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# Large elections with multiple alternatives: A Condorcet Jury Theorem and inefficient equilibria

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# Large elections with multiple alternatives: A Condorcet Jury Theorem and inefficient equilibria

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**Abstract**

We investigate whether the plurality rule aggregates information efficiently in large elections with multiple alternatives, in which voters have common interests. Voters' preferences depend on an unknown state of nature, and they receive imprecise private signals about the state of nature prior to the election. Similar to two-alternative elections (e.g., Myer- son (1998)), there always exists an informationally efficient equilibrium in which the correct alternative is elected. However, we identify new types of coordination failures in elections with more than two alternatives that lead to new types of inefficient equilibria. These can have interesting new properties: Voters may vote informatively, but the correct alternative is not elected.

**Keywords**: efficient information aggregation, simple plurality rule, Poisson games, Condorcet Jury Theorem

**JEL Classification**: C72, D71, D72, D82

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# 1 Introduction

We investigate whether the plurality rule aggregates information efficiently in a large election with multiple alternatives in which voters have common, state-dependent preferences and imprecise information about the state of nature.

This scenario should remind the reader of the Condorcet Jury Theorem (Condorcet (1785)): A jury with common preferences has to elect one of *two* alternatives, convict or acquit. If the defendant is guilty, all jurors prefer to convict. But if the defendant is innocent, all jurors prefer to acquit. However, the jurors do not know precisely whether the defendant is guilty or innocent; each juror has an imprecise private signal about the underlying state of nature, which is independently drawn from the same distribution. Condorcet (1785) finds that a jury elects the correct alternative with a larger probability than a single jury member alone, and with probability converging to one if the number of voters converges to infinity. However, two assumptions are crucial for this theorem: private signals are more likely correct than incorrect in each state of nature and jurors vote informatively for their signal.

Assuming strategic voting, Feddersen and Pesendorfer (1998) and Wit (1998), among others, show that informative voting is not necessarily a Nash equilibrium of the game. Nevertheless, a similar result obtains, even if one relaxes the assumption on private signals. Myerson (1998) shows that there always exists an 'informationally efficient' equilibrium in which the correct alternative is elected with probability converging to one if the number of voters converges to infinity. Other similar models with two alternatives find the same: Information aggregation in a large election is efficient as long as there are two alternatives and voter preferences are not too different (e.g., Feddersen and Pesendorfer (1996, 1997, 1998, 1999), Bouton and Castanheira (2012)).[1] We ask if this result extends to elections with more than two alternatives when voters have common preferences. So, we extend Myerson (1998) to more than two alternatives, and we also allow for abstention.[2]

Goertz and Maniquet (2009, 2011) consider a large election with three alternatives, but in their model voters do not have common preferences. There is a certain fraction of partisan voters who always prefer a certain alternative independent of the state of nature. In Goertz and Maniquet (2011), the voters with common preferences receive private signals from the same distribution; in Goertz and Maniquet (2009), they receive signals from different distributions. Contrary to the previous literature, both articles find that the plurality rule is not informationally efficient because the correct alternative is not elected with probability converging to one. In fact, no scoring rule except approval voting has at least one informationally efficient equilibrium.[3] However, the partisan

[1]Bhattacharya (2012) shows that information aggregation is no longer efficient in two-alternative elections if voter preferences are sufficiently different.

[2]Assuming abstention makes the model more general, but the reader will find that none of the results relies on it.

[3]This is, in some sense, similar to Ahn and Oliveros (2010) who find for a jury with common preferences and an election with three alternatives: "For any finite electorate, the

voters are the cause of the inefficiency. So, the question remains whether inefficient equilibria also exist in elections without partisans and how this depends on the number of alternatives.

We find that an informationally efficient equilibrium always exists. There always exists an unresponsive (inefficient) equilibrium as well. In this equilibrium, all voters vote for the same alternative. However, unresponsive equilibria are not particularly interesting, so we impose a condition on signal distributions and prior probabilities that rules them out. Under this new condition, all equilibria in two-alternative elections are efficient. The same is not true for elections with more than two alternatives.

First of all, inefficient equilibria exist. And they may be such that voters vote informatively. We also identify new types of coordination failures that do not exist with two alternatives. For example, there exist equilibria in which voters consider certain states of nature infinitively more likely than other states (that cannot happen with two alternatives - see our discussion below). In these types of equilibria, the election outcome is inefficient in a state of nature that is disregarded by at least some of the voters. Precisely the voters who cause the inefficiency are among those that disregard it, so they have no incentive to change their vote. Since all voters have the same preferences, the inefficiencies we find do not stem from problems of preference aggregation, but from coordination failures among voters with common preferences and different private information.

In Section 2, we describe the model. In Section 3, we present our results. In Section 4, we conclude.

# 2 The Setting

## 2.1 The Model

We consider an election with a finite number of alternatives $\{A_1, A_2 ...\} = \mathbf{A}$ and states of nature $\{\omega_1, \omega_2 ...\} = \mathbf{\Omega}$. We denote with $q_i$ the prior probability of state $\omega_i$, with $q_i \in (0,1)$ for all $i$. Also, $|\mathbf{A}| = |\mathbf{\Omega}|$. Voters have common, state-dependent, dichotomous preferences. They prefer a particular alternative in each state of nature, and are indifferent between the remaining ones:

$$\begin{aligned} u(A_i|\omega_i) &= 1 \ \forall i, \\ u(A_j|\omega_i) &= 0 \ \forall j \neq i. \end{aligned}$$

Assuming dichotomous preferences simplifies the strategic environment for the voters. With dichotomous preferences, they care only about those election outcomes in which their vote changes the outcome from any of the $|\mathbf{A}| - 1$ disliked alternatives to the preferred alternative; they do not need to consider election

best equilibrium under approval voting is more efficient than...plurality rule...If any scoring rule yields a sequence of equilibria that aggregates information in large elections, then approval voting must do so as well" (p.1). However, they do not discuss whether or not inefficient equilibria exist under any of these two rules.

