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THE ECONOMETRICS OF AIRLINE NETWORK MANAGEMENT

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Abstract

The task of airline network management is to develop new flight schedule variants and evaluate them in terms of expected passenger demand and revenue. Given the industry's trend towards global cooperation, this is especially important when evaluating the potential synergies with alliance partners. From the econometric point of view, this task represents a discrete choice modeling problem in which the analyst has to account for a large number of dependent alternatives. In this paper we discuss the applicability of recently proposed approaches and introduce a new multinomial probit specification designed for the airline network management task. The superior performance of the new model is demonstrated both in a simulation study and in a real-world application using airline bookings data.

Keywords: airline industry, transportation, discrete choice models, multinomial probit model.

JEL Classification: C15, C25, L93.

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1 Introduction

The creation of worldwide alliances such as the British Airways/American Airlines joint venture "One World" and the cooperation between United Airlines, Lufthansa and other major carriers, known as "Star Alliance", emphasized the importance of the network perspective in the airline industry. Rather than perceiving a timetable to be a collection of isolated routes, airlines have realized that their schedules and those of their alliance partners represent a complex network of city-pair connections in origin and destination markets. The number of markets that can be served in a network is a multiple of the limited number of routes that can be offered by any single airline. The potential synergies that airline alliances can generate are therefore mainly caused by the significant additional number of markets that become accessible by harmonizing the alliance partners' schedules. Even before the advent of international joint ventures many carriers already increased their portfolio of served markets by processes called hubbing and banking, i.e. by scheduling incoming and outgoing flights at the airline's home airport (hub) in a way that enables them to penetrate a variety of profitable transfer markets.

In this paper we assess the applicability of the discrete choice models that have been discussed in the literature to the alliance and schedule evaluation problem. We introduce a new multinomial probit (MNP) specification that is motivated by the requirements of airline network management, but also applicable to related problems in which the analyst has to account for a large number of dependent alternatives.

The task of an airline network management department is to create and evaluate flight schedule scenarios in terms of expected passenger demand and revenue. Each schedule redesign or the creation of a joint alliance schedule generates alternative options for passengers wishing to travel from an origin to their desired destination (O&D itineraries). Transfer itineraries may invoke one or more stops, and the legs that build a feasible connection may be offered by the same (online) or different (interline) carriers. Assuming that the utility of an offered itinerary is dependent on the characteristics of the alternative, and that a customer chooses an itinerary that provides her maximum utility, a tailor-made environment is obviously available for the application of discrete choice models. For strategic decisions concerning network design and/or alliance evaluation, the estimated choice probabilities are conceived as market share estimates. Multiplying these by (externally estimated) total O&D market demand and ticket price, the network management department is able to deliver an estimate of revenue and passenger volumes implied by any given schedule scenario.

The application of discrete choice models in the airline industry is facilitated by the commercial availability of worldwide airline schedules. Using timetable information it is straightforward to construct the set of relevant itineraries from which a passenger can choose. The attributes of the alternatives can also be constructed from the schedules (e.g. elapsed time, type of aircraft). Since a large fraction of the passenger demand is recorded in computer reservation systems, the estimation of discrete choice models can be based on high quality data. Commercial software companies, e.g. SABRE decision technologies have specialized in the implementation of schedule evaluation tools that are based on discrete choice

models.

Besides the unique data availability, the requirements that a successful model has to meet are high. The number of alternatives, in terms of offered connections on an O&D market, can be quite large. Whilst the multinomial logit model (MNL) has no problems dealing with large sets of alternatives its application has to be discarded since the independence of irrelevant alternatives (IIA) assumption is violated at a critical level. In fact, the blue bus/red bus paradox, the familiar textbook example to illustrate the IIA problem, becomes a reality in airline network management: On a route with heavy competition between airlines it is often the case that the aircrafts of two carriers will leave from an airport at almost the same time for the same destination. Assuming independence of the utilities in such a situation is definitely a bad idea. Hence, a discrete choice model is required that will allow for dependence between alternatives and that can cope with a situation in which the number of alternatives can become quite large.

In this paper we will discuss the applicability of recently proposed discrete choice models for non-IIA situations and show that most approaches have to be discarded as a result of their restrictive assumptions. After formulating an adapted version of the Generalized Autoregressive (GAR) probit model advanced by Bolduc and Ben-Akiva (1991), Bolduc (1992) and Ben-Akiva and Bolduc (1996), we introduce a new MNP specification that perfectly meets the requirements of airline network management. As in Yai, Iwakura and Morichi's (1997) model our approach bears on a pure attribute based specification of the utility covariance matrix. Yai, Iwakura and Morichi's specification, however, is designed for the specific route planning problem that they investigate, limiting its extension to other fields. The advantage of our approach is that it is applicable to any discrete choice problem in which one has to account for a large number of dependent alternatives. In a simulation study and using airline bookings data, we assess the performance of the standard models and our alternative. We find that the new model clearly outperforms its competitors in terms of predictive performance and the ability to produce economically plausible results when applied to a real world schedule evaluation task.

The remainder of the paper is organized as follows. In the Section 2 we will outline the econometric formulation of the airline network management task (Section 2.1) and discuss the applicability of several models that were recently proposed in the literature (Section 2.2. and 2.3). In Section 2.4 we present an alternative specification. The results of a simulation study, designed to assess the performance of the competing approaches, are discussed in Section 3. An empirical application using airline bookings data is presented in Section 4. We conclude in Section 5.

2 Econometric models

2.1 Discrete choice modeling in airline network management

In this Section we introduce some basic notation, and outline the requirements a discrete choice model must satisfy if it is to be successfully employed for schedule and alliance evaluation.

For an O&D market d , $d = 1, \dots, D$, where D is the number of O&Ds, J_d

alternative itineraries can be constructed. $J = \sum_{d=1}^D J_d$ denotes the total number of itineraries. We are interested in estimating the probability that a passenger n , $n = 1, \dots, N$ chooses alternative i , $i = 1, \dots, J$ where $N = \sum_{d=1}^D N_d$ is the total number of individuals in the sample, and N_d is the number of passengers deciding among the alternatives in market d .

We assume that passengers who want to travel from origin O to destination D are not interested in itineraries offered for other markets.¹ To take this into account we define the function $g(n)$ that assigns an individual n to her O&D market d . Analogously, $f(i)$ is a function that assigns alternative i to the O&D market to which it belongs to. Hence, the probability that individual n chooses alternative i can only be different from zero if $g(n) = f(i)$. We assume that all individuals assigned to a specific O&D market observe the same set of alternatives. This is justified by the passenger's real-life travel decision: Computer reservation systems (CRS) display all the possible connections that are offered for an O&D market. Travelers have access to these CRS screens via travel agencies.

To be precise, we deal with the following probabilistic choice setting

$$y_{in} = \begin{cases} 1 & \text{if } u_{in} \geq u_{jn} \forall j \in \{j = 1, \dots, J \mid f(i) = f(j)\} \text{ and } f(i) = g(n) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$p_{in} = P(y_{in} = 1),$$

where y_{in} is the observed choice of the individual n and u_{in} is the utility that alternative i provides for individual n .

For simplicity, we restrict our attention to a linear utility specification

$$u_{in} = x_i' \beta + \varepsilon_{in}. \quad (2)$$

x_i is a $(K \times 1)$ vector of attributes describing alternative i that may contain alternative-specific constants and alternative-specific covariates. β is a $(K \times 1)$ parameter vector.

The models discussed below imply different specifications of the random utility ε_{in} , but share two common features. First, we assume that the vector β is identical for a subset of the O&D markets (e.g. domestic, short-haul and long-haul market). Second, linear independence between utilities of itineraries on different O&D markets is assumed,

$$\text{cov}(\varepsilon_{in}, \varepsilon_{jn}) = 0 \quad \forall f(i) \neq f(j). \quad (3)$$

In the context of airline network management the covariance matrix of ε_{in} is different from the one that is employed for the standard commuter problem where physically different alternatives are identified by logical names describing the mode of transport, e.g. bus, car or shared ride (nominal identification). In the standard commuter problem, the fact that the bus that consumer n living in region A chooses is not the same vehicle that consumer m living in region B selects does not matter. Although the two buses may possess different attribute levels, the

¹This is a simplifying assumption. It does not take into account, for instance, that an individual living between two airports can choose one as the origin of her trip if the desired destination is offered at both airports.

covariances of the utilities of each bus and car alternative and each bus and shared ride alternative respectively are assumed to be the same.

