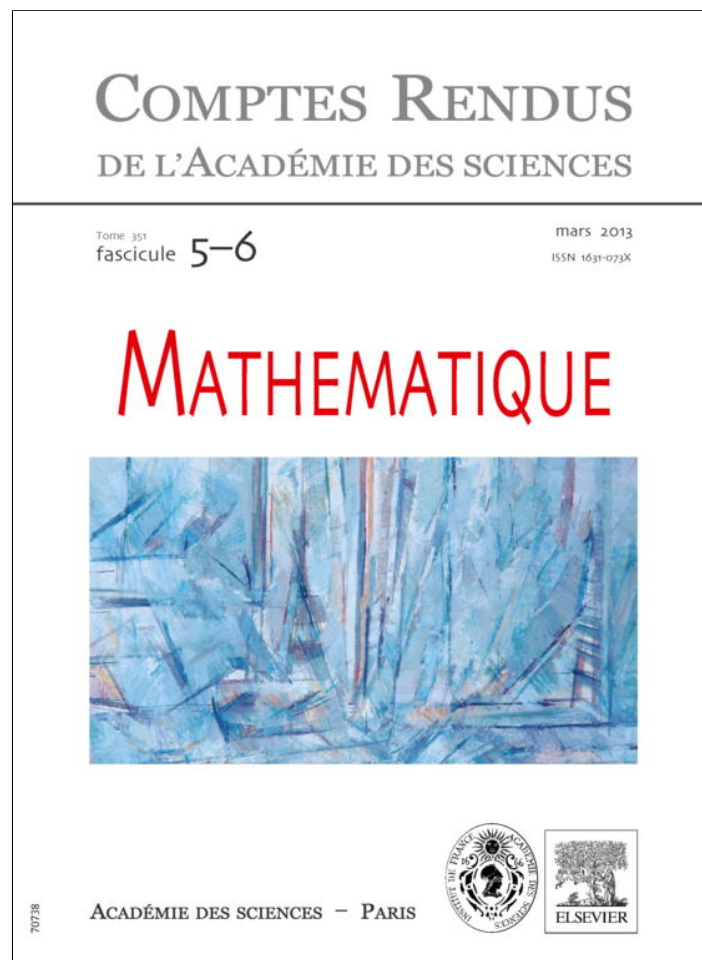




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Geometry/Topology

The L^2 -Alexander invariant detects the unknot

L'invariant d'Alexander L^2 détecte le nœud trivial

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ARTICLE INFO

Article history:

Received 7 February 2013
Accepted after revision 15 March 2013
Available online 10 April 2013

Presented by the Editorial Board

ABSTRACT

The aim of this note is to prove that the L^2 -Alexander invariant, a knot invariant defined using L^2 -torsions, detects the unknot.

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R É S U M É

Le but de cette note est de démontrer que l'invariant d'Alexander L^2 , un invariant de nœuds défini via des torsions L^2 , détecte le nœud trivial.

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Version française abrégée

À la suite des travaux initiés en 1976 par M.F. Atiyah [2], diverses versions L^2 d'invariants topologiques classiques sont apparues, comme les nombres de Betti L^2 ou la torsion L^2 . Suivant ce principe, Li et Zhang [6] ont introduit en 2006 une version L^2 du polynôme d'Alexander usuel, définie à l'aide du calcul de Fox.

