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Essays on Fiscal and Monetary Policy Interactions in a World of Debt

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Contents

List of figures

List of tables

Introduction

"*...every monetary policy action has fiscal consequences.*" says Sims (2016). This statement is particularly relevant in our world of rising public debt levels where policy rate decisions have potentially large effects on the interest bill of governments. Properly accounting for those fiscal consequences involves departing from standard models in monetary economics that study monetary policy as the only game in town.[1] The Ricardian Equivalence assumption that holds in this class of models may be consistent with the fiscal orthodoxy observed during the Great Moderation (from mid-1980s to 2007). It seems far-fetched in the current context of high public debt ratios.

The persistent fall in the natural interest rate in the aftermath of the Great Recession caused a renewed interest for fiscal policy (e.g. Schmidt, 2013, 2016; Nakata, 2016; Eggertsson, 2011). The effective lower bound constraint considerably reduced the margin of monetary policy to stimulate inflation; the Consumer Price Index remained largely subdued for more than a decade despite the massive purchases of public debt by central banks around the world. At the same time, governments grabbed the opportunity offered by the low — sometimes negative — real interest rates to finance expansionary fiscal policies with deficits.

The COVID crisis in the US drastically accelerated this trend and government debt reached levels unprecedented since the Second World War. The exogenous nature of the shock and its uneven impact on households implied that fiscal policy

[1]This literature largely builds on the seminal work of Woodford (2003). See also Galí (2015) and widespread medium-scale dynamic stochastic general equilibrium models in the tradition of Smets and Wouters (2007).

could protect incomes and sustain employer-employee relationships without being plagued by moral hazard concerns. The fiscal loosening was also encouraged by the idea that the output multiplier of government spending is large at the zero lower bound on nominal interest rates (e.g. Woodford, 2011; Christiano et al., 2011; Erceg and Lindé, 2014). This theoretical finding also fostered the hope that the fiscal expansion will to some extent be self-financing because of its multiplicative effects on the denominator of the debt-to-GDP ratio. Evidence about the skyrocketing ratios of public debt is suggesting otherwise.

As a result, monetary and fiscal spheres are becoming increasingly entangled with important policy implications. The ability of a central bank to control inflation is conditional to a specific conduct of fiscal policy. Leeper (1991) shows that when public debt is not clearly backed by future surpluses, the intertemporal solvability condition of the government — rather than monetary policy — guides the dynamics of inflation. For example, Bianchi and Ilut, 2017; Kliem et al., 2016 use this narrative to frame the missing deflation post-2008 as the consequence of a separate regime characterised by a constrained monetary policy and a fiscal policy that temporarily disregards the level of debt to focus on stabilising the economy.

This literature suggests that coordination between fiscal and monetary policy is necessary to produce stable dynamics. In some circumstances, this may translate in a loose fiscal policy combined with an accommodative monetary policy. This is the so-called fiscal theory of the price level (Cochrane, 2001, 2018; Sims, 2011). Some authors even go one step further by suggesting that transitory inflation should be tolerated by the central bank as an instrument to help reducing the debt burden by eroding its real value (Bianchi and Melosi, 2019; Bianchi et al., 2020). However, central bankers generally view this alternative equilibrium as a threat to their independence and have urged to recall the lesson from the stagflation of the 1970s; a central bank subservient to fiscal policy would be incompatible with its mandate of long-term price stability (Draghi, 2018; Waller, 2021). The reality is that attempts to stimulate inflation up to central banks' target have remained largely unsuccessful and committing to unsustainable fiscal policies might prove difficult. There might be signs that this is changing.[2]

Consequently, the course of future fiscal and monetary policy and debt dynamics is subject to substantial uncertainty. To the extent that households are forward-looking, understanding how this uncertainty shapes households' expectations, and how in turn this expectation channel influences observable outcomes, should be a priority on the research agenda in macroeconomics. This thesis contributes to this effort by studying the interactions between fiscal and monetary policy while attributing a meaningful role to the accumulation of government debt for the determination of macroeconomic dynamics. The methodology builds on the workhorse new Keynesian dynamic stochastic general equilibrium (DSGE) model that formulates the agents' forward-looking decision making process as the solution to microfounded optimisation problems. Stochastic shocks disturb the economy and introduce uncertainty in the expectation formation that feeds back into current macroeconomic outcomes. In addition, the general equilibrium nature of the model is well-suited to study the interactions between fiscal and monetary policy, and their influence on agents' behavior. The thesis is composed of three chapters that explore different aspects of those interactions and of the environment in which they unfold.

The first chapter of the thesis studies the role of coordination between fiscal and monetary policy for the desirability of accumulating debt in a liquidity trap (i.e. a situation in which a large negative demand shock is coupled with a binding zero lower bound on nominal interest rates). Two different institutional set-ups are considered; a perfect coordination between the two authorities (the central bank or CB and the fiscal authority or FA) and a strategic noncooperative game consistent with the principle of central bank independence. An analytical example under

[2]As this thesis is going to press, inflation in the US has been surging well above the Fed's target and the transitory nature of this increase is being more and more disputed.

simplifying assumptions reveals that debt accumulation depresses consumption in the liquidity trap when optimal time-consistent monetary policy is conducted independently from fiscal policy. The large debt burden induces an inflationary bias at the exit of the liquidity trap that tightens monetary policy and increases expected real rates. In this context, the optimal time-consistent fiscal policy consists in consolidating debt during the recession to maintain inflation below target at positive interest rates. This strategy leads the FA to provide only a tame — tax-financed — fiscal stimulus and to tolerate a deeper recession in the near-term. This chapter emphasises the importance of expectations about the future degree of coordination between the two authorities for households' consumption and savings decisions during the downturn.