outcomes in which they are pivotal between the $|\mathbf{A}| - 1$ less preferred alternatives. Even in this scenario, we find coordination failures among voters that lead to inefficient equilibria. One can easily imagine that more general assumptions on the preferences would only increase these problems.[4]

Voters are strategic and vote as a function of their expected impact on the outcome of the election. We assume that the electorate is large, but that each voter has a non-negative probability of being the pivotal voter that can change the outcome of the election. We incorporate this feature by assuming that the actual number of voters is uncertain (population uncertainty).[5] This is a fairly common assumption in the literature (e.g., Feddersen and Pesendorfer (1996, 1997, 1999), Myerson (1998, 2000, 2002), Goertz and Maniquet (2009, 2011), among others). Similarly to Myerson (1998, 2000, 2002) we assume that the population size is Poisson-distributed with parameter $n$. The probability that there are exactly $N$ voters is

$$P(N|n) = \frac{e^{-n}n^N}{N!}.$$

Each voter is pivotal with a strictly positive probability even in equilibria in which all voters vote for the same alternative. This implies that these types of equilibria cannot be rule out by ruling out weakly dominated strategies.

Prior to voting, each voter receives some private information about the alternatives in form of an informative, but imprecise, signal about the state of nature. Signals are independent and identically distributed. In this regard, our model is similar to Feddersen and Pesendorfer (1998, 1999), Myerson (1998), and Goertz and Maniquet (2011).[6] Since voters are otherwise identical, the signal determines the type of a voter. For simplicity, we assume that there are only $|\mathbf{\Omega}|$ types, so that $\mathbf{T} = \{t_1, ..., t_{|\Omega|}\}$. We denote by $r_i(t_j)$ the probability that a voter is of type $t_j$ in state $\omega_i$.

A voter's type is informative about the underlying state of nature in the following sense (which is similar to Myerson (1998)):

$$r_i(t_i) > r_j(t_i) \ \ \forall i \neq j. \tag{1}$$

In the original Condorcet Jury Theorem and in, for example, Feddersen and Pesendorfer (1998), signals are 'correct' with a probability larger than 1/2. This means that type $t_i$ is the most likely type in state $\omega_i$. Eq. (1), a relaxed version of this assumption, implies that type $t_i$ is more likely in state $\omega_i$ than in any other state of nature. For two-alternative elections, Myerson (1998) shows that Eq. (1) ensures the existence of an efficient equilibrium. We show the same for

[4]Some of the results, such as the existence of an efficient equilibrium or the existence of an unresponsive equilibrium, still hold if preferences are common and non-dichotomous.

[5]Population uncertainty implies that a voter does not know precisely how many other voters there are in the game. This is different from uncertainty about the number of voters that abstain; some of the voters that are in the game may decide to abstain.

[6]In a second set of papers on information aggregation in large elections, voters receive signals from signal technologies that are differently precise (e.g., Feddersen and Pesendorfer (1996, 1997), or Goertz and Maniquet (2009)).

elections with multiple alternatives. Eq. (1) is essential in several of our proofs.

If type distributions satisfy Eq. (1), there is no aggregate uncertainty in the population. If all private information was public, the state of nature would be known and voters would unanimously vote for the correct alternative (the one that maximizes every voter's utility). We use this as a benchmark and call an election outcome informationally efficient if the elected alternative is the same as the one that would be selected if all information was public.

The voting rule is the plurality rule with the possibility of abstention. Each voter can vote for one alternative or abstain. So, the action space of a voter is $\mathbf{C} = \mathbf{A} \cup \phi$, where $\phi$ denotes abstention. The alternative with the largest number of votes is elected. We assume a particular tie-breaking rule that is without loss of generality: Any tie involving alternatives $A_i$ and $A_j$ is broken in favor of $A_i$ as long as $i < j$; otherwise, it is broken in favor of $A_j$.

An economy in our model is defined by a list $(\mathbf{A}, \mathbf{\Omega}, q, \mathbf{T}, r, n, \mathbf{C})$ that satisfies Eq. (1). For any expected size of the population $n$, a strategy is a function $\sigma_n : \mathbf{T} \rightarrow \Delta(\mathbf{C})$, associating a voter type with a probability distribution over $\mathbf{C}$.[7] Let $\sigma_n^C(t) \geq 0$ denote the probability that a voter of type $t$ chooses action $C \in \mathbf{C}$. It has to be true that $\sum_{C\in\mathbf{C}} \sigma_n^c(t) = 1 \forall t \in \mathbf{T}$. We simplify the notation slightly by denoting with $\sigma_n^i(t)$ the probability that a voter chooses to vote for alternative $A_i$. Suppose that $\sigma_n^*$ is a Bayesian Nash equilibrium of the voting game with $n$ expected voters. We are interested in limit equilibria $\sigma^*$ such that $\sigma_n^* \rightarrow \sigma^*$ as $n \rightarrow \infty$.

Denote by $P_{\sigma_n}(A_i|\omega_j)$ the probability that alternative $A_i$ is elected in state $\omega_j$ given strategy profile $\sigma_n$.

**Definition 1.** ***Informationally Efficient Equilibrium*** *A limit equilibrium $\sigma^*$ of an economy $\mathcal{E}$ is informationally efficient if it is true that $P_{\sigma^*}(A_i|\omega_i) = 1 \ \forall i$.*

Following Goertz and Maniquet (2009, 2011), we call a voting rule informationally efficient if all of its limit equilibria are informationally efficient for all $\mathcal{E}$. A voting rule is called weakly informationally efficient if it has at least one efficient limit equilibrium for each $\mathcal{E}$.

