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**Interaction of wake vortices with the atmosphere and the
ground: advanced numerical simulation and operational
modeling**

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Chapter 1

Introduction

1.1 Scope of the thesis

The wake of an aircraft is composed, when fully rolled-up, of a pair of counter-rotating vortices, which are inherent to the generation of lift by the wing. Those coherent vortices are quite persistent and can live up to several minutes after the aircraft has flown by, depending on the environmental conditions (meteorological and ground proximity conditions). A following aircraft that would enter such two-vortex system could experience a strong rolling moment or a strong downwash velocity that could be hazardous.

The International Civil Aviation Organization (ICAO) hence imposes separation minima between aircraft, based upon this wake turbulence hazard. These separation standards, established in the seventies, divide the aircraft into three categories depending on their Maximum Take-Off Weight (MTOW): Heavy ($\text{MTOW} \geq 136 \text{ ton}$), Medium ($7 \text{ ton} < \text{MTOW} < 136 \text{ ton}$), and Light ($\text{MTOW} \leq 7 \text{ ton}$). The radar separation minima, imposed by ICAO and provided in Table 1.1, are based on two principles: i) heavier aircraft generate more dangerous wakes; ii) heavier aircraft have more control authority to recover in case of a wake vortex encounter.

More recently, an additional category was created for the A380, called Super-Heavy. The separations to be applied are the same as those for the Heavy aircraft except for approach and departure phases of flight, for which they are increased, as reported in Table 1.1.

Nowadays, air traffic increase has made these wake vortex separation standards a limiting factor on the capacity of the busiest airports. Reducing the

Leader/Follower	Heavy	Medium	Light
Heavy (H)	4 NM	5 NM	6 NM
Medium (M)	MRS	MRS	5 NM
Light (L)	MRS	MRS	MRS
Super-Heavy (S-H) (approach or landing)	6 NM	7 NM	8 NM

Table 1.1: ICAO radar wake turbulence separation minima. The Minimum Radar Separation (MRS) is 3 NM if the aircraft is within 40 NM from the antenna, and else 5 NM (the 3 NM can be reduced to 2.5 NM if some requirements on the radar and on the operations are met). (1 NM=1.852 km)

separation minima, while maintaining or even improving the safety level, hence represents a major economic interest. An increase of one aircraft per hour at a busy airport indeed already represents a significant gain. In addition, an optimized Air Traffic Control (ATC) would allow reductions of the delays for approaches and departures, and hence would also reduce the noise generation and the aircraft emissions.

Many large-scale projects in Europe and in the U.S. have been, and are still, dedicated to this problematic. Currently, three major projects attempt to find solutions to improve Air Traffic Management (ATM): the European project SESAR (Single European Sky ATM Research), the FAA project NextGEN and the joint FAA-Eurocontrol project RECAT. To optimize the separation standards, these projects investigate two main solutions:

- *Re-classification of the aircraft*

The current aircraft categories have two major drawbacks. First, they are only based on aircraft mass. This is clearly not realistic as both the wake and the aircraft reactions also depend on the wingspan, the flight speed, the wing configuration, etc. Moreover, the Medium and Heavy categories contain a very broad range of aircraft. Obviously, the wake of the heaviest aircraft of a category is not as dangerous as that of the lightest aircraft of that category.

The RECAT project aims to address this re-categorization. In RECAT Phase I, a metric was developed that takes into account the leader aircraft weight, wingspan and flight speed as well as the wingspan and flight speed of the follower aircraft. It is based on the maximum induced rolling moment coefficient C_R . It hence enables a more realistic classification of

the aircraft, see [29]. Using the developed metric, six categories were defined that aim to optimize the airport capacity gain while maintaining or even improving today's safety. RECAT Phase II should then use the developed metric (possibly improved, see below) to investigate the concept of static pair-wise separations.

- *Dynamic separation depending on the ambient conditions*

The wake vortex behavior is influenced by the atmospheric conditions (wind, turbulence, temperature stratification) and the ground vicinity. Even by using the current categorization, the airport capacity could then be increased by using separation procedures that also depend on the environment. This constitutes the objectives of various projects.

Some of them investigate the feasibility of new separation standards based on the wind. For instance, one of the objectives of the SESAR project¹ is to develop concepts and systems which shall allow controllers to sequence arriving and/or departing aircraft using time-based weather dependent pair-wise wake turbulence separations. The objective of the RECAT project, in Phase III, will also be to establish dynamic weather dependent pair-wise aircraft separations.

Other projects aim to develop a vortex avoidance system that takes into account the air traffic (including the time-based weather dependent separations) and the meteorological conditions to dynamically regulate the aircraft separations. For example, in the framework of the SESAR project, two types of such systems are developed: a ground-based system², and an on-board system³.

A practical application of all these concepts necessarily requires a good understanding and an accurate modeling of: i) the creation of the wake by the aircraft, ii) the evolution of the wake vortices (transport, topology, and decay) depending on their initial properties and on their environment, iii) the following aircraft reactions in case of wake vortex encounter.

This thesis is concerned with the further characterization and modeling of the wake vortex behavior interacting with the atmosphere and/or with the

¹SESAR P6.8.1 "Flexible and Dynamic Use of Wake Vortex Separations", led by Euro-control.

²SESAR P12.2.2 "Runway Wake Vortex Detection, Prediction and Decision Support Tools", under the leadership of Thales.

³SESAR P9.11 "Aircraft Systems for Wake Encounter Alleviation" and P9.30: "Weather Hazards/ Wake Vortex Sensor", led by Airbus.

ground. The results of such modeling hence enable, for a given scenario (given wake vortex system evolving in provided environment), to determine whether an encounter could occur. In case of encounter, it also provides some of the required inputs for the encounter analysis (in a flight simulator for instance).

Finally, it should be noted that this thesis is the continuation of the work performed, by the UCL group since 2001, in the area of wake vortex simulation and modeling, also in the framework of several European projects (the project list, together with a summary description of UCL involvement, can be found in [124]). At the beginning of this thesis, an operational wake vortex prediction software was indeed available. One of the main objectives of this work was to improve the existing software models with a particular attention paid to the modeling of the influence, on wake vortex transport and decay, of turbulent and stratified atmospheres and of ground proximity. The obtained operational software should be more accurate, yet still very fast. It should then be usable as a support to operational real-time systems or off-line studies.

1.2 Contents and achievements of the thesis

- *Chapter 1 - Basics of aircraft wake vortices and simplified models*

In the end of this introduction, we present some fundamentals on aircraft wake vortices. Those concepts are then used throughout this thesis.

We first expose the wake vortex properties and relate them to the aircraft characteristics. Some classical models of vortex circulation, and thus velocity, distributions are also recalled and compared. The parameters that influence the wake vortex evolution are also summarized. Finally, the typical measurements of the wake vortex circulation are presented.

- *Chapter 2 - Large-Eddy Simulations of aircraft wake vortices interacting with the atmosphere*

In this first chapter, we investigate the influence of various atmospheres on the wake vortex behavior, by means of Large-Eddy Simulations (LES) at very high Reynolds number. A particular attention is paid to the realistic representation of both the atmospheric and the wake vortex fields.

We investigate three different types of influences on wake vortex behavior: i) atmospheric stratification alone, ii) combined atmospheric stratification and weak turbulence; iii) atmospheric turbulence, without stratification.

For all cases, the wake vortex transport and decay are fully characterized. The influence of the degree of turbulence and of stratification on the long-wavelength Crow instability is also investigated. Finally, we focus on the case of a two-vortex system evolving in a weakly to moderate turbulent atmosphere and without possible Crow instability development. In those conditions, the vortex system is shown to reach a statistical turbulent equilibrium that is quite “universal”.

Based on these results, a new characterization of the vortex circulation decay, in terms of mean and envelope evolution, is proposed. For weak turbulence and weak stratification levels, this characterization is required for conservative risk analyses. The results obtained in this chapter also constitute a basis for the wake vortex behavior modeling performed in Chapter 3.

- *Chapter 3 - Operational modeling of wake vortex behavior*

This chapter details the operational models that were developed or improved in the framework of this thesis. Those models are integrated in the Deterministic wake Vortex Model (DVM), that uses a simplified vortex particle method to predict the wake vortex behavior in a 2-D plane crossing the flightpath.

The proposed simplified models are based on a physical analysis of the wake vortex behavior interacting with the atmosphere and/or with the ground. The improved model can account for the influences of : i) the atmospheric turbulence on wake vortex circulation decay and on Crow instability development, ii) the atmospheric stratification combined with weak turbulence on wake vortex transport and decay; iii) combined ground proximity and wind (cross and head components) on wake vortex transport and decay.

The DVM, with its improved models, is assessed using the results of LES and of three measurement campaigns.

The DVM itself is also used in a probabilistic predictor, called the Probabilistic wake Vortex Model (PVM). Finally, both the DVM and the PVM are used in the 3-D space + time simulation platform WAKE4D. Example of applications of the PVM and of the WAKE4D platform are also provided for various types of operational studies.

- *Chapter 4 - Analysis of a wake vortex measurement database*

We analyze the EDDF-2⁴ campaign database in depth. The database contains more than 10,000 tracks of wake vortices generated by Heavy and Medium aircraft. Thanks to the very large size of the database, a meaningful statistical analysis of the data is possible.

First, the wake vortex lateral transport is correlated to different crosswind measurements (some being operational). This enables to compute, for a given crosswind value, the temporal evolution of the envelope containing the vortices (with a certain probability level). The envelopes are then used to define separation standards that also depend on the wind.

The vortex circulation decay is also statistically correlated to the wind (crosswind and total wind). This statistical analysis allows the proposition of separation standards for departure that depend on the generating aircraft and on the wind. Those separations guarantee a probability of encounter as low as the one obtained when using the current separation in low wind conditions.

The results of this chapter were also corroborated by those obtained by DLR (Deutsches Zentrum für Luft- und Raumfahrt, German Aerospace Center) and DFS (Deutsche Flugsicherung, German air traffic control organization) in similar studies. This comparison is the object of a common publication [31].

1.3 Basics of aircraft wake vortices and simplified models

We here introduce some concepts related to aircraft wake vortices and that will be used throughout this thesis. In particular, the characteristics of the rolled-up two-vortex system are detailed and are related to the properties of the aircraft that generates them. The parameters that impact the wake vortex behavior are also detailed. Finally, some classical measurements of vortex circulation are recalled.

The wake of a flying aircraft consists in a vortex sheet that emanates from the wing and rolls-up to form, after about 10-15 wingspans, a two-vortex system, see Fig. 1.1 (top). When the vortex sheet is completely rolled-up (e.g., in

⁴EDDF is the ICAO code of the Frankfurt airport

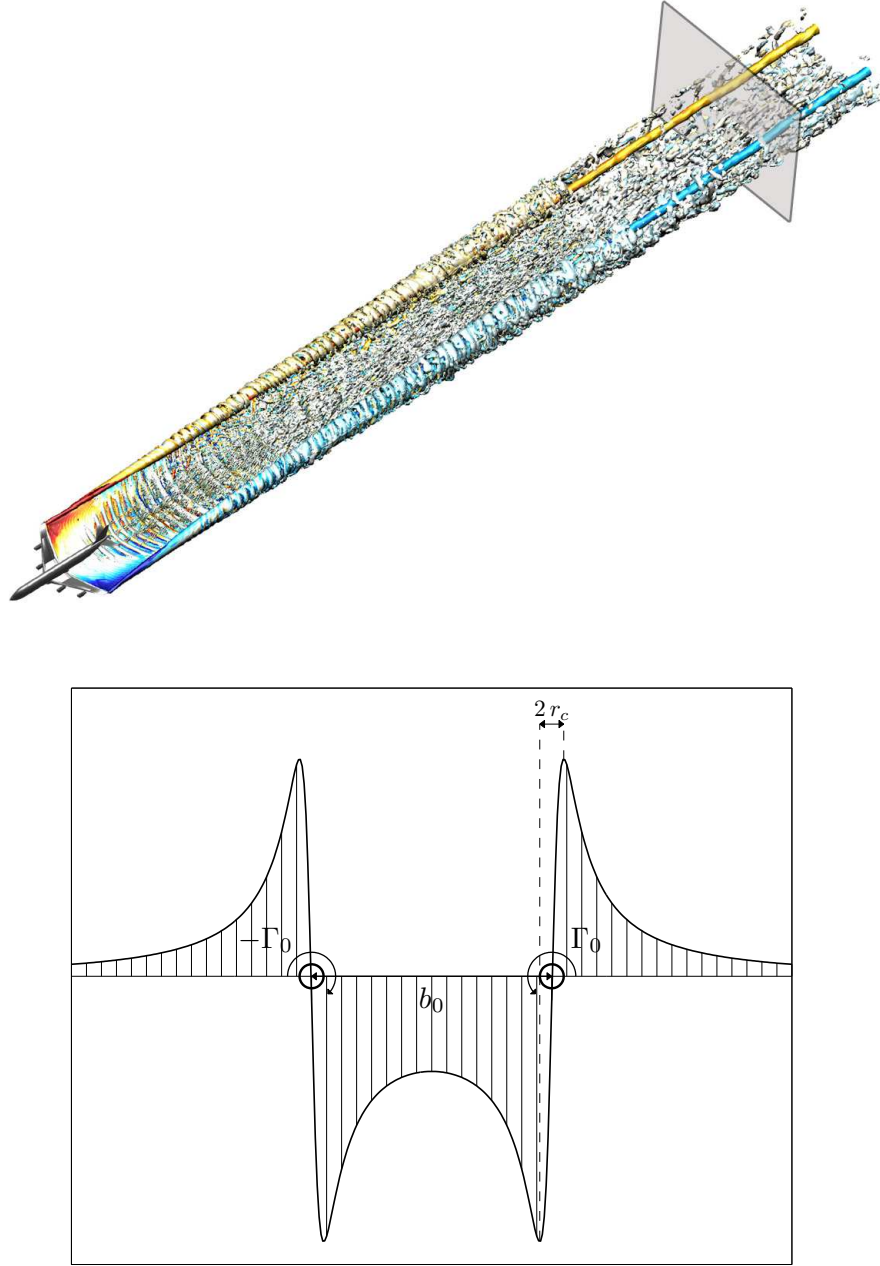


Figure 1.1: Top: Visualization of the turbulent wake vortex roll-up behind an aircraft. Space-developing simulation of a vortex sheet roll-up with velocity deficit, performed by Lonfils [75]. Bottom: Schematic view of the two-vortex system and of the induced velocity profile on a line passing through the centers of the vortices.

the plane depicted in Fig. 1.1 (top)), the vortex system is composed of a pair of counter-rotating vortices with circulations Γ_0 and $-\Gamma_0$ and separated by a distance b_0 , as represented in Fig. 1.1 (bottom). Due to the mutually-induced velocity, the vortex pair sinks at a velocity that is initially close to:

$$V_0 = \frac{\Gamma_0}{2\pi b_0}. \quad (1.1)$$

This also leads to the definition of a characteristic time:

$$t_0 = \frac{b_0}{V_0}. \quad (1.2)$$

Using the coordinate system defined in Fig. 1.2, the vortices are then aligned with the x -axis⁵.

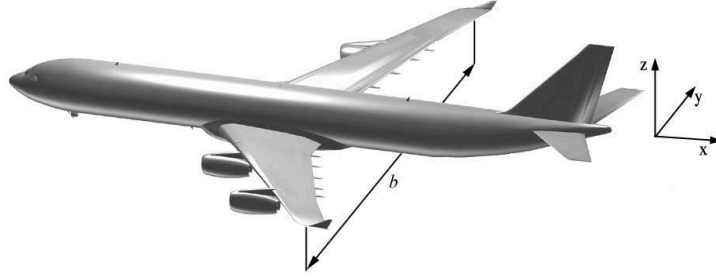


Figure 1.2: Definition of the reference coordinate system and of the wingspan.

The streamlines of a typical rolled-up two-vortex system are shown in Fig. 1.3. One observes that the vortices are contained in a “Rankine-like” oval: i.e., a closed streamline that sinks at a velocity equal to V_0 . Most of the vorticity of the system is contained in that region.

The wake vortex system characteristics can be deduced from the properties of the aircraft that generates them. By conservation of momentum, the lift is indeed related to the circulation distribution of the vortex sheet (we here do not take into account the vortex sheet associated with the horizontal tail plane (HTP); its contribution can nevertheless be significant for an aircraft in approach to landing). If using the coordinate system presented in Fig. 1.2, it

⁵Due to the sink velocity, a small angle is actually observed between the vortices and the x -axis. For cruise conditions (see numbers in Table 1.2), it is estimated to be less than 0.5° .

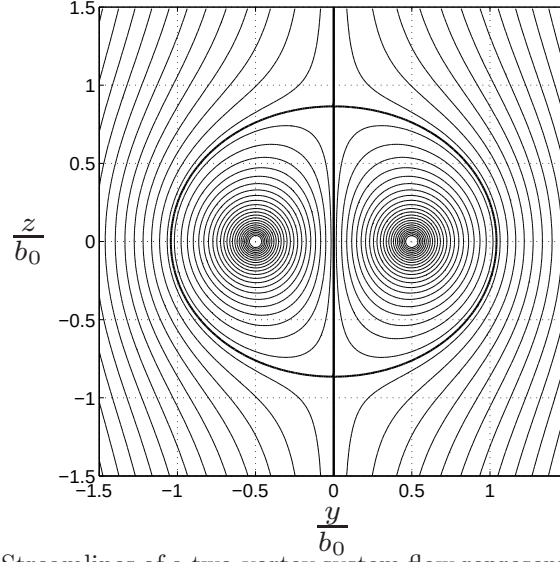


Figure 1.3: Streamlines of a two-vortex system flow represented in a reference frame that sinks at V_0 . The “Rankine-like” oval (i.e., the $\psi = 0$ streamline in that reference frame) is also shown (thick solid).

reads:

$$\begin{aligned}
 L &= \rho V \int_{-b/2}^{b/2} \Gamma(y) dy \\
 &= -\rho V \int_{-b/2}^{b/2} y \frac{d\Gamma}{dy} dy,
 \end{aligned} \tag{1.3}$$

where ρ is the air density, V is the air speed, $\Gamma(y)$ the spanwise circulation distribution, and b the aircraft wing span. One then also defines the loading factor s as:

$$s = \frac{\int_0^{b/2} y \frac{d\Gamma}{dy} dy}{\frac{b}{2} \int_0^{b/2} \frac{d\Gamma}{dy} dy} = \frac{2}{b} \frac{1}{\Gamma_0} \int_0^{b/2} \Gamma(y) dy, \tag{1.4}$$

with $\Gamma_0 = \Gamma(y = 0)$ the root circulation.

The aircraft lift is also related to its weight following (here again neglecting the negative contribution of the HTP):

$$L = M g \cos(\gamma), \tag{1.5}$$

where M is the aircraft mass, g the gravity and γ the flight path angle ($\gamma = 0$ for an horizontal flight, typical values are $\gamma \simeq -3^\circ$ for landing, and $\gamma \simeq 10^\circ$ for

take-off). From Eqs. (1.3) to (1.5), one eventually obtains:

$$L = M g \cos(\gamma) = \rho V b_0 \Gamma_0, \quad (1.6)$$

where $b_0 = s b$. If the s factor is known, the spacing b_0 and the root circulation Γ_0 may be thus deduced from the air density, the lift, the wingspan and the air speed. In case of elliptical circulation distribution, s is equal to $\pi/4$.

Table 1.2 provides, for illustration, the characteristics of the vortices generated by various aircraft in cruise conditions. For all, the mass is taken as $M \simeq \frac{1}{2} (\text{MTOW} + \text{OEW})$ (where OEW stands for Operational Empty Weight), and we assumed that $s = \frac{\pi}{4}$: this is not necessarily the case; yet the obtained numbers are useful as they are close to the real values.

Name	WTC	b	M	V	h	Γ_0	b_0	r_c	V_0	t_0
		[m]	[ton]	[km/h]	[kft]	[m ² /s]	[m]	[m]	[m/s]	[s]
A380	S-H	79.8	420	900	37	749	62.7	3.1	1.9	33
B747-400	H	64.4	290	900	37	641	50.6	2.5	2.0	25
A340-300	H	60.3	200	870	37	488	47.4	2.4	1.6	29
A321-200	M	34.1	70	825	37	319	26.8	1.3	1.9	14
BAE146-200	M	26.1	35	770	29	164	20.6	1.0	1.3	16
Cessna 208B	L	15.9	3	300	12	33	12.5	0.6	0.4	29

Table 1.2: Examples of wake vortex characteristics generated by aircraft of the different Wake Turbulence Categories (WTC) in cruise conditions (i.e., flying at altitude h). The air density is taken from the standard atmosphere, the mass is taken as $M \simeq \frac{1}{2} (\text{MTOW} + \text{OEW})$ and we assume $s = \frac{b_0}{b} = \frac{\pi}{4}$ and $\frac{r_c}{b_0} = 0.05$.

Note that, in approach, the velocities of medium to large aircraft are of the order of $V = 70$ to 80 m/s. At those altitudes, $\rho \simeq 1.2$ kg/m³, the product ρV is thus about the same as that in cruise. Hence, the obtained Γ_0 , and thus V_0 and t_0 , are also about the same (again excluding the effect of the HTP and still assuming $s \simeq \frac{\pi}{4}$).

For such a two-vortex system, the Reynolds number is defined using the initial circulation Γ_0 and the air kinematic viscosity ν :

$$Re_\Gamma = \frac{\Gamma_0}{\nu}. \quad (1.7)$$

For aircraft vortices, it is very high : $Re_\Gamma \simeq 10^7$. In such regime, the molecular viscosity does not play any role except in the inner part of the vortex core and at very small scales.

The vortex system transport, topology and decay is influenced by its environment: the meteorological conditions (wind, wind shear, turbulence, and stratification) and the ground vicinity. The wind (both head-, u , and cross-, v , components) convects the vortices, whereas a wind shear (characterized by a non-zero $\frac{d^2 v}{dz^2}$) induces a tilting of the vortex pair. Atmospheric turbulence (characterized by the Eddy Dissipation Rate ϵ) increases the vortex decay and triggers the long-wavelength Crow instability. Finally, atmospheric stratification (measured through the Brunt-Väisälä frequency N) induces a rebound of the vortices and enhances their decay. In addition, when the vortices are close to the ground, they interact with the secondary vorticity generated at the ground due to the presence of the primary vortices. This interaction induces a rebound of the vortices and an enhanced decay. For more detailed descriptions of the environment influences on wake vortex behavior and of the related researches in the literature, we refer the reader to the introductions of Chapters 2 and 3.

The circulation of each vortex, of the vortex pair, follows a certain distribution $\Gamma(r)$, measured using the velocity $\mathbf{u}$ or the vorticity $\boldsymbol{\omega}$, and defined by:

$$\begin{aligned}\Gamma(r) &= \int_0^{2\pi} \mathbf{u}(r, \theta) \cdot \mathbf{e}_\theta r d\theta \\ &= \int_0^{2\pi} \int_0^r \boldsymbol{\omega}(r', \theta) \cdot \mathbf{e}_x r' dr' d\theta, \end{aligned} \quad (1.8)$$

where r is the distance to the center of the vortex, and with $\Gamma(r \rightarrow \infty) = \Gamma_0$. For an axisymmetric vortex, the distributions of the azimuthal velocity $u_\theta(r)$ and of the axial vorticity $\omega_x(r)$ are hence related to $\Gamma(r)$ following:

$$u_\theta(r) = \frac{\Gamma(r)}{2\pi r}, \quad (1.9)$$

$$\omega_x(r) = \frac{1}{\pi} \frac{d\Gamma(r)}{d(r^2)}. \quad (1.10)$$

The “effective core radius” of a vortex r_c is then defined as the position where the azimuthal velocity reaches its maximum: $u_\theta(r_c) = u_{\theta, \max} = \frac{\Gamma(r_c)}{2\pi r_c}$, see Fig. 1.1. The precise value of the core size is still somewhat undetermined.

It is indeed very difficult to experimentally capture the vortex core details of real aircraft wakes. Moreover, the simulation of the roll-up of the vortex sheet behind an aircraft, at realistic Re_Γ , would require very high resolution and is hence still computationally too expensive. In the present stage of knowledge of aircraft wake vortex physics, it is believed that aircraft wake vortices have a core radius r_c ranging from 3% to 5% of the vortex initial spacing b_0 (the typical values of r_c are also provided in Table 1.2, assuming $\frac{r_c}{b_0} = 0.05$). The vortex spacing b_0 and the core size r_c are, with the circulation Γ_0 , the key parameters that control wake vortex encounter hazard. However, in RECAT Phase I, the developed metric is solely based on the vortex circulation and on the estimate of the induced rolling moment coefficient, C_R , on a follower aircraft (using, in addition, the follower flight speed V_f and wingspan b_f with $C_R \propto \frac{\Gamma_0}{b_f V_f}$). Yet, so far, it does not take into account the vortex spacing b_0 nor the vortex core size r_c . Whether this metric is the most appropriate is thus still a subject of discussion (see e.g., [121]). The metric might hence be adapted, in RECAT Phase II, to take into account b_0 (and maybe r_c).

Different models exist to represent each rolled-up aircraft wake vortex. A first class of those models are characterized by a single degree of freedom and are thus called “one-scale” vortex models. The one-scale models are: the low-order algebraic (LOA) vortex (also called Burnham-Hallock vortex [9]), the high-order algebraic vortex (HOA) [119], the Gaussian vortex (also called Lamb-Oseen vortex), and the top-hat vortex (also called Rankine vortex, which is not physical but is here used for baseline comparison only). In order to compare those models, one defines the dimensionless circulation $\tilde{\Gamma}$, azimuthal velocity $\tilde{u}_\theta$, and vorticity $\tilde{\omega}_x$, using the core radius r_c and the total circulation Γ_0 , as characteristic length and circulation, respectively:

$$\tilde{\Gamma} = \frac{\Gamma}{\Gamma_0} \quad (1.11)$$

$$\tilde{u}_\theta = \frac{u_\theta}{\Gamma_0 / (2\pi r_c)} \quad (1.12)$$

$$\tilde{\omega}_x = \frac{\omega_x}{\Gamma_0 / (\pi r_c^2)} \quad (1.13)$$

The models are recalled in Table 1.3 and compared in Fig. 1.4. One notes that the vortex models are indeed quite different even when properly normalized, as done here, with the same Γ_0 and r_c : the fraction of circulation contained within the circle of radius r_c is quite different, and hence so is the $\tilde{u}_{\theta, \max}$.

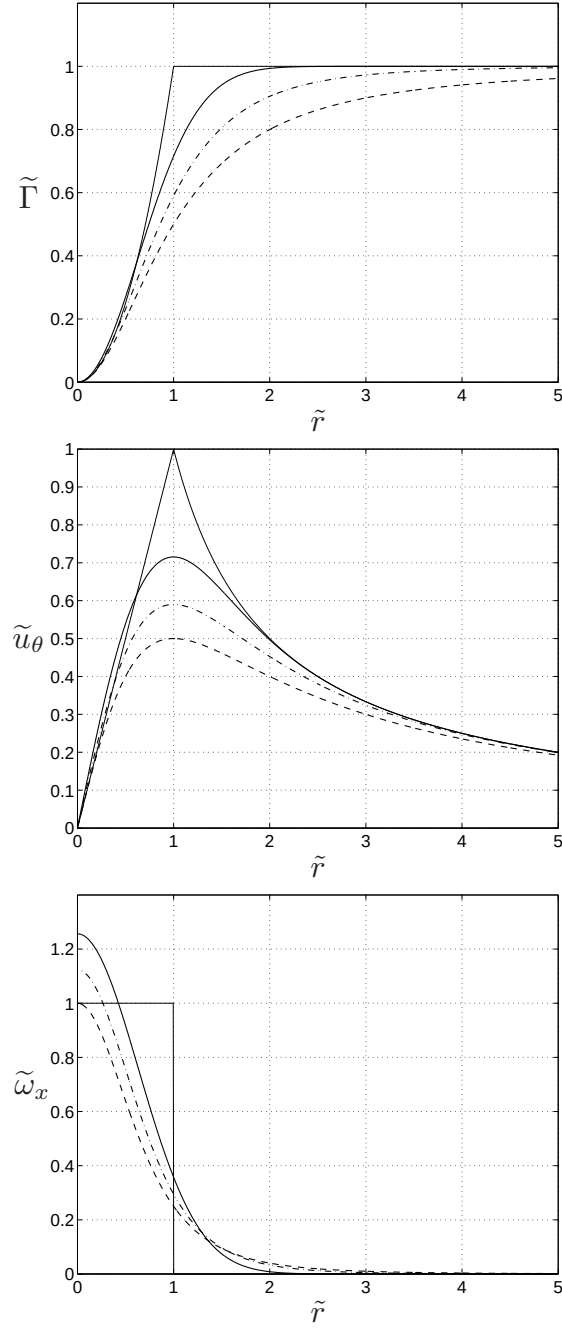


Figure 1.4: Comparison between the Gaussian vortex (thick solid), the low-order algebraic vortex (dash), the high-order algebraic vortex (dash-dot), and the top-hat vortex (thin solid): profiles of the circulation (top), the azimuthal velocity (middle), and the vorticity (bottom).

Model	$\tilde{\Gamma}(\tilde{r})$	$\tilde{u}_\theta(\tilde{r})$	$\tilde{\omega}_x(\tilde{r})$
LOA	$\frac{\tilde{r}^2}{(\tilde{r}^2+1)}$	$\frac{\tilde{r}}{(\tilde{r}^2+1)}$	$\frac{1}{(\tilde{r}^2+1)^2}$
HOA ($\gamma = 1.781$)	$\frac{\tilde{r}^2(\tilde{r}^2+2\gamma)}{(\tilde{r}^2+\gamma)^2}$	$\frac{\tilde{r}(\tilde{r}^2+2\gamma)}{(\tilde{r}^2+\gamma)^2}$	$\frac{2\gamma^2}{(\tilde{r}^2+\gamma)^3}$
Gaussian ($\beta = 1.256$)	$1 - \exp(-\beta \tilde{r}^2)$	$\frac{1}{\tilde{r}} (1 - \exp(-\beta \tilde{r}^2))$	$\beta \exp(-\beta \tilde{r}^2)$
Top Hat	$\begin{cases} 0 \leq \tilde{r} < 1 & \tilde{r}^2 \\ \tilde{r} \geq 1 & 1 \end{cases}$	$\begin{cases} 0 \leq \tilde{r} < 1 & \tilde{r} \\ \tilde{r} \geq 1 & \frac{1}{\tilde{r}} \end{cases}$	$\begin{cases} 0 \leq \tilde{r} < 1 & 1 \\ \tilde{r} \geq 1 & 0 \end{cases}$

Table 1.3: “One-scale” vortex circulation distribution models

“Two-scale” vortex models, with two degrees of freedom, also exist and are reported in Table 1.4. The Proctor-Winkelmanns model is also detailed in Appendix A. Based on the LES results of a roll-up at very high Reynolds num-

Model	$\Gamma(r)/\Gamma_0$	Coefficients
Proctor-Winkelmanns	$1 - \exp\left(-\frac{\beta_i \left(\frac{r}{b_0}\right)^2}{\left(1 + \left(\frac{\beta_i}{\beta_o} \left(\frac{r}{b_0}\right)^{2-q}\right)^p\right)^{1/p}}\right)$	$\beta_o = 10, q = \frac{3}{4}$ $p = 3 \dots 5$
Jacquín [37]	$\frac{\left(\frac{r}{b_0}\right)^2}{\left(\left(\frac{a_1}{b_0}\right)^4 + \left(\frac{r}{b_0}\right)^4\right)^{(1+\alpha)/4} \left(\left(\frac{a_2}{b_0}\right)^4 + \left(\frac{r}{b_0}\right)^4\right)^{(1-\alpha)/4}}$	$\alpha = 0.5 \dots 0.7$ $\frac{a_2}{a_1} = 4 \dots 10$
Regularized Kaden-Winkelmanns	$\left[1 - \exp\left(-\left(\frac{r}{a_1}\right)^{3/2}\right)\right] \frac{\alpha \left(\frac{r}{b_0/2}\right)^{1/2}}{\left(1 + (\alpha-1) \frac{r}{b_0/2}\right)}$	$\alpha \simeq 1.8$

Table 1.4: “Two-scale” vortex circulation distribution models. For each model, typical values of the coefficients are also provided. The coefficient for which no value is provided (i.e., β_i , a_1 and a_1 , respectively) is to be adjusted so that $\frac{r_c}{b_0}$ is set to the desired value.

ber [75], the two-scale models appear to be superior in terms of the azimuthal velocity and the detailed circulation profile. Nevertheless, when it comes to simplified modeling of aircraft wake vortices, the low-order algebraic one-scale model is quite appropriate, better than the high-order algebraic model and certainly better than the Gaussian model. Hence, it is used in the remainder

of this thesis to initialize Large Eddy Simulations .

A quantity of particular importance in wake vortex related hazards is the vortex circulation: at fixed r_c , the higher the circulation, the higher the induced rolling moment on a following aircraft. Its correct measurement is hence of primary importance. The circulation distribution $\Gamma(r)$ is obtained, using Eq. (1.8), by integration of the axial vorticity component on a disk of radius r centered on the vortex position. A good measurement of the total circulation associated with each primary vortex, Γ_{tot} , is the maximum of this function. This reads:

$$\Gamma_{\text{tot}}(t) = \max_r \Gamma(r, t). \quad (1.14)$$

This quantity provides an image of the total circulation but requires a measure of the vortex circulation distribution with a sufficient resolution. This is generally the case in most numerical simulations; but it is very hard to obtain in field measurements. On the contrary, experimental databases using Lidar usually provide “measurements” (in fact quantities derived from measurements and using a model!) of the partially averaged circulation between r_1 and r_2 :

$$\Gamma_{r_1-r_2}(t) = \frac{1}{(r_2 - r_1)} \int_{r_1}^{r_2} \Gamma(r, t) dr. \quad (1.15)$$

The most common averaged circulation definition is Γ_{5-15} defined by:

$$\Gamma_{5-15}(t) = \frac{1}{10 \text{ m}} \int_{5 \text{ m}}^{15 \text{ m}} \Gamma(r, t) dr. \quad (1.16)$$

Γ_{5-15} has been much used as a “good metric” of the vortex hazard of large aircraft wakes. Whether this is indeed the proper metric is still a subject of discussion and research. Γ_{5-15} is in fact typical of what is done for aircraft with $b = 60 \text{ m}$. We then also write:

$$\Gamma_{5-15}(t) = \frac{6}{b} \int_{\frac{b}{12}}^{\frac{b}{4}} \Gamma(r, t) dr. \quad (1.17)$$

When dealing later with wake vortex systems in LES, this is also the definition that will be used. We will then also further assume that $\frac{b_0}{b}$ is known (e.g., $\frac{b_0}{b} = \frac{\pi}{4}$).

Finally, we here introduce a normalization that is used all along the following of this work, in addition to that of Eqs. (1.11) to (1.13). It utilizes the wake vortex properties Γ_0 , b_0 , V_0 , and t_0 as characteristic scales. This normalization

is denoted $(\cdot)^*$. One then defines, for instance, the dimensionless time $t^* = \frac{t}{t_0}$, the dimensionless circulations $\Gamma^* = \frac{\Gamma}{\Gamma_0}$, $\Gamma_{\text{tot}}^* = \frac{\Gamma_{\text{tot}}}{\Gamma_0}$, $\Gamma_{5-15}^* = \frac{\Gamma_{5-15}}{\Gamma_0}$, the dimensionless positions $y^* = \frac{y}{b_0}$, $z^* = \frac{z}{b_0}$, $r^* = \frac{r}{b_0}$, the dimensionless velocity $v^* = \frac{v}{V_0}$, etc.

Chapter 2

Large Eddy Simulations of wake vortices interacting with the atmosphere

The content of this chapter is largely inspired by different papers. Section 2.3 is inspired by an article published in the proceedings of the TSFP-6 conference entitled “Large Eddy Simulations of aircraft wake vortices in a stably stratified atmosphere” [24], whose authors are I. De Visscher, L. Briceux, G. Winckelmans, S. Caliaro and T. Vilbajo. Section 2.4 is based on an article of the proceedings of the ECCOMAS-CFD 2010 conference entitled “LES of aircraft wake vortices evolving in a stably stratified and weakly turbulent atmosphere” [25] and an article submitted for publication in the AIAA journal with title “Aircraft vortices in stably stratified and weakly turbulent atmospheres: simulation and modeling” [23]. The authors of these two articles are I. De Visscher, L. Briceux and G. Winckelmans.

2.1 Introduction

As a consequence of its lift, an aircraft generates a wake that, after roll-up, forms a pair of counter-rotating vortices with circulation Γ_0 and $-\Gamma_0$, and separated by a distance b_0 , that sink under their mutual influence. As the strong rolling moment induced by those wake vortices on a following aircraft entering the wake can be hazardous, their study is of major interest to Air

Traffic Management (ATM). The wake vortex evolution is influenced by its environment: by the atmospheric conditions (wind, turbulence and stratification) and by ground proximity (see, e.g., [108, 43] for a review). Many large scale research projects in the U.S. and in Europe aim at characterizing, numerically or experimentally, the effects of individual or combined parameters of the environment on the wake vortex physics. The objective of this chapter is to address the influence of the stable stratification and the turbulence (also combined with stratification) of the atmosphere on the wake vortex behavior (transport and decay). In this study, the considered vortices are far from the ground. No ground effect is therefore taken into account. The atmospheric turbulence level is characterized using the dimensionless Eddy Dissipation Rate (EDR) $\epsilon^* = \epsilon b_0 / V_0^3$, where ϵ is the EDR. The stratification level is characterized using the dimensionless Brunt-Väisälä frequency $N^* = N t_0$, where N is the Brunt-Väisälä frequency. The Froude number $Fr = 1/N^*$ is also commonly used.

A realistic representation of such a problem is a major challenge for various reasons. The vortices evolving in the atmosphere are at a very high Reynolds number $Re_\Gamma = \Gamma_0 / \nu \sim 10^7$, where ν is the kinematic viscosity. In that regime, the viscous effects are completely negligible, except at very small scales. The instabilities of the flows have a very broad spectrum: the vortex long-wave instability has a wavelength $\sim 8 b_0$, structures created by stratification effects have medium to short wavelengths (from $\sim b_0$ to $\sim 0.1 b_0$) and the core radius of the vortex is $r_c \simeq 0.03 - 0.05 b_0$. In addition, the numerical method used should be kinetic energy preserving and transport vortical structures with limited dispersion errors. The subgrid scale model also has to preserve the large-to medium-scale dynamics, otherwise the model would dissipate too quickly the most relevant vortical structures.

The main influence of the stratification on the wake vortex behavior was first described and modeled by Scorer [104]. It was also the object of many 2-D numerical studies (see [52, 48, 49, 107, 38, 56]). The wake vortex system is essentially contained in a “Rankine-like” oval, which is a closed contour in the 2-D stream function containing the vortex cores (see Fig. 1.3). At early stages of the flow, the vortex circulation and separation are essentially constant. The stratification of the atmosphere tends to reduce the sink velocity due to buoyancy leading to a rebound phenomenon. This behavior is well represented using Saffman’s model [99]. In the meantime, due to horizontal density

gradients created at the interface of the “Rankine-like” oval, baroclinic vorticity is generated around the vortices. For moderate stratification levels, the opposite sign baroclinic vorticity is mostly created outside the “Rankine-like” oval pushing the vortices close to each other and leading to an acceleration of the vortices. This acceleration is taken into account in the simplified models of Crow [17] and Scorer and Davenport (reviewed in [107]). On the contrary, for higher stratification values, the baroclinic vorticity is generated principally below the vortices and tends to push the vortices away from each other. In all the cases, the simplified models are not able to mimic the wake vortex evolution for long times as they do not take into account any 3-D effect. Two main instability types indeed develop in such flows with respectively short- and long-wavelength.

Short-wavelength instabilities are initiated in the baroclinic vorticity and significantly influence the vortex decay. Nomura *et al.* [79] extensively studied those instabilities using Direct Numerical Simulations (DNS). They observed that, for weak to moderate stratification levels, the elliptic short-wave instability development is enhanced due to the decrease in lateral separation distance. For strong stratification, the base flow was observed to quickly become three-dimensional through instabilities not related to the elliptic instability.

In a calm atmosphere without stratification, a long-wavelength symmetric sinusoidal instability develops in the wake vortices. The growth of this instability, known as the Crow instability (first described by Scorer [104] and analyzed by Crow [16]), leads to a reconnection of the vortices generating vortex rings. Using a linear perturbation analysis, Crow showed that a broad range of wavelengths are unstable and that the wavelength with the maximum growth rate is a function of the core radius size. Sarpkaya [101, 103] proposed a model for the wake vortex “time-to-demise” due to turbulence. This demise is either due to Crow linking or due to vortex bursting in strong turbulence. The defined time-to-demise is a decreasing function of the atmosphere turbulence level, measured through ϵ^* .

The influence of the stratification on the Crow instability development was investigated experimentally at moderate Re_Γ [100, 28], numerically, at low Re_Γ , using DNS [30, 39] and at moderate to high Re_Γ (hence closer to realistic conditions), using Large Eddy Simulations (LES) [95, 71, 72, 97, 111, 57, 112, 84, 58]. Those studies highlight that relatively strong ambient stratification alters the growth of the Crow instability development. When the baroclinic vorticity ad-

vects the primary vortices towards each other (for lower stratification levels), it leads to a faster growth while, for higher stratification levels, it retards the growth and prevents significant vortex reconnection from ever starting. Hence, the time of reconnection is reduced for lower stratification levels. A decay in two phases was observed in the results. The starting time of the fast-decay phase was seen to decrease when increasing the stratification level. Different turbulence scenarios only play a significant role for the onset of rapid decay when stratification is neutral to weakly stable. The Crow instability development without stratification was also investigated in various LES studies [96, 46] and experimental works [69, 70].

Stable stratification has a great influence on the atmospheric turbulence. It leads to the formation of quasi-horizontal layers, often described as pancake structures (see [92] for a review). It also significantly affects the length scales and the spectra of the flow. The numerical investigation of stratified turbulence has been a subject of many numerical researches (see, e.g., [91, 74, 4, 34, 114]).

The objective of the present study is to investigate the influence, on wake vortex behavior, of the atmospheric stratification, the atmospheric turbulence and the combination of both stratification and turbulence. For that purpose, LES of wake vortices at very high Reynolds number are performed. The initial wake vortices are represented using a realistic model with a realistic core size ($r_c/b_0 = 0.05$). The simulations are run using a pseudo-spectral incompressible flow solver, in LES mode and with a relatively high resolution, able to capture much of the flow physics, and a subgrid scale model. The interest to work with such a pseudo-spectral code is that it avoids dispersion errors in the very thin structures generated in presence of stratification. Moreover, in order to avoid any spurious numerical dissipation, the used subgrid scale model only acts at very small scales without affecting the large to medium scales of the flow.

First, the development of short-wavelength instabilities solely induced by atmospheric stratification is studied. The results are obtained in a computational domain with limited axial extension so that long-wavelength instabilities are not considered. In these LES, the background field is stratified but not turbulent initially. Some initial disturbances are however introduced in the simulations by means of a random perturbation. In a second part, the effects, on wake vortex behavior, of atmospheric turbulence and atmospheric stratification combined with realistic turbulence are analyzed. In order to investigate the influence of the atmospheric conditions, turbulent fields (stratified or not)

are first generated and are then supplemented to the two-vortex system. These fields are three-dimensional, non-isotropic (for cases with stratification), unsteady and turbulent, so that prior LES are required to generate them. Five stratification levels are investigated, from neutral to very stable, and the associated turbulence level is low in all cases. In addition, the effect of the ambient turbulence level is also investigated for cases without stratification and with low and high stratification levels. In this part, the possible development of the Crow instability is also considered. Finally, we further focus on the case of a vortex system reaching an equilibrium with the ambient turbulence and for which the Crow instability development is not considered. Different turbulence levels are simulated and the influence, on the equilibrium vortex system properties, of the initial vortex circulation distribution profile is also studied.

The realistic representation of the background atmospheric flow and of the two-vortex system, the very high Reynolds number, the relatively high LES resolution, and the absence of numerical dissipation are all factors that contribute to this study establishing a step forward in the realistic simulation of aircraft wake vortices evolving in the atmosphere. Furthermore, the results of this investigation are used to propose and calibrate new simplified models on wake vortex transport and decay, to be used in operational systems, see Chapter 3. Since atmospheric stratification and turbulence have a strong impact on wake vortex transport and decay, an accurate real-time modeling of the phenomenon is indeed of primary importance.

This chapter is organized as follows. In Section 2.2, the governing equations and the numerical method are detailed. Section 2.3 presents the results of the investigation of wake vortices evolving in stratified atmospheres. In Section 2.4, the behavior of wake vortices evolving in turbulent and possibly stratified atmospheres is analyzed in depth. The possible Crow instability development is not considered in the first part of this section whereas it is taken into account in the results analyzed in the second part. Section 2.5 finally focuses on the analysis of the turbulent vortex system at equilibrium. The conclusions drawn are reported in Section 2.6.

2.2 Governing equations and numerical method

2.2.1 Governing equations

The governing equations are the Navier-Stokes equations for incompressible flows, using the Boussinesq approximation for the buoyancy term, and supplemented by a subgrid scale (SGS) model. In a coordinate system with (x, y) the horizontal directions and z the vertical direction, it reads:

$$\nabla \cdot \mathbf{u} = 0, \quad (2.1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} + \nabla \cdot \boldsymbol{\tau}^M + \frac{1}{\Theta_0} \mathbf{g} \tilde{\Theta}, \quad (2.2)$$

$$\frac{\partial \tilde{\Theta}}{\partial t} + (\mathbf{u} \cdot \nabla) \tilde{\Theta} + w \frac{d\bar{\Theta}}{dz} = \alpha \nabla^2 \tilde{\Theta} - \nabla \cdot \mathbf{q}^M, \quad (2.3)$$

where $\mathbf{u} = (u, v, w)$ is the LES velocity field, $P = \frac{p}{\rho}$ the reduced pressure, $\nu = \frac{\mu}{\rho}$ the kinematic viscosity, $\boldsymbol{\tau}^M$ the modeled SGS stress tensor, $\mathbf{g} = g \mathbf{e}_z$ the acceleration due to gravity, Θ_0 the reference temperature, α the thermal diffusivity and $\mathbf{q}^M$ the modeled SGS heat flux (with a unit turbulent Prandtl number). The potential temperature Θ is here decomposed into the mean linear vertical profile and the deviation around this mean:

$$\Theta(\mathbf{x}, t) = \bar{\Theta}(z) + \tilde{\Theta}(\mathbf{x}, t), \quad (2.4)$$

with $\frac{d\bar{\Theta}}{dz}$ constant. This enables to perform the simulations in a periodic domain. The Brunt-Väisälä frequency N , characterizing the stable stratification level, is then defined by:

$$N^2 = \frac{g}{\Theta_0} \frac{d\bar{\Theta}}{dz}. \quad (2.5)$$

2.2.2 Pseudo-spectral Navier-Stokes solver

The Navier-Stokes solver considered here is based on a Fourier-Galerkin pseudo-spectral methodology. The spectral approximation of the velocity vector $\mathbf{u}$ is symbolically written as

$$\mathbf{u}(\mathbf{x}, t) = \sum_{\mathbf{k}} \hat{\mathbf{u}}(\mathbf{k}, t) \exp(i \mathbf{k} \cdot \mathbf{x}), \quad (2.6)$$

where the spatial wavenumber vector is $\mathbf{k} = (k_x, k_y, k_z)$. The “ $\hat{\cdot}$ ” notation is used to identify Fourier transforms.

The time integration of Eqs. (2.2) and (2.3) are carried out in spectral space using a technique in which the convective and SGS model terms are marched explicitly using the 3rd order Williamson scheme [118]. The non-linear terms are evaluated using a pseudo-spectral algorithm and the dealiasing is done using a phase shift procedure as in Canuto *et al.* [10]. The incompressibility constraint of Eq. (2.1) (i.e., the effect of ∇P), is satisfied by reprojection of the velocity field into a divergence free space.

This code was initially developed by the team of D. Carati at the Université Libre de Bruxelles (ULB). It was then modified by the team of G. Winckelmans at UCL in order to be energy conserving and to be able to perform LES with various eddy-viscosity type SGS models (also multiscale models). Further details on the code can be found in [5]. The code was already successfully used for LES of very high Reynolds number flows (see [12, 6, 11]). In the framework of this work, the code was further adapted to take into account the evolution of the temperature using the Boussinesq approximation, following Eqs. (2.2) to (2.3).

For the studied flows, the Reynolds number Re_Γ , defined in Eq. (1.7), is very high so that we here perform LES “in the limit of very high Reynolds number”, where any molecular viscosity effect can be neglected. For all investigated cases, for short times (i.e., before vortex reconnection or break-up), the vortex core size remains approximatively constant. This indicates that the effective Reynolds number, as computed in [41], is also very high. The viscosity and diffusivity are then set to $\nu = \alpha = 0.0$. A comparison of the results of LES obtained using $Re_\Gamma = 10^7$ and $Pr = 1.0$ or using $Re_\Gamma \rightarrow \infty$ (i.e., using $\nu = \alpha = 0$) is performed in a complementary study, presented in Section 2.4.5. No differences are observed in the energy evolution nor in the vortex behavior, confirming that neither the molecular viscosity nor the molecular thermal diffusivity play any significant role in the flow physics of such very high Reynolds number LES.

2.2.3 Subgrid scale model

The numerical solving of the Boussinesq equations for such very high Reynolds flows required a SGS dissipation term acting solely at the small scales of the LES grid, without affecting the large- to medium-scale dynamics. Such a model is mandatory to properly capture the dynamics of the large scale coherent vortices. A high order hyperviscosity (HV) model (as in Borue & Orszag [3])

and Cocle *et al.* [11]) has here been used. This model was already used in various LES studies of stratified turbulence (see, e.g., [50, 115, 116, 74]) and in various very high Reynolds number flows LES studies (see [11], also for wake vortex flows, see [6, 12, 8]). Cocle *et al.* [11] showed that, for such flows, the HV model is superior to the classical Smagorinsky SGS model (defined in [106]). Although it very quickly dissipates the very small scales, it indeed preserves the large- to medium- scale dynamics as it dissipates energy only in a narrow range at the highest wavenumbers; as do multiscale models, such as the regularized variational multiscale (RVM) model (defined in [62] and also investigated in [11]). Moreover, using a pseudo-spectral code, the HV model is numerically much less expensive than the RVM model, as it does not require any additional Fourier transform of the fields. In a complementary study, presented in Section 2.4.5, LES of vortices evolving in a turbulent and stratified atmosphere are performed using, as SGS model, both the RVM model and the HV model. The obtained results show a very good agreement.

High order hyperviscosity SGS model

In the present HV formulation, the SGS stress tensor leads, in spectral space, to:

$$\widehat{\nabla \cdot \boldsymbol{\tau}^M}(\mathbf{k}) = (-1)^p \nu_h k^{2(p+1)} \widehat{\mathbf{u}}(\mathbf{k}), \quad (2.7)$$

where $k = ||\mathbf{k}||$. In a similar way, for the temperature SGS dissipation, we have:

$$\widehat{\nabla \cdot \mathbf{q}^M}(\mathbf{k}) = (-1)^p \alpha_h k^{2(p+1)} \widehat{\Theta}(\mathbf{k}). \quad (2.8)$$

The hyperviscosity coefficient ν_h is taken constant and computed as in Borue & Orszag [3] using $\nu_h (k_{max})^{2(p+1)} \Delta t = 0.5$, with $k_{max} = \pi/h$. The results of this chapter were obtained using $Pr_h = \frac{\nu_h}{\alpha_h} = 1$ and $p = 7$.

Regularized variational multiscale SGS model

The regularized variational multiscale (RVM) models [62] use the high-pass filtered (HPF) velocity field $\mathbf{u}^s$ which is computed from the difference between the LES velocity field, $\mathbf{u}$, and the low-pass filtered field, $\overline{\mathbf{u}}^{(n)}$ obtained using a higher order (order $2n$) filter.

$$\mathbf{u}^{s(n)} = \mathbf{u} - \overline{\mathbf{u}}^{(n)}. \quad (2.9)$$

In Fourier space, the obtained filtered field is

$$\begin{aligned}\overline{\hat{\mathbf{u}}}^{(n)}(\mathbf{k}) &= G^{(n)}(k_x h) G^{(n)}(k_y h) G^{(n)}(k_z h) \hat{\mathbf{u}}(\mathbf{k}) \\ &= (1 - \sin^{2n}(k_x \Delta/2)) \\ &\quad (1 - \sin^{2n}(k_y \Delta/2)) \\ &\quad (1 - \sin^{2n}(k_z \Delta/2)) \hat{\mathbf{u}}(\mathbf{k}).\end{aligned}\tag{2.10}$$

The RVM model is then taken as

$$\tau_{ij}^M = 2\nu_{sgs} S_{ij}^s, \tag{2.11}$$

where S_{ij}^s is the strain rate tensor of the HPF field (written in physical space but computed in Fourier space):

$$S_{ij}^s = \frac{1}{2} \left(\frac{\partial u_i^s}{\partial x_j} + \frac{\partial u_j^s}{\partial x_i} \right). \tag{2.12}$$

This means that the SGS model only acts on the small scale content of the LES field thus preserving the dynamics of the large scales. Here, we consider the subclass of RVM models, noted RVMs, where the SGS viscosity is itself computed using the HPF field (small scale content of the LES field):

$$\nu_{sgs} = \nu_{sgs}^s = C \Delta^2 (2 S_{ij}^s S_{ij}^s)^{1/2}. \tag{2.13}$$

The model coefficient has been calibrated by Cogle *et al.* [11] to $C = 0.11$ with $n = 3$. Concerning the SGS heat flux model, $\mathbf{q}^M$, it is defined as:

$$\mathbf{q}^M = -\alpha_{sgs} \frac{\partial \tilde{\Theta}^s}{\partial x_i}. \tag{2.14}$$

The SGS thermal diffusivity α_{sgs} is here chosen so that $Pr_{sgs} = \frac{\nu_{sgs}}{\alpha_{sgs}} = 1.0$.

2.3 Wake vortices evolving in stratified atmospheres

2.3.1 Numerical set-up

The initial wake vortex flow is composed of a pair of vortices of circulation Γ_0 and $-\Gamma_0$, and separated by a distance b_0 . A low order algebraic velocity profile (see Table 1.3) is used for each vortex. The radius of maximum tangential velocity, is set to $r_c = 0.05 b_0$. In addition to this velocity field, a random isotropic perturbation is added to the velocity field with a maximum amplitude set to $10^{-3} u_{\theta, \max}$ ($u_{\theta, \max} = \frac{\Gamma_0}{4\pi r_c}$). The perturbation will quicken the development of 3-D instabilities without affecting the flow physics. Four simulations are performed using different stratification levels: $N^* = 0.35, 0.75, 1.0$ and 1.4 . Those values are chosen so as to be representative of what can be typically measured in the atmosphere (see also Table 2.1) and to allow comparisons with the literature results. The periodicity lengths are set to $L_x \times L_y \times L_z = b_0 \times 6 b_0 \times 6 b_0$ using $64 \times 384 \times 384$ grid points. The extension in the y and z directions limits the influence of the periodicity and enables to obtain valid results for large times. Moreover, since this study focuses on the development of short-wave instabilities created by stratification effects, we can limit the periodicity length in the axial direction x . The numerical resolution is $\frac{h}{b_0} = \frac{1}{64}$, which ensures that the dynamics of the vortices are properly captured ($r_c/h \geq 3$). It should also be noted that the use of a pseudo-spectral code enables to better handle small-scale structures, compared to other numerical methods such as finite differences.

2.3.2 Characterization of the vortex flow

Flow visualization

We provide, in Figs. 2.1 to 2.5, visualizations of the flow, using iso-surfaces of vorticity, for all investigated cases.

For all cases, at $t^* = 0$, one observes the two vortex cores as well as the initial perturbation. At early stages, the wake vortices descent at a constant speed and the initial separation stays constant (as it will be further quantified hereafter). Later on, due to the vertical mean density gradient, differences in density appear between the fluid carried by the wake vortices and the fluid outside the “Rankine-like” oval. Local density gradients hence appear at the

oval interface. When those gradients are not aligned with the pressure gradient, it then leads to the creation of baroclinic vorticity. Using the Boussinesq approximation, this baroclinic vorticity production term reads:

$$\left. \frac{\partial \boldsymbol{\omega}}{\partial t} \right|_B = \frac{1}{\Theta_0} g \left(\frac{\partial \tilde{\Theta}}{\partial y} \mathbf{e}_x - \frac{\partial \tilde{\Theta}}{\partial x} \mathbf{e}_y \right). \quad (2.15)$$

Those secondary structures are subjected to short-wave instabilities which grow in time, eventually leading to a complex turbulent vortex system. At short times, the baroclinic vorticity creation is mostly due to the first term in the rhs of Eq. (2.15) and is a 2-D effect. Later on, when the flow becomes 3-D, both terms become significant.

For the case with $N^* = 0.35$, significant secondary vorticity structures appear around $t^* = 2.5$. Small-scale instabilities then develop in this secondary vorticity from $t^* = 3.0$. As explained in [104], for vortices sinking in a stratified atmosphere, baroclinic vorticity is created at the boundary of the Rankine oval, and the upper stagnation point is found to move inside the original boundary. Consequently, the fluid between the Rankine oval and the new separating streamline is detrained (i.e., the vorticity is left in a “wake” of the two-vortex system). For $t^* \geq 3.5$, a detrainment of the secondary vorticity is clearly observed. At $t^* = 4.5$, the baroclinic vorticity starts to strongly interact with the primary vortex system. This phenomenon is even more pronounced at $t^* = 5.0$ and induces a fast decay (as characterized hereafter).

For the case with $N^* = 0.75$, secondary baroclinic vorticity structures are already noticeable at $t^* = 1.5$. Short-scale instabilities significantly appear in those structures around $t^* = 2.5$. At $t^* = 3.0$, there is a strong exchange of circulation between the surrounding vorticity and the primary vortex system. Moreover, the secondary vorticity tends to bring the primary vortices closer to each other. This induces a stronger interaction between the vortices, also influencing the descent rate and the decay. Eventually, at $t^* = 3.5$ and later on, the flow is so complex that one can no longer distinguish the vortex cores on the 3-D visualizations. However, on the longitudinally averaged flow, one can still observe coherent primary vortices. The detrainment of the secondary vorticity is seen to be less significant than for the case with $N^* = 0.35$. This is due to the fact that the vortices tend to rebound (as quantified hereafter). The “wake” of the two-vortex system, generated when it sinks, hence remains closer.

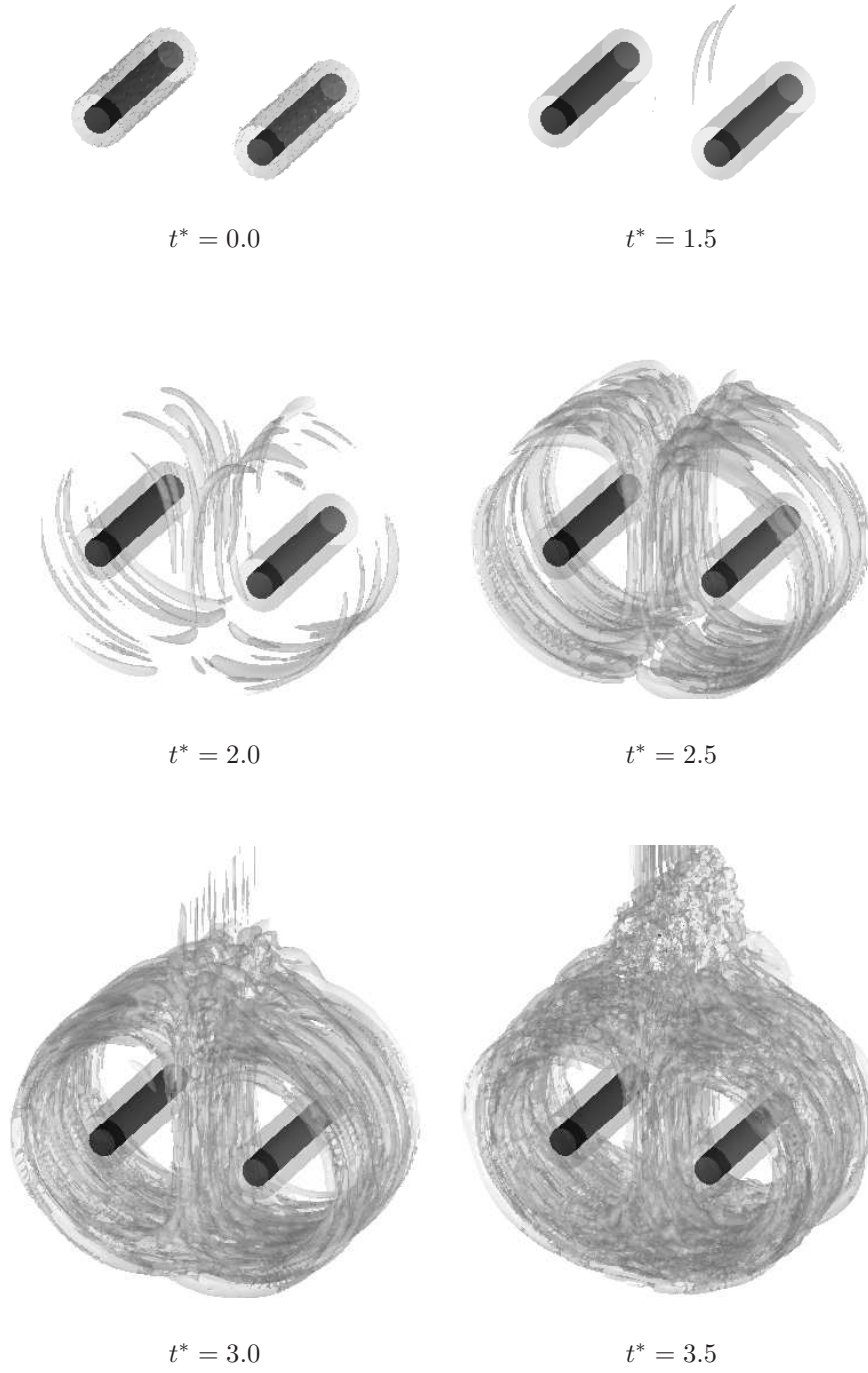


Figure 2.1: Case $N^* = 0.35$: iso-surfaces of $\|\boldsymbol{\omega}\| b_0^2/\Gamma_0 = 1$ and 10 colored by $\omega_x b_0^2/\Gamma_0$ at different times.

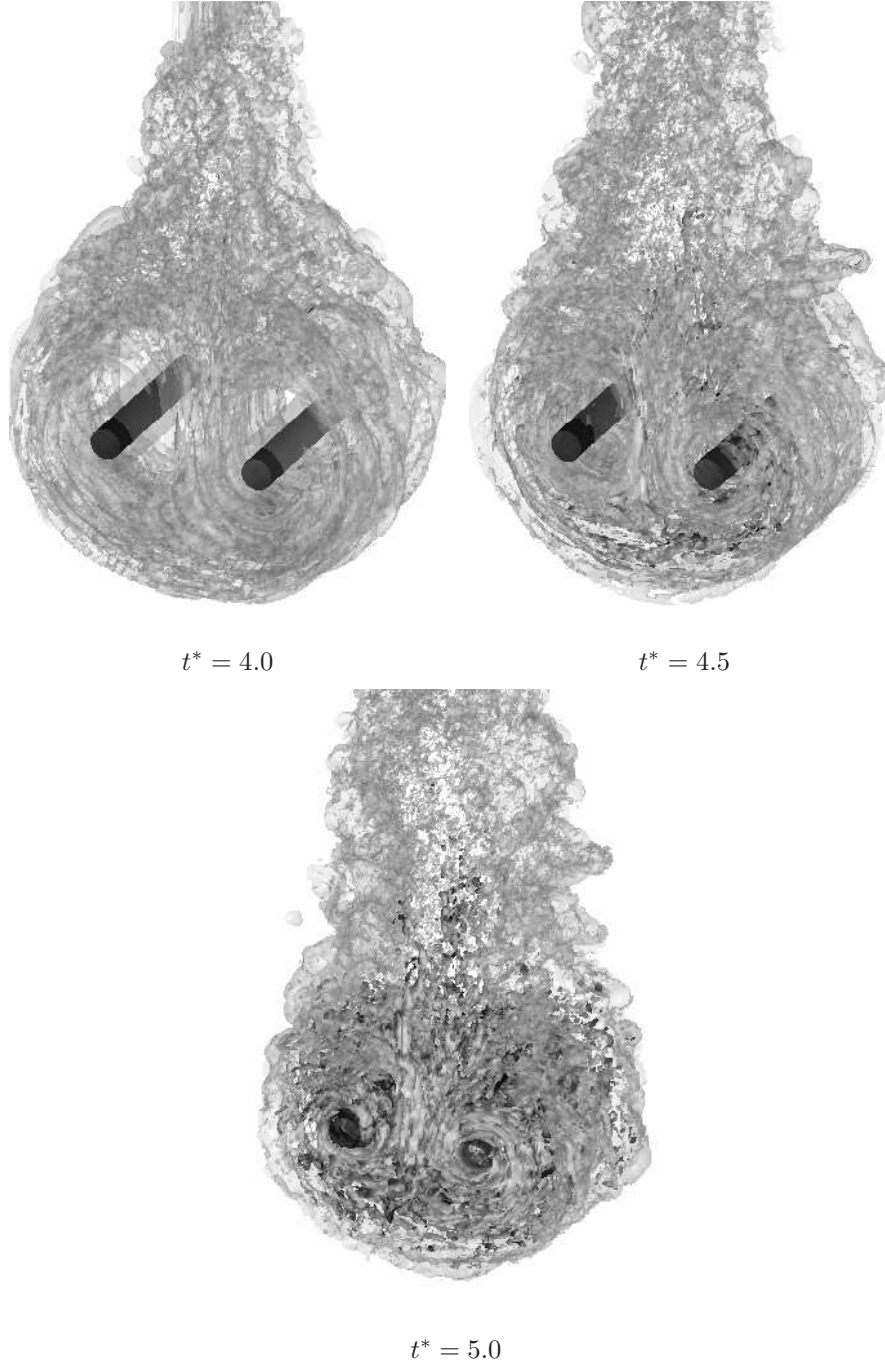


Figure 2.2: Case $N^* = 0.35$: iso-surfaces of $\|\boldsymbol{\omega}\| b_0^2/\Gamma_0 = 1$ and 10 colored by $\omega_x b_0^2/\Gamma_0$ at different times.

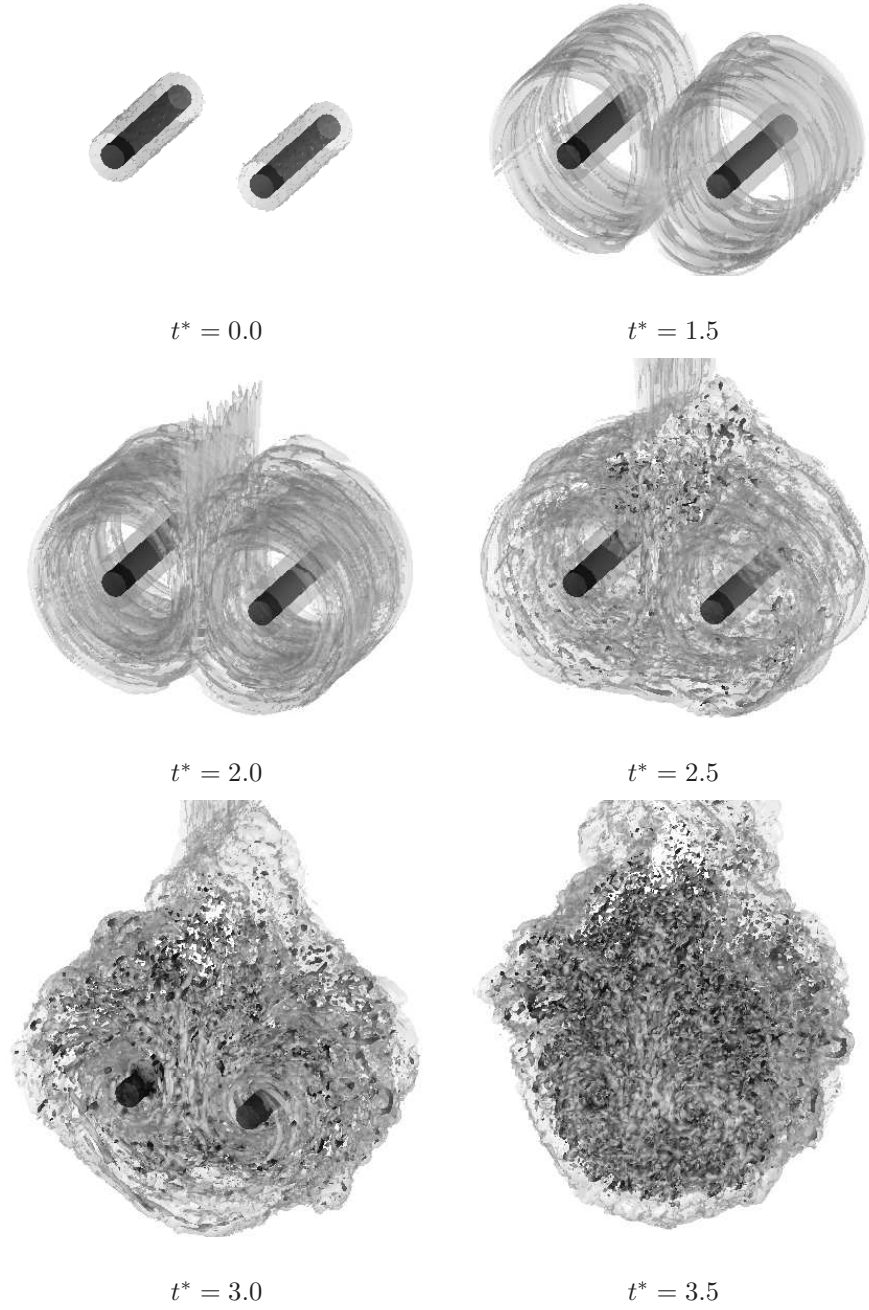


Figure 2.3: Case $N^* = 0.75$: iso-surfaces of $\|\boldsymbol{\omega}\| b_0^2/\Gamma_0 = 1$ and 10 colored by $\omega_x b_0^2/\Gamma_0$ at different times.

For the case with $N^* = 1$, at early times ($t^* \leq 2.0$), one observes the same global behavior as that of the case with $N^* = 0.75$. Of course, as the stratification level is higher, the baroclinic vorticity production is higher. Significant secondary vorticity structures hence appear earlier and are sooner subjected to small-scale instabilities. At $t^* = 2.5$, the primary vortices are still coherent and surrounded by a cloud of secondary turbulent vorticity. For $t^* \geq 2.5$, the strong 3-D interactions between this secondary vorticity and the primary vortices lead to a fast decay of the primary vortex circulation (as further quantified). The secondary vorticity tends also to bring the primary vortices closer to each other for this case.

The physics of the case with $N^* = 1.4$ is a bit different than that of the previous cases. The baroclinic vorticity is indeed generated (at very early stages of the flow) on the Rankine oval mainly in the inner region between the vortices whereas, for the other cases, it was mainly generated in the outer region. This leads the primary vortices to move apart. However, the strong secondary vorticity structures are very unstable and start to strongly interact with the primary vortices for times $t^* \geq 2.0$. The two primary vortices then do not interact with each other but the baroclinic vorticity intensity is very high so that a very strong decay of the vortex is also observed. For the cases with $N^* = 1.0$ and 1.4 , the vortices significantly rebound (as quantified hereafter). A detrainment of vorticity is thus only observed when the two-vortex system sinks.

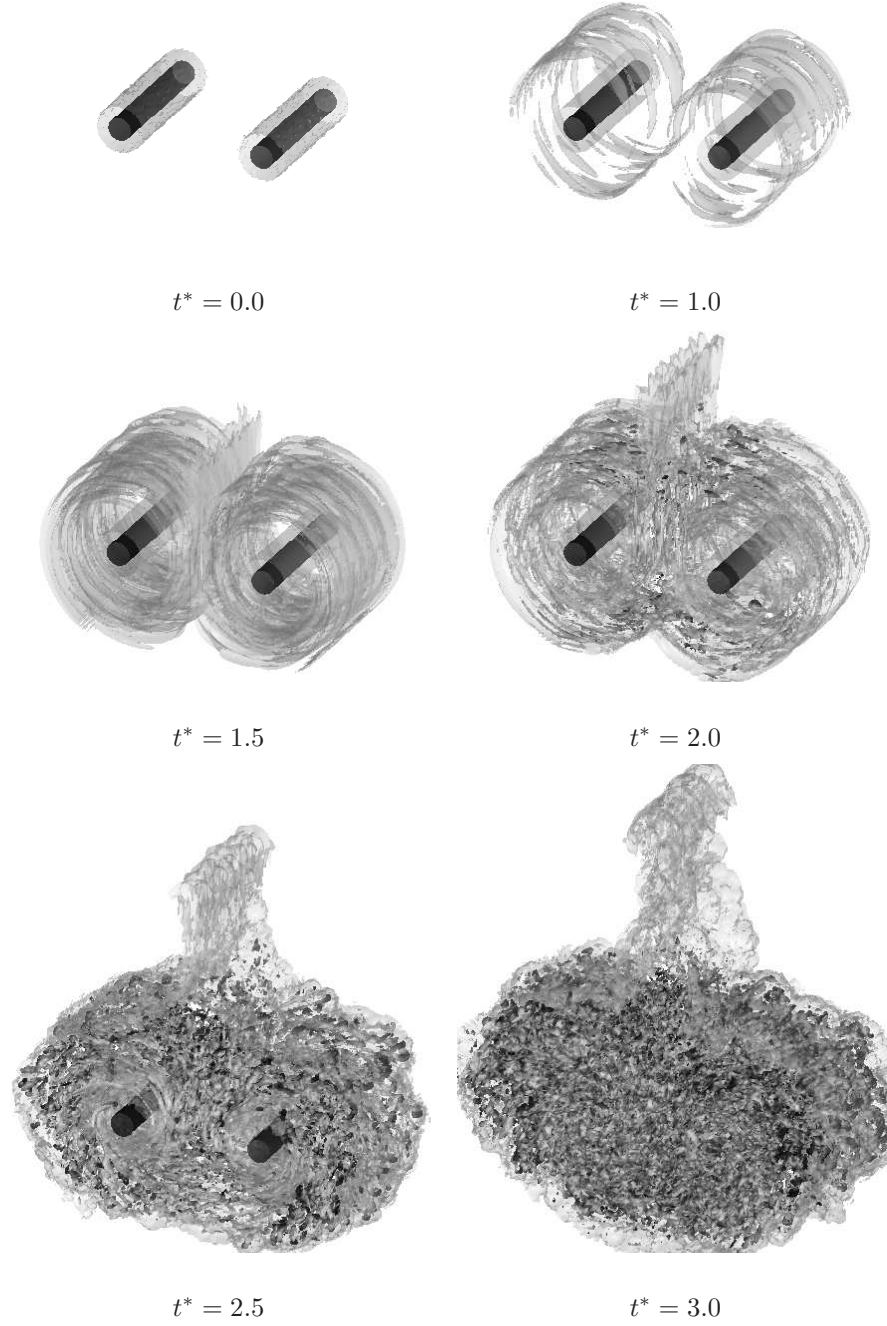


Figure 2.4: Case $N^* = 1.0$: iso-surfaces of $\|\omega\| b_0^2 / \Gamma_0 = 1$ and 10 colored by $\omega_x b_0^2 / \Gamma_0$ at different times.

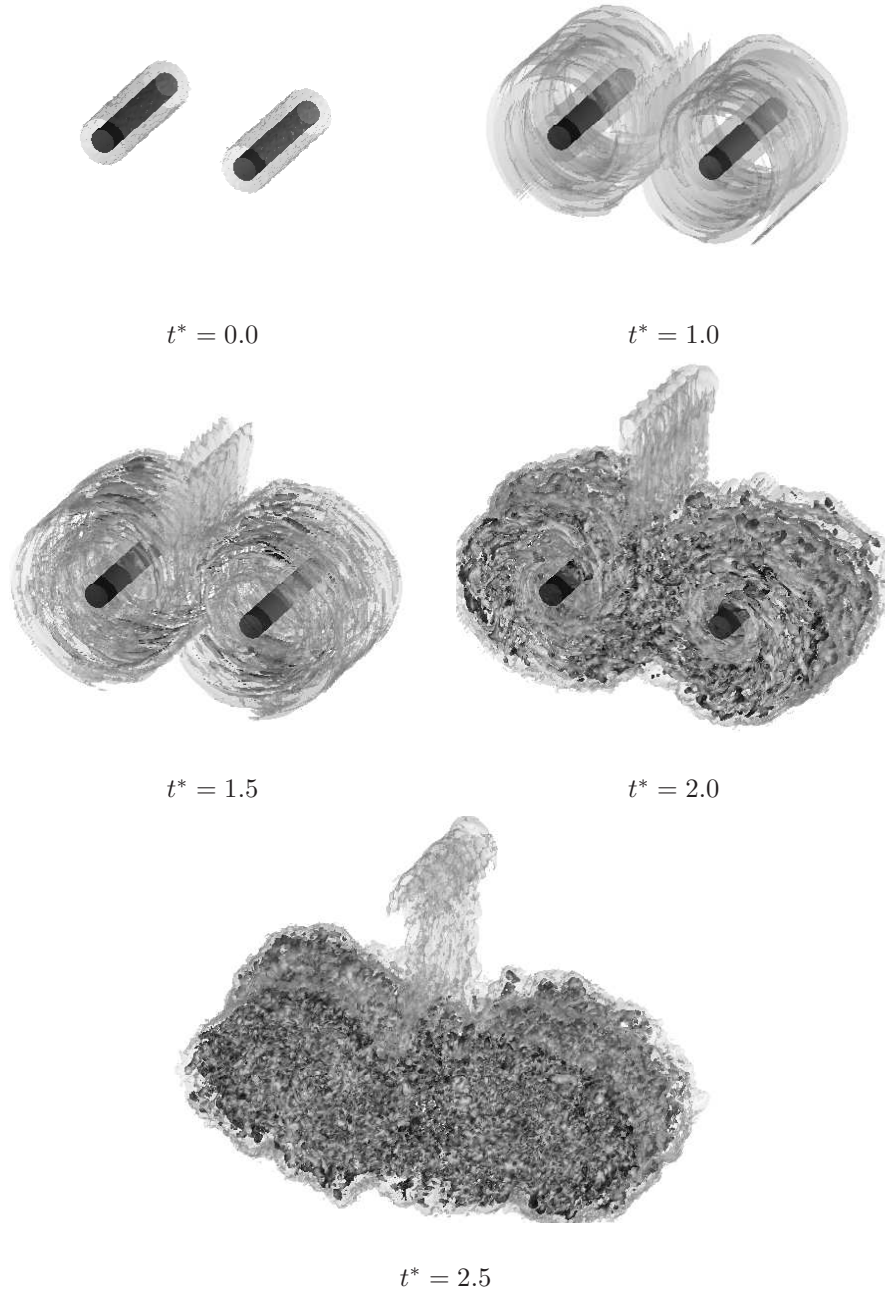


Figure 2.5: Case $N^* = 1.4$: iso-surfaces of $\|\boldsymbol{\omega}\| b_0^2/\Gamma_0 = 1$ and 10 colored by $\omega_x b_0^2/\Gamma_0$ at different times.

Vortex trajectories

For the vortex global characterization, we compute the diagnostics on the longitudinally averaged vorticity fields:

$$\overline{\omega}_x(y, z, t) = \frac{1}{L_x} \int_0^{L_x} \omega_x(\mathbf{x}, t) dx. \quad (2.16)$$

As the domain axial extension is limited ($L_x = b_0$), no long-wave instability can develop in the flow.

The stratification effects have a strong impact on the vortex trajectory. In order to measure the mean displacement of the vortices, we compute the position of the vortex centroid, based on the axially averaged flow.

$$\Gamma = \int_{\Omega} \overline{\omega}_x(y, z, t) d\Omega, \quad (2.17)$$

$$y_c(t) = \frac{1}{\Gamma} \int_{\Omega} y \overline{\omega}_x(y, z, t) d\Omega, \quad (2.18)$$

$$z_c(t) = \frac{1}{\Gamma} \int_{\Omega} z \overline{\omega}_x(y, z, t) d\Omega, \quad (2.19)$$

where Ω is a disk centered on the centroid and with a radius $r = \frac{b_0}{4}$ and Γ is the circulation evaluated on this area. This is thus an iterative procedure.

Figs. 2.6 and 2.7 compare, for the four investigated stratification levels, the descent height, z_c , and the vortex lateral separation, defined by:

$$b_c = y_{c,\text{right}} - y_{c,\text{left}}. \quad (2.20)$$

Apart from the initial perturbation, the initial flow is 2-D. For the four stratification levels, for short times ($t^* \leq 0.6$), the vortices are not disturbed by the stratification effects, hence they sink at a constant velocity V_0 with a constant lateral separation b_0 . This is an indicator that the neighbor vortex pairs (in the y and z directions due to the periodic conditions) are far away enough as they do not affect the descent rate. Later on, we observe a decrease of the descent rate, controlled by the stratification level; the higher N^* , the sooner and the more pronounced is this speed reduction. For $N^* \geq 1$, the vortices even rebound: at $t^* \approx 1.4$, for $N^* = 1.4$, and at $t^* \approx 2.1$, for $N^* = 1.0$. In the $N^* = 0.75$ case, the vortices tend to rebound at time $t^* \approx 3$ but other phenomena are acting. Indeed, as observed in Fig. 2.7, for the cases with $N^* \leq 1$, as the vortices tend to rebound, they come closer to each other ($\frac{b_c}{b_0} <$

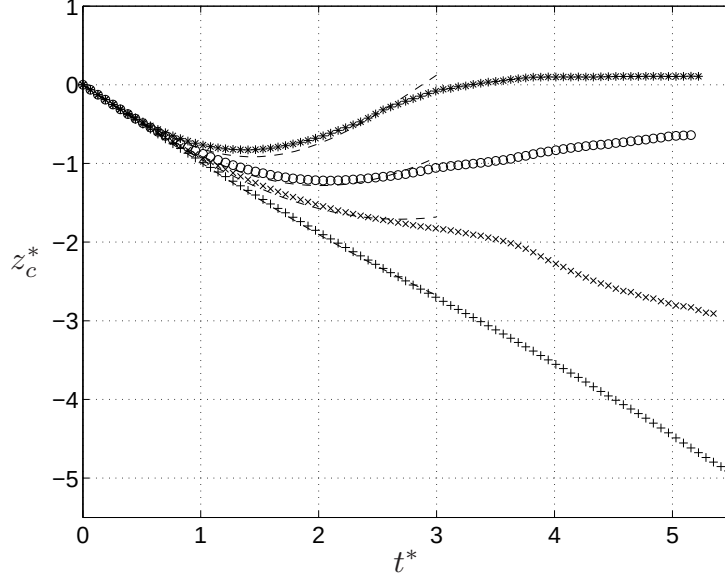


Figure 2.6: Evolution of the vortex height: $N^* = 0.35$ (+), $N^* = 0.75$ (x), $N^* = 1.0$ (o) and $N^* = 1.4$ (*). The results of the adapted Saffman's model, using $\alpha_{str} = 0.78$ in Eq. (2.21), are also shown (dash).

1). This closeness of the vortices balances the stratification effects. Hence, the rebound, with $N^* = 1$, becomes less significant while, with $N^* = 0.75$, the vortices actually increase their sink rate. On the contrary, for $N^* = 1.4$, the two vortices move away from each other ($\frac{b}{b_0}$ up to 2.4), become almost independent, and rebound up to the generation altitude at $t^* \approx 3$. This is in good agreement with previous studies (see, for instance, [57]). It is worth to recall that, for large aircraft, $t_0 \approx 30$ s; hence, with $N^* = 1.4$, the vortices would be back to the flight altitude after 90s and would stay there, although much decayed (see later).

The trajectory can be compared to an adaptation of the simplified trajectory model proposed by Saffman [99] (see further explanations in Chapter 3, Section 3.2.3). In this model, the vortex altitude is described by

$$z_c^*(t^*) = h_0^* - \frac{1}{\alpha_{str} N^*} \sin(\alpha_{str} N^* t^*). \quad (2.21)$$

Saffman proposed $\alpha_{str} = 0.65$, based on geometrical arguments related to the Rankine oval area. Note that this model also corresponds to that proposed by Greene [45]. Based on the present results, $\alpha_{str} = 0.78$ is seen to better fit the

altitude evolution for time t^* up to 3, see Fig. 2.6, and is thus retained.

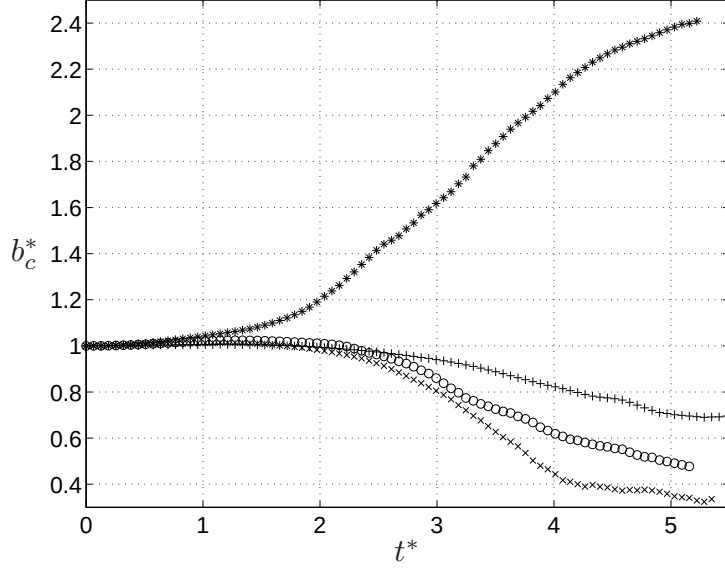


Figure 2.7: Evolution of the vortex spacing: $N^* = 0.35$ (+), $N^* = 0.75$ ($\times$), $N^* = 1.0$ (o) and $N^* = 1.4$ (*).

Circulation decay

To characterize the flow, the half plane total circulation $\Gamma_{\text{HP}}(t)$, i.e., only considering the right-hand side (or rhs) part of the averaged wake vortex system, is computed according to:

$$\Gamma_{\text{HP}}(t) = \int \int_{y \geq 0} \overline{\omega}_x(y, z, t) dy dz. \quad (2.22)$$

A similar definition can be applied to the region $y \leq 0$ and which is identified as the left-hand side (or lhs) plane. Without stratification, the averaged half-plane circulation Γ_{HP} is to remain constant until there is a net exchange of vorticity between the two half planes due to the turbulent interaction between the vortices. The evolution of the half plane circulation is provided in Fig. 2.8. With stratification, when the vortices descend, the production of opposite sign baroclinic vorticity decreases the circulation. The difference in the baroclinic production terms as a function of the stratification level is clearly noticeable. Later on, when the vortex system stops to sink and rebounds, the produced baroclinic vorticity has the same sign as the primary vortex, the half plane

circulation hence re-increases leading to an sinusoidal evolution of Γ_{HP} with a frequency proportional to N^* . The time at which $\Gamma_{\text{HP}} = 0$ corresponds approximately to the time of rebound (or the time at which the vortices tend to rebound for the case with $N^* = 0.75$), observed in Fig. 2.6. When the lateral separation of the vortices significantly decreases (cases with $N^* = 0.75$ and 1.0), the wake vortex mutually-induced velocity increases and there is a strong exchange of circulation between the primary vortices. This hence modifies the sinusoidal shape of the half plane circulation curves at later times. It is important to stress that Γ_{HP} only represents the integral of the vorticity but does not provide any information about its distribution. In particular, a negative Γ_{HP} does not imply a change of sign of the circulation associated to the wake vortices. It only indicates that the integral of the baroclinic vorticity is greater (in absolute value) than the circulation of the wake vortex.

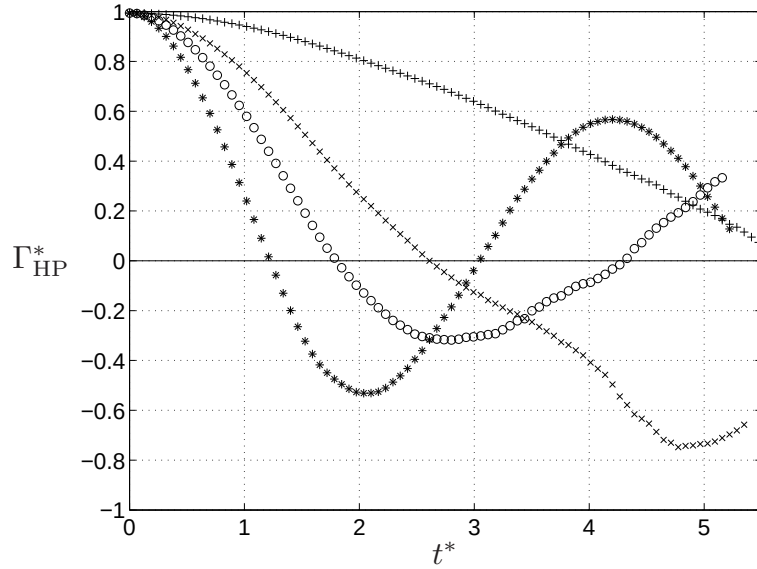


Figure 2.8: Evolution of half plane circulation Γ_{HP} : $N^* = 0.35$ (+), $N^* = 0.75$ (x), $N^* = 1.0$ (o) and $N^* = 1.4$ (*).

A quantity of particular importance in wake vortex related hazards is the vortex circulation: at fixed core radius, the higher the circulation, the higher the induced rolling moment on a following aircraft. The circulation distribution $\Gamma(r)$ is here obtained by integration of the averaged axial vorticity component

on a disk of radius r centered on the vortex centroid:

$$\Gamma(r, t) = \int_0^{2\pi} \int_0^r \bar{\omega}_x(r', \theta, t) r' dr' d\theta. \quad (2.23)$$

Figure 2.9 compares, for the four investigated stratification levels, the circulation distribution at five times. We clearly observe a significant circulation decay, that is also function of the stratification level. The figure also reveals that the baroclinic vorticity is mostly created away from the vortex core (i.e., at the boundary of the Rankine oval). For instance, for the case with $N^* = 0.75$ at $t^* = 2.5$, the circulation associated to the baroclinic vorticity is about the same as that associated to the wake vortices (indeed, $\Gamma_{HP} \simeq 0$, see Fig. 2.8). However, $\Gamma(r)$ starts to decrease only from $r^* \simeq 0.35$, indicating that the opposite sign baroclinic vorticity is significant only for $r \geq 0.35$. This is consistent with the visualization of Fig. 2.3. Later on, the baroclinic vorticity interacts with the wake vortices, see Fig. 2.3, leading to a circulation decay in the region with $r^* \geq 0.1$.

In order to further characterize the obtained results, the wake vortex decay is examined using the evolution of the total circulation, defined in Eq. (1.14). The evolution of Γ_{tot} is provided in Fig. 2.10. The partially averaged circulation Γ_{5-15} , defined in Eq. (1.17), is also computed (assuming $\frac{b_0}{b} = \frac{\pi}{4}$). The Γ_{5-15} circulation values, as well as the region of integration that has been used to obtain them, are also provided in Fig. 2.9. The evolution of the Γ_{5-15} is provided in Fig. 2.10.

Using both Γ_{tot} or Γ_{5-15} , we clearly observe a two phase decay of the mean flow. The first decay phase is very slow, the circulation is almost conserved. This is also an indicator of the quality of the simulation: there is no spurious SGS nor numerical dissipation; the decay is solely due to the 2-D stratification effect, as seen in the $\Gamma(r)$ evolution for short times in Fig. 2.9. The rapid decay phase is due to the complex 3-D and turbulent interactions. The transition time between the two phases, t_d^* , depends on the stratification level ($t_d^* \approx 2$ for $N^* = 1.4$, $t_d^* \approx 2.4$ for $N^* = 1.0$, $t_d^* \approx 2.9$ for $N^* = 0.75$, and $t_d^* \approx 4.5$ for $N^* = 0.35$). The values of those t_d^* are somewhat sensitive to the initial perturbation level, and should thus not be used “as such”, but the difference between the t_d^* for different N^* using the same initial perturbation is relevant. Note that we will have a much better prediction of t_d^* when considering atmospheric stratification with weak, yet realistic, turbulence (see Section 2.4).

The strong decay in the second phase is governed by two phenomena. First,

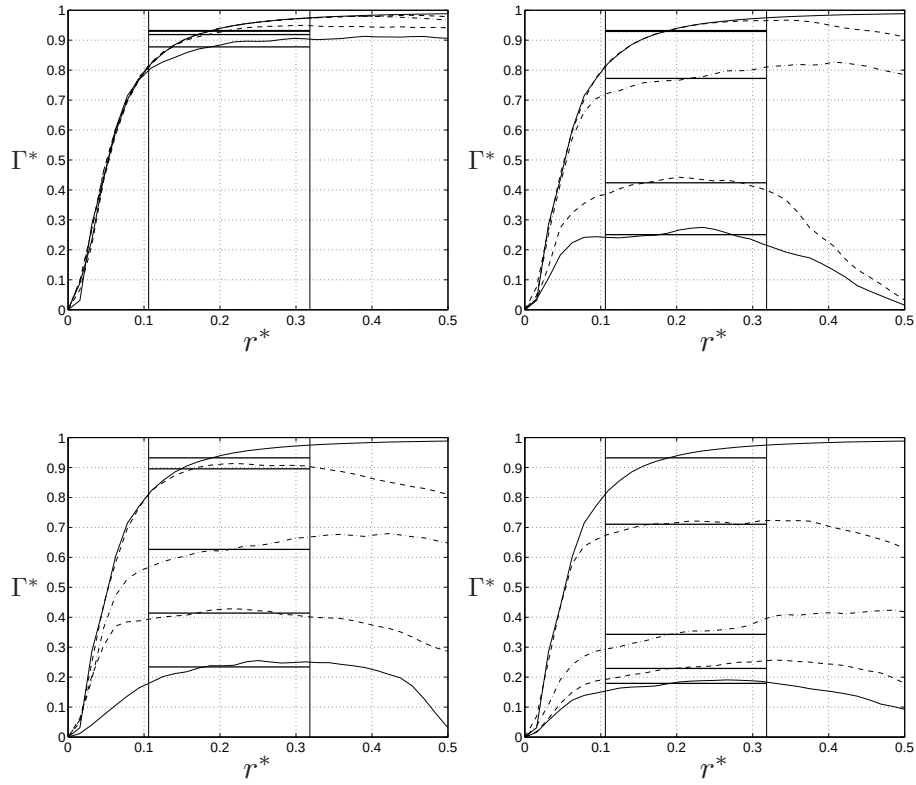


Figure 2.9: Circulation distribution, $\Gamma(r)$, at time $t^* = 0$ (solid), $t^* = 2.5$ (dash), $t^* = 3.5$ (dash dot), $t^* = 4.5$ (dash thin), and $t^* = 5.1$ (thin solid): $N^* = 0.35$ (top left), $N^* = 0.75$ (top right), $N^* = 1.0$ (bottom left) and $N^* = 1.4$ (bottom right). For each time, the corresponding Γ^*_{5-15} value is also shown (thick horizontal line) as well as the region of integration that has been used to obtain the value ($\frac{b}{12} \leq r \leq \frac{b}{4}$)

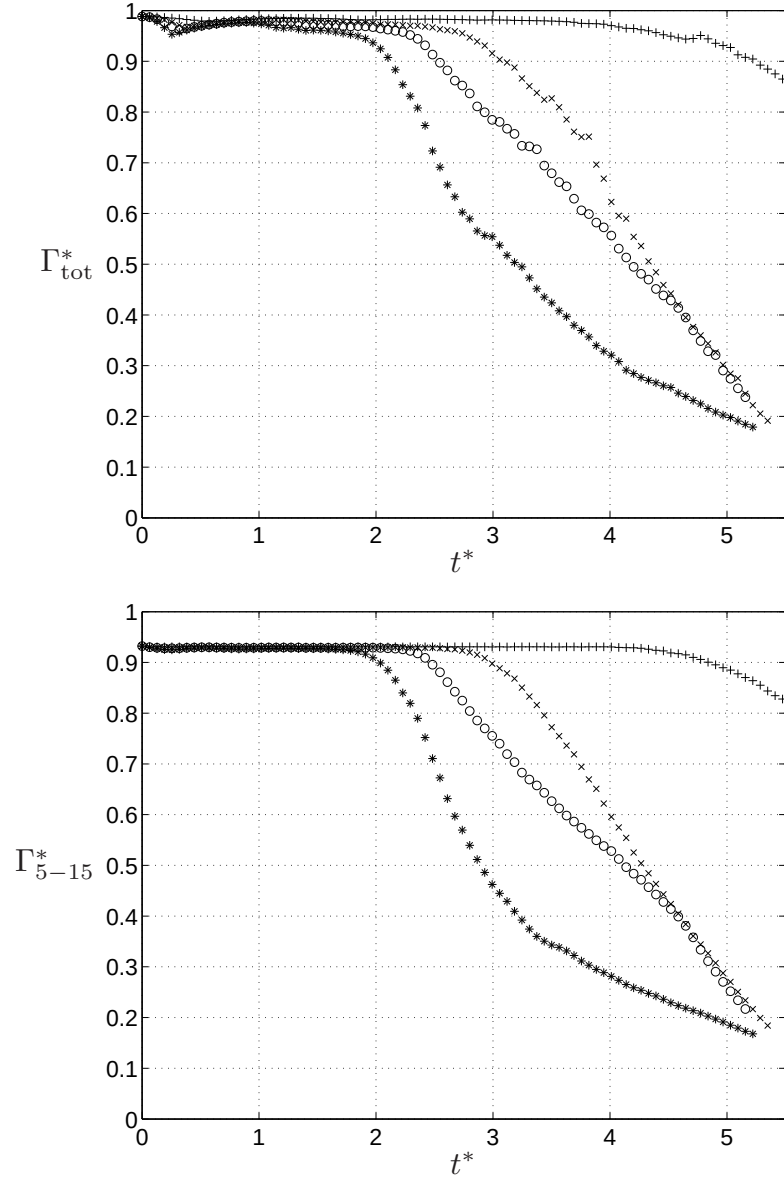


Figure 2.10: Evolution of Γ_{tot} (top) and Γ_{5-15} (bottom): $N^* = 0.35$ (+), $N^* = 0.75$ (x), $N^* = 1.0$ (o) and $N^* = 1.4$ (*).

there is the 3-D interaction between the primary vortices and the baroclinic vorticity, generated by the stratification effects, surrounding them. The decay due to this effect depends on the stratification level. This is the only decay mechanism for the case with $N^* = 1.4$. Then, it is also worth to note that, at $t^* = 3.0$, when the vortices are back to their initial altitude, the maximum circulation is $\Gamma_{\text{tot}} \approx 0.5 \Gamma_0$: the potentially encountered vortices are thus less hazardous than initially. For the cases with $0.75 \leq N^* \leq 1$, the closeness of the primary vortices also leads to strong 3-D interaction between them and enhances the circulation decay. The decay then also depends on the lateral distance between the vortices b_c . Since b_c is smaller for the case with $N^* = 0.75$ than for the case with $N^* = 1.0$, the decay rate is also higher.

