



Université catholique de Louvain

Ecole polytechnique de Louvain

Institut de Mécanique, Matériaux et Génie Civil

INVESTIGATION OF WALL SHEAR STRESS MODELS FOR LARGE EDDY SIMULATION

Olivier Thiry

Composition du jury:

Pr. R. Ronsse (président)

Pr. G. Winckelmans (promoteur)

Pr. P. Chatelain

Dr. M. Duponcheel

Dr. K. Hillewaert

Pr. F. Nicoud

Thèse présentée en vue de
l'obtention du grade de docteur
en sciences de l'ingénieur

Louvain-la-Neuve,
Novembre 2016

Science never solves a problem without creating ten more.
George Bernard Shaw

Remerciements

Avant toute chose, cette thèse a été réalisée grâce à un mandat d'aspirant du Fonds de la Recherche Scientifique-FNRS, ainsi que grâce aux ressources de calculs fournies par le Consortium des Équipements de Calcul Intensif (CÉCI), financé par le Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) sous la convention n° 2.5020.11, et au supercalculateur Tier-1 de la Fédération Wallonie-Bruxelles, infrastructure financée par la Région wallonne sous la convention n°1117545.

Je tiens, en premier lieu, à remercier Grégoire Winckelmans, mon promoteur, qui a su me communiquer sa passion pour la mécanique des fluides et les mathématiques physiques. Son souci du détail et sa minutie sont une permanente source d'inspiration.

Merci ensuite à Matthieu Duponcheel, chez qui j'ai toujours pu trouver les lumières dont j'avais besoin, et qui a su me transmettre, outre les divers codes sur lesquels s'appuie mon travail, sa grande expérience et ses points de vue toujours stimulants et ambitieux.

Je remercie également Philippe Chatelain, Koen Hillewaert, Franck Nicoud et Renaud Ronsse qui ont accepté de faire partie de mon jury et qui ont permis de grandement améliorer la qualité du présent ouvrage par leurs relectures attentives et leurs commentaires sans ambages.

Le camarade de pinaille que j'ai trouvé en la personne de Philippe Parmentier mérite également toute ma reconnaissance pour m'avoir supporté dans son bureau tout ce temps.

L'ambiance générale au TFL et la chaleureuse mais laborieuse atmosphère du Stévin ont toujours créé un climat propice à un travail détendu mais de qualité. Pour ceci, je me dois de remercier ardemment tous ceux qui contribuent ou ont contribué à faire de la cafét' et du laboratoire/kicker un temple de bonne humeur : les furieux (Benjamin, le susnommé Philippe, Pierre), les

jeunes furieux (Thomas, Denis-Gabriel, Caroline, Philippe B.), les bolchéviques (Olivier L., Nicolas), Maud, Elise, Maxime, Gauthier, Simon, Victor, Isabelle, Véronique, Haddy, Hervé, Yann, Franck, Lily, Julien, François, et tous les membres présents ou passés du TFL et du LEFT que j’ai côtoyé.

Je tiens également à remercier ma famille qui m’a toujours soutenu dans mes projets, ainsi que mes amis auprès de qui j’ai toujours pu trouver un instant de détente lorsque j’en avais besoin.

Et enfin, je dois beaucoup à ma merveilleuse femme, Gladys, qui a su m’épauler à travers les hauts et les bas de la thèse et auprès de qui tout paraît si simple.

Mais trêve de verbiages; merci à tous ceux qui se risqueront à parcourir cet ouvrage et à qui je souhaite dès à présent une excellente lecture.

Contents

1	Introduction	1
1.1	Background	2
1.2	Objectives	8
1.3	Outline	10
2	Wall modeling strategies in the BigFlow solver	13
2.1	Introduction	13
2.2	Numerical methods	19
2.2.1	Spatial discretization	19
2.2.2	Time stepping	22
2.3	Testing the flux version of the solver	22
2.3.1	Taylor-Green vortex	23
2.3.2	Channel flow at $Re_\tau = 395$	24
2.4	Handling wall shear stress boundary conditions	40
2.4.1	Numerical details	40
2.4.2	Validation of the implementation of the wall shear stress	45
2.5	Algebraic wall models	56
2.5.1	Averaging procedures	59
2.5.2	Parameter study	59
2.6	RANS sub-layer models	93
2.6.1	Channel flow validation of the RANS sublayer models	95
2.7	Conclusion	110
3	WMLES in the presence of pressure gradients	113
3.1	Introduction	114
3.2	Setup	115
3.3	Flow topology	118

3.4	Wall models accounting for pressure gradient effects	120
3.4.1	Algebraic wall models	131
3.4.2	RANS sub-layer models	132
3.4.3	Validation of the models in classical channel flow	133
3.5	Assessment of the WM in the half-channel	135
3.5.1	Algebraic wall models	135
3.5.2	RANS sublayer models	151
3.5.3	Note about the convective terms of the RANS sublayer	163
3.6	Conclusion	164
4	T-WVS at equilibrium, and IGE: WRLES at $Re_\Gamma = 2.0 \cdot 10^5$	171
4.1	Introduction	172
4.2	Obtention, by LES, of a turbulent 2VS at equilibrium	175
4.3	Turbulent 2VS in ground effect	184
4.4	Results	187
4.4.1	Flow topology	187
4.4.2	Vortex diagnostics	188
4.5	Conclusion	198
5	WMLES of T2VS IGE at $Re_\Gamma = 2.0 \cdot 10^5$	203
5.1	Introduction	203
5.2	Numerical setup	204
5.3	Results	211
5.4	Conclusion	212
6	A mixed multiscale SGS model	217
6.1	Introduction	219
6.2	Reference DNS and behavior of the exact SGS terms	224
6.3	Dynamic mixed model and a priori assessment	231
6.4	A posteriori testing	240
6.5	Conclusion	243
7	Conclusions and perspectives	249
7.1	Conclusions	249
7.2	Perspectives	252
	Bibliography	254

<i>Contents</i>	ix
A Modified Shih law of the wall	271
B Full page volume rendering of vorticity fields	275
C Discrete filters considered	287

Chapter 1

Introduction

From the fluid dynamics of an aircraft to the ocean currents, from you stirring the milk in your coffee to the blood flowing in your veins, turbulence is everywhere, and it is at the heart of many phenomena. As a consequence of this omnipresence, a great deal of flows with industrial or everyday life applications are turbulent. From an industrial point of view, understanding how turbulent flows work, and being able to accurately predict such flows is therefore crucial. Predictions of turbulent flows can come from simplified models, but those quickly fall short of answers as the complexity of the investigated problem increases. One therefore has to resort to numerical computation of the flow of interest (or to experimentation, but this is beyond the scope of this thesis). As we will see however, accurate computation of turbulent flows can be very demanding in terms of computational resources, especially when the turbulence interacts with solid boundaries (often called walls). At the same time, the computing power and the resources available are limited, and even with today's supercomputing facilities, real-life turbulent flows are very expensive and hard to compute. This is due to the inherent physics of turbulence, which, as we will see, involves at the same time very large scales and very small scales of motion. The resolution of all the small scales can become unaffordable. We will see that the cost of a computation can scale as the cube of the Reynolds number, a parameter describing the flow, and whose order of magnitude can be as high as 10^7 or more. Hence the idea of *large eddy simulation* (LES): only resolve the large scales of motion and model the effect of the unresolved scales. Even with this technique, the resolution of wall-bounded flows is still very expensive, for reasons that will become clear in a moment, and the Reynolds

number scaling is only marginally better. Hence, the idea of providing a model for the interaction between a wall and a turbulent flow: a *wall model*. The goal of this work is to investigate a subset of these wall models called wall shear stress models. Precisely, the objective is to assess whether those models allow to significantly reduce the computational cost of the LES of such flows while conserving the accuracy of wall-resolved LES. Various models will be investigated, and we will see that in some cases, it is indeed possible to drastically reduce the cost of the simulation of real-life turbulent flows without great losses of accuracy. To get into the details of these models, we must first understand the basic concepts of turbulence, and a quick overview of the key concept is given below.

1.1 Background

As mentioned hereabove, turbulence consists of a broad spectrum of scales, from very large scales—large eddies—down to very small scales. As an illustration, Fig. 1.1 shows the condensation trail of a Boeing 747, which is rolled up in the wake vortices behind the aircraft. We see that, though the flow is driven by the two very large counter-rotating vortices forming the wake, many small scales of motion are present in the flow, with a very broad range of sizes. To avoid computing all those very small scales phenomena, large eddy simulation uses a model to account for their effect on the large scales we choose to resolve. To derive such a model, we need to have a good understanding of how turbulence behaves. While completely chaotic at first sight, turbulence hides a lot of order, which leaves room for turbulence modeling. To reduce the computational cost of turbulent flow simulations, much effort has to be put into such modeling.

We here study incompressible Newtonian fluids. If we denote the density of the fluid ρ , incompressibility means that the density does not change over time, and hence, we can write $\frac{\partial \rho}{\partial t} = 0$. Since in a fluid, the conservation of mass is written

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0, \quad (1.1)$$

where $\frac{D}{Dt}$ denotes a Lagrangian derivative, hence a derivative taken as is we were *convected by the fluid*. The second term in the left hand side is hence



Figure 1.1: Condensation trail of a Boeing 747: the condensation trail is absorbed by the wake of the airplane and the wake vortices are clearly visible. Image taken from <https://kitskinny.wordpress.com/2013/07/09/jets-clouds-effects/>

simply due to convection. The conservation of mass then simply becomes:

$$\frac{\partial u_j}{\partial x_j} = 0 . \quad (1.2)$$

Since the fluid is Newtonian, the internal stress is written as $\sigma_{ij} = 2\mu S_{ij}$, where $S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ is the strain rate tensor, and μ is the viscosity of the fluid. The conservation of momentum then reads

$$\frac{D\rho u_i}{Dt} = \rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j^2} , \quad (1.3)$$

where the first term in the right hand side is the pressure force, and the second term is the viscous stress. The evolution of the flow is guided by the balance between the convective term $\rho \frac{\partial}{\partial x_j} (u_i u_j)$ and the diffusive—viscous—term $\mu \frac{\partial^2 u_i}{\partial x_j^2}$. Their ratio is characterized by the *Reynolds* number

$$Re = \frac{\rho U^2}{L} \bigg/ \frac{\mu U}{L^2} = \frac{UL}{\nu} , \quad (1.4)$$

where U and L are reference velocity and length respectively of a given flow,

and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid. If the flow is dominated by viscous effects, $Re \leq 1$ the flow is *creeping*. Convection can be completely neglected as it plays no role. When convection and diffusion are of the same order of magnitude, $Re \geq 1$ but still moderate, the flow is laminar. Convection drives the flow but is stabilized by diffusion: no turbulence develops. If Re is large (how large precisely depends on the flow of interest), however, convection dominates at least the greatest scales of the flow, and turbulence can develop. Turbulence develops through the non-linearities of the convective terms and breaks down the eddies of size L to smaller eddies. These smaller eddies are in turn broken down to even smaller eddies. This way, the energy of the largest scales of the flow is drained to smaller and smaller structures, in a process called the energy cascade. The cascade only stops upon reaching a scale that is so small that viscous effects take over and dissipate the energy into heat. The phenomenon was first described by Richardson [109] who elegantly phrased it into a verse:

*Big whirls have little whirls that feed on their velocity,
and little whirls have lesser whirls and so on to viscosity.*

The scale below which viscosity becomes dominant is called the Kolmogorov scale and noted η . If an eddy of size η and of characteristic vorticity ω_η is at the Kolmogorov scale, it means that viscous terms are of the same magnitude as the convective terms, and hence the (vorticity-based) Reynolds number at that scale must be of order 1:

$$Re_\eta = \frac{\eta^2 \omega_\eta}{\nu} = \frac{\eta u_\eta}{\nu} = 1, \quad (1.5)$$

where we defined a velocity scale associated with the characteristic vorticity $u_\eta = \omega_\eta \eta$. Also, since, at this scale, the energy is dissipated into heat, it means that the energy dissipation rate $\epsilon = 2\nu S_{ij} S_{ij}$, scales according to:

$$\epsilon = \nu \frac{u_\eta^2}{\eta^2}. \quad (1.6)$$

From there, and with Eq. (1.5), we can get the Kolmogorov scale:

$$\eta = \left(\frac{\nu^3}{\epsilon} \right)^{\frac{1}{4}}. \quad (1.7)$$

At the Kolmogorov scale, the energy drained from the larger scale is dissi-

pated into heat through the action of the viscous stresses. All that energy is issued by the large scales and drained by the cascade, therefore, all the energy dissipation is actually due to the large scales. We can hence compute the characteristic size of the large structures generating the dissipation rate ϵ , or, vice versa, know what dissipation rate is linked to structures of characteristic size L and speed U by simple dimensional analysis:

$$\epsilon \propto \frac{U^3}{L} . \quad (1.8)$$

These conclusions regarding the large and small scales of the flow L and η have dramatic consequences regarding the computational cost of high Re computations. Indeed, if we are interested in a flow with characteristic size L , at a given Re , and if we want to solve all the scales of the flow, hence from the energetic size L down to the Kolmogorov scales, we can immediately deduce from Eqs. (1.7) and (1.8) that the total cost of the simulation (the number of grid points times the number of time steps required) will scale as

$$N_x N_y N_z N_t \propto \left(\frac{L}{\eta} \right)^4 \propto Re^3 . \quad (1.9)$$

These requirements for the *Direct Numerical Simulation* (DNS) of high Re flows quickly become impossible to meet. The standard industrial way to circumvent these limiting requirements is to simply suppress all fluctuations from the flow, and to only solve for the *Reynolds-averaged* fields. In this formulation, the energy cascade is fully modeled as well as the energetic structures of the flow, and only mean fields are solved for. While, in theory, this average is an ensemble averaging of the statistical fields of the turbulent flow, it is most often understood as a time averaging of the flow. The effect of the turbulent structures and fluctuations is then simply summarized through the action of a *Reynolds* stress tensor, and the resulting equations are the *Reynolds Averaged Navier-Stokes* (RANS) equations. While this technique has proven able to solve a large variety of problems, in some cases, it is of practical interest to be able to solve for the main (time-varying) features of the flow and its energy-bearing structures, especially when the large scale structures play a key role in the flow physics or when large time fluctuations appear. The idea to solve only for the large scales of the flow, while providing a model for the effect, on the large scales, of the smaller more isotropic scales of the flow, down the energy cascade

was first proposed by Smagorinsky [117]. The term *Large-Eddy Simulation* (LES) was coined to describe the technique. It aims at solving the large, energetic, structures (or eddies) of the flow down to some cut-off size in the energy cascade, while the effect of the removed scales is modeled through the action of a *subgrid-scale* (SGS) stress tensor. This tensor models the action of the unresolved scales on the resolved scales. To produce the scale separation, an operator $\tilde{\cdot}$ is applied to all fields, such that the velocity, for instance is written $\mathbf{u} = \tilde{\mathbf{u}} + \mathbf{u}'$, where $\mathbf{u}'$ are the unresolved scales, and $\tilde{\mathbf{u}}$ are the resolved scales. This operator can be of various types, more details on this matter are given in Section 6.1. Since the scales must only be resolved up to some point in the energy cascade, but not up to the Kolmogorov scale, increasing the *Reynolds* number, which reduces the size of the Kolmogorov scale, and hence increases the cost of DNS, has only limited impact on the cost of LES.

This is true in the absence of any solid boundaries. In most industrially relevant cases, however, turbulence interacts with solid boundaries, creating so-called *wall turbulence*. We here only consider smooth boundaries: no—or negligible—surface roughness. In the presence of a solid wall, a thin layer forms wherein the flow velocity drops from the freestream velocity to the wall velocity: the *boundary layer*. This creates a zone of intense shear, and the turbulent structures are modified by this shear. The energetic structures have sizes that scale with the distance to the wall and these smaller and smaller structures are not isotropic anymore. Townsend [136] showed that they form *attached eddies* that tend to align more with the direction of the flow; these tilted eddies are attached to the wall and stretched up to some distance in the boundary layer. Because LES has to solve for these energetic structures, the net cost of LES close to a wall becomes comparable to that of DNS, as we will see later. In the vicinity of the wall, as the structures get smaller and smaller, the viscous effects get bigger with respect to the convection effects, until, very close to the wall, only viscosity matters. This creates a velocity profile in the boundary layer that switches from the outer layer of the boundary layer to the inner layer, which is made of a convection-dominated layer transitioning to a viscosity-dominated layer, and finally to the no slip boundary condition at the solid wall. In the viscosity-dominated layer, and in the absence of a pressure

gradient, only the viscous term of the equations remains:

$$\frac{d}{dy} \left(\nu \frac{du}{dy} \right) = 0 . \quad (1.10)$$

Therefore, we here define a velocity scale linked to the wall shear stress, which provides the only velocity scale available to the inner layer of the boundary layer:

$$u_\tau^2 = \frac{1}{\rho} \tau_w = \nu \left. \frac{du}{dy} \right|_w . \quad (1.11)$$

This velocity scale allows the definition of the standard viscous units and non-dimensionalizations within the boundary layer:

$$u^+ = \frac{u}{u_\tau} , \quad y^+ = \frac{y u_\tau}{\nu} . \quad (1.12)$$

From there, we see that, in the viscous layer of the boundary layer, the velocity profile must be:

$$u^+ = y^+ . \quad (1.13)$$

This layer roughly extends from the wall to $y^+ \simeq 5$. In the convection-dominated part of the inner layer, we must have a velocity profile of the form

$$u^+ = f(y^+) . \quad (1.14)$$

In the outer layer of the boundary layer, the velocity profile must scale as

$$\frac{u - u_e}{u_\tau} = F \left(\frac{y}{\delta} \right) , \quad (1.15)$$

where u_e is the velocity outside of the boundary layer, and δ is the size of the boundary layer. Millikan [83] showed that, if the inner and outer velocity profiles connect smoothly, and if the velocity gradient between the two is continuous, we must have

$$y^+ f'(y^+) = \eta F'(\eta) , \quad (1.16)$$

where $\eta = \frac{y}{\delta}$, and therefore, both sides must be equal to some constant $\frac{1}{\kappa}$. This lead to the logarithmic law of the wall:

$$u^+ = \frac{1}{\kappa} \log(y^+) + C , \quad (1.17)$$

and κ is called the *von Kármán* constant. The velocity profile within the boundary layer is therefore made of several successive contributions, from the wall we have: the linear profile, then a transition layer to the logarithmic profile, and finally the log law. All of those only scale with the viscous inner variable y^+ . Finally, in the outer layer of the flow, the logarithmic profile has a complementary contribution that scales with the outer variable η , which is called the wake function and depends strongly on the nature of the flow: $u^+ = f(y^+) + G(\eta)$, where $G(\eta)$ is a complementary function. The inner part of the velocity profile is very costly to resolve because to accurately compute it, we need a mesh that is able to resolve structures up to the viscous layer. In practice, this means that the cost of the LES of a wall-bounded flow has a Re scaling that can get almost as bad as that of the DNS, most of the computational gain being in the outer layer of the boundary layer, as first quantified by Chapman [22]. These meshing requirements led to the emergence of wall modeling. The idea is that, since the inner layer is, at the same time, very costly to resolve, and less flow-dependent than the outer layer, we should avoid resolving it. Wall-modeled LES (WMLES) uses a model for the inner layer, and performs classical LES in the outer layer of the boundary layer. Several approaches to wall modeling have been proposed over the years. In the present work, we attached to the study a family of such models: wall shear stress models. The idea is to relax the boundary condition at the wall: suppress the no-slip boundary condition. The boundary condition is replaced by a wall shear stress boundary condition. This wall shear stress can be computed in several ways, and is the essence of the model.

1.2 Objectives

The goal of the present work is to implement and assess several wall modeling approaches in a fourth order finite difference code. Many practical questions remain open in the literature regarding wall models. We here aim at testing how the wall models react to various SGS models, to grid refinement and grid aspect ratio variation, etc. The question of what to do regarding wall models when the boundary layer is out of equilibrium is also an open issue, which we plan to address here. For that, we consider two out of equilibrium case: one without boundary layer separation, and one with boundary layer separation.

Such studies will help improve our knowledge of wall models which is a quite active field of study in the current LES community.

The case we use to study wall models for boundary layers out of equilibrium, but without boundary layer separation, is a half channel flow case developed by Duponcheel [35] in the course of his Ph.D. thesis. We here perform a wall-resolved LES of the flow at a moderately high Reynolds number to provide a reference solution for the wall models.

The case we use to study wall models with boundary layer separation is the case of wake vortices in ground effect. Wake vortices, such as those of Fig. 1.1, are an industrially relevant case, that is at the same time a good test-case for turbulence modeling and wall modeling. As an inevitable byproduct of lift production, aircrafts produce a wake that, after a roll-up phase, consists in a pair of counter-rotating vortices that produce a turbulent field around them. Under their own influence, these vortices naturally descend and, if they are produced close to the ground (for instance in take-off or landing phases of flight), they eventually interact with the ground. To accurately represent such situations, the LES has to be able to capture the vortices self-generated turbulence as well as the wall turbulence, the boundary layer separation, zones of high shear, zones of quasi laminar pure rotation in the vortex cores, etc. The SGS model will need to be able to not dissipate the quasi laminar pure rotation parts of the flow, while the wall model will need to accurately capture the boundary layer and its separation. This case therefore presents a challenging case for the tools developed in this work. As a reference for this case, we also produced a wall-resolved LES of this case at a Reynolds number 10 times higher than was previously done in the literature, and with a better physical representation of the vortex system. This simulation is hence analyzed in depth as it represents in itself an achievement.

Finally, a novel SGS model is developed, based on a decomposition of the SGS stress tensor in two separate contributions, and based on the analysis of the behavior of the exact SGS tensor extracted from a DNS. It is tested a priori and a posteriori in decaying homogeneous isotropic turbulence, and compared to DNS results at a moderate Reynolds number. The model is shown to yield accurate energy spectra and to provide significant backward transfer of energy (hence from the unresolved scales to the resolved scales, i.e., negative dissipation) while remaining stable, as does the exact SGS term.

1.3 Outline

This work is attached to the implementation and assessment of wall shear stress models for LES, as well as the development of a new SGS model, in the context of high Reynolds number turbulent flows. The thesis is structured as follows.

Chapter 2

This chapter is attached to the description of wall models and their implementation within the framework of the “BigFlow” code. This code, developed by Georges [42], and improved by Bricteux [13] and Duponcheel [35] is a fourth order finite difference code that is energy-conserving in the absence of viscous or SGS terms. It had to be modified to ease the implementation of the wall shear stress boundary conditions implied by the wall models. The new version of the code is validated against several reference cases. Wall models are then implemented and tested in a channel flow case. This case is used, at moderate Re , to validate the numerical implementation of the wall models. Several numerical parameters are also investigated at higher Re , such as the mesh aspect ratio, the interaction between the wall model and the SGS model, etc.

Chapter 3

The wall models implemented in the previous chapter are then tested in a more challenging test-case. In the channel flow case investigated in Chapter 2, the flow is at equilibrium. We here investigate a case with a mild pressure gradient modifying the flow conditions. The pressure gradient is here rather mild and does not lead to boundary layer separation. The case was already used by Duponcheel [35] to assess SGS models. A wall-resolved LES of the flow at a moderately high Re is performed as a reference solution. Additional wall models explicitly including pressure gradient corrections are implemented and compared.

Chapter 4

In this chapter, a high Re wall-resolved LES of turbulent two-vortex system (T-2VS) in ground effect is performed. It will also be used as a reference solution for the wall-modeled simulations of the next chapter. A simulation of high Re 2VS generating its own turbulent field to become a T-2VS at equilibrium with

its turbulence performed by De Visscher [30] is used as initial condition of the simulation. A method to use this periodic simulation as initial condition for a wall bounded simulation is described. The T-2VS interacting with the ground is analyzed in details. This initial condition is compared to a less physical initial condition obtained from initially non-turbulent vortices perturbed by a random velocity field. The rebound phenomenon is investigated, and the rate of decay of the T-2VS as well as the vortex transport are analyzed and compared to out of ground T-2VS, and to the initially non-turbulent vortices in ground effect.

Chapter 5

The wall resolved simulation of the previous chapter is used as a reference solution to assess the performance of the several wall models implemented in this thesis in the context of T-2VS in ground effect. This case is challenging for wall models because it involves favorable and adverse pressure gradients, leading to boundary layer separation and backflows.

Chapter 6

In this chapter, a new SGS model is proposed. It is based on a decomposition of the SGS tensor into two contributions (the “cross term” and the “Reynolds term”), each of which is modeled separately, also based on an analysis of DNS results. The new model is shown to have a good spectral behavior and to perform well both in a priori and a posteriori analyses in decaying homogeneous isotropic turbulence. It is also shown that the model is able to provide dissipation in the mean, but also to locally provide backward transfer of energy from the subgrid scales to the resolved scales (*backscatter*), as does the exact SGS tensor. The simulations were performed on a pseudo-spectral code (initially developed by the group of D. Carati at Université Libre de Bruxelles (ULB) and by Jeanmart [61] in the course of his Ph.D. thesis).

Chapter 2

Wall modeling strategies in the BigFlow solver

This chapter presents a brief overview of the motivation for wall modeling. A small review of some of the approaches to wall modeling is made and the numerical details of the implementation of wall models in the BigFlow solver are given. The numerical implementation of the wall modeling strategy is validated in a channel flow case. Several wall models are then assessed.

2.1 Introduction

Large eddy simulation techniques are now widely used in the academic community and have in part replaced the RANS simulations. The industrial applications of such techniques, however, are still scarce because of their prohibitive cost, notably for the resolution of turbulent boundary layers (TBL). The issue comes from the fact that, while the outer layer of the TBL has structures scaling with the outer size of the TBL, the inner layer consists of attached eddies (as described by Townsend [136]), the size of which scales with the distance to the wall. As Re increases, those smaller and smaller energy-carrying eddies require finer and finer meshes if one is to resolve them all the way to the wall: WRLES quickly become unaffordable. The objective of wall modeling is to remove the need to resolve the inner layer of the TBL, and hence alleviate the mesh requirements linked to the attached eddies. Chapman [22] was the first to estimate and compare the cost of large-eddy simulations when resolving the

whole TBL versus that of using a wall model, hence only resolving the outer layer of the TBL. These estimates were revisited by Baggett et al. [4] and more recently by Choi and Moin [23]. The latter estimate that the cost of a DNS of a TBL developing in the x direction scales as

$$N_{\text{DNS}} \sim Re_x^{37/14} \approx Re_x^{2.64}. \quad (2.1)$$

This is a prohibitive scaling for high Reynolds number flows. A wall-resolved LES has an estimated scaling of

$$N_{\text{WRLES}} \sim Re_x^{13/7} \approx Re_x^{1.86}, \quad (2.2)$$

which allows for moderate Reynolds number, but is still impractical for industrial applications at high Reynolds number. A wall modeled LES, in contrast, has a cost that scales as

$$N_{\text{WMLES}} \sim Re_x^1. \quad (2.3)$$

This linear scaling is due to the fact that a wall modeled LES must only scale with the outer layer of the TBL and therefore scales with the outer size of the TBL. The alleviation of the constraint due to the need to resolve the inner layer has therefore brought the scaling to an affordable amount, hence pointing towards the need for wall modeling. Much effort has been put over the years into this problem. Many methods have resulted from those efforts, some with more success than others. A few of those will be presented here. More detailed and exhaustive reviews can be found in Cabot and Moin [18], Piomelli and Balaras [98], Sagaut [110], Piomelli [100], and more recently Larsson et al. [69].

The first natural way to solve the issue is to remove the inner layer altogether and provide off-wall boundary conditions at some height above the inner layer, but within the outer layer. The first attempts at off-wall boundary conditions by Baggett [3], Nicoud et al. [92], and Jiménez and Vasco [64], showed that a large amount of information is required for the off-wall Dirichlet boundary condition to work. In particular, the phases and time scales of the simulated turbulence have to be accurately modeled. Despite these unsuccessful attempts, some authors still continue to search for ways to impose off-wall boundary conditions, e.g. Podvin and Faigneau [102], using proper orthogonal decomposition of the inner layer, García-Mayoral et al. [41], exploiting the notion of minimal flow unit of Jiménez and Moin [63], or Mizuno and Jiménez

[86], using rescaled interior data. None of those attempts, however, reached convincing results so far. A slightly different approach, based on a special near-wall SGS model and on an ODE for the wall shear stress, by Chung and Pullin [24], led to slip velocity boundary conditions at a lifted virtual wall. This particular model led to promising results in channel flow.

Another approach to wall modeling is the family of wall shear stress models. The idea traces back to Deardoff [33], who first proposed to suppress the no slip boundary condition at the modeled wall. The question of what to substitute as boundary condition was yet unclear. Schumann [112] later built on the idea and proposed that this substituted boundary condition could be the wall shear stress, computed from local information and a provided mean wall shear stress. Finally, Mason and Callen [79] and Grötzbach [47] suggested that the mean wall shear stress could be computed on the fly by the model, using mean flow information and an assumed velocity profile, based on a law of the wall. This created the family of algebraic wall models, where the model is based on flow information and an algebraic law that links this flow information to the wall shear stress. Many variations have been proposed, notably the models of Piomelli et al. [99], who introduced a shift between the location at which the wall shear stress is computed and the location at which flow information is fetched to represent the tilted vertical structures of the attached eddies of the inner layer as well as a model to represent ejection events from the inner layer. Werner and Wengle [145] based their model on a power law description of the velocity profile. Shih et al. [114] proposed a new law of the wall that explicitly accounts for pressure gradient effects on the velocity profiles. More recently, Yang et al. [151] introduced the integral wall model: the integral method of von Kármán and Pohlhausen is used to include near-wall physics in the determination of the coefficients of an inner layer velocity profile, which is then used to retrieve the wall shear stress. This small list is far from exhaustive: for more details, the reader is referred to the review of Larsson et al. [69].

A more sophisticated and physics-based approach was proposed by Cabot [17] and Balaras et al. [5]: the two-layer models. The idea is to embed a secondary mesh within the LES mesh and to solve thin boundary layer equations supplemented with a RANS model on the embedded mesh. The no slip boundary condition is enforced on the embedded mesh and its upper boundary condition is extracted from the LES. In the thin boundary layer equations, the vertical pressure gradient is zero, and the pressure gradient can therefore also

be extracted from the LES. No pressure field has to be computed inside the embedded mesh, and the computation of the solution on the embedded mesh is therefore rather inexpensive. The wall shear stress from the embedded simulation is in turn fed to the LES as boundary condition. It is a soft RANS-LES coupling: the LES spans all the way to the wall, and the RANS simulation acts as a black box. It is fed flow information and yields a wall shear stress in return. It is therefore a branch of the wall shear stress models family. The main difference between this approach and the algebraic wall models is that, here, the black box is more sophisticated than with an algebraic rule: it can have a “memory” if time derivatives are solved for. When all terms are retained, the model works in unsteady RANS mode, and some adjustments can be performed to improve the results, such as those proposed by Park and Moin [96]. From the beginning, authors have suggested that some terms could be discarded (see Hoffmann and Benocci [52]). When the time derivative and convective terms are dropped, the equations become a series of ODE’s stretching from the wall to the point in the LES at which flow information is fetched. In this particular case, the equations can be integrated from the wall (using a guessed wall shear stress) to yield an algebraic or quasi algebraic representation of the velocity profile. This particular case is therefore from the family of algebraic wall shear stress models. It is the approach of (among others) Wang [142] (and later Wang and Moin [143]), Bocquet et al. [9] in a compressible framework, and Duprat et al. [37].

Spalart et al. [118] proposed a strong RANS-LES coupling in the sense that the LES model reverses to a RANS model below a certain distance from the wall fixed by the grid size. More accurately, it is based on a RANS model with a good wall behavior that is modified to exhibit LES properties above a given ratio of wall distance to grid size. The technique is called Detached-Eddy Simulation (DES). This RANS-LES coupling is a hard coupling in the sense that the same equations are solved on the whole mesh, and only the model term couples the RANS and the LES. This is not a two-layer model with an embedded RANS layer as previously. Nikitin et al. [94] proposed to use an improved version of the DES technique as a wall model. Issues remain but advances have been made with delayed-DES (DDES, see Spalart et al. [121]) who suggested a modification to allow the switch between the RANS and the LES models to be determined by the solution also. Further improvement were

made by Shur et al. [115] mixing various ideas into one improved delayed-DES. A detailed review of DES and related techniques was performed by Spalart [120]. Efforts to couple RANS with LES in a wall modeling framework were also performed by Medic et al. [80]: they imposed the transition point from LES to RANS, and they corrected the RANS viscosity close to the wall to account for the fluctuations contained in the mixed RANS-LES solution. Second generation RANS models (such as the Scale-adaptive model of Menter et al. [82]) also aim at achieving RANS resolution close to walls while maintaining scale resolving capabilities elsewhere. Such advanced RANS models and RANS-LES coupling techniques are beyond the scope of the present work. For a detailed review, the reader is referred to Fröhlich and von Terzi [40].

Fig. 2.1 summarizes the aforementioned methodologies in a graphical way: we see the overlap between the algebraic wall models and the two-layer models, with the family of vertically integrated 1D RANS models. Of course, this is only a small part of the approaches to modeling the inner layer of the turbulent boundary layer. Many other methodologies do not fit into this particular categorization, such as the model based on control theory by Nicoud et al. [93], the dynamic slip velocity model proposed by Bose and Moin [11], who compute a slip velocity at the wall based on a differential filter, the two-level simulation model by Gungor and Menon [48], the one-dimensional turbulence model used by Schmidt et al. [111], etc.

We are here interested in the study of wall shear stress models, both in their algebraic form and with the use of two-layer models. The numerical details, as well as the implementation of a wall shear stress in the framework of the BigFlow solver will be detailed. Once the wall shear stress boundary condition is implemented, various wall models will be tested and compared in a channel flow setup. The algebraic wall models are tested, with various averaging techniques and investigating the effect of the choice of a particular mesh. Interaction with subgrid scales models, in particular, the case of regularized variational multiscale models, will also be discussed. Two-layer models are then considered. Several RANS models are compared, and the correction of Medic et al. [80] is implemented and validated.

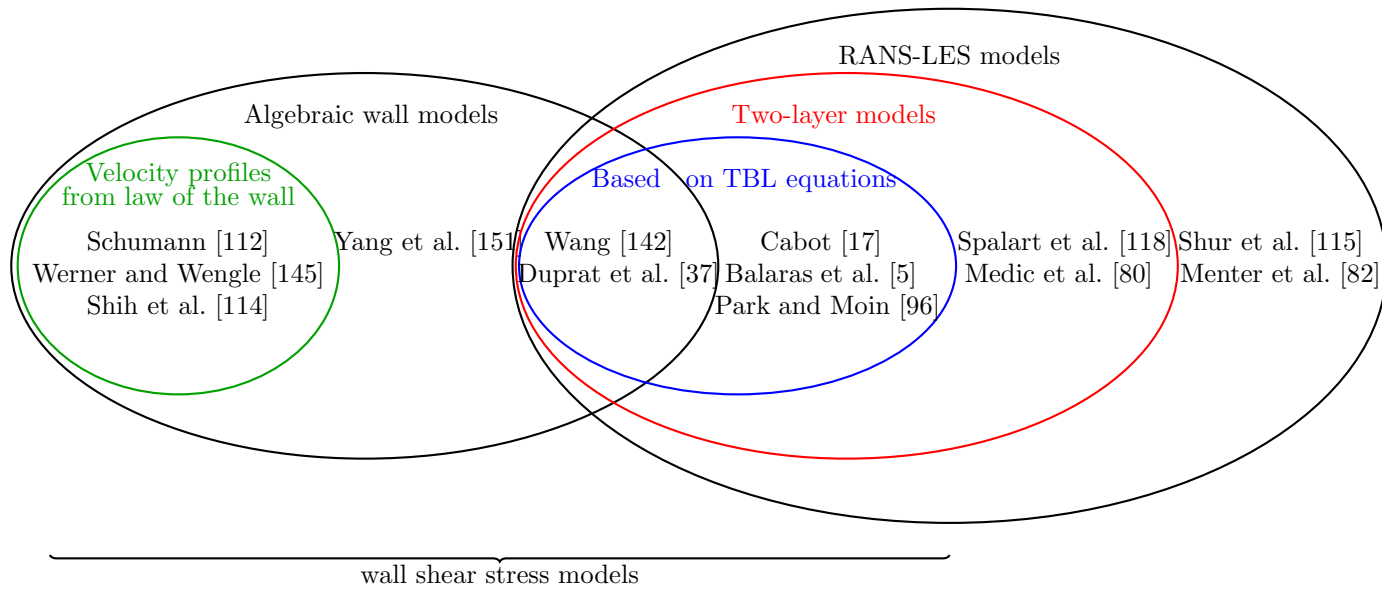


Figure 2.1: Family of approaches to wall modeling and RANS-LES coupling.

2.2 Numerical methods

The numerical solver used here to solve the incompressible Navier-Stokes equations is the BigFlow solver developed by Georges [42], further developed by Bricteux [13] and rewritten by Duponcheel [35], in an effort to improve its modularity and the performance of its Poisson solver. It is a fourth order finite differences solver that uses stretched cartesian meshes. It was originally designed to preserve the discrete kinetic energy exactly. For the purpose of the wall shear stress models, the implementation of the convective term had to be rewritten and the new implementation only preserves the discrete kinetic energy up to fourth order but it conserves the momentum exactly on uniform grids.

2.2.1 Spatial discretization

The Marker and Cell (MAC) arrangement of Harlow and Welsh [49] is used to store the variables on a staggered structured cartesian grid. Staggered grids are used to couple adjacent pressure point when using centered finite difference schemes. When using centered finite difference schemes on collocated grids, the odd pressure points are decoupled from the even pressure points, resulting in spurious pressure oscillations. The pressure is stored at the center of the cell, while the velocity components are stored at staggered points located at the center of the faces whose normal is aligned with the respective velocity component, see Fig. 2.2. The velocity derivatives are therefore available at the center of the edges of the corresponding faces.

The mesh can be stretched by one-dimensional mappings:

$$\frac{x_i}{L_i} = f(\xi_i), \quad (2.4)$$

where the physical coordinate is $x_i \in [0; L_i]$ and the computational coordinate is $\xi_i \in [0; 1]$.

The schemes we here use are those developed by Vasilyev [138], which extend the schemes of Morinishi et al. [88] to stretched grids. To properly write the schemes, a few notations are introduced that will make the schemes readily understandable. Since the grid is staggered, this will require fetching points at half cell distance, and the notation used for differentiation explicitly accounts

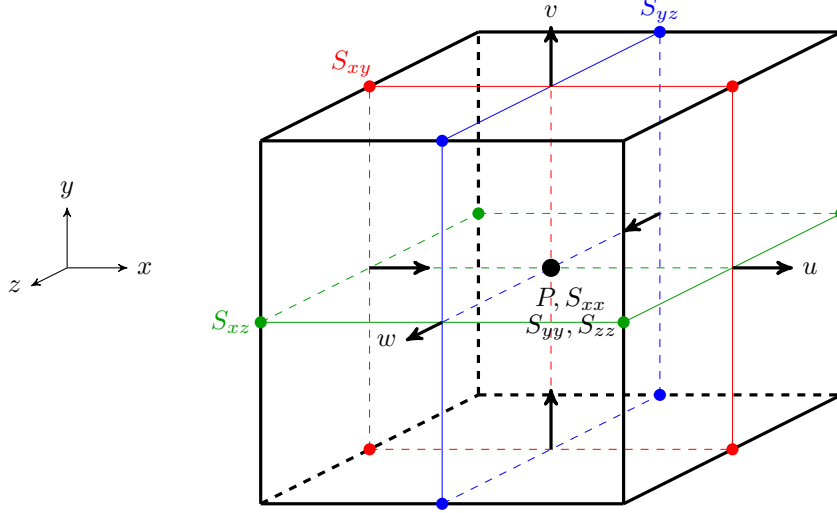


Figure 2.2: Storage of pressure, velocities and shear stress in a typical MAC cell.

for that. The differentiation operator with a stencil of width $n\Delta_1$ is:

$$\frac{\delta_n \phi}{\delta_n \xi_1} \triangleq \frac{\phi(\xi_1 + \frac{n}{2}\Delta_1, \xi_2, \xi_3) - \phi(\xi_1 - \frac{n}{2}\Delta_1, \xi_2, \xi_3)}{n\Delta_1}, \quad (2.5)$$

where Δ_1 is the uniform grid spacing in the computational domain, and in the direction of ξ_1 . Interpolation is also required, and the interpolation operator is defined as follows:

$$\overline{\phi}^{n, \xi_1} \triangleq \frac{1}{2} \left(\phi\left(\xi_1 + \frac{n}{2}\Delta_1, \xi_2, \xi_3\right) + \phi\left(\xi_1 - \frac{n}{2}\Delta_1, \xi_2, \xi_3\right) \right). \quad (2.6)$$

The Jacobians of the mapping in the ξ_i direction are given by

$$J_{i,n} \triangleq \frac{\delta_n x_i}{\delta_n \xi_i}, \quad (2.7)$$

without summation over the i index. We finally define the differentiation operator of order 4, which is a linear combination of first order derivatives with a stencil of width $1\Delta_1$ and $3\Delta_1$:

$$\frac{\partial \phi}{\partial x_i} \approx \frac{9}{8} \frac{1}{J_{i,1}} \frac{\delta_1 \phi}{\delta_1 \xi_i} - \frac{1}{8} \frac{1}{J_{i,3}} \frac{\delta_3 \phi}{\delta_3 \xi_i}. \quad (2.8)$$

Since this pattern is recurrent at order 4, a notation is introduced for it:

$$\left\{ \mathcal{D}^n(\phi) \right\}_n^{1,3} \triangleq \frac{9}{8} \mathcal{D}^1(\phi) - \frac{1}{8} \mathcal{D}^3(\phi), \quad (2.9)$$

where $\mathcal{D}$ is a generic operator. A description of the energy-conserving scheme can be found in Duponcheel [35]. We here describe the version of the convective term that allows for an easy implementation of the wall shear stress boundary condition in the code. The code is based on a momentum flux tensor divergence formulation of the Navier-Stokes equations. In this form, the discretization of the equations is close to a finite volume formulation. The time derivative of the velocity is written as the divergence of a flux tensor:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2.10)$$

$$\frac{\partial u_i}{\partial t} = \frac{\partial \mathcal{F}_{ij}}{\partial x_j}, \quad (2.11)$$

where $\mathcal{F}_{ij} = -u_i u_j - P \delta_{ij} + 2\nu S_{ij}$ is the momentum flux tensor (augmented with viscous stresses and pressure), $P = \frac{p}{\rho}$ is the reduced pressure (i.e. the *kinematic pressure*), and $S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ is the strain rate tensor. The S_{ij} and $\mathcal{F}_{ij}$ tensors are stored component by component at the center of the edges of the MAC cell, as shown in Fig. 2.2. The convective part of the tensor (the actual momentum flux tensor) is computed as

$$\mathcal{F}_{ij}^{conv} = -u_i u_j \approx - \left\{ \overline{u_i}^{n,\xi_j} \right\}_n^{1,3} \left\{ \overline{u_j}^{n,\xi_i} \right\}_n^{1,3}. \quad (2.12)$$

This scheme is equivalent to the scheme Div_S4K of Morinishi et al. [88], already proposed by Kajishima [66], it conserves momentum exactly on uniform grids. It does not conserve energy exactly but still up to fourth order since it is a fourth order accurate scheme. In this formulation, the wall shear stress can simply be expressed as

$$\boldsymbol{\tau}_w = \boldsymbol{\mathcal{F}} \Big|_{\Gamma} \cdot \boldsymbol{n}. \quad (2.13)$$

This is convenient because the wall shear stress can be imposed exactly, as the corresponding components of $\boldsymbol{\mathcal{F}}$ are stored at the wall, see Fig. 2.2. For example, for a wall in the Oxz plane, the respective components $\tau_{w,xy} = \mathcal{F}_{xy}|_{\Gamma}$ and $\tau_{w,zy} = \mathcal{F}_{zy}|_{\Gamma}$ are stored exactly on the wall surface.

2.2.2 Time stepping

The unsteady equations are solved using the fractional-step method in the delta form, see Lee et al. [70]. A predictor velocity increment is computed from a standard time integration and is then projected onto the divergence-free space. The projection is obtained by solving a Poisson equation for the pressure increment. In summary, a time step reads:

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = \nabla \cdot \mathcal{F}(\phi^n, \mathbf{u}^n, \mathbf{u}^{n-1}) \quad (2.14)$$

$$\nabla^2 \phi = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}^* \quad (2.15)$$

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\nabla \phi \quad (2.16)$$

$$\phi^{n+1} = \phi^n + \phi, \quad (2.17)$$

where $\mathbf{u}^* - \mathbf{u}^n$ is the predictor velocity increment. This formulation was shown to be second order accurate in time by Lee et al. [70], and simple boundary conditions can be applied in a consistent fashion:

$$\mathbf{u}^*|_{\Gamma} = \mathbf{u}^{n+1}|_{\Gamma} \quad (2.18)$$

$$\left. \frac{\partial \phi}{\partial n} \right|_{\Gamma} = 0. \quad (2.19)$$

The integration is carried using an Adams-Bashforth 2 scheme for the convective term and an explicit Euler scheme for the viscous term. In practice, Eq. (2.14) reads:

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -\nabla \phi^n + \nabla \cdot \left(\frac{3}{2} \mathcal{F}^{conv,n} - \frac{1}{2} \mathcal{F}^{conv,n-1} + 2\nu \mathbf{S}^n \right). \quad (2.20)$$

The Poisson equation (2.15) is solved using a multigrid solver. Details on the solver can be found in Duponcheel [35] and Georges [42].

2.3 Testing the flux version of the solver

The new version of the code is tested and compared to the energy-conserving scheme. Two cases are tested: first a periodic DNS simulation of the Taylor-Green vortex, demonstrating the ability of the code to run in DNS mode and develop turbulence in a transitional flow; then a wall-resolved LES of channel flow is carried to demonstrate the capability of the code to run in LES mode,

and with a solid wall.

2.3.1 Taylor-Green vortex

The case at hand is a Taylor-Green vortex at moderate Reynolds number. The dimensionless Taylor-Green vortex is defined in a periodic numerical box of size $L_x \times L_y \times L_z = (2\pi)^3$. It is defined as :

$$u(x, y, z) = \sin(x) \cos(y) \cos(z) \quad (2.21)$$

$$v(x, y, z) = -\cos(x) \sin(y) \cos(z) \quad (2.22)$$

$$w(x, y, z) = 0. \quad (2.23)$$

The Reynolds number is here $Re = \frac{1}{\nu} = 1600$. Both versions of the code were run on a uniform mesh with 256^3 grid points and on 8 CPU's. The time step for both simulations was $\Delta t = 10^{-3}$. From the very simple initial condition, the flow naturally evolves to a turbulent state. The transition occurs around $t = 4$, when the kinetic energy dissipation starts increasing significantly. Fig. 2.3 compares the evolution of the kinetic energy dissipation rate of the energy-conserving version of the code and the new version of the code. Both are compared to a reference solution obtained using the spectral solver described in Chapter 6 on the same grid. The transition phase of the three simulations is essentially the same, hence showing the capability of both versions of BigFlow to perform transitional and turbulent simulations. After $t = 8$, when maximum enstrophy is reached, discrepancies appear between the finite difference solutions and the spectral solution, but also between the two versions of the code. The difference between the two versions of the codes, however, remains within the order of magnitude of the difference between the energy-conserving scheme solution and the spectral solution.

Fig. 2.4 displays a 3D visualization of the flow field at $t = 10$. Since the initial symmetry planes of the flow field are maintained, only a zoom on the region $[0; \pi] \times [0; \pi] \times [0; \pi]$ is shown. Turbulent structures are displayed using the λ_2 criterion of Jeong and Hussain [62]. This visualization illustrates the level of complexity of the flow, with many turbulent structures. We also see that the structures developed by the two versions of the code are essentially the same, hence showing that the loss of energy conservation due to the new version of the convective term does not entail the capability of the solver at creating and maintaining proper turbulence, or at least not at this resolution.

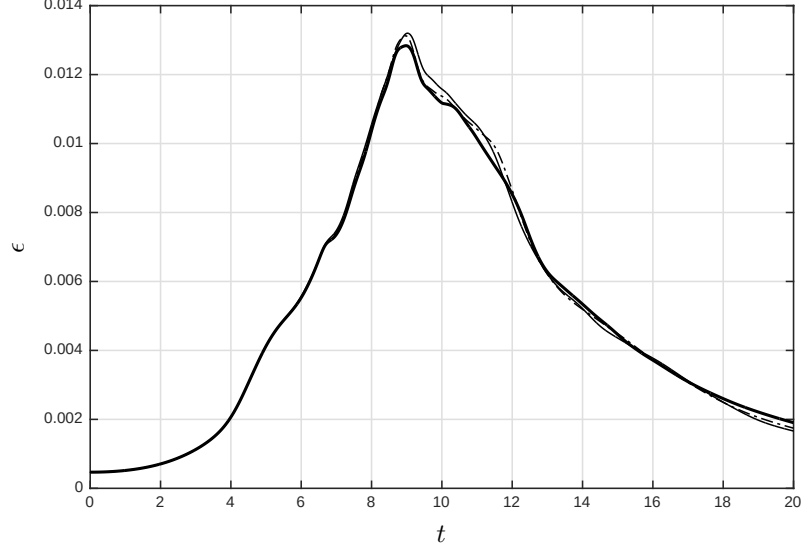


Figure 2.3: Time evolution of the dissipation rate of the **Taylor-Green vortex** at $Re = 1600$. Spectral (thick solid); BigFlow: energy conserving scheme (dashed-dotted) and flux scheme (thin solid).

2.3.2 Channel flow at $Re_\tau = 395$

The code is also tested in wall resolved LES mode with a solid boundary. A turbulent channel flow at $Re_\tau = 395$ is a standard well-known reference case. The DNS solution of Moser et al. [89] will be used as reference solution. The case consists in a flow between two infinite flat plates. The plates are parallel to the Oxz plane and the flow is going in the x direction. The flow is homogeneous in the streamwise (x) and spanwise (z) directions and only varies in the wall normal (y) direction. Numerically, a periodic box of size $L_x \times L_y \times L_z = 2\pi h \times 2h \times \pi h$ is used, see Fig. 2.5. The flow is driven by a forcing pressure gradient $(\frac{dP}{dx})_F$. The wall shear stress yields a velocity scale called the friction velocity:

$$u_\tau \triangleq \sqrt{\frac{\tau_w}{\rho}}. \quad (2.24)$$

The friction Reynolds number is then defined as

$$Re_\tau = \frac{u_\tau h}{\nu} \quad (2.25)$$

which can be linked to the forcing pressure gradient. In such a case, the wall

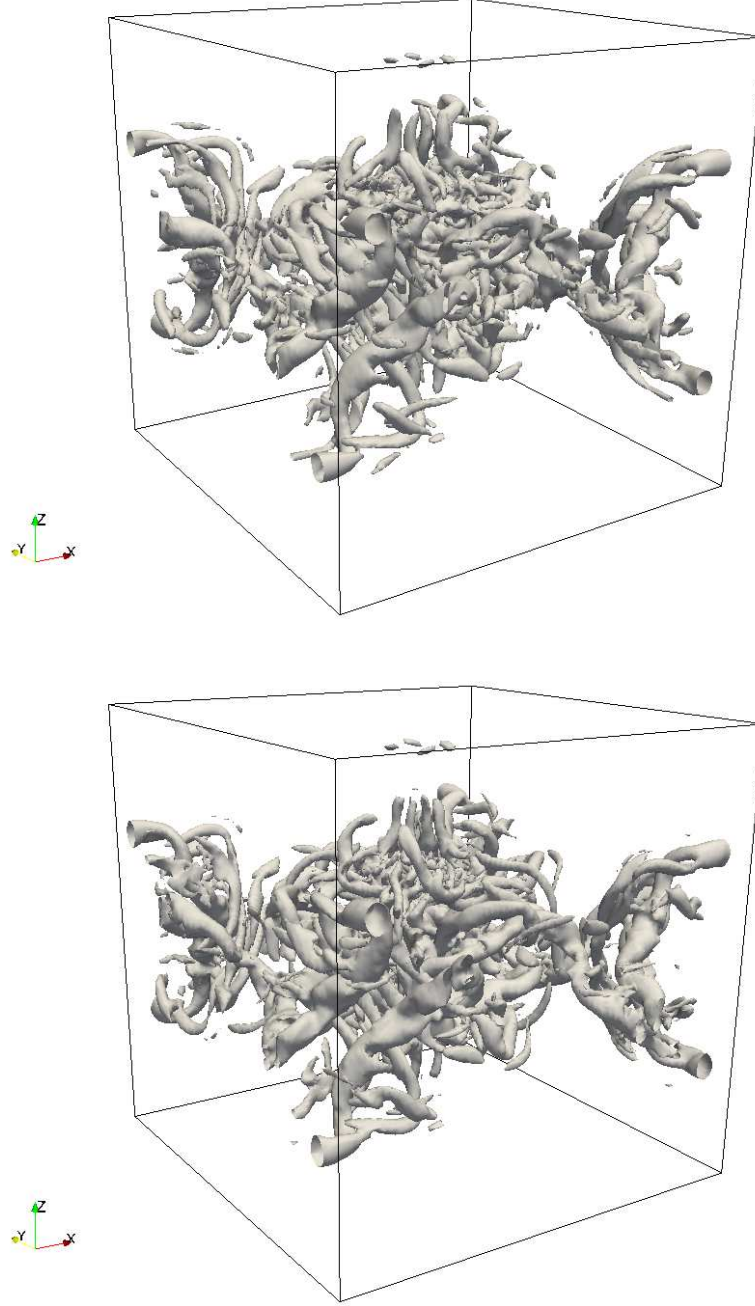


Figure 2.4: Visualization of the *Taylor-Green vortex* at $Re = 1600$ at $t = 10$: isosurfaces of $\lambda_2 = -10$, zoom on the region $[0; \pi] \times [0; \pi] \times [0; \pi]$. Energy-conserving scheme (top) and flux-based scheme (bottom)

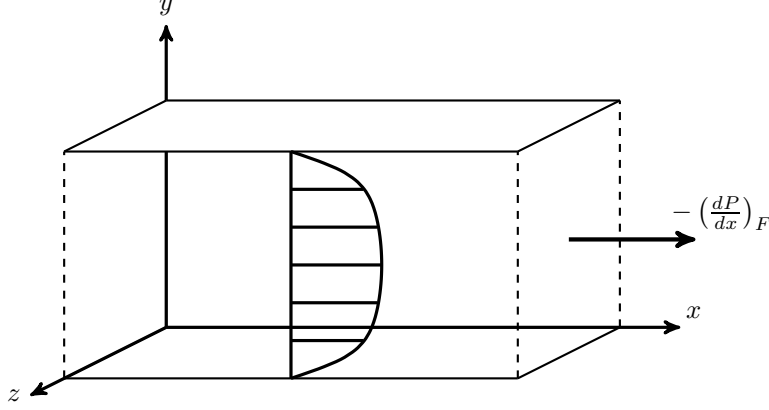


Figure 2.5: Sketch of the channel flow configuration.

friction balances the pressure gradient, such that a momentum budget gives

$$u_\tau^2 = - \left(\frac{dP}{dx} \right)_F h . \quad (2.26)$$

The channel flow case could also be forced using the mass flow rate as control variable, but the Reynolds number is only known a posteriori in that case. In the near-wall region, the variables scale as a function of the viscosity and of the friction velocity. The quantities of interest are then

$$y^+ = \frac{y u_\tau}{\nu} \quad (2.27)$$

$$u^+ = \frac{u}{u_\tau} . \quad (2.28)$$

The grid used here is the same as that used by Bricteux et al. [14]:

$$N_x = 64, \quad N_y = 48, \quad N_z = 48 . \quad (2.29)$$

A hyperbolic tangent stretching is applied in the wall normal direction such that the first cell lies within the viscous sublayer:

$$\frac{y}{h} = 1 + \frac{\tanh(\gamma(\xi - 1))}{\tanh(\gamma)} , \quad (2.30)$$

where $\xi \in [0; 2]$ is the uniform computational variable and $y \in [0; 2h]$ is the physical coordinate. We here use $\gamma = 2.75$ such that the viscous resolution of

the mesh is

$$\Delta x^+ \approx 38.7, \quad \Delta y^+ \approx 0.86 - 45.3, \quad \Delta z^+ \approx 25.8, \quad (2.31)$$

which lies within the bounds of wall-resolved LES proposed by Sagaut [110].

The SGS model is the WALE model of Nicoud and Ducros [91]. It is based on the square of the velocity gradient:

$$\mathcal{S}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} \frac{\partial u_k}{\partial x_j} + \frac{\partial u_j}{\partial x_k} \frac{\partial u_k}{\partial x_i} \right) \quad (2.32)$$

$$= S_{ik} S_{kj} + \Omega_{ik} \Omega_{kj}, \quad (2.33)$$

with $S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, the strain rate tensor, and $\Omega_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$, the rate of rotation tensor. The eddy viscosity is then defined as:

$$\nu^{sgs} = C \Delta^2 \frac{(\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d)^{3/2}}{(S_{ij} S_{ij})^{5/2} + (\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d)^{5/4}}, \quad (2.34)$$

where $\mathcal{S}_{ij}^d$ is the deviatoric part of $\mathcal{S}_{ij}$. Nicoud and Ducros [91] proposed $C = 0.25$. Bricteux et al. [14] used a smaller value of the coefficient $C = 0.20$ with the energy-conserving scheme. We here use an even lower coefficient $C = 0.15$, calibrated by trial and error (for the flux version of the code only). The mean velocity $\langle u \rangle$ is shown in Fig. 2.6. We observe good agreement with the DNS data, in the range of what was observed by Bricteux et al. [14]. The RMS velocity fluctuations $(u'_i)_{rms}$, with $u'_i = u_i - \langle u_i \rangle$ are shown in Fig. 2.7. Again, the agreement with the DNS data, and with the energy-conserving code is good. The streamwise RMS fluctuations reach a peak higher than the DNS, though a bit lower than the energy-conserving code. The spanwise and wall normal fluctuations are very similar, though a bit higher than those obtained with the energy-conserving code, both lying under the level of the DNS. Fig. 2.8 shows the shear stresses and the momentum budget. In the mean, we have

$$\nu \frac{d\langle u \rangle}{dy} + \langle \tau_{xy}^{sgs} \rangle - \langle u'v' \rangle = u_\tau^2 \left(1 - \frac{y}{h} \right). \quad (2.35)$$

We see that again, the budget is essentially the same with the two versions of the code. Since the model coefficient is a bit lower in the flux version of the code than in the energy-conserving version of the code, the SGS contribution

to the budget is a bit lower, and the $\langle u'v' \rangle$ contribution a bit larger. This can also be observed in Fig. 2.9, which displays the budget of dissipation

$$\langle \epsilon \rangle = 2\nu \langle S_{ij} S_{ij} \rangle + \langle \tau_{ij}^{sgs} S_{ij} \rangle. \quad (2.36)$$

where we see that the SGS model contribution to the budget is lower. Finally, the SGS viscosity of the model shown in Fig. 2.10, where, again, we see the effect of the smaller coefficient. Note however that the factor between the two curve is smaller than the ratio between the two coefficients, hence showing that the turbulent structures resolved in the two simulations are not exactly the same, which partly explains why the coefficient had to be lowered.

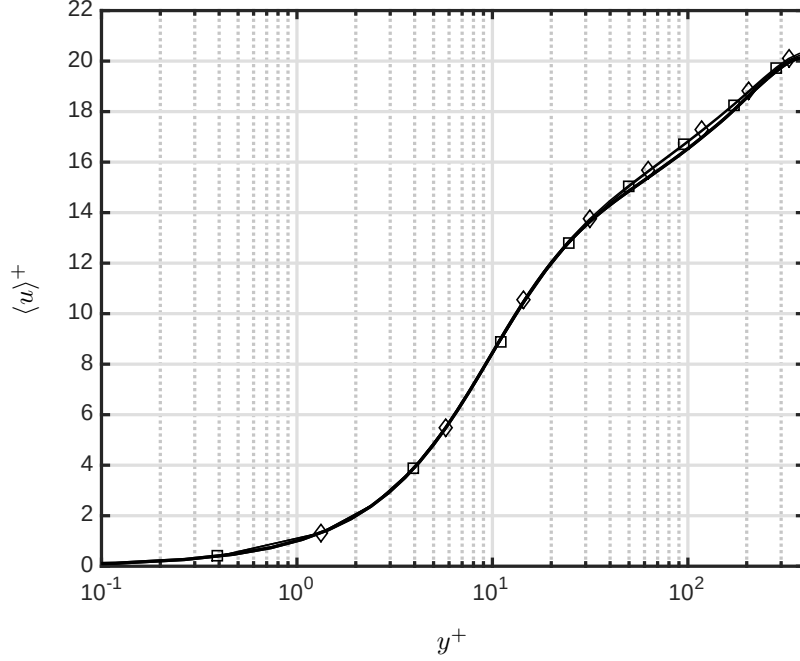


Figure 2.6: Channel flow at $Re_\tau = 395$. **Mean velocity** profile: DNS of Moser et al. [89] (thick solid), LES with the **energy conserving code** (squares), and LES with the **flux version of the code** (diamonds). The markers do not show all grid points for the sake of readability.

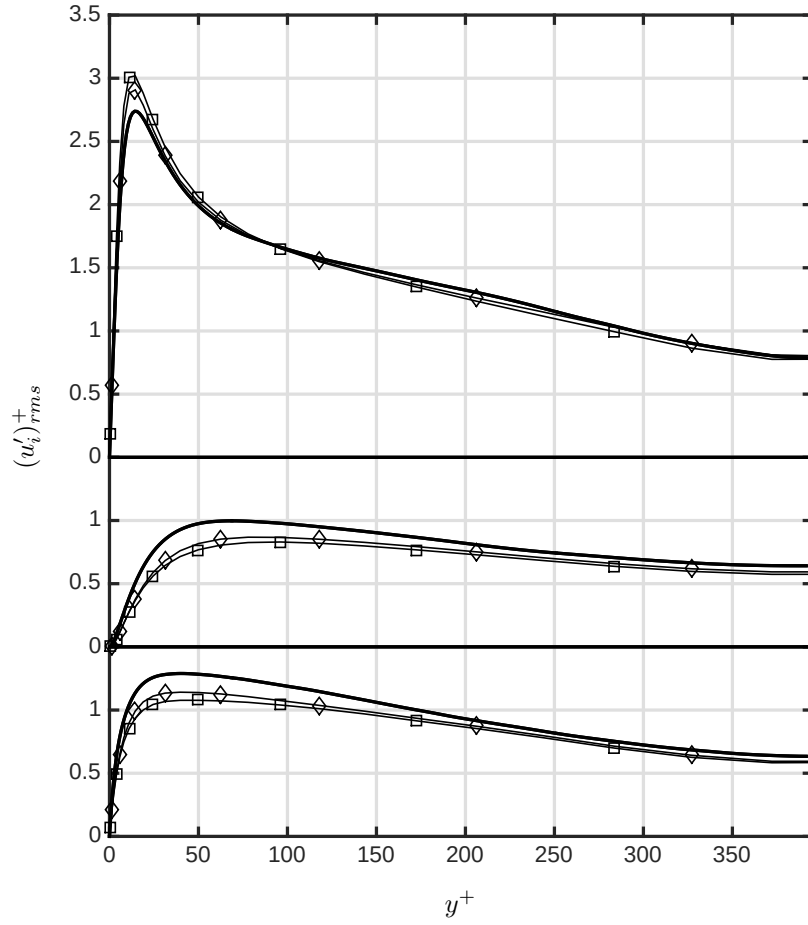


Figure 2.7: Channel flow at $Re_\tau = 395$. **RMS velocity fluctuation** profiles: DNS of Moser et al. [89] (thick solid), LES with the **energy conserving** code (squares), and LES with the **flux version of the code** (diamonds). From top to bottom: $(u')^+_{rms}$, $(v')^+_{rms}$, and $(w')^+_{rms}$. The markers do not show all grid points for the sake of readability.

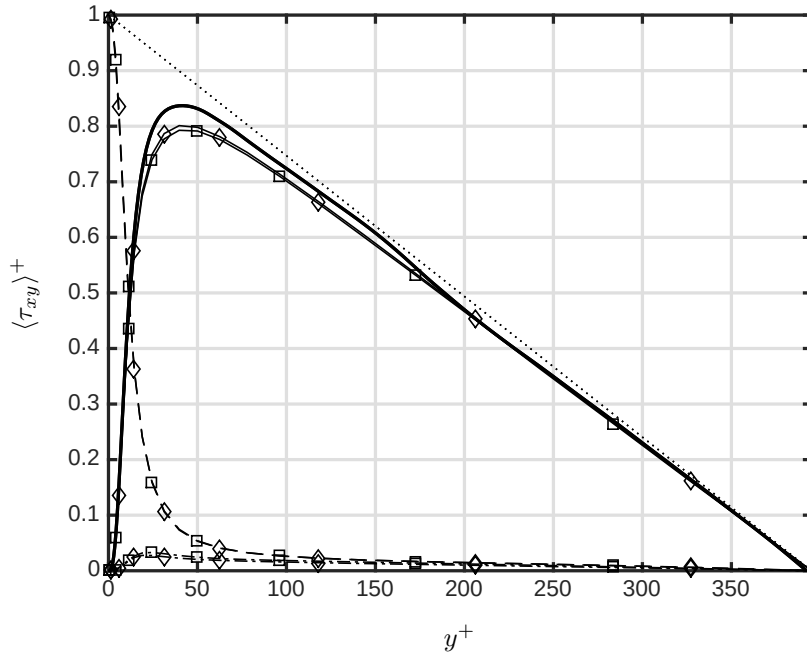


Figure 2.8: Channel flow at $Re_\tau = 395$. **Mean shear stress** profiles: total stress (dotted), viscous shear stress of the LES (thin dashed), SGS model stress of the LES (dashed dotted), resolved turbulent shear stress of the LES (thin solid and markers), and turbulent shear stress of the DNS of Moser et al. [89] (thick solid), LES with the **energy conserving** code (squares), and LES with the **flux version of the code** (diamonds). The markers do not show all grid points for the sake of readability.

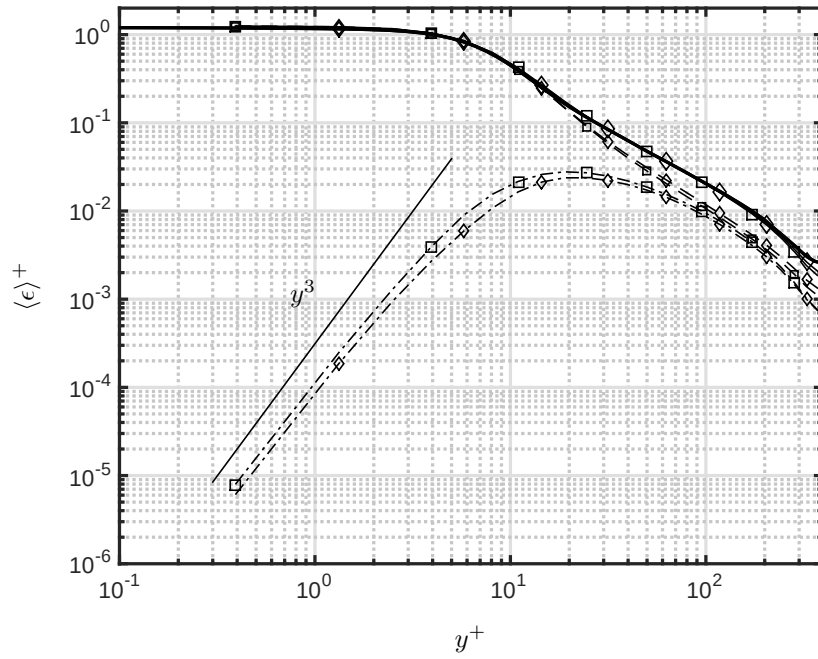


Figure 2.9: Channel flow at $Re_\tau = 395$. **Mean kinetic energy dissipation** profiles: viscous dissipation of the LES (thin dashed), SGS model dissipation of the LES (thin dashed dotted), total dissipation of the LES (thin solid), and total dissipation of the DNS of Moser et al. [89] (thick solid), LES with the **energy conserving** code (squares), and LES with the **flux version of the code** (diamonds). The markers do not show all grid points for the sake of readability.

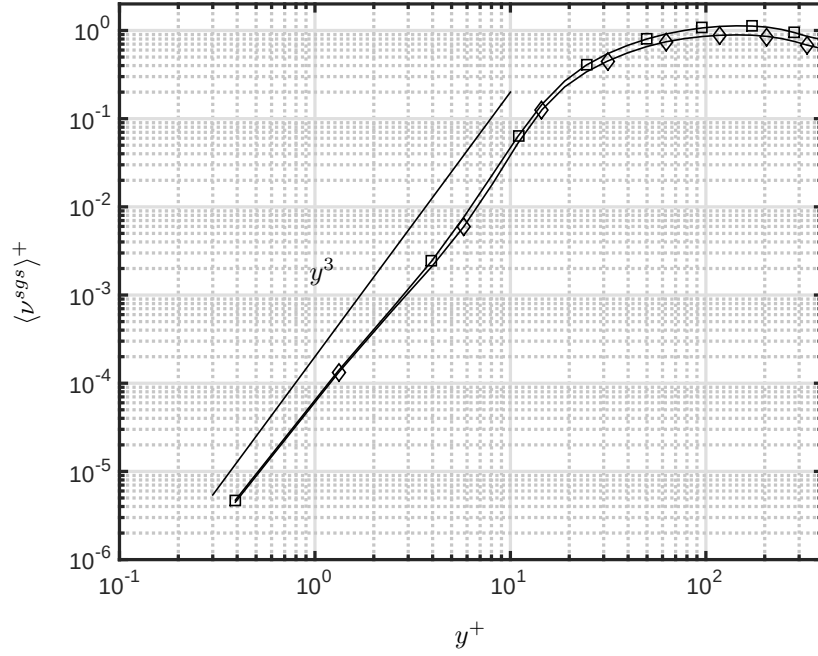


Figure 2.10: Channel flow at $Re_\tau = 395$. **Mean SGS viscosity** profile of the LES with the **energy conserving** code (squares), and LES with the **flux version of the code** (diamonds). The markers do not show all grid points for the sake of readability.

The flux version of the solver required a calibrated coefficient of the SGS model when the WALE SGS model was used. We therefore perform a calibration of the coefficients for several other SGS models when used with the flux version of the solver in wall-resolved LES mode. The velocity profiles are used to perform the calibration. Wall-resolved LES simulations were carried using the Regularized Variational Multiscale (RVM) version of the WALE SGS model. The formulation was first proposed by Winckelmans and Jeanmart [150] and Jeanmart and Winckelmans [59], based on the Variational Multiscale (VMS) model of Hughes et al. [58]. The principle is to decompose the LES field into large scales and resolved small scales:

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}^s, \quad (2.37)$$

and to use the small scales to compute the SGS eddy-viscosity $\nu^{sgs}(\mathbf{u}^s)$. Note that we here only refer to LES fields, hence the LES operator $\sim$ is implied everywhere. Depending on how the small scales are used, several families of models can be developed. The eddy-viscosity computed with the small scales can be applied to the small scales (the “small-small”, or RVMs models):

$$\boldsymbol{\tau}^{sgs} = 2\nu^{sgs}(\mathbf{u}^s)\mathbf{S}^s, \quad (2.38)$$

where $\mathbf{S}^s = \frac{1}{2}(\nabla\mathbf{u}^s + (\nabla\mathbf{u}^s)^T)$ is the strain rate tensor of the small scales. The small scales can also be used only to compute the SGS viscosity only (the “small-all”, which we here call filtered models):

$$\boldsymbol{\tau}^{sgs} = 2\nu^{sgs}(\mathbf{u}^s)\mathbf{S}, \quad (2.39)$$

or in the computation of the strain rate only, on which the full field SGS viscosity is applied (the “all-small” models):

$$\boldsymbol{\tau}^{sgs} = 2\nu^{sgs}(\mathbf{u})\mathbf{S}^s, \quad (2.40)$$

Cocle et al. [26] showed that the use of the small scales strain rate $\mathbf{S}^s$ improves the spectral behavior of the models, and Cocle et al. [25] showed that all RVM-based models have low dissipation in pure rotation flows, which is not the case of all “all-all” models. In the original VMS formulation, the small scales are obtained with a Galerkin projection of the LES field into a restricted functional space. In the RVMs formulation, the small scales are obtained us-

ing an iterated discrete 3-point filter. The details of the filter can be found in Jeanmart and Winckelmans [60]. The extension of the RVMs framework to WALE based models was done by Bricteux et al. [14]. We are here interested in two such models: the RVMs-WALE model, and the filtered WALE model. The calibrated coefficients are $C = 0.35$ for the RVMs-WALE model (which is smaller than the value $C = 0.56$ used by Duponcheel [35]) and $C = 0.25$ for the filtered WALE model. Fig. 2.11 displays the profiles obtained with the calibrated coefficients. They are in good agreement with the DNS data. The level of fluctuations of streamwise velocity obtained with the filtered WALE model is slightly higher than those obtained with the RVMs-WALE model. The wall-normal and spanwise fluctuations are more intense with the RVMs-WALE model than with the filtered WALE model. In Fig. 2.13, we see that more of the turbulent shear stress is resolved when using the RVMs-WALE model than when using the filtered WALE model, because the modeled stress is larger when using S_{ij} instead of S_{ij}^s , as noticed by Bricteux et al. [14]. Despite the slightly smaller SGS viscosity computed with the filtered WALE model (see Fig. 2.15), the total SGS model dissipation is bigger in the case of the filtered WALE model, again because of the use of the complete S_{ij} field (see Fig. 2.14).

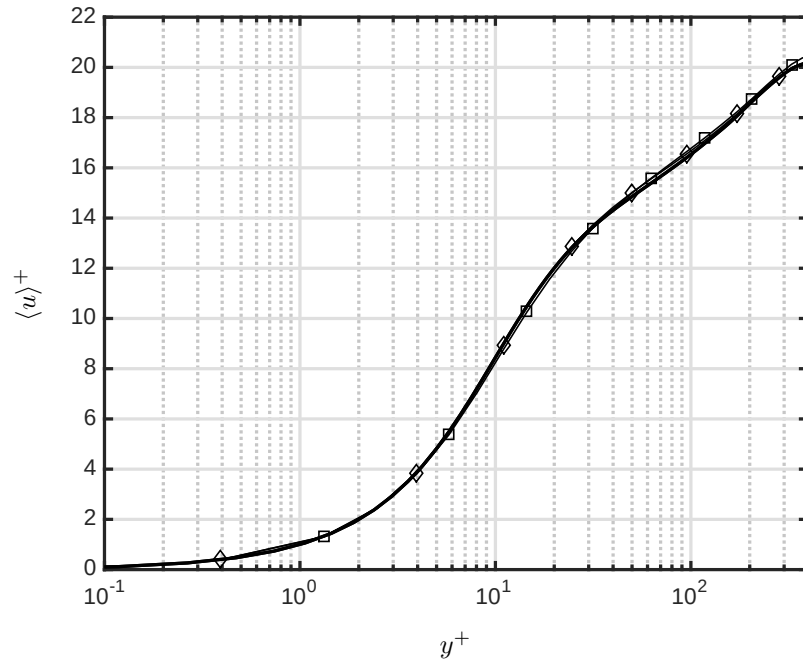


Figure 2.11: Channel flow at $Re_\tau = 395$. **Mean velocity** profile: DNS of Moser et al. [89] (thick solid), and wall-resolved LES with the flux version of the code (thin solid). **RVMs-WALE (diamonds)** and **filtered WALE (squares)**. The markers do not show all grid points for the sake of readability.

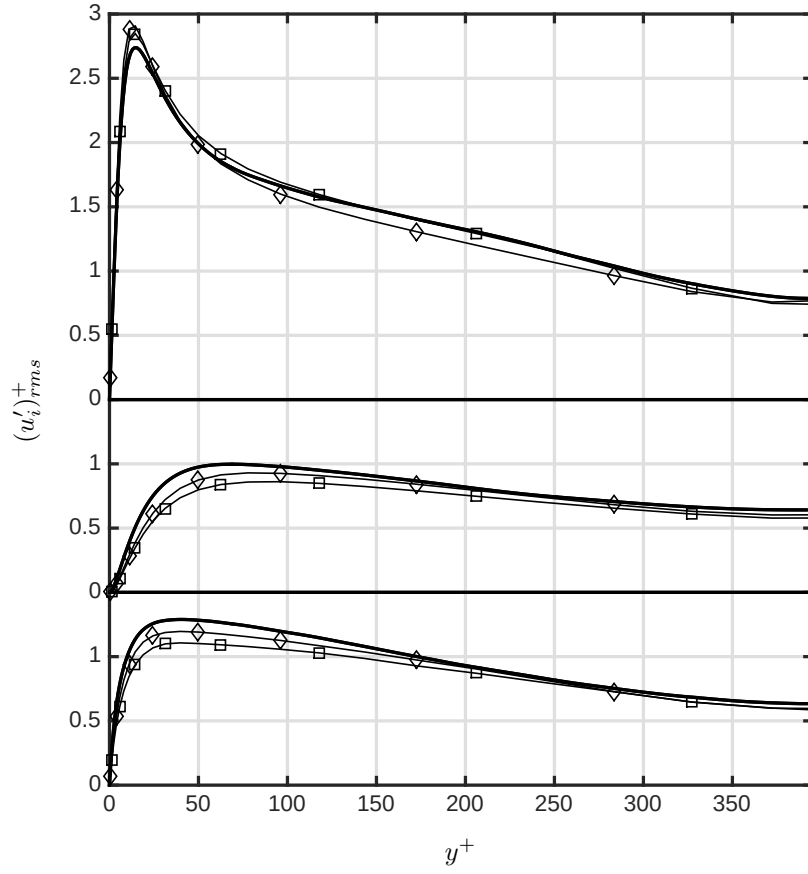


Figure 2.12: Channel flow at $Re_\tau = 395$. **RMS velocity fluctuation** profile: DNS of Moser et al. [89] (thick solid), and wall-resolved LES with the flux version of the code (thin solid). **RVMs-WALE** (diamonds) and **filtered WALE** (squares). From top to bottom: $(u')_{rms}^+$, $(v')_{rms}^+$, and $(w')_{rms}^+$. The markers do not show all grid points for the sake of readability.

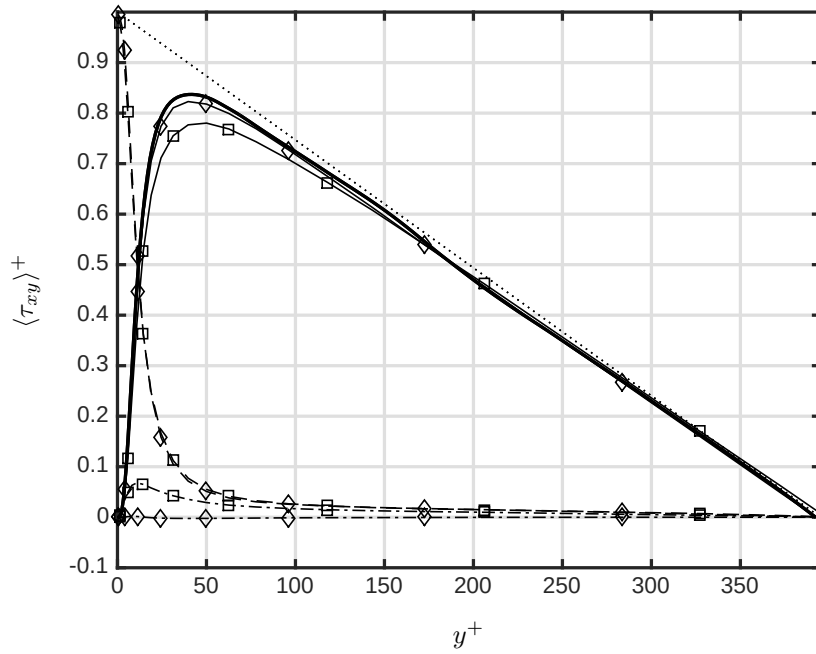


Figure 2.13: Channel flow at $Re_\tau = 395$. **Mean shear stress** profiles: total stress (dotted), viscous shear stress of the LES (thin dashed), SGS model stress of the LES (dashed dotted), resolved turbulent shear stress of the LES (solid and diamonds), and turbulent shear stress of the DNS of Moser et al. [89] (thick solid), wall resolved LES with **RVMs-WALE model** (diamonds) and **filtered WALE model** (squares). The markers do not show all grid points for the sake of readability.

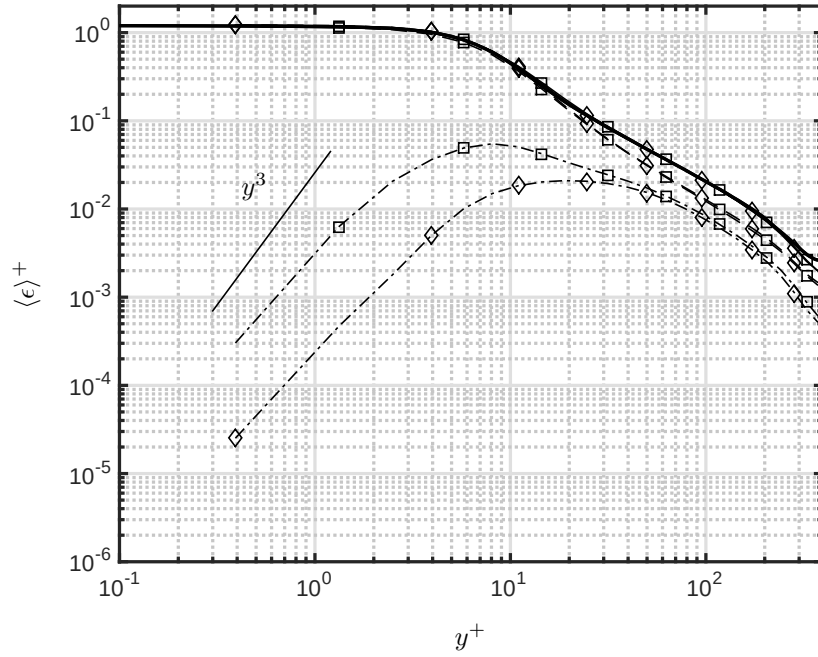


Figure 2.14: Channel flow at $Re_\tau = 395$. **Mean kinetic energy dissipation** profiles: viscous dissipation of the LES (thin dashed), SGS model dissipation of the LES (thin dashed dotted), total dissipation of the LES (thin solid), and total dissipation of the DNS of Moser et al. [89] (thick solid), wall resolved LES with **RVMs-WALE** model (diamonds) and **filtered WALE** model (squares). The markers do not show all grid points for the sake of readability.

