Linearised Bipolar Power Flow for Droop-controlled DC Microgrids

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***Abstract*—The increase in efficiency and easiness to connect renewable energy sources has brought an increased interest into DC microgrids. Bipolar DC microgrids, present advantages over unipolar DC microgrids. Yet, due to their configuration they are prone to suffer from asymmetric loading which results in voltage unbalances among poles. The traditional power flow methods used in AC microgrids do not accurately reflect the steady state of DC microgrids. Equally, The power flow methods used for unipolar DC microgrids are not applicable for bipolar DC microgrids due to the behaviour of the neutral pole. In this paper, a linearised power flow method is proposed for bipolar microgrids. The proposed method allows to calculate the power flow for the positive and negative poles and to obtain the neutral pole voltage and currents resulting from load unbalances. The proposed power flow is suitable for simulating islanded or grid-connected microgrids with droop controlled generators and non-dispatchable generators such as photovoltaic pannels. The simulation results obtained using Python prove the effective performance of the proposed approach.**

***Index Terms*—Bipolar DC microgrid, DC power flow, DC microgrid**

I. Introduction

Microgrids have been proposed as an alternative to the traditional distribution systems due to their decentralized characteristics and their capability to incorporate distributed generators (DG) such as renewable energy sources and energy storage systems (ESS). In recent years, there has been a growing interest in DC distribution systems as they allow to have less power converters when implementing certain renewable sources, provide less line losses than their AC counterpart, and have a reduced system complexity due to the absence of the frequency synchronization [1].

The DC microgrids can be implemented either as a unipolar or a bipolar system. The unipolar system is a two wire system, while the bipolar system is a three wire system. The bipolar system provides a positive and a negative voltage poles that share a neutral pole. The 3 wire system proposes multiple advantages, but it is also due to this that it can suffer from voltage unbalances. The voltage unbalance results in neutral line currents that increase the total line losses of the system. Under balanced operation, the currents in each pole flow in opposite direction and cancel each other in the neutral pole as the currents are of the same magnitude. However, under unbalanced conditions the magnitudes of the currents are not equal and thus do not cancel each other [2].

A power flow analysis is fundamental to get an overall view of the system for its design and operation. It can also be used for component sizing and network optimization [3]. It also provides a view of the steady state voltages, power injections, and line flows of the system and is therefore a key tool for steady state analysis [4]. Considering the nature of the droop control, the power sharing might not be accurate. Thus, based on the power flow result analysis the droop parameter or the control strategy for the grid might need to be adapted [5]. However, the existing AC power flow methods do not reflect the steady state characteristics of a DC system. This is because of the possible absence of a slack bus, and the fact that power injections for droop controlled generators vary with voltage and thus they cannot be specified apriori [4]. When considering the power flow for bipolar microgrids, the neutral pole has to be taken into account as well. Thus, unipolar power flow methods are not able to reflect the whole state of a bipolar system. Therefore, it is of interest to develop a power flow that is well suited for bipolar DC microgrids and that reflects the consequences of the voltage unbalance so that they can be taken into account.This problem has been approached with a variety of different methods. In [6] a power flow approach using the newton-raphson method is proposed. The system considered has a variable-speed diesel-engine generation system as reference unit for the nominal voltage. It also has distributed renewable generation systems and ESSs. It solves the power flow problem and once the voltage, current, and powers are obtained it solves an optimization problem to find the amount of energy the ESSs need to inject to the grid. Then the power flow is solved again, and the process is repeated. [7] also proposes a power flow aproach using the newton-raphson method. The result of this power flow is then used to formulate an optimization problem that aims to reduce the operation cost and the voltage unbalance. A power flow modeling methodology using the Gauss-Seidel iterative method is proposed in [8]. The Gauss-Seidel method is applied to the power flow model and the load model developed. It is coupled with a multi-objective optimization method that minimizes the power loss and voltage unbalance.