2.3.3 Conclusions on wake vortices evolving in stratified atmospheres

We have performed numerical simulations of wake vortices interacting evolving in stably stratified conditions and without initial turbulence (yet with random perturbation of maximum amplitude $10^{-3}u_{\theta,\text{max}}$). Four stratification levels were investigated, from low to very high. Visualizations of the flow were used to help understand the flow dynamics and relevant diagnostics were computed to analyze and compare the wake vortex behavior in the different atmospheric conditions.

As expected, the behavior is first essentially two-dimensional. Later, three-dimensional interactions between the primary vortices and the instabilities generated by the stratification strongly affect the trajectory and induce turbulence, and hence fast decay.

The produced baroclinic vorticity leads the vortices to rebound and, for moderate N^* , to come closer to each other (and hence to enhance their descent rate). For moderate N^* , those effects are competing, leading to an attenuation of the vortex rebound. An adaptation of Saffman's model has also been fitted on the altitude evolution. The model appears to be reliable for time up to $t^* \approx 3$.

A two-phase decay is observed for all cases. The starting time of the rapid decay phase is a function of the stratification level. The decay mechanism is not identical for moderate and high stratification levels. For moderate levels, the decay is also due to the strong interaction between the two primary vortices that have been brought closer by stratification effects; for higher levels, and

after vortex rebound, the vortices move away from each other, and the decay is caused by the interaction with the secondary baroclinic vorticity. In both cases, at late stages of the flow, the baroclinic secondary vorticity is mixed leading to the creation of complex turbulent structures surrounding the primary vortices and then strongly interacting with them (in fact also leading to short wave instabilities developing in the vortex cores).

Those simulation results allow a better understanding and characterization of the flow mechanisms for fairly realistic wake vortices evolving in stratified environments at very high Reynolds number. However, the atmospheres, in which the vortices evolve, are not totally realistic because they are not initially turbulent. The objective of the next section is hence to go a step further in terms of realism by investigating the combined effects of atmospheric stratification and turbulence.

2.4 Wake vortices evolving in stratified and weakly turbulent atmospheres

2.4.1 Objectives, methodology and set-up

The aim of the simulations performed in this section is to investigate the combined effects of atmospheric stratification and weak turbulence on the wake vortex behavior. Compared to the simulations performed in Section 2.3, further attention is paid to the realistic representation of the initial ambient stratified fields.

In order to simulate the wake vortices in realistic conditions, for each stratification level, stratified turbulent fields are first generated, using LES of forced stratified turbulence. The fields are obtained by forcing the low wave numbers of the flow using a forcing with a fixed amplitude $\frac{dE_f}{dt} = \epsilon_f$ and a random phase. The stratified turbulence evolves, with this forcing, until it reaches a statistically converged state (based on energy and dissipation evolutions). This forcing technique was employed for studies of vortices evolving in turbulent (and possibly stratified) atmospheres [46, 111, 84] and for studies of stratified turbulence (see, e.g., [114]).

Once the ambient turbulent (and possibly stratified) field is obtained, a pair of wake vortices is superimposed to the field. The initial wake vortex flow is composed of a pair of horizontal vortices of circulation Γ_0 and $-\Gamma_0$, aligned with the x -direction and separated in the y -direction by a distance b_0 . A low order algebraic velocity profile is used to model each vortex initially (see Table 1.3). The core size r_c is here set to $r_c = 0.05 b_0$, which is fairly realistic for aircraft wake vortices (where $r_c/b_0 \simeq 0.03 - 0.05$). The present vortex system model is representative of the wake of an aircraft after roll-up.

The turbulence level is here characterized through the dimensionless forcing amplitude $\epsilon_f^* = \epsilon_f b_0 / V_0^3$. Two energy forcing levels are investigated: $\epsilon_1^* = 2.42 \cdot 10^{-4}$ and $\epsilon_2^* = 2.42 \cdot 10^{-5}$. For an unstratified atmosphere at equilibrium, the forcing will be equal to the eddy dissipation rate ϵ reached at equilibrium. For vortices generated by a large aircraft with $b_0 = 50$ m and $\Gamma_0 = 400$ m²/s, the forcing levels would then correspond to the eddy dissipation rates $\epsilon_1 = 10^{-5}$ m²/s³ and $\epsilon_2 = 10^{-6}$ m²/s³. Table 2.1 provides typical turbulence levels measured in the atmosphere. The investigated cases hence correspond to a weakly turbulent and a very weakly turbulent atmosphere. The stratification level is here measured by the dimensionless Brunt-Väisälä

frequency N^* . Table 2.1 also provides typical values of stratification levels of the atmosphere. In order to correctly represent the atmosphere and to compare with previous work, five stratification levels are investigated with the forcing $\epsilon_f^* = \epsilon_1^*$, from neutrally stratified to highly stratified ($N^* = 0, 0.35, 0.75, 1$, and 1.4), while two stratification levels are simulated for the lower forcing ($N^*=0.35$ and 1.0).

Turbulence	very weak	weak	moderate	strong	very strong
ϵ [m ² /s ²]	$\leq 10^{-6}$	10^{-5}	10^{-4}	10^{-3}	10^{-2}
ϵ^*	$\leq 2.42 \cdot 10^{-5}$	$2.42 \cdot 10^{-4}$	$2.42 \cdot 10^{-3}$	$2.42 \cdot 10^{-2}$	$2.42 \cdot 10^{-1}$
Stratification	Neutral	weak	moderate	strong	very strong
N [1/s]	0.0	$8.9 \cdot 10^{-3}$	$1.9 \cdot 10^{-2}$	$2.4 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$
N^*	0.0	0.35	0.75	1.0	1.4

Table 2.1: Typical stratification and turbulence levels of the atmosphere.

The computational domain is a cube with side L . The size of the domain is also varied: $L^* = L/b_0 = 8$ and $L^* = 4$. The extension in the y and z directions limits the influence of the periodicity and enables to obtain results valid for large times. The domain $L^* = 8$ enables the development of long-wavelength Crow-like instabilities while the shorter domain with $L^* = 4$ does not. In the following, cases with $L^* = 4$ are marked with a “s” (for “small”, even though it is understood that a domain of $(4b_0)^3$ is not that small; the domain $(8b_0)^3$ is the largest used in this thesis). The grid is uniform with an equal grid size in the three directions: $h/b_0 = 1/64$. This resolution ensures that the most relevant dynamics of the vortices are well captured ($r_c/h > 3$). Table 2.2 summarizes the important parameters of all investigated cases.

2.4.2 Characterization of the obtained stratified and weakly turbulent fields

This section provides an overview of the main characteristics of the obtained stratified turbulence fields. Further detailed analysis of those turbulent fields are found in Appendix B.

To illustrate the topology of this particular flow, a visualization of the vorticity norm in a (y, z) -slice, for different stratification levels, is provided in Fig. 2.11(a-d). This comparison provides, for the largest domain and for the

forcing ϵ_1^* , a first idea of the effect of stratification on the turbulence topology. When stratification increases, from $N^* = 0$ in Fig. 2.11(a) to $N^* = 1.0$ in Fig. 2.11(d), the flow becomes layered; the size of the turbulent structures increases in the horizontal direction and decreases vertically. This layered behavior of the flow, first noticed by Herring and Métais [50], is visible in many stratified turbulence studies (see, e.g., [91]).

Case	N^*	ϵ_f^*	L^*	E_k^*	E_P^*	ϵ^*	ϵ_P^*
0	0.0	ϵ_1^*	8	$9.03 \cdot 10^{-3}$	0.0	$2.37 \cdot 10^{-4}$	0.0
0s	0.0	ϵ_1^*	4	$5.78 \cdot 10^{-3}$	0.0	$2.39 \cdot 10^{-4}$	0.0
1	0.35	ϵ_1^*	8	$7.86 \cdot 10^{-3}$	$2.93 \cdot 10^{-3}$	$1.50 \cdot 10^{-4}$	$9.68 \cdot 10^{-5}$
1s	0.35	ϵ_1^*	4	$4.68 \cdot 10^{-3}$	$1.78 \cdot 10^{-3}$	$1.48 \cdot 10^{-4}$	$10.6 \cdot 10^{-5}$
2	0.75	ϵ_1^*	8	$9.98 \cdot 10^{-3}$	$3.99 \cdot 10^{-3}$	$1.52 \cdot 10^{-4}$	$8.12 \cdot 10^{-5}$
2s	0.75	ϵ_1^*	4	$5.35 \cdot 10^{-3}$	$1.90 \cdot 10^{-3}$	$1.50 \cdot 10^{-4}$	$8.72 \cdot 10^{-5}$
3	1.0	ϵ_1^*	8	$10.4 \cdot 10^{-3}$	$4.50 \cdot 10^{-3}$	$1.58 \cdot 10^{-4}$	$8.00 \cdot 10^{-5}$
3s	1.0	ϵ_1^*	4	$5.70 \cdot 10^{-3}$	$2.04 \cdot 10^{-3}$	$1.50 \cdot 10^{-4}$	$8.38 \cdot 10^{-5}$
4s	1.4	ϵ_1^*	4	$6.19 \cdot 10^{-3}$	$2.88 \cdot 10^{-3}$	$1.54 \cdot 10^{-4}$	$8.14 \cdot 10^{-5}$
5	0.35	ϵ_2^*	8	$2.70 \cdot 10^{-3}$	$1.18 \cdot 10^{-3}$	$1.82 \cdot 10^{-5}$	$9.59 \cdot 10^{-6}$
6s	1.0	ϵ_2^*	4	$2.77 \cdot 10^{-3}$	$0.779 \cdot 10^{-3}$	$1.34 \cdot 10^{-5}$	$6.17 \cdot 10^{-6}$

Table 2.2: Parameters and turbulence characteristics of the investigated cases ($\epsilon_1^* = 2.42 \cdot 10^{-4}$, $\epsilon_2^* = 2.42 \cdot 10^{-5}$).

The kinetic and potential energies, E_k and E_P , are calculated together with the kinetic and potential energy dissipation rates, ϵ and ϵ_P . The potential energy is computed following:

$$E_P = \frac{1}{2} \frac{1}{N^2} \left(\frac{g}{\Theta_0} \tilde{\Theta} \right)^2. \quad (2.24)$$

Figure 2.12 compares the horizontal spectra of the total energy (kinetic + potential) for the forcing level ϵ_1^* in the large computational domain and for four stratification levels. The horizontal wave number k_H is computed according to $k_H^2 = k_x^2 + k_y^2$. For the sake of consistency with the rest of this study, the characteristic scales used for the turbulence without vortices are still related to the wake vortices (i.e., b_0 , V_0 and t_0). The dimensionless energy is then $E^* = E/V_0^2$ and the dimensionless wave number is $k^* = k b_0$. For all cases,

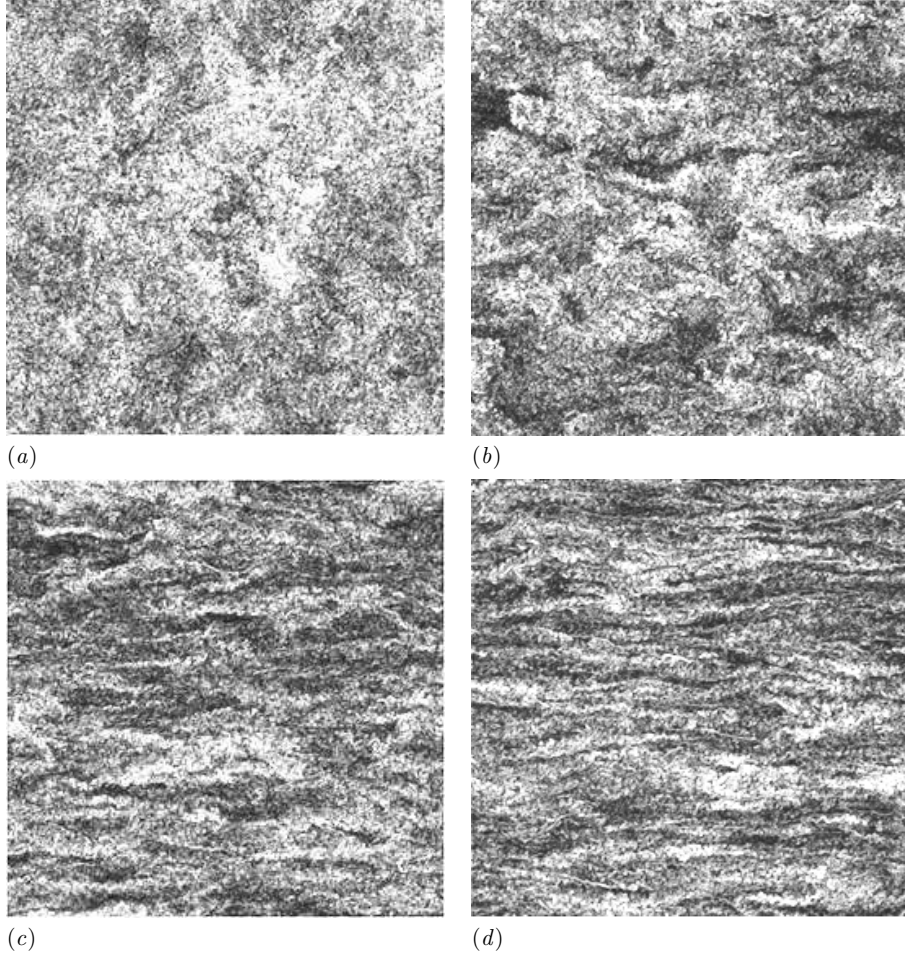


Figure 2.11: Slice of the vorticity field norm, $\|\boldsymbol{\omega}\| b_0^2/\Gamma_0$ in a (y, z) -plane for (a) case 0 ($N^* = 0$), (b) case 1 ($N^* = 0.35$), (c) case 2 ($N^* = 0.75$) and (d) case 3 ($N^* = 1$). For all cases, $\epsilon_f^* = \epsilon_1^*$.

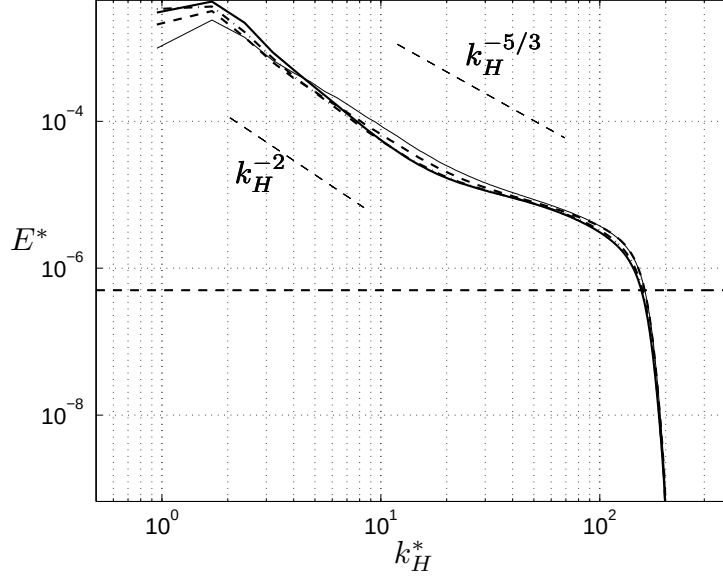


Figure 2.12: Horizontal spectra of the total energy (kinetic+ potential) for cases 0 ($N^* = 0$) (thin solid), 1 ($N^* = 0.35$) (dash), 2 ($N^* = 0.75$) (dash-dot) and 3 ($N^* = 1$) (thick solid). For all cases, $\epsilon_f^* = \epsilon_1^*$. The high wavenumbers damped by the HV SGS model are typically those below the horizontal dashed line and can be discarded. The slopes $k_H^{-5/3}$ and k_H^{-2} are also shown (thin dash).

the forced modes (around $k^* = 2$) are clearly visible. The observed rapid drop of the spectra for the highest wave numbers is typical of the HV SGS model. For the neutral case (case 0), an inertial range, with the characteristic $k^{-5/3}$ evolution, is observed. For the stratified cases, the slope is steeper, close to a k_H^{-2} evolution. This is consistent with the literature results (see e.g., [114]).

Table 2.2 reports the value of the kinetic and potential energies and energy dissipation rates for the investigated cases. Without stratification, the obtained fields have, as desired, a kinetic energy dissipation rate essentially equal to the forcing. When adding a temperature gradient, the total energy (kinetic + potential) increases and part of the forcing is transferred to the potential energy (as seen in Fig. 2.12). The kinetic energy dissipation rate is then reduced: as expected, the turbulence is damped by the stratification. For the stratified cases at equilibrium, the dissipation of the turbulent forcing is spread into a kinetic energy dissipation rate ϵ and a potential energy dissipation rate ϵ_P . When increasing the stratification level at fixed ϵ_f , ϵ slightly increases whereas

ϵ_P slightly decreases accordingly, as $\epsilon_f = \epsilon + \epsilon_P$. The mixing efficiency, as defined in [91], ϵ_P/ϵ slightly decreases from 0.65 to 0.51 when N^* goes from 0.35 to 1.0 for the cases with $L^* = 8$. For the cases with $L^* = 4$, ϵ_P/ϵ varies from 0.72 to 0.53 when N^* goes from 0.35 to 1.4. A value of about 0.4 – 0.5, close to what is here obtained, is found in the literature (see [91, 4]).

The isotropy of the turbulent fields is also characterized by measuring the longitudinal integral length scales L_{ii} (see definition in Appendix B) in the three directions. The results are provided in Table 2.3. Without stratification (cases 0 and 0s), the integral length scales are essentially identical in the three directions as the fields are isotropic. With stratification, the vertical length scale L_{33} is reduced whereas the horizontal length scales (L_{11} and L_{22}) significantly increase, as the stratified fields are anisotropic with large horizontal structures, as observed in Fig. 2.11.

Case	L_{11}^*	L_{22}^*	L_{33}^*	Ri_V	L_O^*	L_b^*
0	0.485	0.497	0.509	0.0	—	—
0s	0.263	0.234	0.260	0.0	—	—
1	0.937	0.736	0.377	0.0374	$4.72 \cdot 10^{-1}$	2.25
1s	0.337	0.314	0.203	0.0376	$4.72 \cdot 10^{-1}$	1.74
2	1.24	1.07	0.427	0.138	$1.50 \cdot 10^{-1}$	1.18
2s	0.486	0.449	0.173	0.144	$1.50 \cdot 10^{-1}$	0.867
3	1.15	1.14	0.405	0.207	$0.977 \cdot 10^{-1}$	0.906
3s	0.475	0.653	0.205	0.224	$0.977 \cdot 10^{-1}$	0.671
4s	0.580	0.465	0.241	0.356	$0.590 \cdot 10^{-1}$	0.499
5	1.66	1.42	0.491	0.131	$1.49 \cdot 10^{-1}$	1.32
6s	1.03	1.48	0.299	0.560	$0.309 \cdot 10^{-1}$	0.468

Table 2.3: Integral length scales and Richardson number of the investigated cases. Cases marked with a “s” are obtained on a domain with $L^* = 4$ whereas the others have $L^* = 8$.

Finally, the volume-averaged Richardson number, that compares the stratification effects to the vertical shear effects associated with the turbulence, is evaluated using:

$$Ri_V = \frac{N^2}{\left\langle \left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right\rangle}. \quad (2.25)$$

Recall that the stratified turbulence considered has a mean imposed temperature gradient, defining N^* , but no mean shear (i.e., no mean velocity gradient). The Richardson number is seen to increase more slowly than N^2 . This again reveals that stratification tends to form horizontal layers, which increases the vertical shear effects.

The comparison of cases 0 to 3 with cases 0s to 3s, respectively, allows the investigation of the result sensitivity to the size of the computational domain. It is worth noticing that, compared to the vortex properties, the stratification and turbulence levels N^* and ϵ_f^* are identical for both domain sizes. However, this is not the case when using the turbulence own characteristics L (i.e., the size of the computational domain) and ϵ_f . One then defines $V = (\epsilon_f L)^{1/3}$ as characteristic velocity. The dimensionless energy $\tilde{E} = E/V^2$ is then related to the dimensionless energy E^* through the relation:

$$\tilde{E} = \frac{E^*}{\left(\epsilon_f^* L^*\right)^{2/3}}. \quad (2.26)$$

Similarly, the dimensionless Brunt-Väisälä frequency $\tilde{N} = N L/V$ is related to the dimensionless Brunt-Väisälä frequency N^* through the relation:

$$\tilde{N} = N^* \left(\frac{L^{*2}}{\epsilon_f^*} \right)^{1/3}. \quad (2.27)$$

The kinetic and potential dimensionless energies E_k^* and E_P^* would then be multiplied by a factor $(1/2)^{2/3} = 0.63$ when comparing a case with $L^* = 8$ to a case with $L^* = 4$. This relationship between the kinetic energies is verified for the unstratified case. One indeed finds $E_{k,4b_0}^* = 0.64 E_{k,8b_0}^*$. It is also noted that the integral length scales are divided by two when L^* is divided by two. Both turbulences are hence similar except at very large scales.

With stratification, compared to the turbulence characteristics, the cases obtained with the small domain are a bit less stratified than the cases obtained with $L^* = 8$. Indeed, the dimensionless Brunt-Väisälä frequencies $\tilde{N}$ are different with $\tilde{N}_{4b_0} = (1/2)^{2/3} \tilde{N}_{8b_0}$. For the present results, the stratified fields are hence not totally identical. The kinetic energies E_k^* are then multiplied by 0.60, 0.54 and 0.55 for the cases with $N^* = 0.35$, 0.75 and 1.0, respectively, whereas the potential energies are multiplied by 0.61, 0.48 and 0.45. Thus, the turbulent fields of cases 1s, 2s and 3s appear to be similar to those of cases 1,

2 and 3, respectively, except at the very large scales (i.e., the integral length scales are approximatively divided by two). They have similar dissipation and stratification properties and a similar degree of anisotropy.

Finally, comparison of cases 5 and 6s with cases 1 and 3s, respectively, allows the investigation of the influence of the forced turbulence level. The potential and kinetic energies are divided by a factor of roughly 2 to 3 when the forcing level is divided by 10. The mixing efficiency is also slightly reduced. Due to the larger influence of the stratification compared to the turbulence, the horizontal integral length scales increase. The Richardson number in both cases also significantly increases.

We note that, in all investigated cases, both the Ozmidov scale $L_O = 2\pi((\epsilon + \epsilon_P)/N^3)^{1/2}$ and the buoyancy scale $L_b = 2\pi U/N$, where U is the rms velocity, are resolved. h/L_O ranges from $3.3 \cdot 10^{-2}$ for case 1 to $1.0 \cdot 10^{-1}$ for case 2. For the cases with $L^* = 8$, h/L_b ranges from $6.9 \cdot 10^{-3}$ for case 1 to $1.2 \cdot 10^{-2}$ for case 5. For the cases with $L^* = 4$, h/L_b ranges from $9.0 \cdot 10^{-3}$ for case 1s to $1.8 \cdot 10^{-2}$ for case 2s. As pointed out by Waite [114], this is of great importance for correct stratified turbulence representation. Waite further noted that a vertical extent of $4.5 L_b$ is large enough to capture the characteristic layers of stratified turbulence. All investigated cases, except cases 1 and 1s, have superior vertical extent (ranging from $4.6 L_b$ for case 2s to 8.8 for case 3). For cases 1 and 1s, $L/L_b = 3.55$ and 2.30 respectively, which are still judged sufficient for the present study.

The analysis of the present results highlights the great care taken to obtain databases of realistic stratified turbulence fields at very high Reynolds number. These 3-D fields are statistically fully converged and their properties are consistent with each other and also with the literature. Moreover, following the criteria, based on horizontal Froude number and Ozmidov length, defined in [4, 74] and recalled in Appendix B (also using hyperviscosity SGS model), the obtained stratified turbulent atmospheres are almost all in the strongly stratified turbulence regime.

2.4.3 Characterization of the vortex system for the cases without Crow instability development

The objective of this section is to characterize the influence of the stratification combined with weak turbulence on the wake vortex behavior. For the cases considered here (cases 0s, 1s, 2s, 3s and 4s), the long-wavelength instability cannot develop due to the limited axial extension of the computational domain ($L^* = 4$). It hence enables to isolate the effect of stratification on the vortices without influence of the Crow instability. The combination of Crow instability and stratification effects is analyzed in a second step in Section 2.4.4.

Flow visualization

We provide, in Figs. 2.13 to 2.17, visualizations of the flow, using iso-surfaces of the λ_2 criterion (as defined in [63], λ_2 is the second eigenvalue of the tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$, with $\mathbf{S}$ the strain-rate tensor and $\mathbf{\Omega}$ the rotation rate tensor), for the cases with $N^* = 0.0$, $N^* = 0.35$, $N^* = 0.75$, $N^* = 1.0$ and $N^* = 1.4$, respectively.

At early times of the flow, one observes the two vortex cores; the wake vortices descent at a constant speed and the initial separation stays constant (as it will be further quantified hereafter). Without stratification (case 0s, see Fig. 2.13), the wake vortex system stretches the vortical structures of the background turbulence. This leads to the generation of transverse vortical structures as observed at $t^* = 2.5$ and $t^* = 3.0$. The turbulence surrounding the wake vortex system is amplified until the system reaches an equilibrium, around $t^* = 5.0$. From that time, the two-vortex system is in equilibrium with the complex surrounding turbulence and slowly decays (as further quantified hereafter). It is also worth noticing that the surrounding turbulence is very little detrained and stays mostly confined in a “Rankine-like” oval. This case is here used as a reference and is further characterized in Section 2.5.

With stratification, due to the vertical mean temperature gradient $\frac{d\bar{\Theta}}{dz}$, local warm and cold zones are formed around the wake vortices. This leads to the creation of local gradients of temperature in the axial, x , and lateral, y , directions. Those gradients are sources of production of baroclinic vorticity, following Eq. (2.15), as explained in Section 2.3.2.

For the case with $N^* = 0.35$ (case 1s), the secondary vortical structures become significant around time $t^* = 2.75$. As the background temperature

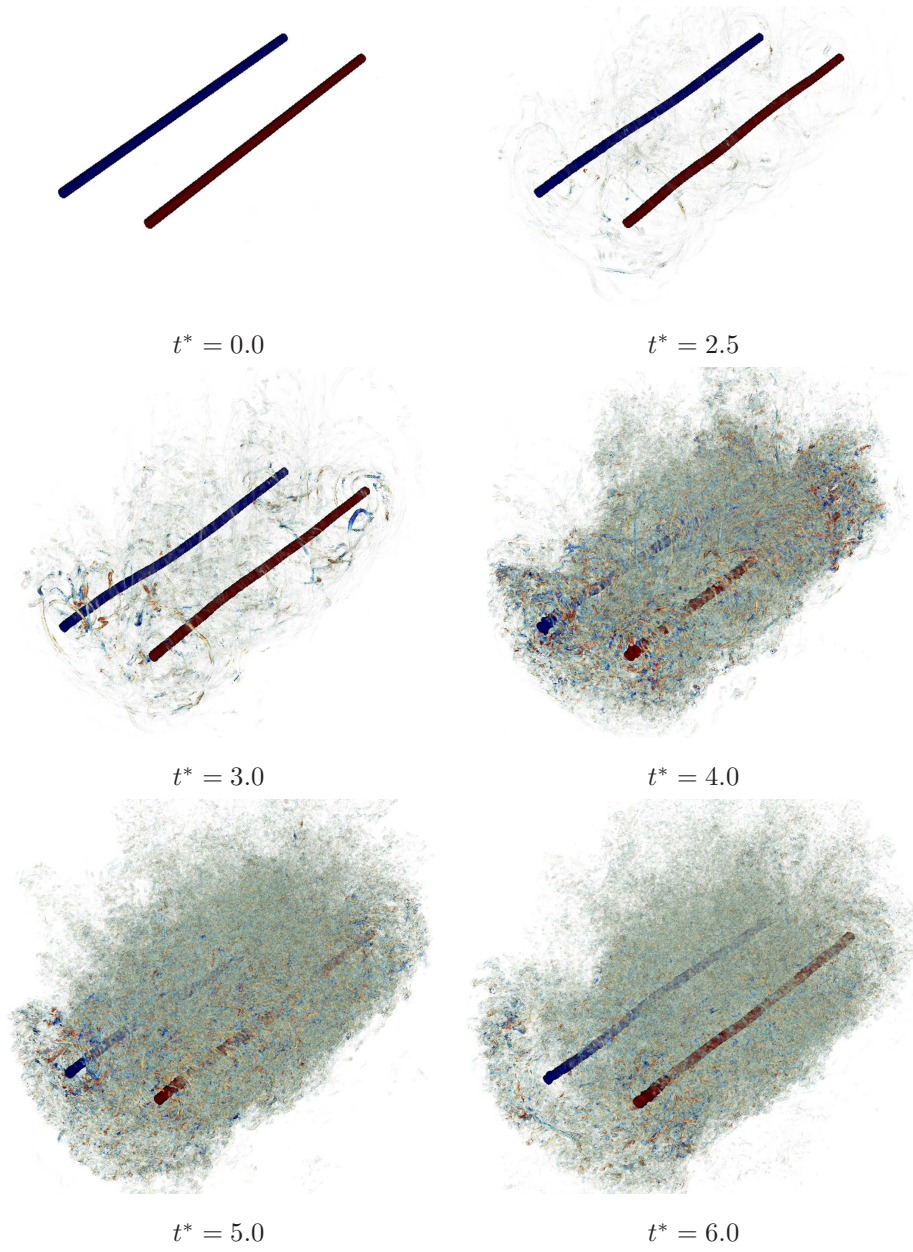


Figure 2.13: Case 0s ($N^* = 0.0$, $\epsilon_f^* = \epsilon_1^*$): iso- λ_2 surfaces at different times colored by the axial vorticity $\omega_x b_0^2 / \Gamma_0$.

field is turbulent, the produced baroclinic vorticity is also immediately turbulent when generated. Small-scale instabilities are then directly visible in those secondary vortical structures. At $t^* = 3.25$, the secondary vortical structures start to strongly interact with the primary vortex cores. This was not the case without stratification. Due to the displacement of the upper stagnation point (see Section 2.3.2), a detraining of the vortical structures is observed. The detraining is seen to be more significant than for the case with $N^* = 0$, confirming that it is related to stratification effects.

The flow physics for the cases with $N^* = 0.75$ (case 2s) and $N^* = 1.0$ (case 3s) is similar to that of the case with $N^* = 0.35$. However, as the production rate of baroclinic vorticity is higher, the transverse secondary vortical structures appear sooner. It is also observed that the produced baroclinic vorticity is indeed of opposite sign than the primary vortex. This is clearly visible at times $t^* \geq 1.75$ for the case with $N^* = 0.75$ and for times $t^* \geq 1.5$ for the case with $N^* = 1.0$. These secondary structures are subjected to short-wave instabilities which grow in time, eventually leading to a complex turbulent vortex system. At late times of the flow, the strong 3-D interaction between this secondary vorticity and the primary vortices leads to a fast decay of the primary vortex circulation (as further quantified). Later on, the flow is so complex that one can no longer distinguish the vortex cores on the 3-D visualizations. In both cases, the baroclinic vorticity production is confined in the Rankine oval of the vortex system. Later on (from $t^* = 2.25$ for $N^* = 0.75$ and from $t^* = 2.0$ for $N^* = 1.0$), some detraining of the vorticity is observed. This detraining appears to be more pronounced for the case $N^* = 0.75$. This is as expected as the rebound is more significant for the case with $N^* = 1.0$.

For the case with $N^* = 1.4$ (case 4s), the secondary transverse structures, generated by baroclinic effect with opposite sign vorticity, are clearly visible at $t^* = 1.25$. Those very turbulent structures, mostly generated in the region between the two vortices, push the vortices away from each other. They also almost immediately interact with the primary vortices ($t^* \geq 1.5$) leading to their very fast decay. As the vortices move away from each other, the secondary vortical structures hence follow the primary vortices and are not detrained.

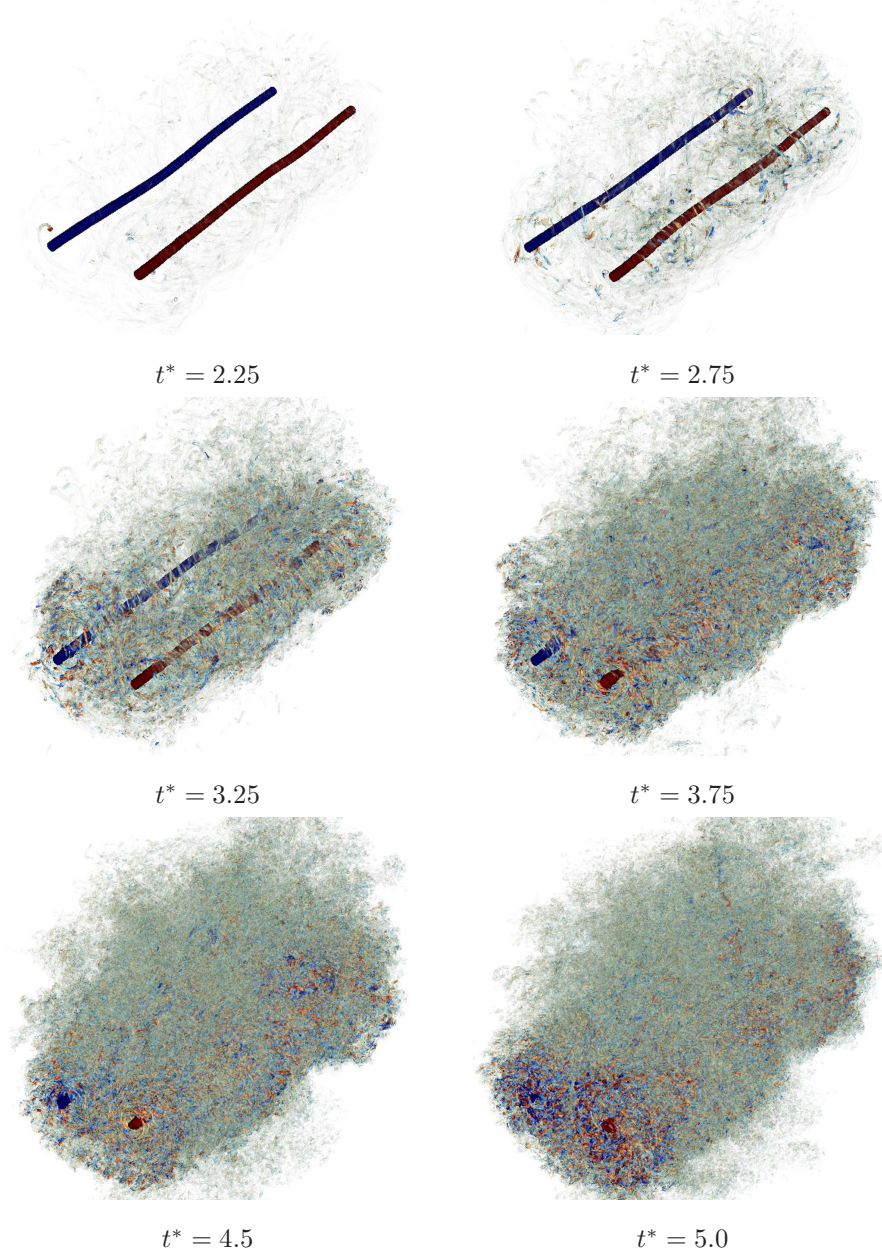


Figure 2.14: Case 1s ($N^* = 0.35$, $\epsilon_f^* = \epsilon_1^*$): iso- λ_2 surfaces at different times colored by the axial vorticity $\omega_x b_0^2 / \Gamma_0$.

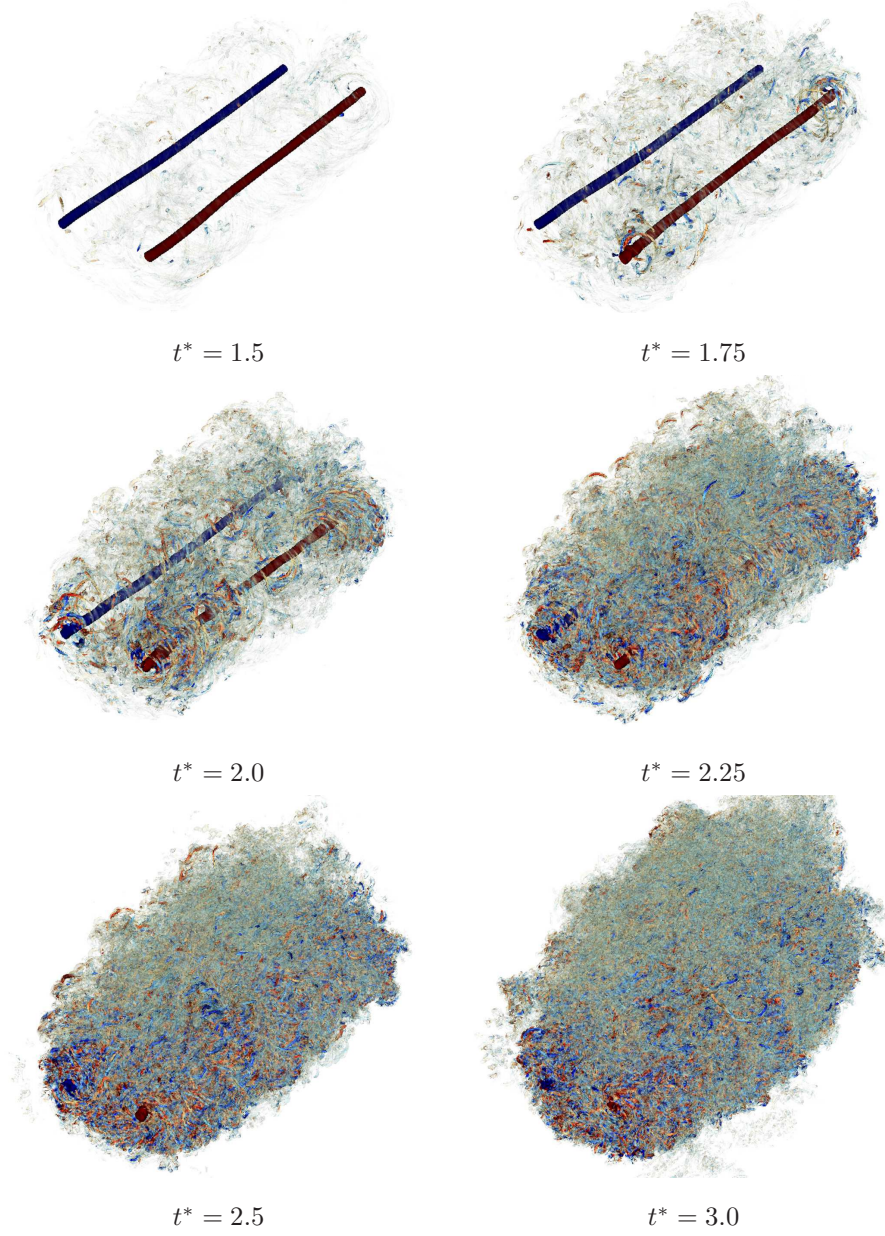


Figure 2.15: Case 2s ($N^* = 0.75$, $\epsilon_f^* = \epsilon_1^*$): iso- λ_2 surfaces at different times colored by the axial vorticity $\omega_x b_0^2 / \Gamma_0$.

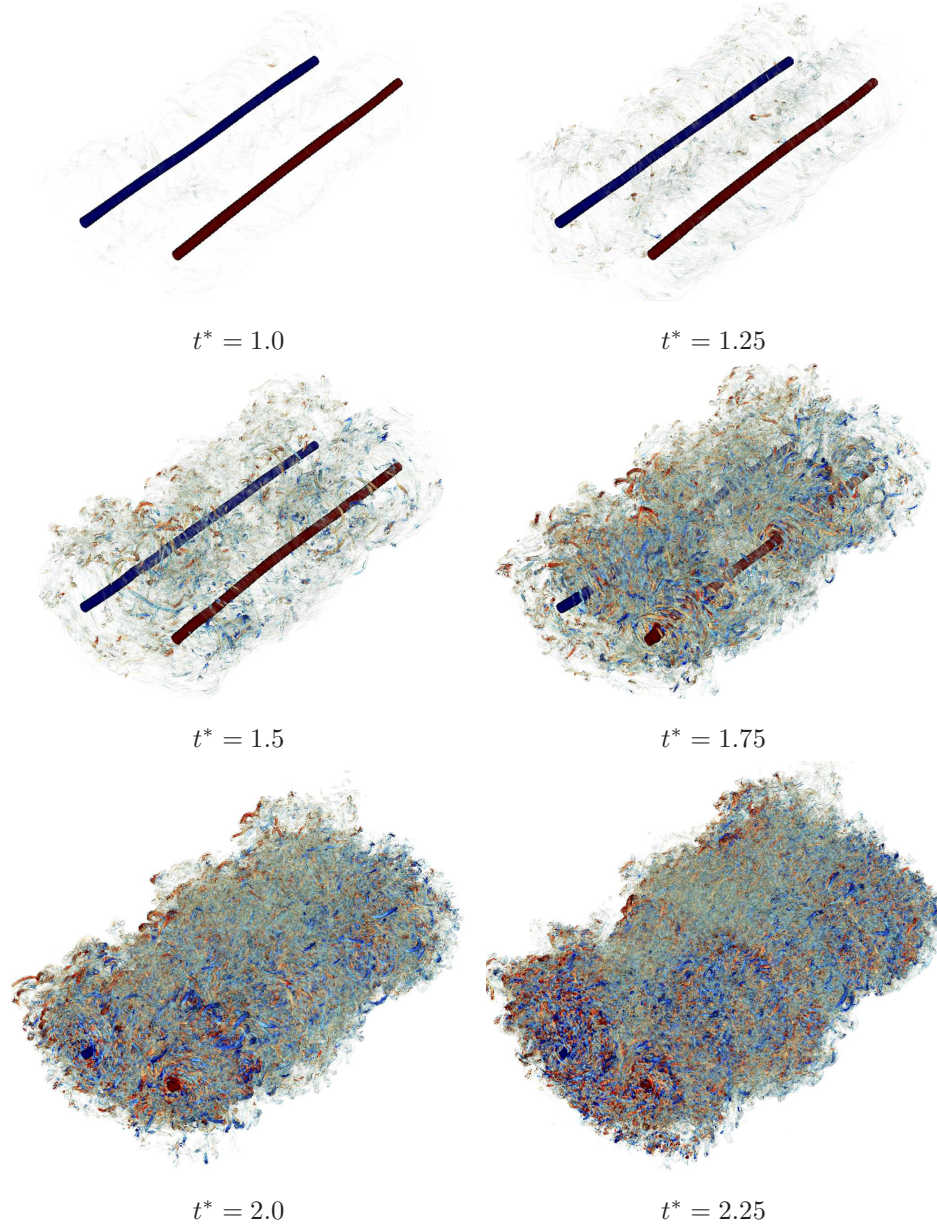


Figure 2.16: Case 3s ($N^* = 1.0$, $\epsilon_f^* = \epsilon_1^*$): iso- λ_2 surfaces at different times colored by the axial vorticity $\omega_x b_0^2 / \Gamma_0$.

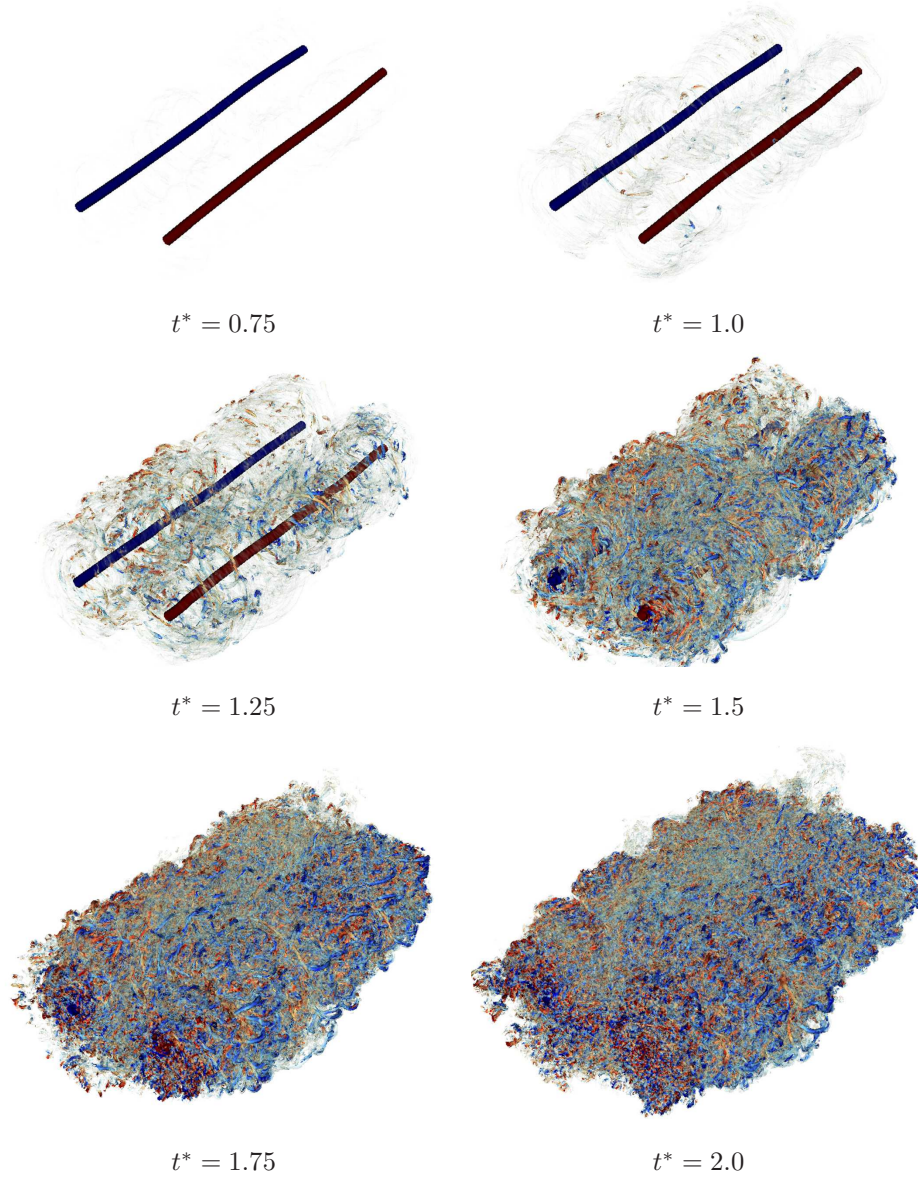


Figure 2.17: Case 4s ($N^* = 1.4$, $\epsilon_f^* = \epsilon_1^*$): iso- λ_2 surfaces at different times colored by the axial vorticity $\omega_x b_0^2 / \Gamma_0$.

Vortex trajectories

For the vortex global characterization, as in Section 2.3, we compute all diagnostics based on the longitudinally averaged vorticity fields defined in Eq. (2.16). As long wavelength instabilities cannot develop in the chosen computational domain, it is expected that this measurement still correctly represents the vortex system.

Figure 2.18 presents the altitude and lateral spacing evolution for the five considered stratification levels. It is to be noted that the curves are stopped when the vortex position is no longer measurable, i.e., when the vortex circulation is too low to be distinguished from the background turbulence. At early times, and for all cases, the vortices descent at a velocity very close to V_0 . This indicates that, as for the simulations of Section 2.3, the neighbor vortex pairs (in the y and z directions due to the periodic conditions) are far away enough. As observed without background turbulence, the stratification level controls the time of rebound. Again, for the case with $N^* = 1.4$, the vortices are almost at the altitude of wake generation after a time $t^* = 3.0$. It is also noted that, for low stratification levels ($N^* = 0.35$), the lateral spacing of the vortices remains almost constant. For cases with $N^* = 0.75$ and $N^* = 1.0$, the lateral vortex separations decrease with a similar evolution, whereas a more pronounced distinction was observed without background turbulence. For $N^* = 1.4$, the vortices move away from each other. Compared to the results obtained without background turbulence (see Fig. 2.7), the variations in the lateral vortex spacing are seen to follow the same trend (i.e., decrease for $0.35 \leq N^* \leq 1.0$ and increase for $N^* = 1.4$). However, the background turbulence is seen to attenuate this effect.

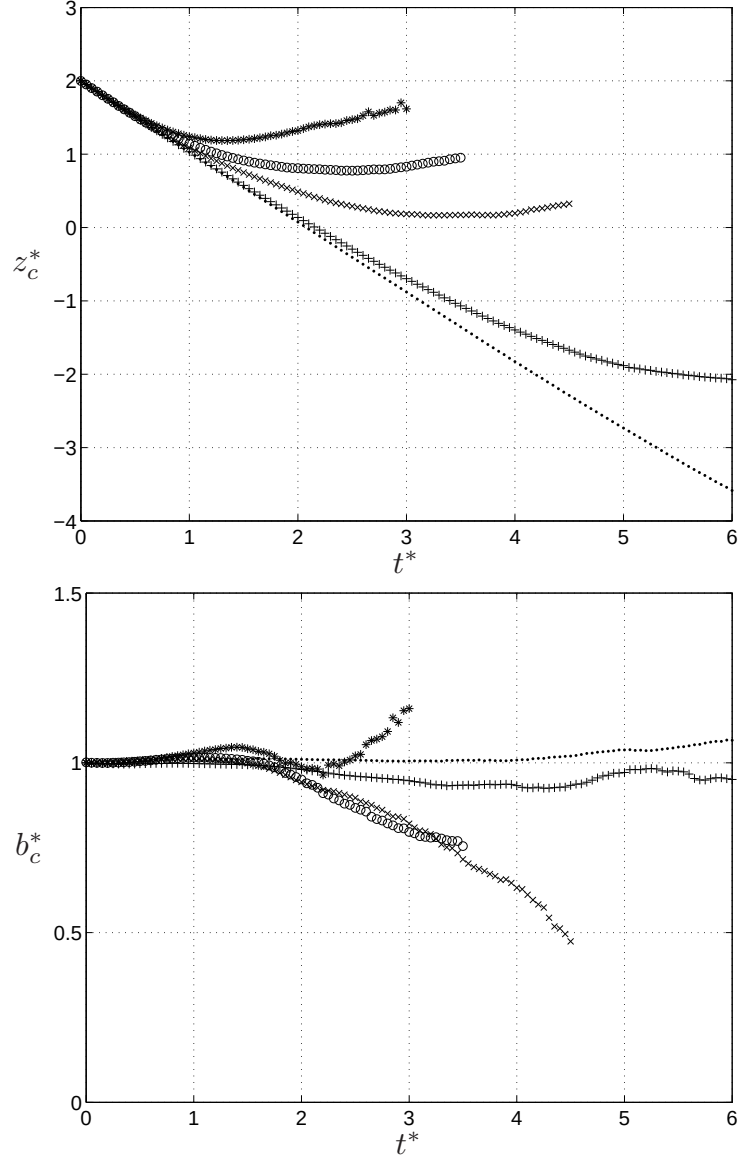


Figure 2.18: Evolution of the vortex altitude (top) and lateral spacing (bottom): $N^* = 0.0$ ($\cdot$), $N^* = 0.35$ (+), $N^* = 0.75$ ($\times$), $N^* = 1.0$ (o) and $N^* = 1.4$ (*).

Circulation decay

In order to characterize the wake vortex decay and to compare it with the results obtained without background turbulence (see Section 2.3), the total circulation Γ_{tot} and the average circulation Γ_{5-15} are computed using the averaged axial vorticity field $\overline{\omega}_x$ defined in Eq. (2.16). The evolution of Γ_{5-15} and Γ_{tot} are provided in Fig. 2.19. Using both circulation measurements, two phases are noted in the decay. During the first phase, a very slow decay phase is observed in the total circulation evolution even for very low or no stratification. This decrease is to be attributed to the presence of turbulence and was not observed in Fig. 2.10. The ambient turbulence indeed favors the exchange of circulation between primary vortices and (possibly baroclinic) turbulence. On the contrary, no decay is observed in the first phase of Γ_{5-15} . The exchange of circulation hence occurs away from the vortex core (at $r > b/4 \simeq 6.4 r_c$). Later on, a second phase of rapid decay appears. The time at which this second phase appears depends on the stratification level. This behavior was also observed without background turbulence (see Fig. 2.10).

For the stratified cases, as observed in Section 2.3, the strong decay phase is caused by two phenomena. First, it is due to the 3-D interactions between the wake vortices and the baroclinic vorticity. Then, for the cases for which the vortices come close to each other, 3-D interactions between the wake vortices themselves occur, which also enhances the circulation decay. As the ambient stratified field is turbulent, the baroclinic vorticity is directly turbulent when generated. This favors the short-scale instability development in the secondary vorticity structures and hence the interactions with the primary vortex, leading to a faster decay of the primary vortices. For the case with $N^* = 0.35$, the lateral spacing remains almost constant and the baroclinic vorticity production is low. The decay in the second phase, mostly due to the interaction with that baroclinic vorticity, is thus the slowest. For the cases with $N^* = 0.75$ and with $N^* = 1.0$, b_c decreases in time leading to an enhancement of the decay rate. Finally, for $N^* = 1.4$, the baroclinic vorticity intensity is very important but the vortices move away from each other, leading to little interaction between them. The net result is that, for the three highest stratification levels studied here, the decay rates are almost identical. It is also worth noting that, for $N^* = 1.4$, at $t^* \approx 3.0$, when the vortices are almost back at the generation altitude, the circulation is then very low: $\Gamma_{\text{tot}} \approx 0.2 \Gamma_0$. The encounter of such vortices would thus be much less hazardous than an encounter with the initial

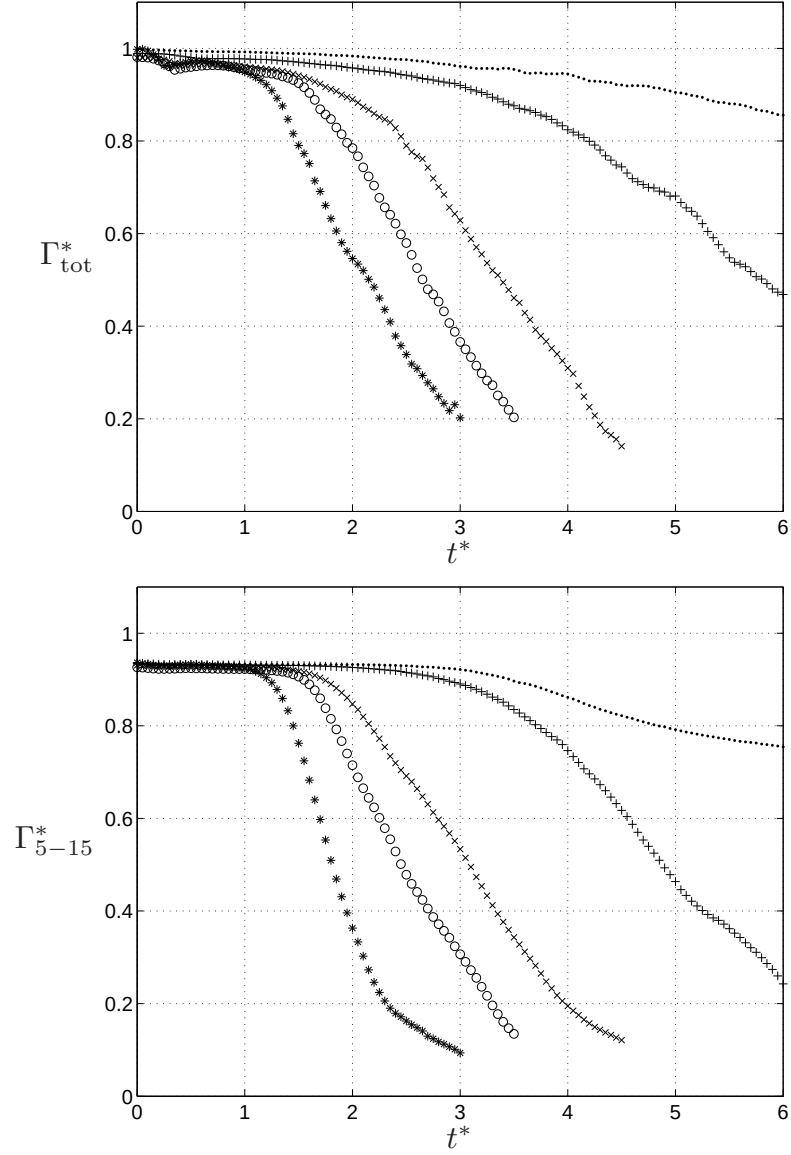


Figure 2.19: Evolution of the vortex circulation Γ_{tot} (top) and Γ_{5-15} (bottom): $N^* = 0.0$ ($\cdot$), $N^* = 0.35$ ($+$), $N^* = 0.75$ ($\times$), $N^* = 1.0$ ($\circ$), and $N^* = 1.4$ ($*$).

vortices. Recall that, without any background turbulence in the atmosphere and/or in the wake vortices themselves (which is not realistic), the circulation, at that time, was still $\Gamma_{\text{tot}} \simeq 0.5 \Gamma_0$.

For the case without stratification, one observes a single phase of slow decay. Recall that the Crow instability cannot develop here since the axial extension of the periodic domain is $L_x = 4b_0$. The decay rate of the case without stratification is thus a useful upper bound of what to expect for vortices without Crow instabilities.

Comparison with the results without background turbulence

The influence of the background turbulence is finally highlighted by comparing, in Fig. 2.20, the evolution of the altitude and Γ_{5-15} for three stratification levels simulated with and without background turbulence. It should be noted that the computational boxes used for the simulations are not identical. However, the influence of the box sizes in the lateral and vertical directions ($6b_0$ versus $4b_0$) is negligible for the considered times. Moreover, the instabilities initiated in the baroclinic vorticity have wavelengths much shorter than b_0 . The influence of the axial domain extension (b_0 versus $4b_0$) is also negligible. It is hence believed that the only significant difference between the simulations of Section 2.3 and 2.4 is indeed the initial turbulence of the ambient stratified fields.

One notices that the rebound, as it is a mainly 2-D effect, is controlled by the stratification and is not influenced significantly by the background turbulence. For short times, the altitude evolutions thus correspond in both cases. The background stratified and weakly turbulent field has, on the other hand, a great influence on the development of the instabilities in the baroclinic vorticity and thus on the circulation decay of the vortices. Indeed, with background turbulence, the temperature field is also turbulent and the baroclinic vorticity field is thus directly turbulent when created. The times of fast decay are hence shorter. As the stratification level increases, the difference between the times of fast decay decreases. Nevertheless, the decay rate during the fast decay phase are essentially identical, with and without background turbulence. Since the circulation decays are different for late times, the altitude evolutions are obviously affected at those times and one can observe a difference in the trajectories.

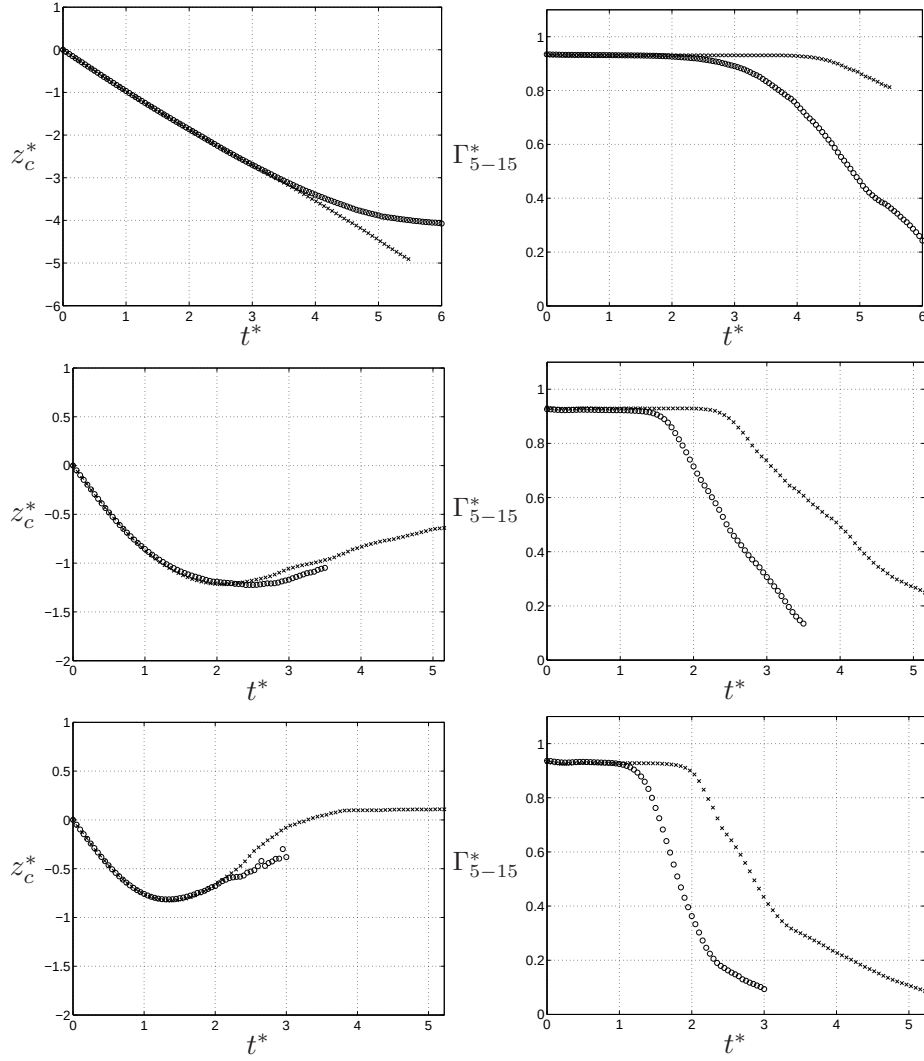


Figure 2.20: Evolution of the vortex altitude (left) and circulation Γ_{5-15} (right) for $N^* = 0.35$ (top), $N^* = 1.0$ (middle) and $N^* = 1.4$ (bottom) for a case with weak background turbulence ($\circ$) and without ($\times$).

2.4.4 Characterization of the vortex flow for the case with possible Crow instability development

The results analyzed in this section concern the investigation of the combined effects of long-wavelength instability, atmospheric turbulence and stable stratification on the wake vortex deformation, transport and decay. The cases analyzed here have thus been simulated in a domain with size $L^* = 8$.

The choice of the domain size is motivated by Crow's theory [16]. Following that theory, a long-wavelength symmetric sinusoidal bending instability indeed develops in the wake vortices with an amplitude that grows exponentially following:

$$A^*(t^*) = A_0^* \exp(\sigma^* t^*), \quad (2.28)$$

where $\sigma^* = \sigma t_0$ is a function of the instability wavelength, $\lambda^* = \lambda/b_0$, and of the core radius, $r_c^* = r_c/b_0$. Crow showed that a large range of long-wavelength instabilities are unstable. The evolution of the growth rate as a function of λ^* , for a vortex pair with $r_c^* = 0.05$, is provided in Fig. 2.21. One observes that the growth rate reaches its maximum close to $\lambda^* = 8$ and that the curve is flat around that point. One also notes that instabilities with wavelengths smaller than $\sim 5.6 b_0$ are stable. For the cases investigated in Section 2.4.3 (with $L^* = 4$), no long-wavelength instability could hence develop. The domain size of the LES analyzed in this section ($L^* = 8$) enables to capture the development of a long-wavelength instability with a growth rate very close to the theoretical maximum. Moreover, for such LES, the number of grid points (i.e., 512^3) is still computationally achievable using the present pseudo-spectral code.

Flow visualization

We provide, in Fig. 2.22 to Fig. 2.25, visualizations of the flow, using iso-surfaces of the scalar parameter λ_2 (as defined in [63]), for the cases with $N^* = 0.0, 0.35, 0.75$ and 1 , respectively. In order to better visualize the Crow instability, the domain is repeated twice in the x -direction. For all cases, at short times (not shown here), the wake vortices descend at a constant speed and the initial separation remains constant. Later on, two main instabilities are observed: a Crow-like long-wave instability, triggered by large scales of the turbulence, and short-scale instabilities initiated in the secondary vorticity structures created due to the stratification effects. At this stage, it is important to distinguish the small-scale instabilities in the secondary vorticity structure from the Widnall

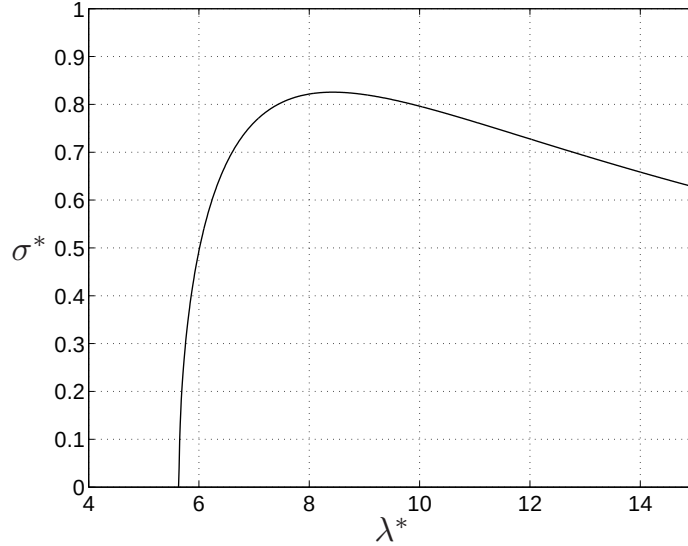


Figure 2.21: Growth rate of the Crow instability as a function of the wavelength, $\lambda^* = \lambda/b_0$, computed following Crow's theory [16]. Case of a vortex pair with a LOA circulation distribution profile and a core size $r_c^* = r_c/b_0 = 0.05$.

instabilities [117] that correspond to the natural short-wave instabilities of a vortex subjected to a strain field.

Figure 2.22 shows the growth of the Crow instability for vortices evolving in a weakly turbulent atmosphere (case 0). Until $t^* = 3.0$, except from some turbulence entrained around the primary vortices, the vortices are not much perturbed and a long sinusoidal deformation of the vortices clearly appears. This instability grows in time and, at $t^* = 4.0$, the vortices are close enough to induce some exchange of vorticity between the left and right parts of the flow. At $t^* = 5.0$, the vortices have just linked, a wave propagates from the reconnection point through the vortices. This phenomenon, observed and studied by Moet *et al.* [77], comes from the sudden change in the vortex core size and structure at the linking location leading to the generation of a pressure wave at both sides of the vortex rings. The two pressure waves propagate toward each other, generating helical instabilities as seen in the box in Fig. 2.22 at $t^* = 5.0$. The vortex bursting occurs when they meet. The vortex bursting leads to the creation of an annular structure around the primary vortex and a reorganization of the vorticity, without change in the circulation (see box in Fig. 2.22 at $t^* = 5.5$). At $t^* = 5.5$, vortex rings are formed surrounded by turbulence.



Figure 2.22: Visualization of the flow for case 0 ($N^* = 0$, $\epsilon_f^* = \epsilon_1^*$), using iso- λ_2 surfaces colored by the axial vorticity $\omega_x b_0^2/\Gamma_0$ at times (from top to bottom): $t^* = 3.0$, $t^* = 4.0$, $t^* = 5.0$ and $t^* = 5.5$.

The flow evolution for case 1 ($N^* = 0.35$) is presented in Fig. 2.23. For short times ($t^* \leq 2$), the flow is almost not influenced by the stratification. At $t^* = 3.0$, the Crow instability is visible with an amplitude slightly lower than without stratification. Some secondary turbulent vorticity structures, generated by baroclinic effects, also appear around the primary vortices. Those structures quicken the reconnection process since at $t^* = 4.0$, the vortices start to link. Due to the baroclinic vorticity surrounding the primary vortices, the linking process is more turbulent than without stratification. At $t^* = 5.0$, vortex rings are still visible but they are much more turbulent. For times $t^* > 5.5$, coherent vortex rings are no longer observable.

For case 2 ($N^* = 0.75$), see Fig. 2.24, at $t^* = 2.0$, baroclinic vorticity structures can already be seen. The long-wave instability amplitude is also already significant. At $t^* = 2.5$, small-scale instabilities in the transverse baroclinic vorticity structures and the long-wave instability in the primary vortices both significantly grow. At $t^* = 3.0$, the vortices link and, at $t^* = 3.5$, some vortex rings are visible surrounded by complex turbulent vorticity. Those vortex rings are almost immediately destroyed by the strong short-scale instabilities generated in the baroclinic vorticity structures.

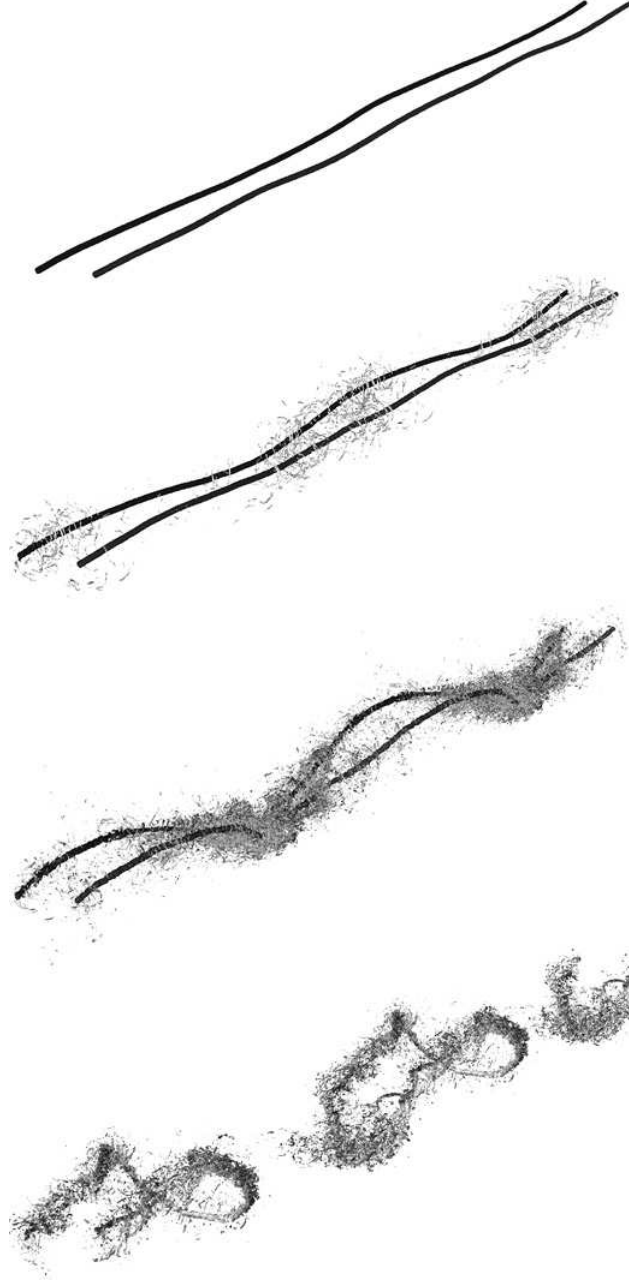


Figure 2.23: Visualization of the flow for case 1 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_1^*$), using iso- λ_2 surfaces colored by the axial vorticity $\omega_x b_0^2/\Gamma_0$ at times (from top to bottom): $t^* = 2.0$, $t^* = 3.0$, $t^* = 4.0$ and $t^* = 5.0$.

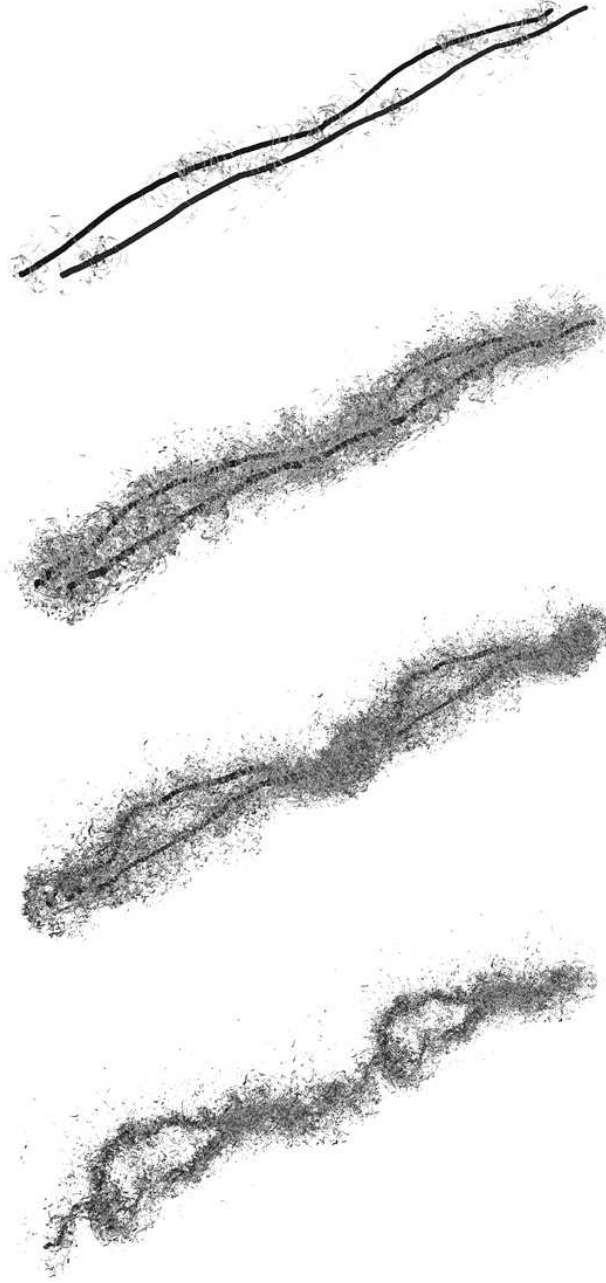


Figure 2.24: Visualization of the flow for case 2 ($N^* = 0.75$, $\epsilon_f^* = \epsilon_1^*$), using iso- λ_2 surfaces colored by the axial vorticity $\omega_x b_0^2/\Gamma_0$ at times (from top to bottom): $t^* = 2.0$, $t^* = 2.5$, $t^* = 3.0$ and $t^* = 3.5$.

A visualization, in Fig. 2.25, of case 3 ($N^* = 1$) using a λ_2 iso-surface with a higher λ_2 value than in the three previous figures enables the observation of the strong baroclinic vorticity structures. At $t^* = 1.0$, the production of the baroclinic vorticity, of opposite sign, leads to the formation of coherent transverse vortical structures. This production is confined in the “Rankine-like” oval of the vortex system. It is important to note that, since the background temperature field is also turbulent, the baroclinic vorticity is also directly turbulent when created. At that time one can also distinguish the slight growth of the Crow instability. The secondary structures are subjected to short-wave instabilities ($t^* = 1.5$). These small-scale structures keep growing in time, eventually leading to a complex turbulent vortex system. At $t^* = 2.0$, the turbulent secondary vorticity interacts with the primary vortices. This strong three-dimensional interaction leads to a fast decay of the primary vortex circulation (as quantified later). At late stages of the flow development ($t^* \geq 2.0$), the flow is so complex that one can no longer distinguish the vortex cores on the 3-D visualizations. However, on the longitudinally averaged flow (i.e., a RANS-like averaging process), one can still observe mean “coherent” primary vortices. For that case, the vortex system “tries” to develop a long-wave instability leading to a linking but the short-wave instabilities destroy the primary vortex structures.

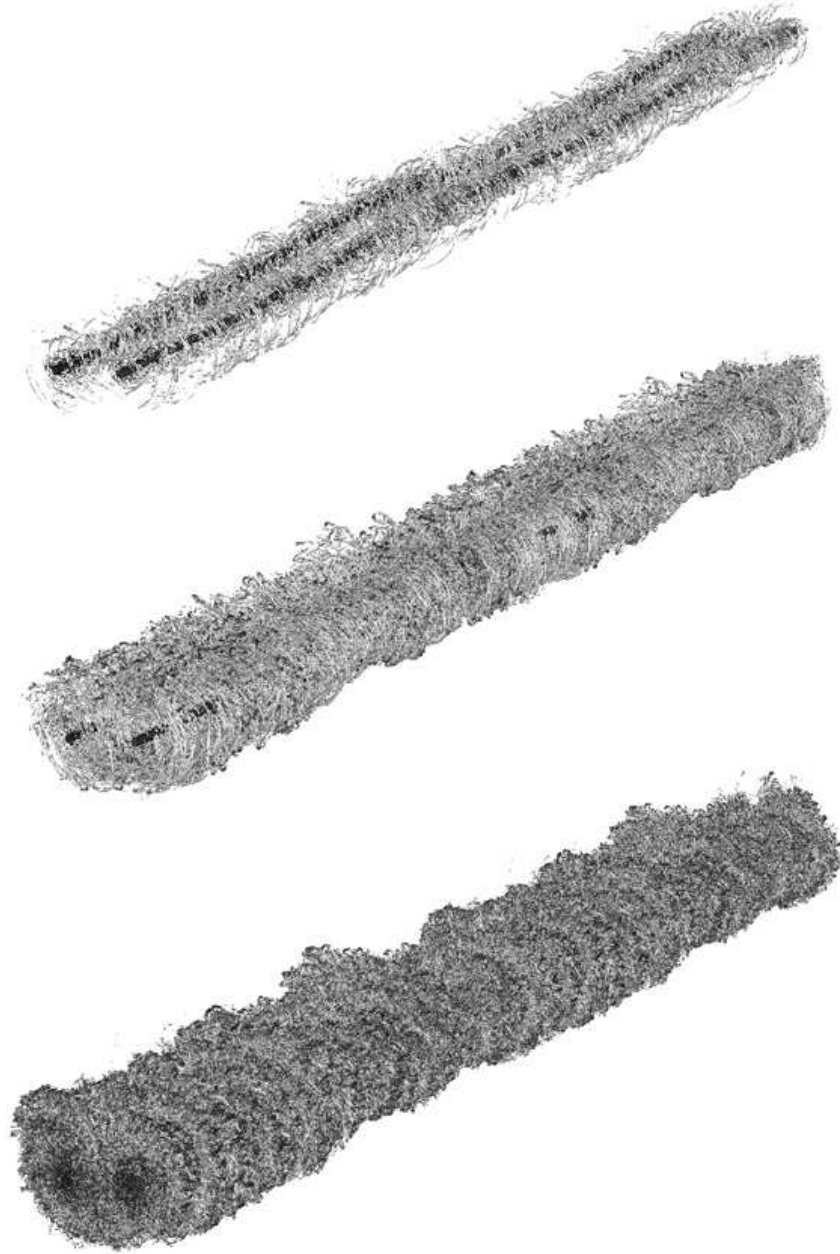


Figure 2.25: Visualization of the flow for case 3 ($N^* = 1$, $\epsilon_f^* = \epsilon_1^*$), using iso- λ_2 surfaces colored by the axial vorticity $\omega_x b_0^2/\Gamma_0$ at times (from top to bottom): $t^* = 1.0$, $t^* = 1.5$ and $t^* = 2.0$.

Characterization of the flow

Due to the long-wave instability, the vortex characteristics can much vary along the vortex. The vortex characteristics are then measured in each x -plane. The lateral and vertical displacement of the vortices are measured by locating, in each x -plane, the (y, z) -positions of maximum vorticity norm in the left and right parts of the domain. This measure takes into account the deformation of the vortex tubes. One then further defines, for the left and right vortices respectively, the average positions by:

$$\bar{y}(t) = \frac{1}{n_x} \sum_{i=0}^{n_x-1} y(x_i, t), \quad (2.29)$$

$$\bar{z}(t) = \frac{1}{n_x} \sum_{i=0}^{n_x-1} z(x_i, t), \quad (2.30)$$

where n_x is the number of grid points in the x -direction. In order to understand the transport mechanisms, we also measure the vortex lateral spacing:

$$\Delta y(x, t) = y_{\text{right}}(x, t) - y_{\text{left}}(x, t). \quad (2.31)$$

Figure 2.26(a) presents the average altitude evolution for the five considered stratification levels. Note that case 4s is added here for completeness of the analysis even if it was computed on a smaller computational domain. Indeed, for such very high stratification level, the flow is dominated by short-scale instabilities generated in the secondary vorticity structures. The results obtained in a computational domain 8 times larger (i.e., with $L^* = 8$) would then be very similar to those obtained in case 4s.

As expected from results of Sections 2.3 and 2.4.3 and of the literature, the stratification level controls the time of rebound. Figure 2.26(b) presents the average lateral spacing evolution $b_c = \overline{\Delta y}$ for the five considered stratification levels. It is interesting to note that, as observed in the previous results, for low stratification levels ($N^* = 0.35$), the lateral spacing of the vortices marginally decreases. For intermediate values ($N^* = 0.75$ and $N^* = 1.0$), the lateral spacing decreases, while, for high values ($N^* = 1.4$), the vortices move away from each other. This behavior is in line with the literature results: for 3-D simulation results with lower axial resolution or with shorter axial extension (see e.g., [111, 57, 79]), and, even more pronounced, for 2-D simulations (see e.g., [56]). Based on the present results, an improvement of Saffman's simplified

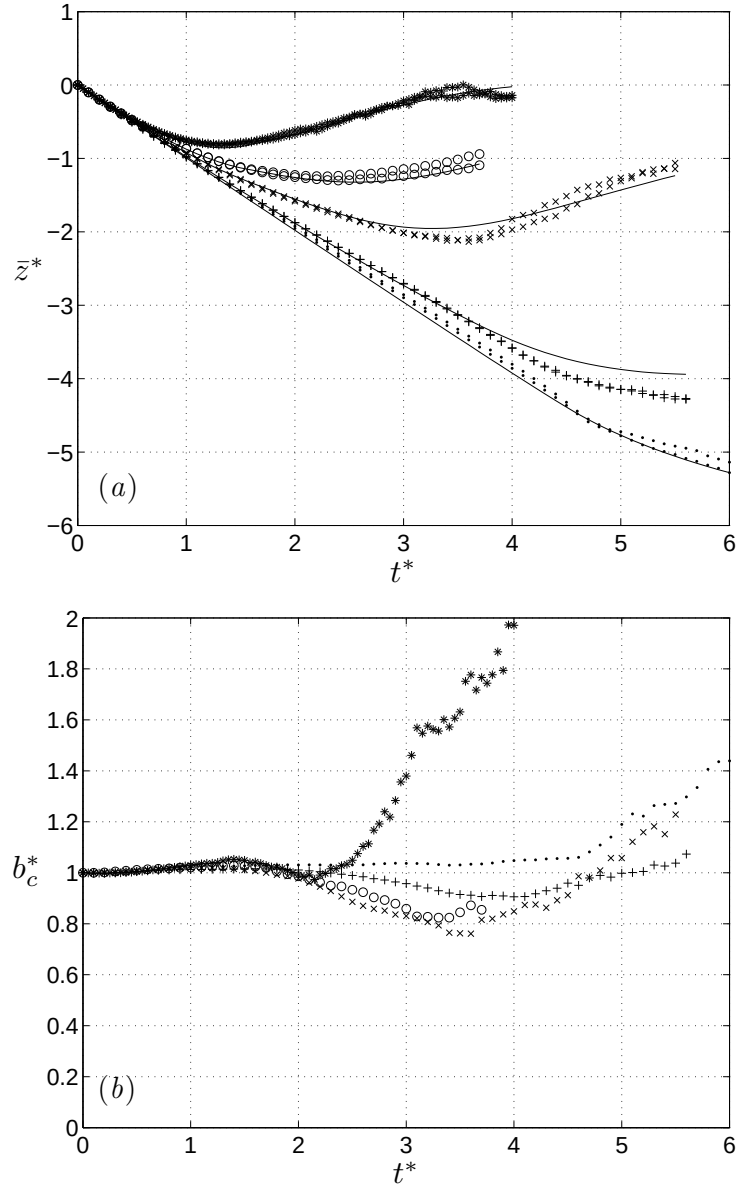


Figure 2.26: (a) Average vertical displacement of both the left and right vortices and (b) lateral spacing evolution for case 0 ($N^* = 0$) ($\cdot$), case 1 ($N^* = 0.35$) ($+$), case 2 ($N^* = 0.75$) ($\times$), case 3 ($N^* = 1$) ($\circ$), and case 4s ($N^* = 1.4$) ($*$). For all cases, $\epsilon_f^* = \epsilon_1^*$. In (a), the results of the model, detailed in Section 3.2.3, are also displayed (solid).

model is proposed and detailed in Section 3.2.3. The results of the model are also shown in Fig. 2.26.

The circulation distribution $\Gamma(r)$ is also obtained in every x -plane by integration of the axial vorticity component on a disk of radius r centered on the vortex position. The total circulation associated with each primary vortex, Γ_{tot} , is then estimated using the maximum of this function. This reads:

$$\Gamma(x, r, t) = \int_0^{2\pi} \int_0^r \omega_x(x, r', \theta, t) r' dr' d\theta, \quad (2.32)$$

$$\Gamma_{\text{tot}}(x, t) = \max_r \Gamma(x, r, t). \quad (2.33)$$

This last quantity is also averaged to provide a general idea of the “average total intensity” of the vortex:

$$\bar{\Gamma}_{\text{tot}}(t) = \frac{1}{n_x} \sum_{i=0}^{n_x-1} \Gamma_{\text{tot}}(x_i, t). \quad (2.34)$$

The evolution of $\bar{\Gamma}_{\text{tot}}$ is provided in Fig. 2.27. One observes two phases in the decay as in the studies of Switzer & Proctor [111, 112] and Holzäpfel *et al.* [57]: a slow-decay phase followed by a rapid-decay phase, the starting time of the second phase depending on the stratification level. However, compared to these studies, we observe a much slower decay in the first phase. There is indeed almost no circulation decay during that phase, indicating that the SGS model does not spuriously dissipate the vortices until they become turbulent. This slow initial decay is seen to slightly depend on the stratification level. Comparing the circulation decay evolution to the 3-D flow visualizations, one concludes that the second phase of decay is due to both the stratification and the linking process induced by the Crow instability, if any. As observed in the results obtained in domain with shorter axial extension, with stratification, the strong decay phase is caused by two phenomena: the three-dimensional interactions between the primary wake vorticity and the secondary baroclinic vorticity and, when the vortices come close to each other ($N^* = 0.75$ and 1), three-dimensional interactions between the wake vortices themselves. Based on the present results, a model for the evolution of the mean total circulation was developed and is detailed in Section 3.2.3. The results of the model are also shown in Fig. 2.27.

The decay is further characterized by measuring the total circulation in various cross-planes, using Eq. (2.33), and then by computing the mean and

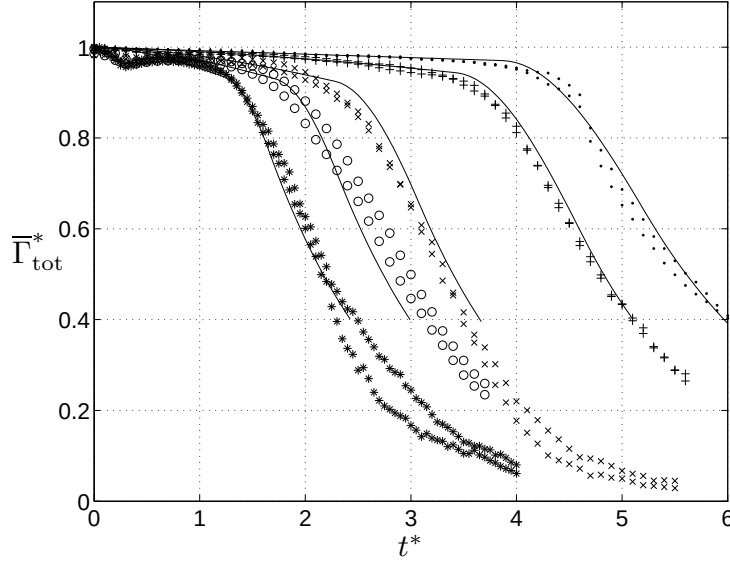


Figure 2.27: Average total circulation evolution of both the left and right vortices for case 0 ($N^* = 0$) ($\cdot$), case 1 ($N^* = 0.35$) (+), case 2 ($N^* = 0.75$) ($\times$), case 3 ($N^* = 1$) ($\circ$), and case 4s ($N^* = 1.4$) (*). For all cases, $\epsilon_f^* = \epsilon_1^*$. The results of the model, detailed in Section 3.2.3, are also displayed (solid).

95% envelope of Γ_{tot} . The uniformity of the circulation decay is observed in Fig. 2.28(a-d). When the stratification level is from medium to high ($N^* \geq 0.75$), the instabilities have short to medium wavelengths and the turbulence integral scale has a weak impact; hence, one obtains a clear two-phase decay that is essentially uniform along the vortex longitudinal direction. The resulting envelopes are relatively tight around the mean. On the other hand, when the stratification is absent or low (e.g., $N^* \leq 0.35$), the long-wavelength Crow-type instability is dominant and this leads to vortex linking. One then observes a sudden increase of the envelope size at the time of linking and the decay is very non uniform along the vortex. Some planes then exhibit a clear and strong second phase of decay starting at the time of vortex linking, while others still only exhibit a weak second phase of decay. It is also to be noted that, even without stratification, the total circulation slightly decreases all along the vortices due to the linking process. In addition, when adding stratification, the interaction with the baroclinic vorticity, of opposite sign, leads to a further decay of the circulation.

In recent LES studies [84, 58], a three-phase decay is observed. On the con-

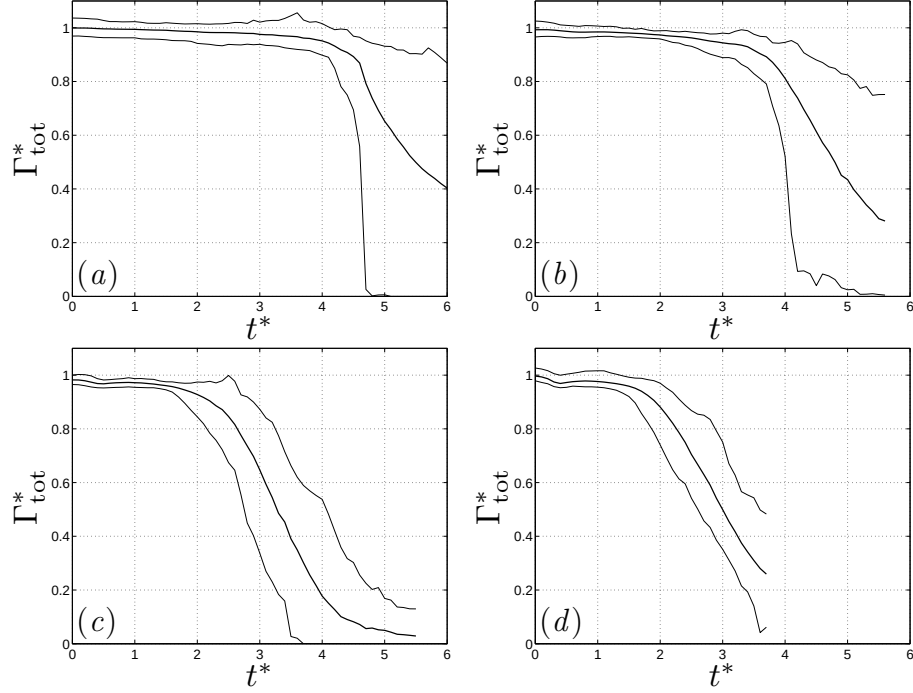


Figure 2.28: Average $\bar{\Gamma}_{\text{tot}}^*$ (thick solid) and 95% envelope (thin solid) of the total circulation evolution for (a) case 0 ($N^* = 0$), (b) case 1 ($N^* = 0.35$), (c) case 2 ($N^* = 0.75$), and (d) case 3 ($N^* = 1$). For all cases, $\epsilon_f^* = \epsilon_1^*$.

trary, in the present investigated cases, a two-phase decay is observed in each x -plane. It is believed that the three-phase decay behavior is a result of the averaged circulation measurement computation technique at and after vortex linking. In Proctor *et al.* [84], in order to only capture the vortex circulation, they average the circulation over every x -plane, except those for which certain thresholds are not met in terms of vorticity, pressure and vortex angle. Following this methodology, after linking, only planes containing the vortex rings are taken into account. In Holzäpfel *et al.* [58], they integrate the circulation in planes that are perpendicular to the vortex filaments. As long as the vortex filament is not perpendicular to the x -direction, the circulation evaluation in planes perpendicular to the vortex filament or in planes perpendicular to the x -direction leads to the same results. Yet, the vortices remain pretty much parallel until vortex reconnection. Just before linking, close to the linking point, the proximity of the other vortex significantly decreases the evaluated circulation value. Just after linking, at the linking point, the vortex filament

is perpendicular to the x -direction. In that particular plane, the vortex ring separation is higher than the vortex separation before linking, leading to an evaluated circulation value higher than before linking. Using either approaches, the obtained three-phase decay successively corresponds to: i) the slow-decay phase, ii) the start of the second fast-decay phase due to vortex reconnection or strong interaction with baroclinic vorticity, iii) the average of the circulation in planes with slowest second phase decay (those containing the vortex ring). The three-phase decay is hence not a new physical vortex behavior, as it is not observed in any x -plane, where the circulation always decays in two phases. It rather is a characteristic of the circulation averaged over planes where the vortex is still “alive”.

The partial circulation averaged between a radius of 5 m and a radius of 15 m, Γ_{5-15} defined in Eq. (1.17) and computed assuming $\frac{b_0}{b} = \frac{\pi}{4}$, is also measured. Figure 2.29 compares the evolution of the x -averaged Γ_{5-15} for the different investigated cases. Compared to Fig. 2.27, one observes globally the same evolution, with a two-phase decay controlled by the stratification level. Some differences are although to be noticed. First, $\bar{\Gamma}_{5-15}$ only contains part of the total circulation, as can be seen by its initial value. It is also interesting to observe that, in the first decay phase, $\bar{\Gamma}_{5-15}$ is even better conserved than $\bar{\Gamma}_{\text{tot}}$ (it is even fully conserved for a significant fraction of the slow decay phase): the vortical structures generated around the primary vortices (due to the stratification or/and the ambient turbulence) do not affect, so far, much the circulation distribution up to $b/4$. On the other hand, for the cases for which reconnection occurs ($N^* \leq 0.75$), close to linking time, $\bar{\Gamma}_{5-15}$ progressively decreases whereas the change in $\bar{\Gamma}_{\text{tot}}$ is more abrupt. This indicates that during the reconnection phase, the circulation distribution is reorganized but the total circulation is not changed yet. This is also consistent with the study of Moet *et al.* [77].

The uniformity of the Γ_{5-15} circulation decay is also observed in Fig. 2.30(a-d). As observed on the Γ_{tot} evolution in Fig. 2.28, a two-phase decay is observed in every x -plane. However the decay rate of this second phase is seen to be higher for Γ_{5-15} than for Γ_{tot} . The spread of the 95% envelopes around the mean observed in Fig. 2.30 is also smaller compared to that in Fig. 2.28.

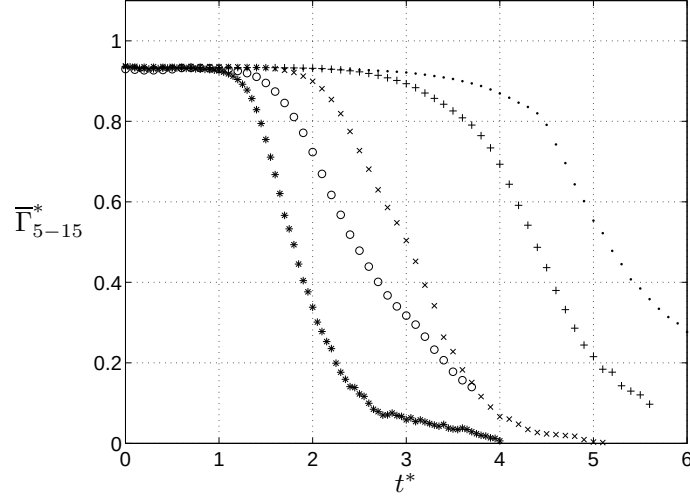


Figure 2.29: Average $\bar{\Gamma}_{5-15}$ circulation evolution for case 0 ($N^* = 0$) ($\cdot$), case 1 ($N^* = 0.35$) ($+$), case 2 ($N^* = 0.75$) ($\times$), case 3 ($N^* = 1$) ($\circ$), and case 4s ($N^* = 1.4$) ($*$). For all cases, $\epsilon_f^* = \epsilon_1^*$.

