Teaching versus Research:
a Model of State University Competition.

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Abstract

This paper analyzes a competition game between two universities that teach and research in the same jurisdiction. The resulting equilibrium is unique and symmetric but differs according to preferences, technologies and public policy. The budget for university finance is exogenously given and consists of a lump-sum amount and a per-student allocation. Under this finance structure, we are able to identify four types of equilibria characterized, respectively, by full-time teaching, full-time research, selective teaching plus research and mass teaching plus research. Conditions for each of them to take place are derived. By manipulating the parameters of the finance scheme the government can, in some cases, determine final levels of research and education quality.

Keywords: education, research, university competition.

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1 Introduction

Public provision of higher education in Europe has traditionally made of this sector a highly immobile and uncompetitive one. Some papers on university behavior (Garvin(80), or Boroah (94)) reflect this view. They view the university as an isolated utility maximizing entity that undergoes teaching and research activities, subject to a budget constraint imposed by the government. In fact, universities and governments in these models can be thought of as the same institution, since the utility maximizing behavior of the university is analogous to that of a government with a social welfare function. Gary-Bobo and Trannoy (1998a) first dissociates these two elements, by modelling a principal-agent relationship between the government and the university that has the local monopoly of higher education provision.

Increasing mobility of students, however, has made that, even in Europe, where the higher education system is still characterized by a prominent role of the government, universities have become more competitive. In particular, some universities have started competing for students in both the national and international markets.

As pointed out by Rothschild and White (1995), students are both inputs and clients for the services provided by a university. Hence, there are two basic reasons why universities may compete for students. As inputs, they are scarce resources required for the production process of universities. As clients students provide, either directly, through fees (USA) or indirectly, via the government (Europe) the funds a university needs to operate.

Of course, the link between funds and enrolments need not exist at all when, like in Europe, higher education is financed by general taxation. But there is evidence that most European countries allocate the global funds to one university in the form of a lump-sum transfer on one side and a per-student allocation on the other.[1] There is a simple reason why this is so. The budget to higher education needs to be allocated among all the operating institutions. Since the universities in one country are not all alike, it does not seem sensible to equally divide the global funds among them. The social optimum would require that each university be endowed with the funds required to cover its costs. However, the calculation of the costs (or the output) of one university is not an easy task. The number of enrolled students turns out to be the easiest way to measure the relative activity, or size, of one university.

The increasing role of competition in higher education has been the object of some academic interest. Gary-Bobo and Trannoy (1998b) is concerned by the welfare effects of increased competition and concludes that whether competition can lead to optimal outcomes or not depends on the incentives given by the government through the finance scheme. In a more positive approach Debande and Demeulemeester (1998) develops a rich model of two-dimensional competition between universities. For simplicity, both these papers concentrate on the education provision role of universities, leaving research out of the scope of their analysis.

In this paper, we consider two universities that teach and research. They operate in the same jurisdiction and are financed by the same government. Following the evidence for Europe, funds are allocated to the universities in the form of a lump-sum and a per-student amount. We are ultimately interested in the implications of this finance scheme. In particular, we investigate the role of the government in resulting levels of research and quantity/quality of education when it can control for the relative weight between these two instruments of finance.

We consider a continuum of students (of mass 1) that differ in location as well

[1] The calculation of the allocations to universities is actually way more complicated than this. But most of the complicated ratios developed to evaluate the cost of a university are based on the number of students. See Kaiser et al.(1992).

as in ability (or ex-ante productivity). University attendance raises this initial productivity in an amount that depends on the average initial productivity of the class and the amount of resources devoted to teaching. At the beginning of the period universities announce by how much they plan to increase initial student productivity. In other words, they announce the quality or teaching they will provide. Given this information, students choose the university they prefer to attend. Funds (or time) not spent in teaching are devoted to research.

Universities care for education quality but they also care for research. We find that, as expected, if the marginal productivity of teaching is high enough they will only teach. In this case, whether funds are received in the form of lump-sum or per student allocations has no impact: global funds are devoted to full-time teaching. If, on the other hand, the marginal utility of research is high and per student allocations small, universities will devote all funds to research. However, if per student allocations are large as compared to lump-sum allocations universities may be forced to teach a maximum number of students if they want to get the funds required to do (more rewarding) research. They may thus engage in competition for quantity of students. All the market will then be covered but the resulting levels of research and teaching quality may be indirectly determined by the government through the choice of the finance scheme.

Finally, the education technology may be such that able students require few investment to obtain large levels of additional productivity. Then, with high preferences for research, it may still be worth teaching a few good students even though the economic support they provide to the institution is not large. The equilibrium admissions level, inferior to the size of the market, cannot be affected by the finance scheme. However, the resulting levels of research and teaching quality ultimately depend once again on the government's will.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyzes in detail the second stage of the game between the universities. Section 4 deals with the first stage of the game and presents the subgame perfect Nash equilibria together with the role of the government. Finally, Section 5 will conclude.

2 The Model

Consider two universities operating in a same jurisdiction. Each of them "produces" N_i graduates of labor productivity (or quality) q_i and do as well some research, R_i.

The funds available to the universities are provided by the government and may be tied to the number of enrolled students. With these funds, the universities can hire teachers that increase the productivity of the students and/or researchers which provide the institution with higher utility levels. The more able the admitted students, the lower the number of teachers required to provide the desired q, and the larger the available funds for research. This is the reason why from the pool of potential students (demand), only the best will be admitted. Failure is assumed away so that enrolments equal final graduate supply.

The timing of the game is the following: In Period 1 each university chooses the level of quality that it wants to provide. Then, at Period 2, applying students sort out between the two universities according to these announced qualities. Finally, in Period 3, the universities will choose the number of admissions from their respective pool of applicants and the proportion of the funds devoted to teaching that will allow them to provide the quality announced in Period 1.

We solve this game by backward induction. Once q_1 and q_2 are known, student's demand and universities' admissions together with relative fund allocation between teaching and research are determined.

2.1 The Students

At the beginning of the second period the universities have already decided on their desired levels of quality q_i $(i = 1, 2)$. Student's demands for the universities are then sorted.

Students are distributed in a two-dimensional space: each student is characterized on the one hand by x, her location along the line (0,1) that separates the two universities, and, on the other, by a, her initial ability or productivity (either determined at birth or by the family background). We assume for simplicity that a and x are uniformly and independently distributed on $[0, 1] \times [0, 1]$.