Dans cette note, nous montrons que cet invariant caractérise le nœud trivial. Rappelons la définition de l'invariant d'Alexander L^2 $\Delta_K^{(2)}$ d'un nœud $K \subset S^3$: considérons $P = \langle g_1, \dots, g_k \mid r_1, \dots, r_{k-1} \rangle$ une présentation de Wirtinger du groupe Γ de K , $F_{P,1} = (\partial r_j / \partial g_i)_{2 \leq i \leq k, 1 \leq j \leq k-1} \in M_{k-1, k-1}(\mathbb{C}\Gamma)$ la matrice de Fox associée (privée de sa première colonne) et $\phi_P : \Gamma \rightarrow \mathbb{Z}$, $g_i \mapsto 1$ le morphisme d'abélianisation. Fixons $t \in \mathbb{R}_+^*$ et considérons le morphisme $\psi_{P,t} : \mathbb{C}\Gamma \rightarrow \mathbb{C}\Gamma$ défini par $\sum_{g \in \Gamma} c_g \cdot [g] \mapsto \sum_{g \in \Gamma} c_g \cdot t^{\phi_P(g)} \cdot [g]$. On considère alors $R_{\psi_{P,t}(F_{P,1})} : l^2(\Gamma)^{k-1} \rightarrow l^2(\Gamma)^{k-1}$, l'opérateur sur $l^2(\Gamma)^{k-1}$ qui agit coordonnée par coordonnée comme Γ agit sur $l^2(\Gamma)$, par multiplication à droite; on suppose, de plus, $R_{\psi_{P,t}(F_{P,1})}$ injectif. Alors, $\Delta_K^{(2)}(t) := \det_{\mathcal{N}(\Gamma)}(R_{\psi_{P,t}(F_{P,1})})$ où $\det_{\mathcal{N}(\Gamma)}$ est le déterminant de Fuglede–Kadison d'un opérateur pour le groupe Γ (cf. [7]). De plus, $\Delta_K^{(2)}(t)$ est supposé strictement positif et est défini à une puissance entière de t près (tout comme le polynôme d'Alexander $\Delta_K(t)$ était défini à $\pm t^{\mathbb{Z}}$ près). Ainsi, Li et Zhang [6] montrent que $K \mapsto \Delta_K^{(2)}$ est un invariant des nœuds de S^3 à isotopie près, à valeur dans l'espace des fonctions de $\mathbb{R}_+^*$ quotienté par les puissances, $\mathcal{F}(\mathbb{R}_+^*, \mathbb{R}_+^*) / \{t \mapsto t^m \mid m \in \mathbb{Z}\}$.

Après le rappel de quelques propriétés de l'invariant, nous annonçons le résultat principal de cette note :

Théorème 0.1. *K est le nœud trivial O si, et seulement si, $\Delta_K^{(2)} = \Delta_O^{(2)}$, avec $\Delta_O^{(2)} = (t \mapsto 1)$.*

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Pour démontrer ce résultat, nous utilisons les mêmes idées que [7, Théorème 4.7], en distinguant le cas où l'extérieur M_K de K possède des composantes hyperboliques dans sa décomposition JSJ (cf. [1]) et le cas où il n'en possède pas. Dans le premier cas, $\Delta_K^{(2)}(1) \neq \Delta_O^{(2)}(1)$ par [7, Théorème 4.6]. Dans le second cas, [9, Lemme 5.5] nous assure que K appartient à la classe de nœuds engendrée par O , la somme connexe et les câblages. Or nous établissons dans les Théorèmes 2 et 3 que $K \mapsto \Delta_K^{(2)}$ vérifie des formules simples de somme connexe et de câblages, formules qui permettent de conclure que $\Delta_K^{(2)} = \Delta_O^{(2)}$ seulement si $K = O$.

1. Introduction

Following Atiyah [2], a new class of invariants has appeared in the last thirty years: the L^2 -invariants. They give new versions of classical invariants like the Betti numbers by using tools of operator algebra. Lück's monograph [7] gives a quasi-exhaustive overview of these new invariants. In particular, the notion of L^2 -torsion has been introduced about twenty years ago by Carey–Mathai, Lott, Lück–Rothenberg, Novikov–Shubin. While the classical Reidemeister torsion was defined using operators with finite spectrum and the usual determinant, the L^2 -torsion involves operators with continuous spectrum, and a new determinant introduced by Fuglede and Kadison.

One fundamental result (cf. [7, Theorem 4.6]) is that the L^2 -torsion of a hyperbolic three-dimensional manifold is proportional to the hyperbolic volume of the manifold (in fact equal up to a factor $-\frac{1}{6\pi}$).

In 1923, Alexander introduced the first polynomial invariant for knots. Since then, many ways of defining it have been discovered. In 1962, Milnor [8] gave an interpretation of the Alexander polynomial in terms of (Abelian) Reidemeister torsions.

In 2006, Li and Zhang [6] introduced an L^2 -Alexander invariant for knots, mirroring the definition of the Alexander polynomial via Fox matrices, but using infinite dimensional operators and the Fuglede–Kadison determinant instead of the usual one. They also observed that the L^2 -Alexander invariant of a knot evaluated at $t = 1$ is precisely the L^2 -torsion of the knot complement.

The aim of this note is to prove two extension formulas for the L^2 -Alexander invariant (Theorems 2 and 3) and deduce that this invariant detects the unknot (Main Theorem).

2. Some background on L^2 -invariants

In this article, all algebras and vector spaces will be over $\mathbb{C}$. Let G be a finitely presented group.

