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Macroscopic modeling of urban flood by a porosity approach

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Introduction

During the last few decades, the world has experienced an increasing number of devastating floods (Figs 1.1, 1.2). Despite the increase in occurrence, the number of people killed remains stable, leading to a decrease in the average flood mortality. Nevertheless, it is important when looking at this trend to keep in mind the influence of the recent major improvements in the reporting of floods and their impacts, especially for the "smaller" events.

Floods are one of the major sources of hardship. Each year on average they kill more than 9000 people and affect more than 115 million others (CRED, 2008). With more than 19 US\$ billions in economic damage, floods are the second most costly natural disasters after windstorms. Asia is the region which pays the highest toll in terms of flood impacts. During the last decade, over 96% of the people affected by floods in the world live in Asia.

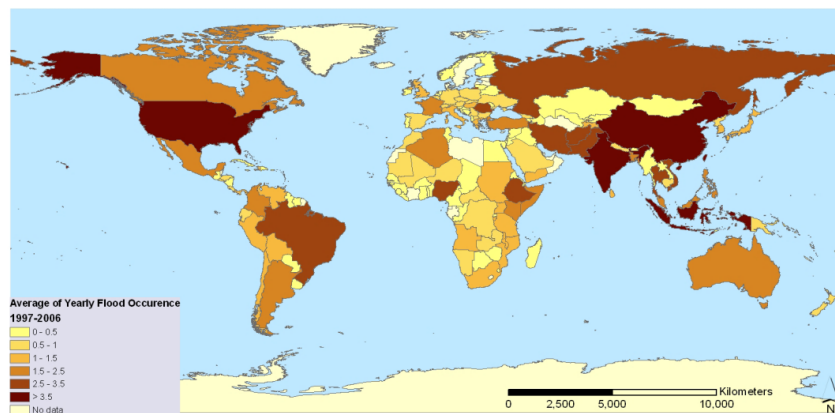


Fig. 1.1: Average of yearly flood occurrence, based on data over the 1997-2006 period (CRED, 2008).

The numbers presented in Fig. (1.3) show that floods in Europe cause high economic damage, but relatively small numbers of victims, while the floods in developing countries usually cause many casualties and comparatively little damage. The Centre for Research on the epidemiology of disasters (CRED) published a map of flood occurrence in the 2000-2009 period in Europe (Fig. 1.4).

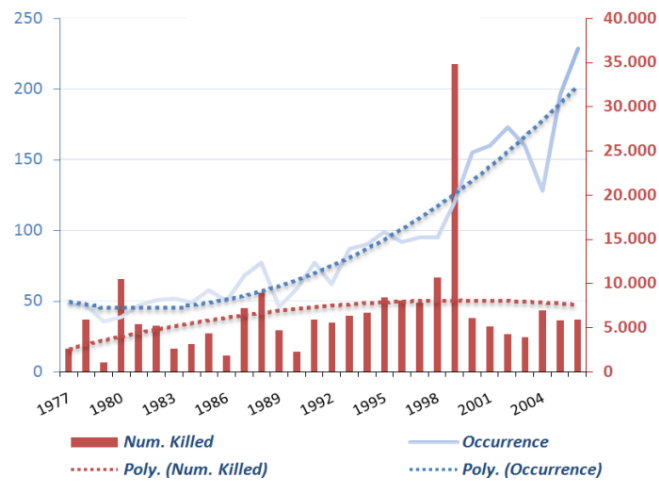


Fig. 1.2: Reported flood occurrence, mortality numbers and polynomial trends from 1977 to 2006 (CRED, 2008).

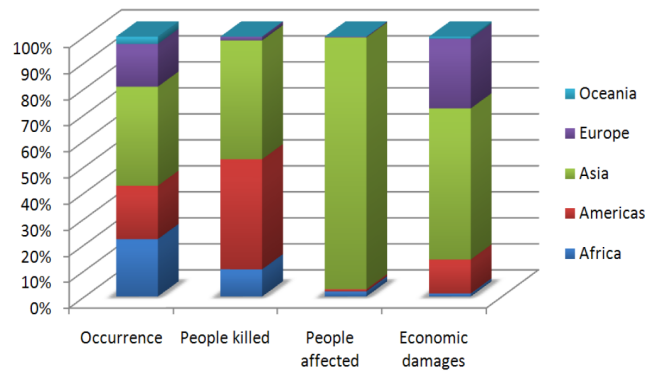


Fig. 1.3: Reported flood occurrence and impacts: regional comparison (CRED, 2008).

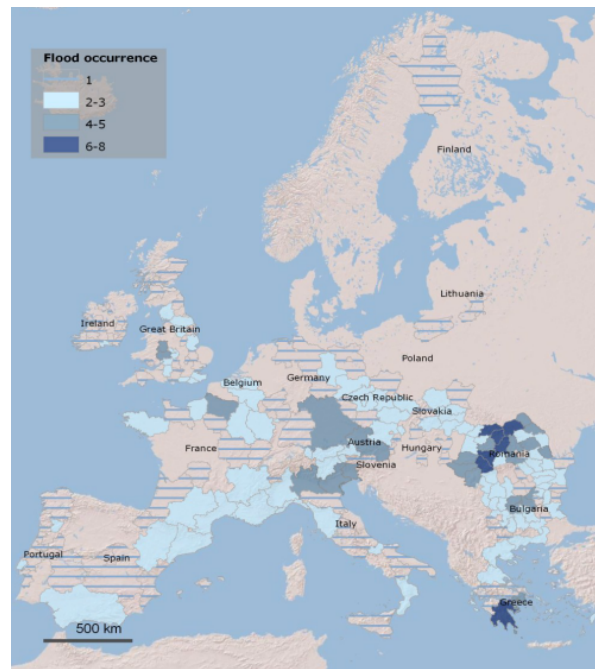


Fig. 1.4: Flood occurrence by sub-national administrative unit in EU: 2000-2009 (CRED, 2010).

1.1 Causes of floods

The floods can be originated by several factors (FLOODsite, www.floodsite.net):

- A flash flood is a flood that occurs in a short period of time after a high intensity rainfall event or a sudden massive snowmelt. It occurs in small catchment basins of a few hundreds square kilometres with steep slopes. This type of basin has a relatively small response time of a few hours. Rising water levels in the river network can reach its peak within minutes to a few hours, leaving an extremely short time for warning. This type of flood can be very dangerous because of the high speed of water that can transport large objects like rocks, trees and cars.
- Coastal floods are caused by a severe storm. The low pressure combined with the wind (this latter having a larger effect) produce

an important raise of the sea level and high waves that can overtop or breach defence works like dunes and dikes. A characteristic of coastal floods is that the water level drops and rises with the tide. At high tide the water may flow in and at low tide it may recede again. When a sea defence is breached, the breach may be repaired during the low tide.

- River floods are originated by overtopping of the banks or breaking of dikes. Rainfalls over an extended period and an extended area can cause major rivers to overflow their banks, and the water can cover enormous areas. Downstream areas may be affected, even when they did not receive much rain themselves. With large rivers the process is relatively slow. It takes time for all the rainwater to reach the river, making the water level to rise slowly, and permitting the officials to decide to evacuate people before the river overflows. When a dike or a dam breaks and a lot of water is released suddenly, the speed of the water at the breach can be compared with the speed of a flash flood.
- Floods in urban areas can be originated by flash floods, coastal floods or river floods, but there is also a specific flood type called urban flooding. Urban flooding occurs when the sewage system capacity is exceeded during a heavy rainfall. The water rises relatively slow and the water level usually does not reach dangerous heights. Urban floods cause a great disturbance of daily life and high economic damage, but the number of casualties is usually low.
- Pluvial floods occur when more rainwater enters a water system than can be stored. It is similar to urban floods, but without the sewage system and in more rural areas. Because of the gradual character people have time to go indoors or to leave the area. The water level is on the order of centimetres and causes no immediate threat to people's lives, but depending on the economic activity developed in the area the economic damage may be important.
- Dam break floods are comparatively seldom, but can cause important damage and loss of life when they occur. The cause of dam failure is diverse: spillway design error, geological instability, poor maintenance, extreme inflow, earthquake, erosion of earthen dams ... The Dartmouth Flood Observatory (Brakenridge) records the flood events in the world and they reported 52 floods caused by dam break since 1985 (Fig. 1.5).

- Floods caused by tsunami are unfrequent but can be very destructive. The waves approaching the coast can have amplitude of the order of 3 or 6 meters high.

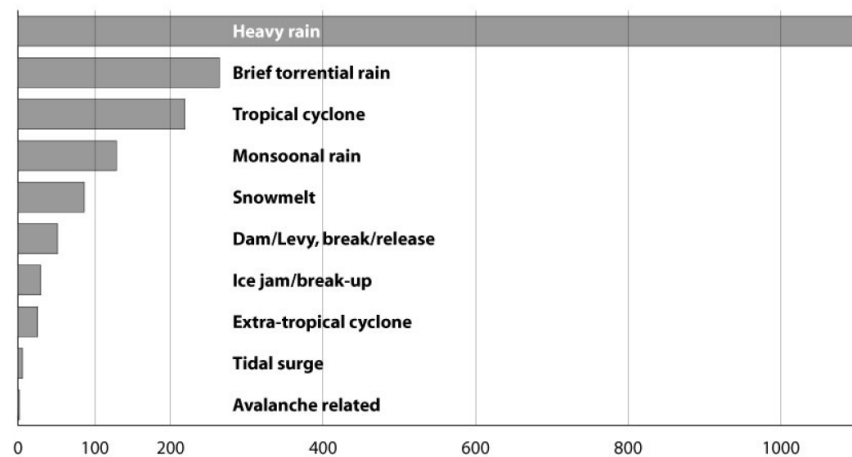


Fig. 1.5: Number of floods originated by different types of hazards, based on worldwide data collected from 1985 up to now (Dartmouth Flood Observatory, Brakenridge).

1.2 Flood risk management

The Intergovernmental Panel on Climate Change (IPCC, <http://www.ipcc.ch>) reports that the increase of the average temperature and the melting of the land based ice caps will raise the sea water level, which will cause coastal erosion and higher risk of coastal flooding. Also, as warm air can hold more water, the IPCC predict more annual potential rain in northern Europe and then a higher discharge in the rivers in the winter season.

To face the increasing threat constituted by floods, it is important to identify the areas exposed to a high risk of flooding. Flood risk can be defined as the combination of the probability of a flood event and of the potential adverse consequences to human health, the environment and economic activity associated with such a flood event (Fig. 1.6).

Flood risk = flood hazard * adverse consequences

where

adverse consequences = value * vulnerability

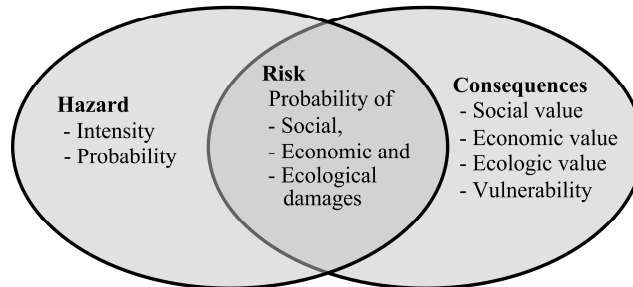


Fig. 1.6: Flood risk can be mathematically calculated as the product of hazard (threatening natural event like storm or rainfall) and the adverse consequences to social, environment and economic activity. The consequences depend on the value of the potentially flooded area, but also on its vulnerability (lack of resistance to damaging forces).

The European Community published a directive on the assessment and management of flood risks (Directive 2007/60/EC of the European Parliament and of the Council of 23 October 2007 on the assessment and management of flood risks), stating that Member States shall undertake a preliminary flood risk assessment for each river basin district and identify areas with potential significant flood risks. For these identified areas, Member States shall prepare flood hazard maps and flood risk maps.

- A flood hazard map is a detailed flood map that shows the type of flood hazard (storm, rainfall), the flood extent, the water depth or water level, the flow velocity or the relevant water flow direction (Fig. 1.7)
- A flood risk map indicates the potential adverse consequences associated with floods, based on range event probabilities. These can be expressed, for example, in terms of indicative numbers of inhabitants potentially affected, in terms of the economic damages or types of economic activity affected, or in terms of location and quantification of accidental pollution caused by the flooding.

Once the risk is determined for vulnerable areas, the government, provinces, municipalities and water authorities together with scientists, citizens and interest groups (such as natural and environmental organizations) decide how high the risk is acceptable. If the risk is found to be too high, measures must be taken to reduce the risk.

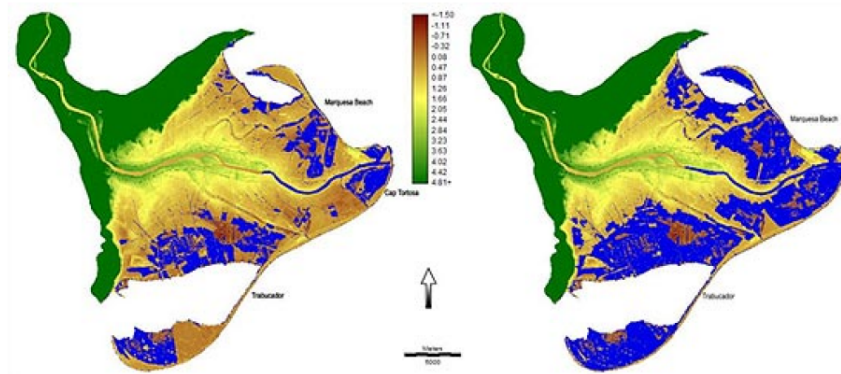


Fig. 1.7: Flood hazard areas (shown in blue) caused by a relative sea level rise of 0.25 m (left) and 0.5 m (right), Ebro delta, Spain (FLOODsite). The colorbar refer to the bed elevation and range from -1.8 m (red) to 4.8 m (green).

The flood risk management consists in taking the appropriate measures to mitigate the risk of floods, either by reducing their probability of occurrence or by reducing their consequences. The probability of flooding can be reduced, for example, by making the dunes or dikes by the sea or the dike along a river higher and/or stronger. A measure to reduce the consequence of a flood would be, for example, building houses on mounds or piles (Fig. 1.8). Measures in the field of spatial planning can also reduce the consequences of flooding.



Fig. 1.8: House raised on piles in New Orleans, as a measure to mitigate flood risk.

A good emergency plan also reduces the impact of a flood. The authorities should prevent and remediate damage to dikes and dunes, and provide information on expected water level. An emergency plan tells which area is to be evacuated, which area is higher and safe and which roads are safe to evacuate people. It also describes the monitoring of dikes or emergency measures when a dike is so weakened that it may break. The water

authorities have extensive information on water levels and can calculate how the water flows in a flood or a higher tide. That information is of great importance for towns that need to determine which areas have to be evacuated and when they should start evacuating.

1.3 Urban Flood Modelling

1.3.1 Why is flood modelling important?

The ability to model floods is important in the process of flood risk management. It is necessary in the design of flood hazard maps and flood risk maps for which the flood extent, the water depth and the main direction of the flow must be known. These maps are designed for hazards (storm, rainfall) of given return periods.

Flood modelling is also crucial when an imminent flood is foreseen and an emergency plan has to be applied. The authorities must then know the extent of flood to know which people they have to start evacuating.

1.3.2 Two levels of accuracy

The presence of urban areas constitutes a challenge in the modelling of the propagation of flood because of the difference of scale between the characteristic size of the streets (~ 10 m) and the characteristic size of the non-inhabited area (~ 100 m). One option is to use a more refined mesh in the streets than in the surrounding floodplain, which implies reliable and precise results but a high computational time. This option would then be suitable for example for the design of flood risk maps in small flooded urban areas

Another option is to use a coarse mesh in the urban area and use a simplified model that accounts for the presence of the buildings through global parameters. This allows a relatively low computational time, but with the drawback that the results are less accurate than with a fine mesh. This option would be suitable for example in emergency situations, when the water extent and averaged water depth are needed but a relatively low precision is required.

In the following of this section, a brief explanation will be given about the detailed models and about two types of simplified models: the "subgrid porosity model" and the "continuum porosity model", this latter being the subject of the present thesis. All these models aim at simulating transient flows with relatively high flow velocities, for example consequent to a dam or a dike break.

1.3.2.a DETAILED MODELS

In a detailed model, the equations governing the propagation of a flood are usually chosen to be the shallow water equations. They are solved on rectangular or unstructured grids with the finite volume method. The inputs of the model are the topography of the studied site and friction parameters depending on the land cover (grass, trees, roads ...).

Topographic data are obtained for example by Light Detection and Ranging (LiDAR) survey¹. The filtering of LiDAR data provides a set of 3D coordinates of the topographic surface called Digital Surface Model (DSM). The DSM contains all the objects on the ground like trees, cars, buildings, bridges, vegetation... It has to be filtered to remove all these objects and obtain the topography of the bare ground only, called the Digital Terrain Model (DTM, Fig. 1.9b). The post-processing of LiDAR data to obtain a DTM is described among others by Mason et al. (2007).

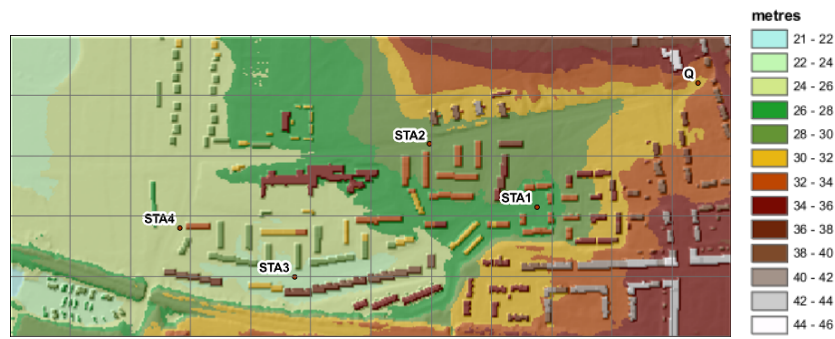
The assignation of resistance parameters to areas of different land use is based on land cover classification maps. These maps are established by imagery and indicate the type of surface of delimitate zones (Fig. 1.9c): grass, trees, buildings, roads... A friction parameter is attributed to each class of affectation. As an example, Table (1.1) gives a range of Manning coefficients for some classes of landcover. These friction parameter values have been estimated empirically and contain large uncertainties. However, Brown et al. (2007) modelled floods of Canvey Island (Thames estuary, UK) due to the overtopping and the failure of sea defences, and observed that the model results were much more sensitive to the boundary conditions like the sea level or the sea defence failure mode than to the bottom friction. Another

¹ LiDAR is an optical remote sensing technology that can measure the distance to a target by illuminating the target with laser. In the present context, it is used for airborne laser altimetry.

factor having a strong influence on the propagation of the flow is sewer and storm drains system, as reported by Gallegos et al. (2009).



(a)



(b)



(c)

Fig. 1.9: Example of input data for flood modelling: (a) orthophoto, (b) LiDAR DTM and (c) landcover classification of an area in Glasgow, Scotland, UK, flooded in 2002 (Schubert et al, 2008).

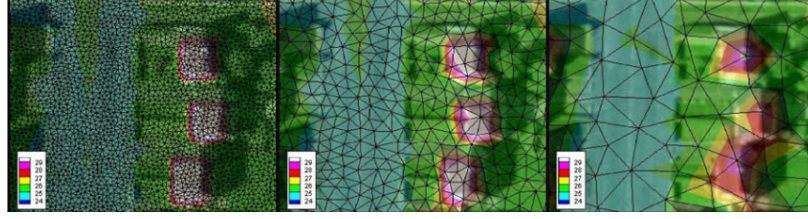


Fig. 1.10: Building-scale view and contour terrain height with (a) 0.8 m, (b) 2 m and (c) 5 m meshes (Schubert et al., 2008), illustrating that with coarse mesh adjacent buildings may merge into one obstacle.

Class	$n_{\min}$	$n_{\max}$
Bare soil	0.012	0.033
Grass	0.025	0.035
Roads	0.010	0.013
Trees/bushes	0.110	0.200

Table 1.1: Landcover classes and minimum and maximum Manning parameters for each class in units of $\text{m}^{-1/3}\text{s}$ (Schubert et al, 2008)

In the detailed models, the buildings are usually represented as a raise of the bottom level. This implies that the mesh has to be fine enough to represent the depressions of the streets. Indeed, when the mesh is too coarse (on the order of a building size), two neighbouring buildings may merge into a single obstacle and lead to erroneous results (Fig. 1.10). Schubert et al. (2008) suggest that a mesh should span streets by approximately 3 cells to achieve a good balance between accuracy and computational effort.

Representing the streets with accuracy is important as the streets create preferential paths for the flow. Indeed, they act like a river thalweg and their resistance is minimal due to a relatively smooth surface (concrete or asphalt) compared to natural surfaces. Then they have a high influence on the direction of flow propagation (Gallegos et al., 2009).

1.3.2.b SUBGRID POROSITY MODELS

The modelling of urban flood by representing the streets in details implies a high computational time. Some investigators then had the idea to coarsen the mesh so that an element size would be of the order of the street width or larger, implying that an element would often be partially occupied by a building. They assigned parameters to each mesh element to capture information about the inner complex geometry. This is the principle of

subgrid models (Fig. 1.11). Mc Millan and Brasington (2007) assigned a porosity parameter to each element to account for the reduction of water storage capacity and the reduced flow conveyance due to blocking by the buildings in a given element. They reported that this approach enabled to reduce the model run-time by an order of magnitude and was a viable method providing a realistic representation of flood evolution. However, Mason et al. (2007) compared the subgrid porosity model with high-resolution simulation of the flood event in Carlisle, UK, 2005, and concluded that this model was inadequate for replicating real urban hydraulics. Sanders et al. (2008) improved the subgrid porosity model by adding two conveyance porosities¹ to capture the anisotropy of the medium, and modelled the resistance exerted by the buildings on the flow by a drag coefficient. They observed that the results depended on the design of the mesh and the elements size, which is a major weakness of the model.

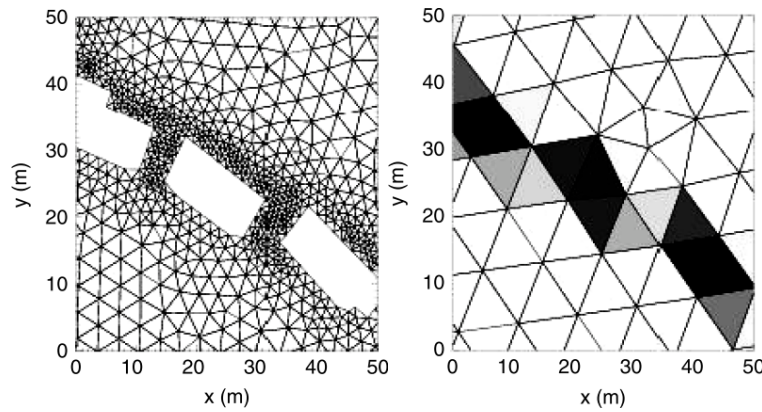


Fig. 1.11: Difference between the meshes (a) used in a detailed model and (b) used in a subgrid porosity model (Mason et al., 2007). In (a), the buildings are represented as inner boundary conditions. In (b), the mesh elements that partly or totally occupied by a building are assigned a porosity parameter equal to the portion of space occupied by the building.

Liang et al. (2007) developed a coupled surface and subsurface flows model. Among others, they modelled floods in urban media by representing each building individually as a raise of porous bottom. Indeed, they considered a building to be a permeable structure through which the water can enter by the windows and they assumed them to behave as a porous bed.

¹ These parameters are called “porosity” and not “permeability” as they intervene in inertia terms, contrarily to permeability that is always put ahead the gradient of pressure and gravity terms.

1.3.2.c CONTINUUM POROSITY MODEL

Another way to consider the problem is to see the urban medium as a continuum and to use a continuum approach as defined in Bachmat and Bear (1986). Continuum models are widely used in the field of flow modelling in porous media. The approach consists in substituting for the heterogeneous porous medium constituted by the fluid and the solid matrix a fictitious homogeneous medium whose parameters account for the small-scale complex geometry. In this type of models, the variables are bulk variables that are representative of the pore-scale variables. The bulk properties are called the macroscopic (or global) properties and the pore-scale properties are referred to as the microscopic (or local) ones. Following these ideas, an urban medium can be assimilated to a large two-dimensional continuous porous medium in which the porosity is continuous all over the urban medium, and the variables of interest are the macroscopic (district-scale) water depth and velocity. The main practical difference with the subgrid models is that the same porosity value is assigned to all the computational elements of the urban medium, no matter how much room is occupied by a building in each element.

The two challenges of the continuum porosity model for urban flood are that:

- the street alignment renders the medium anisotropic and creates preferential flow directions. This anisotropy has to be reflected in the governing equations.
- the presence of the buildings induces hydraulic head (energy) losses that depend on the geometry of the urban area (street width, building dimensions...) and which are difficult to estimate. No reliable expression for these head losses has been proposed so far.

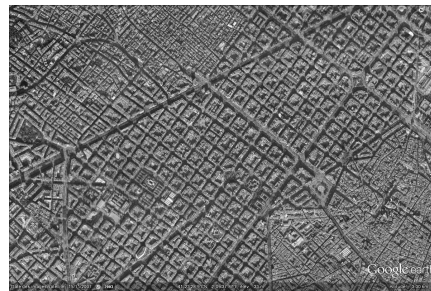


Fig. 1.12: Assimilation of a urban medium to a two-dimensional porous medium, where the streets would be the pores and the buildings the solid matrix (picture of Barcelona, Spain, image from Google Earth)

The governing equations of the porosity model have been first developed by Defina (1994) and Defina (2000) for shallow flows in areas with very irregular topography and partially dry areas. The porosity is defined as the ratio between the wet and the total areas and is dependant on the water depth. Bates (2000) tested the model on a sinusoidal tide on a planar beach to capture the wet/dry front. Hervouet et al. (2000) then adapted the porosity model to the context of urban media. In this model, the porosity is not dependent anymore of the water depth as the buildings walls are considered to be vertical and infinitely high, and the resistance opposed by the buildings is modeled by a drag force. Lhomme (2006) enhanced the model in order to capture the anisotropy of the urban medium. He introduced anisotropic conveyance porosities and he expressed the resistance of the buildings by a law in which the head losses are proportional to the square of the flow velocity and in which tensors allow to capture the anisotropy. The governing equations of this model are:

$$\begin{aligned} \frac{\partial \phi_s h}{\partial t} + \frac{\partial}{\partial x}(\phi_x h U) + \frac{\partial}{\partial y}(\phi_y h V) &= 0 \\ \frac{\partial \phi_s U h}{\partial t} + \frac{\partial}{\partial x} \left(\phi_x h U^2 + \phi_x \frac{g h^2}{2} \right) + \frac{\partial}{\partial y}(\phi_y h U V) &= S_{0x} + S_{fx} \\ \frac{\partial \phi_s V h}{\partial t} + \frac{\partial}{\partial x}(\phi_x h U V) + \frac{\partial}{\partial y} \left(\phi_y h V^2 + \phi_y \frac{g h^2}{2} \right) &= S_{0y} + S_{fy} \end{aligned} \quad (1.1)$$

where g is the gravitational acceleration, h is the water depth, U and V are the x - and y -velocities, respectively, S_{0x} and S_{0y} are the source terms arising from the bottom slopes and the porosity variations in the x - and y -directions, respectively, S_{fx} and S_{fy} are the source terms arising from friction in the x - and y -directions, respectively. The parameter ϕ_s is the storage porosity defined as the ratio of the surface that is available for the flow A_{0f} (i.e. the streets surface) to the total surface A_0 :

$$\phi_s = \frac{A_{0f}}{A_0} \quad (1.2)$$

This parameter accounts for the reduced amount of space available for water storage due to the presence of the buildings. The parameters ϕ_x and ϕ_y are conveyance porosities in the x - and y -directions, respectively, and account for the anisotropy of the medium and the preferential flow paths. Unfortunately, these conveyance porosities are not properly defined and they are taken equal to the storage porosity in the validation step. The source terms S_{0x} and S_{0y} depend on the geometry of the medium. They

account for the bottom slope and the porosity gradients and are given by (Lhomme, 2006)

$$\begin{aligned} S_{0x} &= -\phi_s gh \frac{\partial z_b}{\partial x} + \frac{\partial}{\partial x} \left(\phi_x \frac{gh^2}{2} \right) - \phi_s \frac{\partial}{\partial x} \left(\frac{gh^2}{2} \right) \\ S_{0y} &= -\phi_s gh \frac{\partial z_b}{\partial y} + \frac{\partial}{\partial y} \left(\phi_y \frac{gh^2}{2} \right) - \phi_s \frac{\partial}{\partial y} \left(\frac{gh^2}{2} \right) \end{aligned} \quad (1.3)$$

The source terms S_{fx} and S_{fy} in Eq. (1.1) are the terms accounting for head losses that are due to (i) friction against rough walls and (ii) the eddies due to the presence of the buildings. The friction is modelled by the Manning formula. The total head losses S_{fx} and S_{fy} yield:

$$\begin{bmatrix} S_{fx} \\ S_{fy} \end{bmatrix} = -h\sqrt{U^2 + V^2} \left(\frac{n^2}{h^{4/3}} + \frac{1}{2gh} \right) \begin{bmatrix} M_{xx} & 0 \\ 0 & M_{yy} \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} \quad (1.4)$$

Where n is the Manning parameter accounting for friction, and M_{xx} and M_{yy} are eddy head losses parameters in the x - and y -directions, respectively.

Soares-Frazão et al. (2008) tested the porosity model of Lhomme (2006) (still with $\phi_x = \phi_y = \phi_s$) and assumed that the eddy head losses are due to the successive constrictions and expansions of the flow stream in the streets (Fig. 1.13) and are modelled by a Borda-type formula. They compared the porosity model to a classical shallow water model with a refined mesh and to a simplified model consisting in representing the effect of the buildings by a high friction coefficient. They concluded that the three models reproduced the flow characteristics, the porosity and the friction models being much less time-consuming than the detailed model. The advantage of the porosity model over the friction approach is that it is more physically-based and it does not require calibration.

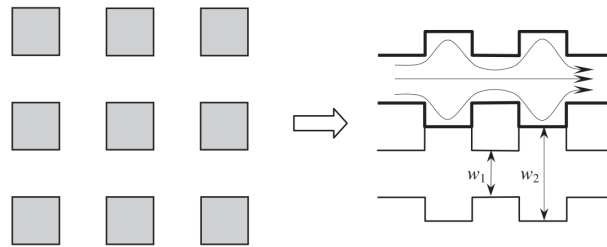


Fig. 1.13: Plan view of an idealized city composed by square buildings and perpendicular streets and scheme of the successive expansions and constrictions of the flow stream in the streets, originating Borda-type head losses in the model of Guinot and Soares-Frazão (2006).

In conclusion, the two main features of the continuum porosity model are that:

- the anisotropy of the urban medium is accounted for by the conveyance porosities ϕ_x and ϕ_y . However, as they are not properly defined, and they are assumed to be equal to the storage porosity, this leads to a model with only one porosity: $\phi_x = \phi_y = \phi_s = \phi$;
- the head losses due to the buildings are considered to be proportional to the square of the flow velocity (Lhomme, 2006) or to the product of the water depth and the square of the velocity (Guinot and Soares-Frazão, 2006). However, no admissible physical explanation was proposed to justify such expressions.

The porosity equations for urban flood developed by Lhomme (2006) and Guinot and Soares-Frazão (2006) are solved by the finite volume method. Guinot and Soares-Frazão (2006) developed a modified HLL Riemann solver on unstructured grids and treat the source terms arising from the bottom slope and the porosity gradient in an upwind fashion. Cea and Vázquez-Cendón (2010) present and compare two different Roe-like numerical discretizations of the porosity equations, both of them being of high-order schemes. Finaud-Guyot et al. (2010) developed an approximate-state solver based on the assumption that both waves of the Riemann problem are rarefactions.

The continuum porosity model was validated on two types of data sets. The first data set consisted in laboratory experiments data conducted at the Civil and Environment Engineering Department of the Université catholique de Louvain, Belgium. The test channel was 36 m long and 3.6 m wide (Fig. 1.14). It was designed to run dam break flows. A gate located in the middle of the flume acted as a dam. The part of the flume upstream the gate, acting as a reservoir, was filled with 0.40 m of water, and a set of 5×5 blocks was placed in the downstream part to act as buildings. The gate is suddenly opened to simulate a dam break. The water flooded then through the downstream reach. The experiment was repeated with several building layouts: the buildings were either aligned with each other, or staggered, and the city was either aligned with the flume or skewed by an angle of 22.5° (Fig. 1.15). Table (1.2a) indicates for each city layout the references of the publications where the experimental data are presented or compared with numerical data.

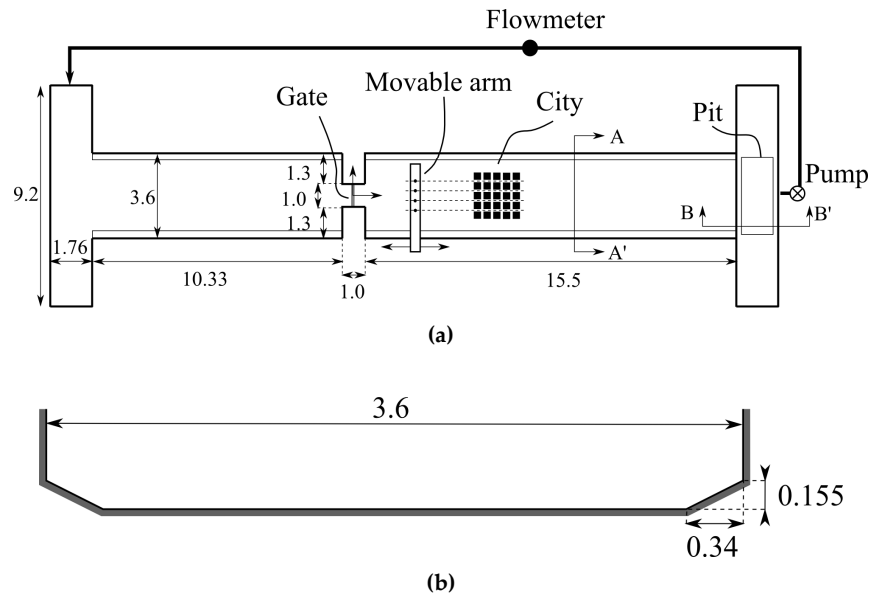


Fig. 1.14: Experimental setup and flume dimensions in (m) in (a) plane view and (b) cross-section.

The second data set used to validate the porosity model consisted in experimental data on a model of the Italian Toce valley at a scale 1:50 at CESI, Italy (Fig. 1.16). A simplified urban area has been placed in the valley using $15\text{ cm} \times 15\text{ cm}$ concrete blocks. The blocks were either aligned with each other, or in a staggered layout. A tank placed upstream of the model apparatus was used to simulate the effect of a severe transient flow over an initially dry floodplain. Table (1.2b) gives the references of the publications where the experimental data are presented and compared to the porosity model.

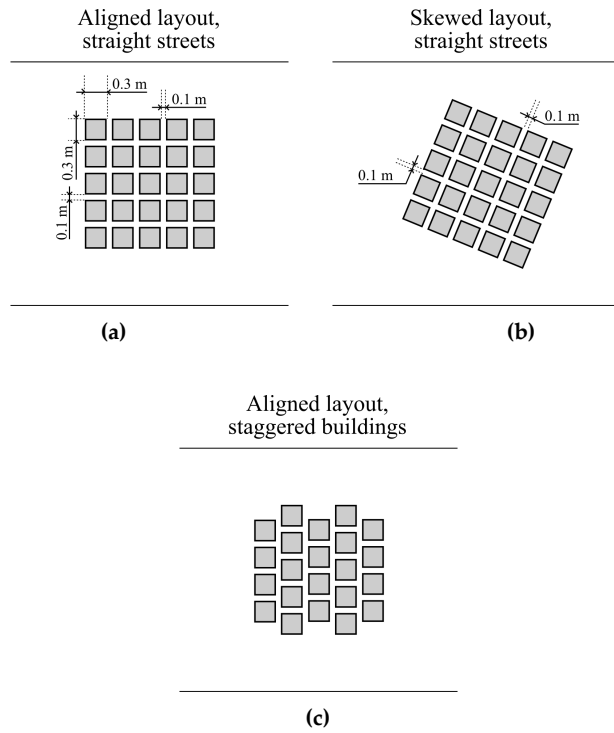


Fig. 1.15: Different city layouts placed in the laboratory flume of Fig. (1.14), used to validate the porosity model: (a) layout aligned with the flume with straight streets, (b) layout skewed by an angle of 22.5° to the flume with straight streets and (c) layout aligned with the flume with staggered buildings.

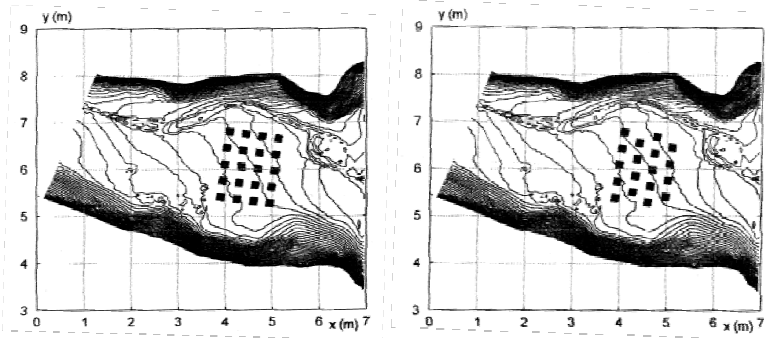


Fig. 1.16: Scale model of the Toce valley (Italy) used to validate the porosity model. Square blocks were placed to simulate a simplified city (a) with straight and perpendicular streets and (b) in a staggered layout.

Rectangular flume (UCL)		
Aligned		Skewed
Straight streets	Lhomme, 2006	Soares-Frazão et al., 2006
	Soares-Frazão et al., 2006	Soares-Frazão and Zech, 2008
	Soares-Frazão and Zech, 2008	Petaccia et al., 2010
	Petaccia et al., 2010	
Staggered	Lhomme, 2006	

(a)

Toce valley (CESI)	
Straight streets	Guinot and Soares-Frazão, 2006
	Lhomme, 2006
	Soares-Frazão et al., 2008
Staggered	Lhomme, 2006
	Soares-Frazão et al., 2008

(b)

Table 1.2: List of publications presenting or using the experimental data of each type of experiment.

Recently, Guinot (2012) developed a multiple-porosity model for urban flood, based on the idea that the porous medium is constituted by dead zones where the flow velocity is negligible and mobile zones with preferential direction. This model will be discussed in the conclusion of this thesis.

1.4 Present thesis

1.4.1 Objectives and requirements

The objective of the present thesis is to improve the continuum porosity model previously developed by Guinot and Soares-Frazão (2006), presented in Section 1.3.2.c and governed by Eq. (1.1).

The two initial objectives of this research project were:

- to define properly the conveying porosities ϕ_x and ϕ_y accounting for preferential flow paths;
- to find the physical origin of the head losses caused by the urban medium and propose a head losses expression that does not require any parameter calibration.

Besides these two initial questions, the porosity model should also satisfy the following general requirements:

- it must be applicable in emergency situations, implying a low computational time, simplicity and a relatively low required level of accuracy (Fig. 1.17);
- it must be suitable for flows with relatively high velocity such as dam break flows;
- it must reflect the preferential flow directions conferred by the alignment of the buildings.

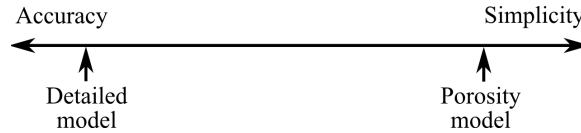


Fig. 1.17: Classification of the models following their precision and simplicity.

1.4.2 Approach and main results

The approach followed was to re-develop the governing equations, relying on the porous media science. Indeed, even if the "classical" porous media are far different from a city viewed as a porous medium, the issues to be solved are similar, like finding an expression for the head losses and taking the

anisotropy effects and preferential flow paths into account in the governing equations.

It was decided to apply the volume averaging method which is based on averaging the fluid equations over a representative elementary volume (REV). Chapter 2 is dedicated to the application of this method to obtain the porosity equations governing the flow in urban media. These porosity equations are close to the ones developed by Guinot and Soares-Frazão (2006), with the differences that:

- there is intrinsically only one type of porosity, corresponding to the storage porosity ϕ_s (Eq. 1.2);
- two terms were omitted in the model of Guinot and Soares-Frazão (2006): the drag force exerted by the buildings on the flow and the mechanical dispersion quantifying the mixing effect due to the chaotic flow paths. In the present thesis, it is proposed to model both phenomena by a single closure expression, which is the object of Chapter 3. This closure expression substitutes an appropriate model for the "head losses" expression in the model of Guinot and Soares-Frazão (2006). It contains a fourth-order tensor, called the "drag/dispersion tensor", whose elements depend on the medium's geometry and characterize the anisotropy of the urban medium.

In continuity with previous works about urban porosity model, this thesis focuses on periodic media as illustrated in Fig. (1.18), with the novelty that the streets may be not equally wide in both directions, in order to study the influence of wider avenues on the flow propagation.

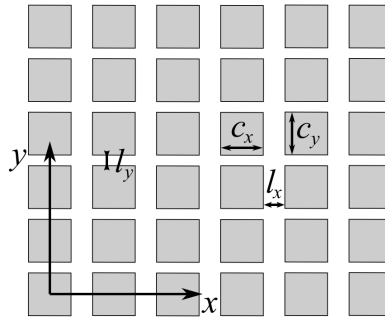


Fig. 1.18: Top view of the urban medium studied in this thesis: a periodic medium made of aligned buildings forming streets that may not be equally wide in both directions.

Numerical experiments in an "infinite medium" made in Chapter 3 show that the flow goes preferentially in the direction of the streets (even if all the

streets are equally wide) and that the flow experiences more resistance if its main direction is not aligned with the street directions (off-axis flow). The resistance opposed by the buildings is then higher if the flow direction is not aligned with the axes of symmetry. The numerical experiments also show that, if the streets in one direction are wider than in the other direction, the flow goes faster along the wider streets. The porosity model reproduces well these trends if the drag/dispersion tensor is appropriately calibrated.

Chapter 4 describes the experimental campaign that has been conducted to validate the porosity model. The experimental setup is shown in Fig. (1.14). It is the same as the one used in previous works about the urban porosity model, but additional building layouts have been studied (Figs. 1.19 and 1.20): layouts in which the streets in one directions are twice wider than in the other direction. Also, all layouts are studied in dam break flow (Fig. 1.19) as well as in steady flow (Fig. 1.20) for various discharges. For the steady flow, obstructing blocks have been placed aside the city to prevent the flow to by-pass it, in order that the difference of water level ahead and behind the city is amplified and the effect of the city on the flow is more visible.

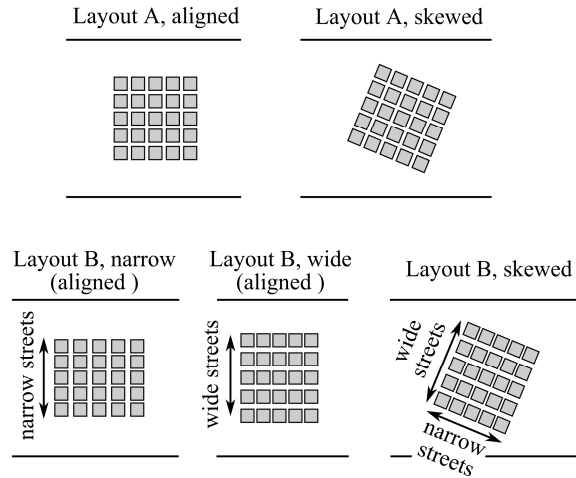


Fig. 1.19: City layouts in dam break flow. In layout A, the streets are all equally wide while in layout B, the streets in one direction are twice wider than in the other direction.

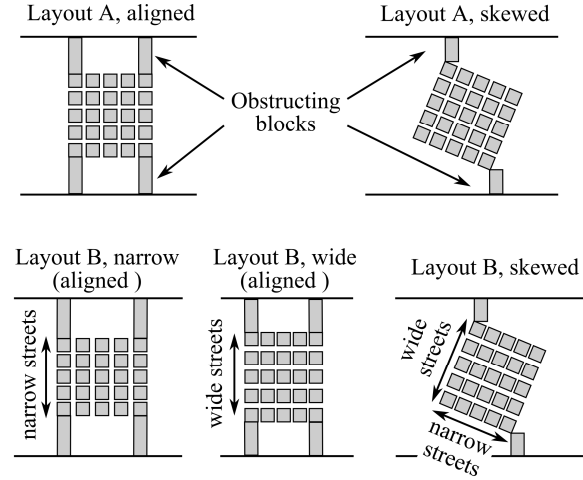


Fig. 1.20: City layouts in steady flow. In layout A, the streets are all equally wide while in layout B, the streets in one direction are twice wider than in the other direction.

Chapter 5 is devoted to the presentation of the numerical solution of the porosity equations by the finite-volume method. Two solvers have been found in the literature and they are briefly explained and compared.

The question of the porosity transition at the city border has never been raised before and is addressed in Chapter 6. Some proposals are provided based on the porous media literature. The porosity model is then tested with the different border porosity transitions in various test cases:

- the steady flow experiments described in Chapter 4 (with obstructing blocks aside the city, Fig. 1.20);
- the steady flow without obstructing blocks (as no experimental data are available for these test cases, the porosity model is compared to a detailed model considered as the reference solution);
- the dam break experiments described in Chapter 4 (Fig. 1.19).

The drag/dispersion tensor used in these test cases has been calibrated in an infinite medium in transient flow in Chapter 3. It results from the test cases that the porosity model gives satisfactory results provided that the porosity at the border of the city is appropriately chosen. Indeed, as the obstructing blocks prevent the flow from by-passing the city, the porosity should be lowered at the border to take the dead zones into account in the transverse streets (in the case of the aligned city layout) and the side effect due to the presence of these blocks. Otherwise, if no side blocks are present, a discontinuous or a linear porosity transition is preferable.

Chapter 2 Shallow water equations with porosity

This chapter provides the development of the macroscopic (district-scale) equations governing the flow in an urban medium. The objective is to provide a clear and comprehensive development using the volume-averaging technique formalized by Bear and Bachmat (1986), Bachmat and Bear (1986) and Whitaker (1999). This technique is usually applied to three-dimensional porous media. It consists in averaging the equations governing the fluid flow over a fictitious portion of the porous medium called a representative elementary volume (REV).

The originality of the present development is that we focus on a medium whose solid matrix is composed of emerged cylinders (the buildings). This requires to re-define the average (macroscopic) variables and to re-develop the averaging theorems that will be used in the development of the macroscopic equations. This is the object of Section 2.1. Then, in Section 2.2, it is explained how to determine the size of the REV and the porosity of the medium. Section 2.3 is devoted to the upscaling of the three-dimensional Navier-Stokes point equations to obtain the macroscopic two-dimensional equations, which are the shallow water equations with porosity.

Once the governing equations for urban flood have been derived, as a complement of information, Section 2.4 briefly presents how the volume averaging technique is applied in classical 3D porous media. The objective is to show that, even if the starting point consists of the same Navier-Stokes equations, developing the equations with the volume averaging technique by making other assumptions and considering another type of REV leads to very different governing equations.

This chapter ends with a conclusion in which the final equations for urban flood modeling are recalled and compared to the former model developed by Lhomme (2006).

2.1 Averaging tools

The volume-averaging technique is an upscaling technique consisting in averaging the quantities and the equations from the pore scale to macroscopic scale. The pore scale is also called the microscopic or the local scale, while the macroscopic scale is called the global scale. In the urban medium context, the local scale is the street scale and the global scale is the district scale.

2.1.1 Averaging surfaces and volumes

The volume averaging technique relies on the concept of a representative elementary volume (REV). The centroid of the REV is called $\mathbf{x}$, as depicted in Fig. 2.1. It encircles several buildings so that a part of its surface, A_f , is free for the flow and the other part is occupied by the buildings. A REV exist at each point of the domain, no matter if that point lies in the free part A_f or in a building, and all the REV have the same total surface A_0 . The porosity ϕ of a REV is defined as the ratio A_f/A_0 . The REV is assumed to have a diameter large enough so that its porosity is the average porosity of the medium. It is then considered to be representative. The buildings have any cross sectional shape. In the following, for the sake of generality, we will use the term "cylinder" instead of "building".

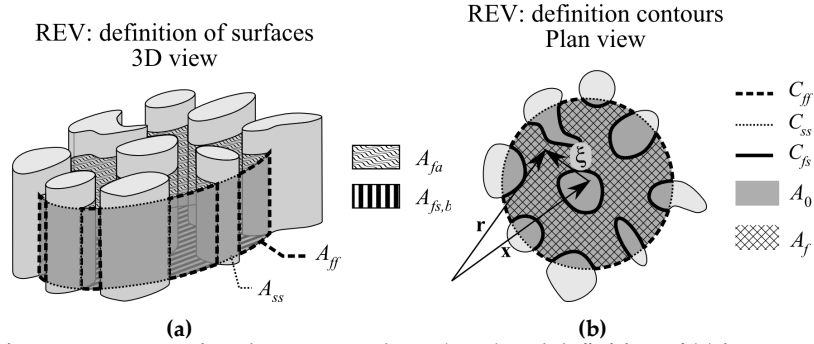


Fig. 2.1: Representative elementary volume (REV) and definition of (a) its contour surfaces A_{fa} , $A_{fs,b}$, A_{ff} and A_{ss} in a 3D view and (b) its contour curves C_{ff} , C_{ss} , C_{fs} and the projected surfaces A_0 and A_f in a plan view. Fig. (2.1b) is the plan view of Fig. (2.1a).

The symbols used to designate the faces and contour curves of the REV are illustrated in Fig. 2.1. Subscript f is used to refer to the fluid phase, subscript

s to the solid phase, subscript fs to a fluid-solid interface, subscript ff to a fluid-fluid interface, fa to a fluid-air interface (the water surface), and ss to a solid-solid interface (the side of the solid cylinders intercepted by the REV). The base of the REV is the bottom $A_{fs,b}$ (b stands for "bottom") and the top is the water surface A_{fa} (these latter are not necessarily horizontal). The REV sides are vertical and intersect either the fluid or the solid phase. The part intercepting the fluid is called A_{ff} and the part intercepting the solid is called A_{ss} . The vertical surfaces consisting of the walls of the set of cylinders inside the REV is called $A_{fs,w}$ where w stands for "walls". The contour curves are illustrated in Fig. 2.1-b. The two-dimensional curve C_{fs} is the set of the bottom footprints of all the cylinders lying inside the REV. The footprint of the REV boundary in the water phase is C_{ff} , while C_{ss} is the part of the REV boundary intercepting the solid phase. The two-dimensional horizontal surfaces A_0 and A_f are the parts of the REV horizontal projection including and excluding the cylinders, respectively. The surface A_f is then the projection of $A_{fs,b}$ (the bottom) and A_{fa} (the water surface) on a horizontal plane. The centroid of the REV is referred to by a two-dimensional position vector $\mathbf{x}$, so that any point inside the REV is characterized by a position vector

$$(\mathbf{r}, z) = (\mathbf{x} + \boldsymbol{\xi}, z) \quad (2.1)$$

where $\mathbf{r}$ is the horizontal polar coordinate of a point, z is its vertical coordinate and $\boldsymbol{\xi}$ is its relative horizontal position with respect to the centroid of the REV (Fig. 2.1-b). We define a distribution function Γ such that

$$\Gamma(\mathbf{r}) = 1 \text{ if the point } (\mathbf{r}, 0) \text{ lies in the fluid phase} \quad (2.2-a)$$

$$\Gamma(\mathbf{r}) = 0 \text{ if the point } (\mathbf{r}, 0) \text{ lies in the solid phase.} \quad (2.2-b)$$

As the cylinders are assumed to have infinite vertical walls, Γ does not depend on the vertical coordinate z . Based on that, we can define the porosity:

$$\phi(\mathbf{x}) = \frac{1}{A_0} \int_{A_0} \Gamma dA = \frac{A_f}{A_0}. \quad (2.3)$$

The Γ function also allows us to define the volume U_f occupied by the fluid phase in the REV:

$$U_f(\mathbf{x}, t) = \int_{U_f} \Gamma dU = \int_{A_f} \left(\int_{z=z_b}^{z=z_w} \Gamma dz \right) dA = \int_{A_f} h dA \quad (2.4)$$

where h is the water depth, z_b is the bottom level and z_w is the water level.

2.1.2 Averaging operators and average quantities

The averaging operators are defined as follows. If e is an extensive local variable defined in the 3D space, its volumetric intrinsic phase average or volume-average, denoted by $\langle e \rangle_U^f$, is defined as:

$$\langle e \rangle_U^f(\mathbf{x}, t) = \frac{1}{U_f} \int_U e(\mathbf{x} + \boldsymbol{\xi}, z, t) \Gamma(\mathbf{x} + \boldsymbol{\xi}) dU = \frac{1}{U_f} \int_{U_f} e(\mathbf{x} + \boldsymbol{\xi}, z, t) dU \quad (2.5)$$

where subscript U in the notation $\langle \rangle_U^f$ is used to recall that the integral is carried out over a volume (and not an area) and superscript f reminds that the integral is divided by U_f , the volume of the fluid phase (and not the total volume U). If E is the depth-averaged value of e , then:

$$E(\mathbf{r}, t) = \frac{1}{h(\mathbf{r}, t)} \int_{z=z_b}^{z=z_w} e(\mathbf{r}, z, t) dz. \quad (2.6)$$

For example, if e is a component of the fluid velocity $\mathbf{v} = (v_x, v_y, v_z)$, then E is the corresponding component of the water depth-averaged velocity $\mathbf{V}$:

$$\begin{aligned} \mathbf{V}(\mathbf{r}, t) &= \begin{pmatrix} V_x(\mathbf{r}, t) \\ V_y(\mathbf{r}, t) \end{pmatrix} = V_i(\mathbf{r}, t) \\ &= \frac{1}{h(\mathbf{r}, t)} \int_{z=z_b}^{z=z_w} v_i(\mathbf{r}, z, t) dz \quad i = 1, 2 \end{aligned} \quad (2.7)$$

we can define the surface average of E , denoted by $\langle E \rangle_A$, or its surface intrinsic average, denoted by $\langle E \rangle_A^f$:

$$\langle E \rangle_A(\mathbf{x}, t) = \frac{1}{A_0} \int_{A_0} E(\mathbf{x} + \boldsymbol{\xi}, t) \Gamma(\mathbf{x} + \boldsymbol{\xi}) dU = \frac{1}{A_0} \int_{A_f} E(\mathbf{x} + \boldsymbol{\xi}, t) dU \quad (2.8-a)$$

$$\langle E \rangle_A^f(\mathbf{x}, t) = \frac{1}{A_f} \int_{A_0} E(\mathbf{x} + \boldsymbol{\xi}, t) \Gamma(\mathbf{x} + \boldsymbol{\xi}) dU = \frac{1}{A_f} \int_{A_f} E(\mathbf{x} + \boldsymbol{\xi}, t) dU \quad (2.8-b)$$

where subscript A in the notation $\langle \rangle_A$ is used to recall that the integral is carried out over a surface (and not a volume). The variables $\langle e \rangle_U^f$, $\langle E \rangle_A$ and $\langle E \rangle_A^f$ are macroscopic variables. Unlike the local variables, they are defined at any point $\mathbf{x}$ of the space, no matter if it stands in the solid or fluid phase. A relation exists between the surface intrinsic average and volumetric intrinsic average quantities:

$$\langle E \rangle_A = \frac{A_f}{A_0} \langle E \rangle_A^f = \phi \langle E \rangle_A^f \quad (2.9)$$

We define the deviations of a two-dimensional variable from its intrinsic average as:

$$\delta E(\mathbf{r}, t) = E(\mathbf{r}, t) - \langle E \rangle_A^f(\mathbf{x}, t). \quad (2.10)$$

By definition, the average of a deviation is zero:

$$\langle \delta E \rangle_A^f = 0. \quad (2.11)$$

If $h(\mathbf{r}, t)$ is the water depth, then the intrinsic averaged water depth (or macroscopic water depth) may be defined as:

$$\langle h \rangle_A^f(\mathbf{x}, t) = \frac{1}{A_f} \int_{A_0} \Gamma(\mathbf{r}) h(\mathbf{r}, t) dA = \frac{1}{A_f} \int_{A_f} h(\mathbf{r}, t) dA = \frac{U_f}{A_f} \quad (2.12)$$

From that definition, and with the definition of the porosity (Eq. 2.3) and of the volume U_f (Eq. 2.4), we may deduce the following relations:

$$U_f = \langle h \rangle_A^f A_f = \phi \langle h \rangle_A^f A_0 \quad (2.13)$$

$\mathbf{V}(\mathbf{r}, t)$ is the local depth-averaged velocity. As an average is meaningful only for extensive quantities, the intrinsic average (or macroscopic average) of velocity is based on the average of momentum:

$$\langle \mathbf{V} \rangle_A^f(\mathbf{x}, t) \equiv \frac{1}{\langle h \rangle_A^f} \frac{1}{A_f} \int_{A_f} \mathbf{V}(\mathbf{r}, t) h(\mathbf{r}, t) dA = \frac{1}{\langle h \rangle_A^f} \langle \mathbf{V}h \rangle_A^f, \quad (2.14)$$

so that we have

$$\langle \mathbf{V}h \rangle_A^f = \langle \mathbf{V} \rangle_A^f \langle h \rangle_A^f. \quad (2.15)$$

Another consequence of the definitions of the porosity (Eq. 2.3), the volumetric intrinsic average (Eq. 2.5), the surface intrinsic averaged water depth (Eq. 2.12) and the relation between U_f and A_f (Eq. 2.13) is that:

$$\frac{1}{A_0} \int_{U_f} e dU = \frac{1}{A_0} \frac{A_f}{A_f} \frac{U_f}{U_f} \int_{U_f} e dU = \phi \langle h \rangle_A^f \langle e \rangle_U^f. \quad (2.16)$$

Eq. (2.16) will be used in Sections 2.3.1 and 2.3.2 in the averaging process.

2.1.3 Averaging theorems

For the averaging process of local equations, averaging theorems are needed to relate the average of a gradient and the gradient of an average. Two types of theorems will be used: the time-derivative theorem and the spatial-gradient theorem. As the present porous medium is two-dimensional, we cannot directly apply the usual theorems from the literature that are written for three-dimensional porous media. These theorems are then adapted to a two-dimensional context and demonstrated in the appendices and only their final forms are listed herebelow.

Appendix A shows how to link the average of a time-derivative and the time-derivative of an average. This is not straightforward as one of the boundaries of the REV is moving: the water surface moves up and down. So the Leibniz rule must be applied by taking the velocity of the water surface into account. By using in addition the definitions and Eqs (2.5 – 2.16), we may write:

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial t} \right\rangle_U^f = \phi \frac{\partial \langle Eh \rangle_A^f}{\partial t} - \frac{1}{A_0} \int_{A_f} e|_{z_w} \frac{\partial h}{\partial t} dA. \quad (2.17)$$

To link the average of a x_i -derivative to the x_i -derivative of an average, the divergence theorem is applied to the REV and gives (see the details in Appendix B.3):

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f = \frac{\partial}{\partial x_i} \left(\phi \langle hE \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i dA \quad (2.18)$$

where n_i is the i -component of the outward unit vector $\mathbf{n}$, A_{fs} is the fluid-solid interface of the REV and A_{fa} is the water surface (Fig. 2.1a). If we decompose E and h into their average and deviation parts (with Eq. 2.10), and if we recall that the total fluid-solid interface is constituted of the bottom and the

cylinders walls ($A_{fs} = A_{fs,b} \cup A_{fs,w}$) and if we notice that the integral over $A_{fs,w}$ is equivalent to the contour integral over C_{fs} of the integral over the water depth h , after some developments this theorem may be written in the following form (Appendix B.4):

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f = \frac{\partial}{\partial x_i} \left[\phi \langle h E \rangle_A^f \right] - \phi \langle e |_{z_w} \nabla z_w \rangle_A^f + \phi \langle e |_{z_b} \nabla z_b \rangle_A^f - \langle E \rangle_A^f \langle h \rangle_A^f \frac{\partial \phi}{\partial x_i} + \frac{1}{A_0} \left[\int_{C_{fs,w}} \delta E \delta h n_i dC \right] \quad (2.19)$$

The spatial derivative theorem, Eq. (2.18), may easily be generalized to the average of a divergence (Appendix B.5):

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e_i}{\partial x_i} \right\rangle_U^f = \frac{\partial}{\partial x_i} \left(\phi \langle h E_i \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e_i n_i dA \quad (2.20)$$

where the Einstein convention is used and i is a dummy index.

2.2 Representative Elementary Volume (REV)

The REV must be large enough for its properties (porosity, permeability, mean drag coefficients...) to be statistically meaningful. Usually, the porosity is the parameter chosen to determine the minimum size of the REV (although this verification should be done for every parameters): if the REV is very small, let us say on the order of a particle size (in which case the REV is not "representative" anymore and should just be called an "EV"), its porosity will not be the same as if the REV were very large.

We could visualize an experiment consisting in calculating the porosity of a succession of REV's (or EV's) of gradually increasing volumes $U_{f1} < U_{f2} < U_{f3} < \dots$ all centered at a certain point of the space $\mathbf{x}_0$, hoping that beyond a certain size the porosity will not change significantly anymore. This is illustrated in Fig. (2.2). For very small values of U , the porosity is one or zero, depending on whether the EV happens to fall in the void space (in a street) or in the solid (in a building). As the EV increases, the porosity presents large fluctuations. Then these fluctuations are gradually attenuated until U

reaches a certain threshold $U_{\min}$ above which the porosity becomes constant. The porosity may then, beyond a certain size $U_{\max}$, either increase or decrease, indicating that the porous medium is heterogeneous. The acceptable range for the REV is:

$$U_{\min} < U_0 < U_{\max} \quad (2.21)$$

Theoretically, if a range satisfying Eq. (2.21) cannot be found, the domain cannot be represented as a continuum.