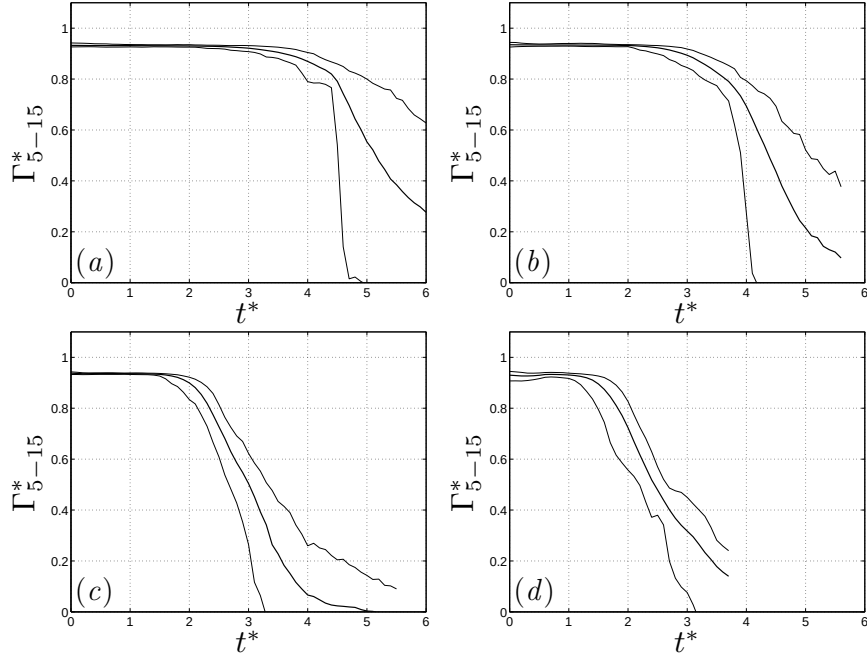


Figure 2.30: Evolution of the average $\bar{\Gamma}_{5-15}^*$ (thick solid) and 95% envelope (thin solid) of the the partially averaged circulation for (a) case 0 ($N^* = 0$), (b) case 1 ($N^* = 0.35$), (c) case 2 ($N^* = 0.75$), and (d) case 3 ($N^* = 1$). For all cases, $\epsilon_f^* = \epsilon_1^*$.

Influence of the stratification on the Crow instability development

Vortices evolving in a turbulent and stratified atmosphere are subjected to both long- and short- scale instabilities. As observed in the flow visualizations in Figs. 2.22 to 2.25, the long-wavelength Crow instability develops up to the vortex linking for cases with $N^* \leq 0.75$. It is interesting to note that Sarpkaya [100] also observed that, for $N^* \geq 0.75$, the vortex first developed the Crow instability and then went on spinning about their axes without linking. According to the LES results of Switzer & Proctor [111], the critical stratification level above which vortices no longer link is between 0.5 and 1.0. The present results propose that linking still occurs at $N^* = 0.75$ (as seen in Fig. 2.24) but certainly no longer occurs at $N^* \geq 1$ (as seen in Fig. 2.25). We thus propose that the critical N^* value could be between 0.8 and 0.9.

The short-wavelength instabilities are initiated in the baroclinic vorticity. The more stratified the atmosphere is, the more energetic will be the high frequency modes. In order to visualize this phenomenon, Figs. 2.31(a) and 2.31(b) compare the time evolution of the axial mode energy for cases 1 and 3. In these figures are shown the rigid mode (mean 2-D flow), representing the energy of the axially averaged field, and the 64 first modes with wavelength varying from $\lambda = 8b_0$ down to $\lambda = 0.125b_0$.

For low stratification level (case 1), for $t^* \leq 1.5$, the long-wave modes are dominant and grow progressively while the 2-D energy is almost conserved. For $1.5 \leq t^* \leq 3$, the short-scale instabilities also start to grow significantly and the 0 mode slightly decreases. For $t^* \geq 3$, the 2-D energy starts to significantly decrease, the long-wave modes reach a plateau whereas the short-scale instabilities continue to grow before reaching a plateau at $t^* = 4$. For high stratification level (case 3), the long-wavelength mode dominates the flow for very short times ($t^* \leq 0.5$). At $t^* = 1.0$, it already reaches a plateau while the short-scale mode energy drastically rises. At $t^* = 1.8$, the 2-D energy starts a phase of rapid decay. At $t^* = 2.2$, the short-scale energies saturate and the long-wavelength energy drastically decreases. The flow has become fully turbulent (recall also Fig. 2.25). Comparing case 1 with case 3, one clearly sees that the stratification level affects the short-wavelength mode development. For low stratification levels, the flow is dominated by the large-scale instabilities with an influence of the high-wavenumber instabilities, whereas, for very high stratification levels, the flow is dominated by the high-wavenumber instabilities with an influence of the large-scale instabilities. Given the low effect of the $8b_0$

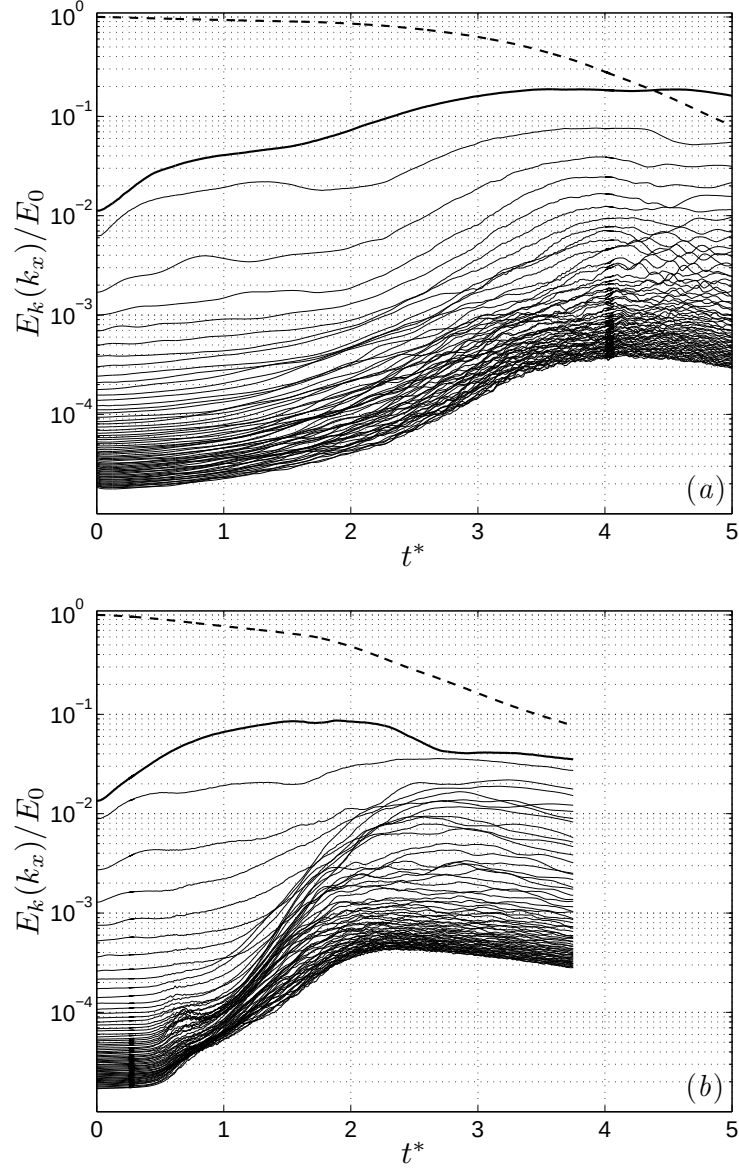


Figure 2.31: Longitudinal modal energy evolution for (a) case 1 ($N^* = 0.35$) and (b) case 3 ($N^* = 1$). For the sake of clarity, only selected modes are displayed: the rigid mode (dash), the $\lambda = 8b_0$ mode (thick solid) and the 63 next modes down to $\lambda = 0.125b_0$ (thin solid). The displayed energy are dimensionless, using the initial rigid mode energy E_0 .

mode on the flow, the results of case 3 and 3s are very similar. This justifies that for case 4s ($N^* = 1.4$), a smaller computational domain ($L^* = 4$) is used.

The development of the Crow instability is first studied by comparing the $\lambda = 8b_0$ mode energy evolution for all investigated cases. The comparison is provided in Fig. 2.32. In all cases, after an adaptation phase, the mode grows exponentially, reaches a plateau and then decreases. For very short times ($t^* \leq 0.5$), the evolutions are very similar. For $N^* = 0.35$ (case 1), the turbulence damped by the stratification appears to have an influence on the early evolution as the mode grows slower compared to case 0. However, the mode saturates earlier (around $t^* = 3.5$ for case 1, against $t^* = 4.0$ for case 0). The instability appears to be first slowed down by the stratification and then accelerated. When increasing the stratification level to $N^* = 0.75$ (case 2), the $8b_0$ mode grows much faster to saturate at around $t^* = 2.0$. As already seen, for further increased stratification levels, $N^* \geq 1.0$, the high-wavenumber modes dominate the flow and the maximum amplitude reached by the $8b_0$ mode does not enable a Crow linking. In order to link, it appears that the mode should reach an energy at least equal to 0.17 of the initial energy of the 2-D vortex system (i.e., the rigid mode energy).

In order to measure the growth rate of the Crow instability, an antisymmetric wave, with a wavelength $\lambda = 8b_0$ and inclined with an angle θ_p , is fitted on the measured vortex 3-D positions at each time, following the formulas:

$$z^*(x^*, t^*) - \bar{z}^*(t^*) = A^*(t^*) \sin(\theta_p(t^*)) \sin\left(\frac{2\pi}{8}(x^* - x_p^*)\right), \quad (2.35)$$

$$y^*(x^*, t^*) - \bar{y}^*(t^*) = \pm A^*(t^*) \cos(\theta_p(t^*)) \sin\left(\frac{2\pi}{8}(x^* - x_p^*)\right), \quad (2.36)$$

where x_p is a possible phase shift, θ_p is the angle formed by the vortices with the horizontal axes, and the $\pm$ sign in Eq. (2.36) stands for the right and left vortices, respectively. This fitting technique enables to only consider the growth of the antisymmetric mode and has three degrees of freedom A , θ_p and x_p . At each time, the data are fitted using this model. The evolution of the fitted amplitude A is displayed in Fig. 2.33(a). In this figure, one observes, for all stratification levels, three phases. First, during the “adaptation” period, the orientation of the vortices progressively changes accompanied by a growth of the amplitude. During the second phase, the orientation angle tends to a plateau and the amplitude grows exponentially. Finally, close to reconnection, the quality of the fit significantly decreases, the vortices tend to form rings not

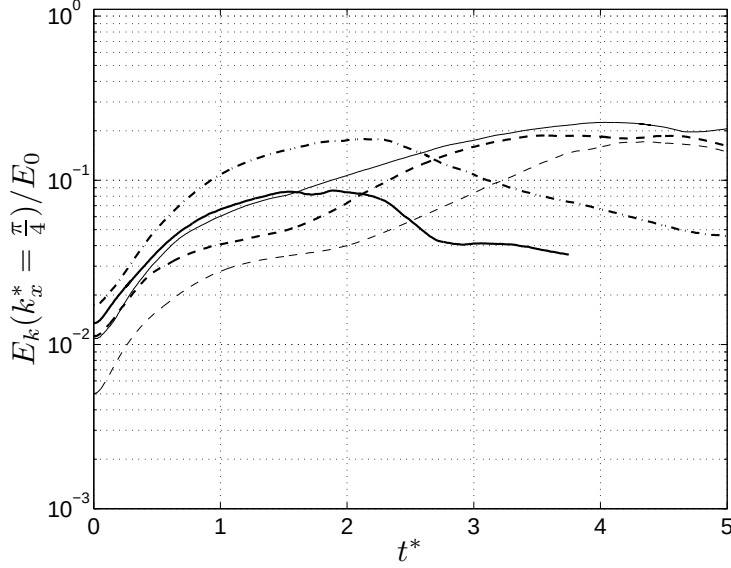


Figure 2.32: Comparison of the $8b_0$ longitudinal modal energy evolution for case 0 ($N^* = 0$, $\epsilon_f^* = \epsilon_1^*$) (thin solid), case 1 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_1^*$) (thick dash), case 2 ($N^* = 0.75$, $\epsilon_f^* = \epsilon_1^*$) (dash-dot), case 3 ($N^* = 1$, $\epsilon_f^* = \epsilon_1^*$) (thick solid), and case 5 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_2^*$) (thin dash). The displayed energies are dimensionless, using the initial rigid mode energy E_0 .

well represented by a sinusoidal wave. A fit is performed on the evolution of the instability amplitude in the second phase, following:

$$\frac{A(t^*)}{b_0} = A^*(t^*) = A_0^* \exp(\sigma^* t^*). \quad (2.37)$$

The fits for the different cases are shown in Fig. 2.33(a). Table 2.4 reports the values of the growth rate σ^* , the initial amplitude A_0^* , and the mean orientation angle θ_p .

The measured growth rate can be compared to that predicted by Crow's theory [16]. In order to properly compare with Crow's model, one considers an equivalent Rankine vortex pair (see e.g. [70]) so that both 2-D flows have the same energy. The energy matching of the Rankine vortex pair (whose distribution is provided in Table 1.3) with that of the low order algebraic vortex pair, used here, leads to:

$$\log\left(\frac{b_0}{a_e}\right) + \frac{1}{4} = \log\left(\frac{b_0}{r_c}\right) - \frac{1}{2}. \quad (2.38)$$

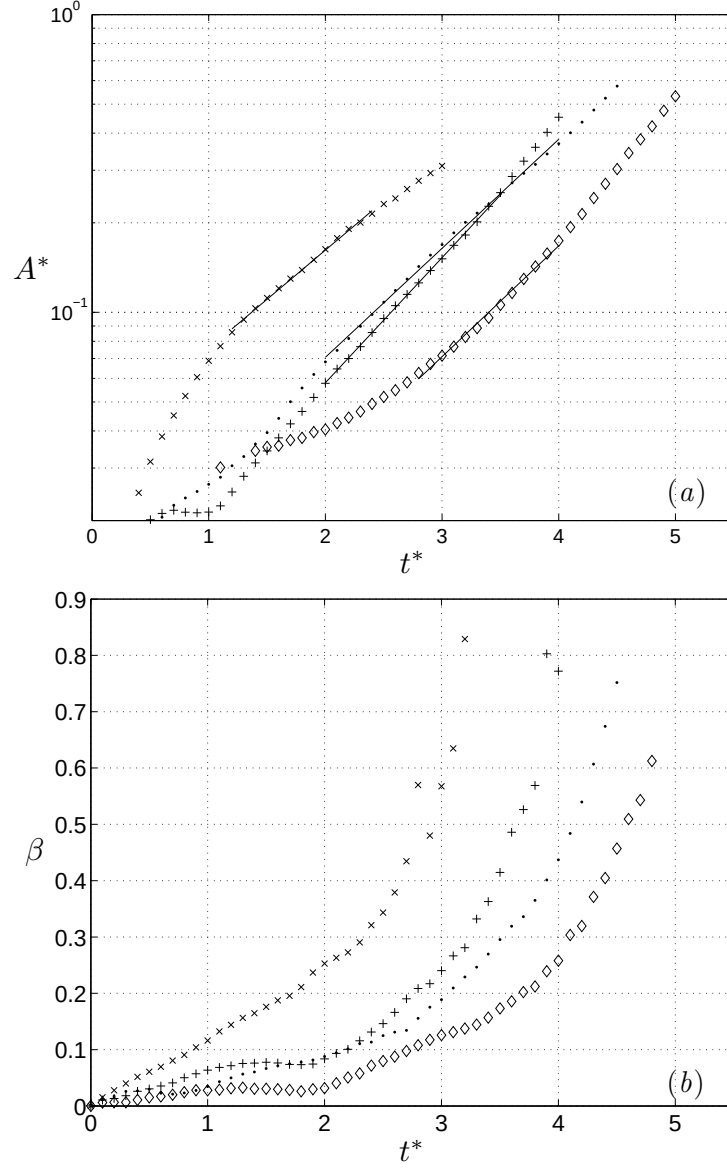


Figure 2.33: Comparison of the temporal evolution of (a) the fitted Crow amplitude mode, using Eqs. (2.35-2.36), (b) the linking ratio β for case 0 ($N^* = 0$, $\epsilon_f^* = \epsilon_1^*$) ($\cdot$), case 1 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_1^*$) ($+$), case 2 ($N^* = 0.75$, $\epsilon_f^* = \epsilon_1^*$) ($\times$), and case 5 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_2^*$) ($\diamond$). In (a), the fit on the amplitude, corresponding to Eq. (2.37), is also displayed (solid).

One then obtains $a_e = 2.12 r_c$. Using the present core size $r_c = 0.05 b_0$, the growth rate and the angle of the instability of wavelength $\lambda = 8 b_0$ can be predicted by Crow's theory. One obtains $\sigma^* = 0.82$ and $\theta_p = 45^\circ$ (see also Fig. 2.21). Without stratification, the growth rate of the instability ($\sigma^* \simeq 0.84$) is very close to that predicted by the theory. However, the instability appears to develop in a plane slightly more vertical than expected. This phenomenon can probably be attributed to the influence of the background turbulence not taken into account in Crow's theory; the vortices have to be in equilibrium with themselves but also with the background turbulence. Robins & Delisi [96], in their numerical simulations, and Sarpkaya [100], in his experiments, also observed angles slightly higher than those predicted by Crow's theory.

With low stratification level (case 1), the background turbulence is damped by the stratification compared to the neutral case (case 0). The long-wavelength instability is therefore less triggered by the turbulence and its growth is lower. However, with stratification, the baroclinic vorticity, produced outside the primary vortices, tends to push the vortices together (see Fig. 2.26(b)), decreasing the actual vortex separation and enhancing the vortex reconnection. Consequently, the initial amplitude is lower but the growth rate is higher. The orientation of the vortices also decreases to a value closer to the theoretical one.

When increasing the stratification level to $N^* = 0.75$ (case 2), the vortices are almost immediately tilted and the Crow instability significantly grows. This is explained by the decrease of the lateral spacing b_c . Moreover, the instabilities coming from the baroclinic vorticity enhance the exchange of circulation between the two sides (see Fig. 2.27) and hence facilitate the reconnection. It should also be noted that, when the vortex lateral spacing decreases, the effective dimensionless wavelength λ/b_c increases, and so does the dimensionless core size r_c/b_c . For case 2, when $b_c/b_0 = 0.8$, $\lambda/b_c = 10$ and $r_c/b_c = 0.0625$. For those values, the theory predicts $\sigma^* = 0.78$ and $\theta_p = 57^\circ$, values closer to the present observations.

In order to take into account the lateral spacing variations, a second measurement of the amplitude growth rate is performed using, at each time, the current vortex spacing as characteristic length scale. This leads to:

$$\frac{A(t^*)}{b_c(t^*)} = A_{0,\text{loc}}^* \exp(\sigma_{\text{loc}}^* t^*). \quad (2.39)$$

The coefficient values of this fit are also provided in Table 2.4. The analysis of the values of σ^* and A_0^* , compared to σ_{loc}^* and $A_{0,\text{loc}}^*$, enables to highlight the influence of the actual vortex spacing $b_c(t^*)$. For neutral stratification, both growth rates and initial amplitudes are identical, as the vortex spacing does not vary. On the other hand, with stratification (e.g., $N^* = 0.75$), the growth rate is significantly higher using this new fit. Therefore, it better reveals the higher Crow instability growth rate with stratification, compared to that of a neutral case. It should also be noted that the characteristic time $t_0 = \frac{2\pi b_0^2}{\Gamma_0}$ also evolves in time due to the variations of both the lateral spacing and the circulation. A local characteristic time could then be computed using $t_{0,\text{loc}} = \frac{2\pi b_c^2}{\Gamma_{\text{tot}}}$. The use of $t_{0,\text{loc}}$, instead of t_0 , to normalize the time does not improve the fit quality of Eq. (2.39). This indicates that the faster Crow instability growth rates of the stratified cases are mostly due to the decrease of lateral separation.

Case	N^*	ϵ_f^*	θ_p	σ^*	A_0^*	σ_{loc}^*	$A_{0,\text{loc}}^*$	t_c^*
0	0.0	ϵ_1^*	57.6°	0.84	$1.3 \cdot 10^{-2}$	0.84	$1.3 \cdot 10^{-2}$	4.6
1	0.35	ϵ_1^*	47.2°	0.97	$8.4 \cdot 10^{-3}$	1.04	$7.2 \cdot 10^{-3}$	4.0
2	0.75	ϵ_1^*	53.7°	0.76	$3.5 \cdot 10^{-2}$	0.85	$3.1 \cdot 10^{-2}$	3.3
5	0.35	$\epsilon_2^* = \epsilon_1^*/10$	53.3°	0.86	$5.4 \cdot 10^{-3}$	0.96	$4.3 \cdot 10^{-3}$	5.1

Table 2.4: Characteristics of the Crow instability evolution for the investigated cases.

Another measurement of the degree of linking of the vortices is the β ratio (initially proposed by Crow & Murman [19] and Moore [78] and used in [100]), defined by:

$$\beta(t) = \frac{\Delta y_{\text{max}}(t) - \Delta y_{\text{min}}(t)}{\Delta y_{\text{max}}(t) + \Delta y_{\text{min}}(t)}. \quad (2.40)$$

β is zero in the absence of perturbation, while β close to 1 corresponds to a reconnection. This ratio has the advantage that it takes into account the actual separation between the vortices. The vortices are considered to link at the time t_c^* when $\beta = 0.8$. The value of the reconnection times t_c^* for all investigated cases are also provided in Table 2.4. Note that Switzer & Proctor [111] used $\beta = 0.85$ as threshold but the current results showed that $\beta = 0.8$ is more appropriate. This new threshold was established so that, for case 0, the obtained time t_c^* corresponds to the time at which the second phase of decay starts (associated with the reconnection). Those values can also be compared to those predicted

by Sarpkaya's model [103] that relates the time-to-demise to the turbulence intensity. Using the kinetic energy dissipation rate measured in Table 2.2, the time-to-demise model gives $t_d^* = 4.5$ for the neutral case, which agrees with $t_c^* = 4.6$ of Table 2.4. Since this model does not take into account stratification, its use for stratified cases would lead to erroneous results. Figure 2.33(b) shows the evolution of β for different stratification levels. One again observes that the linking ratio grows exponentially in time with a growth rate increasing with N^* .

It is interesting to note the value of the mean circulation at reconnection time, see Fig. 2.27. Without stratification, the average total circulation is about 0.86 of its initial value. For $N^* = 0.35$, it decreases to 0.81 whereas for $N^* = 0.75$ it drops to 0.49. The remaining circulation at reconnection thus decreases when the stratification level increases.

The stratification tends to accelerate the vortex reconnection by decreasing both the vortex lateral spacing and the remaining circulation of the vortices to be linked. Moreover, when reconnected, the vortex rings generated in a stratified atmosphere are much more unstable, due to short-scale instabilities in the baroclinic vorticity field, than those generated without stratification. For high enough stratification levels ($N^* \geq 0.8 - 0.9$), the short-wavelength instabilities develop faster than the long-wavelength instability leading to a fast destruction of the vortex pair before reconnection.

Influence of the turbulence level combined with stratification

The influence of the turbulence level for low and high stratification levels is also investigated by comparing case 1 with case 5 (for $N^* = 0.35$) and case 3s with case 6s (for $N^* = 1$). According to Switzer & Proctor [111, 112] and Holzäpfel *et al.* [57] and as observed in Fig. 2.20, it is to be expected that the turbulence intensity has a greater influence for low stratification levels than for higher N^* values. Visualizations of the flow for cases 3s and 5 are provided in Appendix C.

The cases are first compared based on the temporal evolution of their kinetic energy in Fig. 2.34 (a). For all cases, one observes a two-phase decay. In the first phase, the kinetic energy is transferred to the potential energy. The transfer rate only depends on the stratification level. Later on, when small structures are generated in the baroclinic vorticity and/or in the primary vortices (due, e.g., to reconnection), a second phase of energy decay starts. The starting

time of this second phase depends both on the turbulence and stratification levels. For case 3s compared to case 6s ($N^* = 1$), the energy curves are almost superimposed for all times: the physics of the flow are clearly controlled by the stratification. On the other hand, for case 1 compared to case 5 ($N^* = 0.35$), the energy curves are only superimposed up to $t^* \simeq 3$. The fast-decay phase starts a bit later when the turbulence level is lower but the two energy curves are then parallel: the fast-decay rates are the same. The flow behavior appears to be identical, with some delay.

Figure 2.34 (b) compares the kinetic energy dissipation rate ϵ for the four cases. One notes the initial differences of the dissipation rates for the higher (case 1 and 3s) and lower (case 5 and 6s) turbulence levels. One also observes that, for all cases, the initial dissipation rates are higher than those of the ambient turbulent and stratified fields without wake vortices. This initial kinetic energy decay is related to the adaptation of the flow to the superimposition of a two-vortex system to the ambient turbulence. For all cases, these adaptation phases are very short, as the dissipation rate quickly decreases initially. Later on, for $N^* = 1$, the dissipation rate quickly grows, for both turbulence levels, for $t^* \geq 1$, reaching approximately the same maximum value around $t^* = 2.4$. For these cases, the small scales that lead to dissipation are generated in the very strong baroclinic vorticity field. For $N^* = 0.35$, the two curves also grow to reach approximately the same maximum value but with a delay of $\Delta t^* \simeq 1$. The small scales are here generated both in the turbulent baroclinic vorticity (here influenced by the initial turbulence level) and in the primary vortices during the reconnection process (that is also delayed with a lower turbulence level).

In order to characterize the global vortex behavior, a comparison of the circulation decay is shown in Fig. 2.35. For the $N^* = 1$ cases, one again observes that the general vortex behavior is clearly dominated by stratification: both circulation evolution curves are almost superimposed and so are the trajectory curves (not shown). For $N^* = 0.35$, the total circulation evolution is noticeably influenced by the turbulence level only at large times when the Crow instability starts to be more pronounced. For times up to $t^* \simeq 4$, both the circulation and trajectory (not shown) evolutions are almost equivalent.

For $N^* = 0.35$, the turbulence level has an impact on the Crow instability development. In Fig. 2.32, one observes that, for the lower turbulence level of case 5, the $8b_0$ mode follows an evolution parallel to that of case 1, with a delay

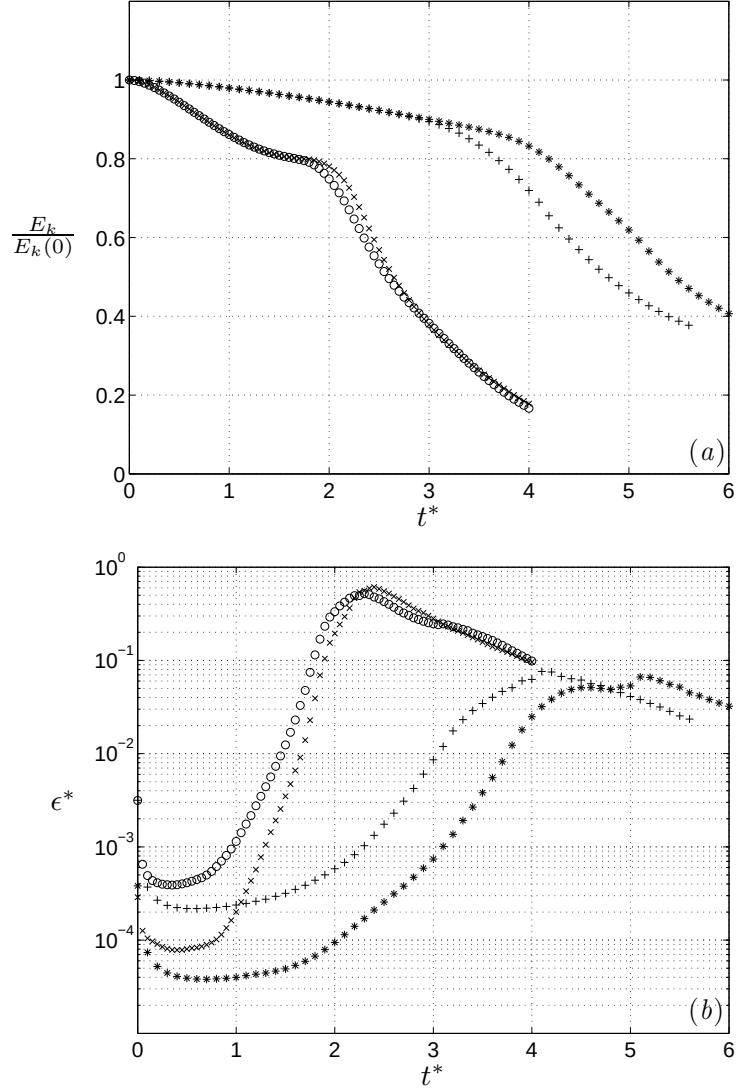


Figure 2.34: Comparison of the time evolution of (a) the kinetic energy and (b) the kinetic energy dissipation rate for case 1 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_1^*$) (+) versus case 5 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_2^*$) (*), and for case 3s ($N^* = 1$, $\epsilon_f^* = \epsilon_1^*$) (o) versus case 6s ($N^* = 1$, $\epsilon_f^* = \epsilon_2^*$) (x).

of $\Delta t^* \simeq 1$. In this case, it reaches a plateau at around $t^* \simeq 4$. Figure 2.33(a) shows the amplitude of the fitted Crow mode (based on Eqs. (2.35) to (2.37)) compared to the other cases and Table 2.4 provides the parameters of the fit. It appears that the Crow instability growth rate is a bit lower compared to that of case 1, but a bit higher than that of case 0. On the other hand, the fitted initial Crow mode amplitude is 1.5 times smaller for case 5 compared to case 1. Using the modified fit of Eq. (2.39), this behavior is even more pronounced with a much higher growth rate, compared to the unstratified case, but a smaller initial amplitude compared to the cases with a higher turbulence level. Based on the β coefficient, shown in Fig. 2.33(b), the linking time is estimated to $t_c^* \simeq 5.1$, hence with a delay $\Delta t^* \simeq 1$ compared to case 1. It again clearly indicates that the stratification increases the Crow instability development by reducing the vortex lateral spacing and the remaining circulation to be reconnected. In addition, the ambient turbulence level also influences the instability development because it determines the initial amplitude of the long-wavelength mode. Due to the combination of stratification and turbulence effects, the influence of the ambient turbulence level on the Crow instability development is expected to be significant for neutrally stratified cases and essentially negligible for cases with $N^* \geq 0.75$.

Influence of the turbulence level without stratification

The influence of the turbulence level of unstratified atmospheres on wake vortex behavior is also investigated. For that purpose, the turbulence field obtained for case 0 ($N^* = 0$, $\epsilon_f^* = \epsilon_1^*$, $L^* = 8$) is re-used to build a strongly turbulent atmosphere. The turbulence level is increased by introducing, in the turbulent field of case 0, a vortex pair with the same separation $b_{0,2} = b_0$ and with a circulation 4 times lower. Hence, one has $V_{0,2} = 1/4 V_0$. Compared to the latter vortex properties, the background field has the same characteristic length scales but is more turbulent with a dissipation rate given by:

$$\epsilon_3^* = \epsilon_1^* \frac{b_{0,2}}{b_0} \frac{V_0^3}{V_{0,2}^3} = 64 \epsilon_1^* = 1.55 \cdot 10^{-2} \quad (2.41)$$

Taking $b_0 = 50$ m and $\Gamma_0 = 400$ m²/s, this case hence corresponds to an atmosphere with an EDR of $6.4 \cdot 10^{-4}$ m²/s³. According to Table 2.1, this represents a case of moderate to strong turbulence. In the following, we will use “strong” to refer to this case and will use “weak” to refer to the case

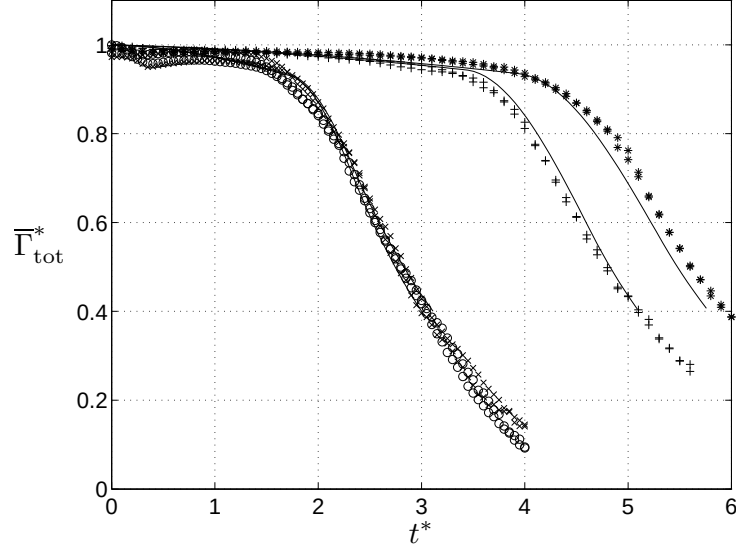


Figure 2.35: Comparison of the average total circulation evolution of both the left and right vortices for case 1 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_1^*$) (+) versus case 5 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_2^*$) (*), and for case 3s ($N^* = 1$, $\epsilon_f^* = \epsilon_1^*$) (o) versus case 6s ($N^* = 1$, $\epsilon_f^* = \epsilon_2^*$) (x). The results of the model, detailed in Section 3.2.3, are also displayed (solid).

$\epsilon_f^* = \epsilon_1^* = 2.42 \cdot 10^{-4}$. The dimensionless energy of the background turbulent field is then $E_k^* = 1.44 \cdot 10^{-1}$.

The influence of this stronger turbulent atmosphere on wake vortex behavior is first visualized in Fig. 2.36, using iso-surfaces of the scalar parameter λ_2 . As the turbulence is more energetic, at short times ($t^* \leq 1$), the $8b_0$ -wavelength instability is seen to quickly develop together with smaller-wavelength instabilities. At $t^* = 2.0$, the Crow instability dominates the flow. A helical instability is also observed on the vortex tube (see the left vortex). At $t^* = 2.5$, the vortices are then very close to link. However, later on, instead of creating vortex rings, medium-scale instabilities become significant and prevent the vortices to reconnect. At $t^* = 3.5$, the vortices have moved away from each other and are destroyed by medium- to small-scale instabilities. Compared to the case with weak turbulence, see Fig. 2.22, the Crow instability, triggered by the more energetic turbulence, is seen to develop much faster but does not lead to vortex linking.

The development of the instabilities is characterized, in Fig. 2.37, by comparing the longitudinal modal energy evolutions of the vortex system evolving

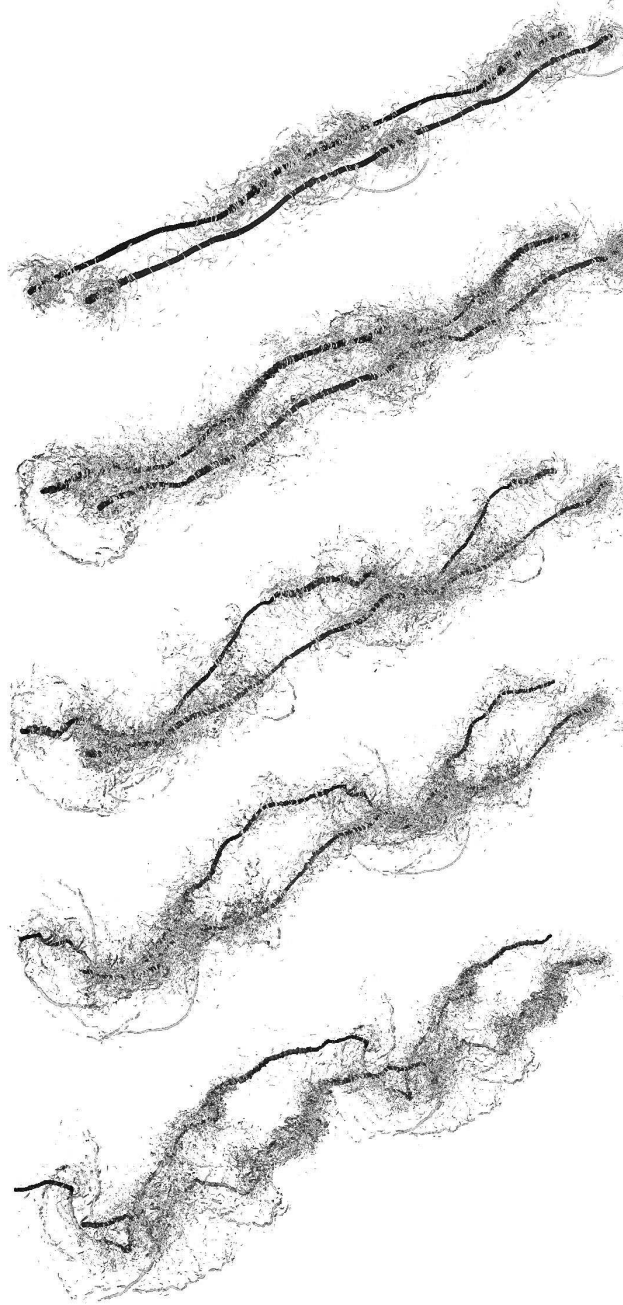


Figure 2.36: Visualization of the flow for case with $N^* = 0$ and $\epsilon_f^* = 1.55 \cdot 10^{-2}$, using iso- λ_2 surfaces colored by the axial vorticity $\omega_x b_0^2 / \Gamma_0$ at times (from top to bottom): $t^* = 1.0$, $t^* = 2.0$, $t^* = 2.5$, $t^* = 3.0$ and $t^* = 3.5$.

in weak and strong turbulence, respectively. In strong turbulence, all mode energies increase in time and saturate much earlier than in weak turbulence. In particular, the $8b_0$ -mode energy already reaches a plateau around $t^* = 0.8$ and then slightly decreases, whereas the $4b_0$ -mode energy slightly increases until $t^* = 3.5$. In weak turbulence, the $8b_0$ -mode clearly dominates the flow. On the contrary, in strong turbulence, the medium- to small-scale instabilities also significantly affect the flow. The 2-D energy evolutions also differ. In weak turbulence, a slow-decay phase is followed by a fast-decay phase starting close to reconnection. In strong turbulence, the 2-D energy decays fast and at a constant rate.

The global characteristics of the wake vortices evolving in both turbulences are measured and compared in Fig. 2.38. In strong turbulence, the vortices are seen to decay more rapidly and to spread laterally. As the total circulation decreases, the increase in lateral separation is to be expected by conservation of the linear impulse. As for the case with weak turbulence, a two-phase decay is observed for the strong turbulence case. As expected from literature results, the fast-decay phase starts much earlier. For $\epsilon^* = 1.55 \cdot 10^{-2}$, Sarpkaya's model (see description in Section 3.2.2) predicts a time of demise of $t_d^* = 2.3$, whereas DLR modification of Sarpkaya's model (see definition in Section 3.2.2) predicts $t_d^* = 1.5$, which is closer to the present results. In strong turbulence, the mean circulation decays faster, as the fast decay phase starts earlier. However, the fast decay rate is a bit lower than that of the weak turbulence case.

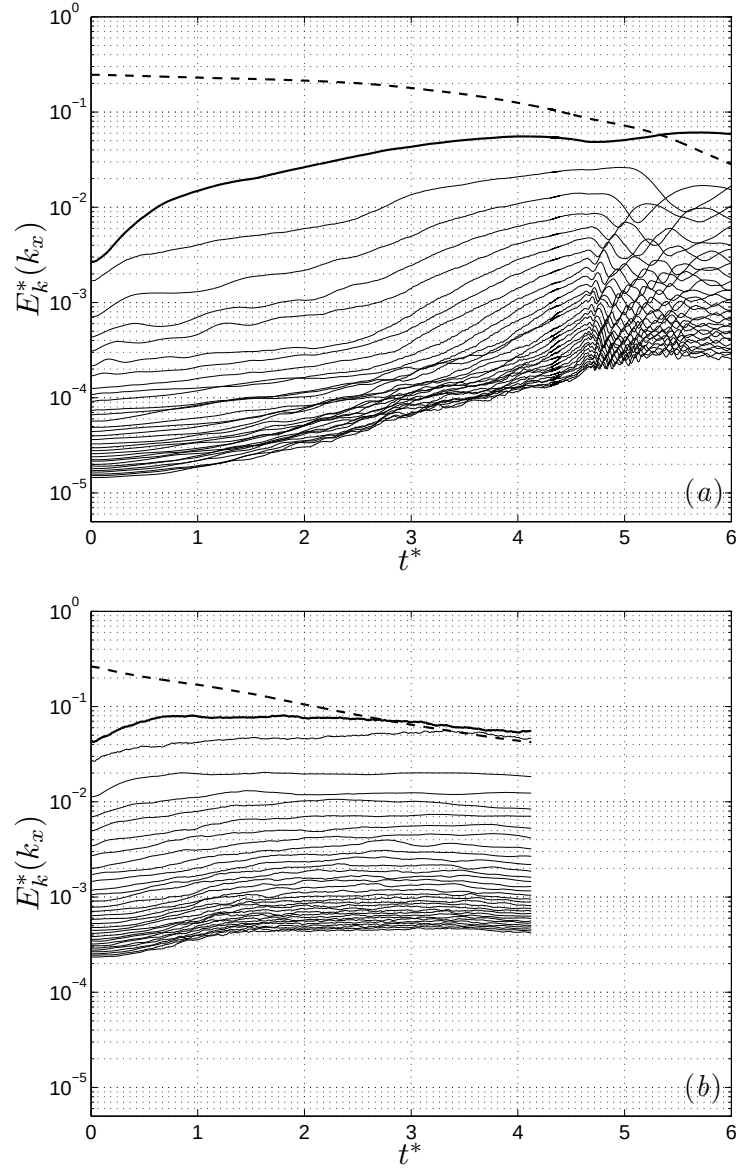


Figure 2.37: Longitudinal modal energy evolution of a vortex pair evolving in unstratified atmospheres with (a) $\epsilon_f^* = \epsilon_1^* = 2.42 \cdot 10^{-4}$ and (b) $\epsilon_f^* = 64 \epsilon_1^* = 1.55 \cdot 10^{-2}$. For the sake of clarity, only selected modes are displayed: the rigid mode (dash), the $\lambda^* = 8$ mode (thick solid) and the 31 next modes with λ^* down to 0.25 (thin solid).

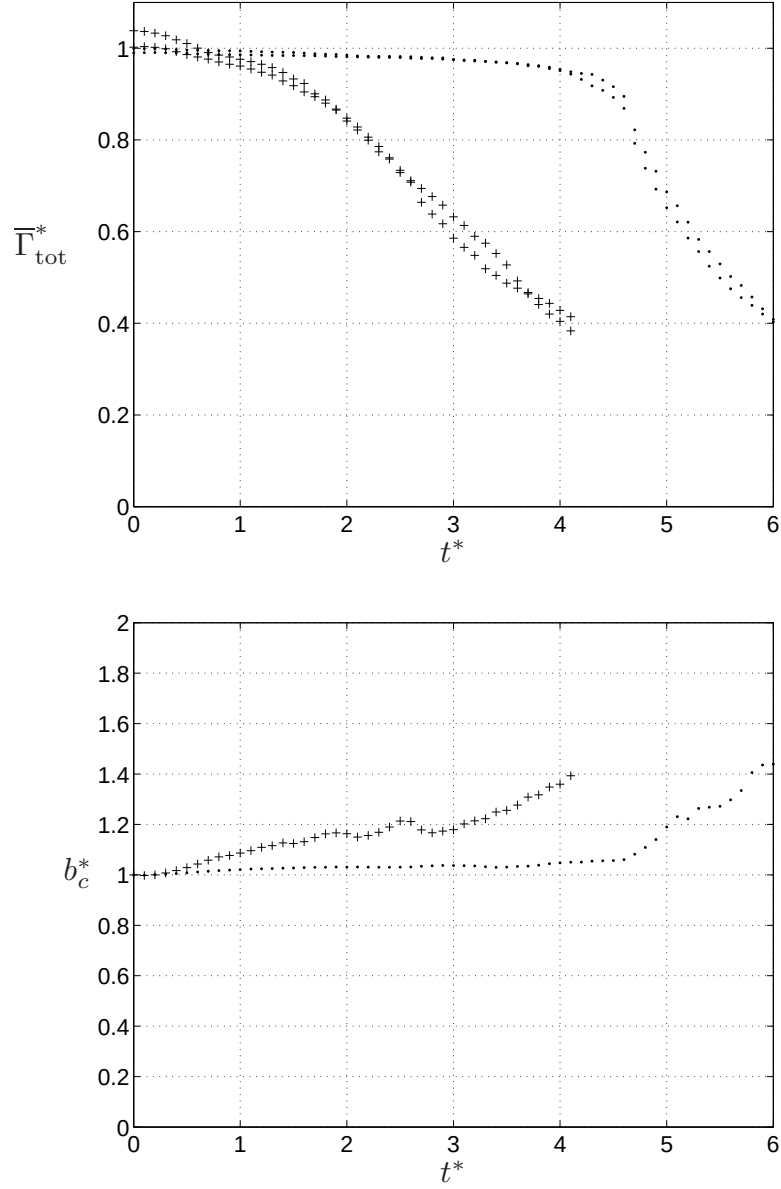


Figure 2.38: Evolution of the total circulation (top) and lateral separation (bottom) of a vortex pair evolving in unstratified atmospheres with: $\epsilon_f^* = \epsilon_1^* = 2.42 \cdot 10^{-4}$ ($\cdot$) and $\epsilon_f^* = 64 \epsilon_1^* = 1.55 \cdot 10^{-2}$ (+). The circulation values are provided for both the left and right vortices.

The circulation decay is further characterized by measuring the evolution of the mean and of the 95%-envelope of the circulation measured in every x -planes. The results are shown in Fig. 2.39. The high turbulence intensity relative to the vortex circulation is clearly observed in the initial size of the circulation envelope. The circulation decay is also seen to be more uniform along the vortex. Very quickly, there is a strong interaction with the ambient turbulence, leading to a fast decay of the vortex circulation all along the vortex. Two decay phases are then observed in the envelope evolution. It is hence worth noting that, in the case of weak turbulence, the fast-decay phase of the mean circulation evolution is related to vortex linking and is not necessarily representative of the vortex hazard (i.e., after reconnection, in some planes, the vortex has still $\sim 85\%$ of its initial circulation although the mean circulation is $\sim 0.4\Gamma_0$, see Fig. 2.28). On the contrary, in the case of strong turbulence, the mean circulation evolution is a good representation of the whole vortex behavior. The mean and 95%-envelope evolution of the partially averaged circulation Γ_{5-15} is also shown in Fig. 2.39. The Γ_{5-15} evolution is similar to that of Γ_{tot} although the envelope is a bit tighter. Γ_{5-15} is also seen to slowly decrease in the first decay phase which is not the case in weak turbulence (even with stratification, see Fig. 2.29). The interaction with strong turbulence hence also occurs in the region $b/12 \leq r \leq b/4$.

These results show that the Crow instability development is much influenced by turbulence. Indeed, for this case of moderate to strong turbulence, very significant large-scale deformations are visible at $t^* = 2.0$. In weak turbulence, see Fig. 2.22, similar Crow instability amplitude are only observed around $t^* = 4.0$. This is of great importance for hazard analysis, as an encounter of a straight vortex pair or of a significantly deformed vortex pair leads to different induced rolling moments. In addition, it appears that, in strong to moderate turbulence, the Crow instability development does not lead to vortex reconnection. Vortex rings will then only be created for cases of weak to moderate turbulence. We showed that linking does occur for $\epsilon^* = 2.42 \cdot 10^{-4}$ but not anymore for $\epsilon^* = 1.55 \cdot 10^{-2}$. The turbulence level threshold that would enable vortex reconnection is then comprised in the interval $2.5 \cdot 10^{-4} < \epsilon^* < 1.5 \cdot 10^{-2}$.

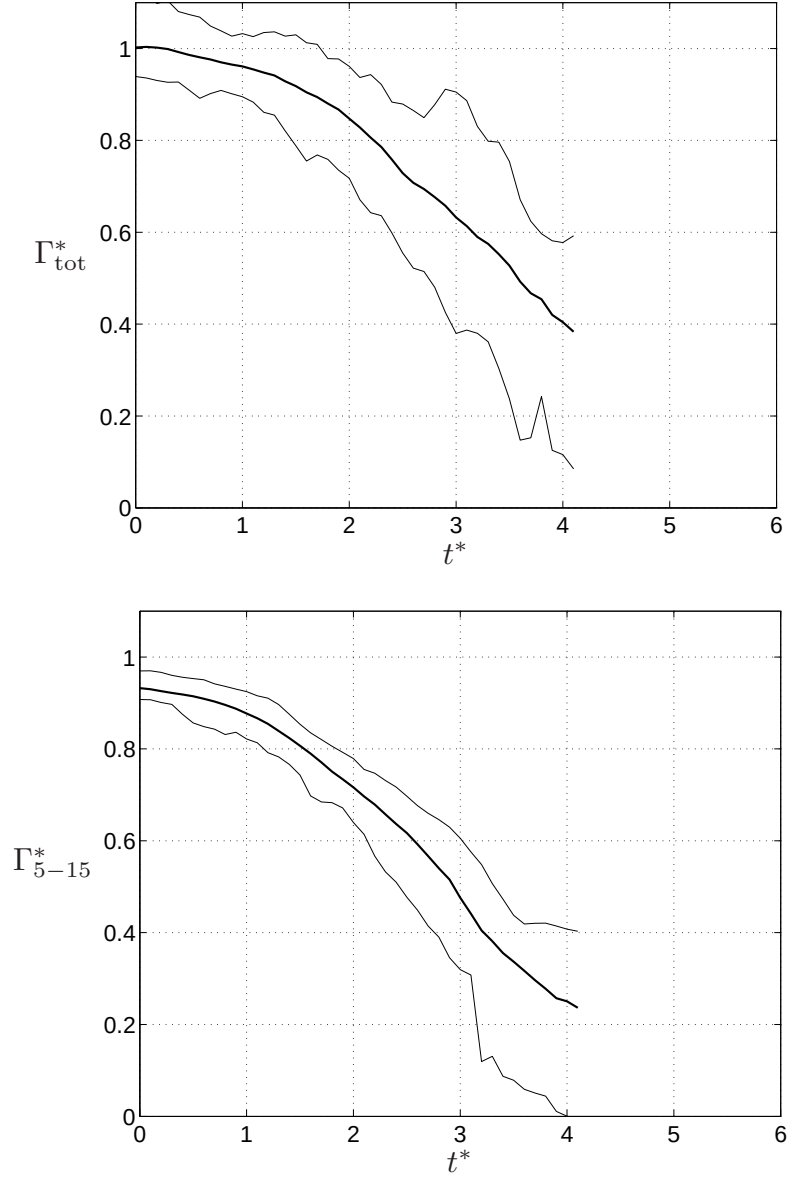


Figure 2.39: Mean (thick solid) and 95% envelope (thin solid) of the circulation evolution of a vortex pair evolving in an unstratified atmosphere with $\epsilon_f^* = 1.55 \cdot 10^{-2}$: total circulation Γ_{tot} (top) and partially averaged circulation Γ_{5-15} (bottom).

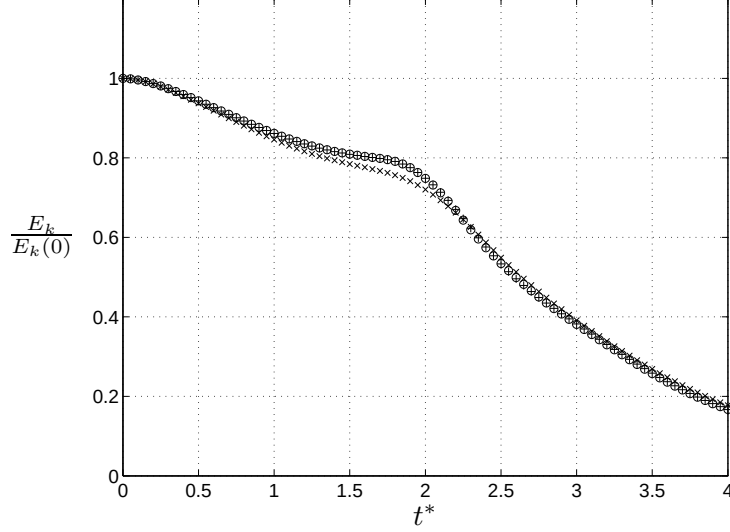


Figure 2.40: Kinetic energy evolution of case 3s using: $Re_\Gamma = 10^7$ ($\nu = 10^{-7}\Gamma_0$) with the HV SGS model ($\circ$), $Re_\Gamma \rightarrow \infty$ ($\nu = 0$) with the HV SGS model ($+$), and $Re_\Gamma \rightarrow \infty$ with the RVM SGS model ($\times$).

2.4.5 Sensitivity to the SGS model and to the Reynolds number

In order to validate the numerical approach followed in this study, two additional simulations are performed for case 3s ($N^* = 1.0$, $\epsilon_f^* = \epsilon_1^*$, $L^* = 4$). In the first one, the molecular viscosity ν is set to 10^{-7} ($Re_\Gamma = 10^7$) and the obtained results are compared to those obtained when using $\nu = 0$ ($Re_\Gamma \rightarrow \infty$). As seen in Fig. 2.40, the two obtained kinetic energy evolution curves are superimposed. The circulation evolutions are also compared in Fig. 2.41. The evolutions of the mean and 95%-envelope of the total circulation are also very similar for both simulation results. The simulations of the present study are therefore run in a regime where the molecular viscosity has no impact on the LES of the flow.

A second complementary LES is also performed for case 3s using the RVM SGS model instead of the HV model. It is important to stress that the turbulent stratified background field is also recomputed in a pre-simulation using this SGS model. The characteristics of the obtained turbulence are provided in Appendix B. Figures 2.40 and 2.41 compare, respectively, the kinetic energy and total circulation evolutions to those obtained when using the HV model.

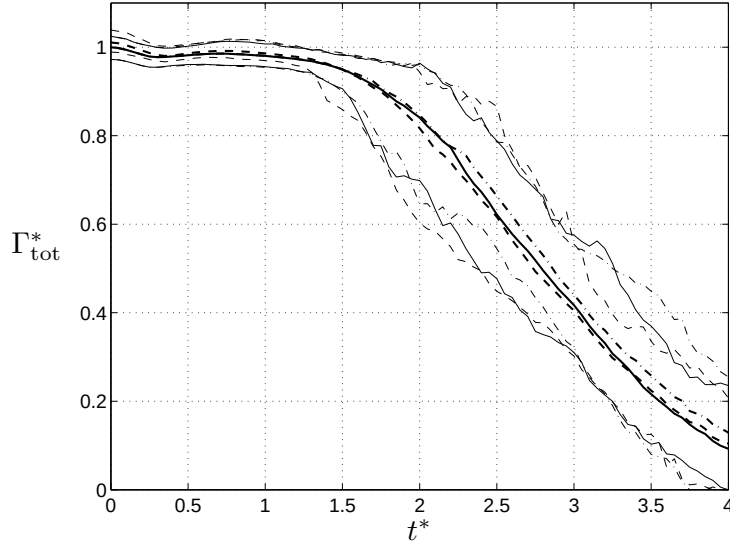


Figure 2.41: Average (thick line) and 95% envelope (thin line) of the total circulation evolution of case 3s using: $Re_{\Gamma} = 10^7$ with the HV SGS model (dash-dot), $Re_{\Gamma} \rightarrow \infty$ with the HV SGS model (solid), and $Re_{\Gamma} \rightarrow \infty$ with the RVM SGS model (dash).

The energy curves are almost identical, even though a marginal difference is observed for t^* around 1.75. This difference is however not visible in the wake vortex behavior as seen in the total circulation evolution. For wake vortex simulations, the HV SGS model appears to provide results of the same quality as those obtained when using a more sophisticated and computationally much more expensive multi-scale SGS model in such pseudo-spectral code.

2.4.6 Conclusions on wake vortices evolving in turbulent and stratified atmospheres

Pseudo-spectral numerical simulations of wake vortices interacting with various atmospheres were performed, using LES at very high Reynolds number. First, eleven turbulent, stratified or not, atmospheric fields were pre-computed. A great effort was put to obtain realistic stratified turbulent fields (both for velocity and temperature). The analysis of the obtained stratified turbulent fields showed that their characteristics are close to those of real atmospheres and are in good agreement with the literature. In each of these fields, we then added a pair of wake vortices, using a model with relatively tight vortex cores

($r_c/b_0 = 0.05$). The resolution was chosen so as to capture the essential physics of both the stratified turbulence and the wake vortex flow ($r_c/h > 3$). The turbulence of the temperature field was seen to have a strong impact on the wake vortex evolution, as the baroclinically generated vorticity, that also strongly interacts with the primary vorticity, is immediately turbulent.

The influence of the stratification on the wake vortex transport was fully characterized. As expected, the behavior is first essentially two-dimensional, the stratification inducing the vortices to rebound.

The circulation decay of each vortex was also characterized in terms of the circulation of the longitudinally averaged field and in terms of the longitudinal average of the vortex circulation. The first measurement only makes sense for cases without long-wavelength deformation of the vortices. Both total circulation Γ_{tot} and partial circulation Γ_{5-15} were measured. The circulation decay was correlated to the ambient atmospheric conditions. Different decay mechanisms related to turbulence and stratification were identified. A clear two-phase decay was observed in all cases. This is consistent with what is found in the literature for studies with lower resolution, higher vortex core size and/or lower effective Reynolds number. On the contrary, as opposed to what is found in the literature, the first decay phase was found to be very slow, yet with a decay rate that can be quantified and modeled, and that depends both on the stratification and the turbulence level. The decay observed in this phase is here not polluted by unwanted SGS model dissipation, as the SGS model only acts on the very small scales, preserving the energy of the large to medium structures. A new model for the vortex circulation decay, with two phases, and that accounts for the influence of both turbulence and stratification, was also calibrated on these results and will be presented in Chapter 3, Section 3.2.3. The circulation decay evolution was also measured in all cross-planes. Its evolution was then characterized in terms of its mean and of the 95% envelope. For low stratification levels and weak turbulence, the variation around the mean behavior is significant, whilst this is not the case for high N^* . This variation in the decay rate along the vortex has an influence on the interpretation of the results in an ATM framework since one cannot trust the mean circulation as a unique diagnostic of the vortex intensity.

We also compared the behavior of wake vortices evolving in stratified atmosphere with and without weak ambient turbulence. It appeared that the turbulent nature of the temperature field has a weak influence on the vortex

rebound, which is mainly governed by buoyancy. On the contrary, the effect on the circulation decay is very high. Indeed, the baroclinic vorticity generated around the primary vortices is directly turbulent and thus sooner interacts in a turbulent way with the vortices. The strong interactions of the turbulent baroclinic vorticity structures with the wake vortices induce a fast decay of the vortex circulation. The decay rate of the fast decay phase is thus essentially identical, with and without background turbulence, but the time at which the fast decay starts is different.

The influence of the ambient conditions on the development of instabilities was investigated, using a modal analysis. This analysis showed that, as expected, the Crow instability growth rate increases with the turbulence level. For weak turbulence, the Crow instability development leads the vortices to link and form vortex rings. On the contrary, in moderate to strong turbulence, the Crow instability quickly develops but strong interactions between the turbulence and the vortices destroy them and prevent the vortices from linking. This is of great interest for encounter analysis as, depending on the turbulence level, both the circulation and the topology of the vortices will be very different. This study also confirmed, at very high Re_Γ , that low to moderate stratification levels enhance the Crow instability development while, for higher N^* values, short-wavelength instabilities dominate the flow and quickly dissipate the wake vortices. For weak turbulence, no linking was seen to occur for N^* above a critical value of 0.8 – 0.9. The study also showed that, with stratification, the vortex rings created after the linking are more turbulent and decay more quickly.

Clearly, stratification efficiently destroys wake vortices, and even more so when combined with some weak ambient turbulence, as is most often the case in real atmospheres. Moderate to strong turbulence also destroys vortices. Both turbulence (moderate to strong) and stratification induce a decay that is very uniform along the vortex tube. On the contrary, in weak turbulence without stratification (or with weak stratification $N^* \leq 0.35$), very little decay is observed along the vortex, before vortex reconnection. Later on, when vortex rings are formed due to Crow instability, a strong decay is seen only in some cross-planes whereas a slow decay is still observed in the others. This fact is of great importance from an operational point of view. Indeed, in weak turbulence, the mean circulation decay has no meaning as it does not represent the decay measured in any cross-plane. One should then use envelopes of circula-

tion evolution. On the other hand, in moderate to strong turbulence and/or with moderate to strong stratification levels, the mean decay well represents the decay along the vortex and is hence a relevant measurement. This section provided a complete characterization of the influence of realistic turbulent and stratified atmospheres on wake vortex behavior: transport, decay and deformation. The present results are of great importance for the further improvement and calibration of operational wake vortex models, as done in Chapter 3.

2.5 Turbulent wake vortex system at equilibrium

2.5.1 Objectives, methodology and set-up

In this section, we further analyze the behavior of a two-vortex system (2VS) evolving in a weak turbulence without possible Crow instability development. This case was already analyzed in Section 2.4.3 and was called case 0s. It was then used as an upper bound of vortex decay in weak turbulence without stratification effects nor Crow instability development. Bricteux, in his PhD thesis [5], also studied this case. He found, using LES of a 2VS evolving in a decaying background turbulence, that the 2VS generates its own turbulence and reaches, after a certain time, a statistical equilibrium. One of the objectives of this section is to refine the previous work (also in terms of analyses). Another objective is to show that this equilibrium is also observed for wake vortices evolving in a weak atmospheric turbulence and is quite universal (in particular, the statistics of the equilibrium is not sensitive to the atmospheric weak turbulence itself).

The methodology used here is the same as that used in Section 2.4 and extensively described in Section 2.4.1. We only here recall the main features. The simulations are performed in two steps. First, the homogeneous isotropic background turbulence field is obtained using LES of forced turbulence reaching a statistically stationary state. The background turbulence has a dissipation rate $\epsilon_f^* = \epsilon_1^* = 2.42 \cdot 10^{-4}$. Then, a pair of wake vortices is added in the turbulence and evolves. The initial circulation distribution of each vortex corresponds to that of the low-order algebraic (LOA) model (see Table 1.3), with a core radius $r_c = 0.05 b_0$. The turbulence level and the initial condition for the 2VS correspond to those studied by Bricteux [5]. The computational periodic domain is a cube of size $L = 4 b_0$, hence not allowing a long-wavelength instability development. The resolution is $h/b_0 = 1/64$.

In the following, the temporal evolution of the 2VS is studied in Section 2.5.2, the wake vortex system at equilibrium is then fully characterized in Section 2.5.3. Finally, in Section 2.5.4, additional simulations are performed to investigate the influence of some parameters, of the initial 2VS and of the background turbulence, on the 2VS characteristics at equilibrium.

2.5.2 Evolution of the 2VS until statistical equilibrium

In the present artificial time-developing simulation, the time required for the 2VS to reach a “turbulent 2VS that has reached a statistical equilibrium” is quite long, due to the very low level forcing by the atmospheric turbulence, and is a bit irrelevant. Time, and weak turbulence continuous forcing, was here used solely as a way to eventually obtain a well-developed turbulent vortex system. In particular, the initial 2VS was not itself turbulent. In reality, the initial 2VS, shortly after roll-up, would also have turbulence in it. This turbulence comes from the turbulent wake of the fuselage (form drag) and from the boundary layers on the wing and the fuselage surfaces. Lonfils indeed showed, in his PhD thesis [75], that the roll-up of a vortex sheet with velocity deficit is unstable and generates turbulence around the formed 2VS. Hence, in reality, the 2VS would attain faster its statistical equilibrium.

Once Γ_0 , b_0 and r_c are defined, the energy of the initial 2VS considered alone (i.e., without the atmospheric weak turbulence added) can be evaluated from the 2-D initial periodic data. This gives:

$$\frac{E_{2D,\text{vort}}}{(V_0 b_0)^2} = \frac{1}{(V_0 b_0)^2} \int_{S_e} \frac{1}{2} (v^2 + w^2) dS = 15.376, \quad (2.42)$$

with S_e the bounded periodic surface. This energy is much larger (more than 100 times larger) than the energy associated with the added and weak initial atmospheric turbulence. The forcing is hence quite weak. This energy can also be compared to the initial energy of the mean flow (made of the superimposition of the two-vortex system and the background turbulence):

$$\frac{E_{2D}}{(V_0 b_0)^2} = \frac{1}{(V_0 b_0)^2} \int_{S_e} \frac{1}{2} (\bar{u}^2 + \bar{v}^2 + \bar{w}^2) dS. \quad (2.43)$$

with $\bar{u} = \frac{1}{L_x} \int u dx$, $\bar{v} = \frac{1}{L_x} \int v dx$ and $\bar{w} = \frac{1}{L_x} \int w dx$, the velocity components of the mean flow. Initially we measure $\frac{E_{2D}}{(V_0 b_0)^2} = 15.414$. The mean flow of the initial background turbulence hence has a non zero (yet relatively small) energy. Another quantity of interest is the total kinetic energy, defined by:

$$\bar{E} = \frac{1}{L_x} \int_{V_e} \frac{1}{2} (u^2 + v^2 + w^2) dV = \int_{S_e} \frac{1}{2} (\bar{u}^2 + \bar{v}^2 + \bar{w}^2) dS \quad (2.44)$$

with V_e the periodic volume. Initially, $\frac{\bar{E}}{(V_0 b_0)^2} = 15.439$, which is very close to the energy of the vortex system without turbulence $E_{2D,\text{vort}}$. Finally, using a

one-scale model, the energy of the 2-D field in an unbounded domain can be theoretically estimated using the formula developed in [21]:

$$E_{2D,\infty} = \int_S \frac{1}{2} (v^2 + w^2) dS = \frac{\Gamma_0^2}{2\pi} \left(\log \left(\frac{b_0}{r_c} \right) - C \right), \quad (2.45)$$

where $C = 0.5$ for a LOA (and $C = -0.05617$ for a Gaussian vortex). This formula has been verified to be very accurate for relatively small cores (i.e., $\frac{r_c}{b_0} \leq 0.1$, which is the case here). Using $r_c/b_0 = 0.05$, one finds $\frac{E_{2D,\infty}}{(V_0 b_0)^2} = 15.68$. Compared to 15.376, this is only 2.0% more. This confirms, at least from the initial energy stand point, that the size of the computationally box is already large enough that the periodically “confined” 2VS is essentially the same as an unbounded 2VS.

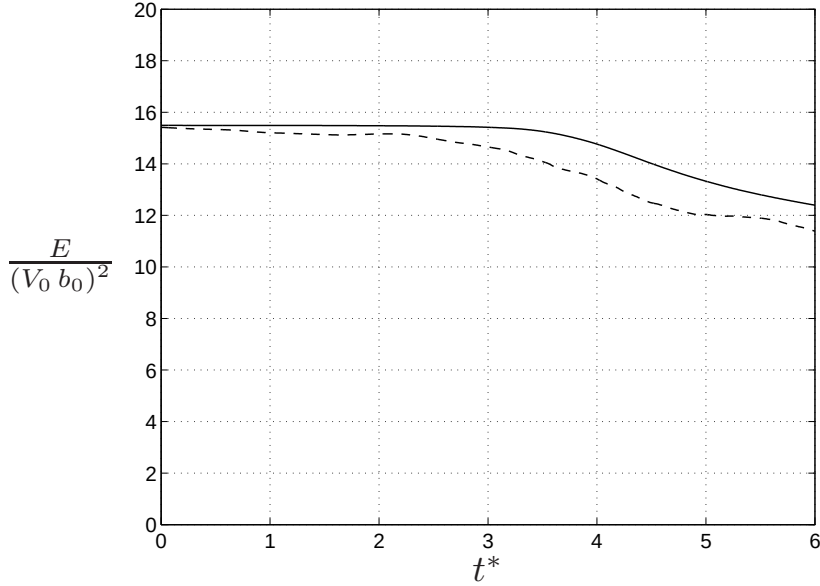


Figure 2.42: Temporal evolution of the kinetic energy: total kinetic energy $\bar{E}$ (solid) and kinetic energy of the mean flow E_{2D} (dash).

The temporal evolution of $\bar{E}$ and E_{2D} are shown in Fig 2.42. The fact that the 2VS evolves into a turbulent 2VS at equilibrium is a process where little, yet some, energy is dissipated. As observed in Fig. 2.13, the wake vortices stretches the vortical structures of the ambient turbulence generating transverse structures around the vortex cores. Those turbulence structures are continuously amplified by non-linear interactions. Eventually, this leads to a 2VS surrounded by complex turbulence. At short times ($t^* \leq 2.5$), no significant transverse vor-

ticity structures are observed. The 3-D energy evolution hence shows no decay. However, the 2-D energy shows a slow decay indicating that part of the base flow energy is transferred to 3-D turbulence. Around $t^* = 3$, the surrounding turbulence generation is seen to accelerate as the 2-D energy shows an increase in its decay rate whereas the 3-D energy stays almost constant. Around $t^* = 4$, the 2VS has reached its statistical equilibrium, as both the 2-D and 3-D energies decay at approximatively the same rate. The turbulent kinetic energy $\bar{k}$, which is the difference between the two curves, is hence almost constant. The “2VS at turbulent equilibrium” is here taken as that at $t^* = 6$. At that time, we have $\frac{\bar{E}}{(V_0 b_0)^2} = 12.396$, $\frac{E_{2D}}{(V_0 b_0)^2} = 11.387$ and hence $\frac{\bar{k}}{(V_0 b_0)^2} = 1.009$ (40 times more than the initial weak atmospheric turbulence!). Relatively, we obtain $\frac{E_{2D}}{\bar{E}} = 0.919$ and $\frac{\bar{k}}{\bar{E}} = 0.081$. Thus, in such 2VS at equilibrium, roughly 8% of the energy is in the turbulence, 92% being still associated with the strong vortical mean flow. Such ratios constitute important findings and are believed to be quite universal of 2VS at high Reynolds number and in equilibrium.

Figure 2.43 compares the 3-D power spectra of the 2VS system at different times. The large scales of the flow (i.e., the left part of the spectra) mostly depend on the wake vortices. Initially, the effect of the added weak atmospheric turbulence is only visible at very small scales. Later on, some energy is transferred from the mean flow to the medium to small turbulent scales. These transfers significantly modify the shape of the spectra until $t^* = 4.0$. Between $t^* = 4.0$ and $t^* = 6.0$, the shape of the spectra no longer changes, again indicating that an equilibrium has been reached. The total kinetic energy decreases slowly, but the energy transfers are still the same.

This energy transfer to the smaller scales of the flow is also visible by examining a zoom on one vortex, see Fig. 2.44. In that figure, one also observes the apparition of a small-scale instability, of Widnall type [117], on the vortex tube.

The wake vortex characteristics (position and circulation) are also evaluated. As done in Section 2.4.4, they are measured in each x -plane and then longitudinally averaged. The center of each vortex (left and right) is here determined using the location of minimum pressure. The pressure field has the advantage of being smoother than the vorticity field and hence enables an easier localization of the vortex center. From the found positions, the circulation profiles are then measured for each vortex and in each x -plane. We used two procedures for integrating the ω_x field: a procedure integrating the field

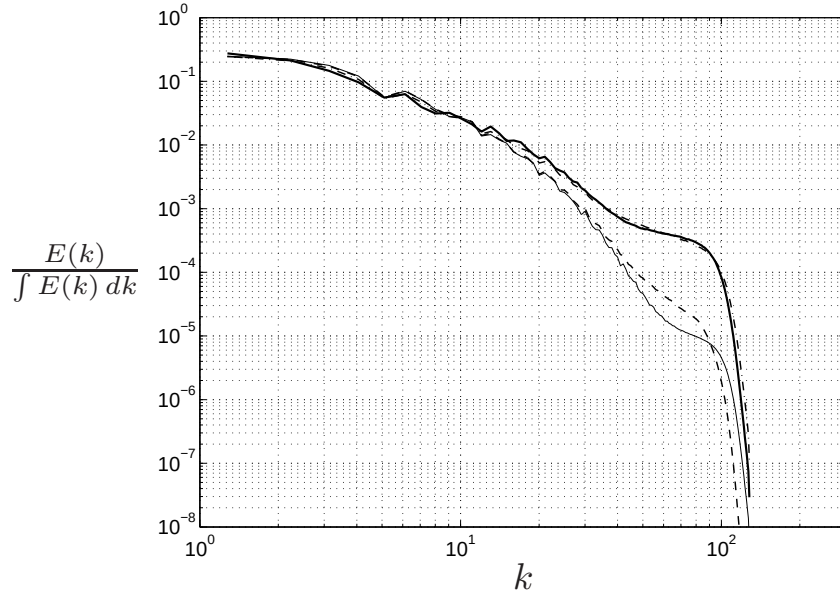


Figure 2.43: Comparison of the energy spectra at $t^* = 0.0$ (thin solid), $t^* = 2.0$ (dash), $t^* = 4.0$ (dash-dot), and $t^* = 6.0$ (thick solid). At each time, the spectrum is normalized by its integral.

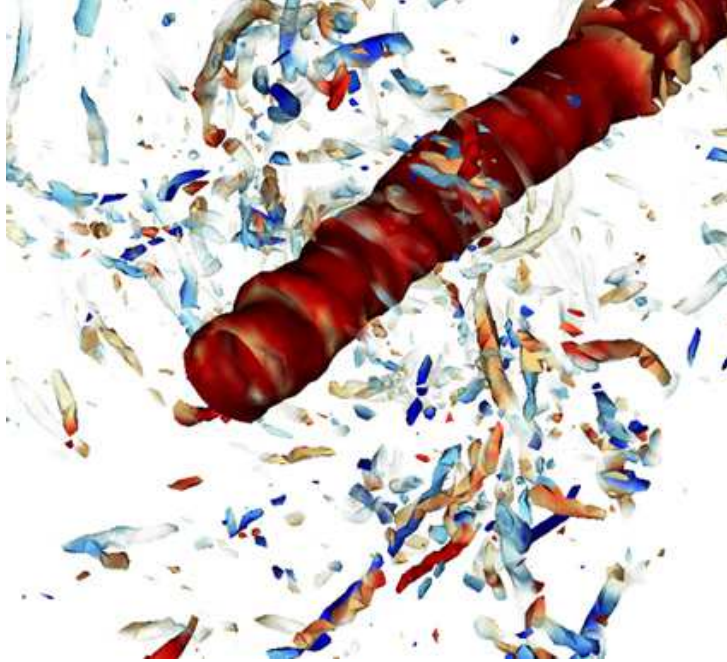


Figure 2.44: Iso-surface of the vorticity norm with $||\tilde{\omega}|| = 0.13$: zoom on one vortex.

from the Cartesian grid “as is”, and an improved procedure where the field is first interpolated on a fine (i.e., finer than the Cartesian data) polar coordinate system centered on the vortex center before doing the integration. From the measured circulation profile $\Gamma(r)$, one computes the total circulation Γ_{tot} , the azimuthal velocity distribution $u_\theta(r) = \Gamma(r)/(2\pi r)$ and the core radius r_c . The integration on the finer polar grid produces more accurate core sizes and slightly smoother profiles.

The evolution of total circulation and vortex separation are provided in Fig. 2.45. The total circulation measurements obtained using the Cartesian and the polar grids are both provided. Almost no difference is observed between both estimates. Until $t^* = 4$, very little circulation is lost. By conservation of the linear impulse, the vortex lateral separation is then also conserved. For $t^* \geq 4$, some circulation decay is observed due to the turbulent interaction between the left and right vortices. For those times, the mean vortex spacing $\bar{b}_{\text{tot}}^*$ hence also increases when the circulation decreases, as it should. Finally, the half plane circulation, computed from Eq. (2.22), evolution is provided in Fig. 2.45. $\bar{\Gamma}_{\text{HP}}^*$ shows an evolution similar to that of $\bar{\Gamma}_{\text{tot}}^*$. However, for $t^* \geq 4$, the total circulation is seen to decay a bit faster than $\bar{\Gamma}_{\text{HP}}^*$. This indicates that the total circulation decay is mostly caused by the exchange of circulation between both wake vortices but is also slightly influenced by the surrounding turbulence (which, by that time, has also become more energetic than initially).

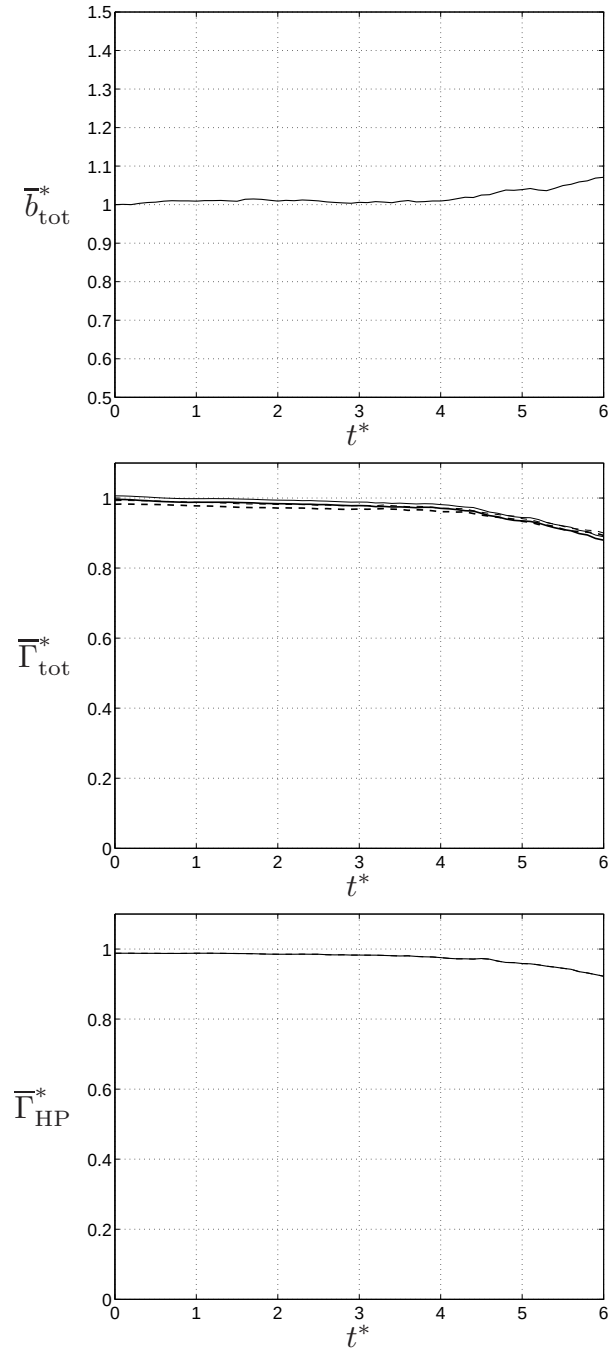


Figure 2.45: Evolution of the longitudinally-averaged vortex lateral separation (top), total circulation (middle), and half plane circulation (bottom). For the total circulation, the results are provided for the left (dash) and the right (solid) vortices using the polar data (thick) or the Cartesian data (thin).

2.5.3 Characterization of the system at equilibrium

We here further characterize the 2VS obtained at $t^* = 6$ as, at that time, the 2VS has reached the equilibrium. At that time, the vortex circulation has decayed (see Fig. 2.45). The half plane circulation is then $\bar{\Gamma}_{\text{HP}}^* = 0.922$ and the total circulation values of the left and right vortices are reported in Table 2.5. The lateral separation has also slightly increased to $\bar{b}_{\text{tot}}^* = 1.071$. It is interesting to note that the core radius has not changed much, if any.

	$\bar{\Gamma}_{\text{tot}}^*$	$\bar{r}_c^*$	Γ_{tot}^*	r_c^*
Left vortex	0.882	0.048	0.883	0.059
Right vortex	0.872	0.050	0.873	0.053

Table 2.5: Vortex properties at equilibrium. The quantities with overbar are the averages of the quantities evaluated in each cross-plane. Those without overbar are obtained from the mean flow.

In the following, two different normalizations of the quantities are used. The first one, denoted by $\cdot^*$, is that already used in this chapter and is related to the initial state of the 2VS using Γ_0 and b_0 as characteristic scales (leading to the definition of $V_0 = \Gamma_0 / (2\pi b_0)$ and $t_0 = b_0 / V_0$). The second normalization, denoted by $\tilde{\cdot}$ and already used in Section 1.3, is related to the current vortex properties and uses $r_c(t)$ and $\Gamma_{\text{tot}}(t)$ as characteristic scales. One then defines $\tilde{r} = \frac{r}{r_c}$, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma_{\text{tot}}}$, $\tilde{u} = u \frac{2\pi r_c}{\Gamma_{\text{tot}}}$, and $\tilde{\omega} = \omega \frac{\pi r_c^2}{\Gamma_{\text{tot}}}$. At equilibrium, only the second normalization is universal as the 2VS slowly decays.

At equilibrium, the 2VS is surrounded by complex turbulence. In a slice of axial vorticity $\tilde{\omega}_x$, as shown in Fig. 2.46, one observes the two vortex cores and the turbulence structures with non negligible intensity compared to the wake vortices (note that, for a LOA vortex, $\tilde{\omega}_x(0) = 1$ and $\tilde{\omega}_x(r_c) = 0.25$). On a view of the mean cross-flow, as also shown in Fig. 2.46, the turbulence is obviously no longer visible and the vortices appear to have slightly larger vortex cores. This is an effect of the axial average of a vortex tube with some global deformation of the centerline (some “meandering”). The vortex core sizes measured on the mean flow are reported in Table 2.5.

The topology of the flow is also well illustrated using a slice of the axial velocity $\tilde{u}_x$ provided in Fig. 2.47. Note that, for a LOA vortex, the maximum azimuthal velocity is $\tilde{u}_\theta(r_c) = \tilde{u}_{\theta,\text{max}} = 0.5$. The axial velocity values hence amount to 10% of the maximum velocity. One also notices that most

of the turbulence is confined in the Rankine-like oval around the 2VS. This is consistent with [5]. It is also observed that the mean flow still presents some axial velocities, as shown in Fig. 2.47. This is because the mean is here the longitudinal averaging over a box of limited size $L = 4b_0$. If we were instead time-averaging a real vortex in a fixed cross-plane, we would obtain zero mean axial velocity, if averaging over a sufficiently long time.

The obtained LES results are of great interest as they provide detailed velocity fields that can be deeply analyzed. In the following, a particular attention is paid to the wake vortex characteristics at equilibrium. In a complementary analysis, presented in Appendix D, we provide the obtained kinetic energy, pressure, and Bernoulli term fields, as examples of what can be obtained from such LES results.

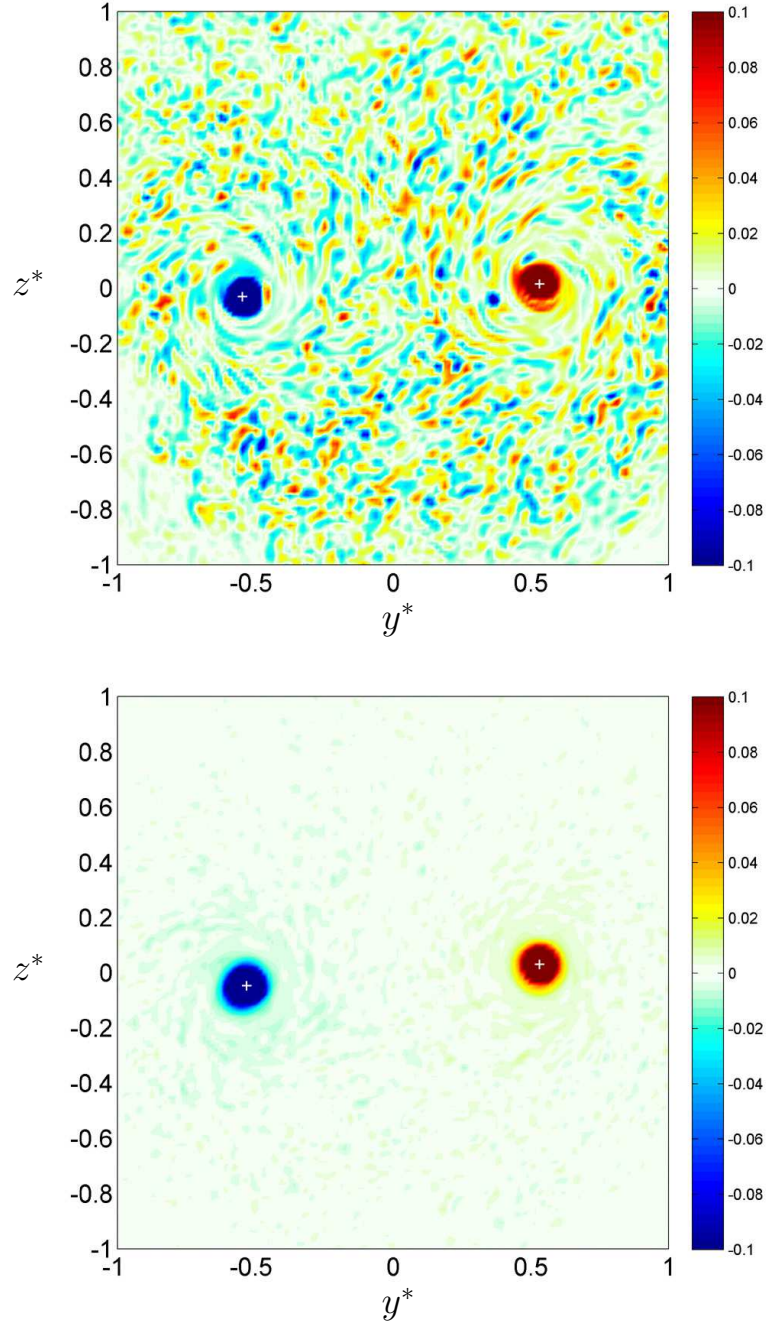


Figure 2.46: Zoom on a slice of axial vorticity $\tilde{\omega}_x$ (top) and on the mean axial vorticity $\overline{\tilde{\omega}_x}$ (bottom) at equilibrium. The positions of the center of the vortices are also displayed (crosses).

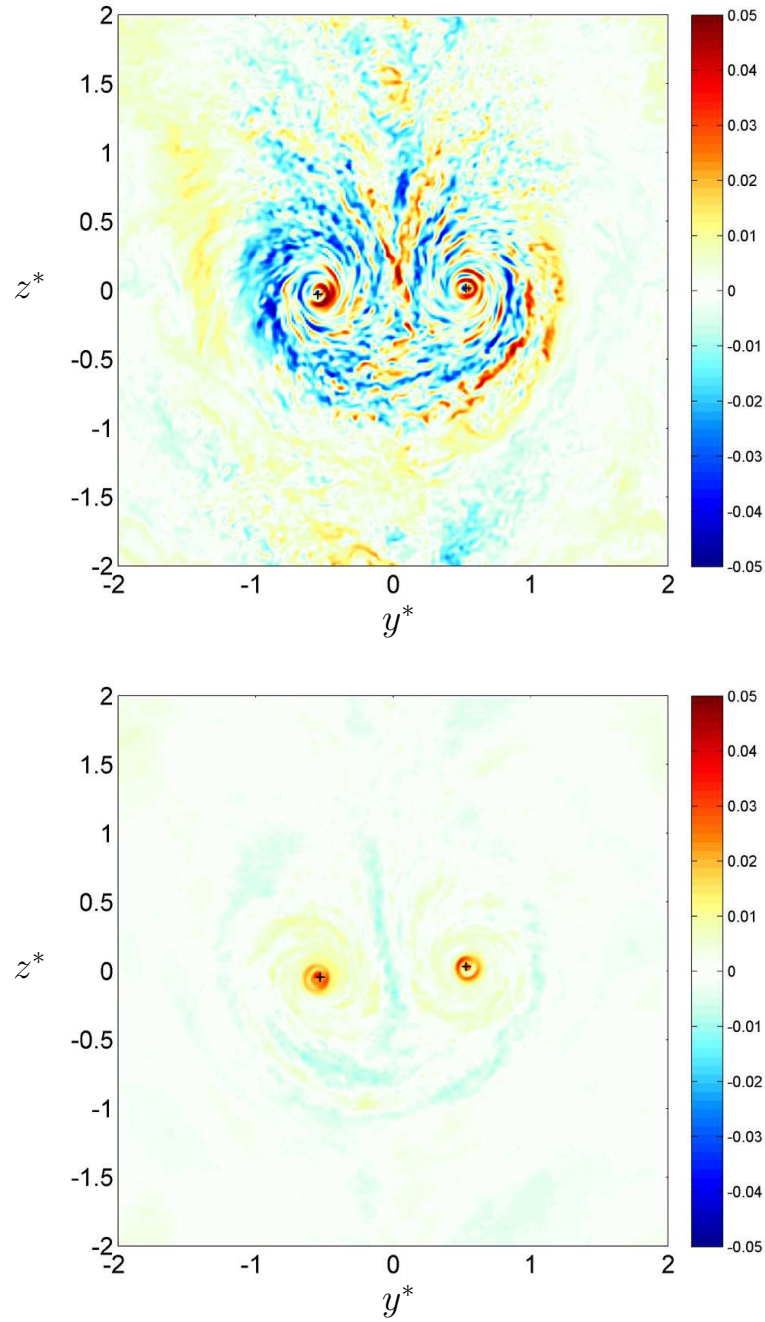


Figure 2.47: Slice of axial velocity $\tilde{u}_x$ (top) and mean axial velocity $\bar{\tilde{u}}_x$ (bottom) at equilibrium. The positions of the center of the vortices are also displayed (crosses).

The circulation and velocity profiles obtained at equilibrium are of great interest. They are provided in Fig. 2.48. Compared to the initial condition, also shown in the figure, the inner core region is seen to be little influenced by the surrounding turbulence as the profiles correspond well. The maximum azimuthal velocity has not much decayed from the initial condition ($u_{\theta, \max} = 10 V_0$ for a LOA vortex with $r_c^* = 0.05$). On the contrary, due to the ambient turbulence around the vortices, away from the core ($r^* > 0.1$), the circulation curve is almost linear and then from $r^* = 0.4$ tends to $\bar{\Gamma}_{\text{tot}}$, which is lower than Γ_0 . In that region the induced velocities are hence lower compared to the initial condition. The circulation and azimuthal velocity profiles obtained from the mean cross-flow (i.e., using $\bar{\omega}_x$) are also displayed in Fig. 2.48. One notes the fatter effective core sizes and hence the lower azimuthal velocities for $0 \leq r^* \leq 0.1$. Far from the core, the obtained curves well correspond to those obtained from the average of profiles measured in each plane. The values of the total circulation measured on the mean flow are also reported in Table 2.5. The values of Γ_{tot} are very close to those of $\bar{\Gamma}_{\text{tot}}$ for both the left and right vortices.

A fair comparison between the circulation and velocity profiles of the initial condition and of the 2VS at equilibrium is made, in Fig. 2.49, using the $\tilde{\cdot}$ normalization. Except in the core, the 2VS at equilibrium clearly has a vorticity distribution that does not correspond anymore to a LOA vortex with a much more progressive increase of the circulation.

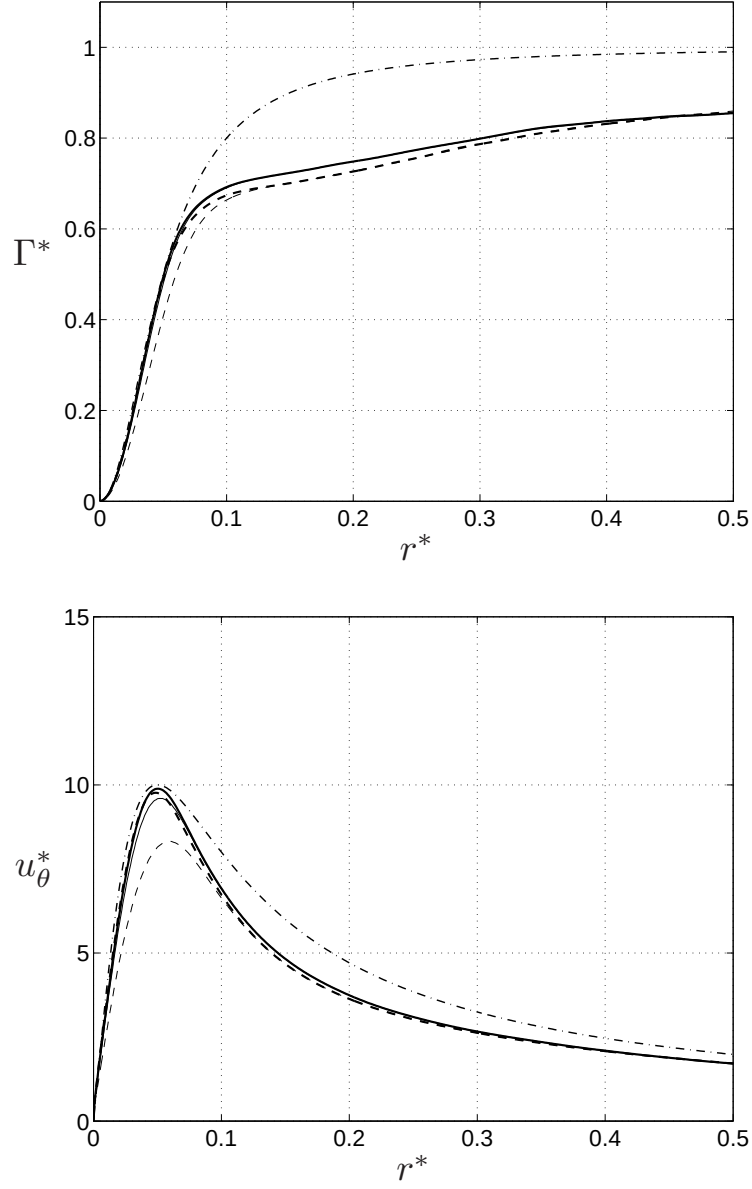


Figure 2.48: Circulation (top) and azimuthal velocity (bottom) profiles at equilibrium: average of profiles measured in each plane (thick) and profiles obtained from the mean flow (thin). The results are provided for the left (dash) and right (solid) vortices. The initial profiles are also displayed for comparison (dash-dot).

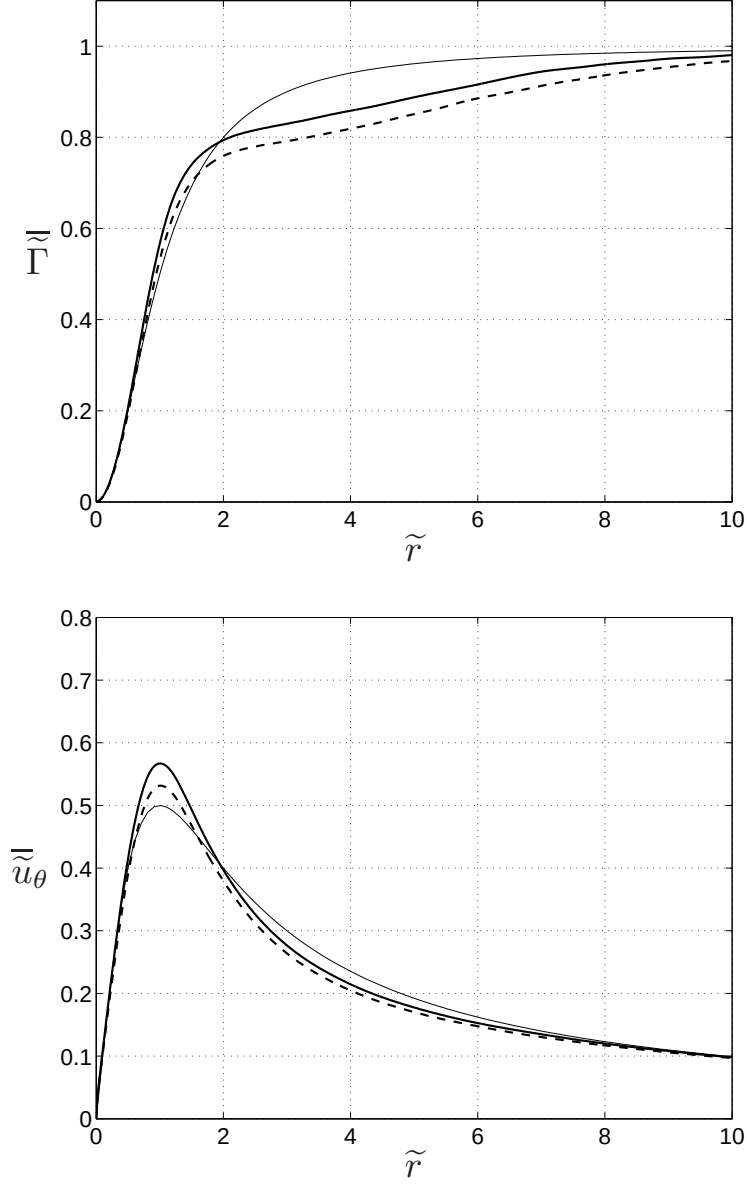


Figure 2.49: Average of profiles measured in each plane for the circulation (top) and the azimuthal velocity (bottom) at equilibrium. The results are provided for the left (dash) and right (solid) vortices. The initial profiles are also displayed for comparison (thin solid).