Feddersen and Pesendorfer (1998) call an equilibrium strategy profile *responsive*, if voters "change their vote as a function of their private information with positive probability" (p. 26). Following them, we call a limit equilibrium *responsive* if $\sigma^*(t_i) \neq \sigma^*(t_j)$ for at least two different $t_i, t_j \in \mathbf{T}$, i.e., if not all voter types vote exactly the same way. In an *unresponsive* equilibrium, all voters vote the same way, independent of their type. We call a limit equilibrium *unresponsive* if $\sigma^*(t_i) = \sigma^*(t_j)$ for all $t_i \neq t_j \in \mathbf{T}$. Following Austen-Smith and Banks (1996), we call a voting profile *informative* if all voter types vote for their type, i.e., if $\sigma^{i*}(t_i) = 1$ for all $i$.[8] Informative voting is responsive, but

[7] In Poisson games, strategies are defined type by type. This corresponds to an assumption of symmetric equilibria in games in which strategies are defined agent by agent.

[8] Note that voting informatively does not necessarily imply utility maximization. A voter who maximizes utility given her type may nor may not vote for her type. If a voter's strategy is utility maximizing, Austen-Smith and Banks (1996) call the voting profile *sincere*.

responsive voting is not necessarily informative. If voters vote informatively, the election outcome accurately reflects the private information in the population. Notice that informative voting is not efficient if signals are less likely correct than incorrect in some state(s) of nature. Surprisingly, we find that it can nevertheless be an equilibrium.

## 2.2 Voter Behavior

Before we can prove any results, we need to consider how rational voters vote in a large Poisson voting game. However, we will only present overarching principles of voter behavior in this section and relegate the specifics to the respective proofs and the Appendix.

Recall that voters derive utility from the outcome of the election alone. A rational voter considers pivotal events in which his or her vote changes the outcome of the election from a less to a more preferred alternative. Pivotal events depend on the particular ballot that a voter wants to submit and, to some extent, on the tie-breaking rule. A voter who considers ballot $A_2$, for example, is pivotal if alternative $A_1$ has the same number of votes as alternative $A_2$ and both have at least as many votes as any other alternative. If a voter considers ballot $A_1$, on the other hand, the voter is pivotal if some alternative has one more vote than alternative $A_1$ and any other alternative is sufficiently behind.

Generally, denote by $E_k^{ij}$ the pivotal event in which one additional vote for alternative $A_i$ changes the outcome of the election from alternative $A_j$ to alternative $A_i$ in state $\omega_k$. And denote by $piv_k^{ij}$ the probability of this pivotal event. The probability of a pivotal event depends on the underlying strategy profile. To save on notation, we will avoid this additional index, if it is not misleading. Denote by $q_i(t)$ the posterior probability of state $\omega_i$ conditional on a voter being of type $t$. If this voter considers voting for alternative $A_i$ rather than abstaining, the expected utility gain can be written as

$$EU(A_i|t) = \sum_{j \neq i} [q_i(t) piv_i^{ij} - q_j(t) piv_j^{ij}]. \tag{2}$$

If $EU(A_i|t)$ is larger than zero, then the voter prefers voting for $A_i$ to abstaining. If, however, the expected utility gain of some other alternative $A_j$ is even larger, then the voter prefers voting for $A_j$ instead.

We need to evaluate equations such as Eq. (2) to construct strategies of the voters. In large elections, the probability of a pivotal event converges to zero, and so do entire equations such as Eq. (2). However, Myerson (2000) shows that probabilities of pivotal events do not converge to zero with the same speed. Events with probabilities that converge to zero faster than others are infinitively less likely and can therefore be ignored. The difference in the speed of convergence of pivotal probabilities makes comparisons between expected utilities from different ballots meaningful.

Myerson (2000) proposes the magnitude as a measure for the speed of convergence in a large Poisson game. The magnitude $\mu$ of the probability of a pivotal event is defined as

$$\mu(E_k^{ij}) = lim_{n\to\infty} \frac{log(piv_k^{ij})}{n}.$$

The magnitude of an event is either zero or negative and can be calculated by solving a maximization problem. Events with larger magnitude converge slower than those with smaller magnitude and are therefore infinitively more likely. In the Appendix, we discuss in a little more detail how magnitudes and precise probabilities of pivotal events are calculated (Magnitude Theorem and Off-Set Theorem from Myerson (2000)) for the cases arising in our proofs. For a more detailed discussion, we would like to refer the reader to Myerson (2000) or to Goertz and Maniquet (2009, 2011).

From Eq. (2) one can immediately deduce some important features of two-alternative elections. In this case,

$$EU(A_i|t) = q_i(t)piv_i^{ij} - q_j(t)piv_j^{ij}$$

Suppose that a voter of type $t_j$ submits a vote for alternative $A_i$ because $EU(A_i|t_j) \geq 0$. Then for a voter of type $t_i$, $EU(A_i|t_i) > 0$ because $q_i(t_i) > q_i(t_j)$ and $q_j(t_i) < q_j(t_j)$. This simple argument tells us that in two-alternative elections, it is necessarily true that 1) a voter of type $t_i$ is at least as likely to vote for alternative $A_i$ as a voter of type $t_j$, and 2) at most one type of voter mixes. This makes analyzing equilibria much simpler because they can only be of a few types: 1) Both voter types vote for the same alternative, or 2) voter type $t_i$ mixes between the two alternatives and voter type $t_j$ votes for $A_j$, or 3) voter type $t_i$ votes for $A_i$ and voter type $t_j$ votes for $A_j$. Unfortunately, with more than two alternatives, we cannot narrow down the set of possible equilibria in a similar way because of the following simple argument. Consider a three-alternative election with alternatives $A_i$, $A_j$, and $A_k$. Then

$$EU(A_i|t_j) = q_i(t_j)piv_i^{ij} + q_i(t_j)piv_i^{ik} - q_j(t_j)piv_j^{ij} - q_k(t_j)piv_k^{ik} \geq 0$$

does not necessarily imply that

$$EU(A_i|t_i) = q_i(t_i)piv_i^{ij} + q_i(t_i)piv_i^{ik} - q_j(t_i)piv_j^{ij} - q_k(t_i)piv_k^{ik} > 0$$

because, for example, $q_k(t_i)$ may be so much larger than $q_k(t_j)$ that voting for alternative $A_i$ yields lower expected utility for type $i$ than abstaining. With two alternatives, strategies exhibit a certain monotonicity in types. They no longer have this property for elections with more alternatives. As a consequence, general and constructive proofs are very hard to come by because we cannot narrow down the set of possible equilibrium strategies. Instead of strategies, we have to characterize equilibria by outcomes. Unfortunately, that leaves a complete characterization of the set of efficient and inefficient equilibria beyond reach.