However, nominal identification is not useful for airline network management purposes. In a typical O&D market one will find a large number of itineraries possessing different attribute levels, e.g. a British Airways nonstop flight departing at 8:00 a.m. with an elapsed time of 6:30 hours, or a Lufthansa/United Airlines interline connection departing at 8:30 a.m. with an elapsed time of 8:00 hours. Although nominal identification would be straightforward, e.g. by distinguishing direct flights, online connections, interline connections etc., this is not helpful. For schedule evaluation purposes an airline is interested in the choice probability of each itinerary, especially in self-offered direct flights and online connections. This implies that it is necessary to account for a specific covariance matrix for each O&D market. Given the independence assumption (3) we have to deal with a block-diagonal, but otherwise unrestricted covariance matrix. Each schedule modification alters the set of relevant itineraries, which changes some or all of the covariance matrix blocks. A model that will be successfully applicable in airline network management must be able to cope with the obvious incidental parameter and identification problems.

2.2 Applicability of standard discrete choice models

The standard specification that has to be considered for discrete choice modeling in airline network management is the Multinomial Logit model (MNL). The MNL is based on the assumption that ε_{in} is i.i.d. and follows a Gumbel distribution. Applied to the framework outlined in the previous subsection we obtain the probability that individual n chooses alternative i :

$$p_{in} = \frac{\exp(x'_i\beta)}{\sum_{j \in N_j} \exp(x'_j\beta)}, \text{ where } N_j = \{j = 1, \dots, J \mid f(i) = f(j)\} \quad (4)$$

The MNL's computational simplicity comes at the cost of the restrictive assumption of independence of irrelevant alternatives (IIA). The IIA assumption implies that the ratio of the choice probabilities of any two alternatives does not depend on the others:

$$\frac{p_{in}}{p_{jn}} = \frac{\exp(x'_i\beta)}{\exp(x'_j\beta)} = \exp(x'_i\beta - x'_j\beta) \quad (5)$$

In airline network management the IIA assumption is violated at a critical level. As is often the case on contested routes the planes of two competitors depart at almost the same time and for the same destination. One should expect that the joint market share on the city-pair connections is lower compared to a situation where the two planes start with some hours departure time difference.²

The nested logit model (NL), the cross correlated logit model (CCL) and other models of the General Extreme Value family (GEV) have been proposed for situations, in which the IIA assumption cannot be maintained (Williams 1977, McFadden 1978, Ortuzar 1982). For a simple two-level NL model the random utility

²This argument assumes the absence of capacity restrictions

ε_{in} is divided into a part that is common to alternatives that belong to the same group and a remaining unobserved utility e_{in} :

$$\varepsilon_{in} = \varepsilon_{g(i),n} + e_{in}. \quad (6)$$

This approach can easily be extended to a multi-level model by further grouping the alternatives within a group and further dividing the error components. A generalization of the NL is the CCL model proposed by Williams and Ortuzar (1982). This approach allows for interaction terms in the covariance matrix and does not require the hierarchical structuring that has to be imposed for NL. However, the model is highly complex and inconsistent with utility maximization, as conceded by the authors.

The core problem of all tree structured models, however, is that a hierarchical structure of the decision process has to be assumed. In the case of the nests being based on continuous variables one has to cut the range of attribute values into pieces. This can lead to implausible discontinuities of choice probabilities, e.g. when changing the departure time of a flight and thereby shifting it from one nest to another. Furthermore, the IIA problem remains present on the level of the nests.

As an example, consider a simple one-level NL model where the departure weekdays are chosen as the nesting criterion. The covariance between an alternative that belongs to a specific nest (e.g. a Tuesday departure) and any other alternative that belongs to a different nest (e.g. a Monday departure or Friday departure) is equal by assumption. For the weekday nesting this is obviously a very doubtful assumption: It is much more reasonable to assume that a Tuesday departure is conceived as being more similar (in terms of unobserved utility) to a Monday departure than it is to a Friday departure.

Bhat (1997) investigates more flexible NL specifications that aim to provide a solution to this problem by introducing covariance heterogeneity between nests based on the individual's characteristics. Yet, because of the lack of information about individuals, this approach is not suitable for airline network management. The same holds true for the MNL-approach recently proposed by Ivaldi and Vignier (1999).

The Multinomial Probit model (MNP) is the natural tool to be applied to non-IIA problems. Recent work on simulation based methods has helped greatly to solve the numerical problems associated with the evaluation of the multidimensional integrals required to compute the choice probabilities. McFadden (1989), Börsch-Supan and Hajivassiliou (1993), Hajivassiliou and McFadden and Ruud (1996) have introduced simulation based methods that permit MNP modeling of choice problems with a larger number of alternatives.

Another, yet unsolved problem associated with the MNP is caused by the abundance of covariance elements that have to be accounted for if the number of alternatives is large. The identifying restrictions needed for MNP application have been extensively discussed in the literature (see Albright, Lerman and Manski (1977) Horowitz, Sparman and Daganzo (1982), Dansie (1985), Bolduc (1992), Bunch (1991), Horowitz (1991), Keane (1992)). Bunch (1991) argues that the identifying restrictions imply assumptions which are equivalent to choosing among hierarchical structures in GEV-type models mentioned above. Horowitz (1991)

and Bunch (1991) conclude that it is questionable whether the performance of MNP is superior to GEV-type models.

Horowitz (1991) has pointed to another problem associated with MNP arguing that since Covariances between new alternatives are unknown, the MNP is inadequate for forecasting purposes. Since market share forecasts for itineraries that are newly generated by schedule redesigns are inevitable in airline network management, this seems to be a devastating critique. We will show in the following that all is not lost for MNP, arguing that a sparse parameterization of the utility covariance matrix is the key to the solution.

Hausman and Wise (1978) were the first to discuss the role of modeling covariances in the MNP model, and subsequent work is built heavily on their basic ideas. In the Hausman and Wise model the utility is specified as

$$u_{in} = x'_{in}\bar{\beta} + x'_{in}\tilde{\beta}_n + e_{in}, \quad (7)$$

where $\bar{\beta}$ contains the average taste parameters and $\tilde{\beta}_n$ represents individual taste variations which are assumed to be iid. $N(0, I)$. The errors e_{in} are assumed to be i.i.d. $N(0, 1)$.

Yai, Iwakura and Morichi (1997) criticize that this approach leads to covariances that are proportional to the product of the attributes of the two alternatives. This is a crucial critique in the context of airline network management: One does not expect two itineraries to be more similar just because, for instance, they both start in the evening instead of in the morning. Yet, this is exactly what Hausman and Wise's approach would imply if we used the departure time as a covariate in (7). Based on their critique Yai, Iwakura and Morichi (1997) introduce the concept of structured covariances in MNP modeling. Their approach towards the modeling of dependencies between alternatives is closely linked to the peculiarities of the route choice problem that they analyze: The covariance of two routes is determined by their common transfer stations and the common parts of the route. Yet, this approach is a rare example of a pure attribute based specification of the covariance structure. In Section 3 we will present a more general approach that contains Yai, Iwakura and Morichi (1997) model as a special case.

2.3 The GAR-MNP adapted for airline network management

In this section we will present an adaption of the Generalized Autoregressive (GAR) MNP that has been introduced by Bolduc and Ben-Akiva (1991) and Bolduc (1992). Ben-Akiva and Bolduc (1996) have extended the concept to a general factor analytic approach, that contains GAR and other specifications as special cases. After outlining its basic idea, we adapt the GAR approach to enable its application in network management.

In Ben-Akiva and Bolduc's (1996) general factor analytic approach, the stochastic utility component is decomposed into an independent random variable ν_{in} and a covariance generating component ϑ_{in} :

$$\varepsilon_{in} = \vartheta_{in} + \nu_{in} \quad (8)$$

The error term ν_{in} is assumed to be i.i.d., either following a normal or a Gumbel

distribution. ϑ_{in} is defined via a factor structure

$$\vartheta_n = F_n \zeta_n, \quad (9)$$

where $\vartheta_n = \vartheta_{1n}, \dots, \vartheta_{L_n n}$ and L_n denotes the number of alternatives in the choice set of individual n . $\zeta_n \sim N(0, I_M)$ is a $(M \times 1)$ random vector that is i.i.d. multivariate normal, where $M \leq L_n$. F_n is a $(L_n \times M)$ matrix for which Ben-Akiva and Bolduc (1996) propose four specifications. Among these only one, the heteroscedastic specification, is suitable for airline network management. Assume

$$\vartheta_n = \rho W_n \vartheta_n + T \zeta_n. \quad (10)$$

where T is a diagonal matrix of alternative-specific standard deviations and $(-1 < \rho < 1)$. The parameter ρ accounts for the overall strength of dependence between alternatives. W_n is a $(L_n \times L_n)$ weighting matrix. Rewrite (10) as

$$\vartheta_n = (I - \rho W_n)^{-1} T \zeta_n. \quad (11)$$

Bolduc (1992) proposes the choice of the i, j 'th element of W_n as

$$w_{ij,n} = \begin{cases} \frac{w_{ij,n}^*}{\sum_{k=1}^{L_n} w_{ik,n}^*}, & \text{if } i \neq j \\ 0, & \text{otherwise} \end{cases}, \quad (12)$$

where $w_{ij,n}^*$ is inversely related to the similarity or proximity of the alternatives i and j . As proposed by Bolduc (1992), $w_{ij,n}^*$ can be defined as a Boolean matrix. This is useful for standard problems using nominal identification, but not for airline network management tasks. As an alternative, Bolduc (1992) suggested to employ a distance function like

$$w_{ij}^* = (\Delta_{ij})^{-\lambda}, \quad (13)$$

where $\lambda > 0$ and Δ_{ij} is a distance measure between i and j . In the following we will adopt these basic ideas in order to provide a GAR variant that is applicable in airline network management.