The second chapter (co-authored with Boris Chafwehé and Rigas Oikonomou) explores the effectiveness of government debt maturity management as an additional margin to stabilise inflation in a situation where monetary policy is subservient to fiscal policy. This environment is interesting because the soaring debt levels may imply that we are already implicitly making a choice about the exit regime at positive interest rates. Assuming that economic growth turns out disappointing in the future, the US government may not be able to generate sufficient surpluses to stabilise its debt and monetary policy would then partially loose control over inflation. We show that in this situation, a properly tailored portfolio of maturity can restore the efficacy of monetary policy under certain conditions about its conduct. In some cases (e.g. when the nominal rate tracks the real rate), a perfect stabilisation outcome can be achieved. We also find that this result extends to an optimal monetary policy under commitment and that the optimal monetary policy rule depends non-trivially on debt maturity management. Finally, we demonstrate the quantitative relevance of our results by estimating a fully-fledged DSGE model on the US data with Bayesian methods.

The third and last chapter of the thesis considers the interactions between fiscal and monetary policy in a liquidity trap caused by negative feedback loops on expectations. Studies about this type of liquidity trap have gained in popularity because of the long expected duration of binding lower bound episodes observed in modern economies. However, a gap exists concerning the output effect of deficit-financed policies in this environment. The chapter thus constraints taxes to labor income and compares stabilisation outcomes of expansionary fiscal policies during an expectations-driven liquidity trap. Government debt turns out to annihilate the efficacy of expansionary government spending or of tax-cuts. In an expectations-driven liquidity trap, spending is deflationary because of expected long-lasting effects on the real interest rate and thus increases the real payout of debt. To avoid a strong monetary tightening at positive rate, the best strategy is then to provide a large tax-cut and to mitigate the accumulation of debt by also cutting government spending. Hence, this chapter suggests that reducing the size of the state produces positive outcomes in the wake of long-lasting deflationary spirals. However, this prescription should be taken with a pinch of salt in case of uncertainty about the type of shock hitting the economy. Cutting taxes and spending is counterproductive if the liquidity trap turns out fundamentally-driven.

Chapter 1

Fiscal and Monetary Policy Interactions in a Liquidity Trap when Government Debt Matters

Abstract What does central bank independence imply for the optimal conduct of time-consistent fiscal and monetary policy in a liquidity trap? To provide an answer, I consider a stochastic noncooperative game in which the lower bound on nominal rates is an occasionally binding constraint and in which government debt serves as a tool to influence future policy trade-offs. I show that a transitory consolidation of debt in the liquidity trap optimally reduces expected real rates and stimulates current consumption and inflation via an expectation channel. The reaction function of the independent central bank at positive interest rates is pivotal in obtaining this result — considering instead coordinated policy produces the opposite effect of an optimal increase in debt. Lengthening the debt maturity allows to mitigate issues related to lack of coordination by insulating bond prices from short-term rate decisions.

> *"Moreover, if central banks were to enter into a form of coordination with fiscal authorities that reduced their independence, it would ultimately be self-defeating."*
>
> – Mario Draghi (2018)

1.1 Introduction

With the COVID-19 pandemic unfolding in the US, the debt-to-GDP ratio has ballooned to levels unprecedented since the Second World War. Conveniently, a form of natural coordination between fiscal and monetary policy has prevailed during the crisis. On the one hand, budget deficits have expanded due to the combined effect of automatic stabilisers and targeted fiscal stimuli. On the other hand, undershooting inflation has prompted a loosening of monetary policy to preserve favorable financing conditions and abundant credit supply for the private sector. Incidentally, the flattening of the yield curve has lowered the interest payments on long-term government debt and provided more space for fiscal policy to support the economy and inflation through the turmoil.

However, this coordination cannot be taken for granted in the future. A strong recovery may come with substantial inflation that would warrant a monetary tightening. If this is so, the objective of curbing inflation should prevail over concerns about the sustainability of government debt for a central bank that is subservient to fiscal policy is incompatible with long-term price stability.[3] Yet, in the aftermath of the COVID-19 crisis, a narrative has emerged in favour of more coordination between fiscal an monetary policy[4] and has led central banks to reaffirm the

[3]This was emphasised by Mario Draghi (2018): "*...if central banks were to enter into a form of coordination with fiscal authorities... Fiscal authorities would have an incentive to use monetary policy to achieve other objectives. And this would end up with monetary policy becoming fiscally dominated, which history shows is inconsistent with price stability in the long run...*"

[4]For example, Bianchi et al. (2020) propose a coordinated strategy aiming at creating a controlled rise of inflation to wear away a targeted fraction of debt. This operation is meant to finance an *emergency budget* to deal with the negative effects of the COVID-19 crisis.

importance of their independence.[5]

Given this ongoing debate, it appears crucial to understand how the degree of coordination at the exit of a liquidity trap (i.e. a situation in which a large negative demand shock is coupled with a binding zero lower bound on nominal interest rates) influences private sector expectations and alters economic outcomes during the recession. This paper seeks to answer this question based on a New-Keynesian framework in which the effective lower bound on nominal rates is an occasionally binding constraint and in which the level of debt matters because it is ultimately financed with distortionary taxes on labor income. In this context, I study the optimal time-consistent policy of a central bank and a government that play a stochastic noncooperative game and I contrast the results with the jointly optimal policy. There are three main insights stemming from this analysis.