# 3 Results

## 3.1 Existence of Efficient and Unresponsive Equilibria

Contrary to a model with partisan voters (as in Goertz and Maniquet (2009, 2011)), the plurality rule is weakly efficient if all voters have common preferences because there always exists at least one efficient limit equilibrium for every economy.

**Theorem 1.** *There exists an informationally efficient limit equilibrium for any economy $\mathcal{E}$ with multiple alternatives if type distributions satisfy Eq. (1).*

The proof is an adaptation of the proof of Theorem 2 in Goertz and Maniquet (2011) to the current model. The result is also closely related to McLennan (1998) who shows that for a common-value environment as ours, the strategy profile that maximizes the expected utility of the players is necessarily a Nash equilibrium of the game.

*Proof.* The proof is divided into two steps. In step 1, we show that for any $\mathcal{E}$ with a finite number of alternatives that satisfies Eq. (1) there exists a sequence of strategy profiles $\sigma_n$ such that $lim_{n\to\infty}P_{\sigma_n}(A_i|\omega_i) = 1$ for all $i$. To guarantee that this is true, it is sufficient (due to the Law of Large Numbers) to verify that the expected fraction of votes for alternative $A_i$ in state $\omega_i$ is larger than the expected fraction of votes for each of the other alternatives in that state.[9] In step 2, we deduce from step 1 that there exists a limit equilibrium $\sigma^*$ that aggregates information efficiently.

Step 1: Consider an economy $\mathcal{E}$ that satisfies Eq. (1). Let $\epsilon_1$, $\epsilon_2$, $\epsilon_3$, ...,$\epsilon_{|\mathbf{A}|}$ be small positive numbers such that

$$\epsilon_1 r_1(t_1) = \epsilon_2 r_2(t_2) = ... = \epsilon_K r_{|\mathbf{A}|}(t_{|\mathbf{A}|}).$$

Consider a sequence of strategy profiles in which for each $t_i \in \mathbf{T}$, $\sigma_n^i(t_i) = \epsilon_i$ and $\sigma_n^{\phi}(t_i) = 1 - \epsilon_i$, i.e., a voter of type $t_i$ mixes between voting for alternative $A_i$ and abstention. Denote by $\lambda_j^i$ the expected fraction of votes for alternative $A_i$ in state $\omega_j$. Given the strategy profile, the expected fractions of votes for the different alternatives in state $\omega_i$ are

$$\begin{aligned} \lambda_i^i &= r_i(t_i)\epsilon_i \\ \lambda_i^j &= r_i(t_j)\epsilon_j \ \ \forall j \neq i. \end{aligned}$$

With Eq. (1), it is true for each $\omega_i$ that $\lambda_i^i > \lambda_i^j \ \forall j \neq i$. So, by the Law of Large Numbers, $lim_{n\to\infty}P_{\sigma_n}(A_i|\omega_i) = 1$ for all $i$ as $n \to \infty$. Notice that $\sigma_n$ does not depend on $n$.

Step 2: Let $\sigma_n^*$ be a sequence of strategy profiles that maximizes the ex-ante utility of the voters. Such strategies exist, as they maximize a continuous function on a compact set. We claim that they are equilibrium strategies. Indeed,

[9] According to the Law of Large Numbers, the whole mass of probability concentrates in arbitrarily close neighborhoods around the expected outcomes as $n \to \infty$.

the existence of a profitable deviation would contradict the fact that $\sigma_n^*(t)$ maximizes expected utilities. Also, it is impossible that the expected utility from $\sigma_n^*(t)$ is lower than the expected utility from $\sigma_n(t)$ as defined above because the expected utility of $\sigma_n(t)$ tends to 1. So, it has to be true that $P_{\sigma_n^*}(A_i|\omega_i) \rightarrow 1$ for all $i$ as $n \rightarrow \infty$, and therefore $lim_{n\rightarrow\infty}\sigma_n^*$ is a limit equilibrium that aggregates information efficiently. $\square$

Unfortunately, not only efficient equilibria exist. Theorem 2 presents a certain type of inefficient equilibrium that exists for economies with certain prior probabilities and type distributions. It is an unresponsive equilibrium because all voters vote for the same alternative. Notice that this type of equilibrium cannot be ruled out by ruling out weakly dominated strategies as, in a Poisson game, all voters have a strictly positive probability of being pivotal.

**Theorem 2.** *For any $\mathcal{E}$ that satisfies Eq. (1), there exists an unresponsive equilibrium in which all voters vote for alternative $A_i$ if and only if $q_i(t) \geq q_j(t)$ for all $i \neq j$ and all $t \in \mathbf{T}$.*

*Proof.* Consider the strategy profile $\sigma^i(t_j) = 1\ \ \forall j$. Then, in all states of nature $\omega_j$, $\lambda_j^i = 1$ and $\lambda_j^k = 0\ \forall j$ and $\forall k \neq i$. This implies that there are only two pivotal events for each voter; each of these has the same probability in each state of nature. The first one is one in which no other voter shows up. In this case, alternative $A_1$ is elected by our tie-breading rule, unless the voter votes for some other alternative. In the other pivotal event, only one other voter shows up and, by assumption, votes for alternative $A_i$. In this case, the voter can change the the outcome of the election by voting for some alternative $A_j$ with $j < i$. Denote by $piv^0$ the probability of the event in which no other voter shows up and by $piv^1$ the probability of the event in which only one other voter shows up. Given the Poisson distribution, $piv^0 = e^{-n}$ and $piv^1 = e^{-n}n$. We need to distinguish two cases: 1) $i = 1$, i.e., all voters vote for alternative $A_1$, and 2) $i \neq 1$.