Consider the following specification for the covariance generating component ϑ_n :

$$\vartheta_n = \rho W \vartheta_n + T \zeta_n. \quad (14)$$

Since individual-specific data are not used for airline management the suffix n is dropped for the weighting matrix. Furthermore, as nominal identification has been discarded, we cannot assign specific standard deviations to the alternatives. Hence, we have $T = \sigma_0 I$. As a consequence (14) reduces to

$$\vartheta_n = \rho W \vartheta_n + \sigma_0 \zeta_n, \quad (15)$$

or, equivalently,

$$\vartheta_n = \sigma_0 (I - \rho W)^{-1} \zeta_n.$$

Conceiving Δ_{ij} in (13) as a general measure for the proximity of the alternatives in terms of one or more attributes, we propose to use the following weighting function:

$$w_{i,j}^* = \exp\left(-\sum_{p=1}^P \alpha_p |z_{i,p} - z_{j,p}|\right), \quad (16)$$

where z_i and z_j are $(P \times 1)$ vectors of attributes that account for the similarity of two alternatives.³ α_p are parameters to be estimated. The weighting function (16) is preferred to (13) because it is also defined for $z_{i,p} = z_{j,p}$.⁴ This specification yields a GAR-MNP in which the covariance matrix depends on the parameters ρ, σ_0 and $\alpha = (\alpha_1, \dots, \alpha_p)$.

2.4 The Attribute Based Covariance MNP

In this section we will introduce an alternative approach towards modeling similarities between alternatives that avoids the drawbacks of the models discussed in the previous sections and that is particularly suitable for application to the airline network management task. The basic idea is to decompose the random utility ε_{in} into error components that are related to attributes describing the alternatives. For simplicity we restrict our attention to the standard linear utility specification

$$u_{in} = x_i' \beta + \varepsilon_{in}. \quad (17)$$

As for GAR and NL, ε_{in} is composed into an i.i.d. $N(0,1)$ error ν_{in} and an attribute-dependent component ϑ_{in}

$$\varepsilon_{in} = \vartheta_{in} + \nu_{in}. \quad (18)$$

Let $z_i = (z_{i1}, z_{i2}, \dots, z_{iP})$ be a vector of attributes describing alternative i , where $z_i \subseteq x_i$. ϑ_{in} is linearly decomposed into P error components. Each of those is associated with a specific attribute in z_i

$$\vartheta_{in} = \vartheta_{in1} + \vartheta_{in2} + \dots + \vartheta_{inP}, \quad (19)$$

where ϑ_{in1} is the error associated with the attribute z_{i1} , ϑ_{in2} is associated with z_{i2} , and so on. We assume that the random utility ϑ_{ink} is linked with the level of the k 'th attribute such that $\vartheta_{ink} = \vartheta_{jnk}$ if $z_{ik} = z_{jk}$. This specification is related to the Hausman and Wise model (7) in the sense that we account for unobserved individual-specific deviations of the utility that are associated with some attributes. However, we do not assume the restrictive linear relation of the random coefficients and attribute levels as is the case in (7). In the following, we will propose three basic variants of (19) by distinguishing non-ordered, ordered categorical and continuous attributes.

Let z_i^b be a dichotomous attribute and ϑ_{in}^b the random utility that is associated with this attribute. Let $\xi_n^{b,1}$ ($\xi_n^{b,0}$) denote the random utility component that is

³Note that the block-diagonal covariance matrix implies that $w_{i,j}^*$ for $f(i) \neq f(j)$.

⁴Whilst a number of alternative weighting functions were tested 16) turned out to be the most robust.

associated with an alternative where the dichotomous variable z_i^b equals one (zero). Consider the following specification:

$$\vartheta_{in}^b = \delta_{z_i^b,0} \cdot \xi_n^{b,0} + \delta_{z_i^b,1} \cdot \xi_n^{b,1}. \quad (20)$$

$\xi_n^{b,0}$ and $\xi_n^{b,1}$ denote i.i.d. random variables, $\xi_n^{b,0} \sim N(0, \sigma_0^2)$ and $\xi_n^{b,1} \sim N(0, \sigma_1^2)$ where $cov(\xi_n^{b,0}, \xi_n^{b,1}) = 0$. $\delta_{s,t}$ is the Kronecker symbol:

$$\delta_{s,t} = \begin{cases} 1 & \text{for } s = t \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

This implies:

$$cov(\vartheta_{in}^b, \vartheta_{jn}^b) = \delta_{z_i^b,0} \delta_{z_j^b,0} \sigma_0^2 + \delta_{z_i^b,1} \delta_{z_j^b,1} \sigma_1^2 \quad (22)$$

In words, the covariance matrix is non-zero only if the two alternatives i and j take on the same level of the dichotomous attribute z_i^b .

To illustrate this specification, consider a binary indicator that equals one if the carrier that offers an itinerary is "American Airlines" and zero if not. The dummy indicator "American Airlines" is assumed to enter the systematic utility as an explanatory variable. However, since people have different experiences when travelling with the airline, it is important to account for individual specific deviations from the average utility: A passenger having experienced an enjoyable (unpleasant) flight with American Airlines will assign a higher (lower) utility to all alternatives operated by this carrier. The dichotomous case is easily extended to deal with polytomous non-ordered variables.

We now turn our attention to ordered polytomous variables. Let z_i^o denote such an attribute taking on the values $m = 1, 2, \dots, M$. We define a $(M \times 1)$ random vector $\xi_n^o = (\xi_{1n}^o, \dots, \xi_{Mn}^o)' \sim N(0, \sigma^2 I_M)'$. ξ_{mn}^o can be interpreted as the intrinsic unobserved utility deviation that is related to the attribute level m . Consider the following specification of a spatial moving average process:

$$\vartheta_{in}^o = \sum_{m=1}^M \xi_{mn}^o \cdot A(m, z_i^o; \lambda). \quad (23)$$

$A(t, s; \lambda)$ is an amplitude function weighting the intrinsic errors ξ_{mn}^o . For obvious reasons we choose an amplitude function that decreases as the distance between z_i^o and m grows. Symmetric functions such as

$$A(m, z_i^o; \lambda) = \Gamma \cdot \exp\left(-\frac{(m - z_i^o)^2}{2\lambda^2}\right) \quad (24)$$

or

$$A(m, z_i^o; \lambda) = \Gamma \cdot (1 + \lambda|m - z_i^o|)^{-2}, \quad (25)$$

where Γ is a normalization constant such that $var(\vartheta_{in}^o) = \sigma^2$, are obvious candidates, but periodic functions may also be suitable. The covariance between ϑ_{in}^o and ϑ_{jn}^o is given by

$$cov(\vartheta_{in}^o, \vartheta_{jn}^o) = \sigma_r^2 \sum_{m=1}^M A(m, z_i^o; \lambda) A(m, z_j^o; \lambda). \quad (26)$$

Details of the derivation of 26 deferred to the appendix. Choosing $A(m, z_i^o; \lambda) = \delta_{m, z_i^o}$ reduces the covariance formula (26) to (22).

To illustrate the ordered case, consider the following example: When modeling passenger choice in airline network management one has to account for weekday departure preferences. The above specification allows us to account for individual-specific deviations from the mean day of week departure preferences in the following way. For some O&D markets in which only a few itineraries per week are offered (e.g. on exotic intercontinental routes), a passenger that is familiar with this constrained supply situation may ask for a flight "in the middle of the week" rather than for a connection on his preferred departure day (e.g. Wednesday). If no connection is available on Wednesday our passenger might prefer a Tuesday or Thursday flight to a flight on Saturday as these days are still in or close enough to his or her favored departure date/time window. This can be accounted for employing the spatial MA specification (23) because the random utility component associated with the departure day of an alternative is also dependent on the intrinsic random utilities of the neighboring days.