Let University 1 be placed at 0 and University 2 at 1. Education raises this individual initial productivity a by q_i, the quality of education provided at the university he attends. A student located at x faces a mobility cost cx if attending University 1 and $c(1 - x)$ if attending University 2. Therefore, the indirect utility function of one student located at x with initial ability a when attending University 1 and University 2 is, respectively

$$\begin{aligned} u_1 &= a + q_1 - cx \text{ , or} \\ u_2 &= a + q_2 - c(1 - x). \end{aligned}$$

Students first apply to the university that provides them with the highest utility level. If not admitted in their first choice university they apply to the other one. The total population of students is normalized to one. Let x^* be the location of the student that is indifferent between attending university 1 or 2:

$$q_1 - cx^* = q_2 - c(1 - x^*) \tag{1}$$

Every student located to the left of x^* will first apply to University 1. The demand (*applications*) for the services provided at this university will be then given by:

$$x^* = \frac{c + q_1 - q_2}{2c} \equiv D_1(q_1, q_2) \tag{2}$$

which is independent of a. Likewise, University's 2 demand is given by

$$1 - x^* = \frac{c - q_1 + q_2}{2c} \equiv D_2(q_1, q_2). \tag{3}$$

Note that the only determinant of demand is location, x. This is due to the fact that ability does not affect students' preferences for universities, since it is given and cannot be influenced by education. Although it does not affect demand, ability plays an important role at the admission level. As we will shortly see, by selecting only the best students among the pool of applicants, universities are able to attain a given quality announcement at a lower cost in terms of teaching. This way ability endows the universities with an additional degree of freedom in their payoff maximization.

Students rejected from their first choice university automatically apply to the other one. If rejected also from the latter, they will keep their initial ability a. In what follows, c is fixed to 1 for simplicity.

2.2 The Universities

Competition for students Consider first a single university, say University 1. Given the number of applicants, this university can control for the average ability of its students by limiting the admissions. It can for example give an exam to all its applicants and admit only the students with the best results[2].

[2] Alternatively, universities can admit all students and select by failure at the end of the first year.

By limiting admissions, University 1 can provide the announced quality at a lower cost in terms of teaching (this point will be made clear later).

Formally, if individual ability varies between zero and one, individuals of ability one will be admitted first. As the number of admissions increases, the ability level of the last entrant will go down. If ability is uniformly distributed between zero and one and $a_1 \in [0,1]$ is the ability of the last student enrolled at University 1, $(1 - a_1)$ will represent the range of abilities corresponding to students admitted to this university. We can also say that a_1 is the grade obtained in the entrance exam by the last student enrolled. Given the number of applications, D_1(i.e., given q_1, q_2), the limiting grade a_1 determines the number of admissions, $N_1 \in [0, D_1]$:

$$N_1 = (1 - a_1)D_1 \tag{4}$$

Hence, the minimum grade that an applicant shall get in the entrance exam in order to be admitted at University 1(or minimum ability of the applicant) is:

$$a_1 = 1 - \frac{N_1}{D_1} \tag{5}$$

and average ability of students enrolled at University 1 will be

$$\overline{a}_1 = \frac{1 + a_1}{2}. \tag{6}$$

When we have two universities, students rejected from their first choice university can still apply to the other. In fact, we assume that students prefer to attend their second choice university rather than to be left out of the higher education system. If rejected from one of the universities they will try to attend the other.

If University 1 is the more demanding university (in terms of required ability) its limiting admission grade is higher and enrollments are determined according to (4). In this case University 2 does not affect average ability of students at 1. If, however, University 1 is the less demanding of the two universities it will attract all the students rejected from 2 who satisfy its own lower requirements. The effective number of admissions to University 1 when it is the less demanding university for given q_1, q_2 will thus be

$$N_1 = (1 - a_1)D_1 + (a_2 - a_1)D_2, \tag{7}$$

where $a_1 \in [0, 1]$, $a_2 \in [0, 1]$, $N_1 \in [0, D_1]$ and $a_2 > a_1$. The average ability of students at the less demanding university will thus depend on how selective the tough university is. Figure 1 provides two graphical examples clarifying this point. It represents the number of students enrolled at each university when $a_1 < a_2$ (with $D_2 > D_1$) and $a_1 > a_2$ (for $D_1 > D_2$). In the sequel, we identify Regimes A and B with the cases $a_1 < a_2$ and $a_1 > a_2$ respectively.

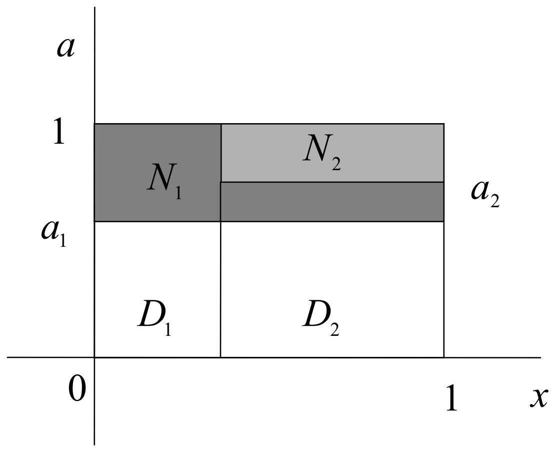

Figure 1.a Regime A: $a_1 < a_2$

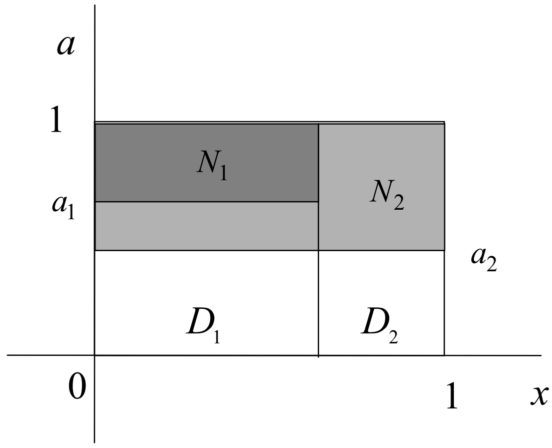

Figure 1.b Regime B: $a_1 > a_2$

Average ability of students at University 1 when it is the less demanding university is then:

$$\overline{a}_1 = \frac{1}{N_1}\left(D_1(1-a_1)\frac{1+a_1}{2} + D_2(a_2-a_1)\frac{a_1+a_2}{2}\right) \tag{8}$$

or, substituting by the values for $a_{1,}$ a_2 obtained from (5) and (7) and simplifying:

$$\overline{a}_1 = \frac{1}{2}\left(\frac{N_2^2}{N_1}\frac{D_1}{D_2} - N_1 + 2 - 2N_2\right). \tag{9}$$

Note that there is a univocal relationship between a and N that makes these two variables interchangeable as decision variables. In particular, *from (4) and (7),* $a_1 < (\geq) a_2$ iff $\dfrac{N_1}{D_1} > (\leq) \dfrac{N_2}{D_2}$.