*Case 1*: In this case, $EU(A_1|t) = 0$ because there is no pivotal event in which a voter can change the outcome of the election to $A_1$. Since $EU(A_1|t) = 0$, no voter prefers abstention. Now consider $EU(A_j|t) = q_j(t)piv^0 - q_1(t)piv^0$ for $j \neq 1$. Voting $A_1$ is a best response for all $t$ if $q_j(t) \leq q_1(t)$.

*Case 2*: Now $EU(A_i|t) = q_i(t)piv^0 - q_1(t)piv^0$. If $j < i$, then $EU(A_j|t) = q_j(t)piv^0 - q_1(t)piv^0 + q_j(t)piv^1 - q_i(t)piv^1$. Voting for $A_i$ i a best response for all $t$ if $piv^0(q_i(t) - q_1(t)) \geq q_j(t)piv^0 - q_1(t)piv^0 + q_j(t)piv^1 - q_i(t)piv^1$. Notice that $lim_{n\rightarrow\infty}\frac{piv^0}{piv^1} = 0$ so that, in the limit, the comparison of utility gains reduces to $0 \geq q_j(t)piv^1 - q_i(t)piv^1$. Voting for $A_i$ is a best response if $q_i(t) \geq q_j(t)$ for all $j < i$. If $j > i$, then $EU(A_j|t) = q_j(t)piv^0 - q_1(t)piv^0$. In this case, voting for $A_i$ is a best response for all $t$ if $q_i(t)piv^0 - q_1(t)piv^0 \geq q_j(t)piv^0 - q_1(t)piv^0$ or if $q_i(t) \geq q_j(t)$ for all $j > i$. Again, no voter prefers abstention. $\square$

Of course, the equilibrium in Theorem 2 is necessarily inefficient because the correct alternative is elected in only one state of nature. This type of equilibrium is not particularly interesting. So, in the remainder, we will restrict our attention to environments in which this type of equilibria does not exist. We assume that prior probabilities and type distributions are such that

$$q_i(t_i) > q_j(t_i) \ \forall i, j \neq i \tag{3}$$

This condition implies that the posterior belief of a voter of type $t_i$ is always such that state $\omega_i$ is more likely than any other state of nature. Now, for two-alternative economies, inefficient equilibria no longer exist.

**Theorem 3.** *For any $\mathcal{E}$ with $|\mathbf{A}| = 2$ that satisfies Eq. (3), all equilibria are efficient.*

We relegate the short proof to the Appendix because two-alternative elections are not our main focus. The proof relies heavily on the fact that in a two-alternative election, it is always true that a voter of type $t_i$ is at least as likely to vote for alternative $A_i$ as type $t_j$, and vice versa for alternative $A_j$. Since strategies no longer have this property with $|\mathbf{A}| > 2$, Theorem 3 does not hold if $|\mathbf{A}| > 2$.

Due to our discussion above, Theorem 3 is the last general result we can obtain for elections with multiple alternatives. To investigate properties of elections with more than two alternatives a bit further, we now restrict our attention to a specific case and present examples that convey a general messages about the types of equilibria and coordination failures that can arise.

## 3.2 Elections with Three Alternatives

In this section, we focus on economies with $|\mathbf{A}| = 3$ that satisfy Eq. (3). First, we characterize the types of equilibria that exist. Due to the fact that strategies no longer exhibit any general properties, we cannot characterize the equilibria in terms of strategies, but do so in terms of outcomes. For an election with three alternatives, the following types of equilibria exist:[10]

1. There exist equilibria in which, conditional on a voter being pivotal, two states of nature are infinitively more likely than the third.

2. There exist equilibria in which, conditional on a voter being pivotal, no state of nature is infinitively more likely than another.

We call the first type of equilibrium type-1 equilibrium. In this equilibrium, pivotal events in two states of nature have the same magnitude, and it is larger than the magnitude of any pivotal event in the third state of nature. In a type-2 equilibrium, there exists pivotal events in each state of nature that have the same (and largest) magnitude. Notice that this characterization also holds if type and prior probabilities satisfy Eq. (1).

There exists no other type of equilibrium. This is quite easy to see: Suppose

[10] A similar type of characterization can be made for elections with more alternatives as well.

that there was an equilibrium in which a pivotal event in $\omega_i$ had the largest magnitude, so that $\omega_i$ was infinitively more likely, conditional on being pivotal, than any other state. This implies that voters ignore their private information and vote for $A_i$. If this was an equilibrium, it would be the one described in Theorem 2 in which the pivotal events in all states of nature have the same magnitude. This is a contradiction to the assumption made above. The equilibrium in Theorem 2 is, in fact, a type-2 equilibrium.

Our first result is that there exist inefficient equilibria with three alternatives even if Eq. (3) holds. In addition, these equilibria can such that voters vote informatively.

**Theorem 4.** *There exists economies with $|\mathbf{A}| = 3$ that satisfy Eq.(3) with inefficient type-1 equilibria in which voters vote informatively.*

*Proof.* We construct $\mathcal{E}$ that satisfies Eq.(3) and an inefficient $\sigma^*$ in which alternative $A_1$ is not elected in state $\omega_1$. Suppose that $q_1 = \frac{1}{3}$, $q_2 = \frac{1}{3}+\epsilon$, $q_3 = \frac{1}{3}-\epsilon$, and $\epsilon$ sufficiently small. Also assume that $r_1(t_1) = 0.2$, $r_1(t_2) = r_1(t_3) = 0.4$, $r_2(t_1) = 0.1$, $r_2(t_2) = 0.5$, $r_2(t_3) = 0.4$, $r_3(t_1) = 0.1$, $r_3(t_2) = 0.4$, $r_3(t_3) = 0.5$. We claim that $\sigma^{i*}(t_i) = 1$ for all $i$ is an equilibrium. The expected fractions of votes are

$$\begin{gathered}\lambda_1^1 = 0.2,\ \ \lambda_1^2 = \lambda_1^3 = 0.4,\\ \lambda_2^1 = 0.1,\ \ \lambda_2^2 = 0.5,\ \ \lambda_2^3 = 0.4,\\ \lambda_3^1 = 0.1,\ \ \lambda_3^2 = 0.4,\ \ \lambda_3^3 = 0.5.\end{gathered}$$

Clearly, $\sigma^*$ is inefficient. Now we need to show that $\sigma^*$ is an equilibrium.