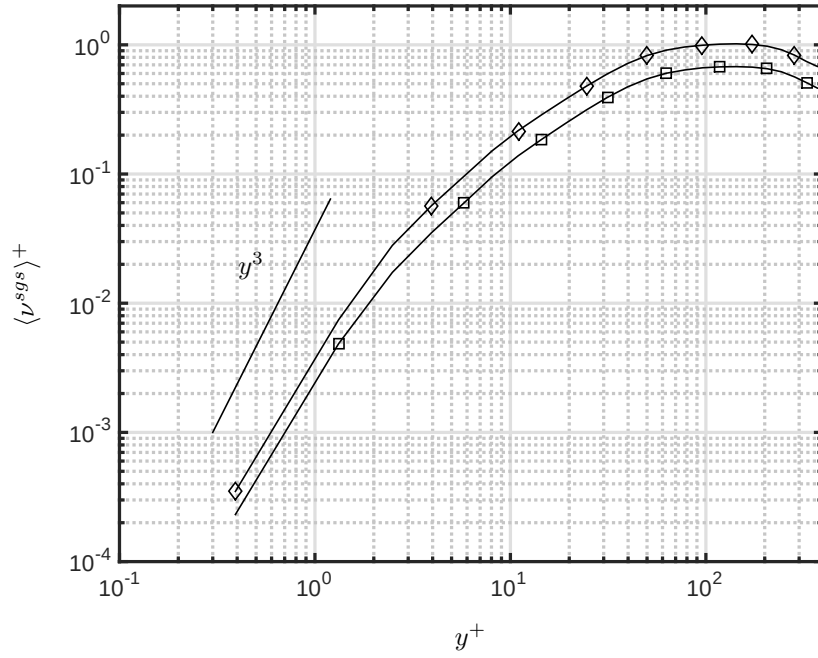


Figure 2.15: Channel flow at $Re_\tau = 395$. Mean SGS viscosity profile of the LES: RVMs-WALE model (diamonds) and filtered WALE model (squares). The markers do not show all grid points for the sake of readability.

2.4 Handling wall shear stress boundary conditions

The objective is now to impose, in the new version of the code, the boundary condition

$$\mathcal{F}\Big|_{\Gamma} \cdot \mathbf{n} = \boldsymbol{\tau}_w \quad (2.41)$$

$$v\Big|_{\Gamma} = 0, \quad (2.42)$$

where we impose a wall shear stress for a wall whose normal is in the y direction, without loss of generality.

2.4.1 Numerical details

Since the code is fourth order accurate, the stencil for the computation of the convective and diffusive terms can involve up to three layers of points beyond the solid wall Γ (*ghost points*), points for which we have to provide values compatible with the boundary condition for the evaluation of the fluxes. This is true for both the energy-conserving version of the code and the flux version of the code. For the wall-normal component of velocity v , we have a Dirichlet boundary condition (2.42) from which we can compute ghost points. For the details of this computation, the reader is referred to Georges [42]. In the case of the wall-parallel components u and w , however, there is no well-defined boundary condition on the velocity field corresponding to Eq. (2.41). The wall shear stress boundary condition can not be written as a Neumann boundary condition in a straightforward fashion, because, in a WMLES framework, the first cell is very coarse. Therefore, the effective ν^{sgs} is non-zero, and we have

$$\tau_{w,xy} = (\nu + \nu^{sgs}) \frac{\partial u}{\partial y}\Big|_{\Gamma}. \quad (2.43)$$

Since $\nu^{sgs}\Big|_{\Gamma}$ is a function of $\frac{\partial u}{\partial y}$, the equation is implicit and deducing a consistent boundary condition for the velocity is tricky. Because of that, even if the values of $\mathcal{F}_{xy}$ and $\mathcal{F}_{zy}$ are properly set at the wall, and with consistent ghost cells of $\mathcal{F}$, the first layer of points above the wall will have incorrect values of $\mathcal{F}_{xy}$ and $\mathcal{F}_{zy}$, because inconsistent information for u and w will have been fetched in ghost cells of velocity. One must therefore refrain from using

information from ghost points of velocity. One way of imposing the boundary condition without involving ghost points is to reduce the order of the wall normal stencils when approaching the wall. The wall normal derivatives are reverted to order 2 at the wall as well as the stencil for the computation of the convective term. The situation is sketched in Fig. 2.16, for the Oxy plane. The situation in the Oyz plane is similar. The computation of $\mathcal{F}_{xz}$, $\mathcal{F}_{xx}$, and $\mathcal{F}_{zz}$ is unchanged since it only involves stencils in wall parallel directions. When $\mathcal{F}_{xy}$ is computed, however, a few changes are necessary. The wall layer of points is computed from the wall model and poses no issue. The third layer of points only involves points that lie inside the physical domain and is therefore computed normally. The second layer of point is computed with second order stencils in the y direction. The convective term reads:

$$\mathcal{F}_{xy}^{conv} \approx -\bar{u}^{1,\xi_2} \left\{ \bar{v}^{n,\xi_1} \right\}_n^{1,3}, \quad (2.44)$$

and the wall normal velocity derivative reads:

$$\frac{\partial u}{\partial y} \approx \frac{1}{J_{2,1}} \frac{\delta_1 u}{\delta_1 \xi_2}, \quad (2.45)$$

such that no ghost point of velocity is required for those computations.

The velocity u is also computed by degrading the divergence scheme close to the wall. The first layer of off-wall points is computed with order 2 only in the wall normal direction:

$$\frac{\partial \mathcal{F}_{xy}}{\partial y} \approx \frac{1}{J_{2,1}} \frac{\delta_1 \mathcal{F}_{xy}}{\delta_1 \xi_2}. \quad (2.46)$$

The switching of the wall normal derivative from order 2 to order 4 scheme is not harmless. Indeed, even on a uniform mesh, such a transition commits an error of order 2 on the momentum budget, and a correction has to be applied to preserve the conservation properties. We show the development on the generic 1D equation in infinite domain

$$\frac{\partial u}{\partial t} = \frac{d\phi(u)}{dy} \quad (2.47)$$

Given a uniform 1D mesh with grid spacing h , and on a staggered grid, (hence with u_i stored at $\frac{h}{2} + ih$ and ϕ_i stored at ih , $i \in \mathbb{Z}$), the transition of order, at

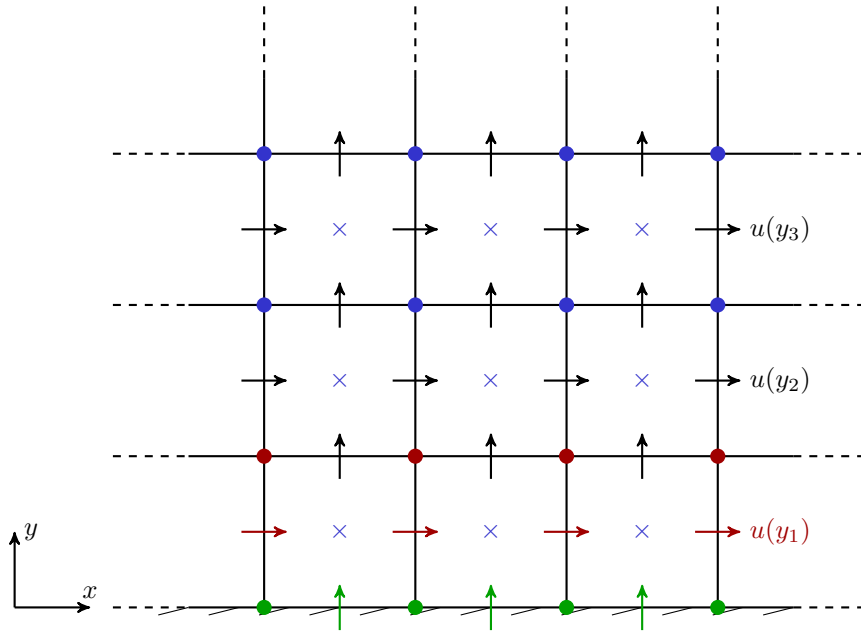


Figure 2.16: Sketch of a cut of the numerical domain in a plane parallel to Oxy and at the middle of the cells in the z direction. Green: computed from boundary condition; Blue: computed with standard stencil; Red: computed with order 2 in the y direction. Circles: points of $\mathcal{F}_{xy}$; Crosses: point of P , $\mathcal{F}_{xx}$, and ν^{SGS} .

point y_I can be written as follows:

$$\frac{d}{dt} \int u dy \approx \frac{d}{dt} \sum_i h u_i \quad (2.48)$$

$$= \sum_i h \frac{du_i}{dt} \quad (2.49)$$

$$= \sum_{i=-\infty}^{I-1} h \frac{\phi_{i+1} - \phi_i}{h} \quad (2.50)$$

$$+ \sum_{i=I}^{\infty} h \left(\frac{9}{8} \frac{\phi_{i+1} - \phi_i}{h} - \frac{1}{8} \frac{\phi_{i+2} - \phi_{i-1}}{3h} \right) \quad (2.51)$$

$$= -\frac{1}{12} \left(\phi_I - \frac{1}{2}(\phi_{I+1} + \phi_{I-1}) \right) \quad (2.52)$$

$$= \frac{1}{24} h^2 \frac{d^2 \phi}{dy^2}(y_I) + \mathcal{O}(h^4). \quad (2.53)$$

This last expression can also be expressed as a Taylor series around $y_{I-1/2}$:

$$-\frac{1}{12} \left(\phi_I - \frac{1}{2}(\phi_{I+1} + \phi_{I-1}) \right) = \frac{1}{24} h^2 \frac{d^2 \phi}{dy^2}(y_{I-1/2}) + \mathcal{O}(h^3). \quad (2.54)$$

With this expression, we see that the error term in the momentum budget is zero up to second order at the last point at which the derivatives are evaluated at order 2 ($y_{I-1/2}$). A way to correct the momentum budget is therefore to subtract the error term while evaluating the derivative at that point:

$$\left(\frac{d\phi}{dy} \right)^h (y_{I-1/2}) = \frac{\phi_I - \phi_{I-1}}{h} + \frac{1}{12h} \left(\phi_I - \frac{1}{2}(\phi_{I-1} + \phi_{I+1}) \right). \quad (2.55)$$

This scheme, however, is only first order accurate, due to the introduce term:

$$\left(\frac{d\phi}{dy} \right)^h (y_{I-1/2}) = \frac{d\phi}{dy}(y_{I-1/2}) - h \frac{1}{24} \frac{d^2 \phi}{dy^2}(y_{I-1/2}) + \mathcal{O}(h^2). \quad (2.56)$$

In conclusion, we can transition from a second order scheme to a fourth order scheme and maintain the conservation of momentum in the discrete sense, if we accept that one layer of points is only first order accurate. This correction is referred to as the “momentum” correction in the following. The transition of the scheme from an order of accuracy to another order of accuracy is therefore non-trivial, and such a correction is required if one desires to maintain the good properties of either scheme (here the conservation of the momentum). In

practice, the first layer of points is computed with the second order scheme, the second layer of points is computed with the second order scheme and the correction is applied when computing the y derivative of $\mathcal{F}$ at y_2 . The rest of the points are computed using the fourth order scheme.

Another issue, raised by Bocquet et al. [9], is that the subgrid-scale model cannot be accurate at the first cell. This is due to several limitations. First, the first cell is so coarse that the subgrid-scale stress model cannot account for all the energy-bearing eddies contained in it. Indeed, it is the principle of wall modeling that the first cell has as size which is significantly larger than the energy-bearing eddies of the inner layer of the TBL, hence the physical content of the first cell does not allow the SGS model to compute an accurate SGS viscosity. Second, the SGS viscosity is computed using inaccurate computation of the strain rate tensor close to the wall, again due to the cell size compared to the mean velocity profile. A better value of the strain rate could probably be extracted from the wall shear stress computed by the wall model, but this value would still be ambiguous, since the SGS viscosity is, here, non-zero at the wall, see Eq. (2.43). The solution they propose is to equate the momentum balance between the first two cells. Thin boundary layer hypothesis leads to $\langle v \rangle = 0$ and $\frac{\partial \langle P \rangle}{\partial y} = 0$. It also allows to neglect all derivative in wall-parallel directions (all but the mean pressure gradient, of course). The mean momentum equation then reads:

$$\frac{\partial \langle u'v' \rangle}{\partial y} = -\frac{d\langle P \rangle}{dx} + \frac{\partial}{\partial y} \left((\nu + \langle \nu^{sgs} \rangle) \frac{\partial \langle u \rangle}{\partial y} \right), \quad (2.57)$$

where we neglected the variations of ν^{sgs} , hence writing $\left\langle \nu^{sgs} \frac{\partial u}{\partial y} \right\rangle \simeq \langle \nu^{sgs} \rangle \frac{\partial \langle u \rangle}{\partial y}$. In the context of channel flow, $\frac{d\langle P \rangle}{dx}$ is the forcing pressure gradient dragging the flow. We can then integrate the momentum equation vertically from the wall:

$$-\langle u'v' \rangle + \left(\nu + \langle \nu^{sgs} \rangle \right) \frac{\partial \langle u \rangle}{\partial y} = \langle \tau_w \rangle + \frac{d\langle P \rangle}{dx} y. \quad (2.58)$$

By equating this momentum balance at the first two layers of points, we get a target viscosity for the first layer of points:

$$\langle \nu^{sgs} \rangle_1^{target} = \frac{(\nu + \langle \nu^{sgs} \rangle_2) \frac{\partial \langle u \rangle}{\partial y} \Big|_2 - (\langle u'v' \rangle_2 - \langle u'v' \rangle_1) + \left(-\frac{d\langle P \rangle}{dx} \right) (y_2 - y_1)}{\frac{\partial \langle u \rangle}{\partial y} \Big|_1} - \nu. \quad (2.59)$$

Bocquet et al. [9] postulated that the momentum and pressure terms could be

neglected because $y_2 - y_1$ is small and hypothesizing that $\langle u'v' \rangle_2 \simeq \langle u'v' \rangle_1$. A more accurate statement would be that the pressure gradient is in equilibrium with the momentum terms, hence allowing to neglect the sum of both terms as a whole. This target mean eddy viscosity is then imposed locally:

$$\nu_1^{sgs,corr} = \frac{\langle \nu^{sgs} \rangle_1^{target}}{\langle \nu^{sgs} \rangle_1} \nu_1^{sgs}, \quad (2.60)$$

where ν_1^{sgs} is the local value computed by the SGS model, $\langle \nu^{sgs} \rangle_1$ is the mean value of the SGS viscosity computed by the SGS model, and $\nu_1^{sgs,corr}$ is the value actually used in the SGS term.

As noted by Cabot and Moin [18] and Nicoud et al. [93], the first layer of off-wall points are always affected by high numerical and modeling error, in a wall-modeled LES. This remains true, even with the correction of Eq. (2.60). This means that the flow information at the first layer of points is of poor quality and is not reliable to use in a wall model. To circumvent this limitation, Kawai and Larsson [67] proposed that flow information at higher points be used to compute the wall model. Indeed, nothing requires the wall model to be computed at the first grid point. They proposed to evaluate the wall model at a given height $h_{wm} = y_j$, with $j \geq 1$. They showed that the results obtained with this practice were much better than with the wall model evaluated at the first grid point y_1 , which was the standard practice at the time. They also showed that converged results can be obtained for $j \geq 3$. In the following, unless stated otherwise, we use $h_{wm} = y_3$.

2.4.2 Validation of the implementation of the wall shear stress

The numerical framework of the wall shear stress model, with the correction for the momentum balance of Eq. (2.55) and with the correction of the SGS viscosity of Eq. (2.60) are tested in a channel flow at $Re_\tau = 395$. The mesh is coarse in the wall normal direction only, to test the implementation of the wall shear stress. The number of points is

$$N_x = 64, \quad N_y = 32, \quad N_z = 48, \quad (2.61)$$

with a stretching in the wall normal direction to coarsen the grid:

$$\frac{y}{h} = 1 + \frac{\arcsin(\delta(\xi - 1))}{\arcsin(\delta)}, \quad (2.62)$$

where $\xi \in [0, 2]$ is the uniform variable in the computational domain, and $y \in [0, 2h]$ is the physical coordinate, and with $\delta = 0.9$. This yields the following viscous resolution:

$$\Delta x^+ = 26, \quad \Delta y^+ = 43 - 20, \quad \Delta z^+ = 26. \quad (2.63)$$

The stretching is illustrated in Fig. 2.17. The stretching is not very strong but sufficient to ensure the point at which the wall model will be evaluated is not in the viscous sub-layer or far in the transitional layer.

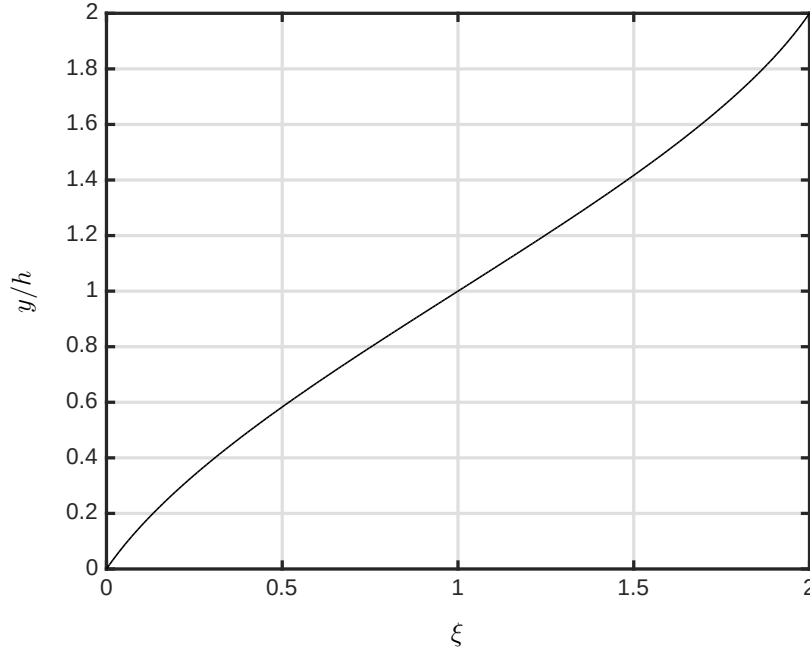


Figure 2.17: Arcsine stretching of Eq. (2.63) to coarsen the first wall cells.

The SGS model used is the WALE model with a coefficient $C = 0.15$, as for the wall-resolved LES. The type of wall model has no practical importance in the present test case since we want to test the wall shear stress implementation. The various wall models implemented are detailed later in Sections 2.5 and 2.6. The model used in this case is a linear algebraic model based on the law of

the wall of Reichardt and evaluated at the third layer of points, as described in Section 2.5. Three simulations are performed: a simulation with the momentum correction and the ν^{sgs} correction, a simulation with the momentum correction only, and a simulation with none of the corrections. An additional simulation is run with all corrections, but where the wall model is evaluated at the first layer of points to demonstrate the need to use informations at y_3 .

Fig. 2.18 compares the mean velocity profiles obtained by the three simulations against the DNS data of Moser et al. [89]. We see that the profiles obtained when using the momentum correction and the ν^{sgs} correction are in good agreement with the DNS, while the profile obtained without corrections is more oscillatory and provides less satisfactory results. Comparing the case with the momentum correction only with the case with both corrections shows that the ν^{sgs} correction here seems to only affect the first layer of points. The RMS velocity profiles are displayed in Fig. 2.19, where we also see a good agreement of the results of the WMLES with the DNS data. The mean shear stresses are shown in Fig. 2.20. We here see that the WMLES without corrections propagates oscillations from the wall up to the bulk of the flow. Oscillations that are much damped by the momentum correction and contained within the first three cells. The momentum correction did not completely suppress the inaccurate behavior of the first two points: a small deficit in $\langle u'v' \rangle$ is observed at the first off-wall point, and a peak is reached immediately after at the second off-wall point. This behavior results in slight oscillations propagating to the bulk of the flow. It is reduced (the reduction however is only marginal) by the ν^{sgs} correction. The ν^{sgs} correction effect is visible in Fig. 2.21, where we see that the correction applied to the ν^{sgs} is here of almost one order of magnitude.

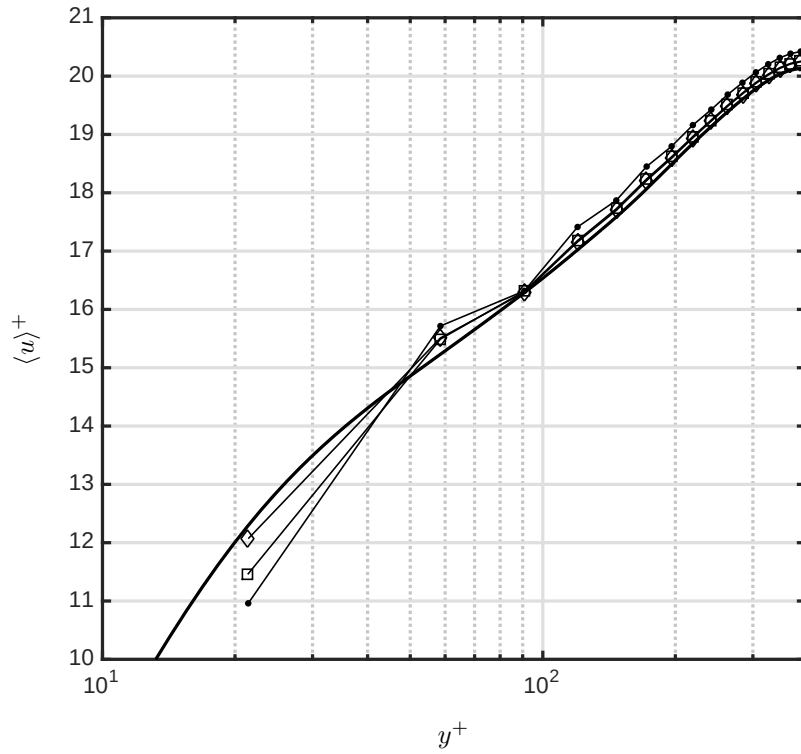


Figure 2.18: Channel flow at $Re_\tau = 395$. **Mean velocity** profile: DNS of Moser et al. [89] (thick solid), and wall-modeled LES, with wall models computed at $y_3^+ = 91$ (thin solid and markers). **No corrections (bullets), momentum correction only (diamonds), and momentum and ν^{SGS} correction (squares).**

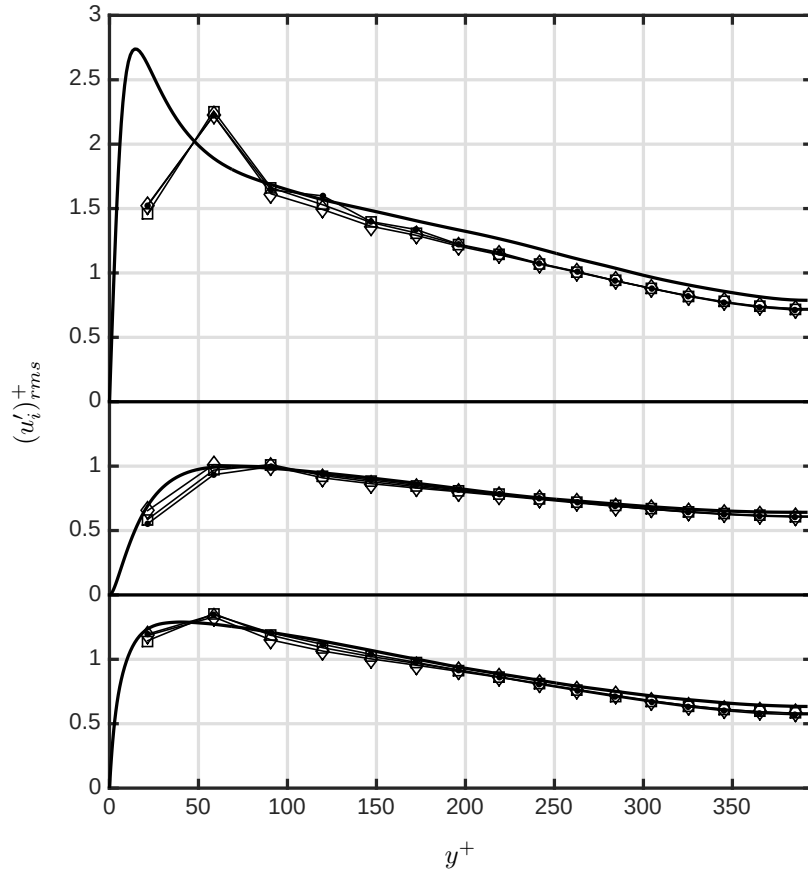


Figure 2.19: Channel flow at $Re_\tau = 395$. **RMS velocity fluctuation** profile: DNS of Moser et al. [89] (thick solid), and wall-modeled LES, with wall models computed at $y_3^+ = 91$ (thin solid and markers). **No corrections (bullets)**, **momentum correction only (diamonds)**, and **momentum and ν^{SGS} correction (squares)**.

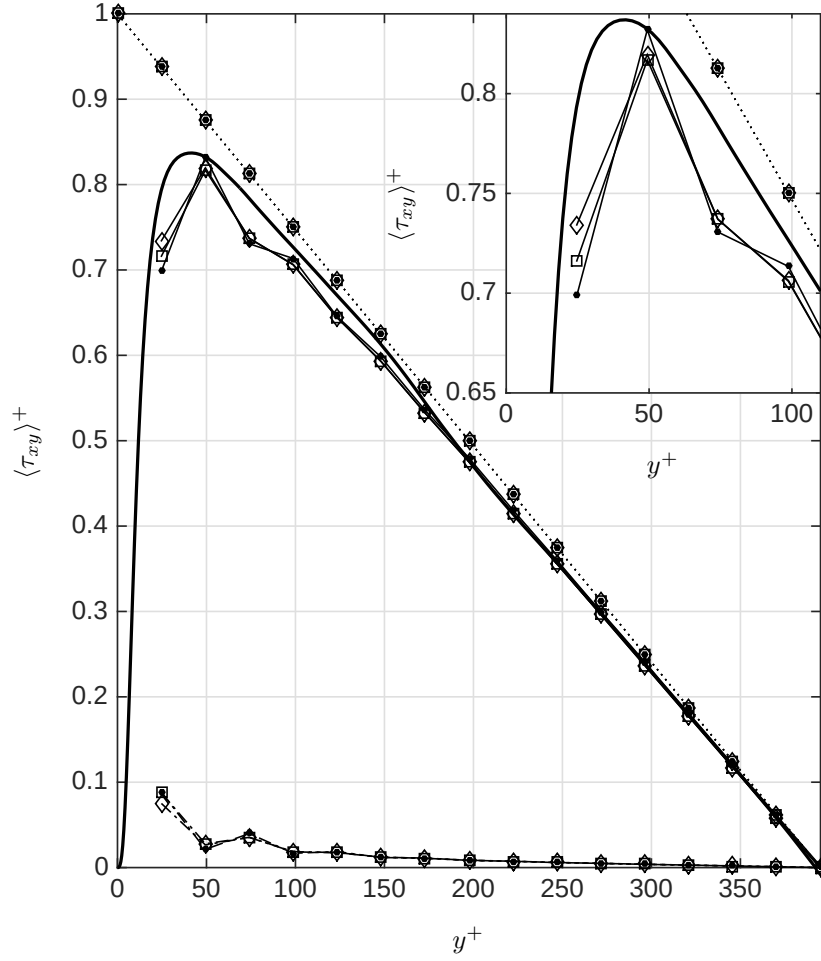


Figure 2.20: Channel flow at $Re_\tau = 395$. **Mean shear stress profiles:** total stress (dotted), SGS model stress of the LES (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), and turbulent shear stress of the DNS of Moser et al. [89] (thick solid). Wall models computed at $y_3^+ = 91$: **no corrections (bullets), momentum correction only (diamonds), and momentum and ν^{SGS} correction (squares).** The upper right corner is a zoom on the turbulent shear stress at the first points.

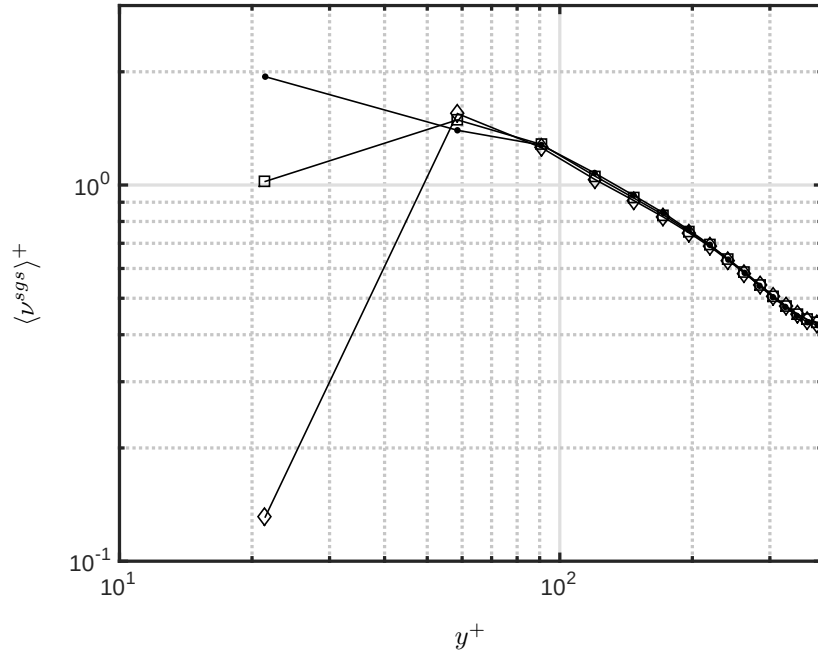


Figure 2.21: Channel flow at $Re_\tau = 395$. Mean SGS viscosity profile of the wall modeled LES, with wall models computed at $y_3^+ = 91$: no corrections (bullets), momentum correction only (diamonds), and momentum and ν^{SGS} correction (squares).

Fig. 2.22 displays the results obtained with all corrections and using either the first or the third layer of points to calculate the wall model. It illustrates the need to use points higher than the first layer of points to compute the wall model: the first few points adapt to the wall shear stress and the very coarse resolution. The reaction results in a gross overestimation of the mass flow rate when the first layer of points is used. In this sense, when the third layer of points is used, the first layers of points can be viewed as a numerical artifact to allow the LES to adapt to the wall shear stress boundary condition. Fig. 2.23 shows the RMS fluctuations obtained by the two runs. We again see that the first few points differ in the two simulations, but the points above y_3 display the exact same results, hinting that the use of y_3 is “enough” to let the numerical solution adapt to the wall shear stress boundary condition, this also shows that the turbulence developed in the simulation is more dependent of the SGS model than of the wall model. The same conclusions can be drawn from the shear stress profiles of Fig. 2.24 and the SGS viscosity profiles of Fig. 2.25.

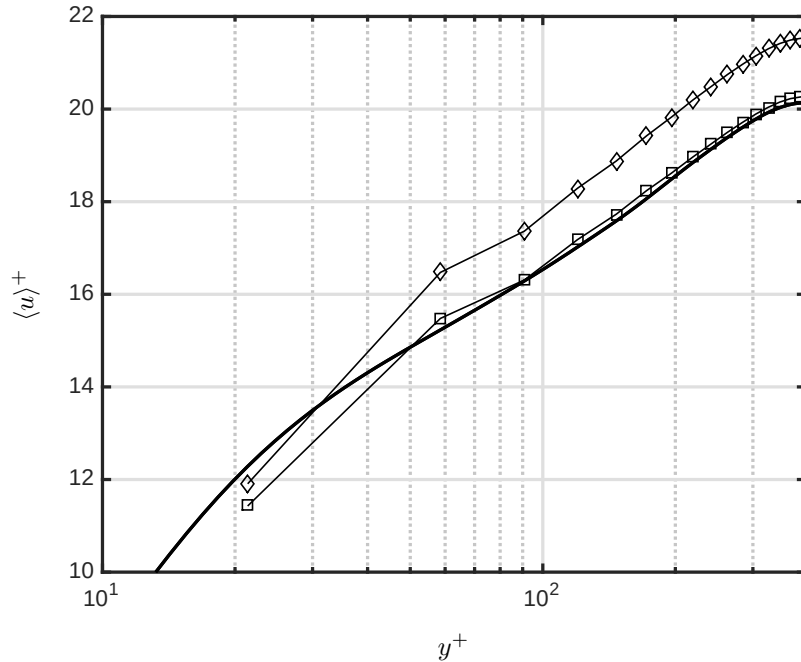


Figure 2.22: Channel flow at $Re_\tau = 395$. **Mean velocity** profile: DNS of Moser et al. [89] (thick solid), and wall-modeled LES (thin solid and markers). **Wall model** computed at $y_1^+ = 21$ (diamonds), Wall model computed at $y_3^+ = 91$ (squares).

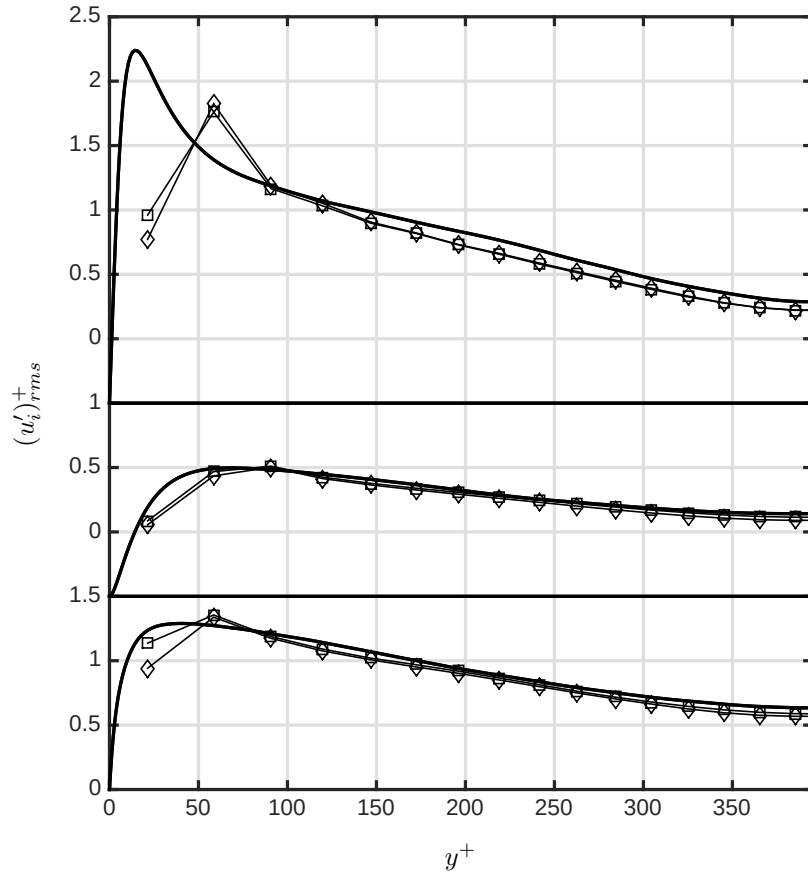


Figure 2.23: Channel flow at $Re_\tau = 395$. **RMS velocity fluctuation profile:** DNS of Moser et al. [89] (thick solid), and wall-modeled LES (thin solid and markers). Wall model computed at $y_1^+ = 21$ (diamonds), Wall model computed at $y_3^+ = 91$ (squares).

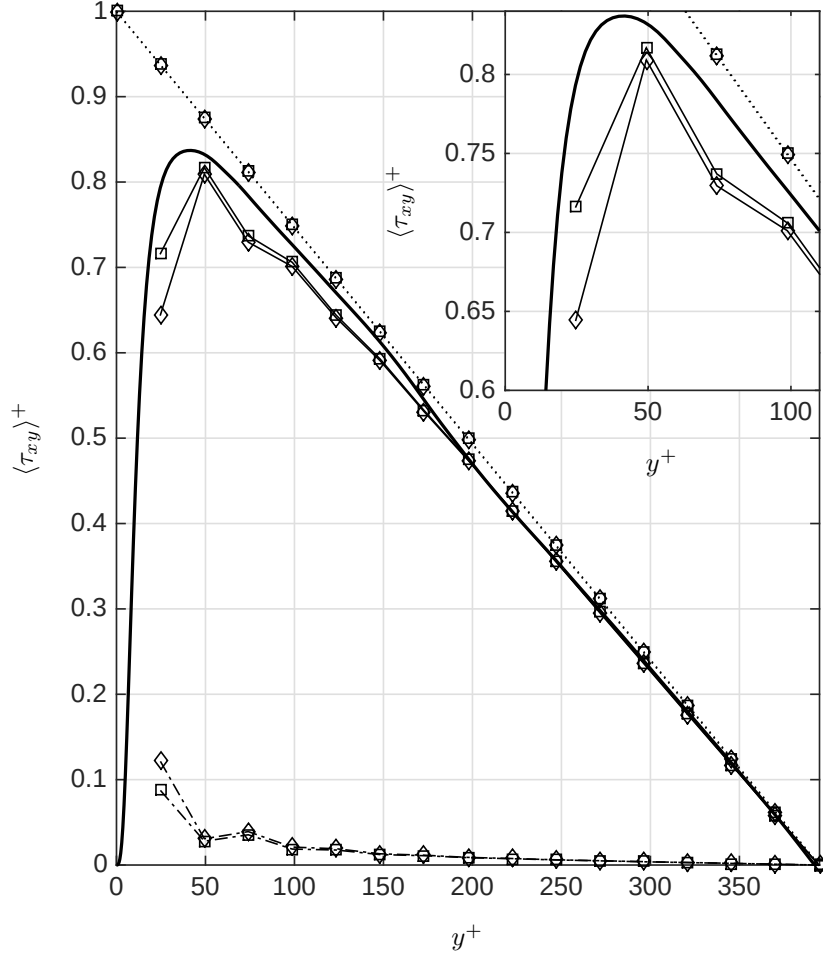


Figure 2.24: Channel flow at $Re_\tau = 395$. **Mean shear stress** profiles: total stress (dotted), SGS model stress of the LES (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), and turbulent shear stress of the DNS of Moser et al. [89] (thick solid). **Wall model computed at $y_1^+ = 21$ (diamonds), Wall model computed at $y_3^+ = 91$ (squares).**

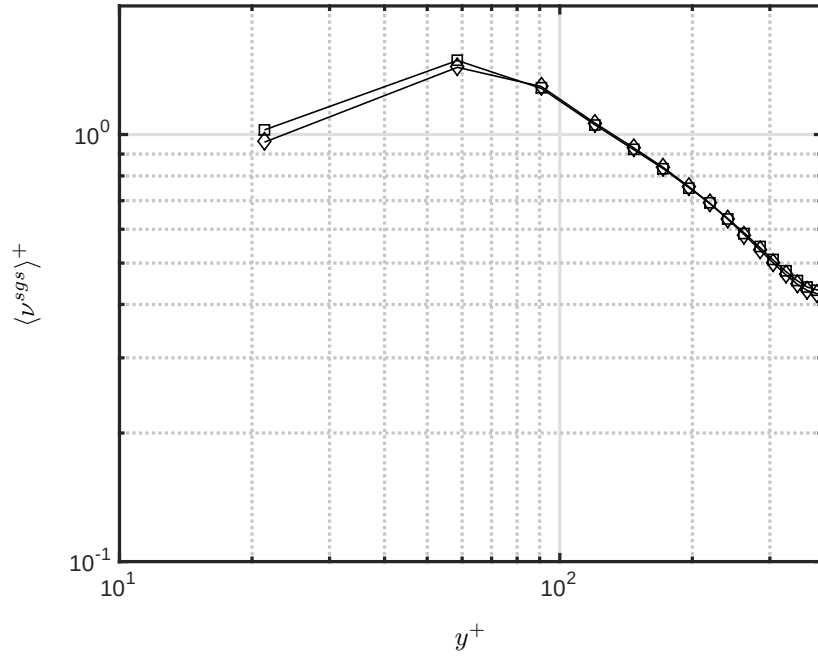


Figure 2.25: Channel flow at $Re_\tau = 395$. Mean SGS viscosity profile of the wall modeled LES: wall model computed at $y_1^+ = 21$ (diamonds), Wall model computed at $y_3^+ = 91$ (squares).

To implement wall shear stress boundary conditions in the BigFlow code, a new version of the code was developed, based on fourth order accurate finite differences and on flux divergence formulation of the momentum equations. The new version of the code was tested in DNS mode, on the Taylor-Green vortex at $Re = 1600$. The results obtained with the new version of the code were in good agreement with the results obtained with the energy-conserving version of the code, and with a spectral reference solution. The code was also tested in wall-resolved LES mode in a channel flow. When SGS models were used, the coefficients of the models needed to be tuned to work well in the new version of the code. The calibration was carried in a channel flow at $Re_\tau = 395$.

The wall shear stress boundary condition has been implemented and tested with success in the flux version of the BigFlow code. Numerical details on how the wall shear stress boundary condition was implemented were given. It was shown that several corrections were required in the vicinity of the wall. First, the order of accuracy of the scheme was reversed to second order at the wall, while the bulk of the flow is still computed with an order of accuracy of four. Second, to handle the transition of the scheme from order two to order four, a correction had to be introduced to conserve the global momentum numerically. This correction is such that one layer of points is only computed at an order of accuracy of one. Finally, a correction was introduced. The next section will attach to the description of the various wall modeling strategies implemented to compute the wall shear stress from flow information. Two main strategies are used. The first family of models is the algebraic wall models family, which is detailed in section 2.5. The second family of models is the family of RANS sublayer models, which is detailed in section 2.6. Both families of models are tested in a channel flow at moderate to high Reynolds number.

2.5 Algebraic wall models

Algebraic wall models aim at computing the wall shear stress directly from flow information using an algebraic formulation that can be either explicit or implicit. The original idea of Schumann [112] was to provide the model with

the mean wall shear stress u_τ^2 and to impose locally:

$$\tau_{w,xy} = \rho u_\tau^2 \frac{u}{\langle u \rangle} \quad (2.64)$$

$$\tau_{w,zy} = \mu \frac{w}{y} . \quad (2.65)$$

Instead of providing the mean wall shear stress, Grötzbach [47] proposed to compute it using $\langle u \rangle$ and a given law of the wall (LOTW) that is considered valid in the mean. From a chosen LOTW, $u^+ = f(y^+)$, and for a given averaged velocity $\langle u \rangle$, the mean friction velocity u_τ can be obtained by solving the nonlinear equation:

$$|\langle u \rangle| - u_\tau f\left(\frac{y u_\tau}{\nu}\right) = 0 , \quad (2.66)$$

averaged or locally averaged flow information is therefore required. The developments are based on a generic local averaging procedure $\langle \cdot \rangle$. Averaging strategies are discussed in Section 2.5.1. Although in theory, this formula is only accurate for mean flow quantities, it can be used locally (pointwise). Both pointwise and averaged models will be considered. Several laws can be used. The standard log law reads:

$$u^+ = \frac{1}{\kappa} \log(y^+) + C . \quad (2.67)$$

This LOTW is of course not valid in the transition layer or below. If we desire a formulation that is robust to fine grids and that can therefore accept points lying in the transition layer, we need to use a LOTW that describes the whole inner layer. Such a law was proposed by Spalding [122]:

$$u^+ = y^+ - \frac{1}{E} \left(e^{\kappa u^+} - 1 - \kappa u^+ - \frac{(\kappa u^+)^2}{2} - \frac{(\kappa u^+)^3}{6} - \frac{(\kappa u^+)^4}{24} \right) . \quad (2.68)$$

This implicit law is equivalent to the log law in the log layer, and connects it smoothly with the viscous sub-layer. An explicit LOTW was also proposed by Reichardt [108]:

$$u^+ = \frac{1}{\kappa} \log(1 + \kappa y^+) + \left(C - \frac{1}{\kappa} \log(\kappa) \right) \cdot \left(1 - \exp\left(-\frac{y^+}{11}\right) - \frac{y^+}{11} \exp\left(-0.33y^+\right) \right) . \quad (2.69)$$

These three laws are displayed in Fig. 2.26, using $\kappa = 0.41$ and $C = 5.0$. We see that both Spalding's and Reichardt's laws are acceptable. Neverthe-

less, Reichardt's LOTW exhibited better convergence properties when solving Eq. (2.66), and is therefore preferred.

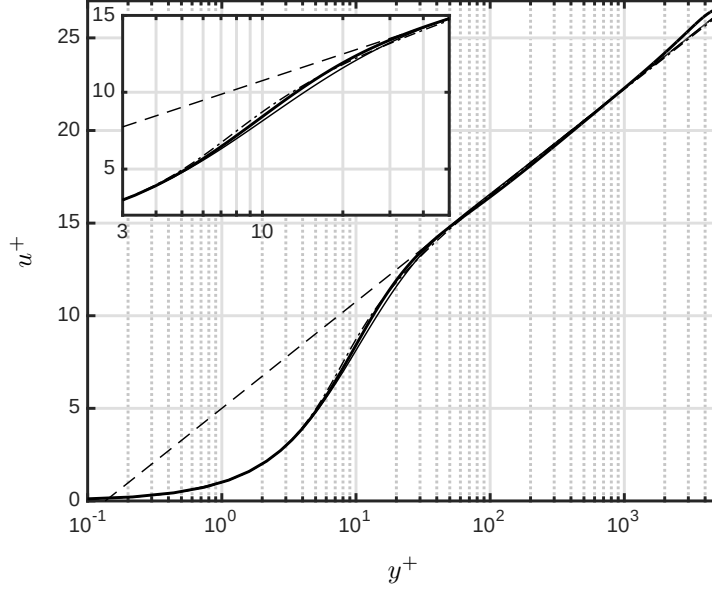


Figure 2.26: Laws of the wall, channel flow at $Re_\tau = 5200$. DNS of Lee and Moser [71] (thick solid), log law (dashed), Spalding's LOTW (dashed-dotted), and Reichardt's LOTW (thin solid).

Once a mean friction velocity is obtained, one must compute a local wall shear stress τ_w . One approach, similar to that of Schumann [112], consists in imposing the wall shear stress in the direction of the local velocity (at the point at which the wall model is evaluated) and with an intensity proportional to the velocity with respect to its local average:

$$\tau_w = \rho \frac{\mathbf{u}}{|\langle \mathbf{u} \rangle|} u_\tau^2, \quad (2.70)$$

This methodology is referred to as the linear algebraic wall model in what follows. The same principle can be applied quadratically (as done by Khanna and Brasseur [68]). The wall shear stress is then computed in the direction of the local velocity but the amplitude is computed based on the square of the velocity:

$$\tau_w = \rho \frac{\mathbf{u} |\mathbf{u}|}{|\langle \mathbf{u} |\mathbf{u}| \rangle|} u_\tau^2. \quad (2.71)$$

The model of Eq. (2.71) can be seen as modeling the inner layer of the TBL

using a drag coefficient C_D . The wall shear stress then reads $\boldsymbol{\tau}_w = \frac{1}{2}\rho C_D |\mathbf{u}| \mathbf{u}$, where the drag coefficient is obtained from mean flow information:

$$C_D = \frac{2u_\tau^2}{|\langle \mathbf{u} | \mathbf{u} | \rangle|} . \quad (2.72)$$

2.5.1 Averaging procedures

The averaging procedure $\langle \cdot \rangle$ can represent many things. In the case of the channel flow, two homogeneous directions exist, stream x and span z direction. Time is also homogeneous in that case. The averaging can be global in the sense that averaging is performed over all homogeneous directions, or local if it is performed only in the spanwise direction or using the closest $n_x \times n_z$ neighbors only. Local averaging can also be performed over a non-homogeneous direction, provided that the averaged field is sufficiently slowly varying with respect to the grid size in that direction. The time averaging is performed using an exponential filtering strategy, so as to avoid having to keep a large amount of data for the averaging procedure. The filter is as follows:

$$\langle u \rangle(t + \Delta t) = (1 - c_{\text{exp}}) \langle u \rangle(t) + c_{\text{exp}} u(t + \Delta t) , \quad (2.73)$$

with $0 < c_{\text{exp}} \leq 1$. The closer c_{exp} is to zero, the more the averaging. The coefficient c_{exp} is tied to the cutoff frequency of the exponential filter f_c at which the modal amplitude is halved. Cahuzac et al. [19] showed that assuming $f_c \Delta t \ll 1$ we get:

$$c_{\text{exp}} \simeq 3.63 f_c \Delta t . \quad (2.74)$$

2.5.2 Parameter study

A turbulent channel flow at $Re_\tau = 5200$ is considered. The DNS data of Lee and Moser [71] is used as reference. The various strategies described above are tested and assessed. The baseline case is a channel of size $L_x \times L_y \times L_z = 2\pi h \times 2h \times \pi h$, with a coarse homogeneous grid of size

$$N_x = 58, \quad N_y = 32, \quad N_z = 48 , \quad (2.75)$$

which yields a viscous resolution of

$$\Delta x^+ = 562, \quad \Delta y^+ = 324, \quad \Delta z^+ = 339 . \quad (2.76)$$

The SGS model used is here the Smagorinsky model:

$$\nu^{sgs} = C \Delta^2 \left(S_{ij} S_{ij} \right)^{1/2}, \quad (2.77)$$

with $C = 0.04$ (note this value is larger than the theoretical value of $C = 0.027$), and using $\Delta^2 = (\Delta_x \Delta_y \Delta_z)^{2/3}$. The wall models with the quadratic and the linear approaches are used, both with Reichardt's LOTW, and using averaging over streamwise and spanwise directions (no time averaging). Fig. 2.27 shows the mean velocity profiles obtained by the two simulations. We see that the two approaches give very similar results and that the WMLES is mostly insensitive to whether the wall shear stress is imposed with the linear or the quadratic approach. This is also true when considering the velocity fluctuations, as shown in Fig. 2.28, as well as the shear stress profiles, see Fig. 2.29. The velocity profiles obtained with the wall-modeled LES, on a very coarse mesh, are in very good agreement with the DNS data, as well as the velocity fluctuations, even though the total kinetic energy is underestimated. As expected from a Smagorinsky model, some of the mean turbulent shear stress is carried by the SGS term, as seen in Fig. 2.29. We also see that some oscillations are visible in the turbulent shear stress close to the wall: this is the LES reacting to the wall model. Fig. 2.30 shows the mean SGS viscosity profiles. Again, we see no difference between the two simulations. As no part of the inner layer is resolved, no y^3 scaling is observed. Since no difference can be observed between the linear or quadratic approach, only the *drag coefficient* quadratic approach is retained for the following.

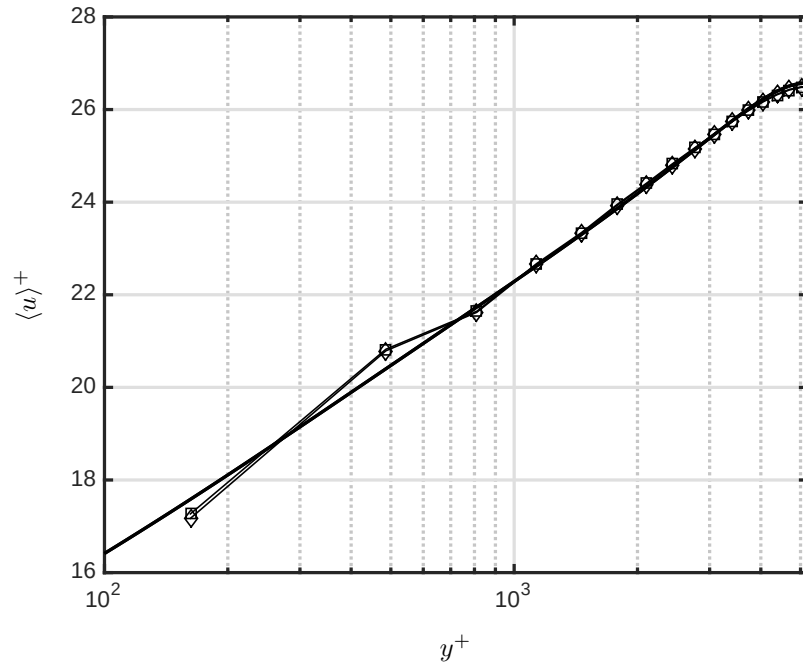


Figure 2.27: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES baseline (thin solid and markers). Quadratic wall model (diamonds) and linear wall model (squares).

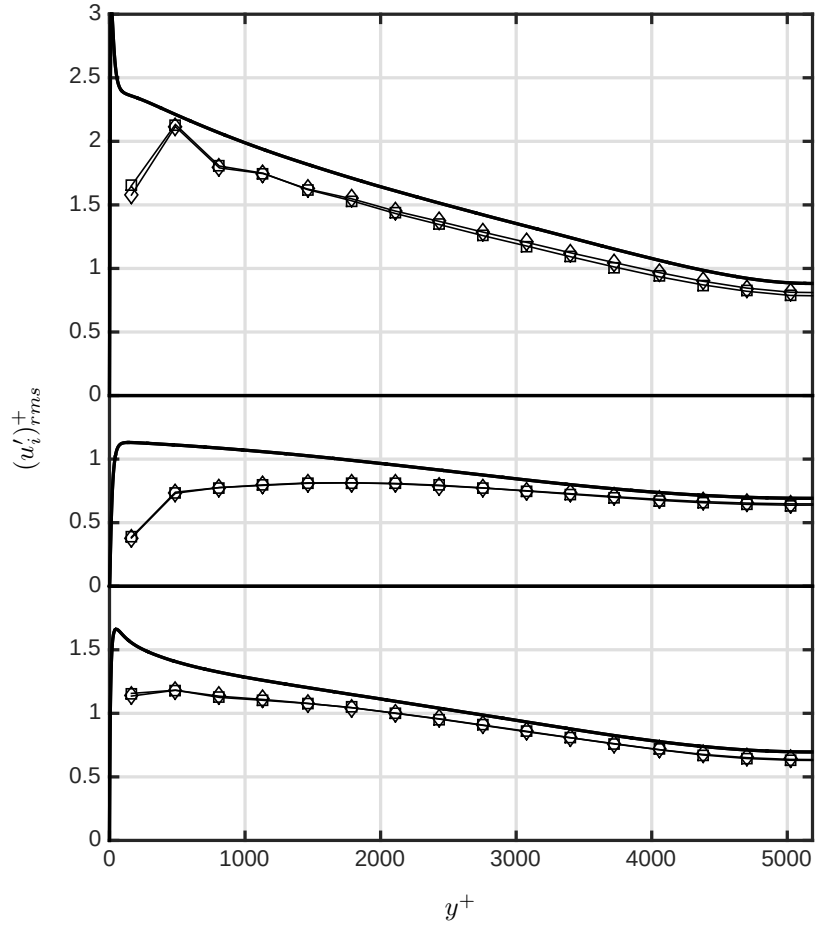


Figure 2.28: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation profile:** DNS of Lee and Moser [71] (thick solid), and wall-modeled LES baseline (thin solid and markers). Quadratic wall model (diamonds) and linear wall model (squares).

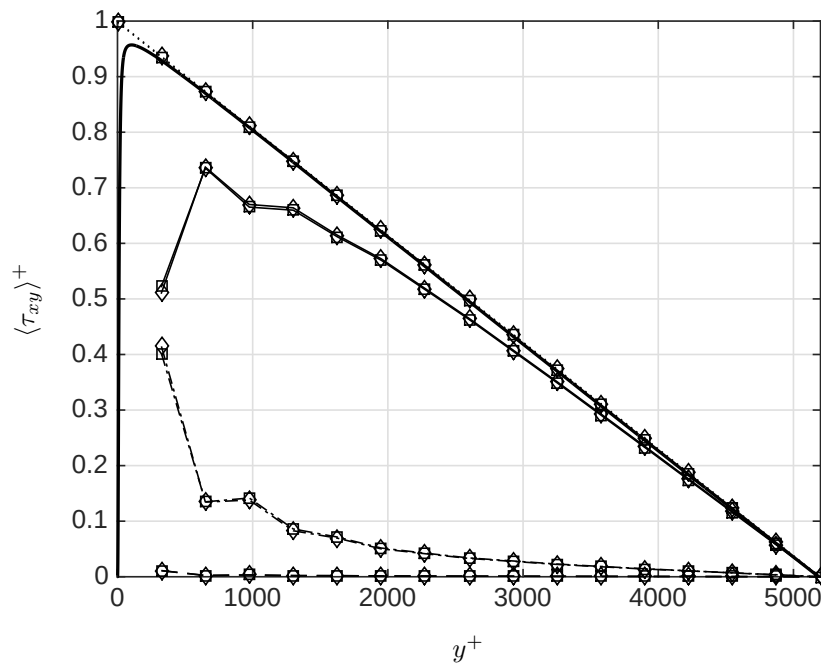


Figure 2.29: Channel flow at $Re_\tau = 5200$. **Mean shear stress profiles:** total stress (dotted), SGS model stress of the LES baseline (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), resolved viscous stress of the LES (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). **Quadratic wall model (diamonds) and linear wall model (squares).**

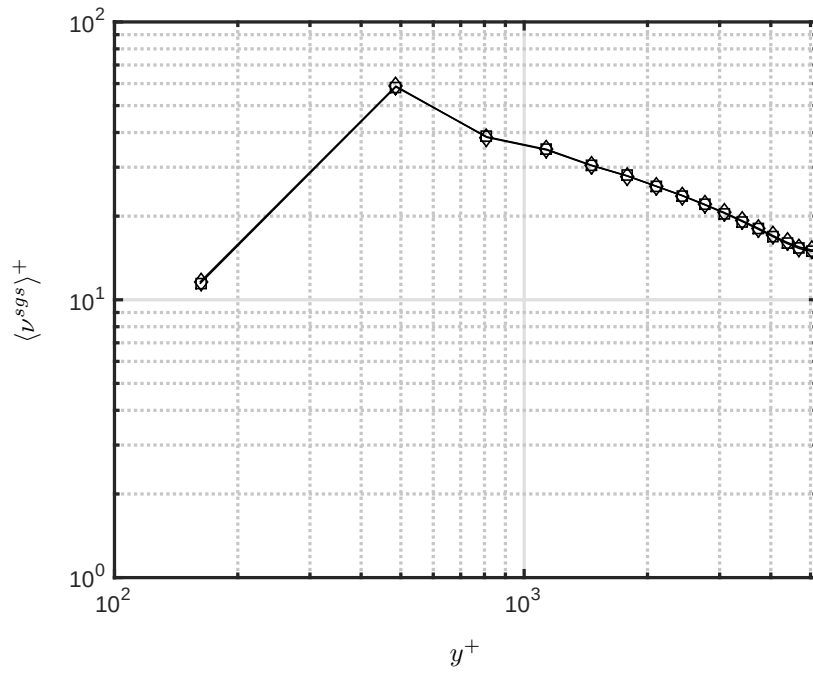


Figure 2.30: Channel flow at $Re_\tau = 5200$. Mean SGS viscosity profile of the wall modeled LES baseline. Quadratic wall model (diamonds) and linear wall model (squares).

The effect of a given mesh and resolution is a non-trivial question when considering wall modeling, as pointed out by Larsson et al. [69]. We here attempt at assessing the effect of mesh refinement and cell aspect ratio, using the same numerics as our baseline case and changing the mesh. Three cases are considered and compared to the baseline case. The meshes are described in Table. 2.1: case 0 is the baseline described above. Case 1 has the same cell aspect ratio as the baseline but with a twice finer resolution. Case 3 is x -refined, such that the mesh is quasi isotropic. Case 2 has the same aspect ratio as the baseline in the Oxy plane, but is isotropic in the wall parallel Oxz plane.

Case	$N_x \times N_y \times N_z$	$\Delta x / \Delta z$	$\Delta x / \Delta y$
0	$58 \times 32 \times 48$	1.66	1.7
1	$116 \times 64 \times 96$	1.66	1.7
2	$116 \times 64 \times 58$	1.0	1.66
3	$192 \times 64 \times 96$	1.05	1.05

Table 2.1: Meshes investigated for WMLES of channel flow at $Re_\tau = 5200$, with quadratic wall model and Reichardt's LOTW.

Surprisingly, the mean velocity profiles of the refined meshes are not as good as that of the coarser baseline, as seen in Fig. 2.31. All cases exhibit a slight overestimation of the global mass flow rate. They still lie within 2% of the DNS curve, however. This is quantified by monitoring the mass flow rate, as seen in Table 2.2. Even though the overall variation of the profiles of the three cases is rather small, the cell aspect ratio seem to play a role in the LES behavior. This can also be seen in Fig. 2.32, where the velocity fluctuations are displayed. Again, the range of variation is not enormous, but we see that case 2 develops more streamwise fluctuations and less wall-normal and spanwise fluctuations than the other cases. The shear stresses are shown in Fig. 2.33. Note that in that figure, the values are computed from interpolated fields at the center of the cells. Because of the wall-modeling framework (resulting in very coarse cells), this interpolation is rather inaccurate close to the wall. In this case, though the actual stresses (which lie on the edges of the cell, see Fig. 2.2) do sum to the expected $1 - y/h$, the interpolated values do not (at the first few points). This was not the case in previous plots. From now on, unless stated otherwise, shear stresses will always be interpolated at the cell center, and the very first points will therefore be inaccurate. The effect of the

mesh can also be seen in Fig. 2.34, which shows the SGS viscosity profiles of the three runs. The difference in SGS viscosity between the cases is due to the effect of the effective cell size Δ , which is impacted by the cell aspect ratio. Case 3 is the most refined, hence with the smallest SGS viscosity, and case 2 is the coarsest, hence with the largest SGS viscosity. This also impacts the shear-stress profiles of Fig. 2.33, where the mean contribution of the SGS model is more important when more SGS viscosity is computed. This also means that, in these coarse simulations, the LES will develop as many fluctuations as the mesh allows, and, in consequence, the topology of the fluctuations will depend on the cell size and aspect ratio. This is exemplified by Fig. 2.35 to Fig. 2.37, showing the turbulent structures developed by each simulation. The isotropic mesh of case 3 develops the most isotropic turbulence, still with some streamwise-coherent structures. Case 1 (coarser in the streamwise direction) has more streamwise-coherent eddies, and less isotropic structures. Case 2 (with $\Delta x = \Delta z = 1.66\Delta y$) has few isotropic structures and mostly consists of streamwise eddies. Overall, these topological differences have a small impact on the velocity profiles (at least with the SGS model here used, which is the simple Smagorinsky model).

Case	Re_b	Relative error (in %)
0	$1.253 \cdot 10^5$	0.2
1	$1.265 \cdot 10^5$	1.2
2	$1.271 \cdot 10^5$	1.7
3	$1.264 \cdot 10^5$	1.1

Table 2.2: Bulk Reynolds number and error with respect to the DNS of Lee and Moser [71] ($Re_b = 1.25 \cdot 10^5$) for the meshes of Table 2.1.

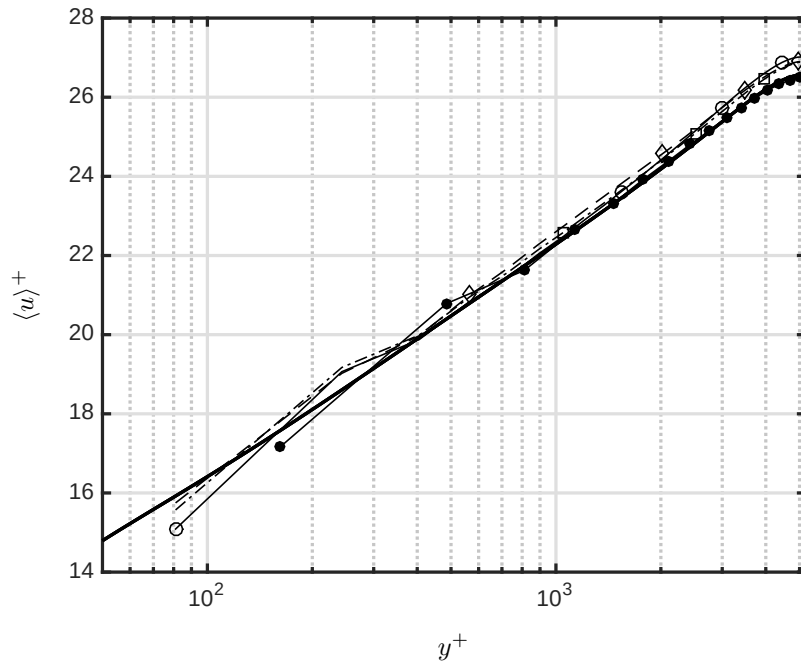


Figure 2.31: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). **Mesh 0** ($N_x \times N_y \times N_z = 58 \times 64 \times 48$, thin solid and bullets), **mesh 1** ($N_x \times N_y \times N_z = 116 \times 64 \times 96$, dashed-dotted and squares), **mesh 2** ($N_x \times N_y \times N_z = 116 \times 64 \times 58$, dashed and diamonds), and **mesh 3** ($N_x \times N_y \times N_z = 192 \times 64 \times 96$, solid and circles). The markers do not show all grid points for the sake of readability.

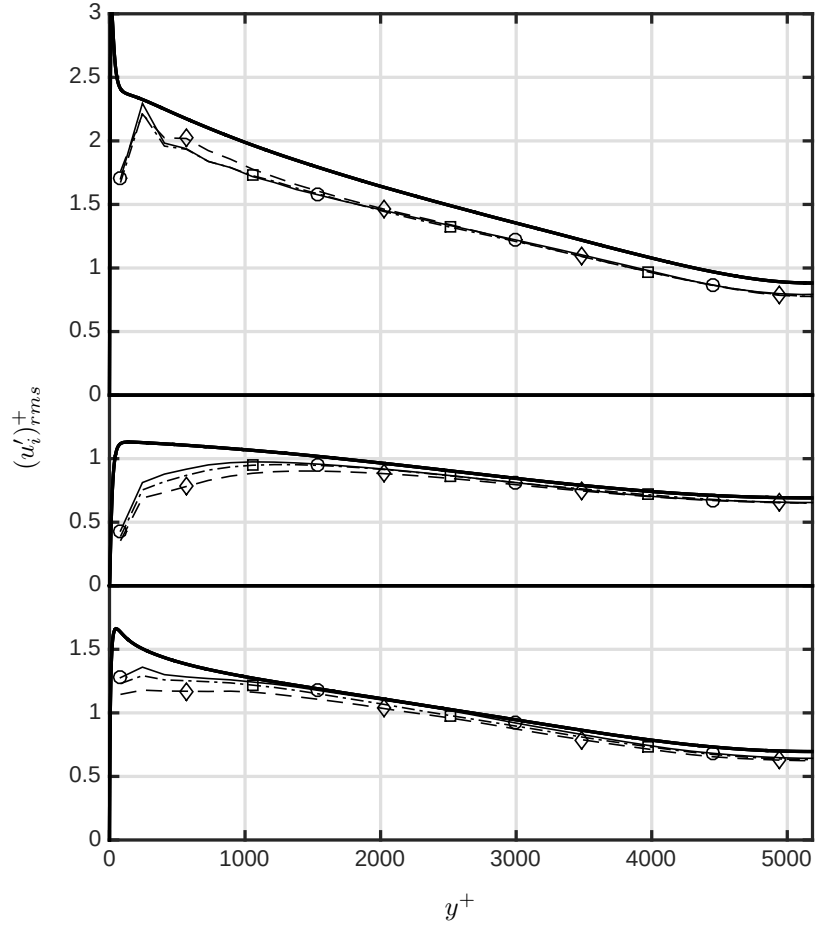


Figure 2.32: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). **Mesh 1** ($N_x \times N_y \times N_z = 116 \times 64 \times 96$, dashed-dotted and squares), **mesh 2** ($N_x \times N_y \times N_z = 116 \times 64 \times 58$, dashed and diamonds), and **mesh 3** ($N_x \times N_y \times N_z = 192 \times 64 \times 96$, solid and circles). The markers do not show all grid points for the sake of readability.

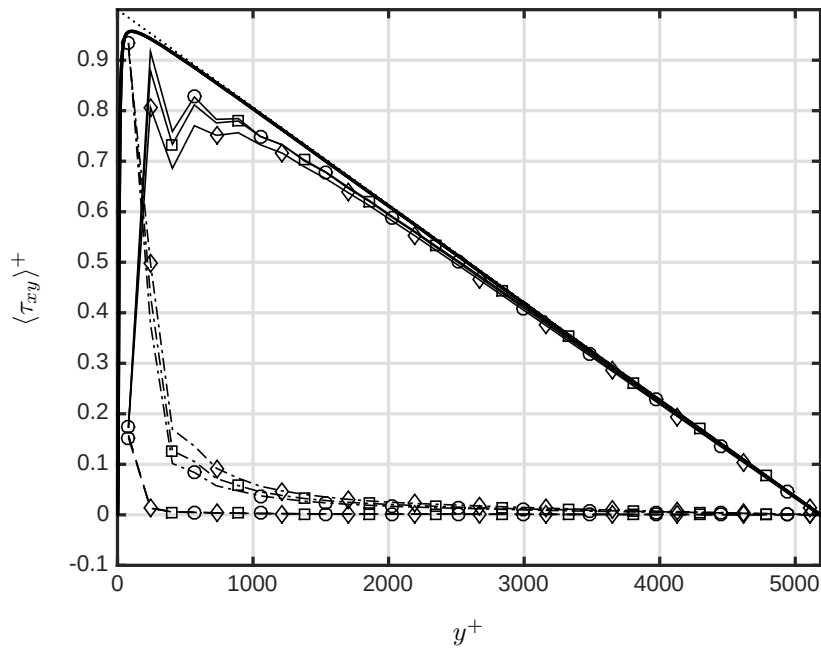


Figure 2.33: Channel flow at $Re_\tau = 5200$. **Mean shear stress** profiles: total stress (dotted), SGS model stress of the LES (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), resolved viscous stress of the LES (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). **Mesh 1** ($N_x \times N_y \times N_z = 116 \times 64 \times 96$, squares), mesh 2 ($N_x \times N_y \times N_z = 116 \times 64 \times 58$, diamonds), and mesh 3 ($N_x \times N_y \times N_z = 192 \times 64 \times 96$, circles). The markers do not show all grid points for the sake of readability.

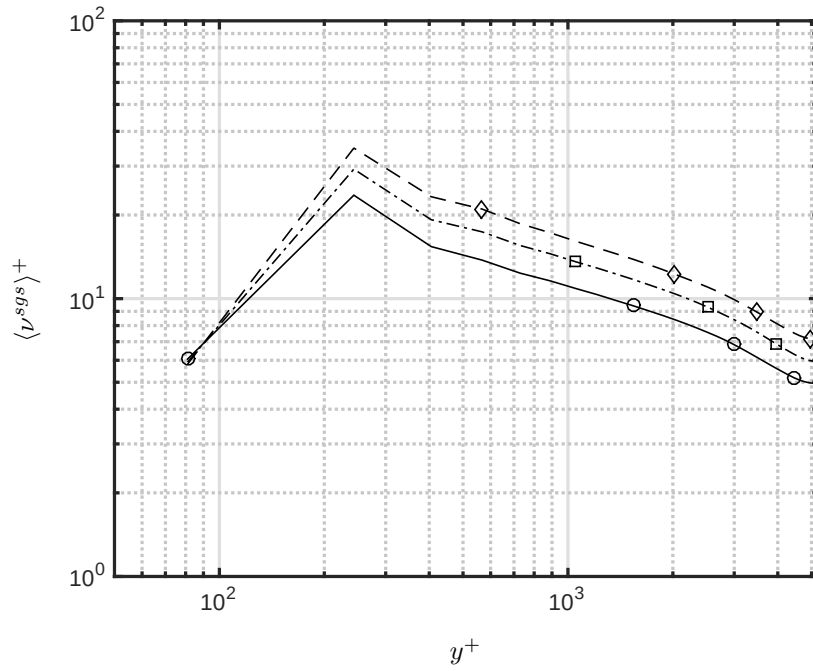


Figure 2.34: Channel flow at $Re_\tau = 5200$. *Mean SGS viscosity* profile of the wall modeled LES. **Mesh 1** ($N_x \times N_y \times N_z = 116 \times 64 \times 96$, dashed-dotted and squares), mesh 2 ($N_x \times N_y \times N_z = 116 \times 64 \times 58$, dashed and diamonds), and mesh 3 ($N_x \times N_y \times N_z = 192 \times 64 \times 96$, solid and circles). The markers do not show all grid points for the sake of readability.

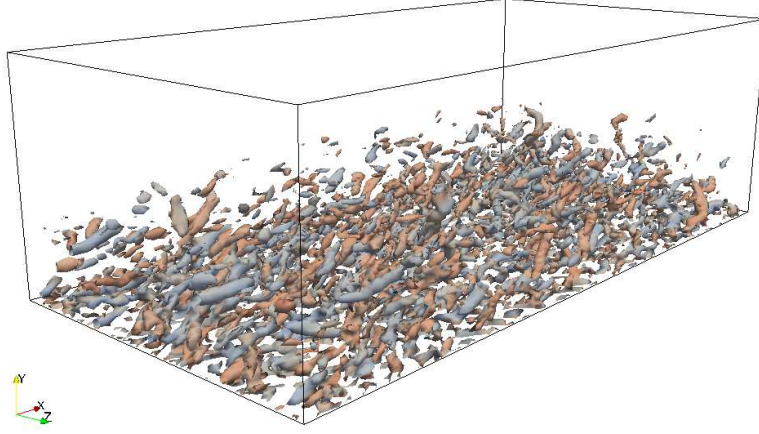


Figure 2.35: Channel flow at $Re_\tau = 5200$. Isosurfaces of $\lambda_2 = -2 \cdot 10^{-5}$ of *case 1* ($N_x \times N_y \times N_z = 116 \times 64 \times 96$), colored by streamwise vorticity.

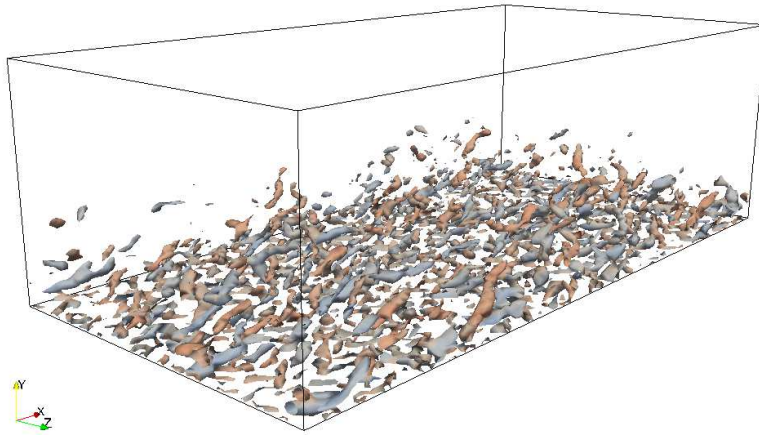


Figure 2.36: Channel flow at $Re_\tau = 5200$. Isosurfaces of $\lambda_2 = -2 \cdot 10^{-5}$ of *case 2* ($N_x \times N_y \times N_z = 116 \times 64 \times 58$), colored by streamwise vorticity.

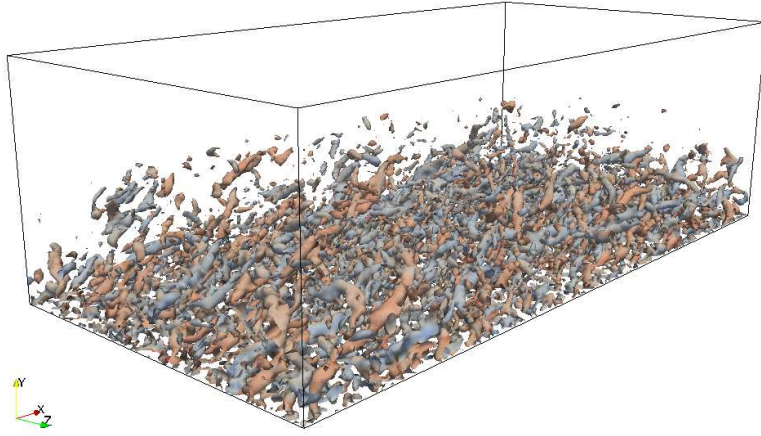


Figure 2.37: Channel flow at $Re_\tau = 5200$. Isosurfaces of $\lambda_2 = -2 \cdot 10^{-5}$ of *case 3* ($N_x \times N_y \times N_z = 192 \times 64 \times 96$), colored by streamwise vorticity.

The effect of the SGS model used is then investigated. The results obtained with the WALE model (with $C = 0.15$, as calibrated in the wall-resolved simulation of section 2.3.2) are shown in Fig. 2.38 to Fig. 2.41, and compared to the baseline, hence using the Smagorinsky model. The results in terms of bulk velocity are shown in Table 2.3. The velocity profiles are in good agreement with the DNS, but fewer velocity fluctuations are produced than when the Smagorinsky model is used. The trends of the curves of velocity fluctuations still compare well with the DNS curves. Though this model is designed for wall-resolved LES, the results obtained in a wall-modeled LES framework are quite good. Fig. 2.40 compares the shear stresses obtained with both models. We see that less shear stress is carried by the SGS term when using the WALE model than when using the Smagorinsky model. This results in the turbulent shear stress of the wall modeled LES with the WALE model being almost equal to the turbulent shear stress of the DNS. The turbulent shear stress profile is also smoother, as no spurious oscillations are observed beyond the third grid point, while some subsists when the Smagorinsky model is used.

SGS model	Re_b	Relative error (in %)
Smagorinsky	$1.253 \cdot 10^5$	0.2
WALE	$1.256 \cdot 10^5$	0.5
RVMs-Smagorinsky	$1.199 \cdot 10^5$	-4.1
RVMs-WALE	$1.233 \cdot 10^5$	-1.3
filtered Smagorinsky	$1.262 \cdot 10^5$	0.9
filtered WALE	$1.259 \cdot 10^5$	0.7

Table 2.3: Bulk Reynolds number and error with respect to the DNS of Lee and Moser [71] ($Re_b = 1.25 \cdot 10^5$) for the SGS models investigated.

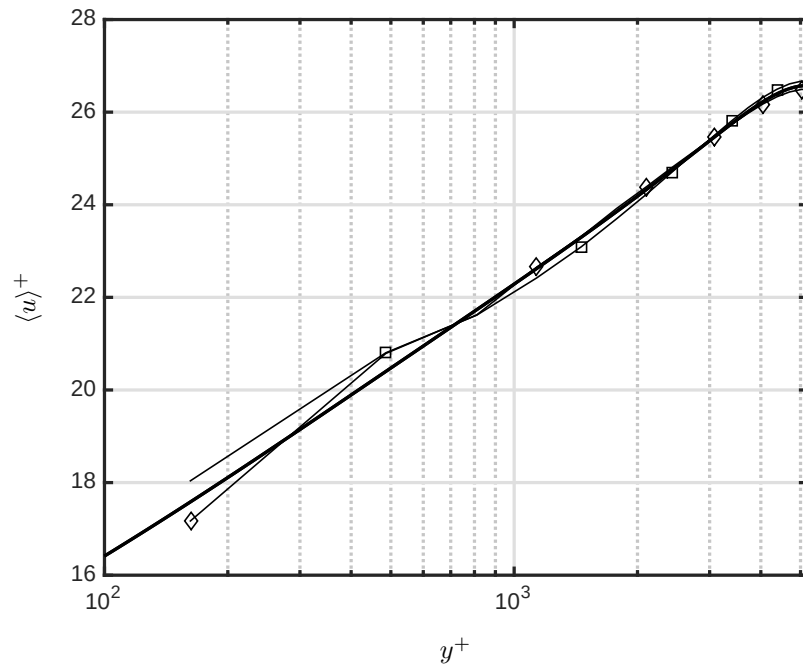


Figure 2.38: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). **Smagorinsky model (diamonds), and WALE model (squares)**. The markers do not show all grid points for the sake of readability.

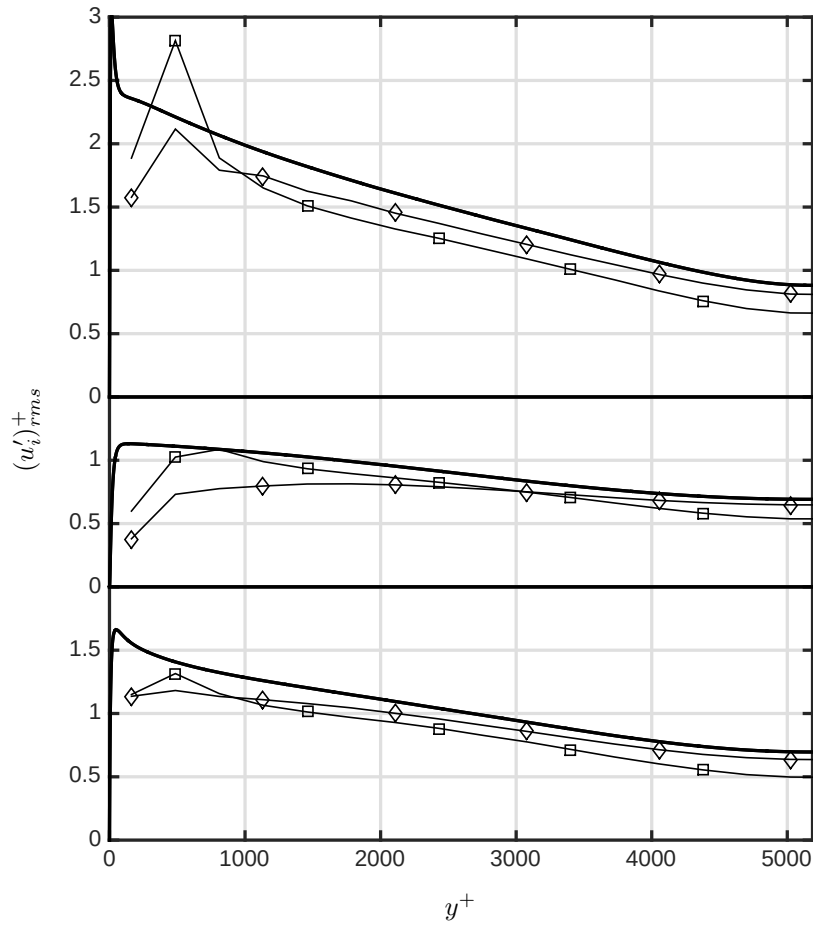


Figure 2.39: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation profile:** DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). **Smagorinsky model (diamonds), and WALE model (squares).** The markers do not show all grid points for the sake of readability.