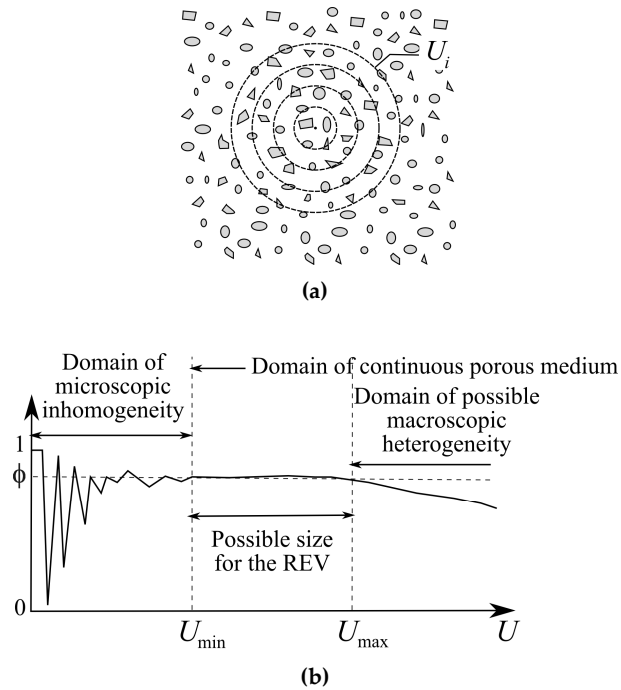


Fig. 2.2: Determination of the REV size by a fictitious experiment: (a) elementary volumes (EV) of growing volume U_i and (b) corresponding porosity.

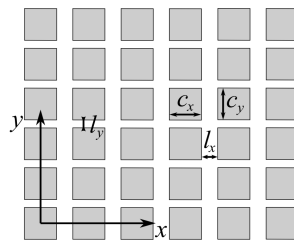


Fig. 2.3: Top view of a periodic medium consisting of rectangular buildings and perpendicular streets.

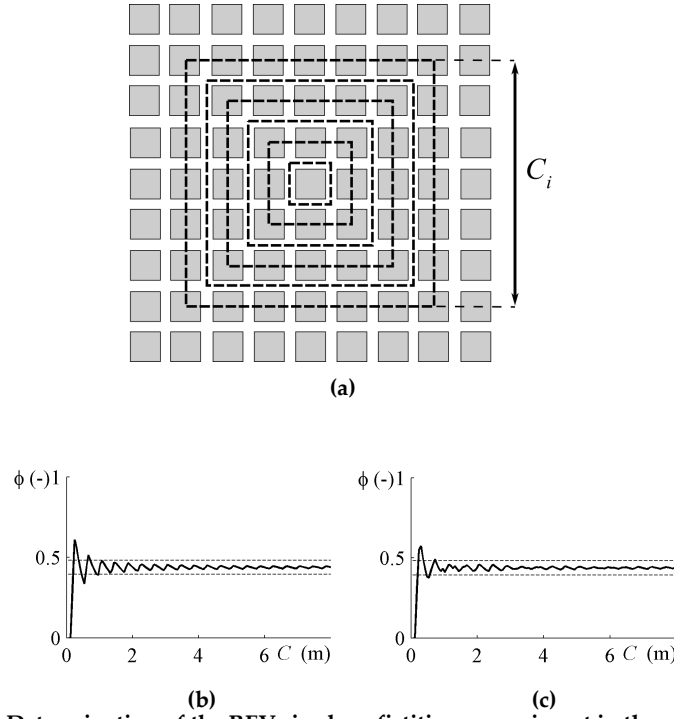


Fig. 2.4: Determination of the REV size by a fictitious experiment in the context of a periodic urban porous medium: (a) square elementary volumes (EV) of growing width C_i (example of layout A), (b) corresponding porosity variation for layout A and (c) corresponding porosity variation for layout B. The two horizontal dashed lines represent the $\pm 10\%$ of the asymptotic value of the porosity.

For a periodic urban medium with perpendicular streets (Fig. 2.3), we could imagine the same experiment of calculating the porosity of EVs of increasing size (Fig. 2.4a). This has been done for the two buildings layouts studied throughout this thesis: layout A for which $l_x = l_y = 0.1$ m and $c_x = c_y = 0.3$ m, and layout B for which $l_x = 0.135$ m, $l_y = 0.0675$ m and $c_x = c_y = 0.3$ m. Fig. (2.4) shows the evolution of porosity as a function of the EV size for layout A (Fig. 2.4a) and layout B (Fig. 2.4b). For both layouts, the EV begins to be representative when its size exceeds 1 m.

In practice, Guinot (2012) showed that the geometry of real urban media is such that the REV dimension exceeds the size of a city, making the REV meaningless. However, he says that this does not make impossible the use of a macroscopic approach, provided that we keep in mind the loss of precision due to the assumption of the existence of a REV.

2.3 Averaged equations

The volume averaging technique consists in averaging on a REV the local equations governing the flow of the liquid phase at the pore scale. The definitions and theorems presented in Section 2.1 are used to derive the macroscopic equations. Section 2.3.1 presents the average of the continuity equation, and Section 2.3.2 the average of the momentum equation. The resulting macroscopic equations will only contain macroscopic variables together with parameters accounting for the geometry of the medium, and some extra terms that will need a closure equation. The local equations are the Navier-Stokes-Boussinesq equations for an incompressible fluid:

$$\frac{\partial v_i}{\partial x_i} = 0 \quad i = 1, 2, 3 \quad (2.23)$$

$$\frac{\partial v_i}{\partial t} + \frac{\partial v_i v_j}{\partial x_j} = \frac{1}{\rho} \frac{\partial \sigma_{ij}}{\partial x_j} + g_i \quad i = 1, 2, 3, \quad j = 1, 2, 3 \quad (2.24)$$

where ρ is the mass density, $\boldsymbol{\sigma}$ is the non-isotropic stress tensor whose elements are σ_{ij} (the i -component of the stress on a face normal to the j -direction), while $\mathbf{g}$ is the gravity acceleration. The Einstein notation convention is adopted. We assume that

$$\sigma_{ij} = -p\delta_{ij} + \sigma_{ij}^t \quad (2.25)$$

where $p\delta_{ij}$ is the isotropic pressure (δ_{ij} being the identity tensor) and σ_{ij}^t the deviatoric stress.

2.3.1 Continuity

The average of the continuity equation writes

$$\frac{1}{A_0} \int_{U_f} \frac{\partial v_i}{\partial x_i} dU = 0, \quad (2.26)$$

and, by Eq. (2.16),

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial v_i}{\partial x_i} \right\rangle_U^f = 0 \quad (2.27)$$

By applying the divergence theorem Eq. (2.20), we directly get

$$\frac{\partial}{\partial x_i} \left(\phi \langle h V_i \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} v_i n_i dA = 0. \quad (2.28)$$

where V_i is the depth-average of the velocity v_i (Eq. 2.7). If we assume that the velocity is tangent to any fluid-solid interface, the integral on A_{fs} in Eq. (2.28) cancels out. The integral on A_{fa} , however, is not zero as the component of the velocity orthogonal to this surface is the water elevation rate, $\partial h / \partial t$. Eq. (2.28) is then equivalent to:

$$\frac{\partial}{\partial x_i} \left(\phi \langle h V_i \rangle_A^f \right) + \frac{1}{A_0} \int_{A_f} \frac{\partial h}{\partial t} dA = 0, \quad (2.29)$$

and, as the surface A_f does not depend on time, the time derivative can be moved outside the integral and finally Eq. (2.29) becomes

$$\frac{\partial \left(\phi \langle h \rangle_A^f \right)}{\partial t} + \frac{\partial}{\partial x_i} \left(\phi \langle h \rangle_A^f \langle V_i \rangle_A^f \right) = 0, \quad (2.30)$$

which is the macroscopic continuity equation. Eq. (2.30) may be rewritten in the developed form:

$$\frac{\partial \left(\phi \langle h \rangle_A^f \right)}{\partial t} + \frac{\partial}{\partial x} \left(\phi \langle h \rangle_A^f \langle V_x \rangle_A^f \right) + \frac{\partial}{\partial y} \left(\phi \langle h \rangle_A^f \langle V_y \rangle_A^f \right) = 0. \quad (2.30a)$$

2.3.2 Momentum equation

The REV-averaged momentum equations reads

$$\frac{1}{A_0} \int_{U_f} \left(\frac{\partial v_i}{\partial t} + \frac{\partial v_i v_j}{\partial x_j} \right) dU = \frac{1}{A_0} \int_{U_f} \left(-\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial \sigma_{ij}^t}{\partial x_j} + g_i \right) dU \quad i = 1, 2; \quad (2.31)$$

or, according to Eq. (2.16),

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial v_i}{\partial t} + \frac{\partial v_i v_j}{\partial x_j} \right\rangle_U^f = \phi \langle h \rangle_A^f \left\langle -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial \sigma_{ij}^t}{\partial x_j} + g_i \right\rangle_U^f \quad i = 1, 2 \quad (2.32)$$

Remark that, as our problem is two-dimensional, only the x - and y -components of the momentum equation will be averaged, explaining why only $i = 1, 2$ is considered. Indeed, the z -component of the momentum balance reduces to the hydrostatic pressure relation. By successively applying theorem (2.17) to the $\partial v_i / \partial t$ term (first term of the left-hand side), the divergence theorem (2.20) to the divergence of $\mathbf{v}\mathbf{v}$ (second term of the left-hand side) and the divergence of $\boldsymbol{\sigma}^t$ (second term of the right-hand side), the spatial derivative theorem (2.19) to the pressure gradient (first term of the right-hand side), and noting that $g_i = 0$ for $i = 1, 2$, we get that:

$$\begin{aligned}
& \phi \frac{\partial \langle V_i h \rangle_A^f}{\partial t} - \frac{1}{A_0} \int_{A_f} v_i \Big|_{z_w} \frac{\partial h}{\partial t} dA \\
& + \frac{\partial}{\partial x_j} \left(\phi \langle B h V_i V_j \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} v_i v_j n_j dA \\
& = - \frac{1}{\rho} \frac{\partial}{\partial x_i} \left[\phi \langle h P \rangle_A^f \right] + \frac{\phi}{\rho} \left\langle p \Big|_{z_w} \nabla z_w \right\rangle_A^f - \frac{\phi}{\rho} \left\langle p \Big|_{z_b} \frac{\partial z_b}{\partial x_i} \right\rangle_A^f \\
& + \frac{1}{\rho} \langle P \rangle_A^f \langle h \rangle_A^f \frac{\partial \phi}{\partial x_i} - \frac{1}{\rho A_0} \left[\int_{C_{fs,w}} \delta P \delta h n_i dC \right] \\
& + \frac{1}{\rho} \left[\frac{\partial}{\partial x_j} \left(\phi \langle h T_{ij} \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} \sigma_{ij}^t n_j dA \right]
\end{aligned} \tag{2.33}$$

where the variables P and T_{ij} are the depth-averages of p and σ_{ij}^t , respectively:

$$P = \frac{1}{h} \int_{z_b}^{z_w} p dz \tag{2.34}$$

$$T_{ij} = \frac{1}{h} \int_{z_b}^{z_w} \sigma_{ij}^t dz, \tag{2.35}$$

and β is a kind of Boussinesq coefficient accounting for the non uniformity of the cross velocity distribution:

$$\beta = \frac{\int_{z_b}^{z_w} v_i v_j dz}{h V_i V_j}. \tag{2.36}$$

We will assume that β is equal to 1, which is admissible in turbulent flows (Graf and Altinakar, 2000). Eq. (2.33) may be further developed. First, the

average of the product in the third term of the left-hand side may be written as:

$$\frac{\partial}{\partial x_j} \left(\phi \langle h V_i V_j \rangle_A^f \right) = \frac{\partial}{\partial x_j} \left(\phi \langle h \rangle_A^f \langle V_i \rangle_A^f \langle V_j \rangle_A^f + \phi \langle d_{ij} \rangle_A^f \right) \quad (2.37)$$

where d_{ij} is the $(i,j)^{\text{th}}$ element of $\mathbf{d}$ and is defined as:

$$d_{ij} = \langle h \rangle_A^f \delta V_i \delta V_j + \delta h \langle V_i \rangle_A^f \delta V_j + \delta h \delta V_i \langle V_j \rangle_A^f + \delta h \delta V_i \delta V_j \quad (2.38a)$$

or, written in another notation:

$$d_{ij} = \mathbf{d} = \begin{pmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{pmatrix} \quad (2.38b)$$

where

$$\begin{aligned} d_{xx} &= \langle h \rangle_A^f (\delta V_x)^2 + 2\delta h \langle V_x \rangle_A^f \delta V_x + \delta h (\delta V_x)^2 \\ d_{xy} &= \langle h \rangle_A^f \delta V_x \delta V_y + \delta h \langle V_x \rangle_A^f \delta V_y + \delta h \delta V_x \langle V_y \rangle_A^f + \delta h \delta V_x \delta V_y \\ d_{yx} &= \langle h \rangle_A^f \delta V_y \delta V_x + \delta h \langle V_y \rangle_A^f \delta V_x + \delta h \delta V_y \langle V_x \rangle_A^f + \delta h \delta V_y \delta V_x \quad (= d_{xy}) \\ d_{yy} &= \langle h \rangle_A^f (\delta V_y)^2 + 2\delta h \langle V_y \rangle_A^f \delta V_y + \delta h (\delta V_y)^2 \end{aligned}$$

The first term of the right-hand side of Eq. (2.37) is the bulk inertia term and represents the convection of momentum by the average velocity. The second term of the right-hand side of Eq. (2.37) is the mechanical dispersion. It arises from the velocity deviations due to the presence of the obstacles. Indeed, the average velocity does not capture the whole momentum transport effect; the local velocity deviations also physically transport momentum and result in a macroscopic mixing effect. A closure has to be defined for this term.

If we assume that the velocity is tangent to any material boundary, the integral of $v_i v_j n_j$ over A_{fs} in the fourth term of the left-hand side of Eq. (2.33) is equal to zero. However, the integral over A_{fa} is not equal to zero:

$$\frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} v_i v_j n_j dA = 0 + \frac{1}{A_0} \int_{A_{fa}} v_i \Big|_{z_w} \frac{\partial h}{\partial t} dA, \quad (2.39)$$

but it will cancel out with the second term of the left-hand side of Eq. (2.33).

For shallow water flows, it is generally assumed that $v_z = 0$, reducing the third momentum Eq. (2.24) to:

$$p(x, y, z, t) = \gamma(z_w(x, y, t) - z). \quad (2.40)$$

We then define:

$$P(x, y, t) = \frac{1}{h} \int_{z_b}^{z_w} \gamma(z_w(x, y, t) - z) dz = \gamma h / 2, \quad (2.41)$$

and

$$p|_{z_w} = 0 \quad (2.42)$$

$$p|_{z_b} = \gamma h. \quad (2.43)$$

In the first term of the left-hand side of Eq. (2.33), we then have

$$\langle hP \rangle_A^f = \langle h^2 \rangle_A^f, \quad (2.44)$$

which we may develop using the decomposition of h into its average and its deviation (Eq. 2.10):

$$\begin{aligned} \langle hP \rangle_A^f &= \langle h^2 \rangle_A^f \\ &= \left\langle \left(\langle h \rangle_A^f + \delta h \right)^2 \right\rangle_A^f \\ &= \left\langle \left(\langle h \rangle_A^f \right)^2 + 2 \langle h \rangle_A^f \delta h + (\delta h)^2 \right\rangle_A^f \\ &= \left(\langle h \rangle_A^f \right)^2 + \langle (\delta h)^2 \rangle_A^f \end{aligned} \quad (2.45)$$

In the present approach, the flow is assumed to be turbulent, implying that the velocity profile varies sharply near the walls but is rather constant inside the pores (the streets), as illustrated in Fig. 2.5. As the turbulent stresses T_{ij} depend on the derivative of the velocity, these are negligible in the inner region of the flow compared to the region close to the walls. We will then make the assumption that

$$\langle hT_{ij} \rangle_A^f$$

is negligible with regard to

$$\frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} \sigma_{ij}^t n_j dA$$

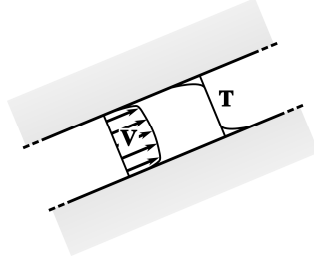


Fig. 2.5: Velocity and stress distribution in a street (top view). The grey areas represent two buildings. The quasi-uniformity of the velocity distribution in the inner part of the street implies that the stress tensor in this inner part is negligible with regard to the stress near the walls.

It is also assumed that the effect of the wind is negligible so that the stress on the water surface A_{fa} in the last term of the right-hand side of Eq. (2.30) is zero. Finally, Eq. (2.33) with the definitions given by Eqs. (2.34 – 2.36), the development of Eq. (2.37) and the assumption of Eq. (2.40) reads

$$\begin{aligned} & \phi \frac{\partial \langle h \rangle_A^f \langle V_i \rangle_A^f}{\partial t} + \frac{\partial}{\partial x_j} \left(\phi \langle h \rangle_A^f \langle V_i \rangle_A^f \langle V_j \rangle_A^f \right) + \frac{\partial}{\partial x_j} (\phi d_{ij}) \\ &= \frac{\partial}{\partial x_i} \left[-\phi \frac{g \langle h \rangle_A^f{}^2}{2} - \phi \frac{g \langle (\delta h)^2 \rangle_A^f}{2} \right] - \phi \left\langle gh \frac{\partial z_b}{\partial x_i} \right\rangle_A^f \\ &+ \frac{g \langle h \rangle_A^f{}^2}{2} \frac{\partial \phi}{\partial x_i} + \frac{1}{A_0} \left[\int_{C_{fs,w}} \frac{g (\delta h)^2}{2} n_i dC \right] + \frac{1}{\rho} \left[\frac{1}{A_0} \int_{A_{fs}} \sigma_{ij}^t n_j dA \right] \end{aligned} \quad (2.46)$$

In Eq. (2.46), the two first terms of the left-hand side are the bulk inertia terms and the third one is the mechanical dispersion. The five first terms of the right-hand side arise from the average of the isotropic pressure term. The two first ones come from the pressure on A_{ff} , the third one is component in the x_i -direction of the reaction against the bottom, the fourth one is due to a difference of hydrostatic pressure exerted by the buildings when these latter are not uniformly distributed, this non-uniformity resulting in a porosity gradient, and the fifth one is the shape drag force exerted by the cylinders that can be viewed as an unbalanced pore-scale pressure on the buildings' walls (Fig. 2.6). The last term of the right-hand side of Eq. (2.46)

represents the friction against the solid-fluid interfaces, i.e. against the bottom and the walls of the cylinders.

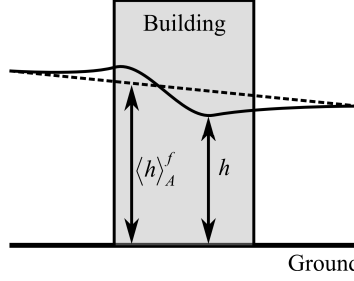


Fig. 2.6: Profile view of the flow in the neighbourhood of a building. The small-scale water depth variation induces an unbalanced pressure on the walls, originating a shape drag force (Sanders et al., 2008).

It is demonstrated in Appendix C that

$$\frac{\partial}{\partial x_i} \left[\phi \frac{g \langle (\delta h)^2 \rangle_A^f}{2} \right] \ll \frac{1}{A_0} \left[\int_{C_{fs,w}} \frac{g (\delta h)^2}{2} n_i dC \right], \quad (2.47)$$

meaning that the gradient of the average of a deviation is negligible compared to the integral of this deviation over the walls of the buildings of the REV. This allows one to neglect the second term of the right-hand side of Eq. (2.46). Moreover, we introduce the following assumption in order to simplify the model:

$$\phi \left\langle gh \frac{\partial z_b}{\partial x_i} \right\rangle_A^f \cong \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial x_i}. \quad (2.48)$$

The friction term of Eq. (2.46) is now expressed as a function of macroscopic quantities. The method here chosen is the one commonly used for a flume with rough bed described in Graf and Altinakar (2000). This method consists in assuming that the head losses produced by friction in transient flow are the same as if the flow were steady and uniform. Writing Eq. (2.46) for steady and uniform flow, i.e. cancelling all the deviations and spatial and time derivatives (the only leaving driving force is the inclination of the bottom and the only resistance force is the friction on A_{fs}) leads to the following form:

$$\frac{1}{\rho A_0} \int_{A_{fs}} \sigma_{ij}^t n_j dA = \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial x_i}. \quad (2.49)$$

It is assumed that, in steady and uniform flow, the bed slope $-\partial z_b / \partial x_i$ is equal to the head loss produced by friction S_{fi} . This allows Eq. (2.49) to be written as:

$$\frac{1}{\rho A_0} \int_{A_{fs}} \sigma_{ij}^t n_j dA = -\phi g \langle h \rangle_A^f S_{fi} \quad (2.50)$$

We now have to link S_{fi} to the variables and parameters of the model. For practical use, the most common way to express the head losses caused by friction is to use the Manning-Strickler coefficient n , that has the advantage of being universally used in open-channel hydraulics. It is linked to the velocity and the head loss S_{fi} in the following way:

$$S_{fi} = \frac{n^2 \langle V_i \rangle_A^f}{R_h^{4/3}} \left| \langle \mathbf{V} \rangle_A^f \right| \quad (2.51)$$

where R_h is the hydraulic radius, that is the ratio between the volume of water in the REV and the total contact surface between fluid and solid:

$$R_h = \frac{\phi \langle h \rangle_A^f A_0}{A_{fs,b} + A_{fs,w}} = \frac{\phi \langle h \rangle_A^f}{\frac{A_{fs,b}}{A_0} + \frac{A_{fs,w}}{A_0}} = \frac{\phi \langle h \rangle_A^f}{\phi + \langle h \rangle_A^f \frac{C_{fs,w}}{A_0}} \quad (2.52)$$

In conclusion, under the assumptions that the velocity is tangent to any material boundary, that the effect of wind is negligible, that $\beta = 1$ (see Eq. 2.36), that the flow is turbulent, that the variation of the bed slope within the REV is negligible (see Eq. 2.48), that the head losses produced by friction in transient state are calculated as if the flow were steady and uniform and that they can be expressed by Eq. (2.51), the macroscopic momentum Eq. (2.46) with Eqs. (2.47), (2.48), (2.50) and (2.51) then becomes

$$\begin{aligned} & \phi \frac{\partial \langle h \rangle_A^f \langle V_i \rangle_A^f}{\partial t} + \frac{\partial}{\partial x_j} \left(\phi \langle h \rangle_A^f \langle V_i \rangle_A^f \langle V_j \rangle_A^f \right) + \frac{\partial}{\partial x_i} \left(\phi \frac{g \langle h \rangle_A^f{}^2}{2} \right) + \frac{\partial}{\partial x_j} (\phi d_{ij}) \\ &= \frac{g \langle h \rangle_A^f{}^2}{2} \frac{\partial \phi}{\partial x_i} - \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial x_i} + \frac{1}{A_0} \int_{C_{fs,w}} \frac{g (\delta h)^2}{2} n_i dC - \phi g \langle h \rangle_A^f S_{fi} \end{aligned} \quad (2.53)$$

where S_{fi} is linked with the macroscopic velocity and hydraulic radius by Eq. (2.51) and $\mathbf{d}$ has been defined in Eq. (2.38). Eq. (2.53) is the final momentum

equation of the porosity model. A closure is defined for the dispersion and the shape drag terms (fourth term of the left-hand side and third term of the right-hand side of Eq. (2.53), respectively) in the next chapter.

Eq. (2.53) may be written in the notation with partial derivatives as:

$$\begin{aligned}
& \phi \frac{\partial \langle h \rangle_A^f \langle V_x \rangle_A^f}{\partial t} + \frac{\partial}{\partial x} \left(\phi \langle h \rangle_A^f \left(\langle V_x \rangle_A^f \right)^2 + \phi \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \right) \\
& + \frac{\partial}{\partial y} \left(\phi \langle h \rangle_A^f \langle V_x \rangle_A^f \langle V_y \rangle_A^f \right) + \frac{\partial}{\partial x} (\phi d_{xx}) + \frac{\partial}{\partial y} (\phi d_{xy}) \\
& = \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \frac{\partial \phi}{\partial x} - \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial x} + \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_x dC - \phi g \langle h \rangle_A^f S_{fx}
\end{aligned} \tag{2.54}$$

$$\begin{aligned}
& \phi \frac{\partial \langle h \rangle_A^f \langle V_y \rangle_A^f}{\partial t} + \frac{\partial}{\partial x} \left(\phi \langle h \rangle_A^f \langle V_x \rangle_A^f \langle V_y \rangle_A^f \right) \\
& + \frac{\partial}{\partial y} \left(\phi \langle h \rangle_A^f \left(\langle V_y \rangle_A^f \right)^2 + \phi \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \right) + \frac{\partial}{\partial x} (\phi d_{yx}) + \frac{\partial}{\partial y} (\phi d_{yy}) \\
& = \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \frac{\partial \phi}{\partial y} - \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial y} + \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_y dC - \phi g \langle h \rangle_A^f S_{fy}
\end{aligned} \tag{2.55}$$

where d_{xx} , d_{xy} , d_{yx} and d_{yy} have been defined in Eq. (2.38b).

2.4 Comparison with 3D porous media

In this section we present the three-dimensional counterpart of the developments made in Sections 2.3.1 and 2.3.2. The volume averaging technique is applied to the Navier-Stokes equations assuming a usual three-dimensional porous medium like depicted in Fig. (2.7). The purpose is to show how, by starting with the same fluid equations as in the urban flood

context, we can get very distinct macroscopic equations by introducing different assumptions. We will see this way that the Darcy, Forchheimer and Brinkman equations (in most usual macroscopic equations in porous media) despite having at first been derived empirically, can all be reconstructed from the same Navier-Stokes equations with a volume averaging technique by making specific assumptions. It is also interesting to see how the closures for the drag force and the dispersion terms are defined according to the flow regime.

Here, only the main equations will be presented based on the work of Loix (2005) and Tucker and Dessenberger (1994), and for further details the reader is referred to these authors. The equations governing the flow of the fluid phase in a three-dimensional porous medium are assumed to be the Navier-Stokes Eqs. (2.23, 2.24). These latter are averaged over a REV as illustrated in Fig. (2.7).

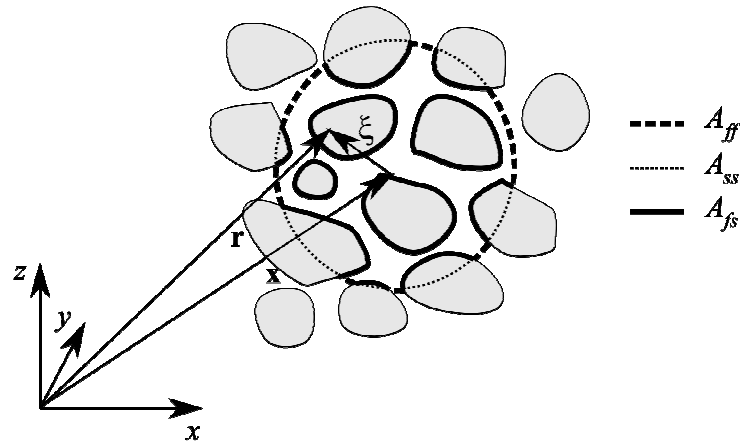


Fig. 2.7: REV in a 3D porous medium and definition of bounding surfaces A_{ff} , A_{fs} and A_{ss}

2.4.1 Theorems and assumptions

The theorems listed hereafter are the 3D counterpart of the theorems presented in Section 2.1.3 for the urban medium. They are simpler because the medium is supposed to be entirely filled with the liquid phase, and the boundary surfaces of the REV are assumed to be stationary.

Gradient theorem : let B be a scalar field on a volume U , A_{fs} be the interface inside this volume between the pores and the matrix, and n_{fs} stand for the unit normal from pores to matrix. Then, the averaged gradient may be expressed as a function of the mean value gradient:

$$\langle \nabla B \rangle_U = \nabla \langle B \rangle_U + \frac{1}{U} \int_{A_{fs}} B n_{fs} dA. \quad (2.56)$$

Extensions also deal with a first- or second-order tensor field $\mathbf{S}$:

$$\langle \nabla \cdot \mathbf{S} \rangle_U = \nabla \cdot \langle \mathbf{S} \rangle_U + \frac{1}{U} \int_{A_{fs}} \mathbf{S} \cdot n_{fs} dA. \quad (2.57)$$

Average phase gradient decomposition: The average fluid-phase gradient of the field B may be rewritten as:

$$\nabla \langle B \rangle_U = \phi \nabla \langle B \rangle_U^f + \langle B \rangle_U^f \nabla \phi \quad (2.58)$$

Porosity gradient: by assuming that $B = 1$ in Eq. (2.58), the porosity gradient may be rewritten as a function of pore-scale details

$$\nabla \phi = -\frac{1}{U} \int_{A_{fs}} n_{fs} dA. \quad (2.59)$$

The basic assumptions are listed below:

- (i) the solid phase is stationary or at least a quasi-static behaviour is assumed;
- (ii) there is no mass exchange between the solid and the fluid;
- (iii) the solid and (assumed incompressible) fluid have constant and uniform density;
- (iv) the fluid viscosity is independent of shear rate (the fluid is Newtonian);
- (v) the moving front is sharply defined; ahead of the front there is no fluid, and behind the front the pores are fully saturated with fluid;
- (vi) the capillary action is negligible;
- (vii) the only body force is gravity.

2.4.2 Mass conservation

The mass balance at the microscopic scale is written as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (2.60)$$

Applying the averaging process, this latter equation transforms into:

$$\left\langle \frac{\partial \rho}{\partial t} \right\rangle_U + \langle \nabla \cdot (\rho \mathbf{v}) \rangle_U = 0, \quad (2.61)$$

The integration order may be inverted in the first term while in the second term the gradient theorem (Eq. 2.58) is applied:

$$\frac{\partial \langle \rho \rangle_U}{\partial t} + \nabla \cdot \langle (\rho \mathbf{v}) \rangle_U + \frac{1}{U} \int_{A_{fs}} \rho \mathbf{v} \cdot \mathbf{n} = 0 \quad (2.62)$$

As the fluid is Newtonian and the solid matrix is stationary, the fluid velocity vanishes on the interface area. The last term is then equal to zero. Finally, since the density is constant, it may be factorized out from the remaining terms and simplified to provide the equation:

$$\frac{\partial \phi}{\partial t} + \nabla \cdot \langle \mathbf{v} \rangle_U = 0 \quad (2.63)$$

In a saturated medium with no consolidation, the first term is equal to zero and the equation finally writes as:

$\nabla \cdot \langle \mathbf{v} \rangle_U = 0 \quad (2.64)$
--

2.4.3 Momentum balance

The classical local momentum equation is written as:

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g}, \quad (2.65)$$

where $\mathbf{v} \mathbf{v}$ means:

$$\mathbf{v}\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \cdot \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} = \begin{bmatrix} v_x^2 & v_x v_y & v_x v_z \\ v_y v_x & v_y^2 & v_y v_z \\ v_z v_x & v_z v_y & v_z^2 \end{bmatrix} \quad (2.65b)$$

The average of Eq. (2.65) is considered:

$$\left\langle \frac{\partial \rho \mathbf{v}}{\partial t} \right\rangle_U + \langle \nabla \cdot (\rho \mathbf{v}\mathbf{v}) \rangle_U = \langle \nabla \cdot \boldsymbol{\sigma} \rangle_U + \langle \rho \mathbf{g} \rangle_U \quad (2.66)$$

The stress tensor can be rewritten by isolating an isotropic contribution p and an extra-stress tensor $\boldsymbol{\tau}$:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau} \quad (2.67)$$

The body force is assumed to be solely due to gravity and hence derives from a potential. Here we assume that this potential is the z coordinate:

$$\mathbf{g} = -\nabla(gz) \quad (2.68)$$

Considering a constant mass density, a stationary solid phase, and applying the gradient theorem and inverting the integration order, the latter equation becomes:

$$\rho \frac{\partial \langle \mathbf{v} \rangle_U}{\partial t} + \langle \nabla \cdot (\rho \mathbf{v}\mathbf{v}) \rangle_U = \nabla \cdot \langle \boldsymbol{\tau} \rangle_U - \phi \nabla \langle P \rangle_U^f + \underbrace{\int_{A_{fs}} \boldsymbol{\sigma} \cdot \mathbf{n} dA}_{-f_d} - \underbrace{\int_{A_{fs}} \rho g z \mathbf{n} dA}_{-f_g} - \underbrace{\langle P \rangle_U^f \nabla \phi}_{f_p} \quad (2.69)$$

where we have defined the modified (or reduced) pressure P as:

$$P = p + \rho g z \quad (2.70)$$

In Eq. (2.69), the last term f_d collects all the interfacial interactions between solid and fluid. Therefore f_d represents the drag force of the fluid acting on the solid along their common interface A_{fs} . Several microscopic features contribute to this macroscopic effect: (i) the stress distribution f_T , (ii) the solid contribution to the fluid buoyancy f_g , and (iii) the effect of the porosity gradient f_p (Fig. 2.8). All these terms represent the effects of the different acting forces: viscous stress, gravity and pressure.

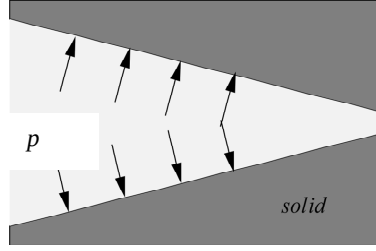


Fig. 2.8: Example of a porous medium with a fluid volume fraction gradient. The hydrostatic pressure in the fluid generates a force on the internal surface that acts in the direction opposite to $\nabla\phi$.

The so-called inertial term is the second one of the left-hand side of Eq. (2.69). It represents the macroscopic transport of momentum through the medium. It can be easily shown that, for a saturated porous medium, the averaged product of $\mathbf{v}\mathbf{v}$ can be rewritten as:

$$\langle \mathbf{v}\mathbf{v} \rangle_U = \phi \langle \mathbf{v} \rangle_U^f \langle \mathbf{v} \rangle_U^f + \phi \langle \delta \mathbf{v} \delta \mathbf{v} \rangle_U^f \quad (2.71)$$

Applying the gradient theorem, the inertial term then becomes:

$$\rho \langle \nabla \cdot (\mathbf{v}\mathbf{v}) \rangle_U = \rho \nabla \cdot \left[\phi \langle \mathbf{v} \rangle_U^f \langle \mathbf{v} \rangle_U^f + \phi \langle \delta \mathbf{v} \delta \mathbf{v} \rangle_U^f \right] + \rho \int_{A_{fs}} \mathbf{v}\mathbf{v} \cdot \mathbf{n} dA \quad (2.72)$$

As previously, the last term vanishes because the fluid is viscous and the solid matrix is stationary. The first term is the bulk inertia term, and the second one is the inertial dispersion term. These two terms were also present in the macroscopic shallow water equations (second and fourth terms in Eq. (2.53), respectively). The bulk inertial term represents the convection of momentum by the average velocity, and the dispersion term takes into account the momentum transport due to the microscopic velocity fluctuations around the average velocity. These microscopic velocity fluctuations result in a mixing effect, analogous to the Reynolds stress in turbulence. However, in the present case, momentum mixing even takes place in laminar flows and is due to the heterogeneous and geometric character of the porous medium. These remarks suggest one to express momentum mixing in the following form:

$$\rho \nabla \cdot \left[\phi \langle \delta \mathbf{v} \delta \mathbf{v} \rangle_U^f \right] = \rho \nabla \cdot \left[\mathbf{M} : \left(\nabla \langle \mathbf{v} \rangle_U + \nabla \langle \mathbf{v} \rangle_U^T \right) \right] \quad (2.73)$$

where $\mathbf{M}$ is a fourth order tensor, that is a function of the average velocity and mostly depends on the medium geometric properties. Finally Eq. (2.73) leads us to considering inertial dispersion as an additional viscosity effect. This approach is in agreement with Brinkman's theory (Section 2.4.3-c) and

observations where an apparent viscosity seems to act in fluids flowing in porous media.

In the decomposition of Eq. (2.69), the extra-stress tensor $\boldsymbol{\tau}$ must be developed by means of an appropriate constitutive equation. Considering a Newtonian fluid, we have:

$$\boldsymbol{\tau} = \eta \left(\nabla \mathbf{v} + (\nabla \mathbf{v})^T \right) \quad (2.74)$$

After having applied the gradient theorem (Eq. 2.57) and considering that the velocity vanishes at the interface A_{sf} , the extra-stress term in Eq. (2.74) can be rewritten as:

$$\nabla \cdot \langle \boldsymbol{\tau} \rangle_U = \nabla \cdot \left[\eta \left(\nabla \langle \mathbf{v} \rangle_U + (\nabla \langle \mathbf{v} \rangle_U)^T \right) \right] \quad (2.75)$$

where η is the dynamic viscosity.

By inserting the results given by Eqs. (2.73) and (2.75) into the momentum balance Eq. (2.69), this later becomes:

$$\begin{aligned} & \rho \frac{\partial \langle \mathbf{v} \rangle_U}{\partial t} + \rho \nabla \cdot \left[\phi \langle \mathbf{v} \rangle_U^f \langle \mathbf{v} \rangle_U^f \right] \\ &= \nabla \cdot \left[(\eta - \mathbf{M}) : \left(\nabla \langle \mathbf{v} \rangle_U + (\nabla \langle \mathbf{v} \rangle_U)^T \right) \right] - \phi \nabla \langle P \rangle_U^f - f_d \end{aligned} \quad (2.76)$$

Darcy's model

The first law governing flows in porous media was proposed by Darcy. From his experiments Darcy deduced the following law:

$$k = -v \frac{e}{\Delta p} \quad (2.77)$$

where v stands for the fluid average velocity, Δp for the total pressure difference across the sand, e for the height of the sand layer and k is defined as the hydraulic conductivity.

This empirical law can be deduced from the general macroscopic momentum balance Eq. (2.76) by assuming that (i) the flow is steady; (ii) the inertial terms (bulk and dispersive) are negligible; (iii) the viscosity is negligible and (iv) the drag force, after a dimensional analysis, is assumed to be given by the following expression:

$$f_d = \frac{\phi\eta}{K} \langle \mathbf{v} \rangle_U \quad (2.78)$$

where K is the permeability. Then the balance momentum Eq. (2.76) becomes

$$\langle \mathbf{v} \rangle_U = -\frac{K}{\eta} \nabla \langle P \rangle_U^f = -\frac{K}{\eta} \nabla \langle p + \rho g z \rangle_U^f. \quad (2.79)$$

This equation is similar to Eq. (2.77). Analyzing this equation shows that the pressure gradient can be seen as the driving force while the drag force is opposed to the fluid motion.

Forchheimer's model

From a synthetic viewpoint, Forchheimer's model consists in introducing an additional term into the drag force:

$$f_d = \underbrace{\frac{\phi\eta}{K} \langle \mathbf{v} \rangle_U}_{\text{Viscous drag}} + \underbrace{\phi C_D \rho \langle \mathbf{v} \rangle_U |\langle \mathbf{v} \rangle_U|}_{\text{Form drag}} \quad (2.80)$$

The first term results from fluid friction on the solid boundary. The second term expresses the form effect of the solid phase on the flow. Hence, the physical phenomenon responsible for the quadratic term in the Forchheimer equation is the form stress exerted on the fluid by any solid surface obstructing the flow path. The parameter C_D (m^{-1}) is related to the geometry of the solid permeable medium.

Under the same assumptions as Dracy's model (steady flow; inertial terms neglected; viscosity neglected) but assuming that the drag force expression includes a shape drag component:

$$f_d = \frac{\phi\eta}{K} \langle \mathbf{v} \rangle_U + \phi \rho C_D \langle \mathbf{v} \rangle_U |\langle \mathbf{v} \rangle_U|, \quad (2.81)$$

the Forchheimer equation yields:

$$\langle \mathbf{v} \rangle_U + \frac{K}{\eta} C_D \rho \langle \mathbf{v} \rangle_U |\langle \mathbf{v} \rangle_U| = -\frac{K}{\eta} \nabla \langle P \rangle_U^f. \quad (2.82)$$

The switch from the Darcy to Forchheimer model occurs when $Re_p \geq 0.1$, where Re_p is the pore-sized Reynolds number:

$$\text{Re}_p = \frac{\rho V \sqrt{K}}{\eta}. \quad (2.83)$$

Brinkman's model

When the form force is negligible and when bulk viscosity effects are important compared to inertial terms, the Brinkman equation is applicable and is written as:

$$\langle \mathbf{v} \rangle_U = -\frac{K}{\eta} \nabla \langle P \rangle_U^f - \frac{K}{\phi \eta} \nabla \cdot \left[\eta' : \left(\nabla \langle \mathbf{v} \rangle_U + \nabla \langle \mathbf{v} \rangle_U^T \right) \right] \quad (2.84)$$

where η' is the modified viscosity. Several authors consider that this viscosity is the same as the real viscosity. However, on the one hand, some experimental results have shown that the "fitting" viscosity is in fact different from the molecular viscosity (Gilver and Altobelli, 1994, cited in Loix, 2005). On the other hand, previous developments on the inertial term have highlighted the existence of a so-called inertial dispersion that could be responsible for the observed increased viscosity.

2.5 Conclusion

In this chapter, the equations governing the flow in urban media have been derived. The urban medium is seen as a porous medium where the pores would be the streets and the solid matrix, the buildings. The porosity is here defined as the ratio between the space available for the flow (i.e, the streets) and the total surface.

The technique chosen to derive the equations governing the flow is the volume averaging technique that is widely used for three-dimensional problems in porous media. It consists in averaging pore-scale equations (i.e. the equations governing the flow at the pore-scale, that are often the Navier-Stokes equations) over a fictitious portion of space of the porous medium called the representative elementary volume (REV). The size of the REV must be such that the REV contains a lot of solid particles (buildings) and is representative of the medium. In the present research, the pore-scale equations are the Navier-Stokes equations, and the REV is depicted in Fig.

(2.1). The buildings that it encircles can have any cross section but always vertical walls, and the above and below bounding surfaces are the water surface and the ground, respectively.

The classical volume averaging technique cannot be applied directly to the urban medium because the "urban" REV is different from the REV of traditional three-dimensional porous media. Its particularities are that the porosity is defined as a ratio of areas and not a ratio of volumes; that the variables of interest are the water depth and the depth-averaged velocity (defined in a 2D plane) and not the pressure and the velocity (defined in the 3D space); and that all its boundaries are not fixed: one of them, the water surface, is moving up and down. The volume averaging technique then had to be adapted: new theorems had to be derived and new variables had to be defined.

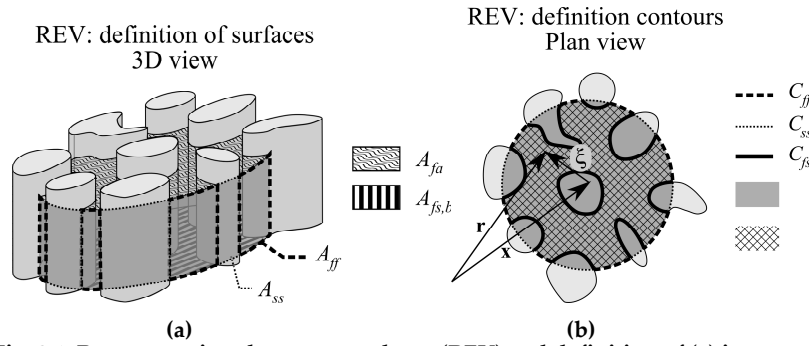


Fig. 2.1: Representative elementary volume (REV) and definition of (a) its contour surfaces A_{fa} , $A_{fs,b}$, A_{ff} and A_{ss} in a 3D view and (b) its contour curves C_{ff} , C_{ss} , C_{fs} and the projected surfaces A_0 and A_f in a plan view. Fig. (2.1b) is the plan view of Fig. (2.1a).

The final equations obtained after applying the volume averaging technique are the equations governing the flow in an urban medium:

$$\phi \frac{\partial \langle h \rangle_A^f}{\partial t} + \frac{\partial}{\partial x_j} \left(\phi \langle h \rangle_A^f \langle V_j \rangle_A^f \right) = 0 \quad (2.30)$$

$$\begin{aligned}
& \phi \frac{\partial \langle h \rangle_A^f \langle V_i \rangle_A^f}{\partial t} + \frac{\partial}{\partial x_j} \left(\phi \langle h \rangle_A^f \langle V_i \rangle_A^f \langle V_j \rangle_A^f \right) + \frac{\partial}{\partial x_i} \left(\phi \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \right) + \frac{\partial}{\partial x_j} (\phi d_{ij}) \\
&= \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \frac{\partial \phi}{\partial x_i} - \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial x_i} + \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_i dC - \phi g \langle h \rangle_A^f S_{fi}
\end{aligned} \tag{2.53}$$

where the Einstein notation is used (for the developed notation of these equations, see Eqs. 2.54 – 2.55); where the notation $\langle \cdot \rangle_A^f$ is used to designate a macroscopic variable (a macroscopic variable being the average of a street-scale variable over the REV, as shown by the definitions given by Eqs. 2.12 and 2.14); δ denotes the deviation of a variable, i.e. the difference between the local (street-scale) and the average (district-scale or REV-scale) values of this variable; ϕ is the porosity (-); h is the water depth (m); $\mathbf{V}$ is the depth-averaged velocity (m/s); g is the gravitational acceleration (m/s²); $\mathbf{d}$ is the dispersion defined in Eq. (2.38a) (m³/s²); z_b is the bottom level (m); A_0 is the cross-sectional area of the REV (m²) (see Fig. 2.1); $C_{fs,w}$ is the set of footprints of the buildings in the REV; $\mathbf{n}$ is the outward unit vector (-) and S_f is the head loss due to friction (-) which is linked to the velocity by:

$$S_{fi} = \frac{n^2 \langle V_i \rangle_A^f}{R_h^{4/3}} \left| \langle \mathbf{V} \rangle_A^f \right| \tag{2.51}$$

where n is the Manning-Strickler friction parameter (s/m^{1/3}); R_h is the hydraulic radius (m) that is the ratio between the volume of water in the REV and the total contact surface between fluid and solid:

$$R_h = \frac{\phi \langle h \rangle_A^f A_0}{A_{fs,b} + A_{fs,w}} = \frac{\phi \langle h \rangle_A^f}{\frac{A_{fs,b}}{A_0} + \frac{A_{fs,w}}{A_0}} = \frac{\phi \langle h \rangle_A^f}{\phi + \langle h \rangle_A^f \frac{C_{fs,w}}{A_0}} \tag{2.52}$$

where $A_{fs,b}$ is the surface of the bottom of the REV; $A_{fs,w}$ is the surface constituted by the walls of the buildings lying in the REV; and $C_{fs,w}$ is the contour line of these buildings in the (x,y) plane. All these contour curves and surfaces are shown in Fig. (2.1).

Eq. (2.30) expresses mass conservation, and Eq. (2.53) momentum balance. The two first terms of the left-hand side of Eq. (2.53) are the bulk inertia terms, the third one is the hydrostatic pressure and the fourth one is the dispersion term. This latter appeared while averaging the inertia terms, and

arose from the fact that the average of a product is not exactly equal to the product of the averages (see Eqs. 2.37 and 2.38); an additional part, the product of deviations, cannot be neglected and is called the inertial dispersion. This term contains the deviations of h and $\mathbf{V}$ from their average value. Physically, it represents the momentum mixing due to the chaotic trajectories of the flow caused by the presence of the buildings. This term contains street-scale variables and thus needs a closure, which is the subject of the next chapter.

The first term of the right-hand side of the momentum Eq. (2.53) accounts for the hydrostatic pressure against the walls of the buildings due to a change in the porosity, and the second term of the right-hand side accounts for the reaction at the bottom when this latter is not horizontal. The third term arises from the average of the hydrostatic pressure against the walls of the buildings and comes from the fact that the average of a product (in the present case, $\langle h^2 \rangle_A^f$) is not equal to the product of averages. This term is the shape drag term and needs a closure, which will be discussed in the next chapter too. The last term of the right-hand side is the friction term. It accounts for the friction against the building walls and the ground. As shown by Eq. (2.51), it is quantified by the Manning-Strickler coefficient n and the hydraulic radius R_h , this latter being the ratio between the volume of water (driving force) and the amount of rough surfaces (resistance force).

In conclusion, the urban medium is represented by the following global parameters:

- the porosity ϕ (-), representing the reduction of available space for the flow, defined as the ratio between the area available for the flow (i.e. the streets) and the total area (Eq. 2.3);
- the Manning-Strickler friction parameter n ($\text{s/m}^{1/3}$), quantifying the friction against the rough surfaces (walls and ground);
- $C_{fs,w}/A_0$ (m/m^2), the amount of walls per unit area, intervening in the wall friction term (for the meaning of $C_{fs,w}$ see Fig. (2.1) above);

and a closure is needed for the two following terms:

- The dispersion $\partial(\phi d_{ij})/\partial x_j$
- The shape drag force $1/(2A_0) \int_{C_{fs,w}} g(\delta h)^2 n_i dC$

Notice that in Eqs. (2.30) and (2.53), there is no parameter having a directional character; the model is then isotropic for the moment. In fact, the anisotropic character will be accounted for in the closure relations for the drag and dispersion terms that will be the object of the next chapter.

Comparison with the classical shallow water equations

In the particular case where the porosity ϕ is constant in space, it can be put outside the x - and y -derivatives of Eqs. (2.30) and (2.53) and its gradient cancels out. In this case, the porosity model reduces to:

$$\frac{\partial \langle h \rangle_A^f}{\partial t} + \frac{\partial}{\partial x_j} \left(\langle h \rangle_A^f \langle V_i \rangle_A^f \right) = 0 \quad (2.85)$$

$$\begin{aligned} & \frac{\partial \langle h \rangle_A^f \langle V_i \rangle_A^f}{\partial t} + \frac{\partial}{\partial x_j} \left(\langle h \rangle_A^f \langle V_i \rangle_A^f \langle V_j \rangle_A^f \right) + \frac{\partial}{\partial x_i} \left(\frac{g \left(\langle h \rangle_A^f \right)^2}{2} \right) + \frac{\partial}{\partial x_j} (d_{ij}) \\ & = -g \langle h \rangle_A^f \frac{\partial z_b}{\partial x_i} + \frac{1}{\phi A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_i dC - g \langle h \rangle_A^f S_{fi} \end{aligned} \quad (2.86)$$

Without the dispersion d_{ij} and the drag force (second term of the right-hand side), Eqs. (2.85) and (2.86) would be equivalent to the classical shallow water equations. This means that, in a medium where there is no change in the porosity, the flow behaves like in a medium without any buildings, except that it experiences dispersion and drag force.

Comparison with the former urban porosity model

The porosity model for urban flood developed by Lhomme, obtained by taking the mass and momentum balances on a control volume was:

$$\phi \frac{\partial \langle h \rangle_A^f}{\partial t} + \frac{\partial}{\partial x} \left(\phi_x \langle h \rangle_A^f \langle V_x \rangle_A^f \right) + \frac{\partial}{\partial y} \left(\phi_y \langle h \rangle_A^f \langle V_y \rangle_A^f \right) = 0 \quad (2.87)$$

$$\begin{aligned}
& \phi \frac{\partial \langle h \rangle_A^f \langle V_x \rangle_A^f}{\partial t} + \frac{\partial}{\partial x} \left(\phi_x \langle h \rangle_A^f \left(\langle V_x \rangle_A^f \right)^2 + \phi_x \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \right) \\
& + \frac{\partial}{\partial y} \left(\phi_x \langle h \rangle_A^f \langle V_x \rangle_A^f \langle V_y \rangle_A^f \right) \\
& = \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \frac{\partial \phi_x}{\partial x} - \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial x} - \phi g \langle h \rangle_A^f S_{fx}
\end{aligned} \tag{2.88}$$

$$\begin{aligned}
& \phi \frac{\partial \langle h \rangle_A^f \langle V_y \rangle_A^f}{\partial t} + \frac{\partial}{\partial x} \left(\phi_y \langle h \rangle_A^f \langle V_x \rangle_A^f \langle V_y \rangle_A^f \right) \\
& + \frac{\partial}{\partial y} \left(\phi_y \langle h \rangle_A^f \left(\langle V_y \rangle_A^f \right)^2 + \phi_y \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \right) \\
& = \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \frac{\partial \phi_y}{\partial y} - \phi g \langle h \rangle_A^f \frac{\partial z_b}{\partial y} - \phi g \langle h \rangle_A^f S_{fy}
\end{aligned} \tag{2.89}$$

where

$$S_{fi} = \left(\frac{n^2}{R_h^{4/3}} + s_i \right) \langle V_i \rangle_A^f \left| \langle \mathbf{V} \rangle_A^f \right| \quad i = x, y \tag{2.90}$$

and where s_i accounted for eddy head losses due to the presence of the buildings and had to be calibrated while the hydraulic radius was assumed to be equal to the water depth:

$$R_h = \langle h \rangle_A^f. \tag{2.91}$$

The eddy head losses in the urban medium were assumed to be proportional to the square of the velocity (Lhomme, 2006) or to the product of the square of the velocity and the water depth (Soares-Frazão et al, 2006).

In Eqs. (2.87) – (2.89), the Einstein notation is not used because the use of ϕ_x and ϕ_y in the inertia terms (second and third terms of the right-hand side of Eqs. 2.87 – 2.89) makes it impossible. The main differences with the equations developed in this thesis are that, in the former model of Lhomme (2006):

- The anisotropy of the medium is accounted for by the directional porosities ϕ_x and ϕ_y . However, these porosities are not properly

defined and set equal to the 'classical' porosity ϕ in the validation of the model. The use of directional porosities is questionable because, to the knowledge of the author, such parameters are never used in the context of porous media. Moreover, the anisotropic parameters usually appear in the pressure, friction and closure terms but not in the inertia terms, and they usually have a tensor form (i.e. at least a second order matrix), like for example the permeability.

- There are no dispersion and no drag force terms. These ones seem to have been neglected.
- The head losses due to the buildings are assumed to be due to the successive expansions and constrictions experienced by the flow (Fig. 2.9).
- The hydraulic radius is assumed to be equal to $\langle h \rangle_A^f$, which supposes that the friction against the walls has been omitted. Indeed, in Eq. (2.52), if $C_{f\text{S},w} = 0$, then $R_h = h$.

Chapter 3 Closure of drag and dispersion

The objective of this section is to define and validate a closure relation for the dispersion term and the drag force term, where dispersion represents a mixing effect due to the chaotic flow path while the drag corresponds to the resistance opposed by the buildings.

In most porous media, inertial effects are negligible and the dispersion terms are either neglected compared to the drag force term, or assumed to be implicitly taken into account through the drag coefficient (Chandesris and Jamet, 2009). However, in a large-scale medium constituted by emerged obstacles like the urban fabric, inertial effects may not be negligible, as reported by Li and Zhang (2010) who analyzed the flow through an array of emerged cylinders. An array of obstacles in this context means a group of many obstacles placed either in ordered (periodic) or unstructured (random) arrangement.

In the literature, the drag force has been more extensively investigated than the mechanical dispersion. Although the drag force exerted by an isolated obstacle is quite well known, the drag exerted by an array of obstacles is not evident to estimate as these latter exert a mutual influence on each other that is difficult to assess. Sanders et al. (2008) expressed the drag force exerted by an array of buildings with a bulk drag coefficient depending on the drag coefficient for an isolated obstacle, assumed to be square, and on the projected frontal area of the buildings. As they reported, the expression they propose presents two main drawbacks. The first one is that the projected frontal area of the buildings is difficult to estimate as it depends on the direction of the flow that is a priori unknown. The second one is that the sheltering effect (exposure and hiding effect) that can have a significant impact in this case, is not taken into account.

Some advances on the bulk drag of a set of obstacles have been performed in the field of vegetated media. Tanino and Nepf (2008) mentioned that the influence between two cylinders begins to be effective as these latter are separated by a distance of 5 diameters or less. Nepf (1999) reported that the downstream cylinder experiences a reduced drag due to the interaction of the wake of the upstream cylinder. In an array of cylinders, the drag of a particular cylinder reduces if the density of the array increases. However, as

the number of obstacles increases as well, we can not a priori tell whether the bulk drag is reduced or enhanced. Nepf (1999) reported that the bulk drag coefficient decreases from 1.2 to 0.6 as the density increases from 0.008 to 0.07, while Tanino and Nepf (2008) have reported that for higher densities, it increases with increasing density.

The resistance coefficients found in the literature are often scalar values, implying that the drag force is assumed to be opposite to the velocity vector. However, an obstacle presenting an asymmetry with regard to the flow direction also experiences a lift force (perpendicular to the drag force). It is the case for rectangular-based cylinder, as shown in Norberg (1993) and Sohankar (1997). Then the ratio between the resistance force and the velocity is not anymore a scalar, but a tensor of at least the second order.

Groups of rectangular-based cylinders have been studied by Bao et al. (2012), who studied the sheltering effect of obstacles in a line (cylinders in row) with different spacing ratio, while Lam et al. (2008) also studied the influence of the angle of attack of the flow. Valencia and Cid (2002) and Yen and Liu (2011) investigated the drag and lift coefficients for two side-by-side square cylinders (close to each other but not adjacent). To our knowledge, the drag and lift forces of an array of square cylinders has not been studied yet.

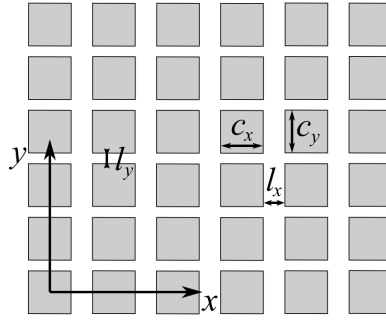


Fig. 3.1: Top view of the studied medium: a periodic medium with rectangular buildings.

In this study, we focus on periodic media composed of a periodic array of rectangle-based buildings, as illustrated in Fig. (3.1). We want to model together the drag and dispersion phenomena into a single expression. We recall that these two terms appear in the momentum Eq. (2.84) and are:

$$\frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_i dC - \frac{\partial}{\partial x_i} (\phi d_{ij}) \quad i = 1, 2,$$

or, written in vector form:

$$\frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} \mathbf{n} dC - \nabla \cdot (\phi \mathbf{d}),$$

or, equivalently,

$$\frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} dC - \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} \phi d_{xx} & \phi d_{xy} \\ \phi d_{yx} & \phi d_{yy} \end{pmatrix}$$

or, in the developed form:

$$\begin{cases} \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_x dC - \left(\frac{\partial \phi d_{xx}}{\partial x} + \frac{\partial \phi d_{xy}}{\partial y} \right) \\ \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_y dC - \left(\frac{\partial \phi d_{yx}}{\partial x} + \frac{\partial \phi d_{yy}}{\partial y} \right) \end{cases}$$

where $\mathbf{d}$ has been defined in Eq. (2.36) and represents the average of the velocity deviations:

$$d_{ij} = \mathbf{d} = \begin{pmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{pmatrix} \quad (2.36b)$$

where

$$\begin{aligned} d_{xx} &= \langle h \rangle_A^f (\delta V_x)^2 + 2\delta h \langle V_x \rangle_A^f \delta V_x + \delta h (\delta V_x)^2 \\ d_{xy} &= \langle h \rangle_A^f \delta V_x \delta V_y + \delta h \langle V_x \rangle_A^f \delta V_y + \delta h \delta V_x \langle V_y \rangle_A^f + \delta h \delta V_x \delta V_y \\ d_{yx} &= \langle h \rangle_A^f \delta V_y \delta V_x + \delta h \langle V_y \rangle_A^f \delta V_x + \delta h \delta V_y \langle V_x \rangle_A^f + \delta h \delta V_y \delta V_x \quad (= d_{xy}) \\ d_{yy} &= \langle h \rangle_A^f (\delta V_y)^2 + 2\delta h \langle V_y \rangle_A^f \delta V_y + \delta h (\delta V_y)^2 \end{aligned}$$

3.1 Proposed expression

To model the drag and dispersion phenomena, we want to take into account all the interactions between V_x and V_y , and therefore it is proposed to adopt a cubic velocity law:

$$\begin{aligned} & \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} \left(\frac{n_1}{n_2} \right) dC - \left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \right) \cdot \begin{pmatrix} \phi d_{11} & \phi d_{12} \\ \phi d_{21} & \phi d_{22} \end{pmatrix} \\ &= -\frac{h}{|\mathbf{V}|} \begin{pmatrix} a_1 V_x^3 + 3a_2 V_x^2 V_y + 3a_3 V_x V_y^2 + a_4 V_y^3 \\ b_1 V_x^3 + 3b_2 V_x^2 V_y + 3b_3 V_x V_y^2 + b_4 V_y^3 \end{pmatrix} \end{aligned} \quad (3.1)$$

In Eq. (3.1) and in the following development, the average symbols $\langle \cdot \rangle_A^f$ have been omitted for the sake of clarity. Notice that the coefficients $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 have a dimension $[L^{-1}]$. Indeed, these coefficients depend on the geometrical characteristics of the medium and have to be calibrated for every considered geometry. Their physical meaning will be investigated in Section 3.3.2. The right-hand side of Eq. (3.1) can be further developed:

$$\begin{aligned} & \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} \mathbf{n} dC - \nabla \cdot (\phi \mathbf{d}) \\ &= -\frac{h}{|\mathbf{V}|} \begin{pmatrix} a_1 V_x^2 V_x + 2a_2 V_x^2 V_y + a_2 V_x^2 V_y + 2a_3 V_x V_y^2 + a_3 V_x V_y^2 + a_4 V_y^2 V_y \\ b_1 V_x^2 V_x + 2b_2 V_x^2 V_y + b_2 V_x^2 V_y + 2b_3 V_x V_y^2 + b_3 V_x V_y^2 + b_4 V_y^2 V_y \end{pmatrix} \end{aligned} \quad (3.2a)$$

$$\begin{aligned} &= -\frac{h}{|\mathbf{V}|} \begin{pmatrix} V_x (a_1 V_x^2 + 2a_2 V_x V_y + a_3 V_y^2) + V_y (a_2 V_x^2 + 2a_3 V_x V_y + a_4 V_y^2) \\ V_x (b_1 V_x^2 + 2b_2 V_x V_y + b_3 V_y^2) + V_y (b_2 V_x^2 + 2b_3 V_x V_y + b_4 V_y^2) \end{pmatrix} \end{aligned} \quad (3.2b)$$

$$\begin{aligned} &= -\frac{h}{|\mathbf{V}|} \begin{pmatrix} a_1 V_x^2 + 2a_2 V_x V_y + a_3 V_y^2 \\ b_1 V_x^2 + 2b_2 V_x V_y + b_3 V_y^2 \end{pmatrix} \begin{pmatrix} a_2 V_x^2 + 2a_3 V_x V_y + a_4 V_y^2 \\ b_2 V_x^2 + 2b_3 V_x V_y + b_4 V_y^2 \end{pmatrix} \cdot \begin{pmatrix} V_x \\ V_y \end{pmatrix} \end{aligned} \quad (3.2c)$$

Eq. (3.2a-c) can be rewritten in the following form:

$$\frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} \mathbf{n} dC - \nabla \cdot (\phi \mathbf{d}) = - \frac{h}{|\mathbf{V}|} (\mathbf{D} : \mathbf{V}\mathbf{V}) \cdot \mathbf{V} \quad (3.3)$$

where

$$\mathbf{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \text{ and } \mathbf{V}\mathbf{V} = \begin{pmatrix} V_x^2 & V_x V_y \\ V_y V_x & V_y^2 \end{pmatrix} \quad (3.4)$$

and where $\mathbf{D}$ is a fourth-order tensor (a tensor of 2^d order whose 4 elements are also of 2^d order tensors):

$$\mathbf{D} = \begin{pmatrix} \begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} & \begin{pmatrix} a_2 & a_3 \\ a_3 & a_4 \end{pmatrix} \\ \begin{pmatrix} b_1 & b_2 \\ b_2 & b_3 \end{pmatrix} & \begin{pmatrix} b_2 & b_3 \\ b_3 & b_4 \end{pmatrix} \end{pmatrix} \quad (3.5)$$

and the matrix product ":" means that:

$$\mathbf{D} : \mathbf{V}\mathbf{V} = \begin{pmatrix} \begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} & \begin{pmatrix} a_2 & a_3 \\ a_3 & a_4 \end{pmatrix} \\ \begin{pmatrix} b_1 & b_2 \\ b_2 & b_3 \end{pmatrix} & \begin{pmatrix} b_2 & b_3 \\ b_3 & b_4 \end{pmatrix} \end{pmatrix} : \begin{pmatrix} V_x^2 & V_x V_y \\ V_y V_x & V_y^2 \end{pmatrix} \quad (3.6a)$$

$$= \begin{pmatrix} \begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} \begin{pmatrix} V_x^2 & V_x V_y \\ V_y V_x & V_y^2 \end{pmatrix} & \begin{pmatrix} a_2 & a_3 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} V_x^2 & V_x V_y \\ V_y V_x & V_y^2 \end{pmatrix} \\ \begin{pmatrix} b_1 & b_2 \\ b_2 & b_3 \end{pmatrix} \begin{pmatrix} V_x^2 & V_x V_y \\ V_y V_x & V_y^2 \end{pmatrix} & \begin{pmatrix} b_2 & b_3 \\ b_3 & b_4 \end{pmatrix} \begin{pmatrix} V_x^2 & V_x V_y \\ V_y V_x & V_y^2 \end{pmatrix} \end{pmatrix} \quad (3.6b)$$

$$= \begin{pmatrix} (a_1 V_x^2 + 2 a_2 V_x V_y + a_3 V_y^2) & (a_2 V_x^2 + 2 a_3 V_x V_y + a_4 V_y^2) \\ (b_1 V_x^2 + 2 b_2 V_x V_y + b_3 V_y^2) & (b_2 V_x^2 + 2 b_3 V_x V_y + b_4 V_y^2) \end{pmatrix} \quad (3.6c)$$

We now define the vector $\mathbf{R}$ as the drag and dispersion vector:

$$\mathbf{R}(\mathbf{V}, h) = \begin{pmatrix} R_x(V_x, V_y, h) \\ R_y(V_x, V_y, h) \end{pmatrix} \equiv - \frac{h}{|\mathbf{V}|} (\mathbf{D} : \mathbf{V}\mathbf{V}) \cdot \mathbf{V} \quad (3.7)$$

The idea of introducing a fourth-order tensor was already proposed by Mal et al. (1998) to model the mechanical dispersion appearing in the energy equation and the species concentration balance.