With the Magnitude Theorem (see Appendix), the magnitudes of the most likely pivotal events are $\mu(E_2^{23}) = \mu(E_2^{32}) = \mu(E_3^{23}) = \mu(E_3^{32}) = -(\sqrt{0.5} - \sqrt{0.4})^2 = -5.75 * 10^{-3}$ and $\mu(E_1^{12}) = \mu(E_1^{13}) = -(\sqrt{0.4} - \sqrt{0.2})^2 = -0.034$. So, $\omega_2$ and $\omega_3$ are infinitively more likely, conditional on a voter being pivotal, than $\omega_1$.

Since the most likely pivotal events in states $\omega_2$ and $\omega_3$ have the same magnitude, we will also need to know the relationship between the actual probabilities of these events. Notice the distribution of votes are the same in $\omega_2$ and $\omega_3$, so that we can use the Off-Set Theorem (see Appendix) even if we compare pivotal probabilities in different states of nature.

Consider $E_2^{32}$ and $E_3^{32}$. These occur if $A_2$ and $A_3$ have the same number of votes. Clearly, it must be the case that $piv_2^{32} = piv_3^{32}$.

Now consider $E_2^{23}$ and $E_3^{23}$. These occur if $A_3$ has one more vote than $A_2$. However, their probabilities are not the same in the two states of nature because $\lambda_3^3 > \lambda_2^3$. The event must be more likely in $\omega_3$. Notice, that the probability of $E_3^{23}$ is the same as the probability of $E_2^{23} - (0,-1,1)$, where $w = (0,-1,1)$ is a vector of votes. So, $E_2^{23} - (0,-1,1)$ is the event in which $A_1$ has the same number of votes as in $E_2^{23}$, $A_2$ has one vote more than in $E_2^{23}$, and $A_3$ has one vote less. Of course, $E_2^{23} - (0,-1,1)$ is not a pivotal event. But we can use it to apply the Off-Set Theorem.

$$lim_{n\to\infty}\frac{piv_3^{23}}{piv_2^{23}} = \prod_i (\lambda_2^i)^{-w(A_i)} = \frac{\lambda_2^2}{\lambda_2^3} \tag{4}$$

So, indeed, the event is more likely in $\omega_3$ than in $\omega_2$. We can now prove that $\sigma^*$ indicated above is indeed an equilibrium.

*Case 1: A voter of type $t_2$.*
Considering only the most relevant pivotal events, $EU(A_2|t_2) = q_2(t_2)piv_2^{23} - q_3(t_2)piv_3^{23}$ and $EU(A_3|t_2) = q_3(t_2)piv_3^{32} - q_2(t_2)piv_2^{32}$. Consider first $EU(A_2|t_2)$. This ballot yields a positive expected utility gain if $q_2(t_2)piv_2^{23} > q_3(t_2)piv_3^{23}$, or if $q_2(t_2)\lambda_2^3 > q_3(t_2)\lambda_2^2$, or if $(\frac{1}{3}+\epsilon)r_2(t_2)\lambda_2^3 > (\frac{1}{3}-\epsilon)r_3(t_2)\lambda_2^2$, which is true. Now consider $EU(A_3|t_2)$. This ballot yields a negative expected utility gain if $q_3(t_2)piv_3^{32} < q_2(t_2)piv_3^{32}$, or if $(\frac{1}{3}-\epsilon)r_3(t_2) < (\frac{1}{3}+\epsilon)r_2(t_2)$, which is true. So, this voter receives a positive expected utility gain from ballot $A_2$ and a negative expected utility gain from ballot $A_3$. In addition, the expected utility gain from ballot $A_2$ is necessarily larger than the expected utility gain from ballot $A_1$ (because of the ranking of pivotal events) or from abstention. Therefore, voting for $A_2$ is a best response for this voter.

*Case 2: A voter of type $t_3$.*
Considering only the most relevant pivotal events, $EU(A_2|t_3) = q_2(t_3)piv_2^{23} - q_3(t_3)piv_3^{23}$ and $EU(A_3|t_3) = q_3(t_3)piv_3^{32} - q_2(t_3)piv_2^{32}$. We use similar arguments as in Case 1 to show: $EU(A_3|t_3) > 0$ because $(\frac{1}{3}-\epsilon)r_3(t_3) > (\frac{1}{3}+\epsilon)r_2(t_3)$ for sufficiently small $\epsilon$. And $EU(A_2|t_3) < 0$ because $q_2(t_3)piv_2^{23} < q_3(t_3)piv_3^{23}$ for sufficiently small $\epsilon$. For similar reasons as above, a voter of type $t_3$ also does not want to submit ballot $A_1$ rather than ballot $A_3$ and also does not want to abstain. So, the best response of a voter of type $t_3$ is to vote $A_3$.