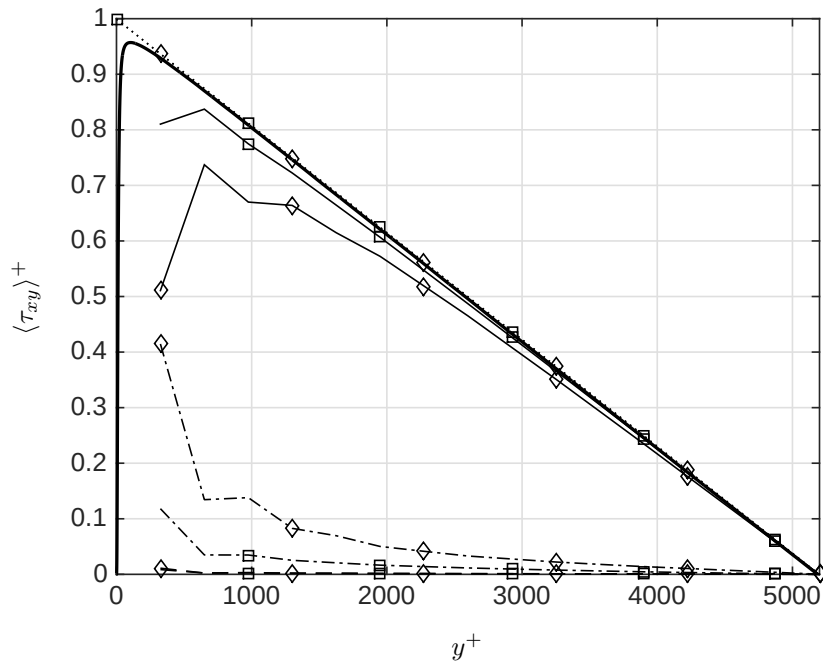


Figure 2.40: Channel flow at $Re_\tau = 5200$. **Mean shear stress** profiles: total stress (dotted), SGS model stress of the LES (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), resolved viscous stress of the LES (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). **Smagorinsky model (diamonds)**, and **WALE model (squares)**. The markers do not show all grid points for the sake of readability. The values are not interpolated at the cell center.

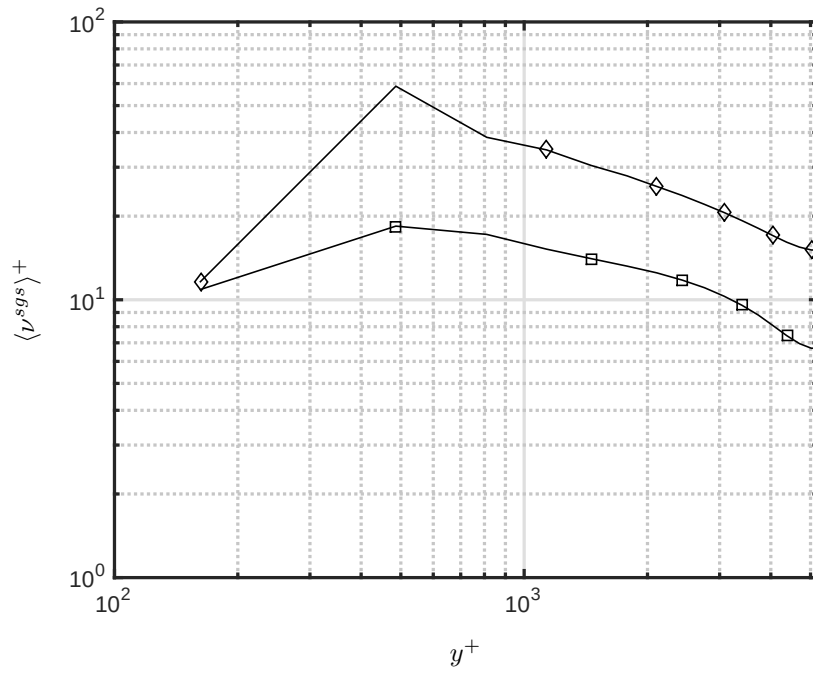


Figure 2.41: Channel flow at $Re_\tau = 5200$. Mean SGS viscosity profile of the wall modeled LES. Smagorinsky model (diamonds), and WALE model (squares). The markers do not show all grid points for the sake of readability.

The wall modeled LES solution seems to be resilient to a modification of the SGS model, since good results were obtained both with the Smagorinsky and with the WALE model. When RVMs models are used, however, the structural changes in the construction of the SGS stress tensor raise a series of issues. First, the SGS viscosity correction of Eq. (2.60) has to be modified to account for the modified structure of the SGS stress tensor. Eq. (2.58) becomes:

$$-\langle u'v' \rangle + \nu \frac{\partial \langle u \rangle}{\partial y} + \langle \nu^{sgs} \rangle \frac{\partial \langle u^s \rangle}{\partial y} = \langle \tau_w \rangle + \frac{d\langle P \rangle}{dx} y, \quad (2.78)$$

and the target viscosity obtained by equating the momentum budget at the first and second layers becomes:

$$\begin{aligned} \langle \nu^{sgs} \rangle_1^{target} = & \frac{1}{\frac{\partial \langle u^s \rangle}{\partial y} \Big|_1} \left(\nu \frac{\partial \langle u \rangle}{\partial y} \Big|_2 + \langle \nu^{sgs} \rangle_2 \frac{\partial \langle u^s \rangle}{\partial y} \Big|_2 - \nu \frac{\partial \langle u \rangle}{\partial y} \Big|_1 \right. \\ & \left. - (\langle u'v' \rangle_2 - \langle u'v' \rangle_1) + \left(-\frac{d\langle P \rangle}{dx} \right) (y_2 - y_1) \right). \end{aligned} \quad (2.79)$$

Second, even with this correction, the interaction of the filters used to compute the small scales with the wall model is non-trivial. The three-point filter used to compute the small scales fetches information in the ghost cells of velocity, which are extrapolated from interior point and do not contain any physically sound information. The small scales computed at the first layers of points are therefore unphysical and unreliable. Since the viscosity of the first layer of points is corrected, the impact on the SGS viscosity is limited. But as RVMs model apply this SGS viscosity to the strain rate tensor of the small scales, the final SGS stress model is impacted by the spurious interaction between the wall model and the small scales filter. In the end, the unphysical content of the first layers of points can pollute the whole simulation. Fig. 2.42 shows the mean velocity profiles obtained using RVMs-Smagorinsky model (with $C = 0.06$) and the RVMs-WALE model (with $C = 0.35$). While the wall model “does the job” of correctly imposing the third layer of points on the DNS curve, the WMLES then fails at predicting the correct outer layer of the velocity profile. The velocity fluctuations of Fig. 2.43, however, come close to the DNS profiles. The issue of the interaction of the SGS model with the wall model is visible on the mean shear stress profiles shown in Fig. 2.44: the prediction of the SGS shear stress is completely off at the first layer of points. The correction of the first layer of points is here insufficient to suppress the spurious interaction of

the filter and the wall model, which propagates to the bulk of the flow, mainly through the action of the SGS term, which uses S_{ij}^s instead of S_{ij} .

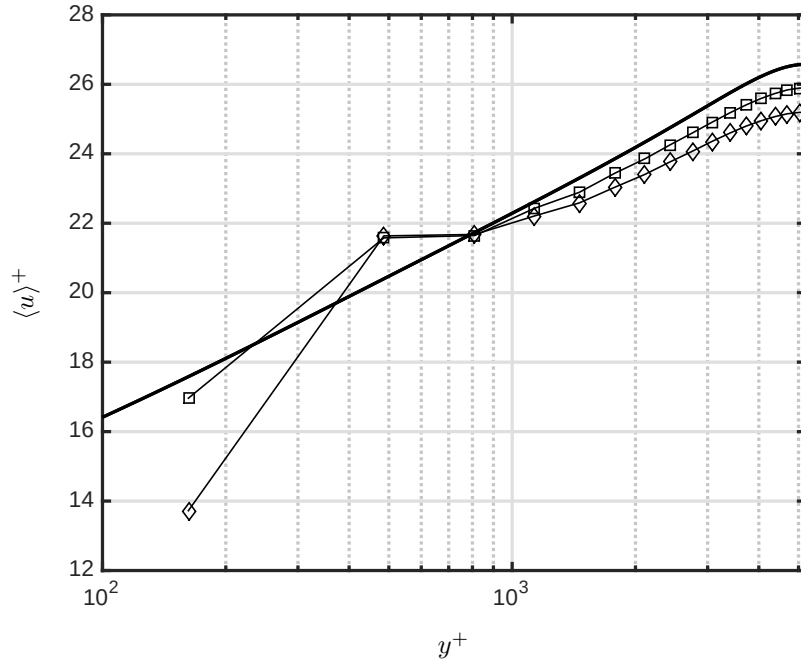


Figure 2.42: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). **RVMs-Smagorinsky** model (diamonds), and **RVMs-WALE** model (squares).

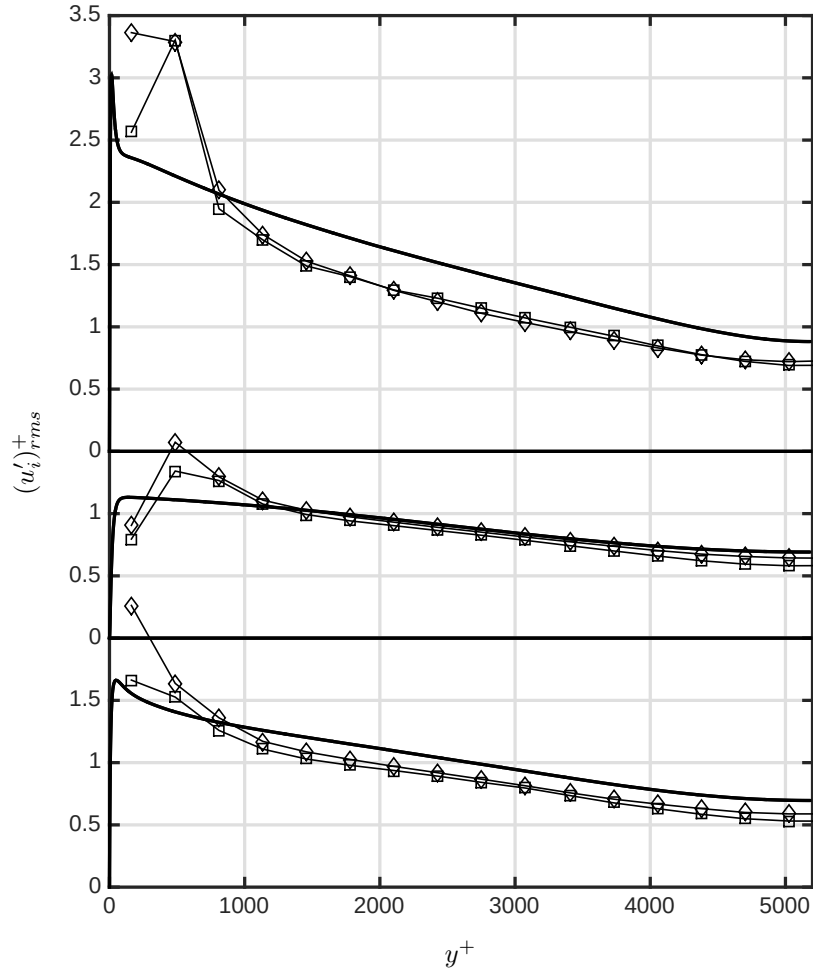


Figure 2.43: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation profile:** DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). RVMs-Smagorinsky model (diamonds), and RVMs-WALE model (squares).

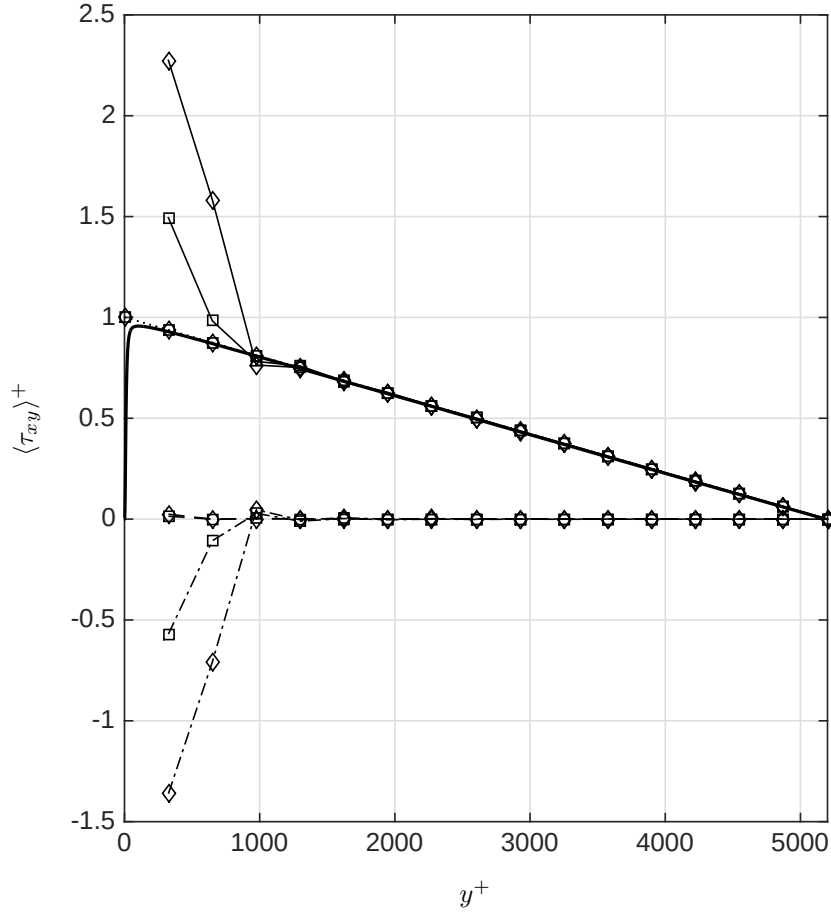


Figure 2.44: Channel flow at $Re_\tau = 5200$. **Mean shear stress** profiles: total stress (dotted), SGS model stress of the LES (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), resolved viscous stress of the LES (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). **RVMs-Smagorinsky model (diamonds), and RVMs-WALE model (squares).** The values are not interpolated at the cell center.

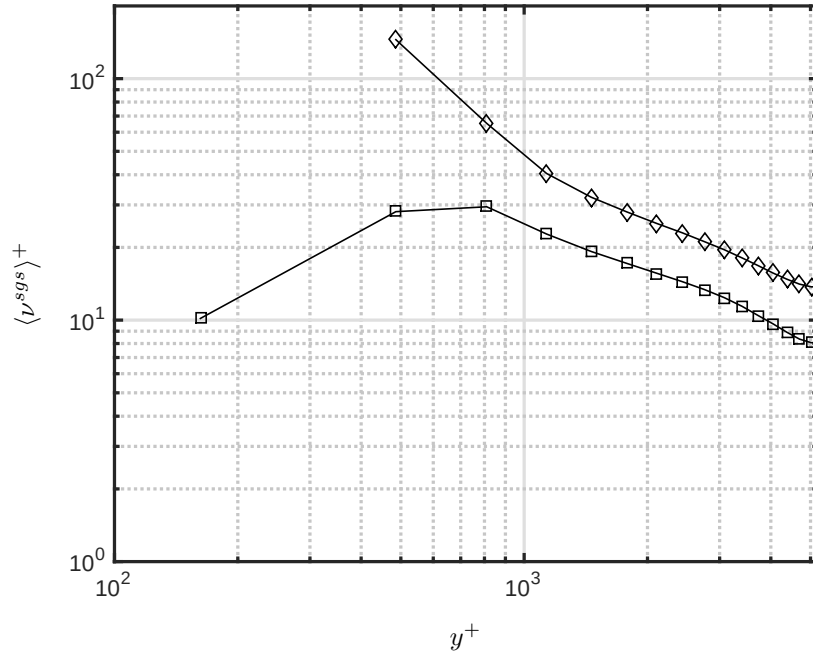


Figure 2.45: Channel flow at $Re_\tau = 5200$. Mean SGS viscosity profile of the wall modeled LES. RVMs-Smagorinsky model (diamonds), and RVMs-WALE model (squares).

The interaction between the filters of the RVMs models and the wall models was seen to result in spurious oscillations. A solution is to employ filtered models, which still use the small scales to compute the SGS viscosity, but apply it to the full LES field strain rate tensor S_{ij} . Fig. 2.46 to Fig. 2.49 show the results obtained with the filtered Smagorinsky model (with $C = 0.05$) and with the filtered WALE model (with $C = 0.25$). In Fig. 2.48, we see that the use of small scales in the computation of the SGS viscosity $\nu^{sgs}(\mathbf{u}^s)$ has an impact in the turbulent shear stress near the wall. Contrarily to the RVMs models which propagated this poor behavior to the bulk of the flow (see Fig. 2.44), the SGS shear stress behaves properly above the first few layers of points. This results in velocity profiles in good agreement with the DNS data, see Fig. 2.46. The velocity fluctuation profiles, shown in Fig. 2.47 again exhibit good overall trends, though they all lie below the levels of the DNS, as did the results with the other models, in Fig. 2.39. The SGS viscosity profiles, however, show that some spurious interactions between the filter and the wall model still exist, as the ν^{sgs} is computed using the small scales. This results in the kink we observe at the second layer of points of the filtered Smagorinsky model, in Fig. 2.49.

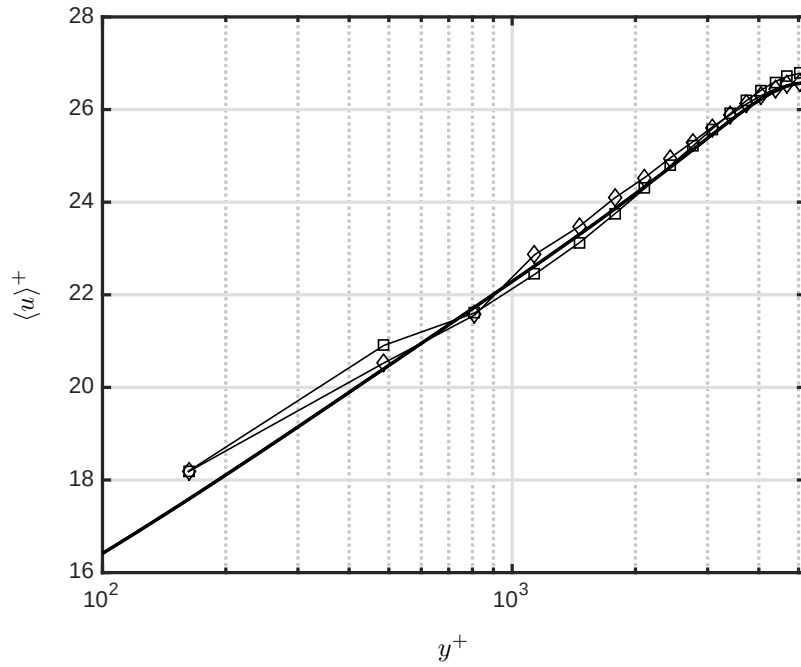


Figure 2.46: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). **filtered Smagorinsky** model (diamonds), and **filtered WALE** model (squares).

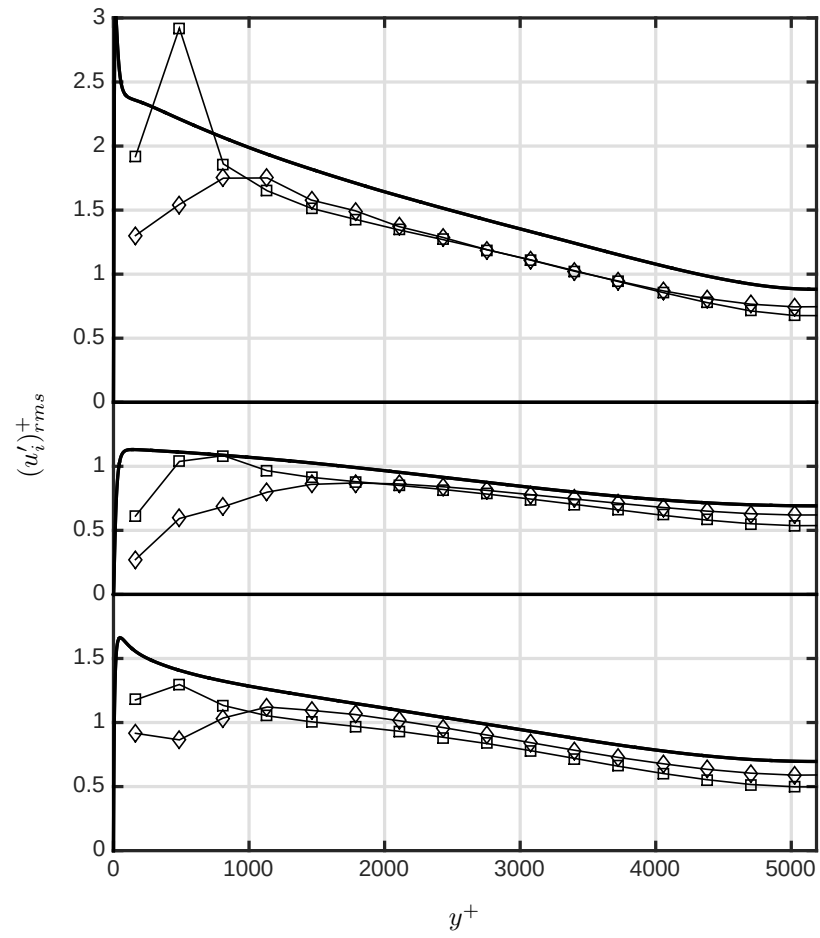


Figure 2.47: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin lines and markers). filtered Smagorinsky model (diamonds), and filtered WALE model (squares).

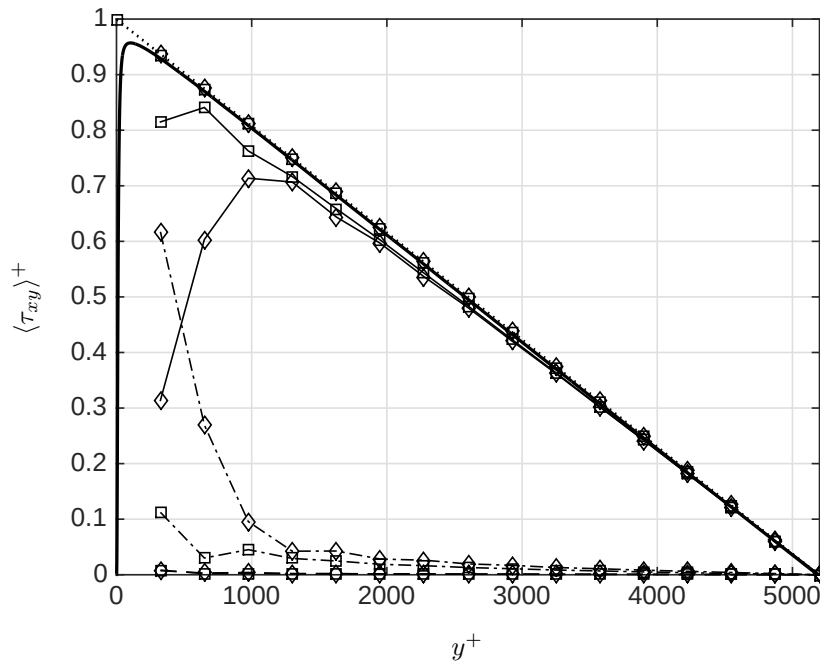


Figure 2.48: Channel flow at $Re_\tau = 5200$. **Mean shear stress profiles:** total stress (dotted), SGS model stress of the LES (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), resolved viscous stress of the LES (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). **filtered Smagorinsky model (diamonds), and filtered WALE model (squares).** The values are not interpolated at the cell center.

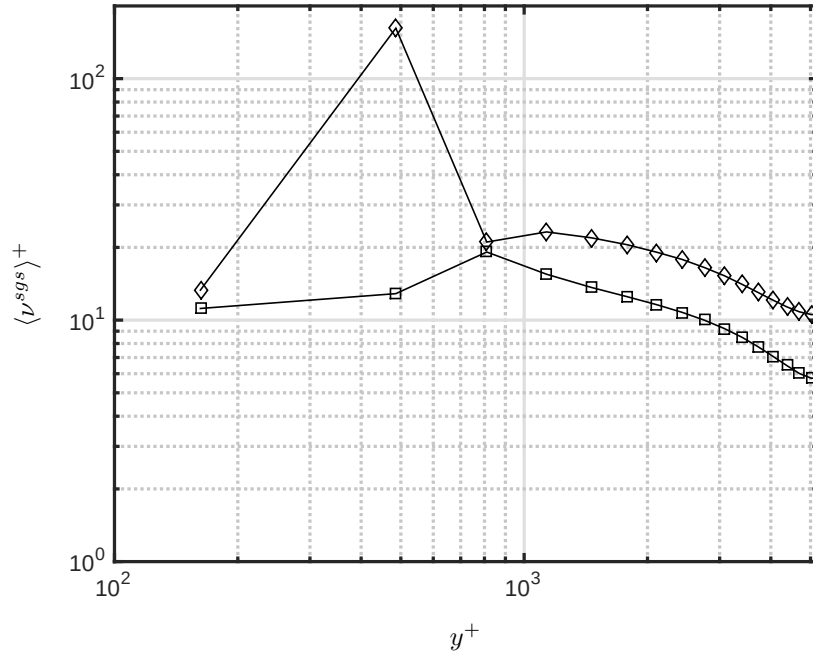


Figure 2.49: Channel flow at $Re_\tau = 5200$. Mean SGS viscosity profile of the wall modeled LES, filtered Smagorinsky model (diamonds), and filtered WALE model (squares).

Finally, we assess the effect of the type of averaging used to compute the wall models (as described in section 2.5.1). The baseline (with averaging over all homogeneous direction x and z , and no time averaging) is used as reference. A simulation is performed using averaging over the $n_x \times n_z = 5 \times 5$ closest points and no time averaging, as well as as a simulation with averaging over time only. A simulation is also performed with the wall models imposed pointwise and instantaneously, hence without averaging. The resulting bulk velocities are given in Table 2.4. The results are displayed in Fig. 2.50 to Fig. 2.53. We see that the averaging procedure has, in this case, little (if any) impact on the results and that even as few as 5×5 points were sufficient to recover the correct mean velocity profile. The velocity fluctuations are also mostly unaffected by the change in averaging strategy. The use of a pointwise wall model has underpredicted the velocity profile by a very small amount, which is barely visible in Fig. 2.50. It however remains as accurate as the other strategies in terms of mass flow rate, as seen in Table 2.4. The velocity fluctuations, shear stress profiles and SGS viscosity profiles are also almost identical, even when the wall model is applied pointwise. Even though the pointwise application of a law of the wall is not theoretically correct, it did not deteriorate the results, and pointwise models were seen to perform with an equal accuracy as averaged models. The need for averaging (or rather the absence of such a need as it is here observed) should also be investigated in a non homogeneous flow. Such a case is investigated in Chapter 3.

$N_x \times N_z$	c_{exp}	Re_b	Relative error (in %)
1×48	1.0	$1.253 \cdot 10^5$	0.2
5×5	1.0	$1.251 \cdot 10^5$	0.1
1×1	0.01	$1.251 \cdot 10^5$	0.1
1×1	1.0	$1.247 \cdot 10^5$	-0.2

Table 2.4: Bulk Reynolds number and error with respect to the DNS of Lee and Moser [71] ($Re_b = 1.25 \cdot 10^5$) for the averaging strategies investigated.

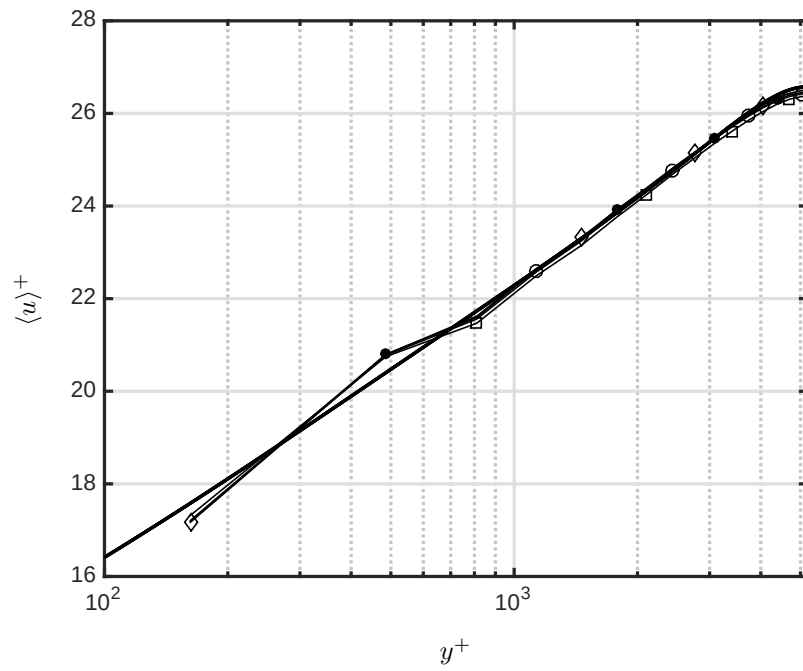


Figure 2.50: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin solid and markers). **Averaging** over x and z (diamonds), averaging over 5×5 points (bullets), averaging over time (circles), and pointwise wall model (squares). The markers do not show all grid points for the sake of readability.

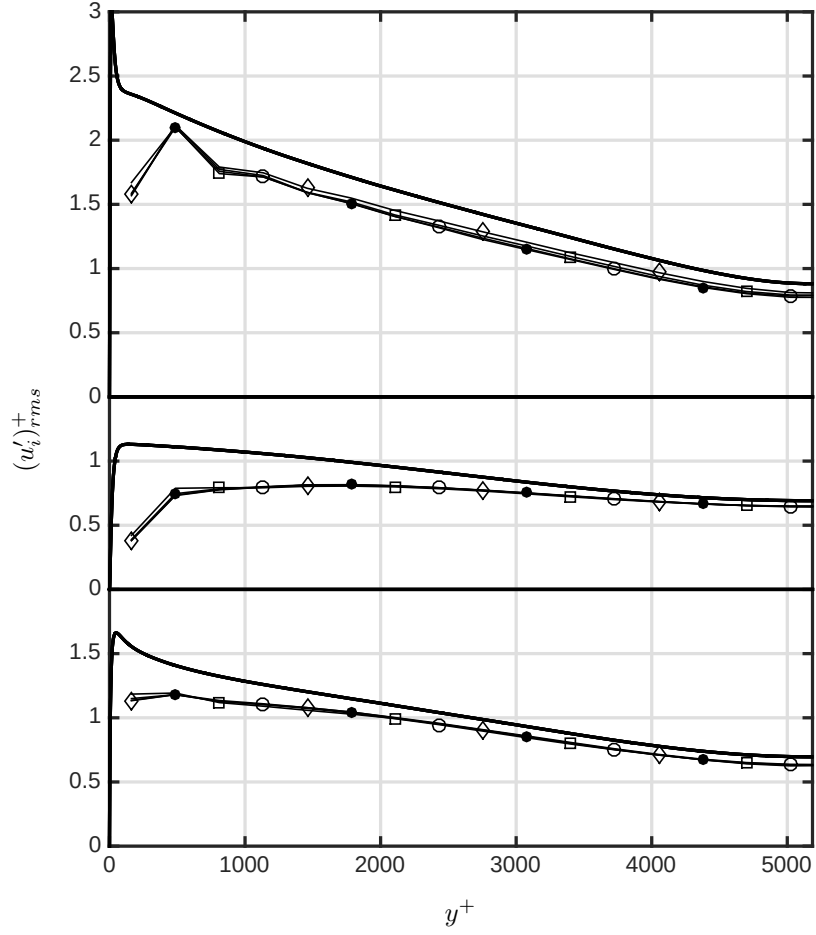


Figure 2.51: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation** profile: DNS of Lee and Moser [71] (thick solid), and wall-modeled LES (thin solid and markers). **Averaging** over x and z (diamonds), averaging over 5×5 points (bullets), averaging over time (circles), and pointwise wall model (squares). The markers do not show all grid points for the sake of readability.

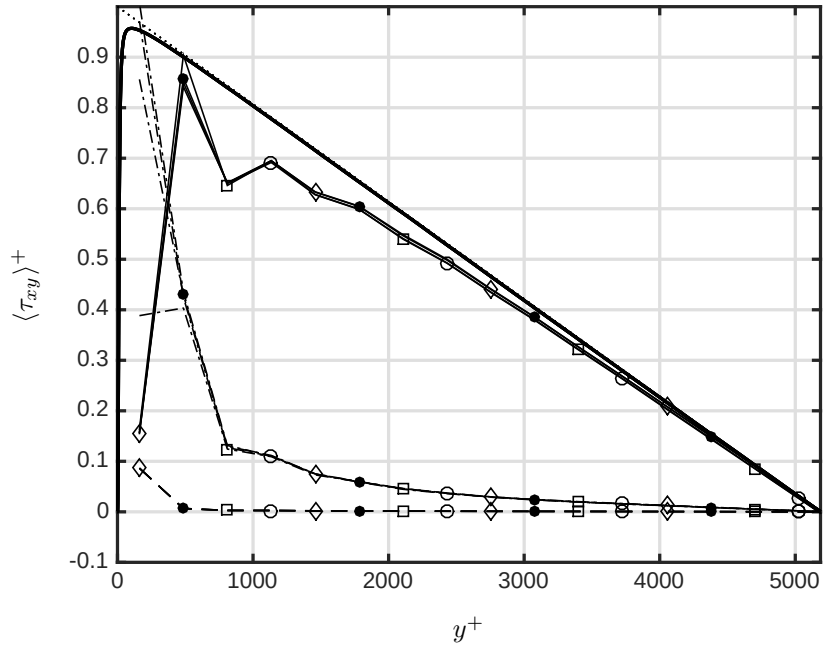


Figure 2.52: Channel flow at $Re_\tau = 5200$. **Mean shear stress** profiles: total stress (dotted), SGS model stress of the LES (dashed dotted and markers), resolved turbulent shear stress of the LES (solid and markers), resolved viscous stress of the LES (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). **Averaging** over x and z (diamonds), averaging over 5×5 points (bullets), averaging over time (circles), and pointwise wall model (squares). The markers do not show all grid points for the sake of readability.

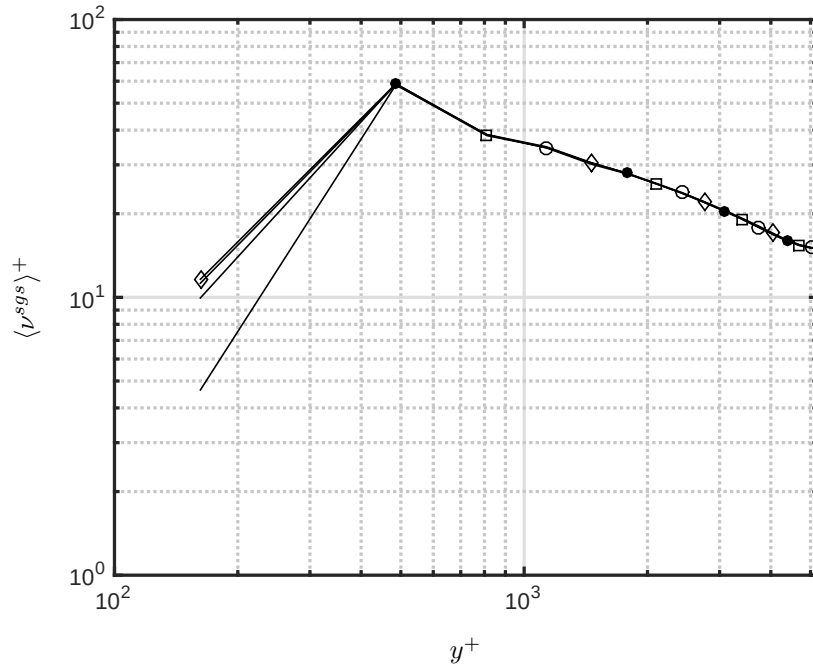


Figure 2.53: Channel flow at $Re_\tau = 5200$. **Mean SGS viscosity** profile of the wall modeled LES. **Averaging** over x and z (diamonds), averaging over 5×5 points (bullets), averaging over time (circles), and pointwise wall model (squares). The markers do not show all grid points for the sake of readability.

2.6 RANS sub-layer models

RANS sub-layer models are then considered. The idea, first proposed by Cabot [17] and Balaras et al. [5], is to solve the turbulent boundary layer (TBL) equations (supplemented by a RANS model) between the point at which the wall model fetches information and the wall. The wall shear stress of the RANS is then fed as boundary condition to the LES locally and instantaneously. An inner mesh is embedded within the first few wall cells, as sketched in Fig. 2.54, and RANS-TBL equations are solved on that mesh with Dirichlet boundary condition at the top, provided by the LES, and no slip boundary condition at the wall. As for the algebraic wall models, the connection from the LES to the RANS could be made at any point $y_{wm} = y_j$, with $j \geq 3$. In practice, unless stated otherwise, the connection is made at the third point.

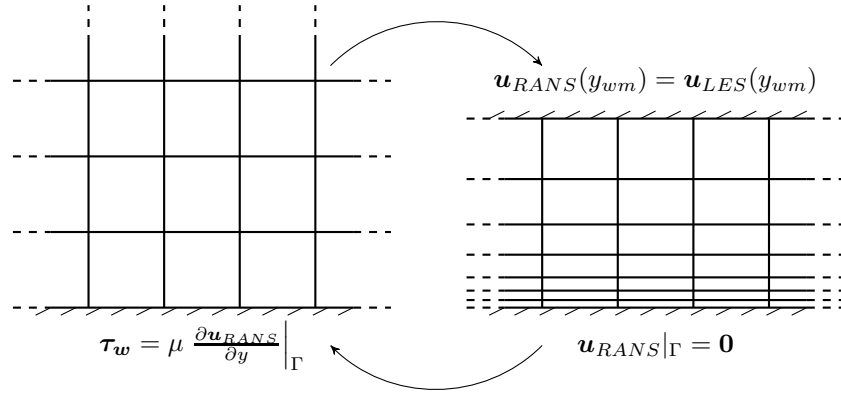


Figure 2.54: Sketch of the embedded inner mesh of the **RANS sub-layer**. On the left is the LES mesh, on the right is the RANS sub-layer mesh. The LES feeds information (in the form of $\mathbf{u}_{LES}$ and $(\nabla P)_{LES}$) to the RANS, and retrieves a wall shear stress boundary condition from the RANS.

The embedded mesh is such that the RANS is wall-resolved: hence, a stretching is applied in the y direction, typically a hyperbolic tangent stretching (see Eq. (2.30)), while the meshing in the wall parallel plane is the same as

the LES. For a wall in the Oxz plane, the generic RANS-TBL equations read:

$$\begin{aligned} \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) &= - \left(\frac{\partial P}{\partial x_i} \right)_F - \frac{\partial P}{\partial x_i} \\ &\quad + \frac{\partial}{\partial x_j} \left(2(\nu + \nu_t) S_{ij} \right), \quad i = 1, 3 \end{aligned} \quad (2.80)$$

$$u_2 = - \int_0^{x_2} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \right) dx'_2 \quad (2.81)$$

$$\frac{\partial P}{\partial x_2} = 0. \quad (2.82)$$

Since $\frac{\partial P}{\partial x_2} = 0$, the wall parallel pressure gradient is also extracted from the LES field. We here solve all terms of the equations, including the time derivative, the convective terms and the pressure term. The pressure and velocity fields of the LES contain many fluctuations, and some local averaging is performed to damp the smallest scales of the LES when feeding it to the RANS. The averaging procedures used for the algebraic wall models, described in Section 2.5.1, are used. The resulting velocity field still contains some resolved fluctuations. An ideal RANS solution, however, is stationary, and all fluctuation are taken into account in the RANS model. To avoid accounting for the resolved fluctuations twice (in the RANS term and in the convective term), we use the correction proposed by Medic et al. [80]. The idea is that the effective corrected term with the addition of the resolved fluctuations should be equal to the RANS term without the convective term:

$$\nu_{t,corr} \frac{du}{dy} - \langle u'v' \rangle = \nu_t \frac{du}{dy}, \quad (2.83)$$

and therefore, we can compute the corrected viscosity such that this is verified:

$$\nu_{t,corr} = \nu_t + \frac{\langle u'v' \rangle}{\frac{du}{dy}}, \quad (2.84)$$

where ν_t is the value computed using the model and $\nu_{t,corr}$ is the corrected value used in the RANS equations (2.80). This approach is also close to (though less general than) that proposed by Park and Moin [96], who used the energy dissipation associated to the full convective and RANS tensors to obtain the corrected RANS term.

Balaras et al. [5] used an eddy viscosity model based on the damping pro-

posed by Piomelli et al. [101], which is of the Van Driest [137] damping type:

$$\nu_t = (\kappa y)^2 |\mathbf{S}| \left(1 - e^{-(y^+/A^+)^3}\right), \quad (2.85)$$

with $A^+ = 25$. Since close to the wall, we have $|\mathbf{S}| \sim 1$, this model exhibit a $\nu_t \sim y^5$ behavior, which is not the expected behavior. This damping was originally proposed to be used to damp a Smagorinsky SGS model (and not a simple RANS model). The Smagorinsky model has a $\nu^{sgs} \sim 1$ behavior toward the wall, and therefore exhibits a $\nu^{sgs} \sim y^3$ behavior after the exponential damping is applied, which is the expected behavior. Cabot [17] used a different damping from the same type:

$$\nu_t = \kappa y u_\tau \left(1 - e^{-y^+/A^+}\right)^2, \quad (2.86)$$

which yields the proper $\nu_t \sim y^3$ behavior. We here use $A^+ = 16$, and $\kappa = 0.38$.

When such a sub-layer is embedded in the LES mesh, stability issues can arise if the same time step is used for the RANS sub-layer as for the LES due to the grid refinement close to the wall. The wall model should not, however, be a limiting factor for the time stepping of the LES. To circumvent this limitation, a smaller time step is used for the RANS sub-layer than for the LES. The process is sketched in Fig. 2.55. At time t^n , we have $\mathbf{u}_{\text{RANS}}^n$, $\mathbf{u}_{\text{LES}}^n$, and P_{LES}^n . A single time step Δt is performed in the LES to obtain $\mathbf{u}_{\text{LES}}^{n+1}$, with the boundary condition provided by the wall shear stress of $\mathbf{u}_{\text{RANS}}^n$, using $\boldsymbol{\tau}_w^n = \mu \frac{\partial \mathbf{u}_{\text{RANS}}^n}{\partial y} \Big|_\Gamma$. In the RANS sublayer, N_{sub} substeps are performed. The upper boundary condition of the RANS is frozen, as well as the pressure gradient: both are provided by the LES field at time t^n . Sub-iterations with a time step $\Delta t/N_{\text{sub}}$ are then performed on the RANS field, until t^{n+1} is reached. After the process is finished, we have obtained $\mathbf{u}_{\text{RANS}}^{n+1}$, $\mathbf{u}_{\text{LES}}^{n+1}$, and P_{LES}^{n+1} , and the next time step can be performed.

2.6.1 Channel flow validation of the RANS sublayer models

The various models are tested in a channel flow setup. The setup for the LES is the same as the one used to validate the algebraic models, hence using a $N_x \times N_y \times N_z = 58 \times 32 \times 48$ uniform mesh in a box of size $L_x \times L_y \times L_z =$

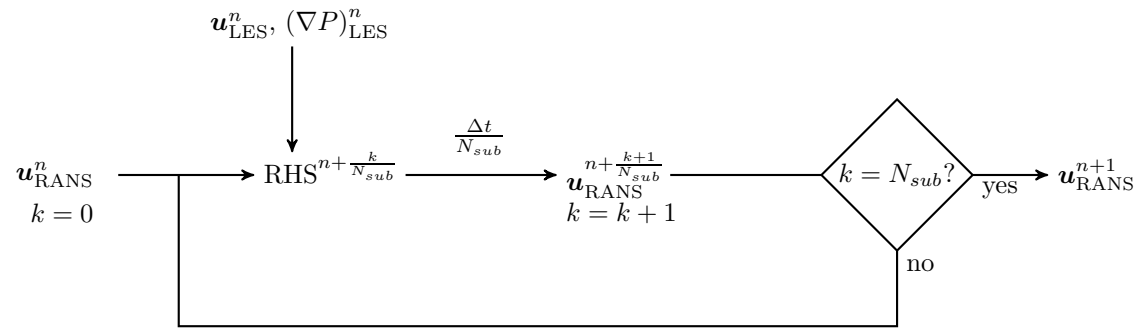


Figure 2.55: Time stepping of the RANS sublayer with N_{sub} substeps. The upper boundary condition taken from the LES is frozen during the sub-iterations. Δt is the time step used by the LES.

$2\pi h \times 2h \times \pi h$. The embedded mesh has 30 points in the vertical direction. It is stretched towards the wall using a hyperbolic tangent stretching, with $\gamma = 2.7$. When convective terms are dropped, the pressure gradient term is dropped as well, and only the forcing pressure gradient is accounted for in the RANS-TBL equations. This is because only the sum of both terms is close to zero, but not each term individually. Table 2.5 summarizes the cases run to validate the RANS sublayer models. Cases 1, 2 and 3 demonstrate the use of the averaging of the LES information fed to the RANS as well as the turbulent viscosity correction when convective terms are retained. Case 4 is the same as case 3, but using the eddy viscosity model with the damping used by Balaras et al. [5], which has the incorrect damping function. Case 5 is run without convective and pressure terms. Case 6 is the same as case 3, with convection and the correction of Eq. (2.84), but with no averaging of the LES data.

Case	RANS model	$N_{AVG,x} \times N_{AVG,z}$	ν_t correction	Convection and pressure terms
1	Cabot	None	No	Yes
2	Cabot	5×5	No	Yes
3	Cabot	5×5	Yes	Yes
4	Balaras et al.	5×5	Yes	Yes
5	Cabot	5×5	No	No
6	Cabot	None	Yes	Yes

Table 2.5: Various channel flow cases investigated using a RANS-TLM with 30 points in the y direction. The LES information fed to the RANS is averaged locally over the closest $N_{AVG,x} \times N_{AVG,z}$ points. When RANS correction is applied, the averaging is performed over the all homogeneous directions. No time averaging is used.

The comparison of cases 3 and 6, shown in Fig. 2.56 shows the need for a bit of averaging when feeding the LES data to the RANS. If the raw LES data is fed to the RANS, spurious oscillations appear due to the ν_t correction. There is too many fluctuations for the RANS mesh, and the ν_t correction becomes unstable and either generates spurious oscillations or blows up numerically. This is also seen in Fig. 2.57, which shows the RANS viscosity for these two cases. We see that the RANS correction yields an oscillatory behavior when no averaging is performed on the LES field fed to the RANS.

Fig. 2.58 shows the profiles obtained with the model of Cabot (cases 1, 2, 3, and 5). They are complemented by the bulk velocities given in Table 2.6.

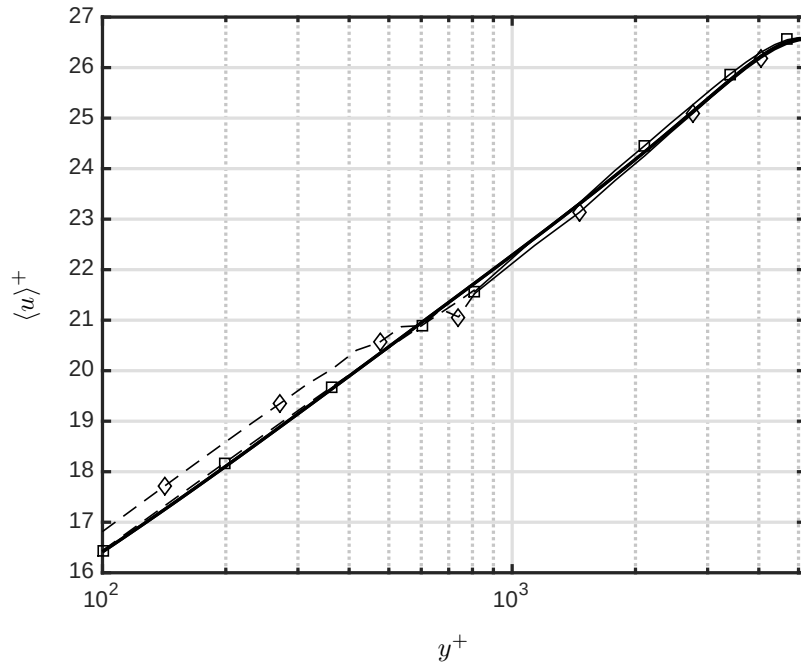


Figure 2.56: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), wall-modeled LES (thin lines and markers), and RANS sublayer with the model of Cabot, with convection and pressure terms and with ν_t correction (dashed and markers). **Case 3 (with averaging, squares), and case 6 (without averaging, diamonds)**. The markers do not show all grid points for the sake of readability.

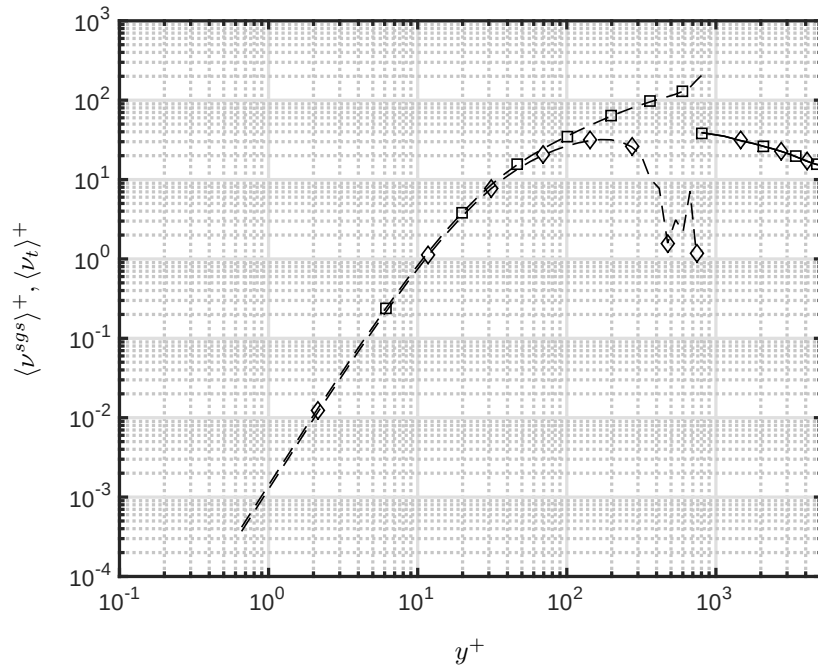


Figure 2.57: Channel flow at $Re_\tau = 5200$. **Mean eddy viscosity** profile: wall-modeled LES (ν^{SGS}) (solid and markers), and RANS sublayer (ν_t) with the model of Cabot, with convection and pressure terms and with ν_t correction (dashed and markers). **Case 3 with averaging, (squares), and case 6 (without averaging, diamonds).** The markers do not show all grid points for the sake of readability.

Clearly, only cases 3 and 5 provide acceptable velocity profiles, hence one should either drop the convective and pressure terms or correct the turbulent viscosity accordingly. Indeed, case 1, with the boundary condition simply extracted without filtering the LES data exhibits an underestimated slope of the velocity profiles in the RANS sublayer. Filtering the LES data (case 2) removed some of the turbulent fluctuations, and therefore improved the resulting profiles, but this was not sufficient to provide the expected velocity profile. Fig. 2.59 shows the turbulent viscosity ν_t used in the RANS term, as well as the SGS viscosity of the LES. As previously, the turbulence developed by the LES is mostly unaffected by the wall model. The net effect of the wall model is therefore to link the wall shear stress and the mass flow rate, but not to generate turbulence. Indeed, when the wall model predictions are off, like in cases 1 and 2, the LES velocity profiles are simply shifted downwards. The viscosity correction in the RANS has a great impact on the ν_t profiles and damps it by a factor of up to 2, resulting in large fluctuation in the slope of the velocity profiles. This mostly happens in the upper part of the RANS domain, and we see that, in case 3, the corrected RANS viscosity does not behave as $\nu_t = \kappa y u_\tau$. The maximum damping is reached close to the RANS-LES matching point, where the most intense turbulent structures lie. All cases provide the same results in the viscous sub-layer, and only start to diverge in the transition layer, when the turbulent shear stress gains importance and turbulent structure are resolved on the embedded RANS-TBL grid. If no correction is made to the turbulent viscosity, this turbulent shear stress is twice accounted for, and the slope of the velocity profiles is underestimated. Indeed, in the mean, we have

$$\frac{d\langle u \rangle}{dy} = \frac{u_\tau^2 \left(1 - \frac{y}{h}\right)}{\nu + \nu_e}, \quad (2.87)$$

where ν_e is the effective turbulent viscosity $\nu_e = \nu_t - \langle u'v' \rangle / \frac{d\langle u \rangle}{dy}$. The correct slope is obtained either when $\langle u'v' \rangle = 0$ (or dropping the convective term) or when ν_t is corrected using Eq. (2.84). We see, that with the correction, the profiles are in good agreement with the DNS data, also within the RANS sublayer. This shows that the RANS model used has the correct behavior in the log layer, but also close to the wall. We indeed see, in Fig. 2.59 that the y^3 scaling is recovered close to the wall. Not accounting for the convective terms (case 5) also yielded good mean velocity profiles, since all the turbulent shear stress is then carried by the RANS model.

Fig. 2.60 shows the velocity fluctuations of cases 3 and 5, hence with and without convective terms. We see that more velocity fluctuations develop inside the RANS sublayer when the convective terms are retained, showing that the embedded mesh has the capability of sustaining some turbulence. Fig. 2.61 shows the shear stresses of cases 3 and 5. We see that, when the convective terms are dropped (case 5), all the turbulent shear stress is carried by the RANS term (the turbulent shear stress of case 5 is also shown, though it is not used in the equations), while, when the convective terms are retained, the contribution of the RANS term is drastically reduced to account for the resolved turbulent stresses.

Case	Re_b	Relative error (in %)
1	$1.137 \cdot 10^5$	-9.4
2	$1.201 \cdot 10^5$	-3.9
3	$1.255 \cdot 10^5$	0.4
4	$1.208 \cdot 10^5$	-3.3
5	$1.249 \cdot 10^5$	-0.08
6	$1.255 \cdot 10^5$	0.1

Table 2.6: Bulk Reynolds number and error with respect to the DNS of Lee and Moser [71] ($Re_b = 1.25 \cdot 10^5$) for the various cases of Table 2.5.

Fig. 2.62 compares the mean velocity profiles obtained with the model of Cabot (case 3) and the model of Balaras et al. (case 4). Clearly, the model of Balaras et al. does not provide the proper profile, and this is due to its incorrect behavior towards the wall, as can be seen in Fig. 2.63, where the wrong y^5 scaling appears in the ν_t profile, despite the correction for the resolved fluctuations. The velocity fluctuations are shown in Fig. 2.64. We see that less turbulent fluctuations are sustained when using the model used by Balaras et al. This is also seen in the turbulent shear stress, shown in Fig. 2.65. This shows that having a RANS model that exhibits the correct y^3 scaling towards the wall is required for the wall model to be able to accurately predict the behavior of the TBL inner layer. This equivalent to requiring the law of the wall implied by the RANS model to be accurate also in the vicinity of the wall. Otherwise, the capability of the model might depend on where the top of the sublayer lies in the TBL.

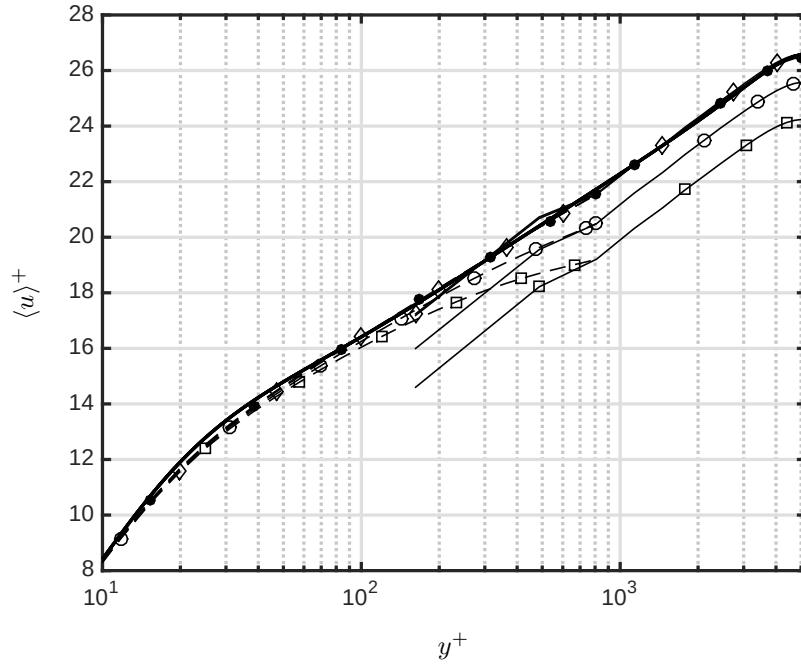


Figure 2.58: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), wall-modeled LES (thin lines and markers), and **RANS sublayer** with the model of Cabot (dashed and markers). Case 1 (without averaging, without ν_t correction, with convection and pressure terms, squares), case 2 (with averaging, without ν_t correction, with convection and pressure terms, circles), case 3 (with averaging, with ν_t correction, with convection and pressure terms, diamonds), and case 5 (with averaging, without ν_t correction, without convection and pressure terms, bullets). The markers do not show all grid points for the sake of readability.

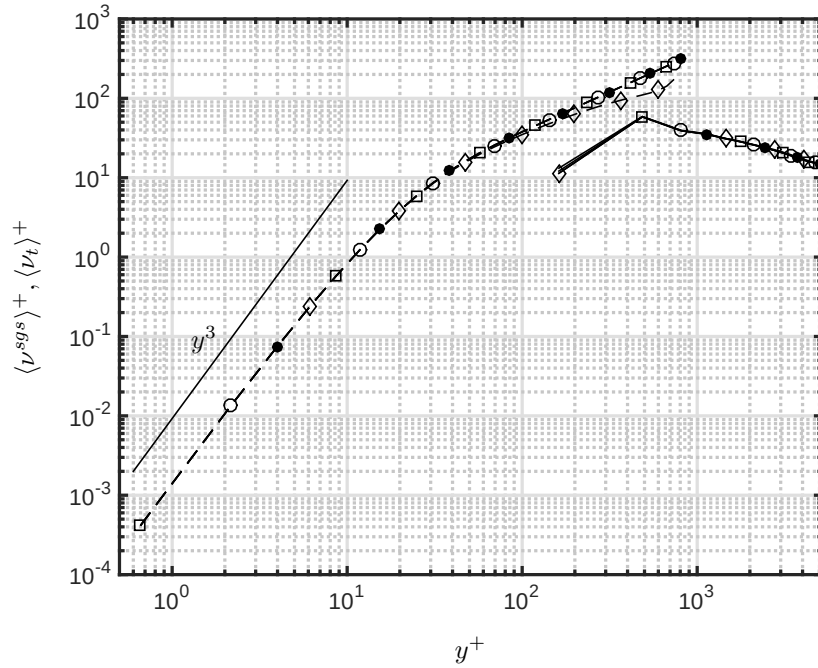


Figure 2.59: Channel flow at $Re_\tau = 5200$. **Mean eddy viscosity** profile: wall-modeled LES (ν^{SGS}) (solid and markers), and **RANS sublayer** (ν_t) with the model of Cabot (dashed and markers). Case 1 (without averaging, without ν_t correction, with convection and pressure terms, squares), case 2 (with averaging, without ν_t correction, with convection and pressure terms, circles), case 3 (with averaging, with ν_t correction, with convection and pressure terms, diamonds), and case 5 (with averaging, without ν_t correction, without convection and pressure terms, bullets). The markers do not show all grid points for the sake of readability.

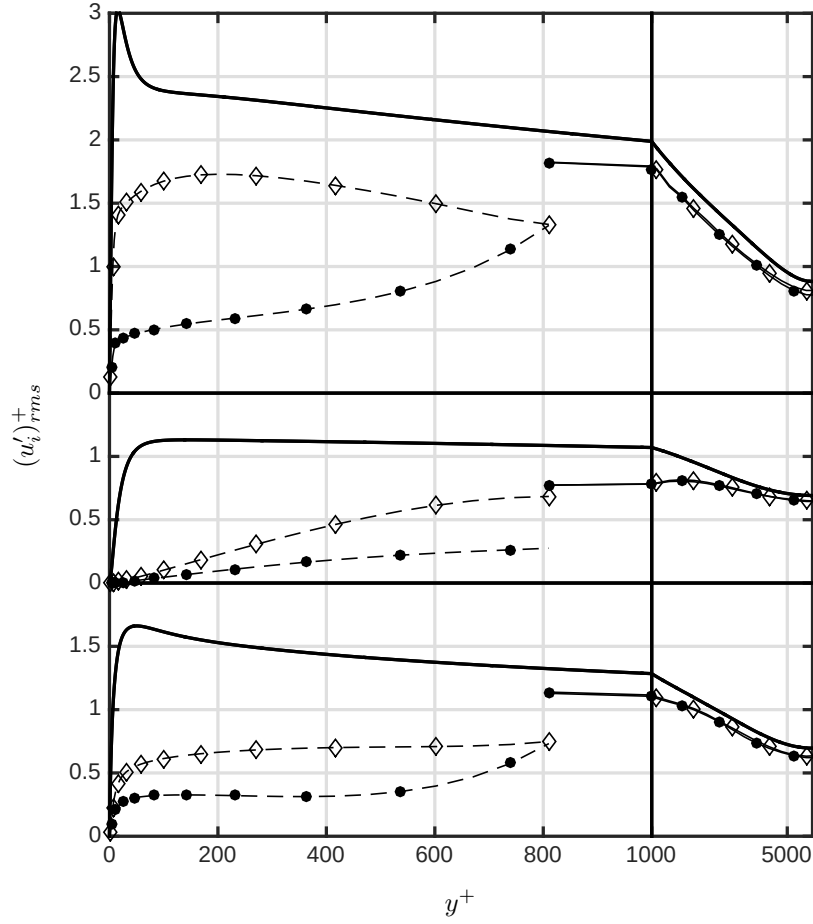


Figure 2.60: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation** profile: DNS of Lee and Moser [71] (thick solid), wall-modeled LES (thin lines and markers), and **RANS sublayer** (dashed and markers). Case 3 (with averaging, with ν_t correction, with convection and pressure terms, diamonds), and case 5 (with averaging, without ν_t correction, without convection and pressure terms, bullets). The markers do not show all grid points for the sake of readability. The first two points of the LES are not shown for the readability of the RANS sublayer.

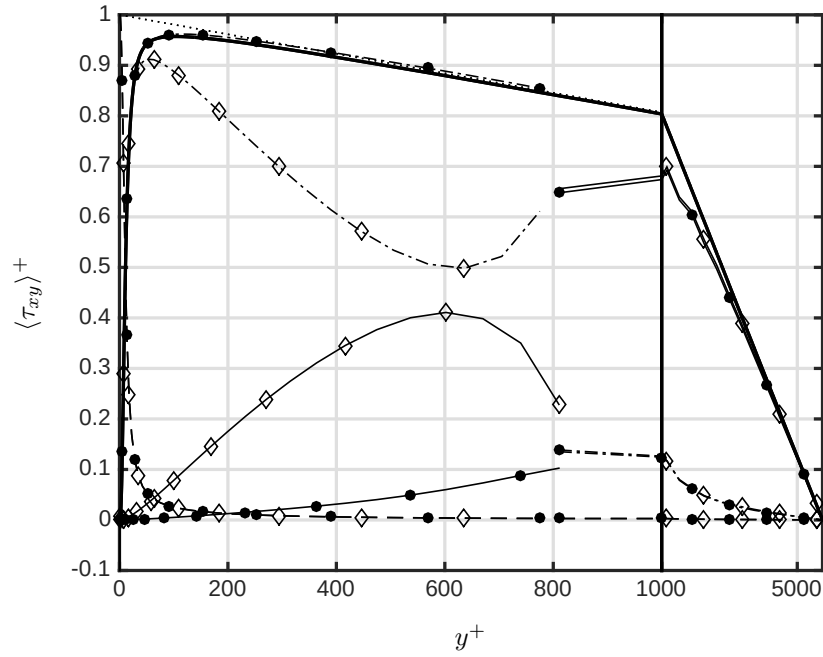


Figure 2.61: Channel flow at $Re_\tau = 5200$. **Mean shear stress** profiles: total stress (dotted), SGS model stress of the WMLES and RANS model stress in the **RANS sublayer** (dashed dotted and markers), resolved turbulent shear stress of the WMLES and of the RANS sublayer (solid and markers), resolved viscous stress of the WMLES and of the RANS sublayer (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). Case 3 (with averaging, with ν_t correction, with convection and pressure terms, diamonds), and case 5 (with averaging, without ν_t correction, without convection and pressure terms, bullets). The markers do not show all grid points for the sake of readability. The first two points of the LES are not shown for the readability of the RANS sublayer.

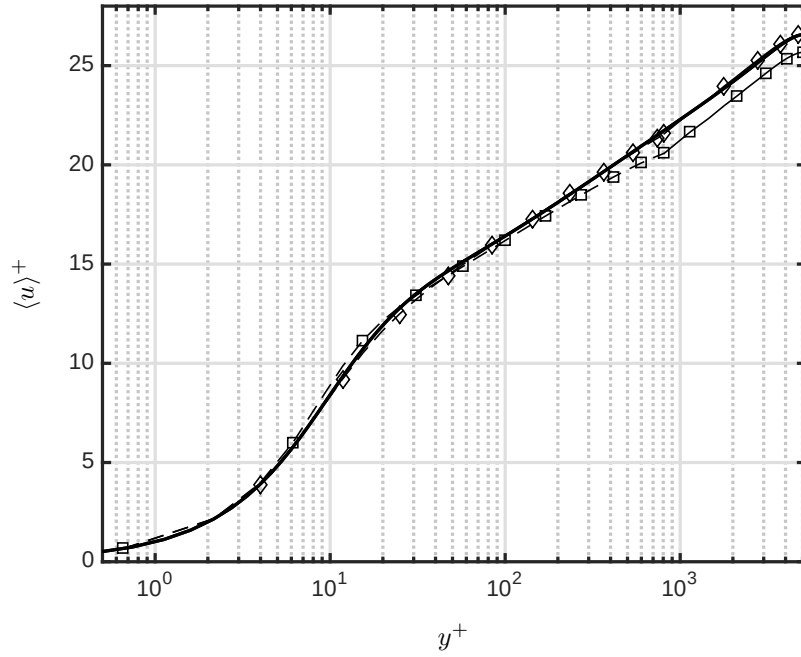


Figure 2.62: Channel flow at $Re_\tau = 5200$. **Mean velocity** profile: DNS of Lee and Moser [71] (thick solid), wall-modeled LES (thin lines and markers), and RANS sublayer (dashed and markers). Case 3 (**model of Cabot**, diamonds), and case 4 (**model of Balaras et al.**, squares). The markers do not show all grid points for the sake of readability. The first two points of the LES are not shown for the readability of the RANS sublayer.

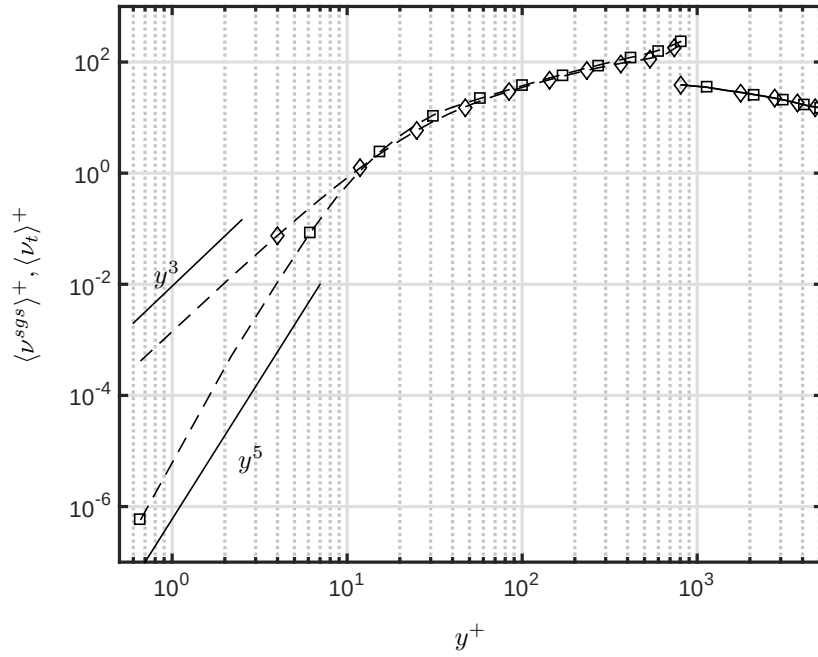


Figure 2.63: Channel flow at $Re_\tau = 5200$. **Mean eddy viscosity** profile: DNS of Lee and Moser [71] (thick solid), wall-modeled LES (ν^{sgs}) (solid and markers), and **RANS sublayer** (ν_t) (dashed and markers). Case 3 (**model of Cabot**, diamonds), and case 4 (**model of Balaras et al.**, squares). The markers do not show all grid points for the sake of readability. The first two points of the LES are not shown for the readability of the RANS sublayer.

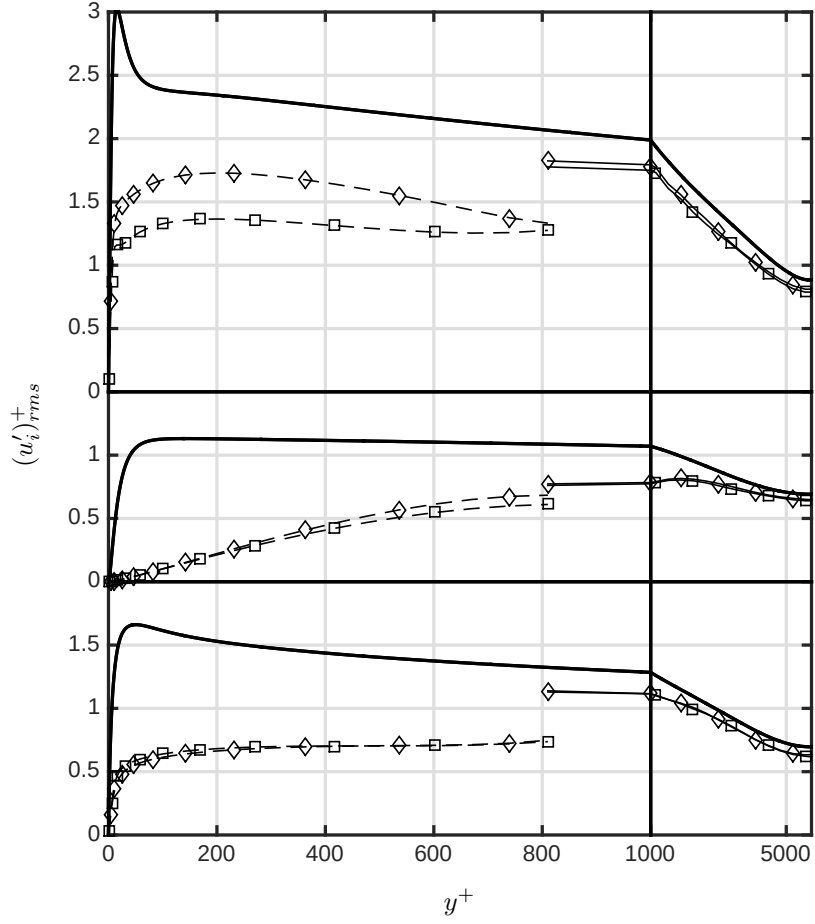


Figure 2.64: Channel flow at $Re_\tau = 5200$. **RMS velocity fluctuation** profile: DNS of Lee and Moser [71] (thick solid), wall-modeled LES (thin lines and markers), and **RANS sublayer** (dashed and markers). Case 3 (**model of Cabot**, diamonds), and case 4 (**model of Balaras et al.**, squares). The markers do not show all grid points for the sake of readability. The first two points of the LES are not shown for the readability of the RANS sublayer.

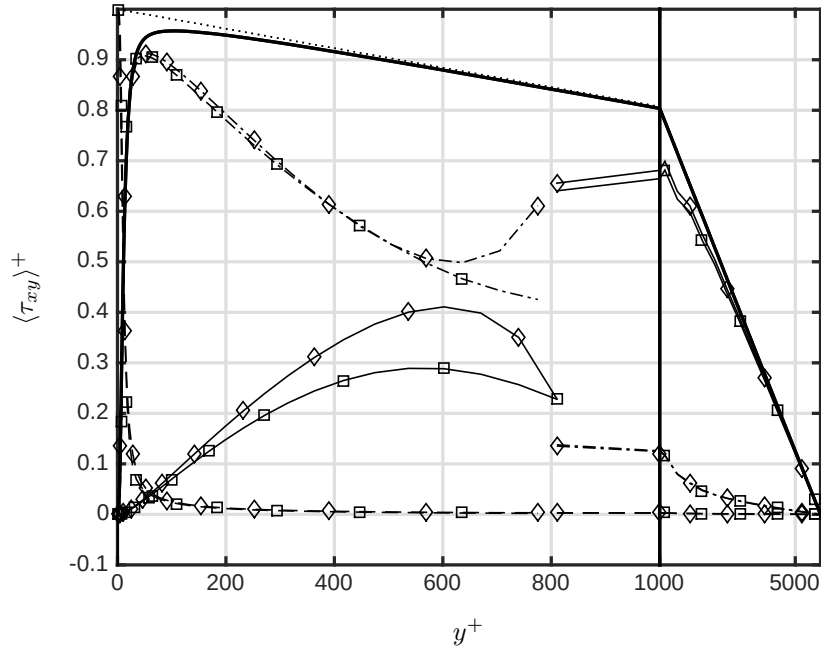


Figure 2.65: Channel flow at $Re_\tau = 5200$. **Mean shear stress** profiles: total stress (dotted), SGS model stress of the WMLES and RANS model stress in the **RANS sublayer** (dashed dotted and markers), resolved turbulent shear stress of the WMLES and of the RANS sublayer (solid and markers), resolved viscous stress of the WMLES and of the RANS sublayer (dashed and markers), and turbulent shear stress of the DNS of Lee and Moser [71] (thick solid). Case 3 (**model of Cabot**, diamonds), and case 4 (**model of Balaras et al.**, squares). The markers do not show all grid points for the sake of readability. The first two points of the LES are not shown for the readability of the RANS sublayer.

2.7 Conclusion

In this chapter, a new version of the BigFlow solver was developed based on a flux divergence formulation of the momentum equation. The new solver was validated against the energy conserving version of the code, and against a spectral reference solution on a Taylor-Green vortex case. It was also validated in turbulent channel flow at $Re_\tau = 395$, against DNS data, in wall-resolved LES mode. The coefficients of some LES models had to be calibrated to be used in the new version of the code. This flux formulation allowed for implementation of wall shear stress boundary conditions, and therefore of wall shear stress models. In practice, the implementation of the wall shear stress boundary condition required that the order of the numerical stencils be lowered to second order of accuracy at the wall. To properly handle the transition between order four and order two, we showed that one layer of point computed at order one is necessary to keep the momentum conservation properties of the scheme. Finally, due to the coarseness of the first cell, the SGS viscosity model had to be corrected in the first off-wall layer of points. These corrections were seen to improve the results, but are a notable complication to the wall-modeling framework.

Two types of wall shear stress models were implemented: algebraic wall models and RANS sublayer models. Both were assessed in channel flow at $Re_\tau = 5200$ against DNS data. The effect of the mesh used was investigated. It was shown that some sensitivity to the aspect ratio of the wall cells exists when using wall modeled LES meshes. Notably, the quality of the results was deteriorated by the use of finer meshes, this is a dangerous issue regarding wall-modeled LES, since, seemingly, no mesh convergence was obtained. It should be noted that the study was here performed by refining the mesh, and evaluating the wall models at the third off-wall grid point, hence the point at which the wall model is evaluated is changing between the meshes, and this is not a proper convergence study. Such a study as well as a thorough study on meshing considerations regarding wall models should be carried. Regardless of the wall model, when varying the aspect ratio of the cells, the turbulence that exists in the LES was shown to depend on the mesh and on the SGS model used, more than on the particular wall model used.

RVMs models were shown to have spurious interactions with the wall models, and part of the solution was to revert to filtered models. This also showed that the interaction of the wall models with SGS models should always be care-

fully examined as they are not always safe. This study is, to the authors' best knowledge, the first study of the interaction of filtered or RVMs SGS models with wall models. It therefore calls for further investigation of such interactions, also in more complex cases.

Various averaging techniques were tested when using algebraic wall models, as well as pointwise evaluation of the wall models. It was shown that very little information was required for the wall model to provide accurate mean profiles, at least in the case of the channel flow. Actually, even when used pointwise, the wall models provided equivalent results than when averaging was used. No difference was observed between the quadratic or the linear algebraic wall models, and only the quadratic wall models were retained.

When RANS sublayer models were used, it was shown that if the convective (and pressure) terms of the equations are computed, the RANS viscosity should be corrected to account for the resolved fluctuations developed by the RANS solution and/or transmitted by the LES to the RANS. It was also shown that some averaging of the LES fields provided to the RANS is necessary for the correction to work efficiently without becoming unstable. This was linked to the fact that an LES supports more fluctuations than a RANS. The averaging effectively damps the finest structures of the LES when communicating with the RANS. The simulations also showed that the RANS model used should have the proper behavior towards the wall, hence with ν_t scaling as y^3 close to the wall. In the end, both algebraic models and RANS-sublayer models were fit to provide accurate results using very coarse meshes, in the channel flow case.

Chapter 3

Wall modeling for LES in the presence of mild pressure gradients

The work presented in this chapter was presented at the 68th Annual Meeting of the American Physical Society's Division of Fluid Dynamics (APS-DFD68) [134], and at the 11th International ERCOFTAC Symposium on Engineering Turbulence Modeling and Measurements (ETMM11) [135], for which a conference paper was published.

In the previous chapter, wall models were used in the homogeneous case of the channel flow. In that case, the boundary layer is at equilibrium and no acceleration or deceleration alters the behavior of the turbulent boundary layer. The wall models use the fact that the boundary layer is at equilibrium, and do not account for potential effects from a pressure gradient generating acceleration and/or deceleration. The algebraic models are based on simple laws that are valid in zero pressure gradient (ZPG) cases. RANS sublayer models do have a pressure gradient term in their formulation, and can therefore account for part of it, but the eddy viscosity model does not account for potential additional pressure gradient effects on the Reynolds stresses. RANS sublayer models can also be used without convective and pressure terms, still maintaining the time derivative. In this case, they are almost equivalent to algebraic wall models. In this study, we call *equilibrium* models, models that

do not include explicit corrections for pressure gradient effects. The RANS sublayer models with simple viscosity models therefore fall into this category, even though they resolve the pressure gradient and convective terms. This is because the law of the wall implied by the viscosity model is not accounting for the pressure gradient. An equally valid nomenclature could consider these models as non-equilibrium models because they resolve the pressure and convective terms. This would be an equally inaccurate nomenclature. Actually, these models are neither at equilibrium nor fully out of equilibrium: pressure gradient effects are partly taken into account and these models therefore fall somewhere in between equilibrium and non-equilibrium. Larsson et al. [69], in their review, listed the case of separated or non-equilibrium boundary layer flows among the main challenges for wall-modeled LES in the future. In particular, they state that no evidence exists to this day to support the superiority of non-equilibrium wall models over equilibrium wall models. In this chapter, we test our equilibrium wall models, as well as a few non equilibrium wall models in the context of non-uniform wall bounded turbulent flow. We analyze the behavior of various wall modeling strategies for LES in the presence of mild pressure gradients, hence, without boundary layer separation. The case of wall-modeled LES with boundary layer separation is treated, in part, in Chapter 5.

3.1 Introduction

The case at hand was motivated by the search of a simple test-case for flows with pressure gradients. The case has to be simple both in terms of geometry and of boundary conditions. The canonical case for such a study is the turbulent boundary layer subject to a pressure gradient. The boundary conditions in this case, however, are not straightforward, both at the inflow and at the outflow. Na and Moin [90] used synthetic turbulence at the inflow, while Skote and Henningson [116] used a periodic domain with a sponge layer to rescale the outflow to the desired inflow profile. Other techniques also exist to generate a proper inflow, such as the interior rescaling technique of Lund et al. [77], but those are beyond the scope of the present work. The way to create the pressure gradient can also vary: Na and Moin [90] used blowing and suction at the top of the numerical box, while Skote and Henningson [116] prescribed the wall normal derivative of the wall normal velocity (which means they implicitly

imposed the mean streamwise velocity profile). Inflow/outflow boundary conditions did not yet exist in the BigFlow solver at the time this investigation was carried, and this case was therefore not an option. The Couette-Poiseuille flow has also been used to study pressure gradient effects, see for instance Johnstone et al. [65], which could also be an interesting test-case for wall modeling in a simple domain, and with simple boundary conditions.

The present case consists of a half-channel flow with periodic acceleration and deceleration. The acceleration-deceleration profile is generated through blowing and suction at the top of the half-channel, instead of a slip boundary condition. The case was developed and described by Duponcheel [35] in his Ph.D. thesis, based on the work of Miyake et al. [85]. In his study, Duponcheel [35] performed several DNS's at $Re_{\tau,F} = 180$, and $Re_{\tau,F} = 395$. These showed large deviations of the velocity profiles from the logarithmic profiles due to the pressure gradient, but he also hinted that those might be low-Reynolds effects. He also performed WRLES with various SGS models at $Re_{\tau,F} = 395$. To a disappointment, the RVMs-WALE model did not out-perform the standard WALE model in predicting the velocity profiles and the wall shear stress. RANS simulations proved unable to provide accurate solutions, probably due to the ad-hoc damping of the models inherent to wall-resolved RANS simulations.

We here aim at first performing WRLES at higher Reynolds number to provide a reference solution to assess several approaches for wall-modeled LES. We first describe the numerical setup of the case. The flow topology is briefly described using the results of the wall-resolved LES. A series of wall models accounting for the pressure gradient is then described. These wall models, as well as the equilibrium wall models described in the previous chapter are used. The performances of the equilibrium and non-equilibrium wall models are compared.

