Algebraically coherent categories: definition, examples and basic properties

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joint work with Alan S Cigoli and James RA Gray

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Quick definition, motivation

A regular category with pushouts is called **algebraically coherent** when the change-of-base functors in the fibration of points preserve jointly strongly epimorphic pairs.

Why this condition?

- A variation on it appears in [S Mantovani & G Metere, 2010].
- Needed in recent work of AS Cigoli–S Mantovani–G Metere, AS Cigoli–A Montoli and M Hartl–TVdL.
- JRA Gray: (LACC) too strong to include all *categories of interest*. Need for a weaker condition which does, while still implying others such as (SH), (NH), strong protomodularity.
- Leads to non-trivial results such as the *Three Subobjects Lemma*.
- A situation where a concept in Categorical Algebra corresponds to a concept in Topos Theory. [G Janelidze, 2012]

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Full definition, basic properties I

In a finitely complete category $\mathbb{X}$, a pair (f, g) is **jointly strongly epimorphic** if and only if it is **jointly extremal epimorphic**: for each commutative diagram

$$\begin{tikzcd} & M \arrow[d, "m"] & \\ X \arrow[ur, "f'"] \arrow[r, "f"'] & Z & Y, \arrow[l, "g"] \arrow[ul, "g'"'] \end{tikzcd}$$

if m is a monomorphism, then m is an isomorphism.

When $\mathbb{X}$ is regular and admits binary sums, this happens if and only if $\binom{f}{g} \colon X + Y \to Z$ is a regular epimorphism.

A functor between regular categories with binary sums is called **coherent** when it preserves finite limits and jointly strongly epimorphic pairs.

A regular category with pushouts is called **algebraically coherent ($\mathbf{C_{Alg}}$)** when the change-of-base functors in the fibration of points are coherent.

Unless mentioned otherwise, we restrict ourselves to semi-abelian $\mathbb{X}$.

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Full definition, basic properties II

Theorem

The following conditions are equivalent:

- **i** $\mathbb{X}$ *is algebraically coherent;*
- **ii** *all kernel functors* $\mathrm{Ker} = !_B^* \colon \mathbf{Pt}_B(\mathbb{X}) \to \mathbb{X}$ *are coherent;*
- **iii** *all* $B\flat(-) \colon \mathbb{X} \to \mathbb{X}$ *preserve jointly strongly epimorphic pairs.*

Theorem

All categories of interest [G Orzech, 1976] *are algebraically coherent.*

Proposition

Algebraic coherence is stable under: taking slices and coslices; taking internal actions; taking presheaves; taking a subcategory, closed under subobjects and products—in particular, a (regular epi)-reflective subcategory; taking internal categories, groups or (pre)crossed modules.

- Groups; associative algebras; Lie, Leibniz, Poisson algebras; n-nilpotent or n-solvable groups, rings, algebras; torsion-free groups, reduced rings; crossed modules of such; n-cat-groups.

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Relation with some other categorical-algebraic conditions

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 & (\mathrm{C}_{\mathrm{Alg}}) \arrow[dl, Rightarrow] \arrow[d, Rightarrow] \\
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(LACC): $\mathbf{Gp}$, $\mathbf{Lie}_R$, $\mathbf{HopfAlg}_{K,\mathbf{coc}}$ are examples

$(\mathrm{C}_{\mathrm{Alg}})$: all *categories of interest* are examples

(SH) + (NH): *internal crossed modules* simplify
cohomology classifies higher central extensions
$[X, X] \lhd X$ is characteristic

(PA): extension is abelianised
by abelianising its kernel

(UCE): good theory of universal central extensions
Grün's Lemma

(LACC) loc. algebraically cartesian closed

[D Bourn & JRA Gray, 2012]

[G Kadjo, 2014]

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(SH) *Smith is Huq* [N Martins-Ferreira & TVdL, 2012][D Rodelo & TVdL, 2012]

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(NH) Higgins commutators of normal subobjects are normal

[AS Cigoli, JRA Gray & TVdL, 2014] [AS Cigoli & A Montoli, 2014]

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Comparing conditions in terms of the fibration of points

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$\mathbb{X}$ finitely complete

$\P_{\mathbb{X}} = \mathrm{cod}\colon \mathbf{Pt}(\mathbb{X}) \to \mathbb{X}\colon \big((p,s)\colon E \rightleftarrows B\big) \mapsto B$ is the **fibration of points**, of which the change-of-base functors are determined by pulling back.

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$$f^*(p,s) \qquad \begin{tikzcd} A \times_B E \arrow[r, dotted] \arrow[d, dotted, shift left, "f^*p"] & E \arrow[d, shift left, "p"] \\ A \arrow[r, "f"'] \arrow[u, dotted, shift left] & B \arrow[u, shift left, "s"] \end{tikzcd} \qquad (p,s)$$

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$$\begin{tikzcd} & A \times_B E \arrow[r, dotted] \arrow[d, dotted, shift left, "f^*p"] & E \arrow[d, shift left, "p"] & \\ f^*(p,s) & A \arrow[u, dotted, shift left] \arrow[r, "f"'] & B \arrow[u, shift left, "s"] & (p,s) \end{tikzcd}$$

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The change-of-base functors…	The category $\mathbb{X}$ is…
reflect isos	