The authors in [4], propose a linearised power flow for unipolar grid-connected or islanded DC microgrids. This method allows the modeling of nodes that can simultaneously contain droop controlled dispatchable generators, non-dispatchable generators, and different type of loads in the power flow. Additionally, it avoids the usage of iterative techniques like the Newton-Raphson method or the trust region method. Since it relies on solving linear equations, it does not require the computation of the Jacobian matrix thus providing a faster technique. The power flow can be solved in a single itteration unless power limits are imposed to the dispatchable generators. The developed approach allows to consider a microgrid operating in grid-connected mode, islanded mode, or connected to a stiff grid.

In this paper, the linearised power flow aproach proposed in [4] is expanded upon. The approach is first modified in order to model the characteristics of the negative pole. The equations that concern the neutral pole are also developed so that they can be integrated to the linearised power flow. The final result is a linearised bipolar power flow that retains all the advantages from [4], and allows to calculate the voltages and currents in the positive and negative pole, the resulting voltage on the neutral pole due to the voltage unbalance among the positive and negative pole, and the currents circulating in the neutral pole.

The paper is organized as follows. Section II presents the unipolar linearised power flow on which the bipolar power flow is based, and the modifications that were done in order to model the negative pole power flow. Section III details the developed bipolar power flow and the modeling of the neutral pole. In section IV the obtained results for the model are shown. Finally the conclusion is given in section V.

II. Unipolar Linearised Power Flow

In order to develop the linearised bipolar power flow, first the linearised unipolar power flow used as its building block is shown. This first method is then adapted for a unipolar system with a negative voltage, that is the negative pole. The elements in the system corresponding to the positive, negative, and neutral pole are represented by the superscripts (+/-/n) respectively.

A. Positive Pole Modeling

The power flow method used here is the one described in [4]. The approach proposed in [4] allows to compute the power flow for DC microgrids including dispatchable DG units operated in droop control mode, non-dispatchable units such as photovoltaic panels (PVs), and different types of loads. For simplicity, the development shown here only shows the constant power loads.

To solve the power flow problem an equation for the current injection at each node is needed. Equation (1) represents the current injection at a node i in a system of NB system buses per pole. The left hand side expresses the relationship between the node current I^+_{ni} and node voltages V^+ expressed through the conductance matrix G^+. The right hand side of the equation is composed of three elements. The first element corresponds to the current injection due to the droop controlled generator where V_0 is the no load reference voltage and Rd^+_i is the droop parameter. The second one is the current injection linked to the non-dispatchable generators, and the last component is the current drawn by the constant power load.

$$I^+_{ni} = \sum_{j=1;j\neq i}^{NB} G^+_{ij}V^+_j + G^+_{ii}V^+_i = \frac{V_{0i} - V^+_i}{Rd^+_i} + \frac{P^+_{ndgi}}{V^+_i} - \frac{P^+_{Pi}}{V^+_i} \tag{1}$$

Equation (1) is not linear due to the V^+_i terms found at the denominator of the elements corresponding to the non-dispatchable generator and the constant power load. In order to linearise the system, the node voltages are expressed in per unit. If expressed in per unit, the node voltage can be described in terms of the voltage deviation ΔV^+_i.

$$V^+_i = 1 - \Delta V^+_i \tag{2}$$

In [4] by defining a function $\phi(\Delta V^+_i)$, and by developing the Taylor series around zero, it is found that the higher order terms can be neglected, and thus it is possible to define (2) as:

$$\frac{1}{1-\Delta V_i} = \frac{1}{V_i} = 1 + \Delta V_i = 2 - V_i \tag{3}$$

By using (3), the right side of (1) can be written as the following linear equation:

$$I^+_{ni} = \frac{V_{0i} - V^+_i}{Rd^+_i} + (P^+_{ndgi} - P^+_{Pi})(2 - V^+_i) \tag{4}$$

Thus, the system can be expressed as a set of linear equations as shown in (5). To solve the power flow, the equations can be set as a $Ax = B$ system where x is the vector of unknown voltages.