Summarizing, average ability of students at University 1 will be:

$$\overline{a}_1 = \begin{cases} 1 - \dfrac{N_1}{2D_1(q_1,q_2)} & \text{if} \quad N_2 \geq \dfrac{D_2(.)}{D_1(.)}N_1 \\ \frac{1}{2}\left(\dfrac{N_2^2}{N_1}\dfrac{D_1(q_1,q_2)}{D_2(q_1,q_2)} - N_1 + 2 - 2N_2\right) & \text{if} \quad N_2 < \dfrac{D_2(.)}{D_1(.)}N_1 \end{cases} \tag{10}$$

Similarly, average ability of students at University 2 is:

$$\overline{a}_2 = \begin{cases} \frac{1}{2}\left(\frac{N_1^2}{N_2}\frac{D_2(q_1,q_2)}{D_1(q_1,q_2)} - N_2 + 2 - 2N_1\right) & \text{if} \quad N_1 < \frac{D_1(.)}{D_2(.)}N_2 \\ 1 - \frac{N_2}{2D_2(q_1,q_2)} & \text{if} \quad N_1 \geq \frac{D_1(.)}{D_2(.)}N_2 \end{cases} \tag{11}$$

Production of education. N_i graduates of equal productivity q_i are produced by University i according to a very simple production function of the form[3]:

$$q_i \leq \alpha\,\overline{a}_i + \beta\,t_i \tag{12}$$

where $\overline{a}_i$ is the average ability of admitted students just described and $t = \frac{T}{N}$ are funds devoted to teaching per student [4]. The parameters α and β thus represent the productivity of average ability of enrolled students and teaching per student respectively.

The funds not allocated to teaching finance research, R. One of the aims of the university is to improve the productivity of its students through teaching. The larger the average ability of students $\overline{a}_i$, the less teaching is required to accomplish this goal and the more funds can be devoted to research.

Note that, even in the case in which universities devote no funds to teaching, mere enrolment can generate an increase in student's ex-ante productivities. This can be interpreted either as a screening or a network effect that makes university attendance always beneficial for our students. In other words q has a positive lower bound $\alpha\overline{a}$.

The university receives funds from a central government in the form of a fixed amount F and a positive per student allocation s that it devotes entirely to paying the teachers a salary in exchange of which they teach and/or research.

[3]This linear specification is bound to yield extreme solutions. Results shall then be interpreted in this manner.

[4]If funds devoted to teaching are used to hire teachers at a wage w normalized to 1, this is the *teacher-to-student* ratio, $t = \frac{1}{w}\frac{T}{N}$.

We can normalize the exogenous wage w to 1 to obtain the simple budget constraint:

$$F + sN_i \geq T_i + R_i.$$

The objective of the universities. Finally, the objective of the university is to maximize the increase in total productivity of its students $N_i q_i$ plus total utility derived from research, γR_i. γ will thus represent the relative weight of research in the payoff of the university. This is all summarized in the following maximization program:

$$\begin{aligned} & Max\ U_i = N_i q_i + \gamma R_i \\ & \text{subject to} \quad \alpha\, \overline{a}_i + \beta\, \tfrac{T_i}{N_i} \geq q_i \\ & \text{and to} \quad F + sN_i \geq T_i + R_i \end{aligned} \tag{13}$$

Both constraints are always binding (if they were not, we could always increase research and thus the resulting payoff) so that we can first substitute T_i from the budget constraint into the production function which yields

$$R_i = F + sN_i + \frac{\alpha}{\beta} N_i\, \overline{a}_i(N_1, N_2, q_1, q_2) - \frac{q_i}{\beta} N_i. \tag{14}$$

Then, substituting R_i in the payoff function gives

$$U_i = N_i \left((1 - \frac{\gamma}{\beta}) q_i + \gamma \frac{\alpha}{\beta}\, \overline{a}_i \right) + \gamma (F + sN_i) \tag{15}$$

where $\overline{a}_i = \overline{a}_i(N_1, N_2, q_1, q_2)$. We are aware that nonnegativity constraints should be taken into account. However, and for the sake of simplicity of exposition, all these constraints will be checked ex-post.

The game between the universities The payoffs of the universities are given by (15). The game has two stages. In the first stage each university simultaneously chooses the quality of education that it wishes to provide q_i. Then, in the second stage, they select from the pool of applicants the number of admissions N_i. We solve the game backwards.

3 Admissions and fund allocation

Due to the nature of the interaction of the universities through the average ability of the admitted students, the way $\overline{a}_i (i = 1, 2)$ is determined differs according to whether $a_1 < a_2$ or $a_1 \geq a_2$ as it has been established above. We can consider then the problem under two different regimes: Regime A corresponding to University 2 being the most demanding ($a_2 \geq a_1$) and Regime B, under which University 1 is the most demanding. It is important to note that, a priori, there is no reason why the most demanding university should be the high quality one.

3.1 University 1

3.1.1 Regime A: $N_2 < N_1 \dfrac{D_2(q_1, q_2)}{D_1(q_1, q_2)}$ equivalent to $a_1 < a_2$.

University 1 takes N_2 as given and its maximization problem is in this case:

$$\underset{N_1}{Max} \left((1 - \frac{\gamma}{\beta}) q_1 + \gamma \frac{\alpha}{\beta} \overline{a}_1^A \right) N_1 + \gamma (F + sN_1)$$

where from (10)

$$\overline{a}_1^A = \frac{1}{2} \alpha (\frac{N_2^2}{N_1} \frac{D_1(q_1, q_2)}{D_2(q_1, q_2)} - N_1 + 2 - 2N_2).$$

Solving this maximization problem gives the optimal number of admissions $N_1^A(N_2, q_1, q_2)$:

$$N_1^A(N_2, q_1, q_2) = \frac{(\alpha + s\beta)\gamma + (\beta - \gamma) q_1}{\alpha \gamma} - N_2 \tag{16}$$

which is valid if

$$N_1^A(N_2, q_1, q_2) > \frac{D_1(q_1, q_2)}{D_2(q_1, q_2)} N_2.$$

This requires that N_2 be not too high, i.e.,

$$N_2 < \frac{(\alpha + s\beta)\gamma + (\beta - \gamma)q_1}{\alpha\gamma} D_2(q_1, q_2) \equiv \overline{N}_2.$$

From (16) the reaction function $N_1^A(N_2, q_1, q_2)$ is decreasing with slope -1 whenever $N_2 < \overline{N}_2$.

3.1.2 Regime B. $N_1 \leq \frac{D_1(q_1, q_2)}{D_2(q_1, q_2)} N_2$ $(a_1 \geq a_2)$.

Average ability of students at University 1 when this is the most demanding institution is:

$$\overline{a}_1^B = 1 - \frac{N_1}{2D_1(q_1, q_2)}$$

So that the maximization problem is now:

$$\underset{N_1}{Max} \quad N_1 \left((1 - \frac{\gamma}{\beta}) q_1 + \gamma (\frac{\alpha}{\beta} (1 - \frac{N_1}{2D_1(q_1, q_2)}) + s) \right) + \gamma F.$$

The solution of this problem yields the optimal number of admissions for University 1 under Regime B

$$N_1^B(q_1, q_2) = \frac{(\alpha + s\beta)\gamma + (\beta - \gamma)q_1}{\alpha\gamma} D_1(q_1, q_2) \tag{17}$$

that is independent of N_2. Recall Figure 1: since at Regime B University 1 is the most demanding institution its choice of a (and hence of N) is unaffected by that of University 2.