According to the two-scale models of Proctor and Proctor-Winckelmans (see Appendix A), the circulation distribution profile can be described in two parts. In the inner core region, the circulation follows:

$$\begin{aligned}\tilde{\Gamma} &= 1 - \exp\left(-\beta_i (r^*)^2\right) \\ &= 1 - \exp\left(-a_i (\tilde{r})^2\right)\end{aligned}\tag{2.46}$$

Far from the core, the circulation distribution then follows:

$$\begin{aligned}\tilde{\Gamma} &= 1 - \exp(-\beta_o (r^*)^q) \\ &= 1 - \exp(-a_o (\tilde{r})^q)\end{aligned}\tag{2.47}$$

The two regions of the circulation distribution are indeed observed by considering the evolution of $\log(-\log(1 - \tilde{\Gamma}))$ as a function of $\log(r^*)$, as shown in Fig. 2.50. The inner scaling distribution of Eq. (2.46) well fits the data for $0 \leq \frac{r}{b_0} \leq 0.04$ (recall that $\frac{r_c}{b_0} = 0.05$) with $\beta_i = 371$ (hence, $a_i = 0.89$). The outer scale distribution of Eq. (2.47) is also seen to fit the data for $0.16 \leq \frac{r}{b_0} \leq 0.5$ with $q = 0.751$ (which indeed corresponds to the typical value used in the models) and $\beta_o = 6.0$ (hence, $a_o = 0.63$).

The two scales are thus observed in the circulation distribution. The q exponent of the second zone (the zone far from the core) corresponds to that observed by Proctor [83], using Lidar observations of several wake vortices measured early in their evolution. This confirms the universality of the obtained profile, at least for this region. In addition, a third zone is observed for $0.04 \leq \frac{r}{b_0} \leq 0.16$ for which the exponent in the exponential slowly decreases, from 2 to a value below q , and then re-increases.

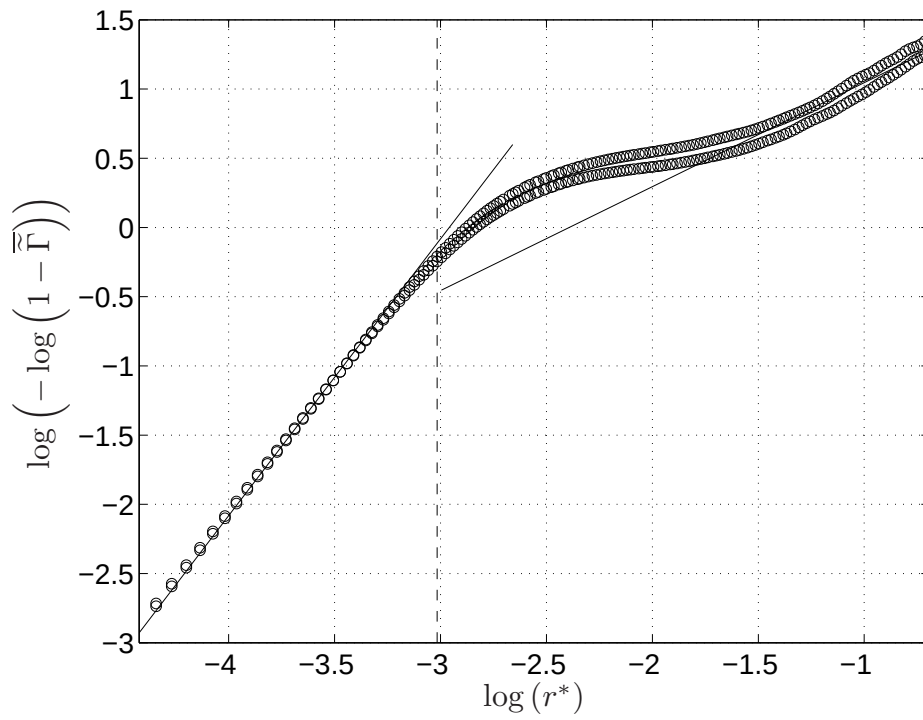


Figure 2.50: Average of the circulation profiles measured in each plane at equilibrium and plotted in a log-scale. The measured profiles of both the left and right vortices are shown ($\circ$) together with the fits in the inner core region and far from the core (solid). The core radius is also indicated (dash).

2.5.4 Sensitivity analysis

We have shown that a two-vortex system evolving in a weak turbulence reaches an equilibrium. It is interesting to investigate the influence of this equilibrium to different parameters of the simulation. In particular, we here investigate the influence of the initialization of the 2VS and the influence of the atmospheric turbulence level. In the following, five additional simulations are then performed (cases 1 to 5) and their results are compared to the baseline case (case 0). The simulation characteristics are detailed in Table 2.6.

Case	r_c^*	$\frac{\Gamma_0}{\Gamma_{0,0}}$	$\frac{E_{2D,\infty}}{E_{2D,\infty,0}}$	$\frac{E_{2D,\infty}}{(V_0 b_0)^2}$	ϵ_f^*	Model
0	0.05	1.00	1.00	15.68	ϵ_1^*	LOA
1	0.1	1.00	0.72	11.33	ϵ_1^*	LOA
2	0.1	1.18	1.00	11.33	$0.61 \epsilon_1^*$	LOA
3	0.05	1.00	1.22	19.18	ϵ_1^*	G
4	0.05	1.00	1.03	16.17	ϵ_1^*	PW
5	0.05	1.00	1.00	15.68	$10 \epsilon_1^*$	LOA

Table 2.6: Vortex and turbulence properties of the investigated cases ($\epsilon_1^* = 2.42 \cdot 10^{-4}$). The investigated vortex circulation profile models are the low-order algebraic (LOA), the Gaussian (G), and the Proctor-Winckelmans (PW) distributions.

Investigation of the influence of the core radius

We here investigate the influence of the initial core radius of the vortices to the obtained equilibrium 2VS. We therefore perform two additional simulations (cases 1 and 2) of wake vortices, with a core radius $r_c = 0.1 b_0$, evolving in the atmospheric turbulence of the baseline case. In case 1, the initial circulation of each vortex Γ_0 is that of case 0 leading to a 2VS that is less energetic (using Eq. (2.45), one finds that $E_{2D,\infty}$ is about 70% of that of the baseline case). In case 2, the circulation is increased so as to have $E_{2D,\infty}$ equal to that of the baseline case, see Table 2.6. Since, in case 2, the vortex circulation is increased, compared to the vortex characteristics, the atmospheric turbulence level is hence relatively a bit lower. For both cases 1 and 2, a low-order algebraic model (LOA) is used for the initial circulation distribution profile. Note that, as the grid remains unchanged, the vortex core is hence twice better resolved

than for case 0 (i.e., $r_c/h = 6.4$).

Figure 2.51 compares the evolution of the core radius for the three cases. The effective core sizes are seen to not vary much (if any) in time. This is typical of such LES at very high Reynolds number and with good quality numerics and SGS model. The turbulence generated by the vortex system also appears not to affect the inner vortex core itself, which is still very “laminar like” at equilibrium.

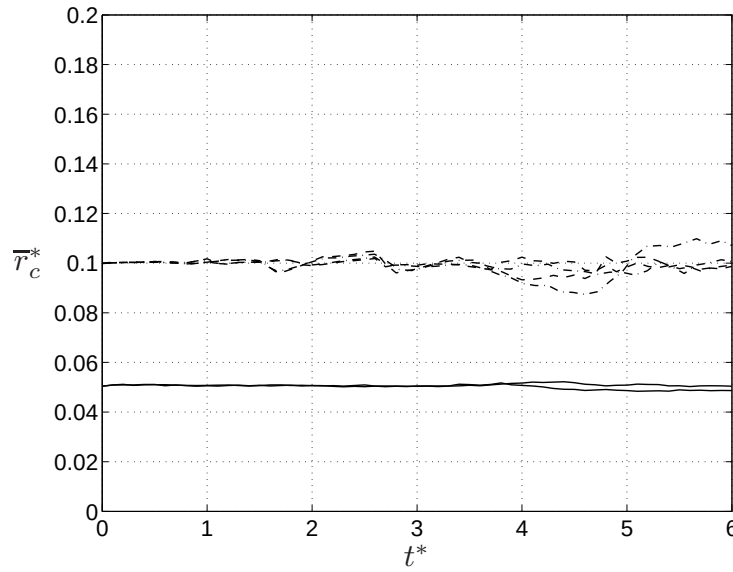


Figure 2.51: Evolution of the core radius for case 0 ($r_c^* = 0.05$) (solid), case 1 ($r_c^* = 0.1$) (dash), and case 2 ($r_c^* = 0.1$, $\Gamma_0 = 1.18 \Gamma_{0,0}$) (dash-dot).

Figure 2.52 compares the energy and dissipation evolution of the three cases. Note that, in the figure, the energy is normalized by the initial energy of the baseline case. The curves of cases 0 and 2 are hence initially superimposed. If the energy were normalized by the vortex properties (Γ_0 and b_0), the energy curves of cases 1 and 2 would then be initially superimposed. In cases 1 and 2, as the vortices are fatter, the stretching on the Rankine-like oval is less significant than for case 0. The creation, by the vortex system, of its own turbulence takes more time and the fast decay of the energy curves appears later.

The dissipation follows the same trends for the three cases. First, turbulence is generated around the vortices, leading to a fast and significant increase (of several decades) of the dissipation. It then reaches a maximum and slowly

decreases to reach a value approximatively constant. It is remarkable that, even if the maximum dissipation rate reached in the three cases are somewhat different, the equilibrium dissipation levels are almost identical. This indicates that the vortex system will generate its own turbulence with similar properties at equilibrium, even though it does not directly affect the inner vortex core. In case 2, as the background turbulence intensity is lower relative to the vortex characteristics, the turbulence also takes more time to reach the equilibrium.

Finally, Fig. 2.53 compares the vortex circulation profiles measured at equilibrium (i.e., at $t^* = 6$). The profiles are first normalized using the global vortex properties Γ_0 and b_0 . One notes that, outside of the core, the circulation curves are parallel. In that region, the turbulent structures created by the 2VS influence the circulation distribution. This influence appears to not depend on the core size. The profiles normalized using the vortex characteristics at the considered time ($\Gamma_{\text{tot}}(t^* = 6)$, $r_c(t^* = 6)$) are also provided in Fig. 2.53. One observes that the vortex inner core regions are identical for both vortex core sizes. This indicates that the vortex core resolution is sufficient to capture the flow physics as neither of the vortex cores diffuse nor shrink. It also again shows that the turbulence does not affect the vortex inner core region. Far from the core, the distribution scales with b_0 and not with r_c . This also shows the interest to work with a two-scale model such as the Proctor-Winckelmans model.

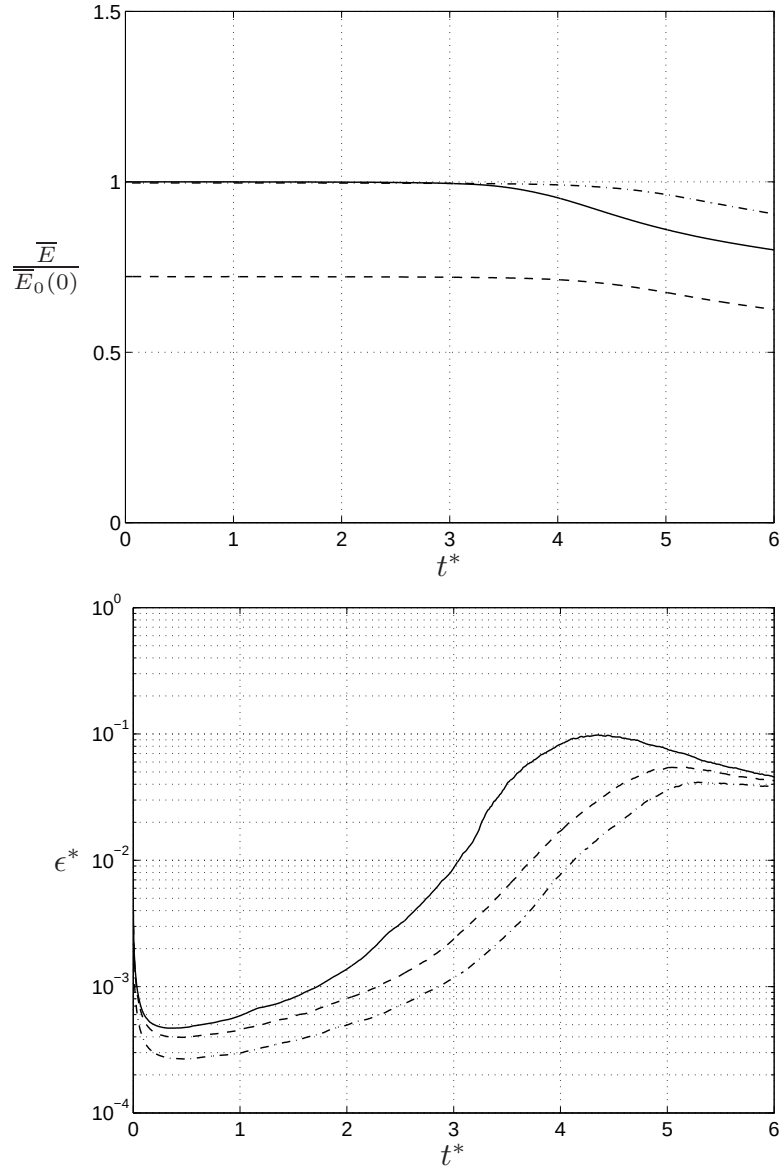


Figure 2.52: Evolution of the energy (top) and the dissipation (bottom) for case 0 ($r_c^* = 0.05$) (solid), case 1 ($r_c^* = 0.1$) (dash), and case 2 ($r_c^* = 0.1$, $\Gamma_0 = 1.18 \Gamma_{0,0}$) (dash-dot).

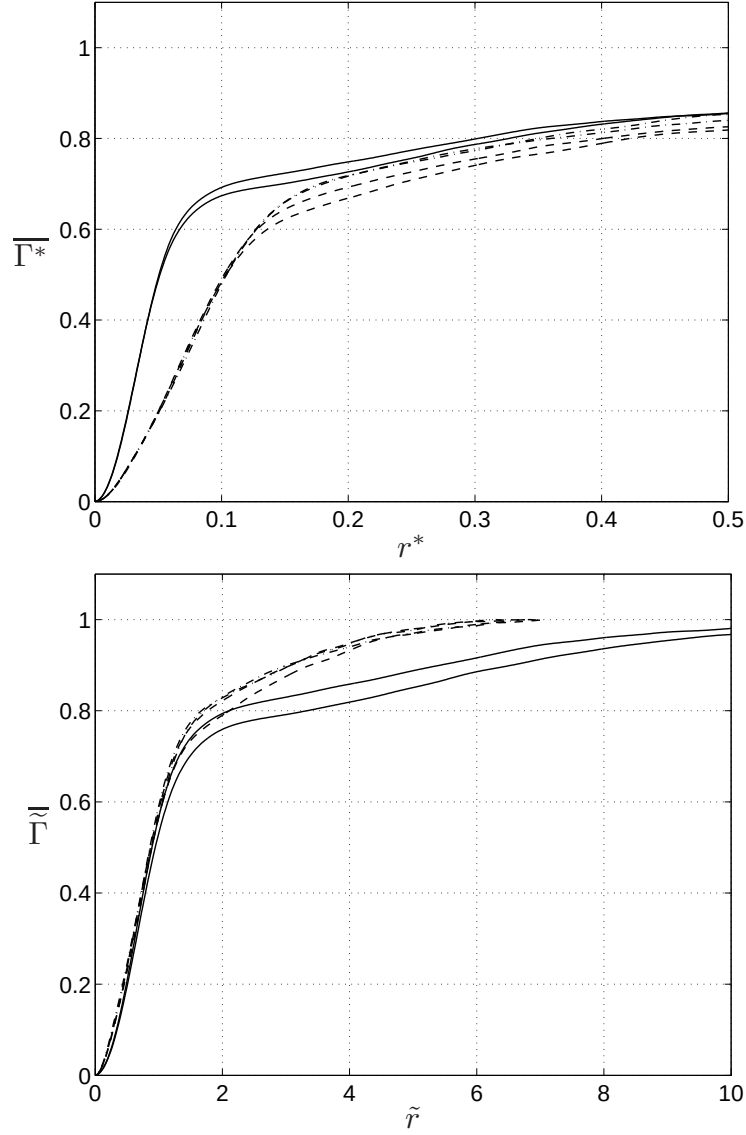


Figure 2.53: Circulation distribution at equilibrium ($t^* = 6$) for case 0 ($r_c^* = 0.05$) (solid), case 1 ($r_c^* = 0.1$) (dash), and case 2 ($r_c^* = 0.1$, $\Gamma_0 = 1.18 \Gamma_{0,0}$) (dash-dot). The circulation profiles are normalized using either the initial vortex characteristics (i.e., Γ_0 and b_0) (top) or the vortex properties at the considered time of equilibrium (i.e., $\overline{\Gamma}_{\text{tot}}(t^* = 6)$ and $\bar{r}_c(t^* = 6)$) (bottom).

Investigation of the influence of the initial circulation distribution

Two additional cases (i.e., cases 3 and 4) are simulated in order to highlight the influence of the initial circulation distribution profile on the obtained 2VS at equilibrium. In both cases, a two-vortex system evolves in the weak atmospheric turbulence obtained for case 0. The core radius r_c/b_0 and the circulation Γ_0 are also kept from case 0. The 2VS is initialized using, as circulation profile, the Gaussian (G) distribution one-scale model (see Table 1.3) or the Proctor-Winckelmans (PW) two-scale model (with $q = 3/4$, $\beta_o = 10$ and $p = 3$, and hence $\beta_i = 365.5$, see Table 1.4), respectively. The choice of the parameter values of the PW distribution, and in particular the value of p , corresponds to those of Lonfils [75] based on a fit on the results of a simulation of space-developing roll-up with velocity deficit. Due to the difference in the circulation distributions, the 2-D energies of the three systems are different and are compared in Table 2.6. Figure 2.54 compares the initial azimuthal velocity profiles of the three cases. One notes the much higher maximum azimuthal velocity of the Gaussian model (case 3). This model is also not a good model of aircraft wake vortices after roll-up.

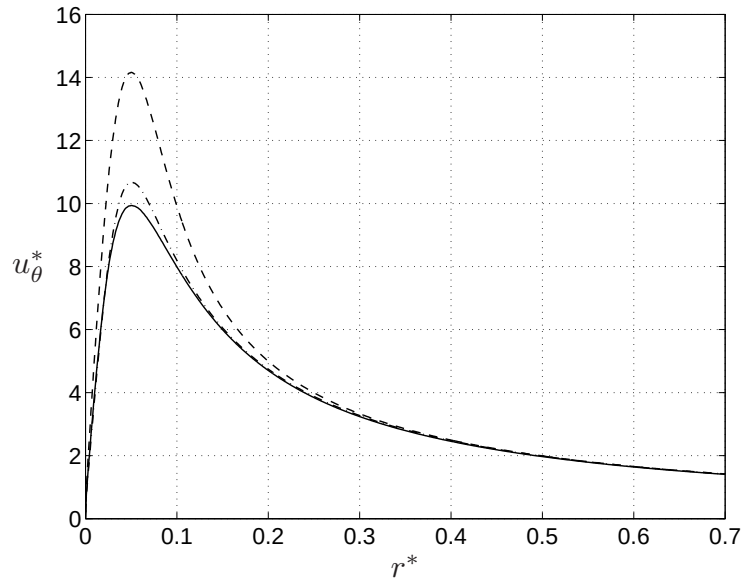


Figure 2.54: Initial azimuthal velocity profile for case 0 (LOA) (solid), case 3 (G) (dash), and case 4 (PW) (dash-dot).

Figure 2.55 compares the evolution of the energy and the dissipation of

the three investigated cases. Although the Gaussian model case has a higher initial energy, it quickly dissipates to reach a energy level similar to the two other cases. The dissipation of case 3 hence quickly rises to reach a plateau around $t^* = 2$. The higher velocity gradients of the Gaussian profile induce more stretching of the turbulence and hence decrease the time needed to reach equilibrium. As the initial velocity profiles of cases 0 and 4 are very close, as expected for the PW with $p = 3$, the evolutions of the 2VS are also close in terms of energy and dissipation. A case with $p = 4$ would already be more different (not investigated). The dissipation level reached at equilibrium is thus seen to be quite universal and does not depend on the initial condition. In the three cases, at equilibrium, ϵ^* indeed oscillates around the same value and the three energy curves are almost parallel.

Figure 2.56 compares the evolution of the longitudinal modal energy. Globally, cases 0 and 4 show very similar evolution whereas some differences are observed for case 3. For the three cases, the 2D energy curves have the same trends as those of the 3-D energy except that they continuously decay as part of the 2-D energy is transferred to turbulence. As for the 3-D energy, the three curves are very close for $t^* \geq 3.5$. The large-scale modes of the flows ($\lambda \geq b_0$) are seen to be unaffected by the initial circulation profile. On the contrary, the small-scale modes ($\lambda \ll b_0$) show a much quicker increase in case 3 (Gaussian profile) than in the two other cases. The small-scale structure energies reach a plateau, in case 3, around $t^* = 2.0$. For each of these modes, the energy levels, reached at equilibrium (i.e., for $t^* \geq 2$, in case 3), are very similar to those obtained in cases 0 and 4 for $t^* \geq 4$. The plateau of each mode is thus quite universal but is reached at a different time (i.e., once the 2VS is at equilibrium).

The circulation and azimuthal velocity profiles measured at equilibrium ($t^* = 6$) are provided in Fig. 2.57. The obtained velocity and circulation profiles are very similar, much more than initially (see Fig. 2.54). However, in the region very close to the vortex core ($r_c \leq r \leq 2r_c$), some remaining influence of the initial circulation model can still be noted. In particular, for the Gaussian model case (case 3), the maximum azimuthal velocity is a bit higher and the circulation curve is somewhat steeper than for the two other cases. This is consistent with what was observed previously: the vortex core, as still very “laminar like”, is not much influenced by the turbulent structures generated around the vortex core.

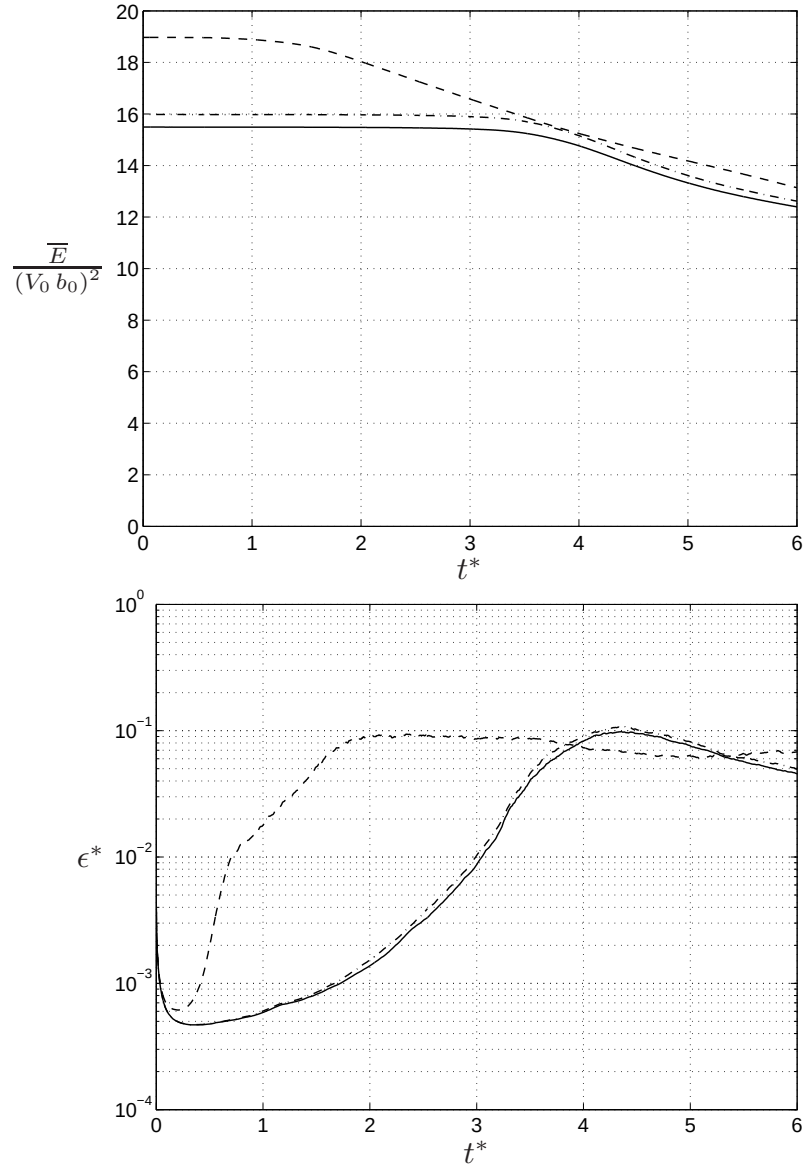


Figure 2.55: Evolution of the mean energy (top) and dissipation (bottom) for case 0 (LOA) (solid), case 3 (G) (dash), and case 4 (PW) (dash-dot).

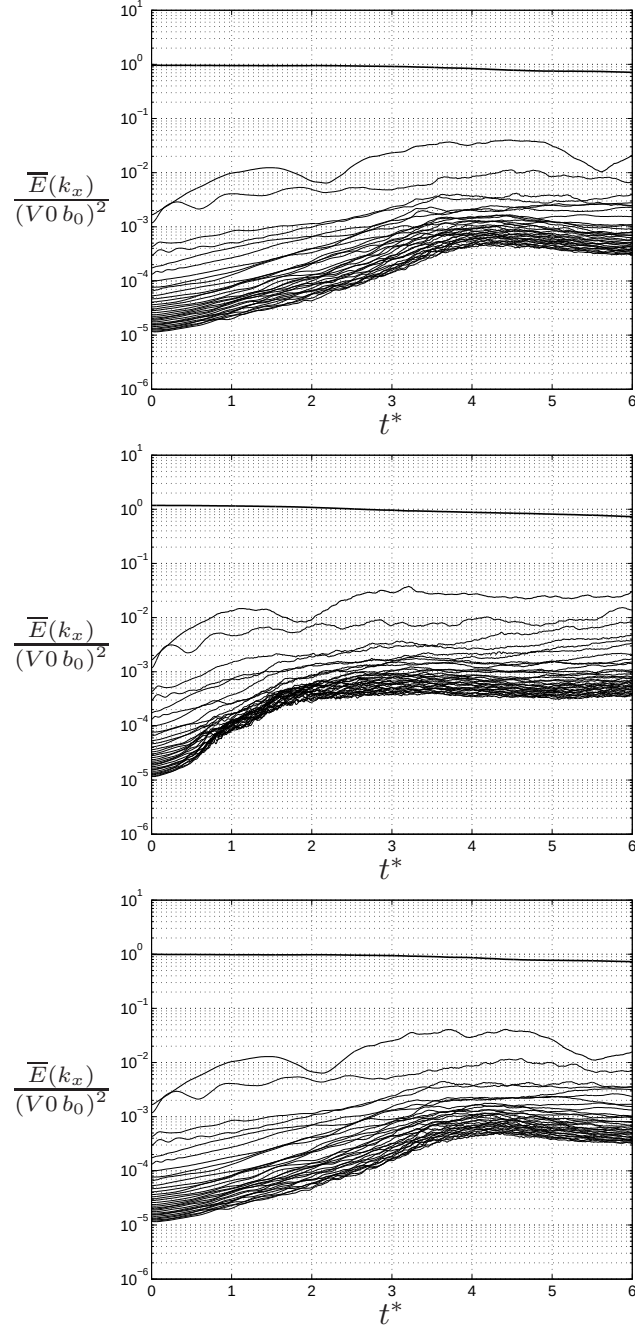


Figure 2.56: Longitudinal modal energy evolution for case 0 (LOA) (top), case 3 (G) (middle), and case 4 (PW) (bottom) . For the sake of clarity, only selected modes are displayed: the rigid mode (thick solid), the $\lambda = 4b_0$ mode (solid) and the 31 next modes down to $\lambda = 0.125b_0$ (thin solid).

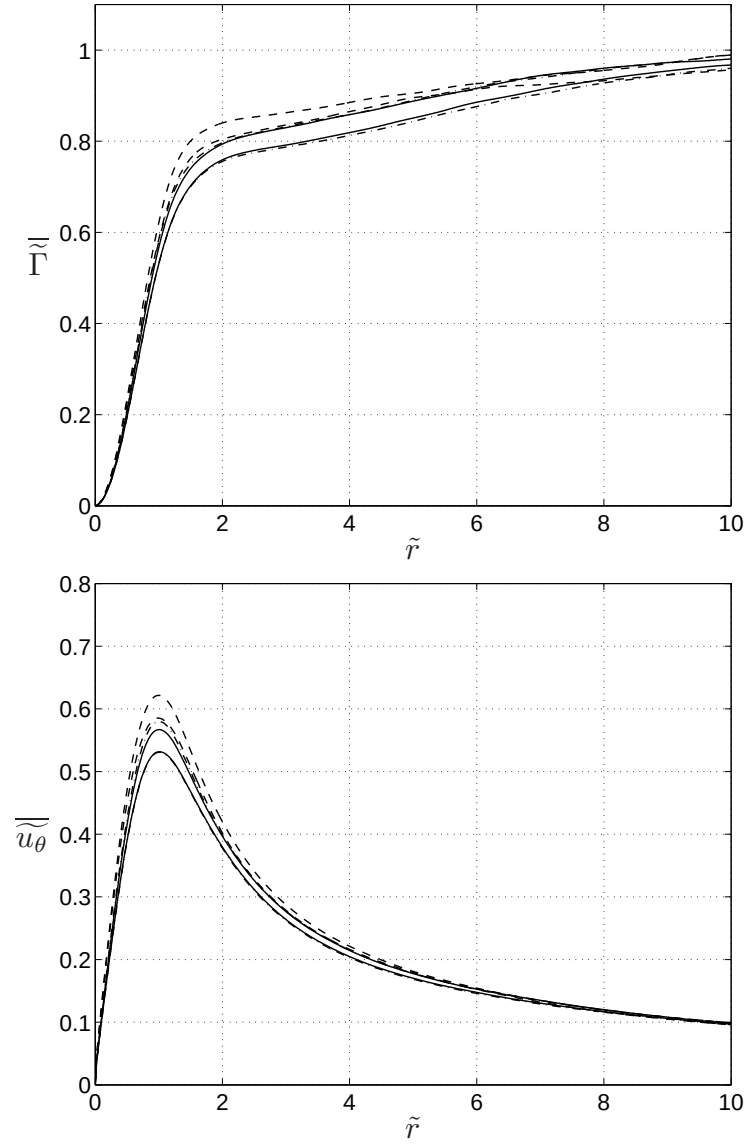


Figure 2.57: Circulation (top) and azimuthal velocity (bottom) profiles normalized by the local vortex properties for case 0 (LOA) (solid), case 3 (G) (dash), and case 4 (PW) (dash-dot).

Investigation of the influence of the atmospheric turbulence level

Finally, the influence of the background turbulence level is also investigated by performing a complementary simulation (case 5) of a 2VS evolving in an atmosphere ten times more turbulent than that of the baseline case. Taking $\Gamma_0 = 400 \text{ m}^2/\text{s}$ and $b_0 = 50 \text{ m}$, this case would then corresponds to an atmosphere with $\epsilon = 10^{-4} \text{ m}^2/\text{s}^3$, which is a case of moderate turbulence, see Table 2.1.

The evolution of the energy and dissipation are provided in Fig. 2.58. One notes that the initial energy is a bit higher in case 5 compared to that of case 0. This is due to the higher energy of the background turbulence. The higher turbulence level is seen to quicken the creation of turbulence around the 2VS. The energy starts indeed its fast decay phase earlier (around $t^* = 2.5$ versus $t^* = 3.5$ for case 0). However, the decay rates of the fast-decay phases are seen to be very close. The dissipation, that starts at a higher level for case 5, first decreases before re-increasing to reach a maximum with the same value as that of case 0 ($\epsilon^* \approx 10^{-1}$). It then slightly decreases to oscillate around a constant value. At $t^* = 6$, one has $\epsilon^* \simeq 6 \cdot 10^{-2}$ in case 5 and $\epsilon^* \simeq 5 \cdot 10^{-2}$ in case 0. Those values are very similar. The equilibrium reached by the system is hence seen to be almost unaffected by the initial atmospheric turbulence level (recall that, initially, we had $\epsilon_{\text{case 5}}^* = 10 \epsilon_{\text{case 0}}^*$).

Figure 2.59 shows the comparison of circulation distribution profiles at equilibrium ($t^* = 6$). It should be noted that, at that time, the total circulation of the vortices for case 5 is a bit lower than that of case 0, as the faster decay phase has started earlier. One has $\bar{\Gamma}_{\text{tot,L}}^* = 0.83$ and $\bar{\Gamma}_{\text{tot,R}}^* = 0.81$ in case 5. Using Γ_0 and b_0 as characteristic scales, the evolutions of $\bar{\Gamma}^*$ are identical in the vortex core and parallel in the outer core region. Using the vortex characteristics ($\bar{r}_c$ and $\bar{\Gamma}_{\text{tot}}$) at the considered time, the obtained circulation profiles are very close. The organization of the vorticity structures around the 2VS hence appears to be independent of the background turbulence intensity.

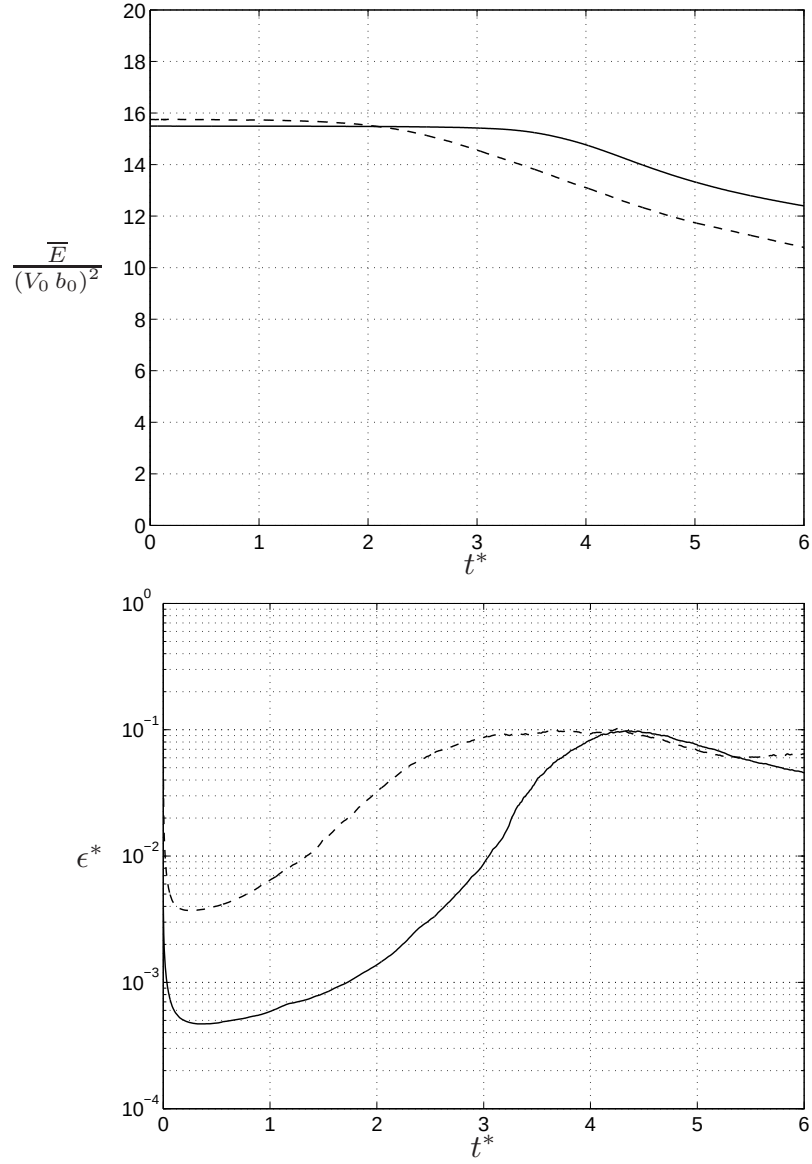


Figure 2.58: Mean energy (top) and dissipation (bottom) evolution for case 0 ($\epsilon_f^* = \epsilon_1^*$) (solid) and case 5 ($\epsilon_f^* = 10 \epsilon_1^*$) (dash).

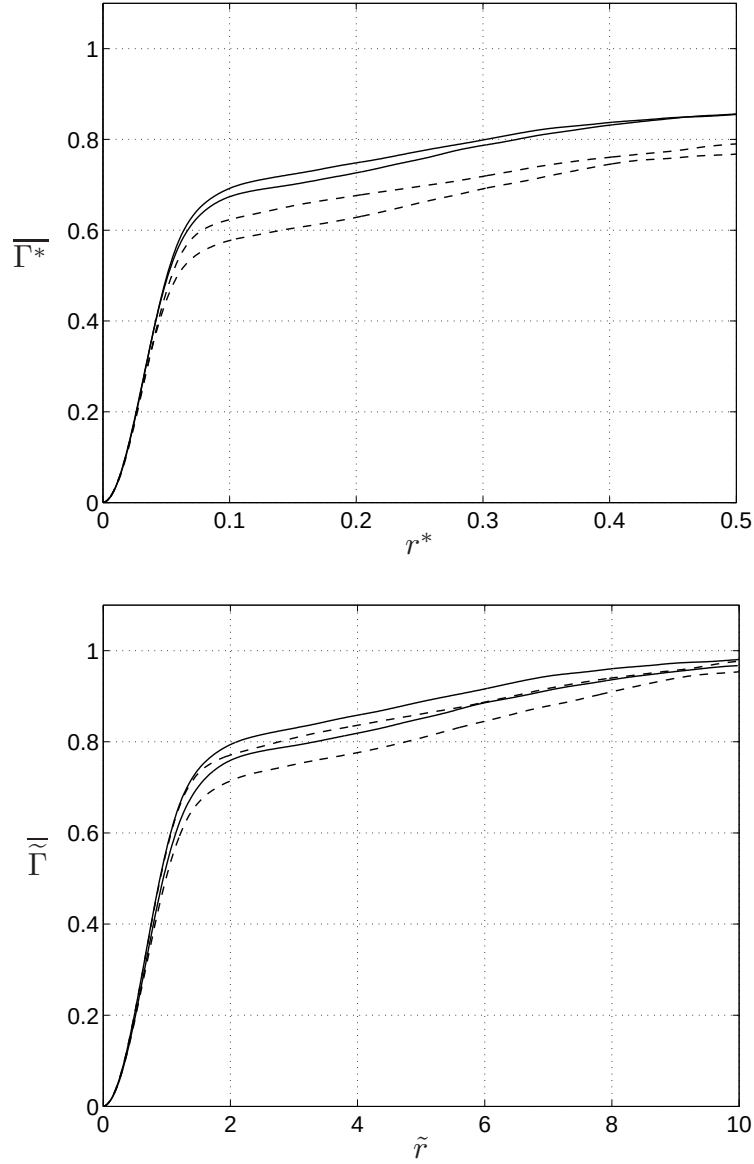


Figure 2.59: Vortex circulation profiles at $t^* = 6$ for case 0 ($\epsilon_f^* = \epsilon_1^*$) (solid) and case 5 ($\epsilon_f^* = 10 \epsilon_1^*$) (dash). The circulation distributions are normalized using either the global vortex properties (i.e., Γ_0 and b_0) (top), or the vortex characteristics at the considered time of the equilibrium (i.e., $\overline{\Gamma}_{\text{tot}}(t^* = 6)$ and $\bar{r}_c(t^* = 6)$) (bottom).

2.5.5 Conclusions on the turbulent wake vortex system at equilibrium

In this section, we have shown that a two-vortex system, evolving in a weak turbulence, generates its own turbulence and reaches, after a certain time, an equilibrium state. At equilibrium, energy is continuously being transferred from the large scales of the flow to the medium to small turbulent scales, in an essentially self-similar way, and the system slowly decays. The time needed to reach the equilibrium depends on the background turbulence and on the initial circulation distribution. It was also shown that this equilibrium state is quite universal: it does not depend on the background turbulence level (at least as long as that one is weak to moderate) nor on the initial condition. A two-vortex system, perturbed by some turbulence, as it is always the case in reality, is progressively transformed in a system with an approximatively constant dissipation rate and universal spectrum. The circulation distribution profile also appears to be universal except in the inner vortex core. It was indeed seen that the inner vortex core remains fairly “laminar-like” and keeps its initial vorticity distribution. Some elliptic instabilities are however observed in the vortex core, as expected.

This study confirmed the possibility for the existence of a turbulent equilibrium of a two-vortex system; with itself and with the weak surrounding turbulence. It also provided its universality: spectrum, circulation profile, decay rate. Recall that the Crow instability was here not taken into account. In reality, the long-wavelength instability, triggered by the turbulence, and with its subsequent vortex deformation and reconnection (as analyzed in Section 2.4.4) will also come in addition to the present physics. In reality, additional turbulence would also come from the boundary layers on the wing and the fuselage surfaces and from the turbulent wake of the fuselage itself. With this additional turbulence, the two-vortex system would then reach faster its equilibrium.

2.6 Conclusions

This chapter focused on the detailed investigation of the behavior of wake vortices interacting with various types of atmospheres, away from the ground. For that purpose, pseudo-spectral Large Eddy Simulations (LES) were run at very high Reynolds number, with a sufficient resolution and using a fairly realistic model of aircraft wake vortices after roll-up. In this study, we are interested in the effects, on wake vortex behavior (transport, instability development and decay), of atmospheric stratification and of turbulence.

The effects of solely stratification was studied first. For that purpose, we performed LES of wake vortices evolving in various stably stratified atmospheres and without initial turbulence (yet with initial random perturbations with maximum amplitude $10^{-3} u_{\theta, \max}$). Although the studied atmospheres are not totally realistic (as not initially turbulent), the characterization, from the LES results, of the vortex transport and decay enabled to confirm, at such very high Reynolds number, some known behaviors of wake vortices in such environment and to identify the decay mechanisms of the vortices. In particular, two phases were observed in the decay. The strong vortex decay was seen to be due to the interaction with opposite sign baroclinic vorticity subjected to small-scale instabilities (for all investigated stratification levels) but also, for moderate stratification levels, to the strong interaction between the two primary vortices themselves, that have been brought closer by the stratification effects.

We then also investigated the effects of combined weak turbulence and stratification on wake vortex behavior. Those cases are typical of real atmospheres because stratified atmospheres are most often also weakly turbulent. We first pre-computed the turbulent, stratified or not, atmospheric fields also using LES. A great effort was put to obtain realistic stratified and weakly turbulent fields (both for velocity and temperature). For short times, the vortex behavior is still essentially two-dimensional. Hence, the results were seen to well correspond to those obtained without initial turbulence. However, the addition of turbulence, even if weak, was seen to strongly affect the results as it quickens the development of instabilities, which favors 3-D interactions and creation of stronger turbulence, and hence enhances the decay of the vortices.

Stratification was also seen to affect the Crow instability development. This instability induces the large deformations of the vortices that eventually leads to vortex reconnection. The characterization of the vortex pair topology is also

of great interest for encounter analysis, because of the difference in the rolling moments induced by a straight vortex pair and by a sinusoidally deformed vortex pair. For low stratification levels and weak turbulence, stratification is seen to significantly decrease the reconnection time whereas, for high stratification levels, the vortices are destroyed before significant deformation can be observed. In addition, turbulence also triggers the Crow instability development. The stronger the turbulence level, the sooner large vortex deformations are visible. However, later on, the vortices are observed to link and form vortex rings only in weak turbulence cases, whereas, in moderate to strong turbulence, smaller wavelength instabilities also significantly develop and prevent the vortices from linking. It hence appears that there is a threshold for the turbulence and for the stratification levels above which the Crow instability does not develop all the way up to vortex reconnection. Based on the present results, the proposed critical stratification level is $N^* = 0.8 - 0.9$. For the cases without stratification, the critical turbulence level is estimated to be in the range $2.5 \cdot 10^{-4} < \epsilon^* < 1.5 \cdot 10^{-2}$.

Stratification, turbulence and the Crow linking process all significantly influence the vortex decay. In weak stratification and weak turbulence, the decay is very slow until the time of vortex reconnection. The Crow linking then induces a decay that is very non uniform along the vortex: some planes exhibit a very fast second phase decay whereas others only see a slight increase of their circulation decay rate. For higher stratification levels, the strong interactions of the turbulent baroclinic vorticity structures with the wake vortices induce a fast decay of their circulation, which is very uniform along the vortices. For cases with moderate to strong turbulence, the smaller wavelength instabilities also induce an additional decay all along the vortex tubes. This quantitative assessment of the circulation decay is new and is very important from an operational point of view. The mean circulation evolution is indeed a relevant measure of the vortex strength (and hence of the related hazard) only for cases of moderate to strong turbulence and/or moderate to strong stratification. For vortices evolving in calm and weakly stratified (or neutral) atmospheres, the use of the complete circulation decay envelope (i.e., as obtained in all cross-planes) is required for any potential encounter analysis.

We finally also investigated the effects, on wake vortex behavior, of weak to moderate atmospheric turbulence, without possible Crow instability development. In those conditions, the vortex system generates its own turbulence,

this generation being triggered by the background turbulence. The turbulent two-vortex system then eventually reaches a statistical equilibrium; both with itself and with the weak surrounding turbulence. This equilibrium was seen to be quite universal in terms of spectrum, circulation profile and decay rate.

In reality, the two-vortex system, when created, is always surrounded by turbulence coming from the atmosphere, from the boundary layers on the wing and the fuselage surfaces and from the turbulent wake of the fuselage itself. In those conditions, the two-vortex system will very quickly reach a statistical equilibrium. As the present results showed that the obtained equilibrium vortex system was little sensitive to the initial condition and to the background turbulence level, it is believed that the obtained equilibrium circulation profile is indeed representative of a turbulent rolled-up vortex system as developed behind a real aircraft.

This chapter provides a quite complete characterization of the influence of various realistic atmospheres on wake vortex behavior: transport, instability development and decay. It therefore provides an improved understanding of the flow physics and an identification of the impact parameters. This is of great importance for accurate operational wake vortex modeling, as done in Chapter 3.

Chapter 3

Operational modeling of wake vortex behavior

The content of this chapter is largely inspired by different papers. Section 3.2.3 is inspired by an article submitted for publication in the AIAA journal with title “Aircraft vortices in stably stratified and weakly turbulent atmospheres: simulation and modeling” [23]. The authors of these two articles are I. De Visscher, L. Bricteux and G. Winckelmans. Section 3.4 is based on an article published in the proceedings of the AIAA ASE conference entitled “The WAKE4D simulation platform for predicting aircraft wake vortex transport and decay : Description and examples of application” [27]. The authors are I. De Visscher, G. Winckelmans, T. Lonfils, L. Bricteux, M. Duponcheel and N. Bourgeois.

3.1 Introduction

The strong moments and/or forces induced by the wake vortices, generated behind an aircraft, on a follower aircraft entering the aircraft wake can be hazardous. This potential hazard is at the origin of the separation standards between aircraft in landing and take-off phases but also in cruise. This separation distance is one of the key factors that limit the capacity of the busiest airports. The study of the dynamics of a pair of counter-rotating vortices has hence been a major subject of research since the seventies. As reviewed in [108], the wake vortex evolution is influenced by its environment: by the atmospheric conditions (wind, turbulence, and stratification) and by the ground proximity.

A sufficiently accurate operational modeling of the wake vortex behavior will help reduce the separation standards, while maintaining or even increasing the safety level. In the scope of this thesis, a particular attention was paid on the modeling of the effects, on wake vortex behavior, of the atmospheric turbulence and stratification, also using the results of Chapter 2. Furthermore, we also focused on modeling the wake vortex behavior in the ground vicinity region, also combined with wind (cross- and headwind components) and the associated turbulence. The final objective of the work presented in this chapter was thus to obtain a software able to predict very fast the transport and decay of wake vortices generated by a given aircraft in provided meteorological conditions. Such platform can then be used as a tool in support to operational real-time systems or off-line studies.

The UCL group has developed an operational tool to predict the wake vortex transport and decay: the Deterministic wake Vortex Model (DVM) [27]. It is based on a method of vortex particles that are transported and decay following various physical models. It accounts for all important impact parameters: atmospheric vertical profiles of wind, turbulence, wind shear and stratification, as well as ground vicinity. The models implemented within the DVM are necessarily simplified; yet they are still based on a physical analysis of the wake vortex flow interacting with the atmosphere and/or with the ground. Currently, in addition to the DVM, different operational wake vortex transport and decay models are found in the literature. Without being exhaustive, one finds: the Vortex Forecast System (VFS) [61], the AVOSS Prediction Algorithm (APA) [98], the Deterministic two-phase model (D2P) [53, 59, 54], the TASS Driven Algorithms for Wake Prediction (TDAWP) [86], the Vortex algorithm Including Parameterized Entrainment Results (VIPER) [89].

The Deterministic wake Vortex Model is, as such, a useful tool for prediction of wake vortex behavior in one cross-plane. However, for many operational applications, a 3-D space prediction and/or a probabilistic forecast are often needed. Two additional operational tools, based on the DVM, were developed in that sense.

The Probabilistic wake Vortex Model (PVM) performs several DVM runs, in a Monte-Carlo approach, with variations on the input impact parameters, i.e., the aircraft characteristics, the meteorological data and also some of the dimensionless coefficients of the physical models. From the DVM result sample, the PVM then enables to obtain the mean evolution of the vortex characteristics

but also the statistics (median, envelopes, etc.) of these quantities.

Furthermore, the DVM and the PVM are integrated in the WAKE4D platform, a software that predicts the evolution of the 3-D wake vortices generated by an aircraft following a given trajectory and evolving in given atmospheric conditions. In this platform, the computational domain is divided in different computational gates, crossing the flight path, separated by a constant time (typically from 2 to 10 seconds). The aircraft crossing one of the gates generates a pair of vortices that are simulated using the DVM (for a deterministic run) or the PVM (for a probabilistic run). From the “gate by gate” computations, the WAKE4D rebuilds, at any given time, the 3-D wake (or the 3-D wake envelope in PVM mode) generated by an aircraft, following a given trajectory (with uncertainties/variations in PVM mode) within given meteorological conditions (with uncertainties/variations in PVM mode). A schematic view of the WAKE4D platform is presented in Fig. 3.1.

In the scope of this thesis, the DVM was improved, assessed, and validated against measurements and LES data. A complete description of the DVM and its submodels can be found in [124]. In what follows, the DVM general modeling approach is first briefly presented in Section 3.2.1 and the submodels that were developed or improved in this thesis are detailed in Sections 3.2.2 to 3.2.4. The DVM is then assessed in Section 3.2.5. The Probabilistic wake Vortex Model (PVM) was partially developed and improved in the framework of this thesis. Section 3.3 details the PVM and its use. Finally, the WAKE4D platform, that was also developed during this thesis, is presented in Section 3.4 together with some examples of applications.

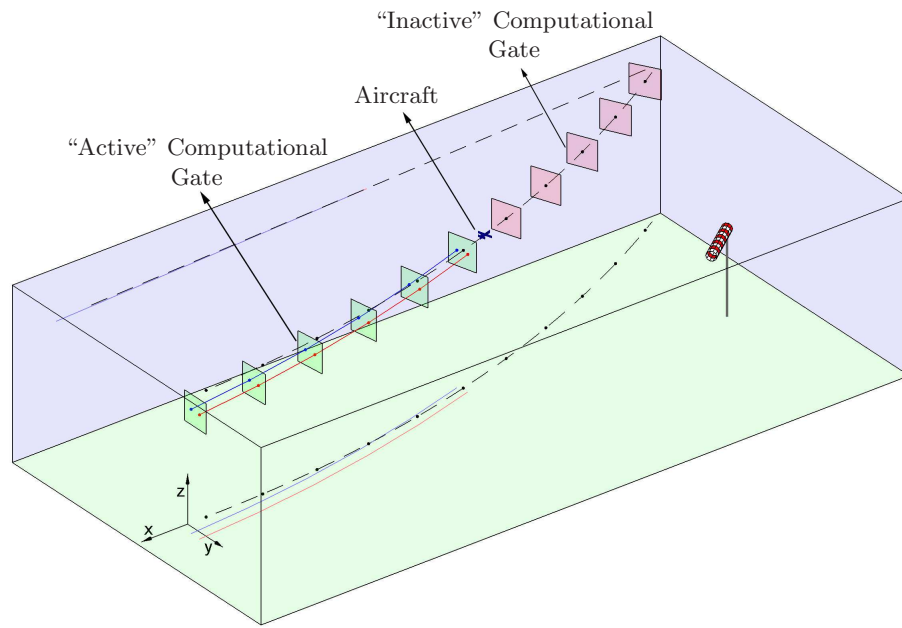


Figure 3.1: Schematic view of the WAKE4D platform. Case of an aircraft flying at high altitude under head- and crosswind conditions. In each “active” Computational Gate (in green), the wake vortex transport and decay is computed using the DVM (for a deterministic prediction) or the PVM (for a probabilistic prediction).

3.2 Deterministic wake Vortex Model (DVM)

3.2.1 Global description of the DVM

The Deterministic wake Vortex Model (DVM) software uses a method of regularized vortex particles (also called, Method of Discrete Vortices, see e.g., [119, 122]). It integrates in time ODEs representing various physical models so as to forecast the transport and decay of the wake vortices in one computational gate (i.e., a 2-D slice of space perpendicular to the flight path), generated by a given aircraft under given meteorological conditions. Some of the physical models of the DVM are taken from the Vortex Forecast System (VFS), that was originally developed by an international team [61], and then further improved and calibrated by UCL from 2001 to 2006 [125], also in the framework of European projects (I-Wake, ATC-Wake [120, 128, 110], WakeNet2-Europe [42, 126]). Those models were then further developed and improved as of 2006, and then solely implemented in the DVM. The DVM was further improved, calibrated, validated, and used in the framework of European projects (AWIATOR, FAR-Wake [55], FLYSAFE [32], CREDOS [22], WakeNet3-Europe [123, 26]).

The initial position, lateral spacing b_0 and circulation Γ_0 of the vortices are calculated using the aircraft characteristics (i.e., position, mass, airspeed, wingspan, lift distribution, and flight angles). Each wake vortex is then represented by one vortex particle placed at the vortex centroid position (y_p, z_p) , with a strength Γ_p and with a certain circulation profile. A one-scale model (see Table 1.3) for the circulation profile $\Gamma(r)$ can be used: e.g., the low-order algebraic model (LOA, Burnham-Hallock):

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{r^2}{(r^2 + r_c^2)}, \quad (3.1)$$

where r_c is the effective core size: $u_\theta = \frac{\Gamma(r)}{2\pi r}$ is maximum at $r = r_c$. Alternatively, one can use the high-order algebraic (HOA) model [119]:

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{r^2 (r^2 + 2\sigma^2)}{(r^2 + \sigma^2)^2}, \quad (3.2)$$

where $\sigma = 1.781 r_c$. A more complex two-scale model (see Table 1.4) can also

be used: e.g., that by Proctor-Winckelmans (PW):

$$\frac{\Gamma(r)}{\Gamma_0} = 1 - \exp \left(- \frac{\beta_i \left(\frac{r}{b}\right)^2}{\left(1 + \left(\frac{\beta_i}{\beta_o} \left(\frac{r}{b}\right)^{5/4}\right)^p\right)^{1/p}} \right), \quad (3.3)$$

with $\beta_o = 10$ and $\frac{\beta_i}{\beta_o}$ that depends on the core size, as $\frac{r_c}{b} \simeq \left(\frac{\beta_o}{\beta_i}\right)^{4/5}$. The Proctor-Winckelmans model is further described in Appendix A. As explained in Chapter 1, the PW model better represents aircraft wake vortex circulation distribution than the LOA model and certainly better than the HOA. It is however computationally more expensive as it requires the evaluation of an exponential. For very fast evaluation, and because we are not interested in the details of the velocity field but just use the model to transport particles, the LOA is preferably chosen. Alternatively, each wake vortex, with its chosen circulation profile, can be discretized using multiple layers of smaller particles (this can be useful for studies where the wake vortex details are themselves discretized in a finer way). In the following, we only consider the first approach, for which each vortex is represented by one vortex particle.

The temporal evolution of the vortex particles is computed using several physical models. Out of ground effect (OGE), the vortex particles are transported by various modeled effects. The velocity induced on each particle by all other vortex particles, with their respective circulation, is calculated using the Biot-Savart law, $u_\theta = \frac{\Gamma(r)}{2\pi r}$. In addition, the particles are convected by both the lateral and axial wind components, as evaluated at the altitude of vortex evolution. The stratification level (measured by the Brunt-Väisälä frequency N) corresponding to the temperature profile is also computed and the thermally induced rebound and decay of the vortices is modeled (see Section 3.2.3). Finally, the shear of the wind profile is computed and its tilting effect on the vortices is also modeled. The circulation Γ_p of the particles also decreases in time, following a two-phase decay: a slow-decay phase followed by a fast-decay phase. The starting time of the fast decay phase is called the time-to-demise and is a function of the turbulence level (i.e., measured by the Eddy Dissipation Rate (EDR= $\bar{\epsilon}$) and scaled using $\epsilon^* = \frac{\bar{\epsilon} b_0}{V_0^3}$) and of the stratification level (N , scaled using $N^* = N \frac{b_0}{V_0} = N t_0$). The time-to-demise itself is evaluated using an “accumulated-damage” approach to take into account the vertically varying turbulence and stratification vertical profile. The decay rate of both the fast- and the slow-decay phases are also function of the turbulence

and of the stratification. Further details on these models are provided in Sections 3.2.2 and 3.2.3. The DVM also contains a model of the space-developing Crow instability for predicting the potential deformation of the wake vortices, detailed in Section 3.2.2.

Finally, in ground proximity, the wake vortex behavior drastically changes, both for the transport and decay. Hence, an additional model is also included. It is described in the Section 3.2.4.

3.2.2 Modeling of turbulence effects on wake vortex decay and on Crow instability development

Modeling of the circulation decay due to turbulence

As observed in Chapter 2, in a turbulent atmosphere, the circulation decay follows a two-phase decay: a slow-decay phase is followed by a rapid-decay phase. The starting time of the fast-decay phase is called the time-to-demise and is a function of the turbulence level (and of the stratification, see Section 3.2.3). In the DVM, the time-to-demise t_d^* is evaluated, in a dimensionless form, as a function of the dimensionless EDR, η , defined by:

$$\eta = \frac{\sqrt[3]{\epsilon b_0}}{V_0} = \sqrt[3]{\epsilon^*}. \quad (3.4)$$

Note that we here use η instead of ϵ^* , to conform with the original model description. Three models of time-to-demise evaluation can be used within the software.

The first model (denoted “SARP0”) is that proposed by Sarpkaya [102, 103] and reads:

$$\begin{aligned} \eta \leq 0.0010 & \quad t_d^* = 9.00 \\ 0.0010 \leq \eta \leq 0.0121 & \quad t_d^* = 9.18 - 180 \eta \\ 0.0121 \leq \eta \leq 0.2535 & \quad \eta = (t_d^*)^{1/4} \exp(-0.70 t_d^*) \\ 0.2535 \leq \eta & \quad t_d^* = \left(\frac{0.7475}{\eta} \right)^{3/4} \end{aligned} \quad (3.5)$$

This model is a modification of the model proposed by Crow & Bate [18] and was fitted on experimental and LES data.

Using new LES results, Holzäpfel [53] proposed a modification of Sarpkaya’s

model (here denoted “SARP1”). This model is obtained by cutting the curve for $\eta \leq 0.0235$, $t_d^* \geq 6.0$ and by using the transformation $t_d^* = 1.20 (t_{d,\text{SARP}}^* - 1.0)$, where $t_{d,\text{SARP}}^*$ is the t_d^* as obtained using the model SARP0. Hence, one obtains:

$$\begin{aligned} \eta \leq 0.0235 \quad t_d^* &= 6.00 \\ 0.0235 \leq \eta \leq 0.2535 \quad \eta &= \left(\frac{t_d^*}{1.2} + 1 \right)^{1/4} \exp \left(-0.7 \left(\frac{t_d^*}{1.2} + 1 \right) \right) \quad (3.6) \\ 0.2535 \leq \eta \quad t_d^* &= \left(\left(\frac{0.7475}{\eta} \right)^{3/4} - 1 \right) 1.2 \end{aligned}$$

A minor problem of this model is that, for large values of η (which are indeed rare), t_d^* becomes negative.

We then here propose a further modification of Sarpkaya’s model (denoted “SARP2”). This further modification is made so as to maintain the model SARP1 for most practical values of η but to adapt the model for the highest values so that it has the proper behavior. The third part of the model is therefore adapted in order to have, at $\eta = 0.2535$, the continuity of t_d^* and $\frac{dt_d^*}{d\eta}$. Hence, the model reads:

$$\begin{aligned} \eta \leq 0.0235 \quad t_d^* &= 6.00 \\ 0.0235 \leq \eta \leq 0.2535 \quad \eta &= \left(\frac{t_d^*}{1.2} + 1 \right)^{1/4} \exp \left(-0.7 \left(\frac{t_d^*}{1.2} + 1 \right) \right) \quad (3.7) \\ 0.2535 \leq \eta \quad t_d^* &= \left(\frac{0.3417}{\eta} \right)^{1.36} \end{aligned}$$

Figure 3.2 shows the comparison between the three models. It is to be noted that Sarpkaya’s model, or one of its modified versions, is a mathematical fit on experimental and numerical data. However, it has the correct physical behavior, as it is a decreasing function of the turbulence intensity, and is commonly used in the literature (see e.g., [46, 84]).

In the DVM, the decay of the vortices is modeled as a change in the circulation of the discrete vortex particle(s) associated with the primary vortices:

$$\frac{d\Gamma_p}{dt} = -C_\eta \frac{\Gamma_p}{t_d}, \quad (3.8)$$

with C_η the decay coefficient of the EDR based decay model and $t_d = t_d^*(\eta) t_0$, the dimensional time-to-demise due to turbulence effects. In the case of a

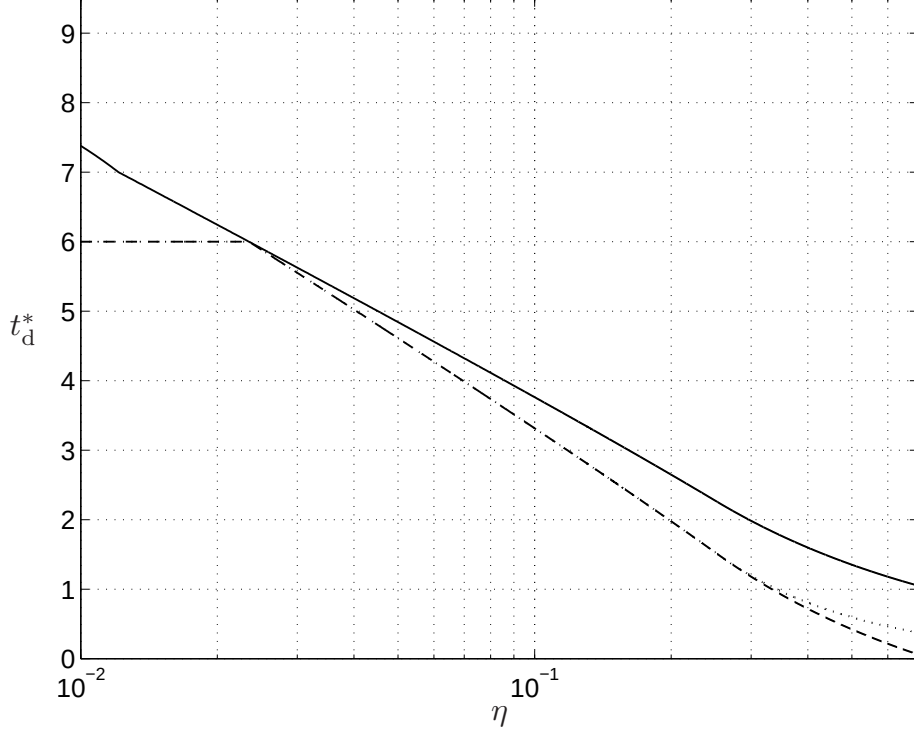


Figure 3.2: Comparison of the time-to-demise evaluation models: SARP0 (solid), SARP1 (dash) and SARP2 (dot).

uniform EDR profile ($\eta = \eta_0$), the model simply integrates:

$$\frac{\Gamma(t^*)}{\Gamma_0} = \exp \left(-C_\eta \frac{t^*}{t_d^*(\eta_0)} \right). \quad (3.9)$$

Furthermore, as time proceeds, the DVM evaluates the “fraction of time-to-demise” attained, f_d , using an “accumulated damage” approach to best make use of the possibly vertically varying EDR profile that the vortex wake encounters as it ages:

$$f_d(t^*) = \int_0^{t^*} \frac{d\tau}{t_d^*(\eta(\tau))}. \quad (3.10)$$

When $f_d(t^*)$ reaches unity, the wake vortices enter the phase of rapid decay. To capture the rapid decay phase, the C_η coefficient of the model is increased significantly. This leads to the “two-phase decay model”: with $C_{\eta,1}$ for the slow decay phase and $C_{\eta,2}$ for the rapid decay phase.

Compared to the results of the simulations obtained in Chapter 2, the best agreement is found using either the model SARP1 or SARP2, for the time-to-demise, and with $C_{\eta,1} = 0.035$ and $C_{\eta,2} = 0.5$. The comparison between LES results and DVM predictions is provided in Fig. 3.3. In the LES results, the average circulation $\bar{\Gamma}_{\text{tot}}$ is seen to decrease quicker for lower turbulence levels which is in contradiction with the present model. To fit the evolution of $\bar{\Gamma}_{\text{tot}}$ for the case with $\eta = 0.0623$, one should indeed increase $C_{\eta,2}$ to 2.75. However, as concluded in Chapter 2, for the case with weak turbulence (here, $\eta = 0.0623$), the fast-decay phase of the average circulation is mostly due to the vortex reconnection, whereas, for the case with moderate to strong turbulence (here, $\eta = 0.2493$), it is due to the strong interaction with the ambient turbulence. In weak turbulence, when the vortices link, some cross-planes exhibit a very fast decay for topological reason (i.e., due to vortex ring formation), while the others show a much slower second decay phase, that is due to the interaction with the ambient turbulence. Therefore, for that case, even if the average circulation is not well represented by the DVM, the evolution of the upper bound of the circulation envelope corresponds well to the model. The proposed model hence represents the mean evolution of the parts of the vortices that are still “alive”. Since a LIDAR can obviously only measure vortices that are alive, it is believed that this model also represents well LIDAR measurements.

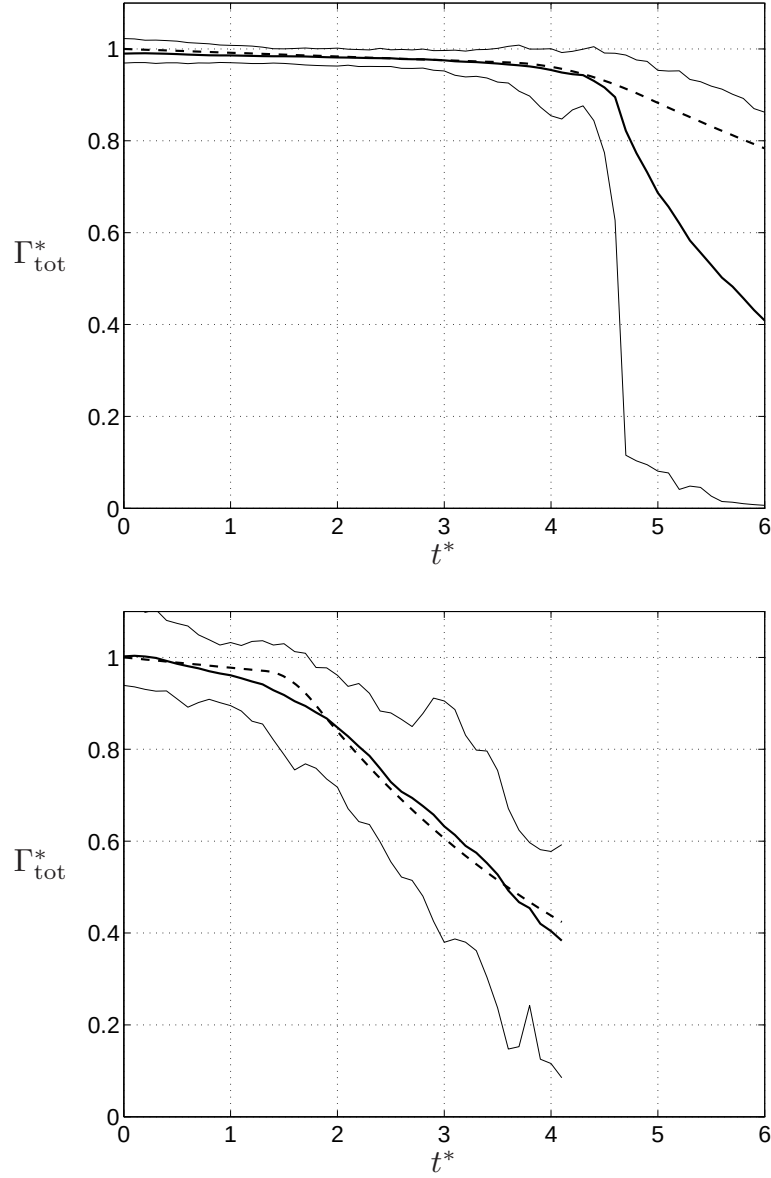


Figure 3.3: Average $\bar{\Gamma}_{\text{tot}}^*$ (thick solid) and 95%-envelope (thin solid) of the total circulation evolution for a case with $\eta = \sqrt[3]{2.42 \cdot 10^{-4}} = 0.0623$ (top) and a case with $\eta = \sqrt[3]{1.55 \cdot 10^{-2}} = 0.2493$ (bottom) obtained using LES results (see Chapter 2, Section 2.4.4). For each case, the DVM prediction is also shown (dash).

Crow instability development modeling

A two-vortex system, evolving OGE in a calm atmosphere, develops a long wavelength instability called the Crow instability, as studied in Chapter 2, Section 2.4.4. Following Crow analysis [16], the instability, with a wavelength $\lambda = \frac{2\pi}{k}$, develops in a plane forming an angle θ_p w.r.t. the line joining the wake vortex centers. Its amplitude A grows exponentially following:

$$A(t) = A_0 \exp(\sigma^* t^*) . \quad (3.11)$$

Crow [16] shows that, for typical aircraft wake vortex core radius, the most unstable wavelength is $\lambda = 8.6 b_0$ with a growth rate $\sigma^* = 0.82$ and an angle $\theta_p = 48^\circ$. The only parameter that is still not characterized at this stage is the initial amplitude A_0 . In order to evaluate this initial amplitude, we suppose that the vortex tube reconnection occurs when $t^* = t_d^*(\eta)$ (so far, we only consider the case of turbulence without stratification). In Chapter 2, we indeed verified that, in weak turbulence, the second decay phase starts when the vortices link. Hence, assuming that the instability remains sinusoidal up to the vortex reconnection:

$$A(t_d^*) = A_0 \exp(\sigma^* t_d^*) = \frac{b_0/2}{\cos(\theta_p)} . \quad (3.12)$$

If we neglect the stratification effects, we find, by dimensional analysis, that $\frac{A_0}{b_0} = f(\eta)$. Three different approaches were then tested to correlate the initial amplitude A_0 to the turbulence level, and thus to find the expression of $f(\eta)$.

In the first approach, we suppose that A_0 is related to the ambient turbulence according to:

$$\frac{A_0(\eta)}{b_0} = C \eta^\alpha . \quad (3.13)$$

The corresponding t_d^* is thus a function of the ambient turbulence obtained as:

$$\begin{aligned} t_d^* &= -\frac{1}{\sigma^*} \log \left(\frac{A_0}{b_0} 2 \cos(\theta_p) \right) \\ &= -\frac{1}{\sigma^*} \log (C \eta^\alpha 2 \cos(\theta_p)) . \end{aligned} \quad (3.14)$$

The values of α and C can be fitted to the different time-to-demise models in the most usual range of turbulence ($\eta \in [0.0235, 0.2535]$). The obtained values are $\alpha = 1.30$ and $C = 0.69$, for the model SARP0, and $\alpha = 1.56$ and

$C = 1.80$, for SARP1 and SARP2 (in this region, both models are identical). The advantage of this correlation is that it is very easy to compute. In the framework of the AWIATOR project, Leweke [68] followed the same kind of approach. He found an exponent $\alpha = 1.5$.

In the second approach, we correlate the A_0 function directly to the time-to-demise models (SARP0, SARP1 or SARP2) for the whole range of η , this leads to:

$$\frac{A_0(\eta)}{b_0} = \frac{1}{2 \cos(\theta_p)} \exp(-\sigma^* t_d^*(\eta)). \quad (3.15)$$

This method is more precise than the previous one but requires, for each computation of A_0 , a (potentially implicit) computation of t_d^* . Figure 3.4 compares the obtained initial amplitude using the two approaches. For large values of η , both approaches lead to unphysically too high initial amplitude.

A third approach is thus chosen using the advantages of the two previous ones. For low η values (i.e., value for which no implicit computation is needed), the amplitude is taken directly from the time-to-demise model. For high turbulence values, an upper threshold is set to a value corresponding to $\frac{A_0}{b_0} = 5\%$. According to the model SARP1 this value is $\eta_{th} = 0.101$, whereas, following the model SARP0, $\eta_{th} = 0.134$. For intermediate η values, in order to avoid to use the implicit relation between η and t_d^* , one fits, on the time-to-demise model, an exponent correlation : $\frac{A_0}{b_0} = C \eta^\alpha$. Using the model SARP0, this leads to:

$$\begin{aligned} \eta \leq 0.001 & \quad \frac{A_0(\eta)}{b_0} = \frac{1}{2 \cos(\theta_p)} \exp(-\sigma^* 9.0) = 4.7 \cdot 10^{-4} \\ 0.001 \leq \eta \leq 0.0121 & \quad \frac{A_0(\eta)}{b_0} = \frac{1}{2 \cos(\theta_p)} \exp(-\sigma^* (9.18 - 180 \eta)) \\ 0.0121 \leq \eta \leq 0.134 & \quad \frac{A_0(\eta)}{b_0} = 0.633 \eta^{1.26} \\ 0.134 \leq \eta & \quad \frac{A_0(\eta)}{b_0} = 5.0 \cdot 10^{-2} \end{aligned} \quad (3.16)$$

When using the model SARP1 (or SARP2), it reads:

$$\begin{aligned} \eta \leq 0.0235 & \quad \frac{A_0(\eta)}{b_0} = \frac{1}{2 \cos(\theta_p)} \exp(-\sigma^* 6.0) = 5.5 \cdot 10^{-3} \\ 0.0235 \leq \eta \leq 0.101 & \quad \frac{A_0(\eta)}{b_0} = 1.63 \eta^{1.52} \\ 0.101 \leq \eta & \quad \frac{A_0(\eta)}{b_0} = 5.0 \cdot 10^{-2} \end{aligned} \quad (3.17)$$

Figure 3.4 also shows the initial amplitude as a function of the turbulence level

using this third model. It is also interesting to compare the results of the model to the initial Crow instability amplitude measured on the LES results in Chapter 2, Table 2.4. In particular, we obtained $A_0/b_0 = 1.3 \cdot 10^{-2}$ for the case 0 ($\eta = \sqrt[3]{2.42 \cdot 10^{-4}} = 6.23 \cdot 10^{-2}$, $N^* = 0$) and $A_0/b_0 = 5.4 \cdot 10^{-3}$ for the case 5 ($\eta = \sqrt[3]{1.82 \cdot 10^{-5}} = 2.89 \cdot 10^{-2}$ and $N^* = 0.35$, thus a weakly stratified case). For these LES, the wake vortices have a core radius $r_c/b_0 = 0.05$ and the instability has a wavelength $\lambda = 8 b_0$. For that conditions, Crow's theory [16] predicts $\sigma^* = 0.82$ and $\theta_p = 45^\circ$. Using approach 3, the model predicts, respectively, $A_0/b_0 = 1.86 \cdot 10^{-2}$ and $A_0/b_0 = 6.29 \cdot 10^{-3}$, using SARP0, and $A_0/b_0 = 2.38 \cdot 10^{-2}$ and $A_0/b_0 = 6.47 \cdot 10^{-3}$, using SARP1 or SARP2. The present model is thus also seen to be in reasonable agreement with the results of Chapter 2.

Finally, recall that, as observed in Chapter 2, the Crow instability development does not necessarily lead to the vortex linking. For high turbulence levels, the instability indeed develops but the vortices are destroyed before reconnection could occur. Based on the results of Chapter 2, the turbulence level threshold was found to be $0.063 < \eta < 0.247$ ($2.5 \cdot 10^{-4} < \epsilon^* < 1.5 \cdot 10^{-2}$). That range is indicated in Fig. 3.4.

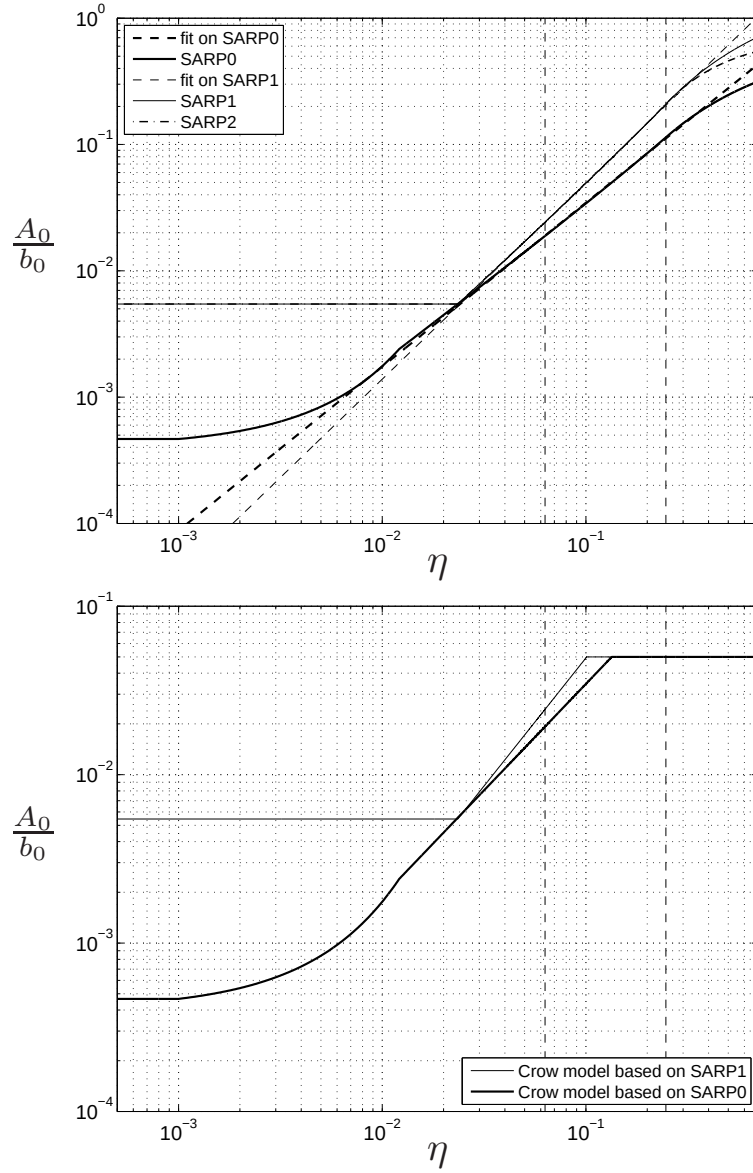


Figure 3.4: Models for the Crow instability initial amplitude as a function of the turbulence level: following approaches 1 and 2 (top), and approach 3 (bottom). Also shown is the range containing the turbulence intensity level threshold above which no Crow linking occurs (thin dash).

3.2.3 Modeling of combined stratification and weak turbulence effects on wake vortex behavior

Based on the LES results presented in Chapter 2, improved simplified models are also proposed for the effects, on the wake vortex transport and decay, of combined stratification and weak turbulence. Those new models are also implemented in the DVM.

Modeling of the vortex transport

An improvement of the simplified trajectory model developed by Saffman [99] is here proposed. In this model, one considers that the vortex altitude evolution is influenced by the self-induced velocity (Biot-Savart effect), v_{BS} , and by an additional vertical velocity due to the stratification, v_{str} :

$$\frac{dz^*}{dt^*} = v_{\text{BS}}^* + v_{\text{str}}^*. \quad (3.18)$$

For the Biot-Savart component, one uses the total circulation measured on the LES results (as defined by Eq. (2.33)):

$$v_{\text{BS}}^*(t^*) = -\Gamma_{\text{tot}}^*(t^*). \quad (3.19)$$

The additional stratification induced velocity is taken as:

$$\frac{dv_{\text{str}}^*}{dt^*} = -(\alpha_{\text{str}} N^*)^2 (z^* - h_0^*) - C v_{\text{str}}^*. \quad (3.20)$$

The first term in Eq. (3.20) corresponds to that of Saffman's model. It is based on the conservation of linear impulse and leads to a sinusoidal rebound controlled by the Brunt-Väisälä frequency. The second part is a new damping term modeling the effect of the reorganization of the baroclinic vorticity around the primary vortices, attenuating the rebound effect. This term is added because the secondary baroclinic vorticity is “turbulently diffused” and “spread” after a certain time leading to a decrease in the rebound. The second term is activated only for time $t^* \geq t_{\text{crit}}^*$, where t_{crit}^* is a parameter of the model. The calibration of the model on the numerical results of Chapter 2 leads to: $\alpha_{\text{str}} = 0.8$, $t_{\text{crit}}^* = 1.6$ and $C = 1.3$. The improved model is seen to fit very well the results for all cases, see Chapter 2, Fig. 2.26(a). A small difference is however still observed for the case with $N^* = 0.35$ for times $t^* \geq 4.0$. Recall

that, for typical aircraft, $t^* = 4.0$ would correspond to a time of 120-160 s, which is already quite long for ATM applications.

Modeling of the vortex circulation decay

For all cases investigated in Chapter 2, one observes a two-phase decay. The starting time of the fast-decay phase t_d^* is seen to depend both on the turbulence and the stratification level. Sarpkaya [103] proposed a model that relates the “time-to-demise” to the turbulence intensity, and that is detailed in Section 3.2.2. Based on Sarpkaya’s model, Holzäpfel [53] suggested an improved model, applied to the Γ_{5-15} decay, that also includes the influence of stratification. However, this model applied to the evolution of Γ_{tot} , does not properly fit the present results. Moreover, for low turbulence levels and high stratification levels, it predicts a time-to-demise that is an increasing function of the turbulence. A new model is then here proposed. This model is an improvement of the time-to-demise model developed by Sarpkaya [103] which, in weak turbulence and without stratification, was verified to correctly represent the starting time of the fast-decay phase (see Section 2.4.4). The new model also takes into account that atmospheric turbulence no longer plays any role for very high stratification levels. It reads:

$$t_d^*(\epsilon^*, N^*) = \begin{cases} t_{d,S}^* \exp\left(-C_1 \left(N^* t_{d,S}^*\right)^a\right) & \text{if } \left(N^* t_{d,S}^*\right) \leq K, \\ \frac{C_2}{N^*} & \text{otherwise,} \end{cases} \quad (3.21)$$

with $t_{d,S}^* = t_{d,S}^*(\epsilon^*)$ computed according to Sarpkaya’s model, and C_1 and a the two model coefficients to be calibrated. The coefficient K is obtained from C_1 and a by requiring that $\left.\frac{\partial t_d^*}{\partial \epsilon^*}\right|_{N^*} \leq 0$, leading to $K = (C_1 a)^{-1/a}$. The coefficient C_2 is then obtained from C_1 and K so that t_d^* is continuous: $C_2 = K \exp(-C_1 K^a)$. Iso-contours of $t_d^*(\epsilon^*, N^*)$ are provided in Fig. 3.5.

Furthermore, and based on this time-to-demise model, a new two-phase model is also proposed for the circulation decay rate. It reads:

$$\frac{d\Gamma_{\text{tot}}^*}{dt^*} = \begin{cases} -\frac{C_1}{t_{d,S}^*} \Gamma_{\text{tot}}^* & \text{if } t^* \leq t_d^* - \Delta t^*/2, \\ -\frac{C_2}{t_{\Gamma}^*} \Gamma_{\text{tot}}^* & \text{if } t^* \geq t_d^* + \Delta t^*/2, \end{cases} \quad (3.22)$$

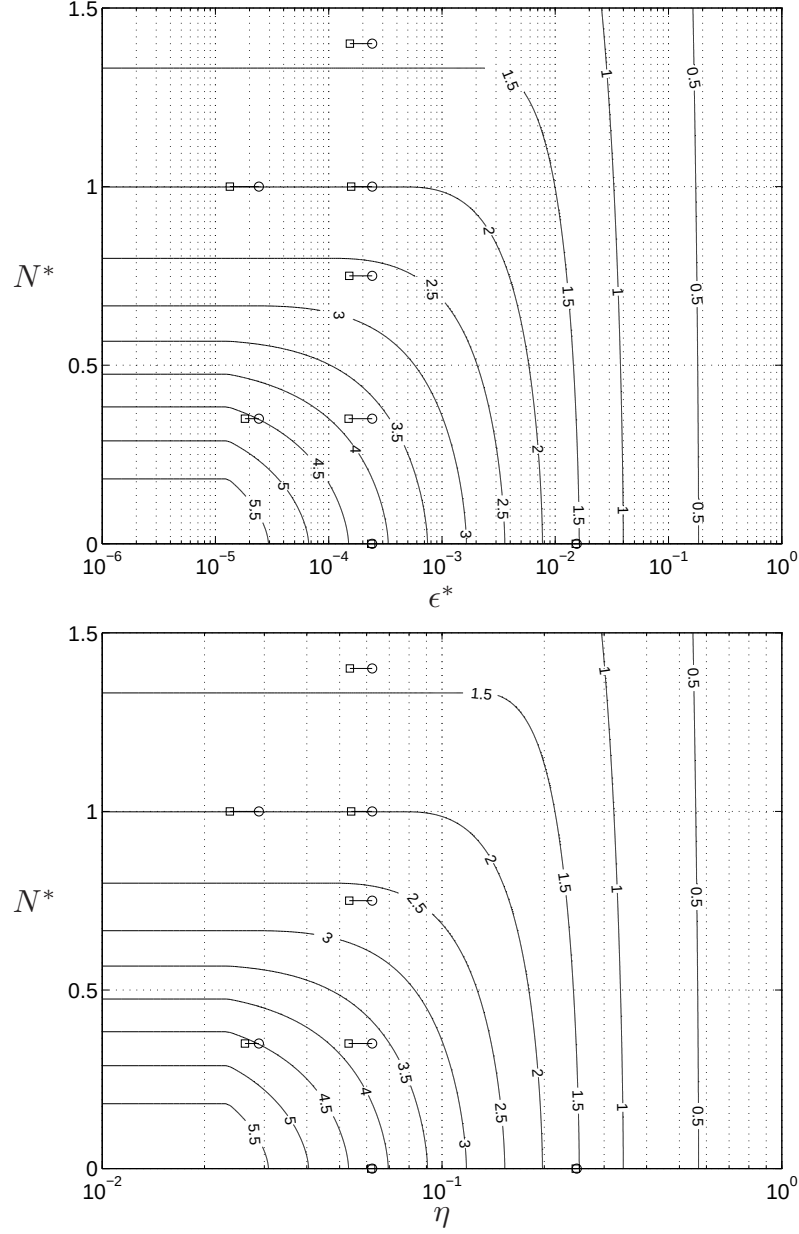


Figure 3.5: Iso-contours of the dimensionless time-to-demise, t_d^* , as a function of the dimensionless EDR, ϵ^* , and the dimensionless Brunt-Väisälä frequency, N^* , and using the Sarpkaya model SARP2 for $t_{d,S}^*(\epsilon^*)$. Also displayed are the cases investigated in Chapter 2 represented by the couple (ϵ_f^*, N^*) ($\circ$). For each case, the resulting kinetic energy dissipation rate ϵ^* ($\square$) is also shown (recall that, for stratified cases, $\epsilon_f^* = \epsilon^* + \epsilon_P^*$). The bottom figure is the same as the top figure, but using η , instead of ϵ^* , as dimensionless EDR.

where c_1 and c_2 are the decay coefficients of the two decay phases, and t_Γ^* is:

$$t_\Gamma^* = \begin{cases} t_{d,S}^* & \text{if } N^* = 0, \\ \sqrt{t_{d,S}^* t_N^*} & \text{otherwise.} \end{cases} \quad (3.23)$$

To even better fit the results, the decay is taken to go smoothly, from phase I to phase II, over an interval $\Delta t^* = \alpha t_d^*$. For $t_d^* - \Delta t^*/2 \leq t^* \leq t_d^* + \Delta t^*/2$, one then uses:

$$\frac{d\Gamma_{\text{tot}}^*}{dt^*} = - \left(\frac{1}{2} \left(\frac{c_2}{t_\Gamma^*} + \frac{c_1}{t_{d,S}^*} \right) + \left(\frac{c_2}{t_\Gamma^*} - \frac{c_1}{t_{d,S}^*} \right) \frac{(t^* - t_d^*)}{\Delta t^*} \right) \Gamma_{\text{tot}}^*. \quad (3.24)$$

Moreover, even at short times, the presence of baroclinic vorticity is seen to induce a small decrease of the total circulation. This enhanced decay is modeled through an additional slow decay model due to stratification, $\left. \frac{d\Gamma_{\text{tot}}^*}{dt^*} \right|_{\text{str}}$. This additional decay also induces a decrease in the vortex descent rate. The model of Eq. (3.20) is hence also further improved:

$$\left. \frac{d\Gamma_{\text{tot}}^*}{dt^*} \right|_{\text{str}} = (1 - \beta_{\text{str}}) (\alpha_{\text{str}} N^*)^2 (z^* - h_0^*), \quad (3.25)$$

$$\frac{dv_{\text{str}}^*}{dt^*} = -\beta_{\text{str}} (\alpha_{\text{str}} N^*)^2 (z^* - h_0^*) - C v_{\text{str}}^*. \quad (3.26)$$

The fitting of the model on the present results provides: $a = 1.6$, $C_1 = 0.076$ (and thus $K = 3.73$ and $C_2 = 2.0$), $c_1 = 0.035$, $c_2 = 2.75$, $\alpha = 0.15$, $t_N^* = 3.0$, $\alpha_{\text{str}} = 0.83$, and $\beta_{\text{str}} = 0.93$. The comparison of the model with the present results are displayed in Chapter 2, Fig. 2.27 and Fig. 2.35. The agreement with the numerical results is very good for all investigated stratification levels (all with weak turbulence). All the fit curves are stopped when $\Gamma_{\text{tot}}^* = 0.4$, which corresponds to the circulation at $t^* = 6$ for the neutral case. For all cases, when $\Gamma_{\text{tot}}^* = 0.4$, Γ_{5-15}^* is below $0.3 - 0.2$ (see Fig. 2.29), which corresponds to the typical minimum threshold of a LIDAR wake vortex measurement ($\Gamma_{5-15, \text{min}} = 100 - 150 \text{ m}^2/\text{s}$).

As observed in Chapter 2, for cases with weak turbulence and weak or neutral stratification levels, the fast-decay phase of the average circulation is mostly due to the vortex reconnection. For those cases, the present model hence represents the evolution of the circulation averaged over all cross-planes (those where the vortices are still “alive” and those where there is no vortex

anymore).

Another model is then here proposed that only represents, for the second decay phase, the circulation evolution of the parts of the vortices that are still “alive”. Without stratification, the model hence corresponds to that of Eq. (3.8) (with $C_{\eta,2} = 0.5$). With stratification, an additional decay term proportional to the Brunt-Väisälä frequency N is added so that, for high stratification levels, the decay corresponds to that of Eq. (3.22). In the second decay phase ($t^* \geq t_d^* + \Delta t^*/2$), the model hence reads:

$$\frac{d\Gamma_{\text{tot}}^*}{dt^*} = -\frac{C_{\eta,2}}{t_{d,S}^*}\Gamma_{\text{tot}}^* - \frac{C_{\text{str}}}{t_{\text{str}}^*}\Gamma_{\text{tot}}^*, \quad (3.27)$$

where

$$t_{\text{str}}^* = \begin{cases} \frac{1}{N^*} & \text{if } N^* \leq N_{\text{lim}}^* \\ \frac{1}{\sqrt{N^* N_{\text{lim}}^*}} & \text{if } N^* \geq N_{\text{lim}}^* \end{cases} \quad (3.28)$$

Using $N_{\text{lim}}^* = 0.8$ and $C_{\text{str}} = 0.6$, the model is seen to fit the LES results. Figure 3.6 displays the evolution of the circulation envelopes measured in the LES results of Chapter 2 together with the predictions using the model of Eq. (3.27). For weak stratification levels ($N^* \leq 0.75$), the predictions are above the average curve, as they only represents the mean evolution of the planes with vortices that are still “alive”. On the contrary, for high stratification levels ($N^* \geq 1$), the predictions corresponds to the evolution of the average circulation as the decay is much more uniform along the vortex.

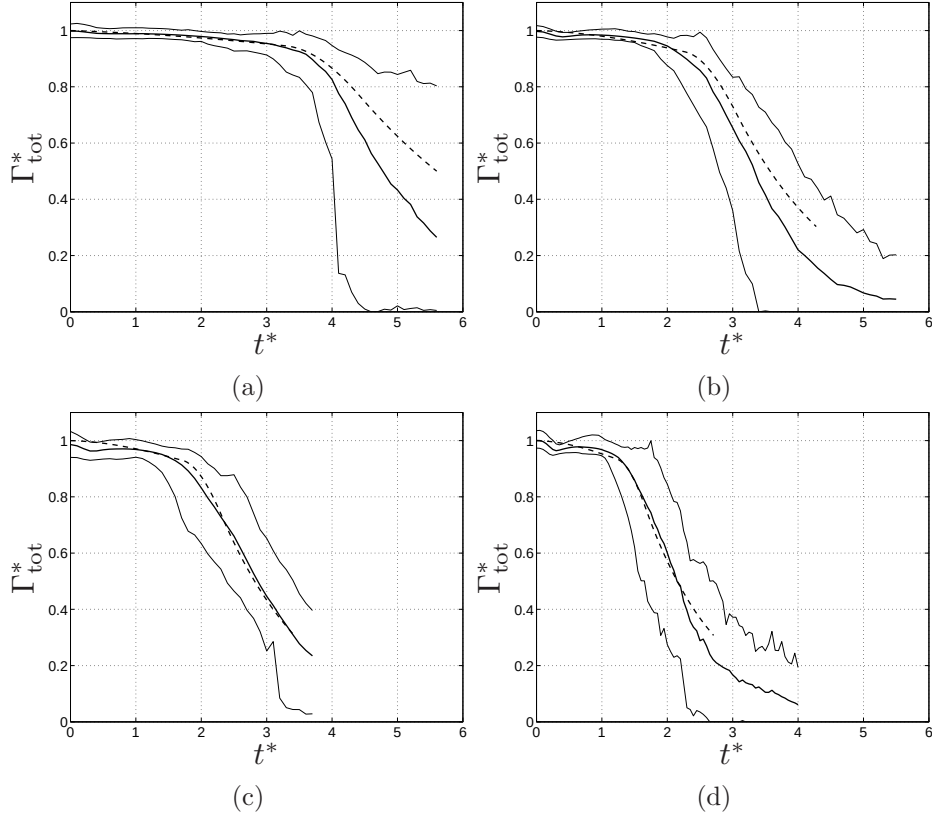


Figure 3.6: Average $\bar{\Gamma}_{\text{tot}}^*$ (thick solid) and 95%-envelope (thin solid) of the total circulation evolution, obtained using LES results (see Chapter 2, Section 2.4.4), for : (a) a case with $N^* = 0.35$, (b) a case with $N^* = 0.75$, (c) a case with $N^* = 1.0$, (d) a case with $N^* = 1.4$. For all cases, $\epsilon^* = 2.42 \cdot 10^{-4}$. For each case, the DVM prediction, using Eq. (3.27), is also shown (dash).

3.2.4 Modeling of ground effects on wake vortex behavior

Description of the ground effects

As already explained, the initial wake vortex system is composed of two vortices with circulation Γ_0 and $-\Gamma_0$, and separated, after roll-up, by a distance b_0 . Consider the case with sufficiently high initial altitude (i.e., $h_0 \geq 4b_0$), the vortices then initially sink under their mutual influence, at a velocity $V_0 = \Gamma_0/2\pi b_0$.

As the vortices approach the ground, in the “near-ground effect” (NGE) region (i.e., for altitudes $z/b_0 \leq 4$), they begin to separate laterally. This inviscid effect was already discussed by Lamb [67].

As they go further down, the wake vortices enter the “in-ground effect” (IGE) region (i.e., for $z/b_0 \leq 1-2$) and start to strongly interact with the ground. Due to the no-slip condition at the ground, “boundary layers” (i.e., here thin layers of vorticity), with opposite sign secondary vorticity, are generated below each primary vortex and starting from the stagnation point. Figure 3.7 and Figure 3.8 display a schematic view of the vortex pair and the velocity profile induced by the vortices just above the thin boundary layer. As seen in these figures, the boundary layer is subjected to an accelerating ($\frac{dv_x}{dy} > 0$), then decelerating ($\frac{dv_x}{dy} < 0$) flow. When the vortices further sink, this effect becomes more pronounced leading the boundary layers to separate. The interaction of the primary vortex with the secondary vorticity leads the vortex to rebound. The rebound phenomenon was first experimentally observed and studied by Harvey & Perry [47] and by Barker & Crow [1]. This behavior, at first essentially two-dimensional, was analyzed in depth in many numerical studies using various numerical methods and Reynolds numbers, see e.g., [81, 80, 113, 90].