$$\begin{aligned} &\sum_{j=1;j\neq i}^{NB} G^+_{ij}V^+_j + (G^+_{ii} + \frac{1}{Rd^+_i} - P^+_{Pi} + P^+_{ndgi})V^+_i \\ &= \frac{V_{0i}}{Rd^+_i} - 2(P^+_{Pi} - P^+_{ndgi}) \end{aligned} \tag{5}$$

B. Negative Pole Modeling

The power flow process for the negative pole is the same as for the positive pole. However, there are modifications that have to be made to take into account the current convention. As in [8] and [7], it is considered that in the negative pole nodes the loads draw current from the neutral, and the current returns to the neutral via the generators. The sign convention for the negative nodes is compared to that of the positive nodes in Fig. 1.

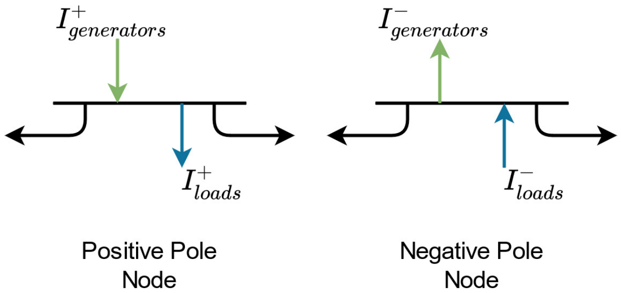

Fig. 1. Current convention at positive and negative nodes

Thus, the sign for the droop controlled source, the non-dispatchable generator, and the load are changed accordingly. These modifications are reflected in (6) through (8).

$$I_{ni}^- = \sum_{j=1;j\neq i}^{NB} G_{ij}^- V_j^- + G_{ii}^- V_i^- = \frac{-V_{0i} - V_i^-}{Rd_i^-} - \frac{P_{ndgi}^-}{V_i^-} + \frac{P_{Pi}^-}{V_i^-} \tag{6}$$

$$I_{ni} = \frac{-V_{0i} - V_i^-}{Rd_i^-} + (P_{Pi}^- - P_{ndgi}^-)(2 + V_i^-) \tag{7}$$

$$\begin{aligned} &\sum_{j=1;j\neq i}^{NB} G_{ij}^- V_j^- + (G_{ii}^- + \frac{1}{Rd_i^-} - P_{Pi}^- + P_{ndgi}^-)V_i^- \\ &= [\frac{-V_{0i}}{Rd_i^-} + 2(P_{Pi}^- - P_{ndgi}^-)] \end{aligned} \tag{8}$$

III. Bipolar Linearized Power Flow

In order to model the bipolar system, the methods described above for the positive pole, and the negative pole are used in conjunction. A third element has to be added which is the representation for the neutral line. The same process described in the previous section is applied to the entire system. Therefore, the objective is to solve the set of linear equations by representing the whole as a system of linear equations in the form of **Ax=B**. The positive and negative pole are independent from each other and thus no further modification is needed. However, the neutral pole voltage depends on the voltage of both the positive and negative pole. Therefore, the equation for the neutral pole has to be expanded upon.

In order to model the neutral pole, the flow of the currents coming from the positive and negative pole has to be modeled. As in [8] and [7] the following convention is adopted. It is considered that the drooped controlled generators and the non-dispatchable generators in the positive pole draw current from the neutral. The loads on the positive pole inject current into the neutral. The opposite is true for the negative pole, the generators inject current into the neutral, and the loads draw from it. The previously described convention is illustrated in Fig. 2.