Transferring these ideas to the continuous case is straightforward. An obvious continuous attribute that has to be considered when modeling discrete choice in airline network management is the departure time associated with an itinerary. Let z_i^c denote a continuous attribute that takes on values in an interval $I \subset \mathbb{R}$. The departure point is the white noise process $\{\xi_n^c(y), y \in I\}$ with variance σ^2 . As above, $\xi_n^c(y)$ denotes a random variable that accounts for deviations from the mean utility associated with the continuous attribute (e.g. departure time). Following the same logic as in the ordered case, we specify a continuous version of the two-sided MA process introduced in equation (23):

$$\vartheta_{in}^c = \int \xi_n^c(y) A(y, z_i^c; \lambda) dy. \quad (27)$$

In appendix A we derive that the covariance of ϑ_{in}^c and ϑ_{jn}^c can be written as:

$$\text{cov}(\vartheta_{in}^c, \vartheta_{jn}^c) = \sigma^2 \int A(y, z_i^c; \lambda) A(y, z_j^c; \lambda) dy. \quad (28)$$

In the following we will use the "normal" weighting function for the two-sided MA process:

$$A(y, z; \lambda) = \sqrt{\frac{2}{\pi}} \frac{1}{\lambda} \exp\left(-\frac{(y - z)^2}{\lambda^2}\right) \quad (29)$$

σ^2 is the amplitude and λ denotes the width of the weighting function. By choosing this weighting function we ensure that $\text{var}(\vartheta_{in}^c) = \sigma^2$. Straightforward algebra yields:

$$\text{cov}(\vartheta_{in}^c, \vartheta_{jn}^c) = \sigma^2 \cdot \exp\left(-\frac{(z_j^c - z_i^c)^2}{2\lambda^2}\right), \quad (30)$$

σ^2 and λ are additional model parameters that have to be estimated. The dichotomous, ordered and continuous specifications can easily be combined. It is only natural to refer to our model as the Attribute Based Covariance-MNP (ABC-MNP). The ABC-MNP is able to defy Bunch's (1991) and Horowitz' (1991) above

mentioned MNP critique in two ways: First, the model does not require a-priori decisions about the hierarchy of the decision process, and is parameterized parsimoniously enough to be empirically tractable. Second, the inclusion of new alternatives poses no problems as long as their attributes are known in advance.

3 Simulation study

In this Section we present the results of a Monte Carlo Study that is designed to compare the performance of the models discussed in the previous Section. The simulated data generating process mimics the passenger behavior that one has to account for in airline network management. For reasons of computational tractability we only consider two O&D markets, i.e. $D = 2$. On each market 10 itineraries are offered which are chosen to represent one day of the week of two typical O&D markets by allowing for a higher density of connections in the morning and in the evening. In each market both nonstop and transfer connections are offered. Table 1 shows the resulting itineraries. O&D market 1 contains some flights that leave almost simultaneously (alternative 1 and 2 or alternatives 3, 4 and 5). In market two the itineraries are more evenly spread throughout the day.

insert table 1 about here

In order to model passenger choice we assume a simplified decision process. Only one attribute is assumed to enter the systematic part of the utility. This variable is one for all simulated nonstop itineraries and zero for all connections. The utility coefficient β is set to one and is assumed to be equal for the two markets. The simulated data generating processes (DGP) produce random utilities that are distributed multivariate normal with covariance matrix Σ that assumes the idiosyncratic block diagonal structure (3),

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix}. \quad (31)$$

With DGP 1 being the sole exception, the covariance of two alternatives depends on their proximity in terms of their departure times t_i and t_j . The covariance generating functions are specified as follows:

DGP 1 (independent): $\sigma_{ij} = \delta_{ij}$

DGP 2 (normal): $\sigma_{ij} = \phi \exp\left(-\frac{(t_i - t_j)^2}{2\gamma^2}\right) + \delta_{ij}$

DGP 3 (triangular): $\sigma_{ij} = \begin{cases} \phi \left(1 - \frac{|t_i - t_j|}{\gamma}\right) + \delta_{ij} & \text{if } |t_i - t_j| \leq \gamma \\ 0 & \text{otherwise} \end{cases}$

DGP 4 (rectangular): $\sigma_{ij} = \begin{cases} \phi + \delta_{ij} & \text{if } |t_i - t_j| \leq \gamma \\ 0 & \text{otherwise} \end{cases}$

DGP 5 (exponential): $\sigma_{ij} = \phi \exp\left(-\frac{|t_i - t_j|}{\gamma}\right) + \delta_{ij}$

where δ_{ij} is the Kronecker symbol. The parameters γ and ϕ are chosen such that positive definiteness of Σ is ensured. For DGPs 2, 3, 4 and 5 ϕ is set to 4. The parameter γ is set to 50 for the normal, 120 for the rectangular, 150 for the

triangular and 30 for the negative exponential function. Figure 1 depicts the implied covariance of two alternatives with respect to departure time differences. With the exception of DGP 1, the covariance between two alternatives decreases with increasing departure time differences.

insert figure 1 about here

Multivariate normal random variables that are needed to compute the stochastic utilities are generated using a Gibbs sampling procedure.⁵ In 100 replications of the process we simulate the random utilities of 1000 passengers for each O&D market. Using (1), we then calculate the itinerary market shares. Having obtained the simulated passenger distribution, we estimate a standard Multinomial Logit, an independent MNP, the adapted GAR-MNP, and the ABC-MNP.

For the GAR-MNP we use the Logit Kernel estimator (LKE) that assumes a Gumbel distribution of ν_{in} in equation (5). 1000 draws are used for the LKE. The independent and the ABC-MNP employ the Geweke, Hajivassiliou-Keane (GHK) simulator with 20 replications and 100 realizations.

For each model, the estimated choice probabilities (=estimated market shares) are compared with the simulated market shares. To measure the forecasting accuracy, we compute the root mean squared error, $RMSE = \sqrt{\frac{1}{20 \cdot 100} \sum_{r=1}^{100} \sum_{i=1}^{20} (\hat{p}_{ir} - p_{ir})^2}$, and the mean absolute error $MAE = \frac{1}{20 \cdot 100} \sum_{r=1}^{100} \sum_{i=1}^{20} |\hat{p}_{ir} - p_{ir}|$, where $\hat{p}_{ir}$ is the estimated choice probability of itinerary i in replication r and p_{ir} the simulated market share of itinerary i in replication r . Figure 2 depicts kernel densities that illustrate the distribution of the forecasting errors for each DGP and model. Table 2 contains the corresponding numerical results.

insert figure 2 about here

insert table 2 about here

Table 2 shows that for DGP 2 to DGP 5 the ABC-MNP outperforms the other approaches in terms of predictive accuracy. The kernel estimates in figure 3 underline this result graphically. This is an expected result for DGP 2 and DGP 3, since these DGPs correspond closely to the ABC-MNP. However, the performance of the ABC-MNP is also outstanding for the DGP 4 and 5 for which this is not the case. Both the GAR- and ABC-MNP are able to adapt easily to a DGP that implies independent utilities.⁶

⁵The Gibbs sampling procedure is based on a Markov chain that utilizes univariate truncated normal densities to construct conditional variates and has the truncated multivariate normal as its limiting distribution (Hajivassiliou, 1992)

⁶For the sake of brevity, we do not compare the efficiency and bias of the estimation of the utility parameter β . However, one result is noteworthy and is important for the interpretation of the estimation results presented in the empirical Section. For the DGP 2, the average estimated utility parameter produced by the independent MNP is 0.40 (standard deviation 0.03). The average ABC-MNP estimated β is 0.97 (standard deviation: 0.21). At first sight one could conceive this as a confirmation of the hypothesis that neglecting dependencies between alternatives will lead to biased utility parameter estimates. Note however, that this deviation is also due to the identifying restrictions that are necessary for the independent MNP formulation: When

4 Empirical Application

4.1 Data

The source for airline demand data are four major computer reservation systems (CRS).⁷ CRS data contain each booking as a separate record, but the origin and destination of each itinerary have to be constructed from the raw data. Standard industry rules were applied to identify an origin and destination of a trip. These rules encompass the maximum allowed stay at an airport that is required to distinguish the destination of a trip from a mere transfer stay. Identical O&D itineraries are identified across the CRS and then consolidated in order to compute the total passenger demand. It is important to note that CRS data do not account for flown but only for booked passengers. The difference is caused by so called no-shows - people that book multiple flights in order to ensure their booking, but do not show up - and so called go-shows, passengers that buy their tickets directly and whose bookings are not recorded in the CRS. If one uses (flown) ticket data instead, these problems would be circumvented. However, airlines have only access to their own ticket data and not to those of their competitors. The main advantage of using CRS data is that CRS data include bookings for all carriers.