3.2 Setup

The “half channel flow” case we here consider is represented in Fig. 3.1. It is set up, with no slip at the bottom ($y = 0$), and with slip and blowing/suction at the top ($y = h$) in order to generate an acceleration-deceleration profile. The flow is periodic in the streamwise (x) and spanwise (z) directions, and the

computational domain size is $L_x \times L_y \times L_z = 4\pi h \times h \times \pi h$. At $y = h$, we impose:

$$\frac{\partial u}{\partial y}(x, h, z) = 0 \quad (3.1)$$

$$\frac{\partial w}{\partial y}(x, h, z) = 0 \quad (3.2)$$

$$v(x, h, z) = v_{max} \sin\left(2\pi \frac{x}{L_x}\right). \quad (3.3)$$

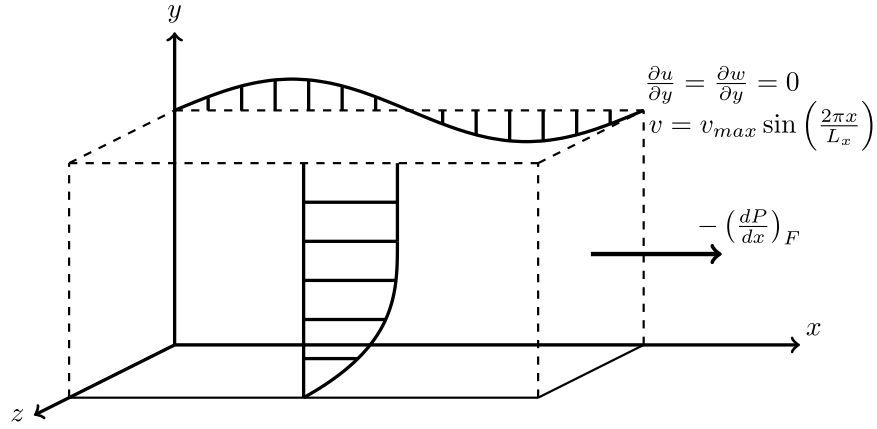


Figure 3.1: Channel flow setup with blowing and suction at the top of the computational domain.

The incompressible Navier-Stokes equations are solved with the addition of a mean pressure gradient forcing term

$$-\left(\frac{dP}{dx}\right)_F = \frac{u_{\tau,F}^2}{h}, \quad (3.4)$$

and in the context of LES, hence with a subgrid scales (SGS) stress tensor,

$$\sigma_{ij}^{sgs} = \tau_{ij}^{sgs} + \frac{1}{3} \sigma_{kk}^{sgs} \delta_{ij}. \quad (3.5)$$

We here only model the deviatoric part of the SGS tensor, the trace being incorporated into the effective reduced pressure,

$$P = \frac{p}{\rho} - \frac{1}{3} \sigma_{kk}^{sgs}. \quad (3.6)$$

The equations then read:

$$\frac{\partial u_j}{\partial x_j} = 0 \quad (3.7)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = - \left(\frac{dP}{dx_i} \right)_F - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (2\nu S_{ij} + \tau_{ij}^{sgs}) . \quad (3.8)$$

We use the filtered WALE SGS model. The forcing pressure gradient is used to perform the global non-dimensionalizations:

$$u^* = \frac{u}{u_{\tau,F}} \quad \text{and} \quad y^* = \frac{y u_{\tau,F}}{\nu} . \quad (3.9)$$

The global Reynolds number is also defined using the forcing pressure gradient:

$$Re_{\tau,F} = \frac{u_{\tau,F} h}{\nu} = 2000 . \quad (3.10)$$

The blowing intensity is set to $v_{max} = 1.25 u_{\tau,F}$, corresponding to a blowing flow rate

$$Q = \int_0^{L_x/2} v(x, h) dx = 5 u_{\tau,F} h . \quad (3.11)$$

The strength of the local pressure gradient is characterized using either p^+ or the pressure velocity u_p :

$$p^+ = \frac{\nu}{u_\tau^3} \frac{dP}{dx} = \frac{u_p^3}{u_\tau^3} , \quad (3.12)$$

where the mean forcing pressure gradient is not accounted for, such that $p^+ = 0$ is the classical half channel flow. A wall-resolved LES (WRLES) is first performed to obtain a reference solution, using a $N_x \times N_y \times N_z = 640 \times 192 \times 256$ mesh and using a hyperbolic tangent stretching in the y direction:

$$\frac{y}{h} = 1 + \frac{\tanh(\gamma(\xi - 1))}{\tanh(\gamma)} , \quad (3.13)$$

with $\gamma = 2.75$, and $\xi \in [0; 1]$. This results in cells of size

$$\Delta x^* = 39 , \quad \Delta y^* = 0.5 - 28 , \quad \Delta z^* = 24 . \quad (3.14)$$

Recall that y^* is here defined using the global friction velocity $u_{\tau,F}$. The

standard viscous variable is $y^+ = \frac{y u_\tau}{\nu}$ defined using the local friction velocity:

$$u_\tau = \left(\frac{|\overline{\tau_w}|}{\rho} \right)^{\frac{1}{2}}, \quad (3.15)$$

where $\overline{}$ stands for averaging over span and time. Since the actual u_τ does not exceed $1.1 u_{\tau,F}$, see Fig. 3.2, these cell sizes lie well within the bounds proposed by Sagaut [110] for WRLES. The simulation was run for a total time $T \overline{u}_{bulk,min}/L_x = 92$, and the statistical averaging started at $t \overline{u}_{bulk,min}/L_x = 55$.

3.3 Flow topology

The flow is periodically accelerated and decelerated by the imposed vertical velocity profile at the top, of Eq. (3.3). This acceleration/deceleration is exactly 90° out of phase with the vertical velocity at the top:

$$\frac{du_{bulk}}{dx} = -\frac{v(y=h)}{h}. \quad (3.16)$$

We see, in Fig. 3.2, that the wall response is not exactly 90° out of phase with the vertical velocity profile. The wall friction first decreases due to the flow deceleration, then increases due to flow acceleration, but the wall friction minimum is reached before the center of the numerical box where the mass flow rate reaches its minimum. This is also visible in Fig. 3.3, where a snapshot of instantaneous wall friction and wall pressure is displayed. The signature of the turbulent structures close to the wall is also visible in Fig. 3.3. Turbulent structures are elongated by the flow acceleration in the second half of the domain. These structures are quickly broken down into streamwise-incoherent structures as the flow is decelerated by the adverse pressure gradient in the first part of the domain. This is consistent with the observations of Na and Moin (1998). Fig. 3.4 also shows a snapshot of instantaneous velocity field at $z/h = 0$ to give some insight into the flow structures. The periodic acceleration/deceleration is visible in the streamwise velocity field u , as well as the broadening of the streamlines resulting from it: the wall layer and its turbulence extends higher in the flow. We also observe, in the spanwise velocity w , that longer and larger structures exist in the accelerated part of the flow, while these structures tend to be smaller in the decelerated part of the flow.

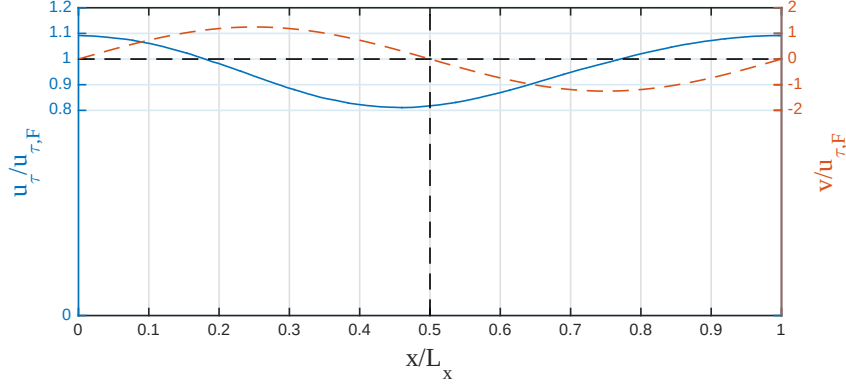


Figure 3.2: Mean friction velocity u_τ^* (solid) and blowing/suction profile $v(y = h)^*$ (dashed).

The mean flow streamlines are shown in Fig 3.5 along with mean velocity profiles at a few streamwise stations. The acceleration/deceleration of the flow through blowing and suction is visible through the broadening of the streamlines at the channel center. Another feature of the flow is the existence of a mean *dividing streamline* above which the flow is essentially irrotational. Downstream of $x/L_x = 0.5$, an “irrotational shower” is blown into the flow, some of which will be sucked back in the suction part of the flow. Most of the flow turbulence is contained below the dividing streamline, as can be seen in Fig. 3.6 as $x/L_x = 0$. Turbulent structures are lifted from the wall between $x/L_x = 0$ and 0.5. At $x/L_x = 0.5$, the mean dividing streamline is at $y = h$, and the turbulence stretches all the way to the top of the box. It is then blown towards the wall between $x/L_x = 0.5$ and 1, where the mean dividing streamline is at its lowest, a little below $y = 0.8h$.

Fig. 3.7 displays the mean velocity profiles, compared to the standard channel flow. The channel flow data are the DNS results of Hoyas and Jimenez [57]. The results are scaled using the forcing pressure gradient velocity, and not the local friction velocity to allow for comparison between the various stations. We here see the amplitude of variation of the profiles with respect to the standard channel flow, which is quite significant. This is the effect of the variation in friction velocity u_τ in the viscous layer. After the transitional layer, the profiles all seem to follow a logarithmic profile, up to a point, where they deviate, around $yu_{\tau,F}/\nu \approx 200 - 300$. The deviation of the profiles can be understood as a wake function that activates earlier and with a bigger amplitude

than in the simple channel flow case. Notice that at station $x/L_x = 0.125$, where $u_\tau/u_{\tau,F}$ is close to 1, the standard channel flow profile is almost recovered. Fig. 3.8 shows the inner layer of the profiles scaled in local viscous units, and compared to the law of the wall of Reichardt. We see that, though the profiles do not lie perfectly on the logarithmic law, the profiles do indeed not depart much from the it. This is due to the fact that the pressure gradient is only mild. Figs. 3.9, 3.10, and 3.11 show the rms velocity fluctuations, and Fig. 3.12 shows the turbulent shear stresses. When examining second order statistics, we see, in Fig. 3.9, that a second smaller peak appears in the u_{rms} profiles that is not present in the standard channel flow case. This secondary peak appears around the minimum wall shear stress and is mostly present in the accelerated part of the flow. Notice also that this peak is present at the height at which the mean velocity profiles start to deviate from a logarithmic behavior. Both could be linked to the formation of new turbulent structures due to the acceleration/deceleration. This modified turbulence close to the wall is also visible in the turbulent shear stress profiles, in Fig. 3.12. We here see that the wall structures are modified by the pressure gradient. This modified turbulent shear stress is also what eventually leads to deformed profiles. The w_{rms} profiles of Fig. 3.11 exhibit the same trends as the u_{rms} profiles. Finally, when examining v_{rms} , in Fig. 3.10, we see, close to the top of the numerical box, the effect of the imposition of the vertical velocity profile: no vertical velocity fluctuations can exist at the top of the half-channel. This does not entail the quality of the simulation below the adaptation zone (i.e. roughly below $y^* \approx 1500$), as Bricteux [13] already showed in a half-channel flow simulation.

3.4 Wall models accounting for pressure gradient effects

Wall-modeled LES is then considered, using a coarse $128 \times 32 \times 48$ and uniform mesh. This mesh has a viscous resolution of $\Delta x^* = 196$, $\Delta y^* = 62.5$, and $\Delta z^* = 130$. An algebraic wall model accounting for the pressure gradient is first detailed. Then, two RANS models corrected for pressure gradient effects are presented and implemented in the RANS sublayer models. Both algebraic and sublayer models are validated in the absence of pressure gradient, in channel flow, then applied to the case at hand, as well as the equilibrium wall models presented in Chapter 2.

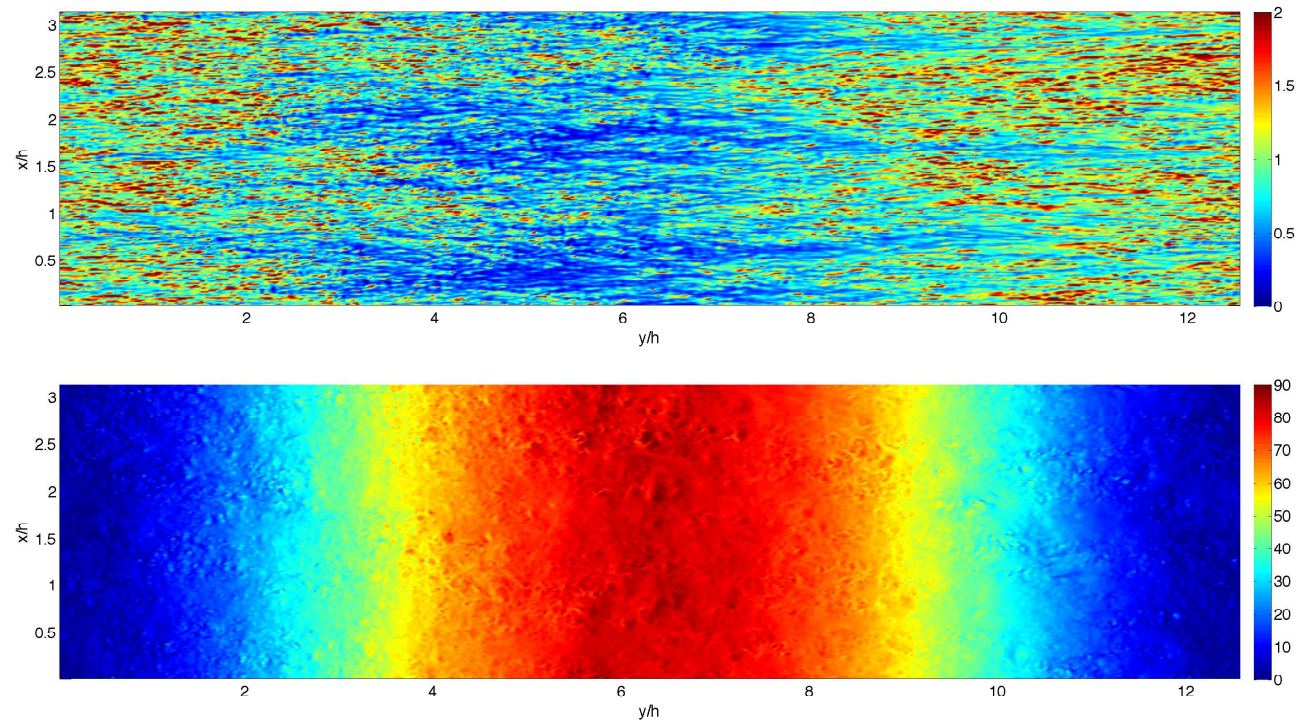


Figure 3.3: Instantaneous wall friction $\tau_{w,x}^*$ (top) and reduced pressure P^* (bottom) at $y/h = 0$.

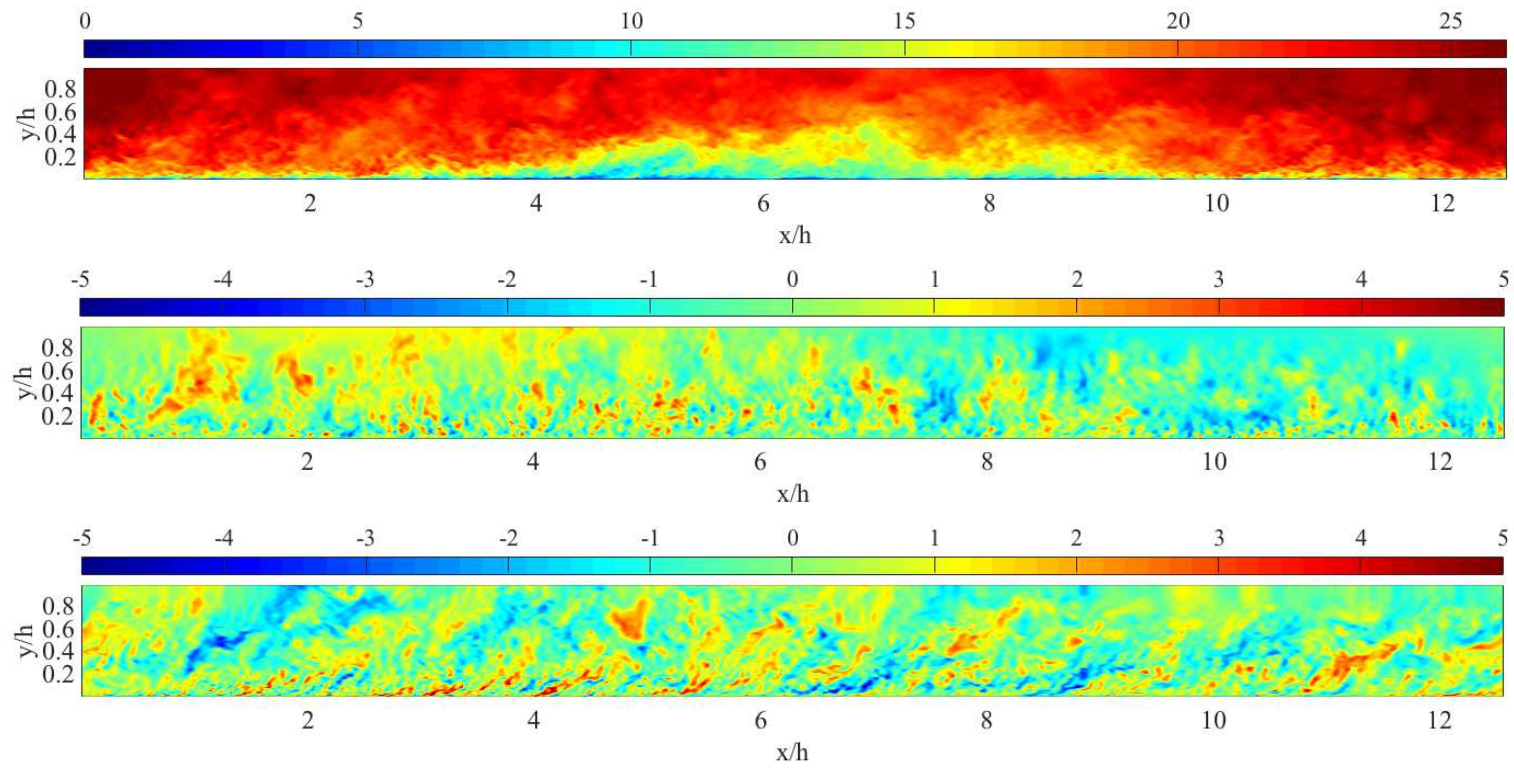


Figure 3.4: Instantaneous velocity field u^* (top), v^* (middle), and w^* (bottom) at $z/h = 0$.

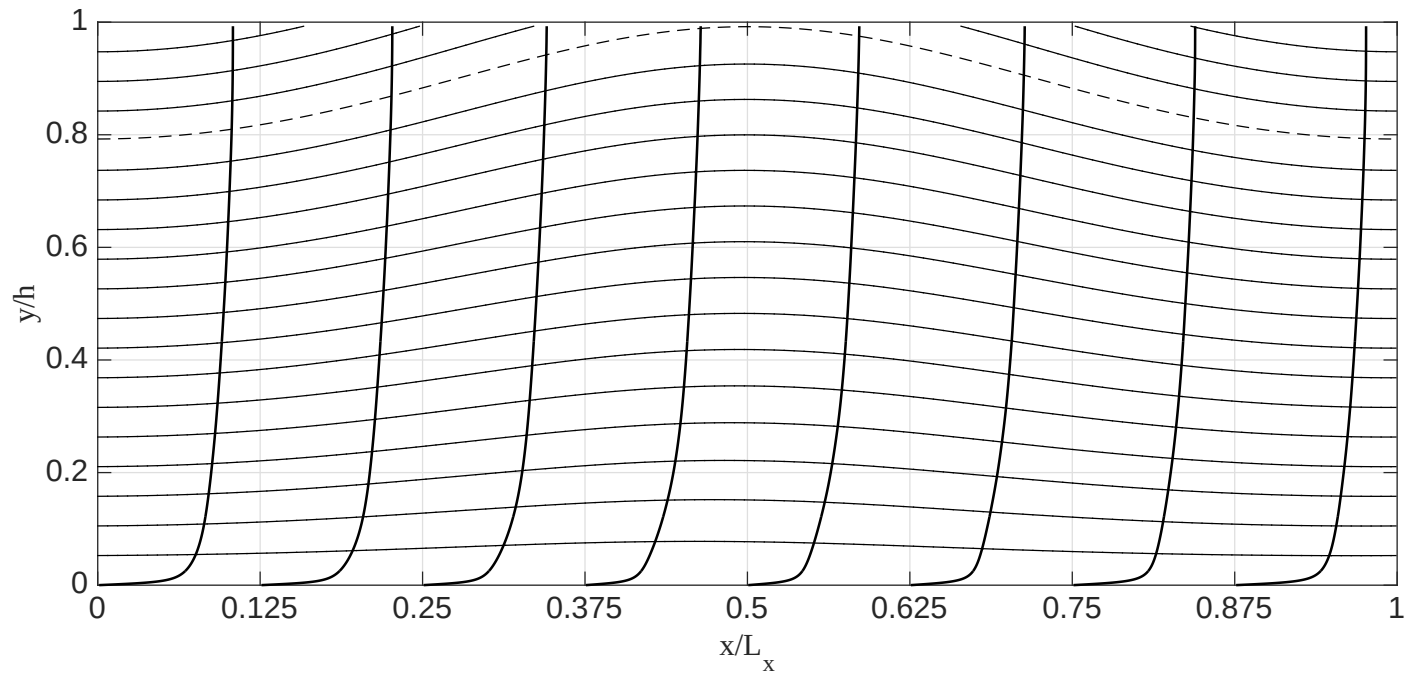


Figure 3.5: Mean streamlines (thin solid; the dividing streamline being dashed) and velocity profiles (thick solid).

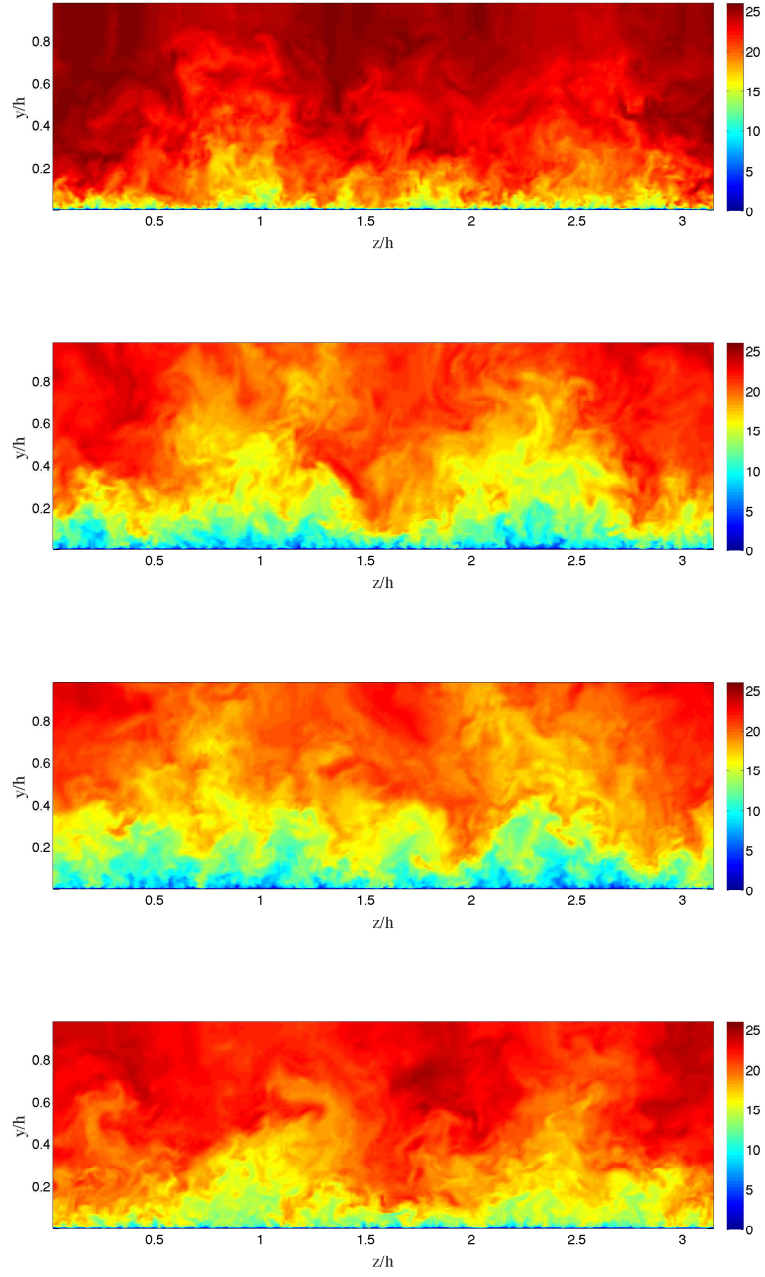


Figure 3.6: *Instantaneous streamwise velocity of the WRLES $u/u_{\tau,F}$ at $x/L_x = 0, 0.25, 0.5$, and 0.75 (from top to bottom).*

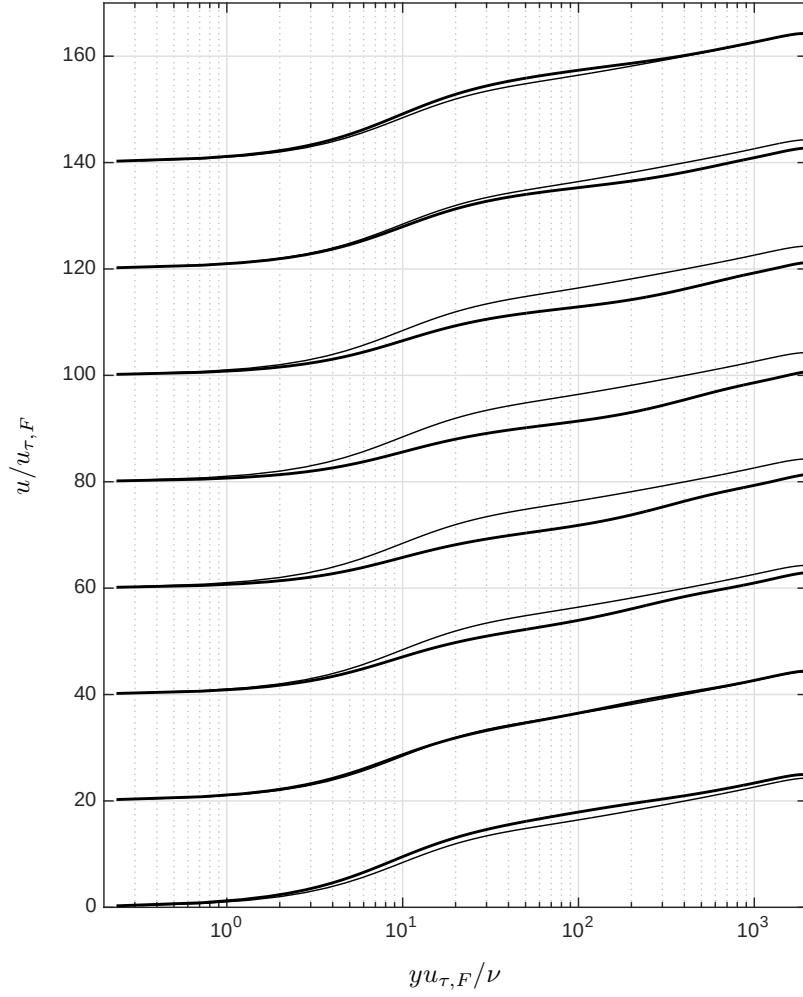


Figure 3.7: Wall-resolved LES. Mean velocity profiles at stations $x/h = 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75,$ and 0.875 (from bottom to top). Each station is shifted upwards by 20 units for the sake of clarity. Blown half-channel WRLES (thick solid), and standard channel flow DNS of Hoyas and Jimenez [57] (thin solid), both scaled using the forcing pressure gradient.

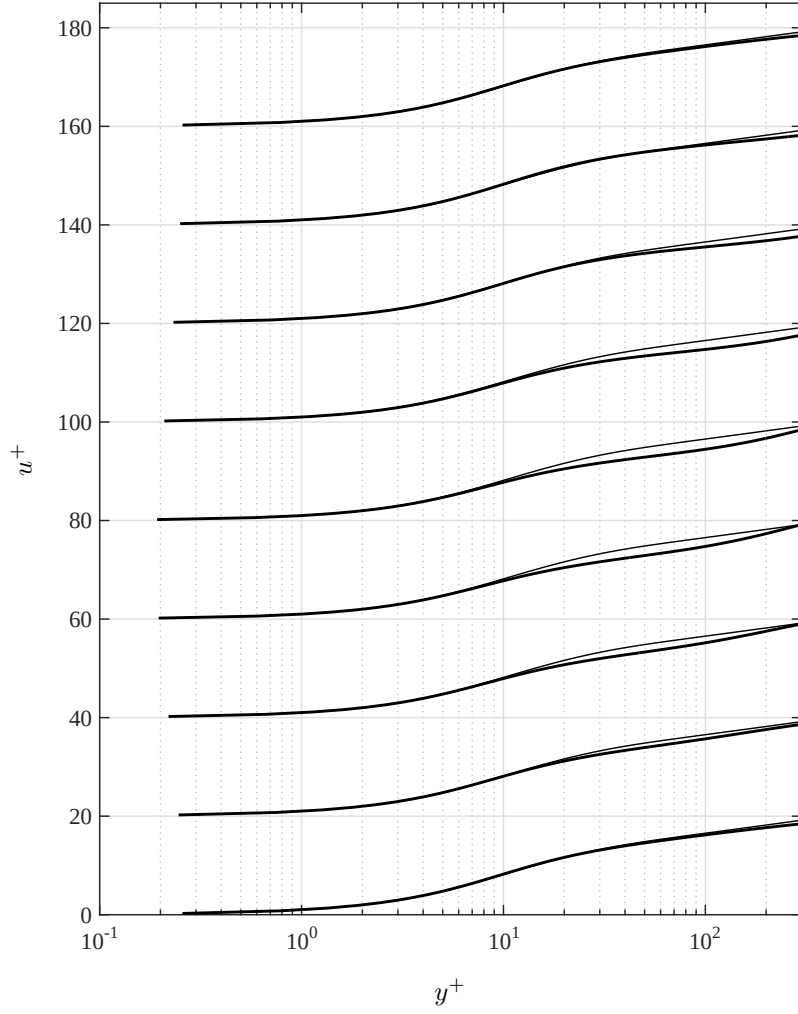


Figure 3.8: Wall-resolved LES. *Inner layer of the mean velocity profiles at stations $x/h = 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75$, and 0.875 (from bottom to top). Each station is shifted upwards by 20 units for the sake of clarity. Blown half-channel WRLES (thick solid), and law of the wall of Reichardt (thin solid), both scaled in local viscous units and zoomed in the range $y^+ < 300$.*

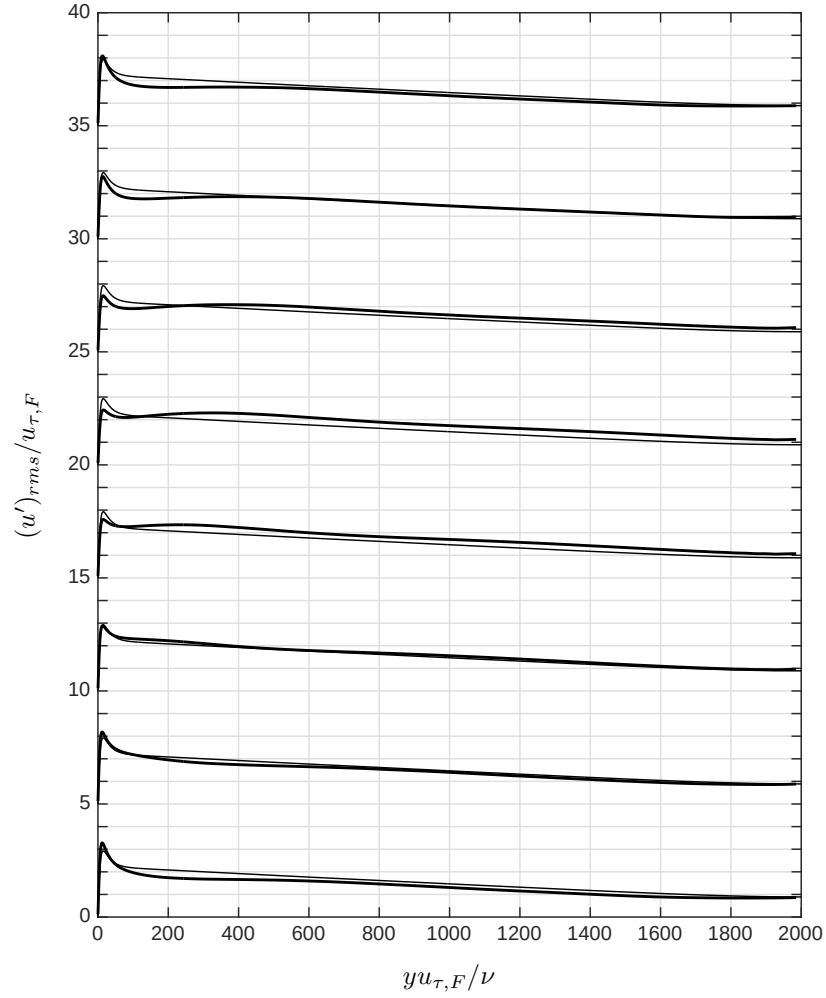


Figure 3.9: Wall-resolved LES. **RMS streamwise velocity fluctuations** profiles at stations $x/h = 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75,$ and 0.875 . Each station is shifted upwards by 5 units for clarity. Blown half-channel WRLES (thick solid), and standard channel flow DNS of Hoyas and Jimenez [57] (thin solid).

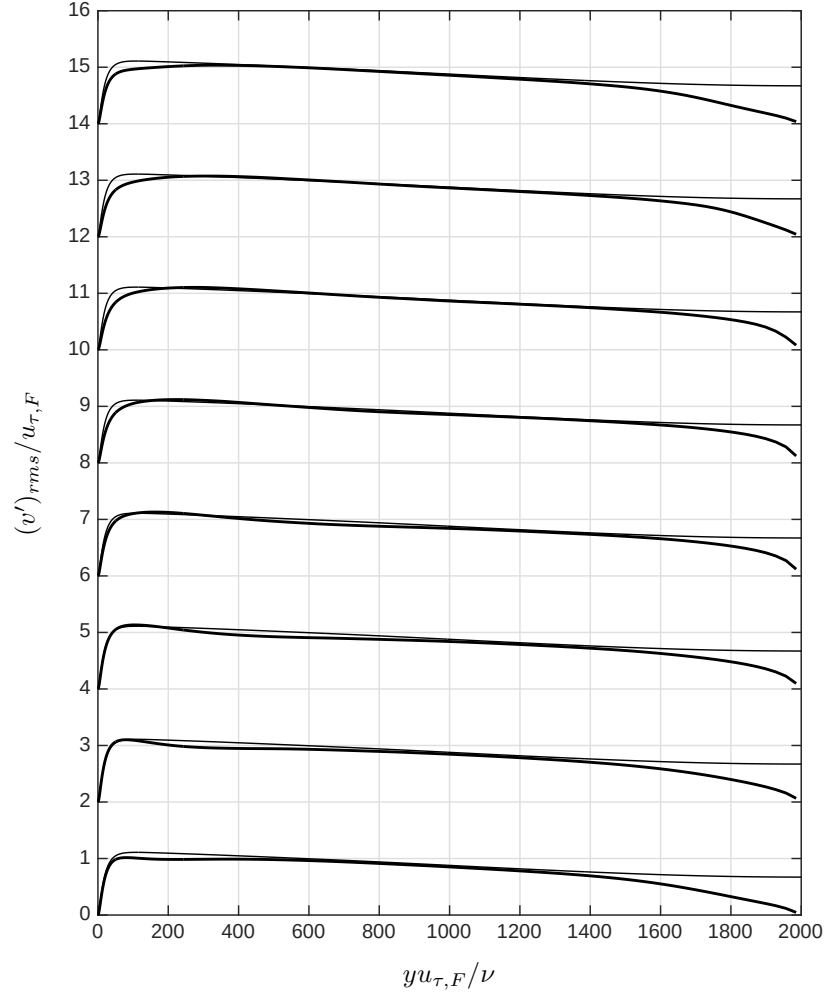


Figure 3.10: Wall-resolved LES. **RMS wall-normal velocity fluctuations** profiles at stations $x/h = 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75$, and 0.875 . Each station is shifted upwards by 2 units for clarity. Blown half-channel WRLES (thick solid), and standard channel flow DNS of Hoyas and Jimenez [57] (thin solid).

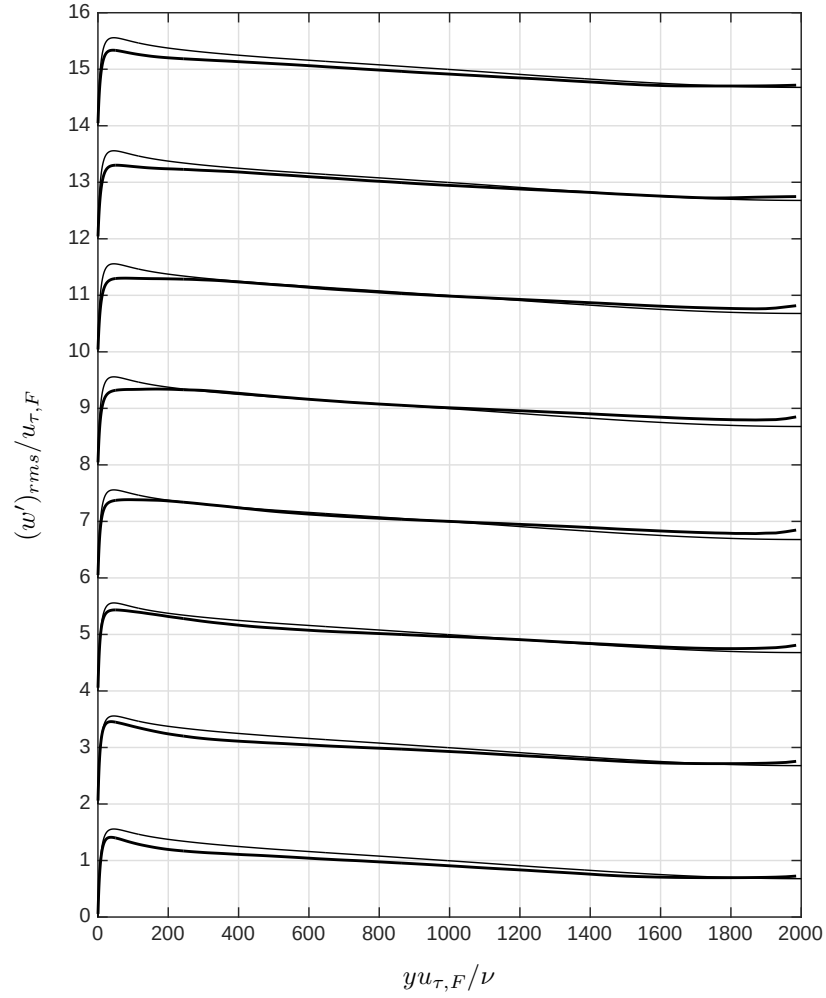


Figure 3.11: Wall-resolved LES. **RMS spanwise velocity fluctuations** profiles at stations $x/h = 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75$, and 0.875 . Each station is shifted upwards by 2 units for clarity. Blown half-channel WRLES (thick solid), and standard channel flow DNS of Hoyas and Jimenez [57] (thin solid).

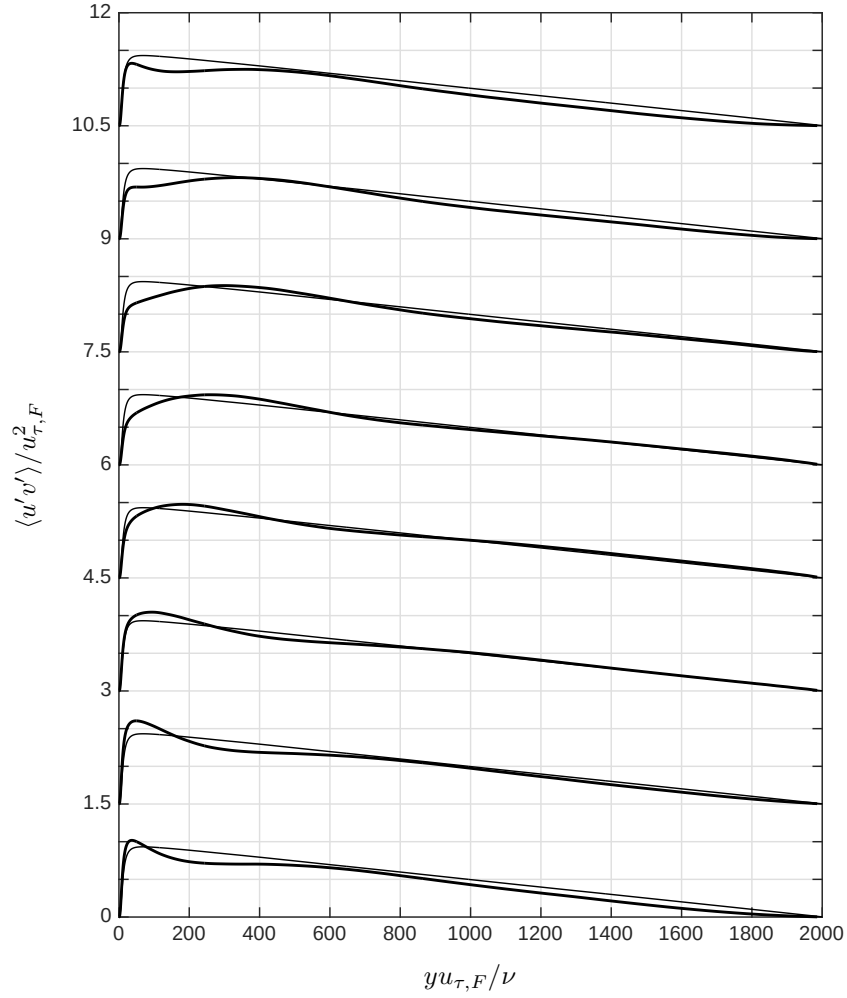


Figure 3.12: Wall-resolved LES. **Turbulent shear stress profiles** at stations $x/h = 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75,$ and 0.875 . Each station is shifted upwards by 1.5 units for clarity. Blown half-channel WRLES (thick solid), and standard channel flow DNS of Hoyas and Jimenez [57] (thin solid).

3.4.1 Algebraic wall models

The algebraic wall models described in Section 2.5 use a law of the wall that only account for wall friction and is therefore independent of pressure gradient effects. When a pressure correction is applied, the same procedure as before can be used, with a slight modification. Shih et al. [113] proposed a modified LOTW to account for both wall friction and pressure gradient. They later proposed a wall model based on that new LOTW, see Shih et al. [114]. The LOTW they propose identifies two separate contributions to the velocity field, one being linked to the wall friction, the other being being linked to the pressure gradient:

$$\mathbf{u} = u_\tau f_1 \left(\frac{y u_\tau}{\nu} \right) \hat{\mathbf{n}}_\tau + u_p f_2 \left(\frac{y |u_p|}{\nu} \right) \hat{\mathbf{n}}_p = \mathbf{u}_1 + \mathbf{u}_2, \quad (3.17)$$

where $\hat{\mathbf{n}}_\tau$ and $\hat{\mathbf{n}}_p$ are unit vectors in the direction of the wall shear stress and of the pressure gradient, respectively. Each of these functions was tuned to be correct when the other is zero. They used Spalding's LOTW for f_1 and proposed a piecewise representation of f_2 , also based on a Spalding-type law to account for both the logarithmic and the viscous layers. Here, we use Reichardt's LOTW for f_1 and the original f_2 , with adjusted coefficients so as to obtain a continuous function. Indeed, the original set of coefficients proposed by Shih et al. [114] yields a discontinuous function, which is not desirable. We therefore propose a slightly modified set of coefficients such that the curve is continuous, see Appendix A. The wall model is then imposed as follows. First, the locally averaged pressure gradient $\langle \nabla P \rangle$ is computed, as well as $u_p = (\nu \langle \nabla P \rangle)^{1/3}$. From this, $\mathbf{u}_2$ is obtained:

$$\mathbf{u}_2 = u_p f_2 \left(\frac{y |u_p|}{\nu} \right) \frac{\langle \nabla P \rangle}{|\langle \nabla P \rangle|}, \quad (3.18)$$

and the velocity $\mathbf{u}_1 = \mathbf{u} - \mathbf{u}_2$ is computed. The *pressure corrected* velocity $\mathbf{u}_1$ is then used to obtain u_τ from

$$|\langle \mathbf{u}_1 \rangle| = u_\tau f_1 \left(\frac{y u_\tau}{\nu} \right). \quad (3.19)$$

The local wall shear stress can then be computed by substituting u_τ and $\mathbf{u}_1$ either in Eq. (2.70) or in Eq. (2.71):

$$\tau_w = \rho \frac{\mathbf{u}_1}{|\langle \mathbf{u}_1 \rangle|} u_\tau^2, \quad (3.20)$$

or

$$\tau_w = \rho \frac{\mathbf{u}_1 |\mathbf{u}_1|}{|\langle \mathbf{u}_1 | \mathbf{u}_1 \rangle|} u_\tau^2. \quad (3.21)$$

This LOTW is strictly equivalent to that of Reichardt in channel flow, since $p^+ = 0$ in that case (recall we do not account for the forcing pressure gradient). The results obtained in channel flow will therefore be strictly similar, provided that the averaging is sufficient to suppress the fluctuating part of the pressure gradient. In fact, we do not need the pressure gradient fluctuations to be suppressed totally, but merely that their remaining part is such that their contribution to the LOTW cancels out on average. As previously, only the quadratic version is used in what follows.

3.4.2 RANS sub-layer models

In the RANS sublayer models we are considering, the pressure gradient term is not neglected, and pressure gradient effects are therefore already naturally present. Nevertheless, the eddy viscosity RANS model ν_t used can account for further effects due to the pressure gradient. Two such models are investigated. The first is the model used by Duprat et al. [37] (and based on the work of Nituch et al. [95]). They defined a compound velocity scale accounting for both pressure gradient and wall friction:

$$u_{\tau,p} = \sqrt{u_\tau^2 + u_p^2} \quad y^{\tau,p} = \frac{y u_{\tau,p}}{\nu}, \quad (3.22)$$

and the ratio of the friction velocity scale to this velocity scale is defined as:

$$\alpha = u_\tau^2 / u_{\tau,p}^2. \quad (3.23)$$

Hence, $\alpha = 0$ corresponds to boundary layer separation, and $\alpha = 1$ is the zero pressure gradient case. The model they used is

$$\nu_t = \kappa y u_{\tau,p} \left(\alpha + y^{\tau,p} (1 - \alpha)^{3/2} \right)^\beta \cdot \left(1 - \exp \left(-\frac{y^{\tau,p}}{1 + A\alpha^3} \right) \right)^2, \quad (3.24)$$

We here used $\kappa = 0.38$, $\beta = 0.78$, and $A = 17$. Again, the forcing pressure gradient is not taken into account while computing ν_t . Note that we here use the eddy viscosity model in the full TBL equations, whereas Duprat et al. [37] used this model without time derivatives nor convective terms and in a verti-

cally integrated approach.

The second is the model proposed by Granville [45], based on the Prandtl mixing length, modified by Granville [46] to obtain the proper $(y^+)^3$ slope:

$$\nu_t = \kappa y u_\tau (1 + ap^+) \left(1 - e^{-\left(\frac{y^+ \sqrt{1+ba p^+}}{A^+} \right)^2} \right) \quad (3.25)$$

where we here used $\kappa = 0.38$, $a = 0.9$, $b = 16.25 - 1.75 \text{sign}(p^+)$, and $A^+ = 22$.

3.4.3 Validation of the models in classical channel flow

The various approaches are first tested in the standard channel flow case of Section 2.5, thus without fluctuating pressure gradient, at $Re_\tau = 2000$, and against the DNS data of Hoyas and Jimenez [57]. In the absence of pressure gradients, the model of Shih et al. reverts to the LOTW of Reichardt. When local averaging is performed, however, small pressure fluctuations may subsist, and we therefore here test whether the effect of such a fluctuating pressure field cancels in the mean or not. We here use the same mesh as for the baseline of Section 2.5, and performed averaging over 5×5 points. When RANS sublayer models were used, the LES field was averaged over 5×5 points, and the turbulent viscosity correction was used. When the model of Duprat et al. was used, additional time filtering was required, with $c_{exp} = 0.01$ to damp the pressure gradient fluctuations. The resulting velocity profiles are displayed in Fig. 3.13. As expected, we see that, in the absence of additional pressure gradient, the results are in good agreement with the DNS data.

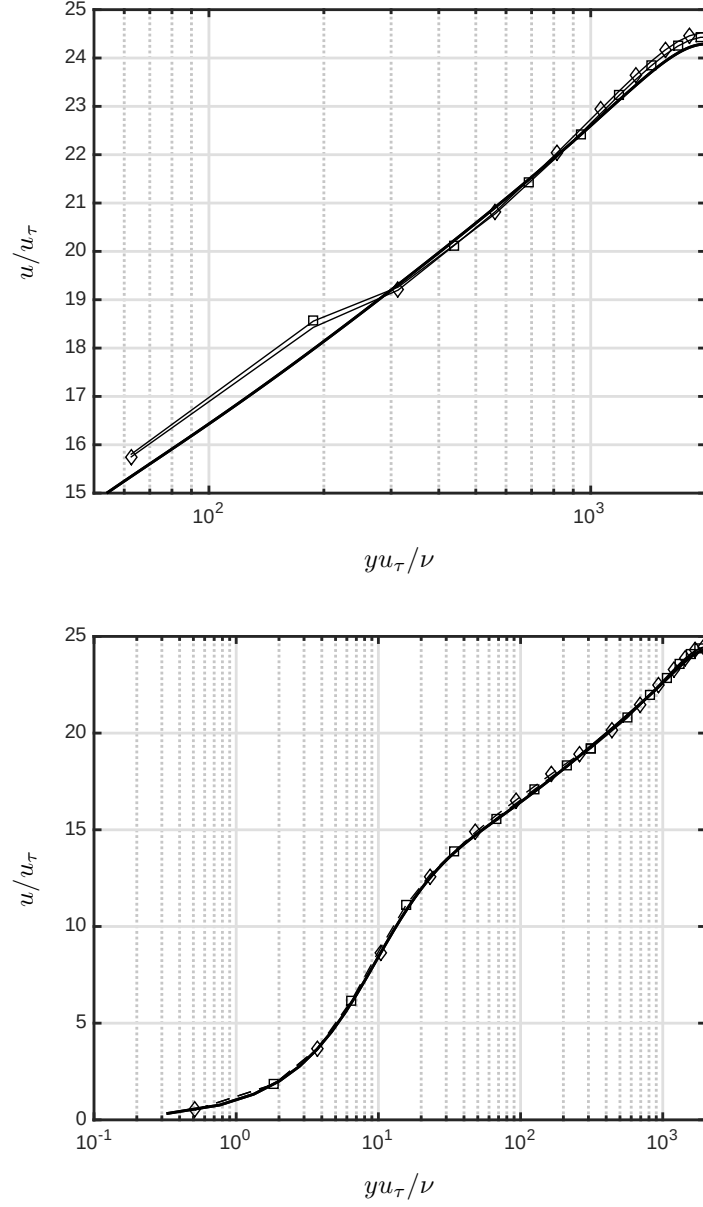


Figure 3.13: Channel flow at $Re_\tau = 2000$: **Velocity profiles.** DNS of Hoyas and Jimenez [57] (thick solid). Top: quadratic wall model with Reichardt's LOTW (squares) and with Shih et al.'s LOTW (diamonds). Bottom: RANS sublayer models with the model of Duprat et al. (diamonds) and with the model of Granville (squares). The markers do not show all grid points for the sake of clarity.

3.5 Assessment of the wall models in the half-channel with blowing and suction

The pressure corrected wall-modeled described above as well as the other wall models described in Sections 2.5 and 2.6 are assessed against the wall-resolved LES. Simulations are performed using the quadratic algebraic wall model, using the law of Reichardt and using the law of Shih et al. RANS sublayer models are also used with the model of Cabot, the model of Duprat et al., and the model of Granville. A simulation with a no slip boundary condition (under-resolved LES) is also run for comparison.

3.5.1 Algebraic wall models

The friction velocity obtained using the algebraic wall models is shown and compared to the wall-resolved and under-resolved LES's in Fig. 3.14. The velocity profiles obtained with the algebraic wall models are shown in Fig. 3.15 and Fig. 3.16. We see that Reichardt's LOTW, with no pressure gradient correction, performs quite well on both the velocity profiles and the wall friction. It follows the u_τ curve of the WRLES accurately, but predicts the minimum wall friction a bit later than the WRLES. The velocity profiles also compare well with the WRLES: the bulk of the flow is very well predicted, but, close to the wall, the velocity is a bit overestimated. The velocity profiles obtained with the model of Shih et al. is also in good agreement with the WRLES, but this is obtained at the cost of a poor wall friction prediction. This might be linked to the fact that the mean (over both spanwise and streamwise directions) wall shear stress is roughly imposed by the forcing pressure gradient. Since the pressure contribution is periodic, it is zero in the mean, and the link between the mean wall shear stress and the mean mass flow rate is therefore controlled by the f_1 part of the model of Shih et al., which is the law of Reichardt. This, in turn, explains why the mass flow rate is well recovered. We see that the model by Shih et al. here fails at predicting u_τ and exhibits a stronger reaction to the pressure gradient than the WRLES. To better understand the situation, an "a priori" analysis is carried: Fig. 3.15 and Fig. 3.16 show the profiles computed using Shih et al.'s LOTW when using the values of u_τ and u_p as extracted from the WRLES. We see that the pressure gradient correction is too intense, hence the predicted profiles are overreacting to the pressure gradient. In the wall-modeled LES, the model adapts its u_τ to recover the profiles, and therefore

fails at predicting u_τ . In comparison, the under-resolved simulation failed to recover either the wall friction or the velocity profiles. The wall models clearly provided major improvements, but the addition of pressure gradient effects, with the law of Shih et al. here proved inefficient. Fig. 3.17 and Fig. 3.18 show the profiles of turbulent shear stresses obtained with the algebraic models. We see that good agreement with the WRLES is reached in the bulk of the flow, but that the predictions are less accurate as we look closer to the wall. The results obtained with the law of Reichardt are a little closer to the WRLES curve than those obtained with the law of Shih et al. Fig. 3.19 and Fig. 3.20 show the rms fluctuations of streamwise velocity. No significant difference is observed between the various runs, even with the under-resolved LES. As in the case of the channel flow, the level of fluctuation is slightly underestimated. None of the cases managed to recover the secondary peak of u_{rms} that is observed in the WRLES, probably due to the coarseness of the mesh of the WMLES. Figs. 3.21 to 3.24 show the rms fluctuations of wall normal and spanwise fluctuations. The profiles of all simulations, including the under-resolved LES are in good agreement with the WRLES. This is consistent with what was observed in the homogeneous channel flow: the second order statistics are a function of the SGS model and of the mesh, but the wall model has only a small impact on them.

To assess the impact of the averaging strategy, additional simulations are performed with different averaging, with various types of space and/or time averaging, as described in Section 2.5.1. A case without averaging, hence with the wall models computed pointwise, is also considered (case 4). The various runs are summarized in Table 3.1.

Case	$N_{AVG,x}$	$N_{AVG,z}$	c_{exp}
0	5	5	1.0
1	3	3	0.4
2	1	25	1.0
3	1	1	0.14
4	1	1	1.0

Table 3.1: Averaging strategies investigated. Averaging can be performed both in space (over $N_{AVG,x} \times N_{AVG,z}$) or in time (if $c_{exp} < 1.0$).

Variation of the averaging strategy did not change the results obtained using Reichardt's LOTW, as seen in Fig. 3.25. In the case of the law of Shih et al., the averaging has some impact on the predicted wall friction, but the velocity profiles, shown in Fig. 3.26 and Fig. 3.27 are unchanged. Time averaging is what provided the greatest improvement to the wall friction, which is probably due to the spacial non-homogeneity of the flow, but the impact is still limited, and the results will not get as good as those obtained with the law of Reichardt, as predicted by the a priori analysis. Notably, in both cases, the best results were obtained when no averaging at all was performed (case 4). This means that, while averaging is in theory required for the law of the wall to be accurate, it is in fact not necessary in practice when wall models are used. This is of practical interest, since averaging can result in a significant increase in the computational cost of the wall model.

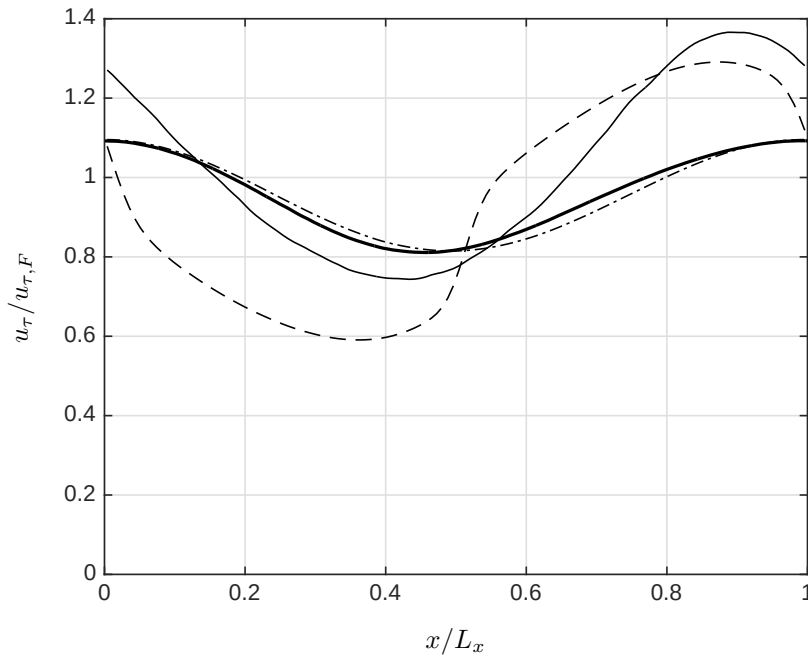


Figure 3.14: Friction velocity: WRLES (thick solid), no wall model (thin solid). Algebraic wall models with Reichardt's LOTW (dash-dot) and with Shih et al.'s LOTW (dash).

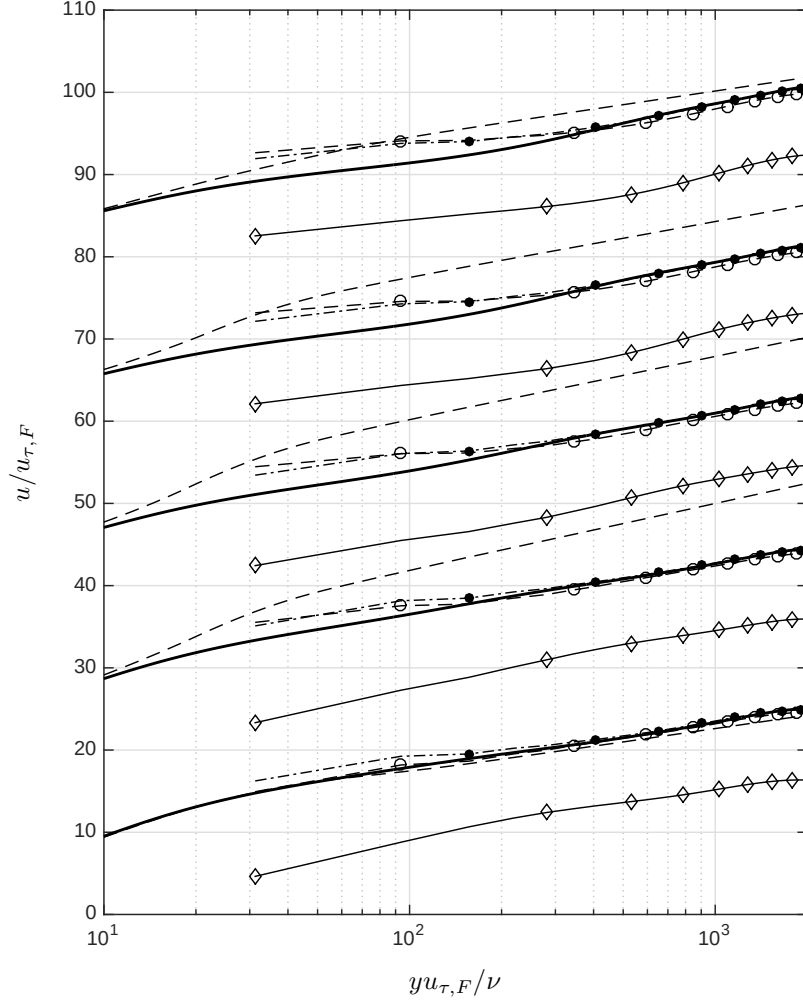


Figure 3.15: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0, 0.125, 0.25, 0.375,$ and 0.5 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), **quadratic algebraic wall model** with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). A priori profiles of Shih et al.'s LOTW, computed with the u_τ and u_p from the WRLES are shown (thin dashed). The markers do not show all grid points for the sake of clarity.

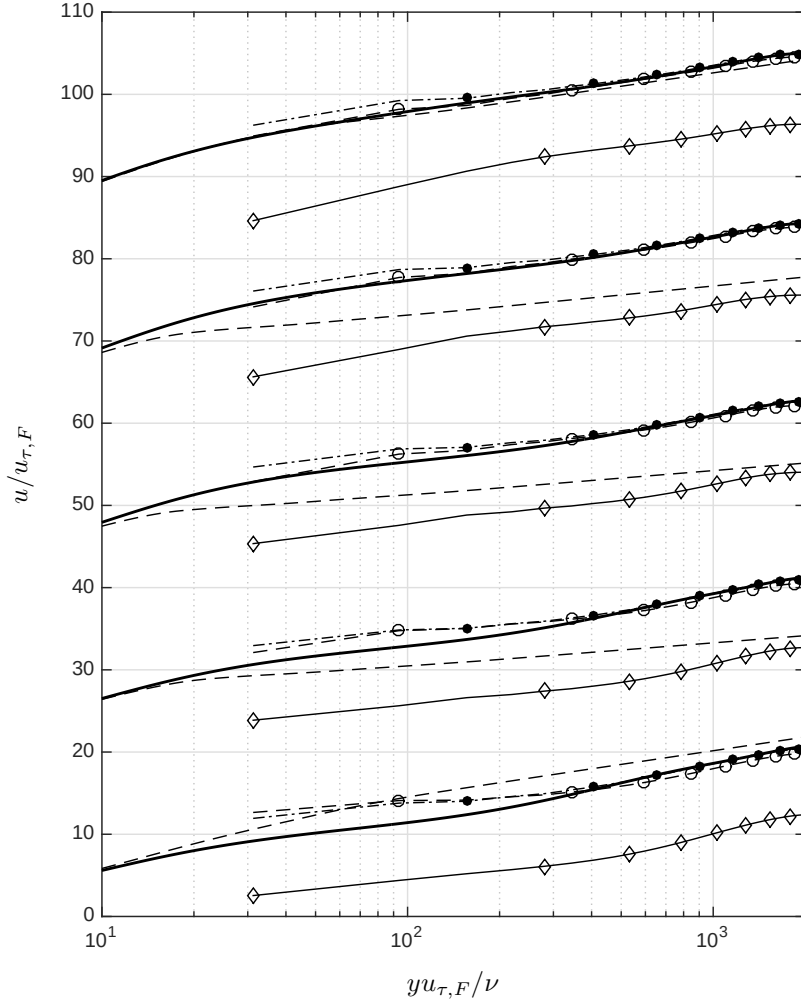


Figure 3.16: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), **quadratic algebraic wall model** with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). A priori profiles of Shih et al.'s LOTW, computed with the u_τ and u_p from the WRLES are shown (thin dashed). The markers do not show all grid points for the sake of clarity.

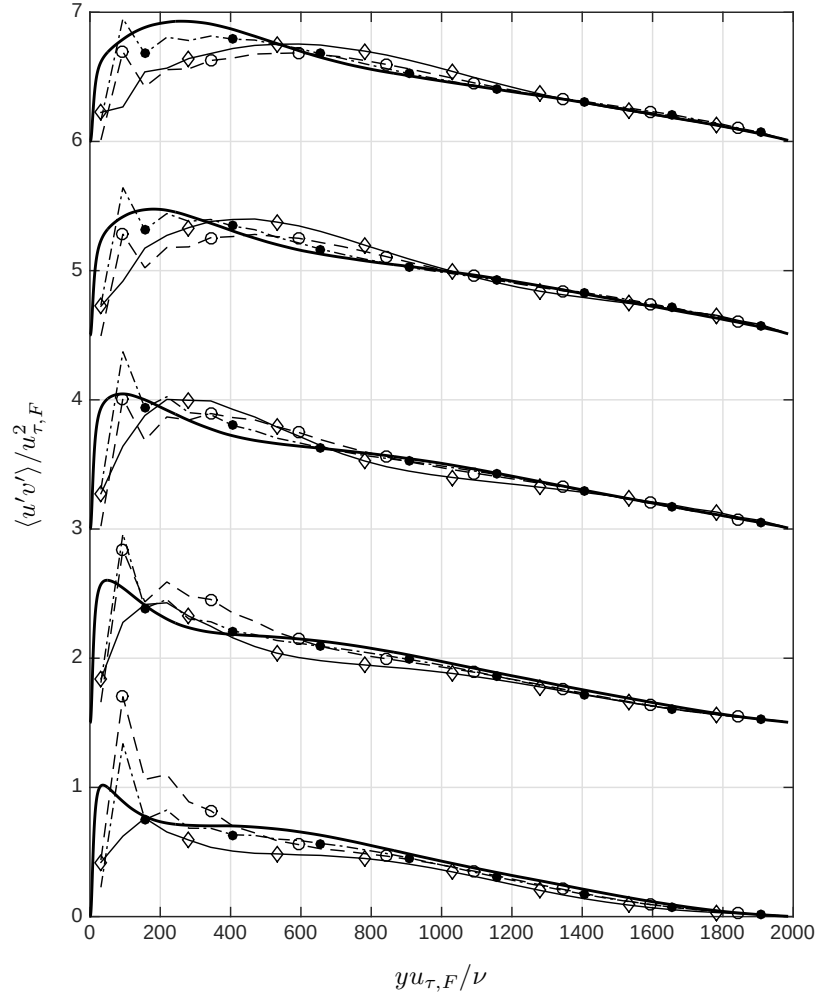


Figure 3.17: Wall-modeled LES of blown half-channel. **Turbulent shear stress** at stations $x/h = 0, 0.125, 0.25, 0.375,$ and 0.5 . Each station is shifted upwards by 1.5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), **quadratic algebraic wall model** with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

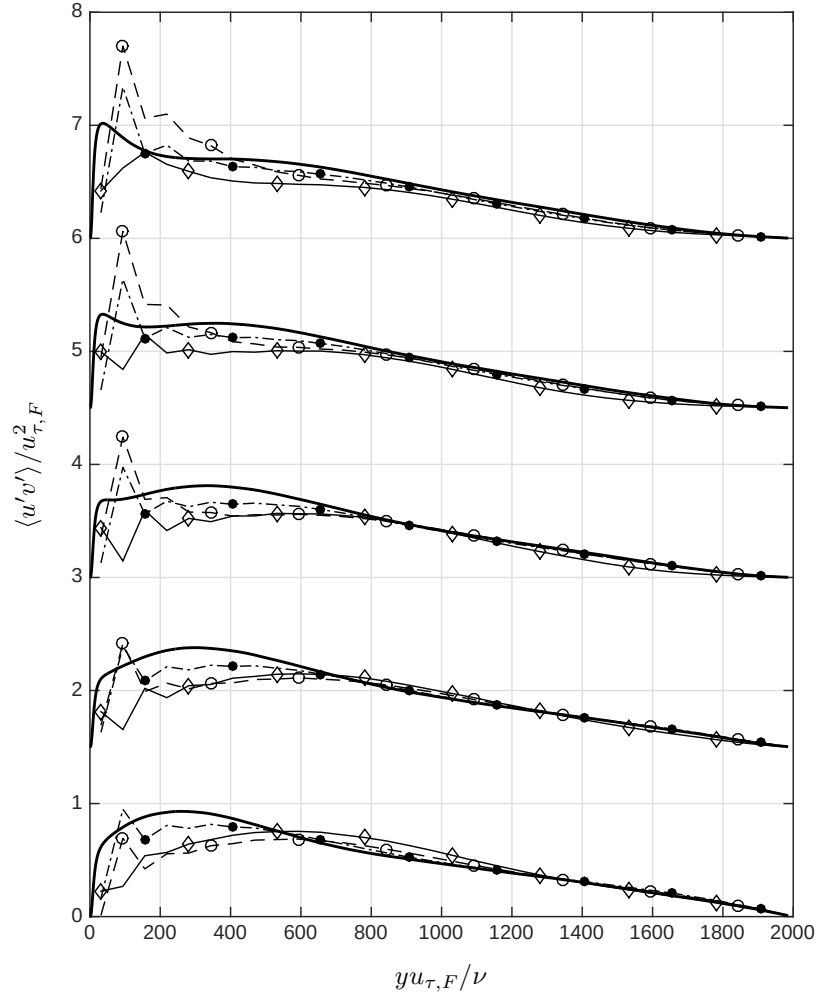


Figure 3.18: Wall-modeled LES of blown half-channel. **Turbulent shear stress** at stations $x/h = 0.5, 0.625, 0.75, 0.875, \text{ and } 1.0$. Each station is shifted upwards by 1.5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), **quadratic algebraic wall model** with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

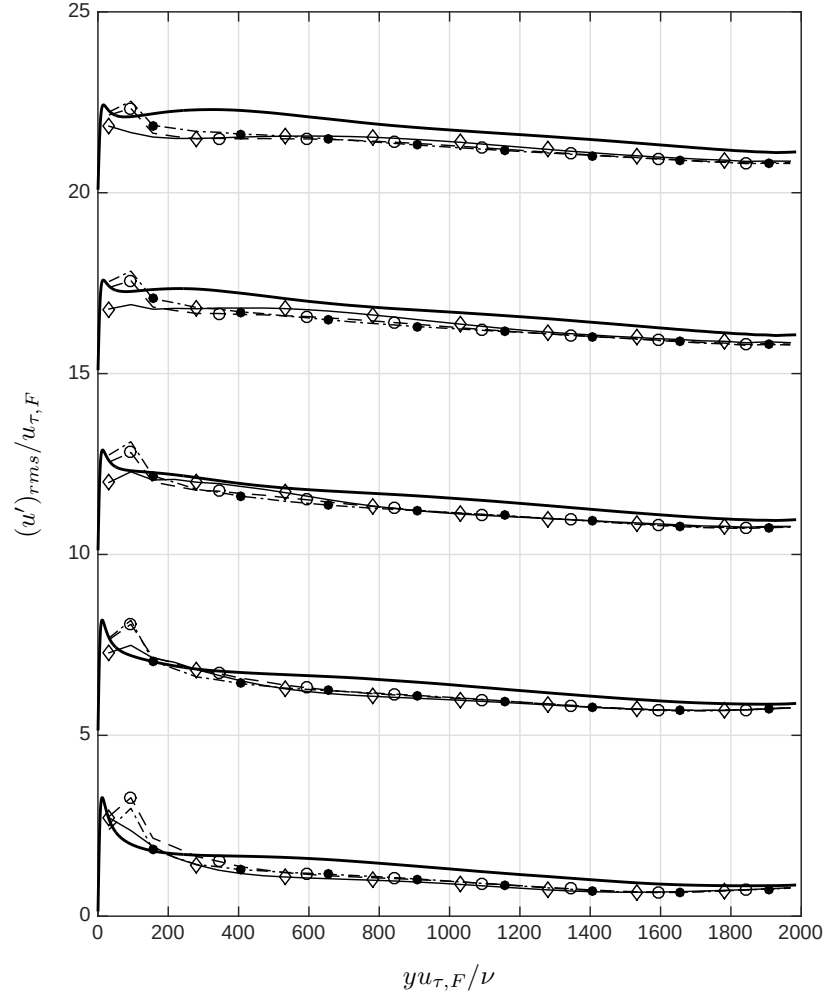


Figure 3.19: Wall-modeled LES of blown half-channel. **Streamwise rms velocity fluctuations profiles** at stations $x/h = 0, 0.125, 0.25, 0.375,$ and 0.5 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), quadratic algebraic wall model with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

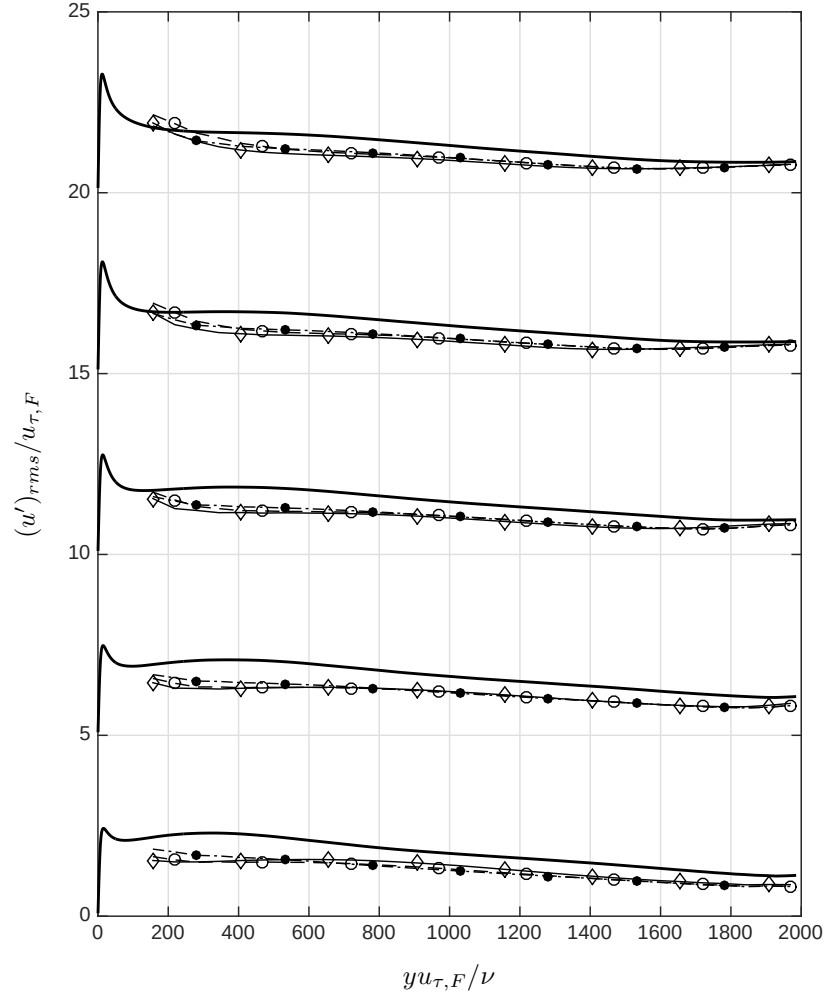


Figure 3.20: Wall-modeled LES of blown half-channel. **Streamwise rms velocity fluctuations profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), quadratic algebraic wall model with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

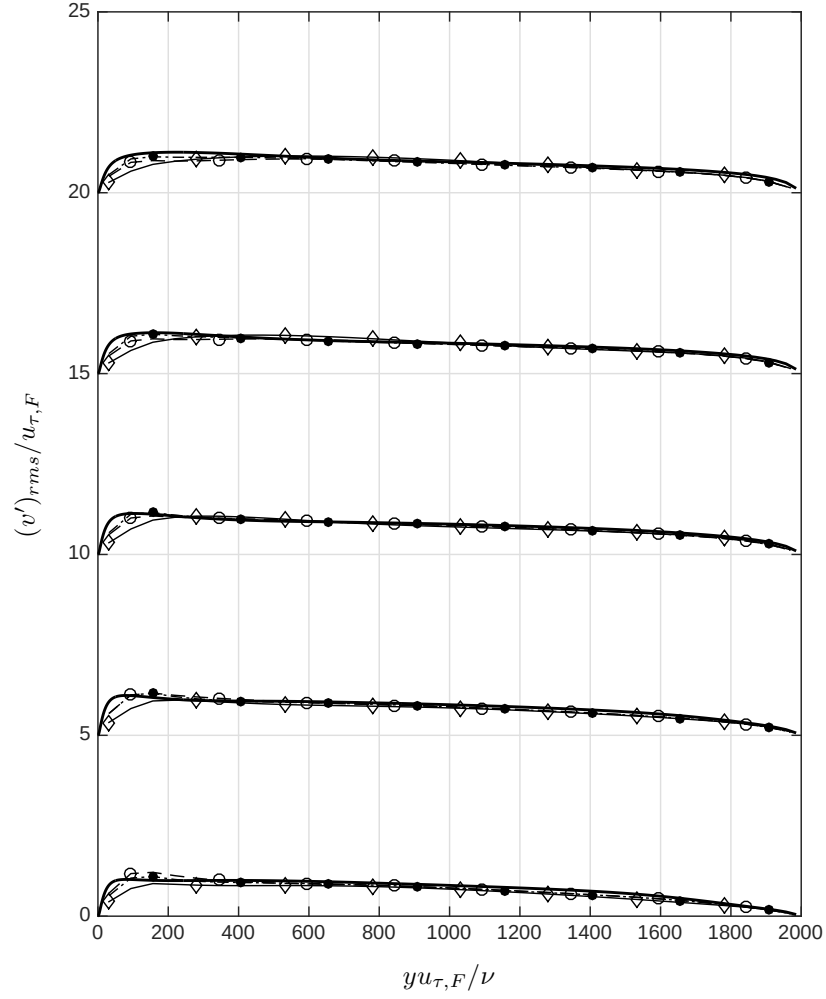


Figure 3.21: Wall-modeled LES of blown half-channel. **Wall normal rms velocity fluctuations profiles** at stations $x/h = 0, 0.125, 0.25, 0.375$, and 0.5 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), quadratic algebraic wall model with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

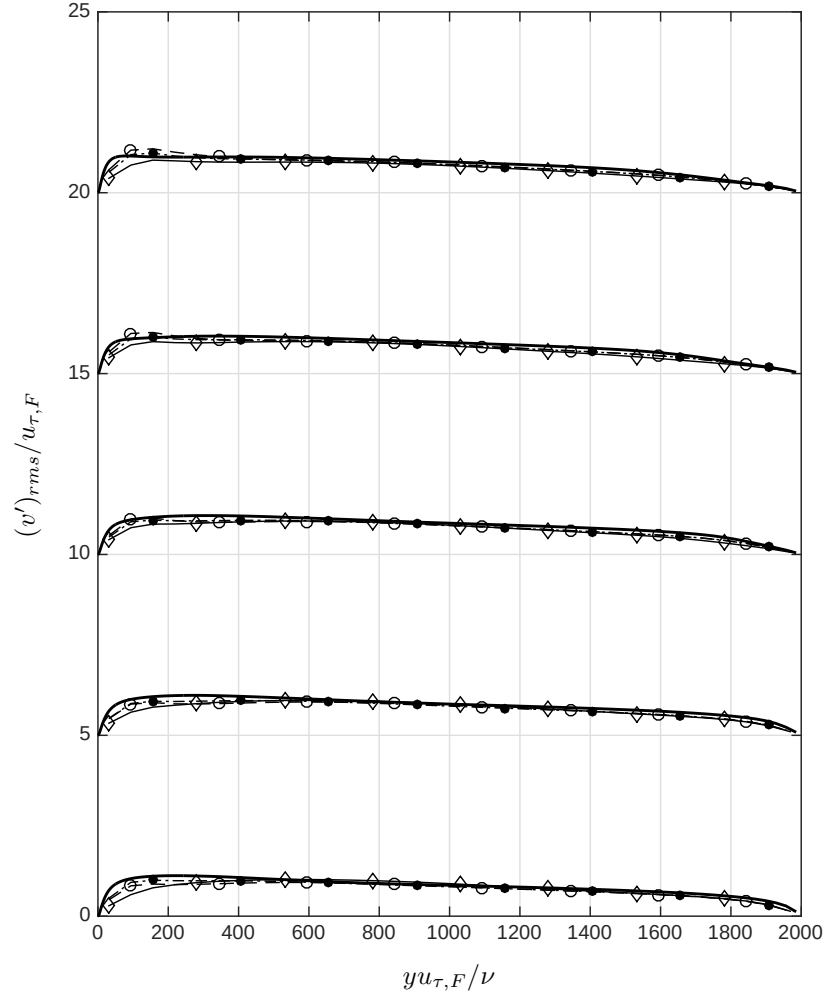


Figure 3.22: Wall-modeled LES of blown half-channel. **Wall normal rms velocity fluctuations profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), quadratic algebraic wall model with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

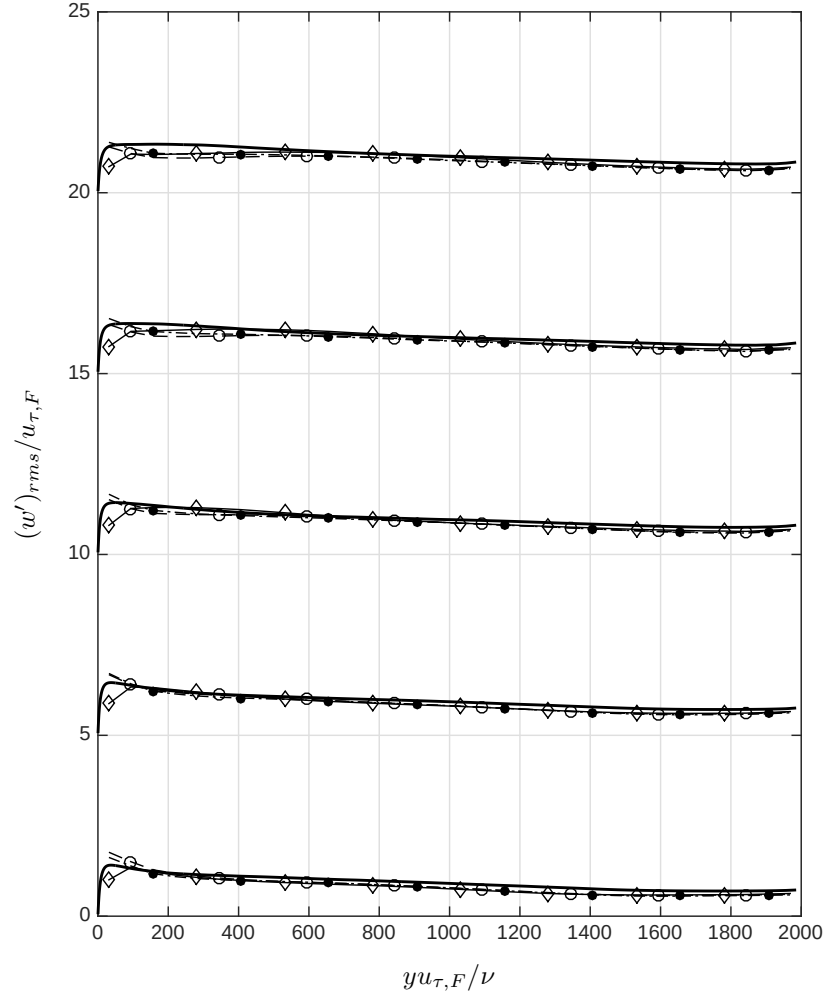


Figure 3.23: Wall-modeled LES of blown half-channel. **Spanwise rms velocity fluctuations profiles** at stations $x/h = 0, 0.125, 0.25, 0.375$, and 0.5 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), quadratic algebraic wall model with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

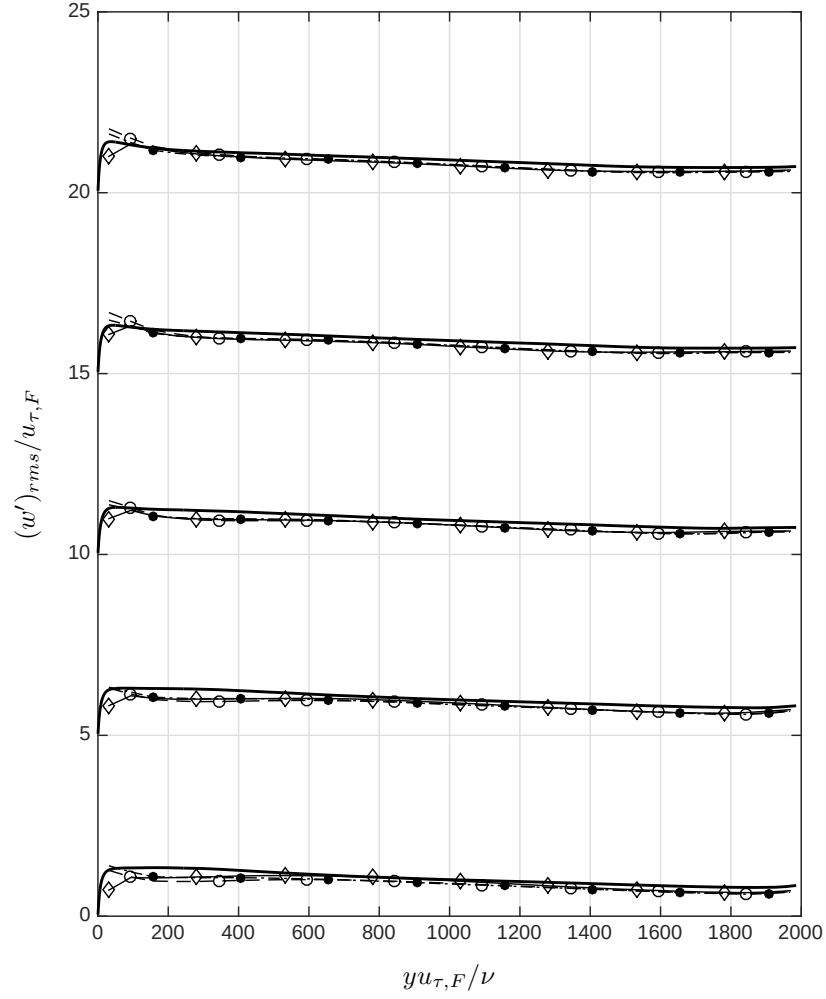


Figure 3.24: Wall-modeled LES of blown half-channel. **Spanwise rms velocity fluctuations profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), quadratic algebraic wall model with Reichardt's LOTW (dashed-dotted and bullets), and with Shih et al.'s LOTW (dashed and circles). The markers do not show all grid points for the sake of clarity.