The crosswind also influences the wake vortex behavior IGE. Other 2-D numerical studies (see [94, 129, 13, 88, 109]) showed that the crosswind favors the secondary vorticity production below one of the two vortices (the downwind vortex) compared to the other one (the upwind vortex). The combination of ground effect and crosswind then leads to an asymmetrical rebound of the vortices. This asymmetry in the vortex rebound should be distinguished from the tilting effect due to the crosswind shear. It has indeed been shown (see e.g., [88, 124]) that the vorticity profile of the crosswind induces an asymmetry in the sinking velocity of the two vortices and hence a tilting of the dipole axis from the horizontal. In ground proximity, a typical crosswind profile (such as a log profile) will induce such tilting. However, the asymmetry in the vortex

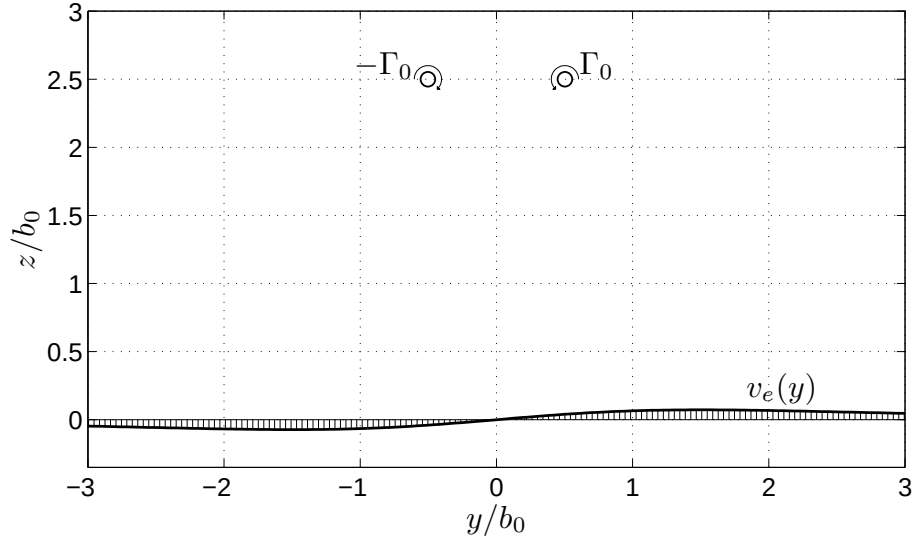


Figure 3.7: Schematic view of the boundary layer creation at the ground, due to the no-slip condition. Here a case NGE with $h_0 = 2.5 b_0$ is shown. The velocity profile just above the boundary layer $v_e(y)$ induced by the wake vortices is also shown.

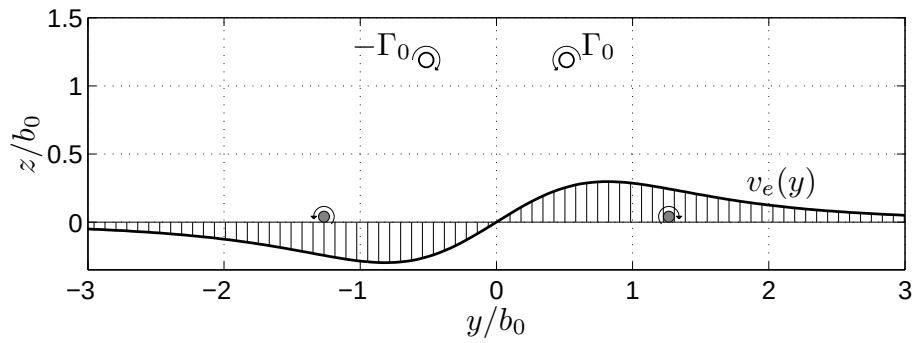


Figure 3.8: Schematic view of the boundary layer creation at the ground, due to the no-slip condition. Here a case IGE with $z_p = 1.2 d_0$ is shown. The velocity profile just above the boundary layer $v_e(y)$ induced by the wake vortices is also displayed. Also shown are the secondary vortex particles created at the ground and that will eventually separate from the ground.

rebound IGE with crosswind is principally due to the difference in intensity of the secondary vorticity generated below the two vortices: even a linear crosswind profile (thus, a profile with zero vorticity gradient) will indeed induce an asymmetrical rebound IGE.

Wake vortices IGE also eventually develop 3-D instabilities. Three-dimensional numerical simulations (see e.g., [76, 66, 87, 85, 35]) revealed that short-wavelength elliptic instabilities appear in the separated secondary vorticity leading to a turbulent interaction with the primary vortices and, hence, an enhanced decay. In addition, a long-wave Crow-like instability could also develop. Large Eddy Simulations (LES) of wake vortices with and without crosswind [5] showed that, with crosswind, an asymmetry between the upwind and downwind vortices also appears in their circulation decay. It is to be noted that some existing operational models (i.e., VFS, APA, and D2P [60]) also contain an IGE model.

As a good modeling of the combined effects of ground and wind is essential for reliable operational predictions, an improved IGE model was developed and was included in the DVM. It is here presented in detail. This model uses a limited set of secondary vortex particles to represent the secondary vorticity generated at the ground. Those secondary vortex particles, whose characteristics depend on the whole vortex system and on the crosswind, interact with the primary vortices (each being itself represented by one or more particles), leading to their rebound and also much enhanced decay. This new model aims at mimicking the whole flow dynamics, including those of the secondary ground-generated vorticity, separating and interacting with the primary vortices. It also accounts for the influence of the wind (cross- and headwind components) on both the primary vortices and secondary vorticity.

Near-Ground Effect behavior

When the vortices are in ground vicinity, a thin vortex sheet (i.e., a “boundary layer”), with a circulation per unit length $\gamma(y)$, is generated at the ground, due to the zero-velocity at the ground. The velocity evaluated just above this vortex sheet is then $v(y, 0^+) = v_e(y) = -\gamma(y)$. As the ground is assumed flat, the velocity field at any position $(y, z \geq 0^+)$ in the upper plane is as easily obtained by using image (mirror) vortex particles below the ground plane instead of the vortex sheet $\gamma(y)$: for each vortex particle of circulation Γ_p , located at (y_p, z_p) with $z_p > 0$, one uses an image particle located at $(y_p, -z_p)$ and of circulation $-\Gamma_p$. We stress that the velocity field obtained using the particles and their

images is fully identical to that one obtains if using instead the particles and the vortex sheet $\gamma(y)$. A schematic view is provided in Fig. 3.9. This Near Ground Effect (NGE) properly captures the inviscid vortex transport in ground proximity so long as there is no separation of the boundary layers. Using solely this model, the vortices would follow a hyperbolic trajectory, as represented in Fig. 3.9. Indeed, without separation of the boundary layers, a vortex pair generated at an altitude h_0 and initially separated by b_0 , would reach the asymptotic altitude $d_0/2$ expressed by [13]:

$$\frac{d_0}{2} = \frac{b_0}{\sqrt{\left(\frac{b_0}{h_0}\right)^2 + 4}}. \quad (3.29)$$

In order to capture the very important effects of the boundary layer separation and of the interaction between the primary vortices and the secondary separated vorticity, a complementary IGE model is implemented.

This IGE model is based on generating “new” secondary vortex particles close to the ground, modeling the separation of the ground-generated boundary layers. Those particles dynamically separate from the ground. They also interact with the primary vortices and induce the rebound. An additional IGE decay model, based on a “Particle Strength Exchange” (PSE) approach, is also further implemented, that enhances the primary vortex decay after rebound due to the interaction with the secondary vorticity that, by then, has become turbulent. The combination of the primary and secondary vortices, also with the enhanced decay model, enables to obtain a realistic representation of the ground effects on wake vortices for both transport and decay, as we shall see.

Generation and transport of the secondary vortex particles

At each time step Δt , for each primary vortex particle with an altitude z_p below a threshold h_{crit} , the model produces a secondary vortex particle so as to represent the separation of the boundary layer generated by the presence of the primary vortices in ground proximity. The height h_{crit} , at which we start to generate such particles, is set to:

$$h_{\text{crit}} = \alpha_s d_0, \quad (3.30)$$

where α_s is a coefficient and d_0 is calculated using Eq. (3.29). The value of the coefficient α_s , as well as those of the other coefficients of the IGE model

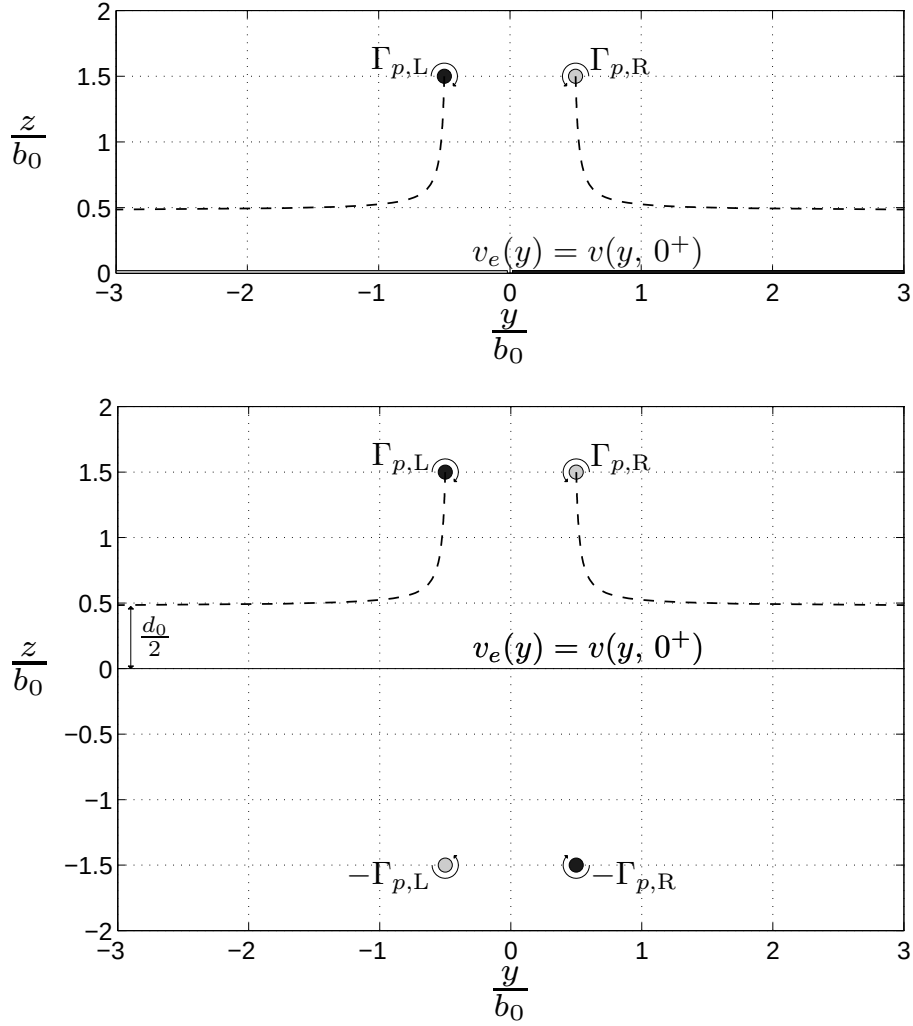


Figure 3.9: General view of the NGE wake vortex behavior. The top figure shows the flow to be modeled with the primary vortex pair and the induced thin boundary layer at the ground. The bottom figure displays the equivalent kinematic model that uses image mirror particles. Also shown is the inviscid trajectory (i.e., case without boundary layer separation) of a vortex pair in ground proximity. The velocities evaluated above the thin boundary layer, $v(y, z \geq 0^+)$, are identical in both cases.

(introduced hereafter), are provided at the end of this section together with an explanation on the way they were obtained.

The generated secondary particle is put at a position (y_s, z_s) , with a regularization size σ_s and a circulation Γ_s . To compute the velocity field induced by such particle, one uses the high-order algebraic circulation profile of Eq. (3.2). For discretization of secondary vorticity with several vortex particles, the use of the high order algebraic function for the velocity evaluation was indeed shown to be more appropriate than the use of the low order algebraic function (see [122]).

As seen in Fig. 3.8, starting from the stagnation point, the velocity norm above the vortex sheet $|v_e(y)|$ increases, reaches a maximum and then decreases. The lateral position y_s is computed so as to correspond to the outboard position (a position located after the maximum) where $|v_e(y_s)|$ reaches a fraction α_v of the maximum of $|v_e(y)|$ measured along the ground, starting from the stagnation point:

$$|v_e(y_s)| = \alpha_v \max_y(|v_e(y)|). \quad (3.31)$$

We, of course, consider the left (port) and right (starboard) problem separately. Figure 3.8 shows the obtained lateral positions y_s for the considered induced velocity profile $v_e(y)$. At that position, the ground-generated boundary layer thickness is noted δ . δ scales with the global scale, b_0 , the Reynolds number, $Re_\Gamma = \frac{\Gamma_0}{\nu}$, and the surface roughness, ϵ . Since, for aircraft applications, Re_Γ is very high, the ground generated boundary layer is “hydraulically” rough (i.e., $\epsilon^+ = \frac{\epsilon u_\tau}{\nu}$, with u_τ the friction velocity, is much greater than 70 even though $\frac{\epsilon}{b_0} \ll 1$) and thus the molecular viscosity no longer plays a role. The boundary layer thickness δ thus scales with the turbulent effective viscosity ν_t (which itself scales with Γ_0) and with t_0 . This finally leads to taking δ scaling with b_0 ($\delta = \alpha_\delta b_0$). The circulation Γ_s is then taken proportional to the vorticity flux of the separating boundary layer:

$$\Gamma_s = \pm \alpha_\Gamma \frac{v^2(y_s, \delta)}{2} \Delta t, \quad (3.32)$$

where $v(y_s, \delta)$ is the lateral velocity induced by the wind and by the vortex particles (primary and secondary, also their images) at the altitude δ . As we only use one vortex particle to model the separating boundary layer, we add a proportionality and calibrated coefficient α_Γ . The sign of Γ_s depends on the sign of $v(y_s, \delta)$. The secondary particle initial altitude and size are then also

scaled using δ :

$$z_s = \frac{\delta}{2}, \sigma_s = \frac{\delta}{2}. \quad (3.33)$$

A sketch of the secondary vortex particle creation is provided in Fig. 3.10.

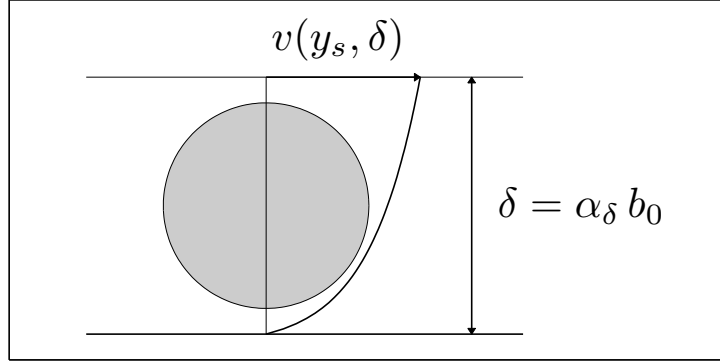


Figure 3.10: Sketch of the secondary vortex particle creation.

Note that, the thin boundary layers still attached to the ground, and due to the presence of both primary and secondary vortex particles, are still taken into account using image particles for both primary and secondary vortex particles, see Fig. 3.11. This again guarantees a correct evaluation of the velocity for every point above the thin attached vortex sheet (i.e., for every $(y, z \geq 0^+)$).

The secondary vortex particles are transported by the velocity induced by the primary and secondary particles (and their images) and by the wind, evaluated at their evolution altitude. The model enables to dynamically mimic the separation of the boundary layer and the roll-up of the secondary vortices around the primary ones. The secondary vortices, with an opposite sign circulation to that of the primary vortices, induce the rebound of the primary vortices. In case of crosswind, the secondary vortex strengths are different for the upwind and downwind vortices, inducing, as it should, an asymmetric rebound. A modification of the wind evaluation is required for the model to be also valid for strong crosswind situations. Due to the wind profile, the wind speed very close to the ground can be much smaller than aloft. In case of strong crosswind, a secondary vortex particle, created at a given time step on the outboard side, would then not “follow” the primary vortex and could be located, at the next time step, on the inboard side. This would lead to the formation of a line of secondary vortex particles generated at the ground and that would never separate because they would lay behind. In order to “give

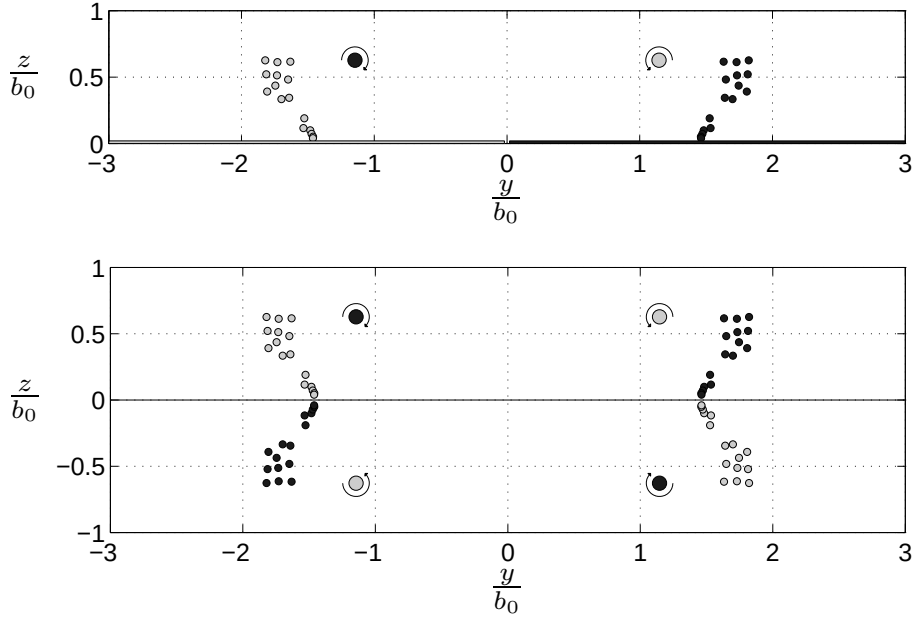


Figure 3.11: General view of the ground effect modeling using vortex particles to model the primary vortices and the detached secondary vorticity generated at the ground. The presence of the thin boundary layers generated at the ground and not separated is still properly captured using image mirror particles.

the time” to the secondary vortex particles to follow the primary vortex and then be able to detach from the ground, the wind speed, used for the secondary particles located below 10 m, is that evaluated at the primary vortex altitude.

Enhanced In Ground Effect decay model

Vortices coming IGE rebound but they also decay faster. An enhanced IGE decay model was therefore established to predict this effect. The model is based on a simplified Particle Strength Exchange (PSE) [119, 122] approach. The primary and the secondary vortices, of opposite sign, are both decaying in such a way that the sum of their circulation is constant. This additional decay model is activated a short time delay Δt_{IGE} after the primary vortex has reached its minimum altitude z_{min} (and thus has started to rebound). Proctor *et al.* [85] estimated this delay to $0.25 t_0$. In the Far-Wake project [127], based on measurements, this delay was seen to range between 0 and $0.2 t_0$. The enhanced decay of the vortex particle (primary or secondary) associated with

each side (port or starboard) is taken as:

$$\frac{d\Gamma_p}{dt} = -C_{\text{IGE}} \frac{\Gamma_p}{\tau}, \quad (3.34)$$

$$\frac{d\Gamma_s}{dt} = C_{\text{IGE}} \frac{\sum_p \Gamma_p}{\sum_s \Gamma_s} \frac{\Gamma_s}{\tau}, \quad (3.35)$$

where C_{IGE} is a coefficient. We stress that this system is applied separately to the port and starboard vortices, primary and secondary vortex particles. Furthermore, the time constant τ itself is taken to depend on the generated secondary vorticity and on the minimum altitude $z_{\min}$:

$$\tau = \frac{(z_{\min})^2}{|\sum_s \Gamma_s|}. \quad (3.36)$$

Recalling that all the secondary particles on one side (port or starboard) have the same sign (opposite to the corresponding primary vortex), one obtains:

$$\frac{|\sum_s \Gamma_s|}{\sum_s \Gamma_s} \sum_p \Gamma_p = - \left| \sum_p \Gamma_p \right|. \quad (3.37)$$

The model then eventually reads:

$$\frac{d\Gamma_p}{dt} = -C_{\text{IGE}} \frac{|\sum_s \Gamma_s|}{(z_{\min})^2} \Gamma_p, \quad (3.38)$$

$$\frac{d\Gamma_s}{dt} = -C_{\text{IGE}} \frac{|\sum_p \Gamma_p|}{(z_{\min})^2} \Gamma_s. \quad (3.39)$$

This model establishes a relative decay of the primary (resp. secondary) vortices proportional to the total secondary (resp. primary) circulation.

In order to ignore secondary particles that are too far from the primary vortex to interact with it, an influence zone is defined around each primary vortex. Outside of this zone, the influence, on the decay, of the secondary vortex particles is neglected.

Agglomeration and reorganization of the secondary vortex particles

This IGE model has the advantage to correctly mimic the physics of the separation of the boundary layer generated at the ground due to the presence of the wake vortices and to the crosswind. However, it entails the use of several secondary vortex particles. Once those secondary vortex particles detach from

the ground, they tend to agglomerate into a larger structure of secondary vorticity. In order to reduce the computational time, once this structure, made of N_s particles, has reached a sufficient size or strength, the N_s secondary vortex particles with positions $(y_{s,i}, z_{s,i})$, circulation $\Gamma_{s,i}$ and regularization size $\sigma_{s,i}$ are replaced by a single vortex particle positioned in (y_s, z_s) with a circulation Γ_s and a regularization size σ_s computed according to:

$$\Gamma_s = \sum_i \Gamma_{s,i}, \quad (3.40)$$

$$y_s = \frac{1}{\Gamma_s} \sum_i y_{s,i} \Gamma_{s,i}, \quad (3.41)$$

$$z_s = \frac{1}{\Gamma_s} \sum_i z_{s,i} \Gamma_{s,i}, \quad (3.42)$$

$$\sigma_s = \sqrt{\sigma_y \sigma_z}, \quad (3.43)$$

where σ_y and σ_z are the sizes of the structure in the lateral and vertical directions, respectively, computed to conserve the second-order moment. It reads:

$$\sigma_y^2 = \frac{1}{\Gamma_s} \sum_i (\sigma_{s,i}^2 + y_{s,i}^2) \Gamma_{s,i} - y_s^2, \quad (3.44)$$

$$\sigma_z^2 = \frac{1}{\Gamma_s} \sum_i (\sigma_{s,i}^2 + z_{s,i}^2) \Gamma_{s,i} - z_s^2. \quad (3.45)$$

The secondary vortex particles are agglomerated when $\sigma_s \geq \sigma_{th}$ (with $\sigma_{th} = \beta_\sigma \sigma_p$) or when $\Gamma_s \geq \Gamma_{th}$ (with $\Gamma_{th} = \beta_\Gamma \Gamma_0$). This technique enables to reduce the number of used secondary particles by a factor 6 to 9, depending on the considered case. This, of course, much increases the model computational efficiency (recall that the cost of the Biot-Savart velocity evaluation scales as the square of the total number of vortex particles).

Considering the results of Large Eddy Simulations (LES) (see [44] and Fig. 3.12) and of experiments in towing tank (see [14]) and also smoke visualization of aircraft wake vortices IGE (e.g., the C-5A flight test by NASA), it is observed that, once the secondary vorticity has rolled-up around the primary ones, after the first rebound, the secondary vorticity is distributed in a “turbulent halo” around the primary vortex due to fast growing 3-D instabilities. At that time, coherent structures of secondary vorticity can no longer be observed around the primary vortex. Moreover, tertiary vorticity becomes important close to the ground. As the secondary vorticity reorganization is a 3-D

effect, it cannot be dynamically captured by the 2-D vortex particle method. On the contrary, in the model, the secondary vortex particles tend to form a large structure that orbits around the primary vortex. A further reorganization of the secondary vortex particles around each primary vortex is hence used in the model to properly mimic the flow physics.

When a primary vortex has rebound, i.e., when the primary vortex would sink again after the first rebound using the model, the N_s secondary particles around the primary vortex are replaced by N_s^r particles distributed uniformly on a circle around the primary vortex. The circulation of the particles is also redistributed: a fraction α^r of the circulation is uniformly distributed to the N_s^r particles so as to have a cloud of uniform secondary vorticity around the primary vortex. The remaining fraction of circulation is given to the particles that are still close to the ground (modeling the tertiary vorticity). This redistribution of the circulation changes the primary vortex trajectory without affecting the calibrated IGE decay model. The circle radius R_s is calculated so as to conserve the relative angular momentum. The model reads:

$$\Gamma_s^r = \alpha^r \frac{1}{N_s^r} \sum_s \Gamma_s, \quad (3.46)$$

$$N_s^r \Gamma_s^r (R_s^2 + 2(\sigma_s^r)^2) = \sum_s (r_{sp}^2 + 2(\sigma_s)^2) \Gamma_s, \quad (3.47)$$

where $r_{sp} = \sqrt{(y_s - y_p)^2 + (z_s - z_p)^2}$ is the distance between a secondary vortex particle and the primary vortex particle. Moreover, in order to obtain a good discretization of the turbulent halo, the regularization size of the new secondary particles is taken proportional to the halo perimeter following:

$$\sigma_s^r = \frac{2\pi R_s}{N_s^r}. \quad (3.48)$$

Combining Eq. (3.47) and Eq. (3.48), R_s is hence computed according to:

$$R_s^2 = \frac{\sum_s \Gamma_s (r_{sp}^2 + 2\sigma_s^2)}{N_s^r \Gamma_s^r \left(1 + 2 \left(\frac{2\pi}{N_s^r} \right)^2 \right)}. \quad (3.49)$$

Figure 3.12 shows the comparison between LES and DVM results, for a case of vortices generating IGE and with a turbulent headwind. The LES is further described and used in Section 3.2.5. One can successively observe the boundary layer separation, the roll-up of the secondary vorticity into a coherent

structure and, then, the almost uniformly distributed secondary turbulent vorticity around the primary vortices at later times, as well as the whole process modeling using vortex particles in the DVM.

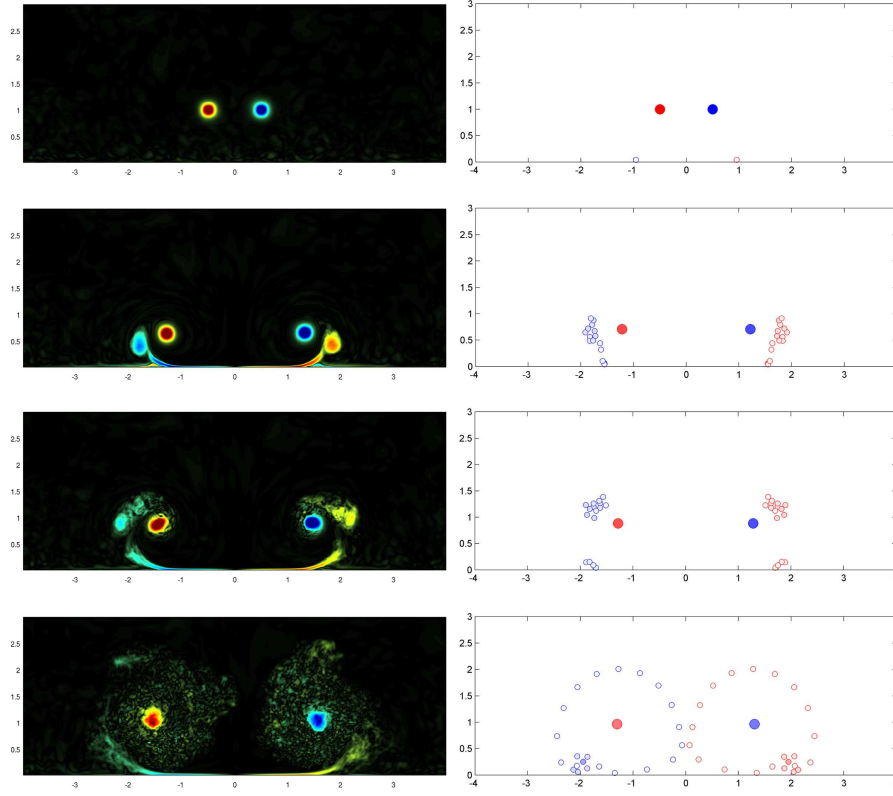


Figure 3.12: Comparison between the longitudinally averaged vorticity field computed by LES [44] (left) and the vortex particles (both primary and secondary) used in the DVM simulation (right) at $t^* = 0.0, 1.6, 2.0$, and 3.0 for a case of a wake generated at $h_0 = b_0$ and with a turbulent headwind. The color intensity is proportional to the vortex particle circulation.

Wind and turbulence profile modeling in ground proximity

The wind and the associated turbulence close to the ground is operationally important since it plays a major role in the transport and decay of the wake vortices IGE. However, in that region, measurements of the profiles of wind and associated turbulence are often not available. A model is therefore added. This model is used to “fill the gap” in the provided meteorological data, when

required.

The velocity profiles (the two horizontal components) and the Eddy Dissipation Rate (EDR = $\bar{\epsilon}$) are represented according to the turbulent boundary layer theory, below the last measurement altitude. It reads:

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \log \left(\frac{z + z_0}{z_0} \right) \quad (3.50)$$

$$\bar{\epsilon} = \frac{u_\tau^3}{\kappa z} \quad (3.51)$$

where $\kappa = 0.4$ is the von Karman constant, z_0 is the roughness length ($z_0 \approx 0.01$ m for grass) and u_τ is the friction velocity, fitted on the available velocity data. In order to be conservative but realistic, the EDR value, used by the DVM in ground proximity (below the lowest measurement altitude), is the maximum between the last measurement point and the value given by Eq. (3.51) at the considered altitude. This EDR model is used down to the altitude δ ; below this altitude, the EDR is damped to reach zero at the ground. Figure 3.13 shows an example of measured profiles of crosswind and EDR and the modeled profiles extrapolated below the last measurement point.

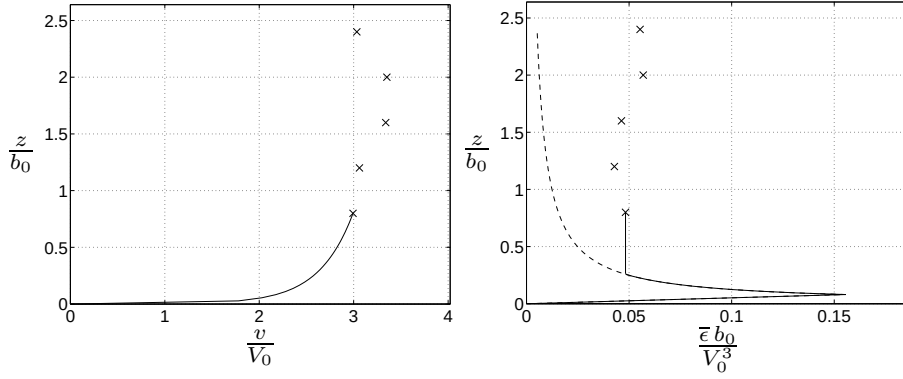


Figure 3.13: Example of profiles of crosswind v (left) and Eddy Dissipation Rate $\bar{\epsilon}$ (right) used in the DVM. The figure shows the provided measurement points ($\times$) and the profiles extrapolated below the last measurement point (solid) using the model. The dash curve in the right figure corresponds to Eq. (3.51). The data are here taken from a measurement case of the WakeFRA 2004 database.

This additional model further increases the turbulent decay close to the ground, also NGE, according to the DVM turbulent two-phase decay model

based on the EDR, see Section 3.2.2. This model is thus in addition to the further enhanced turbulent decay IGE due to the strong interaction between the primary vortices and the ground-generated secondary vorticity orbiting around them.

Model coefficients

The IGE model, used in the DVM, contains some dimensionless coefficients. Those coefficients were determined using the results of LES and of measurement campaigns (see Section 3.2.5). They were also adjusted so that the obtained transport of both primary and secondary vorticity is physically in accordance with the results of the LES and of the measurements.

Since the model is dynamic, the determination of the coefficient values is a sequential process, comparing the model results to those of LES or measurements. The coefficient α_s of Eq. (3.30) is determined so that the secondary vorticity becomes significant at the same time as that observed in the LES results. The initial lateral position of the secondary particles is then adjusted (using the coefficient α_v) to properly reproduce what is observed in the LES for various wind conditions. The coefficients α_δ and α_r that controls the secondary particle intensity and initial altitude are then determined so as to accurately reproduce the vortex rebound. The circulation enhanced decay is then fitted using the coefficients, C_{IGE} and Δt_{IGE}^* . Finally, the adjustment of α^r , the fraction of redistributed secondary particle circulation, enables to properly reproduce the wake vortex rebound at late stages. The coefficients, β_σ and β_Γ , controlling the agglomeration of the secondary vortex particles are determined separately, based on some typical test cases. They are adjusted so as to minimize the number of used secondary particles while maintaining accurate predictions. The obtained coefficient values are provided in Table 3.1.

Description	Symbol	Value
Vortex altitude threshold for secondary particle creation	α_s	1.2
Initial lateral position of secondary particle	α_v	0.82
Boundary layer thickness	α_δ	0.08
Initial circulation of secondary particle	α_Γ	1.5
Time delay before enhanced decay	Δt_{IGE}^*	0.0 – 0.1
Enhanced decay rate	C_{IGE}	0.017
Size threshold for secondary particle agglomeration	β_σ	0.8
Circulation threshold for secondary particle agglomeration	β_Γ	0.1
Fraction of redistributed circulation	α^r	0.2

Table 3.1: Description, symbols and values of the coefficients of the IGE model. The suggested values are based on the comparison of the model results with those of measurement campaigns and LES.

3.2.5 Assessment of the DVM

The DVM is assessed by comparing the DVM predictions to the results of three measurement campaigns and of five LES. Some of those experimental or numerical data were used to improve and calibrate the different versions of the DVM, in the framework of the European projects FAR-Wake and CREDOS. The idea of the present exercise is to compare the performances of the DVM, in its improved version, against all available numerical or experimental data.

For each case used as reference (i.e., one case out of a measurement campaign or the results of a LES), a corresponding DVM simulation is performed. A comparison is then performed using different metrics:

1. For each case, the root mean square (rms) deviation between the prediction and the reference (i.e., measurements or LES results) is computed following:

$$\begin{aligned}
\Delta z_{\text{rms}} &= \sqrt{\frac{1}{N} \sum (z_{\text{DVM}} - z_{\text{ref}})^2}, \\
\Delta y_{\text{rms}} &= \sqrt{\frac{1}{N} \sum (y_{\text{DVM}} - y_{\text{ref}})^2}, \\
\Delta \Gamma_{\text{rms}} &= \sqrt{\frac{1}{N} \sum (\Gamma_{\text{DVM}} - \Gamma_{\text{ref}})^2}.
\end{aligned} \tag{3.52}$$

2. For each case, the mean deviation between the prediction and the reference is also calculated following:

$$\begin{aligned}\overline{\Delta z} &= \frac{1}{N} \sum (z_{\text{DVM}} - z_{\text{ref}}), \\ \overline{\Delta y} &= \frac{1}{N} \sum (y_{\text{DVM}} - y_{\text{ref}}), \\ \overline{\Delta \Gamma} &= \frac{1}{N} \sum (\Gamma_{\text{DVM}} - \Gamma_{\text{ref}}).\end{aligned}\tag{3.53}$$

3. Using a complete database, the statistics (i.e., mean, median, and standard deviation) of the distribution of the rms and mean deviations of all the cases are computed.
4. Using a complete database, the temporal evolution of the rms and mean deviation is also computed. In order to have results interesting from a statistical point of view, the time domain is divided in time bins each of 20 s width. In each time bin, the rms and mean deviation between all predictions and all measurement points are computed.
5. For the vortex position and circulation, the evolution of the mean and the 95% (or 90%)-envelope of the complete database is compared to that of the mean and 95% (or 90%)-envelope of the complete DVM-predicted database. It enables to compare the global database properties to those predicted by the DVM.

For all DVM simulations, the inputs of the model are chosen to be as close as possible to the reference conditions. The model coefficients used in the DVM runs are those provided in this paper, except for Δt_{IGE} (whose value is higher in the LES than in the measurements). It is worth to stress that the present exercise is very sensitive to the first measurement point on the one hand (since it determines the initial mean vortex characteristics), and to the meteorological measurements on the other hand (since they play a major role in the transport and the decay of the vortices). An incorrect initial circulation or altitude, for instance, can induce major differences in the prediction of the wake vortex behavior, even with a very good model. Recall also that, for the present exercise, the measurements are assumed perfect: no uncertainties on the vortex measurements nor on the meteorological conditions are taken into account at this stage.

Use of the results of LES of a two-vortex system IGE with different wind conditions

In order to assess the model validity, the DVM results are compared to those of five Large Eddy Simulations of wake vortices IGE with turbulent head- or cross-wind, and using high quality “wall-resolved” LES. For those simulations, the realistic turbulent winds were also pre-computed using LES. A simulation of a two-vortex system evolving in each obtained wind field was then performed. For cases HW and CW0, the vortices were released IGE, at an altitude $h_0 = b_0$ and with an initial circulation such that the wind amplitude (for the head- or cross-wind, respectively) is approximatively equal to V_0 at the initial altitude ($u(h_0) \approx V_0$). In cases CW1 and CW2, the crosswind amplitude was then varied. Finally, in case CW3, the release altitude of the vortices was decreased. The main characteristics of the investigated cases are reported in Table 3.2. The simulations of cases CW0 and HW were obtained in the framework of the FAR-Wake project and are described in [44, 5]. The results of cases CW1, CW2 and CW3 were obtained by UCL in the framework of a project co-funded by Airbus and are described in [7].

Case	h_0^*	$v^*(h_0)$	$u^*(h_0)$
HW	1.0	0.0	1.14
CW0	1.0	1.14	0.0
CW1	1.0	2.27	0.0
CW2	1.0	0.57	0.0
CW3	0.7	1.08	0.0

Table 3.2: Main characteristics of the investigated LES of a two-vortex evolving IGE with wind.

Five DVM simulations were performed using, as meteorological inputs, the mean wind and EDR profiles measured in the LES. In the LES results, the measured Δt_{IGE} is $1.5 t_0$ for the headwind case, and $0.5 t_0$, for the crosswind cases. Those quite high values, used here in the DVM runs, are due to the fact that the initial two-vortex system is not turbulent in the LES. This induces some additional delay before strong turbulent interactions occur between the turbulent secondary vorticity and the primary vortices. Figures 3.14 to 3.18 compare the obtained results in term of trajectory and circulation evolution.

Moreover, Table 3.3 reports the rms and mean deviation between LES and DVM results for each simulated case and for the port vortex (which corresponds to the upwind vortex for the cases with crosswind), the starboard vortex (which corresponds to the downwind vortex for the cases with crosswind) and for both vortices.

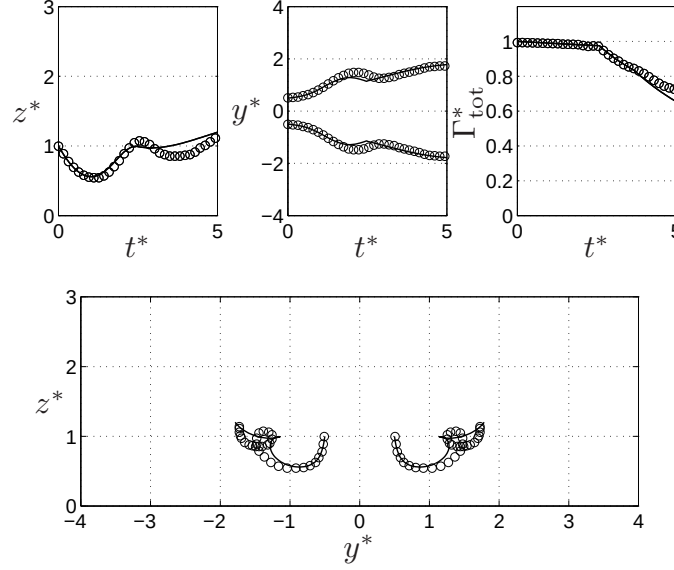


Figure 3.14: Comparison between the results of the DVM (solid) and of a LES (circles) of wake vortices IGE for case HW ($u(h_0) \approx V_0$, $h_0 = b_0$). The figure compares, for both left and right vortices, the evolution of the altitude (top left), the lateral displacement (top center) and the circulation (top right). The trajectory is also displayed (bottom).

The case with headwind (case HW) enables to assess the model capabilities in a realistic turbulent atmosphere and without crosswind. The calibrated model predicts very well the symmetrical lateral and vertical position evolutions. The vortices rebound at an altitude around $0.55 b_0$ and reach a maximum altitude after the first rebound around $1.1 b_0$. The altitude evolution is very slightly overestimated with a rms difference of about $9\% b_0$. The lateral position is predicted without bias with a $\sim 10\% b_0$ accuracy. The circulation decay is also well reproduced by the model ($2\% \Gamma_0$ precision without bias).

The comparison for the cases with crosswind (cases CW0 to CW3) shows the model ability to correctly reproduce the asymmetric rebound of the upwind and downwind vortices for different crosswind amplitudes and altitudes of release. One again observes that the minimum and maximum altitudes of

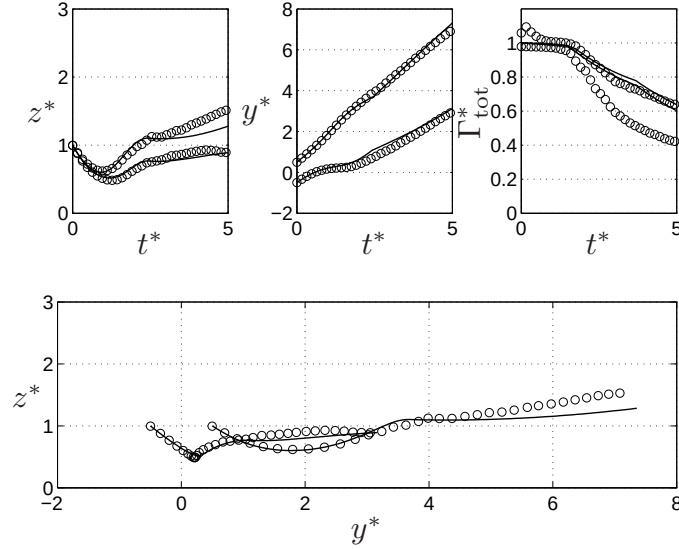


Figure 3.15: Comparison between the results of the DVM (solid) and of a LES (circles) of wake vortices IGE for case CW0 ($v(h_0) \approx V_0$, $h_0 = b_0$). The figure compares, for both left and right vortices, the evolution of the altitude (top left), the lateral displacement (top center) and the circulation (top right). The trajectory is also displayed (bottom).

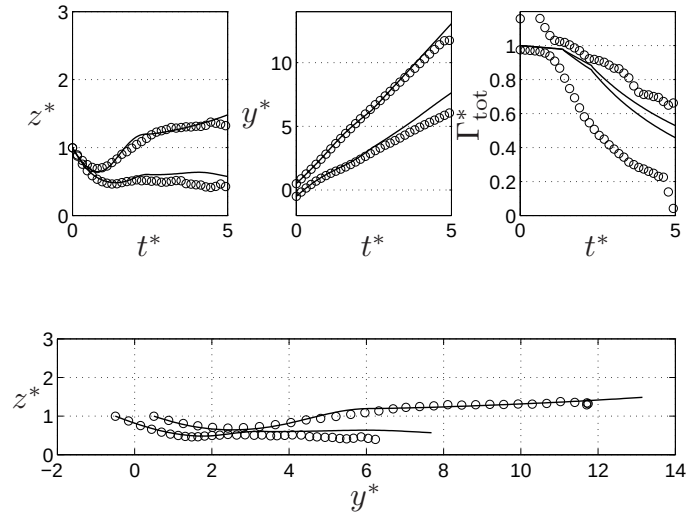


Figure 3.16: Comparison between the results of the DVM (solid) and of a LES (circles) of wake vortices IGE for case CW1 ($v(h_0) \approx 2V_0$, $h_0 = b_0$). The figure compares, for both left and right vortices, the evolution of the altitude (top left), the lateral displacement (top center) and the circulation (top right). The trajectory is also displayed (bottom).

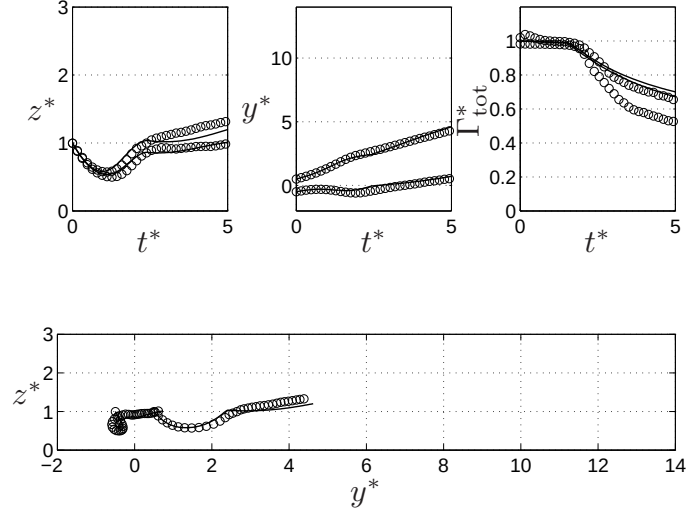


Figure 3.17: Comparison between the results of the DVM (solid) and of a LES (circles) of wake vortices IGE for case CW2 ($v(h_0) \approx 0.5 V_0$, $h_0 = b_0$). The figure compares, for both left and right vortices, the evolution of the altitude (top left), the lateral displacement (top center) and the circulation (top right). The trajectory is also displayed (bottom).

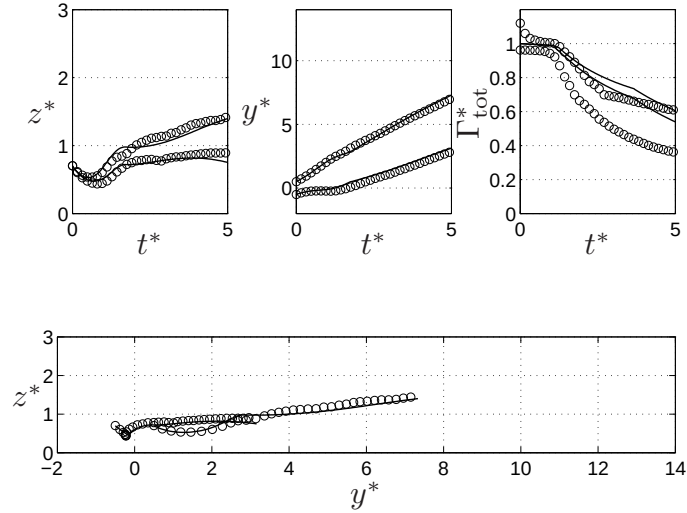


Figure 3.18: Comparison between the results of the DVM (solid) and of a LES (circles) of wake vortices IGE for case CW3 ($v(z = b_0) \approx V_0$, $h_0 = 0.7 b_0$). The figure compares, for both left and right vortices, the evolution of the altitude (top left), the lateral displacement (top center) and the circulation (top right). The trajectory is also displayed (bottom).

Case	$10^{-2} \Delta z_{\text{rms}}^*$			$10^{-2} \Delta y_{\text{rms}}^*$			$10^{-2} \Delta \Gamma_{\text{tot,rms}}^*$		
	port	starb	both	port	starb	both	port	starb	both
HW	8.8	8.8	8.8	10.6	10.6	10.6	2.2	2.2	2.2
CW0	5.2	10.8	8.5	21.0	11.7	17.0	3.8	15.3	11.2
CW1	11.1	5.8	8.9	66.3	33.9	52.6	11.4	25.9	20.0
CW2	4.5	9.3	7.3	18.5	12.8	15.9	3.2	10.0	7.4
CW3	5.5	7.6	6.6	24.6	15.8	20.7	5.7	17.8	13.2

Case	$10^{-2} \overline{\Delta z^*}$			$10^{-2} \overline{\Delta y^*}$			$10^{-2} \overline{\Delta \Gamma_{\text{tot}}^*}$		
	port	starb	both	port	starb	both	port	starb	both
HW	4.9	4.9	4.9	0.97	-0.97	0.0	-0.86	-0.86	-0.86
CW0	-1.3	-5.7	-3.5	17.2	3.3	10.2	0.40	12.5	6.5
CW1	8.7	1.9	5.3	48.9	18.2	33.6	-10.2	22.8	6.3
CW2	0.42	-4.4	-2.0	14.5	0.22	7.4	0.94	7.4	4.2
CW3	-2.1	-2.9	-2.5	21.9	2.5	12.2	2.6	16.0	9.3

Table 3.3: Rms (top) and mean (bottom) deviation between LES results and DVM predictions for the vortex altitude, lateral position and circulation. The results are provided for the port (i.e., the upwind vortex for the cases with crosswind), the starboard (i.e., the downwind vortex for the cases with crosswind) and both vortices.

rebound are well predicted (rms deviation of the altitude ranging from $\sim 5\% b_0$ to $\sim 11\% b_0$ without some significant bias). This shows that the model is in good accordance with the main physics of wake vortices interacting with the ground in a turbulent crosswind. The lateral transport is also well reproduced although the vortices appear to move a bit faster than in the LES results (with a maximum rms deviation of $\sim 0.6 b_0$). It is worth to note that the lateral transport is mainly a function of the wind experienced by the vortex at its altitude of evolution. Yet, close to the ground, the wind significantly varies with altitude (especially for case CW1). A small difference in the altitude evolution hence directly affects the lateral transport. This can thus explain the higher rms deviations (still small differences) in the lateral position prediction. For all crosswind cases, the circulation decay evolution of the upwind vortex is correctly predicted (with a rms deviation ranging from $\sim 3\%$ to $10\% \Gamma_0$) whereas the decay of the downwind vortex is underestimated. However, it was observed that, in the LES results, the difference in decay rate between the upwind and downwind vortices is much more pronounced than what is observed in experimental results (see later).

It is worth noticing that, for all investigated cases, the model reproduces the vortex behavior (transport and decay) for dimensionless times t^* up to 4-5 (recall that, for a typical heavy aircraft, $t_0 \approx 30$ s; hence $t^* = 4 - 5$ corresponds to $t \approx 120 - 150$ s), which are typical of what is required for ATM applications.

Use of the WakeFRA campaign (Frankfurt 2004)

The new DVM IGE model is also assessed using a measurement database (i.e., the WakeFRA 2004 campaign) of 110 wake vortex pairs generated by large aircraft close to the ground (below 70 m). The measurements, funded by Airbus-D, were performed by DLR (German Aerospace Center) at Frankfurt airport between August and December 2004. A $2\text{-}\mu\text{m}$ pulsed LIDAR measured the vortex position (lateral and vertical) as well as the circulation Γ_{5-15} (i.e., the circulation averaged between a radius $r = 5$ m and a radius $r = 15$ m). In addition to the wake measurements, the LIDAR measurements provided EDR profiles while a SODAR/RASS instrumentation measured the wind (three components u, v, w) and the potential temperature (Θ) profiles.

A comparison, case by case, is performed. The time delay between rebound and start of enhanced decay is here set to $\Delta t_{\text{IGE}}^* = 0$, as it yields to good agreement with the measurements. The inputs of the DVM are chosen

so as to be as close as possible to the measured conditions. In particular, the initial vortex position is taken from the first measurement point. The vortex spacing is calculated using the wingspan and assuming an elliptical loading, thus $b_0 = \frac{\pi}{4} b$. The initial averaged circulation is taken from the first measurement point $\Gamma_{5-15}(0) = \frac{1}{2} (\Gamma_{5-15,\text{port}}(0) + \Gamma_{5-15,\text{star}}(0))$. From this value, an estimate of the total initial circulation Γ_0 is computed.¹ The wind, temperature and EDR profiles are rebuilt using the measured data. It is worth noting that, as the wake vortex behavior IGE is very sensitive to the ambient conditions (and, in particular, to the crosswind), errors in the wind measurements will affect the DVM predictions in terms of position and circulation. Similarly, measurement errors on the initial vortex position or circulation, used as DVM inputs, would lead to inaccurate predictions. Figure 3.19 displays the comparison between measurements and predictions for four cases with various crosswind amplitudes. A good agreement is observed for both the position and the circulation evolution for all observation times.

Tables 3.4 and 3.5 provides the statistics of the rms and mean deviations between DVM predictions and measurements, for the 110 cases of the Wake-FRA database. The altitude and circulation evolutions are seen to be well predicted by the DVM : slight overestimation with a $\sim 15\% b_0$ ($= 7.5$ m for typical large aircraft) rms deviation for z and a $\sim 7\% \Gamma_0$ ($= 35$ m²/s for typical large aircraft) rms deviation for Γ_{5-15} . The rms deviation is a bit higher for the lateral displacement: overestimation with a $\sim 60\% b_0$ ($= 30$ m for typical large aircraft) rms deviation. Again, recall that the lateral transport is mainly driven by the crosswind that varies with altitude. Hence, an error in the altitude prediction also induces a deviation in the lateral position prediction. Moreover, the input crosswind is here taken as measured by the SODAR. An error in the crosswind measurement will also induces a deviation in the lateral position evaluation (see below).

A global comparison between the measurement database and the DVM prediction database is also performed in Fig. 3.20. The idea is to compute the temporal evolution of the statistics (i.e., mean and 90%-envelope) of the

¹The initial total circulation $\Gamma_0 = \Gamma_{\text{tot}}(0)$ is greater than the initial “LIDAR processed” circulation $\Gamma_{5-15}(0)$ provided in the database. The physics is governed by the whole vorticity field (not only the portion contained between 5 and 15 m). The DVM is working using $\Gamma_{\text{tot}}(t)$ and outputs $\Gamma_{5-15}(t)$ as post-processing. Assuming a circulation distribution model, the relation between $\Gamma_{\text{tot}}(0)$ and $\Gamma_{5-15}(0)$ is then a function of the wingspan. The distinction between total wake vortex circulation, Γ_{tot} , and “LIDAR processed” circulation, Γ_{5-15} is important for wake vortex behavior prediction. It also enables a better prediction of the vertical displacement of the vortices.

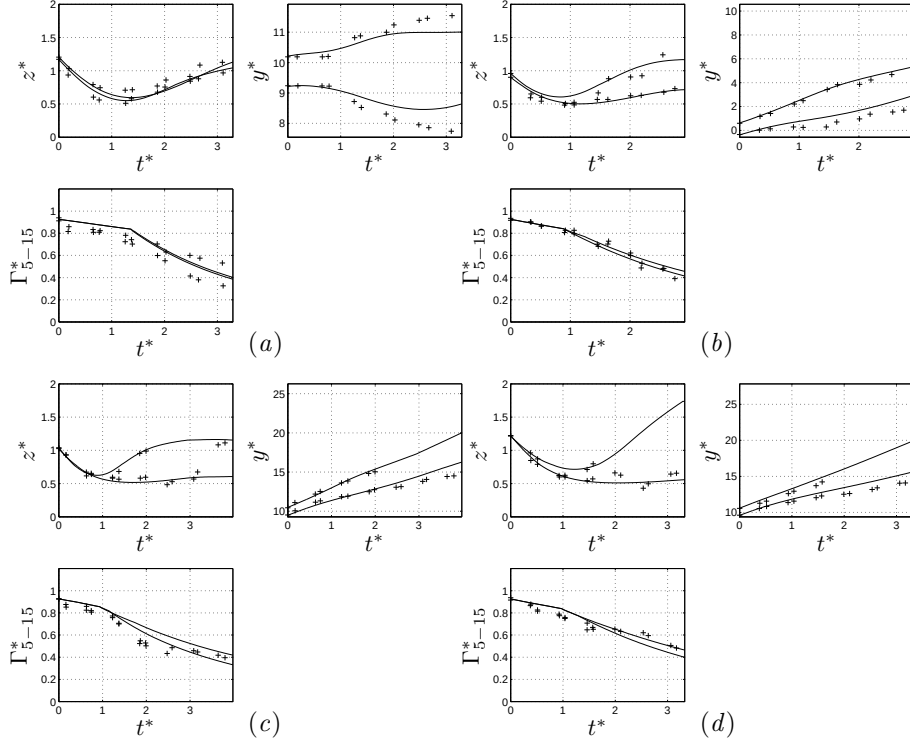


Figure 3.19: Examples of comparison between measurements (+) of the WakeFRA 2004 campaign and DVM predictions (solid) for wake vortices generated IGE with different crosswind levels: (a) $v(h_0) = 0.3 V_0$, (b) $v(h_0) = 1.5 V_0$, (c) $v(h_0) = 2.3 V_0$, (d) $v(h_0) = 2.9 V_0$.

wake vortex characteristics (position and circulation) using all measured or all predicted cases. Figure 3.20(a) shows the comparison between the results. Although the results agree very well for the altitude evolution, some discrepancy is observed for the mean and the envelope of the lateral position and of the circulation. Those differences can be explained by the LIDAR measurement limitations: it measures the vortices in a limited scanning area and too low circulation values cannot be measured. Scanning the database, it indeed appears that the maximum measured lateral position is $y_{\max} \simeq 760$ m ($\approx 15 b_0$ for the considered vortices) and the minimum measured circulation is $\Gamma_{5-15, \min} \simeq 175$ m²/s ($\approx 0.35 \Gamma_0$ for the considered vortices). We then apply an “equivalent LIDAR clipping” to the DVM predictions: every DVM prediction with a circulation $\Gamma_{5-15} < \Gamma_{5-15, \min}$ or with $y > y_{\max}$ is removed from the DVM prediction database. Using this equivalent LIDAR clipping, the agree-

ment between predictions and measurements becomes very good, as shown in Fig. 3.20(b). One also notices that the evolution of the number of “observed” vortices also better agrees when applying this filter. This confirms that some vortices are not measured because they are outside of the measurement area or their circulation is too low. The DVM thus correctly predicts the database statistics, yet it can also do it without limitations in position nor in circulation, contrary to LIDAR.

A quantitative assessment of the DVM is also performed by measuring the differences between the predictions and the measurements as a function of time. For each measurement point, the difference between the measurement and the DVM prediction (interpolated at the corresponding time) is calculated for the altitude, the lateral position and the circulation. In order to obtain statistically interesting results, the time domain is divided in “time bins” of 20 s. In each time bin, the mean and the rms of the differences are computed and the histograms of the results are provided in Fig. 3.21, for the upwind, the downwind and both vortices. The number of samples in each time bin is also displayed. Figure 3.21 shows the good agreement between the DVM simulations and the measurements, both for the upwind and downwind vortices. After a time of about 110 s, the mean differences between DVM results and measurements are $0.34 b_0$ (≈ 17 m for a B747) for the altitude, $1.45 b_0$ (≈ 73 m for B747) for the lateral displacement and $0.09 \Gamma_0$ (≈ 45 m²/s for a B747) for the circulation. A bias is only observed for the lateral position prediction ($\sim 1.15 b_0$ after 110 s). We stress that these results are comparisons between deterministic model results and measurements; no uncertainties, neither on the measurements, nor on the simulation inputs, were taken into account at this stage. In particular, the lateral displacement is mostly a function of the crosswind, here taken as measured by the SODAR. Any crosswind measurement can have an error say: $v = \bar{v} + e_v$, where $\bar{v}$ is the real value and e_v is the error. The rms difference of vortex lateral displacement between measurements and reality or predictions hence grows as $\Delta y = e_v t$. A crosswind measurement error hence induces a deviation in the lateral position prediction that grows linearly with time, as seen in Fig. 3.21. Times above 110 s should here not be considered statistically, due to the too low number of measurements.

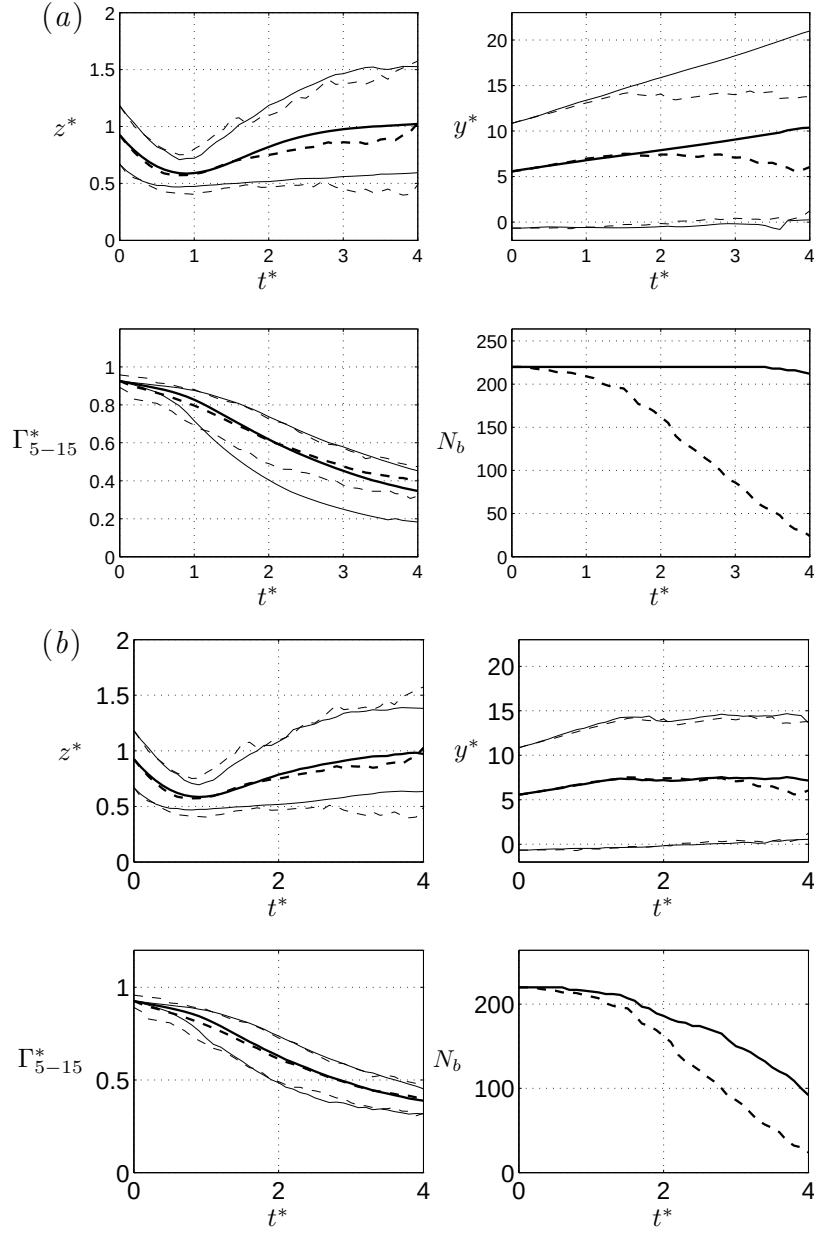


Figure 3.20: (a) Comparison of the database statistics using the WakeFRA 2004 measurements (dash) and DVM predictions (solid) for the dimensionless wake vortex altitude z^* , lateral position y^* and circulation Γ_{5-15}^* . For each quantity, the time evolution of the mean (thick) and the 90%-envelope (thin) are provided. The number N_b of individual vortex remaining in the database at each time is also displayed. (b) Same as (a) but using the “equivalent LIDAR clipping” (for the lateral position and the circulation) of the DVM predictions.

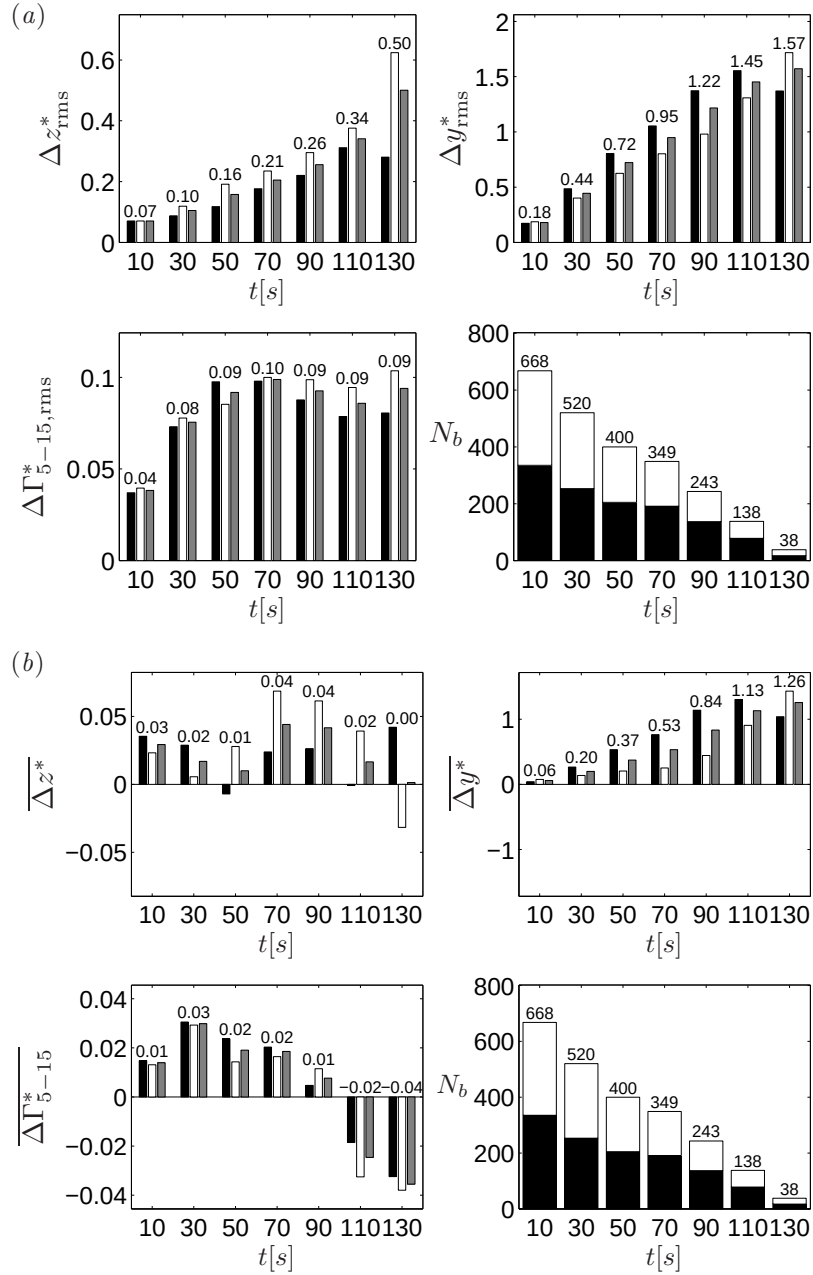


Figure 3.21: Histograms of the rms (top) and mean (bottom) deviations between WakeFRA 2004 measurements and DVM predictions, as a function of time, for the vertical and lateral positions and for the circulation. The number N_b of cases in each time frame (bin on 20 s) is also provided. The results are provided for the upwind (black), the downwind (white) and for both vortices (gray, also with value provided).

Database	Subset	$10^{-2} \Delta z_{\text{rms}}^*$			$10^{-2} \Delta y_{\text{rms}}^*$			$10^{-2} \Delta \Gamma_{\text{rms}}^*$		
		μ	med	σ	μ	med	σ	μ	med	σ
WakeFRA	all	16	14	8.5	66	63	36	7.4	6.7	3.1
	upwind	13	12	6.8	66	58	44	7.0	6.2	3.4
	downwind	16	14	11	49	39	36	6.9	6.4	3.5
EDDF-1	all	44	39	28	92	70	67	15	14	7.0
	upwind	41	34	28	90	69	68	15	14	7.0
	downwind	44	38	32	92	71	68	15	13	7.8
	with complete met	37	30	29	65	47	44	17	14	9.8
EDDF-2	all	68	52	47	85	74	49	21	19	8.7
	OGE	96	83	61	91	77	57	19	17	8.3
	IGE	57	47	36	83	73	45	22	20	8.8
	IGE upwind	52	42	38	83	69	53	22	20	9.6
	IGE downwind	56	47	37	71	58	50	21	20	9.6

Table 3.4: Statistics of the rms deviation between measurement and DVM predictions.

Database	Subset	$10^{-2} \overline{\Delta z^*}$			$10^{-2} \overline{\Delta y^*}$			$10^{-2} \overline{\Delta \Gamma^*}$		
		μ	med	σ	μ	med	σ	μ	med	σ
WakeFRA	all	2.4	2.7	9.8	34	34	46	1.6	2.1	5.1
	upwind	2.7	3.5	8.9	43	38	50	1.9	2.4	5.2
	downwind	2.4	1.7	13	19	15	46	1.9	3.0	5.5
EDDF-1	all	0.2	3.0	40	1.3	-3.2	93	5.7	5.8	10
	upwind	1.1	5.7	39	3.8	1.9	93	6.3	6.2	10
	downwind	-0.6	-0.6	43	-0.9	-6.8	95	5.0	5.0	11
	with complete met	-1.3	6.8	36	14	-3.6	60	6.6	6.9	13
EDDF-2	all	-25	-13	61	-3.1	-4.4	64	7.0	7.2	14
	OGE	-63	-61	67	-4.0	-6.7	75	4.7	5.5	14
	IGE	-9.9	4.0	52	-2.8	-3.8	59	7.8	8.0	14
	IGE upwind	-11	1.5	52	-14	-16	74	9.4	9.7	15
	IGE downwind	-8.5	4.5	54	6.6	7.4	67	7.4	7.8	15

Table 3.5: Statistics of the mean deviation between measurement and DVM predictions.

Use of EDDF-1 database (Frankfurt 2006-2007)

The EDDF-1² database has a total of 138 wake vortex pairs generated by heavy aircraft in ground vicinity ($h_0 \in [2b_0, 8b_0]$). The measurements were performed during 6 days between December 2006 and January 2007 with a pulsed Doppler LIDAR. In addition to the wake measurements, the LIDAR measurements provide EDR profiles while a SODAR/RASS instrumentation measured the wind (three components u, v, w) and the potential temperature (Θ) profiles.

A comparison between EDDF-1 measurements and DVM predictions, case by case, is also performed. The time delay between rebound and start of enhanced decay is here set to $\Delta t_{\text{IGE}}^* = 0$, as for the comparison with the WakeFRA database. The inputs are chosen to be as close as possible to the measured conditions. The procedure is identical to that used for the WakeFRA database. In particular, the initial position (y_0, h_0) is the mean of the first measured port and starboard data points, the initial vortex spacing is calculated from the wingspan and assuming an elliptical distribution $b_0 = \frac{\pi}{4}b$. Regarding the initial circulation, the initial Γ_{5-15} is taken as the average between the first measured port and starboard circulation data. The initial total circulation $\Gamma_{\text{tot}}(0) = \Gamma_0$ is then computed from that value, using the wingspan and assuming a vortex circulation distribution model. The wind, turbulence and temperature vertical profiles are rebuild from the measurements. It is important to note that, for some cases, the available meteorological data do not cover the entire wake vortex trajectory. To simulate the wake vortex behavior at these altitudes, the profiles are extrapolated using a constant value above the last data point provided. This also implies that a constant stratification level is assumed for those heights.

Figure 3.22 shows two examples of comparison between two EDDF-1 measurement cases and the corresponding DVM predictions. In these cases, there is a fairly good agreement between prediction and measurement.

The histograms of the rms and mean deviation are provided in Fig. 3.23. Times after 110 s are not shown due to the low number of data. One observes that the deviation in circulation for short times is three times larger compared to that obtained comparing DVM and WakeFRA measurements (0.04, see Fig. 3.21). This is indicative of the larger spreading of the circulation measurements in the EDDF-1 database. Since the first “measurement” point is

²EDDF is the ICAO code of the Frankfurt airport

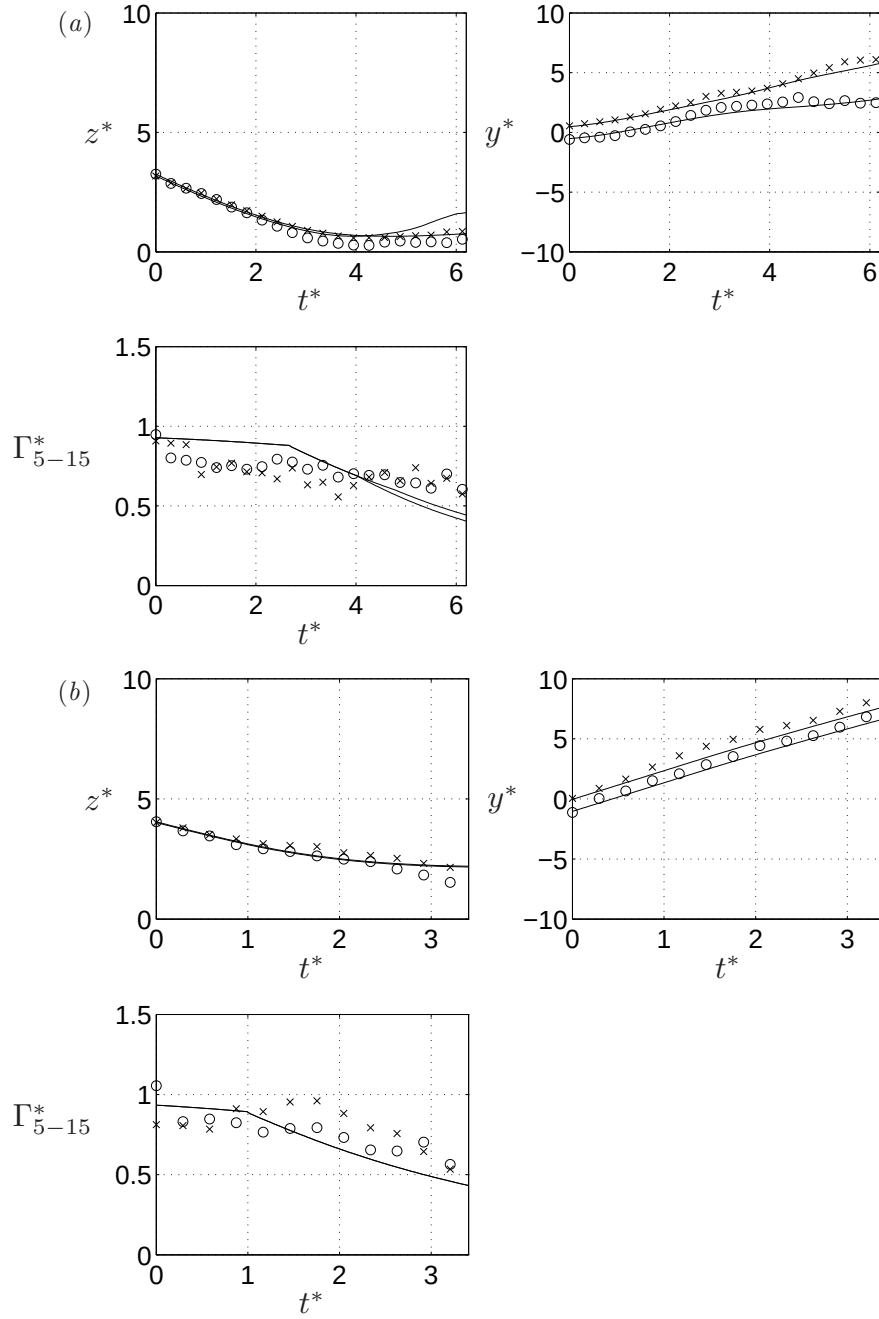


Figure 3.22: Example of comparison between EDDF-1 measurement data for port ($\circ$) and starboard vortex ($\times$) and DVM prediction (solid).

here taken as the input for the DVM, a measurement error on that point much influences the whole simulation, and particularly the circulation and altitude evolutions. One then obtains a rms deviation, after 110 s, of $0.80 b_0$ (≈ 40 m for a large aircraft) for the altitude, $0.14 \Gamma_0$ ($\approx 70 \text{ m}^2/\text{s}$) for the circulation and $1.57 b_0$ for the lateral transport. Considering the histograms of the mean difference in lateral and vertical positions between predictions and measurements, no significant bias is observed. The circulation is seen to be slightly overestimated for short times and slightly underestimated for longer period (with quite small values).

Tables 3.4 and 3.5 report the statistics of the rms and mean deviations between EDDF-1 measurement cases and corresponding DVM predictions. The tables also provide the results separately for the upwind and the downwind vortex. The rms altitude deviations are about twice higher than those of the WakeFRA database, without significant bias. The lateral transport, also unbiased, is also a bit higher than that of the WakeFRA database. The circulation is seen to be slightly overestimated with rms deviations of about $15\% \Gamma_0$. The same analysis is also performed considering only measurement cases for which the meteorological data are available up to the initial altitude of the vortices (41 cases referred to as “with complete met” in the tables). The statistics of the rms and mean deviation distributions are reported in Tables 3.4 and 3.5, respectively. One observes that the rms deviation on the lateral position is decreased by a factor ~ 1.5 . This is as expected since the predicted lateral transport is an image of the input measured crosswind. An extrapolation of the measured crosswind, for altitudes where no data are available, indeed introduces an error. The altitude evolution is also better predicted when the complete meteorological data are available.

Finally, the evolution of the mean and the 90%-envelope of the measured wake vortex characteristics are compared to that of the DVM-predicted wake vortex characteristics. The results are shown in Fig. 3.24 (a). A good agreement between measurements and DVM predictions is observed for the altitude evolution, whilst it is not the case at late times ($t^* > 2$), for the lateral position and the circulation. One also notices the larger size of the measured circulation envelope, at early times ($t^* \leq 1$), compared to that predicted by the DVM. Since each case is normalized using its own initial circulation Γ_0 , the spreading around the mean is indicative of the significant scattering of the measured circulation data at those times. As done when using the WakeFRA

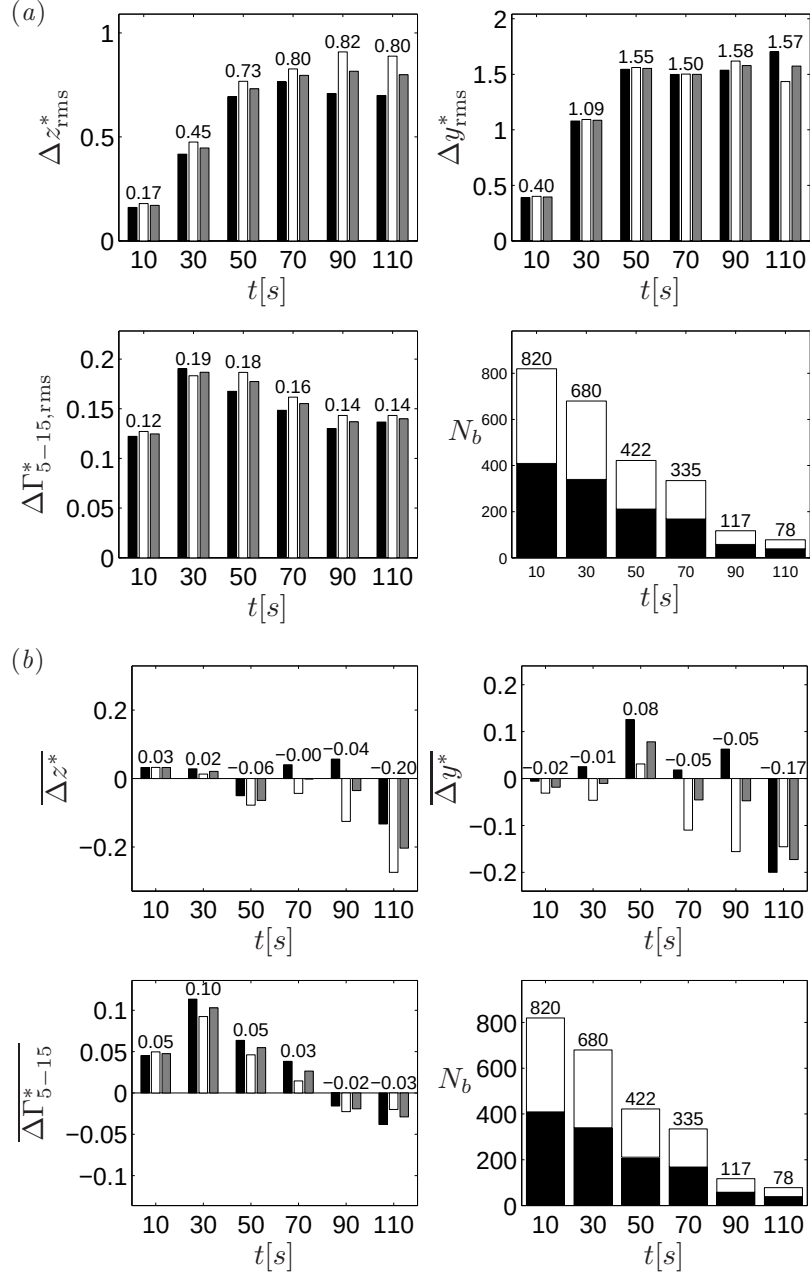


Figure 3.23: Histograms of the rms (top) and mean deviations (bottom) between EDDF-1 measurements and predictions, as a function of time, for the vertical and lateral positions and for the circulation. The number N_b of cases in each time window is also provided. The results are provided for the upwind (black), the downwind (white) and for both vortices (gray also with value provided).

database, an equivalent LIDAR clipping, on the lateral position and on the circulation measurement, is also applied to the DVM results. Indeed, scanning the EDDF-1 database, it appears that the minimum measured circulation is $\Gamma_{5-15,\min} \simeq 180 \text{ m}^2/\text{s}$ and that the lateral positions are measured between $y_{\min} = -450 \text{ m}$ and $y_{\max} = 450 \text{ m}$. Applying the equivalent LIDAR filter to the DVM predictions (i.e., removing from the database all DVM results with $y > y_{\max}$ or $y < y_{\min}$ or $\Gamma_{5-15} < \Gamma_{5-15,\min}$), the agreement between measurements and DVM predictions is much improved, see Fig. 3.24 (b).

Use of EDDF-2 database (Frankfurt 2007)

The EDDF-2 database, that is analyzed in Chapter 4, is the result of a 6-month measurement campaign held in Frankfurt by the Deutsche Flugsicherung GmbH (DFS). The database contains a total of 10442 wake vortex cases generated by heavy and medium aircraft in a height range from near ground up to 150 m. A wind-temperature radar (WTR) supplemented with a radio acoustic sounding system (RASS) provides wind, temperature and turbulence profiles for the height range of 60 – 1650 m. A Wind-tracer LIDAR system provides a detailed crosswind profile for heights 1.65–190 m and the wake vortex positions (lateral and vertical) and circulations. Here, the LIDAR processed circulation data provide an estimation of the total wake vortex circulation, Γ_{tot} . Since the number of cases is large (10442), the whole database was screened to retain a subset of about 6500 cases which were considered relevant for further use.

A complete comparison, case by case, between DVM predictions and measurements is performed. The time delay between rebound and start of enhanced decay is set to $\Delta t_{\text{IGE}}^* = 0$, as done with the two previous databases. The inputs of the model are chosen to be as close as possible to the measured conditions. The initial altitude h_0 and mean lateral position y_0 are set to the average of the first port and starboard measurement points. The vortex initial spacing $b_0 = s b$ is calculated using the known wingspan b and assuming an elliptical loading factor $s = \frac{\pi}{4}$. The initial circulation Γ_0 is taken as the average of the first port and starboard measured circulation. Finally, a bank angle ϕ is added if the measured vortices are not at the same height (if it does not exceed 30°).

Figure 3.25 shows two examples of comparison for a case with the vortices IGE and a case where the vortices are first OGE, then NGE. For these cases, there is a good agreement between predictions and measurements. This database has the advantage to be very large and thus enables a meaningful sta-

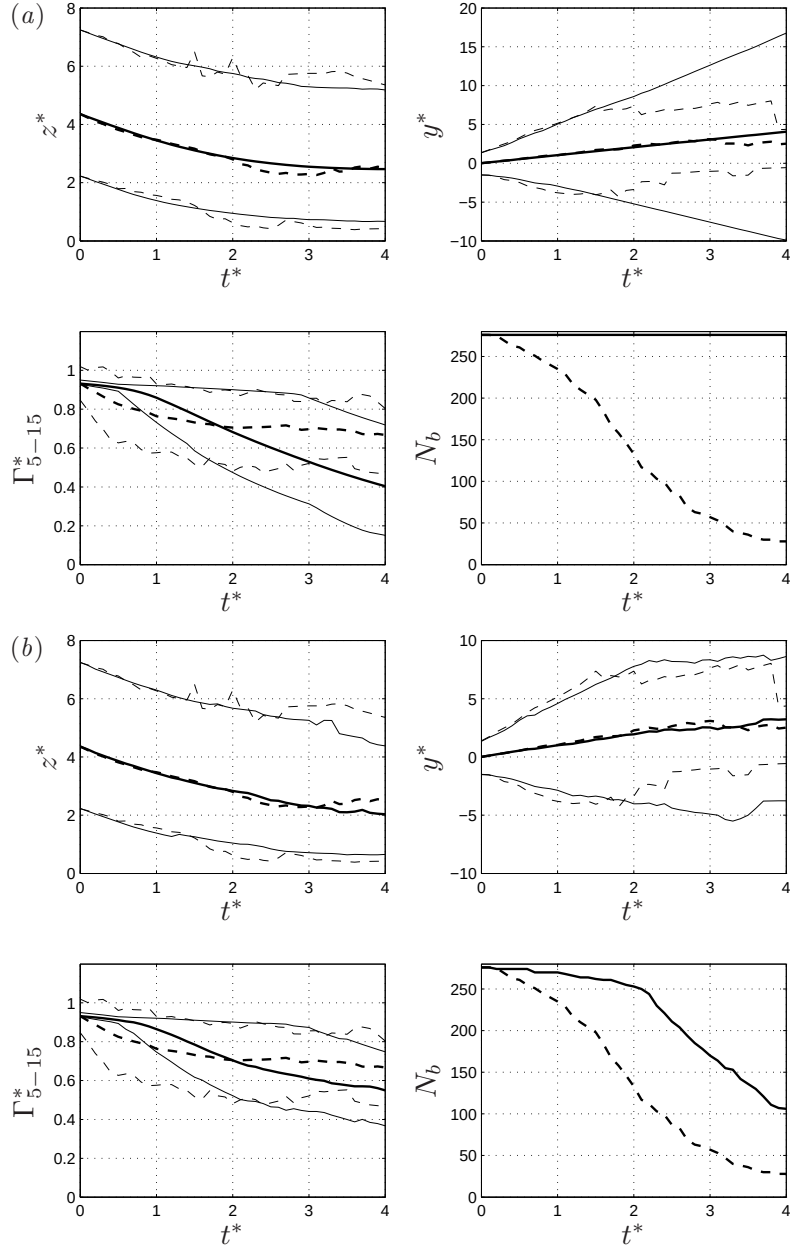


Figure 3.24: (a) Comparison of the database statistics using the EDDF-1 measurements (dash) and DVM predictions (solid) for the dimensionless wake vortex altitude z^* , lateral position y^* and circulation Γ_{5-15}^* . For each quantity, the time evolution of the mean (thick) and the 90%-envelope (thin) are provided. The number N_b of individual vortex remaining in the database at each time is also displayed. (b) Same as (a) but using a “equivalent LIDAR clipping” (for the lateral position and the circulation) of the DVM predictions.

tistical analysis. However, a significant scatter of the measured circulation data is observed in the database, even at early times. It is noted that the scattering level is significantly higher than that observed in the WakeFRA database. Moreover, in EDDF-2, the reported minimum heights reached by the wake vortices are also significantly lower than those observed in other investigations (computational, and experimental, also EDDF-1 and WakeFRA): this indicates that there likely was a problem in the “ground altitude” definition with respect to the LIDAR system.

Tables 3.4 and 3.5 provides the statistics of the distribution of the rms and mean deviations between EDDF-2 measurements and corresponding DVM predictions. The high scattering level in the measured circulations leads to high circulation deviations, higher than those obtained using the two other databases. Compared to the database, the DVM-predicted circulation is a bit overestimated with a rms deviation of about $20\% \Gamma_0$. The altitude is seen to be underestimated with a rms of about $50\% b_0$ whereas the lateral position deviation is about $75\% b_0$ without significant bias. However, it appears that, OGE, the vortex altitude evolution is not consistent with the circulation measurement. Indeed, without stratification, the vortices should initially sink approximatively at a velocity V_0 , which appears not to be the case in the EDDF-2 database. To highlight this phenomenon, a distinction is made between vortices generated above $4b_0$, that are considered OGE (as it will be the case except at late times for which they will be NGE), and those generated below $4b_0$ that are considered IGE (as, by a time of $t^* = 2 - 3$, they will certainly be IGE). Tables 3.4 and 3.5 also reports the statistics of the rms and mean deviation making the distinction between OGE and IGE cases. For the IGE cases, the results are also provided for the upwind, the downwind, and both vortices. For the altitude prediction, the OGE cases are in average underestimated (as they do not corresponds to the expected physics) whereas the rms deviation is almost twice lower for the IGE cases without significant bias. The lateral transport and circulation prediction deviations are about the same for the IGE and the OGE cases.

Figure 3.26 provides the histograms of the temporal evolution of the rms and mean deviations between DVM results and measurements, also with the distinction between IGE and OGE cases. The IGE cases are then further detailed in Fig. 3.27 making the distinction between the upwind and the downwind vortex. The rms of the altitude deviation shows a quick increase for short

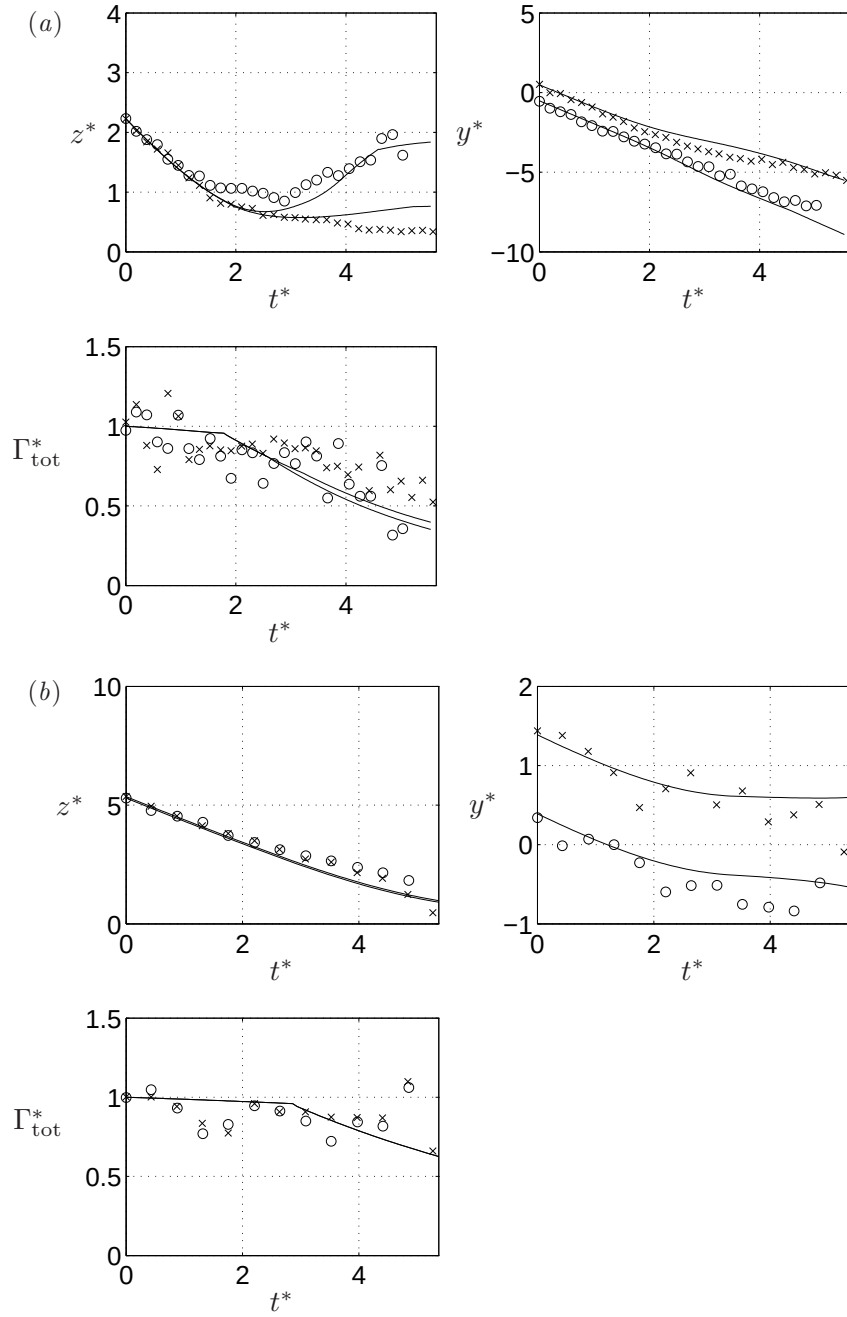


Figure 3.25: Example of comparison between EDDF-2 measurement data for port ($\circ$) and starboard vortex ($\times$) and DVM prediction (solid).

times and then a plateau is reached. It appears, in the mean deviation analysis, that the DVM first underestimates and then overestimates the altitude. However, the mean deviation values are more than three times lower than the rms values. The analysis of the OGE cases in Fig. 3.26 highlights that, for short times, the altitude evolution is much underestimated compared to the IGE cases, although the circulation deviations are of the same order of magnitude. This also affects the results for the lateral transport. The downwind vortex lateral transport is seen to be a bit better predicted than that of the upwind vortex. However, the mean deviation values are close to zero, indicating no systematic bias of the predictions. The rms of the circulation is about constant in time. For short times, the circulation is overestimated while later it is underestimated. Note also that the mean deviation values are more than twice lower than the rms values. It is also worth to stress that, for all the considered quantities, the deviations between measurements and prediction, for very short times, are already quite high. This is due to the high variability in the measurement data. Since the error can only grow in time, it is normal to observe quite large deviations. Compared to the results obtained using the WakeFRA database, we here clearly see the influence of the initial variability of the data.

Finally, Figs. 3.28 and 3.29 compare, respectively for the IGE and the OGE cases, the evolution of the mean and 95%-envelope of the database to the evolution of the mean and 95%-envelope of the DVM-predicted database. Here, the very large size of the database enables to calculate the 95%-envelope instead of the 90%-envelope, which was not the case with the two previous databases. The initial scattering of the circulation measurement is observed in the large size of the circulation envelope for short times. The initial circulation scattering is seen to be even more pronounced than that of the EDDF-1 database. This fact justifies the even higher deviations obtained between measurements and predictions.

As done when using the WakeFRA database, an equivalent LIDAR filtering of the data (i.e., with thresholds of minimum and maximum measurable lateral position and of minimum measurable circulation) is also used in order to apply the same constraints to the DVM results than those of the LIDAR measurements. Scanning the database, the lateral position is seen to be measured for y comprised between $y_{\min} = -350$ m and $y_{\max} = 350$ m, and only circulations above $\Gamma_{\text{tot},\min} = 120$ m²/s are seen to be captured. For the OGE and IGE cases,

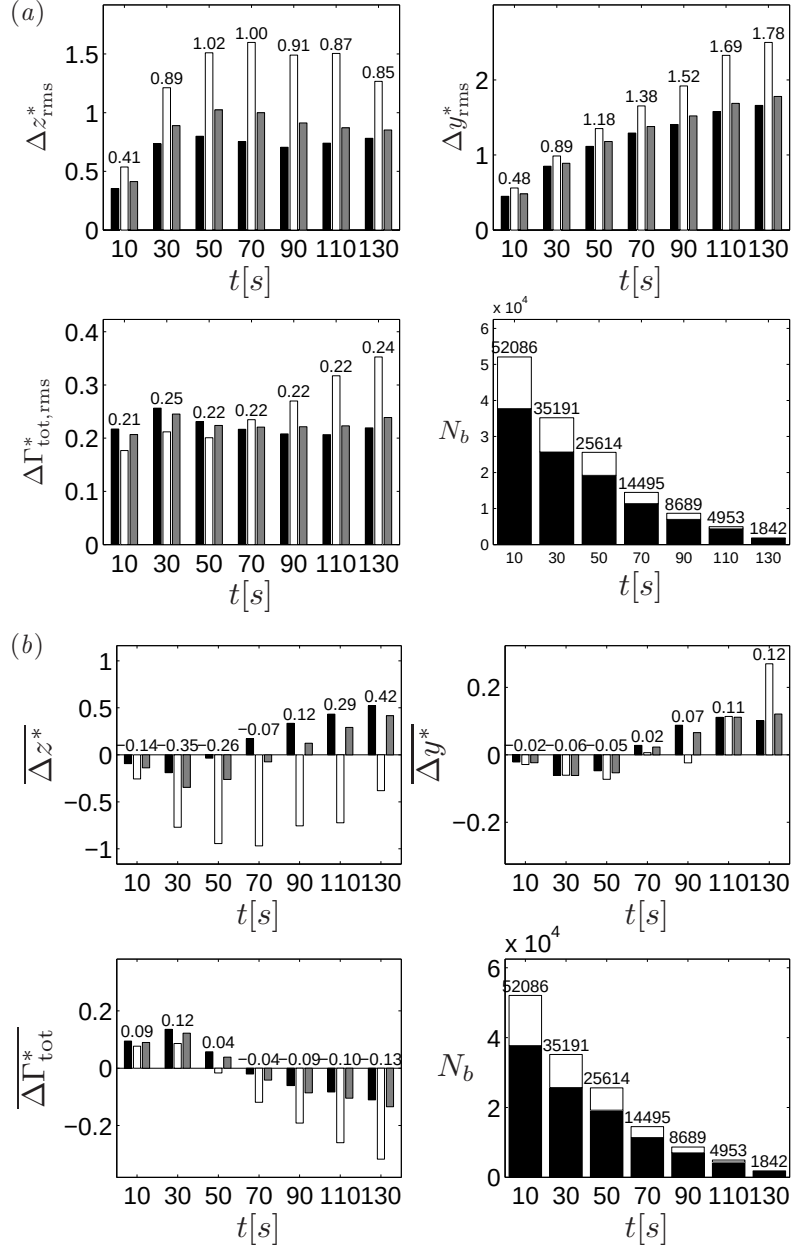


Figure 3.26: Histograms of the rms (top) and mean deviations (bottom) between EDDF-2 measurements and predictions, as a function of time, for the vertical and lateral positions and for the circulation. The number N_b of cases in each time window is also provided. The results are provided for the IGE cases (i.e., those with $h_0^* < 4.0$, black), the OGE cases (i.e., those with $h_0^* > 4.0$, white) and for all cases (gray also with value provided).