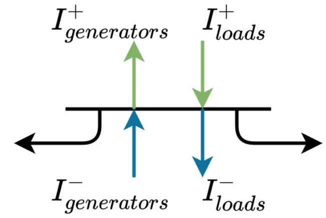

Fig. 2. Current convention at neutral nodes

In order to apply the same process for solving the power flow, an equation for the current at the neutral node similar to the one shown in (1) is needed. Considering the current convention, it is possible to determine that the current at a the neutral node can be described as follows:

$$I_{Neutral} = (I^+_{loads} + I^-_{generators}) - (I^+_{generators+} + I^-_{loads}) \tag{9}$$

To fully express the current of the neutral node, an equation in the shape of (1) and (6) is formulated. However, unlike the positive and negative pole nodes, the neutral nodes do not have any generators or loads themselves. Therefore the resulting equation would be:

$$I_{ni}^n = \sum_{j=1;j\neq i}^{NB} G_{ij}^n V_j^n + G_{ii}^n V_i^n = I_{Neutral} \tag{10}$$

Thus, substituting the right hand side of (10) with (4) and (7) and rearranging one can obtain (11). When substituting (4) and (7) it is important to adjust the signs of the generators and loads according to the neutral node convention described before in (9).

$$\begin{aligned} &\sum_{j=1;j\neq i}^{NB} G_{ij}^n V_j^n + G_{ii}^n V_i^n - (\frac{1}{Rd_i^+} - P_{Pi}^+ + P_{ndgi}^+)V_i^+ \\ &- (\frac{1}{Rd_i^-} - P_{Pi}^- + P_{ndgi}^-)V_i^- = \\ &- (\frac{V_{0i}^+}{Rd_i^+} - 2(P_{Pi}^+ - P_{ndgi}^+)) + (\frac{V_{0i}^-}{Rd_i^-} - 2(P_{Pi}^- - P_{ndgi}^-)) \end{aligned} \tag{11}$$

One final consideration that has to be added to the system is the grounding point. A single neutral node has to be assigned as the grounding point. Therefore, one neutral node has to be modeled according to (12) instead of (11) .

$$V_i^+ + V_i^- - V_i^n = 0 \tag{12}$$

Thus, by taking equations (5), (8), (11), and (12), it is possible to build a system of linear equations into matrices of the form **Ax=B**. This system can be easily solved to find the voltages and currents of the whole bipolar grid.

IV. Results

In order to demonstrate the developed linear model, two different simulations will be carried out. The first one will display how the currents cancel in a balanced grid and how the currents in the neutral appear when there is an unbalance in a microgrid with the same configuration in the positive and negative pole. The second simulation models a microgrid that has different elements in the positive and negative pole. These tests are done in a microgrid that contains 4 nodes in each pole and has a reference voltage of 750 V. The nodes are grouped in 4 sets named from A to D. Each set contains the positive, negative, and neutral nodes that corresponds to them. The line lengths and resistances of the microgrid are given in Table I.

TABLE I
USED LINE PARAMETERS

Line	Length [m]	Resistance [Ω]
A to B	250	0.0265
B to C	200	0.0212
C to D	300	0.0318

A. Unbalance Demonstration

The microgrid configuration for the first test is shown in Fig. 3. The microgrid has two equal droop controlled generators on each pole that feed three loads each. These three loads have an equal power demand as shown in the parameters in Table II.

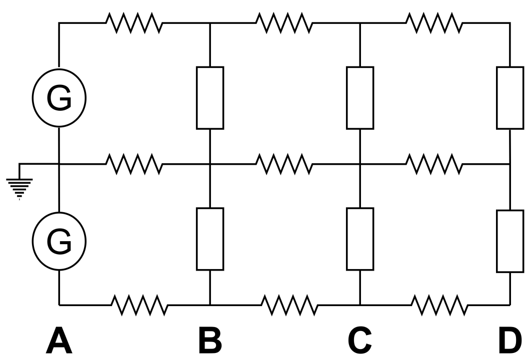

Fig. 3. DC test grid for unbalance demonstration

TABLE II
UNBALANCE TEST GRID PARAMETERS

DG Droop Parameters		
No Load reference voltage	750	[V]
Rd_A^+	0.2296	[Ω]
Rd_A^-	0.2296	[Ω]
Load Parameters		
Base load	10	[kW]

In a first simulation, the loads in the positive and negative pole are kept in a 1:1 ratio which means that each load has a demand of 10 kW. Table III and Table IV show the first results obtained. It is seen that the voltages on both poles are the same and are thus balanced. Due to the symmetry of the microgrid, the currents are the same in each pole. These currents with the same magnitude cancel each other and therefore there is no current on the neutral.