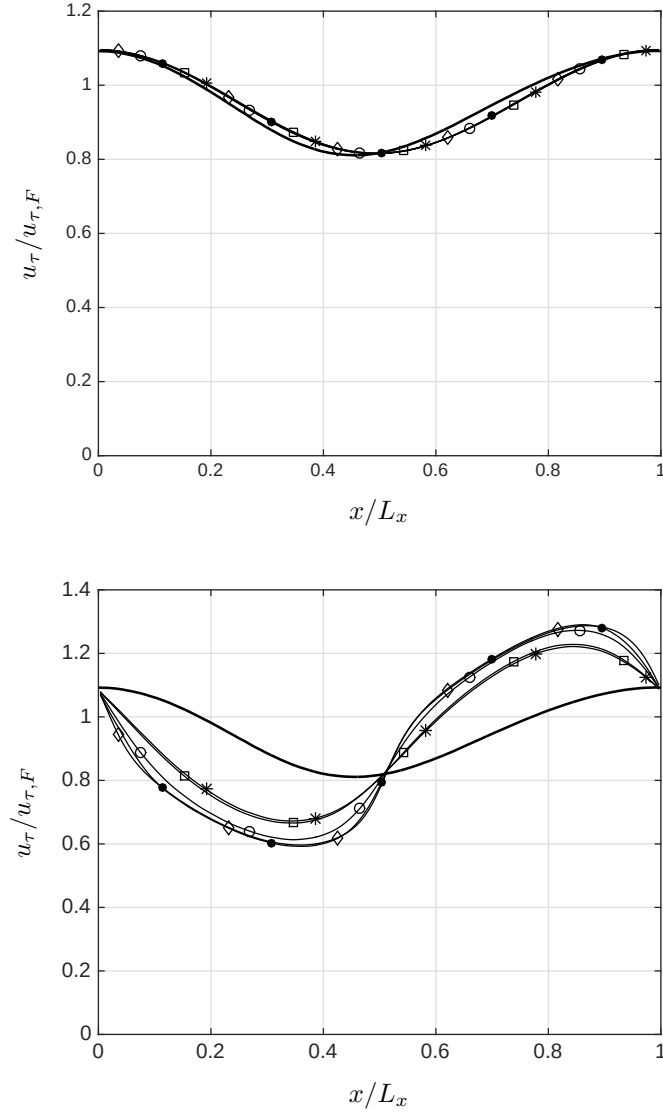


Figure 3.25: Friction velocity: quadratic algebraic wall models with Reichardt's LOTW (top) and with Shih et al.'s LOTW (bottom). Averaging over 5×5 points (diamonds), 3×3 points and $c_{\text{exp}} = 0.4$ (circles), 1×25 points (bullets), $c_{\text{exp}} = 0.14$ (squares), and no averaging (asterisks). The markers do not show all grid points for the sake of clarity.

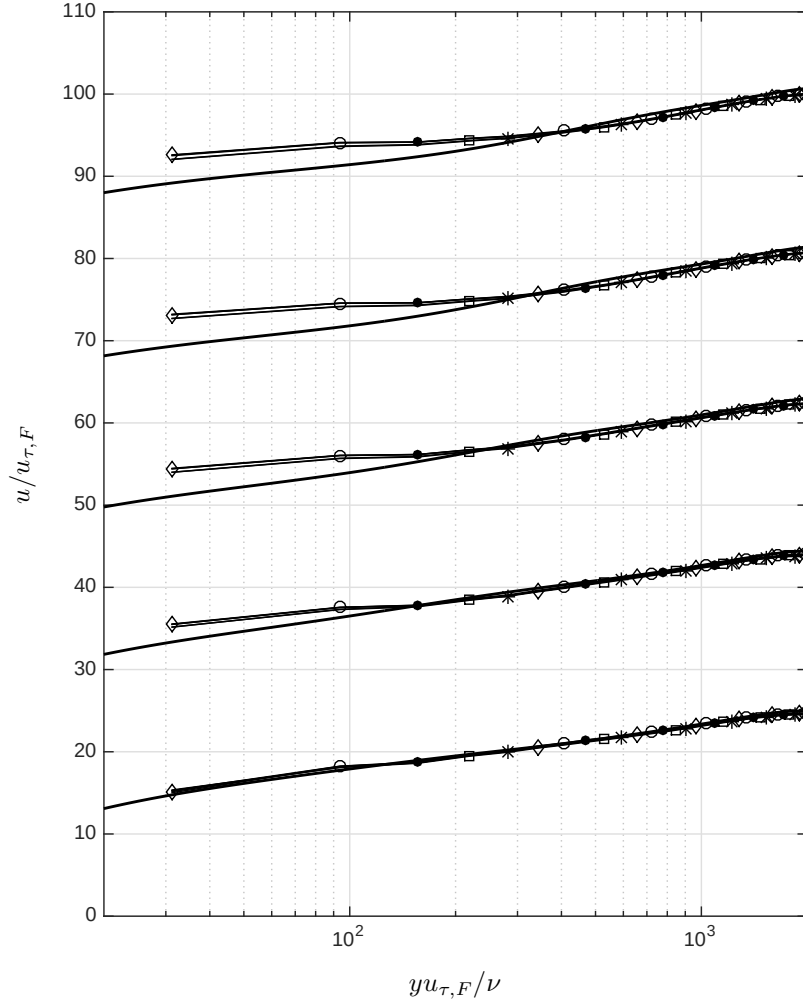


Figure 3.26: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0, 0.125, 0.25, 0.375,$ and 0.5 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), **quadratic algebraic wall model with Shih et al.'s LOTW** (thin solid and markers). Averaging over 5×5 points (diamonds), 3×3 points and $c_{\text{exp}} = 0.4$ (circles), 1×25 points (bullets), $c_{\text{exp}} = 0.14$ (squares), and no averaging (asterisks). The markers do not show all grid points for the sake of clarity.

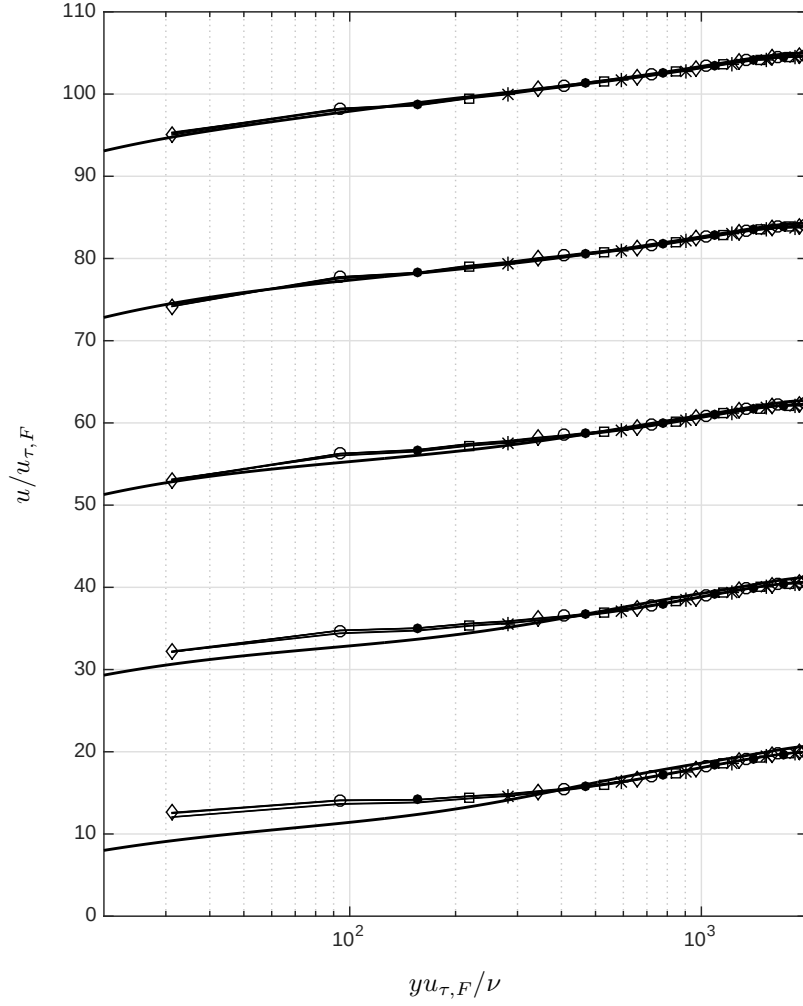


Figure 3.27: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875, \text{ and } 1.0$. Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), **quadratic algebraic wall model with Shih et al.'s LOTW** (thin solid and markers). Averaging over 5×5 points (diamonds), 3×3 points and $c_{\text{exp}} = 0.4$ (circles), 1×25 points (bullets), $c_{\text{exp}} = 0.14$ (squares), and no averaging (asterisks). The markers do not show all grid points for the sake of clarity.

3.5.2 RANS sublayer models

The friction velocity obtained using the RANS sublayer models are shown in Fig. 3.28. Fig. 3.29 and Fig. 3.30 show the velocity profiles obtained using the RANS sublayer models. All RANS sublayer models performed better than the under-resolved LES (i.e., no wall model) on both the velocity profiles and the wall friction. The results provided by the model of Cabot are the closest to the velocity profiles of the WRLES; but the three models tested here follow the trend of the WRLES rather well and remain close to the reference profiles. The model of Duprat et al. underestimates a little the global mass flow rate, resulting in slight underestimation of the velocity profiles. The wall friction obtained using the models of Duprat et al. and Granville are closer to the WRLES than that using the model of Cabot, in the sense that the amplitude of variation of the u_τ curves is smaller when using these models. The pressure gradient corrections have in this case provided some improvement. None of the RANS sublayer models, however, provide results that are better than those obtained using the algebraic wall model with Reichardt's LOTW. As pointed out by Duponcheel [35] at lower Reynolds number, RANS models have a hard time accurately predicting the velocity profiles and the wall friction in this case. This might be due to the low Reynolds number for which the RANS models are not designed. Fig. 3.31 and Fig. 3.32 show the turbulent shear stress profiles obtained with the RANS sublayer models. The same trends as with the algebraic wall models are observed: discrepancies between the WMLES and the WRLES are seen close to the wall, but the predictions are good in the bulk of the flow. In the RANS layer, the contribution of the RANS models dominates the contribution due to the turbulence sustained on the RANS mesh. The model of Granville exhibits the largest deviation from the WRLES. Fig. 3.33 to 3.38 show the rms velocity fluctuations obtained with the RANS sublayer models. We see that, though the turbulent shear stresses are small in the RANS layer, fluctuations are sustained in the RANS layer, and that the convective terms hence play a role in the simulation. As with the algebraic wall models, the WMLES did not recover the secondary peak of streamwise fluctuations, but the wall-normal and spanwise fluctuations are well recovered. This once again confirms that the turbulence developed by the LES is mostly controlled by the SGS model, and not the wall model.

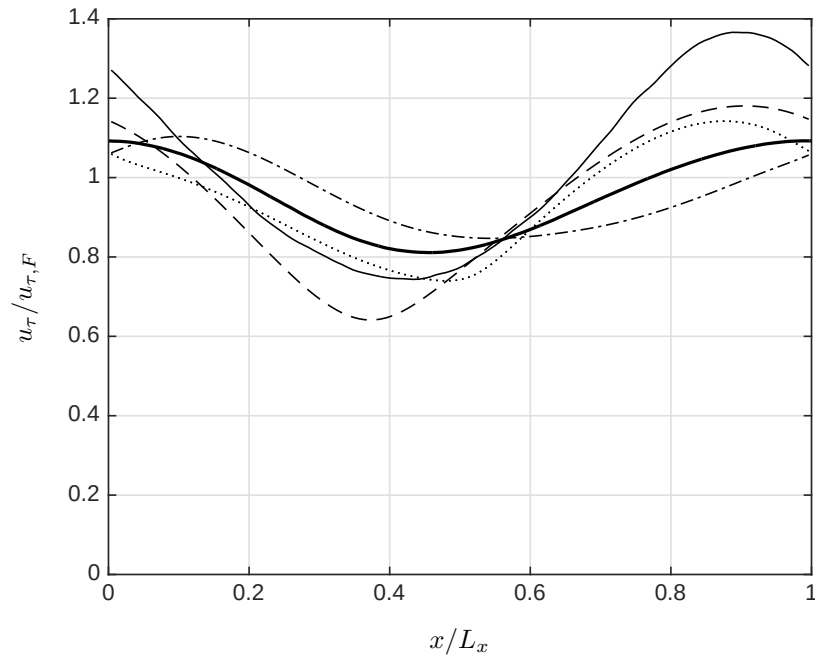


Figure 3.28: Friction velocity: WRLES (thick solid), no wall model (thin solid). RANS sublayer wall models with the model of Cabot (dashed), Granville (dashed-dotted) and Duprat et al. (dotted).

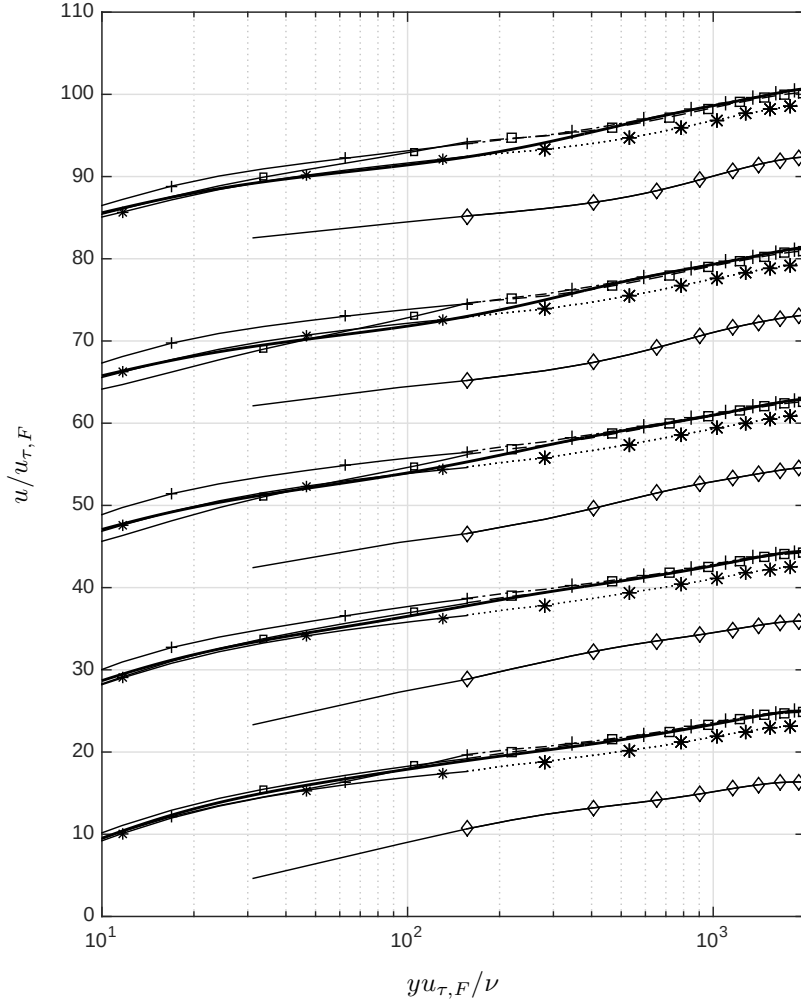


Figure 3.29: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0, 0.125, 0.25, 0.375$, and 0.5 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), **wall-modeled LES with RANS-sublayer**: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

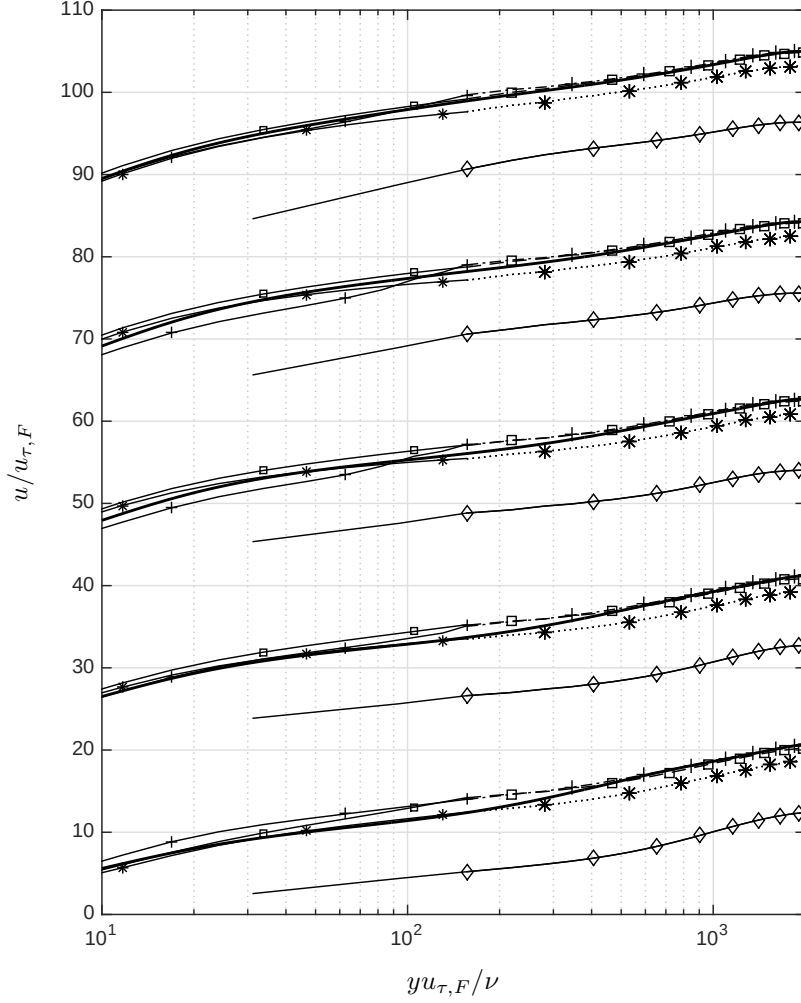


Figure 3.30: Wall-modeled LES of blown half-channel. *Mean velocity profiles* at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), **wall-modeled LES with RANS-sublayer**: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

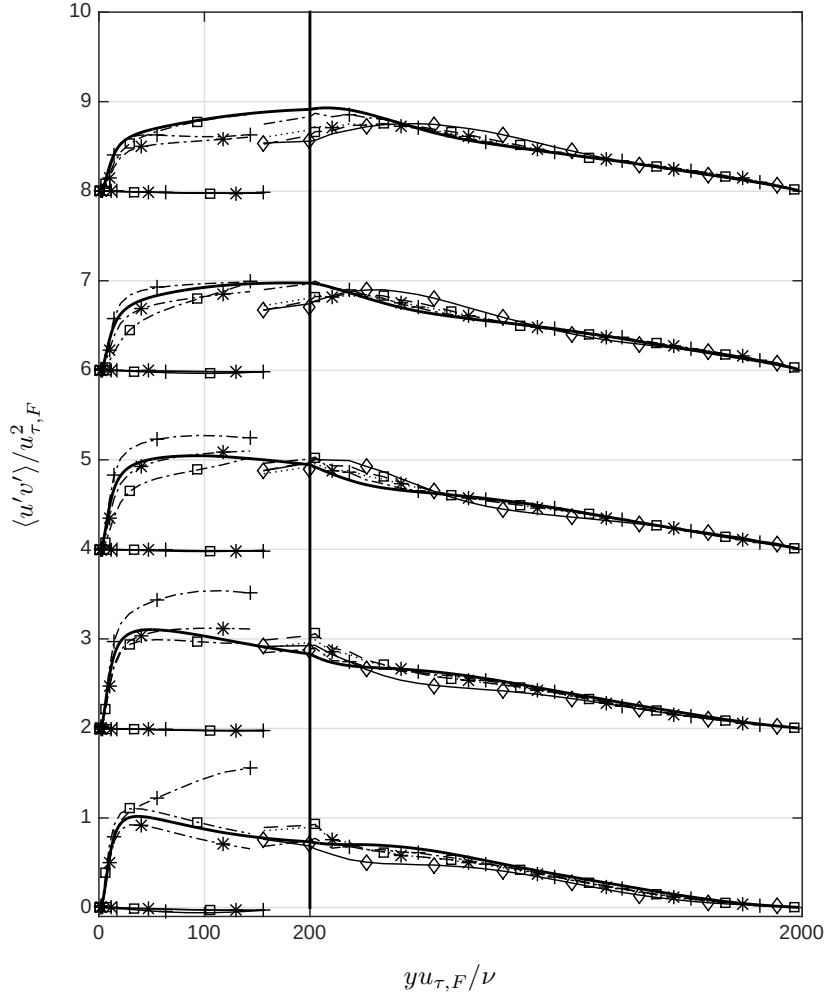


Figure 3.31: Wall-modeled LES of blown half-channel. **Turbulent shear stress** at stations $x/h = 0, 0.125, 0.25, 0.375$, and 0.5 . Each station is shifted upwards by 1.5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

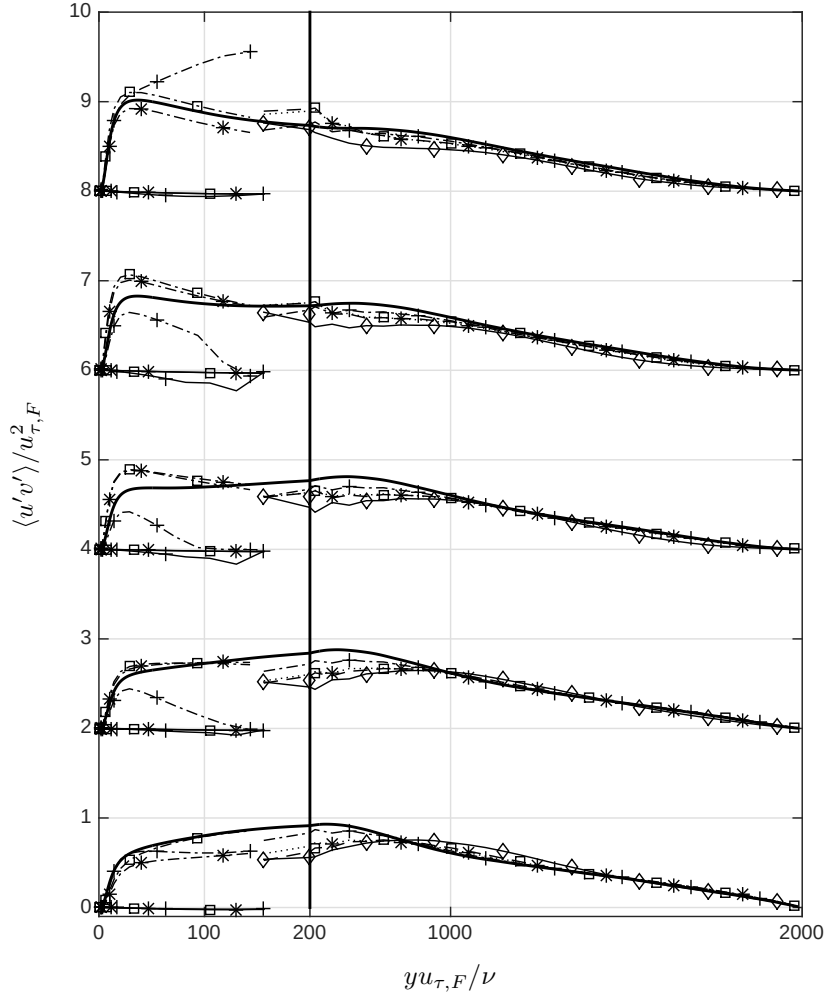


Figure 3.32: Wall-modeled LES of blown half-channel. **Turbulent shear stress** at stations $x/h = 0.5, 0.625, 0.75, 0.875, \text{ and } 1.0$. Each station is shifted upwards by 1.5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

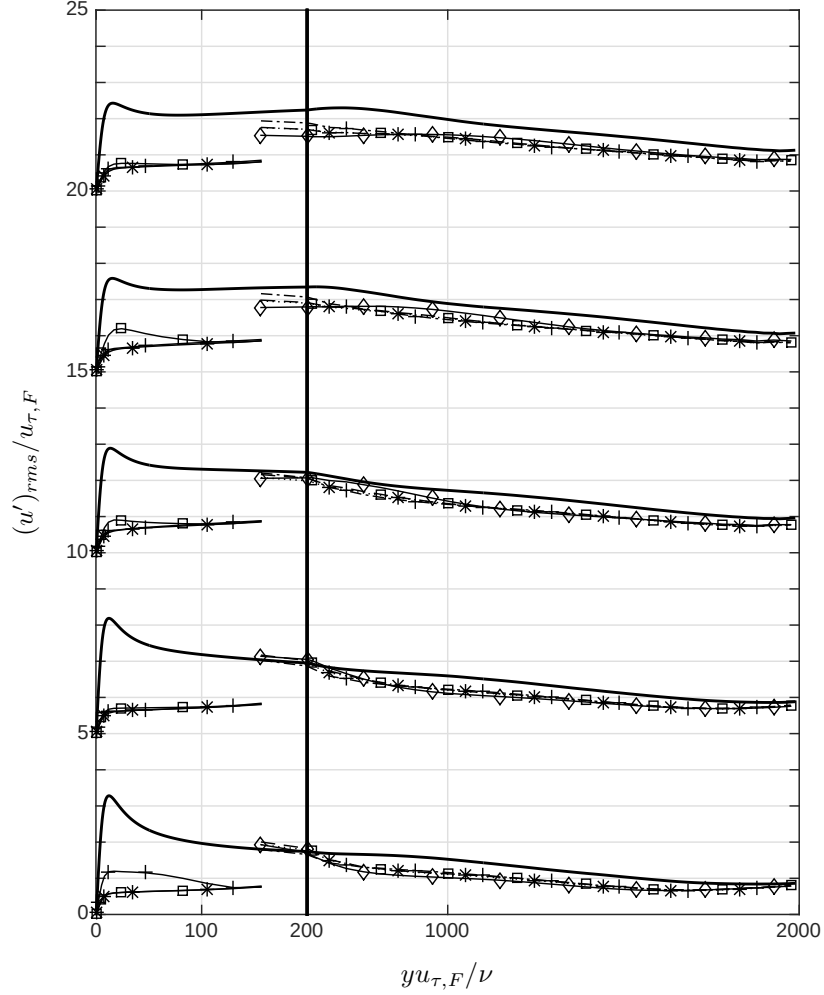


Figure 3.33: Wall-modeled LES of blown half-channel. **Streamwise rms velocity fluctuation profiles** at stations $x/h = 0, 0.125, 0.25, 0.375, \text{ and } 0.5$. Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

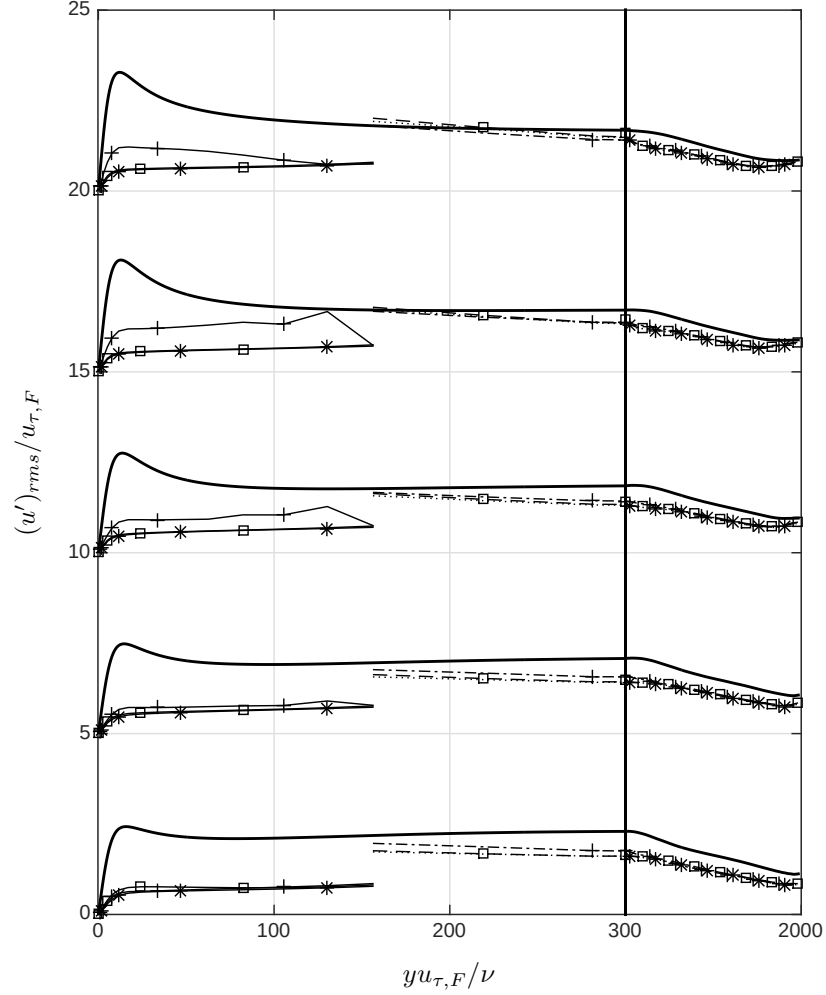


Figure 3.34: Wall-modeled LES of blown half-channel. **Streamwise rms velocity fluctuations profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

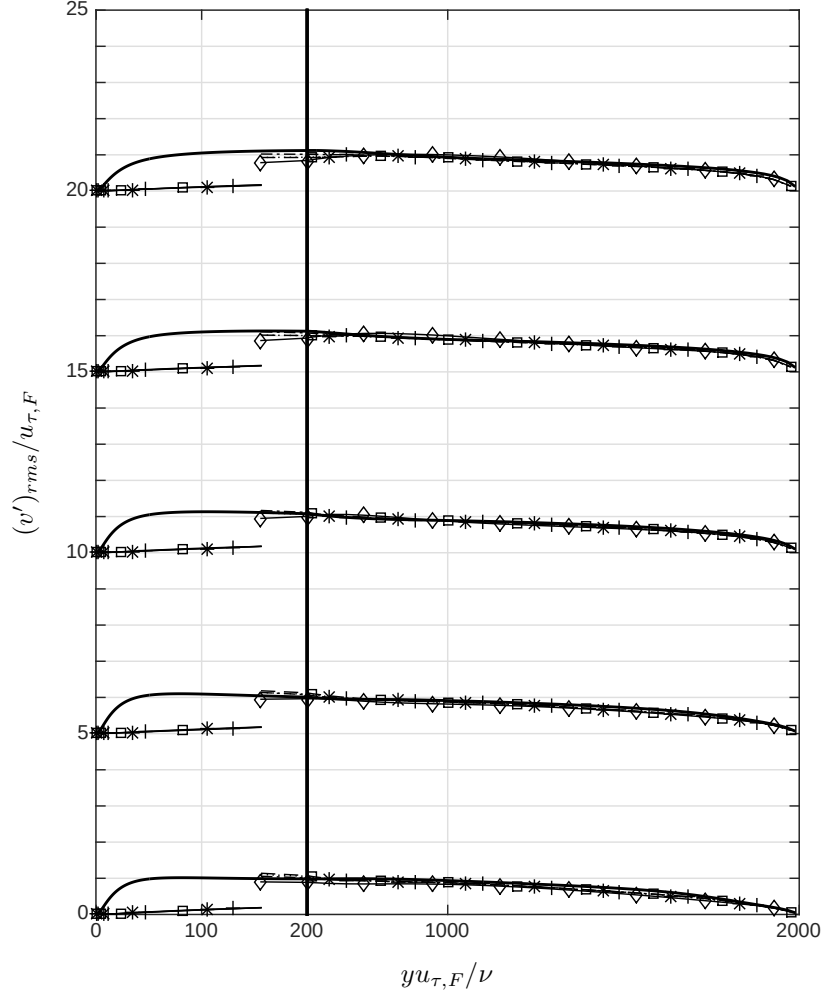


Figure 3.35: Wall-modeled LES of blown half-channel. **Wall normal rms velocity fluctuations profiles** at stations $x/h = 0, 0.125, 0.25, 0.375,$ and 0.5 . Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

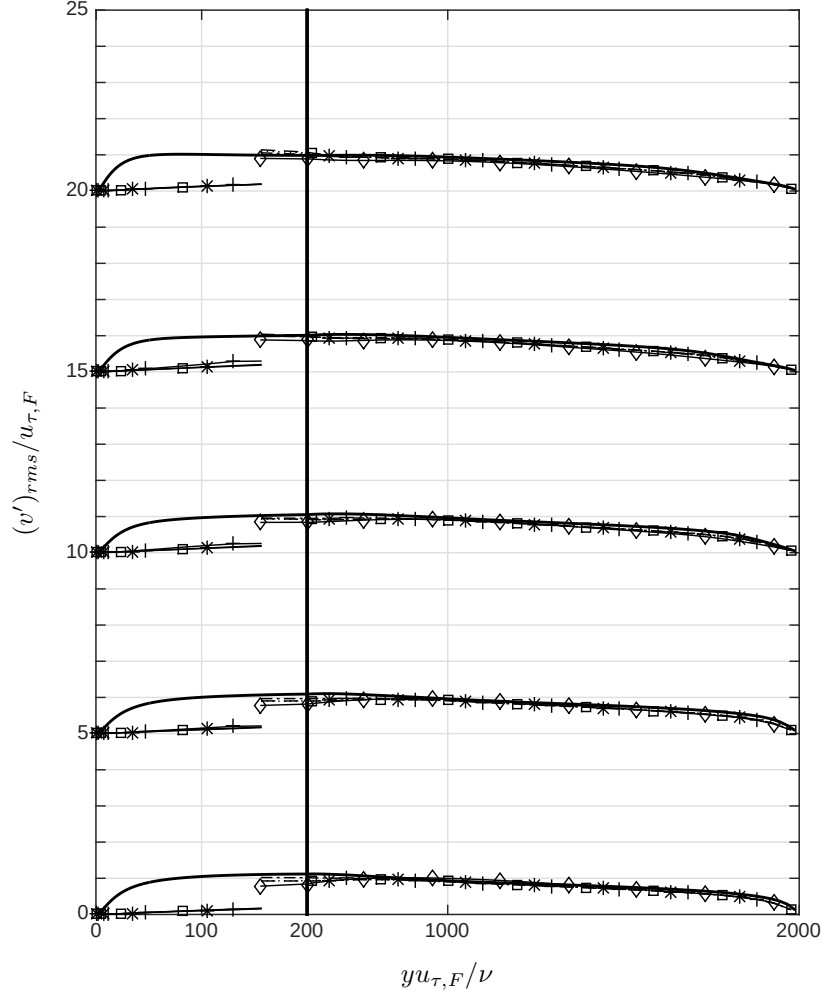


Figure 3.36: Wall-modeled LES of blown half-channel. **Wall normal rms velocity fluctuations profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875, \text{ and } 1.0$. Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

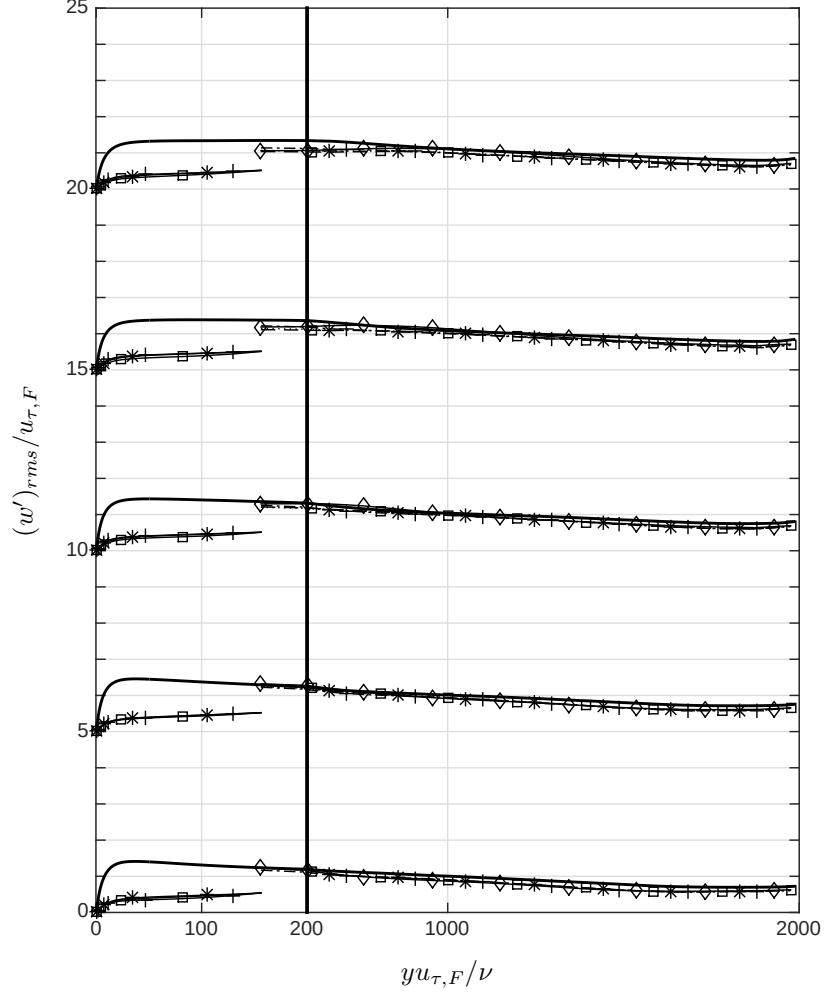


Figure 3.37: Wall-modeled LES of blown half-channel. *Spanwise rms velocity fluctuations profiles* at stations $x/h = 0, 0.125, 0.25, 0.375, \text{ and } 0.5$. Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

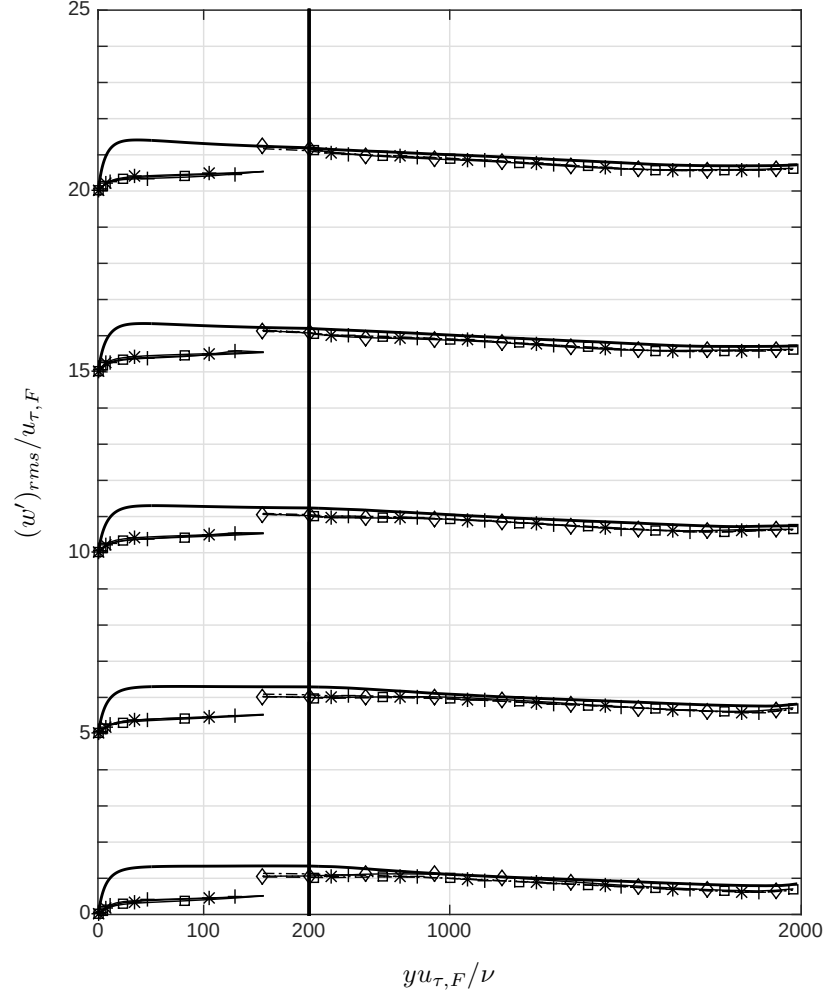


Figure 3.38: Wall-modeled LES of blown half-channel. *Spanwise rms velocity fluctuations profiles* at stations $x/h = 0.5, 0.625, 0.75, 0.875, \text{ and } 1.0$. Each station is shifted upwards by 5 units for the sake of clarity. Wall-resolved LES (thick solid), Under-resolved LES (thin solid and diamonds), wall-modeled LES with RANS-sublayer: model of Cabot (dashed and squares), Duprat et al. (dotted and asterisks), and Granville (dashed-dotted and plusses). The RANS layer is shown in thin solid, the first two LES points are not shown for the sake of readability. The markers do not show all grid points for the sake of clarity.

3.5.3 Note about the convective terms of the RANS sublayer

When the model of Cabot is used in equilibrium mode, hence dropping convective and pressure terms, the equations (2.80) reduce to:

$$\frac{\partial \mathbf{u}}{\partial t} = -(\nabla P)_F + \frac{\partial}{\partial y} \left((\nu + \nu_t) \frac{\partial \mathbf{u}}{\partial y} \right). \quad (3.26)$$

When time derivatives are negligible (or in the mean), this reduces to a one-dimensional ODE, which can be vertically integrated to yield a velocity profile implied by the ν_t model:

$$\mathbf{u}(y) = \int_0^y \frac{\boldsymbol{\tau}_w + y' (\nabla P)_F}{\nu + \nu_t} dy'. \quad (3.27)$$

This is equivalent to imposing a velocity profile $u^+ = f(y^+)$ in the local direction of the flow. Since the velocity profile implied by the ν_t model of Cabot is almost the same as the law of Reichardt, the results obtained with a RANS sublayer using the model of Cabot, and dropping the convective and pressure terms should be similar to those using the algebraic wall model with the law of Reichardt. It is indeed the case, as seen in Fig. 3.39. The only differences are due to the minor difference between the law of Reichardt and the law implied by the model of Cabot, the local variation of direction of $\boldsymbol{\tau}_w$, which is not instantaneously aligned with the forcing pressure gradient, and the time derivative. Even with these small differences, we see that both the wall friction and the velocity profiles are strictly similar (see Fig. 3.40 and Fig. 3.41), demonstrating that the two models are essentially the same.

In the original formulation of the model by Duprat et al. [37], the pressure terms are solved but the convective terms are dropped, as well as the time derivative. It is therefore useful to investigate whether the model leads to better results when either the convective or both the convective and pressure terms are dropped. Fig. 3.42 shows the wall friction obtained with such simulations. The velocity profiles are shown in Fig. 3.43 and Fig. 3.44. We see that the velocity profiles are very similar, but that the predicted wall shear stress changes by large amounts. None of the approaches, however, was able to accurately predict the wall shear stress.

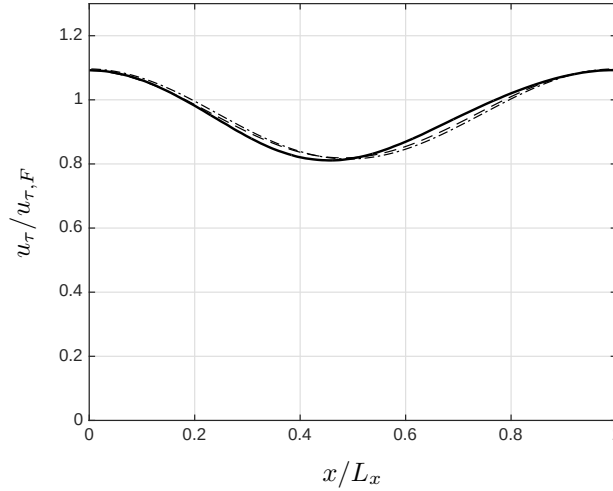


Figure 3.39: Friction velocity: wall-resolved LES (thick solid), **quadratic algebraic wall models** with Reichardt's LOTW (dashed-dotted) and **RANS sub-layer model without convective and pressure terms** and with the model of Cabot (dashed).

3.6 Conclusion

Wall-modeled LES had been investigated for a case with a mild pressure gradient, and using various strategies. Overall, accounting for the pressure gradient effects in the wall models has not provided improvement in this mild pressure gradient case.

When the algebraic wall models were considered, the simplest model (i.e., using Reichardt's LOTW) was the best option, as it recovered fairly well both the velocity profiles and the wall friction. The use of additional pressure information, with the use of the law of Shih et al. deteriorated the results. Several averaging strategies were also investigated, as well as the application of the wall models pointwise (i.e. without averaging). In the end, the use of averages proved useless when the law of Reichardt was used. When the law of Shih et al. was used, the results were even slightly better when the models were computed pointwise. Since in the context of parallel computing, averaging can be costly, it should therefore be avoided whenever possible and this tends to show that averaging could be discarded in this mild pressure gradient case.

When RANS sublayer models were used, the additional pressure corrections included in the turbulent eddy viscosity model provided some improvements;

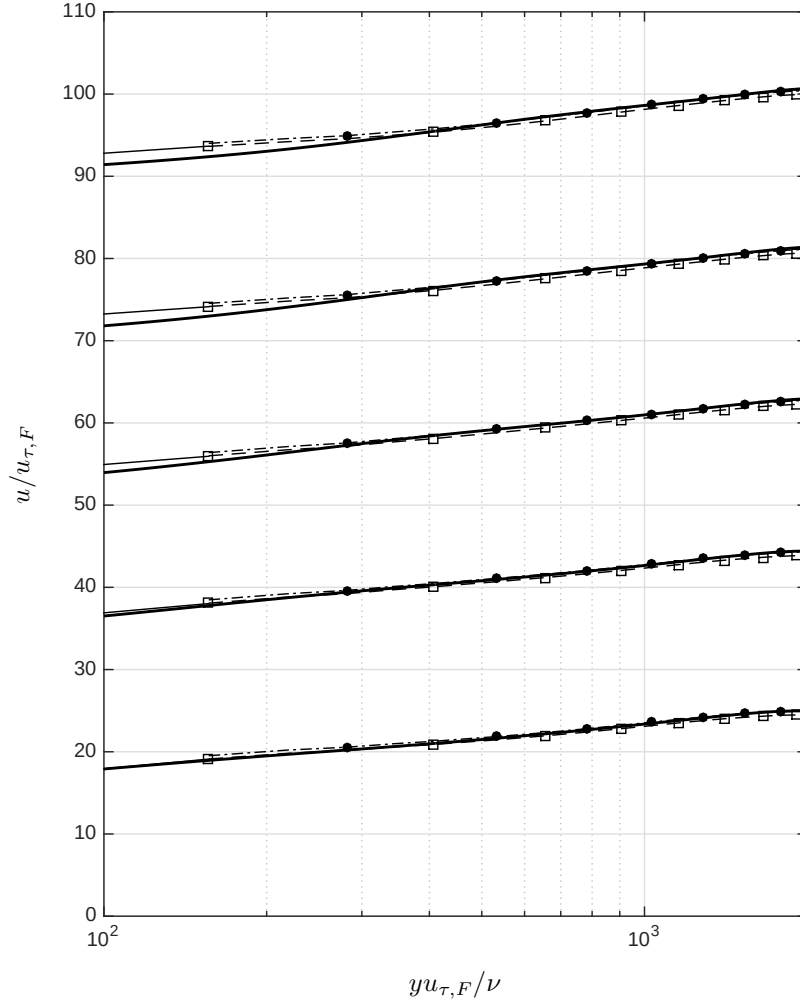


Figure 3.40: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0, 0.125, 0.25, 0.375$, and 0.5 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), **quadratic algebraic wall model with Reichardt's LOTW** (dashed-dotted and bullets), and **RANS sub-layer model without convective and pressure terms** and with the model of Cabot (dashed and squares). The markers do not show all grid points for the sake of clarity.

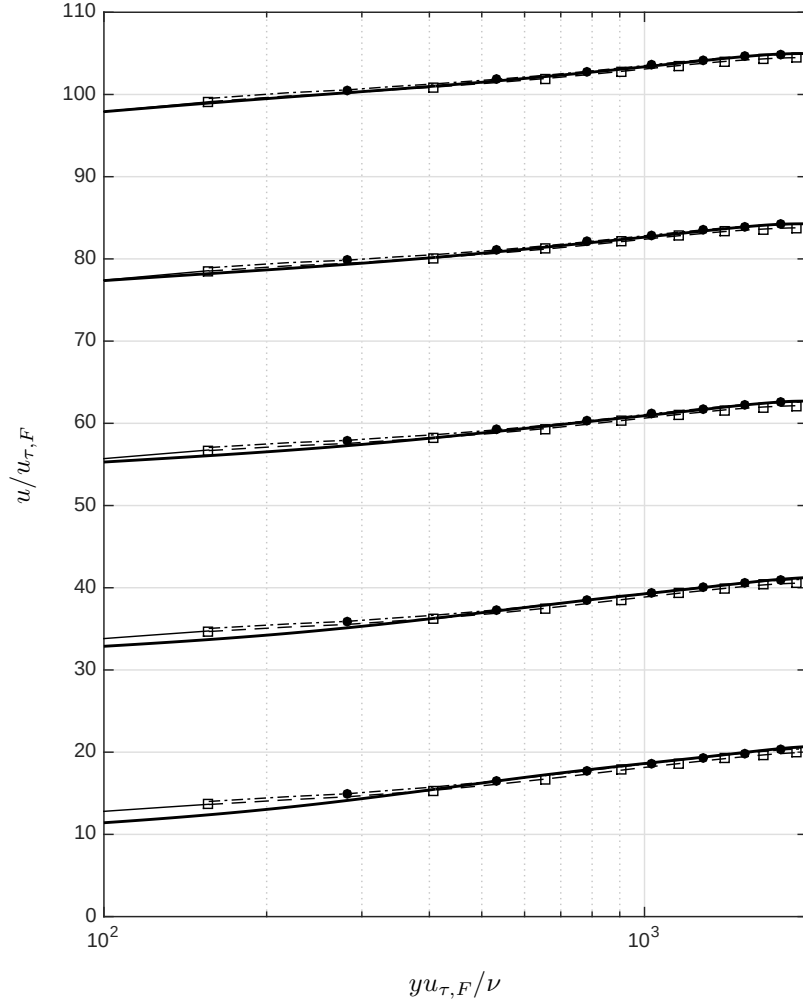


Figure 3.41: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), **quadratic algebraic wall model with Reichardt's LOTW** (dashed-dotted and bullets), and **RANS sub-layer model without convective and pressure terms** and with the model of Cabot (dashed and squares). The markers do not match with grid points.

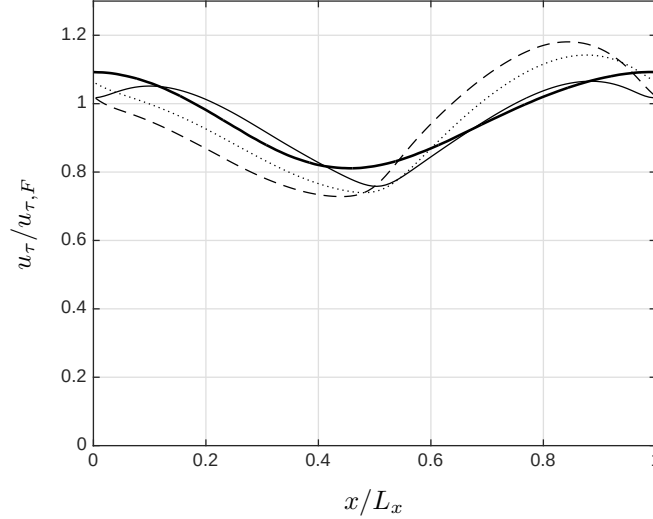


Figure 3.42: Friction velocity: wall-resolved LES (thick solid), **RANS sublayer models with Duprat et al.’s model:** with convective and pressure terms (dotted), without convective and pressure terms (thin solid), without convective terms but with pressure terms (dashed).

yet none of the models could outperform the algebraic wall model using Reichardt’s LOTW. The model by Duprat et al. was also tested without convective terms, both with and without pressure gradient terms, as the original approach was to not account for the convective terms while retaining the pressure term. The predictions of the wall shear stress were slightly improved, and the best results were obtained when both the convective term and the pressure term were dropped. It was also verified that using a RANS sublayer model is equivalent to using an algebraic wall model when the convective and pressure terms are dropped, and when time is homogeneous. Notably, the RANS sublayer model by Cabot reverts to the algebraic model with the law of Reichardt when convective and pressure terms are dropped. This model therefore also yielded better results when discarding the pressure and convective terms. This is of practical interest, since the actual computation of the convective terms can be quite costly, especially in a parallel solver. This means that the RANS sublayer models are much more computationally expensive than the algebraic models. Since none of the RANS sublayer models investigated here was able to provide better results than the algebraic model with the law of Reichardt, they should be discarded, in this case.

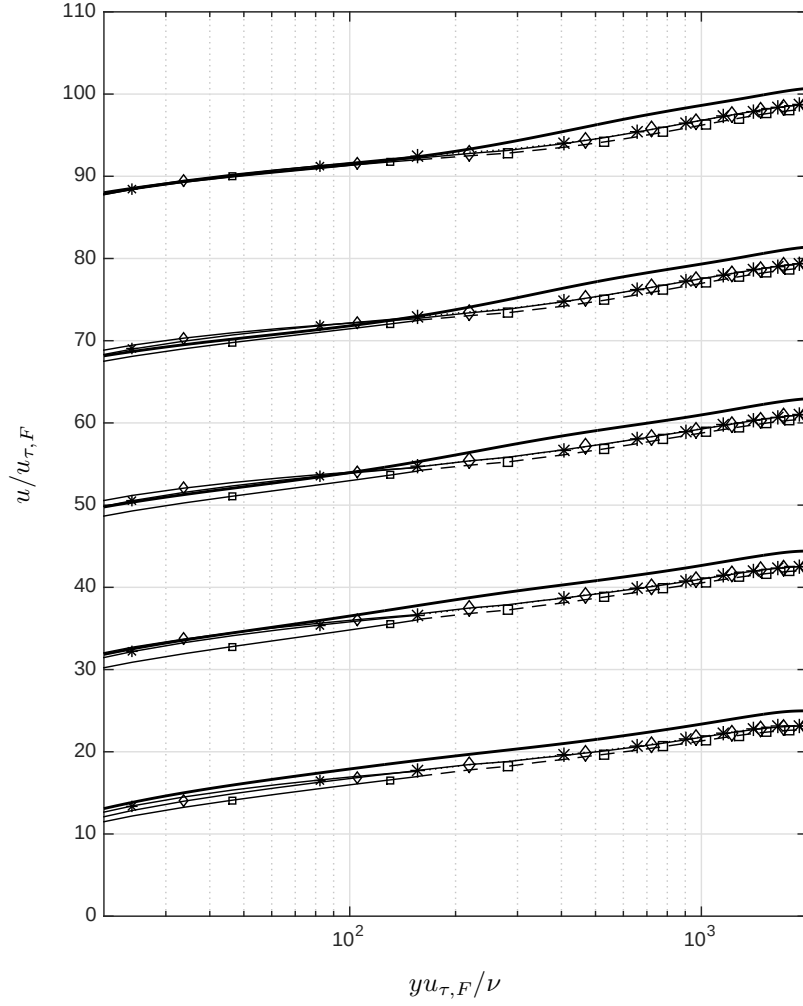


Figure 3.43: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0, 0.125, 0.25, 0.375$, and 0.5 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), **RANS sublayer models with Duprat et al.'s model:** with convective and pressure terms (dotted and asterisks), without convective and pressure terms (thin solid and diamonds), without convective terms but with pressure terms (dashed and squares). Markers do not coincide with grid points.

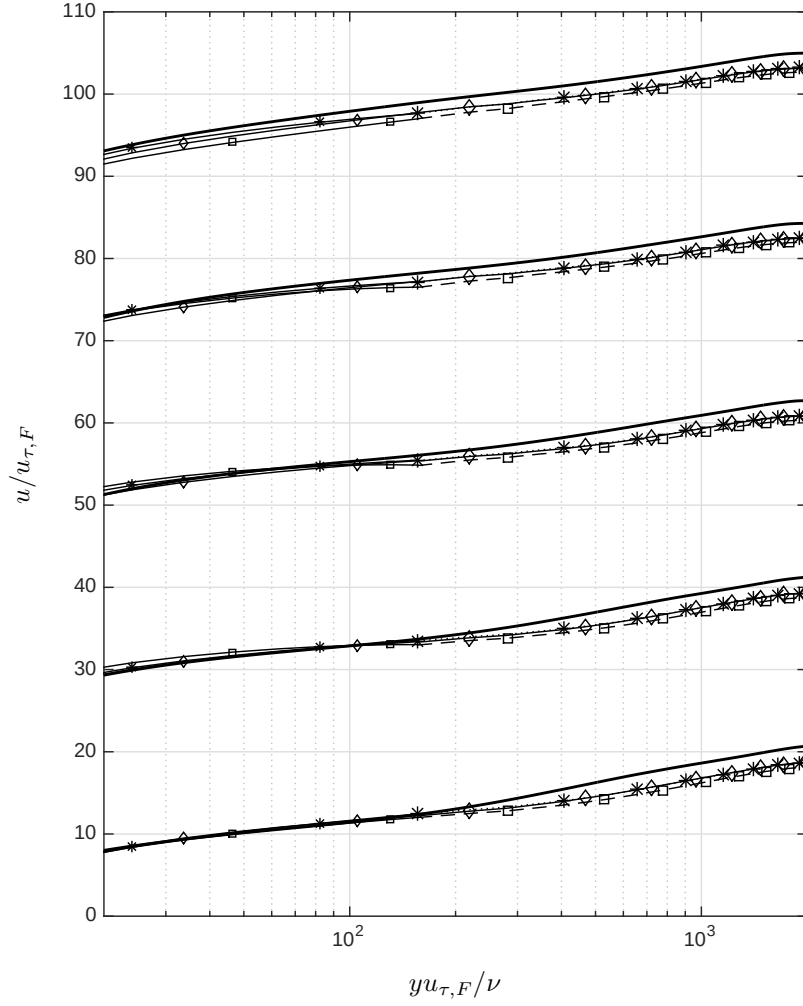


Figure 3.44: Wall-modeled LES of blown half-channel. **Mean velocity profiles** at stations $x/h = 0.5, 0.625, 0.75, 0.875$, and 1.0 . Each station is shifted upwards by 20 units for the sake of clarity. Wall-resolved LES (thick solid), **RANS sublayer models with Duprat et al.'s model:** with convective and pressure terms (dotted and asterisks), without convective and pressure terms (thin solid and diamonds), without convective terms but with pressure terms (dashed and squares). Markers do not coincide with grid points.

Chapter 4

Turbulent wake vortex system at equilibrium, and its interaction with a ground: a wall-resolved LES study at $Re_\Gamma = 2.0 \cdot 10^5$

This chapter is to be submitted for publication. The original authors are Olivier Thiry, Matthieu Duponcheel, Ivan De Visscher, Laurent Bricteux, and Grégoire Winckelmans. The equilibrium T-2VS was obtained by I. De Visscher based on previous work by L. Bricteux and M. Duponcheel.

Abstract

The goal of this work is to study wake vortex systems and their interactions with the ground. First, a realistic “turbulent two-vortex system at equilibrium” (T-2VS) is obtained. This is achieved by putting a system made of two analytical (here Burnham-Hallock model), non turbulent, vortices in a very weak homogeneous isotropic turbulence (HIT) field. After a relatively long transient, the system reaches a statistical equilibrium, where the turbulent kinetic energy is seen to represent roughly 8% of the total kinetic energy, the remaining 92 %

being the energy of the mean flow. The equilibrium T-2VS is then also characterized in terms of its circulation distribution, $\Gamma(r)$. Essentially the same T-2VS is obtained when one changes the analytical model used for the initial condition or the level of the weak HIT perturbation field.

Next, this T-2VS is used as an initial condition to perform a wall-resolved LES of such a wake system interacting with a ground, and at a rather high Reynolds number: $Re_\Gamma = \Gamma_0/\nu = 2.0 \cdot 10^5$ (hence a grid of about 1.5 billion grid points). The use of such a realistic T-2VS is more physically sound than what was used in all previous simulations; those were performed using either a 2VS with analytical model vortices (Burnham-Hallock or Lamb-Oseen model) with some added perturbation, or using a 2VS from a RANS simulation of a near wake rollup. The initial height h_0 of the 2VS is also improved: going from $h_0 = b_0$ in previous works (where b_0 is the spacing between the vortex centers) to $h_0 = 2b_0$ here, such that the T-2VS arrives in ground effect in a more physical fashion. Moreover, the Reynolds number is here 10 times larger than in the previous wall-resolved LES of wake vortex flows in ground effect. The interaction of this T-2VS with the ground is discussed and analyzed in details. It is also compared to that obtained when using, as initial condition, non-turbulent analytical vortices (with white noise perturbation) at the same Re_Γ . The flow topology is discussed for both cases, also with 3D and 2D visualizations. Large differences are observed regarding the separation of the secondary, ground generated, vorticity and the way it interacts with the primary vorticity. The vortices trajectories are also measured, together with the circulation distribution $\Gamma(r)$, and the global circulation, at various times. Large differences are also seen and explained. After the rebound phase, the vortices reach a new “turbulent equilibrium in ground effect”, and the obtained circulation distribution is also provided.

4.1 Introduction

The wake encounter-related hazard is, together with the surveillance capabilities, at the origin of the separations to be applied between landing or departing aircraft. Nevertheless, the increase of air traffic in the recent years makes these separation standards a limiting factor for capacity at the busiest airports. Eurocontrol [38] estimates that, in the most probable scenario, around 12% of the total European flights demands will not be accommodated by 2035. Its ameri-

can counterpart, the Federal Aviation Administration, share the same concerns about airport congestion in the near future (see [1]). Wake vortices are an unavoidable by-product of lift. Because it has lift, a wing generates a vortex wake that rolls up to result in a pair of counter-rotating vortices, which we here call a “two-vortex system” (noted 2VS). The vortex centroids are separated by a distance b_0 , which is related to the wingspan b_w and to the span loading of the wing. The vortices have a total circulation Γ_0 and $-\Gamma_0$, respectively. Under their mutual influence, they sink at a velocity $V_0 = \Gamma_0/(2\pi b_0)$. This also defines the characteristic time $t_0 = b_0/V_0$: the time it takes for the 2VS to descend one unit of b_0 . The Reynolds number is defined as $Re_\Gamma = \Gamma_0/\nu$, where $\nu = \mu/\rho$ is the kinematic viscosity. The 2VS evolution (in terms of vortex location, vortex circulation, etc.) then occurs under the influence of various environmental factors, such as wind intensity and turbulence, stratification, and ground effects. Even when generated in a quiescent environment, the 2VS is surrounded by its own turbulent field, produced while rolling up; we call this turbulent 2VS a T-2VS.

Review on wake vortex issues were made by Spalart [119], and by Gerz et al. [44], also, more recently, by Coustols et al. [27], in an effort to summarize works performed in the framework of various European and American research projects. Recently, Misaka et al. [84], De Visscher et al. [31], among others, also investigated the impact of the atmospheric conditions on the wake behavior.

When the 2VS is close to the ground, several phases are distinguished. Its descent is first altered by the impermeability condition at the ground. The early phase of ground effect is essentially inviscid and the vortices follow a hyperbolic trajectory, this phase can be modeled by the presence of a mirror 2VS (below the ground), this is the so-called *near ground effect* or NGE phase. As the 2VS descends further, the no-slip condition at the ground comes at play and thin boundary layers form below the vortices. Each boundary layer is accelerated until it reaches its maximal outer velocity (roughly below each vortex) then decelerated. At some point, this deceleration leads to the boundary layer separating from the ground. This phase is the *in ground effect* or IGE phase. The separated vorticity rolls up and forms secondary vortices. As the detached secondary vorticity orbits the primary vortices, the primary vortices are dragged upwards by the velocity induced by the secondary vortices: this is the rebound phenomenon. Primary and secondary vortices finally reconnect, causing a rapid circulation decay of the primary vortices: this is the fast decay

phase. Detailed description of those phenomena can be found in Harvey and Perry [51] and Doligalski et al. [34]. Leweke et al. [74] also provided a detailed review of instabilities of vortex pairs, among which wake vortices in ground effect.

At low Reynolds number, Harris and Williamson [50] showed that the secondary vortices interact with the primary vortices in a mechanism similar to that of Crow [28]. At moderate Reynolds number, the secondary vortices become turbulent through short wave instabilities (the elliptic instabilities of Widnall et al. [146]), as shown by numerical simulation by Luton and Ragab [78]. At higher Reynolds number, we will see that the detached secondary vorticity is immediately turbulent and does not form coherent secondary vortices.

Because of its influence on wake decay, ground effect is of major importance in wake vortex systems study. Wakes interacting with the ground are produced by aircrafts in late phases of flight and landing, or in early phases of flight while taking off. Many complex phenomena come into play in such configurations: wind, ground roughness, ambient turbulence, vortex inclination, end effects, etc. Proctor et al. [103] showed that in such configuration, the vortices can reconnect with the ground, and Stephan et al. [125] performed a wall modeled LES at $Re_\Gamma \simeq 5.0 \cdot 10^5$ including tilting and end effects, leading to vortices reconnecting with the ground. They used a stationary RANS to obtain partially rolled up vortices, which they flew and landed in a wall-modeled LES numerical setup. Even though the injected 2VS results from a rollup simulation, it still misses some features of the flow as it is obtained from a stationary RANS simulation and therefore does not contain the vortex self-generated turbulence.

This work focuses on the analysis of the interaction of a T-2VS, hence with its own turbulent field surrounding it, with a flat terrain. This case does not contain more complication such as vortex inclination or end effects, and aims at providing a detailed analysis of the interaction of the T-2VS with the ground. This particular interaction is a key aspect of the vortex circulation decay and deserves thorough investigation, as highlighted by Holzäpfel et al. [55]. In previous works, among which Holzäpfel et al. [55], Holzäpfel et al. [54], Bricteux et al. [15] (both at fairly moderate Reynolds numbers, of the order of $Re_\Gamma \simeq 2.0 \cdot 10^4$), also Proctor and Han [105] (at high Reynolds number, but with wall modeling), wind is used to provide turbulent structures. Stephan et al. [124] showed that in these cases, the near-wall streaks of the

turbulent wind strongly interact with the vortical structures, allowing for transition to turbulence. Crosswind also has the additional impact of transporting the vortices and providing additional shear at the ground, hence resulting in a different rebound for the upwind and downwind vortices. Here, the aim is to provide a realistic 2VS IGE simulation in a quiescent atmosphere, yet with turbulent vortices. This study is done using wall resolved LES at a relatively high Reynolds number of $Re_\Gamma = 2.0 \cdot 10^5$. This study will show that, although there is no wind, and hence no streaks in our case, the secondary vorticity is formed in a turbulent fashion, due to the inherent turbulence of the T-2VS. The vortices “self-turbulence” is also shown to be crucial to the physics of the decay of the 2VS.

The paper is organized as follows: Section 4.2 details how the T-2VS is obtained and provides its main characteristics; Section 4.3 details the numerics of the wall-resolved LES of a T-2VS in ground effect and how the initial condition is obtained from the T-2VS out of ground simulation; Section 4.4 describes the flow and compares its evolution to an analytical 2VS in ground effect simulation and to a T-2VS out of ground effect, both in terms of flow topology and of global diagnostics.

4.2 Obtention, by LES, of a turbulent two-vortex system at equilibrium

For 2VS with vortices of relatively small core size, as will be the case here, the vorticity field of each vortex is essentially axisymmetric. Each mean (i.e., longitudinally-averaged) vortex is then characterized by its circulation distribution, $\Gamma(r)$: the amount of vorticity contained in a disk of radius r centered on the vortex. The vortex-induced velocity is then $u_\theta(r) = \frac{\Gamma(r)}{2\pi r}$. The aforementioned simulations of wake vortices coming into ground effect were all performed using, as initial condition, either an analytical model for $\Gamma(r)$ or the result of a rollup simulation then further fitted using a $\Gamma(r)$ model (as in the case of Stephan et al. [125], using a RANS simulation of rollup); then further perturbed (e.g., using white noise) to excite instabilities. Simple models include the “low order algebraic” vortex model, also called the Rosenhead-Moore model, or the Burnham-Hallock model [16] in the aircraft community (here

noted BH model):

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{r^2}{(r^2 + r_c^2)}, \quad (4.1)$$

and the Gaussian vortex model, also called the Lamb-Oseen model (here noted LO model):

$$\frac{\Gamma(r)}{\Gamma_0} = 1 - \exp\left(-\left(\frac{r}{\sigma}\right)^2\right) = 1 - \exp\left(-a \left(\frac{r}{r_c}\right)^2\right) \quad \text{with } a = 1.25642. \quad (4.2)$$

Both models solely depend on one scale: the effective core radius r_c (defined as the radius of maximum induced velocity). Those simple models have been used in LES studies of wake vortices interacting with a ground, also in the presence of a turbulent wind, see Holzäpfel et al. [54], Bricteux et al. [15], among others. Yet, when considering the detailed description of the circulation distribution $\Gamma(r)$ in an aircraft 2VS, two-scale (or more) models are required. Such models include those by Fabre and Jacquin [39]; or Proctor [104] and further modifications thereof: among which the improved model of Proctor et al. [106] and the Proctor-Winckelmans (PW) model, see de Bruin and Winckelmans [29], and De Visscher et al. [32]. In two-scale models, there is an inner region displaying a rapid growth and scaling with r_c , and an outer region, displaying a slow growth and scaling with b_0 (or b , the wing span). For instance, the inner and outer parts of the improved Proctor model are, respectively for $0 \leq r \leq \alpha r_c$ and for $r \geq \alpha r_c$:

$$\frac{\Gamma(r)}{\Gamma_0} = C \left(1 - \exp\left(-a \left(\frac{r}{r_c}\right)^2\right)\right) \quad \text{and} \quad \frac{\Gamma(r)}{\Gamma_0} = 1 - \exp\left(-\beta \left(\frac{r}{b}\right)^{3/4}\right) \quad (4.3)$$

where $\alpha = 1.4$, $\beta \simeq 10$ (it however varies, depending on the span loading), and where C is then computed so that the model is continuous at αr_c . The PW model consists in a variation, by using a continuous blending between the inner and outer functions. It also uses b_0 instead of b as the reference outer scale:

$$\frac{\Gamma(r)}{\Gamma_0} = 1 - \exp\left(-\frac{a_i \left(\frac{r}{r_c}\right)^2}{\left[1 + \left(\frac{a_i \left(\frac{r}{r_c}\right)^2}{\beta_o \left(\frac{r}{b_0}\right)^{3/4}}\right)^p\right]^{1/p}}\right). \quad (4.4)$$

It is easily verified that it recovers the inner and outer functions of Eq. (4.3) for $0 \leq r \ll r_c$ and $r \gg r_c$ respectively. Of course, $\beta_o = \beta \left(\frac{b_0}{b}\right)^{3/4}$ (hence, for a wing with elliptical loading, $\frac{b_0}{b} = \frac{\pi}{4}$, we obtain $\beta_0 \simeq 0.83\beta$). For each value of r_c/b_0 , the inner coefficient a_i must be adjusted numerically so that the location of the maximum induced velocity is indeed exactly at $r = r_c$ (a good first guess being $a_i \simeq \beta_o \left(\frac{r_c}{b_0}\right)^{3/4}$). The p parameter controls the amplitude of that maximum velocity, with values in the range $p = 3$ to 5 (a typical value being $p = 4$).

Still, the analytical models above only describe the mean vortex; they do not provide any turbulent fluctuations. To produce turbulence, they can be excited either through added perturbations (e.g., using some white noise), and/or by their interaction with some turbulence, such as in the LES of wake vortices IGE and with turbulent wind; all previous works started using an analytical vortex model: as in Holzäpfel et al. [54] using a LO-2VS and Bricteux et al. [15] using a BH-2VS.

In reality, the 2VS quickly develops its own turbulence while rolling up, and it exhibits important turbulent vortical structures, within the Rankine oval and near it. As we will see, the turbulent structures of such fully developed 2VS will also lead to very significant differences when it interacts with a ground, as compared to the previous simulations done using solely a mean vortex analytical model with some added perturbations.

The obtention of a realistic, fully developed, turbulent 2VS is thus of major importance: not only in general, for better characterizing the mean circulation distribution $\Gamma(r)$ and associated turbulence of 2VS; but also when using it as an initial condition for the investigation of a 2VS interacting with a ground. This is even more so when one considers interactions without wind as here; or with weak wind. This section presents the methodology used to obtain a realistic “fully-developed turbulent 2VS” (noted T-2VS).

The simulations were performed using a pseudo-spectral code with phase shift de-aliasing (see Canuto et al. [20]), and where the time integration is carried out using the low-storage third order Runge-Kutta scheme of Williamson [147]. The simulations are performed in a periodic box of size $L^3 = (4b_0)^3$, and using 256^3 points. This box is such that the Crow instability, with maximum (yet slow) growth rate ($L_x \simeq 8b_0$) is not allowed to exist. Indeed, the purpose is here to obtain, by LES, a realistic and fully-developed T-2VS that, in reality, would form within one t_0 or less behind a wing; in such a short time, the Crow

instability would not have time to develop.

The simulation is performed in two steps: first, a forced homogeneous isotropic turbulence (HIT) simulation is performed to obtain a statistically converged HIT field. Then, an analytical 2VS is added to this field. The HIT field energy dissipation rate, ϵ , is characterized relatively to the 2VS by using the characteristic b_0 and V_0 of the 2VS: $(\epsilon b_0)/V_0^3 = 2.42 \cdot 10^{-4}$. We here use a BH-2VS, and with a relatively small core size $r_c/b_0 = 0.050$; as was also used in Bricteux et al. [15]

To assess the intensity of the 2VS with respect to the initial surrounding turbulence, we first compute the kinetic energy of the “2VS field only”, noted (V, W) :

$$E_{2D,0} = \int_{S_e} \frac{1}{2} (V^2 + W^2) dS, \quad (4.5)$$

where $S_e = 4b_0 \times 4b_0$ is the bounded periodic surface. We then further compute the kinetic energy of the total flow field, corresponding to the 2VS field with the superposed HIT field, $(u, v, w) = (u', V + v', W + w')$:

$$E = \frac{1}{L_x} \int_{V_e} \frac{1}{2} (u^2 + v^2 + w^2) dV = \int_{S_e} \frac{1}{2} (\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle) dS, \quad (4.6)$$

where $\langle f \rangle(y, z) = \frac{1}{L_x} \int_0^{L_x} f(x, y, z) dx$ denotes longitudinal averaging. We here obtain $E_{2D,0}/(V_0 b_0)^2 = 15.376$ and $E/(V_0 b_0)^2 = 15.439$. Relatively to the 2VS without turbulence, the turbulent kinetic energy $k = E - E_{2D}$ associated to the added HIT field is therefore very small: the ratio k/E_{2D} is indeed only 0.4 %.

The $E_{2D,0}$ is also compared with the energy of the same vorticity field but in a truly unbounded domain. We then use the formulas derived by de Bruin and Winckelmans [29] (which were also verified to be very accurate for small r_c/b_0 , as is the case here):

$$E_{2D,0}^\infty = \int_{S_\infty} \frac{1}{2} (V^2 + W^2) dS = \frac{\Gamma_0^2}{2\pi} \left(\log \left(\frac{b_0}{r_c} \right) - C \right), \quad (4.7)$$

where S_∞ is the unbounded surface, and $C = \frac{1}{2}$ when using a BH-2VS (for information, one obtained $C = -0.05617$ when using a LO-2VS). We hence obtain $E_{2D,0}^\infty/(V_0 b_0)^2 = 15.681$, which is only 2% more than the value for the periodically confined 2VS; comforting us that the size $(4b_0)^3$ of the periodic domain used here is large enough for the purpose of the present LES. Finally,

we also note that the present BH-2VS has roughly the same energy as that of the vortex sheet that would emanate from a wing with elliptical loading, and which is equal to $E_{elliptical} = \Gamma_0^2 \frac{\pi}{8} = \frac{\pi^3}{2} (b_0 V_0)^2$.

The 2VS develops its own turbulence and reaches an equilibrium. This is seen by examining the time evolution of the 2D and 3D energies (Fig. 4.1), the modal energies (Fig. 4.2), and the energy dissipation rate (Fig. 4.3). Although the HIT is very weak relatively to the 2VS, the combination of this HIT field with the BH-2VS will suffice to trigger instabilities over a wide range of wavenumbers, and that will lead, after a transient, to a “turbulent 2VS at equilibrium”, see Fig. 4.2. Indeed, once the 2VS is introduced in the HIT, it stretches the weak turbulent structures, forming long azimuthal vortex filaments. These azimuthal structures are then much amplified through vortex stretching. Of course, all this takes much more time than what would happen in a real wake developing behind an aircraft. In that case, the turbulent equilibrium would be reached much faster (e.g., in less than one t_0), due to strong instabilities during the wake roll-up process, combined with strong turbulent perturbations (e.g., the turbulent wake of the aircraft fuselage and the boundary layers on the wings). The time axis of Figs. 4.1, 4.2 and 4.3 must thus be considered as an “artificial evolution” axis: the time required for slowly bringing the 2VS to a turbulent equilibrium, when starting with a very weak initial forcing HIT. If we examine the saturation of the high wavenumber modes, in Fig. 4.2, we find that the equilibrium is reached at about $t/t_0 \simeq 4 - 5$. When examining the time evolution of the energy (see Fig 4.1), we see that the energy of the mean flow decreases while the total energy is still essentially conserved, until $t/t_0 \simeq 3.5 - 4$: meaning that a significant amount of energy is transferred, in an essentially inviscid way, to created turbulent structures. Later on, the total energy is seen to also decrease, and with essentially the same rate as that of the mean flow: hence meaning that the turbulent kinetic energy, k , is not decreasing much. The vortex system has therefore reached a fair “statistical equilibrium”, where the mean flow and the turbulence are both established.