We define $\ell^2(G)$ as the Hilbert space completion with respect to the inner product:

$$\left\langle \sum_{g \in G} c_g \cdot [g], \sum_{g \in G} d_g \cdot [g] \right\rangle = \sum_{g \in G} c_g \cdot \overline{d_g},$$

on the complex group ring $\mathbb{C}G$.

We define $\mathcal{N}(G)$, the *von Neumann algebra of G* , as the algebra of all bounded linear endomorphisms of $\ell^2(G)$ that commute with the endomorphisms L_g (where L_g is the left multiplication by $g \in G$).

The *trace* of an element $\phi \in \mathcal{N}(G)$ is defined by $\text{tr}_{\mathcal{N}(G)}(\phi) = \langle \phi([e]), [e] \rangle$, where $[e] \in \mathbb{C}G \subset \ell^2(G)$ denotes the unit element. We can extend this trace to $n \times n$ -matrices over $\mathcal{N}(G)$ by considering the sum of the traces of the diagonal entries.

Let V be a Hilbert space. V is an $\mathcal{N}(G)$ -module if V is equipped with a linear left G -action by isometries such that there exists a $\mathbb{C}G$ -linear embedding (i.e. a linear injective G -equivariant isometry) of V into an orthogonal direct sum of a finite number of copies of $\ell^2(G)$.

The *von Neumann dimension* of an $\mathcal{N}(G)$ -module V is defined by $\dim_{\mathcal{N}(G)}(V) = \text{tr}_{\mathcal{N}(G)}(\text{pr}_V) \in \mathbb{R}_+^*$, where $\text{pr}_V : \bigoplus_{i=1}^k \ell^2(G) \rightarrow \bigoplus_{i=1}^k \ell^2(G)$ denotes the orthogonal projection onto V . The von Neumann dimension does not depend on the choice of the embedding of V into a finite number of copies of $\ell^2(G)$.

A map $f : U \rightarrow V$, which is linear, G -equivariant, and bounded for the respective scalar products of U and V , will be called a *map of $\mathcal{N}(G)$ -modules*.

Let $f : U \rightarrow V$ be a map of $\mathcal{N}(G)$ -modules. The *spectral density function* of f is the function $F(f) : [0; \infty[\ni \lambda \mapsto F(f)(\lambda)$ defined as follows:

$$F(f)(\lambda) = \sup \{ \dim_{\mathcal{N}(G)}(L) \mid L \in \mathcal{L}(f, \lambda) \}$$

where $\mathcal{L}(f, \lambda) = \{ L \text{ sub-}\mathcal{N}(G)\text{-module of } U \mid \forall x \in L, \|f(x)\| \leq \lambda \|x\| \}$.

Observe that $F(f)(\lambda)$ is monotonous and right-continuous, and so defines a measure $dF(f)$ on the Borel set of $[0, \infty[$ solely determined by $dF(f)([a, b]) = F(f)(b) - F(f)(a)$ for all $a < b$.

The *Fuglede–Kadison determinant* of a map of $\mathcal{N}(G)$ -modules $f : U \rightarrow V$ is defined as

$$\det_{\mathcal{N}(G)}(f) = \exp \left(\int_{0+}^{\infty} \ln(\lambda) dF(f)(\lambda) \right)$$

if $\int_{0+}^{\infty} \ln(\lambda) dF(f)(\lambda) > -\infty$ (we then say that f is of *determinant class*) and as 0 otherwise.

The properties of the Fuglede–Kadison determinant used in this note can be found in [6] and [7].

3. The L^2 -Alexander invariants for knots

3.1. Definitions

Let us recall the definition of the L^2 -Alexander invariant for knots (see [6]). Let $K \subset S^3$ be a knot and consider a Wirtinger presentation $P = \langle g_1, \dots, g_k \mid r_1, \dots, r_{k-1} \rangle$ of the knot group $\Gamma = \pi_1(M_K)$, where $M_K = S^3 \setminus V(K)$ denotes the knot exterior ($V(K)$ being an open tubular neighborhood of K).