TABLE III
NODE VOLTAGES 1:1 LOAD RATIO

Set	$V^+[p.u]$	$V^n[p.u]$	$V^-[p.u]$
A	0.9876	0	-0.9876
B	0.9861	0	-0.9861
C	0.9854	0	-0.9854
D	0.9848	0	-0.9848

TABLE IV
LINE CURRENTS 1:1 LOAD RATIO

Line	$I_n^+[A]$	$I_n^n[A]$	$I_n^-[A]$
A to B	40.58	0	-40.58
B to C	13.52	0	-13.52
C to D	0.9	0	-0.9

A second power flow is performed, but the ratio of the loads is changed to 1.2 to 1. This means that the loads on the positive pole represent 12 kW each while the loads on the negative pole represent 10 kW each. The results for this power flow are shown in Table V and Table VI. It is shown that the difference in the ratio results in an unbalance that is reflected in the voltage of the neutral pole. As the poles are independent from each other, the results for the negative pole are the same as before. However, the positive pole has changed due to the load variation. The currents no longer have the same magnitude. It is seen that this results in currents that flow in the neutral cable.

TABLE V
NODE VOLTAGES 1.2:1 LOAD RATIO

Set	$V^+[p.u]$	$V^n[p.u]$	$V^-[p.u]$
A	0.9850	0.0025	-0.9876
B	0.9833	0.0028	-0.9861
C	0.9824	0.0030	-0.9854
D	0.9817	0.0031	-0.9848

TABLE VI
LINE CURRENTS 1.2:1 LOAD RATIO

Line	$I_n^+[A]$	$I_n^n[A]$	$I_n^-[A]$
A to B	48.84	-8.26	-40.58
B to C	32.57	-5.51	-27.06
C to D	16.29	-2.76	-13.54

B. Microgrid Demonstration

The microgrid used for this demonstration is shown in Fig. 4. The positive pole has two PVs which are the non-dispatchable generators, one droop controlled generator, and a load. On the negative pole there are two droop controlled generators and two loads. The parameters used for this power flow are shown in Table VII. In the case of the droop controlled generators on the negative pole, DG_A^- has a smaller droop coefficient and is thus expected to provide more power than the other DG on the negative pole DG_D^-. The line parameters remain as shown in Table I.

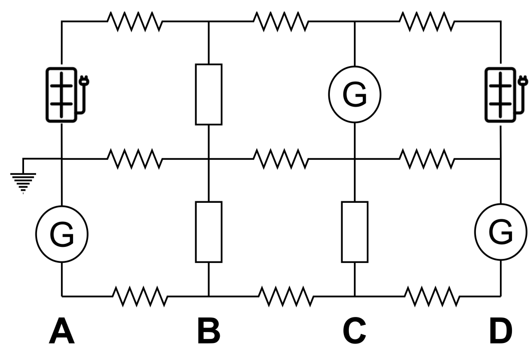

Fig. 4. DC test grid for microgrid demonstration

TABLE VII
TEST MICROGRID PARAMETERS

DG Droop Parameters		
No Load reference voltage	750	[V]
Rd_A^-	1.12	[Ω]
Rd_C^+	5.64	[Ω]
Rd_D^-	4.63	[Ω]
PV Parameters		
P_{ndgA}^+	24	[kW]
P_{ndgD}^+	18	[kW]
Load Parameters		
P_{PB}^+	72.5	[kW]
P_{PB}^-	32.5	[kW]
P_{PC}^-	54	[kW]

The previously described system is then solved using the linearised power flow, and the voltages and currents of the nodes are found. The obtained voltages and currents are shown in Table VIII and Table IX. The resulting voltages for the positive and negative nodes are shown in Fig. 5, and the resulting neutral node voltages are shown in Fig. 6. As the results show, there is degree of unbalance in the microgrid. This results is evidenced by the high currents in the neutral pole.