The obtained “2VS at turbulent equilibrium (T-2VS)” is of course of significant interest in itself, and will here be further characterized. We consider it when well established yet not much decayed. We then obtain that $E_{2D}/E = 0.9186$, and thus, for the turbulent kinetic energy $k = E - E_{2D}$, that $k/E = 0.0814$ and thus $k/E_{2D} = 0.0886$. There is thus much turbulence in this turbulent 2VS: roughly 8% of the total energy lies in the turbulent fluctuations, and 92% lies

in the mean flow. We offer that those obtained ratios could also be representative of what would be found in real 2VS wakes that develop behind wings at high Reynolds number, when the rollup is fully completed and hence when a turbulent equilibrium has been reached.

Another interesting outcome of this simulation is that the evolution towards a T-2VS happens at essentially constant core size. As there is some long wavelength deformation of the vortex centerline (yet here very moderate), it is better to measure the circulation profiles (and hence also obtain the core size) in each of the 256 cross-plane, and then average those. We then obtain that $r_c^e/b_0 = 0.048$ for the left vortex and 0.050 for the right one: indeed essentially the same as the initial condition core size r_c . Up to $t/t_0 \simeq 4$, very little circulation is lost. By conservation of the linear impulse, the vortex lateral separation is then also conserved. Later on, some circulation decay is observed due to the turbulent interactions between the left and the right vortices; hence the lateral spacing also starts to increase.

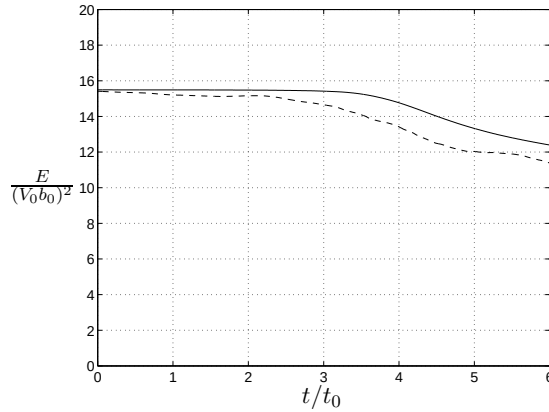


Figure 4.1: Temporal evolution of the kinetic energy for the baseline BH-2VS: total kinetic energy E (solid) and kinetic energy of the mean flow E_{2D} (dashed).

To ensure that the equilibrium reached by the T-2VS is indeed quite universal, a sensitivity analysis was also performed on the parameters of the simulations. Case 0 is the baseline described hereabove. Case 1 corresponds to a variation of the initial circulation profile, using the more realistic two-scales PW model, and with the same Γ_0 and r_c/b_0 as the baseline BH-2VS. The energy of that PW-2VS is then roughly the same as that of the baseline: about 3% higher. Case 2 corresponds to another variation of the circulation profile, using the LO model, also with the same Γ_0 and r_c/b_0 . Here, the energy of the

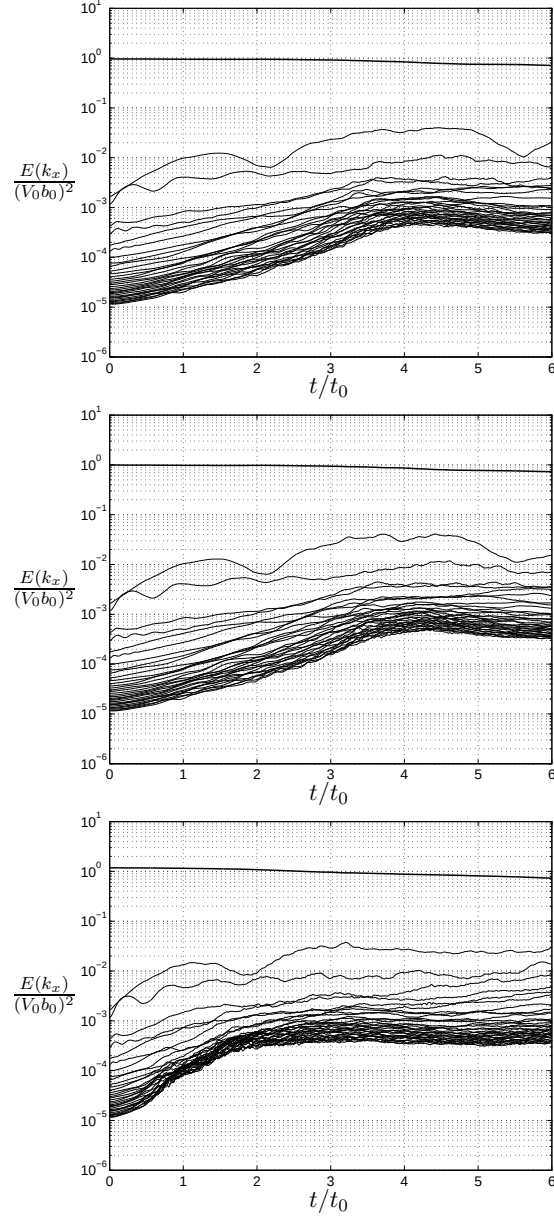


Figure 4.2: Longitudinal modal energy evolution for the BH-2VS (Case 0, top), the PW-2VS (Case 1, middle), and the LO-2VS (Case 2, bottom). For the sake of clarity, only selected modes are displayed: the rigid mode (thick solid), the $\lambda = 4b_0$ mode (solid), and the 31 next modes down to $\lambda = 0.125b_0$ (thin solid).

LO-2VS is significantly higher than that of the baseline: roughly 22% higher (this case could thus correspond to a wake behind a wing which is significantly more outboard loaded than one with elliptical loading). Finally, Case 3 corresponds to the baseline BH-2VS subjected to an HIT where the dissipation rate has been multiplied by a factor of 10: $(\epsilon b_0)/V_0^3 = 2.42 \cdot 10^{-3}$ (note that this is still, relatively, a weak atmospheric turbulence).

Fig. 4.2 displays the evolution of the longitudinal modal energy, and Fig. 4.3 displays the obtained energy dissipation rate, for the various cases. We see that, regardless of the initial circulation distribution, the turbulent equilibrium reached is about the same: compare Cases 0, 1 and 2. Also, and as expected, a HIT forcing with a higher dissipation rate leads to a shorter transient to obtain the same turbulent equilibrium: compare Cases 0 and 3.

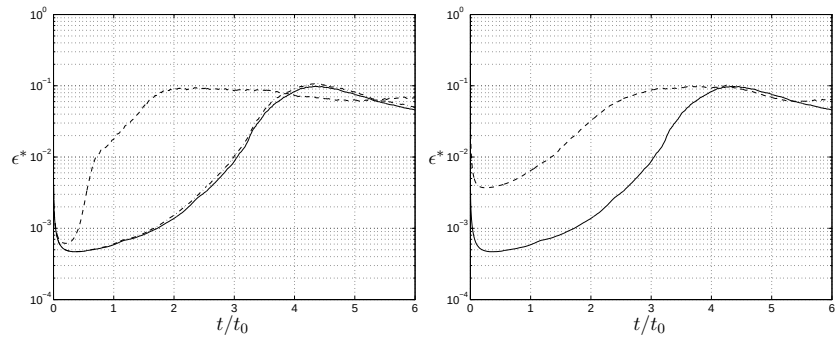


Figure 4.3: Temporal evolution of the energy dissipation rate ϵ : BH-2VS (Case 0, solid). Left: PW-2VS (Case 1, dashed-dotted), and LO-2VS (Case 2, dashed). Right: BH-2VS with 10 times stronger initial perturbation (Case 3, dashed).

The vortex circulation distribution $\Gamma(r)/\Gamma_{tot}$ of the equilibrium turbulent 2VS is reported in Fig. 4.4. We display the measured distribution for the left and right vortices; which also highlights the natural relative variations. It is seen that the circulation distribution displays a two-scale behavior: an inner layer, with a rapid growth of the circulation in the core, and scaling with r_c ; and an outer layer, with a slow growth of the circulation, scaling with b_0 . The obtained circulation distribution is also found to be about the same for all cases. Again, we offer that it is also representative of what would be obtained for real 2VS wakes that develop behind wings at high Reynolds number, when the rollup is fully completed and a turbulent equilibrium has been reached. Such circulation profiles can also be better modeled when using two-scales vortex

models; which again confirms the interest of those models.

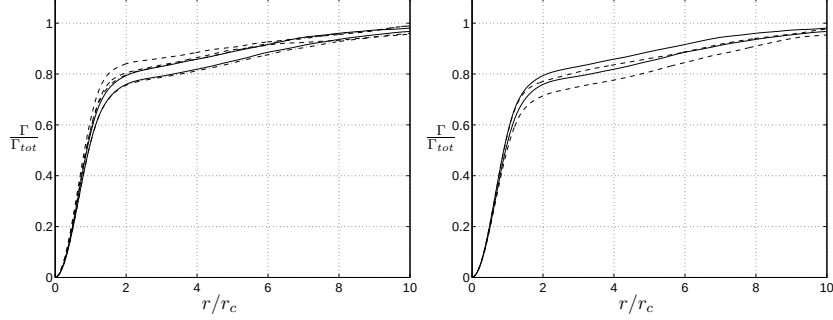


Figure 4.4: Circulation distribution of the 2VS at turbulent equilibrium, scaled using each vortex respective r_c and Γ_{tot} : BH-2VS (Case 0, solid). Left: PW-2VS (Case 1, dashed-dotted), and LO-2VS (Case 2, dashed). Right: BH-2VS with 10 times stronger initial perturbation (Case 3, dashed). The measured distribution are shown for both the left and right vortices.

From the vortex circulation distribution $\Gamma(r)$, one can extract various measures of the “vortex strength”. One is the total circulation Γ_{tot} , which is the maximum of the $\Gamma(r)$ curve. Another one, often used in experimental measurements and in the aircraft community, is the “LIDAR-processed” Γ_{5-15} circulation, defined as:

$$\Gamma_{5-15} = \frac{1}{10} \int_5^{15} \Gamma(r) dr, \quad (4.8)$$

where distance is evaluated in [m], and considering an aircraft of wingspan $b_w = 60$ [m]. The generalization to an aircraft of wingspan b_w is then here taken as:

$$\Gamma_{5-15} = \frac{1}{b_w/6} \int_{b_w/12}^{b_w/4} \Gamma(r) dr. \quad (4.9)$$

Assuming a wing with elliptical loading, similarly to what was done by Stephan et al. [124] and Bricteux et al. [15], one takes $b_0 = \frac{\pi}{4} b_w$. For the turbulent 2VS at equilibrium, we then obtain $\Gamma_{5-15}/\Gamma_{tot} \simeq 0.86$: another interesting ratio that might be similar for real wakes. We note that this is significantly less than the value 0.933 one obtains for a BH-2VS of same r_c/b_0 , see Bricteux et al. [15]. As to a LO-2VS, the vorticity is so confined in the core region that there is essentially no difference between Γ_{5-15} and Γ_{tot} (the ratio is very close to unity); indicating that the LO model alone cannot represent, in terms of circulation distribution, a well developed 2VS (which is also why Proctor et al. [106] introduced an inner LO scaling and an outer different scaling in his

two-scales model, see Eq. (4.3)).

4.3 Turbulent 2VS in ground effect

The turbulent 2VS at equilibrium obtained hereabove, that has realistic and significant turbulence (recall that roughly 8% of the total kinetic energy lies in the turbulent fluctuations, while 92% lies in the mean vortex flow) is further used to perform a study where it interacts with a solid ground. The aim is to perform a high quality LES, thus wall-resolved, and at a Reynolds number that is 10 times larger than what was previously achieved in such wall-resolved LES studies, see Holzäpfel et al. [54], Bricteux et al. [15], among others. The Reynolds number is thus here set to $Re_\Gamma = 2.0 \cdot 10^5$. The T-2VS is also released at a higher altitude of $h_0 = 2b_0$; instead of the initial altitude $h_0 = b_0$ that was used in Bricteux et al. [15]. This corresponds to an almost “out of ground effect” (OGE) altitude: the vortices centroids then initially go almost straight down, start to follow the hyperbolic inviscid trajectory (“near ground effect”) and have plenty of time to come into equilibrium with themselves and with their surrounding by the time they reach the altitude $h(t) = b_0$; later on, they go “in ground effect” (IGE) and then strongly interact with the ground. The initial altitude of release thus allows a smooth transition between the OGE, NGE, and IGE phases. Such a simulation allows for a better physical representation of the phenomena involved in the interaction of a wake 2VS with a solid ground without any further effects, such as cross wind. The higher altitude of release is such that the turbulent boundary layer are given time to develop. The self-generated turbulence of the 2VS will generate and drive the turbulence in the boundary layers. In this simulation, the T-2VS therefore comes into ground effect in a more realistic fashion than in previous studies.

The complete simulation is made in three steps: first, a forced HIT field is generated, using a spectral code, and in a box of size $L^3 = (4b_0)^3$. When the HIT field is converged, a BH-2VS is introduced in the HIT, as described hereabove. The 2VS is then let evolve until it develops its own turbulence. This T-2VS is then used as initial condition for a wall-resolved LES of T-2VS in ground effect in a finite difference code, at $Re_\Gamma = 2.0 \cdot 10^5$. The numerical details of the wall-resolved LES, as well as the methodology employed to transmit the T-2VS field to a wall-resolved LES mesh, and with a ground are described hereunder.

The T-2VS is taken from the above pre-simulation at an earlier time than that of equilibrium, so that it reaches its equilibrium by the time it has gone down to $h \simeq b_0$, and thus still NGE, before going IGE. To carry out such a simulation, a larger computational domain of $L_x \times L_y \times L_z = 4b_0 \times 8b_0 \times 4b_0$ is required due to the rebound of the vortices (see Fig. 4.5). The SGS model is the WALE model, but where the SGS viscosity is computed using the high-passed filtered LES fields. Cocle et al. [25] have shown that such models are adequate for wake vortex flows computation, while the WALE scaling of Nicoud and Ducros [91] exhibits a good behavior in wall-bounded flows. The combination of the two was already applied to wake vortices in ground effect by Bricteux et al. [14] and Bricteux et al. [15]. The simulation is here performed using a fourth order accurate finite difference code using a staggered mesh, and that is energy conserving up to fourth order (see also Morinishi et al. [88], and Vasilyev [138]). The mesh is a $N_x \times N_y \times N_z = 768 \times 1280 \times 1536$ grid (thus a bit more than 1.5 billion grid points) that is uniform in the wall parallel directions (x and y), thus with $\Delta x/b_0 = 1/192$ and $\Delta y/b_0 = 1/160$ and that is stretched in the wall normal direction (z), using the following rule:

$$\frac{z}{L_z} = \frac{(\gamma + 1) - (\gamma - 1) \left(\frac{\gamma+1}{\gamma-1} \right)^{1-\xi}}{1 + \left(\frac{\gamma+1}{\gamma-1} \right)^{1-\xi}}, \quad (4.10)$$

where $\xi \in [0; 1]$ is the uniform numerical coordinate (i.e., $\xi_k = k/(N_z - 1)$ with $k = 0, 1, \dots, N_z - 1$), and γ is the stretching parameter. We use $\gamma = 1.07$ which leads to $\Delta z_{\min}/b_0 \simeq 6.0 \cdot 10^{-4}$ and $\Delta z_{\max}/b_0 \simeq 4.7 \cdot 10^{-3}$.

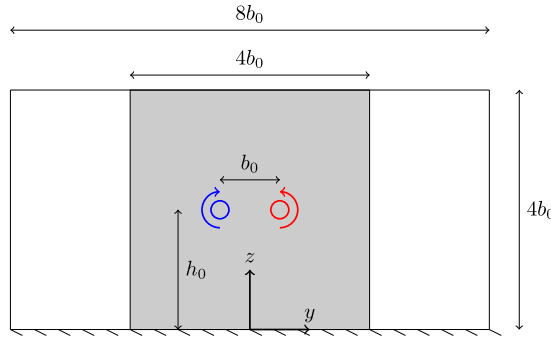


Figure 4.5: Sketch of the computational domain: the grey area is where the vorticity ω is obtained from the pre-simulation of the 2VS; it is set to 0 elsewhere.

To put the T-2VS in this numerical domain, it can not simply be interpolated on the grid because the numerical domain is not the same, and the slip boundary conditions have to be enforced. As the flow is incompressible, the velocity field can be obtained from a streamfunction ψ : $\mathbf{u} = \nabla \times \psi$. The streamfunction is obtained by solving the Poisson equation $\nabla^2 \psi = -\omega$, and the vorticity field ω of the pre-simulated 2VS is interpolated to the new computational grid, where it is available, and set to zero elsewhere, as shown in Fig. 4.5. The Poisson equation is solved with periodic boundary conditions in the wall parallel directions, and with slip conditions at the top and bottom boundaries:

$$\psi_x = \psi_y = \frac{\partial \psi_z}{\partial z} = 0 \quad (4.11)$$

at $z = 0$ and at $z = 4b_0$. The obtained velocity field is then used as initial condition for the new simulation. Of course, at the first time step, and for all others, the no-slip condition is imposed at the lower wall (i.e., the ground). Hence a boundary layer forms in the near wall region. This process is rather smooth thanks to the high altitude of release of the 2VS. The time t is here counted starting from the beginning of the simulation (i.e., $t/t_0 = 0$ is when the 2VS is at the initial altitude $h_0 = 2b_0$).

The maximum wall friction is reached at $t/t_0 \simeq 2.5$, which occurs a bit earlier than when the vortex reaches its lowest altitude (see Fig. 4.15). The mesh is then such that the Δ^+ values attained are $\Delta x_{\max}^+ \simeq 31$, $\Delta y_{\max}^+ \simeq 37$, and $z_{\max}^+ \simeq 1.8$; which are still sufficient for a wall-resolved LES of a turbulent boundary layer flow (see Sagaut [110]); that goes here in the y direction, with a transverse x direction.

As a basis for comparison, another LES using a BH-2VS also released at $h_0 = 2b_0$ is performed on the same mesh and at the same Re_Γ . It is here excited using a random velocity excitation (white noise), whose amplitude is 1.0% of V_0 . The perturbation level is thus small, yet sufficient for the 2VS, when interacting with the ground, to trigger 3D instabilities and, after rebound, lead to a transition to a turbulent wake vortex flow. This is also similar to what was done in the study by Holzäpfel et al. [55], and Holzäpfel et al. [54] in their case without wind, although they used a LO-2VS. Finally, an OGE simulation of the T-2VS is also run using the fourth order finite difference code, thus in a periodic domain of size $L^3 = (4b_0)^3$ and with $N^3 = 512^3$ grid points, and at $Re_\Gamma = 2.0 \cdot 10^5$. Those results are used for comparisons, and using the same

LES code, with the results of the case IGE.

4.4 Results

4.4.1 Flow topology

The temporal evolution of the flow is displayed in Fig. 4.6, through a volume rendering of the vorticity norm. Figure 4.7 shows a detail of the secondary vorticity and its production in the case of the BH-2VS. Fig. 4.8 and Fig. 4.9 display the longitudinally averaged axial vorticity. Both the T-2VS and the BH-2VS are shown. In the BH-2VS case, the primary vortices remain laminar in the core region, until their strong interaction with the secondary vorticity, around $t/t_0 = 3.7$. The secondary vortices form in a laminar and quasi 2D fashion (see Fig. 4.10, and Fig. 4.11). They are also coherent, despite many inhomogeneities (see Fig. 4.6 at $t/t_0 = 3$). The boundary layer separation forms a thin vortex sheet that immediately rolls up to form the secondary vortices. When the vortex sheet is formed, fine scale spanwise structures form (in the longitudinal direction of the vortex). These structures, while the secondary vortices roll up, become the turbulence surrounding the secondary vortices. These turbulent structures are here intrinsically due to the vortices and cannot be related to the near-wall streaks of some atmospheric wind; as was the case in Holzäpfel et al. [54], and Bricteux et al. [15]. The secondary vorticity roll-up process can be seen in Fig. 4.9 at $t/t_0 = 3 - 4$. Note that in previous studies, at lower Reynolds number (among which Bricteux et al. [14]) the secondary vorticity was very coherent and laminar, and only became turbulent through short-wave instabilities, which is not the case here, hence demonstrating the interest of higher Reynolds number studies. A coherent vortex core is formed. As the secondary vortices orbit the primary vortices, azimuthal vortex filaments formed around the secondary vortices are stretched by the primary vortices. Some (but not all) also form hairpin vortices that get wrapped around the primary vortices (see Fig. 4.7). The azimuthal vortex filaments and/or the hairpin vortices finally strongly interact with the core of the primary vortices. The primary vortices are finally surrounded by secondary turbulence, at $t/t_0 = 5$ (see Fig. 4.9).

In the turbulent 2VS case, the primary vortices are in their own turbulent equilibrium starting at $t/t_0 \simeq 1$. The secondary vorticity also behaves

intrinsically differently from the BH-2VS case. It is much less coherent and is continuously incorporated in the primary vortex turbulent field. As seen in Fig. 4.9 and Fig. 4.6, the secondary vorticity never really rolls up to form a coherent core. It therefore quickly and smoothly interacts with the primary vorticity and the turbulence of both parts merge; see also Section 4.4.2 when discussing the vortex circulation decay. The rebound is thus less intense than that of the BH-2VS. The trajectories of the vortices are quite similar up to the time $t/t_0 \simeq 3$ at which the secondary vorticity is detaching from the wall, and they are different after that (see Fig. 4.8, Fig. 4.9 and later, in Fig. 4.15).

4.4.2 Vortex diagnostics

The vortex centers are tracked in a fashion similar to that of Bricteux et al. [15]. It is tracked in each cross plane and the vortex diagnostics are computed in each of these cross planes. The circulation distribution $\Gamma(r)$ is obtained in each plane, from which Γ_{tot} and Γ_{5-15} are deduced. The longitudinally averaged quantities are then obtained by averaging the obtained results over all planes. The result is therefore an average obtained in the frame of reference of the local vortex center, which, when the vortex centerline is deformed, is not equivalent to the values obtained using the longitudinally averaged fields.

Fig. 4.12 displays the total vortex circulation decay over time, $\Gamma_{tot}(t)$. The half-plane circulation *without the boundary layer* is taken as:

$$\Gamma_{HP|z>z_\delta}^+ = \int_{y>0} \int_{z>z_\delta} \langle \omega_x \rangle dS \quad (4.12)$$

$$\Gamma_{HP|z>z_\delta}^- = \int_{y<0} \int_{z>z_\delta} \langle \omega_x \rangle dS \quad (4.13)$$

This diagnostic does not require the aforementioned averaging procedure (“sitting on local vortex center”); it can therefore be evaluated using the longitudinally averaged vorticity field $\langle \omega \rangle$. The final diagnostic is taken as the average over the two vortices: $\Gamma_{HP|z>z_\delta} = \frac{1}{2}(\Gamma_{HP|z>z_\delta}^+ - \Gamma_{HP|z>z_\delta}^-)$. This quantity also has a weak dependence on the choice of the parameter z_δ . It is chosen large enough to exclude the whole boundary layer attached vorticity, yet small enough to quickly detect the detached vorticity. A good tradeoff here is $z_\delta = 0.1 b_0$.

The quantity $\Gamma_0 - \Gamma_{HP|z>0.1 b_0}$ is then an image of the amount of detached secondary vorticity. It is not immediately correlated with the vortex global cir-

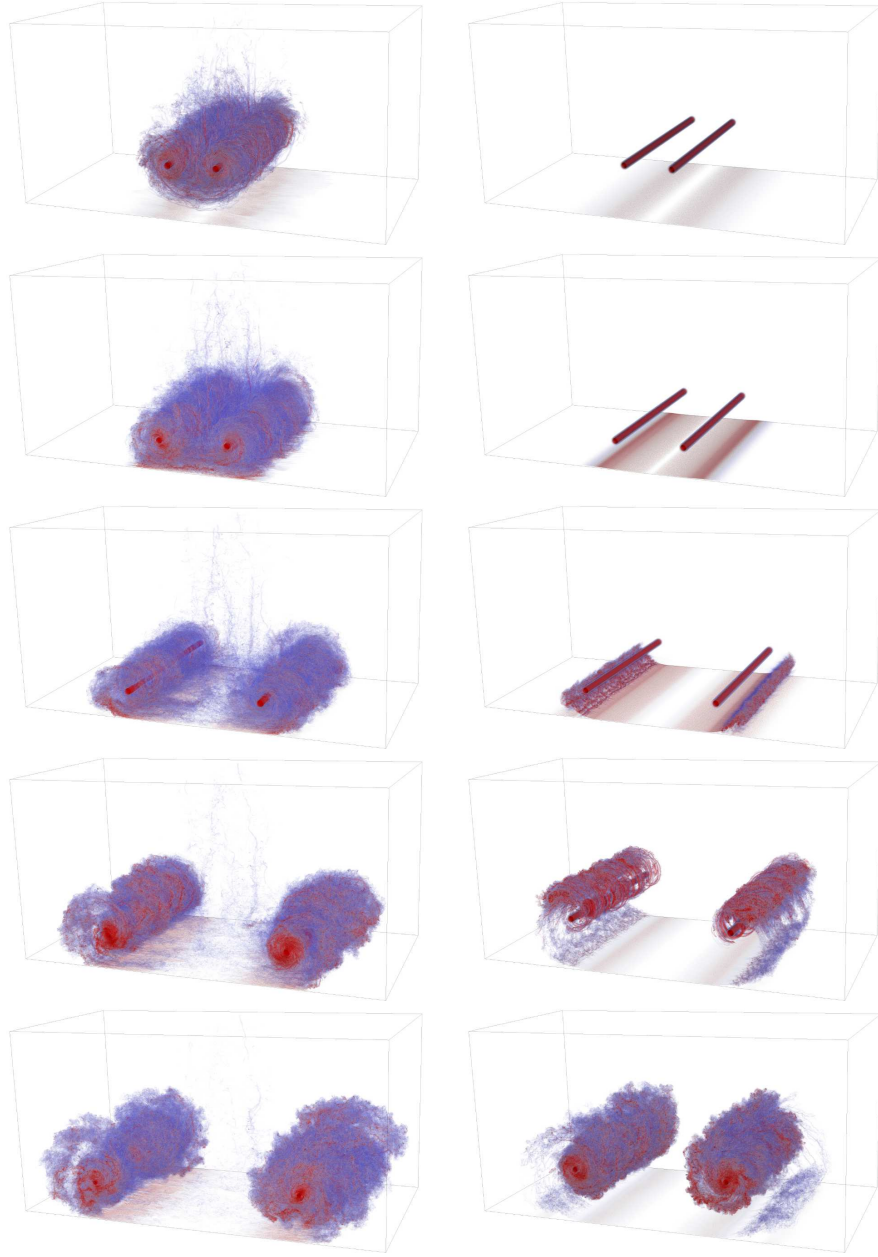


Figure 4.6: Volume rendering of the vorticity magnitude at $t/t_0 = 1, 2, 3, 4$, and 5 . T-2VS (left) and BH-2VS(right), here in small and low resolution, high resolution images are shown in Appendix B.

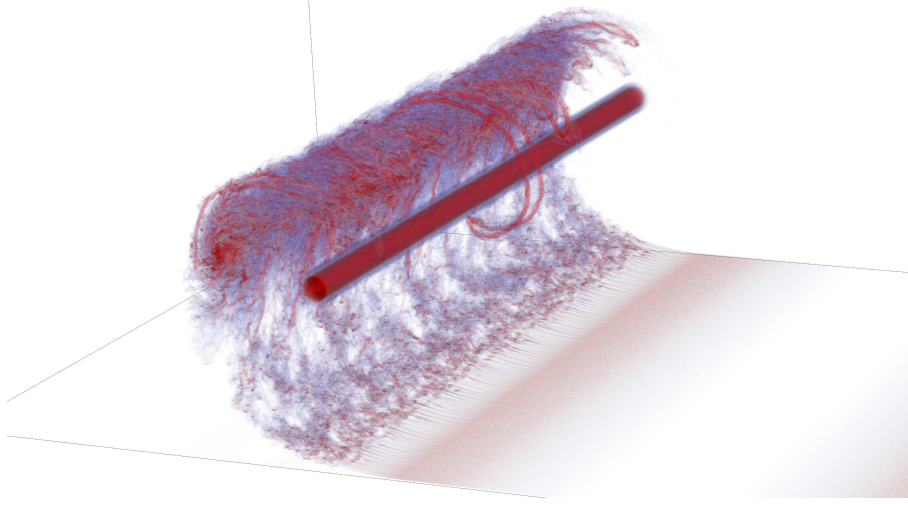


Figure 4.7: Volume rendering of the vorticity magnitude at $t/t_0 = 3.7$ for the BH-2VS: zoom on the left vortex. Some hairpin vortices are also formed and stretched by the primary vortices.

culation decay, since, as seen in the previous section, the detached secondary vorticity does not immediately strongly interact with the primary vortices.

When comparing the two T-2VS cases, we see, in Fig. 4.12, that Γ_{tot} has, at early times, the same decay rate IGE and OGE. The T-2VS IGE curve, however, starts to depart from the OGE curve, and it adopts a slower decay rate between $t/t_0 \simeq 0.5$ and $t/t_0 \simeq 1.5$. This is due to the fact that the vortex spacing b starts to increase significantly, due to the presence of the ground (NGE effect, see Fig. 4.14). The vorticity exchange between the two vortices through the turbulent interactions in the mid plane region is therefore slowed down. The phenomenon can also be observed on the curves of Fig. 4.16 at $t/t_0 = 1$: the OGE $\Gamma(r)$ reaches its maximum around $r/b_0 \simeq 0.7$, then starts decreasing due to the influence of its counter-rotating neighbor; the IGE $\Gamma(r)$ keeps growing up to $r/b_0 \simeq 0.8 - 0.9$. The remainder of this phenomenon can also be observed in the $\Gamma(r)$ curves at $t/t_0 = 2$ through the presence of an overshoot of circulation before the decrease due to the presence of the boundary layer vorticity. Since this is a phenomenon linked to the exchange of vorticity between the turbulent vortices, it only affects the outer part of the vortex. It has very little impact on the vortex inner part evolution. Its impact is hence only visible in the total circulation Γ_{tot} ; but not in the Γ_{5-15} diagnostic which does not account for vorticity beyond $b_w/4$ (see Fig. 4.13).

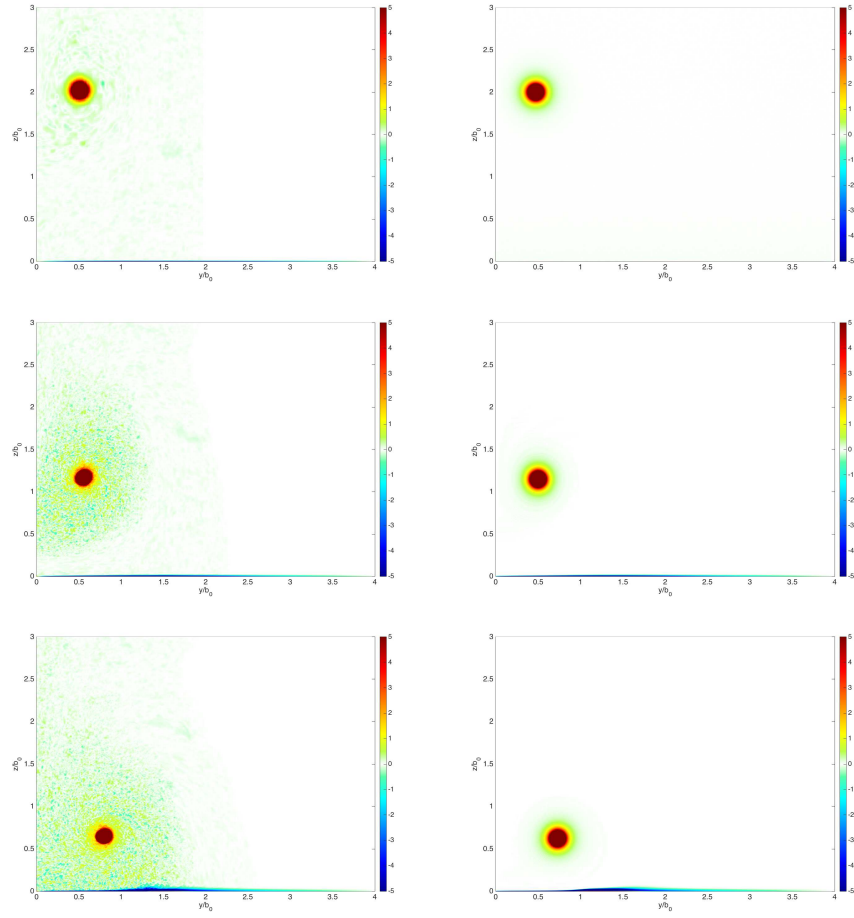


Figure 4.8: Averaged axial vorticity $\langle \omega_x \rangle b_0^2 / \Gamma_0$ at $t/t_0 = 0, 1$ and 2 (only one vortex is shown). T-2VS (left) and BH-2VS (right).

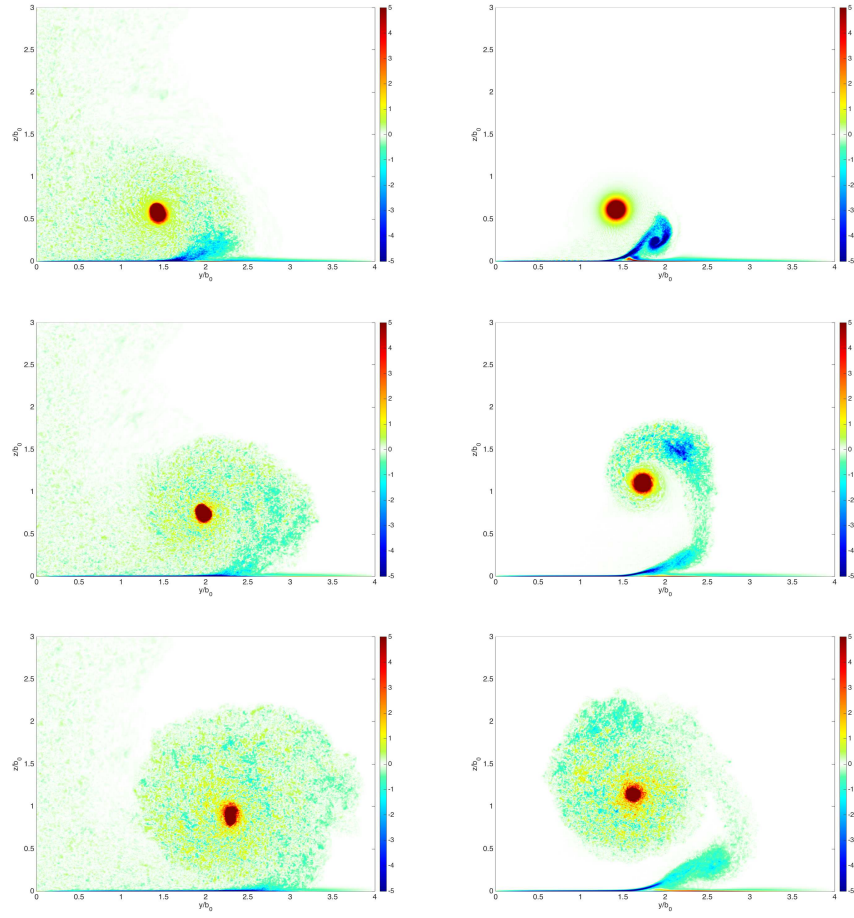


Figure 4.9: Averaged axial vorticity $\langle \omega_x \rangle b_0^2 / \Gamma_0$ at $t/t_0 = 3, 4$ and 5 (only one vortex is shown). T-2VS (left) and BH-2VS(right).

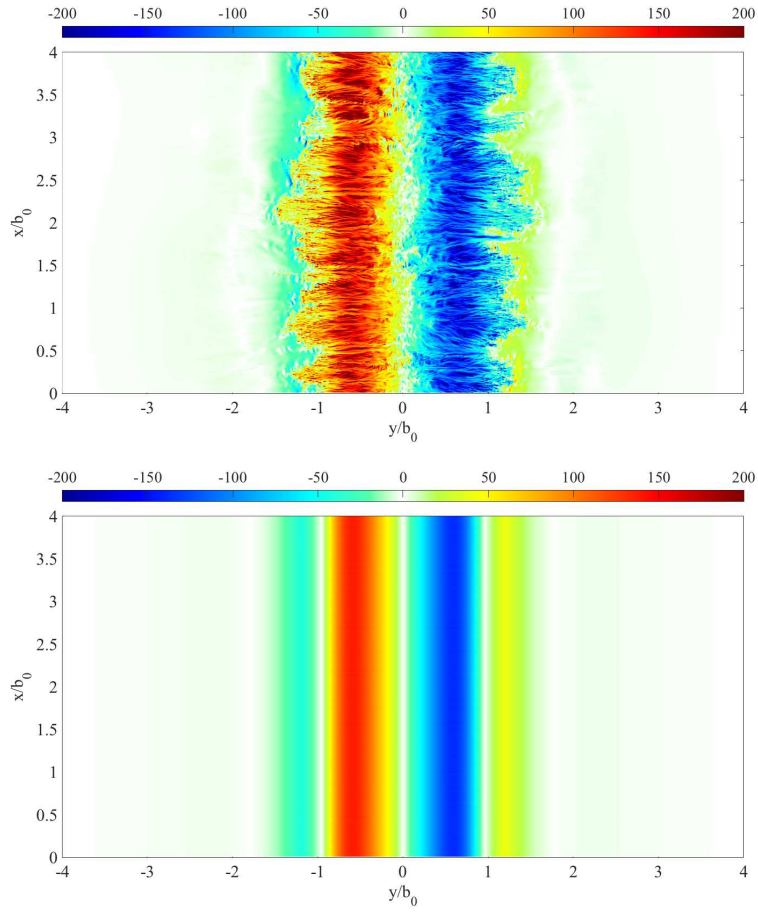


Figure 4.10: Axial vorticity at the wall $\omega_x b_0^2 / \Gamma_0$ at $t/t_0 = 2$, also showing the details of the boundary layer separation. T-2VS (top) and BH-2VS (bottom).

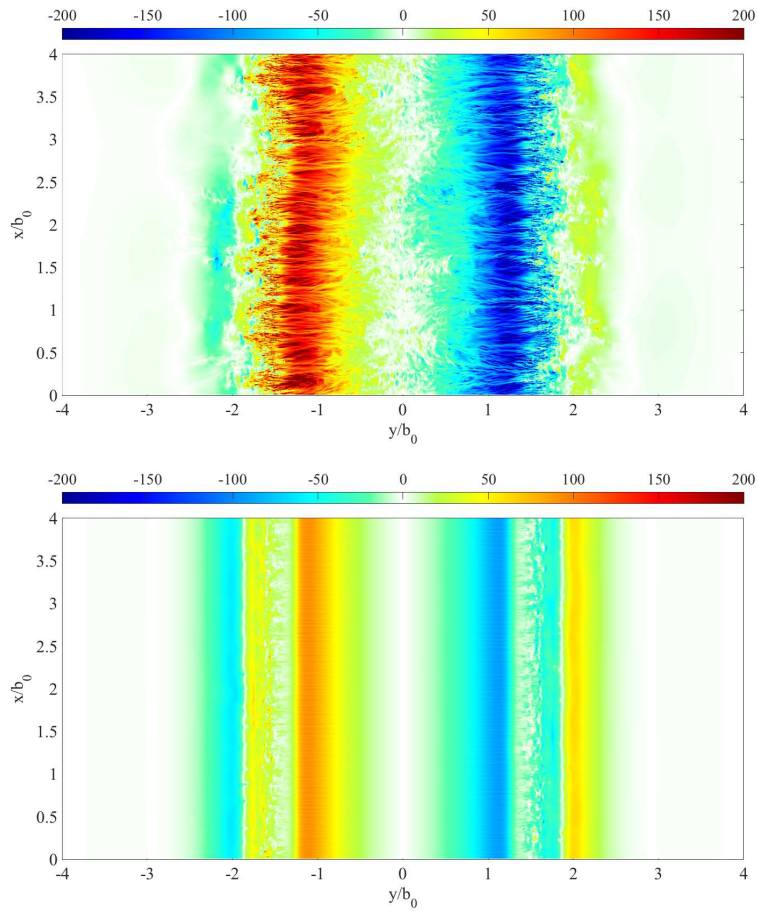


Figure 4.11: Axial vorticity at the wall $\omega_x b_0^2 / \Gamma_0$ at $t/t_0 = 3$, also showing the details of the boundary layer separation. T-2VS (top) and BH-2VS (bottom).

Around $t/t_0 \simeq 1.5$, the vortices reach an altitude of $z/b_0 \simeq 0.8$, and the boundary layers start to interact with them. Recall that $\Gamma(r)$ has a two-scale behavior: rapidly growing in the inner region, then with a slower growth up to its maximum around $r/b_0 \simeq 0.7 - 0.8$ (see Fig.4.16). At that point, the boundary layer vorticity already impacts $\Gamma(r)$, and the result is a immediate decrease of Γ_{tot} , even before any secondary vorticity has separated. When that vorticity has separated, around $t/t_0 \simeq 3$ (see Fig. 4.12), the process is rather smooth. The effect on Γ_{5-15} is delayed until this secondary vorticity reaches the inner part of the vortex, around $t/t_0 \simeq 3.5$, see Fig. 4.13. This can also be seen in the $\Gamma(r)$ distributions of Fig. 4.16: at $t/t_0 \simeq 3$, the secondary vorticity reaches $r/b_0 \simeq 0.5$, the inner part still being unaffected; yet, by $t/t_0 \simeq 4$, it has been incorporated in the inner part and the distributions then much differs from the OGE case. The mixing of primary and secondary vorticity appears more intense when monitoring Γ_{5-15} (see Fig. 4.13) because it mostly accounts for vorticity entering the inner part of the vortex, while Γ_{tot} is a global quantity. We can hence easily distinguish two separate decay phases in the Γ_{5-15} diagnostic, whereas the decay phases are harder to distinguish in the smoother Γ_{tot} evolution.

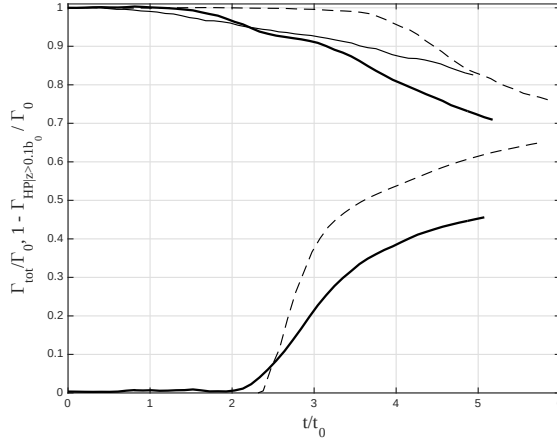


Figure 4.12: Total vortex circulation Γ_{tot}/Γ_0 (top curves) and separated secondary vorticity $1 - \Gamma_{HP|z>0.1b_0}/\Gamma_0$ (bottom curves). T-2VS IGE (thick solid) and OGE (thin solid), and BH-2VS IGE (dashed).

We see that the $\Gamma(r)$ distribution plays a major role in the evolution of vari-

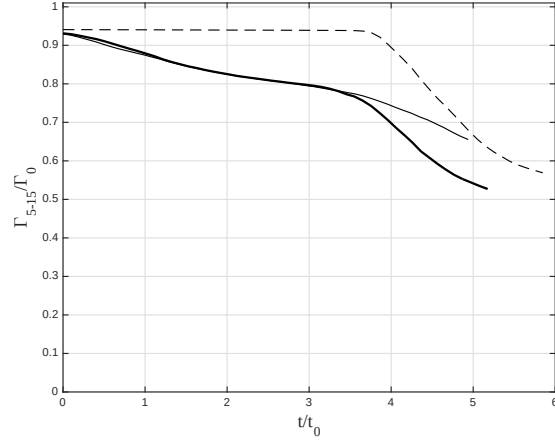


Figure 4.13: Decay of “LIDAR processed” vortex circulation Γ_{5-15}/Γ_0 . T-2VS IGE (thick solid) and OGE (thin solid), and BH-2VS IGE (dashed).

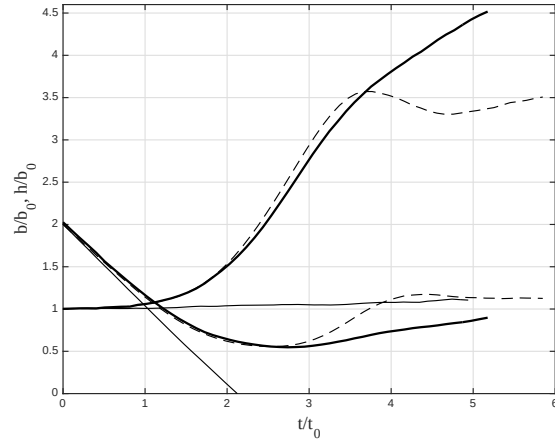


Figure 4.14: Evolution the vortex spacing b/b_0 and of the vortex height h/b_0 . T-2VS IGE (thick solid) and OGE (thin solid), and BH-2VS IGE (dashed).

ous diagnostics, and that the two-scale behavior is of importance to understand the difference in the evolution of the various diagnostics. The BH-2VS has a $\Gamma(r)$ distribution with only one scale, and that grows more rapidly in the inner region than the obtained turbulent vortex, or the PW two-scales model. In this case, we see, in Fig. 4.17 and Fig. 4.12, that at $t/t_0 \simeq 2$, when the ground is seen by the vortex, the boundary layer has little – if any – impact on Γ_{tot} . The effect of the secondary vorticity on the vortex is observed on Γ_{tot} and Γ_{5-15} with almost no delay (see Fig. 4.12 and Fig. 4.13), around $t/t_0 \simeq 3.7$, when the secondary vorticity strongly interacts with the inner part of the vortex, through the process described above in Section 4.4.1 (see also Fig. 4.7). The essentially laminar behavior of the vortex system also plays an important role in the separation mechanism. We see that the rebound is more pronounced than in the T-2VS case (see Fig. 4.15). This is due to several factors. As described in Section 4.4.1, the secondary vorticity separated from the ground is more coherent in the BH-2VS case, due to the absence of vortex turbulence; we also see, in Fig. 4.12 that the secondary vorticity separates very differently in the BH-2VS and in the T-2VS cases. In the T-2VS case, secondary vorticity is being continuously ripped off the ground from $t/t_0 \simeq 2$, and in a turbulent fashion. The resulting secondary vorticity does not form a longitudinally coherent vortex. This secondary vorticity is incorporated in the vortex outer turbulent part and it orbits the vortex for a while within the vortex turbulence before reaching the vortex inner part (see also Fig. 4.6 at $t/t_0 = 2$ and 3). In the BH-2VS case, secondary vorticity is separated from the ground in an essentially laminar fashion and longitudinally coherent fashion (see Fig. 4.6 at $t/t_0 = 3$, also Fig. 4.10 and Fig. 4.11). The result is that more vorticity is separated than in the T-2VS case, as seen in Fig. 4.12. The separation process appears a bit later, around $t/t_0 \simeq 2.4$, but it also happens much faster than in the T-2VS case. This results in the more pronounced rebound, as seen in Fig. 4.14 and Fig. 4.15.

The late phase of decay is finally the same for both cases, since the topology of the vortices is now similar: turbulent vortices with turbulent secondary vorticity being continuously incorporated (see Fig. 4.6). The rate of secondary vorticity production is indeed the same at late times for both the T-2VS and BH-2VS cases (see Fig. 4.12). The dimensionless vortex shape $\Gamma(r/r_c)/\Gamma_{tot}$ is also similar (see Fig. 4.18), indicating, that the turbulence surrounding the vortex core is essentially the same in both cases. As for the the equilibrium

T-2VS OGE, we have reached an other equilibrium two-scales distribution.

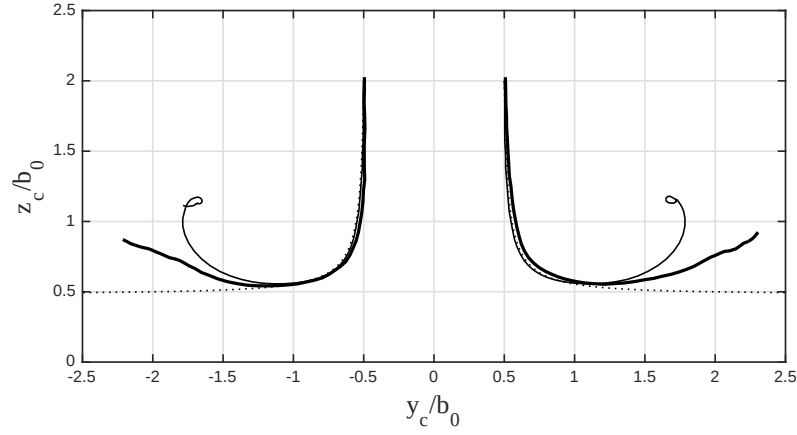


Figure 4.15: Trajectories of the various cases IGE. T-2VS (thick solid), BH-2VS (thin solid) and inviscid hyperbolic trajectory (dotted).

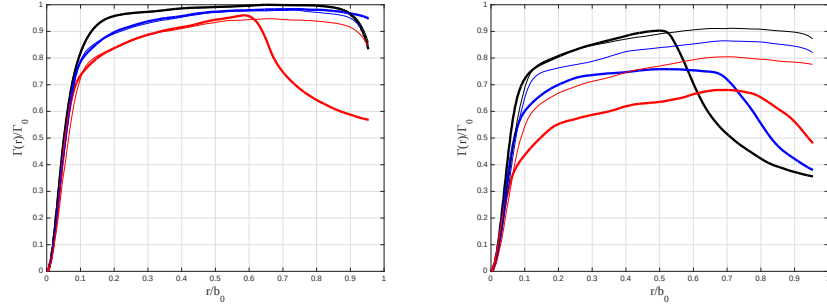


Figure 4.16: Vortex circulation distribution at times $t/t_0 = 0$ (black), 1 (blue), and 2 (red, left) and $t/t_0 = 3$ (black), 4 (blue), and 5 (red, right). T-2VS IGE (thick solid) and OGE (thin solid).

4.5 Conclusion

A wake two-vortex system (2VS) at equilibrium with its own turbulence has first been obtained and characterized. This was achieved by considering a 2VS made of initially analytical vortices (Burnham-Hallock, BH) and evolving them

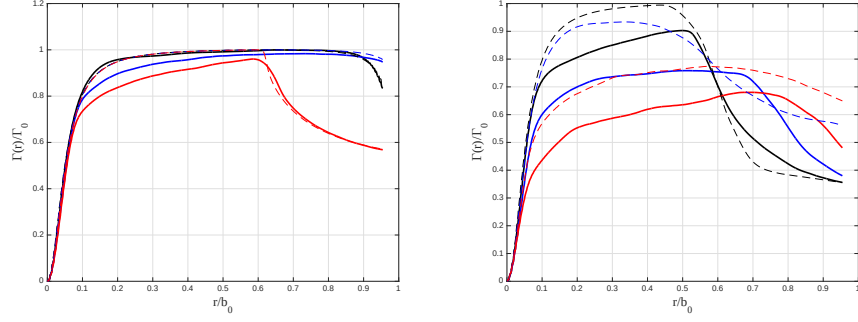


Figure 4.17: Vortex circulation distribution at times $t/t_0 = 0$ (black), 1 (blue), and 2 (red, left) and $t/t_0 = 3$ (black), 4 (blue), and 5 (red, right). T-2VS IGE (thick solid) and BH-2VS IGE (dashed).

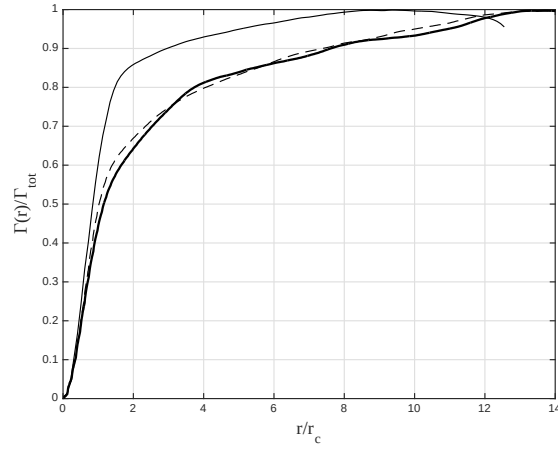


Figure 4.18: Final vortex circulation distribution. T-2VS IGE at $t/t_0 = 5$ (thick solid) and BH-2VS at $t/t_0 = 5.9$ (dashed). For comparison, the circulation distribution of the equilibrium T-2VS OGE is also shown (thin solid).

under the forcing of a very weak atmospheric turbulence. At equilibrium, energy is constantly transferred from the large and mostly 2D scales of the flow to azimuthal medium structures. The kinetic energy contained in the turbulence is found to be rather important, accounting for about 8% of the total kinetic energy. The time to reach the equilibrium depends on the initial level of background turbulence, and on the initial circulation distribution in the 2VS. The equilibrium turbulent 2VS (T-2VS), however, is found to be independent of the initial turbulence level (when it remains in the weak to moderate regime), and of the initial circulation distribution. The dissipation rate of the equilibrium T-2VS is then also the same. The circulation distribution profile is also seen to reach the same equilibrium. The circulation distribution at equilibrium exhibits a two-scales profile: an inner layer, scaling with r/r_c , and an outer turbulent part, scaling with b_0 .

The T-2VS was then used as an initial condition for a wall-resolved LES of such a T-2VS in ground effect (IGE), and at a quite high Reynolds of $Re_\Gamma = 2.0 \cdot 10^5$, using a grid properly sized to the problem (with about 1.5 billion points) together with a state-of-the art SGS model. For comparison, a similar simulation was performed using a BH-2VS model; hence without natural turbulence, yet with some white noise perturbation. For further comparison, the simulation of the T-2VS out of ground effect (OGE) was also performed to obtain the natural decay rate OGE and excluding the Crow instability. The simulations were performed using a fourth order finite difference code. The T-2VS simulation IGE displayed more physical topological features than the BH-2VS IGE, with its own turbulence interacting with the ground, resulting in more continuous and smooth turbulent interaction of the 2VS with the ground, and in a significantly less pronounced rebound of the 2VS. The circulation distribution and decay was also analyzed over time. The T-2VS circulation was shown to behave differently from the BH-2VS when monitoring Γ_{tot} or Γ_{5-15} . The fast decay phase is also more clearly distinguished in the Γ_{5-15} diagnostic, which was explained by the two-scales circulation distribution due to the vortex self-turbulence. The present simulation of a T-2VS in ground effect, using wall-resolved LES and high quality SGS modeling, and at a quite high Reynolds of $Re_\Gamma = 2.0 \cdot 10^5$ (ten times more than previous works), is believed to be physically sound and to properly capture, with a high level of details, the complex flow physics at work. As the initial 2VS is turbulent, and as the interaction with the ground also is, it is believed that the obtained circulation results ($\Gamma(r)$, Γ_{tot} , and Γ_{5-15})

are also, in a good part, applicable to the interaction of a T-2VS with a rough ground; at least for cases with distributed roughness and of scale much smaller than r_c .

Finally, both simulations resulted in a similar “equilibrium turbulent 2VS IGE long after rebound”, which was also characterized in terms of circulation distribution. It differs from the “equilibrium turbulent 2VS OGE”, as it keeps evolving near the ground. Such circulation distribution could also be of use to model wake vortices after rebound (of course before any ground interaction phenomena of wavelength higher than the presently used $L_x = 4 b_0$ computational domain).

Chapter 5

Wall-modeled LES of turbulent 2VS in ground effect at $Re_\Gamma = 2.0 \cdot 10^5$

The content of this chapter, as well as the methodology of Fig. 4.5, were presented at the 67th Annual Meeting of the American Physical Society, Division of Fluid Dynamics (APS-DFD67) [132], and at the WakeNet-Europe 2015 workshop [133].

5.1 Introduction

Chapter 4 attached to the simulation of a turbulent two-vortex system (T-2VS) in ground effect at $Re = 2 \cdot 10^5$, using wall-resolved LES. The aim of the present chapter is to assess the wall modeling approaches developed in Chapters 2 and 3. The pressure-corrected models of Chapter 3 were tested with mixed results in a case with a mild pressure gradient and without boundary layer separation. We are here interested in the comparison of the various approaches in the T-2VS in ground effect case, where boundary layer separation appears, and the amount of vorticity separated from the wall plays a key role in the vortex rebound and decay. The wall-resolved LES of Chapter 4 is used as a reference solution to compare and assess the various wall models.

5.2 Numerical setup

The numerical domain employed is the same as previously and has a size

$$L_x = 4b_0, \quad L_y = 8b_0, \quad L_z = 4b_0. \quad (5.1)$$

The initial T-2VS is obtained and put in the numerical box at height $h_0 = 2b_0$ using the same methodology as previously, see section 4.3, and the sketch of Fig. 4.5. The mesh used is coarser than that used for the WRLES and has a number of points of

$$N_x = 320, \quad N_y = 640, \quad N_z = 512. \quad (5.2)$$

The stretching of Eq. (4.10) is applied in the wall normal direction z , with $\gamma = 1.07$. This mesh yields a maximum value of $z_3^+ \simeq 28$ for the height at which the wall model is computed, as seen in Fig. 5.2. This value is reached around the time of rebound, at $t/t_0 \simeq 2.6$. At this time, the viscous resolution in the wall parallel directions x and y is $\Delta x^+ = \Delta y^+ \simeq 79$. These values are rather moderate for wall-modeled LES. Compared to the wall-resolved LES mesh, the wall parallel resolution is about 2.5 times coarser. The wall normal resolution however is not very coarse. Fig. 5.1 shows the mesh resolution compared to the mesh used for the wall-resolved LES, in units relative to b_0 . Though this resolution is not sufficient for wall-resolved LES, it is still rather fine for wall-modeled LES.

Such a fine resolution is required to resolve the outer part of the thin boundary layers generated near the wall as soon as the T-2VS is put in the domain. Recall that in a wall-modeled LES, only the inner part of the boundary layer is modeled. Even at this rather high Reynolds number of the 2VS, the boundary layers do not reach very high frictional Reynolds numbers. This is illustrated by figures 5.3 and 5.4. We see that the boundary layer is rather small and that a coarser mesh is not possible because it would not have enough points within the boundary layer. A fine resolution is also required in the Oyz plane to resolve the intense gradients in the core of the vortices. The core of each vortex has a radius $r_c/b_0 = 0.05$, while the resolution of the mesh is $\Delta x/b_0 = \Delta y/b_0 = 0.0125$, which means that there are roughly 8 points per diameter in the core of the vortex, which is about as coarse as the mesh can be. With an even coarser mesh, the core would be numerically diffused and/or distorted.

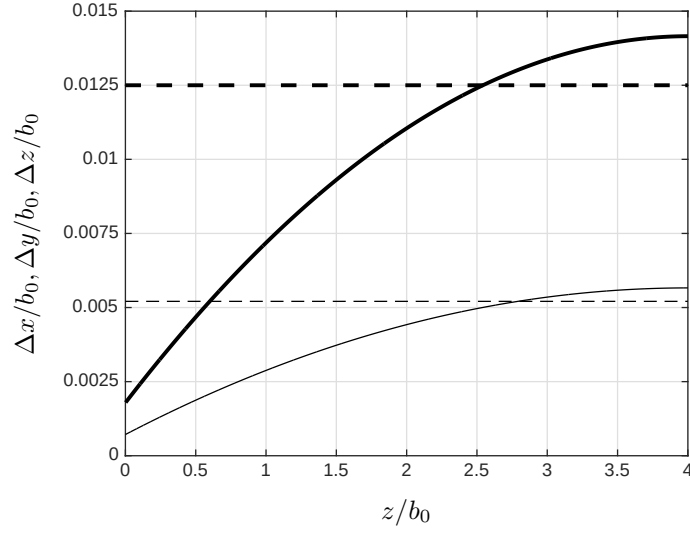


Figure 5.1: Resolution of the meshes used for the wall-resolved LES (thin) and for the wall-modeled LES (thick). Vertical resolution $\Delta z/b_0$ (solid), and wall parallel resolution $\Delta x/b_0 = \Delta y/b_0$ (dashed)

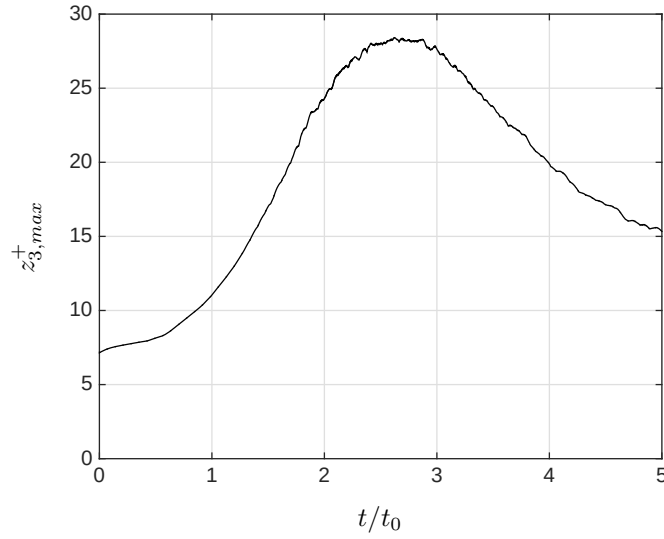


Figure 5.2: Time evolution of the maximum value of z_3^+ for the wall-modeled LES with the algebraic wall model using the law of Reichardt and averaging over 5×5 points.

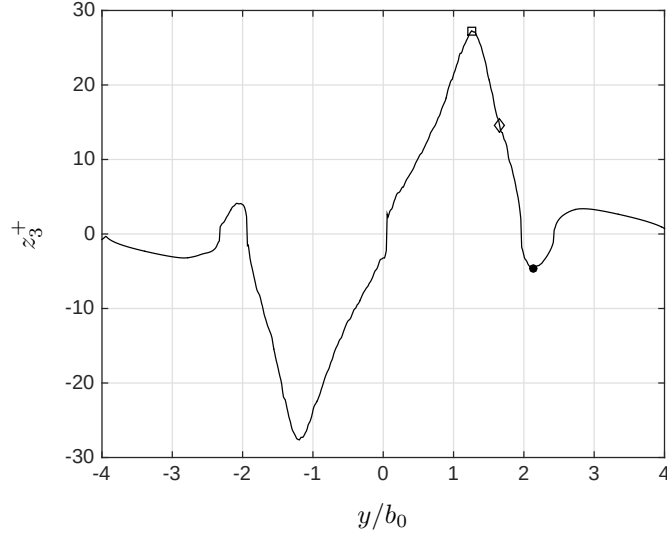


Figure 5.3: Longitudinal average of z_3^+ at $t/t_0 = 3$, for the wall-modeled LES with the algebraic wall model using the law of Reichardt and averaging over 5×5 points. The markers correspond to the various stations at which the velocity profiles are shown in Fig. 5.4.

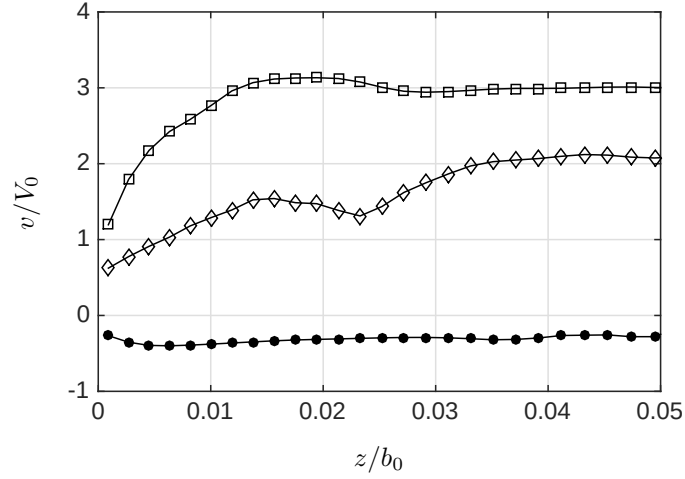


Figure 5.4: Longitudinal average of velocity component $\langle v \rangle(z)/V_0$, for the wall-modeled LES with the algebraic model using the law of Reichardt and averaging over 5×5 points. The stations are given by the corresponding markers in Fig. 5.3.

The filtered WALE SGS model is used. Bricteux [13] showed that filtered (and RVMs) SGS models with the proper wall behavior are able to accurately compute wake vortices in ground effect in wall-resolved LES. Fig. 5.5 shows the vortex decay he obtained at $Re_\Gamma = 2.0 \cdot 10^4$, using such models, the filtered WALE model is also added to demonstrate its capability at performing such simulations. We see that all filtered models with the proper wall behavior (obtained either with the WALE scaling or with a Piomelli et al. [101] damping) are in good agreement. Only the plain Smagorinsky model with Piomelli et al. [101] damping performed poorly, showing that both a good wall behavior and good dissipative properties in pure rotational flow (which the unfiltered Smagorinsky does not have) are required for a SGS model to provide good results in this case.

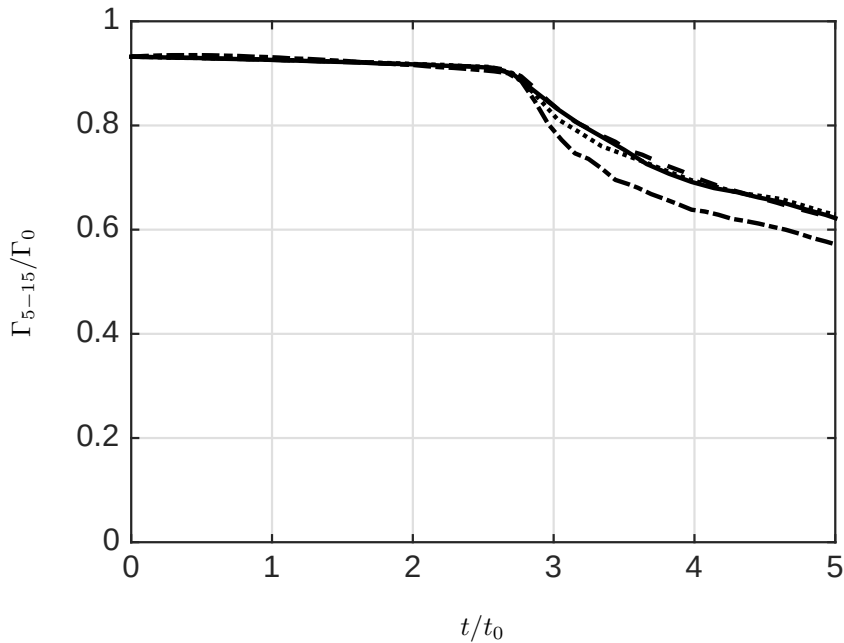


Figure 5.5: Wall-resolved LES of BH-2VS at $Re_\Gamma = 2 \cdot 10^4$: Γ_{5-15} decay over time. RVMs-WALE (solid), filtered Smagorinsky with Piomelli damping (dashed), Smagorinsky with Piomelli damping (dashed-dotted), and filtered WALE (dotted). All but the filtered WALE simulations are taken from Bricteux [13].

Both wall modeling strategies (namely, the algebraic and the RANS sub-layer models) are tested. Algebraic quadratic wall models with the law of Reichardt and with the law of Shih et al. are used. Averaging is performed

over the $N_x \times N_y = 5 \times 5$ closest points, no time averaging is performed. Since the T-2VS is homogeneous in the longitudinal (x) direction, one could therefore be tempted to perform averaging in that direction. Unfortunately, this homogeneity cannot be exploited by the wall models for two main reasons: first, the vortex cores, which initially lie in straight lines start to deform as some long wave instabilities appear; second, the turbulence surrounding the vortices presents structures that impact the boundary layer. The footprint of the vortices on the ground is therefore not longitudinally homogeneous, and the boundary layer separation does not appear on a straight line. To further support this, a simulation was also performed with wall models computed using full longitudinal averaging. Fig. 5.6 illustrates the footprint of the vortices in terms of the vorticity close to the wall (at the first off-wall point) at $t/t_0 = 3$, which is shortly after the rebound. In the simulation with longitudinal averaging, longitudinal lines appear in the footprint, especially close to the boundary layer separation point. This is a spurious effect of the longitudinal averaging, which does not appear in the case with local averaging. No such longitudinal lines were observed in the footprint of the T-2VS in the WRLES in Fig. 4.10 and Fig. 4.11, comforting us that averaging should be performed locally to account for the longitudinal inhomogeneity of the wall flow. Fig. 5.7 shows the total vortex circulation as well as the vorticity separated from the wall for both simulations. While the out-of-ground behavior is unaffected by the type of averaging, the amount of vorticity separated from the wall, as well as the type of interaction of the separated vorticity with the primary vorticity is different. In the early separation, the case with longitudinal averaging tends to separate a little more vorticity than when performing local averaging, then the separation process slows down a bit to finally yield almost the same total amount of secondary vorticity (even a little less) than the case with local averaging strategy. This results in almost the same total circulation decay. The LIDAR-processed Γ_{5-15} however does not exhibit the same trend, as seen in Fig. 5.8. The rate of decay seems to be slightly larger in the case with longitudinal averaging. This means that the way the secondary vorticity interacts with the primary vorticity is changed, hence modifying the $\Gamma(r)$ behavior, and also the ratio of $\Gamma_{tot}/\Gamma_{5-15}$. This is due to the fact that the topology of the separation of secondary vorticity is slightly modified by the averaging procedure employed by the wall models, as seen in Fig. 5.6. The local averaging strategy, which yields a more physical separation, and is consistent with the topology of the flow, is

therefore retained.

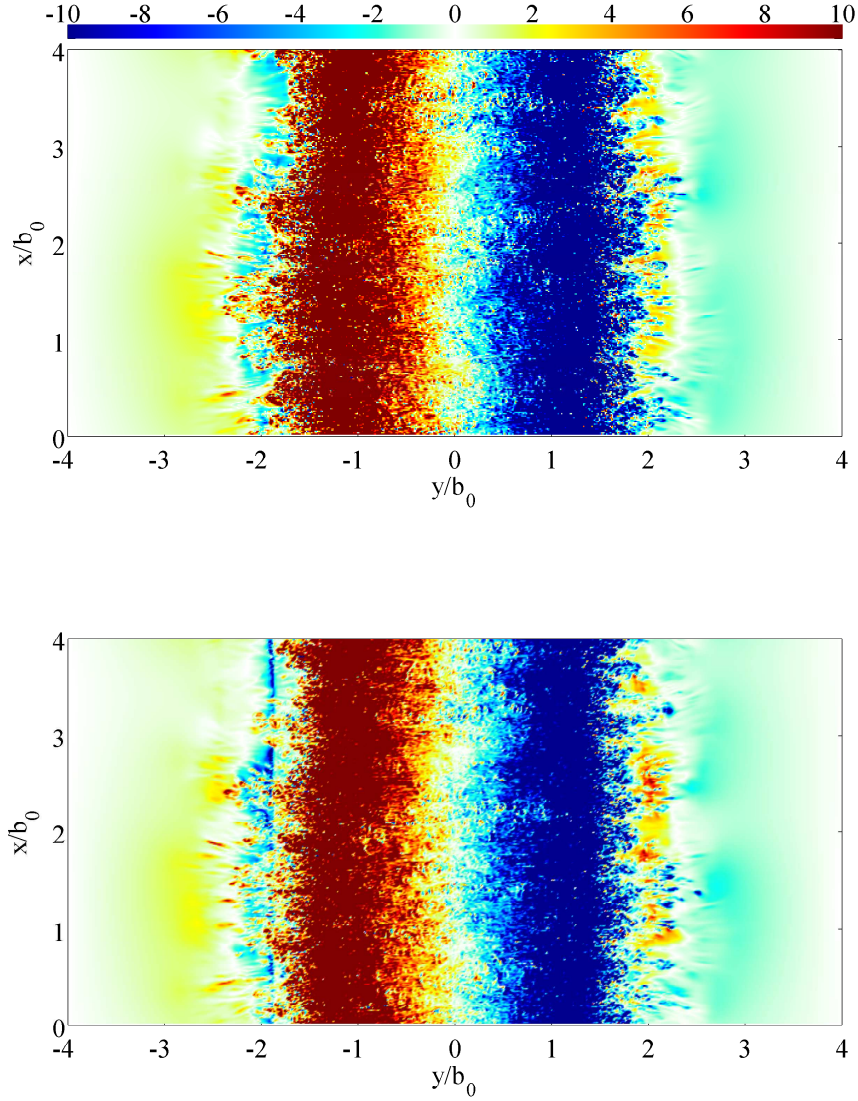


Figure 5.6: Vorticity $\omega_x b_0^2 / \Gamma_0$ at $t/t_0 = 3.0$ at the first off-wall point for the WMLES with an algebraic model and the LOTW of Reichardt, using local averaging over the closest 5×5 points (top) and full longitudinal averaging (bottom).

The RANS sublayer models are also used, on an embedded mesh with 7 vertical points, and with a hyperbolic tangent stretching with $\gamma = 2.0$. Because

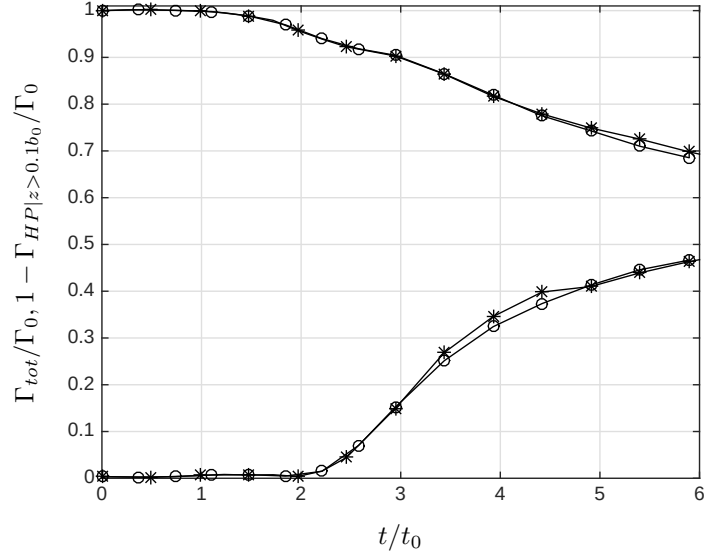


Figure 5.7: Total vortex circulation Γ_{tot} and separated vorticity $1 - \Gamma_{HP|z>0.1b_0}/\Gamma_0$ for the WMLES with algebraic wall models with the law of Reichardt and with longitudinal averaging (asterisks) or local averaging with a 5×5 averaging window (circles).

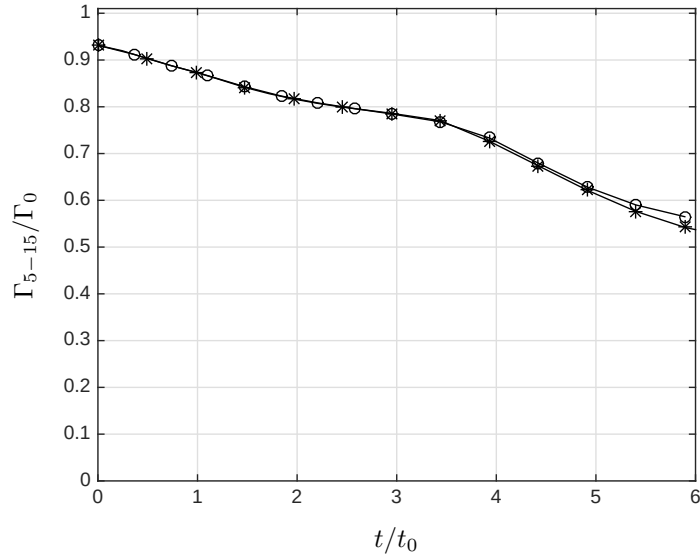


Figure 5.8: LIDAR-processed Γ_{5-15} for the WMLES with algebraic wall models with the law of Reichardt and with longitudinal averaging (asterisks) or local averaging with a 5×5 averaging window (circles).

of stability issues, the models could not be run using the convective terms with the correction of the RANS viscosity. Only the time derivative therefore remains. Contrarily to the conclusions drawn in the statistically stationary case of Chapter 3, the RANS sublayer models without convective terms do not here revert to algebraic wall models, because the solution evolves in time and the time derivatives are resolved. The model therefore has a memory that the algebraic models do not have.

Finally, an underresolved LES is also performed, hence using the wall-modeled LES mesh, but with a no-slip boundary condition instead of a wall model.

5.3 Results

Fig. 5.9 displays the total vortex circulation and the separated secondary vorticity for the wall-modeled cases, and compares it to the wall-resolved simulation and the under-resolved simulation. We see that, while all simulation remain reasonably close to the WRLES total circulation, none quite captures the proper rate of decay. All wall-modeled simulations predicted the boundary layer separation at the correct time, around $t/t_0 \approx 2$, but in all cases, the rate at which secondary vorticity is produced at the wall is slower than in the wall-resolved simulation. After the initial boundary layer separation, after $t/t_0 \approx 3.5$, secondary vorticity is produced at the wall continuously, but at a slower rate, which seems to be the same in all simulation, and roughly equal to that of the WRLES. The higher rate of decay of the total vortex circulation of the WRLES is probably due to the faster initial secondary vorticity production, which is then incorporated in the primary vortex. This is even more visible in Fig. 5.10, in the evolution of the LIDAR-processed circulation Γ_{5-15} : the rate of decay in the fast decay phase, after $t/t_0 \approx 3.5$ is a measure of the amount of secondary vorticity produced and absorbed by the primary vorticity. The rate of decay is initially larger in the WRLES, because more secondary vorticity is produced during the rebound phase and therefore absorbed during the fast decay phase. The initial rate of decay of Γ_{5-15} is slightly larger, between $t/t_0 \approx 2$ and $t/t_0 \approx 3.5$, in the wall-modeled simulations, which might be a sign that the mesh is a bit coarse and that more points are required in the core of the vortices. This might result of a slight diffusion of the cores of the vortices, which is more visible in Γ_{5-15} than in Γ_{tot} . The former only accounts

for vorticity close to the core, while the latter include the whole $\Gamma(r)$ curve and is therefore insensitive to core diffusion.

Comparing the various wall models performances, we see that the algebraic model with the law of Reichardt and the RANS sublayer model with the model of Duprat et al. are the wall models that produce the most secondary vorticity, none producing as much as the WRLES, however. The law of Shih et al., which accounts for pressure gradient effects here again does not perform better than the simpler law of Reichardt. With the RANS sublayer models however, the model of Duprat et al., hence with explicit pressure gradient effects, performed best, and even better than the algebraic wall models. It came closer to the WRLES in terms of secondary vorticity production, and of rate of decay of Γ_{tot} and of Γ_{5-15} . Fig. 5.11 displays the trajectory of the vortices. The slightly less intense rebound observed in the wall-modeled simulations can simply be explained by the smaller secondary vorticity production. Notably, the under-resolved LES performed well and came as close to the wall-resolved LES as the wall-modeled cases. It even performed slightly better in terms of Γ_{5-15} decay and secondary vorticity production. Recall that the wall resolution is quite fine for a wall-modeled simulation, as seen in Fig. 5.2, which means the under-resolved simulation is not very coarse in terms of wall resolution. This partly explains the relative success of this simulation. Furthermore, this case is indeed a great challenge for wall models since it involves a lot of phenomena at the wall: the acceleration, deceleration, and separation of the boundary layer as well as its interaction with the turbulence of the T-2VS.

5.4 Conclusion

The wall modeling strategies developed in the previous chapters were successfully applied to the case of high Reynolds number turbulent two-vortex system in ground effect and compared to the wall-resolved simulation of Chapter 4. All wall models were seen to be able to provide boundary layer separation at the correct time in the simulation. It must be noted however that in this case, the boundary layer separation is probably mainly dictated by the vortex dynamics of the system. In the case of algebraic wall models, it was shown that, when averaging is used, it should be performed locally and that using homogeneous directions is, in this case, not advisable. The averaging over the initially homogeneous longitudinal direction produced spurious longitudinal structures at

the wall which were neither observed when local averages were used, nor in the WRLES. The use of algebraic wall models pointwise should also be investigated. In the case investigated in Chapter 3, the simulation with algebraic wall models based on pointwise information provided similar results than when local averaging was performed. In all simulations the amount of vorticity produced at the wall by the boundary layer separation, however, was seen to be smaller than that obtained in the WRLES. This resulted in a smaller rate of decay of the vortex circulation than in the WRLES. The total vortex circulation curves, however, remain reasonably close to that of the WRLES.

The under-resolved LES, hence with no wall model, provided results comparable to the wall-modeled simulation, even better in some regards. This questions the practical use of wall models in such challenging cases. We acknowledge that the case at hand is quite a challenge for wall models. Nevertheless, the good results provided by the under-resolved LES might be in part explained by the rather fine mesh close to the wall (for a wall-modeled LES). The simulation of this case at a higher Reynolds number would allow for a fairer comparison of the wall-modeled LES's and the under-resolved LES. If wall models prove able to accurately predict such flows at higher Reynolds numbers, this would mean that a simulation at a realistic Reynolds number for a commercial aircraft wake, of the order of $Re_T \simeq 10^7$, could be performed. The results of such a simulation are expected to be close to those obtained at this already high Reynolds number. Validation is however an issue since no reference is available at a higher Reynolds number, the simulation of Chapter 4 being, to the authors' best knowledge, the highest Reynolds number attained using wall-resolved LES.