The reaction function $N_1^B(q_1, q_2)$ is, however, only valid if

$$N_1^B(q_1, q_2) \leq N_2 \frac{D_1(q_1, q_2)}{D_2(q_1, q_2)} \iff \frac{(\alpha + s\beta)\gamma + (\beta - \gamma)q_1}{\alpha\gamma} D_2(q_1, q_2) \equiv \overline{N}_2 \leq N_2.$$

As a result, the best response function $N_1(N_2, q_1, q_2)$ is continuous in N_2.

$$N_1(N_2, q_1, q_2) = \begin{cases} N_1^A(N_2, q_1, q_2) & \text{if} \quad N_2 < \overline{N}_2 \\ N_1^B(q_1, q_2) & \text{if} \quad N_2 \geq \overline{N}_2 \end{cases} \tag{18}$$

The threshold value $\overline{N}_2$ corresponds to the level of N_2 at which universities's marginal abilities equalize ($a_1 = a_2$). When N_2 is smaller than $\overline{N}_2$, $a_2 > a_1$. As N_2 increases from zero, the ability of the last student admitted to 2 is decreasing and, from (7), so is that of University 1 for a given N_1[5]. University 1 is then forced to decrease its admissions (increase the admission grade a_1) in order to maintain a certain average ability. The marginal ability at university 1 thus increases as that of 2 decreases from its maximum value. Once N_2 increases over $\overline{N}_2$ (a_1 becomes larger than a_2) marginal ability at University 2 does no longer affect that of 1(see (4)). Knowing this, University 1 then chooses the number of students that maximizes its payoff under these circumstances and stays there, unaffected by further changes in N_2. The best reply function is illustrated in Figure 2 below.

[5] q_1 and q_2 are given at this stage, hence demand is fixed.

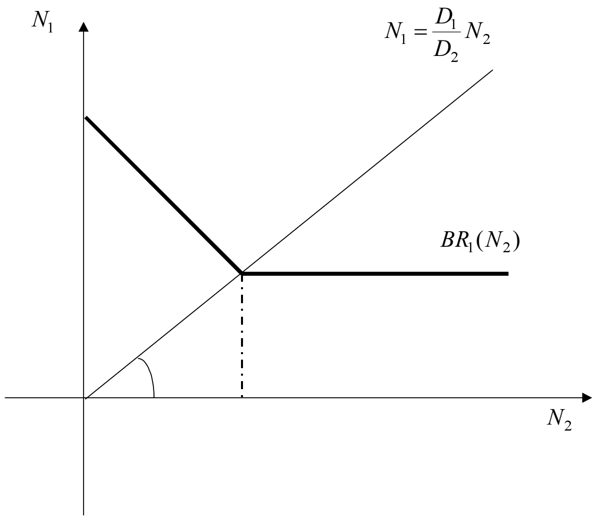

Figure 2

3.2 University 2

Since the two universities are symmetric, the best response of University 2 can be obtained by simply permuting the subscripts 1 and 2. Hence, under Regime A ($a_1 \leq a_2$), while

$$N_1 \leq \frac{(\alpha + s\beta)\gamma + (\beta - \gamma)q_2}{\alpha\gamma} D_1(q_1, q_2) \equiv \overline{N}_1.$$

the best response to N_1 given q_1, q_2 is

$$N_2^A(q_1, q_2) = \frac{(\alpha + s\beta)\gamma + (\beta - \gamma)q_2}{\alpha\gamma} D_2(q_1, q_2).$$

Under Regime B, when $N_1 \geq \overline{N}_1$, the best response is

$$N_2^B(N_1, q_1, q_2) = \frac{(\alpha + s\beta)\gamma + (\beta - \gamma)q_2}{\alpha\gamma} - N_1.$$

Summarizing:

$$N_2(N_1, q_1, q_2) = \begin{cases} N_2^A(q_1, q_2) & \text{if} \quad N_1 \leq \overline{N}_1 \\ N_2^B(N_1, q_1, q_2) & \text{if} \quad N_1 \geq \overline{N}_1 \end{cases} \tag{19}$$

Figure 3 below illustrates the reply function $N_2(N_1, q_1, q_2)$ keeping the same axes as in Figure 2.

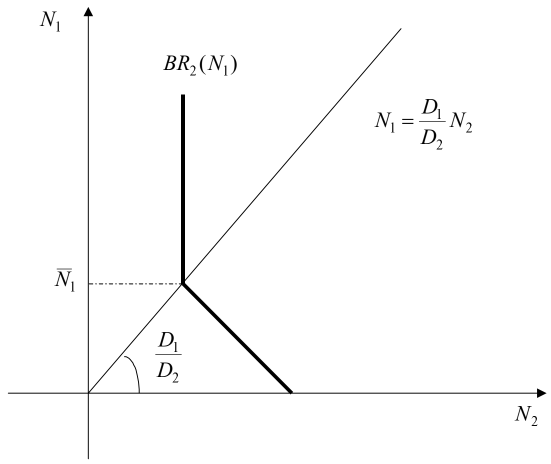

Figure 3

3.3 Nash equilibria of the admissions game

The last section has made clear that the Best Response Functions will be continuous in spite of the changes of regime. There are two candidate equilibria represented in Figure 4.

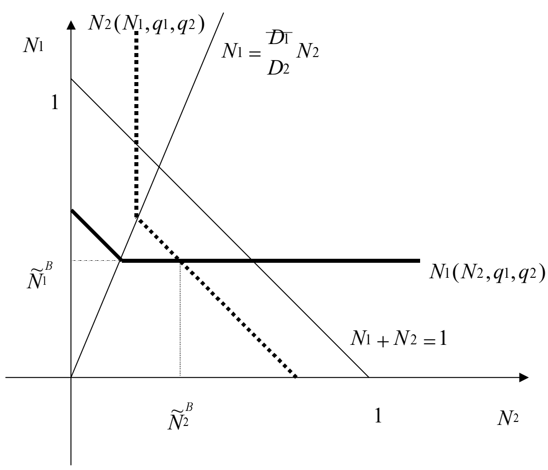

Figure 4.a ($q_1 > q_2$)

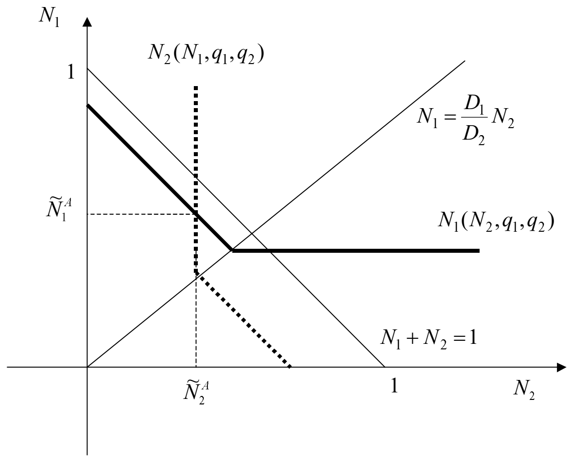

Figure 4.b ($q_1 < q_2$)

Under Regime B ($a_1 > a_2$) the candidate Nash equilibrium is, from (18) and (19)

$$\tilde{N}_1^B = \frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\gamma}D_1, \text{ and} \tag{20}$$

$$\tilde{N}_2^B = \frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_2}{\alpha\gamma} - \frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\gamma}D_1. \tag{21}$$