Comparing conditions in terms of the fibration of points

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Comparing conditions in terms of the fibration of points

$\mathbb{X}$ **semi-abelian**

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The change-of-base functors...	The category $\mathbb{X}$ is...
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Comparing conditions in terms of the fibration of points

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$f^*(p,s)$ $\qquad$ (p,s)

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The change-of-base functors…	The category $\mathbb{X}$ is…
~~reflect isos~~	~~protomodular~~
reflect commuting of normal monos	

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The change-of-base functors…	The category $\mathbb{X}$ is…
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The change-of-base functors…	The category $\mathbb{X}$ is…
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reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular

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The change-of-base functors…	The category $\mathbb{X}$ is…
~~reflect isos~~	~~protomodular~~
reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular
preserve Huq commutators of normal monos	

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The change-of-base functors…	The category $\mathbb{X}$ is…
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reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular
preserve Huq commutators of normal monos	(SH) + (NH)

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The change-of-base functors…	The category $\mathbb{X}$ is…
~~reflect isos~~	~~protomodular~~
reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular
preserve Huq commutators of normal monos	(SH) + (NH)
preserve jointly strongly epic pairs	

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The change-of-base functors…	The category $\mathbb{X}$ is…
~~reflect isos~~	~~protomodular~~
reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular
preserve Huq commutators of normal monos	(SH) + (NH)
preserve jointly strongly epic pairs	algebraically coherent

Comparing conditions in terms of the fibration of points

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\qquad (p,s)
$$

$$
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\end{aligned}
$$

The change-of-base functors…	The category $\mathbb{X}$ is…
~~reflect isos~~	~~protomodular~~
reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular
preserve Huq commutators of normal monos	(SH) + (NH)
preserve jointly strongly epic pairs	algebraically coherent
have right adjoints	

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The change-of-base functors…	The category $\mathbb{X}$ is…
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reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular
preserve Huq commutators of normal monos	(SH) + (NH)
preserve jointly strongly epic pairs	algebraically coherent
have right adjoints	(LACC)

Comparing conditions in terms of the fibration of points

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The change-of-base functors…	The category $\mathbb{X}$ is…
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reflect commuting of normal monos	(SH)
reflect normal monos	strongly protomodular
preserve Huq commutators of normal monos	(SH) + (NH)
preserve jointly strongly epic pairs	algebraically coherent
have right adjoints	(LACC)
are equivalences	

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reflect commuting of normal monos	(SH)
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have right adjoints	(LACC)
are equivalences	abelian

Topos Theory vs. Categorical Algebra

[G Janelidze, 2012]
[PT Johnstone, 2002]

$\mathbb{X}$ regular with pushouts

Topos Theory

basic fibration $\mathbf{Arr}(\mathbb{X}) \to \mathbb{X}$
fibre: slice category $(\mathbb{X} \downarrow B)$

(loc.) cartesian closed
coherent

Categorical Algebra

fibration of points $\mathbf{Pt}(\mathbb{X}) \to \mathbb{X}$
fibre: category of points $\mathbf{Pt}_B(\mathbb{X})$

(loc.) algebraically cartesian closed
algebraically coherent

$$\mathbb{X} \text{ is } \begin{cases} \text{(LCC)} \\ \text{(LACC)} \end{cases} \quad \text{iff all } \begin{cases} f^* : (\mathbb{X} \downarrow B) \to (\mathbb{X} \downarrow A) \\ f^* : \mathbf{Pt}_B(\mathbb{X}) \to \mathbf{Pt}_A(\mathbb{X}) \end{cases} \quad \text{have right adjoints}$$

There are no coherent categories in algebra: unital + coherent $\Rightarrow$ trivial.

"(LACC) $\Rightarrow$ algebraically coherent" just like "(LCC) $\Rightarrow$ coherent"

Are there any other topos-theoretic notions with an interesting categorical-algebraic companion?

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A result, some non-examples, some examples

Theorem

Any pointed algebraically coherent Mal'tsev category is protomodular.

Are semi-abelian but not algebraically coherent:

- (commutative) loops and digroups, which do not satisfy (SH);
- non-associative algebras and Jordan algebras, which need not satisfy (NH);
- Heyting semilattices, which form an arithmetical strongly protomodular category that does not satisfy (SSH).

Are algebraically coherent:

- naturally Mal'tsev categories: all abelian, additive, affine categories;
- all coherent categories—though they cannot be semi-abelian;
- $\mathbf{Haus}^{\mathbb{T}}$, $\mathbf{HComp}^{\mathbb{T}}$ for $\mathbb{T}$ a semi-abelian algebraically coherent theory;
- $\mathbf{Ext}(\mathbb{C}) \simeq \mathbf{NMono}(\mathbb{C})$, for $\mathbb{C}$ semi-abelian algebraically coherent, and $\mathbf{CExt}_{\mathbb{B}}(\mathbb{C})$ for $\mathbb{B} \subseteq \mathbb{C}$ Birkhoff;
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If $K, L, M \lhd X$ in an algebraically coherent semi-abelian category, then

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- (C_{Alg}) captures consequences of (LACC) while still having all *categories of interest* as examples.
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Current questions:

- How to separate (C_{Alg}) from action accessibility?
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