First, considering a noncooperative game overturns the classical policy insight (see e.g. Eggertsson, 2006; Burgert and Schmidt, 2014; Matveev, 2020) that a government with short-term liabilities should provide a large debt-financed fiscal stimulus in the liquidity trap. Instead, in my model, a transitory consolidation of debt financed with labor taxes becomes optimal. To understand this result, I study a simplified example and show analytically that the rise in inflation stemming from a higher debt burden leads the independent central bank to *lean against the wind* when the nominal rate lifts-off the lower bound. Since this contractionary monetary policy is anticipated by the rational households, issuing more debt depresses consumption in the liquidity trap and exacerbates the inflation shortfall. In this context, numerical simulations under plausible calibration reveal that the government provides only a tame — tax-financed — fiscal stimulus and reduces debt to keep inflation under target at the exit of the lower bound. This strategy supports consumption of forward-looking households who expect lower labor taxes and expansionary monetary policy during the recovery. Nevertheless, the subdued

[5]See e.g. the speech of Governor Christopher J. Waller (2021)

response of government spending with respect to the joint policy causes a deeper recession in the near-term.

Second, liquidity trap episodes arise more often when fiscal and monetary policy are conducted noncooperatively. The optimal debt consolidation undertaken at the outset of the liquidity trap weighs persistently on inflation expectations and increases the probability of a binding lower bound. To mitigate this adverse effect, the government accumulates more debt in the risky steady state and sustains a higher nominal rate and inflation rate relative to the coordinated policy.

Third, increasing the maturity of debt alleviates coordination issues. When debt is long-term, the price of government bonds is less sensitive to the short-term interest rate. Hence, monetary policy focuses more on inflation and output gap stabilisation and less on the debt sustainability impact of its policy rate decisions. In other words, the central bank may conduct its policy more independently from fiscal considerations regardless of the specific assumptions about the institutional framework.

This paper is related to the literature on fiscal and monetary policy in a low-rate environment. Standard New-Keynesian models predict that the fiscal multiplier is large (superior to one) at the lower bound (Woodford, 2011; Christiano et al., 2011; Erceg and Lindé, 2014) and optimal fiscal policy is therefore expansionary (Schmidt, 2013; Nakata, 2016). However, those models are mostly silent about the role of government debt in a liquidity trap because they assume lump-sum taxes and Ricardian fiscal policy. Hence, households are indifferent between tax-financing and debt-financing.

Closer to this paper, another strand of the literature attributes a meaningful role to government debt by confining tax instruments to labor income. Under this assumption, the cost-push effect of labor taxes gives rise to an endogenous trade-off between output gap and inflation stabilisation. When the policymaker cannot commit to fiscal and monetary policy, this trade-off provides an incentive to erode the real value of inherited liabilities by frontloading inflation.[6] Consequently, inflation is increasing in the level of government debt; a tendency that Leeper et al. (2020) have labeled the *inflationary bias*.[7]

In a liquidity trap, the inflationary bias of government debt constitutes an important determinant of inflation expectations. With one-period bonds, accommodating inflation yields large benefits for the joint fiscal and monetary policy because the roll-over-cost of government debt depends directly on short-term rate variations. Hence, stabilising debt at a higher pace prevents inflation to rise further in the future. Burgert and Schmidt (2014) demonstrate that the anticipation of this accomodative monetary policy provides fiscal policy with an incentive to accumulate more debt at the outset of the liquidity trap to stimulate consumption of forward-looking agents.

Eggertsson (2013) shows that the effectiveness of this channel tightly depends on the specifics of the institutional set-up. In his model, attributing an independent goal to monetary policy breaks the link between government debt and inflation expectations. This result highlights the importance of policy coordination in making a low for longer interest rate policy time-consistent. Coordination constitutes the usual assumption of the existing literature; monetary and fiscal policy are conducted jointly under a consolidated budget constraint. This framework implies that monetary policy internalises debt sustainability concerns in its interest rate decisions; a conduct at odds with the principle of central bank independence. A growing strand of the literature thus models the interactions between fiscal and monetary policy as a noncooperative game. Some contributions, such as Dixit and Lambertini (2003); Adam and Billi (2008); Blake and Kirsanova (2011) have focused on the gains of appointing a conservative central bank when the lack of

[6] As explained by Niemann et al. (2013), the jump in inflation is akin a lump-sum tax on the financial wealth of the households that eases future policy trade-offs.

[7] Bai et al. (2017) demonstrate that the anticipation of this inflationary bias by private agents creates excessive inflation volatility with respect to the near random walk outcome under commitment (Schmitt-Grohé and Uribe, 2004).

coordination generates policy biases. Gnocchi (2013), and more recently Camous and Matveev (2020), have introduced asymmetry of commitment in a noncooperative game to examine the welfare gains from appointing a central bank that disciplines the fiscal authority.

In line with the above papers, I consider here a central bank and a fiscal authority that play a noncooperative game. I focus on the Markov-Perfect Equilibria under Stackelberg leadership of the non-Ricardian fiscal policy. This institutional set-up is consistent with a central bank that is transparent about its goals and conducts its policy independently from fiscal considerations.[8] Differently from the previous literature, this paper is the first to study a policy game when taxes are distortionary and the lower bound is an occasionally binding constraint.