Similarly, under Regime A ($a_1 \leq a_2$) the candidate equilibrium is:

$$\tilde{N}_1^A = \frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\gamma} - \frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_2}{\alpha\gamma}D_2 \tag{22}$$

$$\tilde{N}_2^A = \frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_2}{\alpha\gamma}D_2. \tag{23}$$

However, these candidate equilibria may not be feasible, as the choices of the admission levels by the universities is limited by the constraint $N_1 + N_2 \leq 1$. By adding up both $\tilde{N}_1^A + \tilde{N}_2^A$ and $\tilde{N}_1^B + \tilde{N}_2^B$ we can see that this constraint will only be satisfied if

$$s\beta\gamma + (\beta-\gamma)q_1 \leq 0 \quad \text{and} \quad s\beta\gamma + (\beta-\gamma)q_2 \leq 0 \tag{24}$$

respectively. If (24) is not satisfied, equilibrium will result at a corner solution. Note that, if $\beta-\gamma \geq 0$, then, $s\beta\gamma+(\beta-\gamma)q_i > 0$ and (24) can *never* be satisfied for any positive q. Only a corner solution is possible when $\beta > \gamma$.

Indeed, it is easy to see, from (20) and (23), that when $\beta > \gamma$ desired admissions will be larger than available demand for each university under any of the two possible regimes. For this reason, they will be constrained in their choices of N by demands D_1 and D_2. At the same time, research will always pay less than teaching so that it will be zero at equilibrium.

Furthermore, as it is shown in Appendix 1, this corner solution is unaffected by government decisions concerning financing schemes. Since we are ultimately interested by the effect of governmental decisions on the university, γ will be assumed larger than β in the rest of the paper.

Coming back to the conditions under which the candidate Nash equilibria of the subgame are feasible, and using $\gamma-\beta > 0$, (24) can be expressed as

$$q_1 \geq \frac{s\beta\gamma}{\gamma-\beta} \text{ and } q_2 \geq \frac{s\beta\gamma}{\gamma-\beta}.$$

An interior solution will result if the given values of q, chosen at the first stage of the game between the universities, are large enough. In this case the universities will be selective and will not accept all applications. However, if the quality levels chosen at stage 1 are too low, the universities will try to enroll as many students as possible and will again be constrained by the total size of the market, at a corner solution.

We know, on the other hand, that, under Regime A, it must be the case that $\widetilde{N}_1^A > \widetilde{N}_2^A \dfrac{D_1(q_1,q_2)}{D_2(q_1,q_2)}$. Hence,

$$\frac{(\alpha+s\beta)}{\alpha}D_1 + \frac{\beta-\gamma}{\alpha\gamma}(q_1 - q_2 D_2) > \frac{(\alpha+s\beta)}{\alpha}D_1 + \frac{(\beta-\gamma)}{\alpha\gamma}q_2 D_1$$

which is equivalent to $q_1 < q_2$ since $D_1 + D_2 = 1$.

Similarly, for Regime B, $\widetilde{N}_1^B \leq \frac{D_1}{D_2}\widetilde{N}_2^B$ implies

$$\frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\gamma}D_2 \leq \frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_2}{\alpha\gamma}-\frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\gamma}D_1$$

now equivalent to $q_1 \geq q_2$, so that *equilibrium will result at Regime A* $(a_1 < a_2)$ *when* $q_1 < q_2$ and at *Regime B* $(a_1 \geq a_2)$ *when* $q_1 \geq q_2$.

This means that, at equilibrium with $\gamma > \beta$, the high quality university has the highest average ability of students. Given q,the high ability of admitted students allows for less teaching, so that more funds will be devoted to research.

Let us now define the constrained best response functions $\widehat{N}_1(N_2)$ and $\widehat{N}_2(N_1)$ and the corresponding constrained equilibria or corner solutions valid when (24) is *not* satisfied. When $q_i < \frac{s\beta\gamma}{(\beta-\gamma)}$, University i would like to enroll a lot of students, but is in fact constrained either by its own demand or by the number of students left in the market by the other university

$$\begin{aligned}\widehat{N}_1(N_2) &= 1-N_2 \quad \text{if} \quad q_1 < \frac{s\beta\gamma}{(\beta-\gamma)} && (25)\\ \widehat{N}_2(N_1) &= 1-N_1 \quad \text{if} \quad q_2 < \frac{s\beta\gamma}{(\beta-\gamma)} && (26)\end{aligned}$$

Note that when both $q_1 < \frac{s\beta\gamma}{(\beta-\gamma)}$ and $q_2 < \frac{s\beta\gamma}{(\beta-\gamma)}$ the constrained equilibrium becomes $\widehat{N}_1(N_2) = D_1$ and $\widehat{N}_2(N_1) = D_2$.

Proposition 1 summarizes our findings concerning the Second Stage of the game. Without loss of generality, $q_1 \geq q_2$.

Proposition 1 *Given* $q_1 \geq q_2$, *(and under the assumption* $\gamma > \beta$*) the unique Nash equilibrium of the subgame is:*

1. $\quad (\widetilde{N}_1^B, \widetilde{N}_2^B) \quad$ if $\quad q_1 \geq q_2 \geq \dfrac{s\gamma\beta}{\gamma - \beta}$

2. $\quad (\widetilde{N}_1^B, 1 - \widetilde{N}_1^B) \quad$ if $\quad q_1 \geq \dfrac{s\gamma\beta}{\gamma - \beta} > q_2$

3. $\quad (D_1, D_2) \quad$ if $\quad \dfrac{s\gamma\beta}{\gamma - \beta} > q_1 > q_2$

Figure 5 below represents the possible Nash equilibria of the subgame at the second stage as functions of qualities.

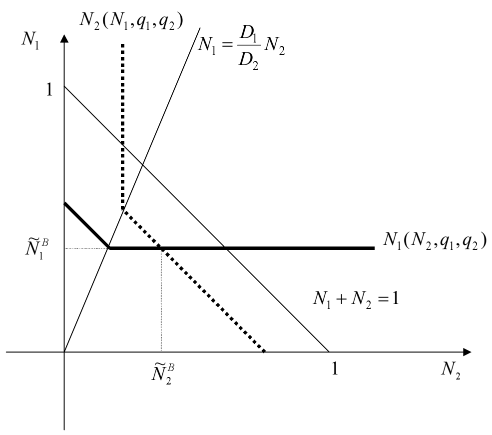

Figure 5

Summarizing, if the announced values of q_i by one university is large enough it will have to be selective in its admissions policy if it wants to be able to provide such an announced level of quality without sacrificing much research. If announced quality is low, so that it can be provided with low use of resources, there is no need to be selective and the university will admit every student left in the market after the other university's choice of admissions. If the other university has announced a low quality as well, then each of them will be constrained by its own demand.