3.2 Small-scale numerical experiment

Numerical experiments have been realized in a medium constituted of a periodic array of buildings with squared base like the medium depicted in Fig. (3.1). The objective of these experiments is to qualitatively observe how the flow behaves in such a medium, and more precisely, to study the influence of the street width. Therefore two buildings layouts are studied; their dimensions are summarized in Table (3.1).

	l_x (m)	l_y (m)	c_x (m)	c_y (m)
Layout A	0.100	0.100	0.300	0.300
Layout B	0.135	0.068	0.300	0.300

Table 3.1: Dimensions of the two building layouts studied in this chapter. For the significance of l_x , l_y , c_x and c_y , see Fig. (3.1).

In the first studied building layout, called layout A, the streets have all the same width of 0.1 m ($l_x = l_y = 0.100$ m), in both directions. In the second layout, called layout B, the streets are twice larger in the y -direction than in the x -direction ($l_x = 0.135$ m and $l_y = 0.0675$ m) in order to evaluate the influence of a difference in the street width in each direction. The base of the buildings in both layouts measures $0.3 \text{ m} \times 0.3 \text{ m}$ ($c_x = c_y = 0.3$ m). The porosity of both media is $\phi = 0.4375$.

The numerical experiment consists in simulating the flow with the same initial conditions in both media. The simulated flow is a "dam-break" flow: the initial water depth is 1 m in a radius of 3.2 m around the origin and 0.01 m elsewhere (Fig. 3.2). The water is initially at rest in the whole domain. The symmetry of this medium with regard to axes x and y allows one to only compute a quarter of the domain. A triangular unstructured mesh has been used whose characteristic size is 0.015 m (there are then on average 4.3, 6.5 and 9 elements across a street width for a street of 0.065 m, 0.1 m, and 0.135 m width, respectively) so that the mesh is fine enough to capture the

detail of the flow at the street level. The flow is governed by the classical shallow water equations (without friction and without bottom slope). These equations are discretized using a finite-volume scheme with a LHLLC scheme (Toro et al., 1994). The buildings are represented by means of inner wall boundary conditions. The external boundary of the medium is also modeled as a wall.

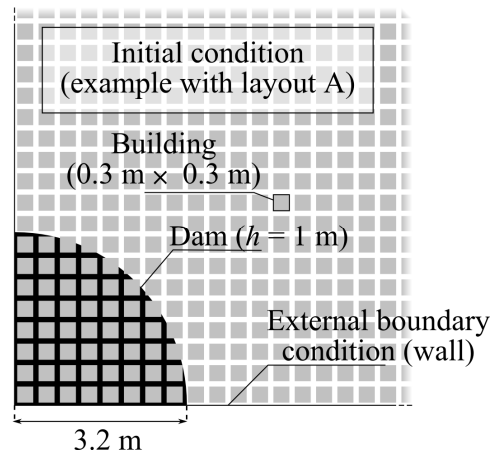
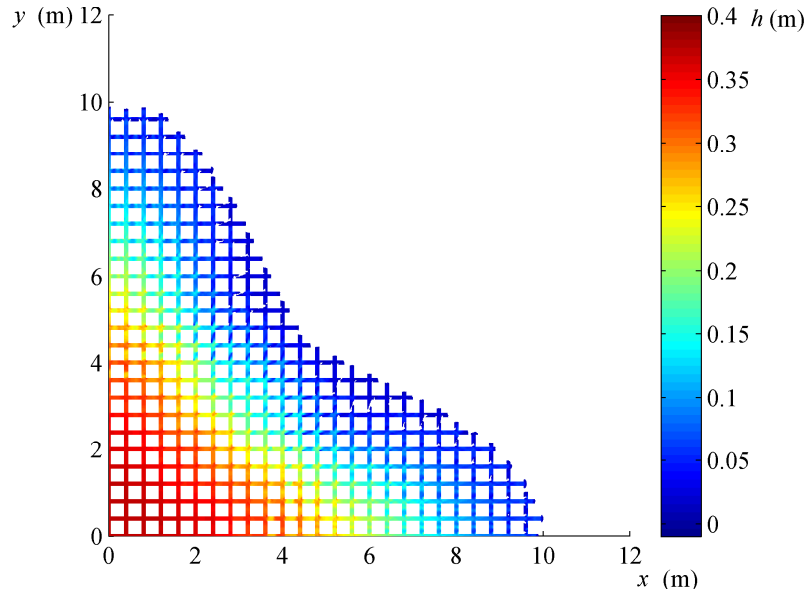


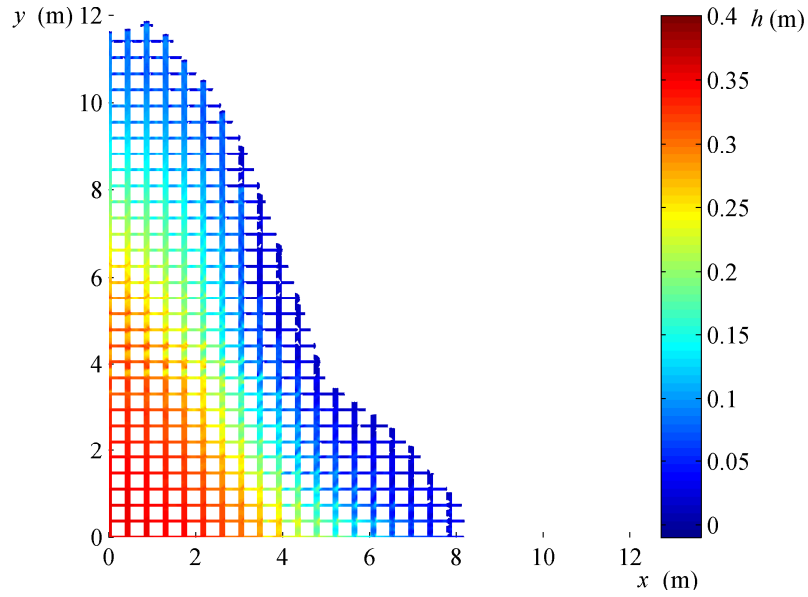
Fig. 3.2: Illustration of the initial condition for the circular dam break experiment: the water depth is 1 m in a radius of 3.2 m around the origin and 0.01 m elsewhere. The water is initially at rest everywhere. The buildings are modeled by means of internal wall boundary conditions. For layout B, the radius and the height of the "dam" are the same, only the street dimensions change.

Fig. (3.3a-b) shows the computed water depth at $t = 3$ s for both media. The water spreads throughout the medium, and it is clearly visible that it flows preferentially in the two main directions, along the streets. For layout A, it presents the highest resistance in the direction forming an angle of 45° with the streets. This angle is not so clear for layout B. It can also be observed that the flow goes faster in the larger streets. The fact that the resistance to the flow is not the same in all directions illustrates the fact that both media are anisotropic, even with layout A for which all streets have the same width.

Breugem and Boersma 2005 studied flow through a 3D periodic array of cubes and paradoxically considered this medium as isotropic. Actually this is not contradictory with the anisotropic behavior of the flow in Fig. (3.3). Indeed, in the present case the flow is mainly governed by inertial forces while in the case of Breugem and Boersma (2005) the flow is essentially governed by viscous forces and inertia was negligible. As explained by Koch and Ladd (1997), when the inertial terms are not negligible, they produce an effect on drag which is not independent of the orientation of the flow with respect to the axes of the array. Koch and Ladd (1997) and Spelt (2005) reported that the resistance to the flow was higher for off-axes flows (flows whose direction is not aligned with the axes of the array) than for on-axis flows (flows whose direction is aligned with the axes of the array). On the contrary, for viscous flows the resistance is independent of the flow orientation.



(a)



(b)

Fig. 3.3: Water surface during the circular dam break experiment at $t = 3$ s, (a) for layout A where all the streets are equally wide and (b) for layout B where the streets parallel to the axis y are twice larger than the streets parallel to the axis x . The initial conditions were the same for both media.

3.3 Drag/dispersion tensor

Following the proposed closure in Section 3.1 for the drag and dispersion terms, the tensor $\mathbf{D}$ (and the vector $\mathbf{R}$) have 8 independent parameters: $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 (Eq. 3.1). In Section 3.3.1, it will be shown that for a medium with rectangular-based buildings and perpendicular streets aligned with axes x and y (i.e. in the medium depicted in Fig. 3.1), the number of independent parameters reduces to 2. Indeed, the expression of $\mathbf{R}$ can be simplified by taking the symmetry of the medium into account. Then, Section 3.3.2 will present a qualitative analysis of the influence of these two independent parameters on the flow in simple cases in order to appreciate their physical meaning. The two parameters will then be calibrated in Section 3.3.3 for layouts A and B studied in Section 3.2. Finally, Section 3.3.4 will illustrate that $\mathbf{D}$ is invariant under rotation, meaning that this tensor is inherent to the medium, regardless of how the medium spatial orientation. A method will also be shown in order to calculate $\mathbf{D}$ for a medium whose streets are not aligned with the axes x and y by means of rotation matrices.

3.3.1 Simplification

The vector $\mathbf{R}$ can be simplified due to the symmetries of the studied medium depicted in Fig. (3.1). The first simplification that we can introduce is to prescribe symmetry with regard to axis y . We postulate that the x -component of $\mathbf{R}$ corresponding to a velocity (V_x, V_y) and the x -component of $\mathbf{R}$ corresponding to a velocity $(-V_x, V_y)$, are opposite, i.e.:

$$R_x^{V_x} = -R_x^{-V_x} \quad (3.8)$$

where the superscripts V_x and $-V_x$ denote whether $\mathbf{R}$ corresponds to the velocity (V_x, V_y) or $(-V_x, V_y)$, respectively. Eq. (3.8) gives the following relation:

$$\begin{aligned} -\frac{h}{|\mathbf{V}|} \left(a_1 V_x^3 + 3a_2 V_x^2 V_y + 3a_3 V_x V_y^2 + a_4 V_y^3 \right) \\ = \frac{h}{|\mathbf{V}|} \left(-a_1 V_x^3 + 3a_2 V_x^2 V_y - 3a_3 V_x V_y^2 + a_4 V_y^3 \right) \end{aligned} \quad (3.9)$$

Eq. (3.9) must hold true for any value of V_x and V_y , leading to the conclusion that

$$\begin{aligned} a_4 &= 0 \\ a_2 &= 0 \end{aligned} \quad (3.10)$$

Following the same procedure, we can deduce that the imposition of symmetry with respect to axis x reads:

$$R_y^{V_y} = -R_y^{-V_y} \quad (3.11)$$

which gives:

$$\begin{aligned} -\frac{h}{|\mathbf{V}|} (b_1 V_x^3 + 3b_2 V_x^2 V_y + 3b_3 V_x V_y^2 + b_4 V_y^3) \\ = \frac{h}{|\mathbf{V}|} (b_1 V_x^3 - 3b_2 V_x^2 V_y + 3b_3 V_x V_y^2 - b_4 V_y^3) \end{aligned} \quad (3.12)$$

leading to

$$\begin{aligned} b_1 &= 0 \\ b_3 &= 0 \end{aligned} \quad (3.13)$$

The resistance $\mathbf{R}$ now reads:

$$\mathbf{R} = -\frac{h}{|\mathbf{V}|} \begin{pmatrix} a_1 V_x^3 + 3a_3 V_x V_y^2 \\ b_4 V_y^3 + 3b_2 V_x^2 V_y \end{pmatrix} \quad (3.14)$$

If the two main directions are equal ($c_x = c_y$ and $l_x = l_y$)

In the special case where the two main directions are equal (i.e. $c_x = c_y$ and $l_x = l_y$ in Fig. 3.1), we can add the condition that $\mathbf{R}$ must be symmetric with respect to axis $x = y$. This can be written as:

$$R_x(h, V_x, V_y) = R_y(h, V_y, V_x) \quad (3.15a)$$

$$R_y(h, V_x, V_y) = R_x(h, V_y, V_x). \quad (3.15b)$$

Eq. (3.15a) (or Eq. 3.15b) leads to prescribing that the following relation must hold true for any V_x and V_y :

$$-\frac{h}{|\mathbf{V}|} (a_1 V_x^3 + 3a_3 V_x V_y^2) = -\frac{h}{|\mathbf{V}|} (b_4 V_x^3 + 3b_2 V_y^2 V_x) \quad (3.16)$$

leading to the conclusion that

$$\begin{aligned} a_1 &= b_4 \\ a_3 &= b_2 \end{aligned} \quad (3.17)$$

With this constraint, $\mathbf{R}$ is reduced to only depend on two independent parameters: a_1 and a_3 :

$$\mathbf{R} = -\frac{h}{|\mathbf{V}|} \begin{pmatrix} a_1 V_x^3 + 3a_3 V_x V_y^2 \\ a_1 V_y^3 + 3a_3 V_x^2 V_y \end{pmatrix}. \quad (3.18)$$

If the medium were isotropic

The periodic medium studied here is anisotropic. However, it is interesting to consider the case of an isotropic medium which would consist of a net of chaotic streets or in a "forest" of cylindrical buildings. By definition, an isotropic medium is a medium for which the properties are equal in all directions. This would only be the case if the $\mathbf{R}$ had the same direction as $\mathbf{V}$, which would be accomplished only if

$$\begin{aligned} a_2 &= a_4 = b_1 = b_3 = 0 \\ a_1 &= 3a_3 = 3b_2 = b_4 \end{aligned} \quad (3.19)$$

and then $\mathbf{R}$ would read:

$$\mathbf{R} = -a_1 h |\mathbf{V}| \begin{pmatrix} V_x \\ V_y \end{pmatrix}. \quad (3.20)$$

After the considerations and simplifications of the present section, the momentum equation of the porosity model finally writes as

$$\begin{aligned} & \phi \frac{\partial \langle h \rangle_A^f \langle \mathbf{V} \rangle_A^f}{\partial t} + \nabla \cdot \left(\phi \langle h \rangle_A^f \langle \mathbf{V} \rangle_A^f \langle \mathbf{V} \rangle_A^f + \phi \frac{g \langle h \rangle_A^f{}^2}{2} \right) \\ &= -\frac{g \langle h \rangle_A^f{}^2}{2} \nabla \phi - \phi \frac{g \langle h \rangle_A^f}{2} \nabla z_b + \mathbf{R} - \phi g \langle h \rangle_A^f \mathbf{S}_f \end{aligned} \quad (3.21),$$

where $\mathbf{R}$ is given by Eq. (3.14) for a periodic medium (reducing to Eq. (3.18) if the two main directions are equal) or Eq. (3.20) for an isotropic medium, and $\mathbf{S}_f$ is given by Eq. (2.51):

$$\mathbf{S}_f = \begin{pmatrix} S_{fx} \\ S_{fy} \end{pmatrix} = \frac{n^2 |\langle \mathbf{V} \rangle_A^f|}{R_h^{4/3}} \begin{pmatrix} \langle V_x \rangle_A^f \\ \langle V_y \rangle_A^f \end{pmatrix} = S_{fi} \quad (i = 1, 2) \quad (2.51)$$

in which

$$R_h = \frac{\phi \langle h \rangle_A^f A_0}{A_{fs,b} + A_{fs,w}} = \frac{\phi \langle h \rangle_A^f}{\frac{A_{fs,b}}{A_0} + \frac{A_{fs,w}}{A_0}} = \frac{\phi \langle h \rangle_A^f}{\phi + \langle h \rangle_A^f \frac{C_{fs,w}}{A_0}} \quad (2.52)$$

Adopting the Einstein convention, Eq. (3.19) writes:

$$\begin{aligned} & \phi \frac{\partial \langle h \rangle_A^f \langle V_i \rangle_A^f}{\partial t} + \frac{\partial}{\partial x_j} \left(\phi \langle h \rangle_A^f \langle V_i \rangle_A^f \langle V_j \rangle_A^f + \phi \frac{g \langle h \rangle_A^f{}^2}{2} \right) \\ &= - \frac{g \langle h \rangle_A^f{}^2}{2} \frac{\partial \phi}{\partial x_i} - \phi \frac{g \langle h \rangle_A^f}{2} \frac{\partial z_b}{\partial x_i} + R_i - \phi g \langle h \rangle_A^f S_{fi} \end{aligned} \quad (3.22)$$

and, written in the developed form:

x-momentum conservation:

$$\begin{aligned} & \phi \frac{\partial \langle h \rangle_A^f \langle V_x \rangle_A^f}{\partial t} + \frac{\partial}{\partial x} \left(\phi \langle h \rangle_A^f \langle V_x \rangle_A^f{}^2 + \phi \frac{g \langle h \rangle_A^f{}^2}{2} \right) \\ &+ \frac{\partial}{\partial y} \left(\phi \langle h \rangle_A^f \langle V_x \rangle_A^f \langle V_y \rangle_A^f \right) \\ &= - \frac{g \langle h \rangle_A^f{}^2}{2} \frac{\partial \phi}{\partial x} - \phi \frac{g \langle h \rangle_A^f}{2} \frac{\partial z_b}{\partial x} + R_x - \phi g \langle h \rangle_A^f S_{fx} \end{aligned} \quad (3.23)$$

y-momentum conservation:

$$\begin{aligned}
& \phi \frac{\partial \langle h \rangle_A^f \langle V_y \rangle_A^f}{\partial t} + \frac{\partial}{\partial x} \left(\phi \langle h \rangle_A^f \langle V_x \rangle_A^f \langle V_y \rangle_A^f \right) \\
& + \frac{\partial}{\partial y} \left(\phi \langle h \rangle_A^f \left(\langle V_y \rangle_A^f \right)^2 + \phi \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \right) \\
& = - \frac{g \left(\langle h \rangle_A^f \right)^2}{2} \frac{\partial \phi}{\partial y} - \phi \frac{g \langle h \rangle_A^f}{2} \frac{\partial z_b}{\partial y} + R_y - \phi g \langle h \rangle_A^f S_{fy}
\end{aligned} \tag{3.24}$$

3.3.2 Qualitative analysis

In this section, the effect of the coefficients of the vector $\mathbf{R}$ is evaluated qualitatively. The test case of a circular dam break is run with the porosity model with various values of a_1 , a_3 , b_2 and b_4 to qualitatively assess the effect of these parameters. The initial condition is a water depth of 3 m in a radius of 1 m around the origin, and 0.01 m of water depth elsewhere. The porosity equations are solved with the finite volume method and a modified HLL flux solver developed by Guinot and Soares-Frazão (2006) (the details of the solver will be explained later in Chapter 5). The results obtained for the porosity model are shown after 0.5 s and are illustrated in Figs. (3.4) and (3.5).

Fig. (3.4a) illustrates the results when $a_1 = 3a_3 = 3b_2 = b_4 = 0.1$. In this case, the flow pattern is circular, conveying that the resistance is isotropic. Indeed, $\mathbf{R}$ has always the same direction as the velocity:

$$\mathbf{R} = - \frac{0.1h}{|\mathbf{V}|} \begin{pmatrix} V_x^3 + V_x V_y^2 \\ V_y^3 + V_x^2 V_y \end{pmatrix} = - \frac{0.1h}{|\mathbf{V}|} \begin{pmatrix} (V_x^2 + V_y^2) V_x \\ (V_x^2 + V_y^2) V_y \end{pmatrix} = -0.1h |\mathbf{V}| \begin{pmatrix} V_x \\ V_y \end{pmatrix} \tag{3.25}$$

Figs. (3.4b-c) illustrates the influence of the ratio a_1/b_4 when $3a_3 = 3b_2 = 0$. The water extent adopts a 'square' shape, indicating that the resistance along the axes x and y is higher than the off-axis resistance. Increasing the ratio a_1/b_4 lets the resistance along axis x higher than the resistance along axis y with the consequence of slowing down the propagation of the wave along axis x (Fig. 3.4c).

Figs. (3.4d-e) illustrates the influence of the ratio $3a_3/3b_2$ when $a_1 = b_4 = 0$. The water extent adopts a 'plus' shape, indicating that the resistance along the

axes x and y is lower than the off-axis resistance. Increasing the ratio $3a_3/3b_2$ lets the flow be preferentially oriented along the axis y (Fig. 3.4e).

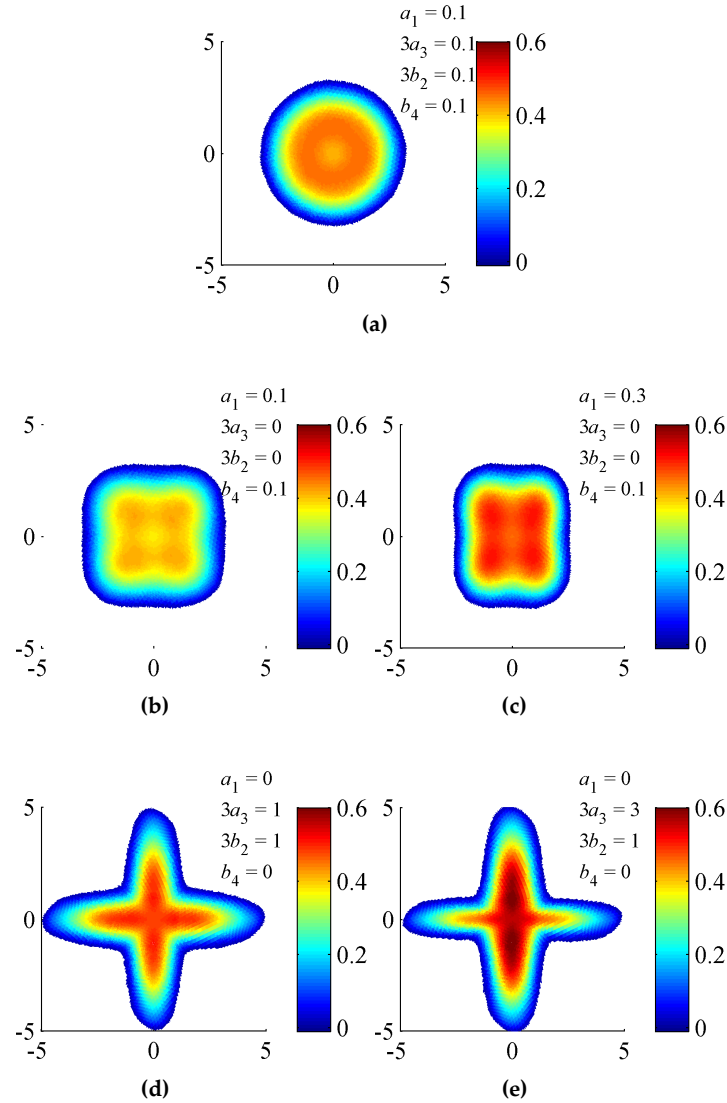


Fig. 3.4: Averaged water depth (m) consequent to a circular dam break of radius 1 m and an initial water depth of 3 m, computed by the porosity model with different values of a_1 , a_3 , b_2 and b_4 (see their values in the pictures) in order to qualitatively study the influence of these parameters. The results are shown at $t = 0.5$ s.

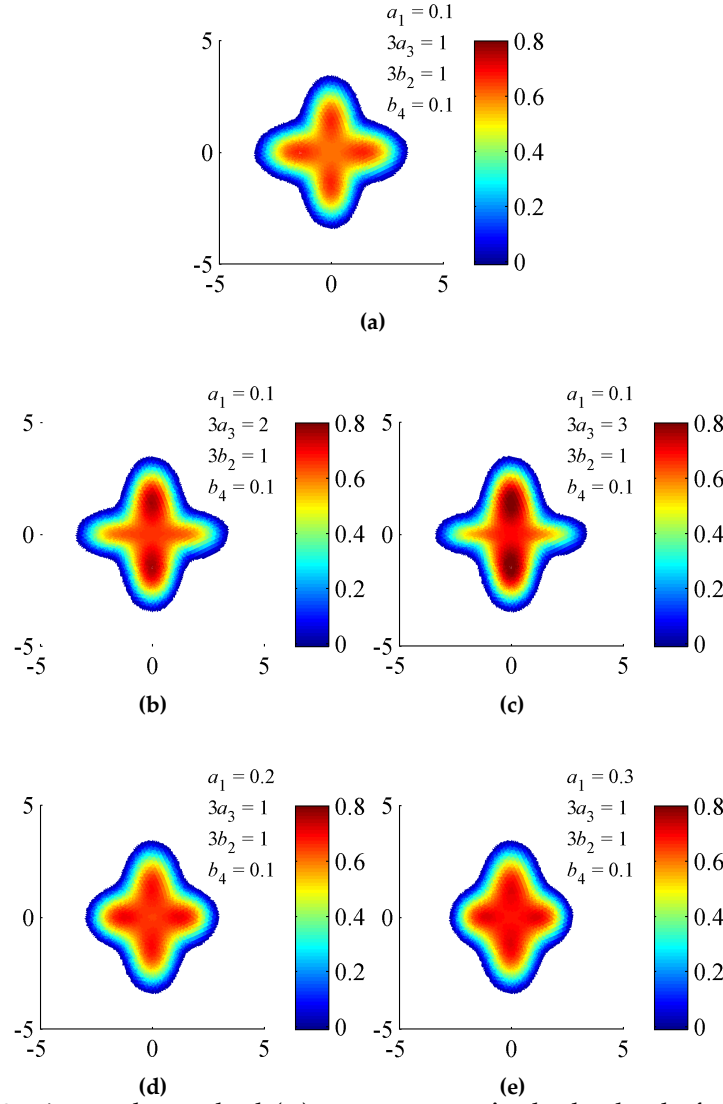


Fig. 3.5: Averaged water depth (m) consequent to a circular dam break of radius 1 m and an initial water depth of 3 m, computed by the porosity model with different values of a_1 , a_3 , b_2 and b_4 (see their values in the pictures) in order to qualitatively study the influence of these parameters. The results are shown at $t = 0.5$ s.

Fig. (3.5a) illustrates that when $a_1/b_4 = 1$ and $3a_3/3b_2 = 1$, the water extent is symmetric under a rotation of 90° . Figs. (3.5b-c) shows the effect of increasing the ratio $3a_3/3b_2$ (keeping the same value for a_1 and b_4) and Figs. (3.5d-e) show the influence of increasing the ratio a_1/b_4 (keeping the same value for $3a_3$ and $3b_2$).

3.3.3 Calibration

The values of a_3 and b_4 are now calibrated for the two special geometries in Section 3.2: layout A where all the streets are equally wide and layout B where the streets parallel to axis y are twice wider than the streets parallel to axis x . The calibration is made by trial and errors so that the results of the porosity model match the results given by the 2D detailed simulation of Section 3.2, shown in Figs. (3.3a-b). It is recalled that Figs. (3.3a-b) show the flow after 3 s from a circular dam of initial water depth of 1 m and a radius of 3.2 m. For more precision, the water depth results are shown along axis x (Figs. 3.6a and 3.7a) along axis y (Fig. 3.7b), and along axis $x = y$ (Figs. 3.6b and 3.7c).

The calibration for layout A is shown in Fig. (3.6). The results are shown for $t = 3$ s. For this layout, $a_1 = b_4$ and $3a_3 = 3b_2$ (Eq. 3.18). Various values for a_1 and $3a_3$ and are tested: $a_1 = 0.1$; 0.15 or 0.2 and $3a_3 = 1$; 1.5 or 2.

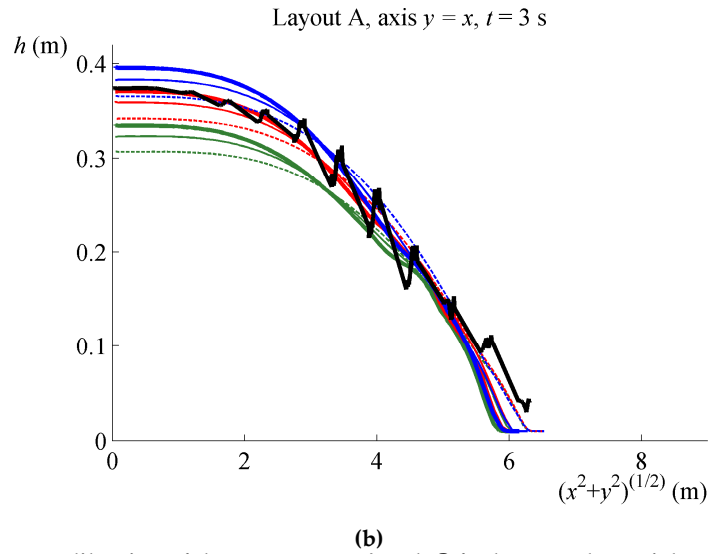
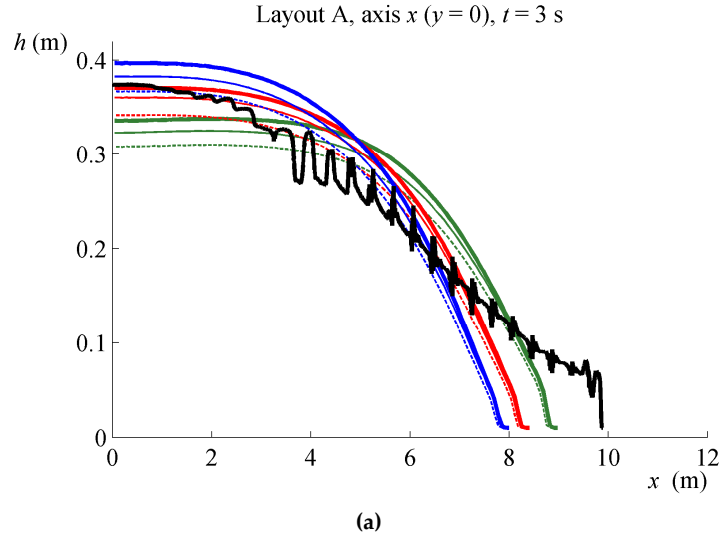
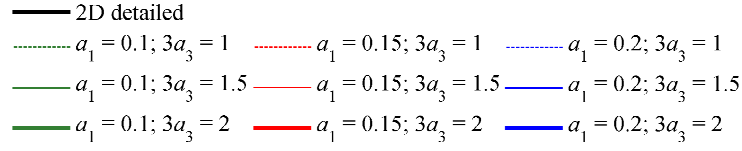


Fig. 3.5: Calibration of the parameters a_1 and a_3 for layout A by a trial and error method: comparison between the 2D detailed model and the results of the porosity model with various values of a_1 and a_3 .

Fig. (3.6a) shows the results along axis x (i.e., for $y = 0$) at $t = 3$ s. In the 2D detailed simulation, the water surface experiments periodic oscillations whose period corresponds to the building width. The wave front reaches $x = 9.9$ m. The oscillations are not reproduced by the porosity model, which is expectable from a simplified model devoted to reproduce the macroscopic tendency of the flow. The wavefront position is underestimated by the porosity model with the coefficients used. It is however interesting to note that it only depends on the coefficient a_1 and not on a_3 : the lowest a_1 , the farthest the wavefront is located. Indeed, this is obvious as along this axis, V_y is close to zero and changing the value of a_1 will not change the value of the resistance $\mathbf{R}$ (Eq. 3.18). The water depth at the origin ($x = 0$) depends on both a_3 and a_1 : the water depth is higher for higher values of a_3 and/or a_1 . In other words, increasing the resistance $\mathbf{R}$ by increasing a_1 will let the wavefront move slower and the water depth at the origin decrease slower too. On the other hand, increasing the resistance $\mathbf{R}$ by increasing a_3 will not influence the propagation of the wavefront along axis x , but will let the water depth at the origin decrease slower.

Fig. (3.6b) illustrates the results along axis $x = y$. It can be seen that the position of the wavefront is better evaluated with the porosity model than for axis $y = 0$. It can also be observed that this result depends more on the value of a_3 than the value of b_4 . Indeed, along this axis, $V_x = V_y = V$, and the expression of the resistance $\mathbf{R}$ reduces to:

$$\mathbf{R} = -\frac{h}{|\mathbf{V}|} \left(\frac{a_1 V_x^3 + 3a_3 V_x V_y^2}{a_1 V_y^3 + 3a_3 V_x^2 V_y} \right), \quad (3.26a)$$

$$= -\frac{h}{|\mathbf{V}|} \left(\frac{(a_1 + 3a_3)V^3}{(a_1 + 3a_3)V^3} \right) \quad (3.26b)$$

$$= -h|\mathbf{V}|(a_1 + 3a_3) \left(\frac{V}{V} \right) \quad (3.26c)$$

Also, as in the present example $3a_3 \gg a_1$ ($3a_3$ is larger than a_1 of an order of magnitude of 10), changing the value of a_1 will not have a significant effect on the results.

Three criteria may be chosen to calibrate the porosity model: the wavefront position along axis x (for $y = 0$), the wavefront position along the axis $y = x$ and the water depth at the origin $(x,y) = (0,0)$. The parameters a_3 and b_4 have been tried to be adjusted by a trial and error method to obey these criteria but so far, no couple (a_1, a_3) has been found to satisfy the three criteria

together at the same time. It has been decided to calibrate the model so that the computed water depth corresponds to the detailed results along axis $y = x$ (i.e. according to the criterion of the wavefront position along $y = x$ and the criterion of the water depth at the origin). Indeed, this will allow one to estimate the resistance in the direction where it is the highest and give a reliable value for a_3 that is the most important parameter as it is much higher than a_1 . The parameter a_3 is the one playing an important role when the flow is not aligned with the streets and, as we want to study the influence of the angle of attack on the resistance $\mathbf{R}$ to the flow, calibrating a_3 with more accuracy than a_1 is a good option. Finally, based on these considerations and on the observations of Figs. (3.6a-b), it has been chosen to adopt for layout A:

$$\begin{cases} a_1 = 0.15 \\ 3a_3 = 1.5 \end{cases} \quad (3.27)$$

Comparison between the water surface computed with the detailed simulation and the water surface computed with the calibrated porosity model is shown in Figs. (3.7a-b).

The calibration for layout B (in which the streets aligned with axis y are twice larger than the streets aligned with axis x) is illustrated in Figs. (3.8a-c). Fig. (3.8a) shows the water depth along axis x (i.e., for $y = 0$); Fig. (3.8b) shows the water depth along axis y (i.e. for $x = 0$) and Fig. (3.8c) shows the results along axis $y = x$. In the 2D detailed simulation, it can be seen that the wavefront travels faster in the y direction (it reaches $y = 11.6$ m after 3 s, see Fig. 3.8b) where the streets are wider, as expected. As previously for layout A, it is difficult to find a set of parameters (a_1, a_3, b_2, b_4) reproducing both the wavefront position and the water depth at the origin. The following combination:

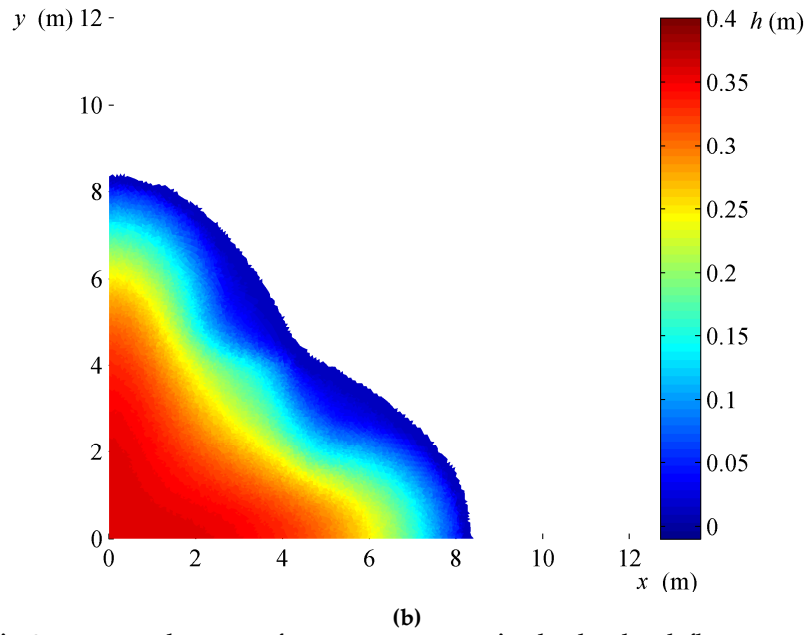
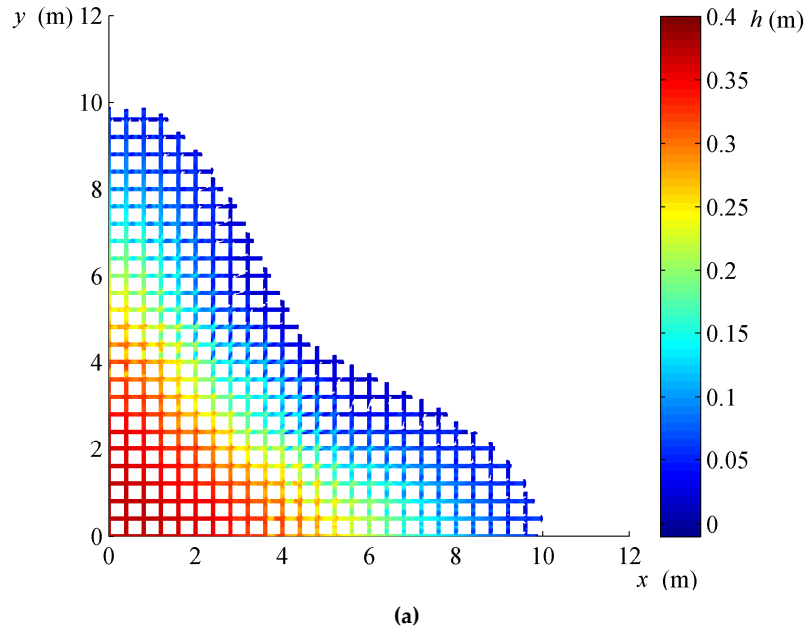
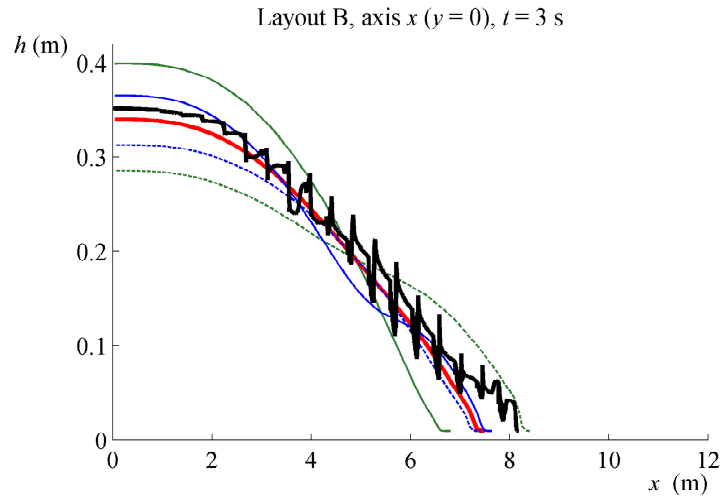
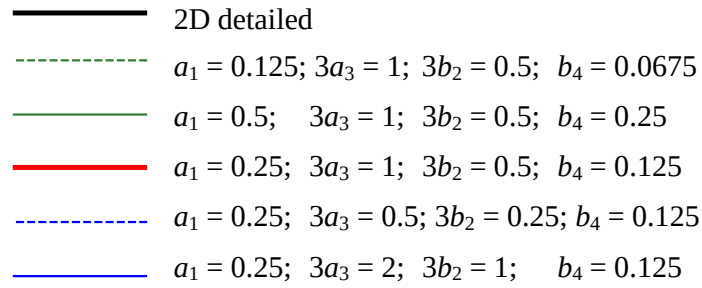


Fig. 3.7: computed water surface consequent to a circular dam break flow centered at $(x,y) = (0,0)$, with an initial water depth of 1 m and radius of 3.2 m, (a) computed with a detailed 2D simulation and (b) with the porosity model with calibrated parameters (Eq. 3.27).

$$\begin{cases} a_1 = 0.25 \\ 3a_3 = 1 \\ 3b_2 = 0.5 \\ b_4 = 0.125 \end{cases} \quad (3.28)$$

has been found to be the best one because it reproduces well the wavefront position on axis $y = x$ (Fig. 3.8c) and also the water depth at the origin ($x = 0$; $y = 0$).



(a)

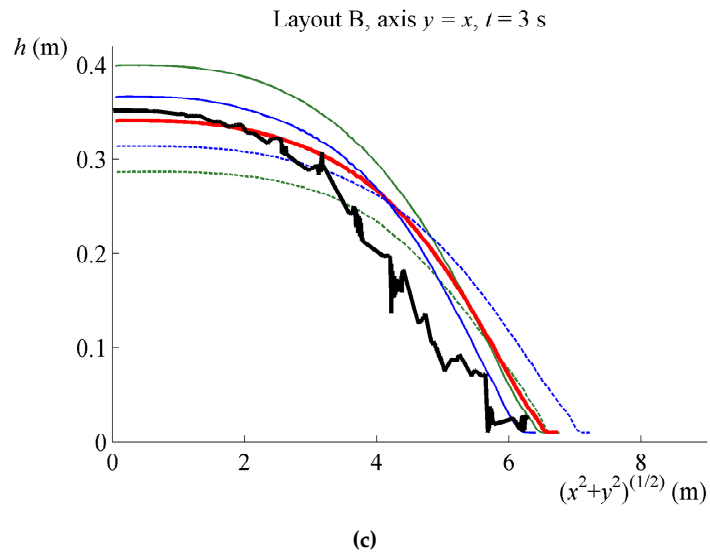
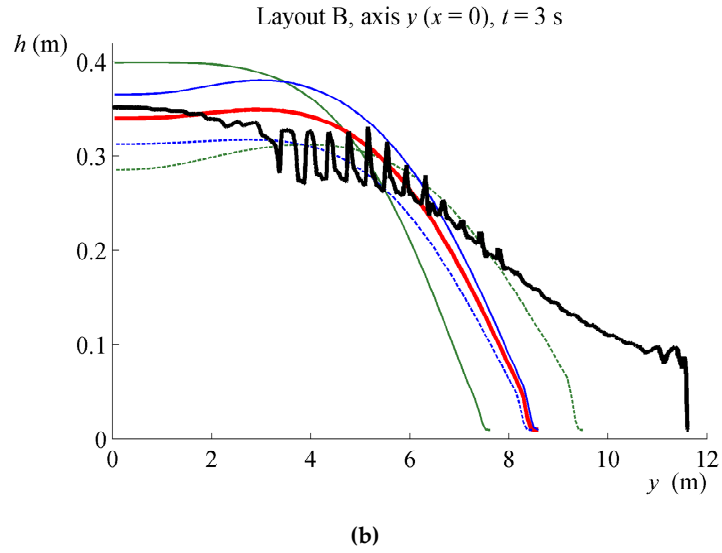


Fig. 3.8: Calibration of the parameters a_1 , a_3 , b_2 and b_4 for layout B by trial and error method: comparison between the 2D detailed model and the results of the porosity model with various values of a_1 , a_3 , b_2 and b_4 .

Comparison between the water surface computed with the detailed simulation and the water surface obtained with the calibrated porosity model is shown in Figs. (3.9a-b).

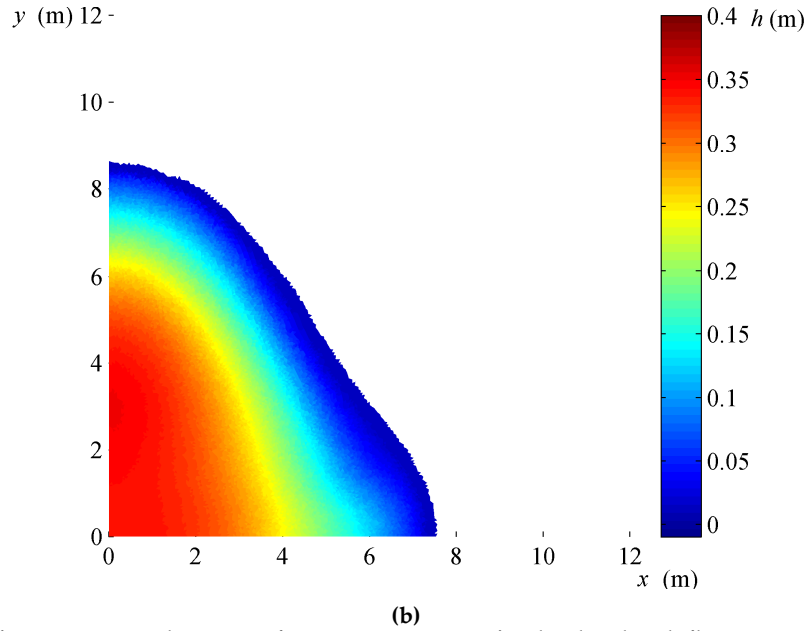
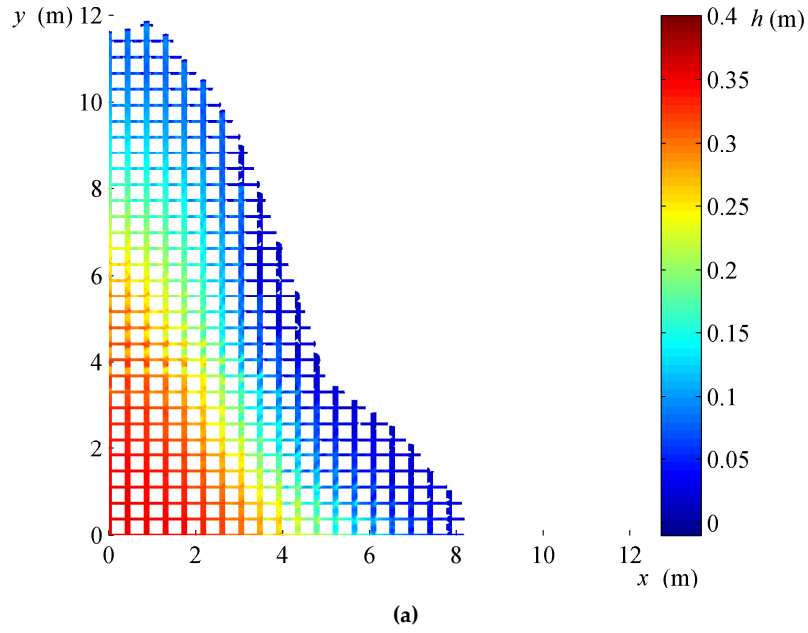


Fig. 3.9: computed water surface consequent to a circular dam break flow centered at $(x,y) = (0,0)$, with an initial water depth of 1 m and radius of 3.2 m, (a) computed with a detailed 2D simulation and (b) with the porosity model with calibrated parameters (Eq. 3.26).

3.3.4 Rotational invariance

It is important for a tensor to have the property to be invariant under rotation. This means that the tensor is inherent to the medium, regardless of how the medium is oriented in space. Practically, it means that if the buildings are oriented such that the streets are not aligned with the axes x and y anymore (like in the Fig. 3.10a), the tensor $\mathbf{D}$ remains the same except that it has to be multiplied by a matrix of rotation $\mathbf{K}$. As $\mathbf{D}$ is a fourth-order tensor, this multiplication is more complicated than for a vector or a second-order tensor. Adopting the Einstein summation convention, it writes as follows:

$$D'_{ijkl} = K_{ip} K_{jq} K_{kr} K_{ls} D_{pqrs} \quad (3.29)$$

where $\mathbf{K}$ is the matrix of rotation equal to

$$[K_{ij}] = \mathbf{K} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (3.30)$$

where θ is the angle of rotation. Eq. (3.29) means that each "sub-matrix" of $\mathbf{D}$ is left-multiplied by $\mathbf{K}^t$ and right-multiplied by $\mathbf{K}$; and that the whole tensor $\mathbf{D}$ is also left-multiplied by $\mathbf{K}^t$ and right-multiplied by $\mathbf{K}$:

$$\mathbf{D}' = \mathbf{K}^t \cdot \left(\begin{pmatrix} \mathbf{K}^t \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \mathbf{K} & \mathbf{K}^t \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \mathbf{K} \\ \mathbf{K}^t \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \mathbf{K} & \mathbf{K}^t \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \mathbf{K} \end{pmatrix} \cdot \mathbf{K} \quad (3.31)$$

As an example, let us calculate $\mathbf{D}'$ associated to the medium of Fig. (3.10a). The components of $\mathbf{D}$ (when the streets are aligned with axes x and y) have been calibrated in Section 3.3.3 to (Eq. 3.28):

$$\mathbf{D} = \begin{pmatrix} \mathbf{D}_{xx} & \mathbf{D}_{xy} \\ \mathbf{D}_{yx} & \mathbf{D}_{yy} \end{pmatrix} \quad (3.32)$$

where

$$\mathbf{D}_{xx} = \begin{pmatrix} 0.25 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{D}_{xy} = \mathbf{D}_{yx} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{D}_{yy} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.125 \end{pmatrix} \quad (3.33)$$

For a rotation of $\theta = 67.5^\circ$,

$$\mathbf{K} = \begin{pmatrix} 0.3827 & 0.9239 \\ -0.9239 & 0.3827 \end{pmatrix} = \begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} \quad (3.34)$$

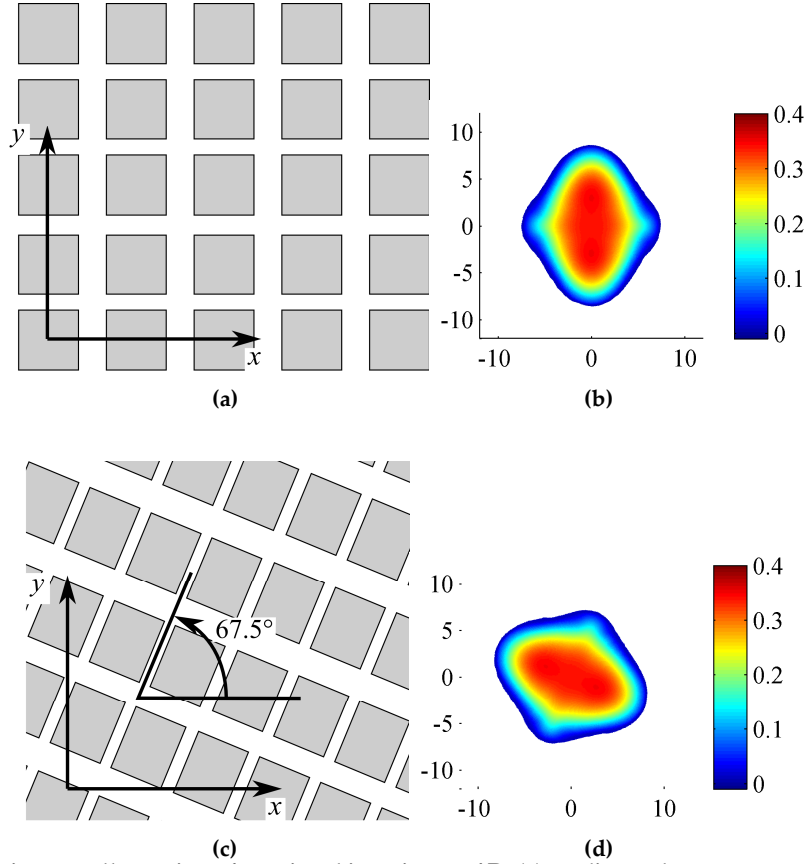


Fig. 3.10: Illustration of rotational invariance of $\mathbf{D}$: (a) medium whose streets are aligned with the axes x and y and (b) corresponding dam break flow computed with the porosity model, with $\mathbf{D}$ given by Eq. (3.32) and (3.33); (c) medium oriented 67.5° with respect to the x direction and (d) corresponding dam break flow computed with the porosity model, with $\mathbf{D}$ given by Eq. (3.42).

So $\mathbf{D}'$ writes:

$$\mathbf{D}' = \mathbf{K}^t \cdot \begin{pmatrix} \mathbf{K}^t \begin{pmatrix} 0.25 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{K} & \mathbf{K}^t \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{K} \\ \mathbf{K}^t \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{K} & \mathbf{K}^t \begin{pmatrix} 0.5 & 0 \\ 0 & 0.125 \end{pmatrix} \mathbf{K} \end{pmatrix} \cdot \mathbf{K} \quad (3.35)$$

$$= \mathbf{K}^t \cdot \begin{pmatrix} \begin{pmatrix} 0.8902 & -0.2652 \\ -0.2652 & 0.3598 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0.1799 & 0.1326 \\ 0.1326 & 0.4451 \end{pmatrix} \end{pmatrix} \cdot \mathbf{K} \quad (3.36)$$

$$= \mathbf{K}^t \cdot \begin{pmatrix} \mathbf{D}'_{xx} & \mathbf{D}'_{xy} \\ \mathbf{D}'_{yx} & \mathbf{D}'_{yy} \end{pmatrix} \cdot \mathbf{K} \quad (3.37)$$

$$= \begin{pmatrix} K_{xx} & K_{yx} \\ K_{xy} & K_{yy} \end{pmatrix} \cdot \left[\begin{pmatrix} \mathbf{D}'_{xx} & \mathbf{D}'_{xy} \\ \mathbf{D}'_{yx} & \mathbf{D}'_{yy} \end{pmatrix} \cdot \begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} \right] \quad (3.38)$$

$$= \begin{pmatrix} K_{xx} & K_{yx} \\ K_{xy} & K_{yy} \end{pmatrix} \cdot \begin{bmatrix} K_{xx} \mathbf{D}'_{xx} + K_{yx} \mathbf{D}'_{xy} & K_{xy} \mathbf{D}'_{xx} + K_{yy} \mathbf{D}'_{xy} \\ K_{xx} \mathbf{D}'_{yx} + K_{yx} \mathbf{D}'_{yy} & K_{xy} \mathbf{D}'_{yx} + K_{yy} \mathbf{D}'_{yy} \end{bmatrix} \quad (3.39)$$

$$= \begin{pmatrix} \mathbf{D}''_{xx} & \mathbf{D}''_{xy} \\ \mathbf{D}''_{yx} & \mathbf{D}''_{yy} \end{pmatrix} \quad (3.40)$$

where

$$\begin{aligned} \mathbf{D}''_{xx} &= K_{xx} (K_{xx} \mathbf{D}'_{xx} + K_{yx} \mathbf{D}'_{xy}) + K_{yx} (K_{xx} \mathbf{D}'_{yx} + K_{yy} \mathbf{D}'_{yy}) \\ \mathbf{D}''_{xy} &= K_{xx} (K_{xy} \mathbf{D}'_{xx} + K_{yy} \mathbf{D}'_{xy}) + K_{yx} (K_{xy} \mathbf{D}'_{yx} + K_{yy} \mathbf{D}'_{yy}) \\ \mathbf{D}''_{yx} &= K_{xy} (K_{xx} \mathbf{D}'_{xx} + K_{yx} \mathbf{D}'_{xy}) + K_{yy} (K_{xx} \mathbf{D}'_{yx} + K_{yx} \mathbf{D}'_{yy}) \\ \mathbf{D}''_{yy} &= K_{xy} (K_{xy} \mathbf{D}'_{xx} + K_{yy} \mathbf{D}'_{xy}) + K_{yy} (K_{xy} \mathbf{D}'_{yx} + K_{yy} \mathbf{D}'_{yy}) \end{aligned} \quad (3.41)$$

By substituting for K_{xx} , K_{xy} , K_{yx} , K_{yy} , D_{xx} , D_{xy} , D_{yx} and D_{yy} the values given in Eqs. (3.33) and (3.34), $\mathbf{D}'$ becomes

$$\mathbf{D}' = \begin{pmatrix} \begin{pmatrix} 0.2839 & 0.0743 \\ 0.0743 & 0.4326 \end{pmatrix} & \begin{pmatrix} 0.2511 & -0.1406 \\ -0.1406 & -0.0301 \end{pmatrix} \\ \begin{pmatrix} 0.2511 & -0.1406 \\ -0.1406 & -0.0301 \end{pmatrix} & \begin{pmatrix} 0.7862 & -0.2069 \\ -0.2069 & 0.3723 \end{pmatrix} \end{pmatrix}, \quad (3.42)$$

and from Eqs. (3.1), (3.3) and (3.7), $\mathbf{R}$ writes as:

$$\begin{aligned}
 \mathbf{R}(\mathbf{V}, h) &= -\frac{h}{|\mathbf{V}|} (\mathbf{D} : \mathbf{V}\mathbf{V}) \cdot \mathbf{V} \\
 &= -\frac{h}{|\mathbf{V}|} \begin{pmatrix} 0.2839V_x^3 + (2 \cdot 0.0743 + 0.2511)V_x^2V_y \\ \quad + (0.4326 - 2 \cdot 0.1406)V_xV_y^2 - 0.0301V_y^3 \\ 0.2511V_x^3 + (0.7862 - 2 \cdot 0.1406)V_x^2V_y \\ \quad - (0.0301 + 2 \cdot 0.2069)V_xV_y^2 + 0.3723V_y^3 \end{pmatrix} \quad (3.43) \\
 &= -\frac{h}{|\mathbf{V}|} \begin{pmatrix} 0.2839V_x^3 + 0.4V_x^2V_y + 0.1514V_xV_y^2 - 0.0301V_y^3 \\ 0.2511V_x^3 + 0.505V_x^2V_y - 0.444V_xV_y^2 + 0.3723V_y^3 \end{pmatrix} \\
 &\quad (3.44)
 \end{aligned}$$

The circular dam break flow computed with the porosity model with $\mathbf{D}$ and $\mathbf{R}$ given by Eqs. (3.32) and (3.44) is shown in Fig. (3.10b) and Fig. (3.10d), respectively. The flow of Fig. (3.10d) corresponds to the medium oriented by 67.5° (Fig. 3.10c), we have then verified that the porosity model was invariant under rotation.

3.4 Conclusion

One of the objectives of this thesis is to investigate how to take into account the preferential flow directions in an urban medium that confers to it an anisotropic character. More precisely, the aim is to find which parameters could take into account the fact that the flow orients preferentially in the direction of wider streets. These parameters should depend on the geometry of the city and reflect the medium anisotropy.