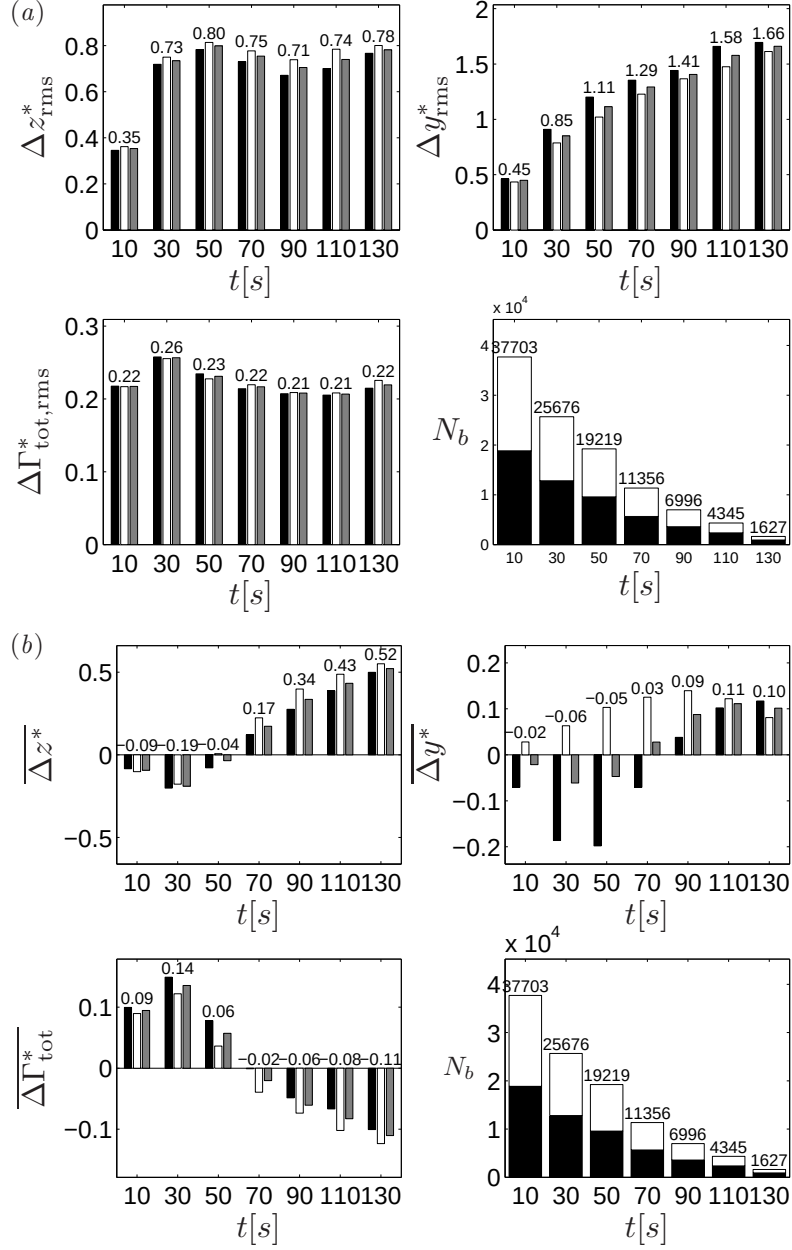


Figure 3.27: Histograms of the rms (top) and mean deviation (bottom) between EDDF-2 measurements and predictions, as a function of time, for the vertical and lateral positions and for the circulation, for the cases IGE (i.e., those with $h_0^* \leq 4$). The number N_b of cases in each time window is also provided. The results are provided for the upwind (black), the downwind (white) and for both vortices IGE (gray also with value provided).

when applying the filter, the lateral transport is very well reproduced. The mean altitude evolution is well reproduced for the IGE cases, whereas it is underpredicted for the OGE cases as explained hereabove. Note that, OGE, the dimensionless mean DVM altitude evolution has an initial slope of about -1 , which is physical, whereas, using the measurements, the slope is about twice higher, which is unphysical, indicating a problem with the initial circulation, or the measured sink rate, or both. IGE, the envelopes built using the DVM or the measurements do not correspond for two reasons. The measured minimum altitude is too low (as explained hereabove), and the vortices far from the ground (those with h_0 close to $4b_0$) sink too slowly compared to V_0 .

Considering the quite high variability of the data, the agreement between predictions and measurements is considered as quite good, for the IGE cases. The OGE cases must probably be excluded, as the circulation and altitude measurements are not consistent with each other.

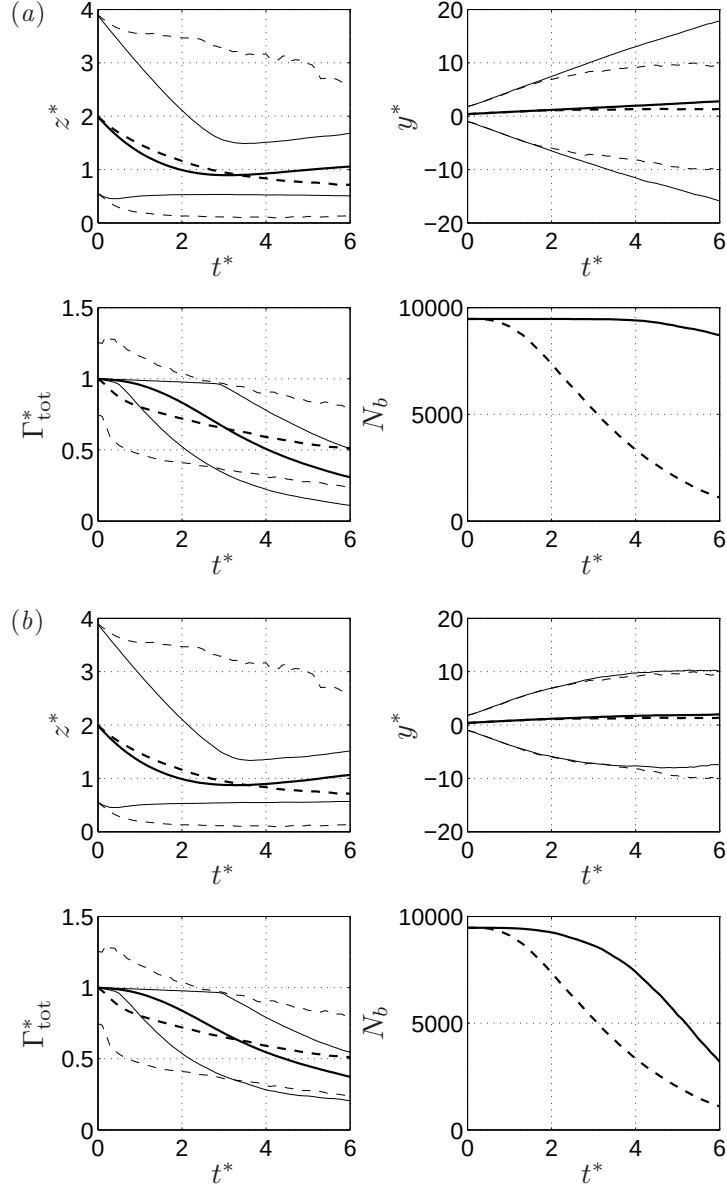


Figure 3.28: (a) Comparison of the database statistics, for the cases IGE ($h_0/b_0 \leq 4$), using the EDDF-2 measurements (dash) and DVM predictions (solid) for the dimensionless wake vortex altitude z^* , lateral position y^* and circulation Γ_{tot}^* . For each quantity, the time evolution of the mean (thick) and the 95%-envelope (thin) are provided. The number N_b of individual vortex remaining in the database at each time is also displayed. (b) Same as (a) but using the “equivalent LIDAR clipping” (for the lateral position and the circulation) of the DVM predictions.

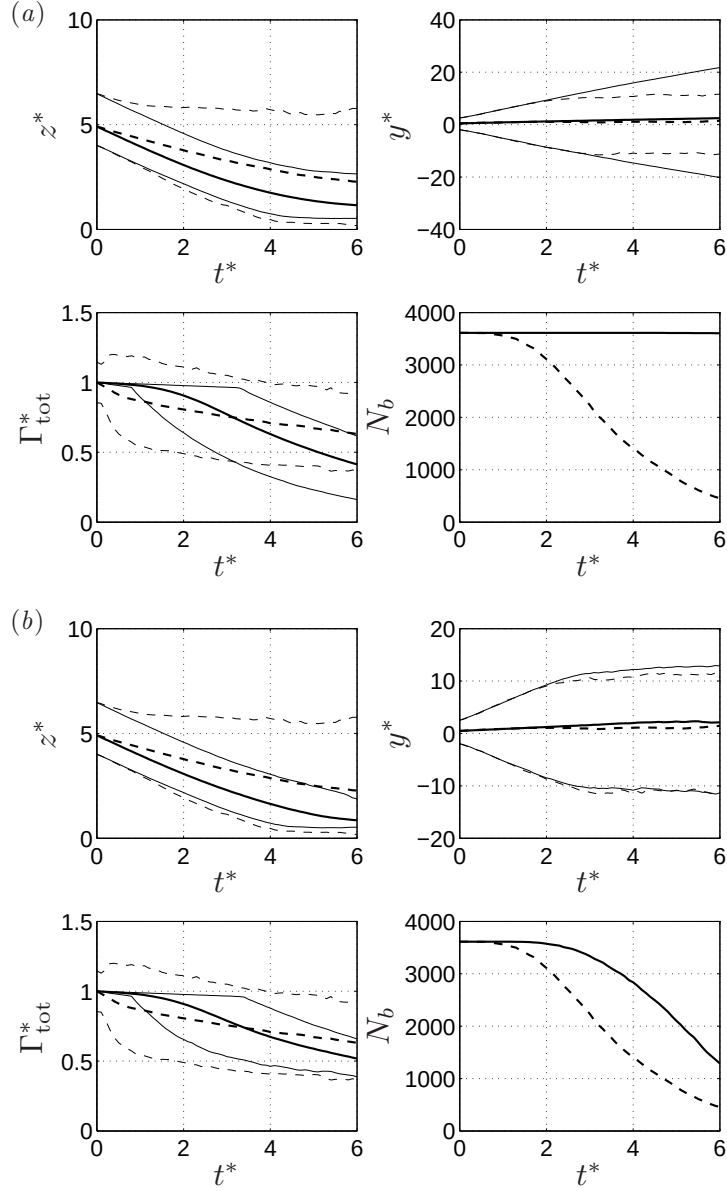


Figure 3.29: (a) Comparison of the database statistics, for the cases OGE ($h_0/b_0 > 4$), using the EDDF-2 measurements (dash) and DVM predictions (solid) for the dimensionless wake vortex altitude z^* , lateral position y^* and circulation Γ_{tot}^* . For each quantity, the time evolution of the mean (thick) and the 95%-envelope (thin) are provided. The number N_b of individual vortex remaining in the database at each time is also displayed. (b) Same as (a) but using a “equivalent LIDAR clipping” (for the lateral position and the circulation) of the DVM predictions.

3.3 The Probabilistic wake Vortex Model (PVM)

The Probabilistic wake Vortex Model (PVM) constitutes an upper software layer which is based on a Monte-Carlo approach, using the DVM as a subtool. It has been developed because probabilistic modeling and assessment of wake vortices is most often required for operational applications. The PVM has also been improved, assessed and used in the framework of several European research projects (ATC-Wake, WakeNet2-Europe, FAR-Wake, FLYSAFE, CRE-DOS, Eurocontrol Time-Based Separation project, WakeNet3-Europe).

3.3.1 Motivation and description of the Monte-Carlo approach

The PVM model is based on a Monte-Carlo approach, using the DVM model as a subtool. It takes into account the uncertainties on the impact parameters (aircraft conditions, meteorological conditions, and coefficients of the DVM physical models). For each PVM run, several deterministic runs with variations of those parameters are performed. The parameters are varied depending on their natural variations (for the meteorological data), their uncertainty (for the meteorological and aircraft data) and their calibration (for the coefficients of the physical models).

All these randomized parameters follow a certain distribution. The user can choose between a normal (Gaussian), a uniform distribution or a experimental input distribution (i.e., distribution from a list of values). A FLIP distribution (Flight Performance using the Frankfurt ILS, see [33]) can also be used for the lateral and vertical aircraft position distribution along the ILS on approach. Each DVM run in the Monte-Carlo approach, uses, for each input parameter, a value taken randomly within that parameter distribution. The total number of DVM runs used is large, yet is not directly related to the number of varied input parameters. One then obtains, in a computational gate at each time, a set of vortex positions and their associated circulations. Due to the non-linearity of the problem, the distribution of the DVM results is not simply an image of the input distributions. To illustrate the PVM results, we consider here two cases. The first case concerns wake vortices generated by a large aircraft far from the ground and evolving in an atmosphere with weak to moderate turbulence (EDR varying from 10^{-4} to 10^{-5} m²/s³), with weak stratification (N^* ranging from about 0.1 to 0.5) and with crosswind (with amplitude of 1.5 to 3.0 m/s

at 1000 m height). The second case considers wake vortices generated by the same aircraft in ground vicinity and evolving in an atmosphere with weak to moderate turbulence (EDR varying from 10^{-4} to 10^{-5} m^2/s^3), without stratification, and with crosswind (with amplitude of 0.5 to 1.0 m/s at 10 m height, the wind profile being the same as that used for the first case). In both cases, the vortex initial position and circulation are also varied. All parameters are assumed to be uniformly distributed. Figure 3.30 compares the evolution of the vortex positions and circulation obtained by multiple DVM runs (i.e., here only 100 DVM runs, for illustration) used for one PVM computation in a Monte-Carlo approach for both cases.

3.3.2 PVM outputs and use

Following the application, the PVM offers various output possibilities. One can choose to output all the DVM results as they are and/or statistical properties of the set of results.

The PVM can thus output the temporal evolution of the vortex circulation and positions of the multiple DVM runs. The user can then post-process the results following its needs. One can, for instance, obtain the Probability Density Function (PDF) for the positions and the circulation computed using the output of the multiple DVM runs. These histograms can be built at each time. If the number of runs is large enough, this provides a good estimate of the wake vortex PDF characteristics at that time. For instance, Fig. 3.31 displays the PDF of the vortex positions and circulation after 180 s for the two investigated cases and using 5000 DVM runs. A mathematical PDF shape can be further assumed and fitted to the obtained distributions of the wake vortex positions and/or circulation.

Using all the DVM runs, one can also calculate the probability for the vortices to be in a prescribed area after a certain time by counting the individual DVM cases. Alternatively, the Complementary Cumulative Density Function of the circulation $\text{CCDF}(\Gamma)^3$ can also be computed, at various times, using the output circulation of the multiple DVM runs. Figure 3.32 compares the obtained $\text{CCDF}(\Gamma_{\text{tot}})$ after 180 s for the two investigated cases. This curve enables to conclude that, for the considered cases, the probability to encounter a vortex with a circulation $\Gamma_{\text{tot}} \geq 250 \text{ m}^2/\text{s}$ is higher for the case OGE than

³the Complementary Cumulative Density Function $\text{CCDF}(X)$ is a function describing the probability to observe $x > X$ as a function X . For the circulation, this curve constitutes a frequency/severity diagram.

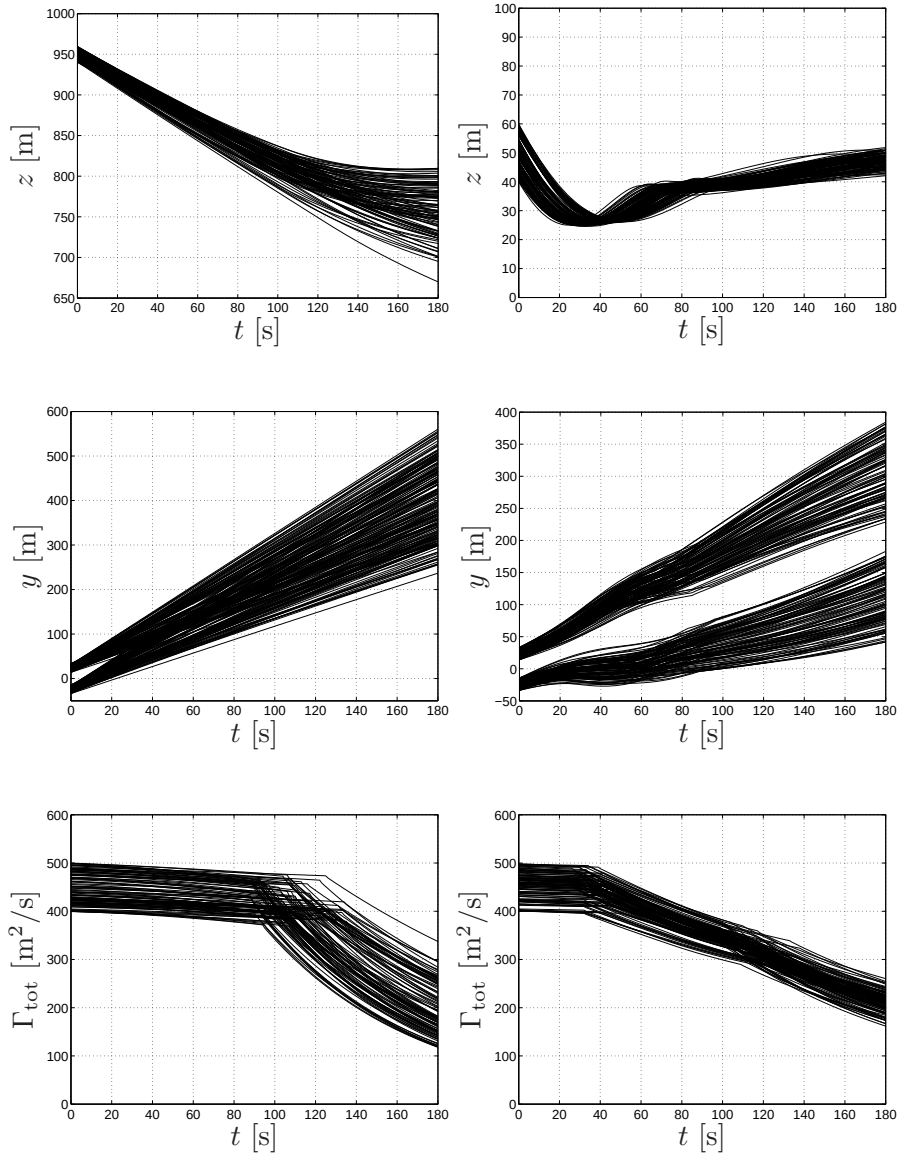


Figure 3.30: Evolution of the altitude (top), lateral position (middle) and total circulation (bottom) of 100 DVM runs used for one PVM computation in a Monte-Carlo approach. The results are provided for the case OGE (left) and the case IGE (right).

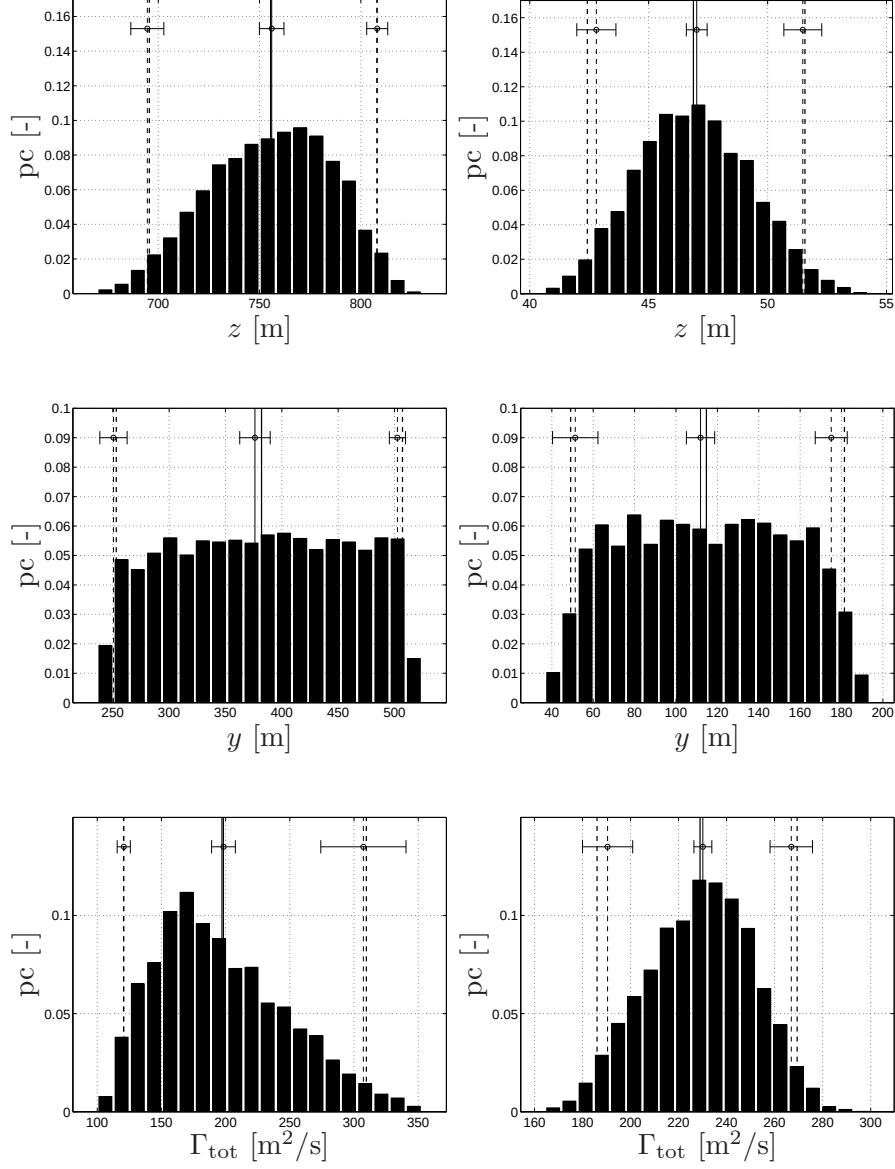


Figure 3.31: Histograms of the vortex altitude (top), lateral displacement (middle) and total circulation (bottom) at $t = 180$ s for the case OGE (left) and the case IGE (right), as predicted by the PVM using 5000 DVM runs. Also shown are the mean (solid) and the 95%-envelope (dash) obtained using 5000 DVM runs (thick) or 200 DVM runs (thin, together with the errorbar providing the bootstrap estimate of the 99%-envelope of the obtained statistics).

for the case IGE but the probability to encounter a vortex with a circulation $125 \text{ m}^2/\text{s} \leq \Gamma_{\text{tot}} \leq 250 \text{ m}^2/\text{s}$ is higher for the case IGE than for the case OGE.

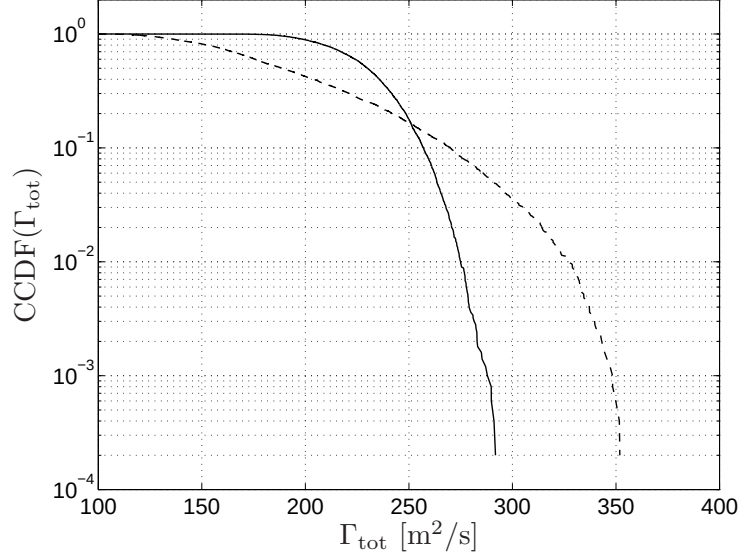


Figure 3.32: Comparison of the CCDF of the total circulation obtained at $t = 180 \text{ s}$ for the case OGE (dash) and the case IGE (solid).

For the 3-D position (x, y, z) and the circulation Γ , the PVM can also output the temporal evolution of statistics: mean μ , median, standard deviation σ as well as percentiles with user-defined probability levels. For the computation of the median and the percentiles, the PVM either assumes the results to follow a Gaussian law (described by $N(\mu, \sigma)$), or uses the experimental distribution of the DVM results as obtained. If a Normal law is assumed, the median is equal to the mean and the percentile is computed analytically. The second solution follows better the results but is computationally more expensive. Moreover, if no distribution is assumed, the number of runs must be sufficient to obtain statistically converged results. Figure 3.33 provides the evolution of the mean and the 95%-envelope of the vortex altitude, lateral position and total circulation for the two considered cases. No assumption was here made on the output distribution (the mean is thus no necessarily centered in the 95%-envelope).

Finally, using those statistics, the PVM can provide the evolution of a “danger area” in which the vortices, generated by an aircraft, have a certain probability to be found. In that danger area, the PVM also provides the circulation distribution of the “potentially encountered” vortex if one was to enter that area. Moreover, in order to obtain fully conservative statistics for

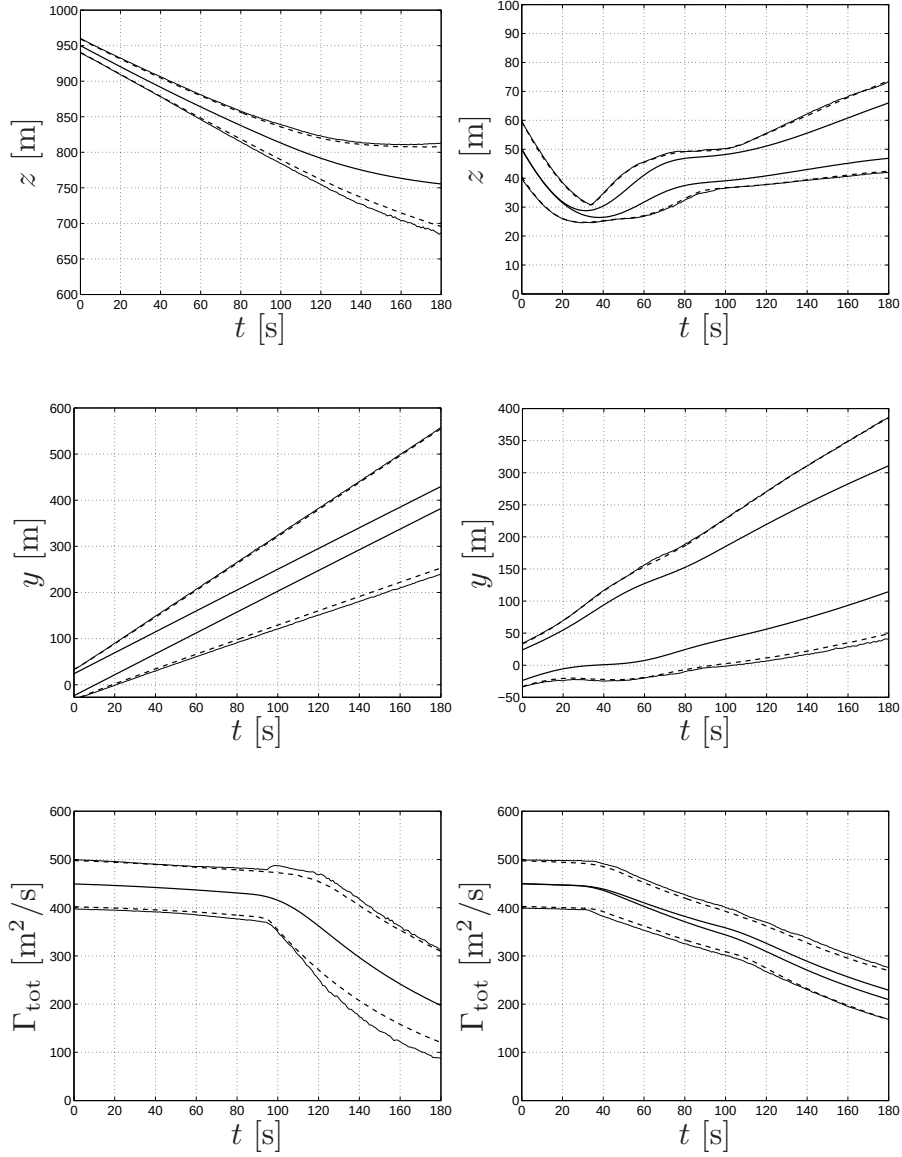


Figure 3.33: Evolution of the mean (thick solid) and the 95%-envelope (dash) of the altitude (top), the lateral position (middle) and the total circulation (bottom) predicted by the PVM using 5000 DVM runs. The results are provided for the case OGE (left) and the case IGE (right). Also shown are the 95%-envelopes obtained using 200 DVM runs and the bootstrap resampling procedure (thin solid).

the mean and the variances (envelopes) that are provided as the final PVM outputs, a resampling method can also be used.

3.3.3 Bootstrap resampling technique

The resampling techniques intend to estimate the precision of sample statistics without conducting new tests. In the case of the PVM, it hence enables to obtain a confidence interval for a statistic (i.e., mean, variance, median or percentile) computed using a given limited number of DVM runs in the Monte-Carlo results. In statistics, two common resampling techniques are found: the jackknife and the bootstrap. Both methods estimate the variability of a statistic from the variability of this statistic between subsamples, rather than from parametric assumptions. The basic idea behind the jackknife method is, in a first step, to build, from a sample of n data, n subsets of $n - 1$ data, leaving out one data at the time and, in a second step, to recompute, for each subset, the statistics estimate. From these n “observations” for the statistics, one calculates estimation of the bias and of the variance of the statistics.

In the bootstrap method [36], one constructs, from a sample $\mathbf{x}$ of size n , B new samples $\mathbf{x}_j^*$ of size n by picking, with replacement, n sample points into the original sample. In other words, the bootstrap method assumes that the original sample contains all the possibly obtained results. Hence, a new campaign of experiments would only lead to a sample made of the same results in a different proportion. For each new sample, the statistic of interest $s_j = s(\mathbf{x}_j^*)$ is then calculated. One obtains a sample of size B of that statistic and evaluates the mean $\bar{s}$ and the standard error σ_s of this distribution by:

$$\bar{s} = \frac{1}{B} \sum_{j=1}^B s(\mathbf{x}_j^*), \quad (3.54)$$

$$\sigma_s = \sqrt{\frac{1}{B-1} \sum_{j=1}^B (s(\mathbf{x}_j^*) - \bar{s})^2}. \quad (3.55)$$

The mean provides a better estimate of the statistic of interest while the standard error allows the computation of a confidence interval on this statistic $s(\mathbf{x})$:

$$s(\mathbf{x}) \in [\bar{s} - \xi_\alpha \sigma_s, \bar{s} + \xi_\alpha \sigma_s]. \quad (3.56)$$

where ξ_α is 1.96 for a confidence level of 95% and 2.57 for a confidence level

of 99% (assuming a Normal distribution of the statistics which is a good approximation). The number B of new samples is independent of the sample size (and typically set to $B = 200$). If the sample distribution is a sufficiently good approximation of the real distribution for the evaluation of the statistics of interest (i.e., if the number of sample is not too low), this method then provides a conservative evaluation of any statistic. Note also that two successive bootstrap evaluations lead to slightly different results, which is not the case using the jackknife method.

In Fig. 3.31, we provide the estimates of the mean and the 95%-envelope of the vortex characteristic distribution computed using 5000 DVM runs without resampling, or 200 DVM runs with bootstrap. The bootstrap estimate of the 99%-envelope of each statistic is also shown. One observes that, for the percentile evaluation, the uncertainty increases in case of long tails (see e.g., Γ_{tot} for the OGE case).

The interest of using the bootstrap resampling technique is also shown in Fig. 3.33. In this figure, we compare the 95%-envelope obtained using 5000 DVM runs to the 95%-envelope obtained using 200 runs increased by the 99%-envelope uncertainty half width (i.e., adding $2.57 \sigma_s$). One observes that the obtained envelopes are very close to the real ones (i.e., those obtained using 5000 DVM runs), yet they are a bit larger.

Finally, the influence of the number of DVM runs on the statistics of the vortex characteristics are investigated in Fig. 3.34, for a case of vortices OGE with wind and turbulence (not the case explained above). One observes the convergence of the confidence envelopes of the statistics. As the number of DVM runs increases, the standard error decreases. For this case, the maximum circulation statistics hence converge from the “safe side”. For instance, an evaluation using 100 DVM runs mean plus the resampling-evaluated standard error indeed leads to a conservative approximation of the real value (as obtained using a few thousand DVM runs).

Of course, in the case where the number of DVM runs becomes very large (e.g., a few thousand), the obtained additional variances provided by either resampling methods tend to zero. It is also found that such statistics on the statistics converge fast. The final and conservative PVM envelopes are then obtained accurately, using a moderate total number of DVM runs. Hence, the PVM approach remains computationally efficient, even for real-time systems.

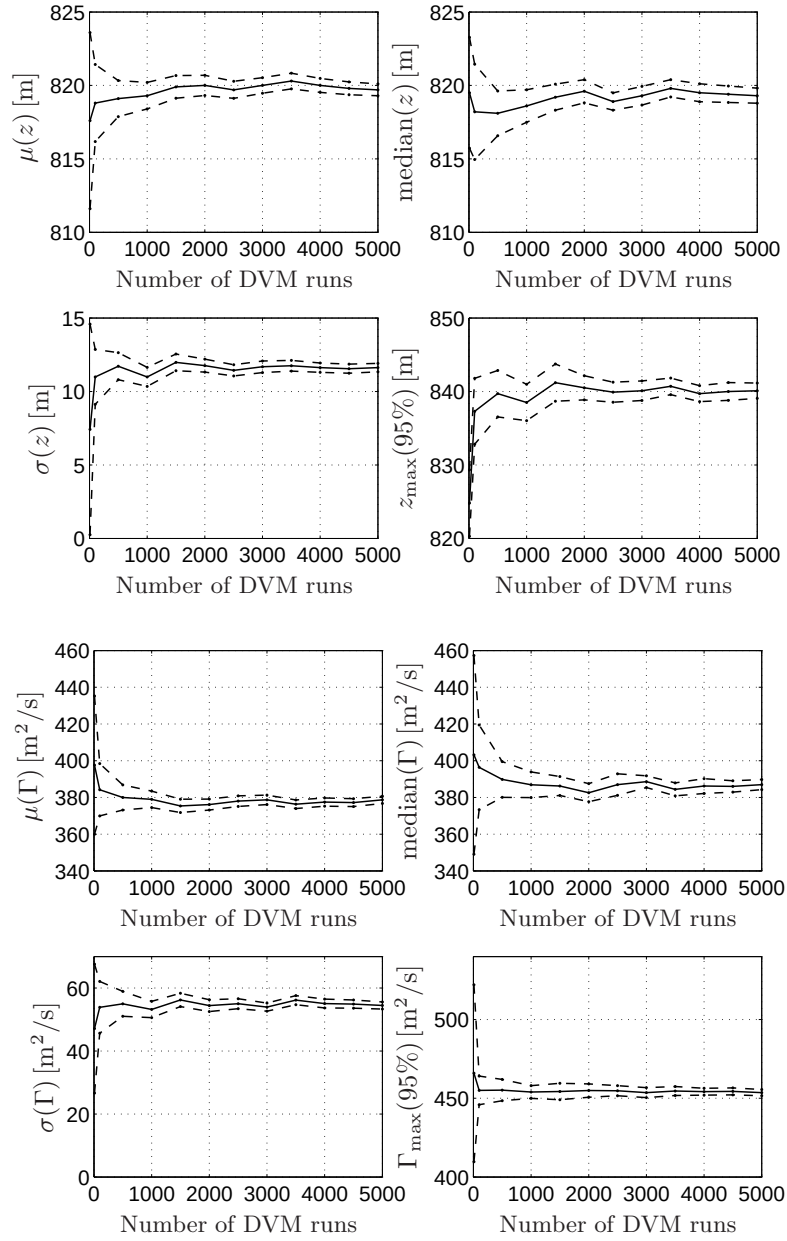


Figure 3.34: Evolution of the statistics for the vortex altitude (top) and circulation (bottom) as a function of the number of DVM runs, for a case OGE with wind and turbulence. The considered statistics are the mean (top left), the median (top right), the standard deviation (bottom left) and the 95%-percentile (bottom right). For each statistic, the mean (solid line) and the 99%-envelope (dash line) are shown.

3.4 The WAKE4D platform and its applications

The WAKE4D is a “3-D space + time” wake vortex prediction platform software that uses the DVM and the PVM. It uses as input the whole “situation”: the aircraft trajectory and the meteorological conditions.

The computational domain is divided in different computational gates, crossing the flight path, separated by a constant time (typically from 2 to 10 seconds). The aircraft crossing one of the gates generates a pair of vortices that are simulated. The platform can be run deterministically (using the DVM in each gate) or probabilistically (using the PVM in each gate). In the following the two modes will be denoted WAKE4D-DVM and WAKE4D-PVM respectively.

In each gate, the DVM (or PVM) computation is initialized using the local aircraft characteristics (with their distribution) and meteorological conditions (with their distribution) at that time and at that location. The gates follow the evolution of the vortices; they thus move in space (also with the axial wind). The wake vortex behavior is simulated, in each gate, for a chosen time (typically 180 s).

From the “gate by gate” DVM computations, the WAKE4D-DVM rebuilds, at any time, the 3-D wake generated by an aircraft, following a given trajectory within given meteorological conditions. An example is presented in Section 3.4.1.

In probabilistic mode, the WAKE4D-PVM builds, at any time, a complete picture of the envelopes containing the wake vortices generated by an aircraft, following a given trajectory (with uncertainties) within given meteorological conditions (with uncertainties). By space interpolation, the entire 3-D vortex of each deterministic run can also be rebuilt, from the corresponding DVM runs in the different computational gates. One then obtains a set of time-varying 3-D vortices corresponding to the number of runs performed in each gate.

The obtained 3-D wake can then be interpolated in multiple fixed (in space) planes, called “control gates”. One then obtains the time evolution of the vortices at those particular locations (i.e., time evolution of position and circulation). A control gate is equivalent to a LIDAR scanning plane. This can, for instance, be used for off-line studies to compare aircraft flying in different configuration and/or in different atmospheric conditions, see example in Section 3.4.2.

In PVM mode, one can also record the events consisting in the presence of

a wake vortex in a region of space (in a given box) as a function of time. This post-processing tool is useful for potential encounter analysis. An example of application for parallel runways is presented in Section 3.4.3.

Finally, in deterministic mode, the 3-D velocity field induced by the wake and the wind can be evaluated from the obtained 3-D wake, as post-processing, in real-time at many points in space. These types of post-processing routines, useful for instance for potential wake vortex encounter analyses in flight simulators, are presented in Sections 3.4.4 and 3.4.5.

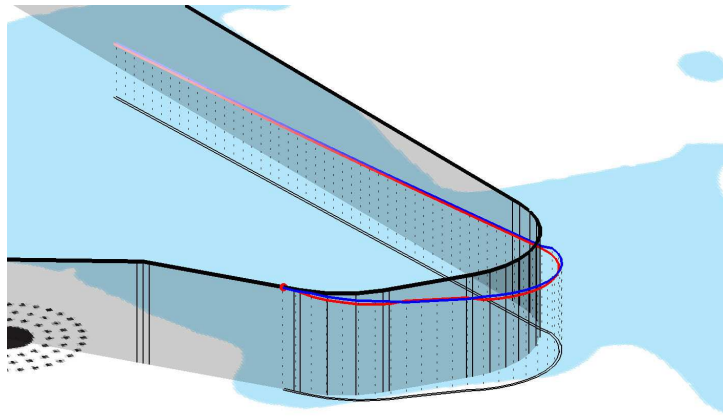
In the following, we present some examples of use of the WAKE4D platform. The applications go from off-line fast-time studies of potential wake vortex encounter to real-time velocity field evaluation for flight simulators.

3.4.1 Example of approach to Marseille-Provence airport

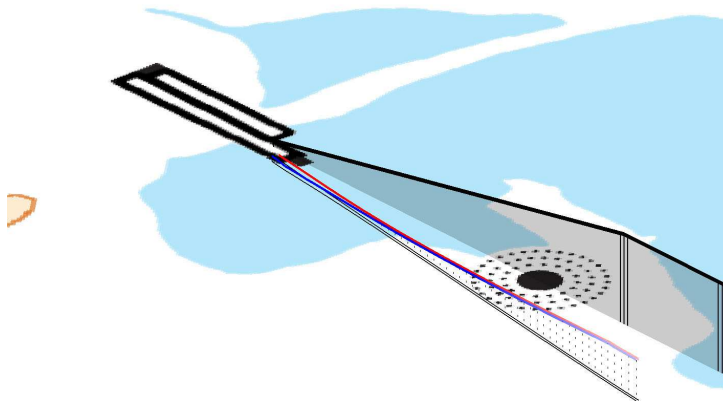
The WAKE4D platform contains a pre-processor that enables to calculate realistic aircraft trajectories depending on some given reference points, given reference airspeed and under given wind conditions. The pre-processor computes the temporal evolution of the aircraft position as well as the attitude angles.

As an example, a typical approach of a heavy aircraft to the Marseille-Provence airport was simulated. A southern typical wind profile was assumed with a moderate turbulence level. The trajectory, obtained using the pre-processor, was discretized using 144 computational gates, generated every 2.5 s along the trajectory. The simulation was run in deterministic mode (i.e., using the DVM in each computational gate), each gate being simulated for 180 s. The evolution of the 3-D wake is then obtained and is shown in Fig. 3.35.

As previously mentioned, multiple control gates, in which the results are interpolated, can be placed anywhere in the landing phase. The trajectory and circulation of the vortices, in those control planes, can then be studied. The 3-D velocity field induced by the predicted wake can also be evaluated, using the routine presented in Section 3.4.4.



a)



b)

Figure 3.35: Snapshots of the 3-D wake, as predicted by the WAKE4D-DVM, for a typical approach of a heavy aircraft to the Marseille-Provence airport. The port (starboard) vortex is represented in blue (red), the color intensity being proportional to the circulation. The axial direction (i.e., the direction parallel to the runway) is compressed for visualization purpose. a) Snapshot of the wake when the aircraft has made its turn. b) Snapshot of the wake with parts IGE when the aircraft has landed. Credit: Matthieu Duponcheel.

3.4.2 Investigation of a Time-Based Separation (TBS) concept using the WAKE4D-PVM

The Eurocontrol Time-Based Separation (TBS) project aims to define and investigate the relevance of a new concept of operation applied to the arrival phase of flights. The actual distance-based separations are to be replaced by time intervals. In the framework of that phase of the Eurocontrol TBS project, the WAKE4D-PVM platform was used for a comparative risk analysis. The wakes generated by various aircraft in various wind conditions were simulated. The characteristics of the potentially encountered vortices were then compared when applying the current separations in still air conditions and when applying the time-based separation.

For that purpose, the wakes, predicted by the platform, were interpolated in several control gates along the flight trajectory. Then, as post-processing, in each of the control gates, the distance between the wake vortices (both left and right) and the following aircraft (with uncertainties on its trajectory) was measured. The PDF of the circulation of the potentially encountered vortex was also computed.

As an example, the arrival of a heavy aircraft is here studied with an aircraft separation of 90 s and for two wind conditions: a case with no mean wind (yet with variations), and a case with a 10 kts (1 kt=0.514 m/s) mean headwind at 10 m height also with wind variations (denoted wind 10). A realistic wind profile was assumed (ICAO standard wind profile), also considering the rotation of the wind profile in the Ekman layer due to the Coriolis effect. A crosswind component thus also appears at higher altitudes. For both cases, a Normal distribution, with a standard deviation $\sigma = 2.5$ kts at 10 m height, was assumed for the wind profile. Figure 3.36 shows, for each wind condition, the histograms of the distance to the ILS and of the circulation of the vortices after 90 s, in two control gates (one in ground proximity, denoted “gate 1”, and the other one far away from the ground, denoted “gate 2”).

Two major trends are observed. Away from the ground, the headwind tends to bring younger vortices in the control gate, hence with higher circulation. However the crosswind component tends to move those vortices away from the ILS. Close to the ground, the headwind enhances the decay IGE and thus decreases the circulation of the potentially encountered vortices. However, the vortices rebound due to IGE and thus do not move away from the ILS.

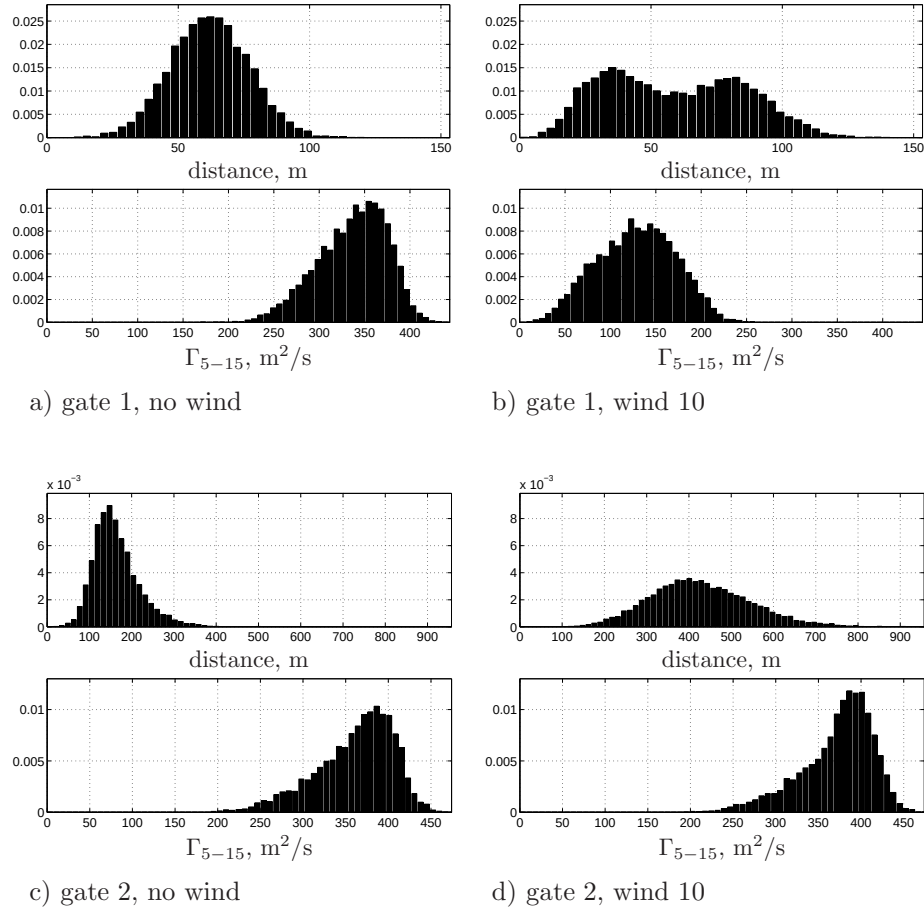


Figure 3.36: Histograms of the distance of the vortices to the ILS and of the circulation, Γ_{5-15} , in two control gates and for two wind conditions, for a 90 s TBS separation. In control gate 1, the ILS altitude is roughly 65 m, whereas, in control gate 2, it is roughly 915 m.

3.4.3 Investigation of potential wake vortex encounter for parallel runways using the WAKE4D-PVM

The WAKE4D-PVM was also used for a fast-time study that aims to quantify the impact on landing medium aircraft safety of wake turbulence generated by heavy aircraft departing on parallel runways. Different scenarios of departure were investigated and the impact on the wake frequency/severity curve of the potentially encountered vortices was studied. The curve is obtained using the complementary cumulative density function (CCDF) of the circulation (i.e., the $\text{CCDF}(\Gamma_{5-15})$ is defined as the probability to observe a vortex with a circulation higher than Γ_{5-15}). This study was performed for the Charles De Gaulle (CDG) airport in the framework of the Eurocontrol-DSNA WIDAO⁴ project. The purpose of the project was first to quantify the increase of frequency and severity of potential wake vortex encounter (WVE) by medium landing aircraft if the departing heavy aircraft are lined up shifted ahead of the currently used runway entry. It also aimed to compare the results with the frequency/severity curve of potential WVE at ICAO separation in trail behind a heavy aircraft. In that project, more than 4000 departing aircraft were considered in about 2000 different wind conditions and for two pairs of runways.

As an example of such application (not a WIDAO case), the impact of the wind on potential WVE is here studied for a pair of parallel runways. The vortices are generated by heavy departing aircraft and the wind direction is such that the vortices reach the landing aircraft region of the parallel runway (as defined by a “control box”). Two cases of wind are investigated: a mean wind profile with a 5 m/s wind at 10 m height and a mean wind profile with 10 m/s wind at 10 m height. The wind direction is at 45° w.r.t. the runway. Variations of 0.5 m/s are assumed around these mean profiles, with a uniform distribution. The frequency/severity curves for these two cases are compared in Fig. 3.37.

As expected, the wind blows the vortices on the arrival runway. A higher wind leads obviously to younger encountered vortices with higher circulation. We stress that, in the present simplified example, the wind direction was fixed and was chosen so that the vortices always come in the arrival zone; hence the probability to observe a vortex in the control box (with any circulation) is almost equal to unity.

⁴Wake-Independent Departure and Arrival Operations

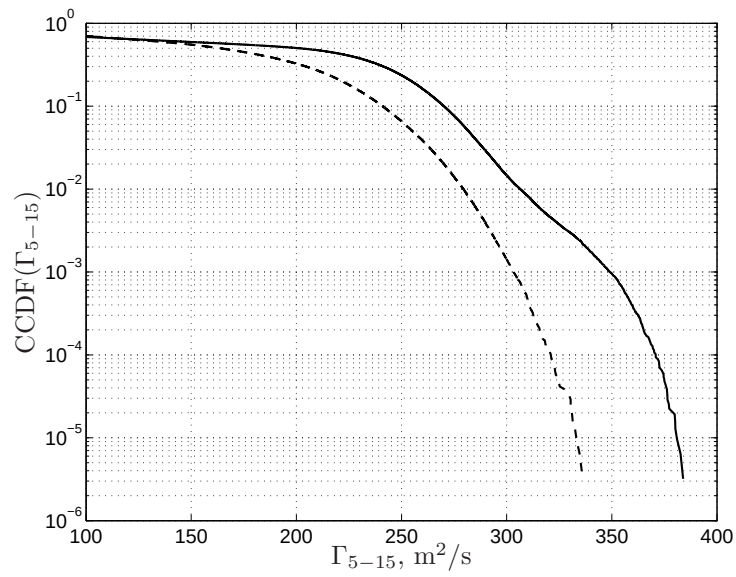


Figure 3.37: Comparison of the frequency/severity curves for potential encounter of wake vortices, generated by heavy departing aircraft, and in a control box in the landing region of the parallel arrival runway: case with a wind at 10 m height of 5 m/s (dash) and 10 m/s (solid).

3.4.4 Evaluation of the induced velocity field using the WAKE4D-DVM results

The WAKE4D-DVM results can also be used to evaluate the 3-D velocity field induced by both the wake vortices and the wind. Using the “gate by gate” computation, the whole velocity field can indeed be rebuilt, as post-processing, using an approach based on vortex tubes segments.

Between each pair of computational gates, an equivalent vortex lattice (i.e., vortex closed loop) is calculated with an average circulation. The total velocity field is calculated by summing the induced velocity of all “active” vortex lattices at a certain point and at a given time. The velocity, at a point (x, y, z) , induced by a vortex tube segment of circulation Γ_p and regularization size σ_p expanding from $\mathbf{x}_1 = (x_1, y_1, z_1)$ to $\mathbf{x}_2 = (x_2, y_2, z_2)$ reads:

$$\begin{aligned} \mathbf{u}_p(x, y, z) &= -\frac{1}{4\pi}\Gamma_p \int_{\mathbf{x}_1}^{\mathbf{x}_2} \frac{g_{\sigma_p}(|\mathbf{x} - \mathbf{x}_p|)}{|\mathbf{x} - \mathbf{x}_p|^3} (\mathbf{x} - \mathbf{x}_p) \times d\mathbf{x}_p \quad (3.57) \\ &= -\frac{1}{4\pi}\Gamma_p \begin{pmatrix} \frac{dz_p}{ds}(y - y_2) - \frac{dy_p}{ds}(z - z_2) \\ \frac{dx_p}{ds}(z - z_2) - \frac{dz_p}{ds}(x - x_2) \\ \frac{dy_p}{ds}(x - x_2) - \frac{dx_p}{ds}(y - y_2) \end{pmatrix} I_p, \end{aligned}$$

where

$$\left(\frac{dx_p}{ds}, \frac{dy_p}{ds}, \frac{dz_p}{ds} \right) = \frac{d\mathbf{x}_p}{ds} = \frac{1}{2}(\mathbf{x}_2 - \mathbf{x}_1), \quad (3.58)$$

and where I_p , using a LOA regularization function (see Table 1.3), is:

$$\begin{aligned} I_p &= \int_{-1}^1 \frac{ds}{(r(s)^2 + \sigma_p^2)^{3/2}} \quad (3.59) \\ &= \left[\left(\frac{d\mathbf{x}_p}{ds} \cdot (\mathbf{x} - \mathbf{x}_2) \right) \left(\frac{-2}{\sqrt{r_2^2 + \sigma_p^2}} + \frac{2}{\sqrt{r_1^2 + \sigma_p^2}} \right) \right. \\ &\quad \left. + \frac{-4 \left| \frac{d\mathbf{x}_p}{ds} \right|^2}{\sqrt{r_1^2 + \sigma_p^2}} \right] \frac{1}{\left(2 \left(\frac{d\mathbf{x}_p}{ds} \cdot (\mathbf{x} - \mathbf{x}_2) \right)^2 - 2 \left| \frac{d\mathbf{x}_p}{ds} \right|^2 (r_2^2 + \sigma_p^2) \right)}, \end{aligned}$$

with $r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$.

Figure 3.38 shows a scheme of the velocity field evaluation technique based on the WAKE4D-DVM results. The figure shows the integration of the velocity

induced by the lattice formed by the vortices predicted in the computational gates $i - 1$ and i . The velocity is obtained by summing the contributions of the following vortex segments: from P_i to P_{i-1} , from P_{i-1} to S_{i-1} , from S_{i-1} to S_i , and from S_i to P_i . A constant circulation is taken for the entire lattice equal to the mean of the four predicted values in the two gates: $\Gamma_p = 1/4 (\Gamma_{P_{i-1}} + \Gamma_{S_{i-1}} + \Gamma_{P_i} + \Gamma_{S_i})$. Close to the ground, the velocity in-

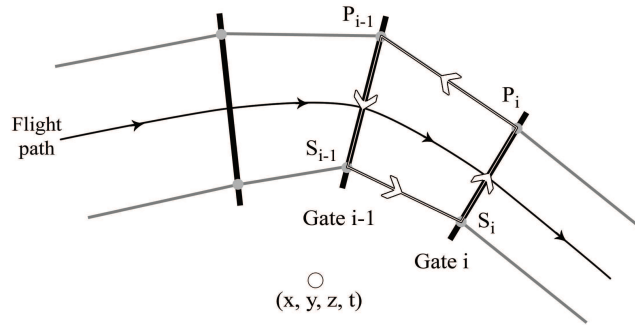


Figure 3.38: Schematic representation of the evaluation of the 3-D velocity field induced by the wake vortices calculated by the WAKE4D-DVM. P and S stands for port and starboard vortex, respectively.

duced by the secondary vorticity, and represented using secondary vortex particles, is interpolated and added. This approach enables the evaluation of the velocity for complex aircraft trajectory scenarios (e.g., take-off, landing, turns), also NGE and IGE. This post-processing routine evaluates the velocity (3 components) at a hundred of points much faster than real-time and is used for wake vortex encounter analysis in flight simulator. It was integrated into two flight simulators (i.e., THOR simulator of Airbus-D and the flight simulator of Technical University of Berlin) for encounter analyses.

Figure 3.39 shows an example of the wake induced velocity field experienced by a following aircraft in take-off with a 90 s separation. The considered wake is generated by a large aircraft following a typical take-off trajectory in pure headwind conditions. The follower is assumed to follow the same trajectory as that of the leader. In Fig. 3.39 (a), the following aircraft is in ground vicinity and the velocity induced by the secondary IGE vorticity is also observed. In Fig. 3.39 (b), the following aircraft is further away from the ground, the vortices are no longer parallel to the ground leading to a significant axial induced velocity.

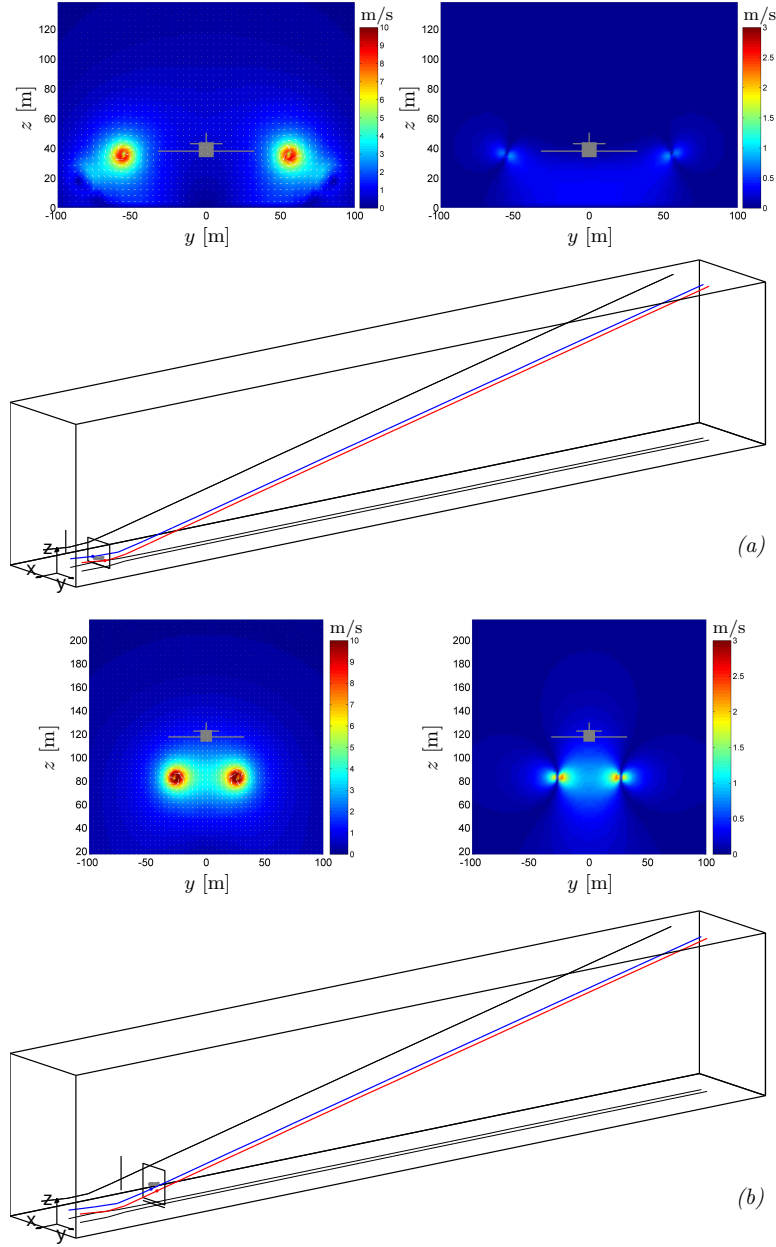


Figure 3.39: Velocity field induced by the wake of a large aircraft in take-off, experienced by a following aircraft with the same trajectory and a 90 s separation. The in-plane velocity component norm (top left), the axial velocity norm (top right) and the 3-D wake topology together with the considered plane (bottom) are shown at two distinct times: (a) when the follower is close to the ground (below 50 m), (b) when the follower is away from the ground (above 100 m).

3.4.5 Evaluation of the induced velocity field using the WAKE4D-DVM results with Crow instability effect

As mentioned in Section 3.2.2, the DVM also contains a simplified model for the growth of the Crow instability, and that depends on the atmospheric turbulence level (and thus the time-to-demise model). In the WAKE4D-DVM, the time and space evolution of the 3-D Crow instability amplitude can then be rebuilt by interpolation, between the computational gates. As post-processing, the velocity field induced by the deformed vortices can also be evaluated, typically at a few hundreds of points, faster than real-time. It uses a vortex filaments approach to compute the 3-D velocity induced by the spatially deformed vortices. A special Biot-Savart integration is performed that enables to obtain the full 3-D field efficiently. This approach is useful for OGE conditions (also cruise), without turns and where Crow effects are important.

An example of the results of a WAKE4D-DVM simulation of cruise flight, with some uniform crosswind and with low turbulence level, is shown in Fig. 3.40. In the near wake of the aircraft (represented in black), the Crow effects are negligible, whereas further away, the Crow instability is fully developed. A cross-plane has been placed in that later zone and the in-plane velocity, as well as the axial velocity, computed by the post-processing routine, are visualized. This routine could also been used for analysis, in flight simulator, of encounter with deformed wake vortices.

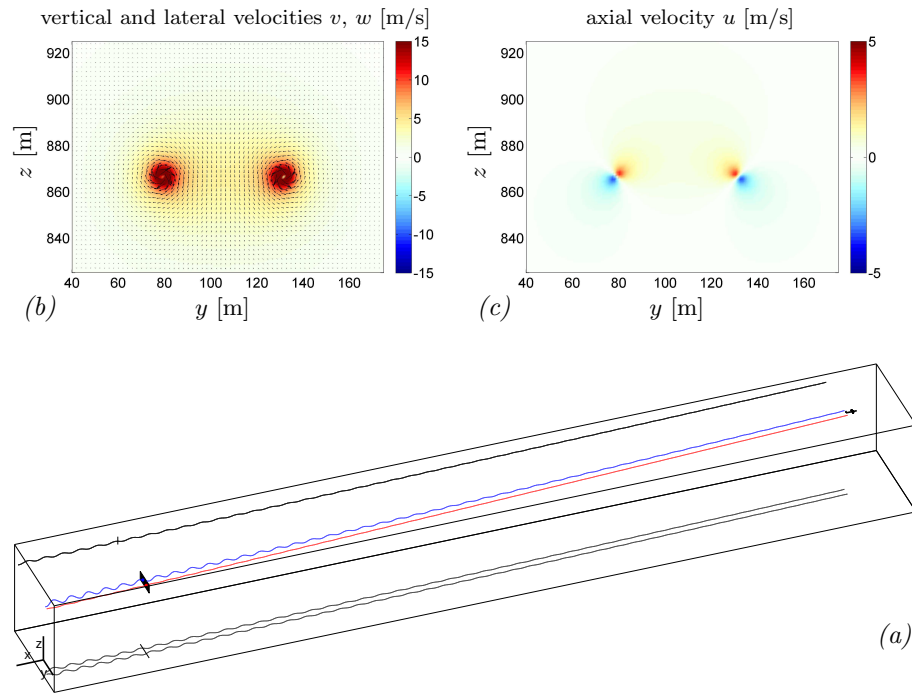


Figure 3.40: a) 3-D view of the deformed wake at a certain time as predicted by the WAKE4D. b) Visualization, in a given cross-plane (shown in gray), of the in-plane velocity components, v and w , colored by the velocity norm. c) Visualization of the axial velocity, u .

3.5 Conclusion

For operational applications, a sufficiently accurate, yet computationally efficient, modeling of the effects, on the wake vortex behavior (transport and decay), of the meteorological conditions and of the ground vicinity is of primary importance. For that purpose, the UCL group has developed the Deterministic wake Vortex Model (DVM), that predicts, in much faster than real-time, the wake vortex behavior generated by a given aircraft evolving in given ambient conditions. The submodels used in the DVM were here further developed, improved and assessed. In particular, we focused on the modeling of the effects of the atmospheric turbulence, the atmospheric stratification (also combined with weak turbulence) and of the ground vicinity.

The modeling of the effect, on the vortex circulation decay, of the atmospheric turbulence without stratification was first investigated. Based on the results obtained in Chapter 2 and using a slightly improved Sarpkaya's time-to-demise model, a two-phase decay model was proposed. The model was seen to reproduce the circulation decay due to the strong interaction between the vortices and the surrounding turbulence. In case of vortex linking due to Crow instability development, the model hence only represents the averaged decay of the parts of the vortices that are still "alive". The effect of the turbulence level on the Crow instability amplitude evolution was also modeled using the time-to-demise model. The model yields a reasonable agreement with the LES results of Crow instability development performed in Chapter 2.

Using the results of the LES of wake vortices evolving in stratified and weakly turbulent atmospheres performed in Chapter 2, an improved model was also proposed for the effects of atmospheric stratification on the wake vortex altitude evolution. The model correctly represents the vortex rebound induced by stratification, even at late stages and for all investigated cases.

Two models were also proposed for the effects of combined stratification and turbulence on the circulation decay. The first model is valid for cases of weak turbulence and for all the investigated stratification levels (from weak to very strong). It represents the decay of the longitudinally-averaged circulation. In case of Crow linking, it then represents the mean decay of the circulation evaluated in all cross-planes (those who cut the vortex ring and those who don't and hence have a zero circulation). The second proposed model only represents the circulation decay of the parts of the vortices that are still alive, thus providing the upper bound. For cases with weak turbulence and weak stratification, it

hence leads to a slower decay than that predicted by the first model. For high stratification levels, as the circulation decay is very uniform along the vortex (see Chapter 2), both models predict similar circulation evolutions, that are also in good agreement with the LES results.

An improved model for wake vortex behavior In-Ground Effect was also developed and validated. It models the flow dynamics using vortex particles both for the primary and secondary ground-generated vorticity. It is able to dynamically reproduce the combined effects, on the wake vortex behavior, of the ground proximity and of the wind (both head- and crosswind components). The agglomeration of secondary separated vortex particles limits the number of used particles (and hence the computational time) while maintaining a sufficiently good representation of the flow dynamics. The further reorganization of the secondary particles, after the primary vortex rebound, also permits to mimic the turbulent reorganization of the secondary vorticity around the primary vortices at those times and its interaction with those, leading to a much enhanced decay IGE.

The DVM, and in particular the IGE model, was calibrated and assessed based on LES results and on measurement data from the WakeFRA 2004 campaign. It was found to properly reproduce the wake vortex behavior both in terms of transport and circulation decay for times up to $t^* = 4 - 5$, the typical times of interest for operational applications (for large aircraft, $t^* = 4$ corresponds approximatively to 120 s). As the model is based on physical concepts, one expects it to be also valid for conditions different from those tested here. The comparison with the EDDF-1 and EDDF-2 databases also highlighted the ability of the DVM to reproduce the wake vortex behavior for a large range of conditions, although those two databases are of significantly lower quality than that of the WakeFRA 2004 campaign.

The DVM itself is also used in a probabilistic predictor, i.e., the Probabilistic wake Vortex Model (PVM). The PVM uses the DVM as a subtool in a Monte-Carlo approach with variations on the impact parameters. It enables to obtain, in addition, the variations around the mean behavior, for the position and the circulation of the vortices. In fact, it provides the whole distribution of behavior. It thus enables to obtain most statistics of interest for operational applications: envelopes (e.g., 90% and 95%), median, mean, variance, etc. Moreover, the bootstrap resampling technique allows the computation of conservative statistics using a limited number of DVM runs in the Monte-

Carlo approach, hence making the PVM also computationally efficient, also for real-time applications.

Finally, both DVM and PVM are used in the 3-D space + time simulation platform WAKE4D. This platform was used for various operational applications such as a support to risk analyses for new operational concepts, and the evaluation of the vortex induced 3-D velocity field for encounter analyses in flight simulator. The WAKE4D platform, together with its improved subcomponents DVM and PVM, then constitute a suitable tool suite for various types of operational (real-time or off-line) studies. Indeed, the WAKE4D platform was, and is still, used in several projects. For instance, in the framework of the WIDAO project, the regulator accepted the implementation of a new concept of operation, at CDG airport, on the basis of a study that used both LIDAR measurements and WAKE4D results. In the ATM history, this was the first time that a concept was accepted based partially on simulation results. In the framework of the SESAR project, the WAKE4D platform will also be used as a wake vortex predictor, in two wake vortex avoidance systems (a ground-based system developed by Thales in SESAR P12.2.2, and an on-board system developed by Airbus in SESAR P9.11).

Chapter 4

Analysis of a wake vortex measurement database (EDDF-2)

The content of this chapter was written within the CREDOS project as the UCL contribution to the Deliverable “D2-5 EDDF-2 database analysis and performance assessment of the Wake Vortex Models on the EDDF-1 and EDDF-2 databases” [22]. Authors are I. De Visscher, L. Briceux, S. Karampelas and G. Winckelmans. This deliverable is reproduced here as it was delivered to the project. Based on this study, an article written in collaboration with DLR and DFS has been published in Meteorological Applications with title “Crosswind Thresholds Supporting Wake-Vortex-Free Corridors for Departing Aircraft” [31] and whose authors are K. Dengler, F. Holzäpfel, A. Wiegele, I. De Visscher, G. Winckelmans, L. Briceux, H. Fisher and J. Konopka.

CREDOS [15], which stands for “Crosswind-Reduced Separations for Departure Operations”, was a research project of the sixth Framework Program that ran between 2006 and 2010. The project aimed to “develop and then demonstrate the feasibility of a concept of operation allowing reduced separations for single runway departures under sufficient crosswind conditions.” In particular, Work Package 2 was dedicated to data analysis and models. A detailed analysis of the EDDF-2 database was then asked to UCL with a particular attention to the correlation between wake vortex behavior and crosswind. This database was also used to assess the DVM (see Chapter 3, Section 3.2.5).

4.1 Introduction

The EDDF-2 database is the result of a long-term wake vortex measurement campaign held in Frankfurt between January and June 2007. The main goal of the campaign was to observe and measure, by experimental means, the wake vortex trajectories and circulations, which are being shed by civil aircraft at Frankfurt airport. In the present database, 10442 cases were tracked by the implemented equipment. The campaign (from the measurements to the collection and delivering of the data) was organized by the Deutsche Flugsicherung GmbH (DFS) company and reported in the CREDOS deliverable D1-2 [64].

Since the number of cases is large (10442), the whole database is screened to retain the cases which were considered relevant for further use within CREDOS. Since the CREDOS concept is based on the crosswind, it is important to properly define it. Five ways of defining the crosswind are presented. These crosswinds are also compared with each other.

The wake vortex lateral transport is then correlated with the crosswind, using the different definitions. For that purpose, one correlates the vortex drift velocity with the crosswind on one hand. The computation of the time evolution of the envelopes containing the wake vortices, with a defined probability level and for a given crosswind, are then also provided. The results show that the time evolution of the envelope width is linear, at least for the times of interest to CREDOS. Based on those results, a potential use of the envelopes is also proposed. As an example, one can compute the crosswind needed for the vortices to be at least at a given distance from the runway centerline after a given time. Finally, an analysis of the wake vortex circulation decay depending on the wind is also presented.

Finally, it should also be noted that a parallel and complementary analysis of the EDDF-2 database was performed by DFS and can be found in the CREDOS deliverable D2-4 [65].

4.2 Description of the equipment and conventions adopted in the measurement campaign

Since a more detailed description of the measurement campaign can be found in [64], we here only recall the major measurement devices. The various sensors which were used in EDDF-2 are:

1. Three anemometers placed at different locations and measuring the wind at 10 m height.
2. A wind-temperature radar (WTR) supplemented with a radio acoustic sounding system (RASS). It provides wind, temperature and turbulence profiles for the height range of 60 – 1650 m.
3. A Wind-tracer LIDAR system (beam intersection with the centerline of the runway). It provides a detailed crosswind profile for heights 1.65 – 190 m. It also provides the measured wake vortex positions (lateral and vertical) and circulations.

The locations of the equipments and the system of reference for the output data are depicted in Fig. 4.1.

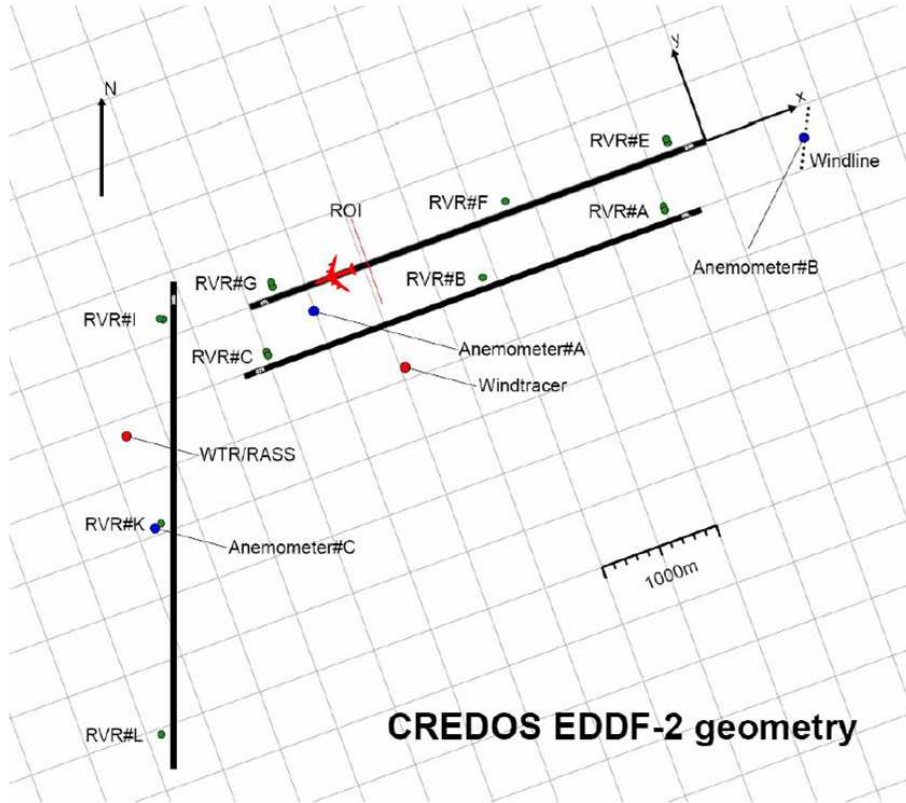


Figure 4.1: Map of the runways and sensor locations.

4.3 Selection of part of the EDDF-2 database for analysis

During the 6 months of measurements, 10442 cases of tracked vortices have been collected. However, we chose to select a subset for the analysis. The size of the obtained dataset is then 6950 cases, which is still very large. The selection procedure, depicted in Fig. 4.2, is the following :

1. We chose to retain the cases where two wake vortices were measured.
2. For those cases, the histograms of the measured initial vortex spacing $b_{0,\text{mes}}$ and bank angle Φ_{mes} were computed, see Fig. 4.3. The analysis showed a mean ratio of vortex spacing divided by wingspan $\frac{b_{0,\text{mes}}}{b} = 0.71$ and a mean bank angle $\Phi_{\text{mes}} = -0.74^\circ$. We then built the 90% envelopes around those means and retained only the cases inside those envelopes. This analysis leads to retain cases where $0.16 \leq \frac{b_{0,\text{mes}}}{b} \leq 1.26$ and $-32.4^\circ \leq \Phi_{\text{mes}} \leq 31.0^\circ$.
3. Moreover, the Wind-tracer algorithm, used to compute the circulation, cannot calculate circulation values above ~ 750 m²/s. For larger values, the algorithm sets the circulation data to exactly 800.0 m²/s (see [65] and histograms in Fig. 4.3). Since this is an artifact of the algorithm, those cases were excluded. Furthermore, circulations of 800.0 m²/s are unrealistically high. Indeed, even a B744 at MTOW ($M = 396.89$ ton) taking off at 181 kts (rotation speed + 10 kts) will produce vortices with a circulation ~ 720 m²/s. Table 4.1 shows the initial circulation of a B744 taking-off, at MTOW in different wing configurations, at rotation speed (which is impossible but constitutes a lower bound) and at the typical takeoff speed (V_2) using the classical formula:

$$\Gamma_0 = \frac{M g}{\rho V s b}$$

Flaps	s	V_R (kts)	$\Gamma_0(V_R)$	V_2 (kts)	$\Gamma_0(V_2)$
		kts	m^2/s	kts	m^2/s
20°	0.72	170	768	181	721
20°	0.78	170	709	181	666
10°	0.72	177	738	188	695
10°	0.78	177	681	188	641

Table 4.1: Estimated initial circulation of a B744 taking-off at MTOW ($M = 396.89$ ton) at an airspeed equal to the rotation speed (V_R) or the take-off speed (V_2). Two flap configurations and two loading factors are considered. The air density is taken as $\rho = 1.25 \text{ kg/m}^3$. (Data source for flaps and corresponding V_R and V_2 is EUROCONTROL).

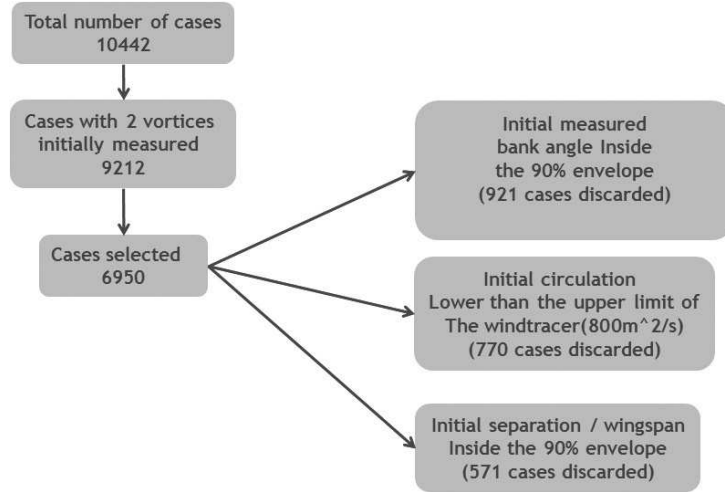


Figure 4.2: Schematic of the dataset selection procedure.

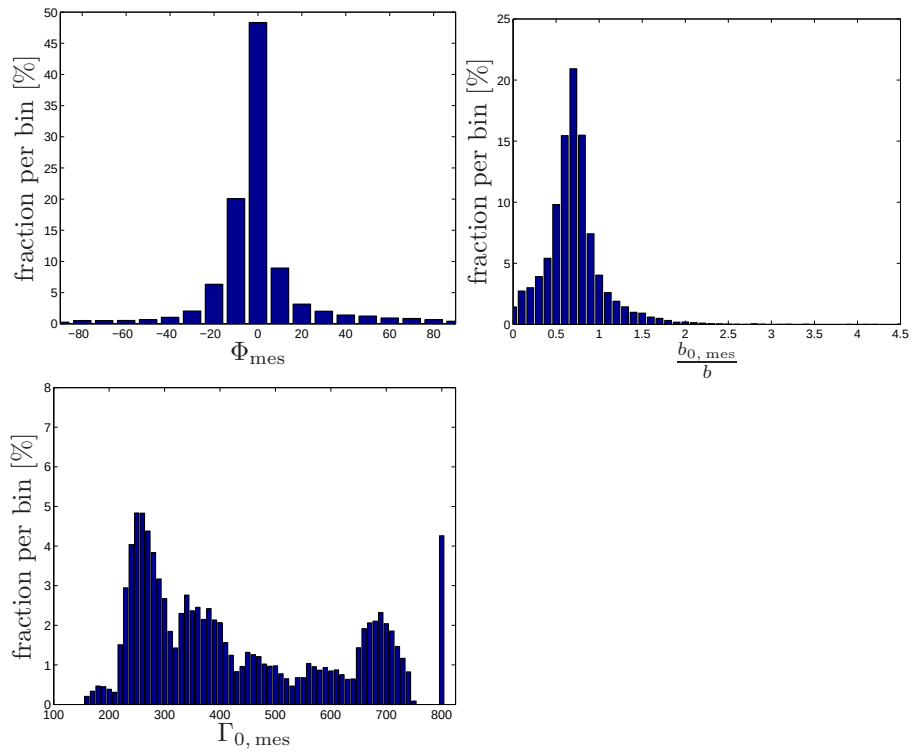


Figure 4.3: Histograms of the initial measured bank angle Φ_{mes} (top left), vortex spacing $b_{0,\text{mes}}$ (top right) and circulation $\Gamma_{0,\text{mes}}$ (bottom). The vortex spacing is divided by the wingspan of the corresponding aircraft.

4.4 Global overview of the dataset

In order to characterize the dataset, Fig. 4.4 shows the histograms of some measurement properties, i.e. the initial and final measurement time, the initial altitude and the initial circulation. For the present sample, the wingspan ranged from $b = 28$ m (light aircraft) up to 64.4 m (heavy aircraft). It is clearly observed, based on the initial scanned height, that most cases are IGE and NGE (h_0 close to b). One also notices the large range of initial circulations, consequence of the large range of aircraft types in the database. Most of the data are measured for 60 s, but some cases are tracked for much longer times. It is interesting to observe the trajectory and decay of the two-vortex

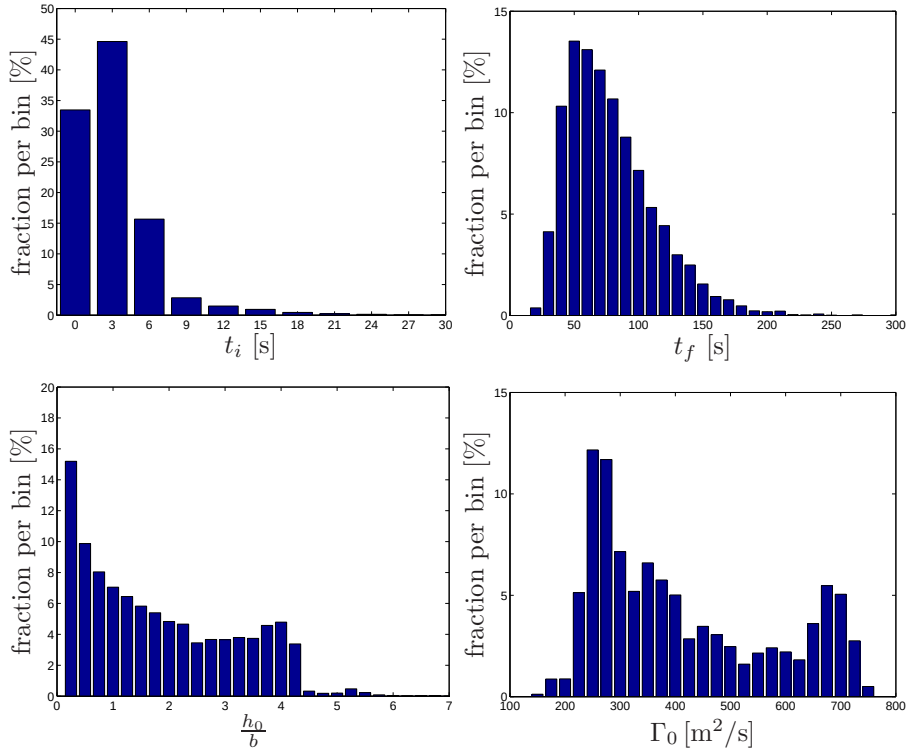


Figure 4.4: Histograms of the initial measurement time (top left), tracking time (top right), generation height (bottom left), and initial circulation (bottom right).

system. This provides the variability range of these parameters. Fig. 4.5 shows the circulation evolution. Fig. 4.6 displays scatter plots of the positions and trajectories of all vortex systems included in the dataset.

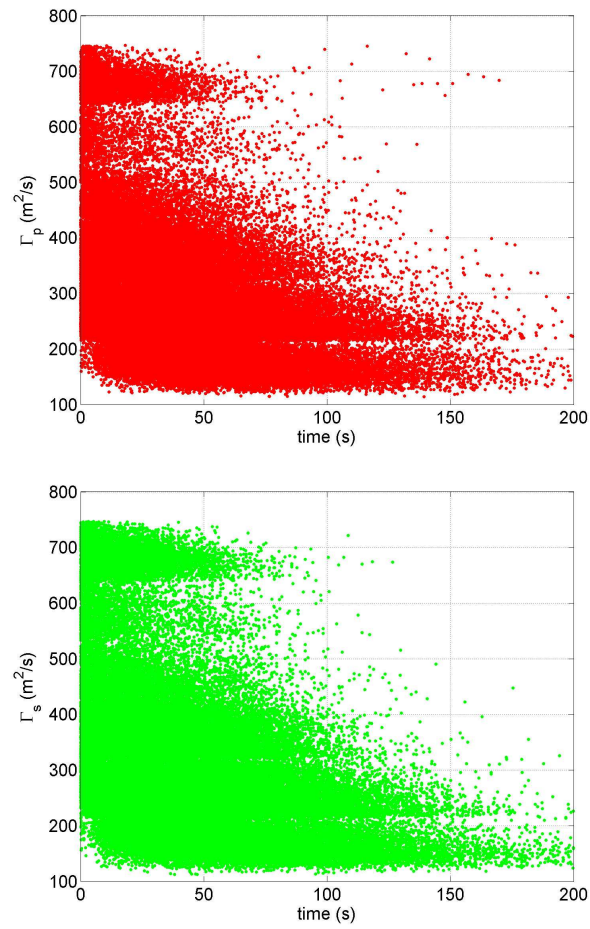


Figure 4.5: Time evolution of the circulation of the port (red) and starboard (green) vortices.

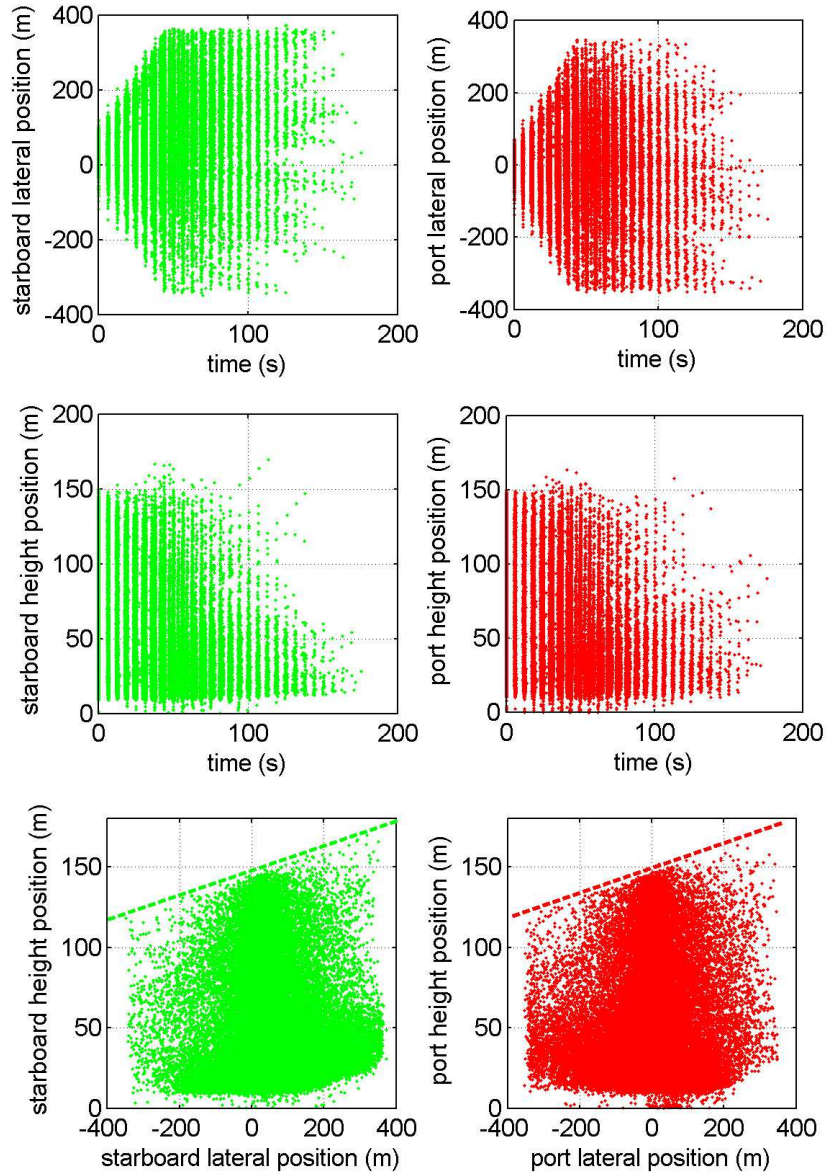


Figure 4.6: Time evolution of the lateral (top) and vertical (middle) position and trajectory (bottom) for the port (red) and starboard (green) vortices. The scanning limit plane is highlighted at the top of the trajectory plots by a dashed straight line.

4.5 Definitions of crosswind and correlations between the definitions

Since the goal of CREDOS is to investigate how the ambient “crosswind” activity affects the vortex dynamics of the aircraft wake, it is important to properly define the crosswind. Five definitions are considered:

- $v(h = 10 \text{ m})$, the crosswind measured at 10 m height. This is what is most convenient as it is already measured today at all airports,
- $v(h_0)$, the crosswind measured at the vortex generation height (i.e., the first measured altitude: $h_0 = z(t_i)$),
- $\bar{v}$, the averaged crosswind up to 100 m, defined by:

$$\bar{v} = \frac{1}{(100 - z_1)} \int_{z_1}^{100 \text{ m}} v(y) dy \quad (4.1)$$

where z_1 is the lowest altitude of measurement (here $z_1 = 1.65 \text{ m}$).

- $v(h_{\text{mean}})$, the crosswind measured at the mean altitude of the wake vortices, defined by:

$$v(h_{\text{mean}}) = v\left(\frac{1}{2}(h_0 + h_f)\right) \quad (4.2)$$

where $h_f = h(t_f)$ is the final altitude of the measured wake vortices.

- v_{traj} , the crosswind as experienced by the vortices along their trajectory, defined by:

$$v_{\text{traj}} = \frac{1}{(t_f - t_i)} \int_{t_i}^{t_f} v(h(t)) dt \quad (4.3)$$

where the subscripts i and f indicate respectively the first and the last vortex measurements.

It is obvious that not all of the defined crosswind velocities are operational. Based on the present equipment of the airports, the most operational (if not the only operational as of today) is $v(h = 10 \text{ m})$. Airport equipped with a wind profiler, can also compute $\bar{v}$. However, in the present measurements, thanks to the wind-tracer output, it is also possible to obtain all the defined crosswind velocities. v_{traj} is considered as the reference, since it gives information of the crosswind activity as experimented at every vortex location, and for the whole vortex trajectory. In Fig. 4.8, the linear fits (of type $y = ax$) between

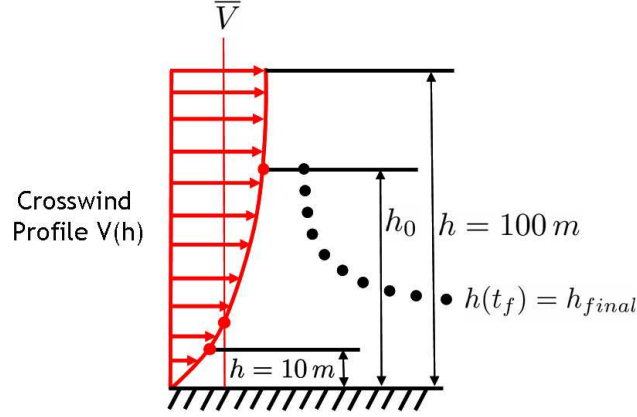


Figure 4.7: Schematic description of the crosswind velocities. The bullets indicate the port vortex trajectory.

Crosswind velocity	$\frac{\text{crosswind}}{v_{\text{traj}}} = a$
$v(h_{\text{mean}})$	1.01
$\bar{v}$	0.96
$v(h = 10 \text{ m})$	0.84
$v(h_0)$	1.04

Table 4.2: Coefficient of the linear fit ($y = ax$) which correlates the defined crosswind velocity and the reference v_{traj} .

the different defined crosswinds and v_{traj} are provided, as obtained from the database analysis. Table 4.2 summarizes the coefficients of the linear fits. As expected, the defined $v(h_{\text{mean}})$ appears to best approach v_{traj} .

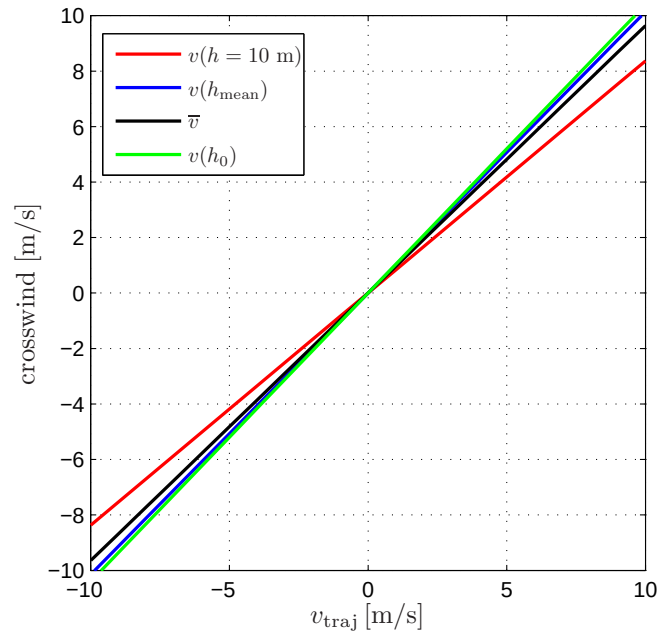


Figure 4.8: Correlation of the defined crosswind velocities with the reference v_{traj} .

4.6 Correlation between vortex lateral transport and crosswind

4.6.1 Drift velocity definition and correlation with the crosswind

The mean lateral transport velocity of the vortices is derived from the first and last measurements. This velocity, denoted here drift velocity v_{drift} , is given by

$$v_{\text{drift}} = \frac{y_f - y_i}{t_f - t_i}. \quad (4.4)$$

It is also interesting to distinguish the transport of the upwind and downwind vortices since, NGE and IGE their trajectories can be slightly different. Figs. 4.9, 4.10, 4.11, 4.12, and 4.13 provide the upwind and downwind drift velocity as a function of the defined crosswind velocities. A linear relation is also fitted on the data points for respectively the upwind, the downwind and both vortices. Since, without wind, the vortices should, in average, not move laterally, one forces the straight line to pass through the origin, leading to:

$$v_{\text{drift}} = a v, \quad (4.5)$$

where v is one of the defined crosswinds. This approach was validated using linear fits with two parameters ($v_{\text{drift}} = a v + b$), leading to b values indeed obtained very close to zero.

The coefficients of the linear fit are listed in Table 4.3. These relations give information on the transport of the downwind and upwind vortices separately. It appears that the downwind vortices are convected a bit faster than the upwind vortices. This is in accordance with combined IGE and crosswind effects, as investigated in FAR-WAKE [40, 44, 55, 127]. It is also worth to notice that:

- As expected, v_{traj} is found to represent best the drift velocity for the ensemble of the cases.
- $v(h_{\text{mean}})$ and $\bar{v}$ slightly underestimates and overestimates, respectively, the vortex transport.
- The vortices are, in average, convected 1.2 times faster than $v(h = 10 \text{ m})$.

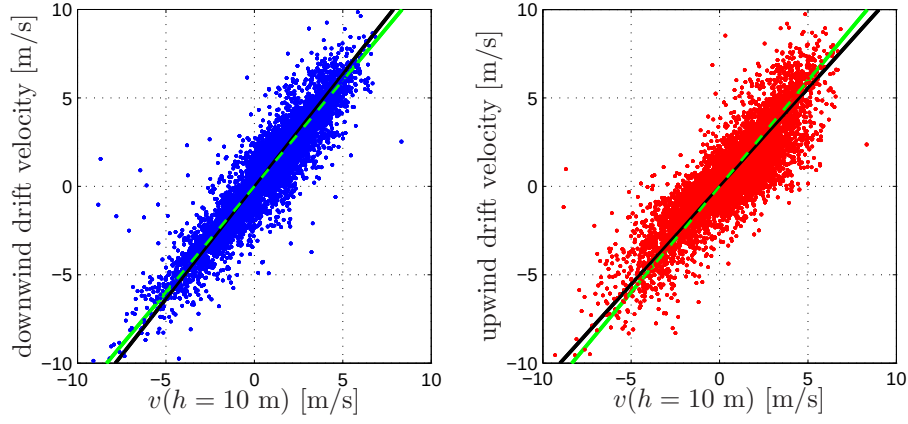


Figure 4.9: Correlation of the upwind (red) and downwind (blue) velocities with $v(h = 10 \text{ m})$, with the corresponding linear fit (black). The linear fit on all vortices is also shown (green).