*Case 3: A voter of type $t_1$.*
Considering only the most relevant pivotal events, $EU(A_2|t_1) = q_2(t_1)piv_2^{23} - q_3(t_1)piv_3^{23}$ and $EU(A_3|t_1) = q_3(t_1)piv_3^{32} - q_2(t_1)piv_2^{32}$. Clearly, $EU(A_2|t_1) < 0$ because $q_2(t_1)\lambda_2^3 < q_3(t_1)\lambda_2^2$ for sufficiently small $\epsilon$. And $EU(A_3|t_1) < 0$ because $q_3(t_1) < q_2(t_1)$. However, with the ranking of magnitudes, $EU(A_1|t_1) = q_1(t_1)piv_1^{12} + q_1(t_1)piv_1^{13} > 0$. The best response of a voter of type $t_1$ is to vote for $A_1$. □

The equilibrium described in the proof is a type-1 equilibrium. It is inefficient because voters of types $t_2$ and $t_3$ would both gain from voting more for alternative $A_1$, but one type will not do so if the other does not as well.

This type of equilibrium can only exist when there are more than two alternatives and states of nature. The fact that two states of nature are infinitively more likely than the third creates competition between the two types of voters that causes the inefficiency in the third state of nature. Both should both vote a bit for alternative $A_1$ as so that this alternative wins in $\omega_1$. However, no single type of voter has an incentive to deviate from the current stragegy: If, say, type $t_2$ voted a bit for $A_1$ and less for $A_2$, but $t_3$ continued voting for $A_3$ only, then type $t_2$ would decrease the probability of $A_2$ winning in $\omega_2$ in order to make sure that $A_1$ is a bit more likely to win in $\omega_1$. This is not a profitable deviation since state $\omega_2$ is much more likely than state $\omega_1$. So, if one type does not vote more for $A_1$, the other one will not either.

While we picked the parameters in the example in the proof to make to make a point, the competition between two types of voters at the expense of the third alternative will always exist in type-1 equilibria. They will be inefficient as long as there are not enough voters of the third type ($t_1$ in the proof). This implies the following:

**Corrollary 1.** *There exist efficient type-1 equilibria. They can be such that voters vote informatively.*

If we change the type distribution in state $\omega_1$ from the one in the proof to $r_1(t_1) = 0.4$, $r_1(t_2) = r_1(t_3) = 0.3$, so that there are now sufficient $t_1$ voters in $\omega_1$, the result obtains. The magnitudes are slightly different as in the proof above, but their ranking carries through, and so do all the other arguments. The only difference is that now alternative $A_1$ is elected in state $\omega_1$. In addition, we can show:

**Corrollary 2.** *There exist efficient type-2 equilibria. They can be such that voters vote informatively.*

*Proof.* Consider $\mathcal{E}$ such that $q_1 = 0.8$, $q_2 = 0.1$, $q_3 = 0.1$ and $r_1(t_1) = 0.8 = r_2(t_2) = r_3(t_3)$, $r_1(t_2) = 0.15 = r_2(t_1) = r_3(t_1)$, $r_1(t_3) = 0.05 = r_2(t_3) = r_3(t_2)$. Arguments and calculations similar to the ones in the proof of Theorem 4 yield that $\mu(E_1^{12})$, $\mu(E_2^{12})$, and $\mu(E_3^{13})$ all have the same magnitude. So, voters use their private information as well when calculating expected gains of the different alternatives. Informative voting is an equilibrium, and it is efficient. $\square$

The reader may have noticed that the construction of these equilibria relies on certain symmetries in prior probabilities and type distributions and may conclude that these results are non-generic. It is, in fact, non-generic that voters vote informatively in these types of equilibria. If parameters in the proof of Theorem 4 are slightly perturbed, voters may no longer vote informatively. We resort to constructing equilibria that are informative because it is very hard to find other equilibria. However, the coordination failures that we find depend more on the type of equilibrium we have than on the fact that voters vote informatively and are therefore generic. For example, a type-1 equilibrium is inefficient because of the fact that two states of nature are infinitively more likely than the third, so that certain types of voters disregard what happens in the third state.

# 4 Conclusion

We investigate elections with common preferences, private information, and multiple alternatives. We show that an efficient equilibrium always exists. We also show that new types of equilibria with new types of coordination failures arise in elections with more than two alternatives. These inefficiencies are caused by coordination failures among voters with common preferences but different private information.

The coordination failures we find are strongly related to the types of equilibria that exist. For example, in elections with more than two alternatives, there exist equilibria in which some states are infinitively more likely than other states, conditional on being pivotal (this is not true with two alternatives). These types of equilibria can be inefficient. If they are, the election outcome is inefficient in the state that voters disregard. Since the voter types who cause the inefficiency disregard this state of nature, they have no incentive to change their vote.

While a complete characterization of the set of efficient and inefficient equililibria is currently beyond reach, there can be no doubt that increasing the number of alternatives beyond two increases the number and types of possible coordination failures between voters immensely and that inefficient equilibria become more and more common. This should in particular be true for environments in which voters do not have dichotomous preferences.


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# Appendix

## A.1 Magnitude Theorem and Off-Set Theorem (Myerson (2000))

The Magnitude Theorem allows us to calculate the magnitude of pivotal events. Since the actual number of voters is uncertain, there is a plethora of subevents that make up a certain pivotal event - each characterized by the fact that two (or more) alternatives tie or are one vote apart but have a different number of actual voters. The Magnitude Theorem states that the magnitude of the entire pivotal event is, in the limit, equal to the magnitude of its most likely subevent. When calculating the magnitude of an event, one first computes this most likely subevent by solving a maximization problem and then uses it to calculate the size of the magnitude.

The Off-Set Theorem allows us to compare the actual probabilities of two pivotal events that are not two different (i.e, differ only by a finite number of votes) and occur in the same state of nature. If two states of nature have the same vote distribution, though, it can be applied as well.

In this Appendix, we only show how to use the Magnitude Theorem and the Off-Set Theorem for the cases needed in our proofs. For a more general exposition, we would like to refer the reader to Myerson (2000) or Goertz and Maniquet (2011).