The porosity equations developed in Chapter 2 contain parameters representing the geometry of the urban medium, such as the porosity or friction parameters, but none of them reflect a trend of the flow to prefer a particular direction. On the other part, these equations are not closed: the drag and dispersion terms still depend on street-scale quantities and then have to be substituted for by an expression exclusively containing macroscopic quantities, called a closure expression. It is then decided to

reflect the anisotropy of the medium in the parameters of this closure expression. It has been proposed in this chapter to group the drag and dispersion terms into a single expression constituted by cubic combinations of V_x and V_y (in the following, the notation $\langle \cdot \rangle_A^f$ designating an average quantity is omitted for the sake of clarity):

$$\begin{aligned} & \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} dC - \left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \right) \cdot \begin{pmatrix} \phi d_{11} & \phi d_{12} \\ \phi d_{21} & \phi d_{22} \end{pmatrix} \\ &= - \frac{h}{|\mathbf{V}|} \begin{pmatrix} a_1 V_x^3 + 3a_2 V_x^2 V_y + 3a_3 V_x V_y^2 + a_4 V_y^3 \\ b_1 V_x^3 + 3b_2 V_x^2 V_y + 3b_3 V_x V_y^2 + b_4 V_y^3 \end{pmatrix} \end{aligned} \quad (3.1)$$

or, written in the vector form:

$$\frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} \mathbf{n} dC - \nabla \cdot (\phi \mathbf{d}) = - \frac{h}{|\mathbf{V}|} (\mathbf{D} : \mathbf{V}\mathbf{V}) \cdot \mathbf{V} \quad (3.3)$$

where $\mathbf{D}$ is a fourth-order tensor (a second order tensor whose four component are also second order tensors) and may be written as:

$$\mathbf{D} = \begin{pmatrix} \begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} & \begin{pmatrix} a_2 & a_3 \\ a_3 & a_4 \end{pmatrix} \\ \begin{pmatrix} b_1 & b_2 \\ b_2 & b_3 \end{pmatrix} & \begin{pmatrix} b_2 & b_3 \\ b_3 & b_4 \end{pmatrix} \end{pmatrix} \quad (3.5)$$

For the meaning of Eq. (3.3), see Eqs. (3.4) to (3.7).

The first term of the left-hand side of Eq. (3.1) or Eq. (3.3) is the "un-closed" drag term, the second one is the "un-closed" dispersion term, and the right hand-side is the closure expression. This expression can represent anisotropic effects as the parameters in the x -momentum equation (a_1 , a_3 , a_3 and a_4) may be different from the parameters in the y -momentum equation (b_1 , b_2 , b_3 and b_4). The tensor $\mathbf{D}$ (Eq. 3.5) groups all these independent parameters and has 8 degrees of freedom. In fact the number of degrees of freedom may be reduced to 4 for periodic media (Fig. 3.1), or even 2 if the streets are equally wide in both directions and the buildings are square. If the studied medium were isotropic (which is not the case for periodic media), there would be a single independent parameter.

In order to appreciate the behavior of the flow in a periodic medium, a numerical experiment has been conducted. This experiment consists in simulating a "circular dam break" numerically (Fig. 3.2), with the basic

shallow water model (without porosity) and a refined mesh in the streets. The experiment has been repeated in two media: a medium where the streets are all equally wide (layout A) and a medium in which the streets are twice wider in one direction than in the other one (layout B). The water depth after 3 s is shown in Fig. (3.3a) for layout A and Fig. (3.3b) for layout B. It can be seen that the flow goes faster along the two directions of the streets, and Fig. (3.3b) shows that it goes faster in the wider streets directions. It can also be seen that when the flow is not aligned with the streets (off-axis flow), it experiences more resistance. These phenomena have to be taken into account in the porosity model through tensor $\mathbf{D}$.

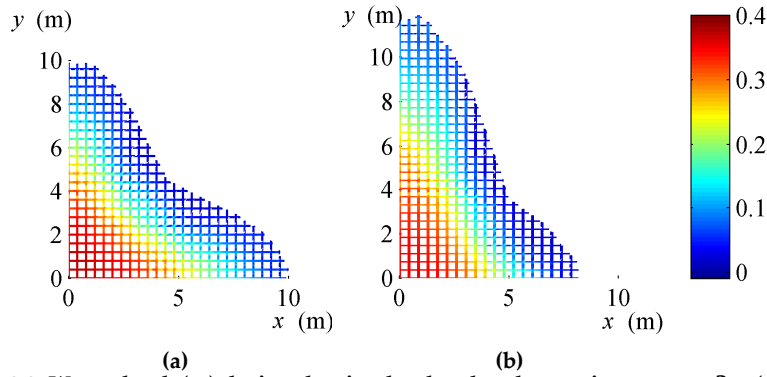


Fig. 3.3: Water depth (m) during the circular dam break experiment at $t = 3$ s, (a) for layout A where all the streets are equally wide and (b) for layout B where the streets parallel to the axis y are twice larger than the streets parallel to axis x . The initial conditions are the same for both media.

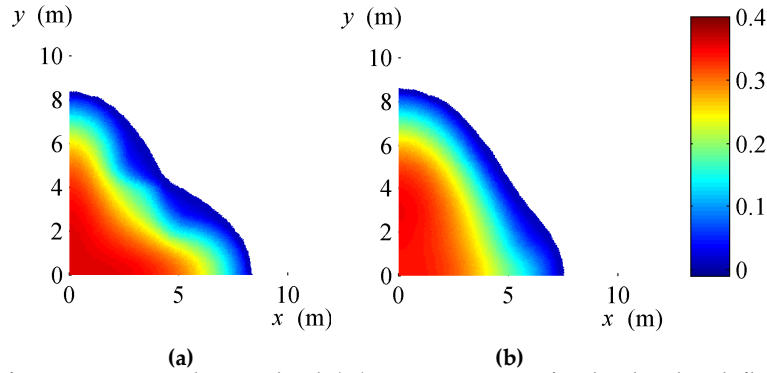


Fig. 3.11: Computed water depth (m) consequent to a circular dam break flow centered at $(x,y) = (0,0)$, with an initial water depth of 1 m and radius of 3.2 m, as calculated with the porosity model (a) with parameters of Eq. (3.27) for layout A; and (b) with parameters of Eq. (3.28) for layout B.

Indeed, it is shown in Section 3.3.3 that it is possible to calibrate the components of $\mathbf{D}$ to reproduce the flow of Fig. (3.3):

$$\begin{cases} a_1 = b_4 = 0.15 \\ 3a_3 = 3b_2 = 1.5 \end{cases} \quad \text{for layout A} \quad (3.27)$$

$$\begin{cases} a_1 = 0.25 \\ 3a_3 = 1 \\ 3b_2 = 0.5 \\ b_4 = 0.125 \end{cases} \quad \text{for layout B} \quad (3.28)$$

The circular dam break flow experiment computed by means of the porosity model with the calibrated values of Eq. (3.27) for layout A and Eq. (3.28) for layout B gives the flow of Fig. (3.11a) and Fig. (3.11b), respectively.

Chapter 4 Experiments

This chapter presents the laboratory experiments that will be used to validate the porosity model developed in Chapter 6. The experiments were conducted in the Laboratoire d'essais mécaniques, structures et génie civil (LEMSC), in the Institute of Mechanics, Materials, and Civil Engineering (iMMC) at the Louvain School of Engineering (EPL) of the Université catholique de Louvain (UCL), in Belgium. They consist in placing arrays of 5×5 square blocks representing buildings in a laboratory flume (Fig. 4.1). The blocks form an idealized city with perpendicular streets and they are laid out in various ways to form streets with different widths and different orientations, in order to study the influence of these parameters on the flow. Two types of flow are tested: steady flow, controlled by a regulation system, and dam break flow consisting in opening suddenly the gate of a reservoir situated upstream of the city.

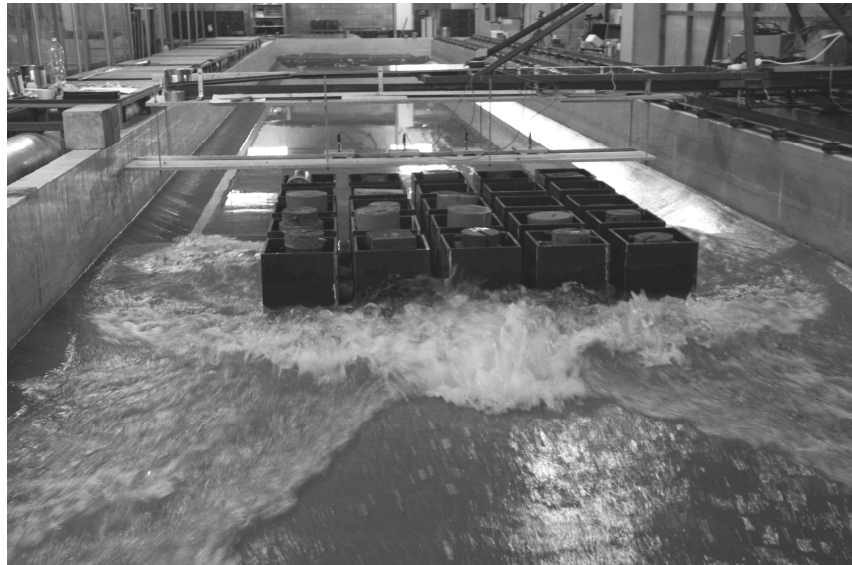


Fig. 4.1: Dam break experiment in a flume where the square-shaped blocks represent buildings.

Section 4.1 presents the 5 city layouts studied in the experimental campaign and the methodology adopted. The experimental setup is described in Section 4.2. Then, the experimental results are presented in Section 4.3.

4.1 City layouts

The different city layouts studied are illustrated in Fig. (4.2) (steady flow) and Fig. (4.3) (dam break flow). For the steady flow experiments, blocks were placed aside the city to prevent the flow from by-passing it. This has the advantage to increase the difference of water level between upstream and downstream the city and then to make the effect of the city more visible. For the dam break flow, though, no obstructing block was placed aside the city. Indeed, the shock of the dam break wave would have been so strong that it would have swept the blocks away. For this practical reason, it was necessary to allow the flow to by-pass the city. All layouts of Figs. (4.2) and (4.3) have the same porosity $\phi = 0.4375$, and differ from each other by the width and orientation of the streets. Layouts (a), (c) and (d) are called "aligned" because their main axes are aligned to axes x and y , and in opposition, layouts (b) and (e) are called "skewed". Layouts (a) and (b) are designated by the letter "A" because the streets all have the same widths (like in Chapter 3), and layouts (c), (d) and (e) are designated by the letter "B".

The base of the buildings has dimensions of $0.3 \text{ m} \times 0.3 \text{ m}$, the streets in layout A are 0.1 m wide, and in layout B, the wide streets are twice wider than the narrow ones: they measure 0.135 m and 0.0675 m respectively. Note that the media (layouts A and B) studied in Chapter 3 had the same dimensions.

The data collected from this experimental campaign will be used to validate the porosity model in Chapter 6. The approach will be to compute the flow with the porosity model with the values of $\mathbf{D}$ calibrated in Chapter 3 for an infinite medium with a circular dam break flow (Fig. 3.11) for layouts A (Eq. 3.27) and B (Eq. 3.28). Recall that $\mathbf{D}$ is the tensor accounting for the drag and the dispersion in the porous medium, and is a key parameter of the porosity model. The objective will be to see if tensor $\mathbf{D}$ remains valid in steady flow and in a "finite" medium (by "finite", we mean that the city is surrounded by a free obstacle flow, just like in the experiments). Also, the rotational invariance of $\mathbf{D}$ will be checked as layouts A and B are either aligned or skewed with respect to the main direction of the flume.

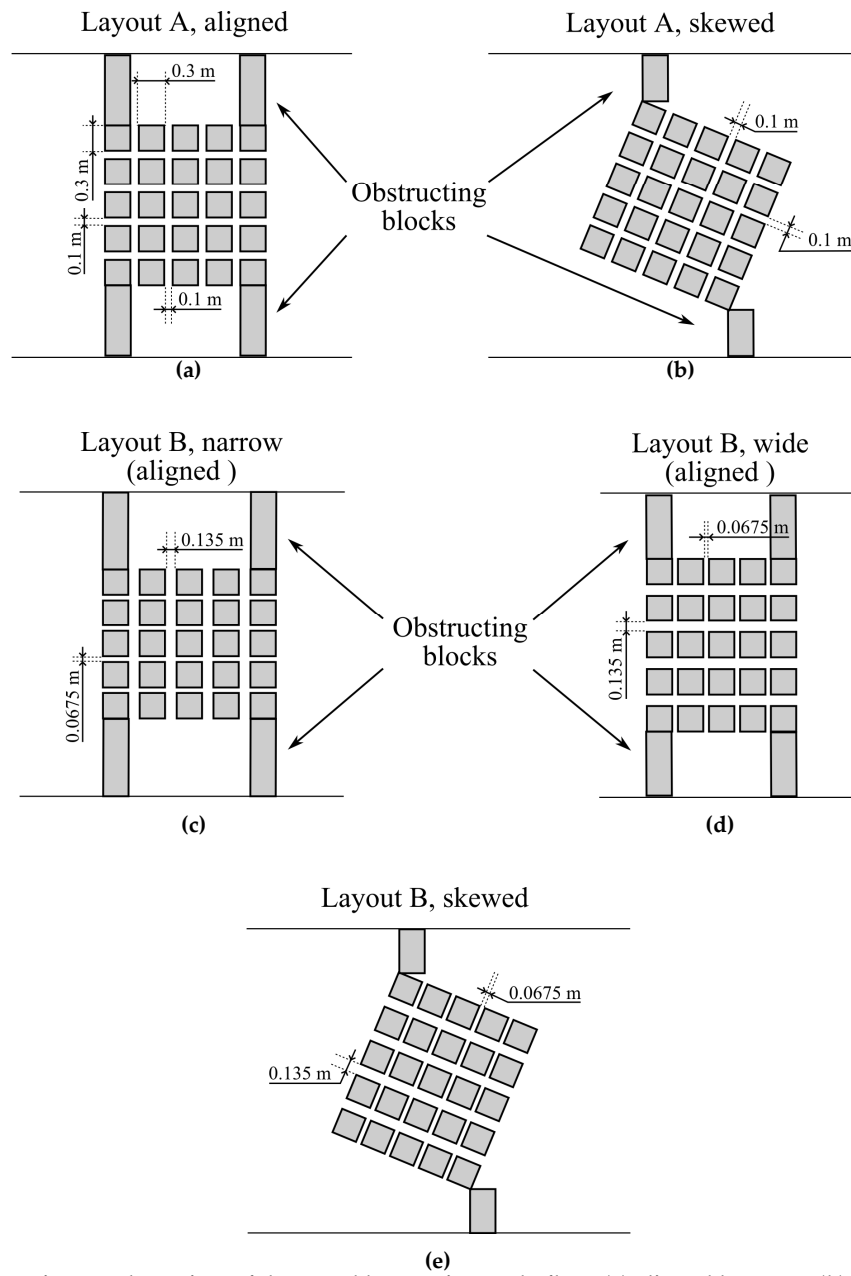


Fig. 4.2: Plane view of the tested layouts in steady flow: (a) aligned layout A, (b) skewed layout A, (c) layout B with the narrow streets aligned with the main direction of the flume, (d) layout B with the wide streets aligned with the main direction of the flume and (e) skewed layout B.

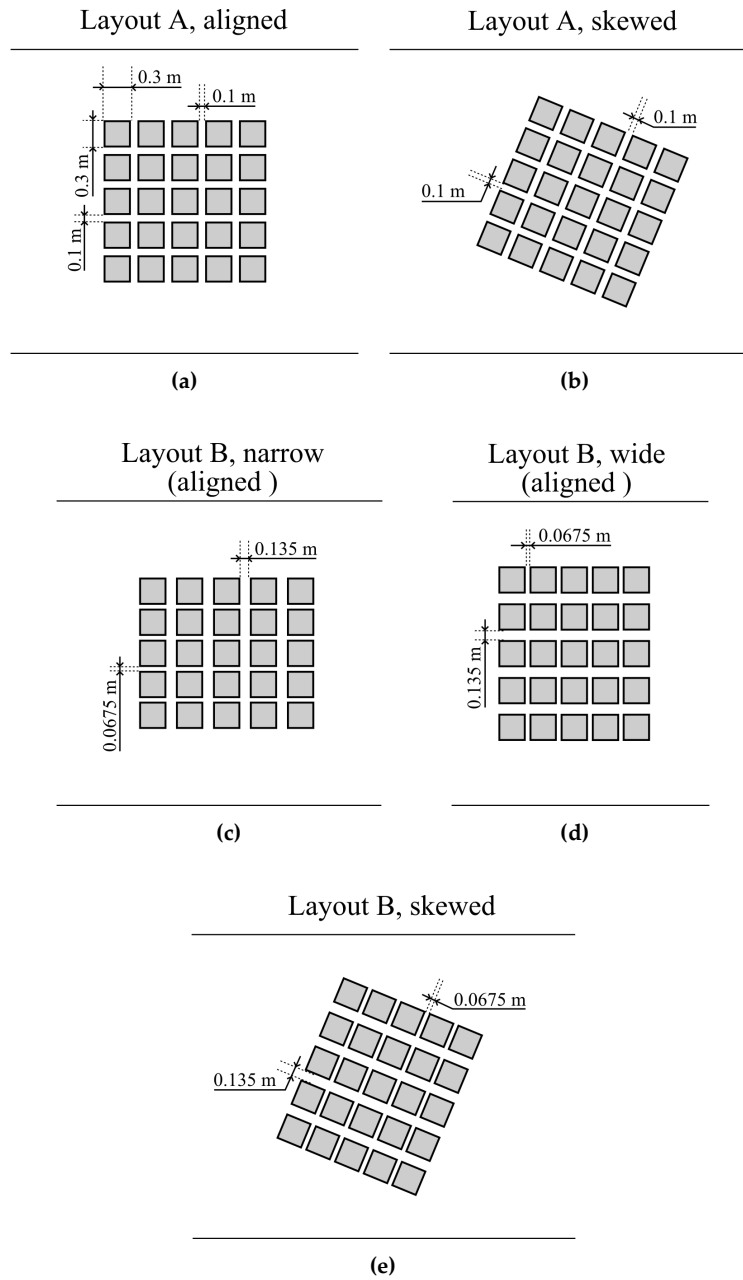


Fig. 4.3: Plane view of the tested layouts in dam break flow: (a) aligned layout A, (b) skewed layout A, (c) layout B with the narrow streets aligned with the main direction of the flume, (d) layout B with the wide streets aligned with the main direction of the flume and (e) skewed layout B.

4.2 Experimental setup

A top view of the experimental flume is shown in Fig. (4.4). It is 30 m long and 3.6 m wide and presents a constriction to 1 m in the middle. Its cross section A-A' is illustrated in Fig. (4.5). Its bed is horizontal, except that it ends with a 2 m-deep pit at the bottom of which is pumped the water (Fig. 4.6). It presents a constriction in the middle where is located a gate. In dam break flow, this gate acts as a dam. It is initially closed, separating the upstream part of the flume from the downstream part. Before the experiment, the downstream part is drained by the aid of the pump and the water is brought into the upstream part. The reservoir of the dam break experiment is then constituted by the part of the flume situated upstream the gate. To simulate the dam break, the gate is suddenly opened by a counterweight system. It originates a water wave spreading throughout the downstream part where is located the "city".

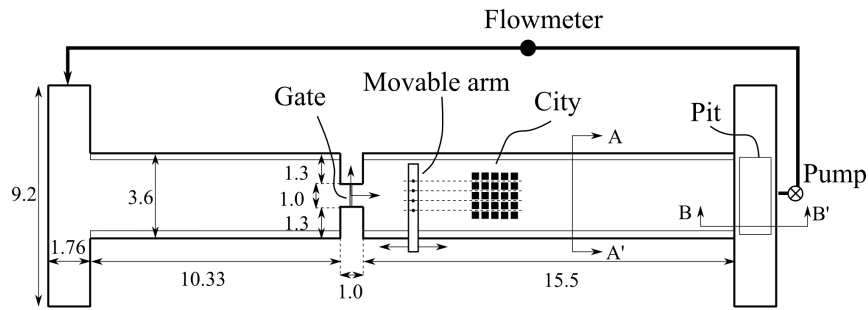


Fig. 4.4: Plane view of the experimental flume.

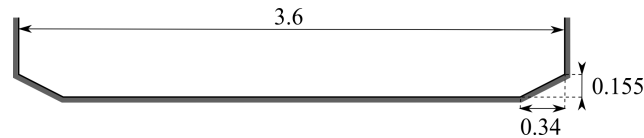


Fig. 4.5: Cross section of the experimental flume (cut AA' in Fig. 4.4).

In the steady flow experiment, the gate remains always open so that it has no influence on the flow, and the discharge is prescribed via a regulation system, consisting of a computer that receives the measured discharge information by flowmeter and prescribes the rotation speed of the pump.

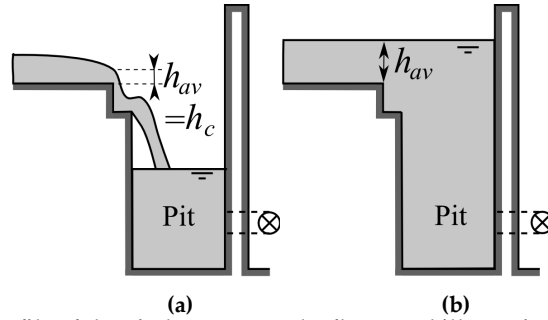


Fig. 4.6: Profile of the pit downstream the flume and illustration of the two possible downstream conditions: (a) spillway and passage by critical depth h_c and (b) imposition of a steady subcritical water depth (cut BB' in Fig. 4.4).

In steady flow, all the experiments are conducted with the same volume of water in the system: when the pipe system is filled, the water depth at rest in the flume is 0.165 m. In steady flow, various discharges are tested on each layout.

To better understand the behavior of the flow in steady flow, an analogy can be made between the hydraulic circuit of Fig. (4.4) and an electrical circuit. Indeed, a hydraulic pump can generate a higher or lower discharge in a hydraulic circuit as a generator can induce a higher or lower current in an electrical circuit. The array of blocks of Fig. (4.4) is analogous to a resistance across which the difference of potential (or the difference of water depth) is proportional to the current (or discharge). By considering this analogy, we may understand why in our hydraulic circuit, the difference of water level ahead and behind the city is higher for higher discharges. This has an implication on the flow regime downstream the city: for low discharges, the water depth downstream the city is higher than the critical depth h_c and the flow is then subcritical, and if the discharge is higher than a certain threshold, the downstream water depth will be lower than h_c and the flow will be supercritical followed by a hydraulic jump.

If we focus on the downstream end of the flume where the pit is located, the flow may be of two types as illustrated in Fig. (4.6). When the discharge exceeds a certain threshold (this threshold being different for each layout), the water flows into the pit and passes by the critical depth h_c (Fig. 4.6a). If contrarily the discharge is lower than a certain threshold, the water depth in the end of the flume exceeds h_c and a very flat axis of type H2 develops in the flume (Fig. 4.6b).

The aligned layout A (Fig. 4.3a) is also tested with different volumes of water, keeping the same discharge, in order to evaluate the influence of the downstream condition under the same discharge.

Two variables are measured: the water level and the pointwise water velocity. The water level is measured with ultrasonic probes. There are four probes fixed to an arm able to move along the flume (Figures 2.6 and 2.9), allowing one to measure the water level at four points simultaneously. In steady flow, the arm is swept along the flume to measure four continuous water profiles. In dam break flow, the arm is placed at a given position so that the probes measure the evolution of the water level as a function of time at four different points. To cover a large area of measurement, the dam break experiment is repeated several times, each time placing the arm at a different place. This is possible thanks to the repeatability of the dam break experiment.

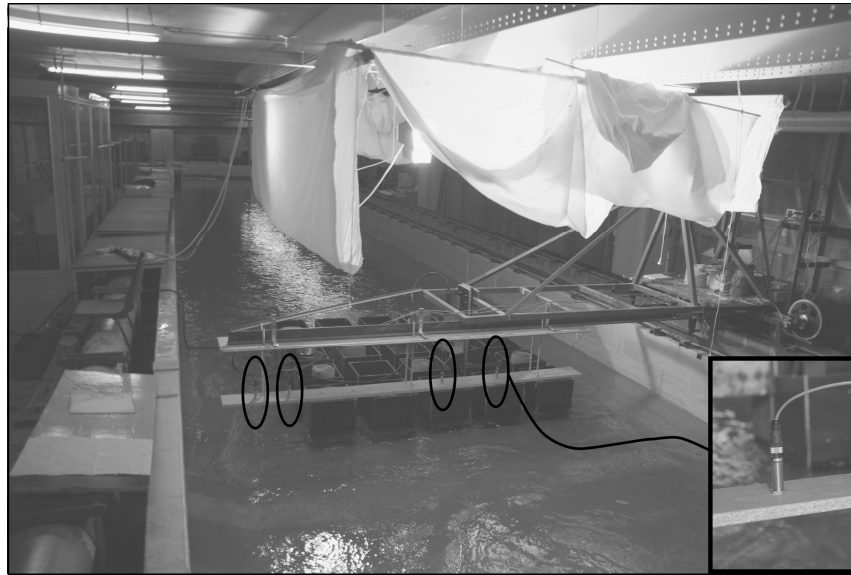


Fig. 4.7: Photograph of the movable arm and (down, right) an ultrasonic probe.

The pointwise water velocity is measured by a propeller (Fig. 4.8). The propeller is plunged into the water at the point of the measurement, and its velocity of rotation is linked to the velocity of water via a calibration curve.



Fig. 4.8: propeller used to measure the water velocity.

4.3 Experimental results

This section is devoted to the presentation of the experimental measurements. Section 4.3.1 presents the experiments conducted on the aligned layouts in steady flow, Section 4.3.2 the skewed layouts in steady flow, and Section 4.3.3 the dam break flow on all layouts.

4.3.1 *Aligned layouts, steady flow*

The aligned layouts of Figs. (4.3a), (4.3c) and (4.3d) (layout A, layout B narrow and layout B wide) are tested for various discharges listed in Table (4.1). The volume of water for each of these experiments is the same, and the downstream condition depends on the discharge as explained above (Fig. 4.6).

Aligned isotropic (a)			
Q (l/s)	h_{av} (m)	h_c (m)	Type of condition
43	0.134	0.028	Subcritical
58	0.100	0.034	Subcritical
63	0.085	0.036	Subcritical
75	0.065	0.040	Subcritical
86	0.044	0.044	Critical depth
103	0.050	0.050	Critical depth

(a)

Aligned anisotropic narrow (c)			
Q (l/s)	h_{av} (m)	h_c (m)	Type of condition
25	0.129	0.019	Subcritical
35	0.090	0.024	Subcritical
41	0.063	0.027	Subcritical
50	0.031	0.031	Critical depth

(b)

Aligned anisotropic wide (d)			
Q (l/s)	h_{av} (m)	h_c (m)	Type of condition
52	0.135	0.032	Subcritical
64	0.120	0.036	Subcritical
75	0.100	0.040	Subcritical
80	0.090	0.042	Subcritical
92	0.070	0.046	Subcritical
99	0.048	0.048	Critical depth

(c)

Table 4.1: Discharge, downstream condition (h_{av}), critical depth (h_c) and type of downstream condition for (a) the aligned layout A, (b) the aligned layout B in the narrow street direction and (c) the aligned layout B in the wide street direction.

Then, the influence of the downstream water level is studied on layout A by keeping the same discharge of 75 l/s but varying the downstream water level by changing the volume of water. Indeed, adding water to the system increases the water level in the pit. These different boundary conditions are listed in Table 4.2.

Aligned isotropic (a)	
h_{av} (m)	Type of condition
0.040	Critical depth
0.088	Subcritical
0.120	Subcritical
0.162	Subcritical
0.198	Subcritical

Table 4.2: Downstream boundary condition (m) and type of condition. The discharge is 75 l/s and the critical depth, 0.040 m.

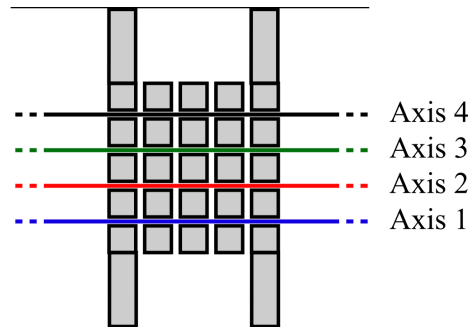


Fig.4.9: Axes of measurement of the four water profiles for the aligned layouts, labelled from 1 to 4.

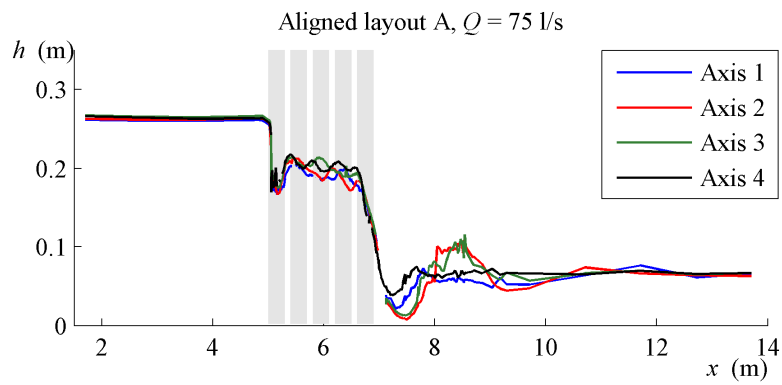


Fig. 4.10: Water profiles measured by the 4 probes for the aligned layout A with a discharge of $Q = 75$ l/s (see the names of axes in Fig. 4.9).

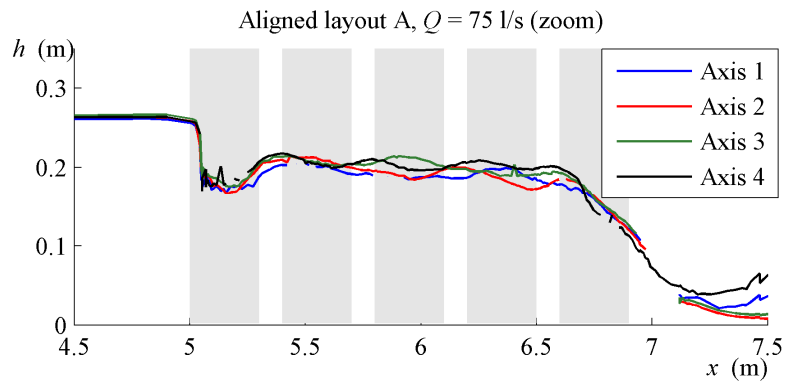


Fig. 4.11: Zoom of Fig. (4.10) focusing on the city.

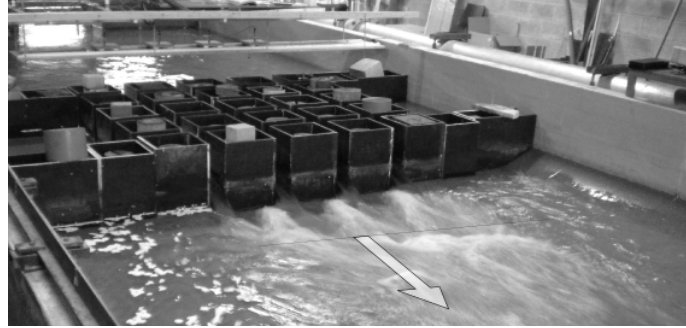


Fig. 4.12: Photograph of the flow downstream the city (layout A, $Q = 75$ l/s).

The water level profiles are measured along the axes of the four streets (Fig. 4.9). Fig. (4.10) shows the water level profiles measured by the four probes for layout A at the discharge $Q = 75$ l/s and Fig. (4.11) shows a zoom focusing on the building area. The grey rectangles represent the buildings. It can be seen that the water surface upstream the city is almost horizontal and then drops suddenly at the entrance of the city at $x = 5$ m. In the urban medium between $x = 5.3$ m and $x = 6.6$ m, the water surface has a slight tendency to decrease. At the exit of the city, the water level drops again below the critical depth (0.04 m) to reach a supercritical flow, followed by a hydraulic jump (at $x = 7.7$ m for probes 1 and 4 and $x = 8.5$ m for probes 2 and 3, see also the photograph in Fig. 4.12) and some perturbations until $x = 10.5$ m, after which it remains quite horizontal again until the downstream section of the flume.

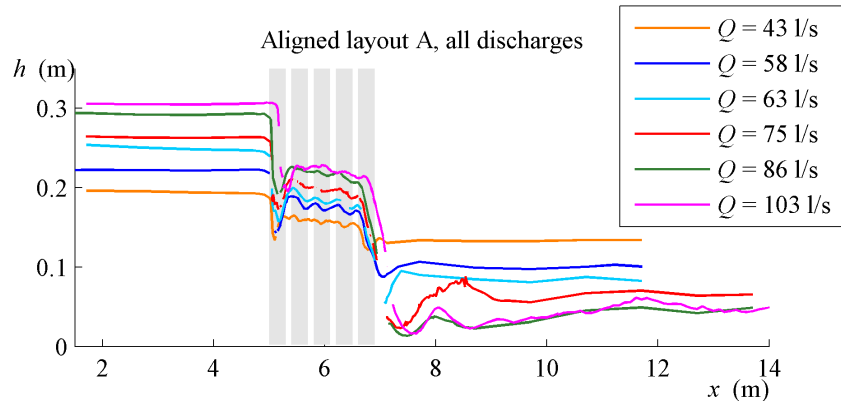


Fig. 4.13: Water profile for the aligned layout A, for all the tested discharges (the four profiles are averaged for each discharge)

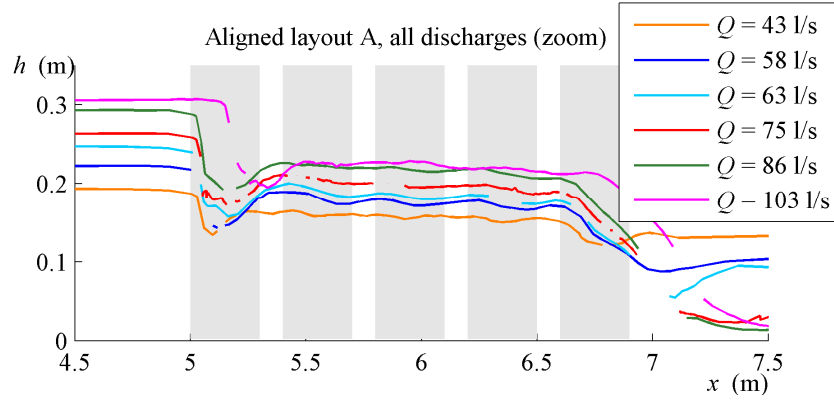


Fig. 4.14: Zoom of Fig. (4.13) focusing on the city

Fig. (4.13) illustrates all the results for the aligned layout A superimposed on each other, for discharges going from 43 l/s to 103 l/s. For the sake of clarity, the water profiles are averaged on the four probes so that there is only one curve per discharge. The difference of water level upstream and downstream the city increases with the discharge. For $Q = 86$ l/s and $Q = 103$ l/s, the water surface downstream the city is similar. In Fig. (4.14), between $x = 5.4$ m and $x = 6.6$ m, the water surface presents similar slope for every discharge.

Figs. (4.15) and (4.16) show the results for layout B in the narrow street direction. The discharges range from 25 l/s to 50 l/s. By comparison with layout A (Figs. 4.13 and 4.14) in which the streets are wider (0.1 m wide while they are only 0.0675 m wide in layout B), we expect the difference of water level between upstream and downstream from the city to be higher with a same discharge. Indeed, it can be seen by comparing the water profile for $Q = 43$ l/s in Fig. (4.14) and the water profile for $Q = 41$ l/s in Fig. (4.16) that this difference is higher even with a lower discharge.

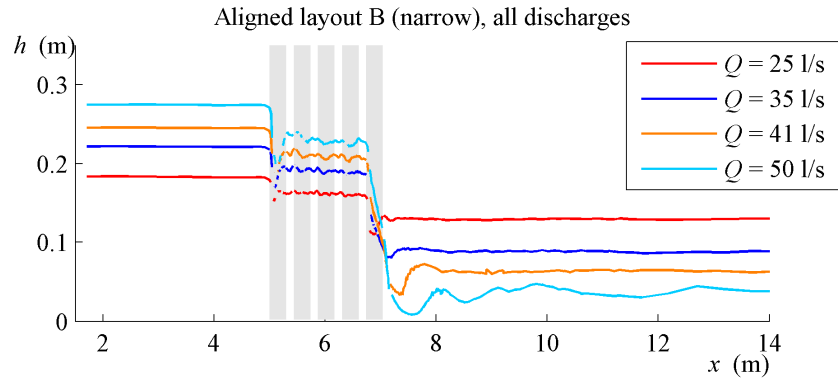


Fig. 4.15: Water profile for the aligned layout B in the narrow direction, for all the tested discharges (the four profiles are averaged for each discharge)

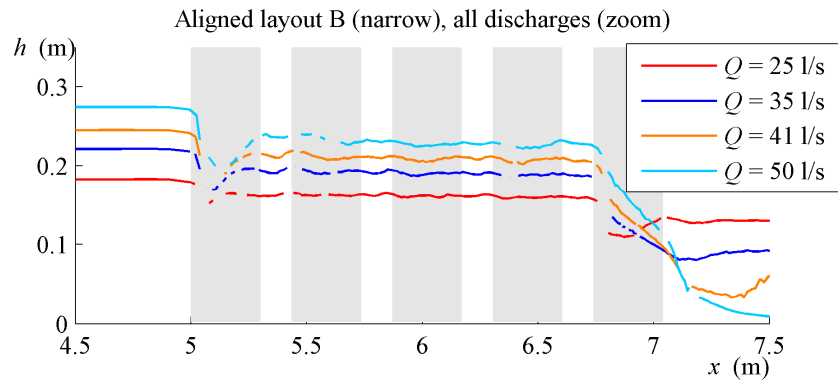


Fig. 4.16: Zoom of Fig. (4.15) focusing on the city

Figs. (4.17) and (4.18) show the results for layout B in the wide street direction. The discharges range from 52 l/s to 99 l/s. The streets are wider than the streets of layout A (0.10 m) and the narrow one of layout B (0.0675 m). By comparison with these two layouts (Figs. 4.13-4.16), similar discharge presents a lower difference of water level between upstream and downstream from the city.

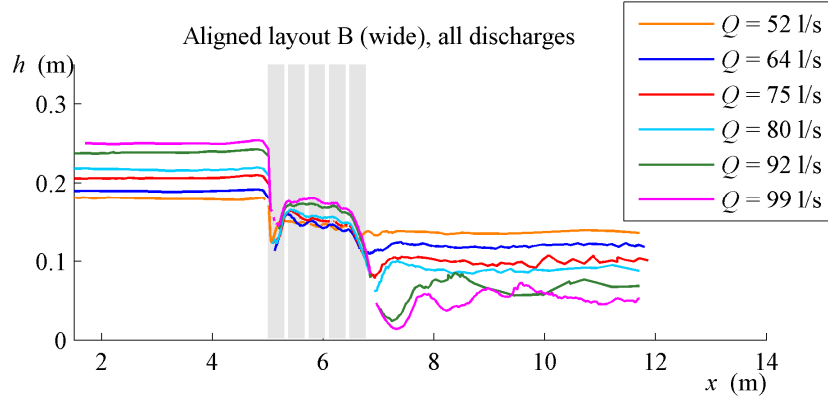


Fig. 4.17: Water profile for the aligned layout B in the wide direction, for all the tested discharges (the four profiles are averaged for each discharge)

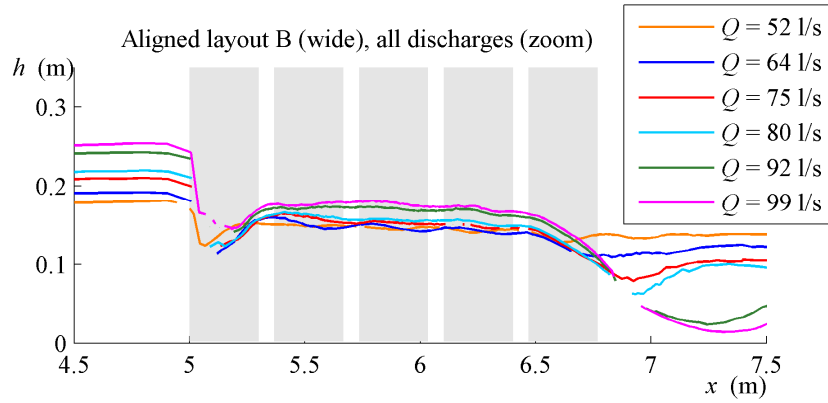


Fig. 4.18: Zoom of Fig. (4.17) focusing on the city

Fig. (4.19) shows the water profiles for the aligned layout A, for the discharge $Q = 75$ l/s and for various downstream conditions (obtained by varying the quantity of water in the system).

For $h_{av} = 45$ mm, the flow regime is subcritical upstream and inside the city, and passes by a critical section at $x = 6.9$ m to become supercritical downstream the city. By imposing a higher water depth condition of $h_{av} = 88$ mm (which is subcritical), a H2 axis develops and a hydraulic jump appears just downstream the city around $x = 7.6$ m. Nevertheless, this hydraulic jump is located too far from the control section at the city exit ($x = 6.9$ m) to influence the flow inside and upstream the city. Contrarily, for $h_{av} = 120$ mm (and higher), the exit of the city starts to be drawn so that h_{av} begins to have a repercussion on the upstream flow.

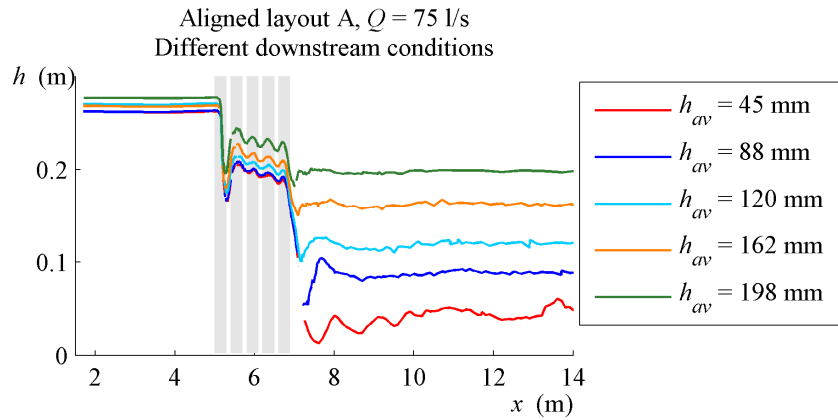


Fig. 4.19: Water profile for the aligned layout A, for $Q = 75$ l/s with different downstream boundary conditions

4.3.2 Skewed layouts, steady flow

The skewed layouts A and B (Figs. 4.3b and 4.3e) are tested in steady flow with a discharge of 75 l/s and 100 l/s. Four water level profiles are measured along the axes indicated in Fig. (4.20). The water profiles for layout A are shown in Fig. (4.21) and the ones for layout B, in Fig. (4.22).

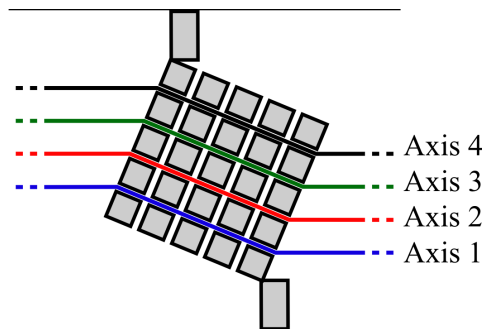


Fig. 4.20: Axes of measurement of the four water profiles for the skewed layouts, labeled from 1 to 4.

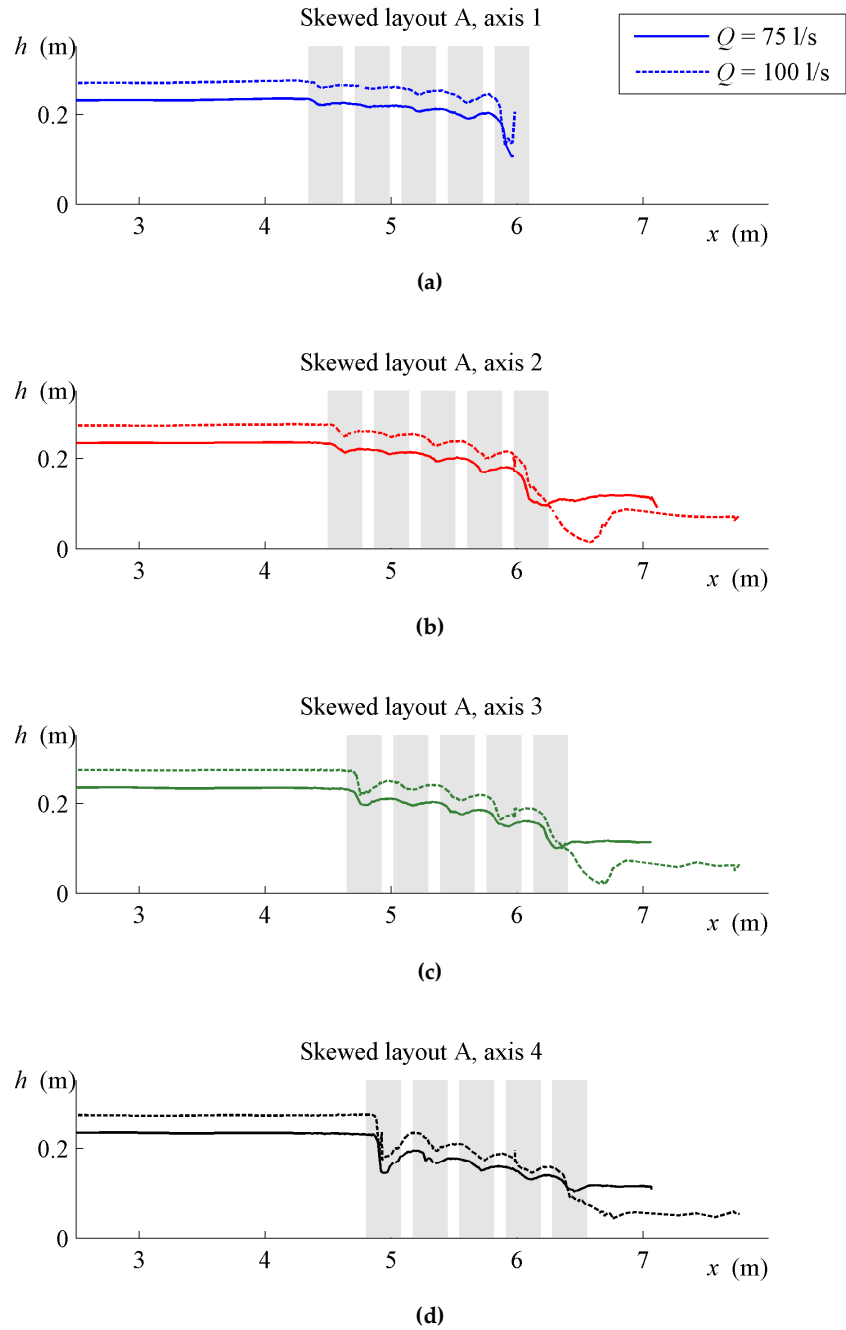


Fig. 4.21: Water level profile for the skewed layout A, along the (a) axis 1, (b) axis 2, (c) axis 3 and (d) axis 4.

For layout A, the water level upstream the city is lower than in the aligned case for the same discharges (Fig. 4.13). This could be explained by the fact that the difference of water level ahead and behind the city is proportional to the resistance that the city offers to the flow: the higher the resistance is, the higher the difference of water level. As in the aligned case the water can flow through only one side of the city, the buildings constitute a higher obstacle than for the skewed case where it can flow through two sides. For layout B, it is more difficult to make such a comparison because there are two aligned layouts (narrow and wide) to compare. But we can observe that the upstream water levels of the skewed layout B are similar to those of the aligned wide layout (Fig. 4.17).

The velocity of the water entering and coming out of the city is also measured with the propeller to obtain the distribution of the discharge entering and coming out of the city. These measurements are performed on the skewed layouts A and B for $Q = 75$ l/s. The sections of measurement are labelled from 1 to 8 (incoming flow) and from 1' to 8' (outcoming flow) (Fig. 4.23). Two velocity profiles, each constituted by 3 or 4 points of velocity measurement, are established in each section of layout A. In layout B, one profile per section is calculated in the narrow streets while 3 profiles are obtained in the wide streets. The velocity profiles are shown in Fig. (4.24) for layout A, and Fig. (4.25) for layout B. The discharge flowing at each section is calculated and is shown in Fig. (4.26) for layout A and in Fig. (4.27) for layout B. The numerical values corresponding to Figs. (4.26) and (4.27) are listed in Table (4.3) (incoming flow) and Table (4.4) (outcoming flow). The sum of the incoming and outcoming discharges in each street is also indicated. They are close to 75 l/s for the outcoming flow and overestimated for the incoming flow. The error is supposed to be due to the inaccuracy of the measurements.

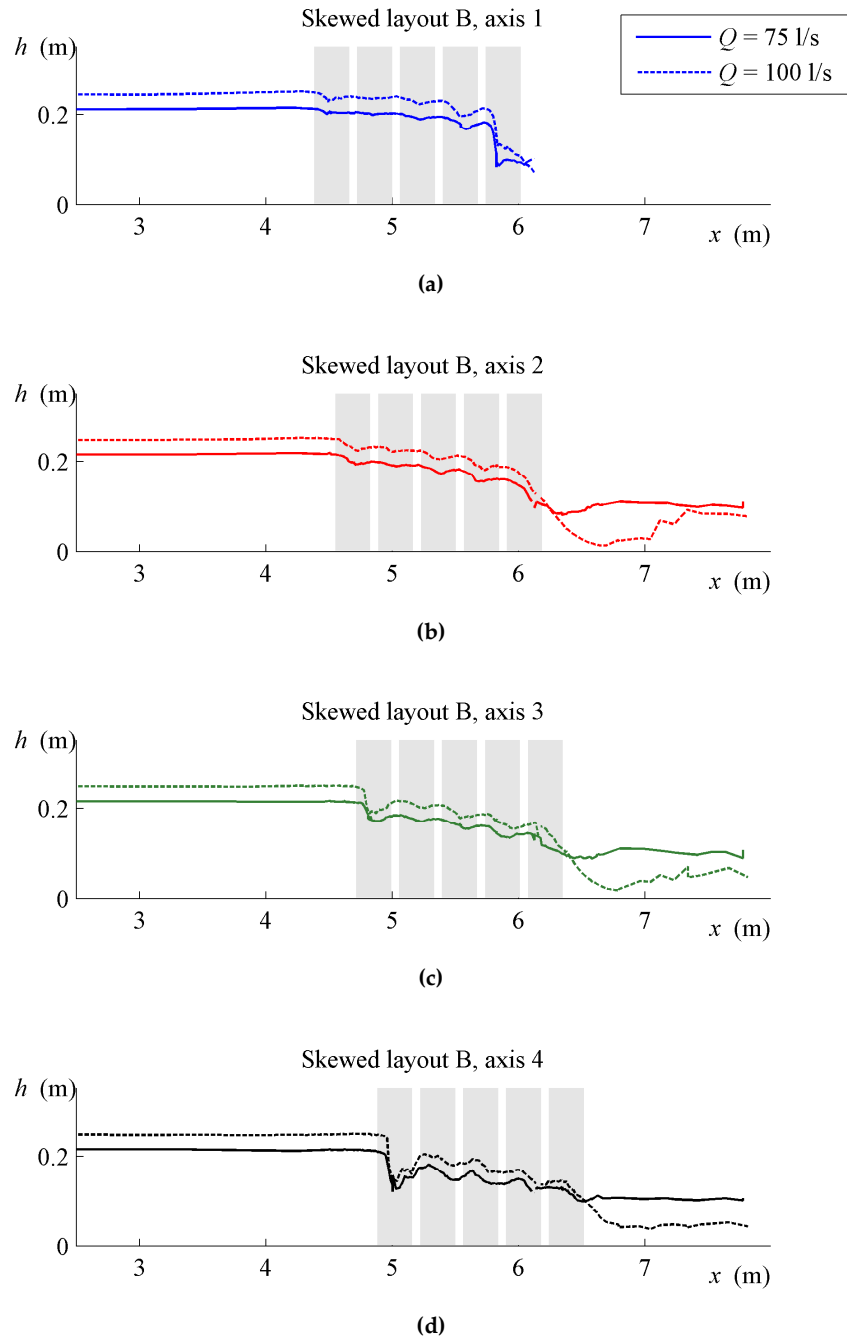


Fig. 4.22: Water level profile for the skewed layout B, along the (a) axis 1, (b) axis 2, (c) axis 3 and (d) axis 4.

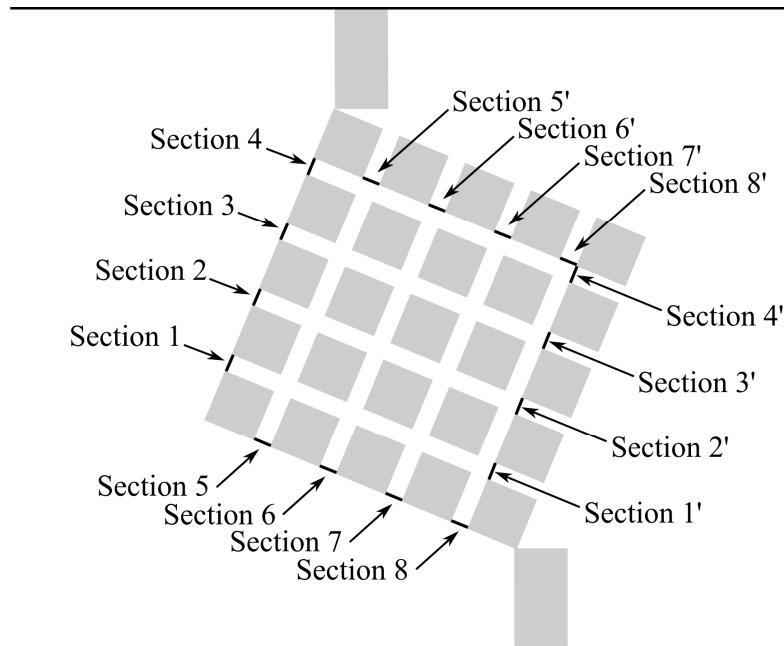


Fig. 4.23: Name of the sections where the velocity profiles are measured. The names of the sections are the same for layout B.

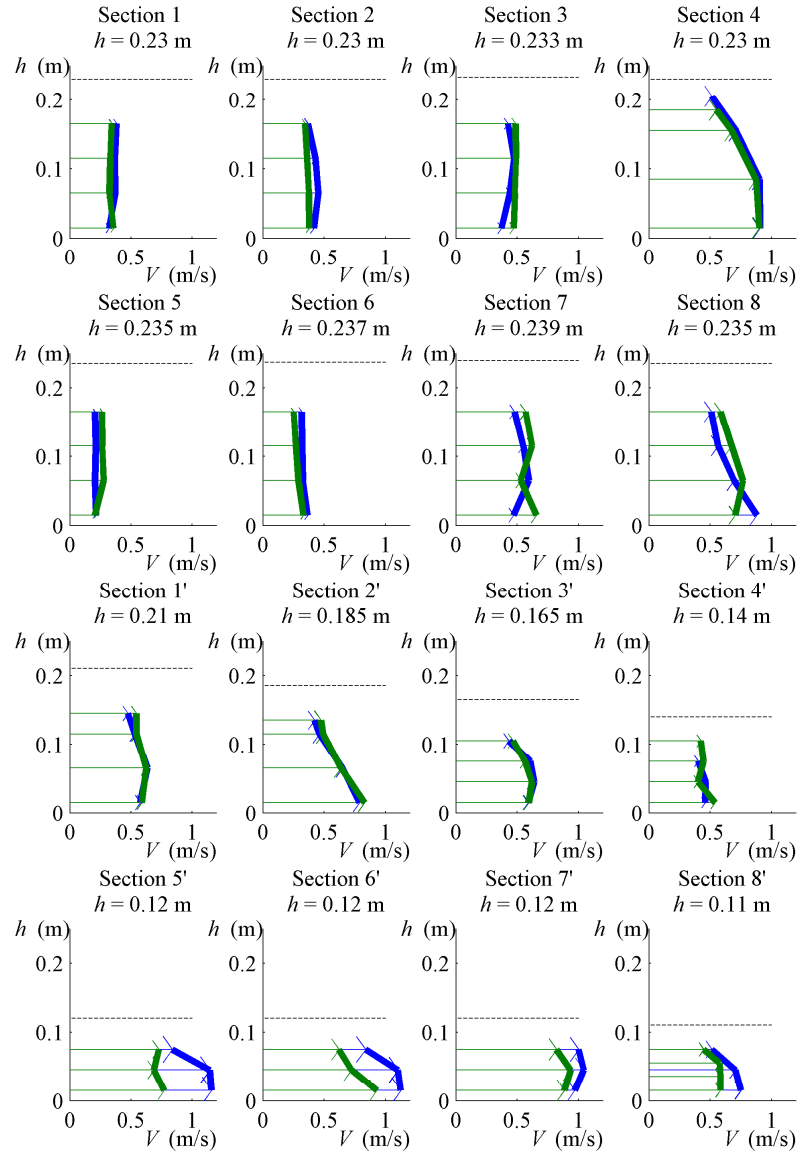


Fig. 4.24: Velocity profiles for layout A. The water level at each section is indicated in the titles and is represented by the black horizontal dotted line in the graph.

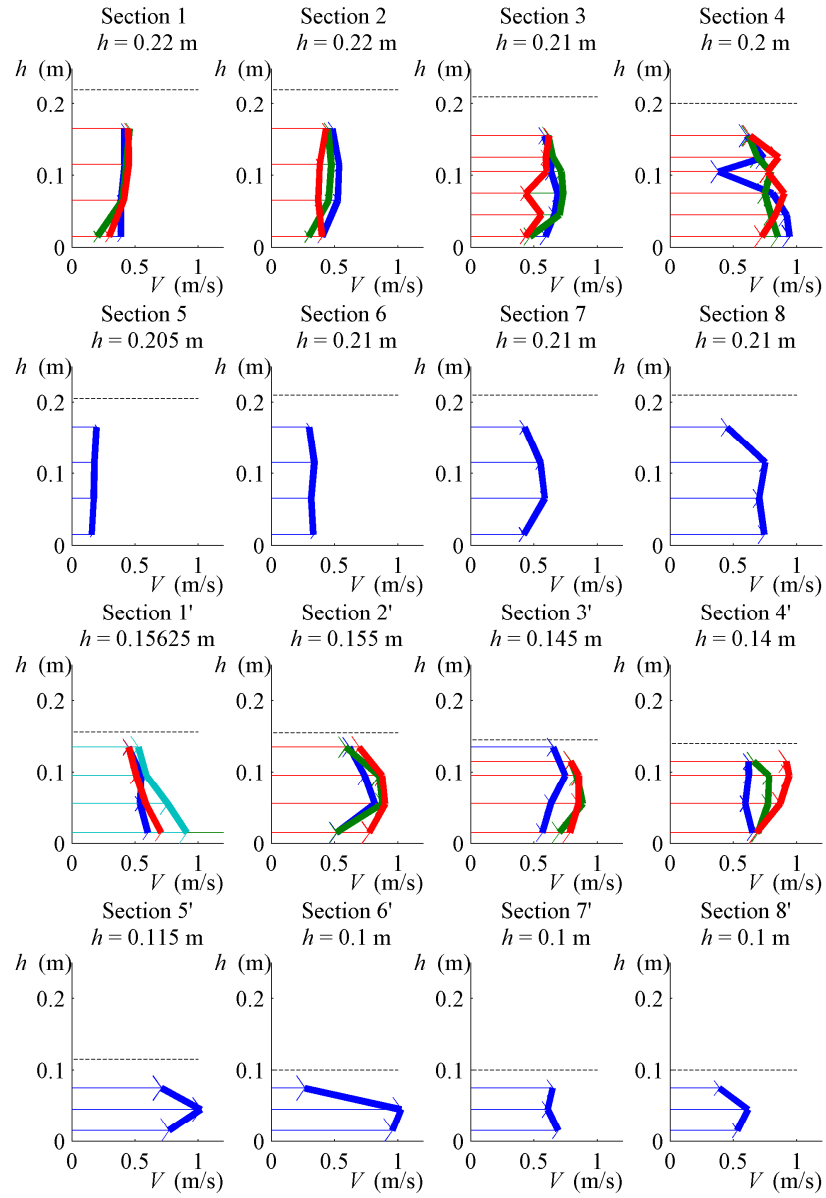


Fig. 4.25: Velocity profiles for layout B. The water level is indicated in the titles and is represented by the black horizontal dotted line in the graph.

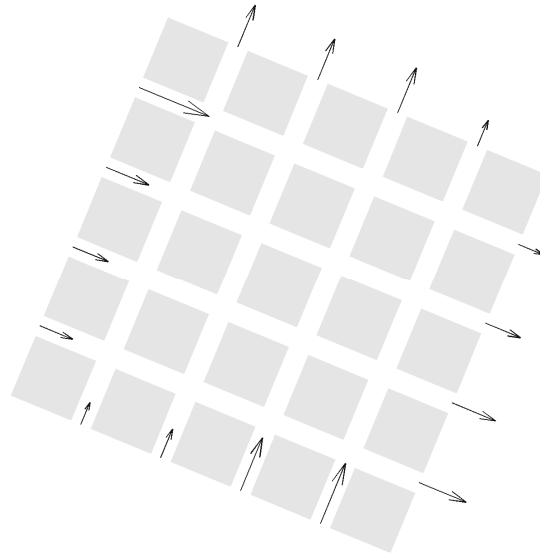


Fig. 4.26: Discharge entering and coming out of the city layout A. Numerical values can be found in Table (4.3).

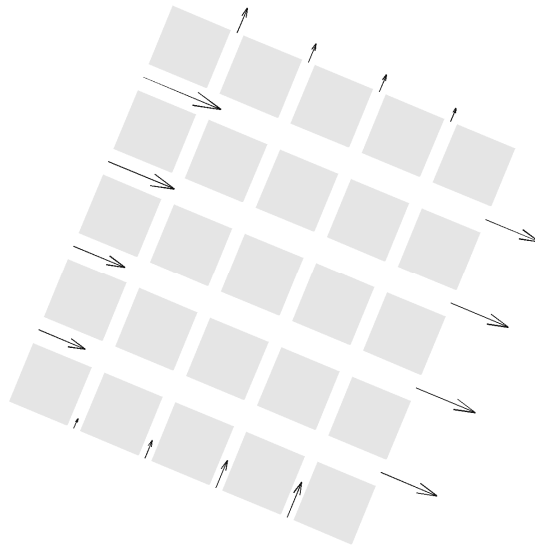


Fig. 4.27: Discharge entering and coming out of the city layout B. Numerical values can be found in Table (4.4).