The network management department of Austrian Airlines provided booking data from the major CRS operating systems Amadeus, Galileo, Sabre and Worldspan, covering the period of October 1 to October 31, 1996. To ensure homogeneity of the markets we focussed on a subset of German domestic O&D markets. The first selection criterion required that Lufthansa (official two letter code LH) and Deutsche BA (official two letter code DI) offered nonstop or connecting flights at least twice a day. In order to reduce computational needs, we restricted our attention to Monday departures that were consolidated, so that they represent a standard weekday. Changes in flight numbers during the four week period were accounted for, and marketing flights were mapped to their corresponding operating flights.⁸ The selection criteria resulted in 10 O&D markets with 229 itineraries attracting non-zero passenger demand. The largest (smallest) number of alternatives in an O&D market was 36 (5). 142 alternatives were nonstop or online connections offered by Lufthansa, 68 alternatives were nonstop or online connections operated by Deutsche BA. The remaining alternatives are direct, online or interline connections offered by other carriers. The total number of booked passengers amounts to 32,246.

4.2 Model specification

The set of explanatory variables that enter the systematic part of the utility function is a standard specification in airline network management. As outlined

estimating the independent MNP we restrict the random utility variances for each alternative to unity. DGP 2, however, generates a random utility variance (homoskedastic) which is five. The independent MNP accommodates to the identifying variance restrictions by reducing the utility parameters.

⁷Major CRS operators are Amadeus, Apollo, Galileo, Sabre, Worldspan.

⁸For marketing reasons, operating flights can be sold under two or more flight numbers (code-sharing). The additional non-operating entries in the schedule are referred to as marketing flights.

above the focus of airline network management is on attributes that describe the itineraries offered. A nonstop connection will *ceteris paribus* provide a higher utility than an itinerary which requires that the passenger changes planes once or twice. In addition, not having to change planes leads to a shorter elapsed time and a higher utility. Another important factor is the role that airline image plays in passenger choice.

Accounting for departure time preferences is a crucial issue when modeling airline passenger demand. In the process of schedule redesign, a flight is often reallocated within the day. Hence, it is important to account for the preferences that are associated with a specific departure time. A departure in the middle of the day is typically inconvenient for business travellers. Early morning and late afternoon fits their schedules better. We advocate a Fourier series approach to model the intra-day pattern (diurnality) of departure time preferences which requires only a few additive terms to fit a meaningful diurnality function,

$$F_Q(t_i; \gamma, \phi) = \sum_{q=1}^Q \gamma_q \sin\left(\frac{2\pi q}{T} t_i + \phi_q\right). \quad (32)$$

t_i denotes departure time, and $\gamma = (\gamma_1, \dots, \gamma_Q)'$ and $\phi = (\phi_1, \dots, \phi_Q)'$ are unknown parameters. T is the maximum departure time of day in minutes (1440).

For all of the estimated models the same specification for the systematic utility is employed. Write the basic specification (1) as $u_{in} = v_i + \varepsilon_{in}$, where $v_i = x_i' \beta$, then we have

$$v_i = \beta_1 \cdot N_i + \beta_2 \cdot E_i + \beta_3 \cdot LH_i + \beta_4 \cdot DI_i + F_3(t_i; \gamma_1, \gamma_2, \gamma_3, \phi_1, \phi_2, \phi_3). \quad (33)$$

E_i denotes the elapsed time if the itinerary i is a nonstop connection, and is zero otherwise. The binary indicator N_i is one for a nonstop connection and zero if otherwise.

For the GAR-MNP

$$w_{i,j}^* = \exp\left(-\left(\alpha_1 \cdot \frac{|t_i - t_j|}{100} + \alpha_2 \cdot |LH_i - LH_j|\right)\right) \quad (34)$$

is used as the weighting function. For the ABC-MNP the covariance generating component is

$$\vartheta_{in} = \int_T \xi_n^c(y) A(y, t_i; \lambda_1) dy + \delta_{LH_i, 0} \cdot \xi_n^{b,0} + \delta_{LH_i, 1} \cdot \xi_n^{b,1}, \quad (35)$$

where $\text{var}(\xi_n^c(t_i)) = \sigma_1^2$ and $\text{var}(\xi_n^{b,0}) = \text{var}(\xi_n^{b,1}) = \sigma_2^2$. This implies that

$$\text{cov}(u_{in}, u_{jn}) = \sigma_1^2 \exp\left(\frac{(t_i - t_j)^2}{2\lambda_1^2}\right) + \delta_{LH, DI} \sigma_2^2. \quad (36)$$

4.3 Estimation results

The maximum likelihood estimation results for the MNL, Independent MNP, GAR-MNP and ABC-MNP are contained in table 3. For the GAR- and ABC-MNP the configuration of the LKE and GHK simulators are the same as in Section 3.

insert table 3 about here

For all specifications the estimates of the parameters that appear in the systematic utility equation (33) have the expected sign and small standard deviations.⁹ The shape of the estimated departure time preference function is as is expected of a typical Monday: There is a high utility associated with an early morning departure and a utility peak in the evening (see figure 3). It is idiosyncratic that the morning peak is higher on Mondays, whereas for Friday departures one would obtain the inverse shape.

insert figure 3 about here

The model parameters that have to take on positive values in order to generate non-zero covariances between alternatives are ρ and σ_0 in the GAR- and σ_1^2 and σ_2^2 in the ABC-MNP. All of these parameters are different from zero at 1% significance. Table 3 also reports the passenger weighted RMSE and the MAE of an in-sample market share forecast. Both the RMSE and MAE are improved by the GAR- and the ABC-MNP which underlines the benefit of accounting for dependencies between utilities. The modified Likelihood Ratio statistics (Horowitz, 1983) in table 4 indicate that both independent MNP and GAR-MNP are rejected in favor of the ABC-MNP.

insert table 4 about here

Improving the predictive performance is certainly an asset of the two newer models, but even more important for the application in network management is the behaviour of the models when employed for schedule scenario evaluation. To test this, we focus on on a single market in our data. The offered connections on this market are described in table 5.

insert table 5 about here

We now investigate the hypothetical situation that Deutsche British Airways network management decides to introduce a new flight which departs at the same time as the nonstop Lufthansa flight at 11:05 a.m. (itinerary number 9). What is the effect of introducing the new connection on the market shares on the Lufthansa flight and the other itineraries? From our derivations in Section 2 we have learned that the GAR-MNP covariance matrix is completely altered when a new alternative is introduced, whereas for the ABC-MNP only a new row and column is inserted. Tables 6, 7 and 8 contain the implied covariance matrices before and after the introduction of the new flight for the ABC-MNP and the GAR-MNP.

insert table 6 about here

insert table 7 about here

⁹The differences in the magnitude of the utility parameters between the I-MNP and ABC-MNP are expected. See Section 3.

insert table 8 about here

We use the parameter estimates of table 3 and estimate the relative change in choice probabilities that are induced by the introduction of the new flight. Figure 4 depicts the outcome for the MNL, GAR-MNP and the ABC-MNP.

insert figure 4 about here

The economically implausible result produced by the MNL is an inevitable consequence of the IIA assumption which implies that the relative change of choice probabilities is identical for all alternatives. By contrast, the ABC-MNP result is much more plausible, since the Lufthansa flight at which the Deutsche BA initiative is aimed, indeed suffers the largest relative reduction in choice probability. Connections with departure times close to 11:05 also experience a greater relative reduction of choice probabilities, but the choice probabilities of early morning or late evening departures are hardly affected at all. In contrast, the GAR-MNP result is very different. The market share of the Lufthansa flight at 11:05 is actually increased after the introduction of the Deutsche BA flight. Tables 6 and 7 reveal that the variance of itinerary number 9 has increased. This variance increase leads, *ceteris paribus*, to a higher choice probability that clearly more than offsets the negative effect on choice probability that is exerted by the non-zero covariance of the Lufthansa 11:05 flight and the newly introduced alternative. One reason for this strange result is that we do not account for alternative-specific error variances in our version of the GAR-MNP. Although this greatly reduces the flexibility of the original GAR-approach, the restriction is inevitable when the model has to be adapted for use in airline network management.