4 Quality Choice

Having fully characterized the game at the second stage, we now consider the choice of q by the universities when $\gamma > \beta$. At the first stage, the two universities simultaneously choose the values of q_1, q_2 that they wish to provide. The chosen qualities will determine the equilibrium at the second stage. We know that according to the chosen q *and* the particular specification of the parameters α, β, γ different Nash equilibria of the subgame at the second stage may result. In particular, and as we will shortly see, the unique equilibrium is symmetric[6] and may involve selective teaching and positive amounts of research (Section 4.1), mass teaching and research (Section 4.2) or only research (Section 4.3).

4.1 Selective Teaching and Positive Research

According to Proposition 1, if $q_1 \geq q_2 \geq \dfrac{s\beta\gamma}{\gamma-\beta}$ the interior Nash equilibrium of the subgame at the second stage is determined by $(\widetilde{N}_1^B, \widetilde{N}_2^B)$. The payoff of University 1 at the first stage is then

$$U_1^S(\widetilde{N}_1^B, \widetilde{N}_2^B) = \widetilde{N}_1^B(q_1, q_2)\ q_1 + \gamma R(\widetilde{N}_1^B(q_1, q_2)).$$

Substituting the equilibrium values of $N_1$and N_2 at the second stage, (20) and (21) into this payoff and rearranging we obtain:

$$\begin{aligned} U_1^S(q_1, q_2) \quad &= \quad \gamma F + \frac{(\alpha + s\beta)^2\gamma}{2\alpha\beta} D_1(q_1, q_2) + \frac{(\beta-\gamma)(\alpha+s\beta)\gamma}{\alpha\beta\gamma} q_1\, D_1(q_1, q_2) \\ &\quad + \frac{(\beta-\gamma)^2}{2\alpha\beta\gamma} q_1^2\, D_1(q_1, q_2). \end{aligned}$$

Similarly, university 2 chooses q_2 that maximizes

$$U_2^S(\widetilde{N}_1^B, \widetilde{N}_2^B) = \widetilde{N}_2^B(q_1, q_2)\ q_2 + \gamma R(\widetilde{N}_2^B(q_1, q_2)).$$

[6] Appendix B shows that no asymmetric equilibrium can take place

Substituting for $\widetilde{N}_1^B, \widetilde{N}_2^B$ and rearranging we obtain the payoff function:

$$\begin{aligned} U_2^S(q_2,q_1) &= \gamma F + \frac{(\alpha+s\beta)^2\gamma}{2\alpha\beta} D_2(q_1,q_2) + \frac{(\beta-\gamma)(\alpha+s\beta)\gamma}{\alpha\beta\gamma} D_2(q_1,q_2)\, q_2 \\ &\quad + \frac{(\beta-\gamma)^2}{2\alpha\beta\gamma}(q_1^2 D_1(q_1,q_2) + q_2^2 - 2q_1q_2D_1(q_1,q_2)) \end{aligned}$$

The equilibrium qualities are those that simultaneously maximize the following payoffs. The first order conditions are:

$FOC(q_1)$:

$$\frac{(\alpha+s\beta)^2\gamma}{4\alpha\beta} + \frac{(\beta-\gamma)(\alpha+s\beta)}{\alpha\beta}(\,D_1(.) + \frac{q_1}{2}) + \frac{(\beta-\gamma)^2}{4\alpha\beta\gamma} q_1(4D_1(.) + q_1) = 0$$

$FOC(q_2)$:

$$\begin{aligned} &\frac{(\alpha+s\beta)^2\gamma}{4\alpha\beta} + \frac{(\beta-\gamma)(\alpha+s\beta)}{\alpha\beta}(D_2(.) + \frac{q_2}{2}) + \\ &\frac{(\beta-\gamma)^2}{4\alpha\beta\gamma}(4q_2 + 2q_1(q_2 - q_1 - 4D_1(.))) = 0 \end{aligned}$$

This system has three solutions of which only the following satisfies the set of second order conditions:

$$q_1 = q_2 = q^S = \frac{(\alpha+s\beta)\gamma}{\gamma-\beta} - 2 \tag{27}$$

which plugged into (20) and (21) give

$$N_1 = N_2 = N^S = \frac{\gamma-\beta}{\alpha\gamma}.$$

Notice that q^S must be larger than or equal to $\dfrac{s\beta\gamma}{\gamma-\beta}$ for this interior equilibrium to exist (Point 1 in *Proposition 1*). A necessary condition is thus

$$\alpha\gamma \geq 2(\gamma-\beta). \tag{28}$$

This means that $\frac{2(\gamma-\beta)}{\alpha\gamma} < 1$ and hence $N^S = \frac{(\gamma-\beta)}{\alpha\gamma} < \frac{1}{2}$: the market is not covered at equilibrium[7]. In other words, the universities are *selective* and admit only some of the applicants. This leads to the following Proposition.

Proposition 2 *Given the assumption $\gamma > \beta$, and if $\alpha \geq 2(1-\frac{\beta}{\gamma})$, a single symmetrical subgame perfect Nash equilibrium exists at* $(q, N) = \left(\frac{(\alpha+s\beta)\gamma}{\gamma-\beta} - 2, \frac{\gamma-\beta}{\alpha\gamma}\right)$. *This equilibrium involves selective teaching and positive research.*

We are now in the position to give the corresponding equilibrium levels of research and teaching at this interior equilibrium. Substituting (20) and (27) into (14) we obtain:

$$R^S = F + \frac{\gamma^2-\beta^2}{\alpha\,\beta\,\gamma^2} - \frac{1}{\gamma} - s\frac{\beta}{\gamma\alpha}. \tag{29}$$

Which plugged into the budget constraint gives the equilibrium level of teaching:

$$T^S = \frac{1}{\gamma} + \frac{s}{\alpha} - \frac{\gamma^2-\beta^2}{\alpha\,\beta\,\gamma^2}. \tag{30}$$

Proposition 3 *At the Selective Teaching and Research Equilibrium the finance scheme is the final responsible for the levels of research and education quality: revenue neutral increases in s, the per-student allocation, reduce research levels and increase teaching and education quality. The admissions policy is however independent from the finance scheme and will remain selective.*

Proof. From (29) and (30) we learn that per student allocation s has a positive effect on teaching (which *does not depend on lump-sum finance*) and

[7] In terms of marginal ability a this condition implies that $a_1 = 1 - N_1 > 0$.

a negative one on research levels. In particular, an increase in the per-student allocation s paired with a reduction in F required to keep the budget balanced[8] has the following effects on the arbitrage between teaching and research:

$$\begin{aligned} \frac{dR^S}{ds} - N^S \frac{dR^S}{dF} &= -\frac{1}{\alpha} < 0 \\ \frac{dT^S}{ds} &= \frac{1}{\alpha} > 0. \end{aligned}$$

The number of admissions is however unaffected by the finance scheme. This is due to the fact that, from the production function (12), when α is large some teaching to good students (with high average ability) is profitable even though $\gamma > \beta$ since it enables to produce a high q with a small part of the funds and still devote a large part to the most rewarding activity in this case: research. The optimal admissions level is thus a function of the relative values of α, β and γ and independent from the finance scheme.