Consider the abelianization homomorphism $\phi_P : \Gamma \rightarrow \mathbb{Z}$ given by $g_i \mapsto 1$. For $t \in \mathbb{C}^*$, we obtain a ring homomorphism:

$$\psi_{P,t} : \mathbb{C}\Gamma \rightarrow \mathbb{C}\Gamma, \quad \sum_{g \in \Gamma} c_g \cdot [g] \mapsto \sum_{g \in \Gamma} c_g \cdot t^{\phi_P(g)} \cdot [g].$$

Let F_{P,i_0} ($1 \leq i_0 \leq k$) be the matrix obtained from the Fox matrix $F_P = (\partial r_j / \partial g_i)$ by removing its i_0 th row: $\psi_{P,t}(F_{P,i_0})$ denotes $(\psi_{P,t}(\partial r_j / \partial g_i))_{i \neq i_0} \in M_{k-1,k-1}(\mathbb{C}\Gamma)$.

The L^2 -Alexander invariant $\Delta_{K,P}^{(2)}(t)$ of the knot K with respect to the Wirtinger presentation P is defined as the Fuglede–Kadison determinant of the map $R_{\psi_{P,t}(F_{P,1})} : l^2(\Gamma)^{k-1} \rightarrow l^2(\Gamma)^{k-1}$, given by right multiplication by the matrix $\psi_{P,t}(F_{P,1})$:

$$\Delta_{K,P}^{(2)}(t) = \det_{\mathcal{N}(\Gamma)}(R_{\psi_{P,t}(F_{P,1})} : l^2(\Gamma)^{k-1} \rightarrow l^2(\Gamma)^{k-1}) \in]0, \infty[.$$

In this definition, we assume that (t, P) satisfies the following property, called Property $\mathcal{W}$: $R_{\psi_{P,t}(F_{P,1})}$ is injective and of determinant class. At the time of writing, we do not know whether there exists a pair (t, P) that does not satisfy Property $\mathcal{W}$.

One has $\Delta_{K,P}^{(2)}(t) = \Delta_{K,P}^{(2)}(|t|)$ for all $t \in \mathbb{C}^*$ (it is not difficult to extend the proof of [6, Theorem 6.1] from $|t| = 1$ to $t \in \mathbb{C}^*$). We can thus see $\Delta_{K,P}^{(2)}$ as a map from $]0, \infty[$ to itself.

The next result – due to Li and Zhang [6] – ensures that the Fuglede–Kadison determinant does not depend on the Wirtinger presentation.

Proposition 3.1. *Let P and P' be Wirtinger presentations of the knot group of K . The pair (t, P) satisfies Property $\mathcal{W}$ if and only if (t, P') satisfies it, and in this case there exists $p \in \mathbb{Z}$ such that*

$$\Delta_{K,P}^{(2)}(t) = \Delta_{K,P'}^{(2)}(t) \cdot t^p.$$

This is proved by generalizing the proof of [6, Proposition 3.4] to the case $t \neq 1$.

The proposition above allows us to define the L^2 -Alexander invariant $\Delta_K^{(2)}$ of the knot K as an element of $\mathcal{F}(\mathbb{R}_+^*, \mathbb{R}_+^*) / \{t \mapsto t^p \mid p \in \mathbb{Z}\}$. We write $f \doteq g$ if $\exists p \in \mathbb{Z}, \forall t \in \mathbb{R}_+^*, f(t) = g(t)t^p$.

Furthermore, by properties of the Fuglede–Kadison determinant and [6, Lemma 3.1] (which also holds for $t \neq 1$), one knows that if $R_{\psi_{P,t}(F_{P,j})}$ is injective (resp. injective and of determinant class) for some j , then it is injective (resp. injective and of determinant class) for all j . Therefore, whether or not $\det_{\mathcal{N}(\Gamma)}(R_{\psi_{P,t}(F_{P,j})})$ is defined and what its value is do not depend on j .

3.2. Properties of $\Delta_K^{(2)}$

The L^2 -Alexander invariant satisfies the following properties:

