which permits reducing the calculation time of the asymptotic interaction without any problem near the origin, as presented in [12]. In our future investigation, we aim to focus on applying the adopted MoM technique to analyze complex antenna designs.

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References

[1] S. N. Jha and C. Craeye, *Fast spectral-domain macro basis function analysis of large arrays of printed antennas*, thesis report, Université catholique de Louvain ,2014.

[2] S. Hubert, *Numerical analysis of strongly coupled antenna arrays and meta surfaces*, Université catholique de Louvain Thesis, March 2018.

[3] A. Latiri, A. Samet and A. Bouallegue , *Étude des structures planaires en très hautes fréquences par une nouvelle formulation de la méthode des moments dans le domaine spatial* , Ann. Télécommun. vol.59, pp 1189–1211 (2004).

[4] Sudarshan R. Nelatury, A. Hoorfar, *Closed form expressions for Green's functions of mi-crostrip structures on a uniaxially anisotropic substrate*, International Journal of RF and Microwave Computer-Aided Engineering 13(2): pp. 161 - 168 .

[5] A. Hoorfar and D. C. Chang, *Closed-form solutions to Green's functions in microstrip problems with a thin substrate*, Radio Science 30 (1995), pp.343–351.

[6] Juan. R. Mosig, *Arbitrarily shaped microstrip structures and their analysis with a mixed potential integral Equation*, IEEE Transactions on microwave theory and techniques, vol. 36, no. 2, February 1988.

[7] Juan R. Mosig and Fred E. Gardiol, *A Dynamical Radiation Model for Microstrip Structures*, Advances in Electronics and Electron Physics Volume 59, Pages 139-237,1982.

[8] Y. .L. Chow , J. J. Yang , D. G. Fang , G. E. Howard , *A closed-form spatial Green's function for the thick microstrip substrate*, IEEE Transactions on Microwave Theory and Techniques , Volume: 39 , Issue: 3 , Mar 1991, pp.588 - 592 .

[9] S. Bhattacharjee, R. S. Kshetrimayum, and R. Bhattacharjee, *Derivation of potential Green functions for ungrounded dielectric slab and its application in full wave analysis of PMAs* ,Journal of Electromagnetic Waves and Applications, 2015.

[10] A. I. Khalil , A. B. Yakovlev and M. B. Steer, *Efficient method-of-moments formulation for the modeling of planar conductive layers in a shielded guided-wave structure* , IEEE Transactions on Microwave Theory and Techniques, Volume: 47, Issue: 9,pp :1730 - 1736, Sep 1999.

[11] F. Mesa and F. Medina, *Numerical computation of the space-domain mixed potential Green's functions for planar layered structures with arbitrarily magnetized ferrites*, IEEE Transaction antenna and propagation-Volume: 52 , Issue11, Nov. 2004.

[12] D. Tihon and C. Craeye , *All-analytical evaluation of the singular integrals involved in the Method of Moments*, IEEE Transactions on Antennas and Propagation , Volume: 66 , Issue: 4 , April 2018 ,pp.1925 - 1936.

[13] D. R.Wilton, S. Rao, A. Glisson, D. Schaubert, O. Al-Bundak and C. Butler, *Potential integrals for uniform and linear source distributions on polygonal and polyhedral domains*, IEEE Trans. Antennas Propag., vol. AP-32, no. 3, pp. 276-281, Mar. 1984.

[14] I. .S. Gradshteyn and I. M. Ryzhik, *"Table of integrals, series, and products*, Seventh Edition, 2007, Elsevier.

[15] H. B. Dwight, *Table of integrals and other mathematical data*, 4th ed.New York: Macmillan, 1961.

[16] D. Tihon and C. Craeye, *Hierarchical implementation of the analytical singularity evaluation technique,* 13th European Conference on Antennas and Propagation (EuCAP), pp. 1-5, 2019.
[17] L. Alatan , M. I. Aksun , K. Mahadevan and M. T. Birand, *Analytical evaluation of the MoM matrix elements* ,IEEE transactions on microwave theory and techniques, Volume: 44, Issue: 4, pp: 519 - 525, Apr 1996.