The remainder of the paper is organised as follows. Section 1.2 describes the key ingredients of the model and outlines the equilibrium conditions in log-linear form. The policy problem and the structure of the noncooperative game are also laid out. Section 1.3 derives the optimality conditions under the benchmark of coordination and addresses the noncooperative game under Stackelberg leadership of the fiscal authority. Section 1.4 presents an analytical example characterising the response of consumption and government spending to a debt increase in the liquidity trap. Section 1.5 solves the model numerically and discusses the main results of the paper and section 1.6 concludes.

1.2 The model

The aim of this paper is to study the strategically optimal behaviour of monetary and fiscal policy at the zero lower bound. The model is a cashless New Keynesian economy in which fiscal policy is non-Ricardian due to the presence of distortionary taxation. I provide an overview of the main ingredients here and leave the details

[8] Sims (2016) states that "*Central bank independence takes on a variety of specific institutional forms, but its aim is to create an institution somewhat insulated from short-run political forces and charged with controlling inflation as a primary duty.*"

to Appendix 1.7.2.

The private sector is composed of an infinitely lived representative household, a representative aggregate good producer and intermediate good producers which compete monopolistically and are subject to costly price adjustments. The public sector is represented by two institutions, a central bank (CB) and a fiscal authority (FA), which I consider independently from one another.

1.2.1 Households and firms

The representative household derives utility from consuming the private good c_t and the public good G_t while it dislikes hours worked h_t. I assume a separable utility function leading to the following expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big[\frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \Big] \tag{1.1}$$

where $\mathbb{E}_t$ is the rational expectations operator conditional on information in period t, β is the time discount factor. Parameters γ_c and γ_g are respectively the intertemporal eslasticy for private and government consumption and, γ_h is the inverse of the Frisch elasticity. I also attach utility weights ν_g and ν_h to characterise preference for government consumption and hours, relatively to private consumption.

The variable ξ_t is an exogenous process characterising the preference for time. Under this specification, time preference between states of two consecutive periods evolves according to $\xi_t/(\beta\xi_{t+1})$. I assume the following autoregressive structure for this process:

$$d_t = \alpha d_{t-1} + \epsilon_t \tag{1.2}$$

where $d_t \equiv \xi_{t+1}/\xi_t$ and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is a normally distributed exogenous shock that will be the source of liquidity trap episodes.

The household sells hours to intermediate firms for a wage W_t net of labor taxes

τ_t and may save via two nominal and non state-contingent assets: a one-period bond B_t^s and a perpetual bond B_t. Following Woodford (2001), the perpetuity yields a coupon with payoff decaying at exponential rate ρ. Consequently, when $\rho = 0$, the short-term bond and the perpetuity have the same one-period maturity.[9] Firm profits yield a dividend $\Pi_{i,t}$ and lump-sum transfers T_t are collected from the government. The household budget constraint (in real terms) reads

$$c_t + \frac{b_t^s}{R_t} + q_t b_t = (1-\tau_t) w_t h_t + \frac{b_{t-1}^s}{\pi_t} + (1+\rho q_t)\frac{b_{t-1}}{\pi_t} + \int_0^1 \frac{\Pi_{i,t}}{P_t} di + \frac{T_t}{P_t} \tag{1.3}$$

where R_t is the gross nominal interest rate, q_t is the price of the perpetual bond and π_t is the gross inflation rate. The household chooses $\{c_t, h_t, b_t^s, b_t\}_{t=0}^{\infty}$ to maximise expected lifetime utility (1.1) subject to (1.3) and no-Ponzi scheme conditions on the two bonds

Intermediate firms operate under monopolistic competition and seek to maximise profits subject to quadratic price adjustment costs à la Rotemberg (1982). From the profit maximisation problem of the final producer, the demand function of the generic firm producing i is given by

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} y_t \tag{1.4}$$

where θ is the marginal rate of substitution between varieties. The program of the firm i is

$$\max_{P_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t \left[\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} y_t \left(\frac{P_{i,t}}{P_t} - (1-s) w_t\right) - \frac{\iota}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)^2 \right] \tag{1.5}$$

where λ_t is the multiplier of budget constraint from the household problem and ι is the price adjustment cost factor.

Parameter s is an employment subsidy which offsets steady state distortions

[9] I only introduce the short-term bond B_t^s because I want to be able to refer to the short-term yield even when the maturity of the perpetuity is superior to one period.

stemming from monopolistic competition and distortionary taxation. For the rest of this paper, this subsidy is kept constant over time and lump-sum transfers are restricted to the sole purpose of financing it.[10] The reason for introducing this subsidy is to make a positive amount of debt sustainable at the steady state. Absent of the subsidy, a time-consistent policymaker wants to reduce any positive amount of liabilities that successive policymakers will inherit. This debt consolidation removes the incentives to inflate debt away in the future and lowers inflation expectations to their efficient level.[11]

1.2.2 Public authorities

There are two authorities exercising policy in this economy: a monetary authority (CB) and a fiscal authority (FA). Each authority uses its own instruments to pursue its objective:

- The CB chooses the sequence of short-term nominal interest rates $\{R_t\}_{t=0}^{\infty}$, while being constrained by a zero lower bound (ZLB).
- The FA chooses the sequence of labor taxes and government spending, $\{\tau_t, G_t\}_{t=0}^{\infty}$, to finance its net debt position.