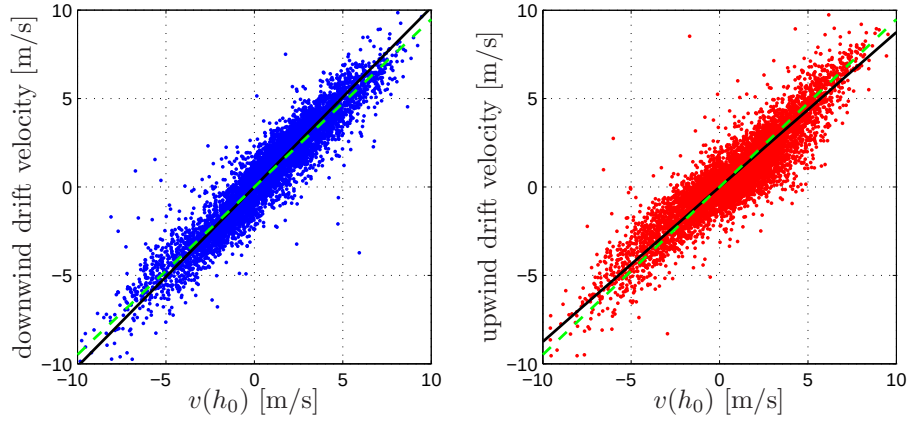


Figure 4.10: Correlation of the upwind (red) and downwind (blue) velocities with $v(h_0)$, with the corresponding linear fit (black). The linear fit on all vortices is also shown (green).

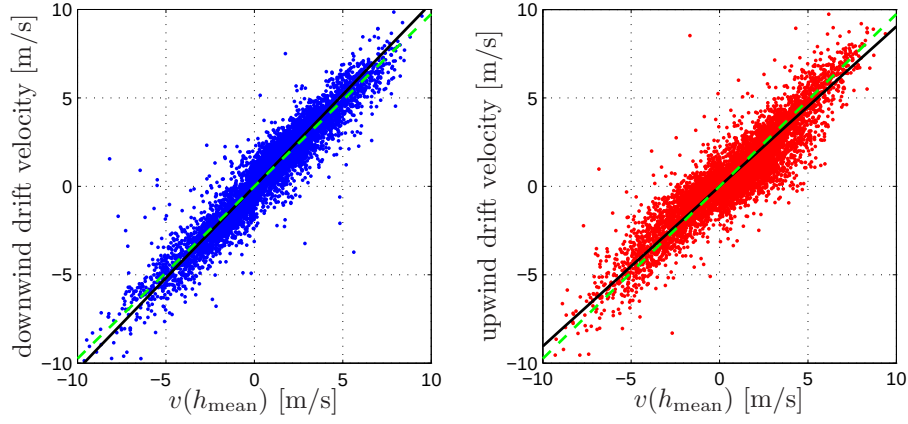


Figure 4.11: Correlation of the upwind (red) and downwind (blue) velocities with $v(h_{\text{mean}})$, with the corresponding linear fit (black). The linear fit on all vortices is also shown (green).

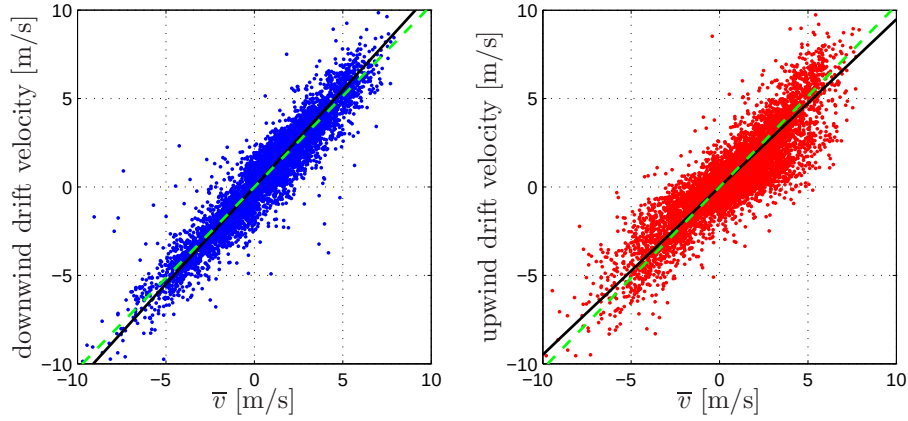


Figure 4.12: Correlation of the upwind (red) and downwind (blue) velocities with $\bar{v}$, with the corresponding linear fit (black). The linear fit on all vortices is also shown (green).

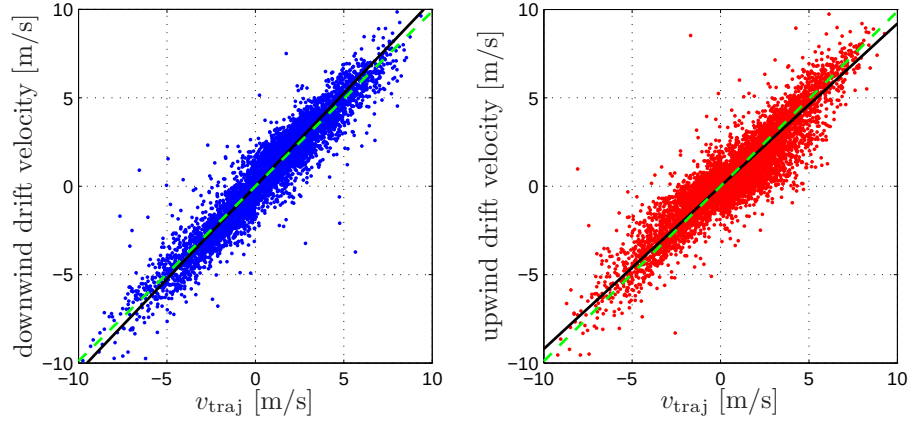


Figure 4.13: Correlation of the upwind (red) and downwind (blue) velocities with v_{traj} , with the corresponding linear fit (black). The linear fit on all vortices is also shown (green).

Crosswind velocity	downwind vortex	upwind vortex	both vortices
$v(h = 10 \text{ m})$	1.27	1.11	1.20
$v(h_0)$	1.02	0.88	0.95
$v(h_{\text{mean}})$	1.04	0.90	0.97
$\bar{v}$	1.10	0.95	1.03
v_{traj}	1.05	0.92	0.99

Table 4.3: Coefficient of the linear fit ($y = ax$) which correlates the defined crosswind velocity with the drift velocity of, respectively, the downwind, the upwind and both vortices.

4.6.2 Computation of the vortex net lateral displacement and correlation with the crosswind

It is also interesting to correlate the crosswind with the vortex lateral displacements. For that purpose, for 5 successive times ($t = 40, 60, 80, 100$ and 120 s), one plots the vortex net lateral displacement, $\Delta y(t) = y(t) - y(0)$, as a function of the crosswind. Figs. 4.14 to 4.18, show the results for the five crosswind definitions. For each crosswind definition, and for each time, a linear relation (again passing through the origin) is fitted on the data. Moreover, one then computes, around this mean behavior, the envelopes containing respectively 90, 95 and 99% of the results. Tables 4.4 to 4.8 list, for the 5 considered times, the coefficients of the linear fits and the envelope widths (for the three probability levels), for the five crosswind definitions.

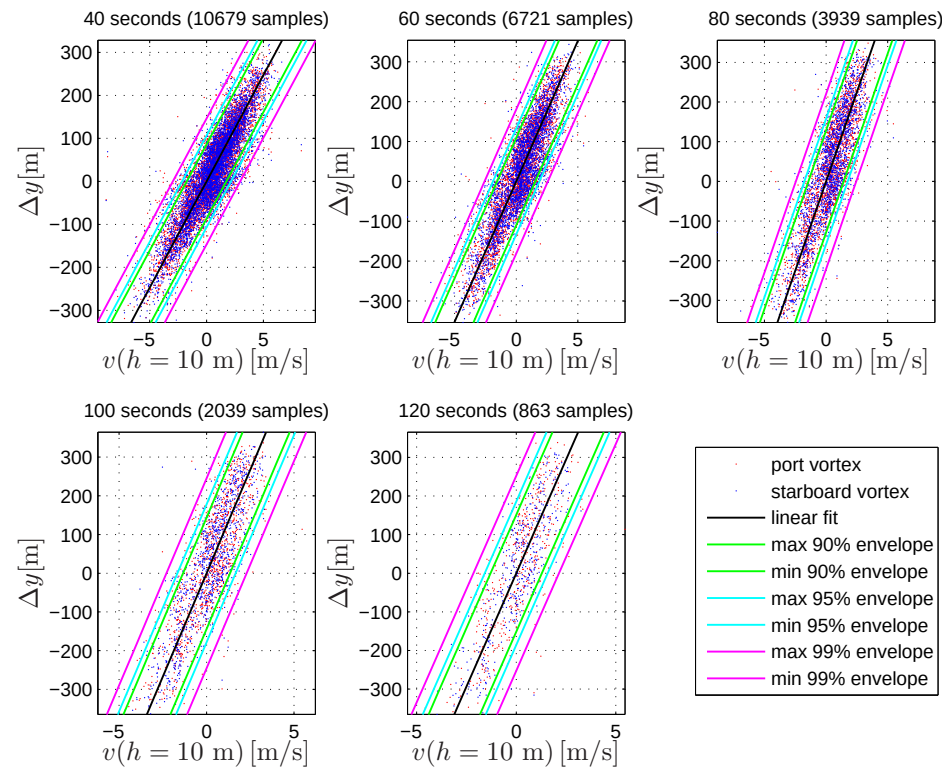


Figure 4.14: Evolution of the vortex net lateral displacement as a function of the crosswind ($v(h = 10 \text{ m})$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (cyan) and 99% (magenta) of the total examined vortices are also drawn.

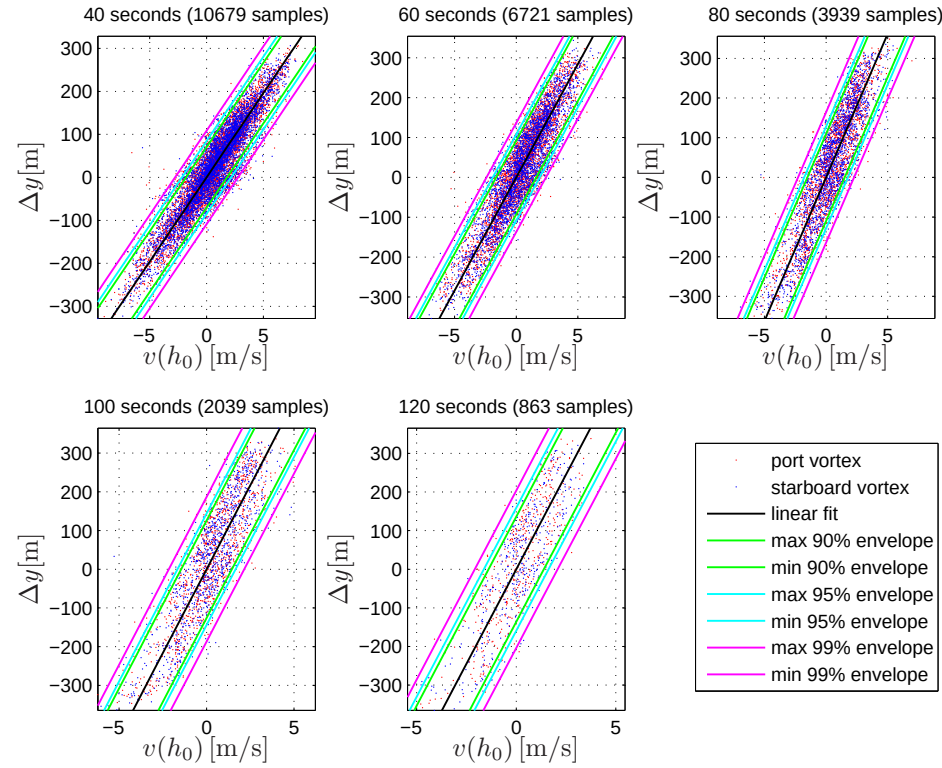


Figure 4.15: Evolution of the vortex net lateral displacement as a function of the crosswind ($v(h_0)$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (cyan) and 99% (magenta) of the total examined vortices are also drawn.

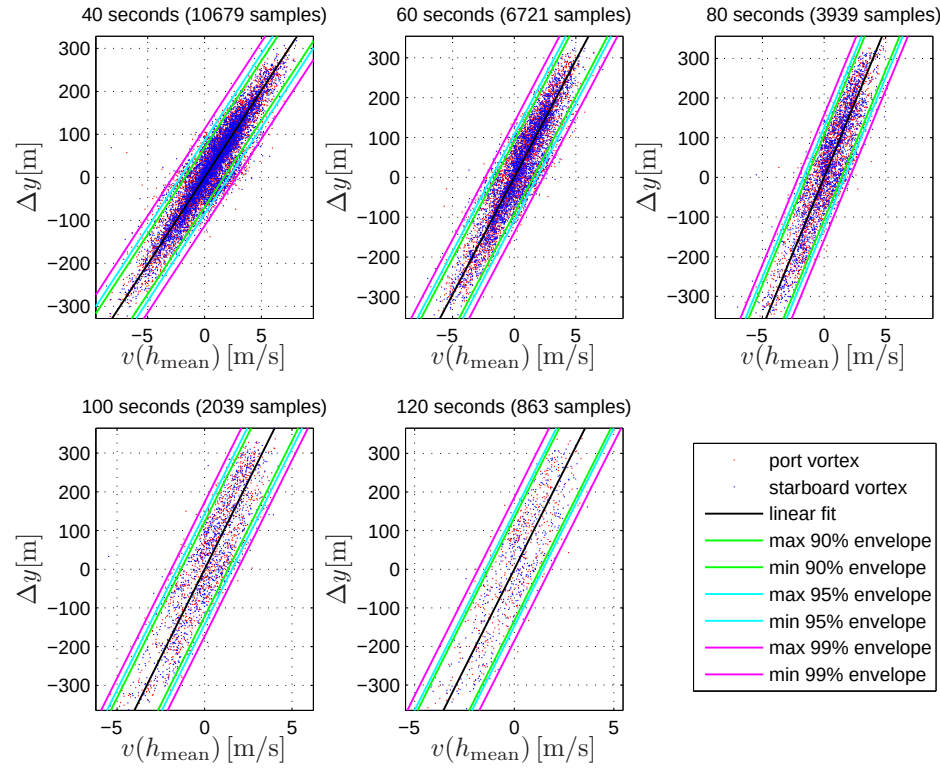


Figure 4.16: Evolution of the vortex net lateral displacement as a function of the crosswind ($v(h_{\text{mean}})$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (cyan) and 99% (magenta) of the total examined vortices are also drawn.

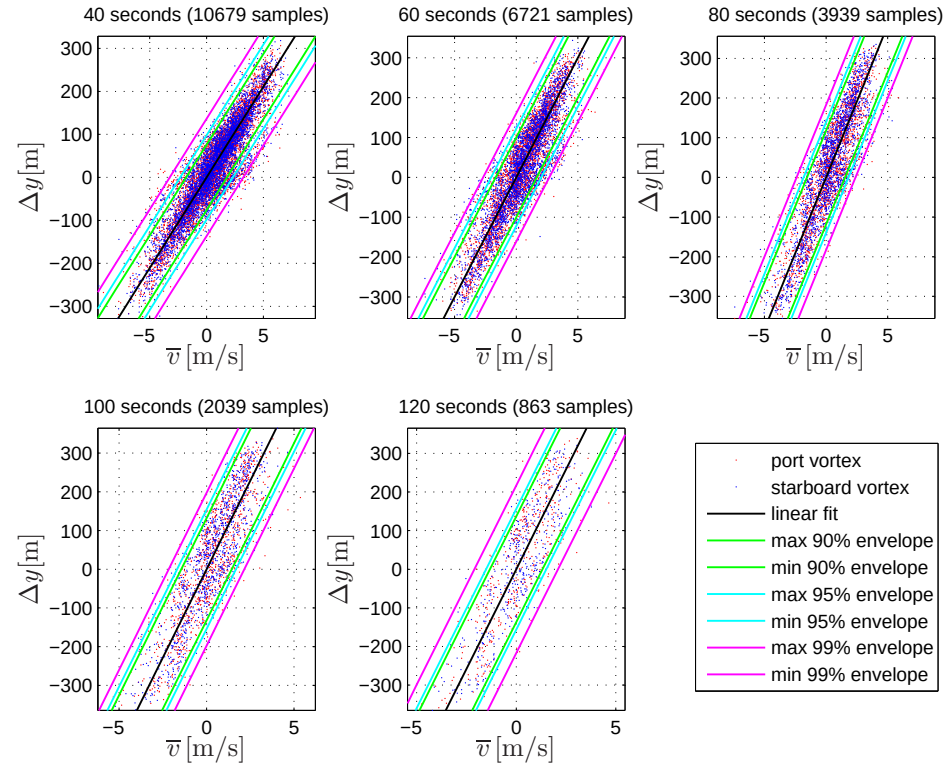


Figure 4.17: Evolution of the vortex net lateral displacement as a function of the crosswind ($\bar{v}$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (cyan) and 99% (magenta) of the total examined vortices are also drawn.

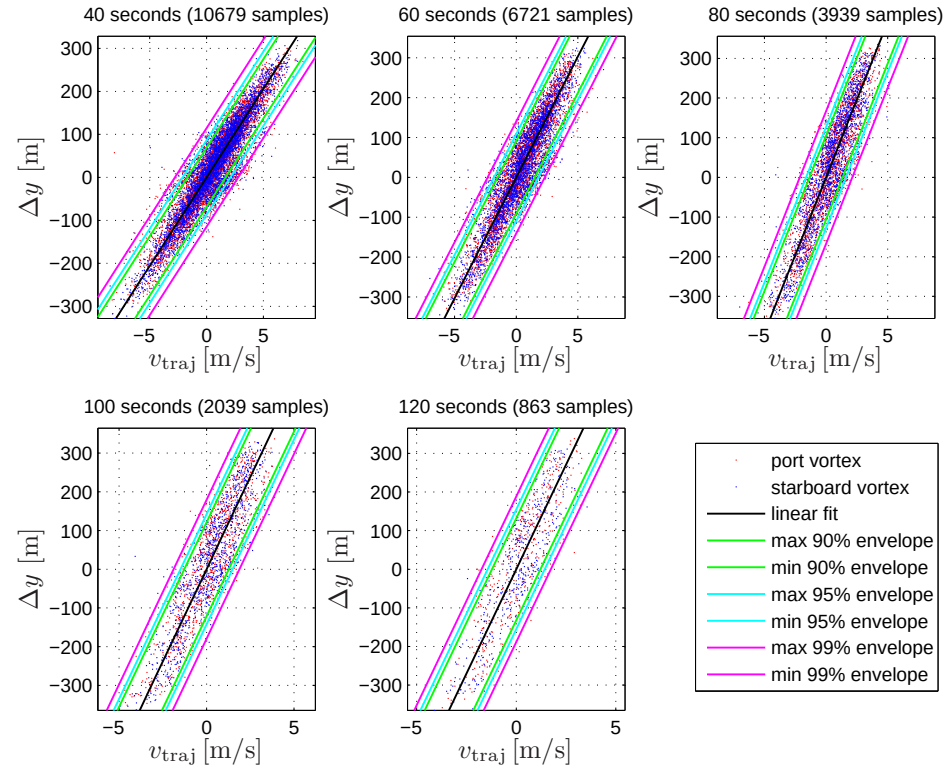


Figure 4.18: Evolution of the vortex net lateral displacement as a function of the crosswind (v_{traj}) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (cyan) and 99% (magenta) of the total examined vortices are also drawn.

t	a	W_{90}	W_{95}	W_{99}
s	s	m	m	m
40	49.5	86.4	106.5	146.3
60	70.5	109.4	132.6	180.7
80	90.1	128.7	158.5	220.5
100	107.0	143.9	178.1	245.3
120	117.5	151.5	185.8	252.4

Table 4.4: Table of the coefficients of the linear fits ($\Delta y = a v$) and envelope half widths, which correlate the relative lateral displacement of all the vortices with $v(h = 10 \text{ m})$ as a function of time.

t	a	W_{90}	W_{95}	W_{99}
s	s	m	m	m
40	39.2	70.6	84.6	108.6
60	56.8	92.5	108.0	136.8
80	72.5	109.6	128.3	164.6
100	86.8	125.8	144.0	186.9
120	97.9	136.2	159.3	203.9

Table 4.5: Table of the coefficients of the linear fits ($\Delta y = a v$) and envelope half widths, which correlate the relative lateral displacement of all the vortices with $v(h_0)$ as a function of time.

t	a	W_{90}	W_{95}	W_{99}
s	s	m	m	m
40	40.4	69.2	85.6	112.7
60	58.8	90.9	108.1	138.3
80	75.5	107.0	125.9	157.6
100	90.9	122.6	142.6	173.0
120	102.6	133.2	147.8	185.7

Table 4.6: Table of the coefficients of the linear fits ($\Delta y = a v$) and envelope half widths, which correlate the relative lateral displacement of all the vortices with $v(h_{\text{mean}})$ as a function of time.

t	a	W_{90}	W_{95}	W_{99}
s	s	m	m	m
40	42.2	76.0	97.8	136.4
60	60.0	99.9	122.0	160.6
80	76.6	118.1	139.5	182.8
100	91.5	132.1	154.1	197.5
120	103.3	136.7	156.4	217.2

Table 4.7: Table of the coefficients of the linear fits ($\Delta y = a v$) and envelope half widths, which correlate the relative lateral displacement of all the vortices with $\bar{v}$ as a function of time.

t	a	W_{90}	W_{95}	W_{99}
s	s	m	m	m
40	41.2	69.0	86.6	114.6
60	60.6	91.1	108.8	140.4
80	78.5	105.6	127.3	167.0
100	95.4	121.4	142.2	179.6
120	108.2	131.9	155.3	188.6

Table 4.8: Table of the coefficients of the linear fits ($\Delta y = a v$) and envelope half widths, which correlate the relative lateral displacement of all the vortices with v_{traj} as a function of time.

4.6.3 Temporal evolution of the mean vortex displacement depending on the crosswind

The mean relative displacement of each vortex centroid is then well approximated using the following linear fit:

$$\Delta y(v, t) = \alpha v t. \quad (4.6)$$

The values of the fit parameters for different times and crosswind are provided in Table 4.9.

v	40s	60s	80s	100s	120s	Mean
$v(h = 10 \text{ m})$	1.24	1.17	1.13	1.07	0.98	1.15
$v(h_0)$	0.98	0.95	0.91	0.87	0.82	0.93
$v(h_{\text{mean}})$	1.01	0.98	0.94	0.91	0.86	0.96
$\bar{v}$	1.06	1.00	0.96	0.92	0.86	0.98
v_{traj}	1.03	1.01	0.98	0.95	0.90	0.99

Table 4.9: Temporal evolution and time average of α parameter of the linear fit, Eq.(4.6), for the different crosswind definitions.

Using $v(h = 10 \text{ m})$, the α factor is seen to decrease with time. To explain this decrease, one has to differentiate three steps. In the first step, the vortices are not IGE yet and sink progressively. Assuming a typical wind profile, the crosswind experienced by the vortices is initially higher than the crosswind at 10 m height and decreases as the vortices sink. The α value decreases thus accordingly. In the second step, the vortices are IGE and interact with the secondary vorticity generated at the ground. With crosswind, the upwind (resp. downwind) vortex travels slower (resp. faster) than the wind. The average displacement speed of the down- and upwind vortices is slightly smaller than the local wind speed. Finally, in the third step, the vortices have rebound and move again with the local wind speed. Since the α factor represents an accumulated effect of the wind on the vortex displacement after a certain time, it is to be expected to observe a slow decrease of α until it reaches a plateau. This trend is also observed in the results of Large Eddy Simulations (LES) of wake vortices IGE experiencing different crosswind profiles, presented and used in Chapter 3, Section 3.2.5. Figure 4.19 shows the α factor computed

from the LES results for a case of a vortex pair released at an altitude equal to the vortex initial spacing $h_0 = b_0$ and with a crosswind profile $v(h)$ such that $v(h_0) \approx 2V_0$ (case CW1). One indeed observes the slow decrease of α .

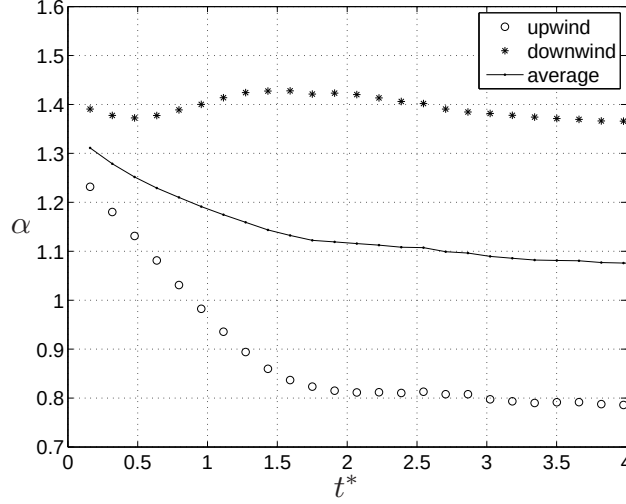


Figure 4.19: Evolution of the α factor (for $v(h = 10 \text{ m})$) measured in a LES of wake vortices generated IGE ($h_0 = b_0$) and with a turbulent crosswind profile such that $v(h_0) \approx 2V_0$ (case CW1).

The values of α also reveal that v_{traj} represents best the wind as experienced by the vortices (value close to 1). Note that times above 100 s have not been considered in the computation of the mean values due to the low number of measurement data. As expected also, it can be observed that $\bar{v}$ slightly overestimates the wind as experienced by the wake vortices while $v(h = 10 \text{ m})$ underestimates it significantly. The α values are also slightly different from those obtained in Table 4.3 since, here, we do not use the measurement at all measured times but only for some chosen times.

4.6.4 Temporal evolution of the envelope width

If one considers the evolution of the envelope widths, as reported in Figs. 4.20, 4.21, and 4.22, one observes a linear behavior for times between 40 s and 100 s. One can thus linearly fit the data using:

$$W(t) = W_0 + \frac{dW}{dt} t. \quad (4.7)$$

Note that the decrease in the envelope width at 120 s is probably due to the low number of data. Since there is no physical reason to explain this decrease, one assumes that the linear growth of the envelope is still valid for times above 100 s, which constitutes a conservative assumption.

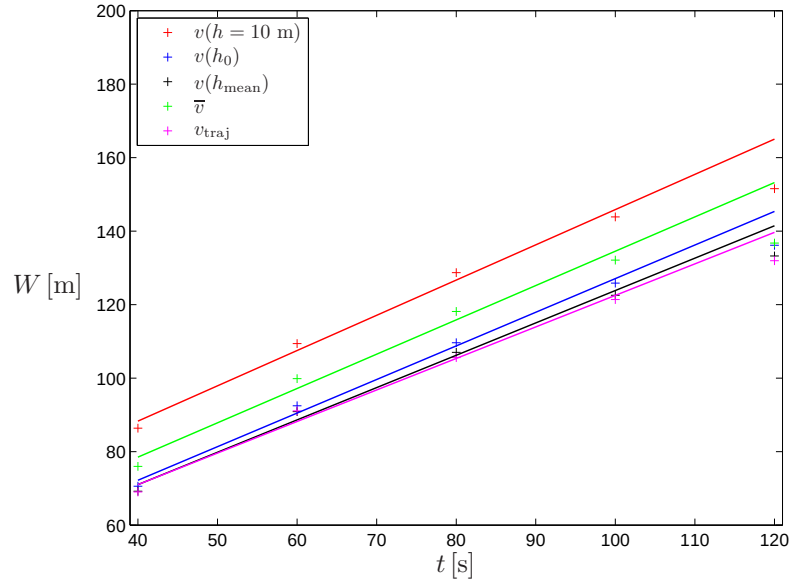


Figure 4.20: Comparison of the envelope half width evolution for the different crosswind definitions and for a probability level of 90%.

Using the results from 40 s to 100 s, the parameters of the linear fit, Eq.(4.7), are evaluated and reported in Table 4.10. As expected, it appears that the envelopes computed using $v(h = 10 \text{ m})$ have the highest growth rate. This shows the interest to use a more sophisticated crosswind definition.

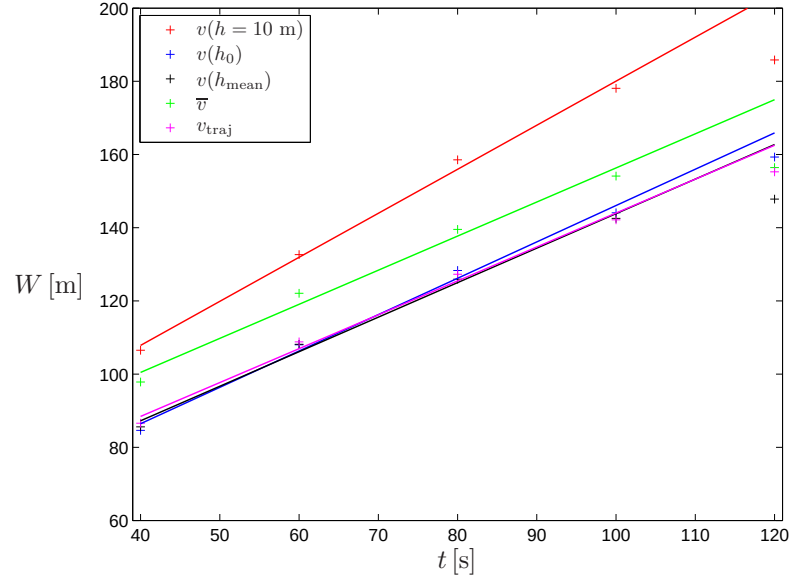


Figure 4.21: Comparison of the envelope half width evolution for the different crosswind definitions and for a probability level of 95%.

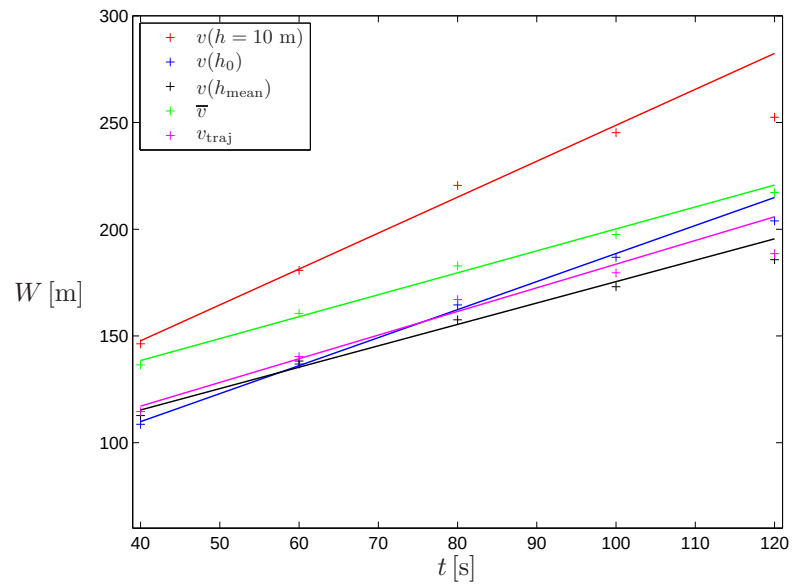


Figure 4.22: Comparison of the envelope half width evolution for the different crosswind definitions and for a probability level of 99%.

		90%	95%	99%
$v(h = 10 \text{ m})$	W_0 [m]	49.9	59.7	80.4
	$\frac{dW}{dt}$ [m/s]	0.96	1.20	1.68
$v(h_0)$	W_0 [m]	35.6	46.7	57.4
	$\frac{dW}{dt}$ [m/s]	0.91	0.99	1.31
$v(h_{\text{mean}})$	W_0 [m]	35.8	49.5	75.2
	$\frac{dW}{dt}$ [m/s]	0.88	0.94	1.00
$\bar{v}$	W_0 [m]	41.2	63.2	97.4
	$\frac{dW}{dt}$ [m/s]	0.93	0.93	1.03
v_{traj}	W_0 [m]	36.7	51.4	72.8
	$\frac{dW}{dt}$ [m/s]	0.86	0.93	1.11

Table 4.10: Parameters of the envelope half width fit, Eq.(4.7), for the different crosswind definitions and for the three probability levels.

4.6.5 Potential use of the computed envelopes

According to the database analysis, after a certain time t , the wake vortices, experiencing a crosswind v , would have traveled a net lateral distance Δy comprised in the interval:

$$\Delta y \in \left[\left(\alpha v - \frac{dW}{dt} \right) t - W_0, \left(\alpha v + \frac{dW}{dt} \right) t + W_0 \right] \quad (4.8)$$

As an example, Fig. 4.23 shows the envelope of the vortex net lateral displacement for a crosswind $v(h = 10 \text{ m}) = 3 \text{ m/s}$.

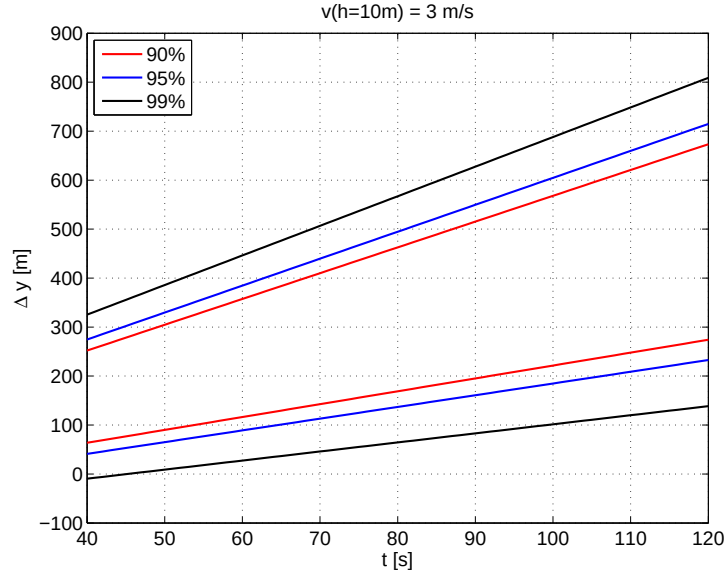


Figure 4.23: Examples of net lateral displacement Δy envelope evolution for given crosswind values $v(h = 10 \text{ m}) = 3 \text{ m/s}$.

One can thus compute the crosswind (for the five definitions) needed for the vortices to travel a certain distance after a certain time, when using the prescribed envelopes (respectively 90%, 95% and 99%). Alternatively, one can compute, for a given crosswind value, the time needed for the vortices to travel a certain distance.

4.6.6 A numerical example

As an example, we propose to compute the value of the crosswind needed for the vortices to be at least at a distance d from the runway centerline after a

separation time t_{sep} , see Fig. 4.24. Here we chose:

- To use, as crosswind definitions, $v(h = 10 \text{ m})$, $\bar{v}$ and $v(h_{\text{mean}})$.
- To consider two distances d : $d_1 \approx \frac{1}{2} \times \text{runway width} + \text{wingspan} = 100 \text{ m}$ and $d_2 \approx \text{wingspan} = 50 \text{ m}$.

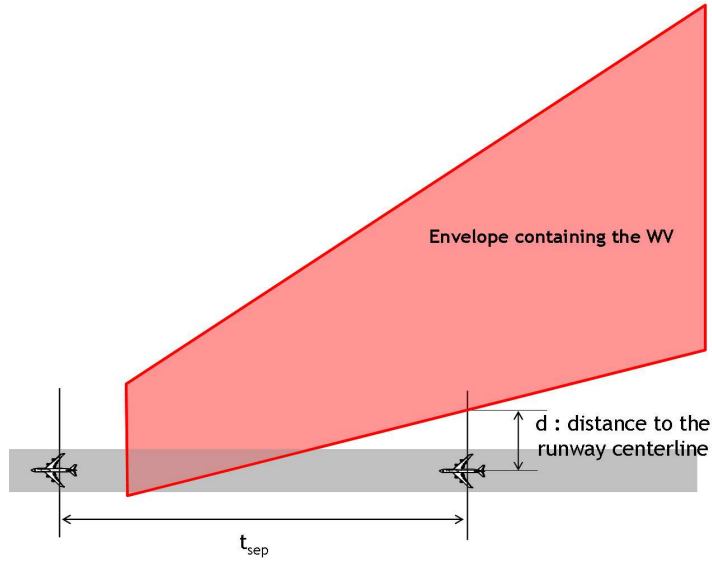


Figure 4.24: Schematic of the envelope definition for the example of application.

As depicted in Fig. 4.25 (left), the crosswind (at 10 m height) needed to blow the WV 100 m away from the runway centerline, when using the 99% envelope, is :

- 4.4 m/s (= 8.6 kts) for a 60 s time separation
- 3.4 m/s (= 6.6 kts) for a 90 s time separation
- 2.9 m/s (= 5.6 kts) for a 120 s time separation

As also shown in Fig. 4.25 (right), the crosswind (at 10 m height) needed to blow the WV 50 m away from the runway centerline, when using the 99% envelope, is:

- 3.7 m/s (= 7.2 kts) for a 60 s time separation
- 3.0 m/s (= 5.8 kts) for a 90 s time separation

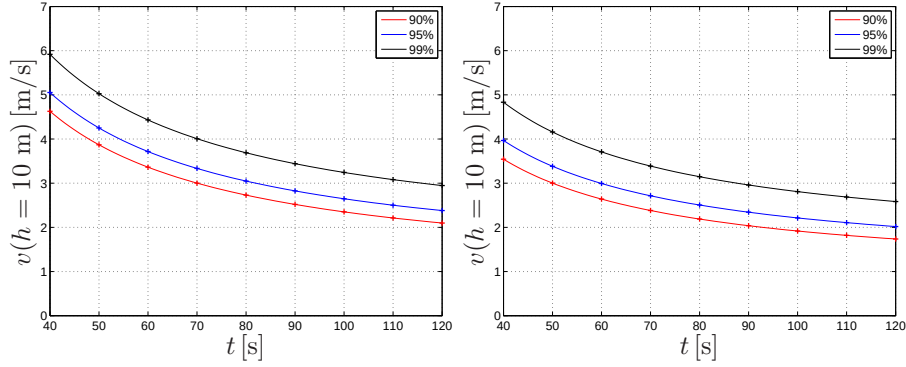


Figure 4.25: Crosswind (at 10 m height) needed to blow the WV away from the runway (here respectively 100 m (left) and 50 m (right) from the runway centerline) when using the 90 (red), 95 (blue) and 99% (black) envelopes.

- 2.6 m/s (= 5.1 kts) for a 120 s time separation

Another way to analyze the results is illustrated in Fig. 4.26. In this case, one reads that the minimum time separation required for the vortices to be blown 100 m away from the runway centerline, when using the 99% envelope, is:

- 116 s for a 3 m/s crosswind (at 10 m height)
- 70 s for a 4 m/s crosswind (at 10 m height)
- 50 s for a 5 m/s crosswind (at 10 m height)
- 39 s for a 6 m/s crosswind (at 10 m height)

And the minimum time separation required for the vortices to be blown 50 m away from the runway centerline, is

- 88 s for a 3 m/s crosswind (at 10 m height)
- 53 s for a 4 m/s crosswind (at 10 m height)
- 38 s for a 5 m/s crosswind (at 10 m height)
- 30 s for a 6 m/s crosswind (at 10 m height)

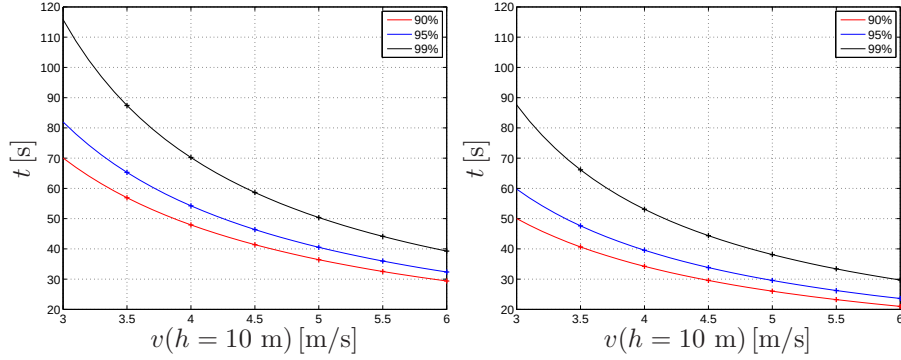


Figure 4.26: Minimum time separation needed to blow the WV away from the runway (here respectively 100 m (left) and 50 m (right) from the runway centerline) when using the 90, 95 and 99% envelopes.

In Fig. 4.27, one observes that the crosswind $\bar{v}$ (averaged over 100m) needed to blow the WV away from the runway (here respectively 100 m and 50 m from the runway centerline), when using the 99% envelope, is :

- resp. 4.8 m/s (= 9.3 kts) and 4.0 m/s (= 7.8 kts) for a 60 s time separation
- resp. 3.6 m/s (= 7.0 kts) and 3.0 m/s (= 5.8 kts) for a 90 s time separation
- resp. 2.9 m/s (= 5.6 kts) and 2.5 m/s (= 4.9 kts) for a 120 s time separation

In the same way, using the crosswind at the mean vortex altitude $v(h_{\text{mean}})$, the value needed to blow the WV away from the runway (here respectively 100 m and 50 m from the runway centerline), when using the 99% envelope, is (see Fig. 4.28):

- resp. 4.5 m/s (= 8.7 kts) and 3.6 m/s (= 7.0 kts) for a 60 s time separation
- resp. 3.4 m/s (= 6.6 kts) and 2.8 m/s (= 5.4 kts) for a 90 s time separation
- resp. 2.8 m/s (= 5.4 kts) and 2.3 m/s (= 4.5 kts) for a 120 s time separation

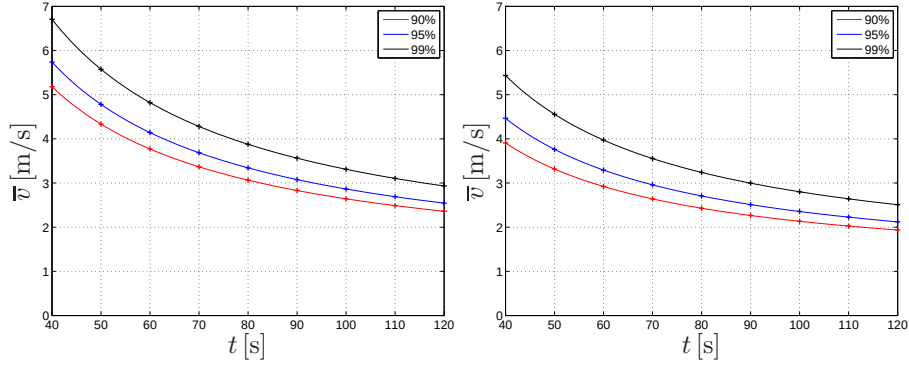


Figure 4.27: Crosswind (averaged over 100m) needed to blow the WV away from the runway (here respectively 100 m (left) and 50 m (right) from the runway centerline) when using the 90, 95 and 99% envelopes.

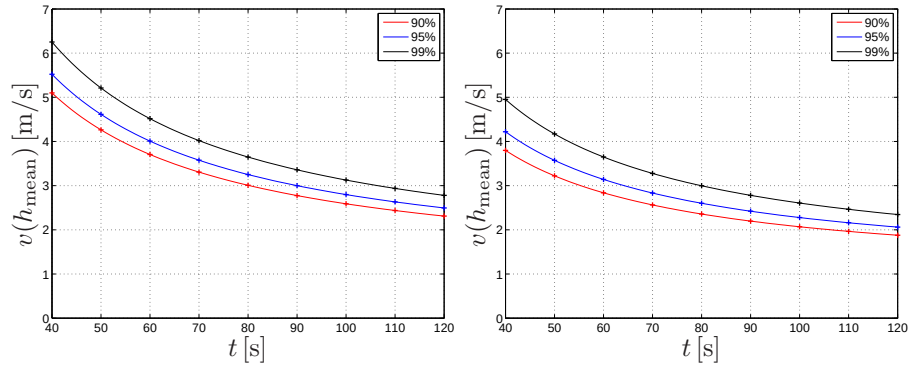


Figure 4.28: Crosswind (estimated at the mean vortex altitude) needed to blow the WV away from the runway (here respectively 100 m (left) and 50 m (right) from the runway centerline) when using the 90, 95 and 99% envelopes.

It should also be recalled that, due to the typical averaged wind profile, $v(h = 10 \text{ m})$ is smaller than $\bar{v}$ and $v(h_{\text{mean}})$. In particular, the ratios between these crosswinds can be calculated according to the measurements, (see Table 4.3). The first ratio is

$$\bar{r} = \frac{\bar{v}}{v(h = 10 \text{ m})} = 1.17. \quad (4.9)$$

This value is consistent with the value predicted using the logarithmic profile from turbulent boundary layer theory, which yields $\bar{r} \approx 1.2 - 1.3$. The second ratio is

$$r_{h_{\text{mean}}} = \frac{v(h_{\text{mean}})}{v(h = 10 \text{ m})} = 1.19. \quad (4.10)$$

t_{sep}	$v(h = 10 \text{ m})$		$v(h = 10 \text{ m})$		$v(h = 10 \text{ m})$	
	from $v(h_{\text{mean}})$		from $\bar{v}$			
s	m/s	kts	m/s	kts	m/s	kts
60	$\frac{4.5}{1.19} = 3.8$	7.4	$\frac{4.8}{1.17} = 4.1$	8.0	4.4	8.6
90	$\frac{3.4}{1.19} = 2.8$	5.4	$\frac{3.6}{1.17} = 3.1$	6.0	3.4	6.6
120	$\frac{2.8}{1.19} = 2.3$	4.5	$\frac{2.9}{1.17} = 2.5$	4.9	2.9	5.6

Table 4.11: Values of the equivalent crosswind at 10 m as calculated from the required $v(h_{\text{mean}})$ and $\bar{v}$ (using the ratios calculated in Eq.(4.9) and (4.10)), and comparison with the required $v(h = 10 \text{ m})$. Here we use the distance $d_2 = 100 \text{ m}$.

Using these ratios, one can compute the corresponding crosswind at 10 m, see Table 4.11. The analysis of these results shows that the use of $\bar{v}$ instead of $v(h = 10 \text{ m})$ is more interesting on an operational basis. Indeed, since $\bar{v}$ better represents the wind experienced by the vortices, the obtained envelopes are tighter than those obtained using $v(h = 10 \text{ m})$. On the contrary, the use of the envelopes computed from $v(h = 10 \text{ m})$ provides over-conservative wind thresholds. For example, for a time separation $t_{\text{sep}} = 90 \text{ s}$, the use of the envelopes computed from $\bar{v}$ predicts that the vortices will be blown away from the runway when the crosswind at 10 m height is greater than 6 kts. If using the envelopes computed from $v(h = 10 \text{ m})$, the minimum required crosswind at 10 m is 6.6 kts. As to the use of $v(h_{\text{mean}})$, it enables to reduce even more the crosswind minima for a given time separation; but its use is not easy in an operational context.

4.6.7 Computation of the vortex net lateral displacement and correlation with the crosswind. Distinction between wake generator aircraft category

The same analysis as that achieved in Sections 4.6.2, 4.6.3 and 4.6.4 is here performed separately for the wake vortices generated by Medium and Heavy aircraft. Again, for 5 successive times ($t = 40, 60, 80, 100$ and 120 s), one plots the vortex net lateral displacement, $\Delta y(t) = y(t) - y(0)$, as a function of the crosswind. Figs. 4.29 to 4.38, show the results for the five crosswind definitions and the two generator aircraft category. A linear relation is fitted on the data and the envelopes containing respectively 90, 95 and 99% of the results are also computed. Tables 4.13 to 4.21 compare, for the 5 considered times, the coefficients of the linear fits and the envelope widths (for the three probability levels) considering the Medium, the Heavy and all generator aircraft, for the five crosswind definitions. It is worth to note that, when we subdivide the database, the number of sample in each category is decreasing: the accuracy of the performed statistics is thus decreasing. This is especially true for the 99% envelopes at late times.

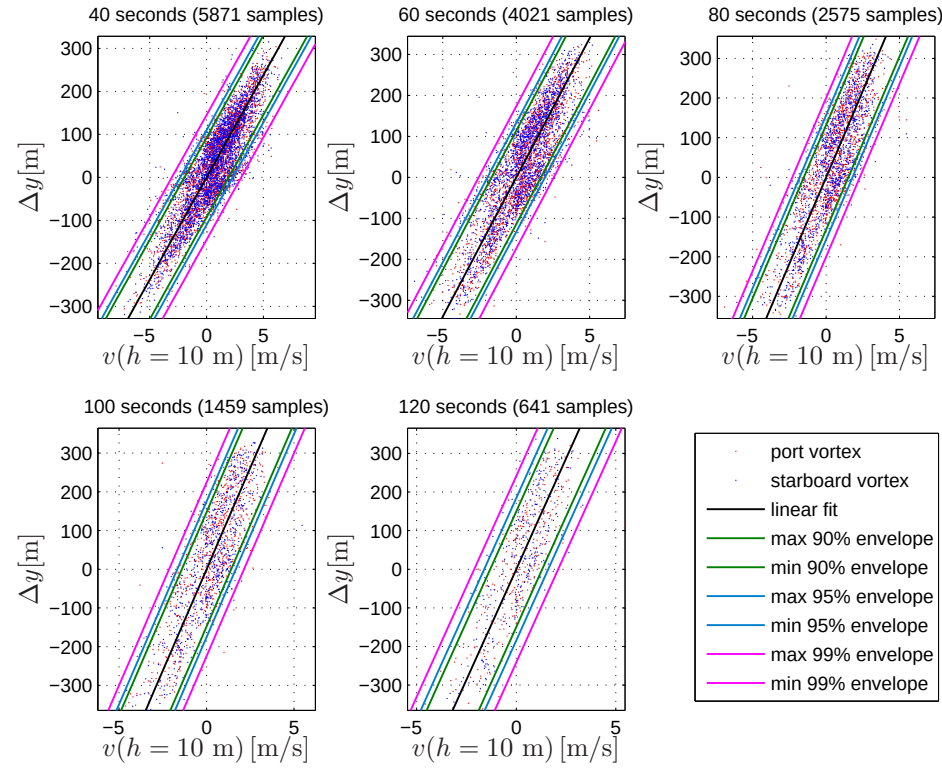


Figure 4.29: Evolution of the net lateral displacement of the vortices, generated by Heavy aircraft, as a function of the crosswind ($v(h = 10 \text{ m})$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

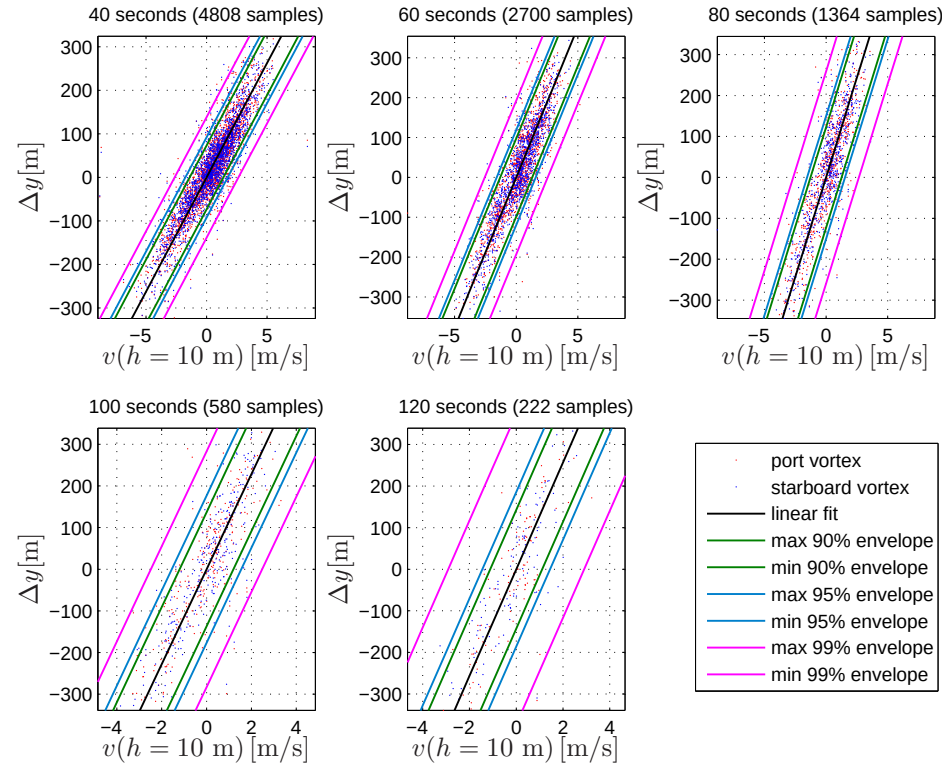


Figure 4.30: Evolution of the net lateral displacement of the vortices, generated by Medium aircraft, as a function of the crosswind ($v(h = 10 \text{ m})$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

t [s]	a [s]		
	M	H	A
40	52.5	47.6	49.5
60	75.5	68.0	70.5
80	97.5	87.3	90.2
100	114.1	104.9	107.1
120	128.4	114.5	117.5

Table 4.12: Linear fit coefficients ($\Delta y = a v$) which correlate the relative lateral displacement of all the vortices with $v(h = 10 \text{ m})$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

t [s]	W_{90} [m]			W_{95} [m]			W_{99} [m]		
	M	H	A	M	H	A	M	H	A
40	73.4	92.6	86.4	92.2	110.2	106.5	138.5	145.0	146.3
60	97.9	114.1	109.4	119.3	133.0	132.6	192.6	173.6	180.7
80	122.8	131.7	128.7	149.9	156.9	158.6	259.2	200.2	220.5
100	137.0	147.7	143.9	176.8	175.2	178.1	282.9	224.5	245.3
120	142.9	149.8	151.6	185.9	184.3	185.8	373.5	241.6	252.4

Table 4.13: Envelope half-widths which correlate the relative lateral displacement of all the vortices with $v(h = 10 \text{ m})$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

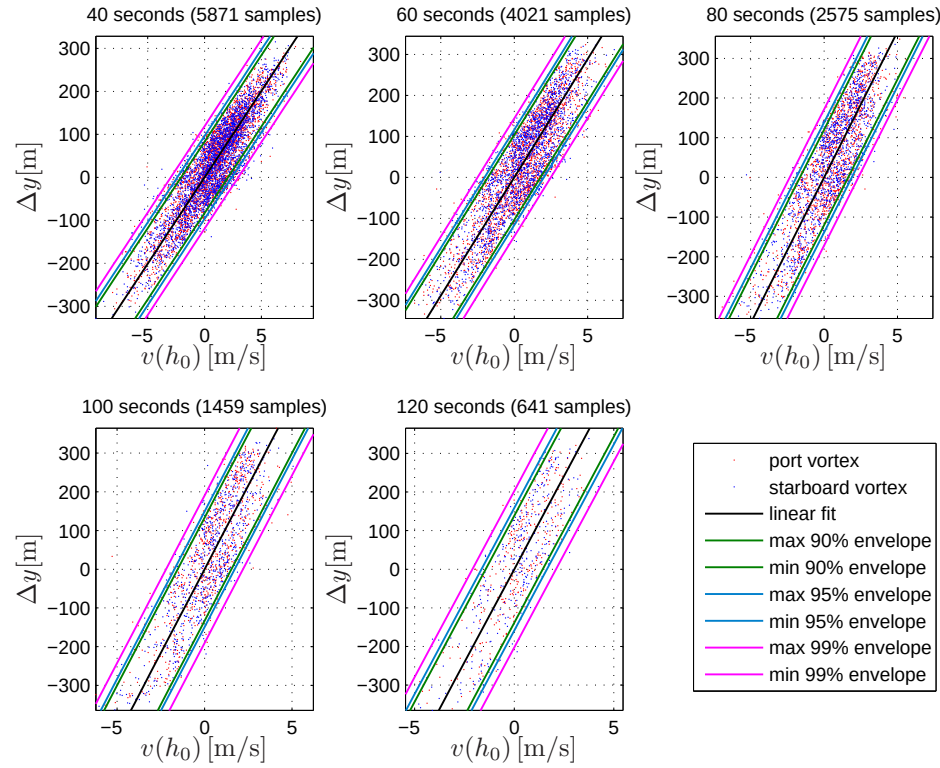


Figure 4.31: Evolution of the net lateral displacement of the vortices, generated by Heavy aircraft, as a function of the crosswind ($v(h_0)$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

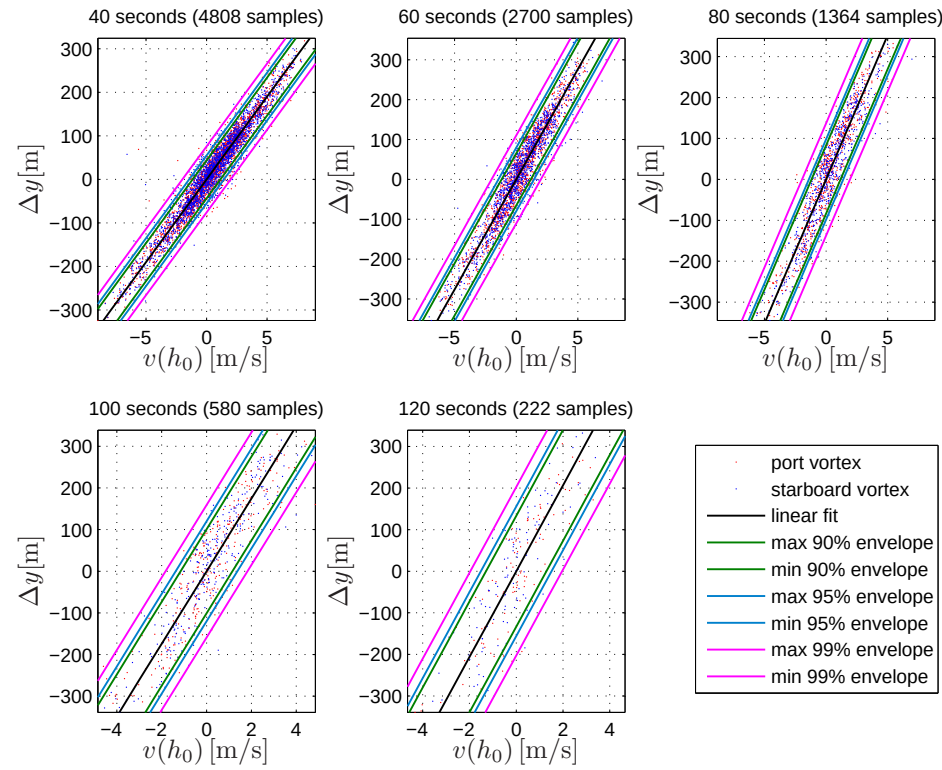


Figure 4.32: Evolution of the net lateral displacement of the vortices, generated by Medium aircraft, as a function of the crosswind ($v(h_0)$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

t [s]	a [s]		
	M	H	A
40	38.0	40.2	39.2
60	55.2	57.8	56.8
80	70.5	73.5	72.5
100	87.1	86.6	86.8
120	103.2	96.4	98.0

Table 4.14: Linear fit coefficients ($\Delta y = a v$), which correlate the relative lateral displacement of all the vortices with $v(h_0)$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

t [s]	W_{90} [m]			W_{95} [m]			W_{99} [m]		
	M	H	A	M	H	A	M	H	A
40	43.7	83.0	70.6	54.9	96.3	84.6	76.0	119.2	108.6
60	65.2	103.4	92.5	79.6	116.5	108.0	110.3	144.1	136.8
80	83.2	120.3	109.6	98.3	138.5	128.3	137.3	171.5	164.6
100	100.7	133.2	125.8	119.8	149.4	144.1	159.1	190.6	186.9
120	133.2	138.5	136.2	155.8	158.6	159.3	202.5	202.6	203.9

Table 4.15: Envelope half-widths which correlate the relative lateral displacement of all the vortices with $v(h_0)$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

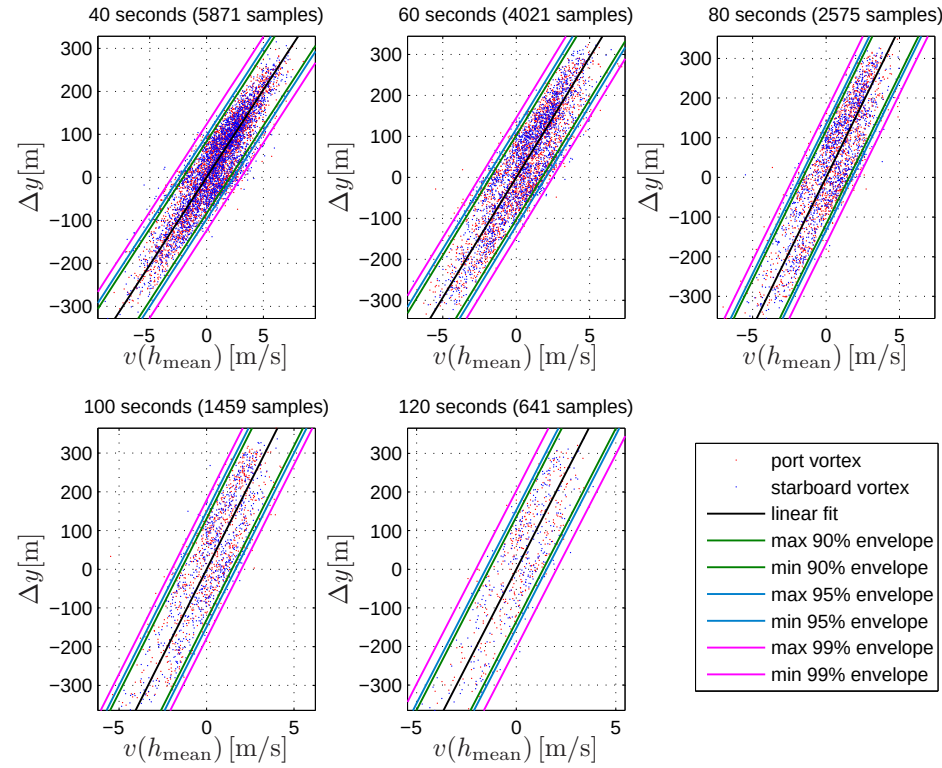


Figure 4.33: Evolution of the net lateral displacement of the vortices, generated by Heavy aircraft, as a function of the crosswind($v(h_{\text{mean}})$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

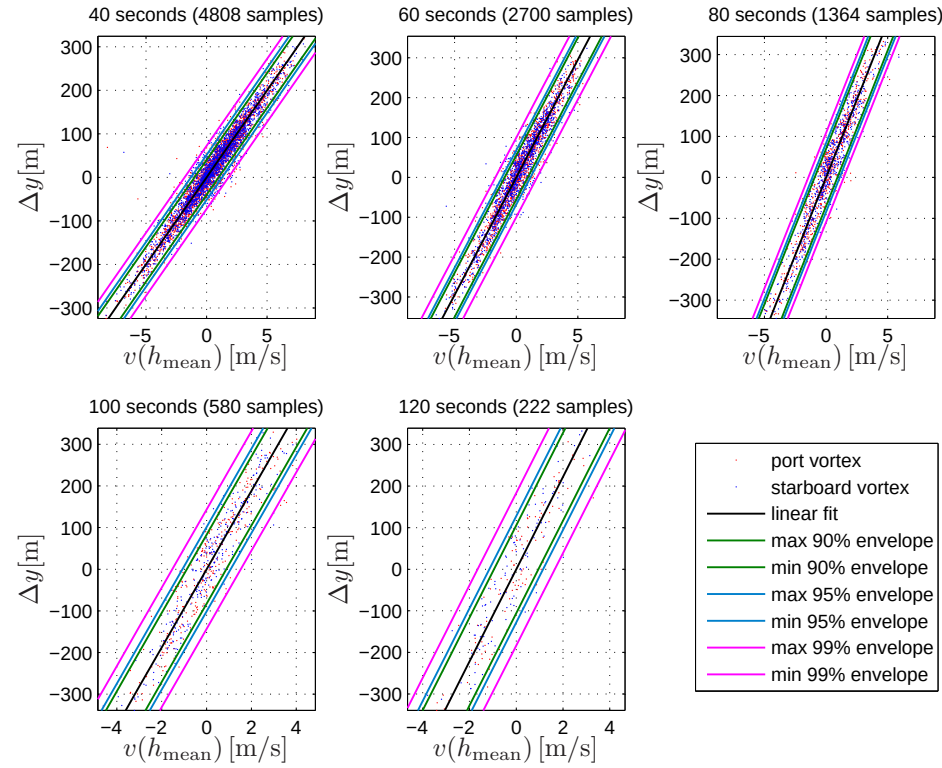


Figure 4.34: Evolution of the net lateral displacement of the vortices, generated by Medium aircraft, as a function of the crosswind ($v(h_{\text{mean}})$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

t [s]	a [s]		
	M	H	A
40	39.9	40.8	40.4
60	59.0	58.7	58.8
80	76.0	75.3	75.5
100	94.0	89.8	90.9
120	111.2	100.1	102.6

Table 4.16: Linear fit coefficients ($\Delta y = a v$) which correlate the relative lateral displacement of all the vortices with $v(h_{\text{mean}})$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

t [s]	W_{90} [m]			W_{95} [m]			W_{99} [m]		
	M	H	A	M	H	A	M	H	A
40	39.9	83.7	69.2	51.5	97.1	85.6	72.5	123.4	112.7
60	55.2	102.9	90.9	68.0	118.1	108.1	99.7	145.8	138.3
80	70.1	118.1	107.0	84.2	133.4	125.9	109.0	166.6	157.6
100	82.8	130.6	122.6	101.1	149.0	142.6	143.5	178.6	173.1
120	105.6	135.7	133.2	127.8	153.1	147.8	183.4	202.9	185.7

Table 4.17: Envelope half-widths which correlate the relative lateral displacement of all the vortices with $v(h_{\text{mean}})$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

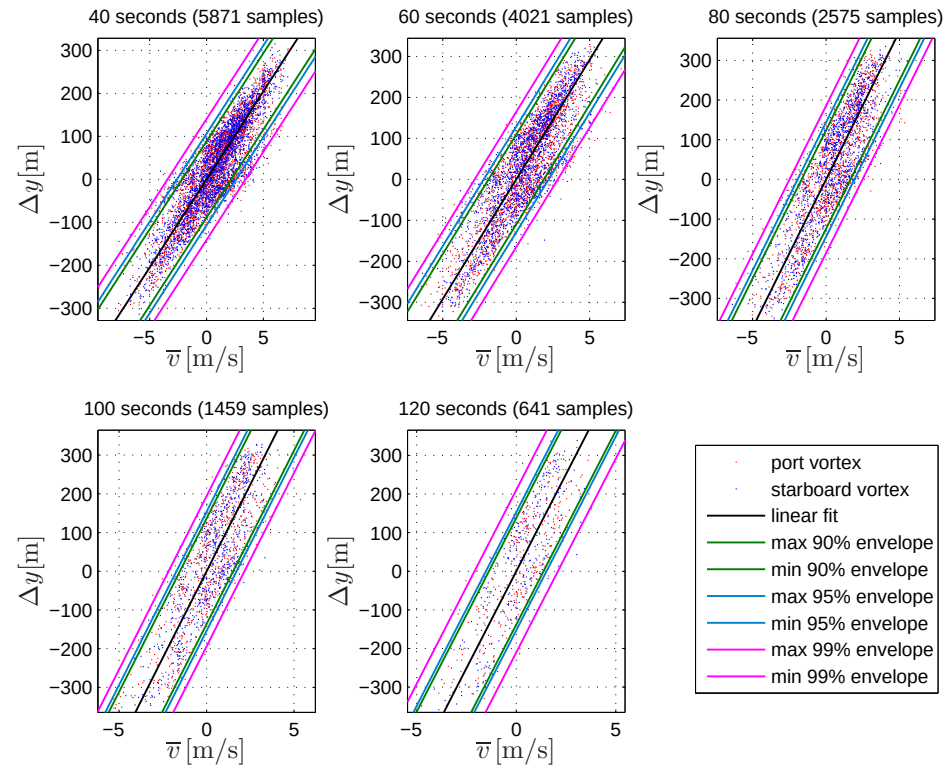


Figure 4.35: Evolution of the net lateral displacement of the vortices, generated by Heavy aircraft, as a function of the crosswind ($\bar{v}$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

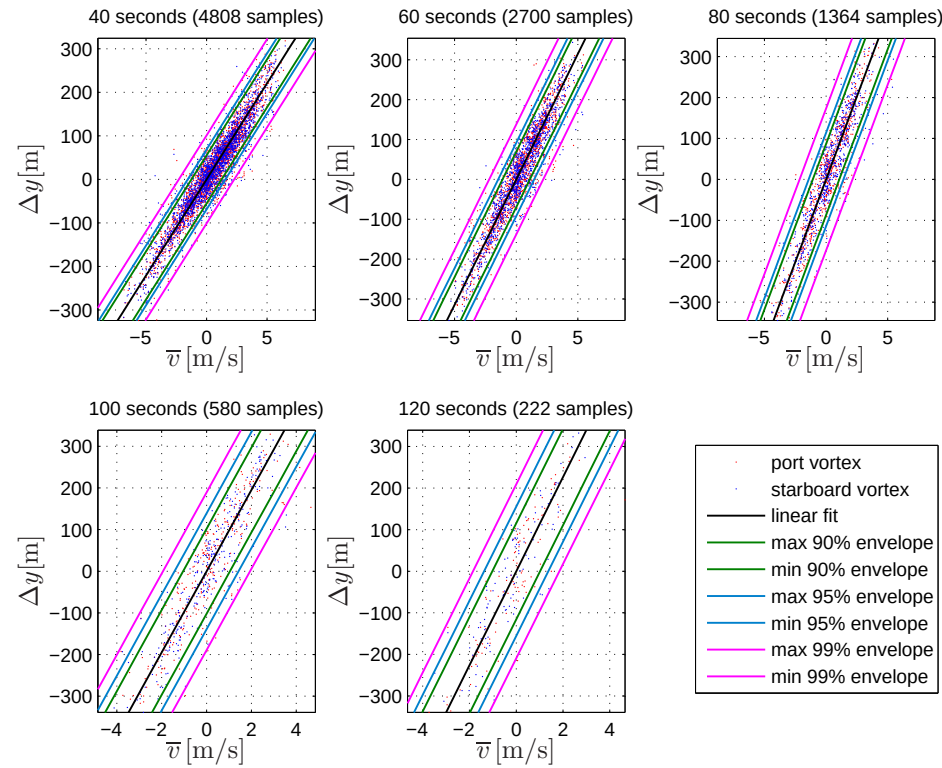


Figure 4.36: Evolution of the net lateral displacement of the vortices, generated by Medium aircraft, as a function of the crosswind ($\bar{v}$) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

t [s]	a [s]		
	M	H	A
40	44.1	41.0	42.3
60	62.9	58.4	60.0
80	81.0	74.7	76.6
100	97.5	89.6	91.5
120	113.2	100.5	103.3

Table 4.18: Linear fit coefficients ($\Delta y = a v$) which correlate the relative lateral displacement of the vortices with $\bar{v}$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

t [s]	W_{90} [m]			W_{95} [m]			W_{99} [m]		
	M	H	A	M	H	A	M	H	A
40	55.3	88.8	76.0	68.7	106.5	97.8	101.2	140.6	136.4
60	69.6	109.7	99.9	91.9	129.0	122.1	137.8	165.0	160.6
80	87.0	124.1	118.2	114.2	144.1	139.6	173.5	185.8	182.8
100	102.0	136.4	132.1	139.2	154.0	154.1	189.3	194.1	197.5
120	115.7	138.1	136.8	155.5	153.2	156.4	209.2	210.5	217.2

Table 4.19: Envelope half-widths which correlate the relative lateral displacement of the vortices with $\bar{v}$ as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

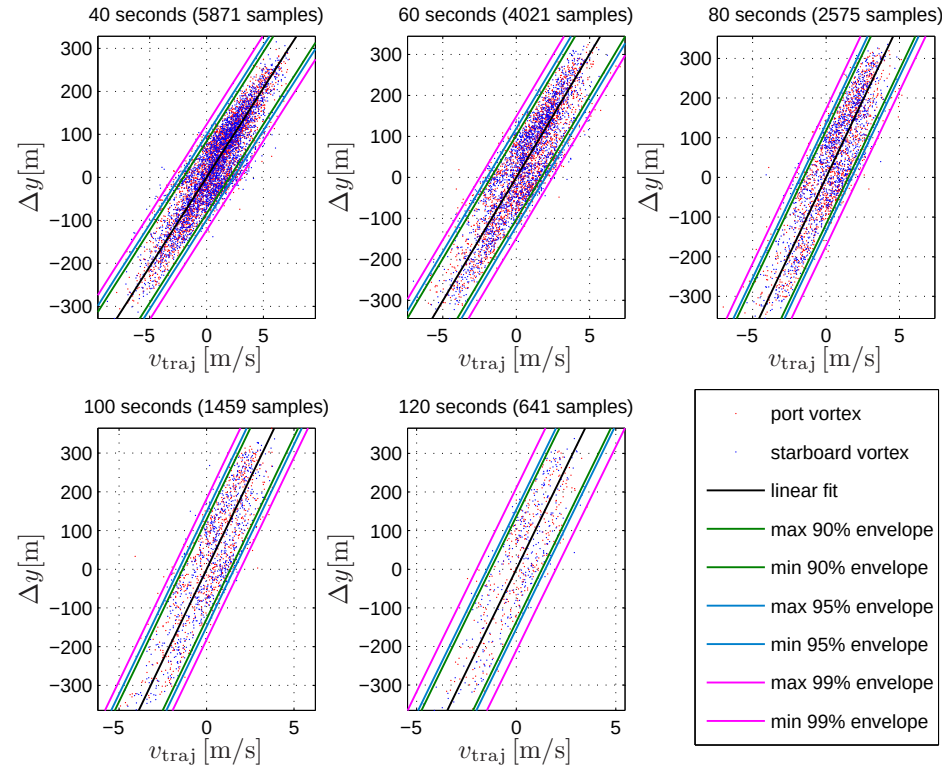


Figure 4.37: Evolution of the net lateral displacement of the vortices, generated by Heavy aircraft, as a function of the crosswind (v_{traj}) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

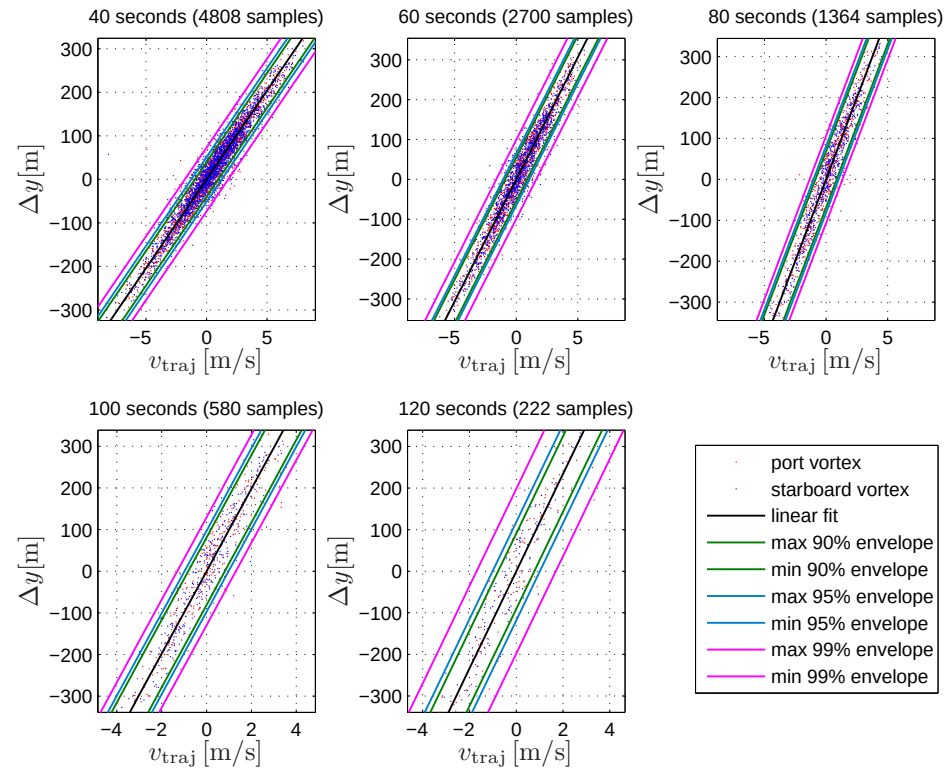


Figure 4.38: Evolution of the net lateral displacement of the vortices, generated by Medium aircraft, as a function of the crosswind (v_{traj}) at different times. At each time, a linear fit (black) shows the mean evolution. Envelopes containing 90% (green), 95% (blue) and 99% (magenta) of the examined vortices are also drawn.

t [s]	a [s]		
	M	H	A
40	40.8	41.6	41.3
60	61.3	60.2	60.6
80	79.8	77.8	78.5
100	99.0	94.2	95.4
120	117.0	105.5	108.2

Table 4.20: Linear fit coefficients ($\Delta y = a v$) which correlate the relative lateral displacement of all the vortices with v_{traj} as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

t [s]	W_{90} [m]			W_{95} [m]			W_{99} [m]		
	M	H	A	M	H	A	M	H	A
40	39.7	84.2	69.0	51.4	98.6	86.6	73.3	123.1	114.6
60	54.6	102.8	91.1	65.0	119.4	108.8	99.1	148.5	140.4
80	68.7	117.9	105.6	81.3	135.4	127.3	106.2	172.4	167.0
100	81.0	128.7	121.4	97.2	148.6	142.2	129.5	182.0	179.6
120	90.6	136.4	131.9	118.4	154.1	155.3	198.2	210.3	188.6

Table 4.21: Envelope half-widths, which correlate the relative lateral displacement of all the vortices with v_{traj} as a function of time. The statistics are performed, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

The mean relative displacement of each vortex centroid is well approximated using the linear fit given by Eq.(4.6). The values of the fit parameters for different times and crosswind are compared in Table 4.22. It appears that the WV generated by Medium aircraft travel a bit faster than those generated by Heavy aircraft. This observation is due to the Near/In Ground Effect. Indeed, close to the ground, the stronger the vortex, the less it is influenced by the wind.

t [s]		$v(h = 10 \text{ m})$	$v(h_0)$	$v(h_{\text{mean}})$	$\bar{v}$	v_{traj}
40	M	1.31	0.95	1.00	1.10	1.02
	H	1.19	1.01	1.02	1.02	1.04
	A	1.24	0.98	1.01	1.06	1.03
60	M	1.26	0.92	0.98	1.05	1.02
	H	1.13	0.96	0.98	0.97	1.00
	A	1.17	0.95	0.98	1.00	1.01
80	M	1.22	0.88	0.95	1.01	1.00
	H	1.09	0.92	0.94	0.93	0.97
	A	1.13	0.91	0.94	0.96	0.98
100	M	1.14	0.87	0.94	0.98	0.99
	H	1.05	0.87	0.90	0.90	0.94
	A	1.07	0.87	0.91	0.92	0.95
120	M	1.07	0.86	0.93	0.94	0.97
	H	0.95	0.80	0.83	0.84	0.88
	A	0.98	0.82	0.86	0.86	0.90
Mean	M	1.23	0.91	0.97	1.03	1.01
	H	1.12	0.94	0.96	0.96	0.99
	A	1.15	0.93	0.96	0.98	0.99

Table 4.22: Temporal evolution and time average of the α parameter of the linear fit, Eq.(4.6), for the different crosswinds. The values are provided, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

As observed in Table 4.9, the values of α again confirm that v_{traj} represents best the wind as experienced by the vortices (value close to 1) for all aircraft category. As previously observed also, $v(h = 10 \text{ m})$ underestimates significantly

the wind experienced by the WV.

Using the results from 40 s to 100 s, the parameters of the linear fit (Eq.(4.7)) are evaluated and reported in Table 4.23. As already observed, the envelopes computed using $v(h = 10 \text{ m})$ have the highest growth rate. This again highlights the potential interest to use a more sophisticated crosswind definition.

		90%			95%			99%		
		M	H	A	M	H	A	M	H	A
$v(h = 10 \text{ m})$	$W_0 \text{ [m]}$	32.3	57.5	49.9	35.0	67.3	59.7	43.3	93.1	80.4
	$\frac{dW}{dt} \text{ [m/s]}$	1.08	0.91	0.96	1.42	1.09	1.20	2.50	1.32	1.68
$v(h_0)$	$W_0 \text{ [m]}$	7.1	51.3	35.6	13.4	61.8	46.7	24.0	71.8	57.4
	$\frac{dW}{dt} \text{ [m/s]}$	0.94	0.84	0.91	1.07	0.91	0.99	1.38	1.21	1.31
$v(h_{\text{mean}})$	$W_0 \text{ [m]}$	11.7	54.2	35.8	18.5	64.6	49.5	28.4	88.3	75.2
	$\frac{dW}{dt} \text{ [m/s]}$	0.72	0.78	0.88	0.82	0.85	0.94	1.11	0.93	1.00
$\bar{v}$	$W_0 \text{ [m]}$	23.4	59.7	41.2	21.6	78.2	63.2	45.4	107.9	97.4
	$\frac{dW}{dt} \text{ [m/s]}$	0.79	0.79	0.93	1.17	0.79	0.93	1.50	0.91	1.03
v_{traj}	$W_0 \text{ [m]}$	12.7	56.3	36.7	20.0	67.4	51.4	40.6	86.4	72.8
	$\frac{dW}{dt} \text{ [m/s]}$	0.69	0.74	0.86	0.77	0.83	0.93	0.88	1.00	1.11

Table 4.23: Parameters of the fit for the envelope half width, Eq.(4.7), using the different crosswind definitions and for the three probability levels. The values are provided, respectively, for the vortices generated by the Medium (M), the Heavy (H) and all (A) aircraft.

The above computed envelopes can also be used in the same way, as proposed in Section 4.6.5. Fig. 4.39 shows, as a function of time, the required crosswind to blow away the vortices generated by Heavy or Medium aircraft. Tables 4.24 and 4.25 compare the required wind for three different time separations considering the WV generated by the Medium, the Heavy and all aircraft, using respectively the 99% and the 95% envelopes.

Using the 99% envelopes, one observes no significant influence of the weight category on the WV transport by the wind. This is probably due to the too low number of data used to compute those envelopes. On the contrary, using the 95% envelopes, the WV generated by Heavy aircraft appear be less influenced by the wind than those generated by Medium aircraft. This is in agreement with the observation of Table 4.22 and is due to the IGE. The envelopes computed using all aircraft WV are in between the two others but are closer to those computed with the Heavy aircraft WV.

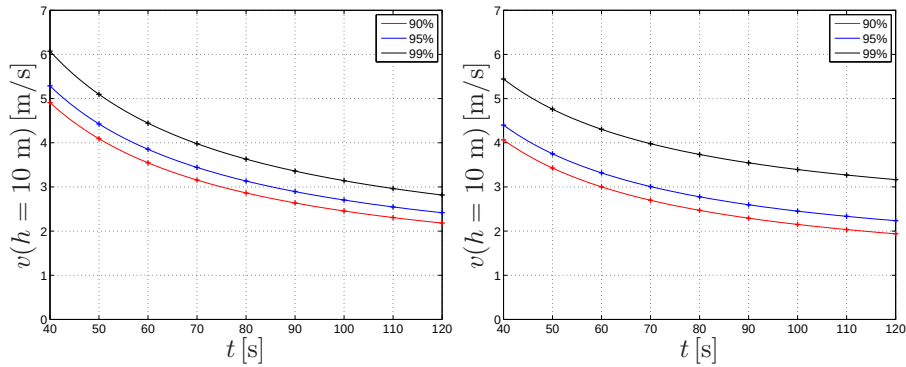


Figure 4.39: Crosswind (at 10 m height) needed to blow away from the runway (here 100 m from the runway centerline) the WV generated by Heavy (left) and Medium (right) aircraft when using the 90, 95 and 99% envelopes.

	Medium			Heavy			All		
t / d	50 m	75 m	100 m	50 m	75 m	100 m	50 m	75 m	100 m
60 s	3.6	4.0	4.3	3.7	4.1	4.4	3.7	3.8	4.4
90 s	3.1	3.3	3.5	2.9	3.1	3.4	3.0	3.0	3.4
120 s	2.8	3.0	3.2	2.4	2.6	2.8	2.6	2.6	2.9

Table 4.24: Table of the crosswind $v(h = 10 \text{ m})$ [m/s] needed to blow away from the runway (here resp. 50 m, 75 m and 100 m from the runway centerline) the WV generated by Medium (left), Heavy (middle) and all aircraft when using the 99% envelopes.

	Medium			Heavy			All		
t / d	50 m	75 m	100 m	50 m	75 m	100 m	50 m	75 m	100 m
60 s	2.6	3.0	3.3	3.1	3.5	3.9	3.0	3.1	3.7
90 s	2.1	2.4	2.6	2.4	2.6	2.9	2.3	2.4	2.8
120 s	1.9	2.1	2.2	2.0	2.2	2.4	2.0	2.1	2.4

Table 4.25: Table of the crosswind $v(h = 10 \text{ m})$ [m/s] needed to blow away from the runway (here resp. 50 m, 75 m and 100 m from the runway centerline) the WV generated by Medium (left), Heavy (middle) and all aircraft when using the 95% envelopes.

4.7 Analysis of the circulation decay depending on the wind

The aim of this further study is to investigate, for different wind conditions, and for different times, the circulation decay compared to the reference low wind situation. The baseline case is chosen as a separation time of 120 s with a total wind amplitude (cross + head) lower than 5 kts. The comparison is based on the Complementary Cumulative Density Function (CCDF) of the measured circulation at the considered time with the considered crosswind. The $\text{CCDF}(\Gamma)$ is defined as the probability to observe a vortex with a circulation higher or equal to Γ . In the computation of the CCDF, when the tracking time is shorter than the considered time and if the last measurement is lower than $150 \text{ m}^2/\text{s}$, one assumes that the vortex was too weak to be measured; the last measured circulation is then used. This is a conservative assumption. It is worth to notice that the CCDF are computed using only the experimental data; no sampling has been considered. Note also that, in this analysis, only vortices generated by Heavy aircraft are taken into consideration.

Five crosswind thresholds are considered: 6 kts, 7 kts, 8 kts, 9 kts, 10 kts. For each of these crosswind situations, a total wind amplitude threshold is also considered (up to 18 kts). Five time separations (60 s, 70 s, 80 s, 90s and 100 s) are tested for the different wind situations. The crosswind, $|v|$, corresponds to the absolute value of the LIDAR crosswind measurement at 10 m height ($v(h = 10 \text{ m})$). The total wind, $|w|$, is taken as the amplitude (head+cross) measured by the anemometer at 10 m height. Obviously, one always has that $|w| > |v|$.

Time sep. / $ v _{\min}$	6 kts	7 kts	8 kts	9 kts	10 kts
100 s	6 kts	7 kts	8 kts	9 kts	10 kts
90 s	9 kts	7 kts	8 kts	9 kts	10 kts
80 s	14 kts	14 kts	12 kts	12 kts	10 kts
70 s	17 kts	17 kts	15 kts	13 kts	12 kts
60 s	18 kts	17 kts	15 kts	13 kts	12 kts

Table 4.26: Table of the total wind $|w|$ needed, for each crosswind and each time separation scenario, to obtain a circulation decay higher or equal to that in the baseline case.

Figs. 4.40 to 4.45 provide the results of the comparison. For information and completeness of the analysis, the CCDF of the measured circulations after 120 s for all wind conditions is also shown on each figure. From these graphs, one could obtain, for each time separation, the wind conditions needed to have a circulation decay higher or equal to that in the baseline situation. In other words, the wind conditions needed to have the CCDF curve, corresponding to the considered time separation, below the CCDF of the baseline case. The results are reported in Table 4.26.

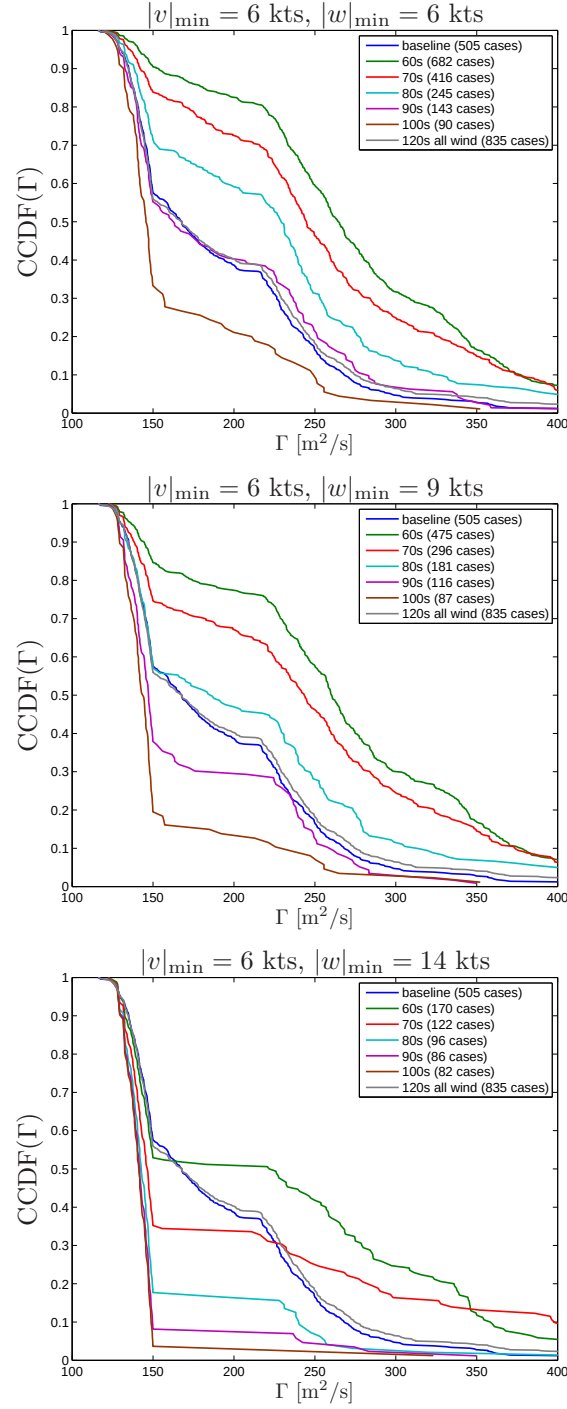


Figure 4.40: Comparison of the CCDF(Γ) obtained for different time separations for a minimum crosswind of 6 kts and a minimum total wind of 6 kts (top), 9 kts (middle) and 14 kts (bottom).

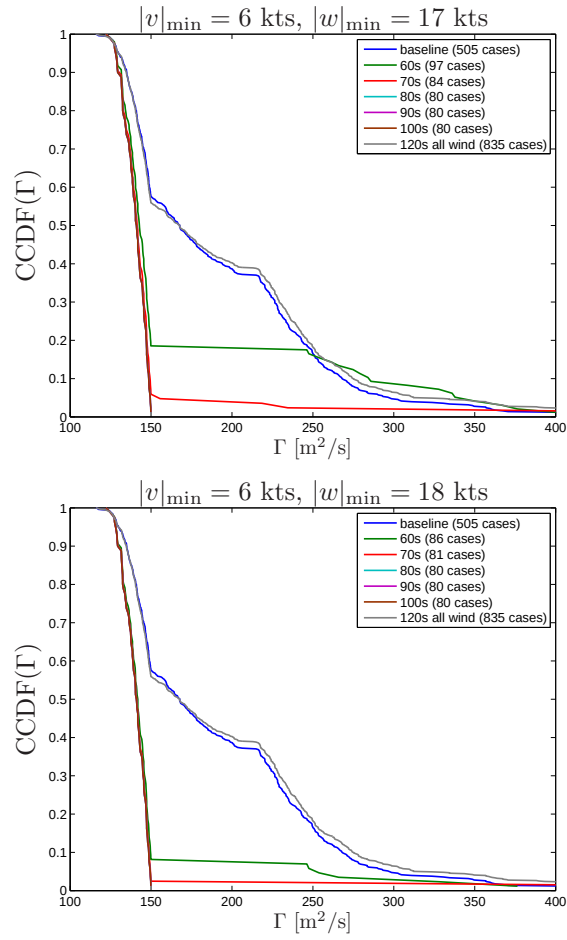


Figure 4.41: Comparison of the $\text{ccdf}(\Gamma)$ obtained for different time separations for a minimum crosswind of 6 kts and a minimum total wind of 17 kts (top) and 18 kts (bottom).

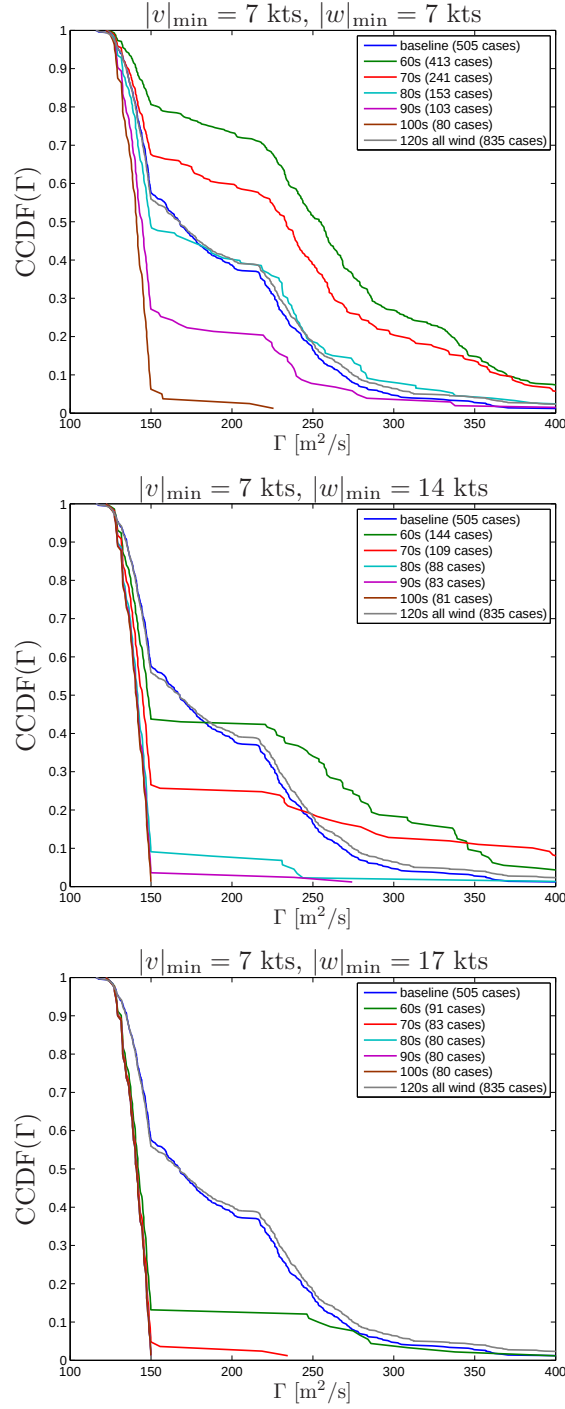


Figure 4.42: Comparison of the CCDF(Γ) obtained for different time separations for a minimum crosswind of 7 kts and a minimum total wind of 7 kts (top), 14 kts (middle) and 17 kts (bottom).

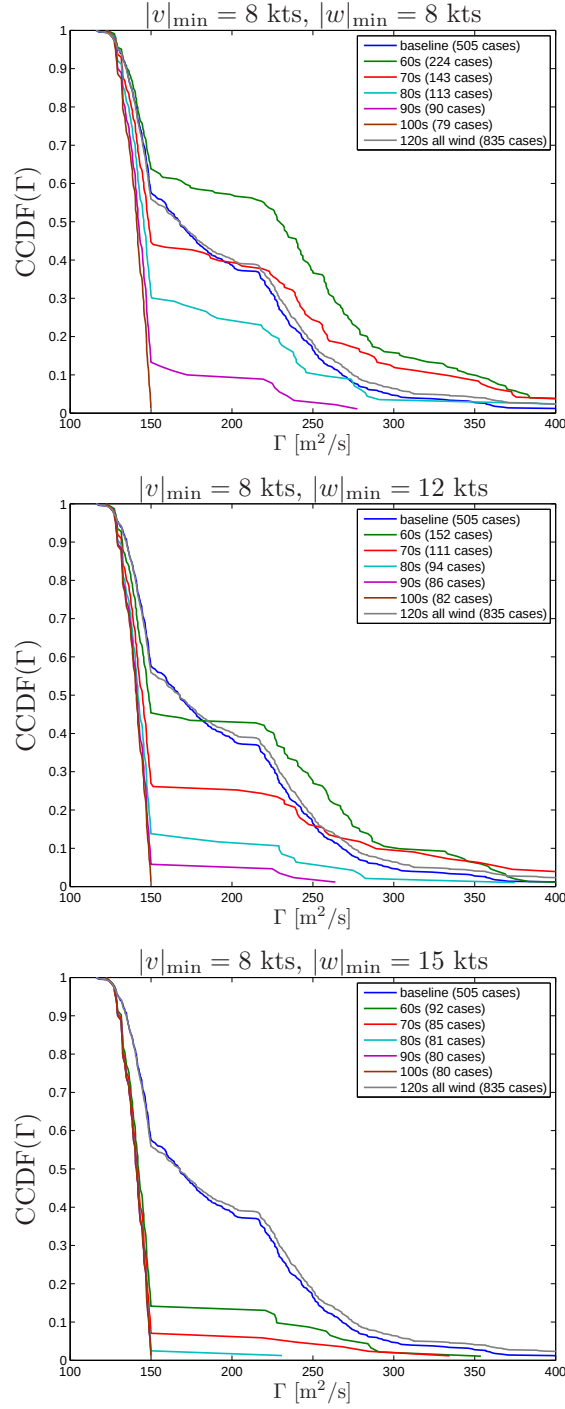


Figure 4.43: Comparison of the CCDF(Γ) obtained for different time separations for a minimum crosswind of 8 kts and a minimum total wind of 8 kts (top), 12 kts (middle) and 15 kts (bottom).