Section	Q_{in} (m ³ /s)	
	Layout A	Layout B
1	0.0081	0.012
2	0.0088	0.0133
3	0.0107	0.017
4	0.0175	0.02
5	0.0055	0.0025
6	0.0071	0.0045
7	0.0132	0.007
8	0.0151	0.0091
Total	0.086	0.0854

Table 4.3: Incoming discharge for layouts A and B (the expected total discharge is 0.075 m³/s)

Section	Q_{out} (m ³ /s)	
	Layout A	Layout B
1'	0.0117	0.0144
2'	0.0108	0.0153
3'	0.0088	0.0148
4'	0.0061	0.0137
5'	0.0104	0.0063
6'	0.0102	0.0047
7'	0.0112	0.0044
8'	0.0064	0.0034
Total	0.0756	0.077

Table 4.4: Outcoming discharge for layouts A and B (the expected total discharge is 0.075 m³/s)

4.3.3 Dam break flow

For the dam break experiment, the gate in the middle of the flume is initially closed. The water is brought upstream of the gate by the pump until the water level reaches 0.40 m, leaving only 0.011 m of water downstream from the gate. The part of the flume located behind the gate constitutes the water reservoir. When it is filled, the gate is suddenly opened by a counterweight system, generating the propagation of a wave throughout the flume. When

the wave reaches the buildings, it is partially reflected and a hydraulic jump forms ahead of the city (Fig. 4.28). This hydraulic jump then recedes towards upstream.

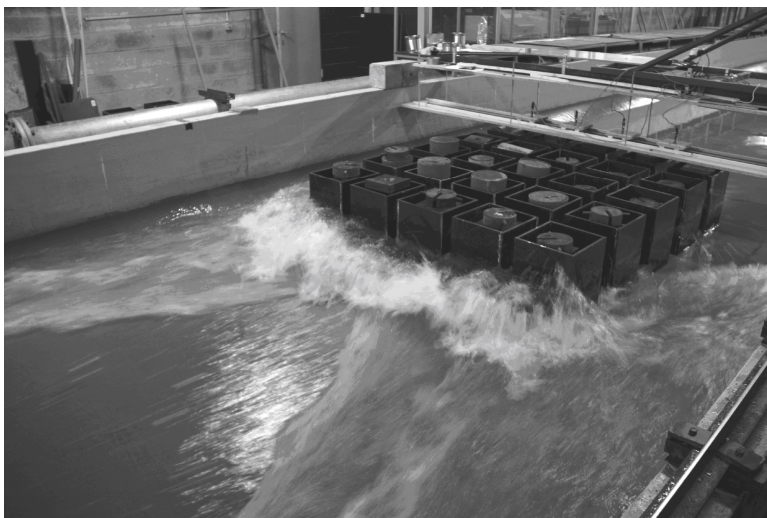


Fig. 4.28: Dam break experiment at approximately $t = 4$ s: formation of a hydraulic jump in front of the city.

The dam break experiment is conducted on the 5 layouts illustrated in Fig. (4.2): layout A aligned (Fig. 4.2a) and skewed (Fig. 4.2b), layout B aligned in the direction of the narrow (Fig. 4.2c) and wide streets (Fig. 4.2d) and skewed (Fig. 4.2e). The water level is measured. During the experiment, the arm supporting the four probes is kept at a fixed place so that we measure the water level as a function of time at four different points. Thanks to the repeatability of the phenomenon, the experiment is repeated various times placing the arm at different spots in order to cover the whole area of interest.

An example of points of measurement is shown in Fig. (4.29) for the aligned layout A. The water level $h(t)$ at the four selected points is shown in Fig. (4.30). It can be seen the water level suddenly increases when the wave reaches the points (between 2 s and 3 s for P1 to P3 and 10 s for P4). Then, between 10 and 65 s, the water level remains quite constant. Indeed, the reservoir is so big that even if the water is running out, the water level in the reservoir varies quite little and the flow in the city behaves as if a constant water level were prescribed at the gate. Then at 65 s, we can see that the water level increases at P3 and P4. This is indeed the moment where the wave that has been reflected against the downstream wall of the flume, has travelled back upstream and reaches the city.

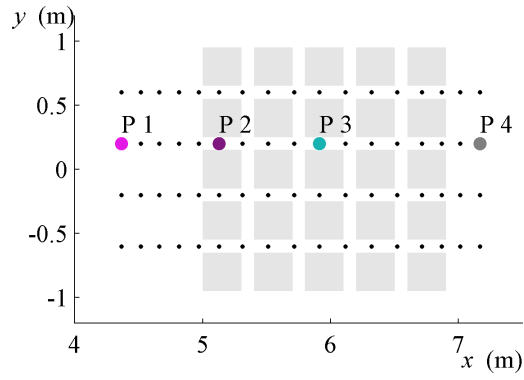


Fig. 4.29: Location of the points of measurement for the aligned layout A (black points) and location of the 4 points for which $h(t)$ is shown in Fig. (4.30) (colored points).

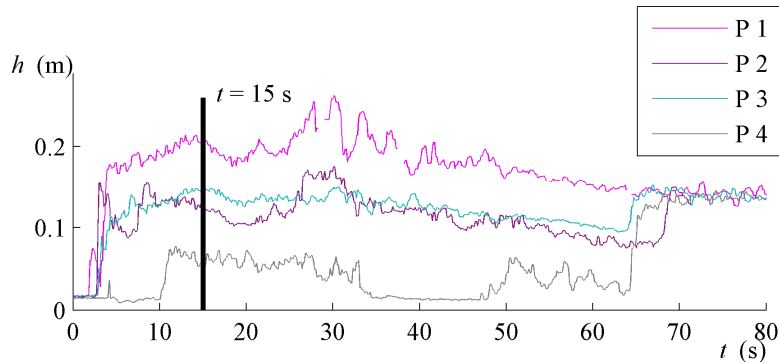


Fig. 4.30: Measurement of water level in function of time at the points P1, P2, P3 and P4 for the aligned layout A (see Fig. (4.29) for the location of points)

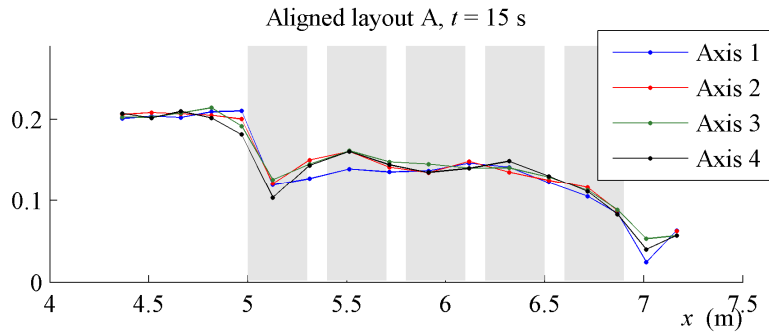


Fig. 4.31: Water level profiles along the four axes at the time $t = 15$ s for the aligned layout A (see Fig. (4.9) for the name of the axes).

The water profiles along the four axes of the aligned layout A (Fig. 4.9) at $t = 15$ s are shown in Fig. (4.31). The water level profiles along the four axes are very similar. We can observe a higher water depth upstream of the city because of the presence of the hydraulic jump and the depression of water level where the flow enters the city (between $x = 5$ m and $x = 5.3$ m).

Figs. (4.32) and (4.33) illustrate the water level profiles at $t = 15$ s for the aligned layout B in the direction of the narrow streets and wide streets, respectively. Like for the aligned layout A, it can be seen that the water level ahead of the city is of the order of 0.20 m, then drops suddenly at the entrance of the city and continues to decrease all along the city. The negative water depth measured by the probes at the exit of the city in Fig. (4.33) is erroneous. Such erroneous measurements are due to the inclination of the water surface at this place, resulting in that the ultrasonic ray emitted by the probe is not reflected vertically and is not intercepted by the probe. Notice that the flow taking place in an aligned city is very self-similar (Figs. 4.31-33), regardless of how wide the streets are. This means that in dam break flow, the city acts as a whole obstacle whose internal geometry plays no important role.

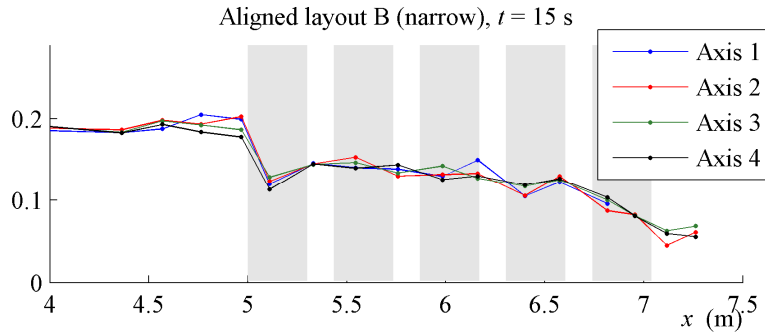


Fig. 4.32: water level profiles along the four axes at the time $t = 15$ s for the aligned layout B in the direction of the narrow streets (see Fig. (4.9) for the label of the axes).

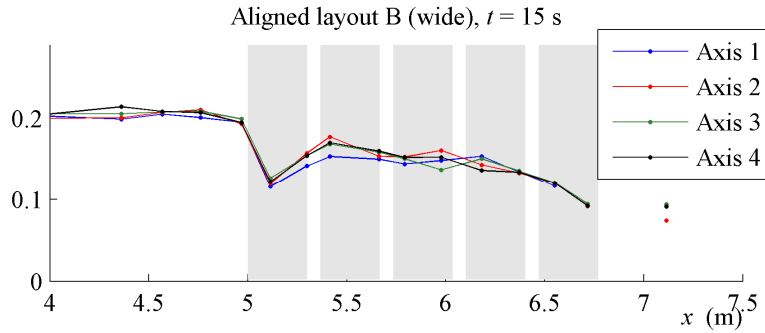


Fig. 4.33: water level profiles along the four axes at the time $t = 15$ s for the aligned layout B in the direction of the wide streets (see Fig. (4.9) for the label of the axes).

Figs. (4.34) and (4.35) illustrate the water level profiles for the dam break experiment on the skewed layouts A and B, respectively, at time $t = 15$ s. The names of the axes are defined in Fig. (4.20). It can be seen that the water level profiles are similar in both layouts meaning that, like for the aligned layouts, the city acts as a whole obstacle poorly influenced by the streets width. The water depth upstream of the city slightly exceeds 0.20 m, indicating that the city constitutes a larger obstacle when it is skewed than when it is aligned. Contrarily to the aligned layouts, we can observe in the graphs of Figs. (4.34) and (4.35) that the water profiles along the four axes have a distinct shape: along axis 1, the water level does not drop at the entrance of the city, then is rather constant until it decreases quite much at the exit of the city; while along axis 4 the water level drops at the entrance and then continues to decrease all along the city.

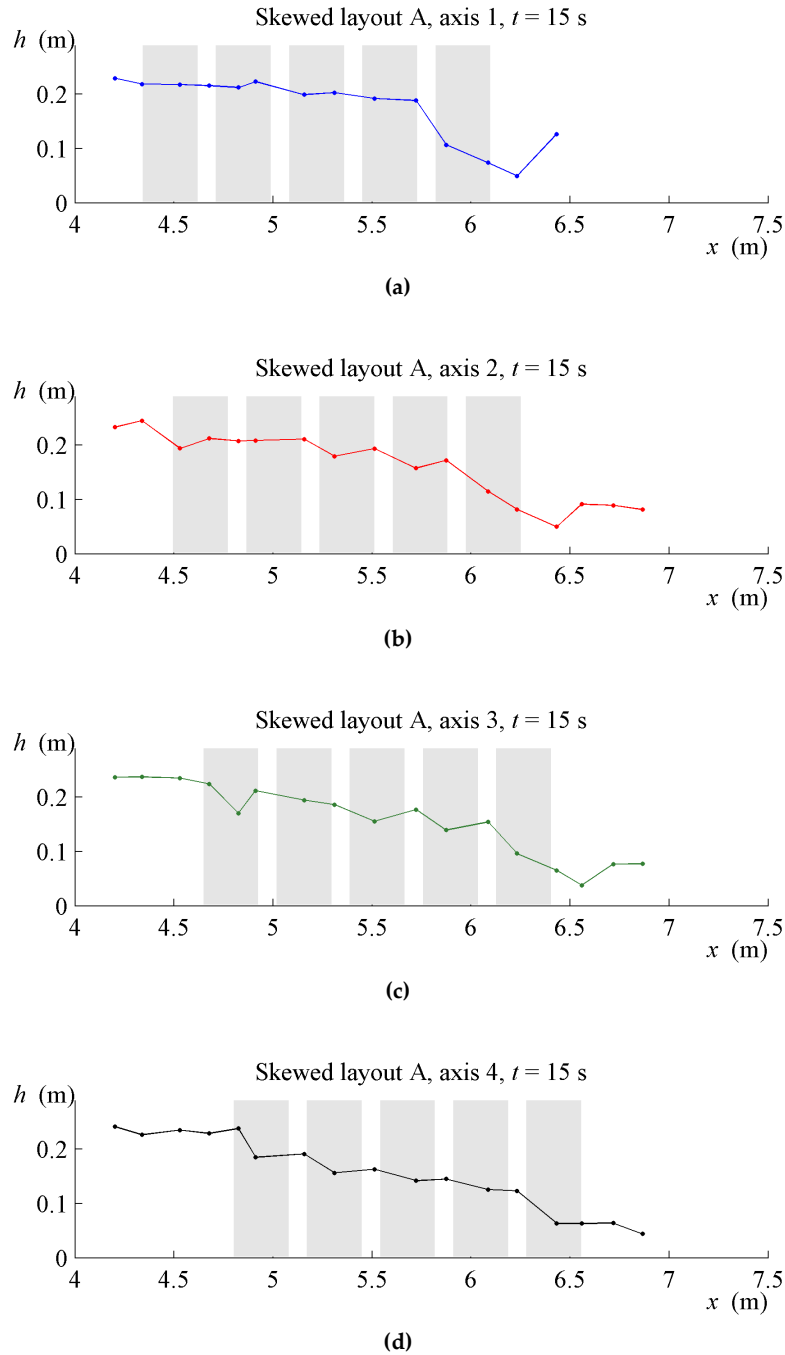
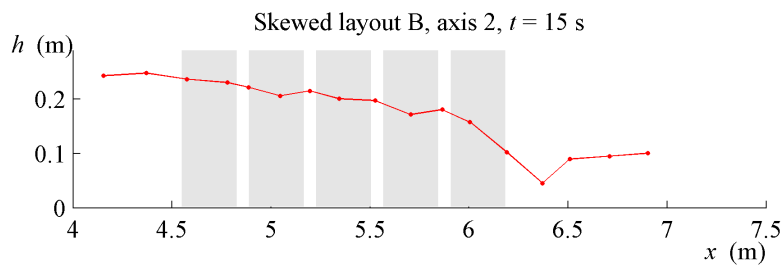


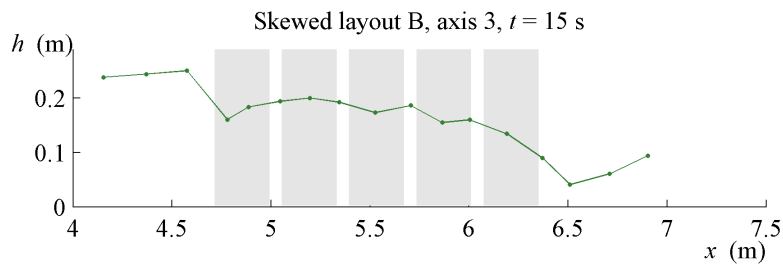
Fig. 4.34: water level at time $t = 15$ s for the skewed layout A along (a) axis 1, (b) axis 2, (c) axis 3 and (d) axis 4 (see Fig. (4.20) for the name of axes).



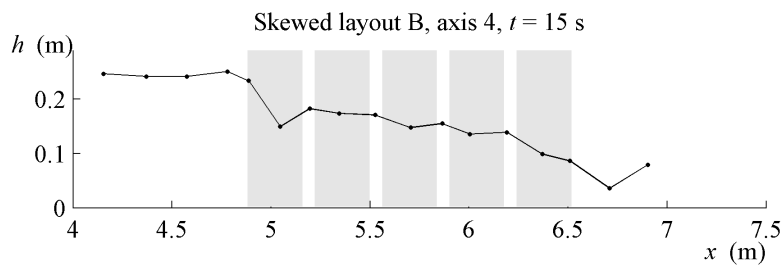
(a)



(b)



(c)



(d)

Fig. 4.35: water level at time $t = 15$ s for the skewed layout B along (a) axis 1, (b) axis 2, (c) axis 3 and (d) axis 4 (see Fig. (4.20) for the name of axes).

4.4 Conclusion

New experiments have been conducted in steady flow and dam break flow on a set of 5×5 square blocks representing a simplified city with perpendicular streets. These blocks have been placed in various layouts, varying the street width and orientation (Figs. 4.2 and 4.3) with the aim of validating the porosity model in Chapter 6. The planar view of the laboratory flume used to make these experiments is shown in Fig. (4.4). There is a gate acting as a dam in the constriction in the middle of the flume. For the dam break experiment, the gate is initially closed and the part upstream of the gate constitutes the water reservoir. The gate is then opened by a counterweight system, generating a dam break wave throughout the flume. For the steady flow experiments, the gate remains always open and a constant discharge is prescribed through a regulation system.

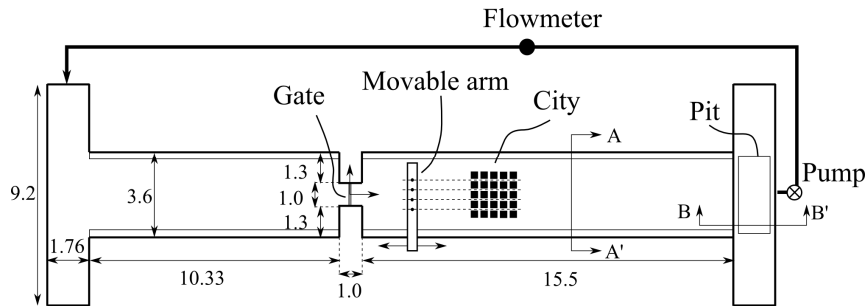


Fig. 4.4: Plane view of the experimental flume.

The water level was measured simultaneously by 4 ultrasonic probes fixed to a movable arm. For the dam break experiment, the arm was immobile during the experiment so that the 4 probes measured the water level as a function of time at 4 points. The experiment is then repeated several times to cover all the area of interest, which is possible due to the repeatability of the dam break phenomenon. For the steady flow experiments, the arm was swept along 4 parallel axes, giving 4 water level profiles. Also, for the skewed layouts, the water velocity was measured with a propeller at the city boundary in order to deduce the repartition of the entering and out-coming discharges in the streets.

In the steady flow experiments, obstructing blocks have been placed aside from the city to force the entire flow to cross the city. Various discharges were tested on layouts with various street widths. It was observed that the water depth is very sensitive to these two parameters. For a given city

layout, the difference of water level upstream and downstream from the city is higher for higher discharges. It should also be observed that for a given discharge and on an aligned layout, the difference of water level upstream and downstream from the city is higher if the streets are narrower. This leads to the expected conclusion that the resistance of the city is higher if the streets are narrower. Remember that this effect had already been highlighted in Chapter 3 with the circular dam break experiments (Figs. 3.3 and 3.11).

In the dam break experiment, it was observed that the flow was influenced by the city orientation but was not so much affected by the street width. This leads to conclude that in dam break flow, the city acts as a whole obstacle whose internal geometry has no big impact on the flow.

Chapter 5 Numerical solution

In this chapter, the numerical method used to solve the shallow water equations with porosity is presented. The equations to solve are:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S}_g + \mathbf{S}_n = \mathbf{S} \quad (5.1)$$

where

$$\mathbf{U} = \begin{bmatrix} \phi h \\ \phi V_x h \\ \phi V_y h \end{bmatrix}; \mathbf{F} = \begin{bmatrix} \phi V_x h \\ \phi V_x^2 h + \phi \frac{gh^2}{2} \\ \phi V_x V_y h \end{bmatrix}; \mathbf{G} = \begin{bmatrix} \phi V_y h \\ \phi V_x V_y h \\ \phi V_y^2 h + \phi \frac{gh^2}{2} \end{bmatrix}; \quad (5.1a)$$

$$\mathbf{S}_g = \begin{bmatrix} 0 \\ -\phi gh \frac{\partial z_b}{\partial x} + \frac{gh^2}{2} \frac{\partial \phi}{\partial x} \\ -\phi gh \frac{\partial z_b}{\partial y} + \frac{gh^2}{2} \frac{\partial \phi}{\partial y} \end{bmatrix} \text{ and } \mathbf{S}_n = \begin{bmatrix} 0 \\ -\phi gh S_{fx} - R_x \\ -\phi gh S_{fy} - R_y \end{bmatrix} \quad (5.1b)$$

where the symbol of the averaging operator $\langle \cdot \rangle_A^f$ has been omitted for sake of clarity; the friction S_{fx} and S_{fy} are given by Eq. (2.51) and the resistance due to the drag/dispersion $\mathbf{R}$ for a periodic medium is given by Eq. (3.14). The term $\mathbf{S}_g$ contains the geometrical source terms (bed slope and porosity gradient) and $\mathbf{S}_n$ the non-geometrical source terms (head losses due to friction, drag and dispersion). The terms $\mathbf{S}_g$ and $\mathbf{S}_n$ are set apart because $\mathbf{S}_g$ contains terms that must exactly balance the momentum under equilibrium conditions (for example, water at rest or steady flow).

Recall that the "former" porosity model, the one developed by Guinot and Soares-Frazão (2006) and Lhomme (2006) (Eq. 2.87-89), was similar to the one given by Eq. (5.1): as far as they finally considered that $\phi_x = \phi_y = \phi_s = \phi$, the only difference with the system of Eq. (5.1) is the expression of the non-geometrical source term $\mathbf{S}_n$ and, as $\mathbf{S}_n$ does not play an important role in the numerical solver, the solvers developed for the former model remain valid

for the present porosity equations. Three different solvers have been found in the literature: a modified HLL solver developed by Guinot and Soares-Frazão (2006), a modified high-order Roe solver developed by Cea and Vázquez-Cendón (2010) and two-rarefactions solver developed by Finaud-Guyot et al. (2010). In this thesis, only the modified HLL and the two-rarefactions solvers will be discussed. They will be explained, and then criticized and compared with each other.

Although these alternatives are not derived in the frame of the present thesis, the originality here consists in explaining how they are theoretically derived in a simple way, by presenting the main steps of the developments and the main assumptions without entering into the detail. Besides the didactic explanation, the originality of the present chapter is that some assumptions made in the development of the two alternatives are criticized.

The equations are solved by the finite-volume method. This method consists in discretizing the domain into computational cells in which the state $\mathbf{U}$ is constant (if we assume that the method is of first order). A difference of state $\mathbf{U}$ in adjacent cells generates a flux at their common interface. This flux depends on the right and left states, $\mathbf{U}_R$ and $\mathbf{U}_L$, respectively, and can be calculated exactly or approximately. For the classical shallow water equations (with $\phi = 1$ and $\mathbf{S} = 0$), an exact flux solver exists, but it is complicated to use in practice because it consists of an iterative computation. Therefore approximate solvers have been developed. For the porosity equations (Eqs. 5.1), an exact solution does not exist and it is necessary to develop approximate solvers.

This chapter begins by explaining the principle of the finite-volume method in Section 5.1. Then, the basics of the exact flux solver for the classical shallow water equations (with $\phi = 1$ and with $\mathbf{S} = 0$) are recalled in Section 5.2. Indeed, despite the equations studied in this thesis are different, knowing how to calculate the exact flux for simpler equations will help understanding the importance of the assumptions further made for the approximate solvers of the porosity equations. Then, Section 5.3 presents two different approximate solvers for the porosity equations, developed by Guinot and Soares-Frazão (2006) and Finaud-Guyot et al. (2010), respectively. These two solvers are based on different assumptions. In the present work, some steps of their theoretical development are criticized. Section 5.4 is devoted to the comparison between the two solvers in dam break flow and in steady flow. This has already been done in Finaud-Guyot et al. (2010), but here the results in dam break flow will only be summarized, while the results in steady flow will be further investigated, and it will be

shown that the discrepancies between the two solutions can be explained by their basic assumptions.

5.1 The finite-volume method

A corresponding integral form of the conservation laws given by Eq. (5.1) is

$$\int_{\mathcal{U}} \left(\frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \frac{\partial \mathbf{G}(\mathbf{U})}{\partial y} + \frac{\partial \mathbf{U}}{\partial t} \right) d\mathcal{U} = \int_{\mathcal{U}} \mathbf{S}(\mathbf{U}) d\mathcal{U}. \quad (5.2)$$

The volume $\mathcal{U}$ is a three-dimensional volume in the space (x, y, t) . It represents a computational cell of the domain (Fig. 5.1), and has then nothing to do with the REV of Chapter 2. Its base is in the plane (x, y) and its height is the computational time step Δt . Noting that the left-hand side of Eq. (5.2) is the divergence of $(\mathbf{F}, \mathbf{G}, \mathbf{U})$ in the space (x, y, t) and by applying the divergence theorem, Eq. (5.2) becomes

$$\int_A ((\mathbf{F} \quad \mathbf{G} \quad \mathbf{U}) \cdot \mathbf{n}) dA = \int_{\mathcal{U}} \mathbf{S}(\mathbf{U}) d\mathcal{U} \quad (5.3)$$

where A is the boundary of the volume $\mathcal{U}$ constituted by the two horizontal surfaces and the vertical surfaces of height Δt , and

$$\mathbf{n} \equiv [n_x, n_y, n_t] \quad (5.4)$$

is the outward unit vector to the boundary A . Fig. (5.1) depicts the situation. Knowing that $\mathbf{n} = (0, 0, -1)$ on the horizontal inferior surface, that $\mathbf{n} = (0, 0, 1)$ on the horizontal superior surface and that $\mathbf{n} = (n_y, n_x, 0)$ on the vertical surfaces, Eq. (5.3) becomes:

$$-\mathbf{U}_i^n A_i + \mathbf{U}_i^{n+1} A_i + \sum_j (\mathbf{F}_{ij}^* n_x + \mathbf{G}_{ij}^* n_y) w_{ij} \Delta t = \mathbf{S} \Delta t A_i \quad (5.5)$$

where $\mathbf{U}_i^n$ is the average value of $\mathbf{U}$ at time n over the computational cell i , A_i is the size of the 2D surface of the computational cell in the plane (x, y) , $\mathbf{F}_{ij}^*$ is the average value of $\mathbf{F}$ on the vertical surface ij , w_{ij} is the length of the j^{th} side of the 2D computational cell, and $\mathbf{S}$ is the average of the source term over $\mathcal{U}$.

The **rotational invariance** property yields that the two-dimensional homogeneous shallow water equations satisfy

$$[n_x, n_y] \cdot [\mathbf{F}(\mathbf{U}), \mathbf{G}(\mathbf{U})]^T = \mathbf{T}^{-1} \mathbf{F}(\mathbf{T}(\mathbf{U})) \quad (5.6)$$

for all vectors $\mathbf{U}$ and for all unit vector (n_x, n_y) (Toro p.65). Here $\mathbf{T} = \mathbf{T}(\mathbf{n})$ is a rotation matrix and $\mathbf{T}^{-1}(\mathbf{n})$ is its inverse, given respectively by

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & n_x & n_y \\ 0 & -n_y & n_x \end{bmatrix}, \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & n_x & -n_y \\ 0 & n_y & n_x \end{bmatrix}. \quad (5.7)$$

With Eq. (5.5), Eq. (5.5) becomes

$$-\mathbf{U}_i^n A_i + \mathbf{U}_i^{n+1} A_i + \sum_j (\mathbf{T}^{-1} \mathbf{F}_{ij}^* (\hat{\mathbf{U}}_i^n)) w_{ij} \Delta t = \mathbf{S} \Delta t A_i \quad (5.8)$$

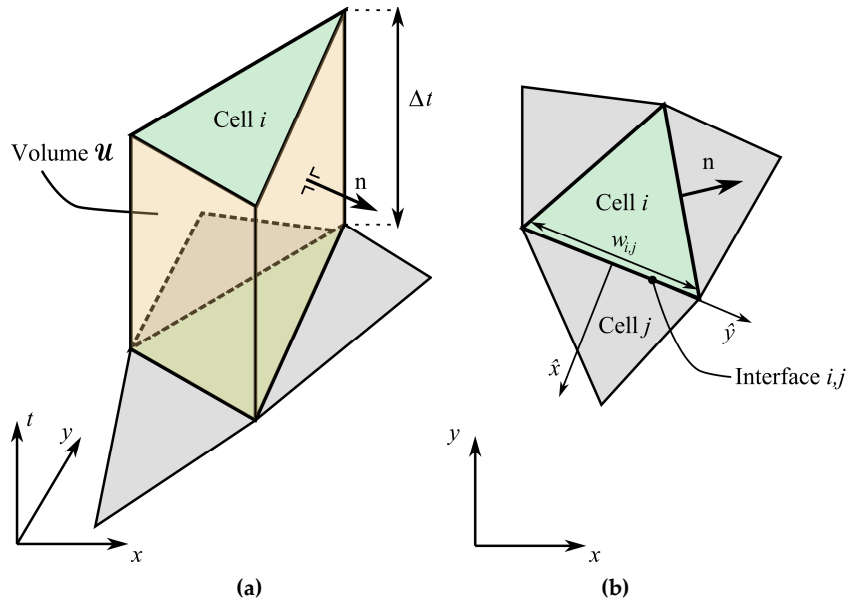


Fig. 5.1: Computational cell of the finite-volume method, (a) in a 3D view, the third dimension being the time t , and (b) in plan view.

where

$$\hat{\mathbf{U}}_i^n = \mathbf{T}_{i,j}(\mathbf{U}_i^n) = [h, h\hat{V}_x, h\hat{V}_y] \quad (5.9)$$

and

$$\begin{aligned} \hat{V}_x &= V_x n_x + V_y n_y \\ \hat{V}_y &= -V_x n_y + V_y n_x \end{aligned} \quad (5.10)$$

Eq. (5.8) gives the general formulation for the finite volume method on unstructured grids:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t}{A_i} \sum_{j \in N(i)} \mathbf{T}_{i,j} \mathbf{F}_{i,j}^{n+1/2} w_{i,j} + \Delta t \mathbf{S}_i^{n+1/2}. \quad (5.11)$$

$\mathbf{U}_i^n$ is the value of $\mathbf{U}$ over the cell i at the time step n , A_i is the area of cell i , $\mathbf{F}_{i,j}^{n+1/2}$ is the value of the flux vector in the direction normal to the interface (i,j) between the time step n and $n+1$, $N(i)$ is the set of neighbor cells of cell i , $\mathbf{T}_{i,j}$ is the matrix that accounts for the coordinate change from the global (x,y) coordinate system attached to the interface, $w_{i,j}$ is the width of the interface (i,j) , $\mathbf{S}_i^{n+1/2}$ is the value in cell i between the time steps n and $n+1$ of the source terms arising from the bottom slope, porosity gradient, friction, drag and dispersion and Δt is the computational time step.

5.2 Exact solver (without porosity)

As the variables are defined inside the computational cells but not at their edges, the difficulty of the finite volume method is the estimation of the fluxes at the interface ($\mathbf{F}_{i,j}^{n+1/2}$ in Eq. 5.11). There are various flux expressions, and most of them arise from the resolution of a Riemann problem for the homogeneous equations ($\mathbf{S} = 0$) at the interface.

5.2.1 The Riemann problem

Definition. The **Riemann problem** is the initial value problem

$$\left. \begin{aligned} \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} &= 0, \\ \mathbf{U}(x,0) &= \begin{cases} \mathbf{U}_L & \text{if } x < 0, \\ \mathbf{U}_R & \text{if } x > 0. \end{cases} \end{aligned} \right\}. \quad (5.12)$$

In the classical shallow water equations, $\mathbf{U}$ and $\mathbf{F}$ are given by Eq. (5.1a) in which $\phi = 1$. The dam break problem is a particular case of the Riemann problem for which the initial values are $V_{xL} = V_{xR} = V_{yL} = V_{yR} = 0$ and $h_R \neq h_L$. Fig. (5.2) shows the initial value of the problem, the solution for time t^* as well as the wave diagram of the process. The solution is a similarity solution

$U(x/t)$, meaning that U depends on the ratio x/t . For the finite volume method, we are interested in computing the flux at the interface, i.e. $F(U(x/t = 0))$.

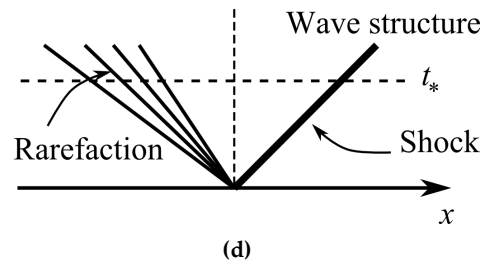
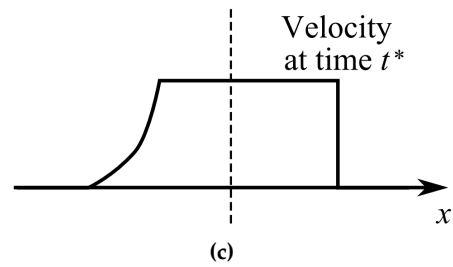
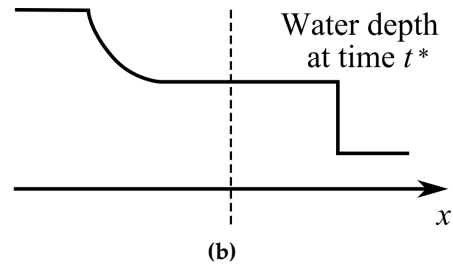
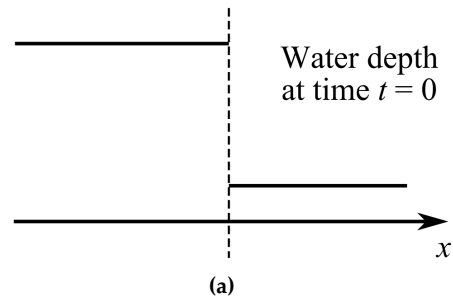


Fig. 5.2: Time evolution of the dam break problem: (a) initial water depth, (b) depth at a later time and (c) corresponding velocity distribution; (d) wave diagram of the full process on the x-t plane.

5.2.2 Characteristic field

Eq. (5.12) can be written as

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}(\mathbf{U}) \frac{\partial \mathbf{U}}{\partial x} = 0 \quad (5.13)$$

where

$$\mathbf{A}(\mathbf{U}) = \frac{\partial F(\mathbf{U})}{\partial \mathbf{U}} = \begin{bmatrix} 0 & 1 & 0 \\ c^2 - V_x^2 & 2V_x & 0 \\ -V_x V_y & V_y & V_x \end{bmatrix} \quad (5.14)$$

where $c = \sqrt{gh}$. The eigenvalues of $\mathbf{A}(\mathbf{U})$ are

$$\lambda_1 = V_x - c, \quad \lambda_2 = V_x, \quad \lambda_3 = V_x + c. \quad (5.15)$$

By expanding the derivatives and after some algebraic manipulations, Eq. (5.13) may also be written

$$\begin{aligned} \frac{\partial h}{\partial t} + V_x \frac{\partial h}{\partial x} + h \frac{\partial V_x}{\partial x} &= 0 \\ \frac{\partial V_x}{\partial t} + g \frac{\partial h}{\partial x} + V_x \frac{\partial V_x}{\partial x} &= 0 \\ \frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} &= 0 \end{aligned} \quad (5.16)$$

The system of Eqs.(5.16) may be written in the vector form

$$\frac{\partial \mathbf{W}}{\partial t} + \mathbf{A}(\mathbf{W}) \frac{\partial \mathbf{W}}{\partial x} = 0 \quad (5.17)$$

where $\mathbf{W}$ is the vector of primitive variables:

$$\mathbf{W} = \begin{bmatrix} h \\ V_x \\ V_y \end{bmatrix} \quad (5.18)$$

and $\mathbf{A}$ is given by

$$\mathbf{A}(\mathbf{W}) = \begin{bmatrix} V_x & h & 0 \\ g & V_x & 0 \\ 0 & 0 & V_x \end{bmatrix}. \quad (5.19)$$

The eigenvalues of $\mathbf{A}(\mathbf{W})$ are

$$\lambda_1 = V_x - c, \quad \lambda_2 = V_x, \quad \lambda_3 = V_x + c. \quad (5.20)$$

The initial discontinuity in the Riemann problem causes the propagation of three waves associated to the three eigenvalues λ_1 , λ_2 and λ_3 .

5.2.3 Wave type

Three types of wave may propagate. The waves associated to the eigenvalue λ_2 is always a contact discontinuity, while the waves associated with λ_1 and λ_3 can be either rarefaction waves or shock waves.

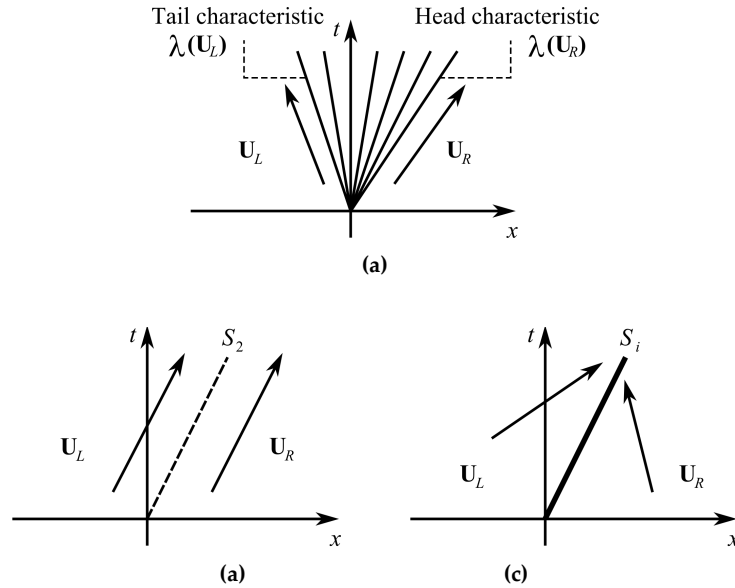


Fig. 5.3: Single wave solution of the Riemann problem and characteristics on both sides of the wave (a) if the wave is a rarefaction, (b) if the wave is a contact discontinuity and (c) if the wave is a shock.

A **rarefaction wave** connects the variables ahead and behind the wave through a smooth transition. A rarefaction wave has a fan structure, centered at the origin, as depicted in Fig. (5.3a). All flow quantities vary

continuously across the wave, at any fixed time. Across the bounding characteristics corresponding to the head and tail, however, all flow variables have a discontinuity in the x -derivative.

A **contact discontinuity** is a wave across which the variables vary discontinuously. The characteristics on both sides of the wave run parallel to the wave path (Fig. 5.3b).

A **shock wave** is also a discontinuity. It has a compressive character: the characteristics ahead and behind the shock wave run into the shock path (Fig. 5.3c).

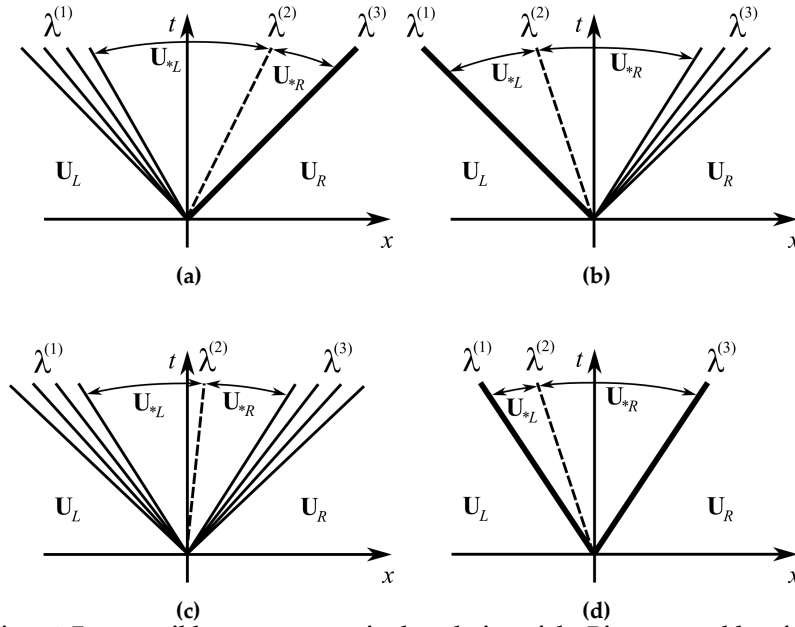


Fig. 5.4: Four possible wave patterns in the solution of the Riemann problem for the x-split two-dimensional shallow water equations.

Then there are 4 possible wave patterns in the Riemann problem, as illustrated in Fig. (5.4). The variables at the left of the left wave are written U_L and the variables at the right of the right wave are written U_R . The region between the right and left waves is referred to as the star region. The variables in the star region that are at the left of the middle wave are called U_{*L} and the ones that are in the star region at the right of the middle wave are called U_{*R} .

5.2.4 Wave relations

The Rankine-Hugoniot jump conditions apply to a discontinuous wave travelling with speed S ; which is related to jumps in conserved variables $\mathbf{U}$ and fluxes $\mathbf{F}(\mathbf{U})$ across the wave as follows :

If

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = 0, \quad (5.21)$$

then

$$\mathbf{F}_{ahead} - \mathbf{F}_{behind} = S(\mathbf{U}_{ahead} - \mathbf{U}_{behind}), \quad (5.22)$$

where S is the shock wave speed.

The generalized Riemann invariants are relations that hold true across a simple wave³ (a wave along which $\mathbf{U}$ is constant). They are defined as

$$d\mathbf{w} = \mathbf{R}^{-1}d\mathbf{U} \quad (5.23)$$

where $\mathbf{R}$ is the matrix of eigenvectors of $\mathbf{A}(\mathbf{U})$. It is possible to show that this definition leads to (Guinot, 2006):

$$\frac{dU_1}{R_1^{(i)}} = \dots = \frac{dU_m}{R_m^{(i)}}. \quad (5.24)$$

Where $R_j^{(i)}$ is the j^{th} component of the eigenvector associated to the wave i . For the shallow water equations, the eigenvectors associated to the eigenvalues $\lambda^{(1)} = V_x - c$, $\lambda^{(2)} = V_x$ and $\lambda^{(3)} = V_x + c$ are:

$$\mathbf{R}^{(1)} = \begin{bmatrix} 1 \\ V_x - c \\ V_y \end{bmatrix}, \quad \mathbf{R}^{(2)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{R}^{(3)} = \begin{bmatrix} 1 \\ V_x + c \\ V_y \end{bmatrix}, \quad (5.25)$$

respectively. Based on these considerations, we can deduce the relations associated to each wave of the Riemann problem (Tables 5.1 and 5.2). To have the detail on how to obtain the wave relations of Table (5.1), see Appendix D.

³ Real-world problems involve very seldom simple waves, which limits the practical utility of the generalized Riemann invariants.

Possible wave type	Left wave		Middle wave	Right wave	
	Rarefaction	Shock	Shear	Rarefaction	Shock
Generalized Riemann invariants	$\frac{dh}{h} = \frac{dU}{-c} = \frac{dV}{0}$ $\Leftrightarrow \begin{cases} U + 2c = cst \\ V = cst \end{cases}$	$\frac{dh}{h} = \frac{dU}{-c} = \frac{dV}{0}$ $\Leftrightarrow \begin{cases} U - 2c = cst \\ V = cst \end{cases}$	$\frac{dh}{0} = \frac{dU}{0} = \frac{dV}{1}$	$\frac{dh}{h} = \frac{dU}{c} = \frac{dV}{0}$ $\Leftrightarrow \begin{cases} U - 2c = cst \\ V = cst \end{cases}$	
Rankine-Hugoniot		$\mathbf{F}(\mathbf{U}_L) - \mathbf{F}(\mathbf{U}_L) = S(\mathbf{U}_L - \mathbf{U}_L)$	$\mathbf{F}(\mathbf{U}_R) - \mathbf{F}(\mathbf{U}_L) = S(\mathbf{U}_R - \mathbf{U}_L)$		$\mathbf{F}(\mathbf{U}_R) - \mathbf{F}(\mathbf{U}_R) = S(\mathbf{U}_R - \mathbf{U}_R)$
Relations between $\mathbf{U}_R$ and $\mathbf{U}_*$ $\mathbf{U}_L$ and $\mathbf{U}_*$	$U_* + 2c_* = U_L + 2c_L$	$U_* = U_L - (h_* - h_L) \sqrt{\frac{1}{2} g \left(\frac{h_* + h_L}{h_* h_L} \right)}$	$\begin{cases} h_{*L} = h_{*R} \\ U_{*L} = U_{*R} \\ V_{*L} \neq V_{*R} \end{cases}$	$U_* - 2c_* = U_R - 2c_R$	$U_* = U_R + (h_* - h_R) \sqrt{\frac{1}{2} g \left(\frac{h_* + h_R}{h_* h_R} \right)}$

Table 5.1: Possible wave relations, depending on whether the left and right waves are shocks or rarefaction waves (for details of how to obtain the wave relations, see Appendix D).

	Rarefaction wave	Shear wave	Shock wave
Variable transition	Smooth	Discontinuous	Discontinuous
Wave relations	Generalized Riemann invariants	Gen. R. invariants Rankine-Hugoniot	Rankine-Hugoniot
Characteristic field	$\lambda(\mathbf{W}_L) < S < \lambda(\mathbf{W}_R)$	$\lambda(\mathbf{W}_L) = S = \lambda(\mathbf{W}_R)$	$\lambda(\mathbf{W}_L) > S > \lambda(\mathbf{W}_R)$

Table 5.2: Features of each type of wave of the Riemann problem (S is the wave velocity).

5.2.5 Exact solution

It is recalled that the left and right waves are either a shock or a rarefaction wave. The middle wave is always a contact discontinuity. The determination of the wave type is part of the solution. Across the right and left waves, both h and V_x change, but V_y remains constant. The relation between the variables on both sides of the waves is given by the generalized Riemann invariants (if the wave is a rarefaction wave) or the Rankine-Hugoniot relation (if the wave is a shock wave). Across the middle wave, h and V_x remain constant but V_y changes discontinuously.

The types of the right and left waves are determined by the following conditions:

$$h_* > h_L : \quad \text{left wave is a shock wave} \quad (5.26a)$$

$$h_* \leq h_L : \quad \text{left wave is a rarefaction wave} \quad (5.26b)$$

$$h_* > h_R : \quad \text{right wave is a shock wave} \quad (5.26c)$$

$$h_* \leq h_R : \quad \text{right wave is a rarefaction wave} \quad (5.26d)$$

The basic step is to derive an equation for the water depth h_* in the star region. This will be achieved by connecting $\mathbf{U}_*$ to data on the left and to data on the right via functions governing the relations of flow quantities across the left and right waves respectively. These relations are the ones listed in the last line of Table (5.2) and are derived in Appendix D. By putting them together, we eliminate V_{x*} and find a relation in which h_* is the only unknown:

$$f(h_*) \equiv f_L(h_*, h_L) + f_R(h_*, h_R) + \Delta U = 0, \quad \Delta U \equiv U_R - U_L \quad (5.27)$$

where the function f_L and f_R are

$$f_L = \begin{cases} 2(\sqrt{gh_*} - \sqrt{gh_L}) & \text{if } h_* \leq h_L \text{ rarefaction} \\ (h_* - h_L) \sqrt{\frac{1}{2} g \left(\frac{h_* + h_L}{h_* h_L} \right)} & \text{if } h_* > h_L \text{ shock} \end{cases} \quad (5.27a)$$

$$f_R = \begin{cases} 2(\sqrt{gh_*} - \sqrt{gh_R}) & \text{if } h_* \leq h_R \text{ rarefaction} \\ (h_* - h_R) \sqrt{\frac{1}{2} g \left(\frac{h_* + h_R}{h_* h_R} \right)} & \text{if } h_* > h_R \text{ shock} \end{cases} \quad (5.27b)$$

There is no general closed-form solution of the non-linear algebraic Eq. (5.27) for the depth in the star region. This equation is therefore solved numerically by an iterative process. Once h_* has been solved, V_{x*} is determined by one of the Eqs. (D.3), (D.7), (D.15) or (D.18) (listed in the last line of Table 5.1).

5.3 Approximate solvers (with porosity)

The previous section was devoted to describe the main steps in the derivation of the exact solution of the Riemann problem for the homogeneous shallow water equations without porosity (Eq. 5.12). The objective was to introduce the mathematical concepts underlying the Riemann problem and the wave structure of the solution. Understanding these concepts, even for the equations without porosity, will help understanding the approximate solvers for the equations with porosity (Eq. 5.1).

Three approximate solvers for the equations with porosity (Eq. 5.1) exist in the literature (Guinot and Soares-Frazão, 2006, Finaud-Guyot et al., 2010, L. Cea and. Vázquez-Cendón (2010)). In this section, two of them are discussed. The first solver is the one derived by Guinot and Soares-Frazão (2006) and consists in assuming that both left and right waves are discontinuities. The expression for the flux is similar to a HLL solver except that it includes the porosity parameter. The second one is the one developed by Finaud-Guyot et al. (2010), consists in assuming that both left and right waves are rarefaction waves. These two solvers will be compared later in Section 5.4.

5.3.1 Modified HLL solver

5.3.1.a WAVE PATTERNS

The flux at the interface between two adjacent cells is computed by assuming that both left and right waves are shocks and the contact discontinuity is ignored, like for the usual HLL solver without porosity.

There are three possible wave patterns (WP). They are schematized in Fig. (5.5): in the first wave pattern (WP1) the flow is directed from left to right and it is supercritical; in the second one (WP2) the flow is either directed from left to right or from right to left and is subcritical; in the third one (WP3) the flow is directed from right to left and is supercritical.

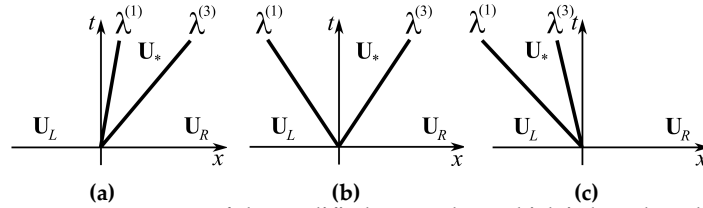


Fig. 5.5: Wave patterns of the modified HLL solver which is based on the assumption that both waves are shocks: (a) supercritical flow with the fluid flowing from left to right, (b) subcritical flow with the fluid flowing from left to right or from right to left and (c) supercritical flow with the fluid flowing from right to left.

The flux $\mathbf{F}_{i+1/2}^{n+1/2}$ in Eq. (5.11) is the flux at $x/t = 0$ and has a different expression depending on the wave pattern:

$$\begin{aligned} \text{WP1} \quad & \text{if } \lambda^{(1)} > 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \mathbf{F}_L \end{aligned} \tag{5.28}$$

$$\begin{aligned} \text{WP2} \quad & \text{if } \lambda^{(1)} < 0 \quad \& \quad \lambda^{(3)} > 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \mathbf{F}_*^{HLL} \end{aligned} \tag{5.29}$$

$$\begin{aligned} \text{WP3} \quad & \text{if } \lambda^{(3)} < 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \mathbf{F}_R \end{aligned} \tag{5.30}$$

For both supercritical wave patterns WP1 and WP3, the flux is equal to the left or right flux, respectively. In subcritical flow (WP2), the flux at the

interface is equal to the flux $\mathbf{F}_*^{\text{HLL}}$ in the star region between both rarefaction waves. The calculation of the star region is the object of Section 5.3.1.b.

5.3.1.b FLUX

It is assumed that a one-dimensional Riemann problem has been defined in the local coordinate system $(\hat{x}, \hat{y})$ attached to the interface (Fig. 5.1) as explained in Section 5.1. In the HLL solver, the second momentum equation is not considered (which explains that there is no contact discontinuity in the solution). The equations to be solved in the direction normal to the interface reduce to

$$\frac{\partial \hat{\mathbf{U}}}{\partial t} + \frac{\partial \mathbf{F}(\hat{\mathbf{U}})}{\partial \hat{x}} = \mathbf{S}_g \quad (5.31)$$

(in which it is recalled that $\mathbf{S}_n$ is neglected), with

$$\hat{\mathbf{U}} = \begin{bmatrix} \phi h \\ \phi \hat{V}_x h \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \phi \hat{V}_x h \\ \phi \hat{V}_x^2 h + \phi \frac{gh^2}{2} \end{bmatrix},$$

$$\mathbf{S}_g = \begin{bmatrix} 0 \\ -\phi gh \frac{\partial z_b}{\partial \hat{x}} + \frac{gh^2}{2} \frac{\partial \phi}{\partial \hat{x}} \end{bmatrix} \quad (5.32)$$

where $\hat{x}$ is the coordinate normal to the interface and $\hat{\mathbf{U}}$ is the vector of variables in this coordinate system.

To establish the flux expression, the source term $\mathbf{S}_g$ is neglected because it will be accounted for in another step of the computation, as it will be explained later. Then the system to solve reduces to:

$$\frac{\partial \hat{\mathbf{U}}}{\partial t} + \frac{\partial \mathbf{F}(\hat{\mathbf{U}})}{\partial \hat{x}} = 0 \quad (5.33)$$

The mass and x -momentum fluxes in the intermediate region derive from the Rankine-Hugoniot relations associated with the $\lambda^{(1)}$ and $\lambda^{(3)}$ waves for Eq. (5.33):

$$\begin{aligned} \mathbf{F}_*^{\text{HLL}} - \mathbf{F}_R &= \lambda^{(3)} (\hat{\mathbf{U}}_* - \hat{\mathbf{U}}_R) \\ \mathbf{F}_L - \mathbf{F}_*^{\text{HLL}} &= \lambda^{(1)} (\hat{\mathbf{U}}_L - \hat{\mathbf{U}}_*) \end{aligned} \quad (5.34)$$

Eliminating $\hat{\mathbf{U}}_*$ in Eq. (5.34) gives

$$\mathbf{F}_*^{HLL} = \frac{1}{\lambda^{(3)} - \lambda^{(1)}} \left[\lambda^{(3)} \mathbf{F}(\hat{\mathbf{U}}_L) - \lambda^{(1)} \mathbf{F}(\hat{\mathbf{U}}_R) + \lambda^{(1)} \lambda^{(3)} (\hat{\mathbf{U}}_R - \hat{\mathbf{U}}_L) \right], \quad (5.35)$$

where $\hat{\mathbf{U}}$ is redefined as

$$\hat{\mathbf{U}} = \begin{bmatrix} \phi z_w \\ \phi \hat{V}_x h \end{bmatrix} \quad (5.36)$$

where the water level $z_w = z_b + h$ replaces h in the first component of Eq. (5.36) to ensure equilibrium when the water is at rest (Nujic, 1995). The wave celerities $\lambda^{(1)}$ and $\lambda^{(3)}$ are the same as for the shallow water equations without porosity and are approximated by:

$$\begin{aligned} \lambda^{(1)} &= \min(\hat{V}_{xL} - c_L, \hat{V}_{xR} - c_R, 0) \\ \lambda^{(3)} &= \max(\hat{V}_{xL} + c_L, \hat{V}_{xR} + c_R, 0) \end{aligned} \quad (5.37)$$

The celerities $\lambda^{(1)}$ and $\lambda^{(3)}$ are bounded by zero to allow the validity of Eq. (5.35) to be extended to supercritical conditions.

It is now shown how and why the mass flux given by the first component of Eq. (5.35) must be corrected. From Eq. (5.35), the mass flux may be written as:

$$\begin{aligned} (\phi \hat{V}_x h)_{i+1/2}^{n+1/2} &= \frac{1}{\lambda^{(3)} - \lambda^{(1)}} \left[\lambda^{(3)} \phi_L \hat{V}_{xL} h_L - \lambda^{(1)} \phi_R \hat{V}_{xR} h_R \right. \\ &\quad \left. + \lambda^{(1)} \lambda^{(3)} (\phi_R \hat{V}_{xR} z_{wR} - \phi_L \hat{V}_{xL} z_{wL}) \right] \end{aligned} \quad (5.38)$$

In steady flow and for a horizontal bottom, the mass flux expression given by Eq. (5.38) would not authorize the water level to vary spatially, which is unrealistic:

$$\text{Following Eq. (5.38):} \begin{cases} \text{if } \begin{cases} \phi_L \hat{V}_{xL} h_L = \phi_R \hat{V}_{xR} h_R = (\phi V_x h)_{i+1/2}^{n+1/2} \\ z_{bL} = z_{bR} \end{cases} \\ \text{then } z_{wL} = z_{wR} \rightarrow \text{not realistic!} \end{cases} \quad (5.39)$$

It is then proposed to modify the mass flux expression given by Eq. (5.38) into:

$$(\phi \hat{V}_x h)_{i+1/2}^{n+1/2} = \frac{\phi_{LR}}{\lambda^{(3)} - \lambda^{(1)}} \left[\lambda^{(3)} \hat{V}_{xL} h_L - \lambda^{(1)} \hat{V}_{xR} h_R + \lambda^{(1)} \lambda^{(3)} (\hat{V}_{xR} z_{wR} - \hat{V}_{xL} z_{wL}) \right] \quad (5.40)$$

in which ϕ_{LR} is estimated by

$$\phi_{LR} \equiv \min(\phi_L, \phi_R) \quad (5.41)$$

This estimation is based on the consideration that the mass exchange is driven by the narrower of the two sections. It has the additional advantage that it accounts automatically for the fact that the water cannot flow into a region of zero porosity.

The transverse momentum flux is calculated with the velocity $\hat{V}_{yL}$ if the velocity $\hat{V}_x$ at the interface is positive and with the velocity $\hat{V}_{yR}$ if the velocity $\hat{V}_x$ at the interface is negative. The following formula accounts for both situations:

$$(\phi h \hat{V}_x \hat{V}_y)_{i,j} = \frac{\phi h \hat{V}_{xi,j} + |\phi h \hat{V}_{xi,j}|}{2} \hat{V}_{yL} + \frac{\phi h \hat{V}_{xi,j} - |\phi h \hat{V}_{xi,j}|}{2} \hat{V}_{yR} \quad (5.42)$$

5.3.1.c SOURCE TERM

The geometrical source term S_g (bottom slope and porosity gradient, defined by Eq. 5.1b) is computed separately from the non-geometrical source term S_n (friction, drag and dispersion, defined by Eq. 5.1b). The source term S_g is evaluated at the interfaces in the same step as the fluxes, and then it is distributed between the two adjacent cells according to the wave celerities. This multi-step algorithm is called the *splitting* method and the distribution of the source term following the celerities is called the *source terms upwinding*. Upwinding the source terms is important to guarantee stability to the solution. In the present case, it allows one to respect the steady-flow condition by imposing that in steady flow and when $S_n = 0$, the source terms S_g must exactly compensate the fluxes. Therefore, the discretization of the source term S_g is conditioned by the Riemann solver used to compute the fluxes.

With the splitting method and the source terms upwinding, the finite-volume discretization given by Eq. (5.11) of two neighboring cells i and j is rewritten as

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \Delta t \sum_{j \in N(i)} \mathbf{T}_{i,j} \left[\frac{w_{i,j}}{A_i} \mathbf{F}_{i,j}^{n+1/2}(\hat{\mathbf{U}}) + (\mathbf{S}_{g,i})_{i,j}^{n+1/2} \right] + \Delta t \mathbf{S}_{n,i}^{n+1/2} \quad (5.43a)$$

$$\mathbf{U}_j^{n+1} = \mathbf{U}_j^n + \Delta t \sum_{i \in N(j)} \mathbf{T}_{i,j} \left[\frac{w_{i,j}}{A_j} \mathbf{F}_{i,j}^{n+1/2}(\hat{\mathbf{U}}) + (\mathbf{S}_{g,j})_{i,j}^{n+1/2} \right] + \Delta t \mathbf{S}_{n,j}^{n+1/2} \quad (5.43b)$$

It can be seen in Eqs. (5.43) that the part of $\mathbf{S}_g$ retrieved from the cell i , $(\mathbf{S}_{g,i})_{i,j}^{n+1/2}$, is not the same as the part of $\mathbf{S}_g$ added to the cell j , $(\mathbf{S}_{g,j})_{i,j}^{n+1/2}$.

These two latter read respectively:

$$(\mathbf{S}_{g,i})_{i,j}^{n+1/2} = \frac{w_{i,j}}{A_i} \left[\frac{0}{-\lambda^{(1)}} \right] (\Delta S_g)_{ij} \quad (5.44a)$$

$$(\mathbf{S}_{g,j})_{i,j}^{n+1/2} = \frac{w_{i,j}}{A_i} \left[\frac{0}{\lambda^{(3)}} \right] (\Delta S_g)_{ij} \quad (5.44b)$$

where $(\Delta S_g)_{ij}$ is the integral of the source term $\mathbf{S}_g$ across the interface and is written in the form

$$(\Delta S_g)_{ij} = -g(\phi h)_{LR}(z_{wR} - z_{wL}) + \frac{g}{2}(\phi_R h_R^2 - \phi_L h_L^2). \quad (5.45)$$

Eq. (5.45) arises from the observation that the source term $\mathbf{S}_g$ may be rewritten as:

$$S_g = -\phi g h \frac{\partial z_b}{\partial \hat{x}} + \frac{g h^2}{2} \frac{\partial \phi}{\partial \hat{x}} = -\phi g h \frac{\partial z_w}{\partial \hat{x}} + g \frac{\partial}{\partial \hat{x}} \left(\frac{\phi h^2}{2} \right) \quad (5.46)$$

In Eq. (5.45), $(\phi h)_{LR}$ has still to be estimated. Indeed, as the parameters and variables are defined inside the cells, their values at the interfaces are not known. Here, they are expressed as a function of the values in the neighboring cells based on some considerations in steady flow for a one-dimensional flow. It must be noticed that in a one-dimensional problem, the shallow water equations with porosity given by Eq. (5.31) are the same equations as the classical shallow water equations in a one-dimensional rectangular flume whose width would be ϕ (Fig. 5.6). The porosity may then be considered as the effective width of the computing cell available for the flow.