5 Conclusions and outlook

With the advent of large international alliances, the network perspective has become even more important in the airline industry. Alliance flight schedules are conceived as a complex network of connections that are offered to passengers who want to travel from origins to desired destinations. For the quantification of alliance synergies or the evaluation of schedule redesigns, econometric discrete choice models play a prominent role which is supported by the availability of high quality data: In the airline industry passenger demand data can be obtained from computer reservation systems, and the commercial availability of worldwide schedule data makes it possible to obtain an exact view of the complete supply side.

Three idiosyncrasies of econometric modeling in airline network management have been emphasized. First, the the independence of alternatives problem is omnipresent: In a competitive market it is likely that the planes of two carriers will start at almost the same time to the same destination. Secondly, nominal identification has to be discarded, since it does not suffice to estimate aggregated choice probabilities, e.g. of all nonstop, interline and online connections. Instead, the airline network management department has to deliver choice probabilities on the level of offered connections. This implies that the number of alternatives that has to be taken into account is large. Third, individual (i.e. passenger) characteristics

are typically not available. Instead, the focus is on schedule related attributes (elapsed time, departure time, etc.) that determine individual utilities. These attributes represent the strategic instruments of an airline to attract passenger demand.

In this paper we have introduced a MNP model that perfectly meets the requirements of airline network management. Beyond that it is suitable for discrete choice modeling in non-IIA situations in which the analyst has to account for a large number of alternatives, and where the focus is on using attribute related covariates. We refer to the specification as Attribute Based Covariance-MNP since we allow for random utility deviations that are associated with the attributes of alternatives. The ABC-MNP adopts elements both from random coefficient models and the Generalized Autoregressive-MNP.

In a simulation study and empirical application using airline bookings data we have shown the ABC-MNP's practical applicability, and demonstrated its superior performance compared to its competitors. However, despite its proven advantages, the ABC-MNP still implies one major drawback. Compared to GEV models and the GAR-MNP, the parameter estimation is much more computer intensive. As a remedy, we intend to apply the method of simulated scores (MSS) as proposed by Hajivassiliou and McFadden (1998). We expect that MSS will provide a significant reduction of the computational burden.

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A Covariances in the ABC-MNP: Ordered and continuous attributes

Let $z_i^o = 1, \dots, M$ be an ordered polytomous attribute of an alternative i and ϑ_i^o be the error component related to that attribute (we drop the index n for the sake of brevity of notation). ϑ_i^o is defined as

$$\vartheta_i^o = \sum_{m=1}^M \xi_m^o \cdot A(m, z_i^o; \lambda),$$

where $A(m, z_i^o; \lambda)$ is an amplitude function and ξ_m^o a random variable representing a white noise process. We have

$$\begin{aligned} E(\xi_i^o) &= 0 \\ E(\xi_i^o \xi_j^o) &= \sigma^2 \cdot \delta_{i,j} \end{aligned} \quad (\text{A.1})$$

In order to derive (26) we use:

$$\text{cov}\left(\sum_i \xi_i^o, \sum_j \xi_j^o\right) = \sum_{i,j} \text{cov}(\xi_i^o, \xi_j^o) \quad (\text{A.2})$$

$$\text{cov}(c \cdot \xi_i^o, \xi_j^o) = \varsigma \cdot \text{cov}(\xi_i^o, \xi_j^o) \quad (\text{A.3})$$

$$\sum_m \delta_{m,m'} \cdot A(m, m') = A(m, m), \quad (\text{A.4})$$

where ς denotes an arbitrary constant and $\delta_{m,m'}$ the Kronecker symbol. We have

$$\text{cov}(\xi_i^o, \xi_j^o) = E(\xi_i^o \xi_j^o) - E(\xi_i^o)E(\xi_j^o) = \sigma^2 \cdot \delta_{i,j} - 0 \cdot 0 = \sigma^2 \cdot \delta_{i,j} \quad (\text{A.5})$$

$$\begin{aligned} \text{cov}(\xi_i^o, \xi_j^o) &= \text{cov}\left(\sum_m \xi_m^o A(m, z_i^o; \lambda), \sum_{m'} \xi_{m'}^o A(m', z_j^o)\right) \\ &= \sum_m \sum_{m'} \text{cov}(\xi_m^o A(m, z_i^o; \lambda), \xi_{m'}^o A(m', z_j^o)) \quad \text{using A.2} \\ &= \sum_m \sum_{m'} A(m, z_i^o; \lambda) A(m', z_j^o) \text{cov}(\xi_m^o, \xi_{m'}^o) \quad \text{using A.3} \\ &= \sum_m \sum_{m'} A(m, z_i^o; \lambda) A(m', z_j^o) \sigma^2 \delta_{m,m'} \quad \text{using A.5} \\ &= \sigma^2 \sum_m A(m, z_i^o; \lambda) A(m, z_j^o) \quad \text{using A.4} \end{aligned}$$

In the following we derive the covariance of the random utility $\vartheta^c(y)$ and $\vartheta^c(z)$ with $y, z \in I$. $\vartheta^c(y)$ is defined in a similar way to equation (27), but we have dropped the subscript i , since only the level of the variable y is needed, i.e. consider $y = x_i^c, z = x_j^c$:

$$\vartheta^c(y) = \int \xi^c(z) A(y, z) dz \quad (\text{A.6})$$

$\{\xi^c(y), y \in I\}$ can be described using Dirac's delta function $\delta(y - z)$:

$$\begin{aligned} E(\xi^c(y)) &= 0 \\ E(\xi^c(y), \xi^c(z)) &= \sigma^2 \cdot \delta(y - z) \end{aligned} \quad (\text{A.7})$$

We will make use of the properties of the Delta function:

$$\int dy \int dz F(y) F(z) \delta(y - z) = \int dy F(y)^2. \quad (\text{A.8})$$

where $F(y)$ is an arbitrary function of y .

By changing the order of integration for the calculation of the expectation and the integral defined in (A.6), we have

$$\begin{aligned} E(\vartheta^c(y)) &= E\left(\int \xi^c(z) A(y, z) dz\right) \\ &= \int E(\xi^c(z)) A(y, z) dz \\ &= \int 0 \cdot A(y, z) dz \\ &= 0 \\ \\ E(\vartheta^c(y) \vartheta^c(s)) &= E\left(\int \xi^c(z) A(y, z) dz \int \xi^c(s') A(s, s') ds'\right) \\ &= \int \int dz ds' A(s, s') A(y, z) E(\xi^c(s') \xi^c(z)) \\ &= \int \int dz ds' A(s, s') A(y, z) \sigma^2 \delta(s' - z) \\ &= \sigma^2 \int dz A(s, z) A(y, z) \\ \\ cov(\vartheta^c(y), \vartheta^c(s)) &= \sigma^2 \int dz A(s, z) A(y, z). \end{aligned} \quad (\text{A.9})$$