Assuming that the short-bond is in zero net supply, the budget constraint of the FA in real terms reads

$$\left(\frac{1+\rho q_t}{\pi_t}\right) b_{t-1} = q_t b_t + \tau_t w_t h_t - G_t - s(w_t h_t - wh)$$

The last term implies that the subsidy is not rebated at the steady state.

[10] This implies that distortions from labor taxation do occur outside the steady state. See Leith and Wren-Lewis (2013) for a similar use of a steady state subsidy.

[11] For a detailed analysis of those dynamics in a real economy and Markov-Perfect Equilibrium, see e.g. Debortoli and Nunes (2013).

1.2.3 Log-linear approximation

Since the optimality conditions of the household are standard, I relegate the details to Appendix 1.7.2 and proceed by showing the log-linear equations directly. Variables without time-subscript represent steady-state values and hated variables are log-deviations from the steady-state.

$$y\hat{y}_t = c\hat{c}_t + G\hat{G}_t \qquad \text{(Resource constraint)} \tag{1.6}$$

$$\hat{\pi}_t = \kappa_t\hat{\tau}_t + \kappa_c\hat{c}_t + \kappa_y\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \qquad \text{(Phillips curve)} \tag{1.7}$$

$$\hat{c}_t = -\frac{1}{\gamma_c}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - d_t) + \mathbb{E}_t\hat{c}_{t+1} \qquad \text{(Euler equation)} \tag{1.8}$$

$$\hat{i}_t = \rho\beta\mathbb{E}_t\hat{q}_{t+1} - \hat{q}_t \qquad \text{(No arbitrage)} \tag{1.9}$$

where $\kappa_t = w\tau(\theta-1)/\iota$, $\kappa_c = \gamma_c(\theta-1)/\iota$ and $\kappa_y = \gamma_h(\theta-1)/\iota$.

The log-linear budget constraint reads

$$\Omega(\hat{b}_t+(1-\rho)\hat{q}_t-\beta^{-1}(\hat{b}_{t-1}-\hat{\pi}_t))-G\hat{G}_t+w\tau y\frac{\theta-1}{\theta}\tau_t-\frac{y}{\theta}(\gamma_c\hat{c}_t+(1+\gamma_y)\hat{y}_t) = 0 \tag{1.10}$$

where $\Omega \equiv qb$ is the steady state market value of debt.

A private-sector rational expectations equilibrium consists of a sequence $x_t \equiv \{\hat{c}_t, \hat{\pi}_t, \hat{b}_t, \hat{y}_t\}$ satisfying equations (1.6)–(1.10), given the policies $p_t \equiv \{\hat{i}_t > -r^*, \hat{G}_t, \hat{\tau}_t\}$, exogenous process $\{\epsilon_t\}$, and initial conditions $\hat{b}_{-1}$.

1.3 The policy problem

The society delegates the same objective to the two authorities which have to pursue it using their respective instruments. I derive this social objective by taking a second-order approximation of the utility function of the household (1.1) around the non-stochastic steady-state. Leaving the details of the derivation to Appendix

1.7.3, the loss function reads

$$\mathcal{L}_t = \frac{1}{2}\Big[\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\hat{\pi}_t^2\Big] \tag{1.11}$$

In minimising this loss function, the policymaker is constrained to choose time-consistent actions. Consequently, it cannot commit beyond the repayment of its debt obligations: promises made to ease current policy trade-offs have a grip on agent expectations only if they are backed by proper incentives to deliver in due time. In other words, I focus on Markov-Perfect Equilibria both between the government and the private sector and between the government in the current period and successive governments.

Notice that, in this framework, time-consistency does not imply that the policymaker has to take expectations as given. When taxes are distortionary, choices about the level of debt inherited by next policymakers influence households' expectations. To account for this possibility, I write expectations in the optimisation problem with the following notation:

$$\mathbb{E}_t\pi_{t+1} \equiv \mathbb{E}_t\Pi(s_{t+1})$$

$$\mathbb{E}_t c_{t+1} \equiv \mathbb{E}_t\mathcal{C}(s_{t+1})$$

$$\mathbb{E}_t q_{t+1} \equiv \mathbb{E}_t\mathcal{Q}(s_{t+1})$$

where $s_t \equiv \{\hat{b}_{t-1}, d_t\}$ is the state vector.

1.3.1 Optimal coordinated policy

Coordination assumes that the policymaker minimises the utility loss stemming from the exogenous demand shocks by choosing the policy instruments available in the economy simultaneously under a consolidated budget constraint. In other words, public authorities internalise the effects of their policy decisions on the constraints and choices of one another. This assumption is standard in the literature and has been studied in a context of optimal time-consistent policy at the ZLB in Burgert and Schmidt, 2014; Matveev, 2020. Hence, this section constitutes a useful benchmark for the following section on noncooperative policies. Given Markov-Perfection, the joint policymaker is represented by a sequence of authorities with identical preferences, each one leading its future selves.