### A.1.1 Magnitude Theorem

Consider some pivotal event $E_k^{ij}$. Recall that $n\lambda_j^i$ denotes the expected number of votes for alternative $A_i$ in state $\omega_j$. Denote by $N_j^i$ the actual number of votes for alternative $A_i$ in state $\omega_j$. The most likely subevent of $E_k^{ij}$ is $N_k^i = N_k^j = n\sqrt{\lambda_k^i \lambda_k^j}$ (or, $N_k^i = n\sqrt{\lambda_k^i \lambda_k^j}$ and $N_k^j = N_k^i + 1$ if the pivotal event requires a one-vote difference with our tie-breaking rule), and $N_k^l = n\lambda_k^l$ for all remaining $l$.[11] Let us know calculate the magnitude of the pivotal event. For our purposes, it is sufficient to consider the following two distinct cases.

*Case 1: Only two alternatives are involved in the close race determining pivotal event $E_k^{ij}$, i.e., there is no alternative $A_s$ such that $n\lambda_k^s > n\sqrt{\lambda_k^i \lambda_k^j}$.*

In this case, the magnitude of the probability of $E_k^{ij}$ can be calculated using the following formula:

$$\mu(E_k^{ij}) = 2\sqrt{\lambda_k^i \lambda_k^j} - (\lambda_k^i + \lambda_k^j), \tag{5}$$

*Case 2: There exists some alternative $A_s$ such that $n\lambda_k^s > n\sqrt{\lambda_k^i \lambda_k^j}$*

In this case, the magnitude of event $E_k^{ij}$ has to be calculated using the formula:

$$\mu(E_k^{ij}) = 3\sqrt[3]{\lambda_k^i \lambda_k^j \lambda_k^s} - (\lambda_k^i + \lambda_k^j + \lambda_k^s). \tag{6}$$

The magnitude is now smaller because $E_k^{ij}$ is less likely. The reason is that the most likely outcome for $N_k^s$ is $n\lambda_k^s$. If this occurred, then a voter would no longer be pivotal between $A_i$ and $A_j$. So, $A_s$ needs to receives less votes.

### A.1.2 Off-Set Theorem

The Off-Set Theorem allows us to compare the actual probabilities of two pivotal events that are not two different, i.e., that have the same magnitude.[12] Consider two pivotal events $E_k^{ij}$ and $E_k^{lm}$ such that $E_k^{ij} = E_k^{lm} - w$, where $w = (w(A_i))_{A_i \in \mathbf{A}}$ is a vector of finite numbers of votes. So, the two pivotal events differ by a finite number of votes for certain alternatives. Then

$$lim_{n\to\infty} \frac{piv_k^{ij}}{piv_k^{lm}} = \prod_i lim_{n\to\infty} (\lambda_k^i)^{-w(A_i)} \tag{7}$$

The Off-Set Theorem implies that $\mu(E_k^{ij}) = \mu(E_k^{ji})$. Note, however, that the Off-Set Theorem can only be used if the two events have the same underlying distribution of votes. So, they either need to occur in the same state of nature, or, if they occur in different states of nature, the vote distributions in the two states needs to be the same. Otherwise, the Off-Set Theorem is not applicable.

[11] There are cases in which the most likely subevent does not take the simple form stated above, but these cases do not arise in our analysis.

[12] The ratio of the probabilities of two pivotal events that have different magnitudes converges to zero or to infinity.

## A.2 Proof of Theorem 3

Consider an election with two alternatives, $A_1$ and $A_2$, and two states of nature, $\omega_1$ and $\omega_2$. Assume that there exists an inefficient equilibrium in which not everyone votes for the same alternative. Also assume wlog that this equilibrium is inefficient because alternative $A_1$ is elected in state $\omega_2$. The same type of argument can be used for any other case.

Recall from our discussion above that a voter of type $t_i$ is always at least as likely to vote for alternative $A_i$ than type $t_j$ in a two-alternative election, i.e., $\sigma^{A_i}(t_i) \geq \sigma^{A_i}(t_j)$, $i \neq \jmath$. This fact, combined with the fact that, by assumption, not everyone votes for $A_1$, allows us to make certain inferences about the expected fractions of votes for the different alternatives in the two states of nature. These lead to inferences about the magnitudes of pivotal events that lead to a contradiction to the assumed voting behavior. In conclusion, the only type of inefficient equilibrium in two-alternative elections is the one in Theorem 2.

In more detail: If alternative $A_1$ is elected in state $\omega_2$, then it has to be true that $\lambda_2^1 \geq \lambda_2^2$. In addition, $\sigma^{A_2}(t_2) > 0$, $\sigma^{A_2}(t_2) \geq \sigma^{A_2}(t_1)$, and $\sigma^{A_1}(t_1) \geq \sigma^{A_1}(t_2)$. So, the differences in expected fractions of votes are

$$\begin{aligned}
\lambda_1^1 - \lambda_1^2 &= r_1(t_1)(\sigma^{A_1}(t_1) - \sigma^{A_2}(t_1)) + r_1(t_2)(\sigma^{A_1}(t_2) - \sigma^{A_2}(t_2)) \\
\lambda_2^1 - \lambda_2^2 &= r_2(t_1)(\sigma^{A_1}(t_1) - \sigma^{A_2}(t_1)) + r_2(t_2)(\sigma^{A_1}(t_2) - \sigma^{A_2}(t_2))
\end{aligned}$$

However, $\sigma^{A_1}(t_1) - \sigma^{A_2}(t_1) > 0$ and $\sigma^{A_1}(t_2) - \sigma^{A_2}(t_2) < 0$. With Eq. (3) (or Eq. (1)) and with $\lambda_2^1 \geq \lambda_2^2$, we have $\lambda_1^1 - \lambda_1^2 > \lambda_2^1 - \lambda_2^2 \geq 0$. This implies that $\mu(piv_2^{21}) > \mu(piv_1^{21})$. Therefore, all voters should vote for alternative $A_2$ because, conditional on being pivotal, $\omega_2$ is infinitively more likely. This is a contradiction to the above assumption that $\sigma^{A_1}(t_1) > 0$. $\square$

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