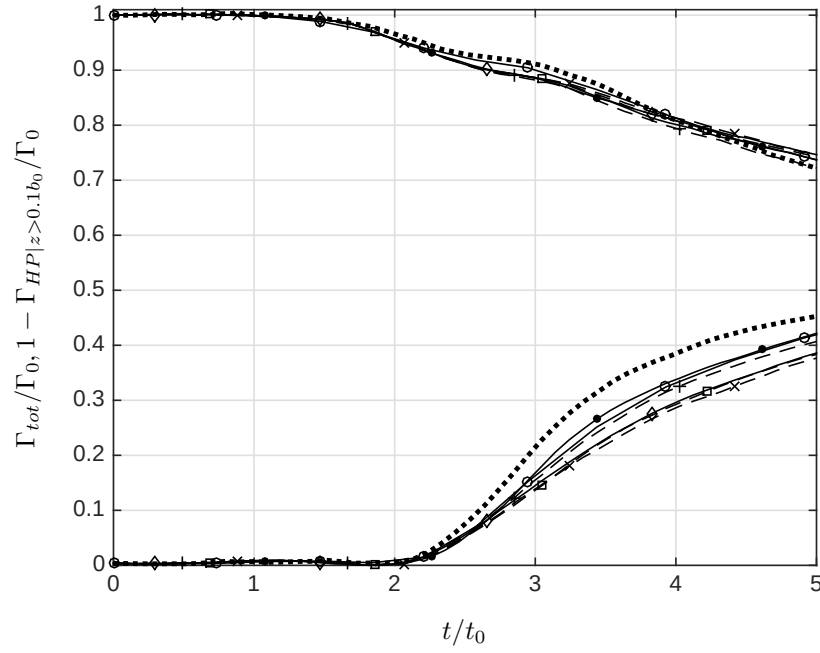


Figure 5.9: Total vortex circulation Γ_{tot} and separated vorticity $1 - \Gamma_{HP|z>0.1b_0}/\Gamma_0$. WRLES (thick dotted), WMLES with the quadratic algebraic wall model (thin solid) and RANS sublayer models (thin dashed). Law of Reichardt (circles) and law of Shih et al. (diamonds). RANS models of Cabot (squares), Duprat et al. (plusses), and Granville (crosses). No wall model (thin solid and bullets)

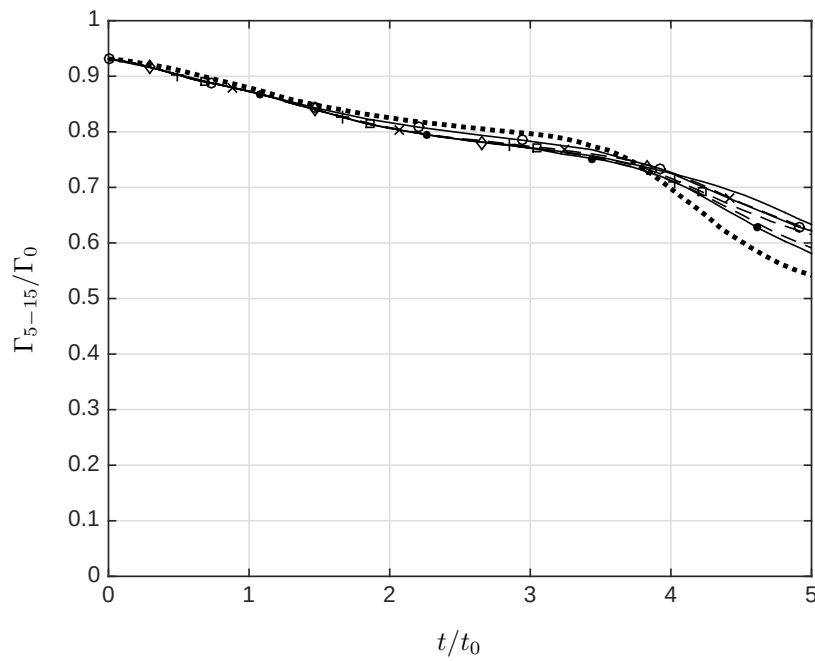


Figure 5.10: LIDAR-processed vortex circulation Γ_{5-15} . WRLES (thick dotted), WMLES with the quadratic algebraic wall model (thin solid) and RANS sublayer models (thin dashed). Law of Reichardt (circles) and law of Shih et al. (diamonds). RANS models of Cabot (squares), Duprat et al. (plusses), and Granville (crosses). No wall model (thin solid and bullets)

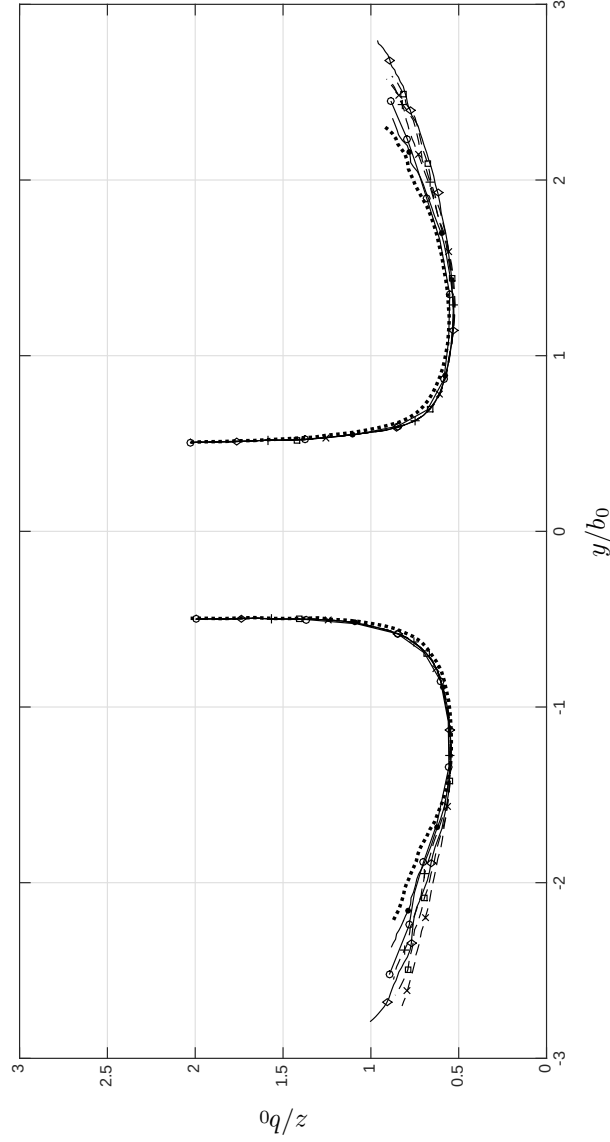


Figure 5.11: Trajectories of the vortices. WRLES (thick solid), WMLES with the quadratic algebraic wall model (thin dotted) and RANS sublayer models (thin dashed). Law of Reichardt (circles) and law of Shih et al. (diamonds). RANS models of Cabot (squares), Duprat et al. (plusses), and Granville (crosses). No wall model (thin solid and bullets)

Chapter 6

A mixed multiscale model better accounting for the cross term of the subgrid-scale stress and for backscatter

This chapter has been published in Physics of Fluids. The original authors are: Thiry and Winckelmans [131]. This work has also been presented in the conference Advances in Computational Mechanics (ACM 2013) – A Conference Celebrating the 70th Birthday of Thomas J.R. Hughes, 17th International Conference on Finite Elements in Flow Problems (FEF 2013) [149], at the 14th European Turbulence Conference (ETC14) [129], and at the 5th European Conference on Computational Mechanics, 6th European Conference on Computational Fluid Dynamics [130]. The version in this chapter is slightly modified to account for comments from the jury.

abstract

In the large-eddy simulation (LES) of turbulent flows, models are used to account for the subgrid-scale stress (SGS). We here consider LES with “truncation

filtering only” (i.e., that due to the LES grid), thus without regular explicit filtering added. The SGS stress tensor is then composed of two terms: the cross term that accounts for interactions between resolved scales and unresolved scales, and the Reynolds term that accounts for interactions between unresolved scales. Both terms provide forward (dissipation) and backward (production, also called backscatter) energy transfer.

Purely dissipative, eddy-viscosity type, SGS models are widely used: models of the Smagorinsky type, or more advanced multiscale-type models. Dynamic versions have also been developed, where the model coefficient is determined using a dynamic procedure. Being dissipative by nature, those models do not provide backscatter. Even when using the dynamic version with local averaging, one typically uses clipping to forbid negative values of the model coefficient, and hence ensure the stability of the simulation; hence removing the backscatter produced by the dynamic procedure.

More advanced SGS model are thus desirable, and that better conform to the physics of the true SGS stress, while remaining stable. We here investigate, in decaying homogeneous isotropic turbulence (DHIT), and using a de-aliased pseudo-spectral method, the behavior of the cross term and of the Reynolds term: in terms of dissipation spectra, and in terms of probability density function (PDF) of dissipation in physical space: positive and negative (backscatter). We then develop a new mixed model that better accounts for the physics of the SGS stress and for the backscatter. It has a cross term part which is built using a scale-similarity argument, further combined with a correction for Galilean invariance using a pseudo Leonard term: this is the term that also does backscatter. It also has an eddy-viscosity multiscale model part, that accounts for all the remaining phenomena (also for the incompleteness of the cross term model), that is dynamic and that adjusts the overall dissipation. The model is tested, both a priori and a posteriori, and is compared to the DNS and to the exact SGS terms, also in time. The model is seen to provide accurate energy spectra, also in comparison to the dynamic Smagorinsky model. It also provides significant backscatter (although four times less than the real SGS stress), while remaining stable.

6.1 Introduction

We consider single phase incompressible and turbulent flows, with constant kinematic viscosity, ν . The Navier-Stokes (NS) equations for the velocity, u_i , and reduced pressure, $P = p/\rho$, fields are:

$$\frac{\partial u_j}{\partial x_j} = 0, \quad (6.1)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) + \frac{\partial P}{\partial x_i} = \frac{\partial}{\partial x_j} (2\nu S_{ij}) = \nu \frac{\partial}{\partial x_j} \frac{\partial u_i}{\partial x_j}, \quad (6.2)$$

where $S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ is the strain rate tensor.

We then consider the large-eddy simulation (LES) of such incompressible turbulent flows. The effective LES equations are here obtained from the NS equations by using a truncation (= projection) operator, here noted $\tilde{\cdot}$, and that restrains the dynamics of the simulated flow to the scales that can be captured by the coarse LES grid, up to the grid cutoff wavenumber $k_c = \pi/h$ (h being the grid size). Here, we do not consider any additional regular explicit filtering. The effective equations for the velocity, $\tilde{u}_i$, and pressure, $\tilde{P}$, LES fields are then obtained as:

$$\frac{\partial \tilde{u}_j}{\partial x_j} = 0, \quad (6.3)$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\widetilde{u_i u_j}) + \frac{\partial \tilde{P}}{\partial x_i} = \frac{\partial}{\partial x_j} (2\nu \tilde{S}_{ij} + \tilde{\sigma}_{ij}^{\text{sgs}}), \quad (6.4)$$

where $\tilde{S}_{ij}$ is the strain rate tensor of the LES velocity field, and where $-\tilde{\sigma}_{ij}^{\text{sgs}} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$ is the subgrid-scale (SGS) stress tensor, to be modeled. All terms in the LES equations are $\tilde{\cdot}$ terms, as they each lie in the projected/truncated space. In that respect, the product in the LES convective term is not $\tilde{u}_i \tilde{u}_j$ as such product would contain scales that lie beyond the cutoff; it is hence $\widetilde{u_i u_j}$. The traceless part of the SGS stress tensor will be noted $\tilde{\tau}_{ij}^{\text{sgs}} = \tilde{\sigma}_{ij}^{\text{sgs},*} = \tilde{\sigma}_{ij}^{\text{sgs}} - \frac{1}{3} \tilde{\sigma}_{kk}^{\text{sgs}} \delta_{ij}$. We also recall that the trace of the SGS stress is related to the energy associated with the subgrid-scales, $\tilde{K}^{\text{sgs}}$: $-\tilde{\sigma}_{kk}^{\text{sgs}} = 2 \tilde{K}^{\text{sgs}}$.

Here, we also use a de-aliased pseudo-spectral method, with sharp Fourier cutoff. It also has no numerical dissipation or dispersion errors. Hence, the method can compute properly, and for all wavenumbers, each term of the effective LES equations written above. The only numerical dissipation comes from the time integration scheme. More details on the method and its conservation

properties can be found in Canuto et al. [20]; see also Duponcheel et al. [36] for a benchmark on a complex Euler flow using various methods and codes, including our pseudo-spectral code.

To capture the physics of the turbulence, the cutoff k_c shouldn't be too small, and lie well within in the inertial range. Considering the simplest case of decaying homogeneous isotropic turbulence (DHIT), and the Kolmogorov scale (= viscous dissipation scale), $\eta = (\nu^3/\epsilon)^{1/4}$ with ϵ the energy dissipation rate, we recall that a direct numerical simulation (DNS) of such turbulence, and using high quality numerics, requires that $k_c \eta \gtrsim 1.5$; thus $h \lesssim 2\eta$.

In the energy spectrum of DHIT, the end of the inertial range / beginning of the viscous range occurs at wavenumbers of the order of $k_{ID} \eta \simeq 0.1 - 0.2$. An LES with cutoff $k_c \eta \lesssim 0.1$ ($h \gtrsim 30\eta$) can be considered as "lying in the inertial range".

For LES with truncation filtering only, as considered here, the SGS stress tensor can be decomposed as the sum of two terms, the cross term, $\tilde{C}_{ij}$, and the Reynolds term, $\tilde{R}_{ij}$:

$$-\tilde{\sigma}_{ij}^{\text{sgs}} = \widetilde{u_i u_j} - \widetilde{\tilde{u}_i \tilde{u}_j} = \underbrace{\left(\widetilde{\tilde{u}_i u'_j} + \widetilde{u'_i \tilde{u}_j} \right)}_{\tilde{C}_{ij}} + \underbrace{\widetilde{u'_i u'_j}}_{\tilde{R}_{ij}} \quad (6.5)$$

where $u'_i \triangleq u_i - \tilde{u}_i$ denote the subgrid scales.

Most popular approaches for the SGS stress modeling are based on a dissipative formulation. Typically, the traceless part of the SGS stress tensor is modeled as $\tilde{\tau}_{ij}^{\text{sgs}} \stackrel{M}{=} 2\nu_{sgs}\tilde{S}_{ij}$, using an effective subgrid-scale viscosity, as first suggested by Boussinesq [12]. The trace part is often not modeled and is simply added to the pressure; so that the pressure numerically obtained represents $\tilde{P} + \frac{2}{3}\tilde{K}^{\text{sgs}}$. The simplest and most classical model is that by Smagorinsky [117], using $\nu_e = C\Delta^2|\tilde{\mathbf{S}}|$ with Δ the local grid size, and $|\tilde{\mathbf{S}}| = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}$. A dynamic version was later proposed by Germano et al. [43] and improved by Lilly [75]. Models designed so as to focus the dissipation on the smallest scales of the LES field were proposed by Hughes et al. [58] with the so-called *variational multiscale* (VMS) model. The scale separation was performed using a sharp Fourier cut-off or variational projections, see also Bazilevs et al. [7] for a thorough analysis of the method. Regularized versions were later developed in order to directly apply the idea using finite differences or finite volumes, e.g. the "filtering analogs to the variational multiscale" models of Vreman [140],

or the "high-pass filtered Smagorinsky" model of Stolz et al. [128], etc. Some attention has also been drawn to the scale partition performed for those models (e.g. Holmen et al. [53], Ramakrishnan and Collis [107]). Among those models, is the *regularized variational multiscale* model (here noted RVMS) by Jeanmart and Winckelmans [60]. Those models apply an additional smooth filter to the LES fields and use the remainder as resolved small scales $\tilde{\phi}^s \triangleq \tilde{\phi} - \bar{\phi}$. Such regularized models have the form $\tilde{\tau}_{ij}^{\text{sgs}} \stackrel{M}{=} 2\nu_{\text{sgs}} \tilde{S}_{ij}^s$. The SGS viscosity ν_{sgs} is itself evaluated using either $|\tilde{\mathbf{S}}|$ or $|\tilde{\mathbf{S}}^s|$. Those models amount to a more complex diffusion operator. They are therefore unfit to provide backward transfer of energy ("backscatter"), as is the case with the true SGS stress. Even though capturing the overall SGS dissipation is often considered as sufficient for an SGS model to provide acceptable results, modeling more finely and more properly the SGS stress, and part of the backscatter, remains desirable. This is the aim of the present chapter.

Before pursuing, it is useful to also recall the theoretical concept where the NS equations are explicitly filtered, using a regular low-pass filter instead of the projection/truncation as considered above (and due to the coarse LES grid). This then corresponds to a regular convolution, $\bar{\phi} \triangleq G * \phi$:

$$\bar{\phi}(\mathbf{x}, t) \triangleq \int G\left(\frac{\mathbf{x} - \mathbf{x}'}{\bar{\Delta}}\right) \phi(\mathbf{x}', t) d\mathbf{x}', \quad (6.6)$$

with $G(\mathbf{x}/\bar{\Delta})$ a regular filter (i.e., one that has a regular Fourier transform, $\mathcal{G}(\mathbf{k}/\bar{\Delta})$, such as the Gaussian filter, the top hat filter, discrete filters, etc.) with filter width $\bar{\Delta}$. The effective equations for the filtered fields are then obtained as:

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0, \quad (6.7)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) + \frac{\partial \bar{P}}{\partial x_i} = \frac{\partial}{\partial x_j} (2\nu \bar{S}_{ij} + \mathcal{Q}_{ij}^{\text{sfs}}), \quad (6.8)$$

This process yields a subfilter-scales (SFS) stress tensor (here noted "form 1"):

$$-\mathcal{Q}_{ij}^{\text{sfs}} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j = (\overline{u_i^s u_j^s} + \overline{u_i^s \bar{u}_j}) + \underbrace{\overline{u_i \bar{u}_j} - \bar{u}_i \bar{u}_j}_{\mathcal{L}_{ij}}. \quad (6.9)$$

where $u_i^s \triangleq u_i - \bar{u}_i$ denotes the subfilter scales, and where $\mathcal{L}_{ij}$ is the so-called "Leonard term". Alternatively, one can write the effective equations with all

terms being filtered terms (i.e., all being τ terms, thus having the same status):

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) + \frac{\partial \bar{P}}{\partial x_i} = \frac{\partial}{\partial x_j} (2\nu \bar{S}_{ij} + \bar{\sigma}_{ij}^{\text{sfs}}) , \quad (6.10)$$

thus with, this time, a “true SFS stress” tensor (here noted “form 2”):

$$-\bar{\sigma}_{ij}^{\text{sfs}} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j = (\overline{u_i u_j^s} + \overline{u_i^s u_j}) + \overline{u_i^s u_j^s} = -(\mathcal{Q}_{ij}^{\text{sfs}} + \mathcal{L}_{ij}) . \quad (6.11)$$

To make filtered NS equations practical for LES, one must consider the combined effect of explicit filtering and LES grid truncation, thus the effective equations for the filtered field and truncated fields, $\tilde{u}_i$ (and typically with the assumption that $\tilde{u}_i = \bar{u}_i$ as the filter is a linear operator). One then obtains effective LES equations with a combination of SFS stresses (that are also projected on the LES grid, and that can be partially deconvolved) and SGS stresses (that can only be modeled). For the details of such equations, see, e.g., Carati et al. [21] and Winckelmans et al. [148]). This approach has been investigated, with some success, by some authors, using various levels of deconvolution of the SFS stress, combined with dissipation-type modeling of the SGS stress: thus mixed modeling approaches; see, e.g., Bardina et al. [6], Liu et al. [76], Vreman et al. [141], Horiuti [56], Winckelmans et al. [148], Morinishi and Vasilyev [87], Stolz et al. [127], etc.

In such approaches, one considers explicit filters such that $\mathcal{G}(\mathbf{k} \bar{\Delta}) > 0$ for all the wavenumbers of the LES, and are thus invertible. In physical space, the non-filtered field, $\tilde{u}_i$, can then be partially deconvolved from the filtered field, $\bar{u}_i$ by using the van Cittert iterative deconvolution method:

$$\begin{aligned} \tilde{u}_i &= \bar{u}_i + (I - G) * \tilde{u}_i + (I - G) * ((I - G) * \tilde{u}_i) + \dots \\ &= \bar{u}_i + (I - G) * (\tilde{u}_i + (I - G) * (\tilde{u}_i + \dots)) . \end{aligned} \quad (6.12)$$

The LES projection of the SFS stress (form 1 or form 2) can then also be partially reconstructed. (When using a spectral method, the non-filtered field can obviously be deconvolved exactly by simply dividing the Fourier transform of the filtered field by $\mathcal{G}(\mathbf{k} \bar{\Delta})$). There is also a way to directly deconvolve the LES projection of the SFS stress (form 1) as a series: see Yeo and Bedford [152], Bedford and Yeo [8] and Leonard [72] for the case of the second order Gaussian filter, $\mathcal{G}(\mathbf{k} \bar{\Delta}) = \exp\left(-\frac{(\mathbf{k} \bar{\Delta})^2}{24}\right)$, and see Carati et al. [21] for the extension to

all second order filters. The first term of this series is then always:

$$\widetilde{\widetilde{u_i u_j}} - \widetilde{\widetilde{u_i}} \widetilde{\widetilde{u_j}} = \frac{\overline{\Delta}^2}{12} \frac{\partial \widetilde{\widetilde{u_i}}}{\partial x_k} \frac{\partial \widetilde{\widetilde{u_j}}}{\partial x_k} + \dots, \quad (6.13)$$

and it corresponds to the so-called “tensor-diffusivity model” (also called the “Clark model”). As to the *scale-similarity* model introduced by Bardina et al. [6], using $-\tilde{\tilde{B}}_{ij} = \widetilde{\widetilde{u_i u_j}} - \widetilde{\widetilde{u_i}} \widetilde{\widetilde{u_j}}$, it too models the LES projection of the SFS stress (form 1). Indeed, the first term in the expansion of $\tilde{\tilde{B}}_{ij}$ is the tensor-diffusivity model. The same conclusion is reached when examining the SFS stress obtained when using only one van Cittert iteration. When tested on explicitly filtered fields, one finds that these models indeed exhibit a large correlation with the true SFS stress. Despite this, they cannot suffice on their own, and additional dissipation is required to model the truncation effects associated with the LES grid (e.g., using an additional eddy-viscosity SGS term as, for example, in Bardina et al. [6] and Winckelmans et al. [148]; or using a SGS relaxation term as in the approximate-deconvolution model by Stolz and Adams [126], see also Stolz et al. [127]); hence leading to mixed models.

Anderson and Domaradzki [2] proposed an asymmetric SGS model that prevents the excessive production of small-scales in the inertial cascade. This is done by subtracting the small scales production terms in the convective term from the equations. They obtained good results in channel flow. The idea of reducing the rate of production of small scales through a regularization of the convective term had already been suggested by Leray [73]. It was revisited and improved more recently by Verstappen [139], who proposed further choices for regularization. Those regularizations indeed reduce the rate of production of small scales, yet they do not prevent them; hence they cannot single-handedly stop the inertial cascade, and a dissipation term must be added, as with scale-similar models.

Speziale [123] and also Horiuti [56] showed that, while the SGS and SFS stress tensors, as well as the Reynolds term are Galilean invariant, the sole cross term contribution is not. Only the sum of the cross term and of the Leonard term is Galilean invariant. This is also true when considering scale-similar models. To circumvent this issue, they had to either cancel the cross term contribution or modify its structure (see also section 6.3 in the present work).

We stress again that the present paper only considers LES with trunca-

tion/projection due to the coarse LES grid, and that it solely focuses on improving the modeling of the SGS stress and of the backscatter. Yet, as we will see, it uses regular explicit filters in order to achieve that; hence the link with the above brief discussion about LES with explicit filtering. Wang and Oberai [144] showed that the spectral behavior of the two SGS terms (cross- and Reynolds terms) is intrinsically different (see also Fig. 6.4 later in this study), and they developed a mixed model within the framework of variational formulations for stabilized finite elements methods. The goal of the present work is to develop a mixed model that is easily implementable in either a pseudo-spectral code or in a finite differences code, that accounts for both terms, that provides significant backscatter while remaining stable, and that uses a dynamic procedure. The cross term model proposed here lies within the family of scale-similarity models. The Reynolds term model is of the RVMS type.

The paper is organized as follows: Section 6.2 presents the reference DNS used and an a priori analysis of the global and local behaviors of the two SGS terms. In Section 6.3, a dynamic mixed model is presented and is tested a priori in the case of decaying homogeneous isotropic turbulence (DHIT). A posteriori tests are then carried out in Section 6.4. The results are discussed in Section 6.5.

6.2 Reference DNS and behavior of the exact SGS terms

In this section, an analysis of the spectral and local behavior of the cross term and of the Reynolds term is carried out, using truncated fields from a DNS of DHIT performed in a box of size L^3 , with $L = 2\pi$, $\nu = 10^{-3}$, and using a 512^3 mesh. The DNS was obtained using a pseudo-spectral code with phase-shift dealiasing. The time integration was carried out using the third order low storage Runge-Kutta scheme of Williamson [147], with $\Delta t = 10^{-3}$. The DNS is considered for the times after which both the energy E and the dissipation rate ε are decreasing (defined as $t = 0$), see Fig. 6.1. We then also define the characteristic rms velocity $\mathcal{U}_0 = \sqrt{\frac{2}{3}E_0}$, the characteristic length $\mathcal{L}_0 = \frac{\mathcal{U}_0^3}{\varepsilon_0}$ and the characteristic time $t_0 = \frac{\mathcal{L}_0}{\mathcal{U}_0}$. We then also have that $Re_{\lambda,0} = \frac{\lambda_0 \mathcal{U}_0}{\nu} = 149$, where $\lambda_0 = \sqrt{\frac{10\nu E_0}{\varepsilon_0}}$ is the transverse Taylor microscale. The numerical value of those quantities is given in Table 6.1. The dissipation rate is here computed as $\varepsilon = 2\nu \langle S_{ij} S_{ij} \rangle$, which does not account for any numerical dissipation due

to the time integration. With the small time step used, ε was always within 0.04% of the actual $-\frac{dE}{dt}$. The time evolution of the Kolmogorov scale and of the Taylor microscale are provided in Fig. 6.2, and can be compared to the two LES cuts performed in this study. We also note that the DNS mesh is such that the minimum value of $k_c\eta = 1.51$ is in agreement with the requirements for pseudo-spectral DNS ($k_c\eta \geq 1.5$). We however recognize that this DNS is significantly box-constrained at the early times. Consequently, the largest scales do not yet display a canonical DHIT behavior, which also impacts some of the features of the flow, such as the dissipation rate. We also notice an inflection point in the early evolution of ε . Only at the times greater than $t \simeq 2.5 t_0$ (i.e., when $Re_\lambda \simeq 80$) are all the scales of the flow such that the simulation can be considered as a canonical DHIT, with then

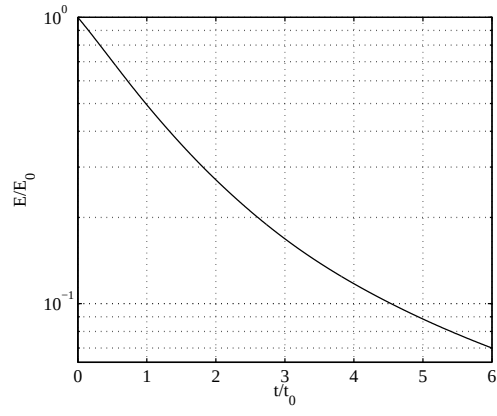
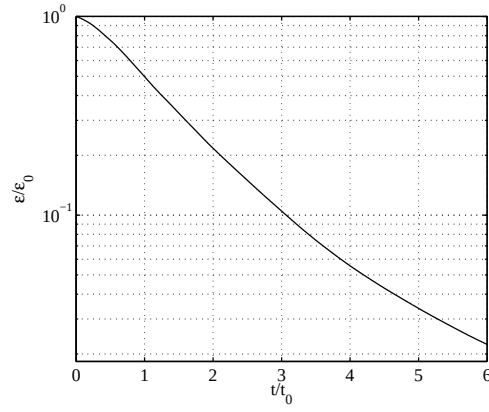
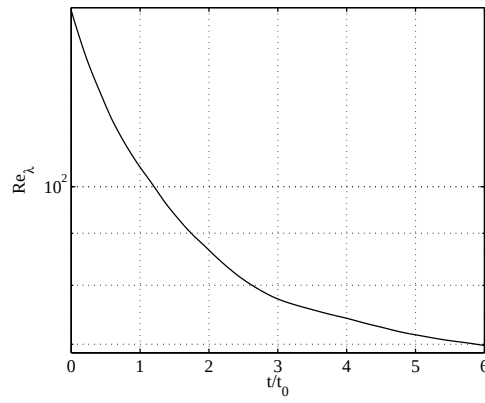
$$\frac{d}{dt} \left(\frac{E}{\varepsilon} \right) = -\frac{E}{\varepsilon^2} \frac{d\varepsilon}{dt} + \frac{1}{\varepsilon} \frac{dE}{dt} = C_2 - 1 = \frac{1}{n}, \quad (6.14)$$

where n is the decay exponent for energy (with $n \simeq 1.30$) and C_2 if the coefficient of the ε decay rate equation, $\frac{d\varepsilon}{dt} = -C_2 \frac{\varepsilon^2}{E}$. Thus, at $t = 0$, we have $Re_{\lambda,0} = 149$ and some small inertial range in the spectrum (see Fig. 6.3), but the DHIT is not canonical; at times greater than $t/t_0 \simeq 2.5$, we have a canonical DHIT but a lower Re_λ and no inertial range. The behavior of the small scales is however not entailed by this limitation at early times, and this simulation is considered suitable for the purpose of the present SGS modeling investigations. We further stress that the performance of the present SGS model will also be tested a posteriori over a wide time range ($t/t_0 = 0.365, 1.46, 2.55$ and 3.65).

E_0	ε_0	$\mathcal{U}_0$	$\mathcal{L}_0$	t_0	λ_0	$Re_{\lambda,0}$
1.620	0.7878	1.039	1.425	1.371	0.1434	149

Table 6.1: Values, at time $t = 0$, of various quantities of the DNS.

The DNS was spectrally truncated to 64^3 and 48^3 LES grids, as shown in Fig. 6.3. The two exact SGS terms were also computed, and the spectrum of the dissipation associated with each term was also obtained. The spectra exhibit a different behavior, as can be seen on Fig. 6.4. The spectrum associated with the Reynolds term quickly reaches a plateau whereas that associated with the cross term increases monotonically, with a peak at the high wavenumbers. We also notice that the energy dissipation due to the Reynolds term is signifi-

(a) E (b) ε (c) Re_λ **Figure 6.1:** Time evolution of the energy E , of the dissipation ε , and of Re_λ .

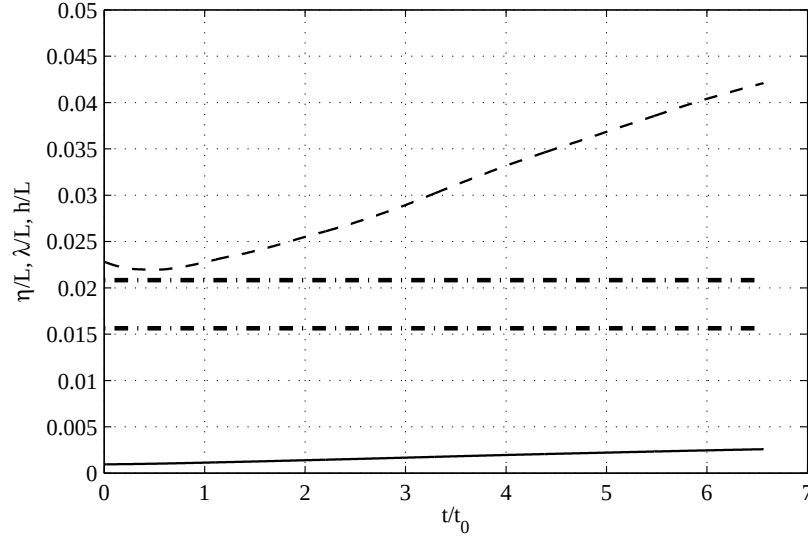


Figure 6.2: Time evolution of the *Kolmogorov scale* (solid) and of the *Taylor microscale* (dashed). The thick horizontal dashed-dotted lines correspond to the LES cuts : $h = L/64$ and $h = L/48$.

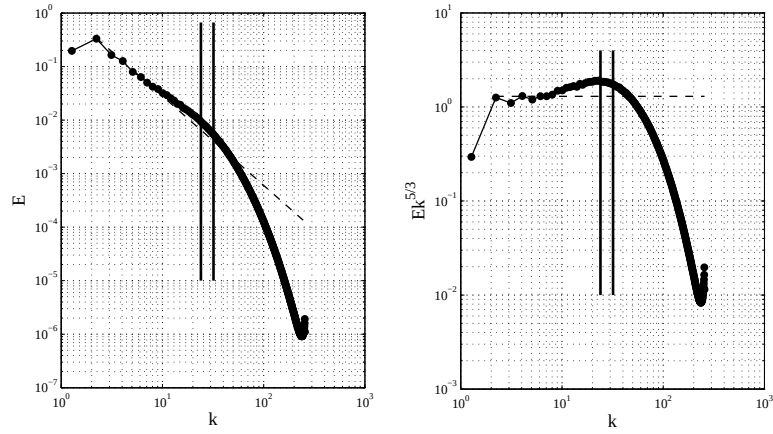


Figure 6.3: *3D Energy spectrum* (left) and *compensated energy spectrum* (right) of the DNS at $Re_\lambda = 149$. The thick vertical curves correspond to 48^3 and 64^3 LES cuts.

cantly smaller than that due to the cross term.

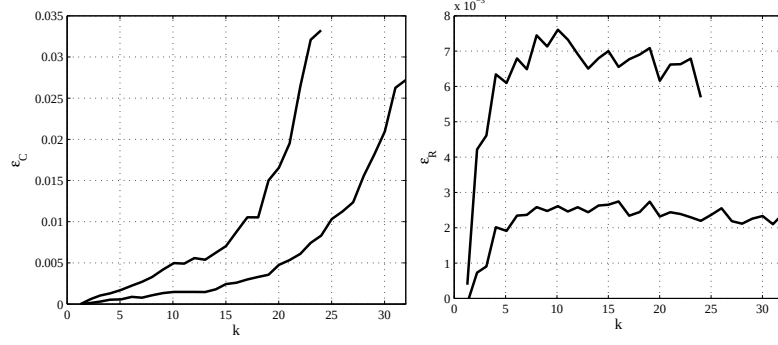


Figure 6.4: A priori analysis of the **dissipation spectrum** due to the cross term (ε_C) and to the Reynolds term (ε_R), for 48^3 and 64^3 fields obtained from truncation of a 512^3 DNS field of DHIT at $Re_\lambda = 149$.

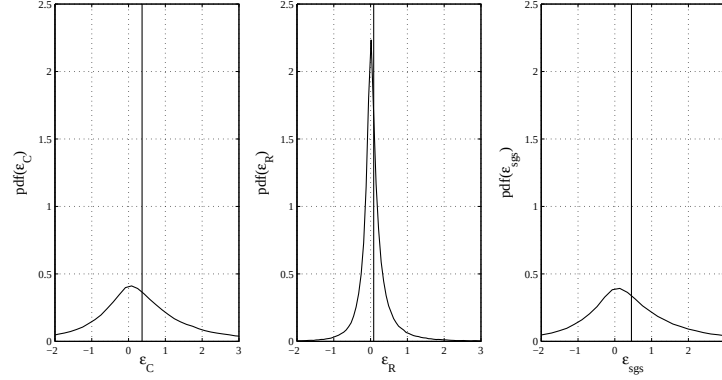


Figure 6.5: **PDF of the dissipation** due to the cross term (ε_C) and to the Reynolds term (ε_R) for a 64^3 field truncated from a 512^3 DNS field of DHIT at $Re_\lambda = 149$. The vertical lines correspond to the mean dissipation: $\langle \varepsilon_C \rangle = 0.3678$, $\langle \varepsilon_R \rangle = 0.0854$, and $\langle \varepsilon^{sgs} \rangle = 0.4532$.

The spectrum of dissipation provides a mean spectral view of the behavior of the term, shell by shell. In that respect, we see that all shells correspond to a positive dissipation. The spectral view thus does not provide any insight on the backscatter. For that, one has to measure the dissipation in physical space, at all locations. The distribution of that dissipation is then condensed into the probability density function (PDF), providing a statistical insight in

physical space. We see, in Fig. 6.5, that both SGS terms provide a significant amount of backscatter. Around 40% of the points are actually gaining energy through the action of the SGS terms, as seen in Table 6.2. Various quantities can be derived from the pdf's of a generic $\varepsilon_* = \tilde{\sigma}_{ij}^* \tilde{S}_{ij}$ dissipation term of a stress tensor $\tilde{\sigma}_{ij}^*$:

$$\int_{-\infty}^0 \text{pdf}(\varepsilon_*) d\varepsilon_* , \quad (6.15)$$

which is the percentage of points that provide backscatter (i.e., negative dissipation),

$$\langle \varepsilon_* \rangle^- = \int_{-\infty}^0 \varepsilon_* \text{pdf}(\varepsilon_*) d\varepsilon_* , \quad (6.16)$$

which is the total amount of backscatter. Notice, in Table 6.2, that the total amount of backscatter of the sgs tensor is not necessarily the sum of the amount provided by each sgs term taken individually. This is because both terms do not always provide backscatter at the same time, and the backscatter from one term can sometimes be compensated by the forward dissipation of the other. We are also interested in

$$\frac{1}{\langle \varepsilon_* \rangle} \left(\int_{-\infty}^{\infty} (\varepsilon_* - \langle \varepsilon_* \rangle)^2 \text{pdf}(\varepsilon_*) d\varepsilon_* \right)^{1/2} , \quad (6.17)$$

which is the ratio of the standard deviation of the considered ε_* dissipation term to the mean value

$$\langle \varepsilon_* \rangle = \int_{-\infty}^{\infty} \varepsilon_* \text{pdf}(\varepsilon_*) d\varepsilon_* . \quad (6.18)$$

	N_{LES}	C	R	sgs
Percentage of backscatter in ε_*	48	41%	38%	38%
(Eq. 6.15)	64	41%	43%	40%
Amount of backscatter in ε_* (Eq. 6.16)	48	-0.439	-0.104	-0.415
	64	-0.428	-0.103	-0.425
Ratio of standard deviation to mean	48	5.07	3.84	3.85
value in ε_* (Eq. 6.17)	64	5.24	6.50	4.60

Table 6.2: Percentage of **backscatter**, amount of backscatter and ratio of standard deviation to mean value in the PDF's of dissipation.

The mean dissipation associated with the Reynolds term is smaller than that associated with the cross term, yet sensitive to the LES cut: $\langle \varepsilon_R \rangle / \langle \varepsilon_C \rangle = 0.232$ for the 64^3 truncation, and 0.472 for the 48^3 truncation. The PDF of the Reynolds term is also tighter than that of the cross term; yet it has a comparable ratio of standard deviation to mean value, and roughly the same percentage of points that provide backscatter, see Table 6.2. We nevertheless notice that, relatively, the values obtained for the Reynolds term are more sensitive to the LES cut than those obtained for the cross term. This behavior will also serve to support the choice that will be made in what follows, i.e., that the coefficient of the cross term model will be static (i.e., calibrated), whereas that of the Reynolds term model will be determined dynamically.

Clearly, a purely dissipative SGS model cannot reproduce this behavior; at best, it can only be “accurate in the mean”, in the sense of providing the proper global dissipation. The time evolution of the SGS terms dissipation is provided in Fig. 6.6. It is seen that the cross term is the main driver in the SGS dissipation. Yet, we see again that, relatively, the contribution of the Reynolds term increases when the cut is coarser. As the 48^3 cut corresponds to $k_c \eta = 0.14$ at $t = 0$, we can consider that this relative contribution, of $\langle \varepsilon_R \rangle / \langle \varepsilon_C \rangle \simeq 0.5$, is fairly well “established”: in the sense that it would be roughly the same for a cut with $k_c \eta \lesssim 0.1$, thus here requiring a larger scale DNS at higher Re_λ ; something we cannot afford with the present code (as to here considering an even coarser cut, say of 32^3 , we refrained from doing that as such LES would no longer have enough modes in the inertial range to be of interest in the present investigation). For the same reason that Re_λ is here moderate, we also observe that the viscous dissipation is not negligible for both LES cuts. Nevertheless, we see that, up to $t/t_0 \simeq 3$ for the 48^3 cut, and up to $t/t_0 \simeq 1.5$ for the 64^3 cut, the total SGS dissipation is larger than the viscous dissipation. We also see that, when this is no longer the case, then the dissipation of the Reynolds term, ε_R , is negligible compared to that of the cross term, ε_C .

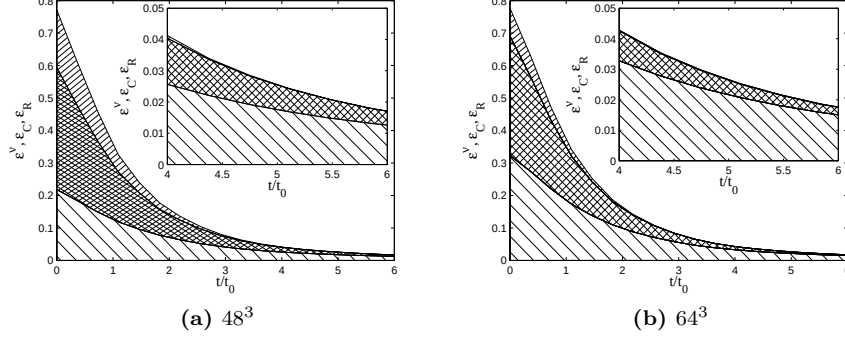


Figure 6.6: Time evolution of the *dissipation associated with each term*: resolved viscous term ($\backslash\backslash$), cross term ($\times \times \times$) and Reynolds term ($///$).

6.3 Dynamic mixed model and a priori assessment

A natural way to model the SGS stress tensor is to use separate modeling for each term, as their behavior is intrinsically different:

$$\tilde{C}_{ij}^M + \tilde{R}_{ij}^M = C_c \tilde{c}_{ij}^M + C_r \tilde{r}_{ij}^M, \quad (6.19)$$

where C_c and C_r are coefficients. The cross term model is chosen so as to mimic the exact term behavior, and based on scale-similarity. The Reynolds term model, and some additional dissipation (as the cross term model is incomplete), are taken from the RVMS family. We first discuss the cross term model. A regular filter is applied on the LES velocity field so as to obtain a “small resolved scales” field $\tilde{\mathbf{u}}^s$ and a “large resolved scales” field $\overline{\mathbf{u}}$, such that $\tilde{\mathbf{u}} = \overline{\mathbf{u}} + \tilde{\mathbf{u}}^s$, see Fig. 6.7.

The simplest cross term model is then

$$\tilde{c}_{ij}^{M_0} = \widetilde{\overline{u}_i \tilde{u}_j^s} + \widetilde{\tilde{u}_i^s \overline{u}_j}. \quad (6.20)$$

Now: in all that follows, we will refrain from putting another $\sim$ symbol above the product of two quantities evaluated on the LES grid: this is to make the notation easier to read. It is however understood that the product of any two LES quantities is also on the LES grid and thus that the symbol is implied.

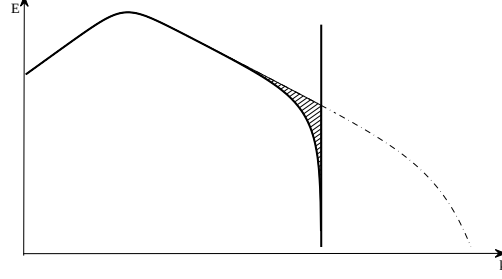


Figure 6.7: Schematic of the *energy spectrum* : unresolved small scales $\mathbf{u}'$ (dashed-dotted), resolved small scales $\tilde{\mathbf{u}}^s$ (hashed), and resolved large scales $\bar{\mathbf{u}}$ (thick solid).

We thus rewrite the model as:

$$\tilde{c}_{ij}^{M_0} = \bar{u}_i \tilde{u}_j^s + \tilde{u}_i^s \bar{u}_j. \quad (6.21)$$

Such a model, however, does not verify Galilean invariance, i.e., invariance under the change of frame $(t, \mathbf{u}) \rightarrow (t + T, \mathbf{u} + \mathbf{U})$ for any constant $(T, \mathbf{U})$. Therefore, the model has to be modified to yield a proper SGS term. Note that this problem does not arise when considering the exact cross term: as $\tilde{u}'_i = 0$, one obtains that $\widetilde{(\tilde{u}_i + U_i)u'_j} = \widetilde{\tilde{u}_i u'_j}$ and the exact cross term is indeed invariant. Various modifications of the model have here been investigated in order to respect Galilean invariance. The simplest way to achieve that is to subtract the mean field $\langle \bar{\mathbf{u}} \rangle$ (which, of course, is zero in DHIT) from the filtered large scales in the M_0 model. The resulting model (noted M_1) is then

$$\tilde{c}_{ij}^{M_1} = (\bar{u}_i - \langle \bar{u}_i \rangle) \tilde{u}_j^s + \tilde{u}_i^s (\bar{u}_j - \langle \bar{u}_j \rangle). \quad (6.22)$$

Recalling that, in theoretical explicitly filtered LES, only the addition of the cross term and of the Leonard term is invariant in the SFS stress tensor (see e.g. Speziale[123]), another possible modification of the M_0 model is obtained by filtering it and by adding a pseudo Leonard term, to obtain another model (noted M_2):

$$\tilde{c}_{ij}^{M_2} = \overline{\tilde{c}_{ij}^{M_0}} + \left(\overline{\bar{u}_i \bar{u}_j} - \bar{u}_i \bar{u}_j \right) = \overline{\left(\tilde{c}_{ij}^{M_0} + \bar{u}_i \bar{u}_j \right)} - \bar{u}_i \bar{u}_j = \dots = \overline{\bar{u}_i \tilde{u}_j^s - \tilde{u}_i^s \bar{u}_j} - \bar{u}_i \bar{u}_j. \quad (6.23)$$

This pseudo Leonard term is a pure subfilter quantity and it should therefore improve the model performance at the subfilter scales.

Finally, and taking some distance from the scale-similarity assumption, the cross term model can be viewed as needing an emulation of the unresolved scales to approximate their effect on the resolved scales. From there, we can propose another model that uses $\tilde{\mathbf{u}}^s$ as the emulated unresolved scales but that still uses the resolved scales $\tilde{\mathbf{u}}$. The argument being that, a priori, there is no good reason not to use all the information we have about the resolved scales. The resulting model is then also made invariant by filtering it and by adding a pseudo Leonard term to it; and we finally obtain for that model (noted M_3):

$$\tilde{c}_{ij}^{M_3} = \overline{\tilde{u}_i \tilde{u}_j^s} + \overline{\tilde{u}_i^s \tilde{u}_j} + (\overline{\tilde{u}_i \tilde{u}_j} - \tilde{u}_i \tilde{u}_j) . \quad (6.24)$$

The three models above, M_1 , M_2 and M_3 , are compared a priori to assess their spectral behavior in comparison to the exact term extracted from the DNS. Various filters are also investigated. The fundamental filters are one-dimensional filters: they are successively applied in the three directions, producing tensor product filters. The filters considered are: the sharp Fourier cut-off filter, explicit discrete filters (here a family of filters that can also be efficiently obtained in physical space by using iterations of a compact 3 points filter), and implicit discrete filters (which, in physical space, would require solving a tridiagonal system in each direction). These filters are detailed in Appendix C. Each time, the coefficient C_c is calibrated so that the total dissipation of the model equals that of the exact term. The comparison in Fig. 6.8 exhibits two trends. Firstly, the M_1 model exhibits a poor behavior at the high wavenumbers: it starts decreasing where it should actually reach a maximum. The addition of the pseudo-Leonard term is thus required to obtain a proper behavior at high wavenumbers (this term is a pure subfilter quantity and is therefore only active at those wavenumbers). Secondly, The M_3 model provides a less convincing curve shape than the M_2 model: it is too flat, and will therefore provide excessive dissipation at the low and medium wavenumbers and insufficient dissipation at the high wavenumbers. Both models M_1 and M_3 provide a plateau of excessive dissipation in the low and medium wavenumber range, whereas the M_2 model provides a rather good behavior at those wavenumbers. The level of this plateau could be lowered by reducing the model coefficient. Note the low and medium wavenumbers are the most important wavenumbers because a non-spectral method would disperse the high wavenumbers. Overall, the shape of the curve and its behavior in the low and medium wavenumber range is the determining factor. In this regard, the M_2

model is seen to perform best, regardless of the type of filter used, though the improvement of M2 over the other models is not always massive. Finally, only the M_2 model is retained. It will be noted M in the following when there is no ambiguity.

The spectral behavior of this retained model is then further investigated for the various filter types. The a priori analysis using the sharp Fourier filters displays a very promising behavior of the model, but also a sensitivity to the amount of wavenumbers cut off by the filter, especially in the high wavenumbers range, as seen in Fig. 6.9. The calibrated values of the coefficient C_c for Fig. 6.9 and Fig. 6.10 are given in Table 6.3.

A practical SGS model should be directly applicable in physical space. This constraint dictates the use of discrete filters: either explicit or implicit. Those filters are not as good as a sharp Fourier cutoff, and hence it is more difficult to model the cross term, as seen in Fig. 6.10. We see that the use of implicit filters only provides moderate improvement over the use of explicit filters. Yet, like explicit filters, they provide too much dissipation at the small to medium wavenumbers, which are the most important modes for LES. When used in physical space, implicit filters are far more costly as they require solving a tridiagonal system in each direction. They are therefore discarded in favor of the explicit filters. The present explicit filters are obtained by using an efficient iterative scheme that is local and compact (it only uses the nearest neighbors): the filter of order $2n$ being obtained using n iterations of that compact operator. The use of such filters with $n = 3$ and $n = 5$ is something very practical and that is now routinely used in the RVMS model, see for instance Bricteux et al. [14], Cocoli et al. [26], etc. Using $n = 8$ iterations is a bit “pushing it”, yet it is here also considered for the sake of completeness of the investigation. In the rest of the paper, we solely use explicit filters with $n = 3$ and $n = 5$; yet we do not dismiss that using implicit filters, although far more costly, could also be further investigated.

When the model is evaluated using the present explicit discrete filters, we observe a lack of dissipation at the smallest scales. These are the scales used to compute the resolved small scales $\tilde{\mathbf{u}}^s$. Such a behavior was already apparent when a spectral cut-off was used: the model loses efficiency at the scales used to obtain the resolved small scales. It is however much more visible when using discrete filters because they tend to retain more scales in the small scales field. This is taken into account in the modeling of the rest of the SGS stress, called

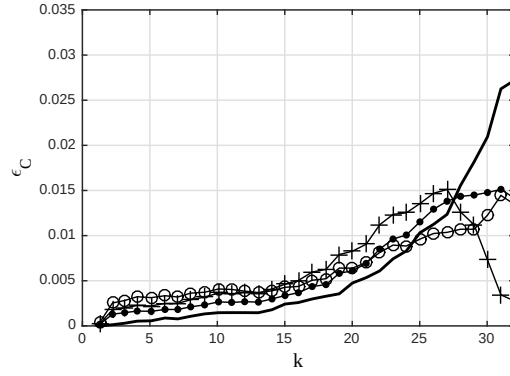
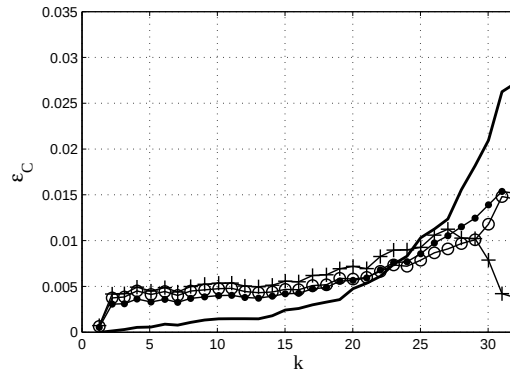
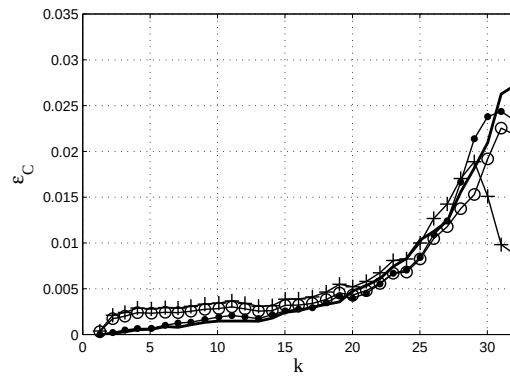
(a) Explicit filters of order 16 ($n=8$)(b) Implicit filter of order 2 with $k_c h / \pi = 0.90$ (c) Sharp Fourier cut-off at $k_c h / \pi = 28/32$

Figure 6.8: Dissipation of the cross term (Galilean invariance analysis) for a 64^3 cut of the reference DNS: exact term (thick solid), M_1 ($- + -$), M_2 ($- \bullet -$), and M_3 ($- o -$). The three models were computed using a calibrated coefficient C_c such that the total dissipation of the model equal that of the exact term. See Appendix C for a description of the filters.

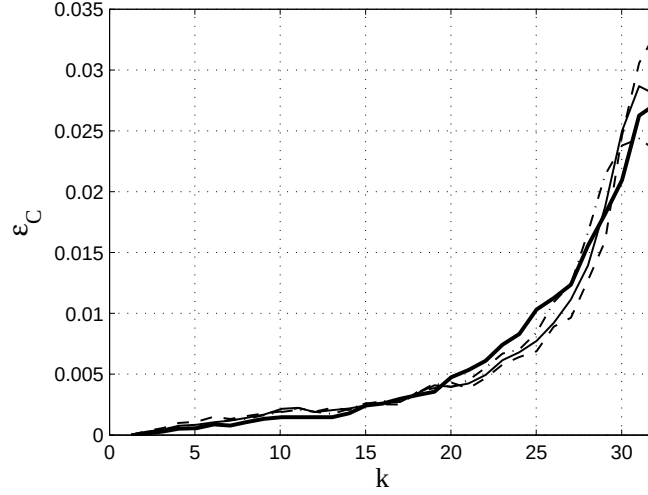
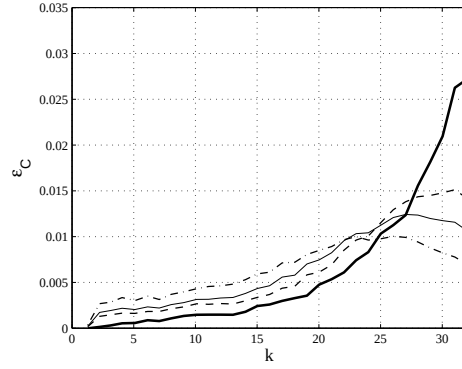


Figure 6.9: Dissipation of the cross term for a 64^3 cut of the reference DNS: exact term (thick solid), model with sharp Fourier cutoff at $k_ch/\pi = 30/32$ (dashed), 29/32 (solid) and 28/32 (dashed-dotted), using calibrated C_c .

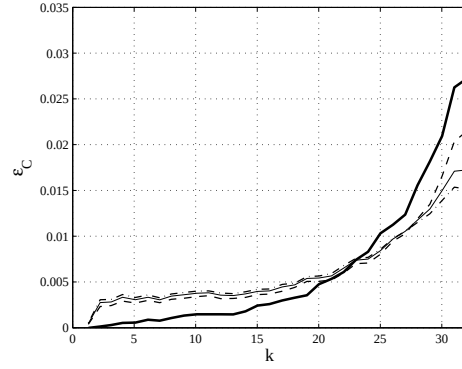
Sharp cutoff k_ch/π	C_c	Explicit filter order	C_c
28/32	0.76	6 ($n = 3$)	0.35
29/32	1.09	10 ($n = 5$)	0.41
30/32	1.66	16 ($n = 8$)	0.51

Implicit filter cut-off k_ch/π	C_c (order 2)	C_c (order 4)
0.90	0.73	0.78
0.92	0.91	0.97
0.95	1.54	1.64

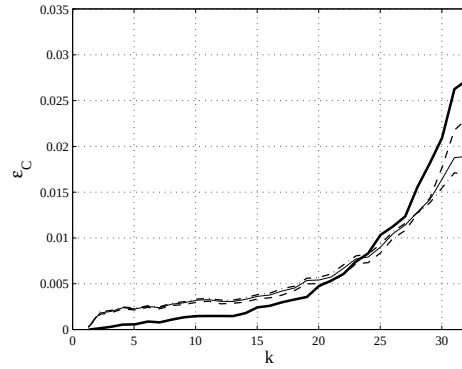
Table 6.3: Calibrated C_c values for the results of Figure 6.10



(a) Explicit filters of order 6 (dashed-dotted), 10 (solid) and 16 (dashed)



(b) Implicit filter of order 2 with $k_c h / \pi = 0.90$ (dashed-dotted), 0.92 (solid) and 0.95 (dashed)



(c) Implicit filter of order 4 with $k_c h / \pi = 0.90$ (dashed-dotted), 0.92 (solid) and 0.95 (dashed)

Figure 6.10: Dissipation of the cross term (filter analysis) for a 64^3 cut of the reference DNS: exact term (thick solid), and model using discrete filters and calibrated C_c .

R_{ij}^M . This second part of the model therefore has to account for both the effect of the Reynolds term and for the additional dissipation required at the highest wavenumbers of the mesh. This model is here taken as a RVMS model:

$$\tilde{R}_{ij}^M = C_r \tilde{r}_{ij}^M = -2(C_r \Delta^2) |\tilde{\mathbf{S}}| \tilde{S}_{ij}^s, \quad (6.25)$$

where $\tilde{S}_{ij}^s$ is obtained using the same filter as that used for the cross term model. The dissipation spectrum of this model is reasonably close to that of the Reynolds term in the range of large to medium scales, and it provides extra dissipation at the smallest scales (see also Jeanmart and Winckelmans [60], Cogle et al. [26], Bricteux et al. [14]). Of course, the RVMS model only provides very little backscatter. So, most of the backscatter is provided by the $\tilde{C}_{ij}^M$ model. We stress that we do not ask that each model term mimics each exact term (cross term and Reynolds term); we only ask that the global model, $\tilde{C}_{ij}^M + \tilde{R}_{ij}^M$, mimics the exact global SGS stress, $\tilde{C}_{ij} + \tilde{R}_{ij}$. As an example, Fig. 6.11 shows the dissipation provided by the Reynolds term model initially for the 64^3 LES and with the filter of order 10. We see than, though the model is rather good in the low wavenumber range, its is much larger than that of the Reynolds term in the high wavenumber range. This is because it also accounts for the missing dissipation of the cross term model. And we accept that the total amount of backscatter that will be provided by the model, although significant, will still be less than the total amount of backscatter provided by the exact SGS stress.

Finally, the mixed model reads

$$-C_c \tilde{c}_{ij}^M - C_r \tilde{r}_{ij}^M = -C_c (\overline{\tilde{u}_i \tilde{u}_j} - \overline{\tilde{u}_i^s \tilde{u}_j^s} - \overline{\tilde{u}_i} \overline{\tilde{u}_j}) + (C_r \Delta^2) 2 |\tilde{\mathbf{S}}| \tilde{S}_{ij}^s. \quad (6.26)$$

In the present approach, the C_c coefficient is obtained by pre-calibration, and a procedure similar to that proposed by Park and Mahesh [97] is used to dynamically compute the C_r coefficient. Thus, we here decided to stay away from applying the full dynamic procedure on the whole model, with dynamic C_c and C_r . This is because, as the present model uses an extensive amount of explicit filtering, its evaluation on a test filtered field would become quite costly; and moreover potentially inaccurate because of a too coarse mesh with only a few modes left in the inertial range. The procedure by Park and Mahesh [97] relies on the experience that the dynamic Smagorinsky model, despite its poor spectral behavior, yields the proper amount of total SGS dissipation (as also shown

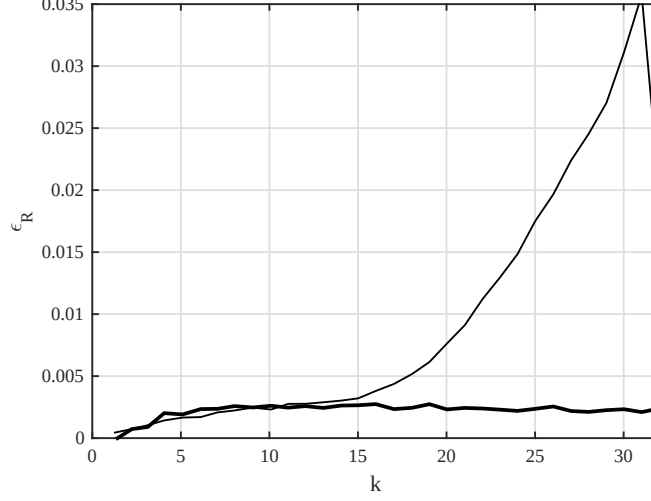


Figure 6.11: Dissipation of the Reynolds term for a 64^3 cut of the reference DNS: exact term (thick solid), model with explicit filter of order 10 (thin solid), with the dynamically computed C_r .

by Meneveau and Katz [81] in a mixing layer). So, here, the C_c coefficient is fixed and the C_r coefficient is computed dynamically by requiring that the global model dissipates on average as much as the dynamic Smagorinsky model applied on the same LES field:

$$\left\langle \left(C_c \tilde{c}_{ij}^M - (C_r \Delta^2) 2 |\tilde{\mathbf{S}}| \tilde{S}_{ij}^s \right) \tilde{S}_{ij} \right\rangle = \left\langle - (C_s \Delta^2) 2 |\tilde{\mathbf{S}}| \tilde{S}_{ij} \tilde{S}_{ij} \right\rangle, \quad (6.27)$$

where $\langle \cdot \rangle$ denotes averaging over the homogeneous directions and where $(C_s \Delta^2)$ is obtained using the Germano identity and Lilly's procedure on the Smagorinsky model, and hence

$$C_r \Delta^2 = \frac{(C_s \Delta^2) \langle |\tilde{\mathbf{S}}|^3 \rangle + C_c \langle \tilde{c}_{ij}^M \tilde{S}_{ij} \rangle}{2 \langle |\tilde{\mathbf{S}}| \tilde{S}_{ij}^s \tilde{S}_{ij} \rangle}. \quad (6.28)$$

We also note that a further investigation of the present model could be to use dynamic procedure to determine both C_c and C_r (but, this would require more filtering and thus a larger scale LES, and thus also a larger scale reference DNS). A larger scale LES, and a higher Reynolds number would also provide a C_c calibration less dependent on the LES cut.

6.4 A posteriori testing

The mixed model was tested on 48^3 and 64^3 LES cuts of the initial 512^3 DNS. Sixth ($n = 3$) and tenth ($n = 5$) order explicit filters were used. In the 64^3 case, the C_c coefficient was the value from the a priori testing; in the 48^3 case, a slightly lower value than that from the a priori calibration was used, to obtain better a posteriori results. The values of the “best” determined C_c coefficient are provided in Table 6.4. We see that the ranges obtained are then not that large: $0.35 - 0.30$ when using the explicit filter of order 6 ($n = 3$), and $0.41 - 0.35$ when using the explicit filter of order 10 ($n = 5$). Hence, we also offer that using any C_c value in each range is an acceptable choice when running that mixed model in LES run using only one dynamic coefficient (C_r).

An LES using the now classical dynamic Smagorinsky model (DSM) was also run as a basis for comparison. All simulations were able to reproduce the time evolution of the energy and of the dissipation, see Fig. 6.12 and Fig. 6.13; even though, at 48^3 , the model somewhat lacks dissipation in the early stages of the simulation.

At the very first time step, the dissipation is calibrated to be equal to that of the exact cross term, but the dissipation quickly decreases, over the course of only a few time steps (not visible in Fig. 6.13). We also see, in Fig. 6.13, that the $\tilde{C}_{ij}^M$ contribution to the dissipation budget is lower than that of the exact cross term dissipation of Fig. 6.6. As discussed previously, this is partly due to the fact that $\tilde{C}_{ij}^M$ does not model the whole cross term contribution, and that part of it has to be taken into account in the $\tilde{R}_{ij}^M$ contribution. The global model dissipation is however quite satisfactory and the $\tilde{C}_{ij}^M$ term contribution is significant. Recalling that this term also corresponds to significant backscatter as seen on Fig. 6.14, the fact that this term does not blow up numerically and that it also contributes significantly to modeling the SGS stress already constitutes, in itself, an achievement.

The backscattering behavior of the model is then assessed by comparing the normalized PDF of the dissipation produced by the model term $\tilde{C}_{ij}^M$ to that produced by the exact cross term from the DNS, $\tilde{C}_{ij}$: see Fig. 6.14. We observe that the normalized PDF have the same shape, and that this remains true for long times: hence, the cross term model provides nearly the same percentage of points with backscatter as the exact term. Yet, and as seen previously, the global dissipation of the cross term model $\tilde{C}_{ij}^M$ is lower than that of the exact cross term. As the contribution of the R_{ij}^M term has no backscatter (in fact,

Filter order	Mesh size	C_c
6	64^3	0.35
6	48^3	0.30
10	64^3	0.41
10	48^3	0.35

Table 6.4: Calibrated C_c values used for the a posteriori tests, using explicit filters of order 6 and 10.

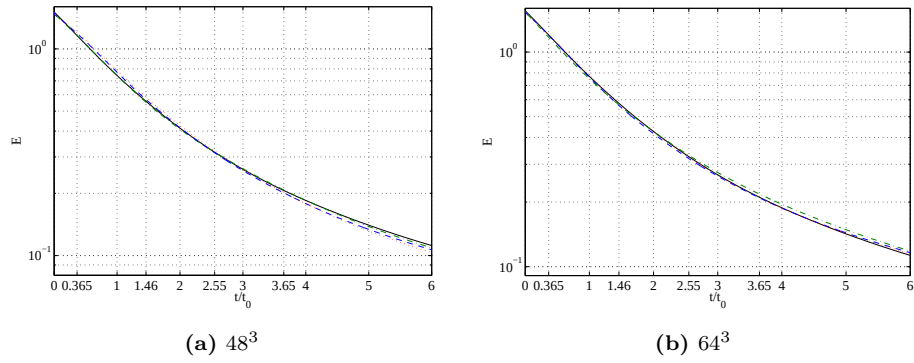


Figure 6.12: Time evolution of the **resolved energy**: truncated DNS (solid), dynamic Smagorinsky model (dashed-dotted), present dynamic mixed model, using the explicit filter of order 6 (dashed) and 10 (dotted).

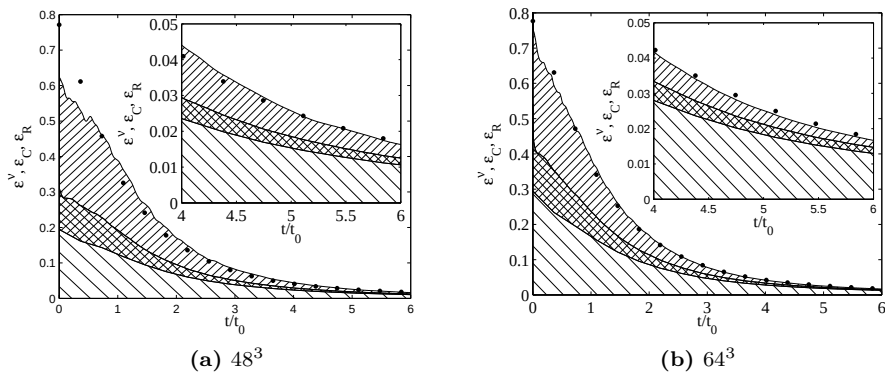


Figure 6.13: Time evolution of the **dissipation contributions** for the dynamic mixed model using the explicit filter of order 10: viscous term ($\backslash$), C_{ij}^M term ($\times \times$) and R_{ij}^M term ($//$). The bullets show the global dissipation for the truncated DNS.

being an RVMS model, it has some, but very little), the mixed SGS model has a smaller percentage of points with backscatter than the exact SGS stress. The values are reported in Table 6.5: we obtain that, with the mixed model, 28-30% of the points provide backscatter; to be compared to the 40% for the exact SGS stress.

The PDF of the total SGS dissipation of the mixed model is also compared to that of the exact stress in Fig. 6.15. The values of the SGS dissipation, $\langle \varepsilon^{\text{sgs}} \rangle$, and of the amount of backscatter, $\langle \varepsilon^{\text{sgs}} \rangle^-$, are also provided in Table 6.5: we obtain that the amount of backscatter provided by the model is roughly 22 – 25 % of that of the exact SGS stress (which itself is roughly equal to $-\langle \varepsilon^{\text{sgs}} \rangle$). Fig. 6.15 and Table 6.5, together with Fig. 6.13 compared to Fig. 6.6, really summarize what this mixed model does: about 30% of the points provide backscatter, and the amount of backscatter, changed sign, corresponds to about 20% of the total dissipation.

$t/t_0 = 0.365$	exact	model
ε_C	41%	42%
ε^{sgs}	40%	30%
$\langle \varepsilon^{\text{sgs}} \rangle$	0.3564	0.3612
$\langle \varepsilon^{\text{sgs}} \rangle^-$	-0.3183	-0.0700

$t/t_0 = 2.55$	exact	model
ε_C	42%	41%
ε^{sgs}	40%	28%
$\langle \varepsilon^{\text{sgs}} \rangle$	0.03761	0.0474
$\langle \varepsilon^{\text{sgs}} \rangle^-$	-0.03600	-0.00946

Table 6.5: Percentage of points with **backscatter**, and amount of SGS dissipation and of SGS backscatter, in the case of the 64^3 LES with 10th order filter: exact term versus model term, at short time (left) and at long time (right).

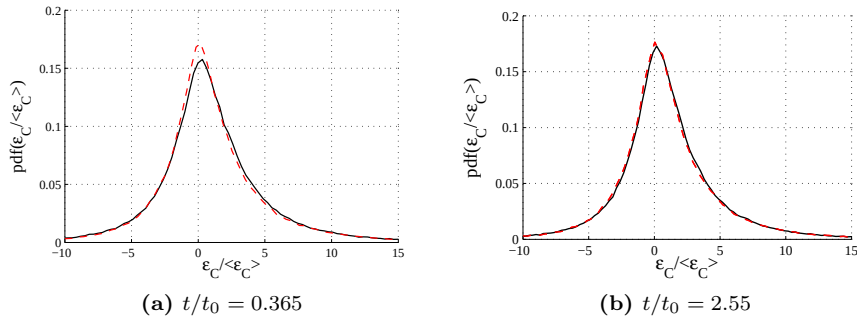


Figure 6.14: Normalized **PDF of cross term dissipation** for a 64^3 LES: exact $\tilde{C}_{ij}$ from DNS (solid), model $\tilde{C}_{ij}^M$ using the explicit filter of order 10 (dashed).

Finally, we examine the energy spectra obtained when using the dynamic

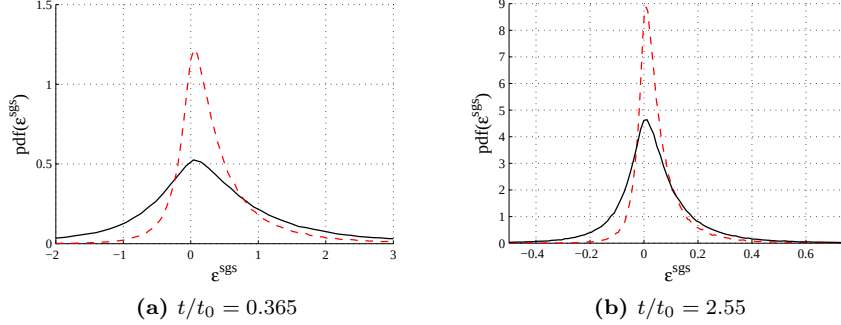


Figure 6.15: PDF of total SGS dissipation for a 64^3 LES: exact from DNS (solid), mixed model using the explicit filter of order 10 (dashed).

mixed model, and the dynamic Smagorinsky model, see Fig. 6.16. Both models yield spectra that compare fairly well with the DNS. Over the low wavenumbers range, the mixed model produces spectra that appear slightly better. For the 64^3 LES, the spectra remain close to the reference DNS for almost all wavenumbers, and throughout the simulation. Yet, for the 48^3 LES, we observe, at short times, a build-up of energy in the range $k/k_c \geq 10/24$. This build-up is related to the lack of dissipation that we already observed in the model at the early times, see Fig. 6.13. This effect lingers, and it is significantly damped after $t/t_0 = 1.5$. This effect is likely due to some backscattering from the $\tilde{C}_{ij}^M$ term of the model that is not compensated by sufficient dissipation from the $\tilde{R}_{ij}^M$ term. Indeed, recall that the C_r coefficient is here determined dynamically, and so that the global model has the same dissipation as the dynamic Smagorinsky model; hence, we did not allow for C_r to be varied independently from C_c . In that respect, some further investigations could include a version of the mixed model where both coefficients are determined dynamically.

Finally, the effect of the filter order is displayed in Fig. 6.17. Some improvements are observed but the overall effect of the filter order appears to be small. The lower filter order ($n = 3$) is therefore recommended.

6.5 Conclusion

The decomposition of the SGS stress tensor, as the cross term and the Reynolds term, was here investigated in detail in the context of LES with grid filtering only. We considered decaying homogeneous isotropic turbulence (DHIT), and

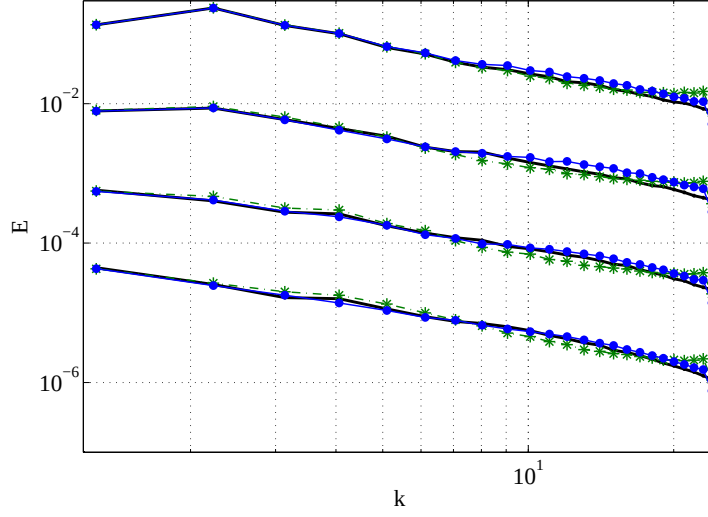
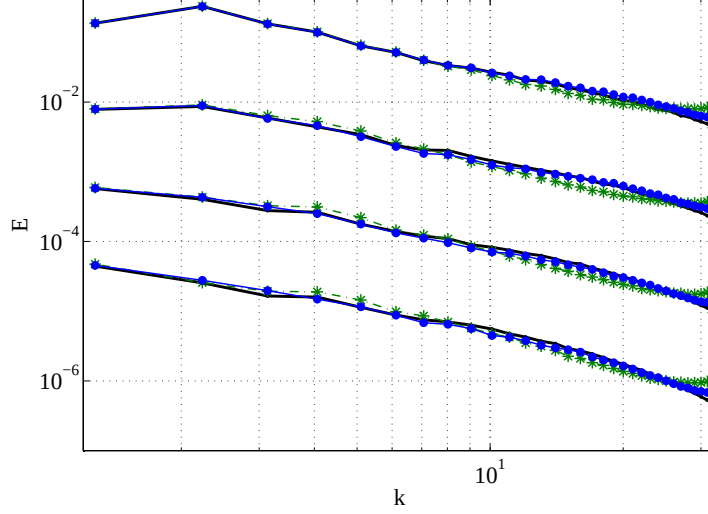
(a) 48^3 (b) 64^3

Figure 6.16: Energy spectra at the times $t/t_0 = 0.365, 1.46, 2.55$ and 3.65 (for visibility, they are shifted downward by one decade): DNS (thick solid), **dynamic Smagorinsky** (*), **dynamic mixed model** using the explicit filter of order 6 ($\bullet$).

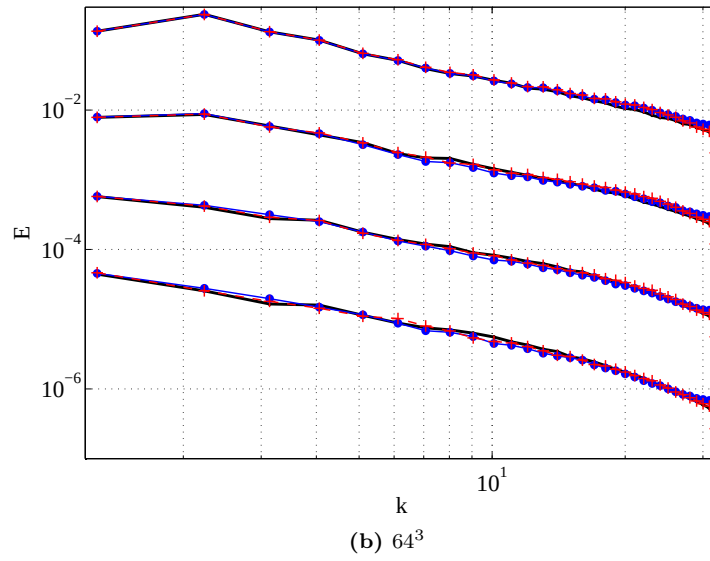
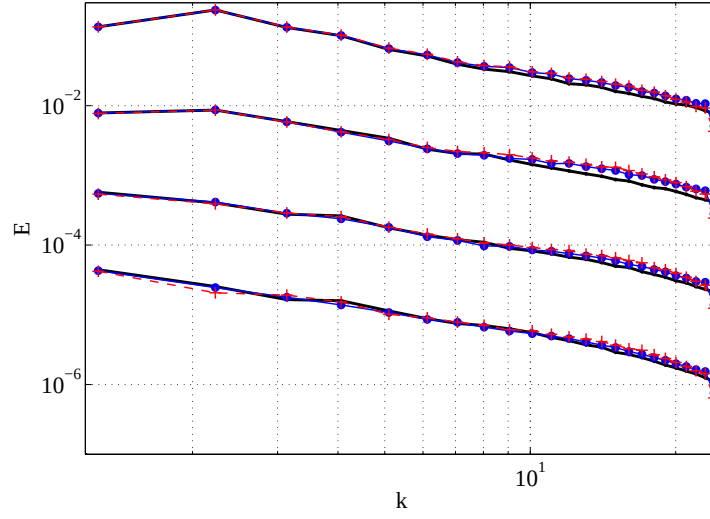


Figure 6.17: *Energy spectra at the times $t/t_0 = 0.365, 1.46, 2.55$ and 3.65 (for visibility, they are shifted downward by one decade): DNS (thick solid), dynamic mixed model using the explicit filter of order 6 ($\bullet$) and 10 ($+$).*

we used a de-aliased pseudo-spectral method with sharp Fourier truncation. A 512^3 DNS of DHIT, at $Re_\lambda = 149$, was truncated to 64^3 and 48^3 LES grids, and was run further in time. Both exact SGS terms were evaluated and analyzed: spectrum of dissipation, as well as distribution of dissipation and of backscatter (negative dissipation) in physical space, through the PDF. The spectral view shows that both terms provide dissipation at all wavenumbers, yet their spectral behavior is very different. The cross term is seen to dominate the Reynolds term: yet, this dominance is reduced relatively when the LES cut is coarser: for the 48^3 grid (with $k_c \eta \simeq 1.5$), the cross term dissipates roughly twice as much as the Reynolds term. Despite a different spectral behavior, both terms exhibit a significant amount of backscatter. The analysis of the PDF shows that about 40% of the points provide backscatter, and that the global amount of backscatter is roughly equal to the global dissipation; indeed, the budget is $2 + (-1) = 1$.

A new mixed model that better accounts for the physics of the SGS stress and for the backscatter was then proposed. It has a cross term part which is based on a scale-similarity argument, and combined with a correction for Galilean invariance (using a pseudo Leonard term): this is the term that also provides backscatter. It also has an eddy-viscosity multiscale model term, of RVMS type, that accounts for all the remaining phenomena (also for the incompleteness of the cross term model), that is dynamic, and that adjusts the overall model dissipation so as to conform to that of a dynamic Smagorinsky model. Both terms make use of the small-scales field and the large-scales field, here obtained by using high order explicit filters. Those filters are themselves efficiently obtained by using n iterations of the compact second order discrete filter (i.e., that solely uses the nearest neighbors 3^3 points).

The model was tested, both a priori and a posteriori, and was compared to the DNS and to the exact SGS terms, over a wide time range. It is seen to predict energy spectra with good accuracy, also when compared to the reference dynamic Smagorinsky model. It is however seen to be under-dissipative at the largest wavenumbers in the coarser 48^3 LES, and at early times. The analysis of the PDF's of the terms of the SGS model showed that the physics of the SGS stress are better captured than for purely dissipative models. The model is also seen to remain stable while about 30% of the points participate to backscatter. The fact that this term provides negative energy dissipation almost a third of the time without numerically blowing up is in itself an achievement. The global

amount of backscatter corresponds to roughly 20% of the total dissipation, with the budget $1.2 + (-0.2) = 1$: to be compared to the above budget for the exact SGS stress; thus relatively moderate, yet significant, backscatter.

As to the calibration of the cross term model coefficient, C_c , it could deserve further investigations, using LES at higher Reynolds number, and thus using a larger DNS; yet we see that the ranges calibrated so far are not that large: 0.35–0.30 when using the explicit filter of order 6 ($n = 3$), and 0.41–0.35 when using the explicit filter of order 10 ($n = 5$). As to the dynamic procedure, it would also be interesting to investigate the case where both coefficients are determined dynamically. It is however probable that at higher Reynolds number, and with a larger scale LES, the calibration of the model coefficient C_c will be independent of the LES cut, as long as it stays well within the inertial range.

The backscattering and spectral behavior of the model deserves further investigation. Notably, a wall-bounded flow, such as the channel flow would be of interest to assess the capability of the model to predict the log layer accurately. Since the mean dissipation of the model is set to equate that of a dynamic Smagorinsky model, the model should be able to reproduce the mean flow accurately. The hope here is that the backscattering of energy provided by the model will help the model better predict the velocity fluctuations, which are often underestimated by SGS models. The wake vortex system in ground effect would also be a challenging case, since it involves laminar zones, thin boundary layers and turbulent boundary layer separation. How this model would handle non-homogeneous flows (such as a wall-bounded flow, boundary layer separation, transition to turbulence, free vortex flows, etc.) is an open question.

Chapter 7

Conclusions and perspectives

This thesis has been dedicated to the implementation and assessment of various wall modeling approaches for large eddy simulation (LES) in the framework of the “BigFlow” code—a fourth order finite differences solver—and to the development of a novel subgrid scales (SGS) stress model. Advancements were made regarding wall modeling for LES, but questions were also raised as to their applicability to industrial cases. The understanding of the SGS terms was also improved and the backscattering of energy was partly taken into account by the SGS model proposed in this work.