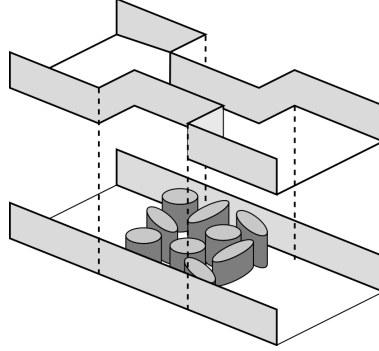


Fig. 5.6: Analogy between a rectangular flume and a flume occupied by obstacles.
This analogy comes from the observation that the governing equations are the same in both cases, the porosity substituting the flume width.

Based on the comparison with a one-dimensional flume, we may say that, in steady flow and for a one-dimensional problem:

- (i) The discharge in each cell is equal to the prescribed discharge:

$$(\phi \hat{V}_x h)_L = (\phi \hat{V}_x h)_R = q. \quad (5.47)$$

Note that Eq. (5.47) is valid in 1D but does not hold true anymore in 2D.

- (ii) The momentum balance may be formulated as:

$$\frac{\partial \Sigma}{\partial \hat{x}} = S_g \quad (5.48)$$

$$\text{with } \Sigma = \phi \hat{V}_x^2 h + \phi \frac{gh^2}{2} \quad (5.49)$$

where Σ is called the specific force. Eq. (5.48) means that the difference of specific force Σ between two adjacent cells differs if the source term is not equal to zero. Conversely, if there is no porosity or bottom level discontinuity at the interface between two adjacent cells, Σ is equal in both cells because in this case $S_g = 0$. Taking into account Eq. (5.45), Eq. (5.48) may be discretized as:

$$\begin{aligned} \Sigma_R - \Sigma_L &= (\Delta S_g)_{LR} \\ &= -g(\phi h)_{LR}(z_{wR} - z_{wL}) + \frac{g}{2}(\phi_R h_R^2 - \phi_L h_L^2) \end{aligned} \quad (5.50)$$

where $(\Delta S_g)_{LR}$ is the discretization of the source term at the interface between the left and right cells and $(\phi h)_{LR}$ is the value of ϕh at the interface that we are looking for.

- (iii) The source term $(\Delta S_g)_{LR}$ of Eq. (5.50) may be seen as a difference of hydrostatic pressure on the interface between two adjacent cells caused by a change in the porosity (Fig. 5.7). This difference of hydrostatic pressure may be expressed as a function of ϕ_L and ϕ_R (the bottom is assumed to be horizontal: $z_{bL} = z_{bR}$). The porosity of the left cell may be either higher or lower than the porosity of the right cell. Two cases have then been considered. This leads to (for more details, the reader is referred to Guinot and Soares-Frazão, 2006):

$$\begin{aligned} \Sigma_R - \Sigma_L &= \frac{gh_L^2}{2} (\min(\phi_L, \phi_R) - \phi_L) \\ &\quad - \frac{gh_R^2}{2} (\min(\phi_L, \phi_R) - \phi_R) \end{aligned} \quad (5.51)$$

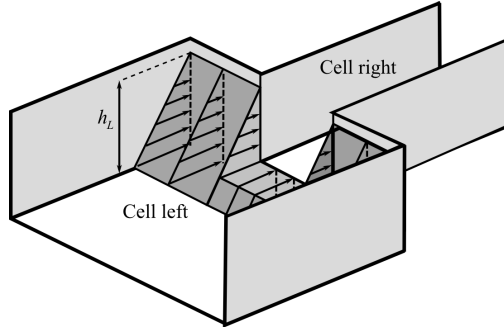


Fig. 5.7: Hydrostatic pressure distribution at the interface between right and left cells in the case that $\phi_L > \phi_R$ (the porosity being analogous to the width of a flume, see Fig. 5.6)

Eqs. (5.47), (5.50) and (5.51) allow to deduce an expression for $(\phi h)_{LR}$:

$$(\phi h)_{LR} = \frac{h_L + h_R}{2} \min(\phi_L, \phi_R) \quad (5.52)$$

5.3.2 PorAS solver

The PorAS solver, developed by Finaud-Guyot et al. (2010), is based on the assumption that both left and right waves are rarefaction waves. 'Por' stands for 'porosity' and 'AS' stands for 'approximate-state'. The geometrical source term S_g is treated in an upwind manner, as for the HLL solver presented in Section 5.3. The general finite volume discretization is then also given by Eq. (5.43).

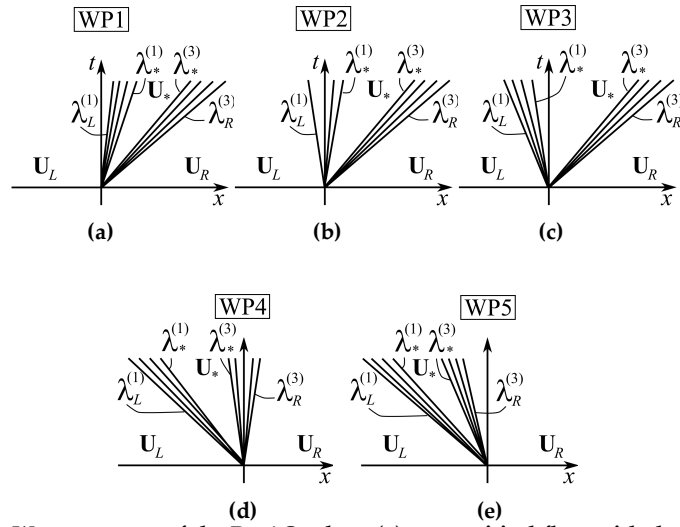


Fig. 5.8: Wave patterns of the PorAS solver: (a) supercritical flow with the fluid flowing from left to right, (b) critical flow with the fluid flowing from left to right, (c) subcritical flow with the fluid flowing from left to right or from right to left, (d) critical flow with the fluid flowing from right to left and (e) supercritical flow with the fluid flowing from right to left.

5.3.2.a WAVE PATTERNS

There are five possible wave patterns (WP). They are schematized in Fig. (5.8): in the first wave pattern (WP1) the flow is directed from left to right and it is supercritical; in the second one (WP2) the flow is directed from left to right and is critical, implying that the head and tail characteristics of the left rarefaction wave are positive and negative, respectively ($\lambda_L^{(1)} < 0$ and $\lambda_*^{(1)} > 0$); in the third one (WP3) the flow is either directed from left to right or from right to left and is subcritical (all the characteristics of the left rarefaction wave are negative and all the characteristics of the right rarefaction wave are positive); in the fourth one (WP4) the flow is directed from right to left and the flow is critical, implying that the head and tail

characteristics of the right rarefaction wave are positive and negative, respectively ($\lambda_*^{(3)} < 0$ and $\lambda_R^{(3)} > 0$); in the fifth one (WP5) the flow is directed from right to left and is supercritical.

The flux $\mathbf{F}_{i+1/2}^{n+1/2}$ in Eq. (5.11) is the flux at $x/t = 0$ and has a different expression for each wave pattern:

$$\begin{aligned} \text{WP1} \quad & \text{if } \lambda_L^{(1)} > 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \mathbf{F}_L \end{aligned} \quad (5.53)$$

$$\begin{aligned} \text{WP2} \quad & \text{if } \lambda_L^{(1)} < 0 \quad \& \quad \lambda_*^{(1)} > 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \frac{1}{\lambda_3 - \lambda_1} \left(\lambda_3 \mathbf{F}_L - \lambda_1 \mathbf{F}_R - \lambda_1 \lambda_3 (\hat{\mathbf{U}}_L - \hat{\mathbf{U}}_R) \right) \end{aligned} \quad (5.54)$$

$$\begin{aligned} \text{WP3} \quad & \text{if } \lambda_*^{(1)} < 0 \quad \& \quad \lambda_*^{(3)} > 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \mathbf{F}_*^{PA} \end{aligned} \quad (5.55)$$

$$\begin{aligned} \text{WP4} \quad & \text{if } \lambda_L^{(1)} < 0 \quad \& \quad \lambda_*^{(1)} > 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \frac{1}{\lambda_3 - \lambda_1} \left(\lambda_3 \mathbf{F}_L - \lambda_1 \mathbf{F}_R - \lambda_1 \lambda_3 (\hat{\mathbf{U}}_L - \hat{\mathbf{U}}_R) \right) \end{aligned} \quad (5.56)$$

$$\begin{aligned} \text{WP5} \quad & \text{if } \lambda_R^{(3)} < 0 \\ & \text{then } \mathbf{F}_{i+1/2}^{n+1/2} = \mathbf{F}_R \end{aligned} \quad (5.57)$$

The expression of $\lambda_*^{(1)}$ and $\lambda_*^{(3)}$ arise from the generalized Riemann invariants across the rarefaction waves. Following the relations of Table (5.2), we may write that

$$\begin{aligned} \hat{V}_{x*} + 2c_* &= \hat{V}_{xL} + 2c_L \\ \hat{V}_{x*} - 2c_* &= \hat{V}_{xR} - 2c_R \end{aligned} \quad (5.58)$$

and solving the system of equations leads to

$$\begin{aligned} \hat{V}_{x*} &= \frac{\hat{V}_{xL} + \hat{V}_{xR}}{2} + c_L - c_R \\ c_* &= \frac{\hat{V}_{xL} - \hat{V}_{xR}}{4} + \frac{c_L + c_R}{2} \end{aligned} \quad (5.59)$$

The celerities λ_1^* and λ_3^* are computed as follows:

$$\begin{aligned}\lambda_*^{(1)} &= \hat{V}_{x*} - c_* \\ \lambda_*^{(3)} &= \hat{V}_{x*} + c_*\end{aligned}\tag{5.60}$$

In the flux expressions of Eqs. (5.54) and (5.56), $\lambda^{(1)}$ and $\lambda^{(3)}$ are not defined by Finaud-Guyot et al. It is supposed here that they are given by Eq. (37) such as for the HLL solver.

For both supercritical wave patterns WP1 and WP5, the flux is equal to the left or right flux, respectively. In subcritical flow (WP3), the flux at the interface is equal to the flux $\mathbf{F}_*^{PA}$ (the superscript 'PA' stands for the solver 'PorAS') in the star region between both rarefaction waves. The calculation of $\mathbf{F}_*^{PA}$ is the object of Section 5.3.2b. The flux expression for both critical flows WP2 and WP4 are in fact the HLL formulation. Finaud-Guyot et al. (2010) justify this by saying that the characteristic relationships across rarefaction waves may be approximated by jump relations, and cite Dukowicz (1985) and Lax (1957) as references.

5.3.2.b FLUX

The numerical fluxes $\mathbf{F}_*^{PA}$ are calculated by solving the Riemann problem in a coordinate system $(\hat{x}, \hat{y})$ attached to the interface:

$$\frac{\partial \hat{\mathbf{U}}}{\partial t} + \frac{\partial \mathbf{F}(\hat{\mathbf{U}})}{\partial \hat{x}} = \mathbf{S}_g\tag{5.61}$$

with

$$\hat{\mathbf{U}} = \begin{bmatrix} \phi h \\ \phi \hat{V}_x h \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \phi \hat{V}_x h \\ \phi \hat{V}_x^2 h + \phi \frac{gh^2}{2} \end{bmatrix}, \mathbf{S}_g = \begin{bmatrix} 0 \\ -\phi gh \frac{\partial z_b}{\partial \hat{x}} + \frac{gh^2}{2} \frac{\partial \phi}{\partial \hat{x}} \end{bmatrix}\tag{5.62}$$

Note that, contrarily to what was done in the modified HLL scheme in Section 5.3.1b, the source term $\mathbf{S}_g$ is not neglected while deriving the flux expression. In the following, the characteristic relations of the system of Eqs. (5.61) will be established and then used to compute $\mathbf{F}_*^{PA}$. To do so, Finaud-Guyot et al. (2010) show that by some algebraic manipulations, the system of Eqs. (5.61) is equivalent to:

$$\frac{\partial \mathbf{W}}{\partial t} + \mathbf{\Lambda} \frac{\partial \mathbf{W}}{\partial \hat{x}} = \mathbf{\Lambda} \mathbf{S}'\tag{5.63}$$

where

$$d\mathbf{W} = \frac{1}{2c} \begin{pmatrix} \lambda_3 dF_1 - dF_2 \\ -2cVdF_1 + 2cdF_3 \\ -\lambda_1 dF_1 + dF_2 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \hat{V}_x - c & 0 & 0 \\ 0 & \hat{V}_x & 0 \\ 0 & 0 & \hat{V}_x + c \end{pmatrix} \quad (5.64a)$$

$$\mathbf{S}' = \frac{1}{2c} \begin{pmatrix} -\frac{gh^2}{2} \frac{\partial \phi}{\partial \hat{x}} \\ 0 \\ \frac{gh^2}{2} \frac{\partial \phi}{\partial \hat{x}} \end{pmatrix} \quad (5.64b)$$

The system of Eqs. (5.63) leads to the following characteristic relations (which are in fact the Riemann invariant⁴):

$$\frac{dW_k}{dt} = \sum_{j=1}^3 \Lambda_{kj} S'_j \quad \text{along} \quad \frac{dx}{dt} = \lambda_k, \quad k=1,2,3 \quad (5.65)$$

where subscript k corresponds to the k^{th} component of the vector in Eq. (5.64). Eqs (5.65) are used to calculate $\mathbf{F}_*^{PA}$, the fluxes at the interface ($\hat{x} = 0$) at time step $n+1/2$. Indeed, if we consider the three characteristics leading to the point $(0, n+1/2)$ (point M in Fig. 5.9) and originating either from the left side or from the right side of the interface, the system of Eqs. (5.65) leads to the following set of equations:

⁴ Note that the *Riemann invariants* should not be confused with the *generalized Riemann invariants* presented in Section 5.2.4: the Riemann invariants hold true *along* characteristic curves while the generalized Riemann invariants hold true *across* characteristic curves if these latter are simple waves.

for $\frac{dx}{dt} = \lambda_1 :$

$$\lambda_3 (F_{*1}^{PA} - F_{1R}) - (F_{*2}^{PA} - F_{2R}) = \frac{-\lambda_1}{\lambda_3 - \lambda_1} \frac{g(h_{LR})^2}{2} (\phi_R - \phi_L)$$

for $\frac{dx}{dt} = \lambda_2 :$

$$-V(F_{*1}^{PA} - F_{1A}) + (F_{*3}^{PA} - F_{3A}) = 0$$

for $\frac{dx}{dt} = \lambda_3 :$

$$-\lambda_1 (F_{*1}^{PA} - F_{1L}) + (F_{*2}^{PA} - F_{2L}) = \frac{\lambda_3}{\lambda_3 - \lambda_1} \frac{g(h_{LR})^2}{2} (\phi_R - \phi_L) \quad (5.66)$$

with

$$F_{iA} = \begin{cases} F_{iL} & \text{if } \lambda_2 > 0 \\ F_{iR} & \text{if } \lambda_2 < 0 \end{cases} \quad (5.67)$$

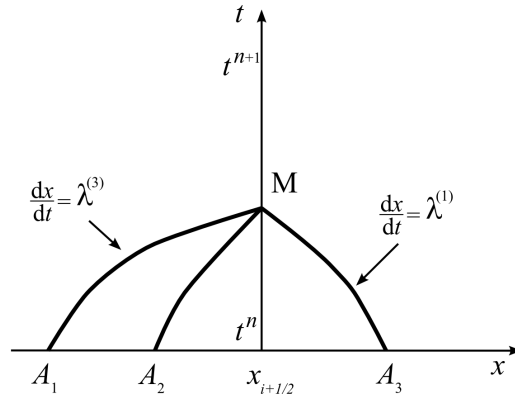


Figure 5.9: Definition sketch for the integration path along the characteristic

In Eq. (5.66), h_{LR} is the water depth at the interface and F_{*i}^{PA} are the fluxes at the interface. Eqs. (5.66) give the solution for F_{*i}^{PA} :

$$\begin{aligned}
F_{*1}^{PA} &= \frac{1}{\lambda_3 - \lambda_1} (F_{2L} - F_{2R} - \lambda_1 F_{1L} + \lambda_3 F_{1R}) \\
&\quad + \frac{1}{\lambda_3 - \lambda_1} \left(\frac{g(h_{LR}^{(1)})^2}{2} (\phi_R - \phi_L) - g(\phi h)_{LR}^{(1)} (z_{bR} - z_{bL}) \right) \\
F_{*2}^{PA} &= \frac{1}{\lambda_3 - \lambda_1} (\lambda_3 F_{2L} - \lambda_1 F_{2R} - \lambda_1 \lambda_3 (F_{1L} - F_{1R})) \\
&\quad + \frac{(\lambda_1 + \lambda_3)}{\lambda_3 - \lambda_1} \left(\frac{g(h_{LR}^{(2)})^2}{2} (\phi_R - \phi_L) - g(\phi h)_{LR}^{(2)} (z_{bR} - z_{bL}) \right) \\
F_{*3}^{PA} &= V(F_1^* - F_{1A}) + F_{3A}
\end{aligned} \tag{5.68}$$

In Eq. (5.68), the variables h and ϕh at the interface are written as $(h)_{LR}$ and $(\phi h)_{LR}$, respectively. The values intervening in the expression of F_{*1}^{PA} are noted with superscript (1) and the ones intervening in the expression of F_{*2}^{PA} are noted with superscript (2). These variables are not defined at the interface; therefore an expression as a function of the right and left states has to be established for each of them. This expression depends on the discretization of the source term and therefore it will be presented in the next section.

5.3.2.c SOURCE TERM

The expression for the fluxes (Eq. 5.68) takes the source terms $\mathbf{S}_g$ into account along the characteristics. However, it must be stated that the source terms must still be evaluated in the conservation step ($\mathbf{S}_i^{n+1/2}$ in Eq. (5.11) or, if the source terms are split, $(\mathbf{S}_{g,i})_{i,j}^{n+1/2}$ in Eq. 5.43). Indeed, the flux expressions (Eq. 5.68) only take into account the influence of the source terms on the characteristics. This does not replace the finite-volume integration of the source term over the computational cell $\mathbf{S}\Delta t$ (Fig. 5.1).

For the PorAS scheme, the source terms are split, just like for the modified HLL scheme. The general finite-volume discretization is then also given by Eq. (5.43) (for an explanation of this equation, the reader is referred to Section 5.3.1c):

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \Delta t \sum_{j \in N(i)} \mathbf{T}_{i,j} \left[\frac{w_{i,j}}{A_i} \mathbf{F}_{i,j}^{n+1/2}(\hat{\mathbf{U}}) + (\mathbf{S}_{g,i})_{i,j}^{n+1/2} \right] + \Delta t \mathbf{S}_{n,i}^{n+1/2} \quad (5.43a)$$

$$\mathbf{U}_j^{n+1} = \mathbf{U}_j^n + \Delta t \sum_{i \in N(j)} \mathbf{T}_{i,j} \left[\frac{w_{i,j}}{A_j} \mathbf{F}_{i,j}^{n+1/2}(\hat{\mathbf{U}}) + (\mathbf{S}_{g,j})_{i,j}^{n+1/2} \right] + \Delta t \mathbf{S}_{n,j}^{n+1/2} \quad (5.43b)$$

And, as for the modified HLL solver, the part of $\mathbf{S}_g$ retrieved from the cell i , $(\mathbf{S}_{g,i})_{i,j}^{n+1/2}$, is not the same as the part of $\mathbf{S}_g$ added to the cell j , $(\mathbf{S}_{g,j})_{i,j}^{n+1/2}$:

$$(\mathbf{S}_{g,i})_{i,j}^{n+1/2} = \frac{w_{i,j}}{A_i} \left[\begin{array}{c} 0 \\ -\lambda^{(1)} \\ \lambda^{(3)} - \lambda^{(1)} \end{array} \right] (\Delta S_g)_{ij} \quad (5.44a)$$

$$(\mathbf{S}_{g,j})_{i,j}^{n+1/2} = \frac{w_{i,j}}{A_i} \left[\begin{array}{c} 0 \\ \lambda^{(3)} \\ \lambda^{(3)} - \lambda^{(1)} \end{array} \right] (\Delta S_g)_{ij} \quad (5.44b)$$

where $(\Delta S_g)_{ij}$ differ from the one used in the modified HLL solver in Section 5.3.1c and is given by:

$$(\Delta S_g)_{ij} = -g(\phi h)_{LR}^{(S)}(z_{wR} - z_{wL}) + \frac{g}{2} (h_{LR}^{(S)})^2 (\phi_R - \phi_L). \quad (5.69)$$

Eq. (5.69) arises directly from the expression of $\mathbf{S}_g$ given by Eq. (5.62). Notice that the interface value of h and ϕh also intervenes in the source term expression (Eq. 5.69) and are noted with the superscript (S). An expression for them as a function of the left and right states has also to be established.

Finaud-Guyot et al. derive the expressions for $(h)_{LR}^{(1)}$, $(\phi h)_{LR}^{(1)}$, $(h)_{LR}^{(2)}$, $(\phi h)_{LR}^{(2)}$, $(h)_{LR}^{(S)}$ and $(\phi h)_{LR}^{(S)}$ by artificially forcing the set of Eqs. (5.68) and Eq. (5.69) to verify steady-flow conditions listed hereafter (a similar procedure has been followed in Section 5.3.1c for the modified HLL solver). Note that by doing this, the variables $(h)_{LR}^{(1)}$, $(h)_{LR}^{(2)}$ and $(h)_{LR}^{(S)}$ may be different from each other, and $(\phi h)_{LR}^{(1)}$, $(\phi h)_{LR}^{(2)}$ and $(\phi h)_{LR}^{(S)}$ may also be different from each other. If we consider a one-dimensional flow that is comparable to a rectangular flume (Fig. 5.6), the steady-flow conditions are:

- (i) The discharge in each cell is equal to the prescribed discharge:

$$F_{1L} = F_{1R} = F_{*1}^{PA} = q \quad (5.69)$$

- (ii) The momentum balance may be formulated as:

$$\frac{\partial \Sigma}{\partial \xi} = S_g \quad (5.70)$$

where the specific force Σ is defined in Eq. (5.49). Eq. (5.70) may be discretized as:

$$\begin{aligned} \Sigma_R - \Sigma_L &= (\Delta S_g)_{LR} \\ &= -g(\phi h)_{LR}^{(S)}(z_{bR} - z_{bL}) + \frac{g(h_{LR}^{(S)})^2}{2}(\phi_R - \phi_L) \end{aligned} \quad (5.71)$$

where $(\Delta S_g)_{LR}$ is the discretization of the source term (Eq. 5.69) at the interface between the left and right cells.

- (iii) The source term $(\Delta S_g)_{LR}$ of Eq. (5.71) may be seen as a difference of hydrostatic pressure on the interface between two adjacent cells caused by a change in the porosity (Fig. 5.7) and a change in the bottom level (the difference of bottom level was neglected in Section 5.3.1c). This difference of hydrostatic pressure may be expressed as a function of ϕ_L , ϕ_R , z_{bL} and z_{bR} . The porosity of the left cell may be either higher or lower than the porosity of the right cell, and the bottom level of the left cell may also be higher or lower than the bottom level of the right cell. Four cases have then been considered. This leads to (for more details, the reader is referred to Finaud-Guyot et al., 2010):

$$\begin{aligned} \Sigma_R - \Sigma_L &= \frac{g(h_{LR}^{(0)})^2}{2}(\phi_L - \phi_R) \\ &\quad + g\phi_{LR}^{(0)}(z_{bR} - z_{bL})\left(z_{LR}^{(0)} - \frac{z_{bR} + z_{bL}}{2}\right) \end{aligned} \quad (5.72)$$

where

$$h_{LR}^{(0)} = \begin{cases} h_L & \text{if } \phi_L > \phi_R \\ h_R & \text{if } \phi_L < \phi_R \end{cases} \quad (5.73a)$$

$$z_{LR}^{(0)} = \begin{cases} z_L & \text{if } z_{bL} < z_{bR} \\ z_R & \text{if } z_{bL} > z_{bR} \end{cases} \quad (5.73b)$$

$$\phi_{LR}^{(0)} = \min(\phi_L, \phi_R). \quad (5.73c)$$

Note that if $z_{bL} = z_{bR}$, Eq. (5.72) is equivalent to its counterpart Eq. (5.51) found in Section 5.3.1c.

- (iv) If we consider a computational cell i surrounded by the cells $i-1$ and $i+1$, and if we take into account that in steady flow, $\mathbf{U}_i^{n+1} = \mathbf{U}_i^n$, the second component of the finite-volume discretization (Eq. 5.43) writes as:

$$\left(F_{*2}^{PA} + \frac{\lambda^{(3)}}{\lambda^{(3)} - \lambda^{(1)}} \Delta S_g \right)_{i-1/2}^{n+1/2} - \left(F_{*2}^{PA} + \frac{\lambda^{(1)}}{\lambda^{(3)} - \lambda^{(1)}} \Delta S_g \right)_{i+1/2}^{n+1/2} \quad (5.74)$$

By combining the conditions (i) (Eq. 5.69) and (iii) (Eq. 5.72), after some manipulations, it results that (for the details, see Finaud-Guyot et al., 2010)

$$(\phi h)_{LR}^{(1)} = \phi_{LR}^{(0)} \left(z_{LR}^{(0)} - \frac{z_{bR} + z_{bL}}{2} \right) \quad (5.75)$$

$$h_{LR}^{(1)} = h_{LR}^{(0)}. \quad (5.76)$$

Then, by combining the conditions (ii) (Eq. 5.71) and (iii) (Eq. 5.72), it results that

$$(\phi h)_{LR}^{(S)} = (\phi h)_{LR}^{(1)} \quad (5.77)$$

$$h_{LR}^{(S)} = h_{LR}^{(1)}. \quad (5.78)$$

Finally, by combining the conditions (ii) (Eq. 5.71), (iii) (Eq. 5.72) and (iv) (Eq. 5.74), it results that

$$(\phi h)_{LR}^{(2)} = 0 \quad (5.79)$$

$$h_{LR}^{(2)} = 0. \quad (5.80)$$

Now that all the interface values of h and ϕh have been determined, the PorAS solver is completely defined.

5.4 Test cases

The modified HLL and PorAS solvers have been validated against various test cases, in dam break flow (Guinot and Soares-Frazão, 2006; Finaud-Guyot et al., 2010) and in steady flow (Finaud-Guyot et al., 2010). In the following, the main conclusions of the test cases in dam break flow are briefly recalled (Section 5.4.1). Then, we focus on the steady flow and show how the reference solution is derived (Section 5.4.2) and discuss the results (Section 5.4.3).

5.4.1 Dam break flow

The test cases in dam break in Guinot and Soares-Frazão (2006) and Finaud-Guyot et al. (2010) flow include:

- (i) A one-dimensional dam break flow where the porosity is linear: $\phi(x) = 0.01 x$. The reference solution for this test case is given by the results along an axes of a 2D simulation of a circular dam break without porosity. Indeed, it was found that the porosity equations for $\phi(x) = kx$ (where k is a constant) are equivalent to the shallow water equations without porosity written in polar coordinates.
- (ii) A one-dimensional dam break flow where the porosity is discontinuous: $\phi = 1$ in the reservoir (in which the initial water depth is $h_{0L} = 10$ m) and $\phi = 0.5$ downstream from the dam (in which the initial water depth was $h_{0R} = 1$ m). The reference solution is given by the exact Riemann solver.
- (iii) A two-dimensional circular dam break flow for which the porosity is given by:

$$\phi(x, y) = \frac{\phi_{\max}}{\max(r, r_0)} \quad (5.81)$$

where r_0 is the center of the circular dam, r is the distance from this center and $\phi_{\max} = 1$. The reference solution for this test case is given by the one-dimensional classical shallow water equations without porosity. Indeed, it may be shown that the 2D porosity equations with the porosity distribution given by Eq. (5.81) written in cylindrical coordinates are equivalent to the 1D shallow water equations without porosity.

- (iv) A two-dimensional unsteady flow on a laboratory scale model of the Italian Toce valley at CESI (Italy). The model, the cross-section of which is roughly trapezoidal, is 7 m long and 3.5 - 5 m wide depending on the location. Square blocks representing buildings were placed in the scale model. The porosity model was compared with experimental data as well as with a very refined solution given by the classical model without porosity. The porosity model gave good results provided that the head losses due to the buildings were correctly calibrated.

Finaud-Guyot et al. (2010) compared both solvers in all the abovementioned test cases and observed that, besides the fact that both solvers use very different expressions for the fluxes and the source terms, they give similar results. Still, small discrepancies can be highlighted. In dam break, the PorAS solver presents an erroneous slight depression where the flow is critical. This may be explained by the fact that for critical flow, the solver is based on a shock relation (Eqs. 5.54 and 5.56).

5.4.2 Reference solution in steady flow

Here we derive two reference solutions, both being equivalent. The first one is the Bernoulli equation, and the second one is the one used by Finaud-Guyot et al. (2010) to validate their PorAS solver. The reference solution in one dimension in steady flow is obtained by writing the shallow water equations with porosity where the y -derivative and the time-derivatives cancel out and where $V_y = 0$:

$$\frac{\partial(\phi V_x h)}{\partial x} = 0 \quad (5.82)$$

$$\frac{\partial \Sigma}{\partial x} = S \quad (5.83)$$

where the specific force Σ is defined as:

$$\Sigma = \phi V_x^2 h + \phi \frac{gh^2}{2} \quad (5.84)$$

and

$$S = -\phi gh \frac{\partial z_b}{\partial x} + \frac{gh^2}{2} \frac{\partial \phi}{\partial x} \quad (5.85)$$

As it was already highlighted in Section 5.3.1c, Eqs. (5.82) and (5.83) are equivalent to the shallow water equations for a flow in a one-dimensional flume of rectangular cross section, where the width of the cross section would be ϕ . Eqs. (5.82) and (5.83) may be rewritten as:

$$\frac{\partial Q}{\partial x} = 0 \quad (5.86)$$

$$\frac{\partial}{\partial x} \left(\frac{Q^2}{A} + \phi \frac{gh^2}{2} \right) = -gA \frac{\partial z_b}{\partial x} + \frac{gh^2}{2} \frac{\partial \phi}{\partial x} \quad (5.87)$$

where we have defined

$$\begin{cases} Q = \phi V_x h \\ A = \phi h \end{cases} \quad (5.88)$$

Note that because of Eq. (5.86), Q is constant and may be put outside the x -derivative. Eq. (5.87) may be developed as:

$$\frac{\partial}{\partial x} \left(\frac{Q^2}{A} + \phi \frac{gh^2}{2} \right) = -\phi gh \frac{\partial z_b}{\partial x} + \frac{gh^2}{2} \frac{\partial \phi}{\partial x} \quad (5.89a)$$

$$\Leftrightarrow \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + gA \frac{\partial h}{\partial x} = -gA \frac{\partial z_b}{\partial x} \quad (5.89b)$$

$$\Leftrightarrow \frac{1}{gA} \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x} \quad (5.89c)$$

$$\Leftrightarrow -\frac{Q^2}{gA} \frac{1}{A^2} \frac{\partial A}{\partial x} + \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x} \quad (5.89d)$$

$$\Leftrightarrow \frac{Q^2}{2g} \frac{\partial}{\partial x} \left(\frac{1}{A^2} \right) + \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x} \quad (5.89e)$$

$$\Leftrightarrow \frac{\partial}{\partial x} \left(\frac{Q^2}{2gA^2} + h + z_b \right) = 0 \quad (5.89f)$$

Eq. 5.89f is the first reference solution. It is the reference that will be used in the present thesis. It is the Bernoulli equation and it expresses the invariance of the head H :

$$H \equiv \frac{Q^2}{2gA^2} + h + z_b \quad (5.90)$$

It may be discretized in the following way:

$$\Leftrightarrow \left(\frac{Q^2}{2gA^2} + h + z_b \right)_{i+1} = \left(\frac{Q^2}{2gA^2} + h + z_b \right)_i \quad (5.91)$$

where the only unknown is h_{i+1} as Q is a data of the problem.

Finaud-Guyot et al. (2010) do not use the Bernoulli Eq. (5.91) but another one. We will show here that this other equation may also be derived from the momentum Eq. (5.89a) and is consequently equivalent to the Bernoulli equation. We start from the fourth step of the development, Eq. (5.89d):

$$-\frac{Q^2}{gA} \frac{1}{A^2} \frac{\partial A}{\partial x} + \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x} \quad (5.92a)$$

$$\Leftrightarrow -\frac{V_x^2}{g(\phi h)} \frac{\partial(\phi h)}{\partial x} + \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x} \quad (5.92b)$$

$$\Leftrightarrow -\frac{V_x^2}{g(\phi h)} \left(h \frac{\partial \phi}{\partial x} + \phi \frac{\partial h}{\partial x} \right) + \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x} \quad (5.92c)$$

$$\Leftrightarrow \left(1 - \frac{V_x^2}{gh} \right) \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x} + h \frac{V_x^2}{\phi gh} \frac{\partial \phi}{\partial x} \quad (5.92d)$$

$$\Leftrightarrow \frac{\partial h}{\partial x} = -\frac{1}{\left(1 - \frac{V_x^2}{gh} \right)} \frac{\partial z_b}{\partial x} + h \frac{V_x^2}{\phi (gh - V_x^2)} \frac{\partial \phi}{\partial x} \quad (5.92e)$$

The reference solution given by Eq. (5.92e) is discretized in the following way if we assume that there is no bottom slope ($\partial z_b / \partial x = 0$):

$$h_{i+1} = h_i + h_i \frac{V_{x,i}^2}{\phi (gh_i - V_{x,i}^2)} (\phi_{i+1} - \phi_i). \quad (5.93)$$

Finaud-Guyot et al. (2010) do not specify whether ϕ in Eq. (5.93) is equal to ϕ_i or ϕ_{i+1} , or a combination of both. For both reference solutions, the flow has to be computed from upstream to downstream (supercritical flow) or from downstream to upstream (subcritical flow). Note that Eqs. (5.89f) and (5.92e)

are equivalent to the strong formulation given by Eqs. (5.82-83) and assume that U and F are derivable and then continuous. However, in the presence of a discontinuity such as a hydraulic jump, strong formulations cannot be used anymore; the flow must be computed by the weak formulation given by Eq. (5.2) (or equivalently Eq. 5.11) that is more general. As a consequence, Eq. (5.89f) will be used as the reference solution where the flow is continuous and in the presence of a hydraulic jump, the equality of specific force will be used to compute its position.

5.4.3 Results in steady flow

We now compare both solvers HLL and PorAS in one-dimensional steady test cases. This comparison has been done in Finaud-Guyot et al. (2010), where the comparison of h and $V_x h$ has been made with the reference solution. In the present thesis, we also show $\phi V_x h$ and H that are supposed to be constant (except H in the presence of a discontinuity), and Σ . Three test cases are examined: a subcritical flow, a supercritical flow and a transcritical flow. The transcritical flow consists of a flow which is subcritical upstream, then passes through a control section to reach a supercritical regime, followed by a hydraulic jump (Fig. 5.13). The porosity distribution is the same for the three test cases. It is given by:

$$\phi(x) = \begin{cases} ax^2 + bx + c & \text{for } x \in [x_m - L_0; x_m + L_0] \\ \phi_0 & \text{elsewhere} \end{cases} \quad (5.95)$$

$$a = \frac{\phi_0 - \phi_m}{L_0^2}, \quad b = -2ax_m, \quad c = \phi_0 + a(x_m^2 - L_0^2) \quad (5.96)$$

The parameters used are the following:

$$\begin{aligned} \phi_0 &= 1 \\ \phi_m &= 0.6 \\ L_0 &= 25 \text{ m} \\ x_m &= 50 \text{ m} \end{aligned} \quad (5.97)$$

The porosity profile is shown in Fig. (5.10). For all test cases, the upstream boundary condition is a constant discharge of $Q = 1 \text{ m}^3/\text{s}$. For the subcritical test case, the downstream boundary condition is a constant water level of $h = 1 \text{ m}$. For the supercritical case, the upstream water level is prescribed to

$h = 0.2$ m. For the transcritical test case, a water level of $h = 0.811$ m is prescribed as a downstream boundary condition.

Fig. (5.11) shows the comparison between the modified HLL and PorAS solvers and the reference solution for the subcritical case.

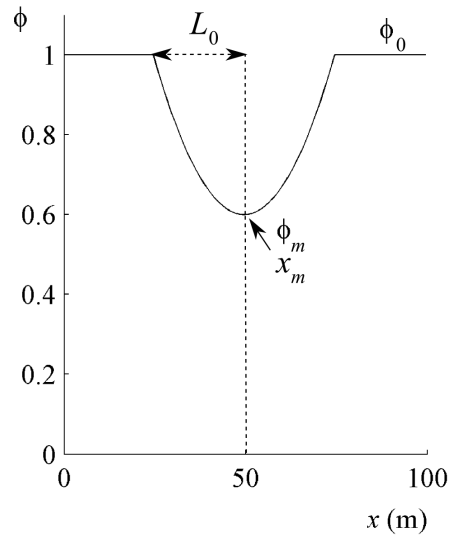


Fig. 5.10: Porosity distribution for the steady-flow test cases

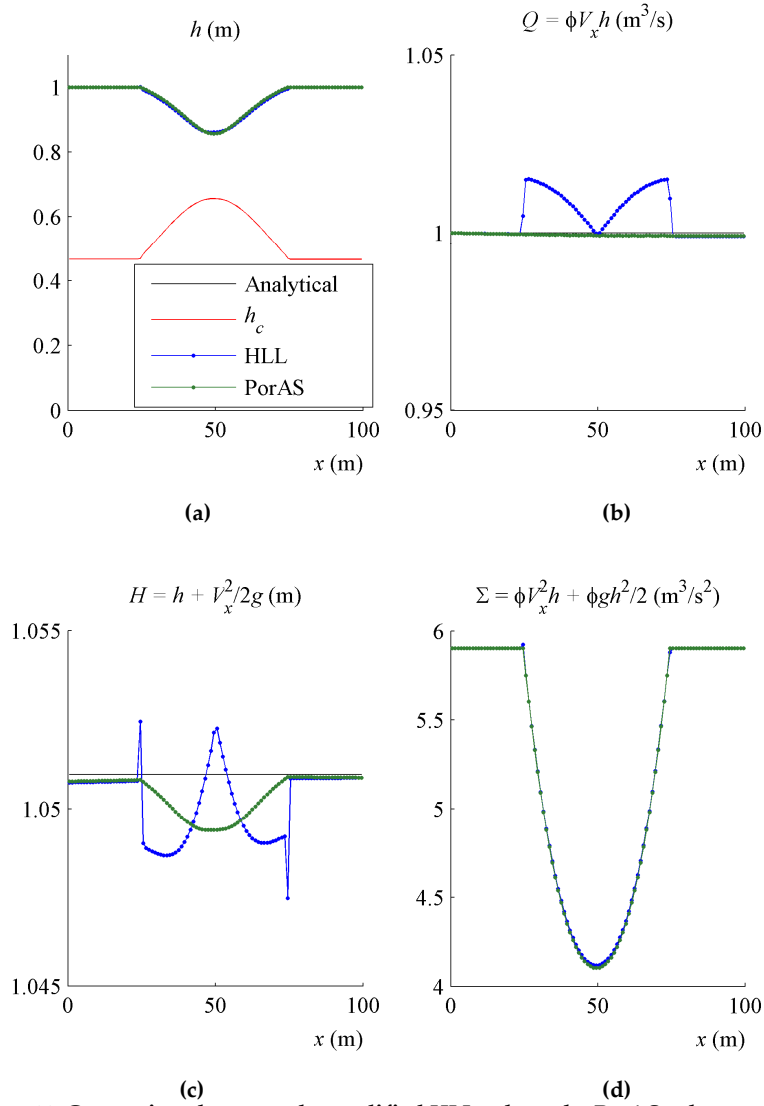


Fig. 5.11: Comparison between the modified HLL solver, the PorAS solver and the reference solution in subcritical flow with $\Delta x = 1$ m: (a) water depth, (b) discharge, (c) head and (d) specific force.

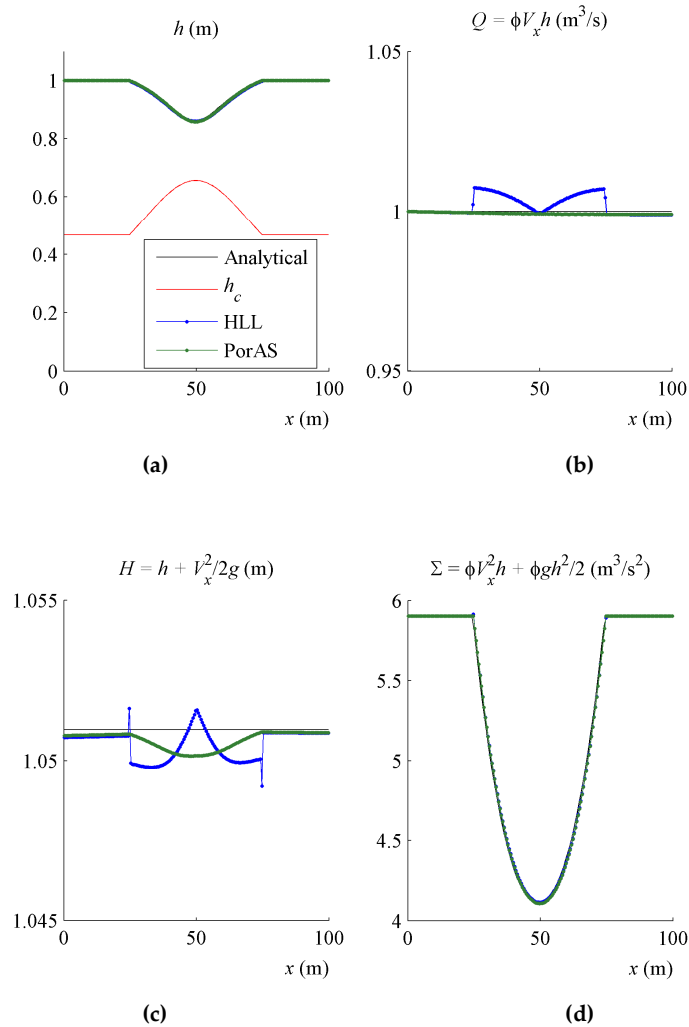


Fig. 5.12: Comparison between the modified HLL solver, the PorAS solver and the reference solution in subcritical flow with $\Delta x = 0.5$ m: (a) water depth, (b) discharge, (c) head and (d) specific force.

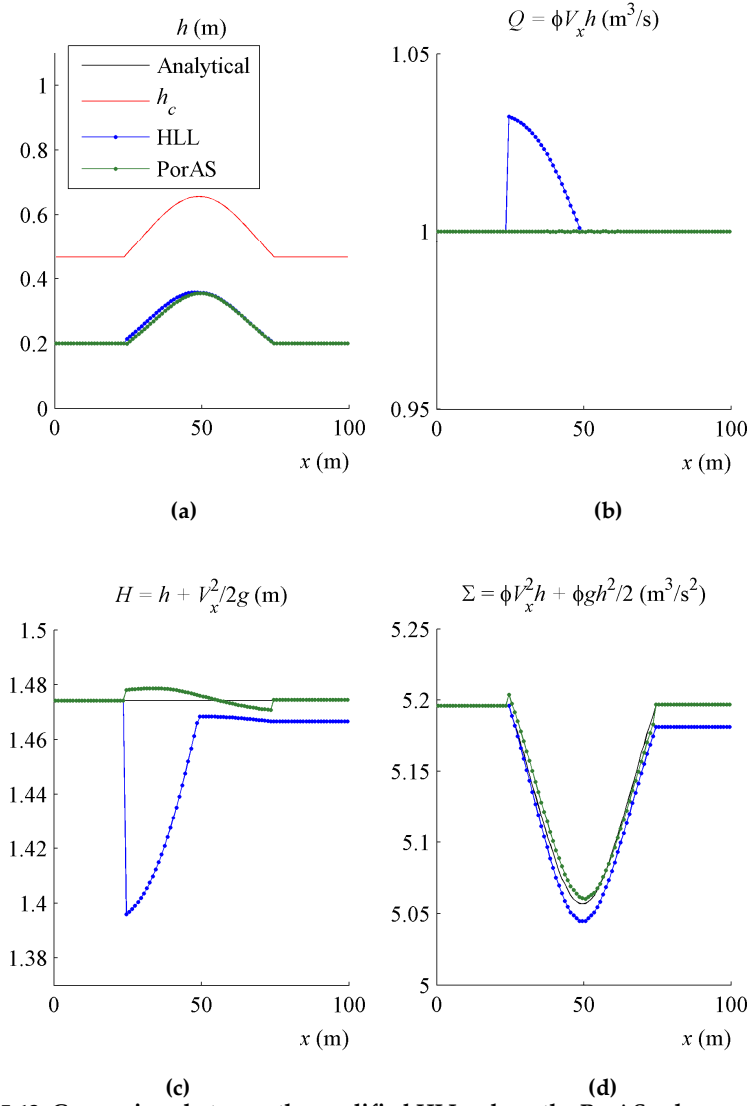


Fig. 5.13: Comparison between the modified HLL solver, the PorAS solver and the reference solution in supercritical flow: (a) water depth, (b) discharge, (c) head and (d) specific force.

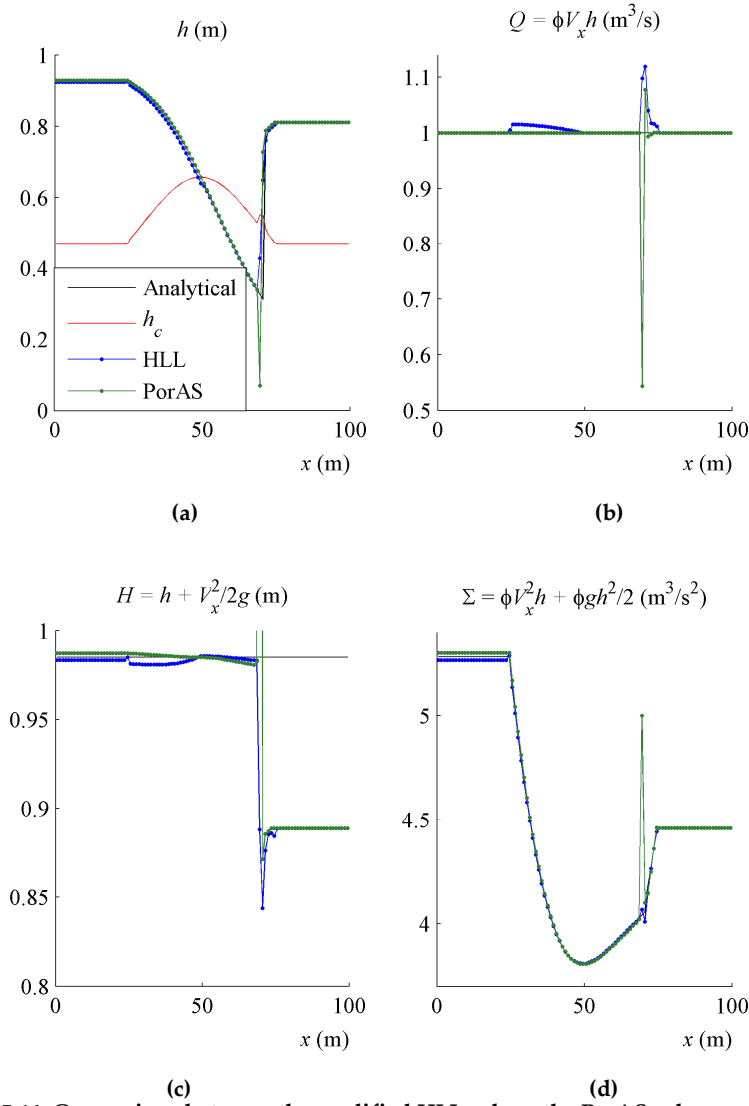


Fig. 5.14: Comparison between the modified HLL solver, the PorAS solver and the reference solution in transcritical flow: (a) water depth, (b) discharge, (c) head and (d) specific force.

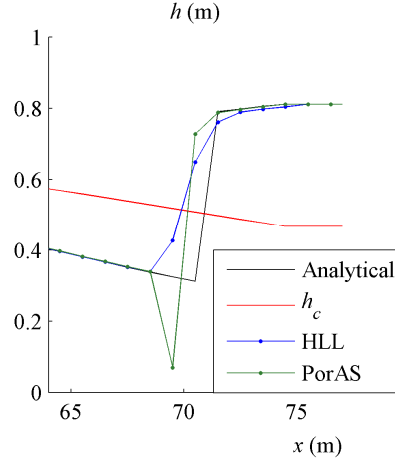


Fig. 5.15: Detail of Fig. 5.13a showing the water depth calculated by both solvers in the neighborhood of a hydraulic jump.

Fig. (5.11) show the results for the subcritical flow. In Fig. (5.11a), it can be seen that both solvers give similar results for the computed water depth, and that these latter are similar to the reference solution. Note that the water level is the same upstream and downstream the change of porosity, as predicted by the reference solution. The critical depth h_c is also shown in the graph. Fig. (5.11b) shows that the discharge $Q = \phi h V_x$ that is supposed to be constant, is well computed by the PorAS solver but not by the HLL solver. This can be explained by the mass flux formula. Indeed, the mass flux of the PorAS solver was derived based on the condition that:

$$(\phi V_x h)_i = (\phi V_x h)_{i+1} = F_{1,i+1/2} = q. \quad (5.98)$$

Such a condition is not respected by the mass flux formula of the HLL solver, which explains that the discharge ($Q_i = \phi V_x h$) computed by the HLL solver is not constant. However, one may question whether it is important for Q_i to be invariant. Indeed, it is a chance that in our case the second component of the conserved variable $\mathbf{U}$, Q_i , is equal to the flux of the first component of $\mathbf{U}$, $Q^*_{i+1/2}$. Some researchers consider that the one that has to be constant is the flux $Q^*_{i+1/2}$, but no such requirement has to be imposed on the conserved variable Q_i (Petaccia et al., 2009). Fig. (5.11c) illustrates that none of the solvers gives a constant head H in the portion of the domain where the porosity varies. The graph of the reference solution for the specific force Σ (Fig. 5.11d) illustrates that the specific force is the same upstream and downstream of the change of porosity. There is no loss of specific force through the porous portion of the domain.

Fig. (5.12) shows the results in the same conditions than for the subcritical flow but with a finer discretization ($\Delta x = 0.5$ m in Fig. (5.12) while $\Delta x = 1$ m in Fig. (5.11)), and it can be seen that the errors on Q (Fig. 5.12b) and H (Fig. 5.12c) reduce with refinement, leading to the conclusion that both solvers tend to the exact solution.

The results for the supercritical flow are shown in Fig. (5.13). Once again, the results of the computed water depth are very similar, and very close to the reference solution. Fig. (5.13b) illustrates the computed discharge and shows that the discharge computed with the PorAS solver is constant all over the domain. The discharge computed with the HLL solver, on the contrary, is not constant when the porosity decreases. This can be explained by the mass flux expression:

$$F_{*1}^{HLL} = \frac{\phi_{LR}}{\lambda^{(3)} - \lambda^{(1)}} \left[\lambda^{(3)} \hat{U}_L h_L - \lambda^{(1)} \hat{U}_R h_R + \lambda^{(1)} \lambda^{(3)} (\hat{U}_R z_{wR} - \hat{U}_L z_{wL}) \right] \quad (5.40)$$

In supercritical case, $\lambda^{(3)} = V_x + c$ and $\lambda^{(1)} = 0$ (by Eq. 5.37), and the mass flux Given by Eq. (5.40) simplifies into:

$$F_{*1}^{HLL} = \phi_{LR} [\hat{U}_L h_L] \quad (\text{HLL, supercritical flow}) \quad (5.99)$$

On the other hand, according to the assumed wave pattern (Fig. 5.5a), the flux must be equal to the left flux in supercritical flow. Following the mass flux Eq. (5.41), it is the case if $\phi_L < \phi_R$ (the porosity increases) but not if $\phi_L > \phi_R$ (the porosity decreases). This explains why the discharge is invariant when the porosity increases but not if the porosity decreases. Fig. (5.13c) shows that none solver preserve the invariance of the head H through the porosity variation and that the HLL solver behaves worse than the PorAS solver. Fig. (5.13d) shows that the PorAS solver conserves the specific force between the upstream and the downstream section, but the HLL solver does not.

Fig. (5.14) show the results for the transcritical flow. In Fig. (5.14a), the computed water depth presents erroneous peaks in the vicinity of the hydraulic jump that did not appear in the graphs of Finaud-Guyot et al. (2010) This may be due to the fact that in Finaud-Guyot et al. (2010), the results were represented only every 10th cell in the graph for the sake of clarity and the erroneous points were not shown. In the vicinity of the hydraulic jump, the PorAS solver presents an erroneous point $h = 0.069$ m (Fig. 5.15) while the HLL solver behaves better. Both solvers present erroneous peaks in the discharge in the vicinity of the hydraulic jump (Fig. 14b). The head is not invariant either, and both solvers compute a sharp

head loss at the place of the hydraulic jump (Fig. 14c), as expected. As the porosity ϕ varies little in the vicinity of the hydraulic jump, this head loss can be compared with the theoretical head loss in a rectangular channel of constant width:

$$\Delta H_{th} = \frac{(h_2 - h_1)^3}{4h_1h_2} = \frac{(0.79 - 0.34)^3}{4 \cdot 0.79 \cdot 0.34} = 0.085 \text{ m} \quad (5.100)$$

The head loss obtained in the test case is then higher than the theoretical one:

$$\Delta H_{1,2} = H_1 - H_2 = 0.981 - 0.888 = 0.093 \text{ m} > \Delta H_{th}. \quad (5.101)$$

Fig. (5.14d) shows that an erroneous peak in the specific force appears in the vicinity of the hydraulic jump.

5.5 Conclusion

Three approximate-state Riemann solvers exist for the shallow water equations with porosity (Guinot and Soares-Frazão, 2006, Finaud-Guyot et al., 2010, Cea and Vázquez-Cendón, 2010). In this chapter, two of these solvers have been presented: the solver developed by Guinot and Soares-Frazão (2006) based on the assumption that the two waves of the Riemann problem are shocks and the solver developed by Finaud-Guyot et al. (2010) based on the assumption that the waves are rarefactions.

In order to appreciate the simplifications provided by these solvers, this chapter begins by recalling the exact solution for the classical shallow water equations (without porosity). Then, it is explained how this exact solution is simplified in the case of the equations with porosity and on which assumptions the two approximate solvers are built.

The first solver is based on the assumption that the left and right waves of the Riemann problem are shocks. It consists then in an adaptation of the classical HLL flux to the porosity equations, and it is called "modified HLL solver". The second solver is based on the assumption that the left and right waves are rarefaction waves. It is called "PorAS", standing for "PORosity" and "Approximate State". The fact that the two solvers are based on a different wave pattern assumption makes their flux expression different. Another difference is that in the PorAS solver, the source terms are taken

into account in the derivation of the flux expression, which is not the case for the modified HLL solver.

The modified HLL and PorAS solvers have in common to treat the geometrical source term S_g by the splitting method, consisting in evaluating S_g at interface rather than inside the computational cells. The splitting method presents the advantage that in equilibrium conditions (in this case: water at rest or steady flow), the source terms S_g balance exactly the inertia and pressure terms ($\partial \mathbf{U} / \partial t + \partial \mathbf{F} / \partial x$). The difficulty of calculating the source terms at the interface is that the parameters and variables such as porosity and water depth are defined in the cells and not at the interfaces. So an expression for the interface value of h and ϕ is derived as a function of the left and right states by forcing the fluxes and source terms to satisfy some steady-flow conditions.

The two solvers have been compared in dam break and steady flow in Finaud-Guyot et al. (2010). In the present work the main conclusions of the dam break test cases are summarized. In Finaud-Guyot et al. (2010), the test cases in steady flow are compared to a reference solution. This reference solution may be deduced from the shallow water equations with porosity by assuming a steady unidirectional flow without discontinuity. It is shown that it is in fact equivalent to the Bernoulli equation, stating that the hydraulic head must be invariant.

In Finaud-Guyot et al. (2010), the results of the water depth h and the discharge $Q = \phi V_x h$ in steady-flow are compared for both solvers, and it was observed that both solvers gave a very similar water depth. In contrast, it has been highlighted in the present investigation that the PorAS solver gives spurious water depth in the vicinity of a hydraulic jump that was not pointed out in Finaud-Guyot et al. (2010) probably because of the sampling of the results shown in the graphs. The modified HLL solver seemed on the contrary to be more robust. Finaud-Guyot et al. (2010) also showed that the PorAS solver preserves better the invariance of the discharge Q than the HLL solver. In the present work, the hydraulic head H and the specific force Σ are also compared, and it has been observed that none solver give a constant H (even in continuous flow), but it was shown that the error decreased with the refinement of the mesh, leading to the conclusion that both solvers converge to the reference solution.

Chapter 6 Validation

The porosity model was already tested in an infinite homogeneous porous medium in Chapter 3 (Fig. 3.11) for the study of the closure expression. However, in practice, the porous medium is only a part of the computational domain, surrounded by areas where there are no obstacles and where the governing equations are the classical shallow-water equations without porosity. This region free of obstacles will be referred to as the "undeveloped region" in the following. Actually, only one set of equations may be used for the whole domain: indeed, the macroscopic porous shallow water Eqs. (2.30, 2.53) are equivalent to the classical shallow-water equations as far as the porosity is taken equal to one outside the urban area and the head loss parameters due to the presence of the buildings are equal to zero. The value of the porosity in the transition between the urban area and the free fluid region is a difficult problem that has not been investigated yet in the context of urban flooding. Should the porosity vary progressively or discontinuously? Should it have a different value from the porosity inside the porous urban medium? In Section 6.1 different possible ways to model this transition are presented based on the literature on porous media.

These different porosity transitions are tested in Sections 6.2 to 6.4 on test cases that consist mainly in the experimental data described in Chapter 4. The drag/dispersion tensor $\mathbf{D}$ used in the porosity model is the one calibrated in Chapter 3 for layout A (where the streets are all 0.1 m wide) and layout B (where the streets in one direction are 0.135 m wide and in the other direction 0.0675 m wide). We will then see if these calibrations (Eqs. 3.27 and 3.28), made in circular dam break flow (Fig. 3.11), remain valid in steady flow. The results of the porosity model will be compared to the experimental data as well as fully detailed simulations (i.e. simulations with the classical shallow water equations where a refined mesh is used to compute the flow at the street level).

In Sections 6.2, the porosity model is tested in the steady flow. Recall that the steady flow experiments described in Chapter 4 consisted in imposing various discharges through a set of 5×5 buildings, and that obstructing blocks were placed aside to prevent the flow to by-pass the city (Fig. 4.2). The advantage of preventing the flow to by-pass the city was to increase the difference of water level upstream and downstream the city so that the effects of the city were more visible. During the experimental campaign (Chapter 4), it was observed that the water level was sensitive to the

prescribed discharge, to the streets' width and the city orientation. Among others, it was observed that for the aligned layouts (Figs. 4.13, 4.15 and 4.17), the resistance of the city to the flow was higher if the streets were narrower, which is a phenomenon that was already observed in Chapter 3 with the circular dam break experiments. We will then see if the porosity model with the calibrated tensor $\mathbf{D}$ represents correctly this tendency in steady flow.

In Section 6.3, the porosity model is tested in steady flow conditions but without obstructing blocks aside the city in order to assess the impact of these latter on the flow. No experimental data are available for these test cases; the porosity model will only be compared to fully detailed simulations, considered as the reference solution. Only layout A is studied. It is laid out either aligned, skewed by 22.5° or skewed by 45° to the main flow direction.

In Section 6.4, the porosity model will be compared to experimental data in dam break flow.

6.1 Porosity transition at the border

For the development of the porosity equations in Chapter 2, we defined two scales: the local or microscopic scale and the global or macroscopic scale. For the modeling of the interface between a porous medium and a homogeneous fluid, Chandersis and Jamet (2007) introduce a third scale: the mesoscopic scale that is intermediate between the others two. At this scale, the fluid and solid phases are seen as two continua (like at the macroscopic scale), and the parameters like porosity vary strongly but continuously across the interface that is seen as a smooth and diffuse transition (Fig. 6.1). The governing equations are the porosity equations in which the parameters and variables vary smoothly at the interface. On the contrary, at the macroscopic level, the interface is represented as a discontinuity, across which the parameters and variables vary abruptly. This interface is then considered as an internal boundary and requires a boundary condition. So, depending on the considered scale, the porosity distribution can be defined either at a mesoscopic level as a continuous transition, or at the macroscopic level as a discontinuity with a boundary condition.

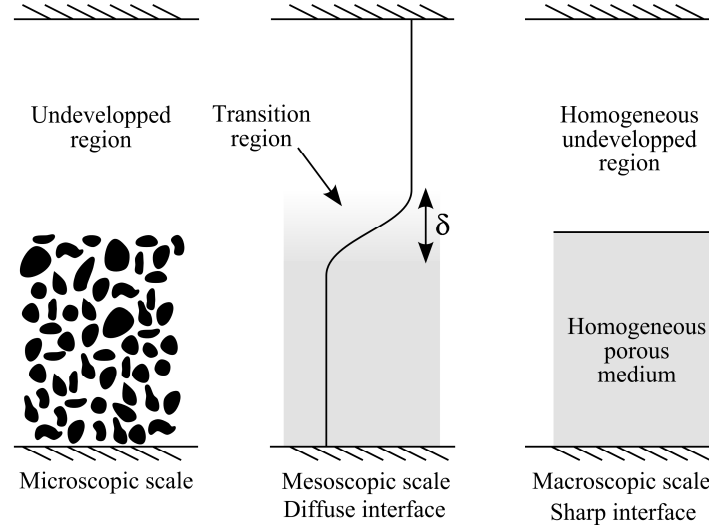


Fig. 6.1: Different scales of representation of the transition region between a porous medium and a medium free of obstacles (undeveloped region): (a) the microscopic scale, at which the pores are explicitly represented; (b) the mesoscopic scale, at which the porous medium is seen as a continuum and the transition is diffuse; and (c) the macroscopic scale, at which the transition is seen as a sharp discontinuity.