TABLE VIII
OBTAINED MICROGRID NODE VOLTAGES

Set	$V^+[p.u]$	$V^n[p.u]$	$V^-[p.u]$
A	0.9840	-0.0053	-0.9787
B	0.9829	-0.0066	-0.9763
C	0.9848	-0.0092	-0.9756
D	0.9858	-0.0082	-0.9776

TABLE IX
OBTAINED MICROGRID LINE CURRENTS

Line	$I_n^+[A]$	$I_n^n[A]$	$I_n^-[A]$
A to B	32.51	36.93	-69.44
B to C	-65.81	90.89	-25.08
C to D	-24.34	-24.34	48.68

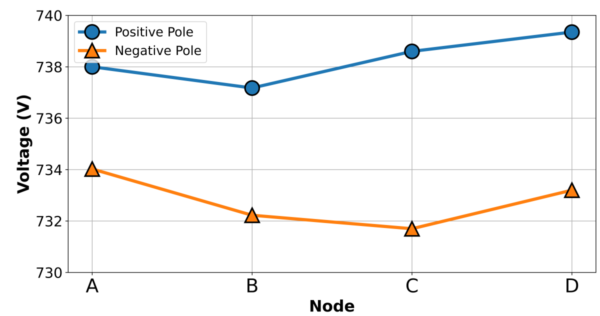

Fig. 5. Positive and negative pole voltages

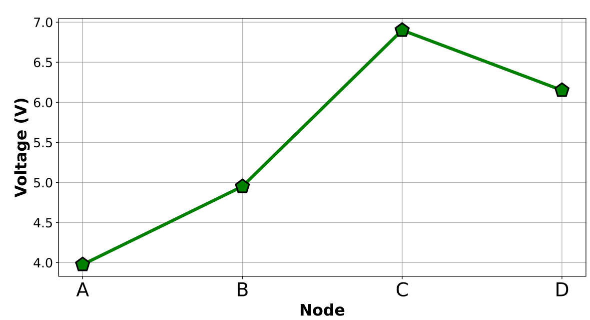

Fig. 6. Neutral pole voltages

The power injected by each droop controlled source is shown in Table X. On the negative pole, it is shown that DG_A^- injects more power than DG_D^- as it was specified by the droop parameters. On the positive pole DG_C^+ provides less power than what the load is demanding as the remaining part is complemented by the PV pannels P_{ndgA}^+ and P_{ndgD}^+.

TABLE X
OBTAINED DG POWER INJECTION

DG Power Injection		
DG_A^-	50.97	[kW]
DG_C^+	30.63	[kW]
DG_D^-	35.7	[kW]

The system shown here is simplified for demonstration purposes. In a more complex system, each node could have loads of different types, a droop controlled generator, and a non-dispatchable generator. The proposed linearised powerflow method is able to solve these types of systems. Furthermore, it is possible to impose power limits to the dispatchable generators if the power flow is allowed to iterate. Due to the complexity of bipolar mirogrids, it is thus of interest to develop tools as the one shown that allow to study the steady-state of the system.

V. CONCLUSION

The linearised power flow presents an approach that allows for easily solving the system of linear equations that model the bipolar microgrid. It allows for the inclusion of dispatchable generators, non-dispatchable generators, and different types of loads. In this paper, the linearised power flow was successfully adapted in order to model a bipolar DC microgrid. The modifications needed for the negative pole were shown, and the modeling of the neutral pole was detailed. The results show a successful implementation of the power flow, and its importance in the study of bipolar systems. Future work includes the utilization of the described method systematically for the sizing and analysis of microgrids. This will involve the inclusion of batteries while taking into account the state of charge of them. The systematic approach would allow to evaluate the evolution of the energy sharing among the different elements in the microgrid, and how does the interconnection benefit the members of it in terms of auto-consumption and self-sufficiency.

VI. ACKNOWLEDGEMENT

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