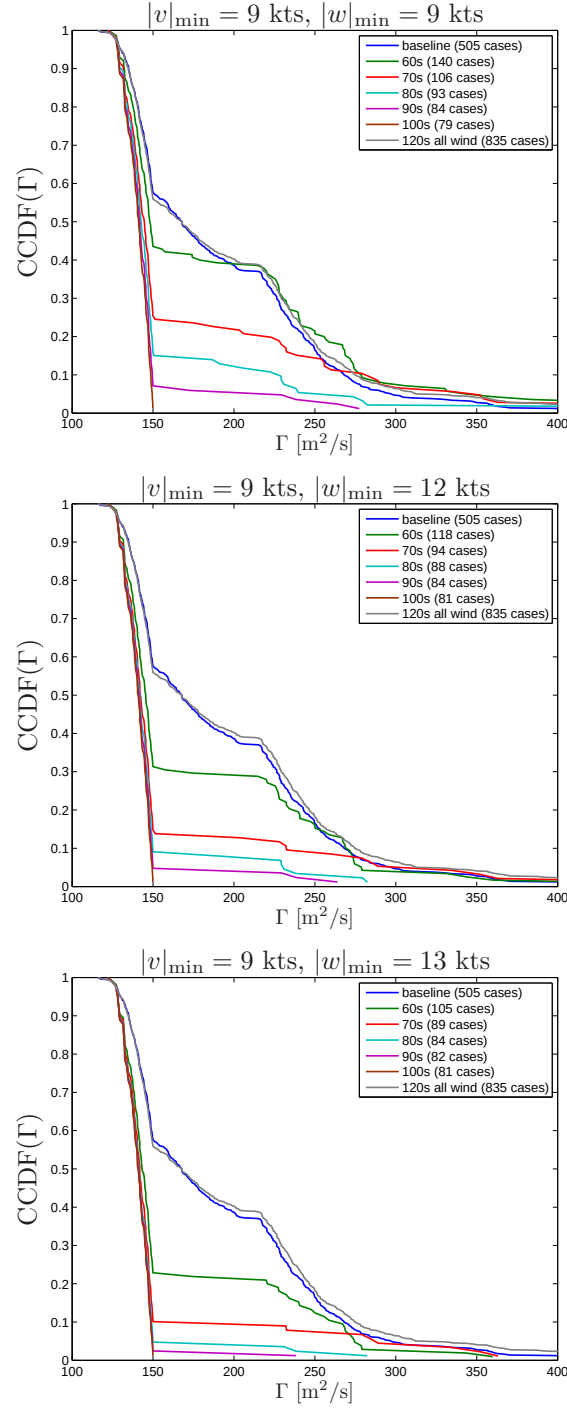


Figure 4.44: Comparison of the CCDF(Γ) obtained for different time separations for a minimum crosswind of 9 kts and a minimum total wind of 9 kts (top), 12 kts (middle) and 13 kts (bottom).

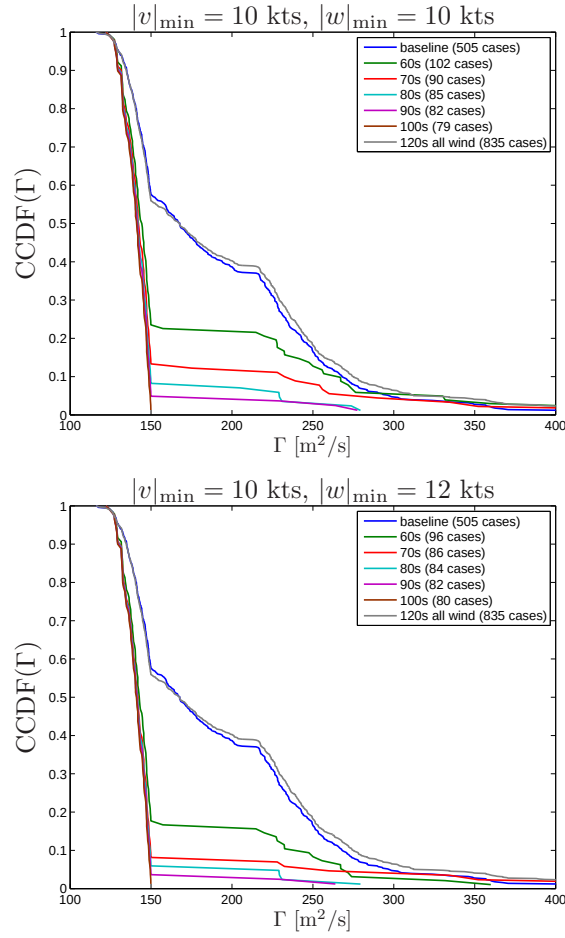


Figure 4.45: Comparison of the CCDF(Γ) obtained for different time separations for a minimum crosswind of 10 kts and a minimum total wind of 10 kts (top) and 12 kts (bottom).

4.8 Conclusions

The EDDF-2 database was extensively analyzed. The database provides significant insight for the investigation of wake vortices (transport and decay), as thousands of cases have been tracked successfully, providing measurements for OGE (few) NGE and IGE cases. After screening of the database, 6950 cases were here chosen for the analysis, following physically defined criteria.

Five definitions of crosswind were proposed, going from the most operational one to the most representative of the wind as experienced by the vortices: $v(h = 10 \text{ m})$, the velocity evaluated at 10 m height; $\bar{v}$, the crosswind averaged over the first 100 meters; $v(h_0)$, the crosswind evaluated at the vortex initial altitude, $v(h_{\text{mean}})$, the crosswind evaluated at the vortex mean altitude; and v_{traj} , the crosswind integrated along the vortex trajectory.

Those crosswinds were compared with each other. The vortex lateral transport was then compared to the crosswind, using the vortex drift velocity. The downwind vortices were, as expected, observed to be convected a bit faster than the upwind vortices. This trend was observed for all possible crosswind definitions and is due to the NGE/IGE effects (as also investigated in FAR-WAKE). This analysis showed that $v(h_{\text{mean}})$ and $\bar{v}$ are good estimators, and $v(h_0)$ a reasonable estimator, of the “net crosswind activity” as experienced by the vortices (v_{traj}). On the contrary, $v(h = 10 \text{ m})$ (i.e., the main crosswind measurement used in today’s airport) poorly correlates with the vortex drift velocity.

We then correlated the vortex net lateral displacement to the crosswind, at different times. The mean correlation and the envelopes containing 90, 95 and 99% of the results were also computed. The envelopes were observed to grow linearly in time, at least for the times of interest to CREDOS. The envelopes computed using $v(h = 10 \text{ m})$ were seen to have the highest growth rate. This is to be expected since $v(h = 10 \text{ m})$ is not a good estimate of the crosswind experienced by the vortices. Using the mean evolution and the envelope growth, it was possible to build the envelopes containing the vortices at any time and for any crosswind.

The computation of the envelopes leads to a proposal for potential use of the results. As an example, the crosswind needed for the vortices to be at least at a certain distance from the runway after a given time was computed. The envelope analysis also showed the interest to use a more sophisticated definition of the crosswind than $v(h = 10 \text{ m})$. In that respect, $\bar{v}$ is interesting as it is

also still operationally feasible to obtain $\bar{v}$ from a wind profiler installed at the airport.

The vortex circulation decay related to the wind was also analyzed. We compared the potentially encountered vortex circulations, after 120 s and in low wind conditions (≤ 5 kts), to those for shorter times and with higher wind conditions. It enables to find, for each time separation (from 60 s to 100 s), the crosswind and total wind thresholds that ensure the encounter of vortices with a circulation lower or equal to that measured in the baseline case (120 s with low wind).

Finally, in the framework of a common publication with DLR and DFS [31], the crosswind thresholds obtained in this work were compared to those obtained in similar analyses. DLR correlated the wake vortex transport to the wind using the EDDF-1 database (see Section 3.2.5 for a database description), whereas DFS also analyzed the EDDF-2 database. In both analyses, they used the crosswind measured at 10 m height by a SONIC anemometer. The comparison of the numbers obtained in the three analyses are provided in Table 4.27, for three time separations and two corridor widths. The results were found to be in good agreement. The differences between the obtained numbers are explained by two factors. First, they are due to the measurement device and location: a LIDAR scanning in the plane of wake vortex evolution provides more accurate crosswind measurements than a SONIC anemometer measuring further away. They are also related to the altitude of wake vortex evolution: the lateral transport of wake vortices evolving in ground proximity (as most of the cases of the EDDF-2 database) is also influenced by ground effect, whereas the lateral transport of vortices evolving at higher altitudes (such as most of the cases of the EDDF-1 database) better correlated with the wind.

Nr	Database	Wind measurement device	d [m]	Clearance time [s]			
				50	60	90	120
1	EDDF-1	Dedicated SONIC	75	4.1	3.7	3.1	2.8
3	EDDF-2	Operational SONIC	75	-	4.6	-	3.6
2	EDDF-2	LIDAR	75	4.0	3.5	2.6	2.2
1	EDDF-1	Dedicated SONIC	100	4.8	4.3	3.5	3.1
3	EDDF-2	Operational SONIC	100	-	5.0	-	3.8
2	EDDF-2	LIDAR	100	-	3.9	2.9	2.4

Table 4.27: Comparison of the crosswind thresholds obtained by the different approaches: Nr=1: DLR analysis, Nr=2: present results, Nr=3: DFS analysis. (Source: Dengler *et al.* [31]).

Chapter 5

Conclusion

This thesis is dedicated to the characterization and the modeling of the behavior of aircraft wake vortices interacting with the atmosphere and/or with the ground.

We performed high-quality Large Eddy Simulations (LES) of wake vortices evolving in various atmospheric conditions, with a particular attention paid to the realistic representation of both the vortices and the atmosphere. The detailed analysis of the simulation results brought new insights that enable a better understanding of the flow physics governing the wake vortex behavior.

Using LES results (including those performed in this thesis) and experimental measurement databases, we extracted the physics of the investigated flows and were able to identify the critical parameters. This physical understanding was then used to develop and improve physics-based operational models. Those models are of course simplified, so that they can be run faster than real-time. Yet, they were verified to correctly represent the transport and decay of wake vortices interacting with the atmosphere and/or with the ground.

The simplified models were then integrated into the Deterministic/ Probabilistic wake Vortex Model (DVM/PVM) that constitute the subcomponents of the operational platform WAKE4D. The model improvements and developments have made the WAKE4D platform a sufficiently mature software so that it is now used as a tool for operational off-line studies and real-time systems. WAKE4D simulation results were indeed used to support the acceptance of a new concept of operation at Charles De Gaulle airport, in the framework of the WIDAO project. This was the first time, in ATM history, that a regulator decision was based, in part, on simulation results. The WAKE4D is also currently

used as wake vortex predictor in two vortex avoidance platforms developed in the framework of the SESAR projects.

In the following, we first summarize the main accomplishments of this thesis. We then propose some extensions and potential improvements for future work.

Achievements

The problems handled in this thesis can be divided into two categories.

a) Wake vortices evolving OGE

The first category is concerned with the evolution of wake vortices away from the ground: in the Out of Ground Effect (OGE) region. At those altitudes, in addition to the wind that simply convects the vortices, the main impact parameters are the atmospheric stratification and turbulence.

The influence of atmospheric stratification combined with weak turbulence was studied. Those cases are more realistic than solely stratification, because stratified atmospheres are most often also weakly turbulent. Stratification was verified to induce a rebound of the vortices and a two-phase circulation decay. The addition of turbulence, even if weak, was seen to strongly affect the results as it quickens the development of instabilities, which favors 3-D interactions and creation of stronger turbulence, and hence enhances the decay of the vortices. Based on those results, improved models were proposed for the vortex rebound and circulation decay.

The influence of stratification and turbulence on the long-wavelength Crow instability development was also studied. In weak turbulence, the instability growth rate was seen to increase for cases with weak to moderate stratification. On the contrary, for higher stratification levels, the vortices were observed to be destroyed before the deformation becomes significant. We then proposed a stratification level threshold $N^* = 0.8 - 0.9$, above which the Crow instability does not significantly develop (in particular, there is no reconnection of the vortex tubes).

The atmospheric turbulence was also verified to trigger the Crow instability. The stronger the turbulence level, the sooner large vortex deformations are visible. Later on, the vortices were observed to link and form vortex rings only in weak turbulence cases, whereas, in moderate to strong turbulence, smaller wavelength instabilities also significantly develop and prevent the vortices from linking. Without stratification, the turbulence level threshold, above which no

vortex rings are formed, was estimated to be in the range $2.5 \cdot 10^{-4} < \epsilon^* < 1.5 \cdot 10^{-2}$ (i.e., $0.063 < \eta < 0.25$). A simplified model for the Crow instability development depending on the atmospheric turbulence level was also proposed.

Stratification, turbulence and the Crow linking process all significantly influence the vortex decay. They all induce a two-phase decay. We proposed an improved model for the starting time of the fast-decay phase (i.e., the time-to-demise) as a function of the atmospheric turbulence and stratification. In moderate to strong turbulence ($\epsilon^* \geq 10^{-2}$) and/or for high stratification levels ($N^* \geq 0.8$), the second-phase of decay was seen to be very uniform along the vortex. This is not the case in weak stratification and weak turbulence: some planes then exhibit a very fast second phase decay whereas others only see a slight increase of their circulation decay rate. For those cases, the measurement of the mean circulation evolution is hence not a good image of the whole vortex hazard. A better representation is provided by the mean evolution of the parts of the vortices that are still “alive”. Models of the effects of turbulence and stratification were proposed for the evolution of the mean circulation of both all the vortices and the parts of the vortices that are still “alive”. The LES results obtained in this thesis will also be used for wake vortex encounter analyses in the framework of an ongoing project at the Technical University of Berlin (TUB, group of prof. R. Luckner). The project objective is to fly, in a simulator, into the obtained 3-D LES velocity fields and to characterize the wake vortex encounter severity, for various turbulence and stratification levels (hence, different wake vortex intensities and topologies).

Finally, we analyzed the statistical equilibrium state reached by a two-vortex system evolving in a weak to moderate turbulent atmospheres and without possible Crow instability development. This equilibrium was seen to be quite universal in terms of spectrum, circulation profile and decay rate.

b) Wake vortices evolving NGE/IGE

The second category of problems, investigated in this thesis, is concerned with the evolution of wake vortices close to the ground: in the Near- or In-Ground Effect (NGE/IGE) region. At those altitudes, the vortices are strongly influenced by the wind and the ground proximity. For those cases, existing LES results and measurement databases were analyzed and used to improve and develop models.

An improved model for wake vortex behavior IGE was proposed and validated. It is able to dynamically reproduce the combined effects, on the wake

vortex behavior (transport and decay), of the ground proximity and of the wind (both head- and crosswind components). The model was shown to represent well the flow dynamics even at late stages (after the vortex rebound), while maintaining a limited computational time.

The DVM, and in particular the IGE model, was calibrated and assessed based on LES results and on the measurements of three campaigns of wake vortices IGE and NGE. It was found to properly reproduce the wake vortex behavior both in terms of transport and circulation decay for times up to $t^* = 4 - 5$, the typical times of interest for operational applications (for large aircraft, $t^* = 4$ corresponds approximatively to 120 s).

Finally, we analyzed the EDDF-2 database where the transport and decay of many wake vortices generated NGE or IGE were measured. The wake vortex lateral transport was first statistically correlated to the crosswind. From this analysis, we were able to propose a model for the evolution of the envelope containing the vortices (with a certain probability) as a function of the crosswind. The vortex circulation decay was then also correlated to the wind (both cross- and total wind). Both analyses eventually provide tools to establish separation standards for departure operations, that also depend on the wind.

Perspectives

Based on the conclusions of this thesis, we here propose some recommendations for extensions of the present study.

a) LES of wake vortices

Nowadays, quality Large Eddy Simulations appear to be the main way (if not the only way) to obtain realistic and detailed physics of aircraft wake vortices OGE. LES indeed enables to control and “measure” both the wake vortex system and the atmospheric conditions. On the contrary, Lidars, and even more so radars, provide reasonable measurements of the vortex position but often fail to correctly post-process the circulation. Moreover, as, OGE, the circulation may significantly vary along the vortex, a single circulation measurement can be misleading.

In this thesis, we attempted to build a (ϵ_f^*, N^*) -plane, shown in Fig. 5.1, for the influences of the atmospheric conditions on the wake vortex behavior. The cases to be investigated in the future should aim to fill the remaining gaps in this plane. Some interesting simulations would then, for instance, be cases

with $\epsilon_f^* \simeq 2 \cdot 10^{-3}$ with neutral to moderate stratification levels ($0 \leq N^* \leq 0.75$). This would allow a better determination of the critical ϵ^* above which no Crow linking is observed. It would also be interesting to study cases with weak turbulence and very weak stratification levels ($\epsilon_f^* = 2.42 \cdot 10^{-4}$ and $N^* \simeq 0.1$). This would then enable to assess, for very small N^* , the models proposed in this thesis. The suggested cases to be investigated are also shown in the (ϵ_f^*, N^*) -plane of Fig. 5.1.

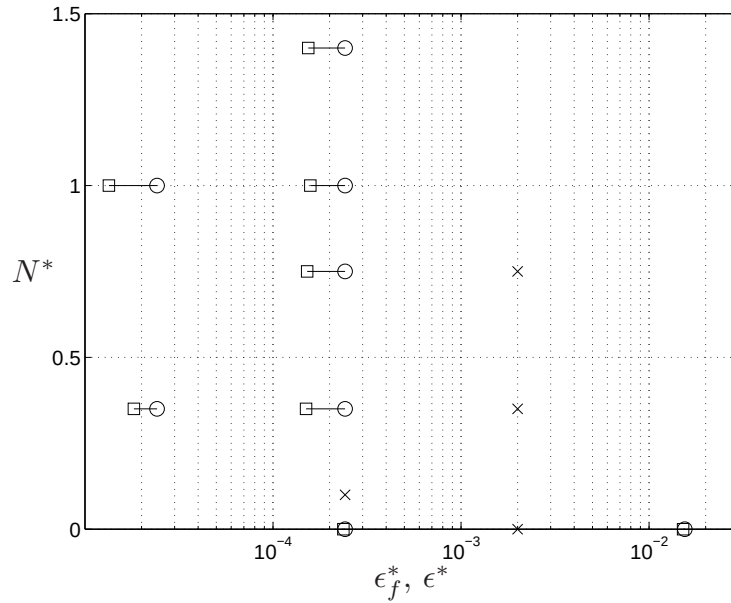


Figure 5.1: (ϵ_f^*, N^*) -plane of the cases investigated in this thesis. Each case is represented by the forcing amplitude ϵ_f^* ($\circ$) and the Brunt-Väisälä frequency N^* . For every case, we also show the resulting kinetic energy dissipation rate ϵ^* ($\square$) (recall that, for stratified cases, $\epsilon_f^* = \epsilon^* + \epsilon_P^*$). The cases that could be interesting to investigate in future studies are also indicated ($\times$).

For future LES studies, we would also recommend to use a more realistic initial condition for the wake vortex pair. A good candidate would then be the “turbulent vortex at equilibrium”. We indeed showed that, a two-vortex system surrounded by turbulence reaches a quite universal equilibrium state. As the roll-up behind an aircraft is always surrounded by turbulence, it is then believed that the obtained equilibrium circulation profile is representative of a turbulent rolled-up vortex system as developed behind a real aircraft.

Another parameters of interest for further investigation is the core radius. Its exact value for aircraft wake vortices is indeed still somewhat undetermined.

In the present stage of knowledge of aircraft wake vortex physics, it is believed that $0.03 \leq \frac{r_c}{b_0} \leq 0.05$. It would hence be interesting to examine the behavior, and in particular the Crow instability development, of vortices with smaller r_c . Those LES would however require a higher resolution.

Finally, the pseudo-spectral code used in this thesis could also be improved to allow simulations in larger domains or with increased resolutions. Currently, the code is parallelized in only one direction, each processor handling a slice of the domain. For large problems, the computational time is then much affected by the operations (including the 2-D FFT) that each processor has to perform in its sub-domain. A parallelization of the code in the three directions (as in [2]) would enable to solve even larger problems, still in an acceptable time, using more processors.

b) Operational modeling of wake vortex behavior

The operational models contained in the WAKE4D have reached a sufficient maturity level to be trusted and hence used in operational studies. The use of WAKE4D would furthermore allow a better definition of the operational requirements and hence of the required improvements of the input and output definitions. New databases (experimental or numerical) would also be of great interest for further calibration and assessment of the models.

We finally provide here three potential improvements to the WAKE4D and its subcomponents.

- *Parallelization*

The architectures of the PVM and of the WAKE4D (with multiple independent runs), easily enable parallel computing, which would significantly reduce the computational time. This was already partially tested using OpenMP and was seen to be successful.

- *Use of GPU architecture*

General-Purpose computation on Graphics Processing Units (GPGPU) presents a significant advantage on CPU computing for algorithms where processing (with simple operations) of large blocks of data is done in parallel. One could thus imagine to efficiently use GPGPU in the WAKE4D, each computational gate being simulated on some GPU processor cores. GPGPU would also present an interest in the DVM, for the computation of transport IGE, that requires the evaluation of the Biot-Savart law with several particles (primary and secondary).

- *Correlation between the varied parameters*

An interesting functionality, that could be added to the PVM, is the imposition of constraints between the varied parameters. Currently, in the PVM, all randomized parameters are indeed picked independently. Some future developments could concentrate on developing a sampling method that could enforce some correlations between the inputs. For instance, the correlation between the turbulence and the stratification levels would restrict our exploration of the (ϵ^*, N^*) -plane to a physically-relevant region (i.e., the lower left part of the plane).

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Appendix A

Description of the Proctor and of the Proctor-Winckelmans two-scale models

The Proctor model (see [83]) assumes a circulation distribution in two parts, both parts being based on an exponential model. In the part $0 \leq r \leq \alpha r_c$, the circulation distribution profile corresponds to that of the Gaussian model (see Table 1.3). In the part $r > \alpha r_c$, the circulation distribution no longer scales with r_c but with the wingspan b and the exponent in the exponential is reduced. The model reads:

$$\begin{aligned}\tilde{\Gamma}(r) &= C \left(1 - \exp \left(-\beta \left(\frac{r}{r_c} \right)^2 \right) \right) & \text{for } 0 \leq r \leq \alpha r_c \\ \tilde{\Gamma}(r) &= 1 - \exp \left(-\beta_o \left(\frac{r}{b} \right)^q \right) & \text{for } r \geq \alpha r_c\end{aligned}\tag{A.1}$$

where $\beta = 1.256$ (see Table 1.3) and C is computed so that the model is continuous at $r = \alpha r_c$. Hence :

$$C = \frac{(1 - \exp(-\beta_o (\frac{\alpha r_c}{b})^q))}{(1 - \exp(-\beta \alpha^2))}$$

Originally, Proctor [83] uses $\beta_o = 11.88$, $q = 3/4$ and $\alpha = 1.0$, and thus $C = 1.398 \left(1 - \exp \left(-11.88 \left(\frac{r_c}{b} \right)^{3/4} \right) \right)$ (the model with this set of coefficients is denoted “P1”). The advantage of this model is that it takes into account two scales in the vortex representation. However its derivative is discontinuous at $r = \alpha r_c$. More recently, Proctor [84] slightly improved the parameters values with $\beta_o = 10$, $q = 3/4$ and $\alpha = 1.4$, hence $C = 1.093 \left(1 - \exp \left(-10 \left(\frac{1.4 r_c}{b} \right)^{3/4} \right) \right)$. The model with this new set of coefficients is denoted “P2”. For typical core radius size values ($r_c/b = 0.03 \cdots 0.05$), using P2, the derivative is indeed almost continuous at $r = 1.4 r_c$. The profiles using P1 and P2 are compared in Fig. A.1.

The idea of the two-scale model of Proctor-Winckelmans (denoted “PW”) is to maintain the two-scale approach and to use a function that goes smoothly from part I to part II. The two parts are written:

$$\begin{aligned} \tilde{\Gamma}(r) &= 1 - \exp \left(-\beta_i \left(\frac{r}{b_0} \right)^2 \right) = 1 - \exp (-a_i \tilde{r}^2) \\ \tilde{\Gamma}(r) &= 1 - \exp \left(-\beta_o \left(\frac{r}{b_0} \right)^q \right) = 1 - \exp (-a_o \tilde{r}^q) \end{aligned} \quad (\text{A.2})$$

The model, that uses only one equation, is established in such a way that the distribution corresponds to part I for small r and to part II for large r . It reads:

$$\begin{aligned} \tilde{\Gamma} &= 1 - \exp \left(- \frac{\beta_i \left(\frac{r}{b_0} \right)^2}{\left(1 + \left(\frac{\beta_i}{\beta_o} \left(\frac{r}{b_0} \right)^{(2-q)} \right)^p \right)^{1/p}} \right) \\ &= 1 - \exp \left(- \frac{a_i \tilde{r}^2}{\left(1 + \left(\frac{a_i}{a_o} \tilde{r}^{(2-q)} \right)^p \right)^{1/p}} \right), \end{aligned} \quad (\text{A.3})$$

where the additional parameter p enables to adjust the maximum azimuthal velocity, without varying much the core radius r_c/b_0 . For aircraft wake vortices, typical parameters values are: $\beta_o = 10$, $q = 3/4$, $p = 3$ to 5, and β_i adjusted depending on the chosen value for $\frac{r_c}{b_0}$. Using this formulation, it is to be noted that because $\frac{\beta_i}{\beta_o} \approx \left(\frac{r_c}{b_0} \right)^{(q-2)}$, we also have $a_i \approx a_o$. Figure A.2 provides the model results for two values of p and compares the obtained profiles to that of Proctor model P2. It appears that the P2 model is very similar to the PW model using $p = 6$.

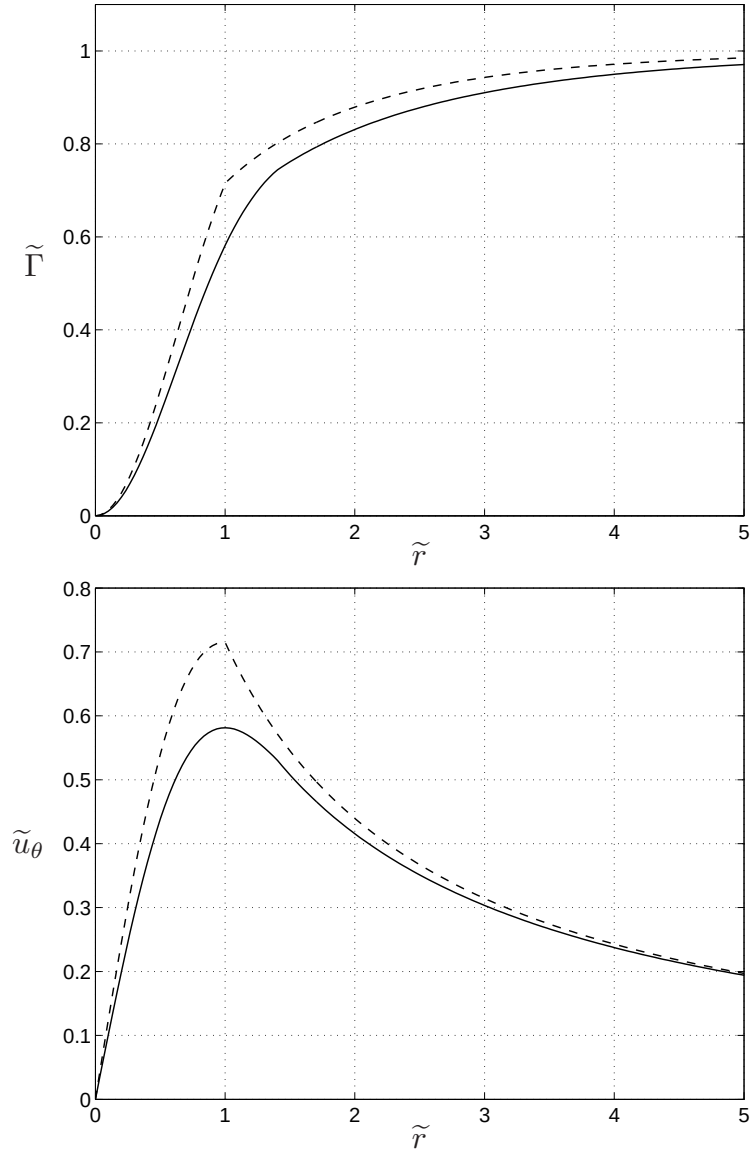


Figure A.1: Circulation (top) and azimuthal velocity (bottom) distribution profiles using Proctor's models P1 (dash) and P2 (solid).

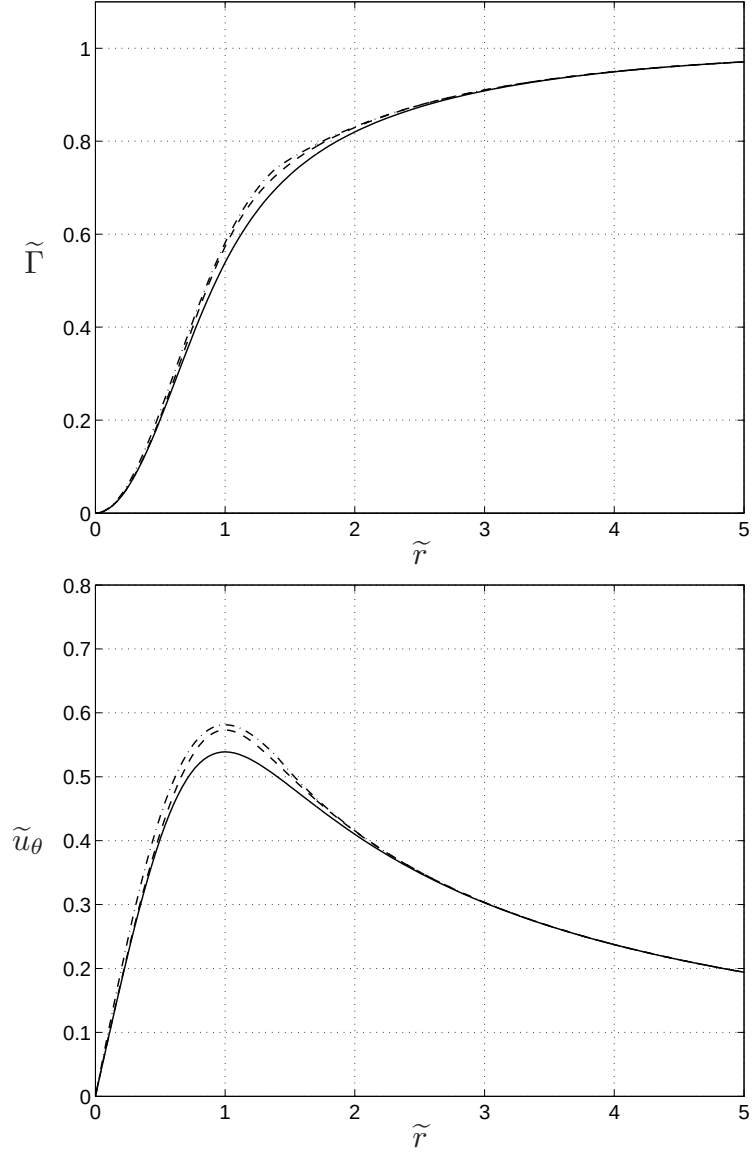


Figure A.2: Circulation (top) and azimuthal velocity (bottom) distribution profiles using the PW model for vortices with $r_c/b_0 = 0.05$. The results are obtained using $q = 3/4$, $\beta_o = 10$ (hence, $a_o = 1.057$) and with : $p = 3$ (hence, $\beta_i = 365.5$, $a_i = 0.914$) (solid) and $p = 5$ (hence, $\beta_i = 369.8$, $a_i = 0.925$) (dash). The results of the P2 model are also displayed for comparison (dash-dot).

Appendix B

Stratified and weakly turbulent atmospheres: complementary analysis

In this appendix, we further present and analyze the simulation of the atmospheric stratified and weakly turbulent fields used in Chapter 2, Section 2.4. Those fields were already partially analyzed in Section 2.4.2. We here provide some additional properties that are also compared with the literature.

In a simulation of forced stratified turbulence, the equations of conservation of momentum and temperature read:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + b \mathbf{e}_z + \mathcal{D}_{\mathbf{u}} + \mathcal{F}, \quad (\text{B.1})$$

$$\frac{\partial b}{\partial t} + \mathbf{u} \cdot \nabla b = -N^2 w + \mathcal{D}_b, \quad (\text{B.2})$$

where $b = \frac{g}{\Theta_0} \tilde{\Theta}$ is the buoyancy, $\mathcal{D}_{\mathbf{u}}$ and $\mathcal{D}_b$ are the dissipation operators of velocity and buoyancy, respectively, and $\mathcal{F}$ is the forcing.

Defining the horizontal and vertical kinetic energy $E_H = \frac{1}{2} \langle u^2 + v^2 \rangle$ and $E_V = \frac{1}{2} \langle w^2 \rangle$, and the potential energy $E_P = \frac{1}{2} \frac{1}{N^2} \langle b^2 \rangle$, the evolution equations

of the domain-averaged energy components are then written:

$$\begin{aligned}
\frac{dE_H}{dt} &= \Phi_H - \epsilon_H + \epsilon_{f,H}, \\
\frac{dE_V}{dt} &= \Phi_V + \text{BF} - \epsilon_V + \epsilon_{f,V}, \\
\frac{dE_P}{dt} &= -\text{BF} - \epsilon_P,
\end{aligned} \tag{B.3}$$

where $\Phi_H = \left\langle P \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right\rangle$ and $\Phi_V = \left\langle P \left(\frac{\partial w}{\partial z} \right) \right\rangle$ are the pressure-strain correlation terms, $\text{BF} = \langle b w \rangle$ is the buoyancy flux, ϵ_H and ϵ_V are the kinetic energy dissipations in the horizontal and vertical directions, ϵ_P is the potential energy dissipation; and $\epsilon_{f,H}$ and $\epsilon_{f,V}$ are the kinetic energy forcing in the horizontal and vertical directions. With stratification, the vertical kinetic energy is thus partially transferred to the potential energy. This leads to a system that is less energetic in the vertical direction. Hence, stratification tends to create large horizontal structures. This layering of the flow, already seen in Fig. 2.11, is further observed in the 3-D visualizations provided in Fig. B.1 for a case weakly stratified (case 1) and a case strongly stratified (case 3).

As, by continuity, one has $\Phi_H + \Phi_V = 0$, the total energy ($E_{\text{tot}} = E_H + E_V + E_P = E_k + E_P$) hence evolved following:

$$\begin{aligned}
\frac{dE_{\text{tot}}}{dt} &= -\epsilon_H - \epsilon_V - \epsilon_P + \epsilon_f, \\
&= -\epsilon - \epsilon_P + \epsilon_f,
\end{aligned} \tag{B.4}$$

with $\epsilon = \epsilon_H + \epsilon_V$ and $\epsilon_f = \epsilon_{f,H} + \epsilon_{f,V}$. We also define $\eta = \frac{\epsilon_P}{\epsilon}$, the mixing efficiency. When the turbulence reaches a statistically stationary state one has, on average, $\frac{dE_{\text{tot}}}{dt} = 0$. The energy forcing ϵ_f is then balanced by the total dissipation $\epsilon + \epsilon_P$. Table B.1 provides the values of the different energies at statistical equilibrium. One notes that, for a fixed forcing level, as the stratification level increases, the total energy of the system E_{tot} and the potential energy increase. The repartition of the kinetic energy between the horizontal and the vertical directions also changes. For the isotropic case, $2/3$ of E_k is contained in E_H and $1/3$ in E_V , as expected. For the stratified cases, the horizontal kinetic energy contains between 78% (for case 1) to 86% (for case 6s) of the total kinetic energy.

The values of the energy dissipations measured at equilibrium are provided in Table B.2. With stratification, the kinetic energy dissipation is reduced

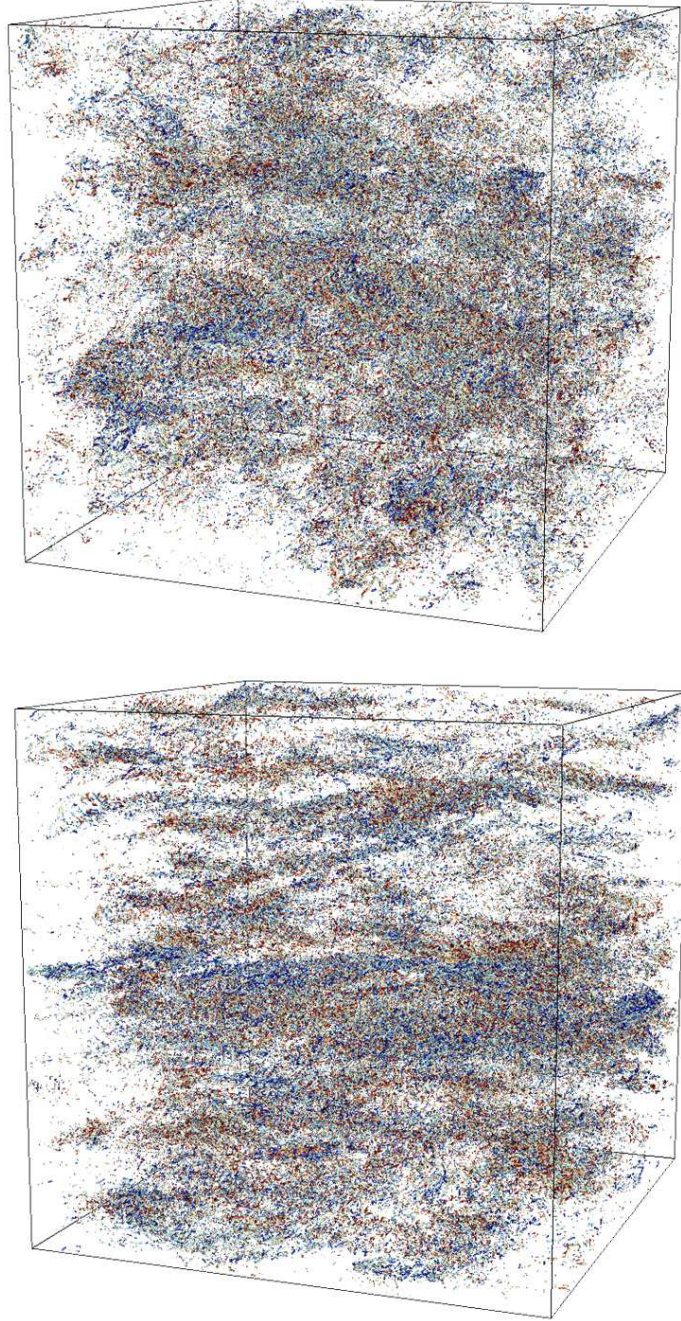


Figure B.1: Visualization of the stratified turbulent fields using iso-surfaces of λ_2 colored by the horizontal vorticity component ω_x for case 1 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_1^*$, top) and case 3 ($N^* = 1$, $\epsilon_f^* = \epsilon_1^*$, bottom).

Case	N^*	ϵ_f^*	E_H^*	E_V^*	E_k^*	E_P^*	E_{tot}^*
0	0.0	ϵ_1^*	$5.98 \cdot 10^{-3}$	$3.05 \cdot 10^{-3}$	$9.03 \cdot 10^{-3}$	0.0	$9.03 \cdot 10^{-3}$
0s	0.0	ϵ_1^*	$3.84 \cdot 10^{-3}$	$1.94 \cdot 10^{-3}$	$5.78 \cdot 10^{-3}$	0.0	$5.78 \cdot 10^{-3}$
1	0.35	ϵ_1^*	$6.11 \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$	$7.86 \cdot 10^{-3}$	$2.93 \cdot 10^{-3}$	$10.79 \cdot 10^{-3}$
1s	0.35	ϵ_1^*	$3.59 \cdot 10^{-3}$	$1.09 \cdot 10^{-3}$	$4.68 \cdot 10^{-3}$	$1.78 \cdot 10^{-3}$	$6.46 \cdot 10^{-3}$
2	0.75	ϵ_1^*	$7.98 \cdot 10^{-3}$	$2.00 \cdot 10^{-3}$	$9.98 \cdot 10^{-3}$	$3.99 \cdot 10^{-3}$	$13.97 \cdot 10^{-3}$
2s	0.75	ϵ_1^*	$4.36 \cdot 10^{-3}$	$0.99 \cdot 10^{-3}$	$5.35 \cdot 10^{-3}$	$1.90 \cdot 10^{-3}$	$7.25 \cdot 10^{-3}$
3	1.0	ϵ_1^*	$8.13 \cdot 10^{-3}$	$2.27 \cdot 10^{-3}$	$10.40 \cdot 10^{-3}$	$4.50 \cdot 10^{-3}$	$14.90 \cdot 10^{-3}$
3s	1.0	ϵ_1^*	$4.58 \cdot 10^{-3}$	$1.12 \cdot 10^{-3}$	$5.70 \cdot 10^{-3}$	$2.04 \cdot 10^{-3}$	$7.74 \cdot 10^{-3}$
3 _{SRVM}	1.0	ϵ_1^*	$5.09 \cdot 10^{-3}$	$1.17 \cdot 10^{-3}$	$6.26 \cdot 10^{-3}$	$1.99 \cdot 10^{-3}$	$8.25 \cdot 10^{-3}$
4s	1.4	ϵ_1^*	$4.81 \cdot 10^{-3}$	$1.38 \cdot 10^{-3}$	$6.19 \cdot 10^{-3}$	$2.88 \cdot 10^{-3}$	$9.07 \cdot 10^{-3}$
5	0.35	ϵ_2^*	$2.20 \cdot 10^{-3}$	$0.499 \cdot 10^{-3}$	$2.70 \cdot 10^{-3}$	$1.18 \cdot 10^{-3}$	$3.88 \cdot 10^{-3}$
6s	1.0	ϵ_2^*	$2.39 \cdot 10^{-3}$	$0.385 \cdot 10^{-3}$	$2.77 \cdot 10^{-3}$	$0.779 \cdot 10^{-3}$	$3.35 \cdot 10^{-3}$

Table B.1: Energies obtained at statistical equilibrium for the investigated cases ($\epsilon_1^* = 2.42 \cdot 10^{-4}$, $\epsilon_2^* = 2.42 \cdot 10^{-5}$).

as part of the forcing is dissipated through the potential energy. It is also interesting to examine the ratio $\frac{\epsilon_H}{\epsilon}$, which also constitutes a measure of the anisotropy of the field. This ratio is seen to increase with the stratification level, again indicating that the flow has become layered with large horizontal structures, as seen in Fig. 2.11. The values of the mixing efficiency $\eta = \frac{\epsilon_P}{\epsilon}$ are also reported in Table B.2 and are discussed in Section 2.4.2.

In addition to the spectra provided in Section 2.4.2, Fig. 2.12, we compare, in Fig. B.2, the obtained kinetic energy spectra for the forcing level ϵ_1^* in the large computational domain and for four stratification levels. For all cases, the forced modes (around $k^* = 2$) are clearly visible. The observed rapid drop of the spectra for the highest wave numbers is typical of the HV SGS model. For the neutral and weakly stratified cases (cases 0 and 1), an inertial range, with the characteristic $k^{-5/3}$ evolution, is observed. For higher stratification levels, the anisotropy of the turbulence makes difficult any interpretation of such 3-D spectrum.

To further characterize the spectral behavior of the stratified turbulence, a decomposition of the flow is performed between vortical mode and internal wave components. This decomposition, introduced by Riley *et al.* [93] and

Case	N^*	ϵ_f^*	ϵ_H^*	ϵ_V^*	ϵ^*	ϵ_P^*	ϵ_H/ϵ	η
0	0.0	ϵ_1^*	$1.58 \cdot 10^{-4}$	$0.79 \cdot 10^{-4}$	$2.37 \cdot 10^{-4}$	0.0	0.666	0.0
0s	0.0	ϵ_1^*	$1.59 \cdot 10^{-4}$	$0.80 \cdot 10^{-4}$	$2.39 \cdot 10^{-4}$	0.0	0.666	0.0
1	0.35	ϵ_1^*	$1.01 \cdot 10^{-4}$	$0.490 \cdot 10^{-4}$	$1.50 \cdot 10^{-4}$	$0.968 \cdot 10^{-4}$	0.676	0.645
1s	0.35	ϵ_1^*	$0.988 \cdot 10^{-4}$	$0.492 \cdot 10^{-4}$	$1.48 \cdot 10^{-4}$	$1.06 \cdot 10^{-4}$	0.668	0.714
2	0.75	ϵ_1^*	$1.07 \cdot 10^{-4}$	$0.450 \cdot 10^{-4}$	$1.52 \cdot 10^{-4}$	$0.812 \cdot 10^{-4}$	0.705	0.535
2s	0.75	ϵ_1^*	$1.04 \cdot 10^{-4}$	$0.460 \cdot 10^{-4}$	$1.50 \cdot 10^{-4}$	$0.872 \cdot 10^{-4}$	0.691	0.582
3	1.0	ϵ_1^*	$1.14 \cdot 10^{-4}$	$0.440 \cdot 10^{-4}$	$1.58 \cdot 10^{-4}$	$0.800 \cdot 10^{-4}$	0.723	0.506
3s	1.0	ϵ_1^*	$1.06 \cdot 10^{-4}$	$0.440 \cdot 10^{-4}$	$1.50 \cdot 10^{-4}$	$0.838 \cdot 10^{-4}$	0.706	0.560
3SRVM	1.0	ϵ_1^*	$1.01 \cdot 10^{-4}$	$0.580 \cdot 10^{-4}$	$1.59 \cdot 10^{-4}$	$0.702 \cdot 10^{-4}$	0.639	0.443
4s	1.4	ϵ_1^*	$1.12 \cdot 10^{-4}$	$0.420 \cdot 10^{-4}$	$1.54 \cdot 10^{-4}$	$0.814 \cdot 10^{-4}$	0.731	0.530
5	0.35	ϵ_2^*	$1.29 \cdot 10^{-5}$	$0.530 \cdot 10^{-5}$	$1.82 \cdot 10^{-5}$	$0.959 \cdot 10^{-5}$	0.707	0.526
6s	1.0	ϵ_2^*	$1.08 \cdot 10^{-5}$	$0.260 \cdot 10^{-5}$	$1.34 \cdot 10^{-5}$	$0.617 \cdot 10^{-5}$	0.804	0.461

Table B.2: Energy dissipations obtained at statistical equilibrium for the investigated cases ($\epsilon_1^* = 2.42 \cdot 10^{-4}$, $\epsilon_2^* = 2.42 \cdot 10^{-5}$).

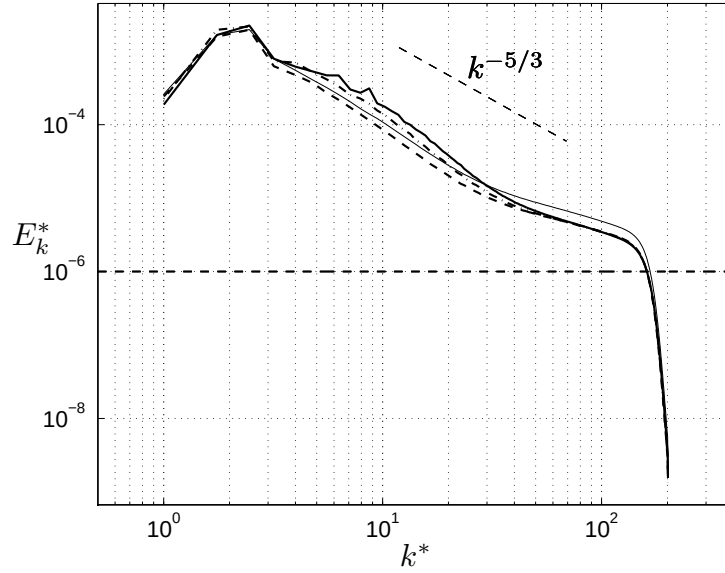


Figure B.2: Turbulent kinetic energy spectra for cases 0 (thin solid), 1 (dash), 2 (dash-dot) and 3 (thick solid). The very high wavenumbers damped by the HV SGS model are typically those below the horizontal dashed line. The slope $k^{-5/3}$ is also shown (thin dash).

Lilly [73], is based on the fact that, in spectral space, due to mass conservation ($\nabla \cdot \mathbf{u} = 0$), the velocity vector is perpendicular to the wave number vector. The velocity can then be decomposed into two components by defining a new coordinate system in Fourier space:

$$\mathbf{e}'_1(\mathbf{k}) = \frac{\mathbf{k} \times \mathbf{g}}{\|\mathbf{k} \times \mathbf{g}\|} = \frac{(-k_y, k_x, 0)}{\sqrt{k_x^2 + k_y^2}}, \quad (\text{B.5})$$

$$\mathbf{e}'_2(\mathbf{k}) = \frac{\mathbf{k} \times \mathbf{k} \times \mathbf{g}}{\|\mathbf{k} \times \mathbf{k} \times \mathbf{g}\|} = \frac{(-k_z k_x, -k_z k_y, (k_x^2 + k_y^2))}{\sqrt{k_x^2 + k_y^2 + k_z^2} \sqrt{k_x^2 + k_y^2}}, \quad (\text{B.6})$$

$$\mathbf{e}'_3(\mathbf{k}) = \frac{\mathbf{k}}{\|\mathbf{k}\|} = \frac{(k_x, k_y, k_z)}{\sqrt{k_x^2 + k_y^2 + k_z^2}}. \quad (\text{B.7})$$

The projection of the spectral velocity field on this coordinate system leads to:

$$\hat{\mathbf{u}} = \hat{\phi}_v(\mathbf{k}) \mathbf{e}'_1 + \hat{\phi}_w(\mathbf{k}) \mathbf{e}'_2, \quad (\text{B.8})$$

where $\hat{\phi}_v$ is the vortex mode (the projection of the field on the horizontal plane) and $\hat{\phi}_w$ is the wave mode, the third component being zero by continuity ($\mathbf{k} \cdot \hat{\mathbf{u}} = 0$).

The horizontal kinetic energy spectra of the vortex and wave modes of the flow are compared in Fig. B.3. The horizontal wave number is $k_H = \sqrt{k_x^2 + k_y^2}$. As expected, for isotropic turbulence (case 0), the energy spectra are the same for the wave and vortex modes, and display a large inertial range. Stratification changes the energy transfer mechanism compared to the neutral case. For the stratified cases, the vortex mode spectra show a buoyant range with a k^{-3} decay for medium wave numbers and tend to an inertial range with a $k^{-5/3}$ evolution for larger k . The transition from the buoyant to the inertial range depends on the Ozmidov length scale. The buoyant range width is hence larger for higher stratification levels. On the other hand, for the wave mode, the spectra show a k^{-2} evolution at large scales (moderate k) and an inertial range for larger k . The presence of an inertial range in the horizontal spectra is consistent with the literature, see, e.g., [91, 74, 4]. Moreover, the spectra obtained by Herring & Kimura [51] using DNS with a larger grid (1024^3), yet of course at much lower Reynolds number than the present LES, display the same tendency with an evolution in k^{-3} followed by $k^{-5/3}$ for the vortex mode and an evolution in k^{-2} followed by $k^{-5/3}$ for the wave mode. The here obtained spectra are therefore representative of stratified turbulence.

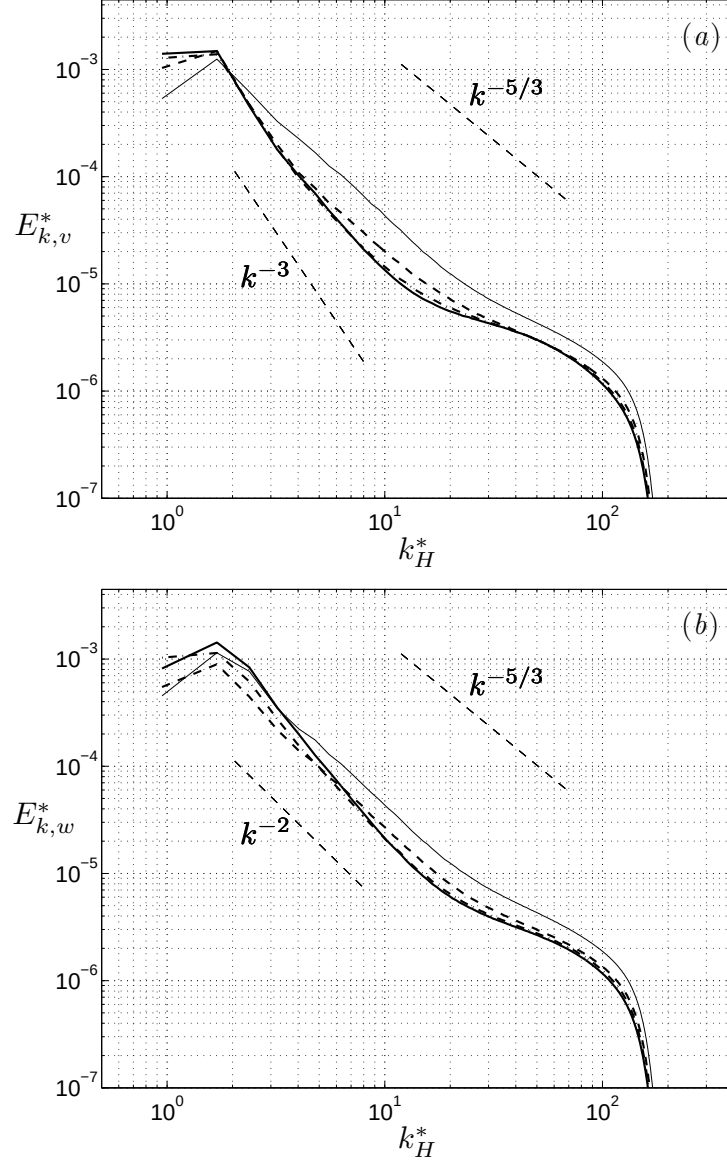


Figure B.3: Kinetic energy spectra of (a) the vortex mode $E_{k,v}$ and (b) the wave mode $E_{k,w}$ as a function of the horizontal wave number k_H for cases 0 (thin solid), 1 (dash), 2 (dash-dot) and 3 (thick solid). The slopes $k^{-5/3}$, k^{-2} and k^{-3} are also shown (thin dash).

The analysis of the second-order longitudinal structure function in the horizontal and vertical directions allows a characterization of the size of structures in the stably stratified field. The second-order longitudinal structure function, D_{ii} , is defined by (see, e.g., [82, 20]):

$$D_{ii}(r) = \left\langle (u_i(\mathbf{x} + r \mathbf{e}_i) - u_i(\mathbf{x}))^2 \right\rangle, \quad (\text{B.9})$$

where $\langle \cdot \rangle$ is the volume average, and with $i = 1$ or 2 for the horizontal direction and $i = 3$ in the vertical direction. It represents an indicator of the energy contained in structures of size $\sim r$, or less. Moreover, it is shown that (see, e.g., [82]), in isotropic turbulence, for sufficiently large r , $D_{ii} = C (\epsilon r)^{2/3}$. Figure B.4 (a, b) compares the spatial evolution of $D_{11}(r_x)$ (as characteristic horizontal direction, $D_{22}(r_y)$ being similar) and $D_{33}(r_z)$ for the various stratification levels and for the forcing ϵ_1^* . As expected, the isotropic case displays a clear $r^{2/3}$ evolution in both horizontal and vertical directions followed by a plateau. For the stratified cases, at small scales, the evolution of D_{ii} is similar to that in a neutral atmosphere, in both horizontal and vertical directions, although less energetic. The turbulence appears to be damped by the stratification effects. On the other hand, the horizontal large-scale structures are more energetic when adding stratification. As already observed in Fig. 2.11, stratification leads to the formation of large horizontal structures. In the vertical direction, the structure function follows the $r^{2/3}$ evolution on a much broader range of radii. D_{33} slightly decreases for the very large radii, indicating a small reduction in the vertical structure size.

The obtained fields are finally further characterized by measuring various characteristic length scales. We first measure the classical turbulent length scale L_t that characterizes the large eddies of the flow and obtained by dimensional analysis using E_k and ϵ (see e.g., [82]):

$$L_t = \frac{E_k^{3/2}}{\epsilon}. \quad (\text{B.10})$$

In order to measure the degree of isotropy of the field, we also define the turbulent length scale in the horizontal direction $L_{t,H}$ and in the vertical direction $L_{t,V}$ using, E_H and ϵ_H or E_V and ϵ_V , respectively. A factor is added in the definition, that takes into account the dimension of the considered field (i.e., 3-D for L_t , 2-D for $L_{t,H}$ and 1-D for $L_{t,V}$), so that, in isotropic turbulence, the

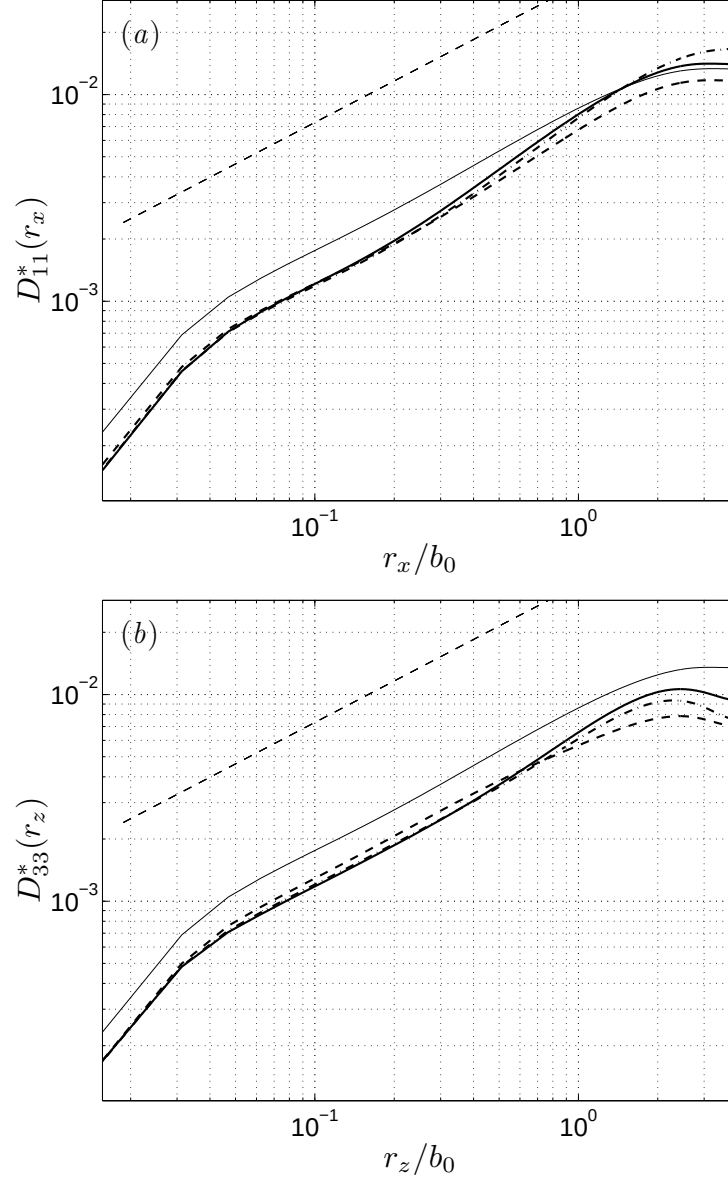


Figure B.4: Second-order longitudinal structure functions in (a) the horizontal direction and (b) the vertical direction for cases 0 (thin solid), 1 (dash), 2 (dash-dot) and 3 (thick solid). The evolution in $r^{2/3}$ is also shown (thin dash). The dimensionless structure function is defined as $D_{ii}^* = D_{ii}/V_0^2$.

three length scales are equal. It finally reads:

$$\begin{aligned} L_t &= \frac{1}{\sqrt{3}} \frac{E_k^{3/2}}{\epsilon}, \\ L_{t,H} &= \frac{1}{\sqrt{2}} \frac{E_H^{3/2}}{\epsilon_H}, \\ L_{t,V} &= \frac{E_V^{3/2}}{\epsilon_V}. \end{aligned} \quad (\text{B.11})$$

Table B.3 reported the values of the length scales for the investigated cases. For the isotropic cases (0 and 0s), the obtained length scales are, as expected, equivalent in the vertical and the horizontal directions. For the stratified cases, the horizontal scales are much larger than the vertical ones. For instance, for the stratified cases with $L^* = 8$ and $\epsilon_f^* = \epsilon_1^*$, the large eddies in the horizontal direction are roughly twice larger than those in the vertical direction ($\frac{L_{t,H}}{L_{t,V}}$ ranging from 1.9 to 2.4). This ratio also increases when the turbulence level is reduced: $\frac{L_{t,H}}{L_{t,V}} = 2.7$ for case 5 compared to 2.2 for case 1.

Case	L_t^*	$L_{t,H}^*$	$L_{t,V}^*$	F_H	L_w^*	L_d^*
0	2.09	2.07	2.13	—	—	—
0s	1.06	1.06	1.07	—	—	—
1	2.68	3.34	1.49	0.045	$16.9 \cdot 10^{-2}$	$21.9 \cdot 10^{-2}$
1s	1.25	1.54	0.73	0.078	$13.3 \cdot 10^{-2}$	$17.0 \cdot 10^{-2}$
2	3.79	4.71	1.99	0.016	$8.42 \cdot 10^{-2}$	$11.9 \cdot 10^{-2}$
2s	1.51	1.96	0.68	0.030	$5.93 \cdot 10^{-2}$	$8.21 \cdot 10^{-2}$
3	3.88	4.55	2.46	0.011	$6.72 \cdot 10^{-2}$	$9.47 \cdot 10^{-2}$
3s	1.66	2.07	0.85	0.021	$4.73 \cdot 10^{-2}$	$6.39 \cdot 10^{-2}$
3SRVM	1.80	2.54	0.69	0.018	$4.83 \cdot 10^{-2}$	$6.30 \cdot 10^{-2}$
4s	1.83	2.11	1.22	0.014	$3.75 \cdot 10^{-2}$	$5.42 \cdot 10^{-2}$
5	4.45	5.66	2.11	0.015	$9.02 \cdot 10^{-2}$	$13.9 \cdot 10^{-2}$
6s	6.28	7.65	2.85	0.0035	$2.78 \cdot 10^{-2}$	$3.95 \cdot 10^{-2}$

Table B.3: Characteristic length scales of the investigated cases.

In Section 2.4.2, the isotropy of the turbulent fields was also characterized by measuring the two-point correlation R_{ij} , the longitudinal autocorrelation function $f_i(r)$ and, finally, the longitudinal integral length scales L_{ii} in the

three directions:

$$R_{ij}(\mathbf{r}) = \langle u_i(\mathbf{x} + r \mathbf{e}_i) u_j(\mathbf{x}) \rangle, \quad (\text{B.12})$$

$$f_i(r) = \frac{R_{ii}(r \mathbf{e}_i)}{R_{ii}(0)}, \quad (\text{B.13})$$

$$L_{ii} = \int_0^{L/2} f_i(r) dr, \quad (\text{B.14})$$

where L is the periodicity length. We also recall that, for isotropic turbulence, the integral length scale can also be computed from the 1-D spectrum using:

$$\int_0^\infty \frac{E_k(k_i)}{k_i} dk_i = \frac{4}{3\pi} E_k L_{ii}. \quad (\text{B.15})$$

Figure B.5 compare the obtained longitudinal autocorrelation functions $f_i(r)$, in the horizontal and vertical directions, obtained for various stratification levels. The values of the obtained integral length scales are reported in Table 2.3.

Two other classical length scales of stratified turbulence are the wave and the displacement scales. The wave scale L_w represents the vertical displacement of a fluid element that would occur if all of the vertical kinetic energy in the flow was transformed into potential energy:

$$L_w = \frac{(2 E_v)^{1/2}}{N}. \quad (\text{B.16})$$

The displacement scale L_d quantifies the rms vertical displacement of a fluid parcel from its equilibrium position:

$$L_d = \frac{\langle \tilde{\Theta}^2 \rangle^{1/2}}{\frac{d\tilde{\Theta}}{dz}} = \frac{(2 E_p)^{1/2}}{N}. \quad (\text{B.17})$$

The values of the wave and displacement scales are provided in Table B.3. One finds that the ratio L_d/L_w ranges from 1.28 to 1.54. This is consistent with Siegel *et al.* [105] who found a ratio ranging from 1.2 to 1.5. As they noted, this indicates that the rms displacement of isothermal surfaces is greater than that only produced by the vertical kinetic energy.

Brethouwer *et al.* [4] derived a scaling enabling the distinction between different regimes in stratified turbulence. The flow is characterized by the

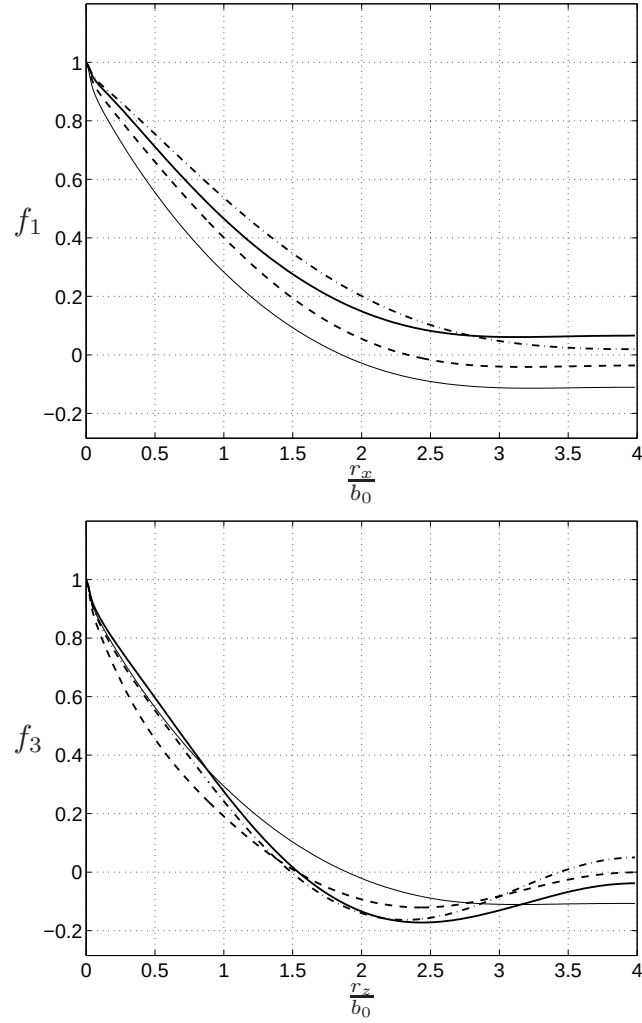


Figure B.5: Longitudinal autocorrelation function in the horizontal direction (top) and the vertical direction (bottom) for cases 0 (thin solid), 1 (dash), 2 (dash-dot) and 3 (thick solid)

Reynolds number and the horizontal Froude number F_H defined by:

$$F_H = \frac{\epsilon + \epsilon_P}{N u_{H,rms}^2}, \quad (\text{B.18})$$

where $u_{H,rms}$ is the r.m.s. of the horizontal velocity fluctuations. Following [74], stratified turbulence can be observed if $F_H < 0.02$ (although they noted that the latter value is merely an estimate). The values of F_H for the investigated cases are also provided in Table B.3. For the cases with $L^* = 8$, the criterion is met except for case 1 ($N^* = 0.35$, $\epsilon_f = \epsilon_1$) with $Fr = 0.045$. For the cases with $L^* = 4$, the criterion is also satisfied except for cases 1s ($N^* = 0.35$, $\epsilon_f = \epsilon_1$) with $Fr = 0.078$ and 2s ($N^* = 0.75$, $\epsilon_f = \epsilon_1$) with $Fr = 0.03$.

Moreover, following [4], the atmosphere is in the strongly stratified turbulence regime when an inertial range is found in the horizontal spectrum. Using the present hyperviscosity dissipation model, the criterion for the flow to be in that regime is:

$$\left(\frac{L_O}{\eta^{(p)}} \right)^{(6p-2)/3} F_H^{1-p} \gg 1, \quad (\text{B.19})$$

where $\eta^{(p)} = (\nu_h/\epsilon^{1/3})^{3/(6p-2)}$ is the effective Kolmogorov scale based on the hyperviscosity SGS model. For all stratified investigated cases, this condition is totally met.

For completeness of the analysis, we also provide some turbulence properties normalized using the turbulence characteristics (i.e., the energy forcing ϵ_f and the size of the computational domain L), instead of the wake vortex characteristics Γ_0 and b_0 . The Brunt-Väisälä frequency is then normalized using Eq. (2.27) whereas the energies are normalized following Eq. (2.26). The results are reported in Table B.4. Using this normalization, cases 0 and 0s are simulations of the same turbulence using different resolutions. The dimensionless energies and length scales of these two cases are hence very close. With stratification, the turbulence obtained with $L^* = 8$ are seen to be a bit more stratified than those obtained with $L^* = 4$. We thus observe some differences in the dimensionless energies and length scales for cases that have the same N^* and ϵ_f^* .

In section 2.4.5, the sensitivity of the results to the used SGS model was also investigated. For that purpose an additional simulation of case 3s ($N^* = 1.0$, $\epsilon_f^* = \epsilon_1^*$, $L^* = 4$) was performed using, as SGS model, the more sophisticated

Case	$\tilde{N}$	$\tilde{E}_k$	$\tilde{E}_P$	$\tilde{\epsilon}$	$\tilde{\epsilon}_P$	$\tilde{L}_t$	$\tilde{L}_{11}$	$\tilde{L}_{22}$	$\tilde{L}_{33}$
0	0.0	0.581	0.0	0.979	0.0	0.261	$6.06 \cdot 10^{-2}$	$6.21 \cdot 10^{-2}$	$6.36 \cdot 10^{-2}$
0s	0.0	0.591	0.0	0.988	0.0	0.265	$6.58 \cdot 10^{-2}$	$5.85 \cdot 10^{-2}$	$6.50 \cdot 10^{-2}$
1	22.5	0.506	0.189	0.620	0.400	0.335	$11.7 \cdot 10^{-2}$	$9.20 \cdot 10^{-2}$	$4.71 \cdot 10^{-2}$
1s	14.2	0.478	0.182	0.612	0.438	0.312	$8.43 \cdot 10^{-2}$	$7.85 \cdot 10^{-2}$	$5.08 \cdot 10^{-2}$
2	48.1	0.642	0.257	0.628	0.336	0.473	$15.5 \cdot 10^{-2}$	$13.4 \cdot 10^{-2}$	$5.34 \cdot 10^{-2}$
2s	30.3	0.547	0.194	0.620	0.360	0.377	$12.2 \cdot 10^{-2}$	$11.2 \cdot 10^{-2}$	$4.33 \cdot 10^{-2}$
3	64.2	0.670	0.290	0.653	0.331	0.484	$14.4 \cdot 10^{-2}$	$14.3 \cdot 10^{-2}$	$5.06 \cdot 10^{-2}$
3s	40.4	0.582	0.208	0.620	0.346	0.414	$11.9 \cdot 10^{-2}$	$16.3 \cdot 10^{-2}$	$5.13 \cdot 10^{-2}$
3SRVM	40.4	0.640	0.203	0.657	0.290	0.450	$13.8 \cdot 10^{-2}$	$23.6 \cdot 10^{-2}$	$4.45 \cdot 10^{-2}$
4s	56.6	0.633	0.294	0.636	0.336	0.456	$14.5 \cdot 10^{-2}$	$11.6 \cdot 10^{-2}$	$6.03 \cdot 10^{-2}$
5	48.4	0.807	0.353	0.752	0.396	0.556	$20.8 \cdot 10^{-2}$	$17.8 \cdot 10^{-2}$	$6.14 \cdot 10^{-2}$
6s	87.1	1.314	0.370	0.554	0.255	1.57	$25.8 \cdot 10^{-2}$	$37.0 \cdot 10^{-2}$	$7.48 \cdot 10^{-2}$

Table B.4: Turbulence characteristics of the investigated cases normalized using ϵ_f and L as characteristic scales.

regularized variational multiscale (RVM) SGS model instead of the simple high-order hyperviscosity (HV) SGS model (both were described in Section 2.2.3). This case is denoted “case 3SRVM”. The atmospheric stratified and weakly turbulent field was then also obtained using the RVM model. The characteristics of the obtained stratified turbulence are reported here and compare to those obtained when using the HV model (case 3s). Figure B.6 compares the obtained 1-D spectra in the x -direction (as characteristic horizontal direction, the spectrum in the y -direction being similar) and in the vertical direction. One notes that, using the RVM model, the energy is dissipated at even smaller scales than when using the HV model, thus leaving more wavenumbers to capture relevant physics before SGS dissipation. The “bump” in the spectrum before the fast dissipation is also less significant. However, at medium to large scales, the spectra show similar trends for both cases. Those findings are in accordance with those of the investigation by Cocle *et al.* [11] in homogeneous isotropic turbulence at very high Reynolds number.

The values of the energies, energy dissipations and some characteristic length scales are reported in Table B.1, Table B.2 and Table B.3. Globally, the turbulence characteristics obtained at equilibrium for case 3SRVM are similar to

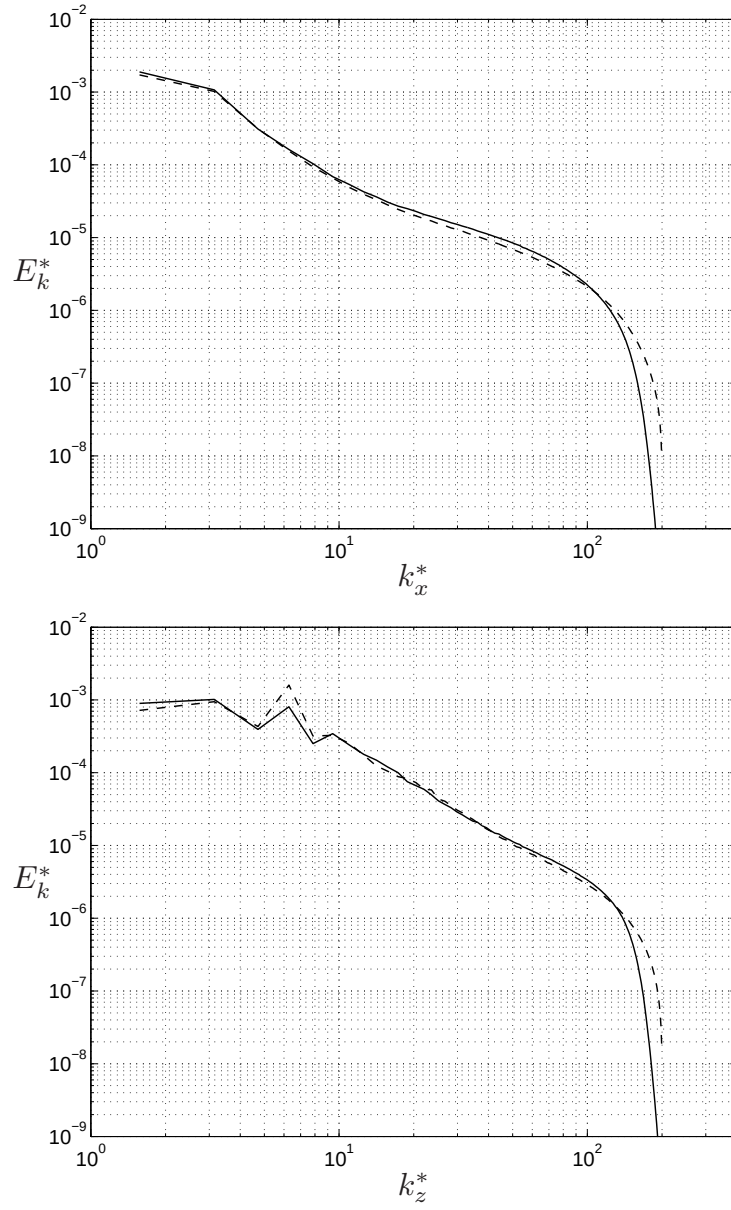


Figure B.6: Comparison of the 1-D power spectra in the x -direction (top) and the vertical direction (bottom) for a case with $N^* = 1.0$, $\epsilon_f^* = 2.42 \cdot 10^{-4}$, $L^* = 4$ and using: the HV SGS model (solid) or the RVM SGS model (dash)

those of case 3s. Yet, some small differences are observed. The potential energies are almost equal in both cases while the kinetic energy is a bit higher when using the RVM model. The kinetic energy dissipation is also seen to be a bit higher for case 3_{RVM}, the potential energy dissipation being hence a bit lower as, at equilibrium, $\epsilon_f = \epsilon + \epsilon_P$. The obtained mixing efficiency is hence also lower. The obtained turbulence length scales are close in both cases, yet $L_{t,H}$ is seen to be a bit larger and $L_{t,V}$ a bit lower. The tendency is also observed using the integral length scales. For the case 3_{RVM}, one finds $L_{11}^* = 0.551$ (against $L_{11}^* = 0.475$ for case 3s), $L_{22}^* = 0.945$ (against $L_{22}^* = 0.653$) and $L_{33}^* = 0.178$ (against $L_{33}^* = 0.205$). The obtained Richardson numbers and buoyancy scales are similar: $Ri_V = 0.207$ (against $Ri_V = 0.224$), $L_b^* = 0.703$ (against $L_b^* = 0.671$). Finally, the obtained wave and displacement scales are equivalent.

The use of either the high-order hyperviscosity SGS model or the RVM SGS model thus provides stratified turbulent fields with similar characteristics in terms of energies and length scales. Both models only affects, yet in a different way, the very small scales of the LES without significantly influencing the length scales of interest. The RVM model however preserves more wavenumbers at the high end of the LES spectrum; this is a quality, as those wavenumbers can then be used to capture more of the fine scale physics. The price to pay is of course that the RVM model is significantly more expensive than the HV model in such pseudo-spectral code.

Appendix C

Visualizations of wake vortices in stratified and very weakly turbulent atmospheres

This appendix provides visualizations of the flow fields of wake vortices evolving in very weakly turbulent atmospheres ($\epsilon_f^* = \epsilon_2^* = 2.42 \cdot 10^{-5}$) and for two stratification levels. The results were obtained in Chapter 2, Section 2.4. Figure C.1 shows the flow evolution for a case weakly stratified ($N^* = 0.35$), while Fig. C.2 concerns a case with strong stratification ($N^* = 1$).

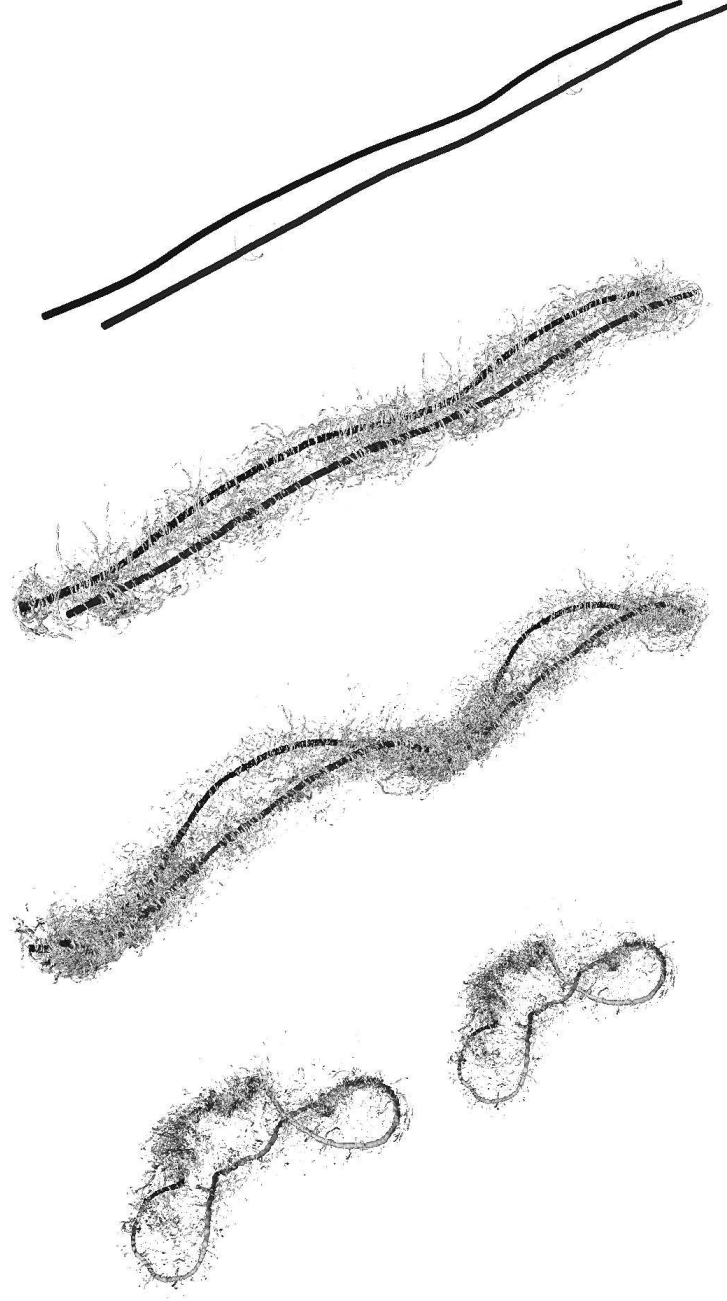


Figure C.1: Visualization of the flow for case 5 ($N^* = 0.35$, $\epsilon_f^* = \epsilon_2^*$, $L^* = 8$), using iso- λ_2 surfaces colored by the axial vorticity $\omega_x b_0^2/\Gamma_0$ at times (from top to bottom): $t^* = 3.0$, $t^* = 4.0$, $t^* = 5.0$ and $t^* = 6.0$.

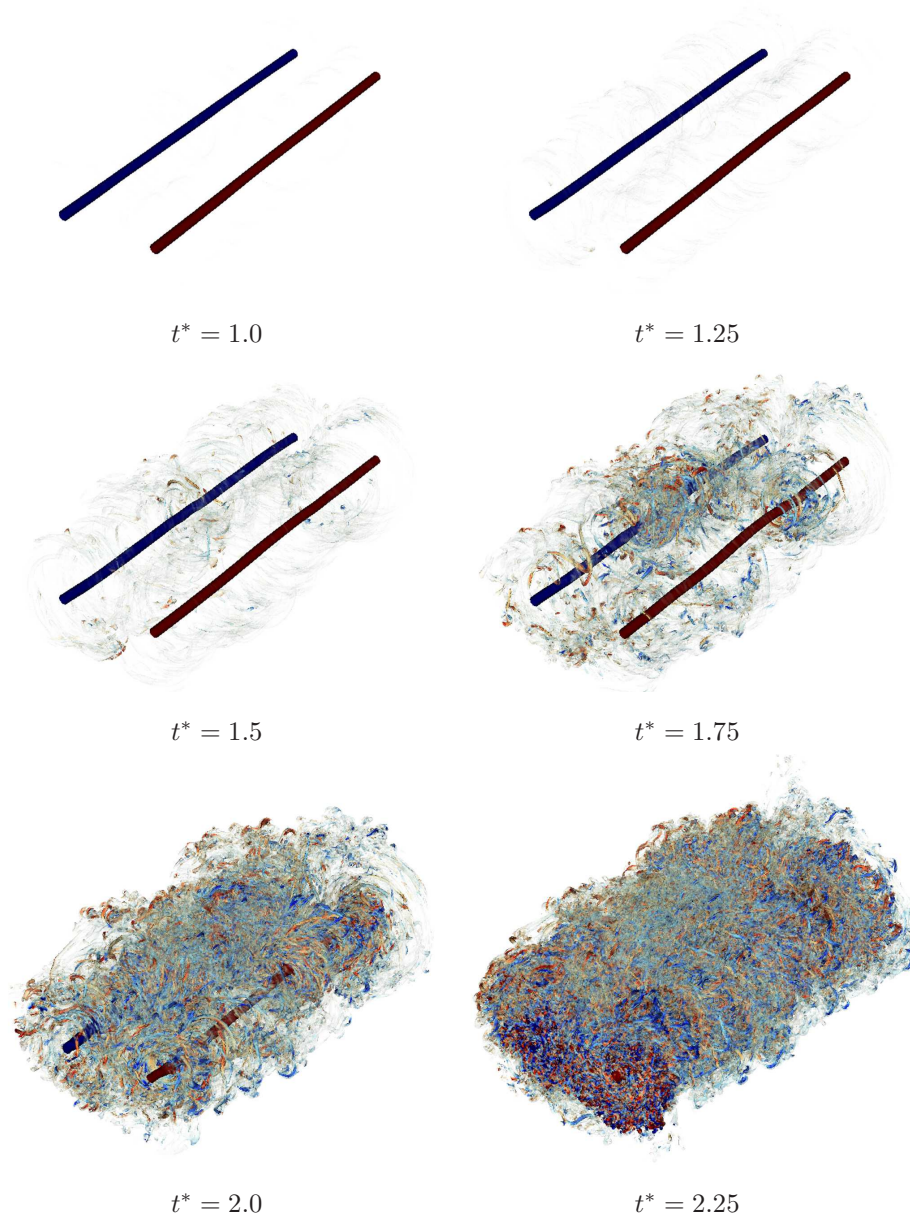


Figure C.2: Visualization of the flow for case 6s ($N^* = 1$, $\epsilon_f^* = \epsilon_2^*$, $L^* = 4$), using iso- λ_2 surfaces colored by the axial vorticity $\omega_x b_0^2 / \Gamma_0$ at different times.

Appendix D

Complementary analysis of the turbulent two-vortex system at equilibrium

We here further analyze the LES fields of the turbulent two-vortex system at equilibrium obtained in Chapter 2, Section 2.5. In particular, we investigate the evolution of the pressure P , the kinetic energy k , and the Bernoulli term B , related through:

$$B = P + k , \quad (\text{D.1})$$

where $k = \frac{1}{2} (u^2 + v^2 + w^2)$. From the Navier-Stokes equations, one obtains that the Bernoulli term satisfies the following Poisson equation:

$$-\nabla^2 B = \nabla \cdot (\boldsymbol{\omega} \times \mathbf{v}) . \quad (\text{D.2})$$

Here we solve the Poisson equation for $B_e - B(x, y, z)$. Because of the lateral periodicity, B_e is defined as the longitudinal average of the $B(x, y, z)$ field, averaged over the four edges (i.e., outer surfaces) of the periodic box (the 2VS being of course re-centered vertically in the periodic box, so that the periodic upper and lower surfaces are as far away as possible). The field $B_e - B(x, y, z)$ is then normalized using V_0^2 . A cut of that normalized field, in an arbitrary x cross-plane, is shown in Fig. D.1.

The average of the field k over the four outer surfaces of the periodic box, k_e , is also not zero. We also evaluate the longitudinal average of that term, by

averaging it over the four outer surfaces of the periodic box. The pressure P_e is then obtained by subtracting k_e from B_e . This pressure cannot however be associated with the pressure P_∞ that the same 2VS would have if it were in an unbounded domain in y and z . With that, we proceed to compute the field $P_e - P(x, y, z)$ from the previous results and definitions:

$$P_e - P = (B_e - B) + (k - k_e) . \quad (\text{D.3})$$

The field $P_e - P(x, y, z)$ is then also normalized using V_0^2 . A cut of that normalized field, in an arbitrary x cross-plane (yet same plane as above), is shown in Fig. D.2. A cut of the field, $k(x, y, z) - k_e$, normalized using V_0^2 and in an arbitrary x cross-plane (also same plane as above), is also shown in Fig. D.3. This field can be compared to the field, $P_e - P(x, y, z)$. Those fields differ significantly; they differ by the field $B_e - B(x, y, z)$.

Finally, one can average all fields above to obtain the normalized mean 2-D fields: $B_e - \overline{B}(y, z)$, $P_e - \overline{P}(y, z)$ and $\overline{k}(y, z) - k_e$, all normalized by V_0^2 . Those mean fields are also shown in Figs. D.1, D.2, and D.3. The last two fields differ significantly: by $B_e - \overline{B}(y, z)$; the integral of which is however zero. Indeed, the Bernoulli term is computed from a Poisson equation whose right-hand-side is the divergence of something. Hence, the volume integral of the Bernoulli term is exactly zero. Thus, although the Bernoulli term $B_e - B$ is clearly a non-zero field, its longitudinally-averaged integral is exactly zero:

$$\int_{S_e} (B_e - \overline{B}) dS = \frac{1}{L_x} \int_{V_e} (B_e - B) dV = 0 . \quad (\text{D.4})$$

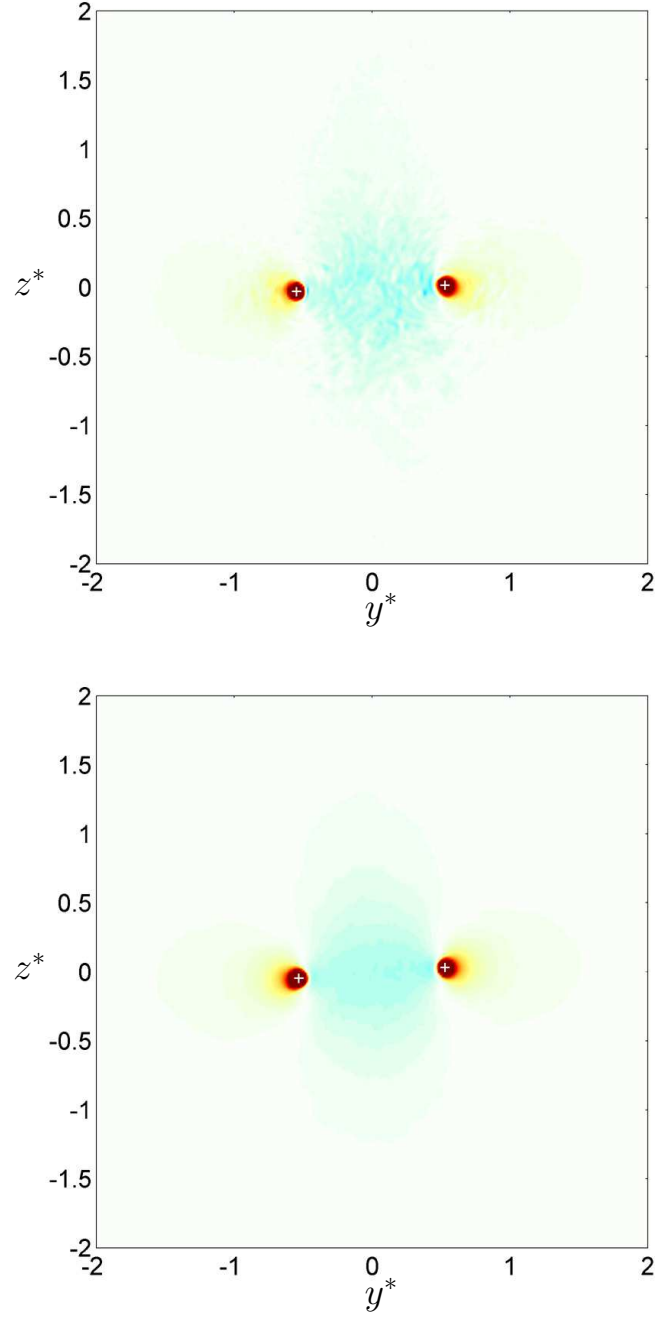


Figure D.1: Slice (top) and longitudinal average (bottom) of the Bernoulli term field, $(B_e - B)/V_0^2$ at equilibrium. The positions of the center of the vortices are also displayed (crosses).

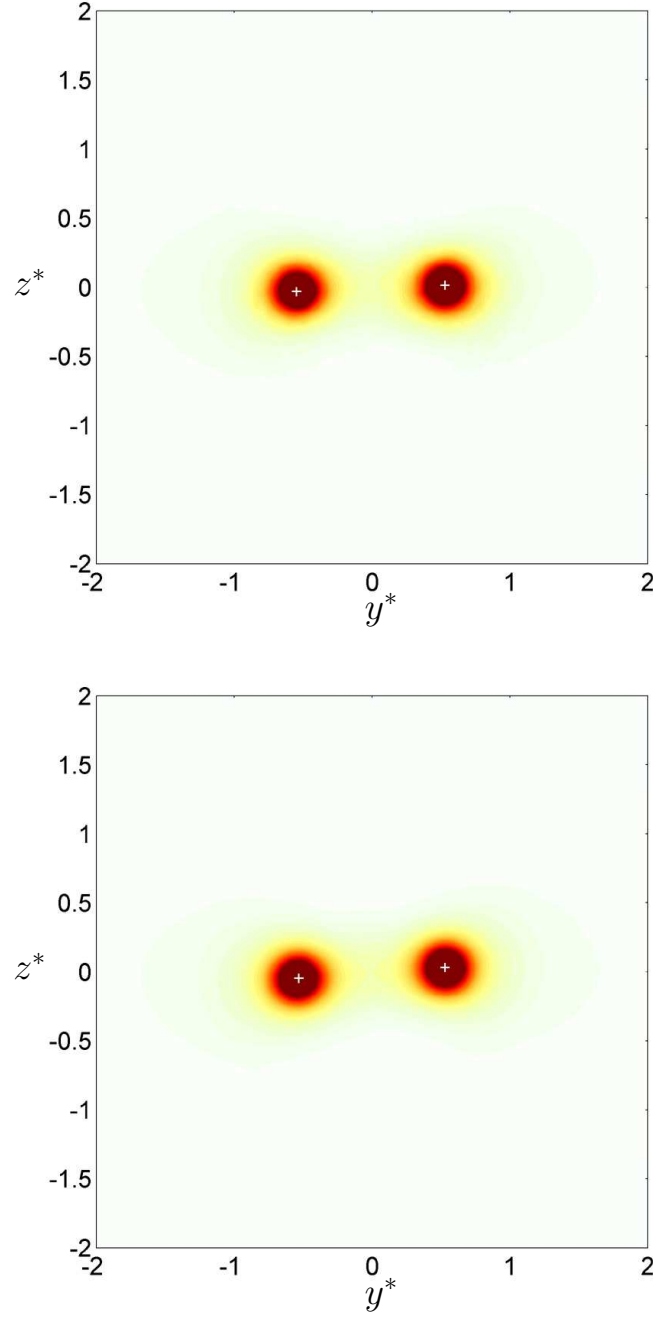


Figure D.2: Slice (top) and longitudinal average (bottom) of $(P_e - P)/V_0^2$ at equilibrium. The positions of the center of the vortices are also displayed (crosses).

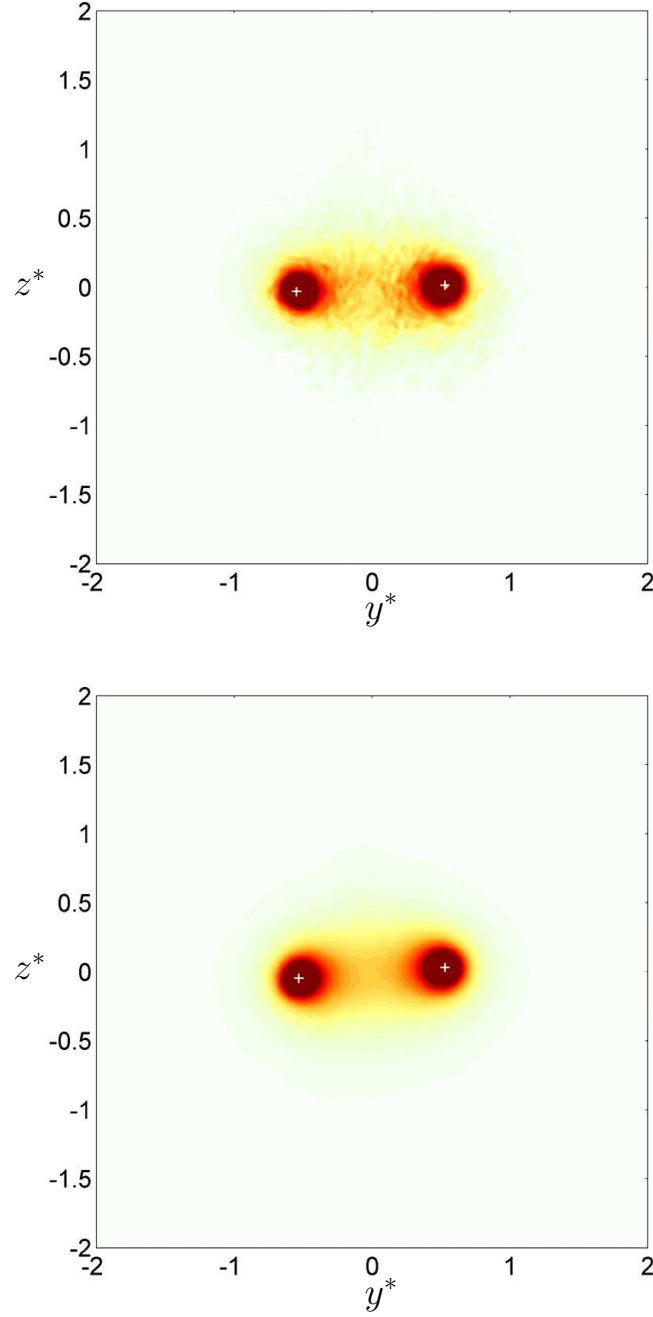


Figure D.3: Slice (top) and longitudinal average (bottom) of the the kinetic energy field, $(k - k_e)/V_0^2$ at equilibrium. The positions of the center of the vortices are also displayed (crosses).