Breugem and Boersma (2005) studied the representation of transition region for a periodic medium at the mesoscopic scale. The medium they worked on was constituted by a 3D Cartesian grid of cubes. To evaluate the porosity transition at the border, they calculated the porosity of a cubical EV (elementary volume) displaced across the border. The size of the EV is of one period so that it is not representative of the medium and cannot be called REV. They found that the porosity could be approximated by a 5th-order polynomial function. This porosity distribution was close to the one proposed by Quintard and Whitaker (1994), who computed it by means of a weighting function. It is also close to a linear relation. Ochoa-Tapia and Whitaker (1995) reported that the width of the interfacial region (δ in Fig. 6.1) should be of the order of a particle size (a particle being a building in our context). Based on these considerations, we propose to use in the present study the following linear porosity distribution for the mesoscopic scale of representation (Fig. 6.2)

$$\begin{aligned}
 \phi(x') &= \phi_p \quad \text{if} \quad x' < -\delta/2 \\
 \phi(x') &= \frac{\phi_p + 1}{2} + \frac{1 - \phi_p}{\delta} x' \quad \text{if} \quad -\delta/2 < x' < \delta/2 \\
 \phi(x') &= 1 \quad \text{if} \quad x' > \delta/2
 \end{aligned} \tag{6.1}$$

Where ϕ_p is the porosity of the homogeneous porous medium, x' is the distance to the interface, positive outward the city, and δ is the length of the transition region and is equal to

$$\delta = c + l = 0.3\text{m} + 0.1\text{m} = 0.4\text{m} \quad (\text{layout A}) \quad (6.2a)$$

$$\delta = c + l = 0.3\text{m} + 0.0675\text{m} = 0.3675\text{m} \quad (\text{layout B, narrow streets direction}) \quad (6.2b)$$

$$\delta = c + l = 0.3\text{m} + 0.135\text{m} = 0.435\text{m} \quad (\text{layout B, wide streets direction}) \quad (6.2c)$$

where c is the length of a side of a building and l the size of a street (see Fig. (3.1) and Table (3.1) for the values of c and l for layouts A and B).

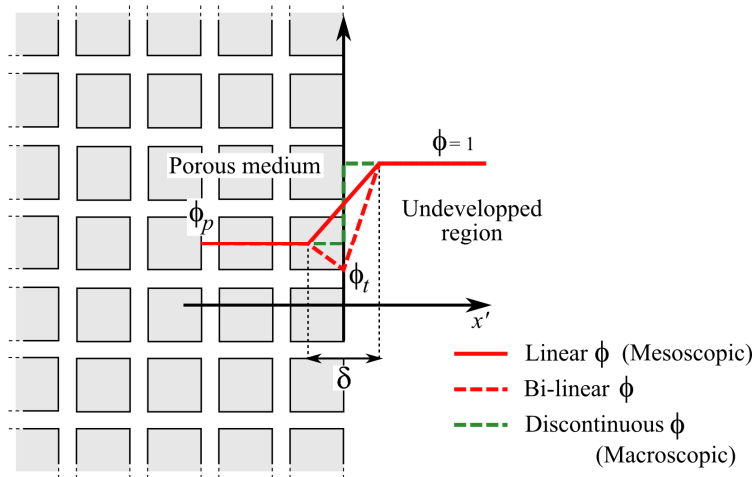


Fig 6.2: Types of tested porosity transitions at the interface between the urban porous medium and the free fluid region: the linear transition (mesoscopic representation), the variable porosity transition on the basis of Ochoa-Tapia and Whitaker (1995), and the discontinuous transition (but without any jump condition) as in the previous works on the porosity model (macroscopic representation).

At the macroscopic scale, the interface is represented as a discontinuous boundary across which must be applied an inner boundary condition linking the parameters and variables of both sides. In the literature, this inner boundary condition is called a jump relation. However, in the present text, we will keep the term "inner boundary condition" to avoid confusion with hydraulic jump that also appears in this context. The inner boundary condition is not easy to derive, and it is still a subject on on-going research. Chandesris and Jamet (2006) derived analytically a relation for laminar flows

with two jump parameters depending on the mesoscopic variation of porosity and permeability in the interface region. For turbulent flows, Chandesris and Jamet (2009) obtained an inner boundary condition in which some quantities were unknown and had to be closed by using the results of pore-scale simulations. All these inner boundary conditions are restricted to the case where the flow is tangent to the porous border. As it is not the case in the present study (in our case the flow could cross the border), we can not apply these relations.

An alternative to the use of a jump condition has been proposed by Ochoa-Tapia and Whitaker (1995), who proposed to adopt a variable porosity model in the interface region. This model relies on the idea that the shape of the REV would become thinner as it would lie closer to the interface, until it becomes only one-particle-thick at the border (Fig. 6.3). Note that in our present case (and in Fig. 6.3), the volume should better be called EV rather than REV because its size is too small to be representative of the medium. Considering an EV that is “deformable” in the interfacial region has repercussion on the value of porosity that is not the same for the thin interfacial EV (ϕ_t) than for the ‘usual’ EV of the homogeneous porous medium (ϕ_p), as illustrated in Fig. (6.3). The variable porosity model has the advantage that it is very simple. The porosity distribution corresponding to this concept is bi-linear, as illustrated in Fig. (6.2), and is given by:

$$\begin{aligned} \phi(x') &= \phi_p \quad \text{if } x' < -\delta/2 \\ \phi(x') &= \phi_t - \frac{2(\phi_p - \phi_t)}{\delta} x' \quad \text{if } -\delta/2 < x' < 0 \\ \phi(x') &= \phi_t + \frac{2(1 - \phi_t)}{\delta} x' \quad \text{if } 0 < x' < \delta/2 \\ \phi(x') &= 1 \quad \text{if } x' > \delta/2 \end{aligned} \quad (6.3)$$

Where ϕ_t is the porosity of the interfacial REV (Fig. 6.3):

$$\phi_t = \frac{l}{c+l} = \frac{0.1}{0.3+0.1} = 0.25 \quad (\text{layout A}) \quad (6.4a)$$

$$\phi_t = \frac{l}{c+l} = \frac{0.0675 \text{ m}}{0.3 \text{ m} + 0.0675 \text{ m}} = 0.1837 \quad (\text{layout B narrow}) \quad (6.4b)$$

$$\phi_t = \frac{l}{c+l} = \frac{0.135 \text{ m}}{0.3 \text{ m} + 0.135 \text{ m}} = 0.3103 \quad (\text{layout B wide}) \quad (6.4c)$$

The porosity ϕ_t is then smaller than ϕ_p , implying that the boundary of the city is more impermeable than its inside part. Lowering the porosity at the

boundary makes the porous medium more difficult to penetrate, the extreme case being $\phi_t = 0$ for a totally impermeable boundary.

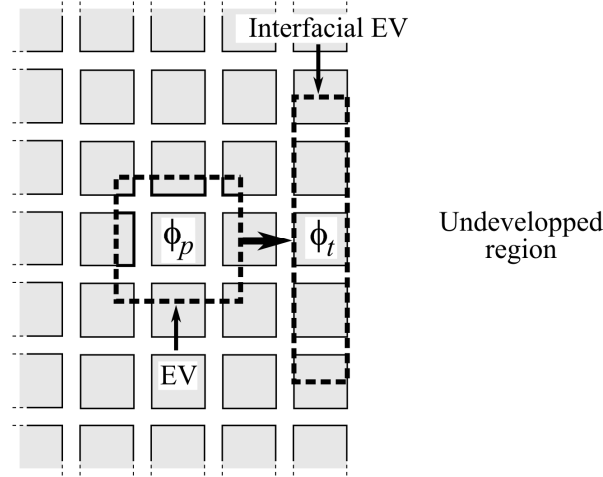


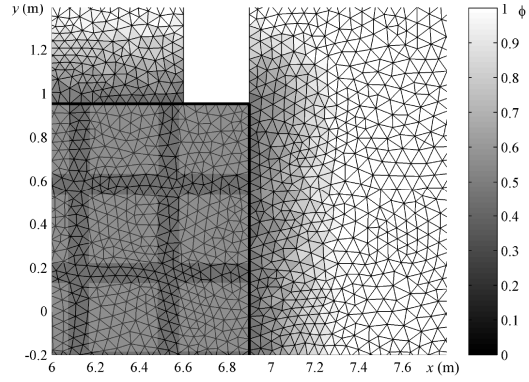
Fig. 6.3: Illustration of the bi-linear porosity concept to represent the transition between the porous medium and the free fluid region. At the border of the porous medium, the EV is considered to be only one-building thick, which has the consequence to change the value of the porosity at the border.

In the previous works about urban porosity, the question of how to represent the porosity at the border has never been raised. The porosity was discontinuous at the interface between the urban area and the undeveloped region. This option is also tested in the present work (Figs. 6.2 and 6.4c).

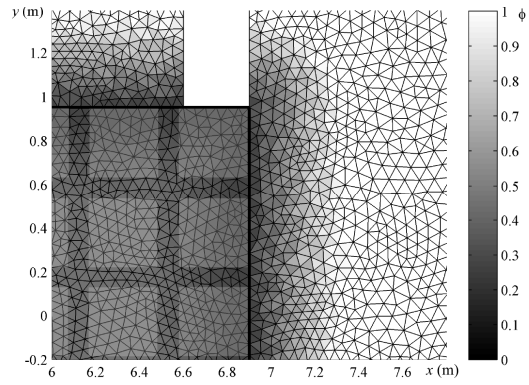
Fig. (6.4a), Fig. (6.4b) and Fig. (6.4c) show the mesh and the porosity transition at the city's border for layout B, assuming a linear transition, a bi-linear transition and a discontinuous transition, respectively. It should be highlighted that none of the three porosity boundary transitions illustrated in Fig. (6.4) guarantees that the integral of the porosity over the domain is equal to the total pore space of the domain:

$$\int_{A_f} \phi dA \neq A_f. \quad (6.5)$$

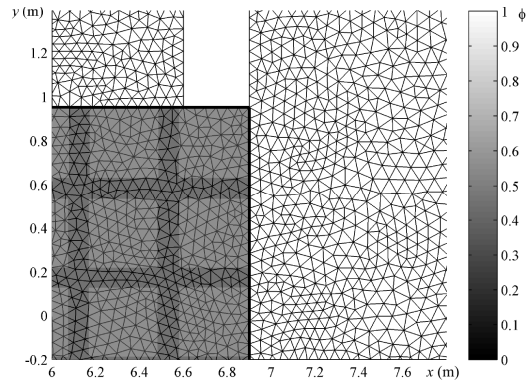
The consequence is that the total mass of water in the system computed with the porosity model will not be equal to the real mass of water computed by the detailed model. Anyway, the error is small and it can be considered to be negligible.



(a)



(b)



(c)

Fig. 6.4: Porosity map and mesh for layout B, assuming (a) a linear porosity transition, (b) a bi-linear porosity transition and (c) a discontinuous porosity transition. The shaded squares represent the buildings as a reference.

6.2 Steady flow with side obstacles

In this section, the porosity model is compared to the steady flow experimental data presented in Chapter 4. The scheme of the five city layouts that were studied is recalled in Fig. (6.5), and the names of the axes of measurement are recalled in Fig. (6.6).

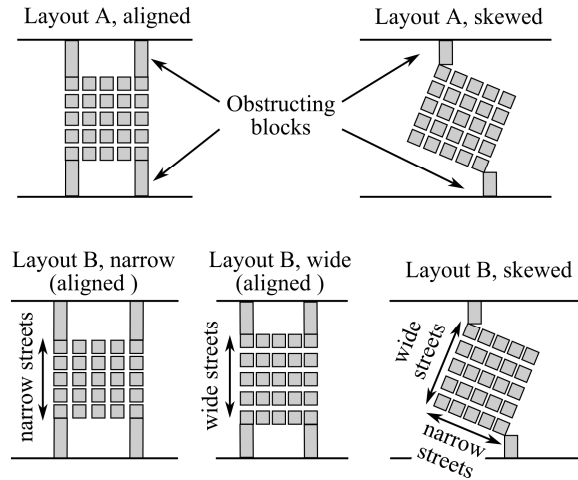


Fig. 6.5: First type of test cases: steady flow with obstructing blocks.

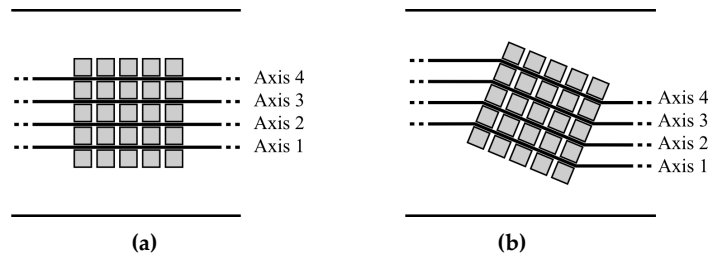


Fig. 6.6: Recall of the names of the axes for (a) the aligned layouts and (b) the skewed layouts.

The objective is to see if the value of $\mathbf{D}$, calibrated in Section 3.3.3, allows reproducing correctly the fact that the resistance that the city opposes to the flow is higher if the water flows in narrower streets. It is recalled that, for layout A (Eq. 3.27):

$$\mathbf{D}_{0^\circ}^A = \begin{pmatrix} \begin{pmatrix} 0.15 & 0 \\ 0 & 1.5 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1.5 & 0 \\ 0 & 0.15 \end{pmatrix} \end{pmatrix} \quad (6.6)$$

where the superscript 'A' stands for layout A and the subscript '0°' stands for the aligned layout (or, which is the same, skewed by 0°). For layout A skewed by 67.5° (Fig. 6.5b), $\mathbf{D}_{0^\circ}^A$ must be multiplied by a matrix of rotation just like in the example shown in Section 3.3.4 for layout B. It gives:

$$\mathbf{D}_{67.5^\circ}^A = \begin{pmatrix} \begin{pmatrix} 0.4875 & 0.3375 \\ 0.3375 & 1.1625 \end{pmatrix} & \begin{pmatrix} 0.3375 & -0.3375 \\ -0.3375 & -0.3375 \end{pmatrix} \\ \begin{pmatrix} 0.3375 & -0.3375 \\ -0.3375 & -0.3375 \end{pmatrix} & \begin{pmatrix} 1.1625 & -0.3375 \\ -0.3375 & 0.4875 \end{pmatrix} \end{pmatrix} \quad (6.7)$$

For layout B with the narrow streets aligned with the axis x , the following calibration value was obtained in Section 3.3.3 (Eq. 3.28):

$$\mathbf{D}_{narr}^B = \begin{pmatrix} \begin{pmatrix} 0.25 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0.5 & 0 \\ 0 & 0.125 \end{pmatrix} \end{pmatrix}. \quad (6.8)$$

If the wide streets are the one aligned with the axis x ,

$$\mathbf{D}_{wide}^B = \begin{pmatrix} \begin{pmatrix} 0.125 & 0 \\ 0 & 0.5 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 0.25 \end{pmatrix} \end{pmatrix}. \quad (6.9)$$

For the skewed layout B (Eq. 3.42),

$$\mathbf{D}_{67.5^\circ}^B = \begin{pmatrix} \begin{pmatrix} 0.2839 & 0.0743 \\ 0.0743 & 0.4326 \end{pmatrix} & \begin{pmatrix} 0.2511 & -0.1406 \\ -0.1406 & -0.0301 \end{pmatrix} \\ \begin{pmatrix} 0.2511 & -0.1406 \\ -0.1406 & -0.0301 \end{pmatrix} & \begin{pmatrix} 0.7862 & -0.2069 \\ -0.2069 & 0.3723 \end{pmatrix} \end{pmatrix}. \quad (6.10)$$

It is recalled that the porosity in the homogeneous porous medium is (Eq. 2.22):

$$\phi_p = 0.4375. \quad (6.11)$$

Fig. (6.6) shows the mesh used for the porosity model with the example of the aligned layout A. In steady flow, a constant discharge Q is prescribed upstream the flume and a constant water level h_{av} is prescribed downstream. If the water pours into the pit (Fig. 4.6), the bottom level is lowered to $z_b = -1$ m for $14.5 \text{ m} < x < 15.5 \text{ m}$ and a water level of $h_{av} = -0.5$ m is prescribed at $x = 15.5$ m, so that the critical depth h_c is automatically obtained by the model at 14.5 m.

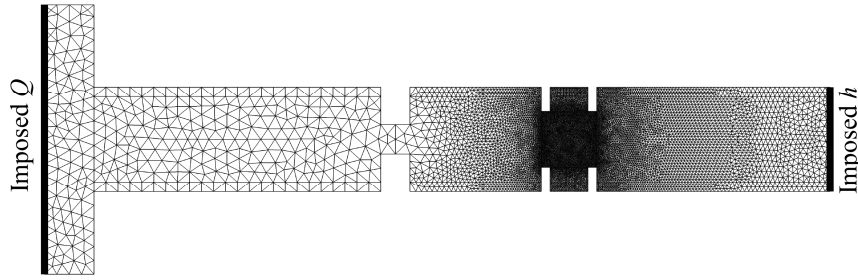


Fig. 6.6: Numerical modeling of the experiment in steady flow (example with the aligned layout A and the representation of the city as a porous medium): a constant discharge is prescribed upstream and a constant water depth is prescribed downstream.

The porosity model is tested with the three porosity border transitions (linear, bi-linear and discontinuous) in order to appreciate the implication of these assumptions. Fig. (6.7a) shows the mesh used in the porosity simulations in the city area, and the example of the porosity value under the assumption of a bi-linear porosity transition. The characteristic mesh size is 0.05 m in the porous area, 0.2 m in the undeveloped area¹ and 0.4 m upstream the gate.

The porosity model is also compared with a fully 2D detailed model consisting of the classical shallow water model (without porosity) and a refined mesh in the streets to that the flow is computed in details at the street scale. Fig. (6.7b) shows the mesh used in these detailed simulations. The characteristic size of the mesh is 0.01 m in the streets, 0.2 m in the undeveloped area and 0.4 m upstream the gate. The buildings are modeled as internal wall boundary conditions.

¹ The mesh in the area outside the porous medium between the two obstructing blocks has a characteristic size of 0.05 m (like in the porous medium), explaining why the mesh seems fine in that region.

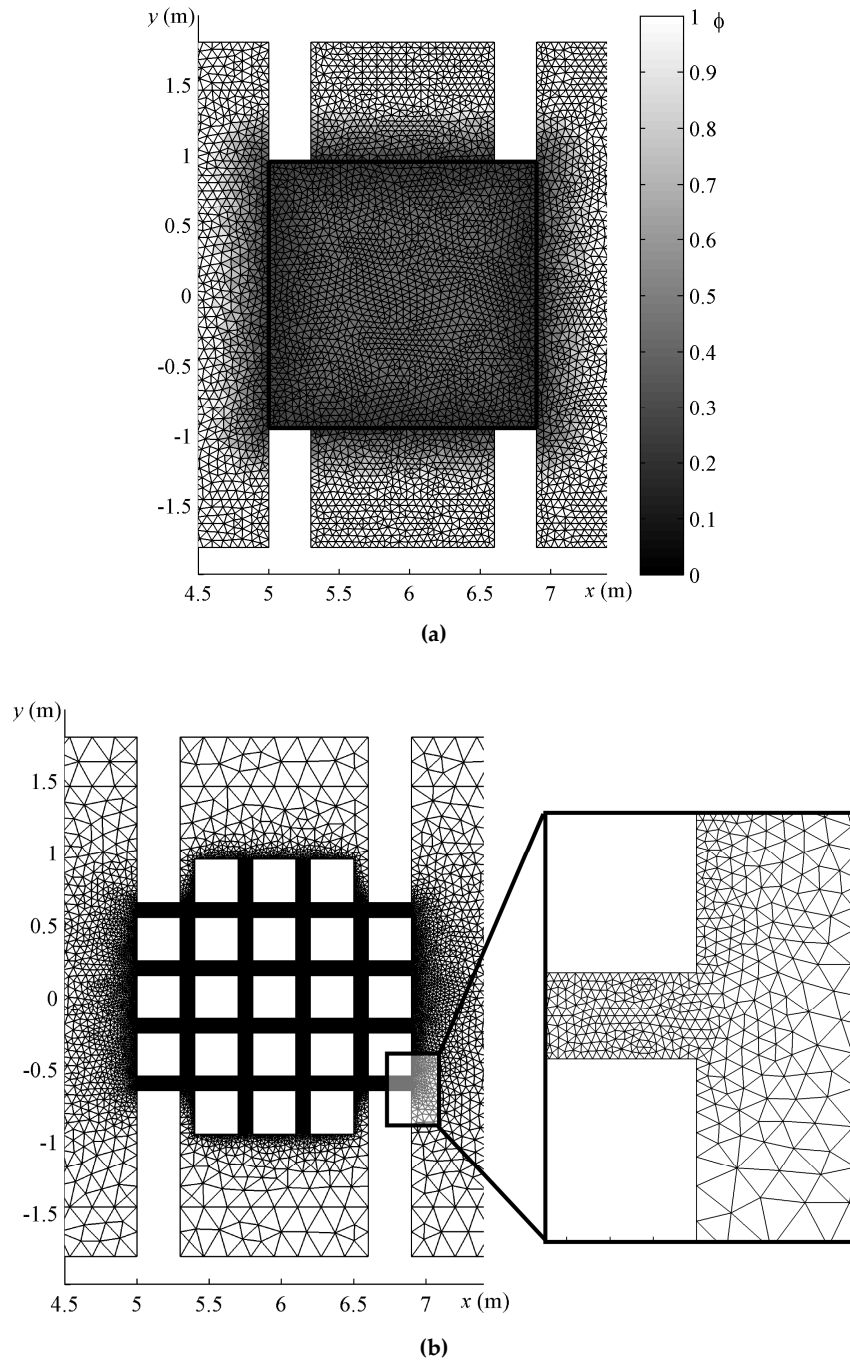


Fig. 6.7: Mesh and representation of the aligned city layout A (a) with the porosity model assuming a bi-linear porosity border transition (b) with a detailed model where the buildings are modeled as internal wall boundary conditions.

The flow was solved with a HLL scheme, despite its diffusivity. Indeed, as the porosity equations are solved with the modified HLL scheme presented in Section 5.3.1, it is interesting to compare it to detailed simulations solved with the classical HLL scheme.

Figs. (6.8) and (6.9) shows the comparison between the porosity model with the three border transition assumptions (discontinuous, linear or bi-linear porosity), the detailed simulation and the experimental data for the aligned layout A in steady flow, for all the tested discharges ($Q = 43$ l/s; $Q = 58$ l/s; $Q = 63$ l/s; $Q = 75$ l/s; $Q = 86$ l/s; $Q = 103$ l/s). The grey rectangles represent the buildings. The detailed simulation reproduce quite well the experimental data for $Q = 43$ l/s (Fig. 6.8a), but for higher discharges it overestimates the water level inside and upstream the city. Indeed, the basic assumption of the shallow water model is that the horizontal length scale is much greater than the vertical length scale. However, in the present case, this assumption is violated as the water surface slope at the boundaries of the city is very high, implying that the pressure is locally not hydrostatic anymore. However, this is only a local feature and the shallow water model is still appropriate for this kind of flow. The porosity model with the linear and discontinuous porosity transitions give very similar results, except that with a discontinuous porosity the water level presents a spurious drop at the city's border (see inset of Fig. 6.8c), probably caused by the absence of inner boundary condition. Both underestimate the water level upstream the city and then the resistance of the city. On the contrary, the porosity model with the bi-linear transition gives a satisfying upstream water level, although it overestimates it inside the city, similarly to the detailed model. Indeed, the bi-linear model presents a reduced porosity at the city's border ($\phi_t = 0.25$ while in the homogeneous porous medium $\phi_p = 0.44$) which makes the city more impermeable so that it is more difficult for the flow to go through, and then confers it a higher resistance. It seems that this higher resistance is necessary to correctly reproduce the flow.

In the following of this chapter, the discontinuous porosity model will be omitted as it can be considered as a limit case of the linear porosity model.

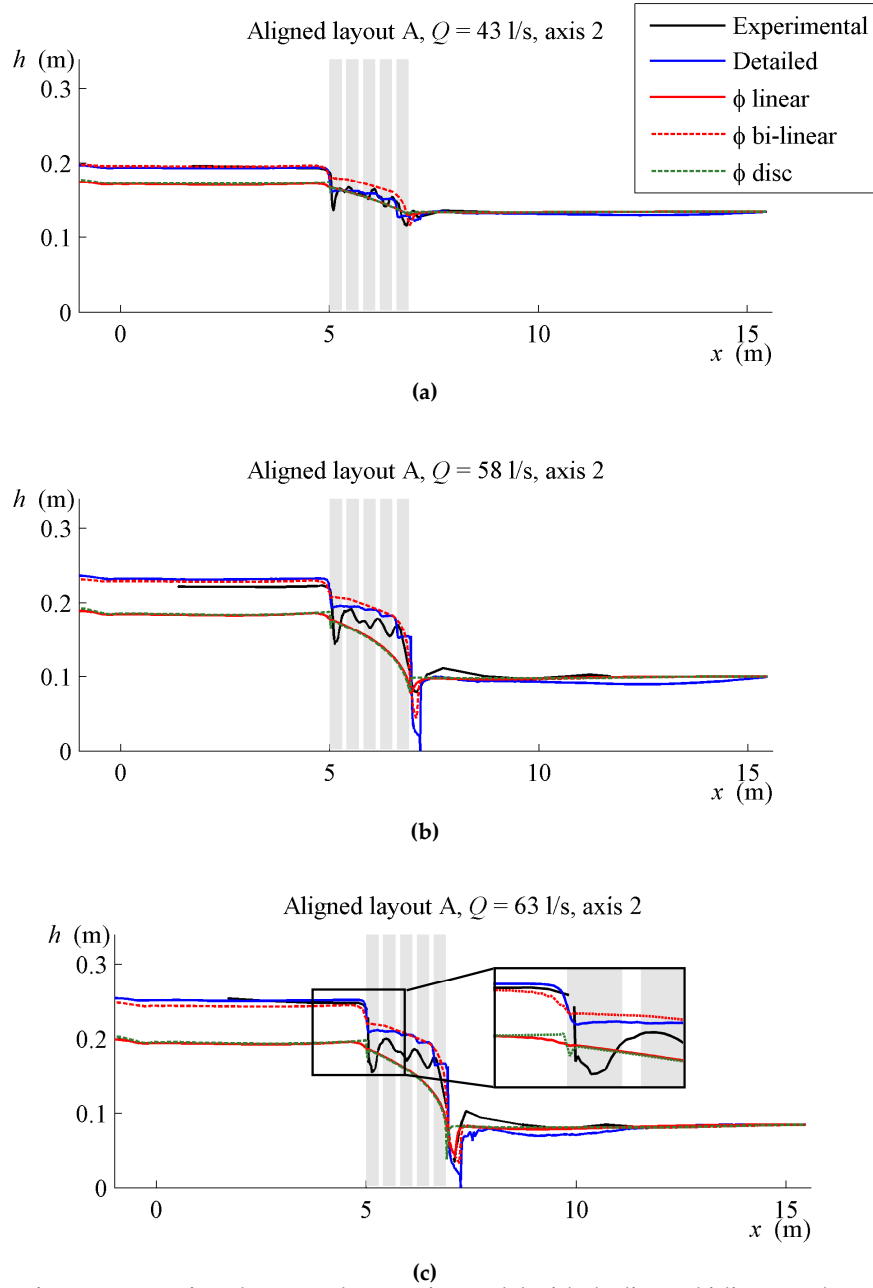


Fig. 6.8: Comparison between the porosity model with the linear, bi-linear and discontinuous ('disc') porosity transition assumptions, the detailed simulation and the experimental data for the aligned layout A in steady flow at (a) $Q = 43$ l/s; (b) $Q = 58$ l/s and (c) $Q = 63$ l/s.

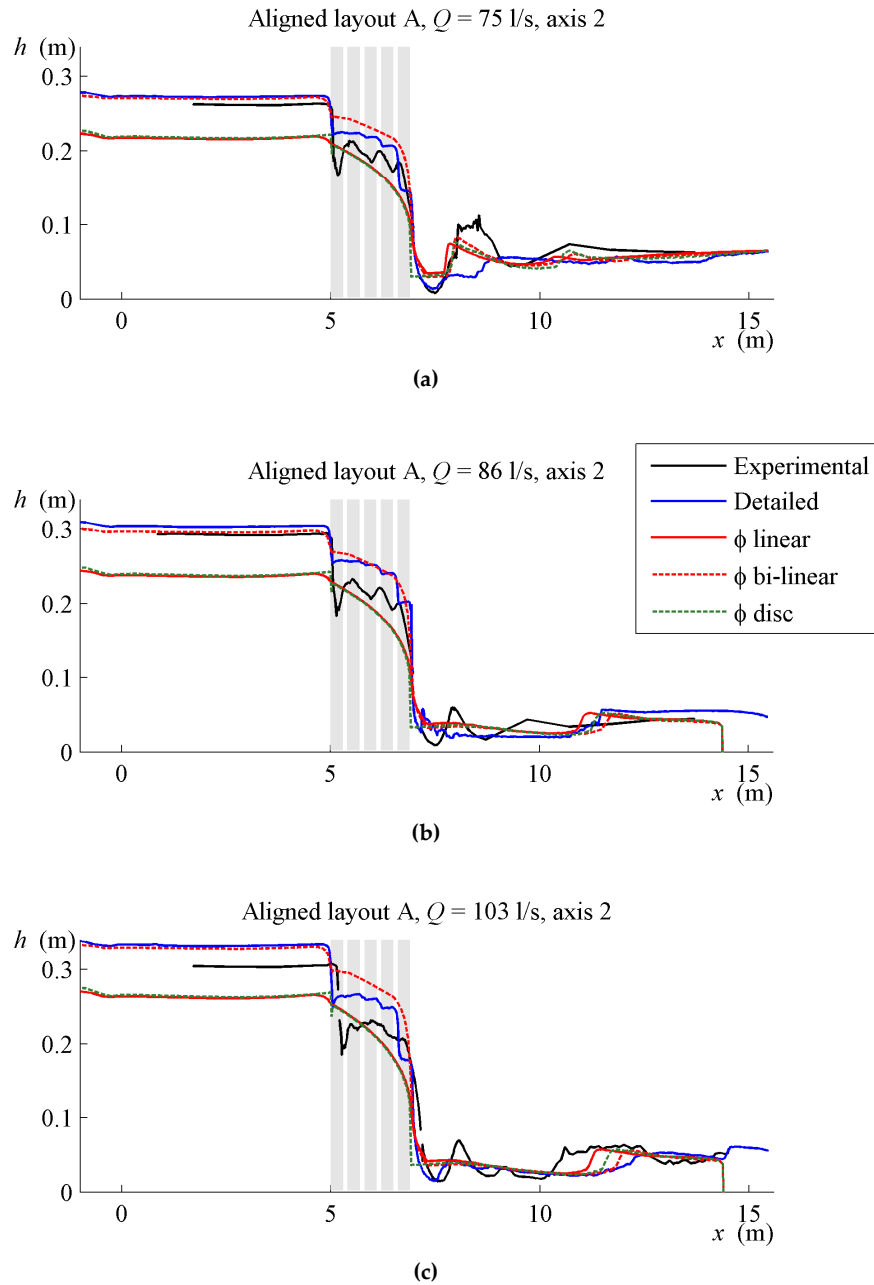
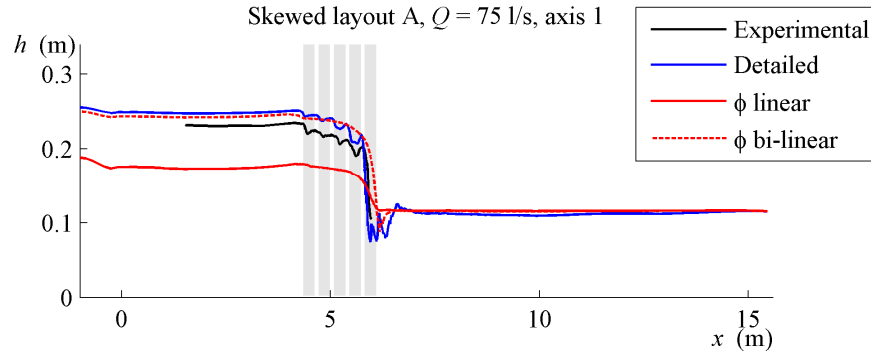
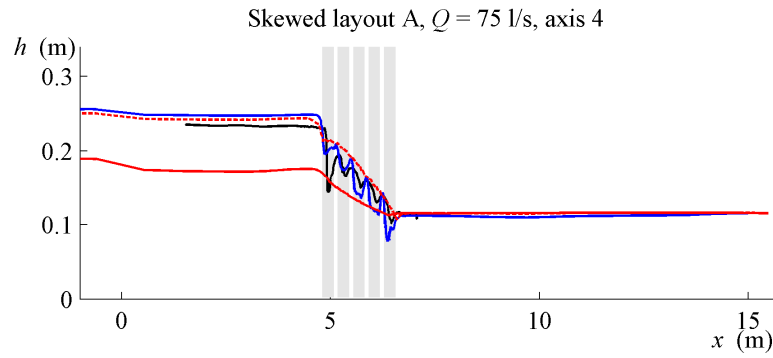


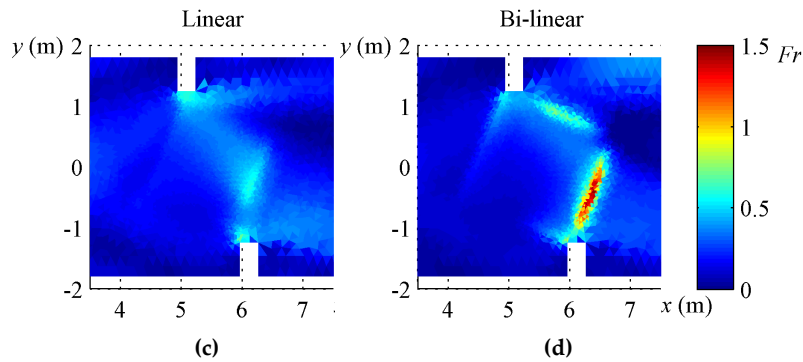
Fig. 6.9 (following of Fig. 6.8): Comparison between the porosity model with the linear, bi-linear and discontinuous ('disc') porosity transition assumptions, the detailed simulation and the experimental data for the aligned layout A in steady flow at (a) $Q = 75$ l/s; (b) $Q = 86$ l/s; (c) $Q = 103$ l/s.



(a)



(b)



(c)

(d)

Fig. 6.10: Comparison between transition models with the linear and bi-linear porosity transition assumptions, respectively: , (a-b) detailed simulation and experimental data for the skewed layout A in steady flow at $Q = 75$ l/s, (a) along the axis 1 and (b) along the axis 4; (c-d) Froude number (c) for the linear and (d) the bi-linear models.

Figs. (6.10) and (6.11) illustrate the comparison between the porosity model (with the linear and bi-linear porosity transitions), the detailed simulation and experimental data for the skewed layout A, for $Q = 75$ l/s along the axis 1 (Fig. 6.10a) and along the axis 4 (Fig. 6.10b), and $Q = 100$ l/s along the axis 1 (Fig. 6.11a) and the axis 4 (Fig. 6.11b).

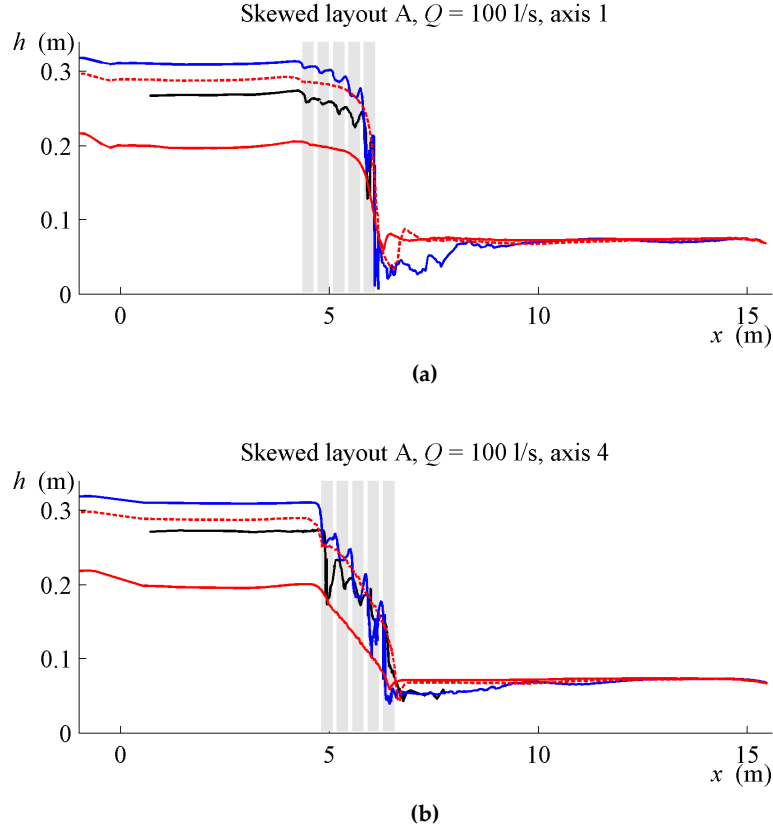


Fig. 6.11: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions, the detailed simulation and the experimental data for the skewed layout A in steady flow at $Q = 100$ l/s, (a) along the axis 1 and (b) along the axis 4.

The detailed simulations slightly overestimates the water depth for $Q = 75$ l/s (Figs. 6.10a, b) and overestimates much more for $Q = 100$ l/s (Figs. 6.11a, b). It is clear that the porosity model with the linear transition underestimates the resistance of the city, like it was the case previously for the aligned layout. But if we assume a bi-linear border transition of the porosity, the results agree much better with the experimental data. Then, lowering the porosity

at the border seems a good option to increase the resistance offered by the city to the flow.

In Fig. (6.10b) (and in Figs. 6.15b and 6.16b), the water profile computed by the linear and bi-linear transition models present distinct slopes inside the urban fabric and distinct local head losses at the entrance and the exit of the city. This latter difference is explained by the transition model itself that leads to very distinct velocities (note in Figs 6.10c and 6.10d that the flow can become supercritical at the exit in bi-linear transition model). The difference in slope is probably due to the influence of the boundary conditions on the discharge distribution between street axes (the difference in slope in axis 4 is more important than in axis 1).

For the aligned and skewed layout B, only a part of the results are shown in the present text, and the rest is presented in Appendix E for the sake of conciseness. Fig. (6.12a) shows the results for the aligned layout in the narrow streets direction at $Q = 50$ l/s; Fig. (6.12b) shows the results for the aligned layout in the wide streets direction at $Q = 99$ l/s and Fig. (6.12c) shows the results for the skewed layout at $Q = 100$ l/s.

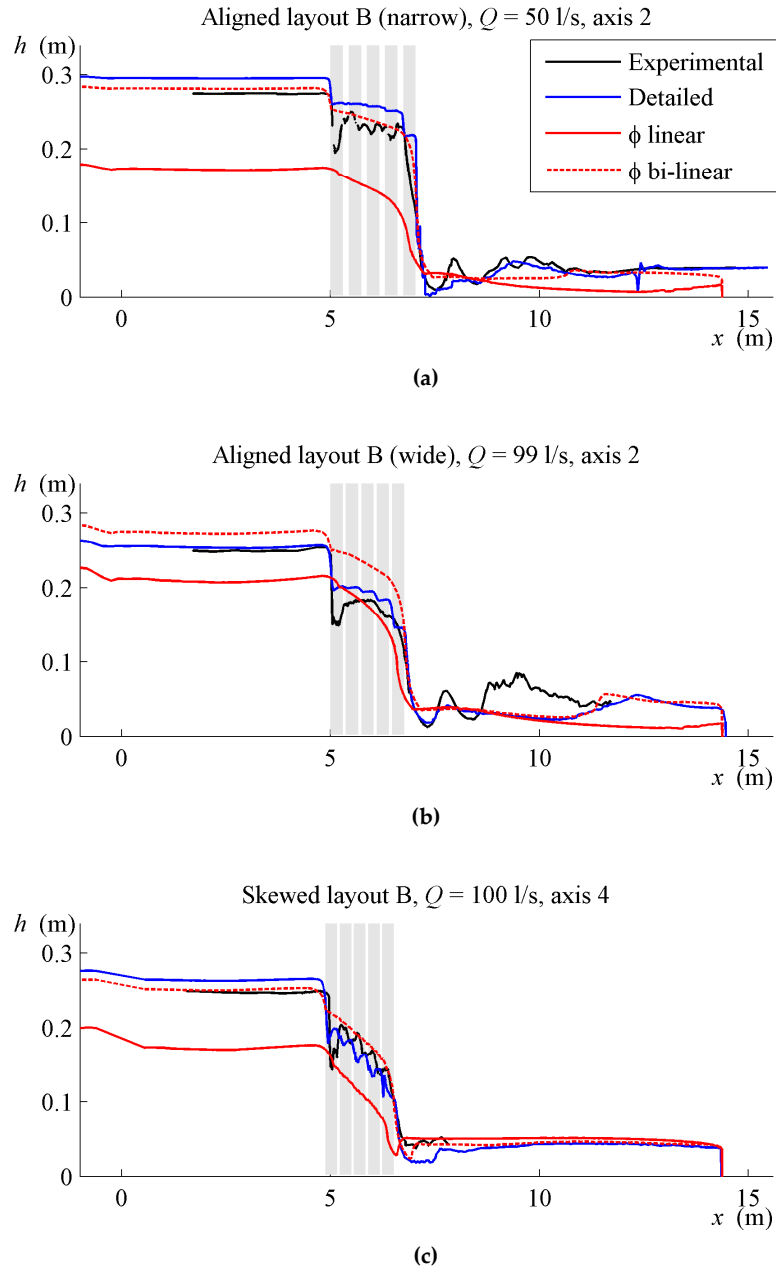


Fig. 6.12: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions, the detailed simulation and the experimental data for layout B (a) aligned in the direction of the narrow streets at $Q = 50$ l/s, (b) aligned in the direction of the wide streets at $Q = 99$ l/s and (c) skewed at $Q = 100$ l/s.

The observations are the same as for layout A: the detailed simulations overestimate the water level inside and upstream the city, and the porosity model presents a good agreement with experimental data if the porosity is lowered at the city's border as made by the bi-linear model.

It can be observed in Figs. (6.12a) and (6.12b) that the detailed model and the two transition models give different water levels downstream of the city, for $7.04 \text{ m} < x < 15 \text{ m}$ and for $6.77 \text{ m} < x < 15 \text{ m}$, respectively. The flow regime is supercritical in that region so that the water level is highly influenced by the control section at the exit of the city (at $x = 7.04 \text{ m}$ and $x = 6.77 \text{ m}$, respectively). The discrepancies come from the fact that in a detailed representation, the flow coming out of the city is localized at the four streets while in the porosity model the discharge is distributed all along the boundary of the porous medium. Also the denser grid used for detailed model could explain some discrepancies. The difference between the two porosity-transition models (linear and bi-linear) is more difficult to explain. Lateral jumps could explain why accurate location of water depth changes is not guaranteed.

The conclusion of this section treating about layouts A and B in steady flow with obstructing side blocks is that the porosity model with the calibrated drag/dispersion tensors $\mathbf{D}$ (given by Eqs. 6.6-10) agrees well with the experimental data provided the porosity is lowered at the city's border. This has the effect to make the city more impermeable, and then to increase its resistance to the flow. For layout A, the porosity at the border is $\phi_t = 0.25$ (Eq. 6.4a), and for layout B, $\phi_t = 0.18$ in the direction of the narrow streets (Eq. 6.4b) and $\phi_t = 0.31$ in the direction of the wide streets (Eq. 6.4c).

Unexpectedly, the detailed simulations do not perform so well. They usually give a water level too high inside and upstream the city, maybe due to the fact that the basic assumption of the shallow water model (i.e. the horizontal length scale is much greater than the vertical length scale) is violated in the present test cases. The bi-linear porosity model gives results of a similar quality, for a much better running time (on the order of 10 times faster).

6.3 Steady flow without side obstacles

In this section the porosity model is tested on layout A in steady flow, but without obstructing side blocks, and oriented by 0° , 67.5° and 45° with the main flow direction (Fig. 6.13). The objectives are to evaluate how the flow

behaves without side obstructions and to test again the validity of the drag/dispersion tensor $\mathbf{D}$ calibrated in Section 3.3.3. For layout A skewed by 67.5° , $\mathbf{D}$ is given by Eq. (6.7). , and for layout A skewed by 45° , it writes, after multiplying by the corresponding matrix of rotation:

$$\mathbf{D}_{45^\circ}^A = \begin{pmatrix} \begin{pmatrix} 0.8250 & 0 \\ 0 & 0.8250 \end{pmatrix} & \begin{pmatrix} 0 & -0.6750 \\ -0.6750 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -0.6750 \\ -0.6750 & 0 \end{pmatrix} & \begin{pmatrix} 0.8250 & 0 \\ 0 & 0.8250 \end{pmatrix} \end{pmatrix} \quad (6.12)$$

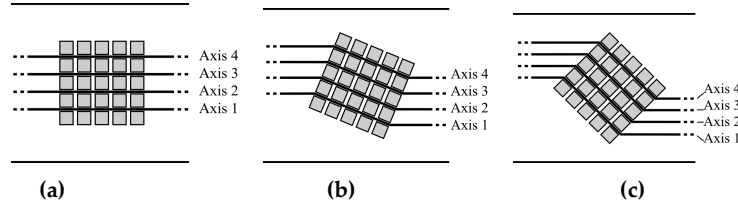


Fig. 6.13: City layouts A studied in steady flow oriented by (a) 0° , (b) 67.5° and (c) 45° with the flume main direction and name of axes.

For these test cases, there no experimental data available, therefore the detailed simulation will be considered as the reference solution. The three city layouts of Fig. (6.13) are tested under a constant discharge of $Q = 200$ l/s prescribed at the upstream section and a prescribed water level of $h = 0.12$ m in the downstream section. The comparison between the porosity model (with the bi-linear and linear porosity transitions) and the detailed simulation are shown in Figs. (6.14-16).

It can be observed that the linear porosity model agrees better with the reference solution than the bi-linear model, contrarily to the previous test cases with the presence of obstructing obstacles. The bi-linear porosity model overestimates the city's resistance. However, the porosity transition at the border should be an intrinsic property that does not depend on whether there are obstructing blocks or not. A possible explanation for this paradox is then that the closure for drag and dispersion may be inappropriate. Indeed, the two phenomena of drag and dispersion are modeled here by a single expression, which may be a too crude assumption in the present case. Future investigation should concentrate in establishing an independent closure for each of the two phenomena.

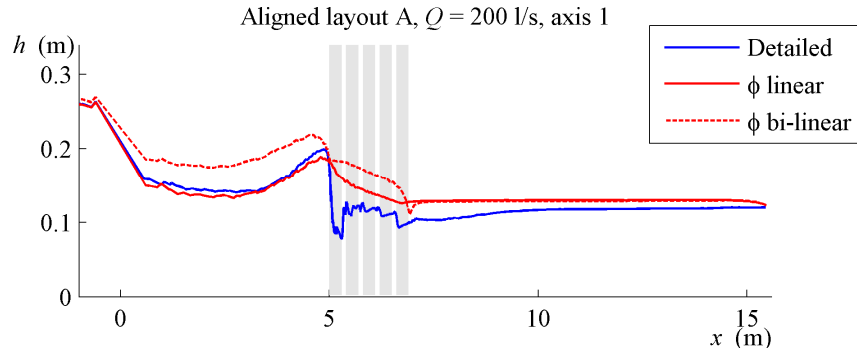
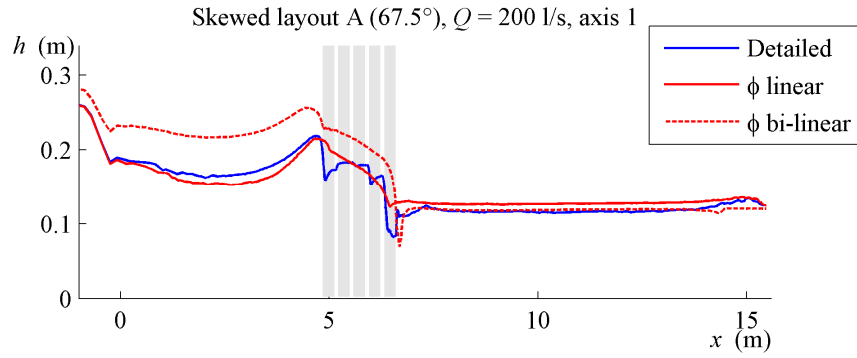
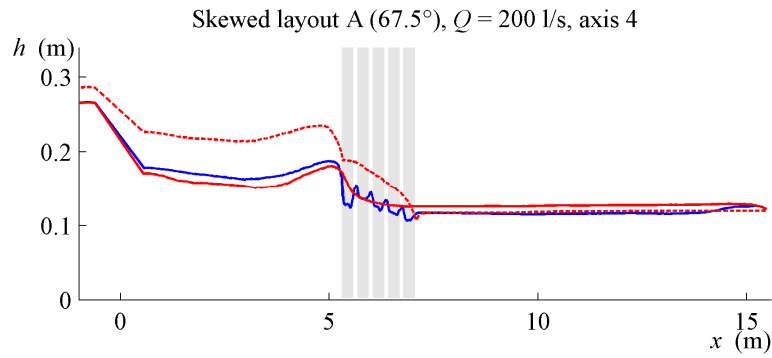


Fig. 6.14: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions and the detailed simulation for the aligned layout A along the axis 1 at $Q = 200$ l/s.



(a)



(b)

Fig. 6.15: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions and the detailed simulation for the 67.5° -skewed layout A at $Q = 200$ l/s (a) along the axis 1 and (b) along the axis 4.

By comparing the results of Figs. (6.14-16) for increasing angle of orientation (0° to 45°), we can observe that the results of the bi-linear porosity model get worse as the orientation angle increases: the discrepancies with the reference solution is higher for the 45° -skewed layout than for the aligned one. This can be explained by the fact that the aligned layout extends less in the cross section of the flume than the 45° -skewed layout: the aligned one extends from $y = -0.95$ m to $y = 0.95$ m while the 45° -skewed one lies between $y = -1.34$ m and $y = 1.34$ m. This explains why increasing the city's resistance for the 45° -skewed layout has higher consequences than for the aligned one.

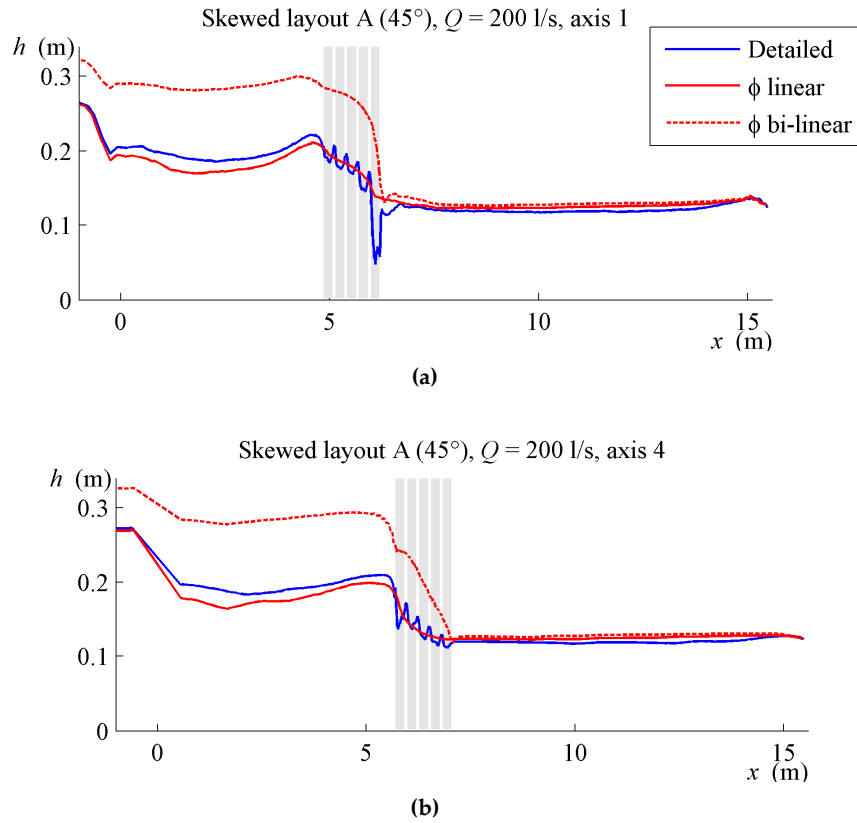


Fig. 6.16: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions and the detailed simulation for the 45° -skewed layout A at $Q = 200$ l/s (a) along the axis 1 and (b) along the axis 4.

6.4 Dam break without obstacles

This section presents the comparison between the porosity model, the detailed simulations and the experimental data in dam break flow on layout A. The two studied layouts are the ones of Fig. (6.13a, b). To simulate the dam break experiment, all the boundaries of the flume are modeled as reflective walls; there is no more prescribed discharge or water level. The initial water level is 0.40 m upstream the gate (i.e. for $x < 0$) and 0.01 m elsewhere.

The results for the aligned layout A at the time $t = 15$ s are shown in Fig. (6.17). It can be seen that the detailed simulation underestimates the water level inside the city and upstream the city. The detailed model seems then unable to represent the resistance that the city exerts to the flow. The porosity model with the bi-linear transition gives results in accordance with the experimental data, even better than the detailed simulation. This observation is surprising as the porosity model is a more simplified model than the detailed model that takes all the geometrical details of the city into account. So, in the best case, it should tend to the detailed simulation. It is then a coincidence here that the porosity model performs better than the detailed simulation. At the contrary, the linear porosity model is in good agreement with the detailed simulation, except that the hydraulic jump is situated slightly too far upstream. The tendencies of all the models (detailed, linear and bi-linear) hold during the entire experiment.

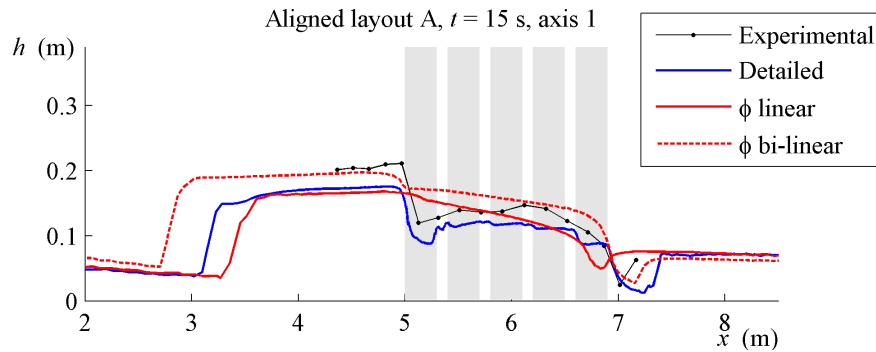


Fig. 6.17: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions and the detailed simulation for the aligned layout A in dam break flow along the axis 1.

Fig. (6.18) shows the results for the skewed layout A at the time $t = 15$ s, along the axes 1 and 4. Here again, the bi-linear porosity model performs

better than the detailed simulation, and the linear porosity model agrees well with the detailed simulation, this latter underestimating the water level inside and upstream the city. We can deduce from these observations that the detailed model is not able to represent correctly the resistance opposed by the city in a dam break flow. A higher-order model would maybe be better appropriated. If we expect the porosity model to be in accordance with the detailed simulations, then a linear porosity border transition is preferable. On the other part, if we expect that it is in accordance with the reality, the porosity should be lowered at the border to make the city more impermeable.

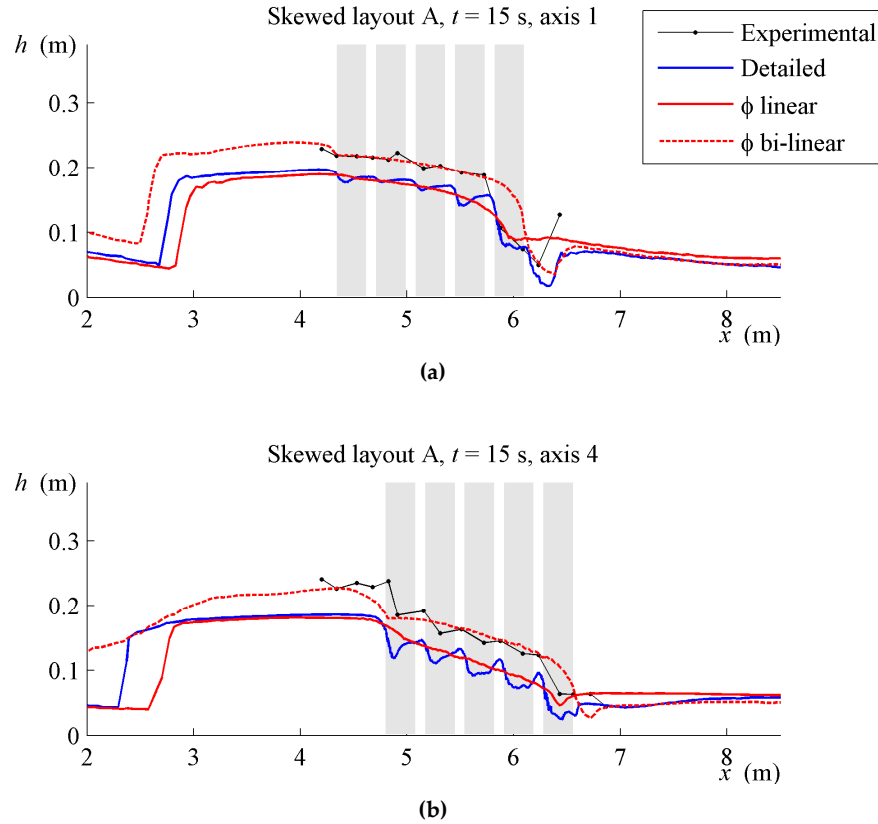


Fig. 6.18: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions and the detailed simulation for the skewed layout A in dam break flow (a) along the axis 1 and (b) along the axis 4.

6.5 Conclusion

This chapter began with the question of how varied the porosity at the border of a city if this latter is surrounded by an area without obstacles and where consequently $\phi = 1$, like in a real case or like in the experiments presented in Chapter 4. Three options have been proposed based on the literature:

- the porosity varied linearly at the border (Fig. 6.4a);
- the porosity varied bi-linearly at the border (Fig. 6.4b). The bi-linear porosity transition consists in fact in reducing the porosity at the border.
- the porosity varied discontinuously at the border (Fig. 6.4c);

The first objective of this chapter was to test these three porosity transitions on various test cases consisting mainly in the experimental data presented in Chapter 4, and to investigate the implication of these choices. The second objective was to check if the value of the drag/dispersion tensor $\mathbf{D}$ calibrated in Chapter 3 remained valid in steady flow, knowing that it was calibrated in dam break flow.

Three types of test cases were examined. The first type (investigated in Section 6.2) consisted in the steady flow experiments presented in Chapter 4, recalled in Fig. (6.19). In these experiments, a set of 5×5 square blocks was placed in a flume to represent a city, and obstructing blocks were placed aside to prevent the flow to by-pass the city. The set of 5×5 blocks was laid out to form either streets of equal width (layout A), or streets twice wider in one direction than in the other (layout B). Each layout was placed either aligned or skewed with respect to the main direction of the flume.

The porosity model with the three border porosity transitions was compared to experimental data and also detailed simulations consisting in solving the classical shallow water equations (without porosity) on a very refined mesh (~ 0.01 m) at the street scale. It could be seen in most cases that the detailed model overestimated the water level upstream the city, probably because the solver used is only of the first order. It was also observed that the porosity model with linear and discontinuous porosity transitions gave very similar results, both underestimating the water level upstream the city (Figs. 6.8-6.10). At the contrary, the bi-linear model, consisting in reducing the

porosity at the border of the city, gave quite good agreement with the experimental data. Indeed, decreasing the porosity at the border makes the city more impermeable, more difficult to flow through, and it is then a manner to increase its resistance to the flow.

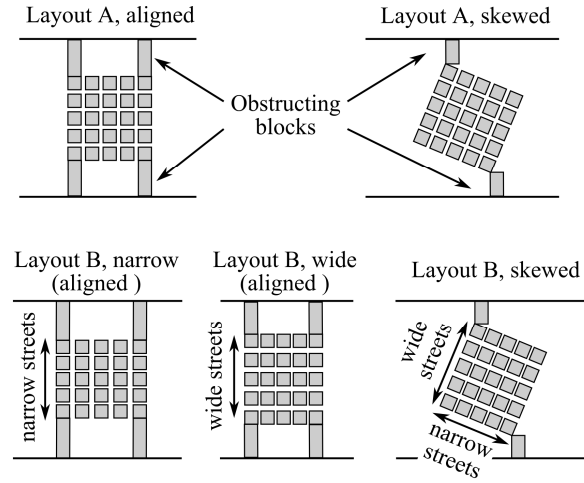


Fig. 6.19: First type of test cases (Section 6.2): steady flow with obstructing blocks.

The tensor $\mathbf{D}$ reproduced well the fact that the resistance was higher when the water flowed in the direction of the narrow streets. It is then applicable to steady flow despite it was calibrated in dam break flow.

The second type of test cases (investigated in Section 6.4) consisted in steady flow on layout A but without obstructing blocks (Fig. 6.20), in order to assess the influence of these latter on the behavior of the flow. As no experimental data were available, the reference solution for these cases was given by the detailed simulations.

This time, the linear porosity model gave better agreement with the reference solution than the bi-linear porosity model (Figs. 6.15-6.15), which is in contradiction with the conclusions of the test cases of Fig. (6.19) with the presence of obstructing blocks. This paradox was explained by the fact that the closure for the drag and dispersion phenomena could be inappropriate. In the future, an independent closure equation for each one of the two phenomena should be established.

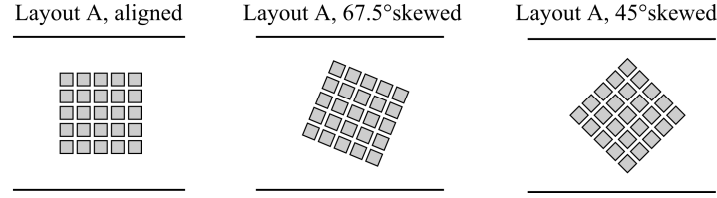


Fig. 6.20: Second type of test cases (Section 6.3): steady flow without obstructing blocks.