Since the admissions level is unaffected, the effect of compensated changes in s on the resulting quality is

$$\frac{dq^S}{ds} = \frac{\beta\gamma}{\gamma - \beta} > 0.$$

Notice also that $\dfrac{dq^S}{ds} = \dfrac{\beta}{N^S}\dfrac{dT^S}{ds}$: the direct effect on q_i of the change in teaching induced by s.

Therefore, at the Selective Teaching and Positive Research Equilibrium, lump-sum university finance favors research whereas per-student allocations enhance teaching activities and hence resulting education quality. ■

4.2 Mass Teaching and Positive Research

We have seen in the previous Section that each University will try to enroll as many students as possible if their choice of quality lies below $\dfrac{s\beta\gamma}{\gamma - \beta}$. If

[8] Global funds are exogenously given.

$\frac{s\beta\gamma}{\gamma-\beta} > q_1 \geq q_2$ the payoff functions at the first stage are

$$\begin{aligned} U_1^M &= D_1(.)q_1 + \gamma R_1(D_1(.), q_1, q_2) \\ &= D_1(q_1, q_2)\, q_1(1-\frac{\gamma}{\beta}) + \gamma(F + D_1(q_1, q_2)\,(s + \frac{\alpha}{2\beta})) \end{aligned}$$

and

$$\begin{aligned} U_2^M &= D_2(.)q_2 + \gamma R_2(D_2(.), q_1, q_2) \\ &= D_2(q_1, q_2)\, q_1(1-\frac{\gamma}{\beta}) + \gamma(F + D_2(q_1, q_2)(s + \frac{\alpha}{2\beta})). \end{aligned}$$

From these payoff functions we obtain the first order conditions:

$$\begin{aligned} FOC(q_1) &: \frac{1}{2}q_1(1-\frac{\gamma}{\beta}) + D_1(q_1, q_2)(1-\frac{\gamma}{\beta}) + \gamma\frac{1}{2}(s + \frac{\alpha}{2\beta}) = 0 \\ FOC(q_2) &: \frac{1}{2}q_2(1-\frac{\gamma}{\beta}) + D_2(q_1, q_2)(1-\frac{\gamma}{\beta}) + \gamma\frac{1}{2}(s + \frac{\alpha}{2\beta}) = 0 \end{aligned}$$

The solution of this system reveals that a single subgame perfect Nash equilibrium results at the symmetric values

$$q_1 = q_2 = q^M = \frac{(\alpha + 2s\beta)\gamma}{2(\gamma - \beta)} - 1 \tag{31}$$

with the two universities equally sharing the market, that is fully served.

The second order conditions are satisfied:

$$\frac{\partial^2 U_1}{\partial q_1^2} = \frac{\partial^2 U_2}{\partial q_2^2} = 1 - \frac{\gamma}{\beta} < 0 \text{ when } \gamma > \beta$$

Thus, this single solution is a maximum.

At this equilibrium, from *Proposition 1*, all demand is satisfied. *Proposition 1* also shows that for it to be feasible we need that $q^M < \frac{s\beta\gamma}{\gamma-\beta}$. This implies

$$\alpha\gamma < 2(\gamma - \beta). \tag{32}$$

On the other hand, q is limited at the bottom by the minimum levels of quality generated at university which are larger than zero (see Section 2.2). Since q is minimum when $T = 0$, then $q = \alpha\ \overline{a}$. Average ability of students when all applicants are accepted is $\frac{1}{2}$. Hence $q_{min} = \frac{\alpha}{2}$. Note that

$$\frac{\alpha}{2} \leq q^M \Longleftrightarrow s \geq \frac{2(\gamma-\beta)-\alpha\beta}{2\beta\gamma}. \tag{33}$$

Proposition 4 *Given the assumption $\gamma > \beta$ and if both $\alpha < 2(1-\frac{\beta}{\gamma})$ and $s \geq \frac{2(\gamma-\beta)-\alpha\beta}{2\beta\gamma}$, a single symmetrical subgame perfect Nash equilibrium exists at $(q, N) = \left(\frac{(\alpha+2s\beta)\gamma}{2(\gamma-\beta)} - 1, \frac{1}{2}\right)$. In other words, equilibrium involves mass teaching (all the market is served) and positive amounts of research.*

Let us now determine the corresponding levels of research and teaching at this corner solution. The following values are obtained from substitution of (31) and $N^M = \frac{1}{2}$ in (14) and the budget constraint.

$$R^M = F + \frac{1}{2\beta} - \frac{\alpha+2s\beta}{4(\gamma-\beta)} \tag{34}$$

$$T^M = \frac{\alpha+2s\gamma}{4(\gamma-\beta)} - \frac{1}{2\beta} \tag{35}$$

Proposition 5 *At the Mass Teaching and Research Equilibrium the finance scheme is once again responsible for the resulting levels of research and education quality: revenue neutral increases in s, the per-student allocation, will reduce research levels and increase teaching and education quality. All applications are admitted independently from s, the per student allocation.*

Proof. From (34), (35) the effect of compensated increases of s on research and teaching is:

$$\begin{aligned} \frac{dR^M}{ds} - N^M \frac{dR^M}{dF} &= -\frac{\gamma}{2(\gamma-\beta)} < 0 \\ \frac{dT^M}{ds} &= \frac{\gamma}{2(\gamma-\beta)} > 0. \end{aligned}$$

As for quality, the effect of compensated $ds > 0$ equals again the direct effect on q_i of the induced change in teaching $\frac{\beta}{N^S}\frac{dT^S}{ds} = \frac{\beta\gamma}{\gamma-\beta}$.

■

Summarizing the findings in this section: since α is low and $\gamma > \beta$ a positive q relies entirely on the ability of s to make teaching activities profitable enough. Then, if s is large, all students will be admitted (selection of the best would not *pay* much since α is *small*) and provided with extra productivity q. Research, on the contrary, becomes less and less rewarding as s increases.

4.3 Full-time Research

What happens then if s is low ((33) not satisfied)? In this case, q will be pushed to the corner solution $q = \frac{\alpha}{2}$ that, recall, involves no allocation to teaching. All the funds are then devoted to research. As in the previous case, corresponding to Point 3 in *Proposition 1*, enrolments equal global demand and are equally shared by the universities.

Proposition 6 *Given the assumption $\gamma > \beta$ and if $\alpha < 2\left(1 - \frac{\beta}{\gamma}\right)$ and $s < \frac{2(\gamma-\beta) - \alpha\beta}{2\beta\gamma}$, at the single symmetrical subgame perfect Nash equilibrium all funds are devoted to research.*

Marginal compensated changes in s will not be able to change this outcome: all funds are devoted to research and only the global amount, and not how it is allocated, will matter.

However, by relying more on s, the government can induce the universities to teach all students in the market, transforming this equilibrium into the previous one (mass teaching and some research).

5 Concluding comments

In this paper, we have analyzed a two stage game between two competing universities that allocate their funds to research and teaching activities. The challenge was important as previous literature had never considered both problems at the same time. Indeed, the amount of variables and interrelations made the analytical solution of a general model impossible. Simplification was required to reproduce in a systematic way some sensible limiting cases. Let us briefly summarize these findings.