The Bellman equation of the coordinated policymaker reads

$$V(s_t) = \min_{\{\hat{c}_t, \hat{G}_t, \hat{y}_t, \hat{\pi}_t, \hat{\tau}_t, \hat{i}_t, \hat{b}_t\}} \frac{1}{2}(\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\hat{\pi}_t^2) + \beta V(s_{t+1})$$

such that

$$y\hat{y}_t = c\hat{c}_t + G\hat{G}_t \tag{1.12}$$

$$\hat{\pi}_t = \kappa_t\hat{\tau}_t + \kappa_c\hat{c}_t + \kappa_y\hat{y}_t + \beta\mathbb{E}_t\Pi(s_{t+1}) \tag{1.13}$$

$$\hat{c}_t = -\frac{1}{\gamma_c}(\hat{i}_t - \mathbb{E}_t\Pi(s_{t+1}) - d_t) + \mathbb{E}_t\mathcal{C}(s_{t+1}) \tag{1.14}$$

$$\hat{i}_t = \rho\beta\mathbb{E}_t\mathcal{Q}(s_{t+1}) - \hat{q}_t \tag{1.15}$$

$$\hat{i}_t > -r^* \tag{1.16}$$

$$0 = \Omega(\hat{b}_t + (1-\rho)\hat{q}_t - \beta^{-1}(\hat{b}_{t-1} - \hat{\pi}_t)) - G\hat{G}_t + w\tau y\frac{\theta-1}{\theta}\hat{\tau}_t - \frac{y}{\theta}(\gamma_c\hat{c}_t + (1+\gamma_y)\hat{y}_t) \tag{1.17}$$

where $r^* \equiv ln(\beta^{-1})$.

I attach multipliers $\Lambda_t^r, \Lambda_t^p, \Lambda_t^q, \Lambda_t^i, \Lambda_t^{zlb}$ and Λ_t^b to constraints (1.12)-(1.17), respectively. The first order necessary conditions of the program can be found in Appendix 1.7.4.

1.3.2 Optimal noncooperative policy

Noncooperative game assumes that the CB and the FA play a noncooperative Markov-perfect game. The game is described formally as follows:

Timing of the game — For every period $t \geqslant 0$, the timeline of events is: a) exogenous demand shock realises and is observed by both the authorities and the private sector; b) the FA chooses its fiscal instruments; c) the CB chooses the nominal rate; d) economic variables realise. Let vector $\upsilon_t \equiv (d_t, \hat{G}_t, \hat{\tau}_t, \hat{i}_t, x_t)$ gather chronologically the intraperiod events. Then, I can define the history of the game as $\phi_t \equiv (\upsilon_t, \phi_{t-1})$ for $t > 0$ and $\phi_0 \equiv (\upsilon_0, \hat{b}_{-1})$ for $t = 0$.

Strategy of the players — The FA leads the CB and private sector within each period with histories $\phi_t^{FA} \equiv (\hat{b}_{t-1}, d_t)$.[12] I denote its strategy by $\omega_{FA} = \{\hat{G}_t(\phi_t^{FA}), \hat{\tau}_t(\phi_t^{FA})\}_{t\geqslant 0}$. The CB then follows with histories $\phi_t^{CB} \equiv (\hat{b}_{t-1}, d_t, \hat{G}_t, \hat{\tau}_t)$ and chooses the policy rate according to strategy $\omega_{CB} = \{\hat{i}_t(\phi_t^{CB})\}_{t\geqslant 0}$. Finally, private agents face histories $\phi_t^x \equiv (\hat{b}_{t-1}, d_t, p_t)$ and follow decision rule $\omega_x = \{x_t(\phi_t^x)\}_{t\geqslant 0}$. Given this sequence, I define the continuation strategies as $\omega_{FA}^t = \{\hat{G}_r(\phi_r^{FA}), \hat{\tau}_r(\phi_r^{FA})\}_{r\geqslant t}$ and $\omega_{CB}^t = \{\hat{i}_r(\phi_r^{CB})\}_{r\geqslant t}$. Hence, starting from any level of inherited debt $\hat{b}_{t-1}$, and given a sequence of exogenous demand shocks for $r \geqslant t$, fiscal and monetary strategies generate policies denoted by $\{p_r\}_{r\geqslant t}$ for any possible path of government debt. This systematic response allows the private sector to form expectations. Then, knowing history ϕ_t^x, continuation private decisions are set according to $\omega_x^t = \{x_r(\phi_r^x)\}_{r\geqslant t}$. Consequently, fixed strategies generate a continuation competitive equilibrium $\mathcal{C}_t = \{x_r, p_r\}_{r\geqslant t}$ from any level of inherited debt $\hat{b}_{t-1}$.

Notice that, since the FA is the first intraperiod mover, it takes strategy ω_{CB} as a constraint in its optimisation program. This leadership structure reflects the empirical nature of the interactions between fiscal and monetary policy. Central banks have a very stable mandate that is revised only occasionally. Moreover, they communicate regularly about their strategy as expectation management is part of their toolkit. In contrast, political decision process is often less transparent and subject to changes in the composition of the government that cannot be perfectly

[12] I restrict here histories of the FA to the inherited debt and exogenous shock. Hence, I avoid issues related to multiple reputational equilibria as in King et al. (2008).

anticipated. As a follower, the CB is also not constrained by debt sustainability when choosing its strategy ω_{CB}. Hence, this sequence is consistent with the principle of CB independence observed in the US and other advanced economies.[13] Finally, I want to focus on the interactions between the CB and the FA and so I do not consider households and price setters as a player of the game interacting strategically. Instead, private agents optimality conditions constitute a constraint delimiting the set of implementable solutions.