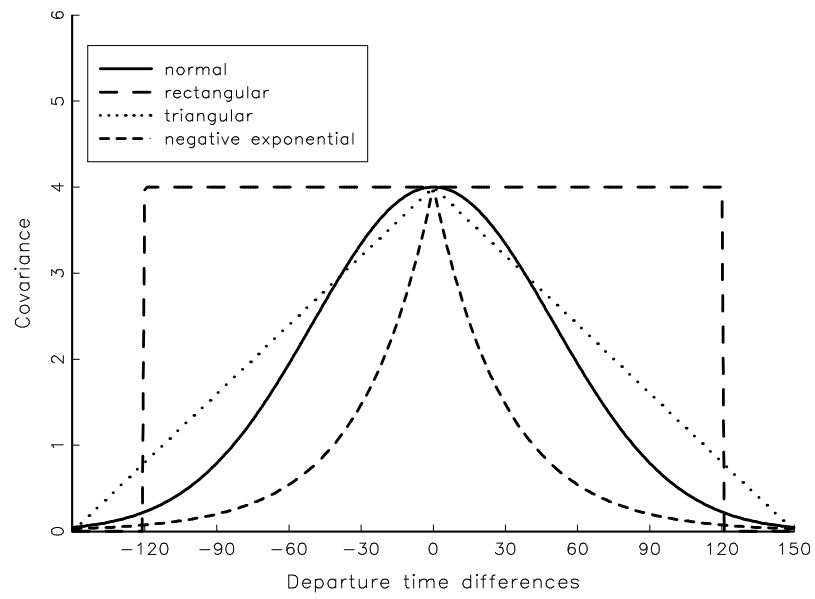


FIGURE 1.— Departure time difference dependent covariances

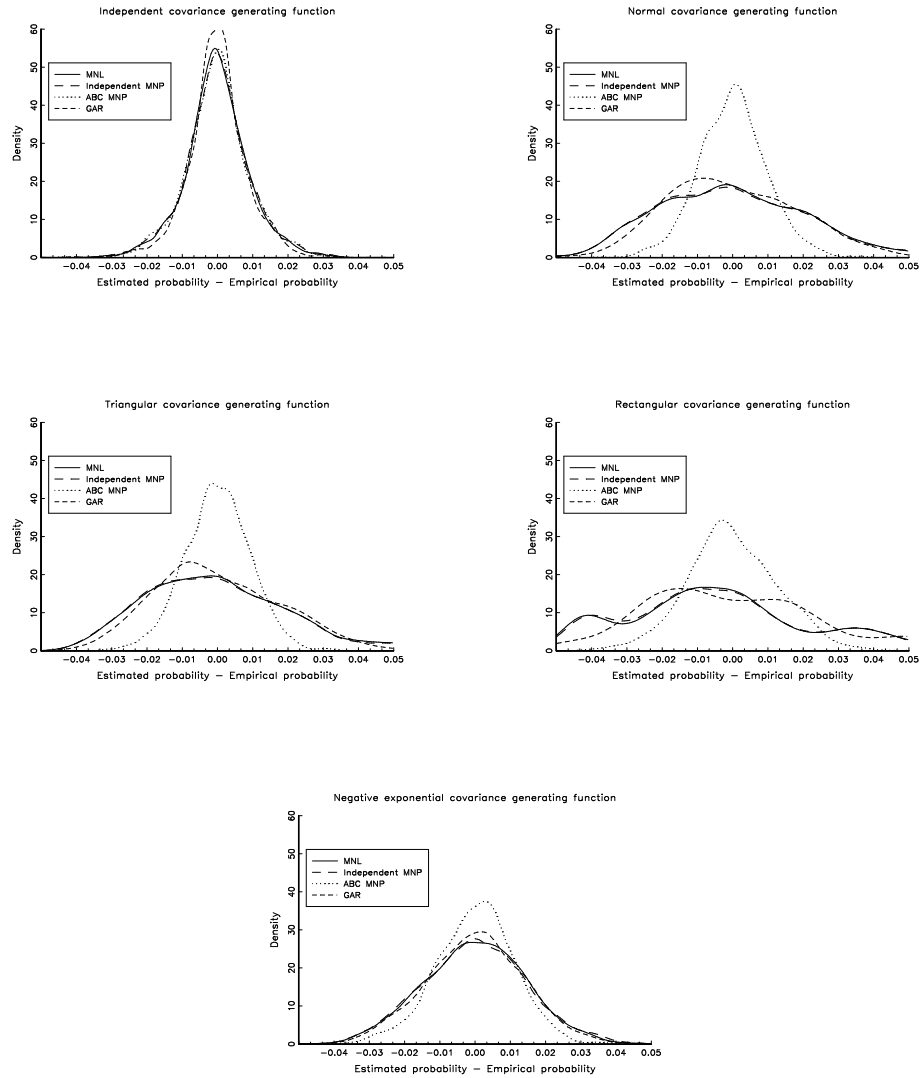


FIGURE 2.— Monte Carlo Results^a: Density plots for difference between actual vs. forecasted market shares

^a Note: Gaussian kernels with bandwidth as proposed by Silverman (1986) p. 48.

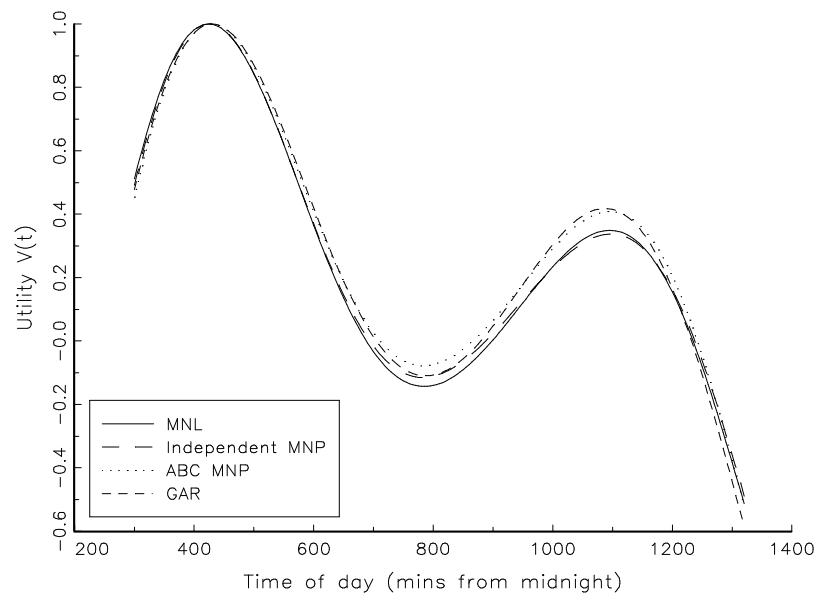


FIGURE 3.— Diurnality: Time of day preferences

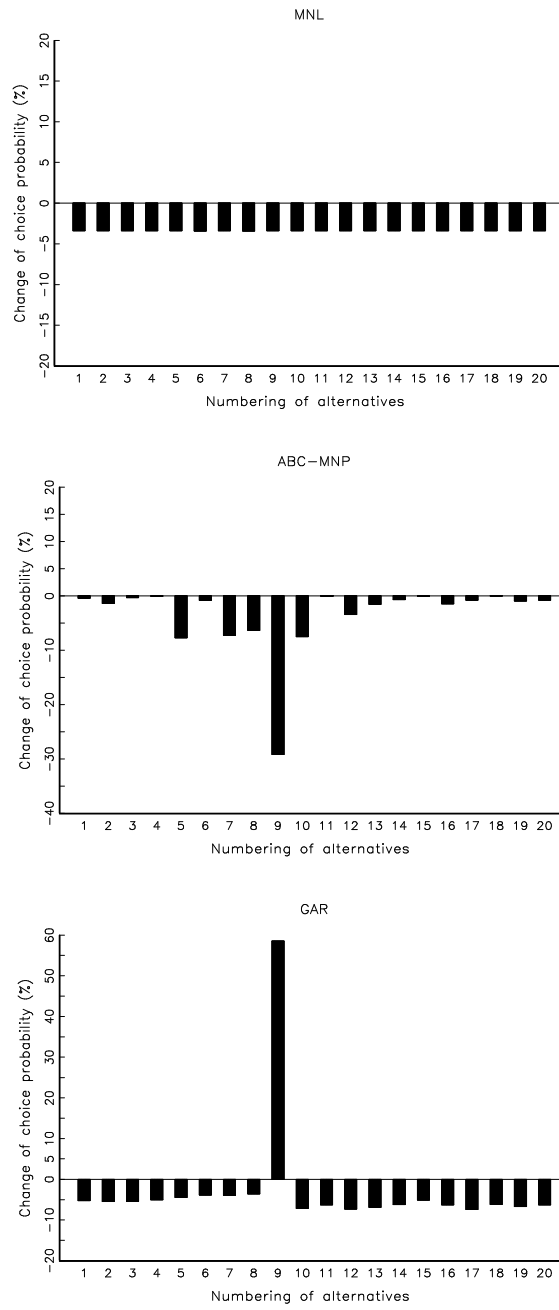


FIGURE 4.— Change of choice probabilities

TABLE 1
SAMPLE DESIGN MONTE CARLO STUDY

O&D Market 1			O&D Market 2		
Alternative	Nonstop- Indicator	Dep. Time (min. after midnight)	Alternative	Nonstop- Indicator	Dep. Time (min. after midnight)
1	1	390	1	1	390
2	1	390	2	1	420
3	0	480	3	1	450
4	1	485	4	0	600
5	0	500	5	0	620
6	1	735	6	0	800
7	0	900	7	0	960
8	1	1120	8	0	1140
9	1	1140	9	1	1170
10	1	1200	10	1	1200

TABLE 2
MONTE CARLO RESULTS

DGP	Model	Mean	Std.Dev.	Skewness	Kurtosis	RMSE	MAE
1	MNL	0.0000	0.0090	0.0001	0.0001	0.0090	0.0070
	I-MNP	0.0000	0.0092	0.0001	0.0001	0.0090	0.0070
	ABC-MNP	0.0000	0.0092	0.0002	0.0001	0.0090	0.0070
	GAR	0.0000	0.0082	-0.0001	0.0000	0.0080	0.0060
2	MNL	0.0000	0.0206	0.0047	0.0010	0.0210	0.0170
	I-MNP	0.0000	0.0209	0.0062	0.0011	0.0210	0.0170
	ABC-MNP	0.0000	0.0096	0.0001	0.0001	0.0100	0.0070
	GAR	0.0000	0.0183	0.0030	0.0006	0.0180	0.0150
3	MNL	0.0000	0.0203	0.0091	0.0011	0.0200	0.0160
	I-MNP	0.0000	0.0206	0.0108	0.0012	0.0210	0.0160
	ABC-MNP	0.0000	0.0093	-0.0001	0.0000	0.0090	0.0070
	GAR	0.0000	0.0176	0.0035	0.0005	0.0180	0.0140
4	MNL	0.0000	0.0319	0.0644	0.0085	0.0320	0.0240
	I-MNP	0.0000	0.0322	0.0707	0.0092	0.0320	0.0240
	ABC-MNP	0.0000	0.0128	0.0005	0.0002	0.0130	0.0100
	GAR	0.0000	0.0251	0.0132	0.0023	0.0250	0.0200
5	MNL	0.0000	0.0144	0.0001	0.0003	0.0140	0.0120
	I-MNP	0.0000	0.0148	0.0006	0.0003	0.0150	0.0120
	ABC-MNP	0.0000	0.0111	-0.0004	0.0001	0.0110	0.0090
	GAR	0.0000	0.0138	0.0000	0.0002	0.0140	0.0110

$R = 100$ replications, $J = 20$ itineraries.