- (1) $K \mapsto \Delta_K^{(2)}$ is a knot invariant. It is proved in [6] by using strong Tietze equivalence between group presentations.
- (2) If K is the trivial knot O , then Property $\mathcal{W}$ is always satisfied and $\Delta_O^{(2)} \doteq (t \mapsto 1)$.
- (3) We can compute the L^2 -Alexander invariant for a non-Wirtinger presentation [5]: let K be a knot in S^3 , Γ its group, P a Wirtinger presentation of Γ with associated maps $\phi_P : \Gamma \rightarrow \mathbb{Z}$ and $\psi_{P,t} : \mathbb{C}\Gamma \rightarrow \mathbb{C}\Gamma$. Let $P' = \langle g'_1, \dots, g'_l \mid r'_1, \dots, r'_{l-1} \rangle$ be a further presentation of Γ (not necessarily a Wirtinger presentation) with deficiency one. If $t > 0$, then (t, P') satisfies Property $\mathcal{W}$ if and only if (t, P) satisfies Property $\mathcal{W}$, and in this case, for every $1 \leq i \leq l$, we have $\Delta_K^{(2)}(t) = \det_{\mathcal{N}(\Gamma)}(R_{\psi_{P,t}(F_{P,i})}) \cdot \max(1, t)^{1-\phi_P(g'_i)}$.
- (4) (Cf. [5]) If K is the (p, q) -torus knot, then for any $t > 0$ and any presentation P , the pair (t, P) satisfies Property $\mathcal{W}$, and $\Delta_{T_{p,q}}^{(2)} \doteq (t \mapsto \max(1, t)^{(p-1)(q-1)})$.
- (5) (Cf. [7, Theorem 4.6, Theorem 4.9], [4]) If K is not the trivial knot, then the 3-manifold M_K is irreducible and, according to the JSJ-decomposition (see for example [1]), splits along disjoint incompressible tori into pieces that are Seifert manifolds or hyperbolic manifolds. Furthermore $(1, P)$ satisfies Property $\mathcal{W}$ for any presentation P , the hyperbolic pieces $M_1, \dots, M_h$ have all finite hyperbolic volume vol , and

$$\Delta_K^{(2)}(1) = e^{\frac{1}{6\pi} \sum_{i=1}^h \text{vol}(X_i)}.$$

The aim of this note is to prove the following result:

Theorem 1 (Main Theorem). *Let K be a knot in S^3 . The L^2 -Alexander invariant $\Delta_K^{(2)}$ detects the unknot in the sense that: $t \mapsto \Delta_K^{(2)}(t) \doteq t \mapsto 1$ if and only if K is the trivial knot.*

The main ideas of the proof are the following: a knot K either has hyperbolic pieces in its JSJ-decomposition, or it does not. The first case is treated by Luck's fundamental theorem, recalled in Property (5). In the second case, we use the same ideas as Murakami and Murakami in [9]; we therefore need to prove that the L^2 -Alexander invariant satisfies formulas for the operations of connected sum and cabling. This is the object of the next two sections.

3.3. Connected sum formula

Here we prove that the L^2 -Alexander invariant is multiplicative under connected sum as the usual Alexander polynomial is.

Let K_1, K_2 be knots in S^3 and K their connected sum. Let us denote $\Gamma, \Gamma_1, \Gamma_2$ the respective groups of K, K_1, K_2 . There exist $P_1 = \langle x_1, \dots, x_k \mid r_1, \dots, r_{k-1} \rangle$ and $P_2 = \langle y_1, \dots, y_l \mid s_1, \dots, s_{l-1} \rangle$ Wirtinger presentations of Γ_1 and Γ_2 such that $P = \langle x_1, \dots, x_k, y_1, \dots, y_l \mid r_1, \dots, r_{k-1}, s_1, \dots, s_{l-1}, x_k y_l^{-1} \rangle$ is a Wirtinger presentation of Γ .

Theorem 2. *Using the previous notation, let $t > 0$. If Property $\mathcal{W}$ is satisfied by (t, P_1) and (t, P_2) , then (t, P) satisfies Property $\mathcal{W}$ and $\Delta_K^{(2)}(t) = \Delta_{K_1}^{(2)}(t) \Delta_{K_2}^{(2)}(t)$.*

The gist of the proof is this: the previous choice of P gives us F_P as mostly a direct sum of F_{P_1} and F_{P_2} , and the properties of the Fuglede–Kadison determinant allow us to conclude.

3.4. Cabling formula

First let us recall the definition of a satellite knot (see for example [3]):

Let C be a knot in S^3 , and P a knot inside a solid torus T_P (T_P being also embedded in S^3). Let T_C be a tubular neighborhood of C , and $h_{PC} : T_P \rightarrow T_C$ a homeomorphism between the two solid tori. We assume that C is not the trivial knot and that P meets every meridional disk of T_P . $S_{C,P} := h_{PC}(P)$ is a knot in S^3 and is called the *satellite knot of companion C and pattern P* . We call $n_P \in \mathbb{Z}$ the linking number between P and a meridian of T_P . If P is a torus knot $T(p, q)$, we call $S_{C,P}$ a *cable knot*, or the (p, q) -cable of C .

We prove that the L^2 -Alexander invariant of a cable knot can be computed from the invariant of its companion. Let us consider the cable knot S of companion C and pattern $T(p, q)$.