Central bank reaction function Consistently with the literature on central bank independence, the CB is constrained by its own balance sheet in choosing the path of nominal interest rates. Del Negro and Sims, 2015; Benigno and Nisticò, 2020 show that remittances to the Treasury thigh the CB balance sheet to the government budget constraint. However, in the absence of quantitative easing as in this paper, the CB does not issue reserves or hold government bonds, and so remittances are zero at all time and the CB can choose the interest rate without being concerned by the solvency risk inherent to maturity mismatch.[14] Hence, the balance sheet constraint of the CB holds trivially and its Bellman equation reads

$$U(s_t) = \min_{\{\hat{c}_t, \hat{y}_t, \hat{\pi}_t, \hat{i}_t\}} \frac{1}{2}(\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\hat{\pi}_t^2) + \beta U(s_{t+1})$$

with $\{\hat{c}_{t+j}, \hat{y}_{t+j}, \hat{\pi}_{t+j}, \hat{i}_{t+j} > -r^*, \hat{G}_{t+j-1}, \hat{\tau}_{t+j-1}\}$ given for $j \geq 1$ and such that

$$\begin{aligned}
&y\hat{y}_t = c\hat{c}_t + G\hat{G}_t \\
&\hat{\pi}_t = \kappa_t\hat{\tau}_t + \kappa_c\hat{c}_t + \kappa_y\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \\
&\hat{c}_t = -\frac{1}{\gamma_c}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - d_t) + \mathbb{E}_t\hat{c}_{t+1} \\
&\hat{i}_t > -r^*
\end{aligned}$$

[13] For a similar leadership structure, see Bai et al. (2017) and Chen et al. (2019).

[14] See Reis (2017) for a parallel between the standard New-Keynesian model and the literature on central bank independence.

Rearranging the FONCs of this problem yields the following reaction function of the CB (CBRF)

$$\gamma_h \hat{y}_t + \gamma_c \hat{c}_t = \gamma_c c^{-1} \mu_t^{zlb} - \Psi \hat{\pi}_t \tag{1.18}$$

where μ_t^{zlb} is the lagrange multiplier associated with the ZLB constraint and $\Psi \equiv \iota y (c^{-1}\kappa_c + y^{-1}\kappa_y)$. Outside the lower bound, this rule corresponds to a usual *leaning against the wind* policy of a time-consistent CB whereby any rise in inflation is dampened by creating a negative consumption and output gap.[15] Notice that, since taxes are distortionary, it would not be possible for the CB to reach a perfect stabilisation outcome (the so-called "divine coincidence") by closing both the inflation and output gap. Moreover, the trade-off is exacerbated by the lower bound constraint.[16]

Optimisation problem of the fiscal leader The fiscal leader optimises taking the CBRF (1.18) as a constraint. Its Bellman equation reads

$$W(s_t) = \min_{\{\hat{c}_t, \hat{G}_t, \hat{y}_t, \hat{\pi}_t, \hat{\tau}_t, \mu_t^{zlb}, \hat{b}_t\}} \frac{1}{2}(\gamma_c c \hat{c}_t^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \iota y \hat{\pi}_t^2) + \beta W(s_{t+1})$$

such that (1.12)-(1.17) as well as the reaction function (1.18) and $\mu_t^{zlb} < 0$. I attach multipliers λ_t^r,λ_t^p,λ_t^q,λ_t^i,λ_t^{zlb},λ_t^b,λ_t^m and λ_t^s to those constraints, respectively.

[15] See e.g. Chapter 5 of Galí (2015).

[16] Adam and Billi (2007) show that the cost of facing a binding ZLB is higher under discretion than under commitment because of a reinforcement loop between discretionary policy and private expectations.

The FONCs for the policy problem read

$$\partial \mathcal{L}/\partial \hat{c}_t \equiv \quad 0 = \gamma_c c \hat{c}_t + \lambda_t^r c + \lambda_t^p \kappa_c - \lambda_t^q \gamma_c - \lambda_t^b \frac{y}{\theta} \gamma_c - \lambda_t^m \gamma_c \tag{1.19}$$

$$\partial \mathcal{L}/\partial \hat{G}_t \equiv \quad 0 = \gamma_g G \hat{G}_t + \lambda_t^r G - \lambda_t^b G \tag{1.20}$$

$$\partial \mathcal{L}/\partial \hat{y}_t \equiv \quad 0 = \gamma_h y \hat{y}_t - \lambda_t^r y + \lambda_t^p \kappa_y - \lambda_t^b \frac{y}{\theta}(1 + \gamma_h) - \lambda_t^m \gamma_h \tag{1.21}$$

$$\partial \mathcal{L}/\partial \hat{\pi}_t \equiv \quad 0 = \iota y \hat{\pi}_t - \lambda_t^p + \lambda_t^b \Omega \beta^{-1} - \lambda_t^m \Psi \tag{1.22}$$

$$\partial \mathcal{L}/\partial \hat{\tau}_t \equiv \quad 0 = \lambda_t^p \kappa_\tau + \lambda_t^b w \tau y \frac{\theta - 1}{\theta} \tag{1.23}$$

$$\partial \mathcal{L}/\partial \hat{q}_t \equiv \quad 0 = \lambda_t^q + \lambda_t^b \Omega (1 - \rho) - \lambda_t^{zlb} \tag{1.24}$$

$$\partial \mathcal{L}/\partial \mu_t^{zlb} \equiv \quad 0 = \lambda_t^m \gamma_c c^{-1} + \lambda_t^s \tag{1.25}$$

$$\partial \mathcal{L}/\partial \hat{b}_t \equiv \quad 0 = \lambda_t^p \beta \Theta_{1,t} + \Omega(\lambda_t^b - \mathbb{E}_t \lambda_{t+1}^b) + \lambda_t^q \Theta_{2,t} + \lambda_t^{zlb}(\Theta_{3,t} - \Theta_{2,t}) \tag{1.26}$$