Mean AE (mean absolute error): $\frac{1}{RJ} \sum_{r=1}^R \sum_{i=1}^J |\hat{p}_{ir} - p_{ir}|$

RMSE (root mean squared error): $\left[\frac{1}{RJ} \sum_{r=1}^R \sum_{i=1}^J (\hat{p}_{ir} - p_{ir})^2 \right]^{0.5}$

TABLE 3
ESTIMATION RESULTS

	MNL	I-MNP	GAR-MNP	ABC-MNP
β_1	6.25 (0.233)	2.67(0.109)	6.55 (0.273)	4.16 (0.235)
β_2	-0.03 (0.003)	-0.01 (0.001)	-0.03 (0.003)	-0.01 (0.003)
β_3	0.96 (0.065)	0.45 (0.061)	1.00(0.072)	1.07 (0.067)
β_4	0.87 (0.063)	0.41 (0.063)	0.82(0.070)	0.77 (0.056)
γ_1	0.61 (0.065)	0.32 (0.065)	0.92 (0.078)	4.01 (0.334)
γ_2	0.76 (0.053)	0.39 (0.026)	1.08 (0.079)	3.67 (0.244)
γ_3	0.10 (0.027)	0.06 (0.014)	0.11 (0.038)	0.32 (0.043)
ϕ_1	3.48 (0.098)	3.45 (0.093)	3.43 (0.085)	3.13 (0.016)
ϕ_2	2.73 (0.035)	2.71 (0.035)	2.74 (0.036)	2.60 (0.011)
ϕ_3	1.87 (0.140)	1.81 (0.119)	2.01 (0.186)	1.32 (0.116)
ρ			0.63 (0.020)	
σ_0			1.07 (0.100)	
α_1			6.74 (-0.422)	
α_2			2.77 (-0.220)	
σ_1^2				37.69 (5.833)
λ_1				144.59 (6.02)
σ_2^2				1.24 (0.162)
$\log \mathcal{L}$	-95500.78	95461.77	-95084.84	-94891.32
MAE	0.285	0.283	0.267	0.253
RMSE	0.836	0.847	0.695	0.68

MAE (mean absolute error): $\left[\frac{1}{N} \sum_{i=1}^J \left| \hat{N}_i - N_i \right| \right]$

RMSE (root mean squared error): $\left[\frac{1}{N} \sum_{i=1}^J \left(\frac{\hat{N}_i - N_i}{N_i} \right)^2 \right]^{0.5}$

N is the total number of passengers, N_i the number of passengers on itinerary i

$\hat{N}_i = \hat{p}_i N_{d(i)}^D$ is the estimated demand for itinerary i , where $\hat{p}_i$ is the estimated choice probability and $N_{d(i)}^D$ is the total passenger volume on O&D market $d(i)$.

Robust standard errors in parantheses

TABLE 4
MODIFIED LIKELIHOOD RATIO STATISTICS

	LR	MLR
GAR-MNP vs. MNL	832	414
ABC-MNP vs. I-MNP	1141	569
ABC-MNP vs. GAR-MNP		192

Modified LR statistic (MLR) as proposed by Horowitz (1983) and Horowitz et al. (1986)

TABLE 5
FLIGHTS IN EXEMPLARY MARKET

i	Dep. time	Nonstop	Elap. time	Airline 1	Airline 2
Original example market					
1	06:25:00	1	65	1	0
2	06:30:00	1	65	0	1
3	07:55:00	1	65	0	1
4	08:00:00	1	65	1	0
5	09:15:00	1	65	0	1
6	09:15:00	0	185	1	0
7	09:45:00	1	65	1	0
8	09:45:00	0	185	1	0
9	11:05:00	1	65	1	0
10	13:20:00	1	70	1	0
11	14:20:00	1	65	1	0
12	14:25:00	1	65	0	1
13	15:50:00	1	65	0	1
14	16:15:00	1	65	1	0
15	17:35:00	1	65	1	0
16	17:40:00	1	65	0	1
17	18:25:00	1	65	1	0
18	19:05:00	1	65	0	1
19	19:25:00	1	65	1	0
20	20:50:00	1	65	0	1
Inserted flight:					
9+	11:05:00	1	65	0	1

TABLE 6
COVARIANCE MATRIX IMPLIED BY ABC-MNP

	1	2	...	5	6	7	8	9	10	11	12	13	14	...	20	9+
1	40															
2	38	40														
3	31	33	...													
4	32	31	...													
5	19	21	...	40												
6	20	20	...	38	40											
7	16	15	...	37	38	40										
8	16	15	...	37	38	39	40									
9	7	6	...	28	29	34	34	40								
10	2	1	...	9	10	14	14	26	40							
11	1	0	...	4	5	7	7	16	36	40						
12	0	1	...	5	4	6	6	14	34	38	40					
13	0	1	...	2	1	2	2	5	22	31	33	40				
14	1	0	...	1	2	2	2	5	19	29	28	37	40			
15	1	0	...	0	1	1	1	2	9	16	16	29	34	...		
16	0	1	...	1	0	0	0	1	7	14	16	29	32	...		
17	1	0	...	0	1	1	1	2	5	10	10	21	26	...		
18	0	1	...	1	0	0	0	0	2	5	7	16	19	...		
19	1	0	...	0	1	1	1	1	3	5	4	12	17	...		
20	0	1	...	1	0	0	0	0	0	1	2	6	6	...	40	
9+	6	7	...	29	28	32	32	38	24	15	16	7	4	...	1	40

TABLE 7
COVARIANCE MATRIX IMPLIED BY GAR-MNP

	1	2	...	5	6	7	8	9	10	11	12	13	14	...	20
1	1.8														
2	1.1	1.8													
3	0	0	...												
4	0	0	...												
5	0	0	...	1.5											
6	0	0	...	0.8	1.5										
7	0	0	...	0.3	0.3	1.5									
8	0	0	...	0.3	0.3	0.8	1.5								
9	0	0	...	0.2	0.2	0.6	0.6	1.1							
10	0	0	...	0	0	0	0	0	1.1						
11	0	0	...	0	0	0	0	0	0	1.8					
12	0	0	...	0	0	0	0	0	0	1.1	1.7				
13	0	0	...	0	0	0	0	0	0	0	0	1.7			
14	0	0	...	0	0	0	0	0	0	0	0	1	1.7		
15	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
16	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
17	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
18	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
19	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
20	0	0	...	0	0	0	0	0	0	0	0	0	0	...	1.1

TABLE 8
COVARIANCE MATRIX IMPLIED BY GAR-MNP (AFTER ADDING NEW FLIGHT)

	1	2	...	5	6	7	8	9	10	11	12	13	14	...	20	9+
1	1.8															
2	1.1	1.8														
3	0	0	...													
4	0	0	...													
5	0	0	...	1.5												
6	0	0	...	0.8	1.5											
7	0	0	...	0.3	0.3	1.5										
8	0	0	...	0.3	0.3	0.8	1.5									
9	0	0	...	0	0	0	0	1.8								
10	0	0	...	0	0	0	0	0	1.1							
11	0	0	...	0	0	0	0	0	0	1.8						
12	0	0	...	0	0	0	0	0	0	1.1	1.7					
13	0	0	...	0	0	0	0	0	0	0	0	1.7				
14	0	0	...	0	0	0	0	0	0	0	0	1	1.7			
15	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
16	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
17	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
18	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
19	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
20	0	0	...	0	0	0	0	0	0	0	0	0	0	...	1.1	
9+	0	0	...	0	0	0	0	1.1	0	0	0	0	0	...	0	1.8