If the productivity of teaching is larger than the preferences for research, each university will serve its own demand and devote all funds to teaching with complete independence of the finance scheme (only *total* funds matter).

If, on the other hand, preferences for research are high and the average ability of students is a productive input to the education process, it may be optimal for the universities to select a few good students who will not require much teaching (thus leaving enough resources to research) independently of the size of per-student allocations. The cost in terms of forgone research will be lower than the increase in the payoff through admissions and education production since the productivity of students is large.

Conversely, if student productivity is small but per-student allocations are large when preferences for research are high, all applicants will be admitted as a way to ensure the funds required to research. At equilibrium the symmetric universities will equally share the market and undertake positive levels of research.

Finally, if both student quality and per-student allocations are low when preferences for research are high, universities will devote all funds to research.

To conclude, we shall not forget the specification of the education production function that has generated our results. Its separability and excess of linearity were bound to lead to extreme solutions. In this sense we should interpret the results as magnified tendencies. An extension to this piece of work may eventually allow for a larger degree of concavity of the production function. Another interesting extension could consider several periods of university education. This way, the student selection process could take place at the end of each year. Each period several classes would coexist, which would add an additional dimension to the university finance problem.

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A Appendix: Full-time teaching

Consider the payoff of University 1 at the first stage when the equilibrium of the subgame at the second stage involves the admission of all applicants (as is indeed the case if $\beta > \gamma$):

$$U_1 = D_1(q_1, q_2)\, q_1(1 - \frac{\gamma}{\beta}) + \gamma(F + D_1(q_1, q_2)(s + \frac{\alpha}{2\beta})).$$

The effect of raising q_1 on the payoff of University 1,

$$\frac{\partial U_1}{\partial q_1} = (1 - \frac{\gamma}{\beta})(\frac{q_1}{2} + D_1(q_1, q_2)) + \frac{\gamma}{2}(s + \frac{\alpha}{2\beta}),$$

can only be negative if we allow for negative demands. Assuming this possibility away we can conclude that the payoff is increasing and convex:

$$\frac{\partial^2 U_1}{\partial q_1^2} = (1 - \frac{\gamma}{\beta}) > 0.$$

The level of quality that will under these circumstances maximize the payoff is the maximum quality available to this university: all funds will be devoted to teaching and no research will take place at equilibrium. We know, on the other hand, that all applicants are admitted, so that average ability of enrolled students is $\frac{1}{2}$. Final quality is then:

$$q_{\max} = \frac{\alpha}{2} + \beta(\frac{F + s\frac{1}{2}}{\frac{1}{2}})$$

Since quality is determined by global funds, compensated changes in the finance scheme imposed by the government do not affect resulting levels of quality (nor research, that remains zero independently of the finance scheme).

B Appendix: Asymmetric Equilibria

Consider now the corner solution $q_1 \geq \frac{s\beta\gamma}{\gamma - \beta} > q_2$. We know that in this case the equilibrium of the subgame at the second stage is $(N_1^B, 1 - N_1^B)$. University 1 chooses the number of admissions that maximizes its payoff independently of University 2 and the latter enrolls the rest of the students.

The corresponding payoffs are:

$$
\begin{aligned}
U_1(q_1,q_2) &= \gamma F + \frac{(\alpha+s\beta)^2\gamma}{2\alpha\beta}D_1(.) + \frac{(\beta-\gamma)(\alpha+s\beta)\gamma}{\alpha\beta\gamma}D_1(.)\,q_1 \\
&\quad + \frac{(\beta-\gamma)^2}{2\alpha\beta\gamma}D_1(.)\,q_1^2.
\end{aligned}
$$

$$
\begin{aligned}
U_2(q_1,q_2) &= \gamma F + (\gamma s + q_2(1-\frac{\gamma}{\beta}))(1-\frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\beta\gamma}D_1(.)\,) + \\
&\quad + \frac{\alpha\gamma}{2\beta}(D_2(.) + \frac{(s\beta\gamma+(\beta-\gamma)q_1)^2}{\alpha^2\gamma^2}D_1(.))
\end{aligned}
$$

The first order conditions are:

$Foc(q_1)$:

$$
\begin{aligned}
&\frac{(\alpha+s\beta)^2\gamma}{4\alpha\beta} + \frac{(\beta-\gamma)(\alpha+s\beta)\gamma}{\alpha\beta\gamma}(\,D_1(q_1,q_2)+\frac{q_1}{2}) \\
&+\frac{(\beta-\gamma)^2}{4\alpha\beta\gamma}q_1(4D_1(q_1,q_2)+q_1) = 0
\end{aligned}
$$

$Foc(q_2)$:

$$
\begin{aligned}
&(1-\frac{\gamma}{\beta})(1-\frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\gamma}D_1(.)) - \frac{(s\beta\gamma+(\beta-\gamma)q_1)^2}{4\alpha\gamma\beta} \\
&+\frac{\gamma}{\beta}\frac{\alpha}{4} + \frac{1}{2}(s\gamma+q_2(1-\frac{\gamma}{\beta}))\frac{(\alpha+s\beta)\gamma+(\beta-\gamma)q_1}{\alpha\gamma} = 0
\end{aligned}
$$

The solution of this system yields two solutions of which the following is a maximum:

$$
\begin{aligned}
q_1^A &= \frac{-(\gamma-\beta)+3\gamma(\alpha+s\beta)-\sqrt[2]{\gamma-\beta}\sqrt[2]{\gamma-\beta+12\alpha\gamma}}{3(\gamma-\beta)} \\
q_2^A &= \frac{(\gamma-\beta)+2\gamma(\alpha+s\beta)-\sqrt[2]{\gamma-\beta}\sqrt[2]{\gamma-\beta+12\alpha\gamma}}{2(\gamma-\beta)}.
\end{aligned}
$$

Since $q_1 \geq \frac{s\beta\gamma}{\gamma-\beta} > q_2$ the following conditions have to be satisfied for feasibility of this equilibrium:

$$
\frac{-(\gamma-\beta)+3\gamma(\alpha+s\beta)-\sqrt[2]{\gamma-\beta}\sqrt[2]{\gamma-\beta+12\alpha\gamma}}{3(\gamma-\beta)} - \frac{s\beta\gamma}{\gamma-\beta} > 0
$$

and

$$\frac{(\gamma - \beta) + 2\gamma(\alpha + s\beta) - \sqrt[2]{\gamma - \beta}\sqrt[2]{\gamma - \beta + 12\alpha\gamma}}{2(\gamma - \beta)} - \frac{s\beta\gamma}{\gamma - \beta} < 0.$$

Solving for α we find that these two conditions are equivalent to

$$\begin{aligned} \alpha\gamma &> 2(\gamma - \beta) \\ \alpha\gamma &< 2(\gamma - \beta) \end{aligned}$$

respectively, which can clearly not be satisfied simultaneously.

Therefore, *the asymmetric corner solution is non-feasible.*