There exists $P_C = \langle a_1, \dots, a_k \mid r_1, \dots, r_{k-1} \rangle$ a Wirtinger presentation of $\Gamma_C = \pi_1(S^3 \setminus C)$ such that $P_S = \langle a_1, \dots, a_k, x, y, \lambda \mid r_1, \dots, r_{k-1}, x^p = \lambda^p y^q, \lambda^{-1} W(a_i), a_k^{-1} y \rangle$ is a presentation of $\Gamma_S = \pi_1(S^3 \setminus S)$, with $W(a_i)$ a word in the a_i , $i = 1, \dots, k$.

Theorem 3. *Using the previous notation, let $t > 0$. If (t^p, P_C) satisfies Property $\mathcal{W}$, then (t, P_S) satisfies Property $\mathcal{W}$, and $\Delta_S^{(2)}(t) = \Delta_C^{(2)}(t^p) \max(1, t)^{(p-1)(q-1)} = \Delta_C^{(2)}(t^p) \Delta_{T(p,q)}^{(2)}(t)$.*

3.5. Details of Main Theorem's proof

The Main Theorem states that the L^2 -Alexander invariant can detect the unknot. Let us give the details of its proof.

First, let K_0 be an arbitrary knot. If the exterior of K_0 has hyperbolic pieces in its JSJ-decomposition, then $\Delta_{K_0}^{(2)}(1) \neq 1$, by Property (5). Therefore, let us assume that $\tilde{K}$ is a knot whose exterior does not have hyperbolic pieces and such that $\Delta_{\tilde{K}}^{(2)} \doteq (t \mapsto 1)$. Let us prove that $\tilde{K}$ is the unknot.

[9, Lemma 5.5] tells us that if we call $\mathcal{K}$ the class of knots generated by the unknot, the connected sum operation, and all cabling operations (for all torus knot patterns), then $\tilde{K} \in \mathcal{K}$.

Let us prove that for each $K \in \mathcal{K}$, $\Delta_K^{(2)} \doteq (t \mapsto \max(1, t)^{n_K})$ where n_K is an integer. From Property (3), it is true for the unknot and $n_O = 0$. Secondly, if the property is true for K_1 and K_2 in $\mathcal{K}$, then, by Theorem 2, it is true for $K_1 \# K_2$ and $n_{K_1 \# K_2} = n_{K_1} + n_{K_2}$. Finally, if the property is true for $C \in \mathcal{K}$ and S is the (p, q) -cable of C , then, by Theorem 3, it is true for S and $n_S = p \cdot n_C + (p-1)(q-1)$.

Observe that $n_{K_1 \# K_2} = 0$ if and only if $n_{K_1} = n_{K_2} = 0$, and $n_S = 0$ if and only if $n_C = 0$ and $p = 1$ (i.e. the cabling operation is trivial). Therefore, the subclass $\mathcal{K}'$ of knots K' in $\mathcal{K}$ such that $n_{K'} = 0$ is exactly the class generated by O , the connected sum, and the trivial cabling operation. But this class is reduced to O . Therefore, for $K \in \mathcal{K}$, $n_K = 0$ if and only if $K = O$.

Thus, if $\tilde{K}$ is a knot whose exterior does not have hyperbolic pieces and such that $\Delta_{\tilde{K}}^{(2)} \doteq (t \mapsto 1)$, then $\tilde{K}$ is the unknot. The theorem follows.

By taking into account the indetermination in $t^{\mathbb{Z}}$ in the definition of $\Delta_K^{(2)}(t)$, we therefore only need three values of $\Delta_K^{(2)}$ to know if K is the unknot:

First in $t = 1$, we can see if M_K has any hyperbolic pieces. If it is the case, then K is not the unknot.

If there are no hyperbolic pieces, we then look at one $t_2 < 1$, for example $t_2 = 1/2$: it will give $\Delta_K^{(2)}(t_2) = t_2^m$, and we will then know the uncertainty of the form $t^{\mathbb{Z}}$ due to the particular group presentation that was chosen.

Finally, we compute $\Delta_K^{(2)}$ in $t_3 > 1$, for example $t_3 = 2$, it will be $t_3^{m+n_K}$, and we can now check whether or not $n_K = 0$, and therefore whether or not K is the unknot.

Acknowledgement

I would like to offer my special thanks to my advisor Jérôme Dubois, who taught me all about L^2 -invariants and always showed me the right mathematical direction to explore.

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