7.1 Conclusions

A modified convective term discretization was implemented in the BigFlow solver to allow for the implementation of wall shear stress boundary conditions. The new version of the code was validated and LES coefficients had to be tuned to work in the new version of the code. The implementation of wall shear stress boundary condition was successfully carried out and validated in a moderate Reynolds number channel flow case. Several numerical tweaks were employed to apply the wall shear stress boundary condition. First, the numerical schemes were modified at the first off-wall points such that the wall-normal stencils are reduced to second order, while keeping the fourth order in the bulk of the flow. It was shown that for the scheme to transition from order two

to order four while maintaining the conservation of momentum in the discrete sense, a correction had to be applied when evaluating the derivatives at the transition point. At the transition point, where the correction is applied, the scheme is, however, reduced to order one. The correction was shown to reduce spurious oscillations that were present in the simulation without the correction. Then a correction for the SGS viscosity at the first layer of points was introduced. The correction is required because the physical content of the first cell is such that the SGS model cannot produce accurate results. The correction is based on the mean momentum budget in the thin boundary layer equations between the first two layers of points. With those corrections (though the SGS viscosity correction only provided marginal improvements), the wall shear stress boundary condition was shown to provide accurate results in channel flow at a much reduced computational cost, when compared to wall-resolved LES. Two branches of the wall shear stress family were investigated: algebraic wall models and RANS sublayer models. Both were tested with success in a high Re channel flow setup. Several parameters were investigated. Worryingly, the results obtained on finer meshes were of lesser quality than those obtained on coarse meshes. The aspect ratio of the cell was also seen to play a role in the results. Nevertheless, the velocity profiles obtained on the finer meshes remained within 2% of the DNS profiles. The wall models were seen to have spurious interactions with the filters used by RVMs models, making their use rather impractical. To alleviate this limitation, the use of filtered models instead of RVMs models was preferred, hence only using the filtered velocity when computing the SGS viscosity. The effect of the type of averaging used for the algebraic wall models was seen to be very small. Locally computed (“point-wise”) wall models were also seen to perform with an accuracy similar to that of averaged wall models. When RANS sublayer models were considered, it was shown that one should either drop convective and pressure terms altogether, or correct the RANS term to account for resolved fluctuations. When that was done, the results were in good agreement with the DNS results.

The wall modeling approaches were then assessed in the presence of a mild pressure gradient. The test-case was a half channel flow with periodic acceleration and deceleration generated by blowing and suction at the top of the numerical box, which resulted in a periodic pressure gradient. Additional wall models were implemented to explicitly account for additional pressure gradient effects. In the end, results in good agreement with the wall-resolved LES

were obtained by the algebraic wall model without any additional correction for pressure gradient effect. None of the “out-of-equilibrium” wall models yielded better results than the algebraic wall model with the law of Reichardt—hence the simplest of all. The impact of the averaging procedure was investigated, and it was shown that, again, the results obtained with pointwise wall models were of equal quality than those obtained with averaged wall models. The conclusion is that in the presence of mild pressure gradients, the simplest models, without any correction for pressure gradient effects, perform best and should be retained.

Another achievement of this work has been the analysis of turbulent two-vortex systems (T-2VS) in ground effect. The equilibrium T-2VS (out of ground effect) obtained by De Visscher [30] was used. This equilibrium T-2VS was put near ground effect and its interaction with the ground was analyzed using wall-resolved LES. We demonstrated that such a turbulent initial condition allowed for better representation of the flow physics than using an initially non turbulent 2VS as initial condition. In particular, in the early phases of the simulation, before strong interaction with the ground, we observed a decay in the LIDAR-processed vortex circulation Γ_{5-15} but not in the total circulation Γ_{tot} , which is a sign of turbulent redistribution of the vorticity. This is also observed experimentally, but it was not observed in previous simulations performed using analytical vortices with some perturbations to trigger instabilities and turbulence. The rather high Reynolds number of the simulation (10 times higher than previous simulations) also allowed for a better representation of the flow physics, with the secondary vorticity containing more turbulence, even in the case with an initially non turbulent 2VS. The decay of the T-2VS, which is linked to the production of detached secondary vorticity, was also shown to behave differently when using a T-2VS than when using an initially non turbulent initial condition. The interaction between the detached secondary vorticity and the vortex self-turbulence was shown to play a role in the vortex circulation decay. This demonstrated that the use of a realistic 2VS (hence a T-2VS instead of an analytical 2VS) as initial condition leads to a more physically accurate simulation. The present simulation therefore presents an upper bound for circulation decay, in the absence of further disturbance to the 2VS, such as wind, surface roughness, vortex tilting, etc.

This setup was then used to assess the wall models in a context with boundary layer separation. Here, the wall-modeled simulations remained in reason-

able agreement with the wall-resolved LES when observing the Γ_{tot} curve. While all wall models predicted boundary layer separation at the correct time, none managed to produce as much secondary vorticity through boundary layer separation as the wall-resolved LES. This insufficient secondary vorticity production resulted in a slowed circulation decay. None of the models therefore recovered the rate of decay of either Γ_{tot} or Γ_{5-15} . Some models still performed better than others, and here the best results were obtained using a RANS sub-layer model with pressure correction, namely, the model of Duprat et al. [37]. The other wall models gave rather close solutions. Notably, the simplest model (the algebraic wall model with the law of Reichardt) performed reasonably well when compared to more sophisticated models. A simulation without wall models, on the wall-modeled LES mesh was also run (thus an under-resolved LES), with a no-slip boundary condition. The results of this simulation were of equal quality than those using wall models. This was explained partly by the relatively fine resolution of the mesh at the wall. When algebraic wall models were used, the method of averaging was shown to result in topologically incorrect results when averaging over the whole longitudinal direction. Local averaging was therefore used, but pointwise wall models would probably also provide good results, as in the other cases investigated in this thesis.

Finally, a SGS model was developed based on a decomposition of the SGS stress tensor into a cross-term and a Reynolds term. The cross-term model is based on the argument of scale-similarity. It is supplemented by a dynamic RVMs model for the Reynolds term. Overall, the model was shown to have good dissipation properties. It was seen to provide backscatter while remaining stable, which is in itself an achievement. It also exhibited a good spectral behavior in decaying homogeneous isotropic turbulence.

7.2 Perspectives

Overall, several advances were made during the course of this thesis, but this also raised a series of questions for future research:

- A wall modeling framework for LES was developed within the framework of the “BigFlow” solver and tested with success. The wall models were seen to exhibit some sensitivity to the mesh and to its aspect ratio. This question should be investigated in more details and also in more challenging cases than the channel flow case.

- The issue of pressure gradient and boundary layer separation were investigated with mixed success. The blown half-channel test-case should be investigated with a more severe acceleration/deceleration to increase the impact of the pressure gradient. Whether the conclusions, that the best results are obtained by the simplest model, will be maintained or not is an open question.
- The disappointing results of the wall models in the T-2VS case could be due to the rather fine mesh close to the wall. The investigation of this case at higher Reynolds number is also a path for the future, since the points at which the wall model would be evaluated could be well in the log layer of the boundary layers, which was not an option, here.
- The boundary layer is also transitional, in this case. The development of a wall model capable of handling accurately the transition between laminar and turbulent regimes could bring improvement. Such a model would also be of use in a broad variety of applications. Such a model was already proposed by Bodart and Larsson [10], and this constitutes an interesting direction of research. More investigations on wall modeling of boundary layers with acceleration, deceleration and/or separation should be carried.
- More advanced wall models such as those recently proposed by Yang et al. [151] or Bose and Moin [11] also constitute a promising path for further research. The former is also from the wall shear stress model family and would be therefore easy to implement in our framework. The latter explores a new type of wall models, based on a slip velocity boundary condition, and its implementation in our framework is also accessible. Some of the conclusions drawn here will probably extend to those models, and the issue of the grid convergence should also be studied for those.
- The proposed SGS model showed promising results in DHIT, and its testing in more challenging (also wall-bounded) cases, such as the channel flow, or the T-2VS (in ground effect or not) could demonstrate its viability in non homogeneous flows. In the case of wall-bounded flows, the question of the averaging employed for the dynamic procedure should be investigated. The determination of the coefficient of the model should also be carried in a case at higher Reynolds number, as it is expected to be independent of the LES cut at sufficiently high Reynolds number.

Bibliography

- [1] Fact3: Airport capacity needs in the national airspace system. Technical report, Federal Aviation Administration, January 2015.
- [2] B. W. Anderson and J. A. Domaradzki. A subgrid-scale model for large-eddy simulation based on the physics of interscale energy transfer in turbulence. *Physics of Fluids*, 24:065104, 2012.
- [3] J.S. Baggett. Some modeling requirements for wall model in large eddy simulation. *Annual Research Briefs, Center for Turbulence Research*, pages 123–134, 1997.
- [4] J.S. Baggett, J. Jiménez, and A. G. Kravchenko. Resolution requirements in large eddy simulations of shear flows. *Annual Research Briefs, Center for Turbulence Research*, pages 51–66, 1997.
- [5] Elias Balaras, Carlo Benocci, and Ugo Piomelli. Two-layer approximate boundary conditions for large-eddy simulations. *AIAA Journal*, 34(6), 1996.
- [6] J. Bardina, J. H. Ferziger, and W. C. Reynolds. Improved subgrid-scale models for large eddy simulation. *AIAA Paper*, 80:1357–1367, 1980.
- [7] Y. Bazilevs, V.M. Calo, J.A. Cottrell, T.J.R. Hughes, A. Reali, and G. Scovazzi. Variational multiscale residual-based turbulence modeling for large eddy simulation of incompressible flows. *Computer Methods in Applied Mechanics and Engineering*, 197:173–201, 2007.
- [8] K. W. Bedford and W. K. Yeo. Conjunctive filtering procedures in surface water flow and transport. In B. Galperin and S. A. Orzag, editors, *Large eddy simulation of complex engineering and geophysical flows*, pages 513–537. Cambridge University Press, 1993.

- [9] S. Bocquet, P. Sagaut, and J. Jouhaud. A compressible wall model for large-eddy simulation with application to prediction of aerothermal quantities. *Physics of Fluids*, 24:065103, 2012.
- [10] Julien Bodart and J. Larsson. Sensor-based computation of transitional flows using wall-modeled large eddy simulation. *Annual Research Briefs, Center for Turbulence Research*, page 229, 2012.
- [11] S. T. Bose and Parviz Moin. Dynamic slip boundary condition for wall modeled large eddy simulation. *Physics of Fluids*, 26:015104, 2014.
- [12] J. Boussinesq. *Théorie de l'écoulement tourbillonnant et tumultueux des liquides dans les lits rectilignes a grande section*. Comptes rendus des séances de l'académie des sciences, Paris: Gauthier-Villars, 1897.
- [13] L. Bricteux. *Simulation of turbulent aircraft wake vortex flows and their impact on the signals returned by a coherent Doppler LIDAR system*. PhD thesis, Université catholique de Louvain, 2008.
- [14] L. Bricteux, M. Duponcheel, and G. Winckelmans. A multiscale subgrid model for both free vortex flows and wall-bounded flows. *Physics of Fluids*, 21(10):105102, 2009.
- [15] L. Bricteux, M. Duponcheel, I. De Visscher, and G. Winckelmans. Les investigation of the transport and decay of various-strengths wake vortices in ground effect and subjected to a turbulent crosswind. *Physics of Fluids*, 28(6):065105, 2016.
- [16] D. C. Burnham and J. N. Hallock. Wake vortex decay. In *Chicago monostatic acoustic vortex sensig system*. National Technical Information Services, Springfield, VA, 1982.
- [17] W. Cabot. Large-eddy simulations with wall models. *Annual Research Briefs, Center for Turbulence Research*, pages 41–50, 1995.
- [18] W. H. Cabot and P. Moin. Approximate wall boundary conditions in the large-eddy simulation of high reynolds number flows. *Flow, Turbululence and Combustion*, 63(269-291), 1999.
- [19] A. Cahuzac, J. Bouden, P. Borgnat, and E. Lévêque. Smoothing algorithms for mean-flow extraction in large-eddy simulation of complex turbulent flows. *Physics of Fluids*, 22:125104, 2010.

- [20] Claudio Canuto, M. Yousuff Hussaini, Alfio Quarteroni, and Thomas A. Zang. *Spectral Methods: Fundamentals in Single Domains*. Scientific Computation. Springer-Verlag Berlin Heidelberg, 2006.
- [21] Daniele Carati, G. Winckelmans, and H. Jeanmart. On the modelling of the subgrid-scale and filtered-scale stress tensors in large-eddy simulation. *Journal of Fluid Mechanics*, 441:119–138, 2001.
- [22] D. R. Chapman. Computational aerodynamics development and outlook. *AIAA Journal*, 17(12):1293–1313, 1979.
- [23] H. Choi and P. Moin. Grid-point requirements for large-eddy simulation: Chapman’s estimates revisited. *Annual Research Briefs, Center for Turbulence Research*, pages 31–36, 2011.
- [24] D. Chung and D. I. Pullin. Large-eddy simulation and wall modelling of turbulent channel flow. *Journal of Fluid Mechanics*, 631:281–309, 2009.
- [25] R. Cocle, L. Dufresne, and G. Winckelmans. Investigation of multiscale subgrid models for les of instabilities and turbulence in wake vortex systems. *ecture Notes in Computational Science and Engineering : Complex effects in large eddy simulations*, 56:141–159, 2007.
- [26] R. Cocle, L. Bricteux, and G. Winckelmans. Scale dependence and asymptotic very high reynolds number spectral behavior of multiscale subgrid models. *Physics of Fluids*, 21(8):085101, 2009.
- [27] E. Coustols, L. Jacquin, and G. Schrauf. Status of wake vortex alleviation in the framework of european collaboration: validation using tests and cfd results. In J. Périaux, P. Wesseling, and E. Oñate, editors, *ECCOMAS CFD 2006: Proceedings of the European Conference on Computational Fluid Dynamics, Egmond aan Zee, The Netherlands*. Delft University of Technology; European Community on Computational Methods in Applied Sciences, September 5-8, 2006.
- [28] S. C. Crow. Stabilities theory for a pair of trailing vortices. *AIAA Journal*, 8:2172–2179, 1970.
- [29] A. de Bruin and G. Winckelmans. Cross-flow kinetic energy and core size growth of analytically defined wake vortex pairs. Awiator research project d 1.1.4-24, Netherlands Aerospace Centre and Université catholique de Louvain, 2005.

- [30] I. De Visscher. *Interaction of wake vortices with the atmosphere and the ground: Advanced numerical simulation and operational modeling*. PhD thesis, Université catholique de Louvain, May 2012.
- [31] I. De Visscher, L. Bricteux, and G. Winckelmans. Aircraft vortices in stably stratified and weakly turbulent atmospheres: Simulation and modeling. *AIAA Journal*, 51(3):551–566, 2013. URL <http://dx.doi.org/10.2514/1.J051742>.
- [32] Ivan De Visscher, Grégoire Winckelmans, Thimothée Lonfils, Laurent Bricteux, Matthieu Duponcheel, and Nicolas Bourgeois. *The WAKE4D Simulation Platform for Predicting Aircraft Wake Vortex Transport and Decay: Description and Examples of Application*. American Institute of Aeronautics and Astronautics, 2016/10/27 2010.
- [33] James W. Deardoff. A numerical study of three-dimensional turbulent channel flow at large reynolds numbers. *Journal of Fluid Mechanics*, 41(2):453–480, 1970.
- [34] T. L. Doligalski, C. R. Smith, and J. D. A. Walker. Vortex interactions with walls. *Annual Review of Fluid Mechanics*, 26(1):573–616, 1994. doi: 10.1146/annurev.fl.26.010194.003041. URL <http://dx.doi.org/10.1146/annurev.fl.26.010194.003041>.
- [35] M. Duponcheel. *Direct and large-eddy simulation of turbulent wall-bounded flows: further development of a parallel solver, improvement of multiscale subgrid models and investigation of vortex pairs in ground effect*. PhD thesis, Université catholique de Louvain, 2009.
- [36] M. Duponcheel, P. Orlandi, and G. Winckelmans. Time-reversibility of the euler equations as a benchmark for energy conserving schemes. *Journal of Computational Physics*, 227(19):8736–8752, 2008.
- [37] C. Duprat, G. Balarac, O. Métais, P.M. Congedo, and O. Brugière. A wall-layer model for large-eddy simulations of turbulent flows with/out pressure gradient. *Physics of Fluids*, 23(1):015101, 2011.
- [38] Eurocontrol. Challenges of growth. Summary report, Eurocontrol, 2013.
- [39] D. Fabre and L. Jacquin. Short-wave cooperative instabilities in representative aircraft vortices. *Physics of Fluids*, 16(5):1366–1378, May 2004.

- [40] J. Frölich and D. von Terzi. Hybrid les/rans methods for the simulation of turbulent flows. *Progress in Aerospace Sciences*, 44(5):349–377, 2008.
- [41] R. Garcià-Mayoral, B. Pierce, and J Wallace. Off-wall boundary conditions for turbulent simulations from minimal flow units in transitional boundary layers. *Annual Research Briefs, Center for Turbulence Research*, pages 87–96, 2012.
- [42] Laurent Georges. *Development and validation of a LES methodology for complex wall-bounded flows. Application to high order structured and industrial unstructured solvers*. PhD thesis, Université catholique de Louvain, 2007.
- [43] M. Germano, U. Piomelli, P. Moin, and W.H. Cabot. A dynamic subgrid-scale eddy viscosity model. *Physics of Fluids A: Fluid Dynamics*, 3: 1760–1765, 1991.
- [44] Thomas Gerz, Frank Holzäpfel, and Denis Darracq. Commercial aircraft wake vortices. *Progress in Aerospace Sciences*, 38(3):181 – 208, 2002. ISSN 0376-0421. doi: [http://dx.doi.org/10.1016/S0376-0421\(02\)00004-0](http://dx.doi.org/10.1016/S0376-0421(02)00004-0).
- [45] P. S. Granville. A modified van driest formula for the mixing length of turbulent boundary layers in pressure gradients. *Journal of Fluids Engineering*, 111(1):94–97, 1989.
- [46] Paul S. Granville. A near-wall eddy viscosity formula for turbulent boundary layers in pressure gradients suitable for momentum, heat, or mass transfer. *Journal of Fluids Engineering*, 112:240–243, June 1990.
- [47] G. Grötzbach. Direct numerical and large eddy simulation of turbulent channel flows. *Encyclopaedia of Fluid Mechanics*, 6:1337–1391, 1987.
- [48] Ayse Gul Gungor and Suresh Menon. A new two-scale model for large eddy simulation of wall-bounded flows. *Progress in Aerospace Sciences*, 46:28–45, 2010.
- [49] F. H. Harlow and J. E. Welsh. Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface. *Physics of Fluids*, 8(10):2182–2189, 1965.

- [50] D. M. Harris and C. H. K. Williamson. Instability of secondary vortices generated by a vortex pair in ground effect. *Journal of Fluid Mechanics*, 700:148–186, 2012.
- [51] J. K. Harvey and F. J. Perry. Flowfield produced by trailing vortices in the vicinity of the ground. *AIAA Journal*, 9(8):1659–1660, 1971.
- [52] G. Hoffmann and C. Benocci. Approximate wall boundary condition for large eddy simulations. In R. Benzi, editor, *Advances in Turbulence V*, pages 222–228. Kluwer Academic Publishers, Dordrecht, 1995.
- [53] Jens Holmen, Thomas J. R. Hughes, Assad A. Oberai, and Garth N. Wall. Sensitivity of the scale partition for variational multiscale large-eddy simulation of channel flow. *Physics of Fluids*, 16(3):824–827, 2004.
- [54] Frank Holzäpfel, Nikola Tchipev, and Anton Stephan. Wind impact on single vortices and counterrotating vortex pairs in ground proximity. *Flow, Turbulence and Combustion*, 97(3):829–848, 2016. doi: 10.1007/s10494-016-9729-2.
- [55] Frank N. Holzäpfel, Nikola Tchipev, and Anton Stephan. *Wind Impact on Single Vortices and Counter-Rotating Vortex Pairs in Ground Proximity*. American Institute of Aeronautics and Astronautics, 2015. doi: 10.2514/6.2015-3174.
- [56] K. Horiuti. A new dynamic two-parameter mixed model for large-eddy simulation. *Physics of Fluids*, 9:3443, 1997.
- [57] S. Hoyas and J. Jimenez. Scaling of velocity fluctuations in turbulent channels up to $Re_\tau = 2000$. *Physics of Fluids*, 18:011702, 2006.
- [58] T. J. R. Hughes, A. A. Oberai, and L. Mazzei. The multiscale formulation of large eddy simulation: Decay of homogeneous isotropic turbulence. *Physics of Fluids*, 13:505–512, 2001.
- [59] H. Jeanmart and G. Winckelmans. Comparison of recent dynamic subgrid-scale models in turbulent channel flow. *Proceedings of CTR Summer Program*, pages 105–116, 2002.
- [60] H. Jeanmart and G. S. Winckelmans. Investigation of eddy-viscosity models using discrete filters: A simplified "regularized variational mul-

- tiscale model” and an ”enhanced field model”. *Physics of Fluids*, 19(5):055110, 2007.
- [61] Hervé Jeanmart. *Investigation of novel approaches and models for large-eddy simulation of turbulent flows*. PhD thesis, Université catholique de Louvain, 2002.
- [62] J. Jeong and F. Hussain. On the identification of a vortex. *Journal of Fluid Mechanics*, 285:69–94, 1995.
- [63] J. Jiménez and Parviz Moin. The minimal flow unit in near wall turbulence. *Journal of Fluid Mechanics*, 225:213–240, 1991.
- [64] J. Jiménez and C. Vasco. Approximate lateral boundary conditions for turbulent simulations. *Annual Research Briefs, Center for Turbulence Research*, pages 399–412, 1998.
- [65] Roderick Johnstone, Gary N. Coleman, and Philippe R. Spalart. The resilience of the logarithmic law to pressure gradients: evidence from direct numerical simulation. *Journal of Fluid Mechanics*, 643:163–175, 2010.
- [66] T. Kajishima. Conservation properties of finite difference methods (in japanese). *Trans. JSME*, 60(574):2058–2063, 1994.
- [67] Soshi Kawai and Johan Larsson. Wall-modeling in large eddy simulation: length scales, grid resolution, and accuracy. *Physics of Fluids*, 24(015105), 2012.
- [68] S. Khanna and J. G. Brasseur. Analysis of monin-obukhov similarity from large-eddy simulation. *Journal of Fluid Mechanics*, 345:251–286, 1997.
- [69] Johan Larsson, Soshi Kawai, Julien Bodart, and Ivan Bermejo-Moreno. Large eddy simulation with modeled wall-stress: recent progress and future directions. *Mechanical Engineering Reviews*, 3(1):15–00418, 2016.
- [70] M. J. Lee, B. D. Oh, and Y. B. Kim. Canonical fractional-step methods and consistent boundary conditions for the incompressible navier-stokes equations. *Journal of Computational Physics*, 168(1):73–100, 2001.

- [71] Myoungkyu Lee and Robert D. Moser. Direct numerical simulation of turbulent channel flow up to $Re_\tau \approx 5200$. *Journal of Fluid Mechanics*, 774:395–415, 2015.
- [72] A. Leonard. Energy cascade in large-eddy simulations of turbulent fluid flows. *Advances in Geophysics*, 18:237, 1974.
- [73] J. Leray. Sur le mouvement d’un liquide visqueux emplissant l’espace. *Acta mathematica*, 63:193–248, 1934.
- [74] Thomas Leweke, Stéphane Le Dizès, and Charles H.K. Williamson. Dynamics and instabilities of vortex pairs. *Annual Review of Fluid Mechanics*, 48(1):507–541, 2016. doi: 10.1146/annurev-fluid-122414-034558.
- [75] D. K. Lilly. A proposed modification of the germano subgrid-scale closure method. *Physics of Fluids A: Fluid Dynamics*, 4:633–635, 1992.
- [76] S. Liu, C. Meneveau, and J. Katz. On the properties of similarity subgrid-scale models as deduced from measurements in a turbulent jets. *Journal of Fluid Mechanics*, 275(83-119), 1994.
- [77] T.S. Lund, X. Wu, and K. D. Squires. Generation of turbulent inflow data for spatially-developing boundary layers. *Journal of Computational Physics*, 140(2):233–258, 1998.
- [78] J.A. Luton and S.A. Ragab. Three-dimensional interaction of a vortex pair with a wall. *Physics of Fluids*, 9(10):2967–2980, 1997.
- [79] P. J. Mason and N. S. Callen. On the magnitude of the subgrid-scale eddy coefficient in large-eddy simulations of turbulent channel flow. *Journal of Fluid Mechanics*, 162:439–462, 1986.
- [80] G. Medic, G. Daeninck, J.A. Templeton, and G. Kalitzin. A framework for near-wall rans/les coupling. *Annual Research Briefs, Center for Turbulence Research*, 2005.
- [81] C. Meneveau and J. Katz. Scale-invariance and turbulence models for large-eddy simulation. *Annual Review of Fluid Mechanics*, 32:1–32, 2000.
- [82] F. Menter, B. Bender, and M. Kuntz. A scale-adaptive simulation model for turbulent flow predictions. In *41st AIAA Aerospace sciences meeting and exhibit*, Jan.s 2003.

- [83] Clark B. Millikan. A critical discussion of turbulent flows in channels and circular tubes. In J. P. den Hartog and H. Peters, editors, *Proc. 5th Int. Cong. Applied Mechanics*, pages 386–392. Wiley/Chapman and Hall, New York-London, 1938.
- [84] T. Misaka, Frank Holzäpfel, I. Henneman, T. Gerz, M. Manhart, and F. Schwertfirm. Vortex bursting and tracer transport of a counter-rotating vortex pair. *Physics of Fluids*, 24(2):025104, 2012.
- [85] Yutaka Miyake, Koichi Tsujimoto, and Hideki Beppu. Direct numerical simulation of a turbulent flow in a channel having periodic pressure gradient. *International Journal of Heat and Fluid Flow*, 16:333–340, 1995.
- [86] Yoshinori Mizuno and J. Jiménez. Wall turbulence without walls. *Journal of Fluid Mechanics*, 723:429–455, 2013.
- [87] Y. Morinishi and O. Vasilyev. A recommended modification to the dynamic two-parameter mixed subgrid scale model for large eddy simulation of wall bounded turbulent flow. *Physics of Fluids*, 13(11):3400–3410, 2001.
- [88] Y. Morinishi, T.S. Lund, O. V. Vasilyev, and Parviz Moin. Fully conservative higher order finite difference schemes for incompressible flow. *Journal of Computational Physics*, 143(1):90–124, 1998.
- [89] Robert D. Moser, John Kim, and Nagi N. Mansour. Direct numerical simulation of turbulent channel flow up to $re_\tau = 590$. *Physics of Fluids*, 11(4):943, 1999.
- [90] Y. Na and Parviz Moin. Direct numerical simulation of a separated turbulent boundary layer. *Journal of Computational Physics*, 374:379–405, 1998.
- [91] F. Nicoud and F. Ducros. Subgrid-scale stress modelling based on the square of the velocity gradient tensor. *Flow, Turbulence and Combustion*, 62(3):183–200, 1999.
- [92] F. Nicoud, G. Winckelmans, D. Carati, and J. Baggett. Boundary conditions for les away from the wall. *Proceedings of CTR Summer Program*, 1998.

- [93] F. Nicoud, J.S. Baggett, Parviz Moin, and W. Cabot. Large eddy simulation wall-modeling based on suboptimal control theory and stochastic estimations. *Physics of Fluids*, 13(10):2968–2984, 2001.
- [94] N. V. Nikitin, F. Nicoud, B. Wasistho, K. D. Squires, and Philippe R. Spalart. An approach to wall modeling in large eddy simulation. *Physics of Fluids*, 12(7):1629–1632, 2000.
- [95] M. J. Nituch, S. Sjolander, and M. R. Head. An improved version of the Cebeci-Smith eddy-viscosity model. *Aeronautical Quarterly*, 29:207–225, August 1978.
- [96] George Ilhwan Park and Parviz Moin. An improved dynamic non-equilibrium wall-model for large eddy simulation. *Physics of Fluids*, 26(1):015108, January 2014.
- [97] N. Park and K. Mahesh. A velocity-estimation subgrid model constrained by subgrid scale dissipation. *Journal of computational Physics*, 227:4190–4206, 2008.
- [98] U. Piomelli and E. Balaras. Wall-layer models for large-eddy simulations. *Annual Review of Fluid Mechanics*, 34(1):349–374, 2002.
- [99] U. Piomelli, J. Ferziger, P. Moin, and J. Kim. New approximate boundary conditions for large-eddy simulations of wall-bounded flows. *Physics of Fluids A: Fluid Dynamics*, 1(6):1061–1068, 1989.
- [100] Ugo Piomelli. Wall-layer models for large-eddy simulations. *Progress in Aerospace Sciences*, 44:437–446, 2008.
- [101] Ugo Piomelli, T. A. Zang, C. G. Speziale, and M. Y. Hussain. On the large-eddy simulation of transitional wall-bounded flows. *Physics of Fluids A: Fluid Dynamics*, 2(2):257–265, 1990.
- [102] Bérengère Podvin and Yann Faigneau. Synthetic wall boundary conditions for the direct numerical simulation of wall-bounded turbulence. *Journal of Turbulence*, 12(4):1–26, 2011.
- [103] F. H. Proctor, D. W. Hamilton, and H. Han. Wake vortex transport and decay in ground effect: Vortex linking with the ground. In *38th AIAA Aerospace Sciences Meeting and Exhibition*, page 0757, Reno, NV, USA, January 10-13 2000.

- [104] Fred Proctor. *The NASA-Langley wake vortex modelling effort in support of an operational aircraft spacing system*. American Institute of Aeronautics and Astronautics, 1998.
- [105] Fred Proctor and Jongil Han. *Numerical study of wake vortex interaction with the ground using the terminal area simulation system*. American Institute of Aeronautics and Astronautics, 1999.
- [106] Fred Proctor, Nash'at Ahmad, George Switzer, and Fanny Limon Duparcmeur. *Three-Phased Wake Vortex Decay*. American Institute of Aeronautics and Astronautics, 2010.
- [107] Srinivas Ramakrishnan and S. Scott Collis. Partition selection in multi-scale turbulence modeling. *Physics of Fluids*, 18(7), 2006.
- [108] H. Reichardt. Vollständige darstellung der turbulenten geschwindigkeitsverteilung in glatte leitungen (complete representation of the turbulent velocity distribution in smooth pipes). *Zeitschrift für Angewandte Mathematik und Mechanik (Journal of Applied Mathematics and Mechanics)*, 31(7):208–219, 1951.
- [109] Lewis Fry Richardson. *Weather Prediction by Numerical Process*. Cambridge University Press, 1922.
- [110] Pierre Sagaut. *Large-Eddy simulation for Incompressible Flows: an introduction*. Springer, 3rd edition, 2006.
- [111] Rodney C. Schmidt, Alan R. Kerstein, Scott Wunsch, and Vebjorn Nilsen. Near-wall les closure based on one-dimensional turbulence modeling. *Journal of Computational Physics*, 186:317–355, 2003.
- [112] U. Schumann. Subgrid scale model for finite differences simulations of turbulent flows in plane channels and annuli. *Journal of computational Physics*, 18:376–404, 1975.
- [113] Tsan-Hsing Shih, Louis Povinelli, Nan-Suey Liu, Mark Potapczuk, and J. L. Lumley. A generalized wall function. Technical report, Institute for Computational Mechanics in Propulsion, NASA, 1999.
- [114] Tsan-Hsing Shih, Louis A. Povinelli, Nan-Suey Liu, and Kuo-Huey Chen. Generalized wall function for complex turbulent flows. Technical report, Institute for Computational Mechanics in Propulsion, NASA, 2000.

- [115] M. L. Shur, Philippe R. Spalart, M. Kh. Strelets, and A. Travin. A hybrid rans-les approach with delayed-des and wall-modelled les capabilities. *International Journal of Heat and Fluid Flow*, 29:1638–1649, 2008.
- [116] M. Skote and D. S. Henningson. Direct numerical simulation of a separated turbulent boundary layer. *Journal of Fluid Mechanics*, 471:107–136, 2002.
- [117] J. Smagorinsky. General circulation experiments with the primitive equations. *Monthly Weather Review*, 91:99–164, 1963.
- [118] P. R. Spalart, W.-H. Jou, M. Stretlets, and S. R Allmaras. Comments on the feasibility of les for wings and on the hybrid rans/les approach. In Greyden press C. Liu, Z. Liu, editor, *Advances in DNS/LES, First AFOSR International Conference on DNS/LES*, pages 137–148, 1997.
- [119] Philippe R. Spalart. Airplane trailing vortices. *Annual Review of Fluid Mechanics*, 30(1):107, 1998. ISSN 00664189.
- [120] Philippe R. Spalart. Detached-eddy simulation. *Annual Review of Fluid Mechanics*, 41:181–202, 2009.
- [121] Philippe R. Spalart, S. Deck, M. L. Shur, K. D. Squires, M. Kh. Strelets, and A. Travin. A new version of detached-eddy simulation, resistant to ambiguous grid densities. *Theoretical Computational Fluid Dynamics*, 20:181–195, 2006.
- [122] D. B. Spalding. A single formula for the “law of the wall”. *Journal of Applied Mechanics*, 28(3):455–458, 1961.
- [123] C. G. Speziale. Galilean invariance of subgrid-scale stress in large eddy simulation. *Journal of Fluid Mechanics*, 156:55–62, 1985.
- [124] A. Stephan, F. Holzäpfel, and T. Misaka. Aircraft wake-vortex decay in ground proximity – physical mechanisms and artificial enhancement. *Journal of Aircraft*, 50(4):1250–1260, 2013.
- [125] Anton Stephan, Frank Holzäpfel, and Takashi Misaka. Hybrid simulation of wake-vortex evolution during landing on flat terrain and with plate line. *International Journal of Heat and Fluid Flow*, 49:18–27, October 2014.
- [126] S. Stolz and N.A. Adams. An approximate deconvolution procedure for large-eddy simulation. *Physics of Fluids*, 11:1699–1701, 1999.

- [127] S. Stolz, N.A. Adams, and L. Kleiser. An approximate deconvolution model for large-eddy simulation with application to incompressible wall-bounded flows. *Physics of Fluids*, 13:99–1015, 2001.
- [128] S. Stolz, P. Schlatter, and L. Kleiser. High-pass filtered eddy-viscosity models for large-eddy simulations of transitional and turbulent flow. *Physics of Fluids*, 17(6), 2005.
- [129] Olivier Thiry and Grégoire Winckelmans. A mixed multiscale dynamic sgs model accounting for the cross-term. In *European Turbulence Conference 14 (ETC14)*, Lyon, France, 2013.
- [130] Olivier Thiry and Grégoire Winckelmans. A mixed multiscale model accounting for the cross term of the sub-grid scale stress. In *5th European Conference on Computational Mechanics, 6th European Conference on Computational Fluid Dynamics*, Barcelona, Spain, 2014.
- [131] Olivier Thiry and Grégoire Winckelmans. A mixed multiscale model better accounting for the cross term of the subgrid-scale stress and for backscatter. *Physics of Fluids*, 28(2):025111, 2016.
- [132] Olivier Thiry, Grégoire Winckelmans, and Matthieu Duponcheel. High re wall-modeled les of aircraft wake vortices in ground effect. In *Bulletin of the American physical society*, volume 59, no. 20, page 456, 2014.
- [133] Olivier Thiry, Matthieu Duponcheel, and Grégoire Winckelmans. Investigation of "vortex self turbulence" and reynolds number effects for wake decay and transport ige. In *WakeNet-Europe 2015 workshop*, Amsterdam, The Netherlands, 2015.
- [134] Olivier Thiry, Grégoire Winckelmans, and Matthieu Duponcheel. Investigation of pressure gradient aware wall modeling in les. In *Bulletin of the American Physical Society*, volume 60, no. 21, pages 200–201, Boston, MA, USA, 2015.
- [135] Olivier Thiry, Matthieu Duponcheel, and Grégoire Winckelmans. Investigation of various wall modeling approaches for les in the presence of mild pressure gradients. In *11th International ERCOFTAC Symposium on Engineering Turbulence Modelling and Measurements (ETMM11)*, Palermo, Sicily, Italy, 2016.

- [136] A. Townsend. *The structure of turbulent shear flows*. Cambridge University Press, 1976.
- [137] E. R. Van Driest. On turbulent flow near a wall. *Journal of Aeronautical Science*, 23(11):1007–1011, 1956.
- [138] Oleg V. Vasilyev. High order finite difference schemes on non-uniform meshes with good conservation properties. *Journal of Computational Physics*, 157:746–761, 2000.
- [139] R. Verstappen. On restraining the production of small scales of motion in a turbulent channel flow. *Computers and fluids*, 37:887–897, 2008.
- [140] A. W. Vreman. The filtering analog of the variational multiscale method in large-eddy simulation. *Physics of Fluids*, 15(8), 2003.
- [141] B. Vreman, B. Geurts, and H. Kuerten. On the formulation of the dynamic mixed subgrid-scale model. *Physics of Fluids*, 6(12):4057–4059, 1994.
- [142] Meng Wang. Les with wall models for trailing-edge aeroacoustics. *Annual Research Briefs, Center for Turbulence Research*, pages 355–364, 1999.
- [143] Meng Wang and Parviz Moin. Dynamic wall modeling for large-eddy simulation of complex turbulent flows. *Physics of Fluids*, 14(7):2043–2051, 2002.
- [144] Z. Wang and A. A. Oberai. A mixed large eddy simulation model based on the residual-based variational multiscale formulation. *Physics of Fluids*, 22:075107, 2010.
- [145] H. Werner and H. Wengle. Large-eddy simulation of turbulent flow over and around a cube in a plate channel. In Springer, editor, *Turbulent Shear flows 8*, volume 8, 1993.
- [146] S. E. Widnall, D. B. Bliss, and C.Y. Tsai. The instability of short waves on a vortex ring. *Journal of Fluid Mechanics*, 66:35–47, 1974.
- [147] J. H. Williamson. Low-storage runge-kutta schemes. *Journal of computational Physics*, 35:48–56, 1980.

- [148] G. S. Winckelmans, A. A. Wray, O. V. Vasilyev, and H. Jeanmart. Explicit filtering large-eddy simulation using the tensor-diffusivity model supplemented by a dynamic smagorinsky term. *Physics of Fluids*, 13: 1385–1403, 2001.
- [149] Grégoire Winckelmans. Some progress in efficient multi-scale subgrid modeling for turbulence simulation. In *Advances in Computational Mechanics (ACM 2013) – A Conference Celebrating the 70th Birthday of Thomas J.R. Hughes, 17th International Conference on Finite Elements in Flow Problems (FEF 2013)*, San Diego, CA, USA, 2013.
- [150] Grégoire Winckelmans and Hervé Jeanmart. Assessment of some models for les without and with explicit filtering. In Bernard Geurts, Rainer Friedrich, and Olivier Métais, editors, *Direct and Large-Eddy Simulation IV*, volume 8 of *ERCOTAC Series*, pages 55–66. Kluwer Academic Publishers, Dordrecht, 2001.
- [151] X. I. A. Yang, J. Sadique, R. Mittal, and Charles Meneveau. Integral wall model for large eddy simulations of wall-bounded turbulent flows. *Physics of Fluids*, 27:025112, 2015.
- [152] W. K. Yeo and K.W. Bedford. Closure-free turbulence modeling based upon conjunctive higher order averaging procedure. In H. Niki and M. Kawahara, editors, *Computational Methods in Flow Analysis*, volume 2, pages 844–851. Okayama University of Science, 1988.

Appendix A

Modified Shih law of the wall

The law of the wall proposed by Shih et al. [113] is composed of two parts, one of which is the standard law of the wall linked to wall shear stress, the other being due to pressure gradient effects. This second part reads:

$$\mathbf{u}_2 = u_p f_2\left(\frac{y u_p}{\nu}\right) \hat{\mathbf{n}}_p. \quad (\text{A.1})$$

They proposed a piecemeal representation of that function of the form:

$$f_2(y_p) = \begin{cases} a_2 y_p^2 + a_3 y_p^3, & y_p \leq 4 \\ b_0 + b_1 y_p + b_2 y_p^2 + b_3 y_p^3 + b_4 y_p^4, & 4 < y_p \leq 15 \\ c_0 + b_1 y_p + c_2 y_p^2 + c_3 y_p^3 + c_4 y_p^4, & 15 < y_p \leq 30 \\ 5.0 \log(y_p) + 8.0, & y_p > 30. \end{cases} \quad (\text{A.2})$$

The coefficient they proposed, however, yields a discontinuous function. It is therefore not well suited for practical applications. We here propose a slightly different set of coefficients to fix that, see Table A.1. We see in Fig. A.1 that the original and the corrected function agree well. The discontinuities seen in Fig.A.2 are however suppressed by the corrected coefficients.

	Value in Shih et al. [113]	Corrected value
a_2	0.5	0.5
a_3	$-7.31 \cdot 10^{-3}$	$-4.87194 \cdot 10^{-3}$
b_0	-15.138	-15.369812959090913
b_1	8.4688	8.487744359772728
b_2	-0.81976	-0.81976
b_3	$3.7292 \cdot 10^{-2}$	$3.7292 \cdot 10^{-2}$
b_4	$-6.3866 \cdot 10^{-4}$	$-6.3866 \cdot 10^{-4}$
c_0	11.925	11.678746216689236
c_1	0.93400	0.946926756387385
c_2	$-2.7805 \cdot 10^{-2}$	$-2.7805 \cdot 10^{-2}$
c_3	$4.6262 \cdot 10^{-4}$	$4.6262 \cdot 10^{-4}$
c_4	$-3.1442 \cdot 10^{-6}$	$-3.1442 \cdot 10^{-6}$

Table A.1: Original and corrected coefficients for the f_2 function in the Shih law of the wall.

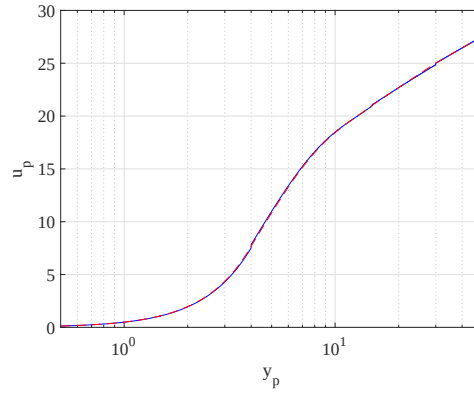


Figure A.1: f_2 function, as proposed in Shih et al. [113] (thin solid), and corrected for continuity (dashed).

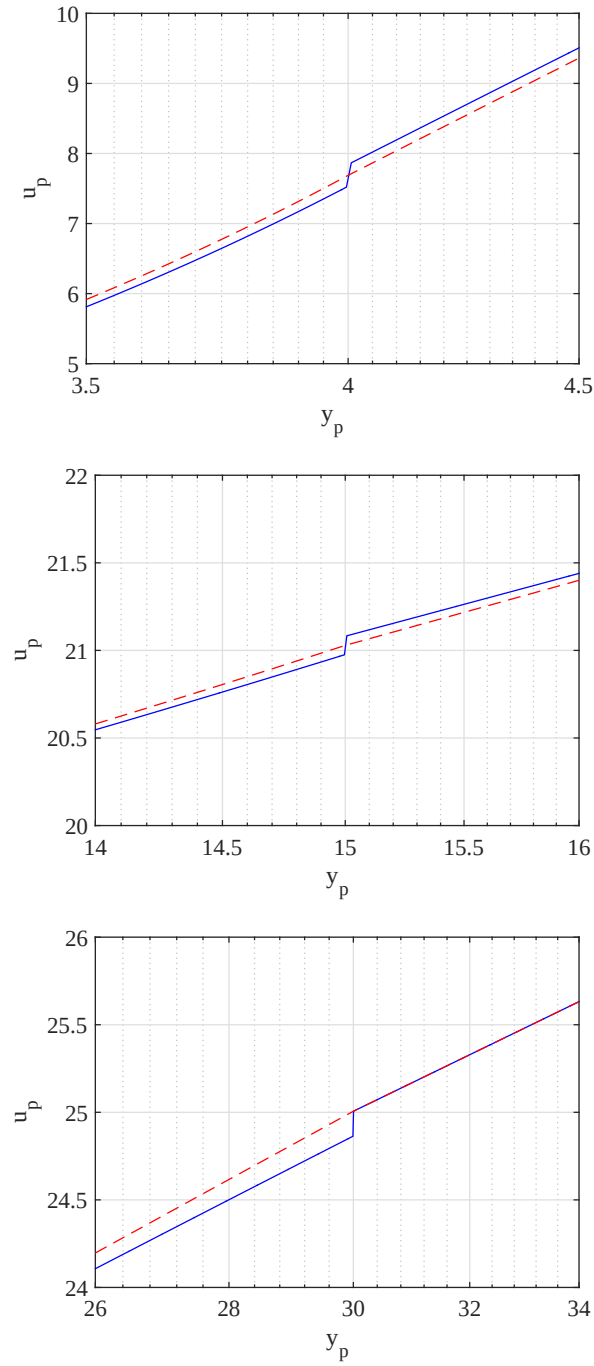


Figure A.2: Zoom on transition points of the f_2 function, as proposed in Shih et al. [113] (thin solid), and modified for continuity (dashed).

Appendix B

Full page volume rendering of vorticity fields

We here reproduce, in full page and high definition the volume rendering of the vorticity norm for the wall-resolved LES of the T-2VS and of the BH-2VS in ground effect, shown in low resolution in Fig. 4.6.

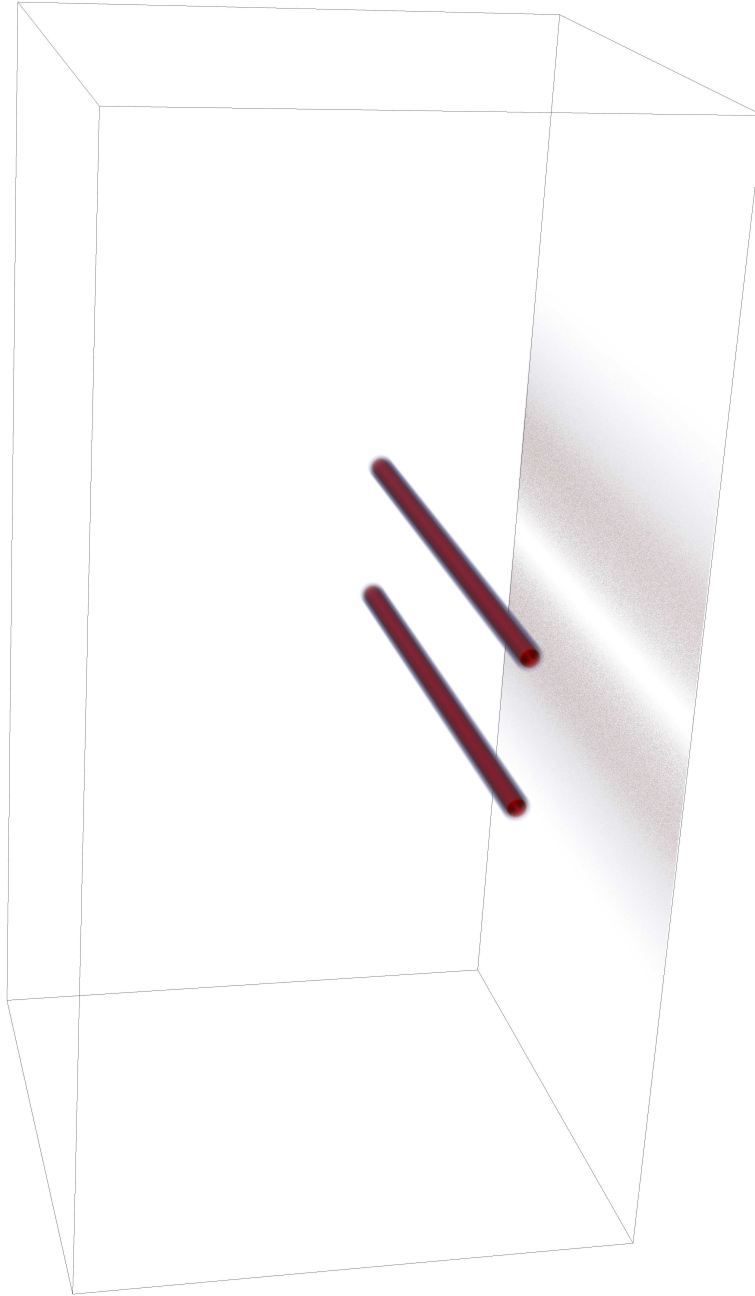


Figure B.1: Volume rendering of the vorticity magnitude of the BH-2VS at $t/t_0 = 1$.

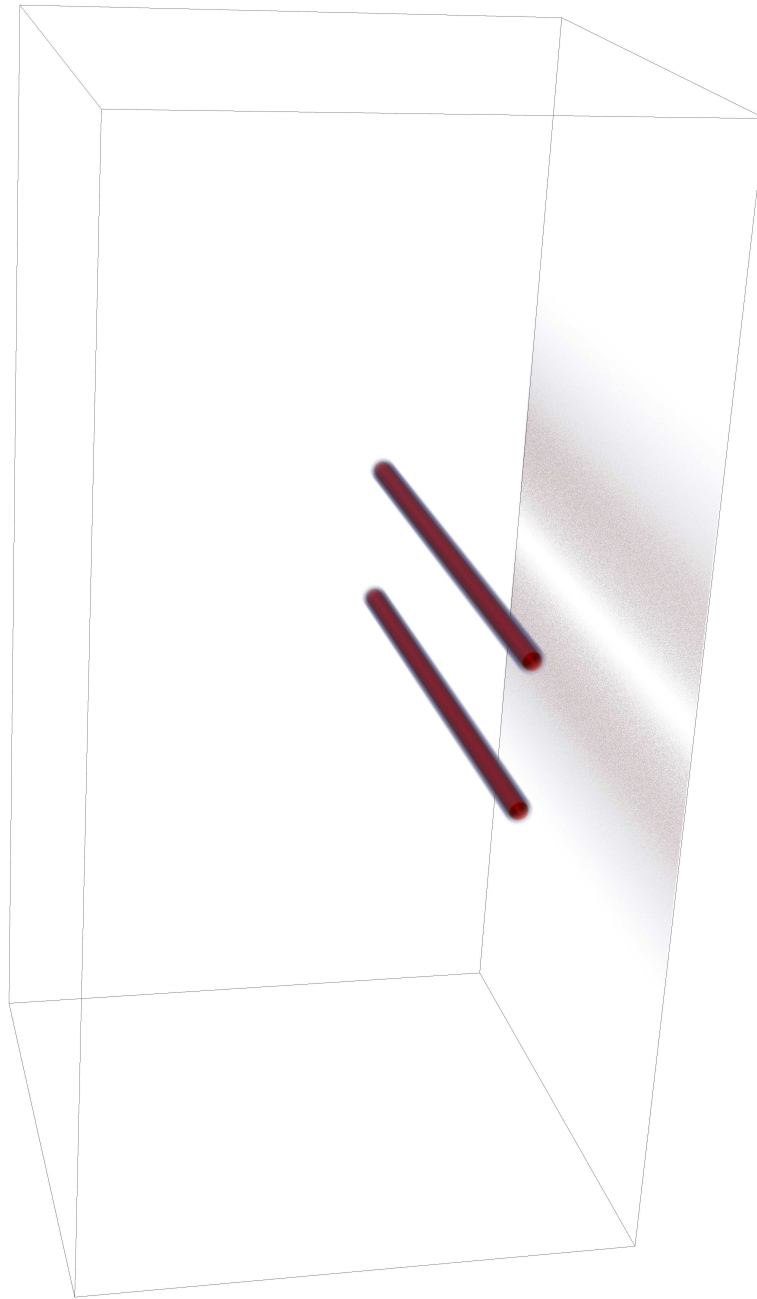


Figure B.2: Volume rendering of the vorticity magnitude of the BH-2VS at $t/t_0 = 2$.

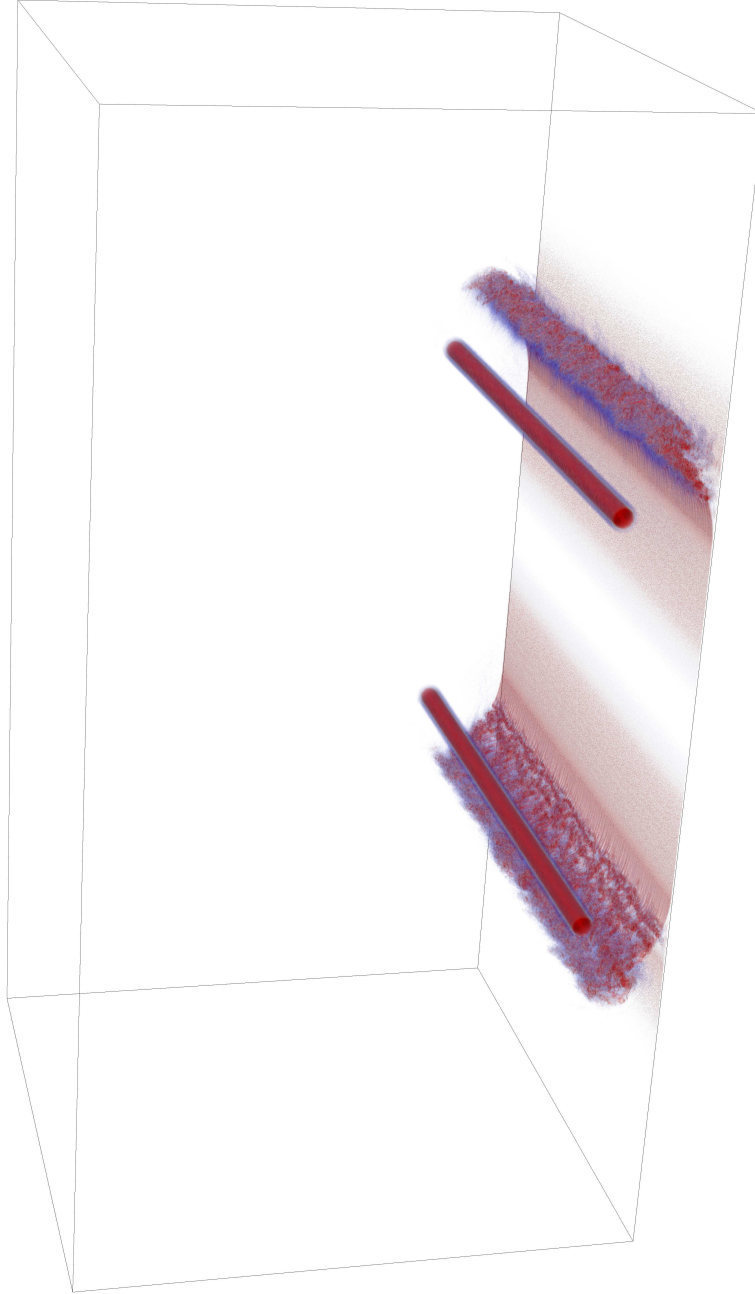


Figure B.3: Volume rendering of the vorticity magnitude of the BH-2VS at $t/t_0 = 3$.

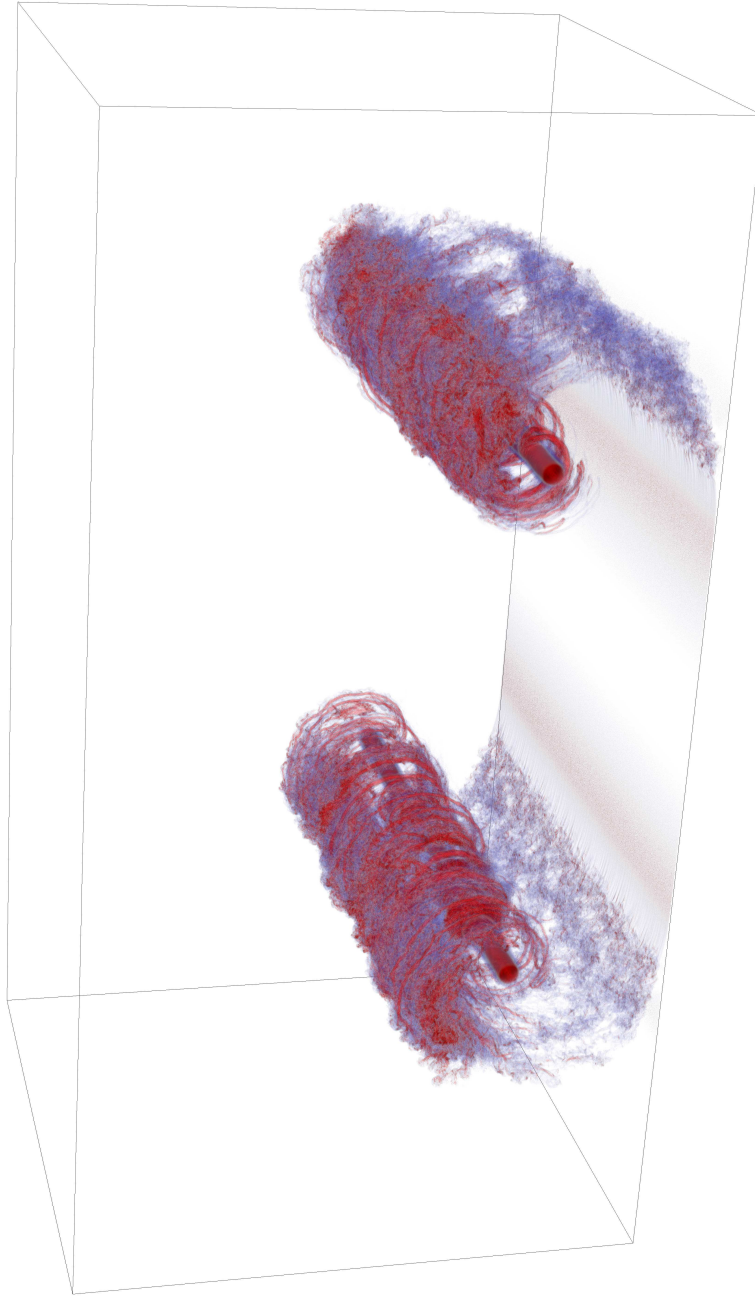


Figure B.4: Volume rendering of the vorticity magnitude of the BH-2VS at $t/t_0 = 4$.

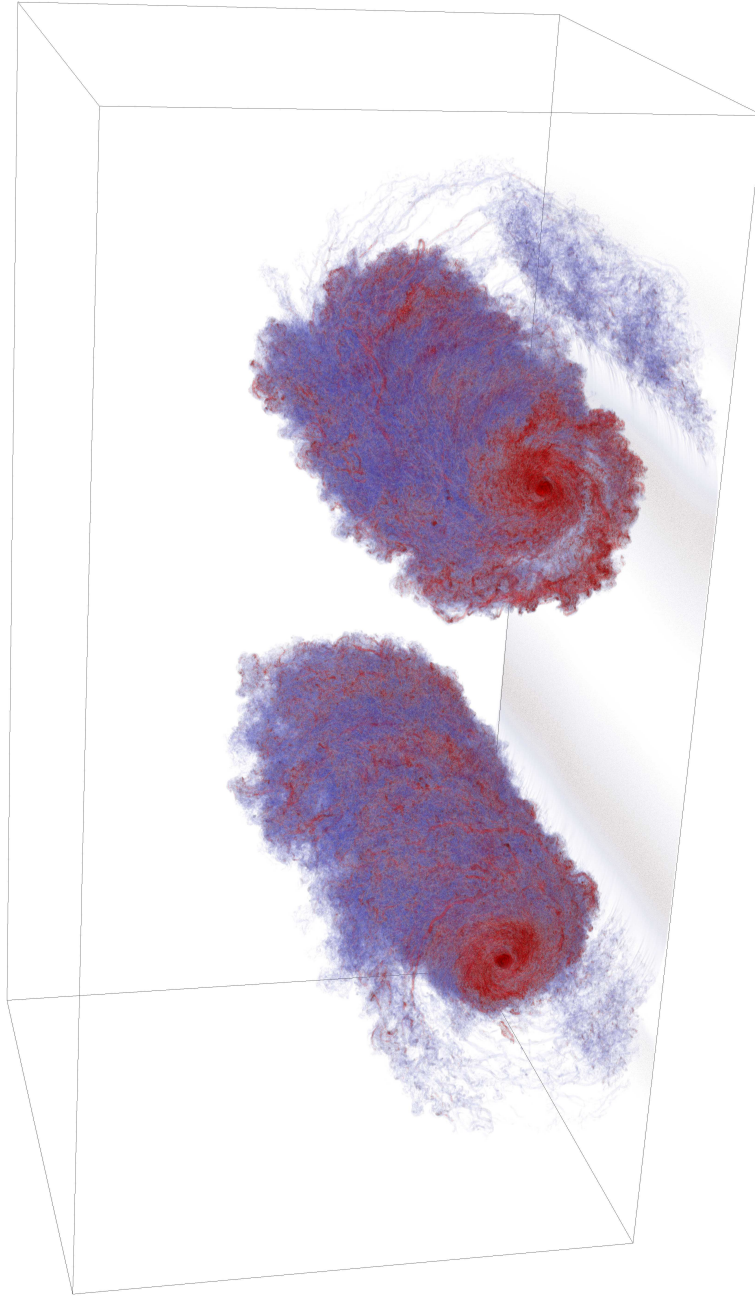


Figure B.5: Volume rendering of the vorticity magnitude of the BH-2VS at $t/t_0 = 5$.

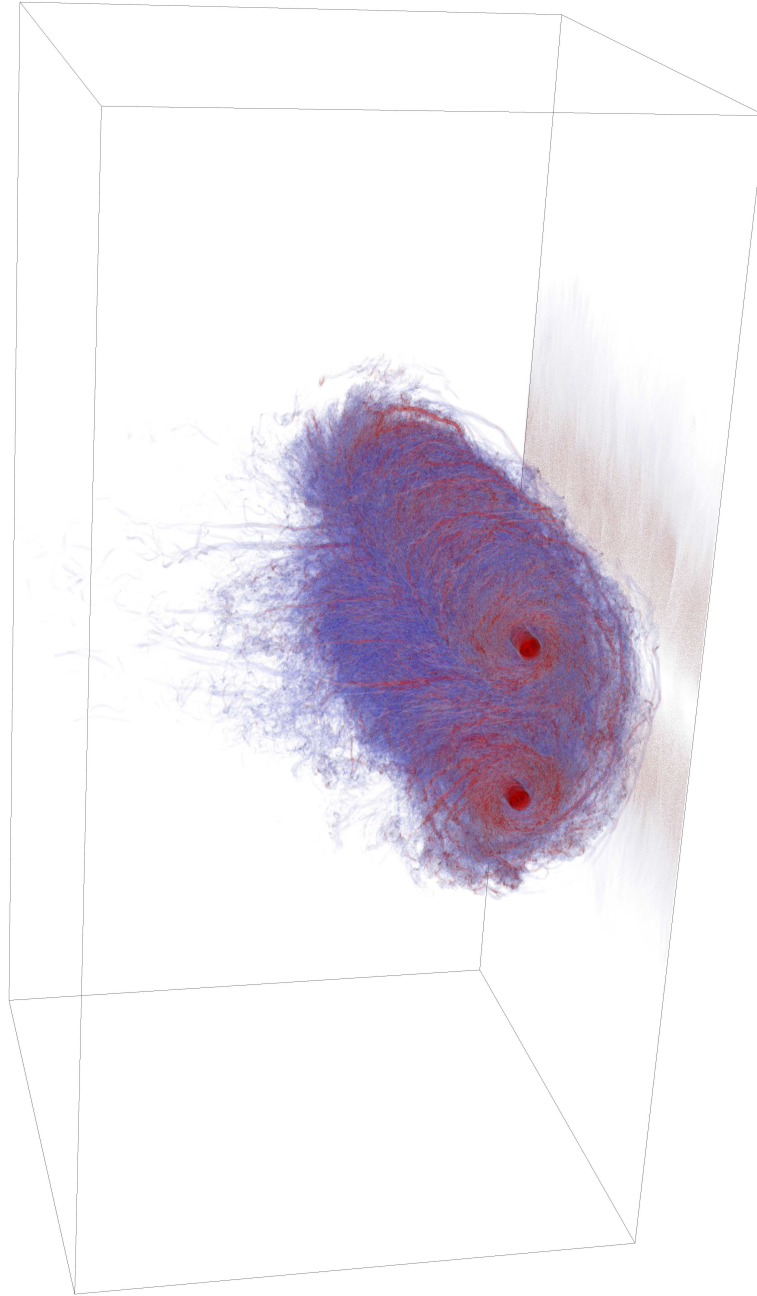


Figure B.6: Volume rendering of the vorticity magnitude of the T-2VS at $t/t_0 = 1$.

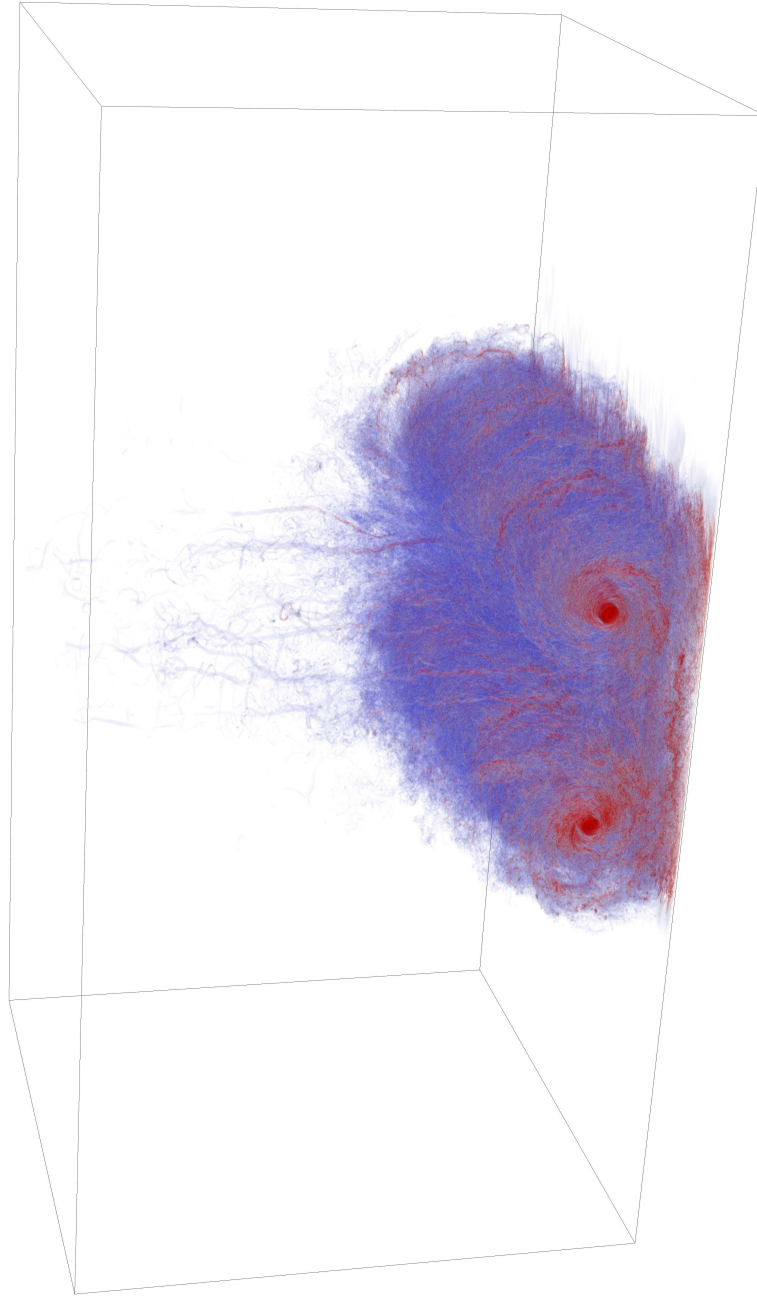


Figure B.7: Volume rendering of the vorticity magnitude of the T-2VS at $t/t_0 = 2$.

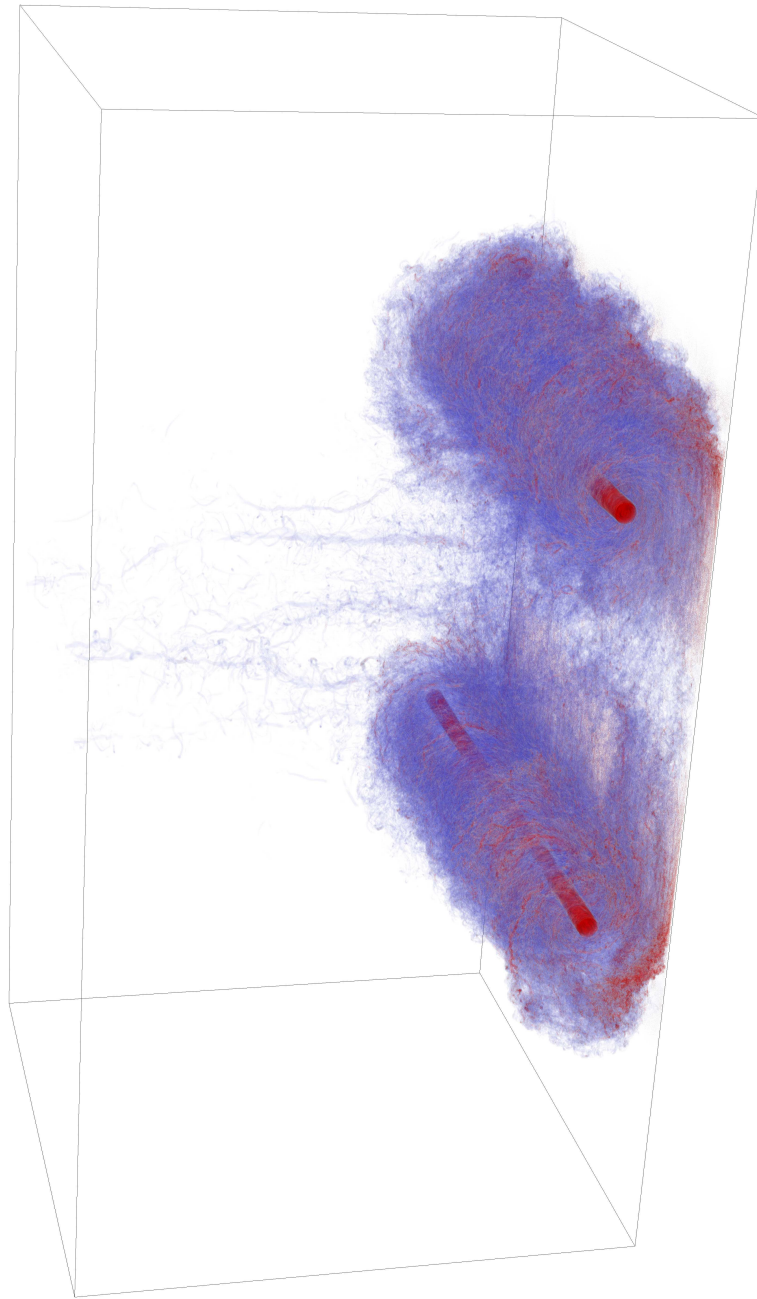


Figure B.8: Volume rendering of the vorticity magnitude of the T-2VS at $t/t_0 = 3$.

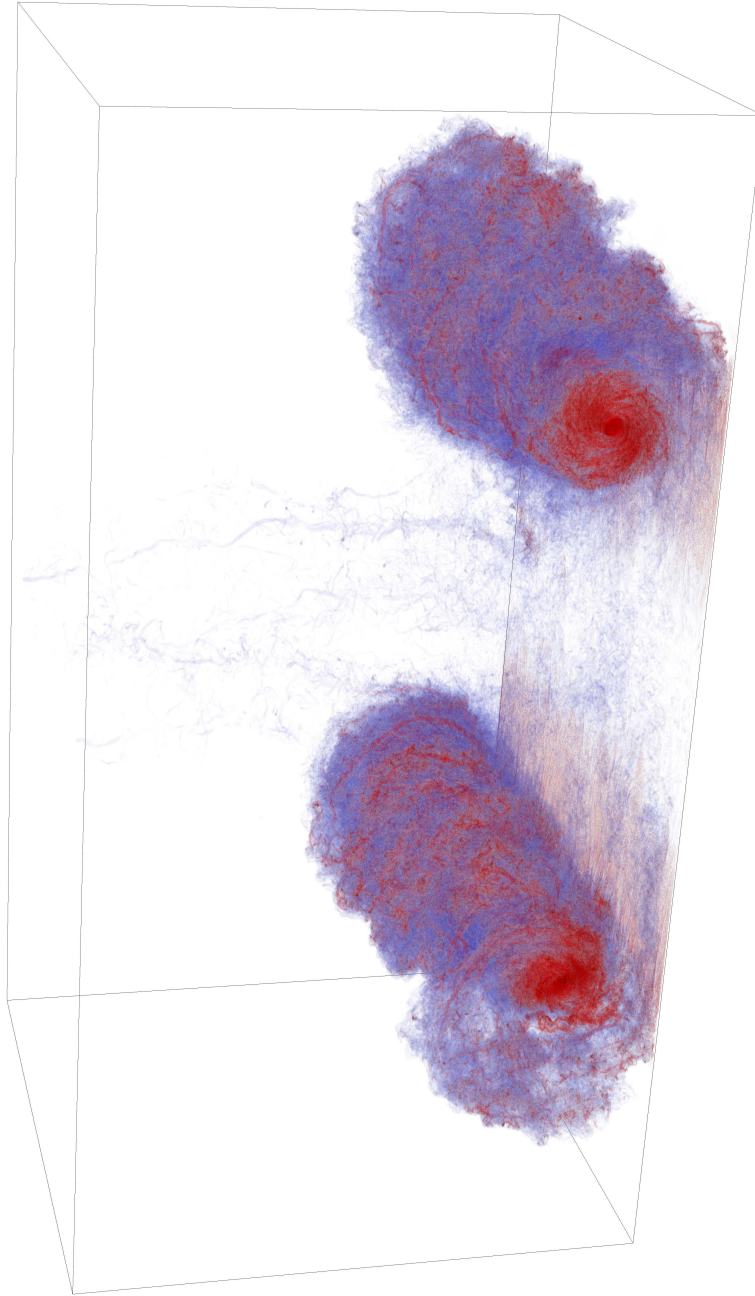


Figure B.9: Volume rendering of the vorticity magnitude of the T-2VS at $t/t_0 = 4$.

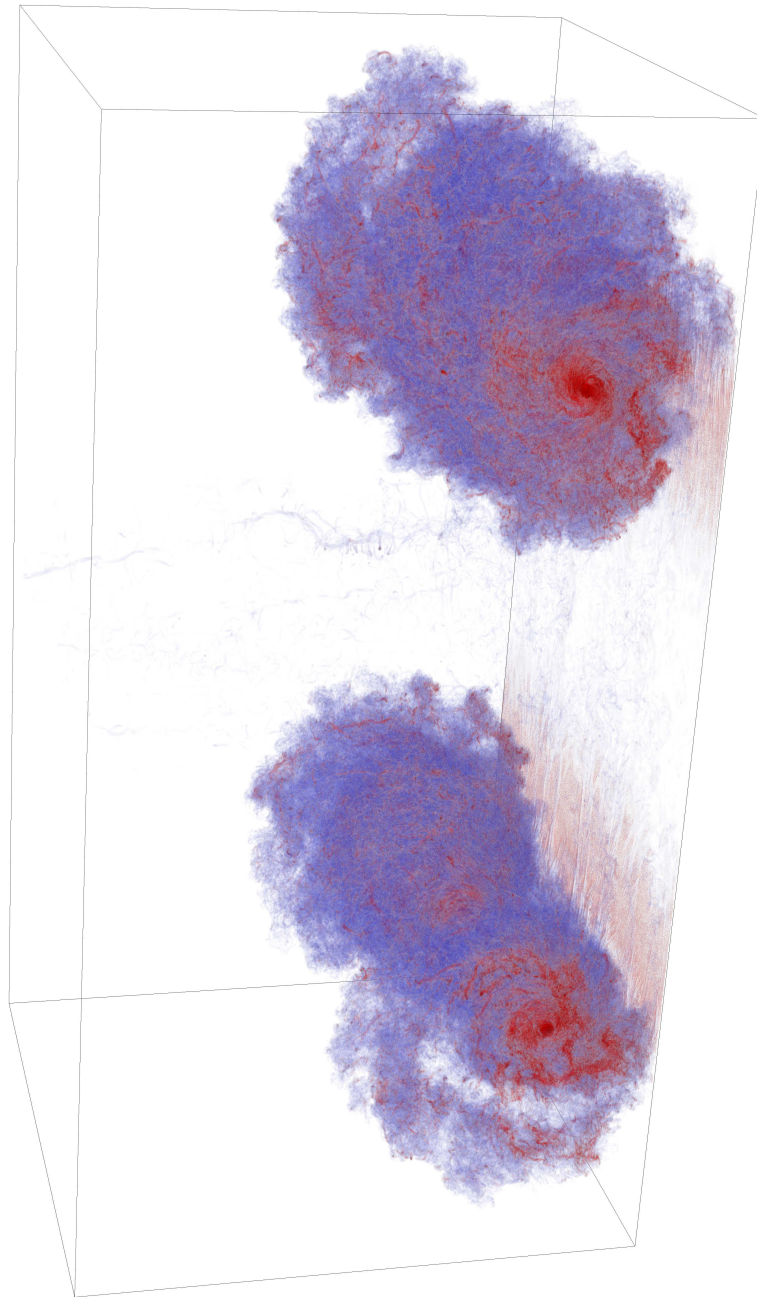


Figure B.10: Volume rendering of the vorticity magnitude of the T-2VS at $t/t_0 = 5$.

Appendix C

Discrete filters considered

Various discrete filters were used in this study. They are divided into two categories: the explicit filters and the implicit filters.

The second order explicit filter is

$$\bar{f}_i = f_i + \frac{1}{4}(f_{i-1} - 2f_i + f_{i+1}) \triangleq f_i - \underbrace{\left(-\frac{\delta^2}{4}\right)f_i}_{f_i^s}, \quad (\text{C.1})$$

where the notation $-\frac{\delta^2}{4}$ denotes the second order small scales operator. The transfer function (Fourier transform) is

$$\mathcal{G}(kh) = 1 - \frac{1}{2}(1 - \cos(kh)) = 1 - \sin^2(kh/2), \quad (\text{C.2})$$

thus with $\mathcal{G}(kh = 0) = 1$ and $\mathcal{G}(kh = \pi) = 0$. The $-\frac{\delta^2}{4}$ operator, of transfer function $\sin^2(kh/2)$, can be iterated to produce higher order filters. The transfer function of the filter of order $2n$ obtained by n iterations of the operator is then $\mathcal{G}^{(n)}(kh) = 1 - (\sin^2(kh/2))^n$. The “effective cutoff wavenumber”, k_c , here defined as $\mathcal{G}(k_ch) = 1/2$, is thus obtained as $k_ch = 2 \arcsin(1/2^{1/(2n)})$.

The implicit filters considered here are of the form:

$$\bar{f}_i + a \frac{\bar{f}_{i-1} + \bar{f}_{i+1}}{2} = bf_i + c \frac{f_{i-1} + f_{i+1}}{2} + d \frac{f_{i-2} + f_{i+2}}{2} + \dots \quad (\text{C.3})$$

Thus, they only require solving a tridiagonal system. Their transfer function

is

$$\mathcal{G}(kh) = \frac{(b + c \cos(kh) + d \cos(2kh) + \dots)}{(1 + a \cos(kh))} . \quad (\text{C.4})$$

The coefficients are determined by requiring that $\mathcal{G}(kh = 0) = 1$, $\mathcal{G}(kh = \pi) = 0$, and $\mathcal{G}(k_ch) = 1/2$ for some chosen cutoff wavenumber $k_c < \pi/h$. The remaining coefficients, if any, are determined by setting the initial derivatives of $\mathcal{G}$ to zero. The second order filters are then

$$\mathcal{G}(kh) = \frac{(1 + \alpha)(1 + \cos(kh))}{2(1 + \alpha \cos(kh))} \text{ with } \alpha = -\cos(k_ch) \quad (\text{C.5})$$

the fourth order filters are

$$\mathcal{G}(kh) = \frac{(5 + 3\alpha) + 4(1 + \alpha) \cos(kh) - (1 - \alpha) \cos(2kh)}{8(1 + \alpha \cos(kh))} \quad (\text{C.6})$$

$$\text{with } \alpha = \frac{(-1 - 4 \cos(k_ch) + \cos(2k_ch))}{(3 + \cos(2k_ch))} \quad (\text{C.7})$$

and the sixth order filters are

$$\mathcal{G}(kh) = \frac{(22 + 10\alpha) + (15 + 17\alpha) \cos(kh) - 6(1 - \alpha) \cos(2kh) + (1 - \alpha) \cos(3kh)}{32(1 + \alpha \cos(kh))} \quad (\text{C.8})$$

$$\text{with } \alpha = \frac{(6 + 15 \cos(k_ch) - 6 \cos(2k_ch) + \cos(3k_ch))}{(-10 - \cos(k_ch) - 6 \cos(2k_ch) + \cos(3k_ch))} . \quad (\text{C.9})$$

Examples of those filters are displayed in Fig. C.1.

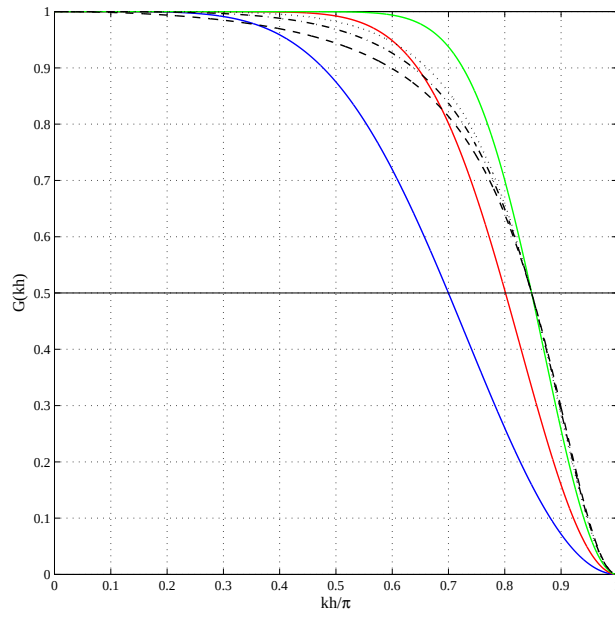


Figure C.1: Transfer function of discrete filters: explicit filters of order 6 ($n = 3$), 10 ($n = 5$) and 24 ($n = 12$) (solid); implicit filters of order 2 (dashed), 4 (dashed-dotted) and 6 (dotted), and with cutoff equivalent to that of the explicit filter of order 24 ($k_ch = 2 \arcsin(1/2^{1/24}) = 0.8477 \pi$).