The third type of test cases (investigated in Section 6.5) consisted in the dam break experiments on layout A presented in Chapter 4. No obstructing blocks were placed aside the city (Fig. 6.21).

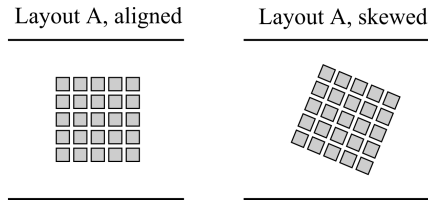


Fig. 6.20: Third type of test cases (Section 6.4): dam break flow without obstructing blocks.

The porosity model was confronted to the experimental data as well as detailed simulations. The detailed simulation underestimated the water depth of the hydraulic jump upstream the city (Figs. 6.17-6.18), probably because the three dimensional effects were not negligible and could obviously not be taken into account by a two-dimensional model. The linear porosity model gave results very close to the detailed simulation. It was surprising to observe that the bi-linear porosity model presented better agreement with the experimental data than the detailed model. Indeed, lowering the porosity at the border of the city may be a way to artificially increase the resistance of the city and then to take into account other effects that are not modeled by simple two-dimensional models.

In conclusion, we may say that the tensor $\mathbf{D}$ calibrated in circular dam break flow in homogeneous porous media in Chapter 3 remains valid in steady flow in non-homogeneous media, provided that the border of the porous medium is modeled in an appropriate way. If the border is modeled with a lower porosity, it has the effect of making the city more impermeable and then more resistant to the flow. It is an appropriate choice in case of steady confined flow ('confined' meaning here that the flow is not allowed to by-pass the city) or in dam break flow. On the contrary, in steady 'open' flow ('open' meaning here that the flow is allowed to by-pass the city, as it is almost always the case in practice) it is appropriate to adopt a linear porosity

transition. A discontinuous porosity transition may also be admissible although some spurious peaks in the water depth will arise at the city's border.

Conclusion

In emergency situations when an imminent dam or dike break is forecast, the propagation of the flood wave and the detection of the potentially flooded areas must be assessed within a few hours. The reduced time available for the numerical simulation of the water propagation requires models that are able to run very fast, and then models very simple to use and that are able to run on coarse meshes. In this spirit, the porosity model has arisen with the objective to quantify the effect of urban areas on the flood wave propagation with few parameters and little computational effort.

Successive developments of the porosity model have been achieved in the past. At the beginning of the present work, the two main questions to be solved were:

- How could the preferential flow directions due to the alignments of the buildings be taken into account in the porosity model? The porosity model contained then two parameters called the directional porosities that were supposed to take the anisotropy of the urban medium into account but no one had ever achieved to define them properly.
- What is the origin of the head losses produced by the urban medium? Indeed, it had been shown that it was necessary to introduce head losses specific to the urban medium in order to enhance the results of the porosity model.

The methodology to answer these questions was to re-develop the porosity equations in another way, hoping at first that the definition of directional porosities and the theoretical origin of the head losses would arise by themselves. The volume averaging method, commonly applied in three-dimensional porous media, was then applied here to a peculiar REV in which the "solid matrix" was constituted by the buildings. It has been shown that way that all the porosities were all equal to the storage porosity (i.e. the ratio between the surface available for the flow and the total surface) so that there was no need to define directional porosities. It has also been shown that the drag and dispersion terms had been omitted in the former porosity equations and could be the origin of the head losses that we were looking for. These two terms depend on street-scale quantities that are unknown. It was chosen here to substitute them by an only closure expression which is function of macroscopic quantities only.

The anisotropy of the urban medium is accounted through this drag/dispersion closure expression. The parameters of this closure are the components of a fourth-order tensor (second-order tensor whose components are also second-order tensors). This tensor contains 8 independent parameters, but it has been shown that this number reduced to 4 if the medium was periodic, or even two if both main direction were identical. If the medium were isotropic (which is not the case of a periodic medium), there would remain only one independent parameter. This closure expression has been validated against numerical test cases consisting of a circular dam break in an "infinite" periodic medium constituted by buildings regularly aligned, forming perpendicular streets. It has been shown that when the parameters are calibrated appropriately, the drag/dispersion tensor was able to reproduce the fact that the water flowed preferentially along both axes of the streets and the resistance to the flow was higher if for off-axis flow (i.e. when the general flow direction was not aligned to any of the axes of the streets). It also reproduced the fact that the flow went preferentially in wider streets in the case that the streets were wider in a direction than in the other.

An experimental campaign has been conducted to validate the new porosity model and the closure expression. A set of 5×5 square blocks was laid out in a laboratory flume to represent an idealized city. These blocks were either aligned with the main direction of the flume, or skewed by 22.5° . This city was submitted to steady flow and dam break flow. For the steady flow experiments, obstructing blocks have been placed aside the city to prevent the flow to by-pass this latter. This had the advantage to increase the difference of water depth ahead and behind the city, and then made the effects on the flow more visible. In dam break flow, no obstructing blocks have been placed because these ones would have been swept away under the shock of the dam break wave. One of the observations of the steady flow experiments was that the city opposed much more resistance in the direction of the narrower streets.

Before the validation of the model, the question of how to represent the porosity transition at the border of the city was addressed. Three types of porosity transitions have been proposed and tested: a discontinuous transition, a linear transition and a bi-linear transition. The bi-linear transition consisted in lowering the porosity at the border, making then the city more impermeable. It was in fact a manner to increase the city's resistance to the flow.

The porosity model with the three porosity transitions at the border has been confronted to the experimental data and to the results of detailed

simulations. The conclusions were that the drag/dispersion tensor that had been calibrated in dam break flow remained valid in steady flow provided that the city border was correctly modeled. It was observed that porosity model with the linear and discontinuous border porosity transitions gave similar results, except that spurious water level peaks arose in the discontinuous case. In steady flow with obstructing blocks aside the city, the bi-linear porosity transition gave better results. But without these obstructing blocks, the linear porosity model seemed to be more adapted. This paradox was assumed to be due to the fact that the drag and dispersion closure was inappropriate. In dam break flow, the detailed simulation underestimated the water level inside and upstream the city. The linear porosity model showed a good agreement with the detailed simulations, while the bi-linear porosity model, surprisingly, gave results closer to the experimental data than the detailed simulations.

In conclusion, the porosity model developed in the present work characterizes the effect of a city on a flow by the porosity ϕ and by a set of 8 drag/dispersion parameters grouped in a fourth-order tensor to quantify the head losses. The number of head losses parameters reduce to 4 for a periodic medium, to 2 if both main directions of the periodic medium have the same properties, and to 1 if the medium is isotropic. Three different porosity transitions at the city border were tested: a linear, a discontinuous and a bi-linear transition. The bi-linear transition consisted in lowering the porosity on the border, which had the impact to increase the resistance opposed by the city to the flow. It was observed that in some cases a discontinuous or linear porosity transition gave better results, while in other cases the bi-linear model gave better agreement. However, the border porosity transition should be an intrinsic property of the medium that does not depend whether the flow regime is steady or transient. This leads to the conclusion that the drag/dispersion closure may be inappropriate; the two phenomena of drag and dispersion should maybe be modeled by independent relations.

Single porosity model versus multiple porosity model

In the present thesis, the porosity model has been validated against laboratory experiments that are still very far from real cases, where the urban medium is not periodic, the avenues channeling the flow are not necessarily perpendicular to each other and there are dead zones that do not participate to the flow.

An alternative porosity model has been recently developed by Guinot (2012), taking into account dead zones and the preferential flow directions prescribed by the presence of wide avenues. This model is called "multiple porosity model" and consists in considering that the urban medium is constituted by various superimposed porous media of different porosities. By opposition, the porosity model developed in the present thesis will be called "single porosity model".

The multiple porosity model would consider that the periodic media studied in this thesis (Fig. 3.1) is the superimposition of two porous media, the first medium associated to the streets aligned with the a given direction and the second medium to the streets aligned with the other direction. The first medium would be characterized by a porosity ϕ_1 , the space ratio occupied by the streets of the first direction and the second medium would have a porosity ϕ_2 , the space ratio occupied by the streets of the other direction. A flow direction angle would be associated to the porosities ϕ_1 and ϕ_2 . For the aligned layouts, these angles would be 0° and 90° , and for the 22.5° -skewed layouts, 22.5° and $22.5^\circ + 90^\circ = 112.5^\circ$. The equations governing the flow in each medium would be the porosity equations similar to the ones developed in this these except that the drag/dispersion tensor would be substituted by source terms expressing the mass and momentum transfer that both media can exchange between each other.

This model was compared to the single porosity model in various test cases of idealized cities consisting of periodic media with perpendicular and not perpendicular streets, and also periodic media with dead zones. It resulted that the multiple porosity model gave better results than the single porosity model.

Does that mean that the single porosity model developed in the present work is obsolete? This question arises another one: what level of simplicity do we expect from a simplified model?

Indeed, a simplified model is always a compromise between precision and simplicity: the more precise we want the model to be, the higher the number of parameters is needed, and the more complex the model becomes (Fig. 7.1). The multiple porosity model is comparatively more precise but more complex than the single porosity model. Indeed, even if it is simple to use in an idealized periodic medium, it might be more difficult to implement in a real medium like the one depicted in Fig. (7.2). It would require an image analysis processor capable to distinguish the dead zones, the "low-velocity" zones and the channeling avenues, and to classify these latter in function of their orientation. Or, if such processor does not exist, it would require enough time to perform this parameterization manually. Moreover, in a real

urban medium, the porous media associated to the channeling avenues of a given orientation might have a very low porosity, close to zero, which could be difficult to handle numerically. These difficulties make the multiple porosity model more complex to use in practice than the single porosity model.

In a practical application, the preference of a model over the other will depend on the required precision and the available time to prepare the simulation. For example, for risk analysis, the multiple porosity model would be preferable as it would give more reliable results, but for emergency situations the single porosity model would be more appropriate as it would require the knowledge of less parameters.

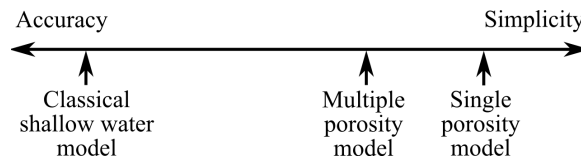


Fig. 7.1: Classification of the models following their precision and simplicity.

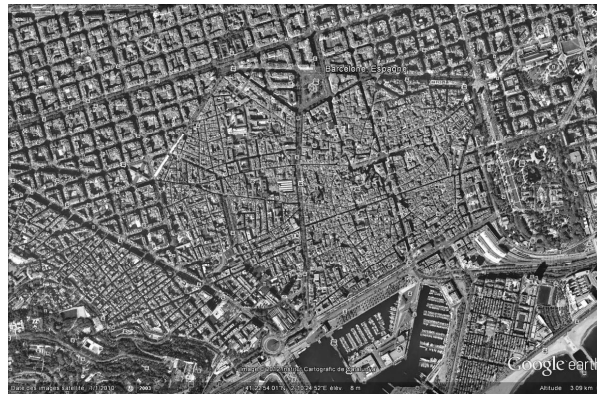


Fig. 7.2: Aerial view of Barcelona, Spain.

Perspectives

In the elaboration of a model, the underlying physics must be investigated and fully understood before the model is applied to a practical case. The present thesis has been devoted to the identification, description and

quantification of the physical mechanisms underlying the flow in urban media.

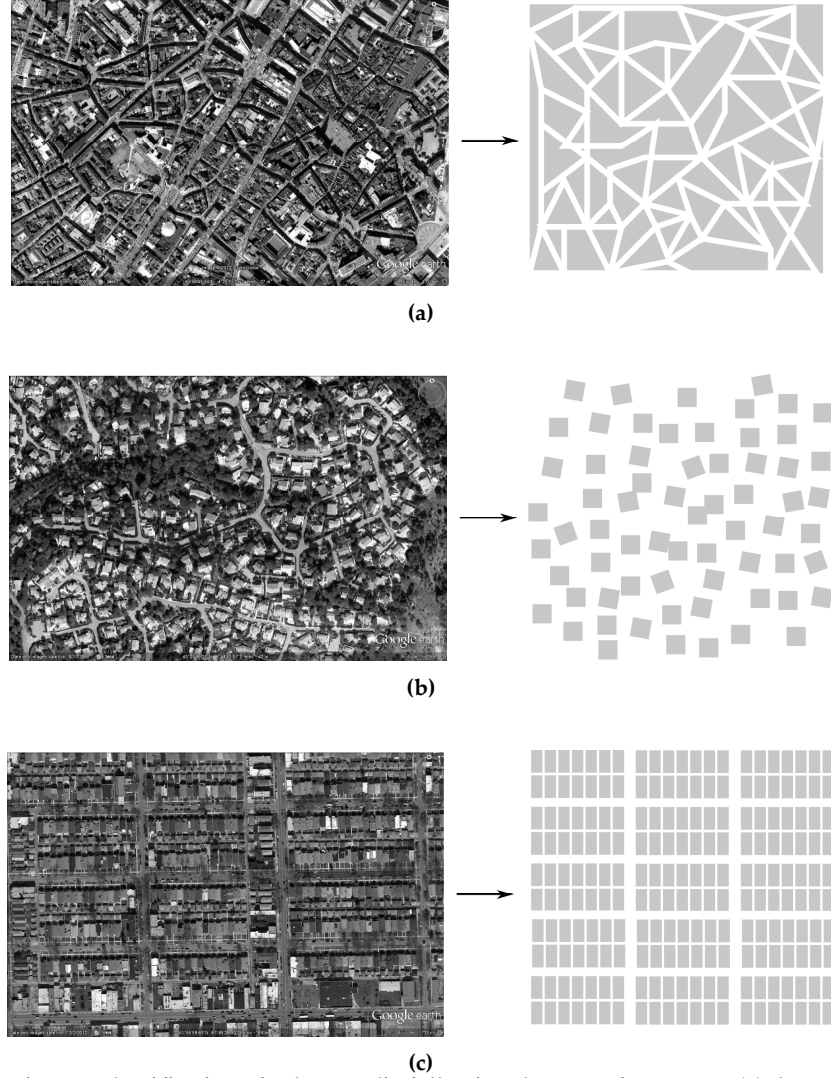


Fig. 7.3: Classification of urban media following the type of geometry: (a) dense and chaotic net of connected streets (example of Brussels, Belgium), (b) scattered and chaotic set of buildings (example of Fréjus, France) and (c) periodic set of rectangle buildings (example of Chicago, USA).

Future efforts should be oriented to making the porosity model applicable to real cases without the need of calibration of the drag/dispersion tensor. A possible strategy would be to draw a parallel with the multiple porosity

model, as this latter substitutes the drag and dispersion by terms expressing mass and momentum exchanges. Another strategy would be to calibrate the parameters for typical urban media like the ones illustrated in Fig. (7.3): a chaotic and dense net of interconnected streets typical of European cities (Fig. 7.3a), a chaotic and scattered set of square buildings typical of small towns (Fig. 7.3b) and a periodic set of rectangle buildings typical of American cities (Fig. 7.3c). The parameters would be calibrated on these three types of geometry within a realistic range of porosity in order to give plausible input parameters for the porosity model.

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Appendix A Time-derivative theorem

Suppose an extensive quantity $e(\mathbf{r},t)$ defined in the fluid phase of a porous medium. We want to write the average of a time-derivative as the time-derivative of an average. As $dU = dAdz$, we may write:

$$\frac{1}{A_0} \int_{U_f} \frac{\partial e}{\partial t} dU = \frac{1}{A_0} \int_{A_f} \left(\int_{z_b}^{z_w(t)} \frac{\partial e}{\partial t} dz \right) dA \quad (\text{A.1})$$

where z_b is the bottom level and $z_w(t)$ is the water level. We apply the Leibniz rule on the inner integral taking into account that, as the bottom is supposed to be fixed, the boundary z_b does not depend on time:

$$\frac{1}{A_0} \int_{U_f} \frac{\partial e}{\partial t} dU = \frac{1}{A_0} \int_{A_f} \left(\frac{\partial}{\partial t} \left(\int_{z_b}^{z_w(t)} e dz \right) - e|_{z=z_w} \frac{\partial z_w}{\partial t} \right) dA \quad (\text{A.2})$$

By definition of a depth-averaged quantity (Eq. 2.6), and by the fact that the time derivative of z_w is equal to the time derivative of h , Eq. (A.2) becomes

$$\frac{1}{A_0} \int_{U_f} \frac{\partial e}{\partial t} dU = \frac{1}{A_0} \int_{A_f} \left(\frac{\partial}{\partial t} (Eh) - e|_{z=z_w} \frac{\partial z_w}{\partial t} \right) dA, \quad (\text{A.3-a})$$

$$= \frac{1}{A_0} \int_{A_f} \left(\frac{\partial}{\partial t} (Eh) \right) dA - \frac{1}{A_0} \int_{A_f} \left(e|_{z=z_w} \frac{\partial z_w}{\partial t} \right) dA \quad (\text{A.3-b})$$

$$= \frac{1}{A_0} \int_{A_f} \left(\frac{\partial}{\partial t} (Eh) \right) dA - \frac{1}{A_0} \int_{A_f} \left(e|_{z=z_w} \frac{\partial h}{\partial t} \right) dA \quad (\text{A.3-c})$$

$$= \frac{\partial}{\partial t} \left(\frac{1}{A_0} \int_{A_f} (Eh) dA \right) - \frac{1}{A_0} \int_{A_f} \left(e|_{z=z_w} \frac{\partial h}{\partial t} \right) dA \quad (\text{A.3-d})$$

$$= \frac{\partial}{\partial t} \left(\frac{1}{A_0} \frac{A_f}{A_f} \int_{A_f} (Eh) dA \right) - \frac{1}{A_0} \int_{A_f} \left(e|_{z=z_w} \frac{\partial h}{\partial t} \right) dA \quad (\text{A.3-e})$$

$$= \frac{\partial}{\partial t} \left(\phi \frac{1}{A_f} \int_{A_f} (Eh) dA \right) - \frac{1}{A_0} \int_{A_f} \left(e|_{z=z_w} \frac{\partial h}{\partial t} \right) dA \quad (\text{A.3-f})$$

$$= \phi \frac{\partial}{\partial t} \left(\frac{1}{A_f} \int_{A_f} (Eh) dA \right) - \frac{1}{A_0} \int_{A_f} \left(e|_{z=z_w} \frac{\partial h}{\partial t} \right) dA \quad (\text{A.3-g})$$

$$= \phi \frac{\partial \langle Eh \rangle_A^f}{\partial t} - \frac{1}{A_0} \int_{A_f} e|_{z_w} \frac{\partial h}{\partial t} dA \quad (\text{A.3-h})$$

and, by Eq. (2.16),

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial t} \right\rangle_U^f = \phi \frac{\partial \langle Eh \rangle_A^f}{\partial t} - \frac{1}{A_0} \int_{A_f} e|_{z_w} \frac{\partial h}{\partial t} dA, \quad (\text{A.4})$$

which is the time-derivative theorem used in the present study.

Appendix B Spatial derivative theorem

The spatial derivative theorem makes the link between the average of a x_i -derivative and the x_i -derivative of an average. First, in Appendix B.1, the expression of the derivative of an average is simplified. Then, a useful geometric relation involving the gradient of the porosity is presented in Appendix B.2. The spatial derivative theorem can then be deduced in Appendix B.3, as well as a variant in Appendix B.4. This theorem is then generalized to a kind of divergence theorem in Appendix B.5.

B.1 Derivative of an average

Suppose a scalar quantity $e = e(\mathbf{r}, t)$ defined in the fluid phase of a porous medium as illustrated in Fig. (B.1). From the volume-average definition given by Eq. (2.5), the depth-average definition given by Eq. (2.6) and the relation between U_f and A_0 given by Eq. (2.13) about porosity, we may deduce the following relationships:

$$\frac{\partial}{\partial x_i} [\phi \langle h \rangle_A^f \langle e \rangle_U^f] = \frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \frac{1}{U_f} \int_{U_f} e \, dU \right] \quad (\text{B.1-a})$$

$$= \frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \frac{1}{U_f} \int_{A_f} \left(\int_{z_b}^{z_w} e \, dz \right) dA \right] \quad (\text{B.1-b})$$

$$= \frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \frac{1}{\phi \langle h \rangle_A^f A_0} \int_{A_f} E h \, dA \right] \quad (\text{B.1-c})$$

$$= \frac{\partial}{\partial x_i} \left[\frac{1}{A_0} \int_{A_f} E h \, dA \right] \quad (\text{B.1-d})$$

$$= \frac{1}{A_0} \frac{\partial}{\partial x_i} \left[\int_{A_f} E h \, dA \right] \quad (\text{B.1-e})$$

where A_0 was put outside the derivative because, as it is the total surface of the REV, it does not depend on x_i (the REV has the same radius all over the domain, no matter where is its centroid). By definition of the partial derivative, Eq. (B.1e) is equivalent to

$$\frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \langle e \rangle_U^f \right] = \frac{1}{A_0} \lim_{\Delta x_i \rightarrow 0} \frac{\int_{A_{f2}} E h \, dA - \int_{A_{f1}} E h \, dA}{\Delta x_i}. \quad (\text{B.2})$$

Fig. (B.1) that is the plan view of the REV, illustrates the surfaces A_{f1} and A_{f2} , centered at the points x_1 and x_2 , respectively. As we consider a x_i -derivative, the points x_1 and x_2 are very close to each other, and it can be seen that both surfaces overlap. Their intersection is also shown in Fig. (B.1). The difference of the integrals over A_{f1} and A_{f2} in Eq. (B.2) cancels on the overlapping part, and it remains only a difference between the non-overlapping parts of each surface, called A_{ff1} and A_{ff2} , respectively. For $\Delta x_i \rightarrow 0$, the integrals over A_{ff1} and A_{ff2} tend to be a contour integral over the curve C_{ff} (Fig. 2.b) in such a way that:

$$\frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \langle e \rangle_U^f \right] = \frac{1}{A_0} \int_{C_{ff}} E h n_i \, dC \quad (\text{B.3})$$

Eq. (B.3) will be used in the development leading to the theorem of spatial derivative, in Appendix B.3.

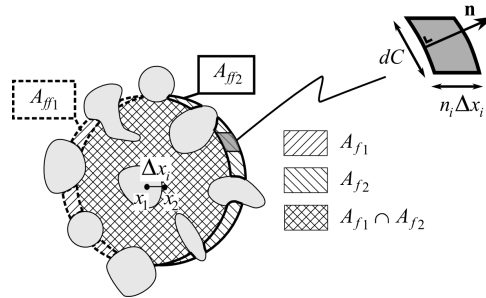


Fig. B1: The derivative of a macroscopic variable is the limit when $\Delta x_i \rightarrow 0$ of the difference between two macroscopic variables defined at x_1 and x_2 . The two REV's centered at x_1 and x_2 overlap and as the difference of the two macroscopic variables is reduced to the difference over the non-overlapping areas A_{ff1} and A_{ff2} that tend to be contours when zero $\Delta x_i \rightarrow 0$.

B.2 Divergence theorem in 2D

In this section, we are going to derive a 2D form of the divergence theorem. We start by writing it in its well-known 3D form for any vector $\mathbf{u}$:

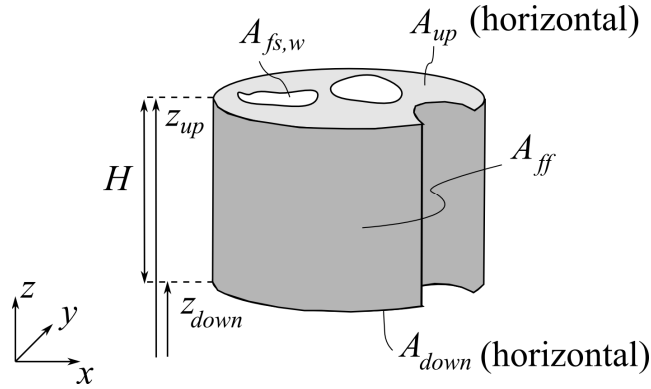


Fig. B.2: Volume used to derive a 2D form of the divergence theorem. The volume is formed by the surfaces A_{up} , A_{down} (both horizontal), A_{ff} and $A_{fs,w}$ (both vertical).

$$\int_U \nabla \cdot \mathbf{u} dU = \int_A \mathbf{n} \cdot \mathbf{u} dA \quad (\text{B.4})$$

Now, if we consider the special case where $\mathbf{u} = (0,1,0)$ or $\mathbf{u} = (1,0,0)$ and where the volume U is the one depicted in Fig. (B.2), we may write that:

$$\int_A n_i dA = 0 \quad i = 1, 2. \quad (\text{B.5-a})$$

$$\underbrace{\int_{A_{up}} n_i dA + \int_{A_{down}} n_i dA}_{=0} + \int_{A_{fs,w}} n_i dA + \int_{A_{ff}} n_i dA = 0 \quad i = 1, 2, \quad (\text{B.5-b})$$

The first two terms of the left-hand side of Eq. (B.5b) are equal to zero because on the surfaces A_{up} and A_{down} , $n_1 = n_2 = 0$ (because these surfaces are horizontal) and $u_3 = 0$ (by assumption). Then, as the surfaces $A_{fs,w}$ and A_{ff} are vertical, we may write that $dA = dz dC$ (where dC is a curve in the (x,y) plane):

$$\int_{C_{fs,w}} \left(\int_{z_{down}}^{z_{up}} dz \right) n_i dA + \int_{C_{ff}} \left(\int_{z_{down}}^{z_{up}} dz \right) n_i dA = 0 \quad i = 1, 2 \quad (\text{B.6-a})$$

$$\int_{C_{fs,w}} H n_i dA + \int_{C_{ff}} H n_i dA = 0 \quad i = 1, 2 \quad (\text{B.6-b})$$

$$\int_{C_{fs,w}} n_i dA + \int_{C_{ff}} n_i dA = 0 \quad i = 1, 2 \quad (\text{B.6-c})$$

where $C_{fs,w}$ and C_{ff} are, the curves in the (x,y) plane formed by the projections of the surfaces $A_{fs,w}$ and C_{ff} , respectively, on the (x,y) plane.

B.2.1 Porosity gradient

Let us now express the derivative of the porosity:

$$\frac{\partial \phi}{\partial x_i} = \lim_{\Delta x_i \rightarrow 0} \frac{A_{f2}/A_0 - A_{f1}/A_0}{\Delta x_i} = \frac{1}{A_0} \lim_{\Delta x_i \rightarrow 0} \frac{A_{f2} - A_{f1}}{\Delta x_i}. \quad (\text{B.7})$$

where A_{f1} and A_{f2} are the fluid surfaces of two REV's centered in x_1 and x_2 , respectively. Although the areas of these two surfaces are different (because the portion occupied by the cylinders is slightly different for both REV's), the area of the surface A_0 is the same by definition of the REV. As $\Delta x_i \rightarrow 0$, the difference between A_{f1} and A_{f2} reduces to the contour integral of n_i over the curve C_{ff} (Fig. B1):

$$\frac{\partial \phi}{\partial x_i} = \frac{1}{A_0} \int_{C_{ff}} n_i dC \quad (\text{B.8})$$

By using Eq. (B.6c), we get also that:

$$\frac{\partial \phi}{\partial x_i} = -\frac{1}{A_0} \int_{C_{fs}} n_i dC. \quad (\text{B.9})$$

This last expression is interesting in the sense that it relates on one part, the detailed geometrical description of the medium in the right-hand side and on the other part, a macroscopic quantity on the left-hand side. This relation will be used in the variant of the spatial derivative theorem that will be applied in the equations to the pressure term.

B.3 Spatial derivative theorem

Suppose an extensive quantity $e(x_i, t)$, ($i = 1, 2, 3$) defined in the fluid phase of a porous medium. Using the definition of a volume-average Eq. (2.5) and Eq. (2.13), we may write:

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f = \frac{\phi \langle h \rangle_A^f}{U_f} \int_{U_f} \frac{\partial e}{\partial x_i} dU = \frac{1}{A_0} \int_{U_f} \frac{\partial e}{\partial x_i} dU \quad (\text{B.10})$$

By applying the divergence theorem (the classical one, in 3D) to the right-hand side of Eq. (B.10), this latter reads

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f = \frac{1}{A_0} \int_{A_{tot}} e dA \quad (\text{B.11-a})$$

$$= \frac{1}{A_0} \int_{A_{ff} \cup A_{fs} \cup A_{fa}} e n_i dA \quad (\text{B.11-b})$$

$$= \frac{1}{A_0} \int_{C_{ff}} E h n_i dC + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i dA \quad (\text{B.11-c})$$

where A_{tot} is the total bounding surface of the volume U_f , consisting of A_{ff} , the vertical part of this boundary lying in the fluid phase, A_{fs} , the solid-fluid interface (constituted by the bottom and the walls of the cylinders), and A_{fa} , the water surface. By substituting the first term of the right-hand side of Eq. (B.11c) using Eq. (B.3), Eq. (B.11c) becomes:

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f = \frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \langle e \rangle_U^f \right] + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i dA \quad (\text{B.12})$$

On the other part, by the definition of the volume average of e (Eq. 2.5), by Eq. (2.13) and by the definition of a water-depth average E (Eq. 2.6), we may write that:

$$\langle e \rangle_U^f = \frac{1}{U_f} \int_{U_f} e dU = \frac{1}{\langle h \rangle_A^f A_f} \int_{A_f} \left[\int_{z_b}^{z_w} e dz \right] dA = \frac{\langle E h \rangle_A^f}{\langle h \rangle_A^f}, \quad (\text{B.13})$$

By inserting Eq. (B.13) into Eq. (B.12), this latter writes

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f = \frac{\partial}{\partial x_i} \left(\phi \langle h E \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i dA, \quad (\text{B.14})$$

which is the spatial derivative theorem.

B.4 Variant

The theorem given by Eq. (B.14) may be further developed. In the first term of the right-hand side, the average of the product hE may be further decomposed. If we recall that the local value of E and h may be decomposed into their average part and their deviation part (noted δ), like suggested by Eq. (2.10), we may write that:

$$\langle hE \rangle_A^f = \left\langle \left(\langle h \rangle_A^f + \delta h \right) \left(\langle E \rangle_A^f + \delta E \right) \right\rangle_A^f \quad (\text{B.15-a})$$

$$= \left\langle \langle h \rangle_A^f \langle E \rangle_A^f + \delta h \langle E \rangle_A^f + \delta E \langle h \rangle_A^f + \delta h \delta E \right\rangle_A^f \quad (\text{B.15-b})$$

$$= \left\langle \langle h \rangle_A^f \langle E \rangle_A^f \right\rangle_A^f + \underbrace{\left\langle \delta h \langle E \rangle_A^f \right\rangle_A^f}_{=0} + \underbrace{\left\langle \delta E \langle h \rangle_A^f \right\rangle_A^f}_{=0} + \left\langle \delta h \delta E \right\rangle_A^f \quad (\text{B.15-c})$$

$$= \langle h \rangle_A^f \langle E \rangle_A^f + \langle \delta h \delta E \rangle_A^f \quad (\text{B.15-d})$$

On the other hand, the surface integral of the second term of the right-hand side of Eq. (B.14) may be decomposed as

$$\frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} en_i dA = \frac{1}{A_0} \left[\int_{A_{fs,w}} en_i dA + \int_{A_{fs,b}} en_i dA + \int_{A_{fa}} en_i dA \right] \quad (\text{B.16})$$

where we have used the fact that the solid-fluid interface A_{fs} is composed of the bottom $A_{fs,b}$ and the wetted part of the cylinders' walls $A_{fs,w}$. If we admit that (i) $\mathbf{n}$ is equal to the bottom surface gradient in the integrals over $A_{fs,b}$ ($\mathbf{n} = \nabla z_b$), that (ii) $\mathbf{n}$ is the opposite of the water surface gradient in the integrals over A_{fa} ($n_i = -\nabla z_w$), and (iii) that the integral over $A_{fs,w}$ the wetted part of the cylinders' walls, is equal to the integral over C_{fs} of the integral over the water depth, Eq. (B.16) becomes

$$\frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} en_i dA = \frac{1}{A_0} \left[\int_{A_{fs,w}} en_i dA + \int_{A_{fs,b}} e \frac{\partial z_b}{\partial x_i} dA - \int_{A_{fa}} e \frac{\partial z_w}{\partial x_i} dA \right] \quad (\text{B.17-a})$$

$$= \frac{1}{A_0} \left[\int_{C_{fs,w}} \left[\int_{z_b}^{z_w} e \, dz \right] n_i \, dC + \frac{A_f}{A_f} \int_{A_{fs,b}} e \frac{\partial z_b}{\partial x_i} \, dA - \frac{A_f}{A_f} \int_{A_{fa}} e \frac{\partial z_w}{\partial x_i} \, dA \right] \quad (\text{B.17-b})$$

$$= \frac{1}{A_0} \int_{C_{fs,w}} E h n_i \, dC + \frac{1}{A_0} \frac{A_f}{A_f} \int_{A_{fs,b}} e \frac{\partial z_b}{\partial x_i} \, dA - \frac{1}{A_0} \frac{A_f}{A_f} \int_{A_{fa}} e \frac{\partial z_w}{\partial x_i} \, dA \quad (\text{B.17-c})$$

$$= \frac{1}{A_0} \int_{C_{fs,w}} E h n_i \, dC + \phi \frac{1}{A_f} \int_{A_{fs,b}} e \frac{\partial z_b}{\partial x_i} \, dA - \phi \frac{1}{A_f} \int_{A_{fa}} e \frac{\partial z_w}{\partial x_i} \, dA \quad (\text{B.17-d})$$

$$= \frac{1}{A_0} \int_{C_{fs,w}} E h n_i \, dC + \phi \left[\frac{1}{A_f} \int_{A_{fs,b}} e \Big|_{z_b} \frac{\partial z_b}{\partial x_i} \, dA \right] - \phi \left[\frac{1}{A_f} \int_{A_{fa}} e \Big|_{z_w} \frac{\partial z_w}{\partial x_i} \, dA \right] \quad (\text{B.17-e})$$

$$= \frac{1}{A_0} \int_{C_{fs,w}} E h n_i \, dC + \phi \left\langle e \Big|_{z_b} \nabla z_b \right\rangle_A^f - \phi \left\langle e \Big|_{z_w} \nabla z_w \right\rangle_A^f, \quad (\text{B.17-f})$$

and, by decomposing E and h into their average and deviation following Eq. (B.15), we have that

$$\frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i \, dA = \frac{1}{A_0} \int_{C_{fs,w}} E h n_i \, dC + \phi \left\langle e \Big|_{z_b} \nabla z_b \right\rangle_A^f - \phi \left\langle e \Big|_{z_w} \nabla z_w \right\rangle_A^f \quad (\text{B.18-a})$$

$$= \frac{1}{A_0} \int_{C_{fs,w}} \left(\langle E \rangle_A^f \langle h \rangle_A^f + \langle E \rangle_A^f \delta h + \delta E \langle h \rangle_A^f + \delta E \delta h \right) n_i \, dC + \phi \left\langle e \Big|_{z_b} \nabla z_b \right\rangle_A^f - \phi \left\langle e \Big|_{z_w} \nabla z_w \right\rangle_A^f \quad (\text{B.18-b})$$

$$\begin{aligned}
&= \frac{1}{A_0} \int_{C_{fs,w}} \langle E \rangle_A^f \langle h \rangle_A^f n_i dC + \underbrace{\frac{1}{A_0} \langle E \rangle_A^f \int_{C_{fs,w}} \delta h n_i dC}_{\equiv 0} \\
&\quad + \frac{1}{A_0} \langle h \rangle_A^f \underbrace{\int_{C_{fs,w}} \delta E n_i dC}_{\equiv 0} + \frac{1}{A_0} \int_{C_{fs,w}} \delta E \delta h n_i dC \\
&\quad + \phi \left\langle e \Big|_{z_b} \nabla z_b \right\rangle_A^f - \phi \left\langle e \Big|_{z_w} \nabla z_w \right\rangle_A^f
\end{aligned} \tag{B.18-c}$$

$$\begin{aligned}
&= \frac{1}{A_0} \left[\langle E \rangle_A^f \langle h \rangle_A^f \int_{C_{fs,w}} n_i dC + \int_{C_{fs,w}} \delta E \delta h n_i dC \right] \\
&\quad + \phi \left\langle e \Big|_{z_b} \nabla z_b \right\rangle_A^f - \phi \left\langle e \Big|_{z_w} \nabla z_w \right\rangle_A^f
\end{aligned} \tag{B.18-d}$$

And, because of Eq. (B.9), Eq. (B.18d) may be written

$$\begin{aligned}
\frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i dA &= -\langle E \rangle_A^f \langle h \rangle_A^f \frac{\partial \phi}{\partial x_i} + \frac{1}{A_0} \int_{C_{fs,w}} \delta E \delta h n_i dC \\
&\quad + \phi \left\langle e \Big|_{z_b} \nabla z_b \right\rangle_A^f - \phi \left\langle e \Big|_{z_w} \nabla z_w \right\rangle_A^f
\end{aligned} \tag{B.19}$$

The spatial-derivative theorem Eq. (B.14) may be re-written, using Eq. (B.15) for the right-hand first term, and Eq. (B.19) for the second term:

$$\begin{aligned}
\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f &= \frac{\partial}{\partial x_i} \left(\phi \langle h E \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i dA \\
&= \frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \langle E \rangle_A^f + \phi \langle \delta h \delta E \rangle_A^f \right] + \frac{1}{A_0} \int_{A_{fs} \cup A_{fa}} e n_i dA
\end{aligned} \tag{B.19-a}$$

(B.19-b)

$$\begin{aligned}
\phi \langle h \rangle_A^f \left\langle \frac{\partial e}{\partial x_i} \right\rangle_U^f &= \frac{\partial}{\partial x_i} \left[\phi \langle h \rangle_A^f \langle E \rangle_A^f + \phi \langle \delta h \delta E \rangle_A^f \right] \\
&\quad - \langle E \rangle_A^f \langle h \rangle_A^f \frac{\partial \phi}{\partial x_i} + \frac{1}{A_0} \int_{C_{fs,w}} \delta E \delta h n_i dC \\
&\quad + \phi \left\langle e \Big|_{z_b} \nabla z_b \right\rangle_A^f - \phi \left\langle e \Big|_{z_w} \nabla z_w \right\rangle_A^f
\end{aligned} \tag{B.19-c}$$

which is a variant of the spatial derivative theorem given by Eq. (B.14).

B.5 Divergence theorem

If e_i is a three-dimensional vector, the average of its divergence results directly from Eq. (B.14):

$$\phi \langle h \rangle_A^f \left\langle \frac{\partial e_i}{\partial x_i} \right\rangle_U^f = \frac{\partial}{\partial x_i} \left(\phi \langle h E_i \rangle_A^f \right) + \frac{1}{A_0} \int_{A_{fs} \cup A_{fd}} e_i n_i dA. \quad (\text{B.20})$$

where the Einstein convention is adopted.

Appendix C Effect of depth deviation on hydrostatic pressure

A proof similar to the ones made in Appendix B.1 or B.2, linking the derivation of a surface integral over A_f to a contour integral over C_{ff} , would lead to

$$\frac{\partial}{\partial x_i} \left(\frac{1}{A_0} \int_{A_f} \frac{g(\delta h)^2}{2} dA \right) = \frac{\partial}{\partial x_i} \left(\phi \left\langle \frac{g(\delta h)^2}{2} \right\rangle_A^f \right) \quad (C.1-a)$$

$$= \frac{1}{2} g \lim_{\Delta x \rightarrow 0} \left[\phi_2 \frac{1}{A_{f2}} \int_{A_{f2}} (\delta h)^2 dA - \phi_1 \frac{1}{A_{f1}} \int_{A_{f1}} (\delta h)^2 dA \right] \quad (C.1-b)$$

$$= \frac{1}{2} g \lim_{\Delta x \rightarrow 0} \left[\frac{A_{f2}}{A_0} \frac{1}{A_{f2}} \int_{A_{f2}} (\delta h)^2 dA - \frac{A_{f1}}{A_0} \frac{1}{A_{f1}} \int_{A_{f1}} (\delta h)^2 dA \right] \quad (C.1-c)$$

$$= \frac{g}{2A_0} \lim_{\Delta x \rightarrow 0} \left[\int_{A_{f2}} (\delta h)^2 dA - \int_{A_{f1}} (\delta h)^2 dA \right] \quad (C.1-d)$$

$$= -\frac{g}{2A_0} \int_{C_{ff}} (\delta h)^2 n_i dC \quad (C.1-e)$$

$$= -\frac{1}{A_0} \int_{C_{ff}} \frac{g(\delta h)^2}{2} n_i dC \quad (C.1-f)$$

The contour C_{ff} is much smaller than the contour $C_{fs,w}$ because it is one part of the external boundary of the REV while $C_{fs,w}$ is the specific surface of the obstacles inside the REV that is far longer:

$$\frac{1}{A_0} \int_{C_{ff}} \frac{g(\delta h)^2}{2} n_i dC \ll \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_i dC \quad (C.2)$$

This implies that the integral over C_{ff} is negligible compared to the integral over $C_{fs,w}$.

$$\frac{\partial}{\partial x_i} \left(\frac{1}{A_0} \int_{A_f} \frac{g(\delta h)^2}{2} dA \right) \ll \frac{1}{A_0} \int_{C_{fs,w}} \frac{g(\delta h)^2}{2} n_i dC \quad (\text{C.3})$$

Appendix D Wave relations

D.1 Right rarefaction

Assume first a right rarefaction. This wave is associated with the $\lambda^{(3)}(\mathbf{U})$ characteristic field. Its head characteristic is given by $dx/dt = V_{xR} + c_R$ and its tail characteristic is given by $dx/dt = V_{x*} + c_*$. The generalized Riemann invariants read

$$\frac{dh}{1} = \frac{d(V_x h)}{V_x - c} = \frac{d(V_y h)}{V_y}. \quad (\text{D.1})$$

The first equality of Eq. (D.1) leads to, after some manipulations:

$$\begin{cases} V_x - 2c = cst \\ V_y = cst \end{cases} \quad (\text{D.2})$$

We obtain then a relationship between the variables ahead and behind the wave:

$V_{x*} - 2c_* = V_{xR} - 2c_R, \quad V_{y*} = V_{yR} \quad (\text{D.3})$

As the characteristics diverge in a rarefaction wave, we have also that:

$$\lambda^{(3)}(\mathbf{W}_*) < \lambda^{(3)}(\mathbf{W}_R) \text{ or } V_{x*} + c_* < V_{xR} + c_R. \quad (\text{D.4})$$

D.2 Left rarefaction

Assume a left rarefaction. This wave is associated with the $\lambda^{(1)}(\mathbf{U})$ characteristic field. Its head characteristic is given by $dx/dt = V_{x*} - c_*$ and its tail characteristic is given by $dx/dt = V_{xL} - c_L$. The generalized Riemann invariants read

$$\frac{dh}{1} = \frac{d(V_x h)}{V_x + c} = \frac{d(V_y h)}{V_y}, \quad (D.5)$$

Leading to, after some manipulations:

$$\begin{cases} V_x + 2c = cst \\ V_y = cst'' \end{cases} \quad (D.6)$$

We obtain then a relationship between the variables ahead and behind the wave:

$$\begin{aligned} V_{x*} + 2c_* &= V_{xL} + 2c_L \\ V_{y*} &= V_{yR} \end{aligned} \quad (D.7)$$

As the characteristics diverge in a rarefaction wave, we have also that:

$$\lambda^{(1)}(\mathbf{W}_L) < \lambda^{(1)}(\mathbf{W}_*) \text{ or } V_{xL} - c_L < V_{x*} - c_*. \quad (D.8)$$

D.3 Contact discontinuity

Assume a contact discontinuity. This wave is associated with the $\lambda^{(2)}(\mathbf{U})$ characteristic field. The generalized Riemann invariants read

$$\frac{dh}{0} = \frac{d(V_x h)}{0} = \frac{d(V_y h)}{1}, \quad (D.9)$$

Leading to:

$$\begin{cases} h = cst \\ V_x = cst \\ V_y \neq cst \end{cases} \quad (D.10)$$

In terms of initial data, we have:

$$\begin{aligned} h_{*L} &= h_{*R} \\ V_{x*L} &= V_{x*R} \\ V_{y*L} &\neq V_{y*R} \end{aligned} \quad (D.11)$$

As the contact discontinuity is a discontinuous wave, we can apply also the Rankine-Hugoniot relation:

$$\mathbf{F}(\mathbf{U}_{*R}) - \mathbf{F}(\mathbf{U}_{*L}) = S(\mathbf{U}_{*R} - \mathbf{U}_{*L}) \quad (\text{D.12})$$

Moreover, the parallel characteristic condition yields:

$$\lambda^{(2)}(\mathbf{U}_{*L}) = \lambda^{(2)}(\mathbf{U}_{*R}) = S \text{ or } V_{x*L} = V_{x*R} = S. \quad (\text{D.13})$$

D.4 Left shock wave

Assume a left shock. This wave is associated with the $\lambda^{(1)}(\mathbf{W})$ characteristic field. Its speed is noted S_L that is unknown. The Rankine-Hugoniot relation writes

$$\mathbf{F}(\mathbf{U}_{*L}) - \mathbf{F}(\mathbf{U}_L) = S_L(\mathbf{U}_{*L} - \mathbf{U}_L). \quad (\text{D.14})$$

In order to facilitate the application of Eq. (D.14), this latter is rewritten in a frame of reference moving at speed S_L , cancelling that way its right-hand side. After some manipulations (whose details can be consulted in Toro), the following relation is obtained:

$$V_{x*} = V_{xL} - (h_* - h_L) \sqrt{\frac{1}{2} g \left(\frac{h_* + h_L}{h_* h_L} \right)} \quad (\text{D.15})$$

Note that the wave speed S_L is undetermined. However, because of the compressive character of the characteristics on both sides of the shock wave, we may write that:

$$\lambda^{(1)}(\mathbf{W}_*) < S_L < \lambda^{(1)}(\mathbf{W}_L) \text{ or } V_{x*} - c_* < S_L < V_{xL} - c_L. \quad (\text{D.16})$$

The condition given by Eq. (D.16) is called the entropy condition.

D.5 Right shock wave

Assume a right shock. This wave is associated with the $\lambda^{(3)}(\mathbf{W})$ characteristic field. Its speed is noted S_R that is unknown. The Rankine-Hugoniot relation writes

$$\mathbf{F}(\mathbf{U}_{*R}) - \mathbf{F}(\mathbf{U}_R) = S_L (\mathbf{U}_{*R} - \mathbf{U}_R). \quad (\text{D.17})$$

In order to facilitate the application of Eq. (D.17), this latter is rewritten in a frame of reference moving at speed S_L , cancelling that way its right-hand side. After some manipulations (whose details can be consulted in Toro), the following relation is obtained:

$$V_{x*} = V_{xR} + (h_* - h_R) \sqrt{\frac{1}{2} g \left(\frac{h_* + h_R}{h_* h_R} \right)} \quad (\text{D.18})$$

Note that the wave speed S_R is undetermined. However, because of the compressive character of the characteristics on both sides of the shock wave, we may write that:

$$\lambda^{(3)}(\mathbf{W}_R) < S_R < \lambda^{(3)}(\mathbf{W}_*) \text{ or } V_{xR} + c_R < S_R < V_{x*} + c_*. \quad (\text{D.19})$$

The condition given by Eq. (D.19) is called the entropy condition.

Appendix E Results of the porosity model

This appendix is the complement of Section 6.2 in which the results of the porosity model are confronted to the experimental data and to the results of detailed simulations. Fig. (E.1) recalls the name of the axes for the aligned layouts and skewed layouts.

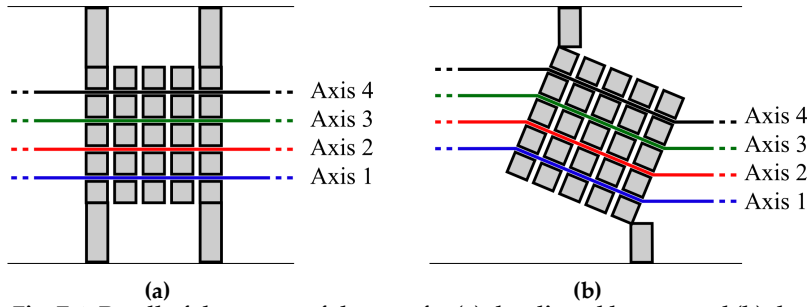


Fig. E.1: Recall of the names of the axes for (a) the aligned layouts and (b) the skewed layouts.

Fig. (E.2) shows the results for the aligned layout B in which the narrow streets are aligned with x , for $Q = 25$ l/s, $Q = 35$ l/s and $Q = 41$ l/s, along the axis 2 (Fig. E.1).

Figs. (E.3) and (E.4) show the results for the aligned layout B in which the wide streets are aligned with x , for $Q = 52$ l/s, $Q = 64$ l/s, $Q = 75$ l/s, $Q = 80$ l/s and $Q = 92$ l/s, along the axis 2 (Fig. E.1).

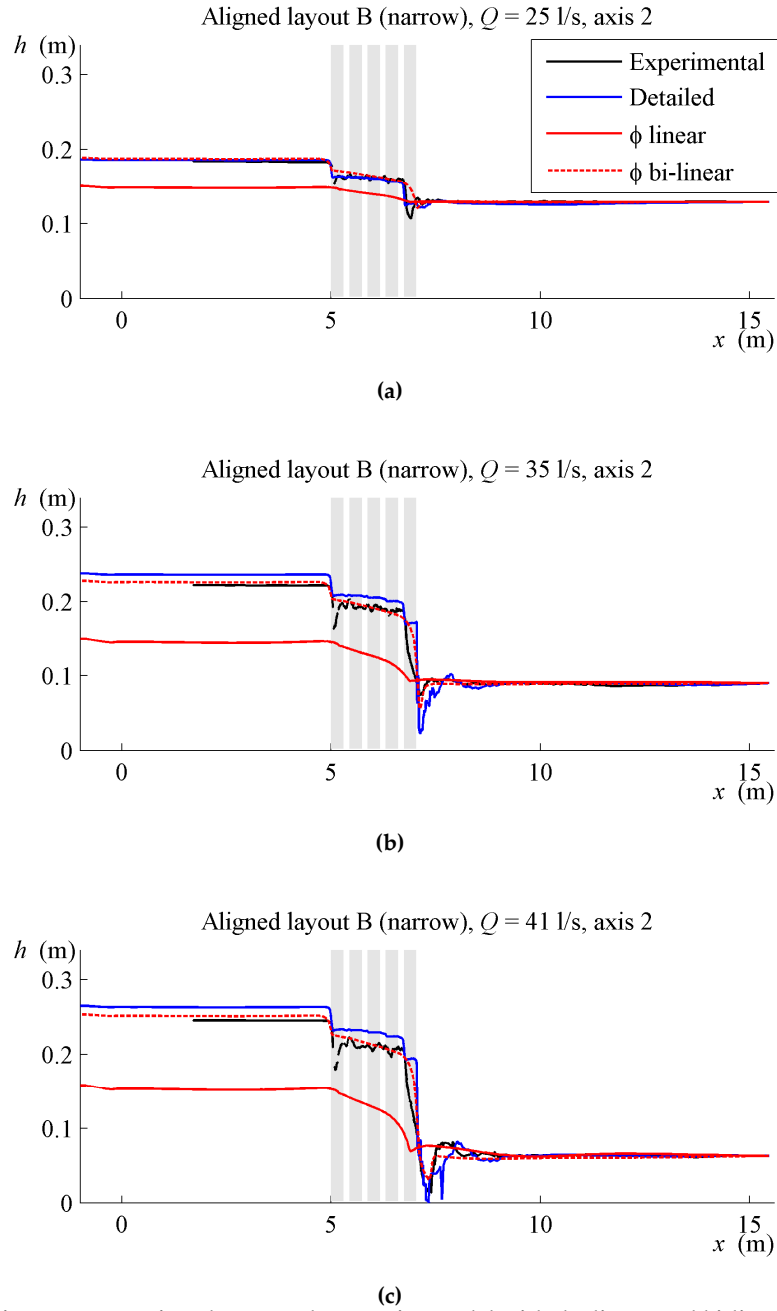


Fig. E.2: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions, the detailed simulation and the experimental data for layout B aligned in the direction of the narrow streets (a) at $Q = 25$ l/s, (b) at $Q = 35$ l/s, (c) at $Q = 41$ l/s.

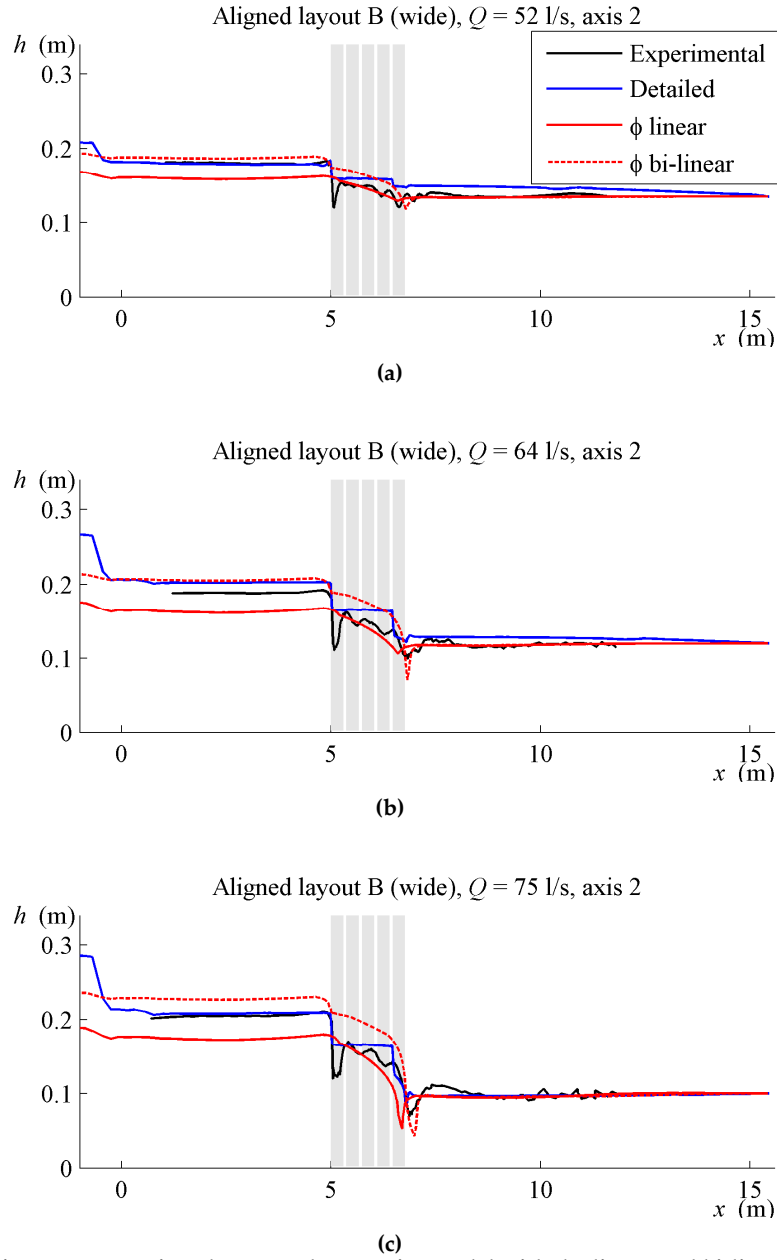


Fig. E.3: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions, the detailed simulation and the experimental data for layout B aligned in the direction of the wide streets (a) at $Q = 52$ l/s, (b) at $Q = 64$ l/s, (c) at $Q = 75$ l/s, (d) at $Q = 80$ l/s, (e) at $Q = 92$ l/s.

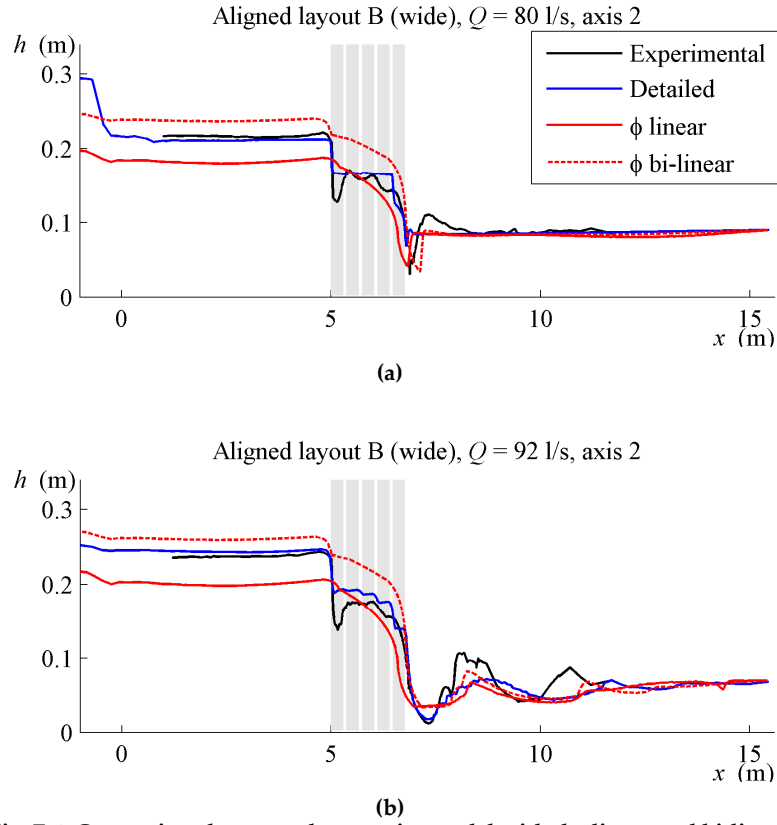
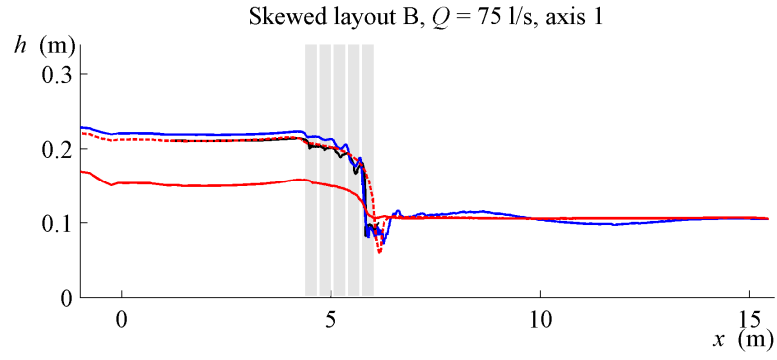
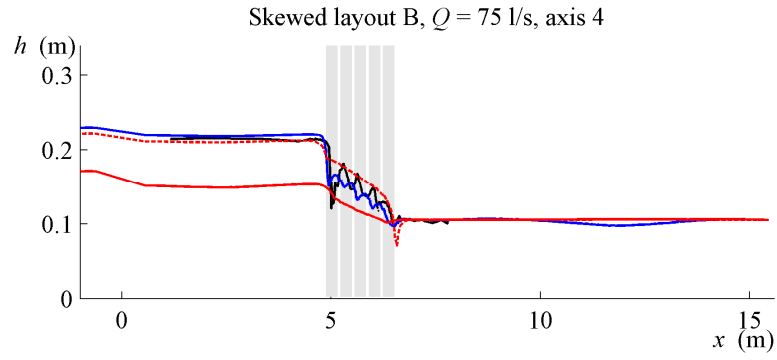


Fig. E.4: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions, the detailed simulation and the experimental data for layout B aligned in the direction of the wide streets (a) at $Q = 80$ l/s, (b) at $Q = 92$ l/s.

Fig.s (E.5) and (E.6) show the results for the skewed layout B for $Q = 75$ l/s along the axis 1 (Fig. E.5a) and along the axis 4 (Fig. E.5b), and $Q = 100$ l/s, along the axis 1 (Fig. E.6a) and along the axis 4 (Fig. E.6b).

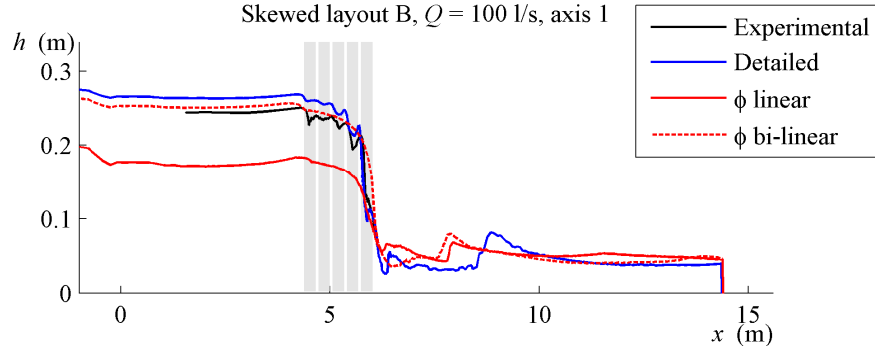


(a)

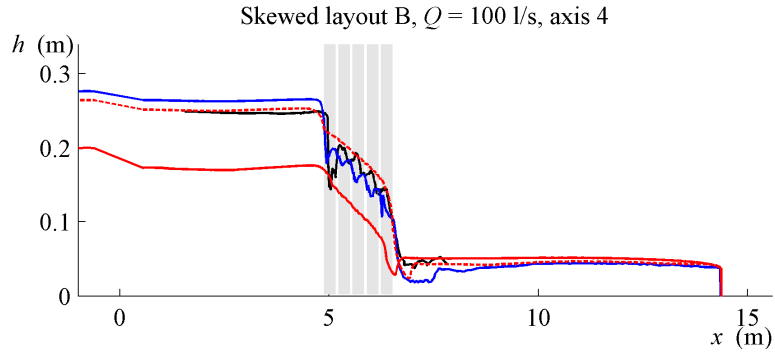


(b)

Fig. E.5: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions, the detailed simulation and the experimental data for the skewed layout B at $Q = 75$ l/s (a) along the axis 1 and (b) along the axis 4.



(a)



(b)

Fig. E.6: Comparison between the porosity model with the linear and bi-linear porosity transition assumptions, the detailed simulation and the experimental data for the skewed layout B at $Q = 100$ l/s (a) along the axis 1 and (b) along the axis 4.