and the lower bound constraints $\mu_t^{zlb} < 0$, $\rho\beta\mathbb{E}_t\mathcal{Q}(s_{t+1}) - \hat{q}_t > -r^*$ and, complementary slackness conditions $\lambda_t^s \mu_t^{zlb} = 0$, $(\rho\beta\mathbb{E}_t\mathcal{Q}(s_{t+1}) - \hat{q}_t)\lambda_t^{zlb} = 0$. Moreover, the following definitions apply

$$\begin{aligned}
\Theta_{1,t} &\equiv \frac{\partial \mathbb{E}_t \Pi(s_{t+1})}{\partial \hat{b}_t} \\
\Theta_{2,t} &\equiv \frac{\gamma_c \partial \mathbb{E}_t \mathcal{C}(s_{t+1})}{\partial \hat{b}_t} + \frac{\partial \mathbb{E}_t \Pi(s_{t+1})}{\partial \hat{b}_t} - \rho\beta \frac{\gamma_c \partial \mathbb{E}_t \mathcal{Q}(s_{t+1})}{\partial \hat{b}_t} \\
\Theta_{3,t} &\equiv \frac{\gamma_c \partial \mathbb{E}_t \mathcal{C}(s_{t+1})}{\partial \hat{b}_t} + \frac{\partial \mathbb{E}_t \Pi(s_{t+1})}{\partial \hat{b}_t}
\end{aligned}$$

Those terms summarise the derivative of expectations with respect to debt and imply that, despite the lack of commitment, the FA can influence households' expectations through the level of debt. Moreover, the derivatives vary across time because of the lower bound that introduces non-linearities in the policy functions.

Finally, notice that the value of the Lagrange multiplier attached to the ZLB constraint is not necessarily the same for the CB and the FA i.e. $\mu_t^{zlb} \neq \lambda_t^{zlb}$. The shadow price depends on the instruments available to each authority to deal with the constraint. Clearly, the marginal utility of relaxing the constraint is higher for the CB because the interest rate peg at zero undermines the effectiveness of its unique instrument. On the other hand, the FA may still rely on its fiscal instruments, the tax rate and the public spending, and manage expectations about future interest rates and the long-bond price through its choice of government debt. In the noncooperative game considered here, the ZLB muliplier of the CB is thus a choice variable of the fiscal leader.

1.4 Analytical results for a special case

I present here an analytical example with short-term one period debt to demonstrate the main mechanisms under the following simplifying assumptions:

Assumption 1 *The shock is discrete with two states* $\{d_H, d_L\}$ *and occurs only once and for all in the initial period. We have* $d_t = d_L$ *if* $t = 0$ *and* $d_t = d_H$ *otherwise. Moreover,* $d_L << 0$ *such that the ZLB binds.*

Assumption 2 *An outstanding amount of debt above the steady state level tightens the budget constraint of the government at positive interest rates. Formally, if* $\hat{b}_{t-1} > 0$ *and* $\hat{i}_t > r^*$*, then* $\lambda_t^b < 0$*.*

Assumption 3 *The following parameter restrictions apply:*

1. $(\gamma_c G + \gamma_g c)\eta_y > -\gamma_g y \eta_c$

2. $(\gamma_h G + \gamma_g y)\eta_c > -\gamma_g c \eta_y$

where $\eta_c \equiv \frac{\iota y}{\theta}\kappa_c + \frac{y}{\theta}\gamma_c - c - \gamma_c\Omega < 0$ *and* $\eta_y \equiv y + \frac{\iota y}{\theta}\kappa_y + \frac{y}{\theta}(1+\gamma_h) > 0$.

Notice that Assumption 1 is without loss of generality since the initial period could be interpreted as the last one in which the ZLB is binding. Moreover, considering an i.i.d shock has no qualitative effects. Assumption 2 is intuitive when looking at the budget constraint (1.17) and, like Assumption 3, holds under baseline calibration.[17]

[17]See Table 1.1 for the baseline calibration.

After the initial period, the environment becomes deterministic. Hence, the decision rules for the set of endogenous variables are a function of the inherited level of debt only. Given the linear-quadratic structure of the problem, this function is known to be linear with constant coefficient and, for each variable, will be denoted as follows: $\hat{\pi}_t = \Pi_b \hat{b}_{t-1}$, $\hat{c}_t = \mathcal{C}_b \hat{b}_{t-1}$, $\hat{y}_t = \mathcal{Y}_b \hat{b}_{t-1}$, etc.

1.4.1 The role of inflation expectations and the CBRF

When the objective function of the two authorities is the same, the first order necessary conditions *in the liquidity trap* are the same for the coordination and noncooperative game. The sequential policy problem can be seen as a coordinated policy problem in which the instrument of the CB is taken as given and equal to the lower bound. However, this does not mean that the equilibrium is the same, since control variables are chosen taking into account expectations outside the ZLB which are determined by the outcome of the game when the CB regains control of its instrument. Outside the ZLB, the following proposition applies

Proposition 1 *Issuing more debt in the initial period leads to more inflation at the exit of the liquidity trap. Formally, if $\hat{\pi}_t = \Pi_b \hat{b}_{t-1}$ for $t > 0$, then we have $\Pi_b > 0$.*

Proof. Outside the ZLB, the target rule for the budget constraint multiplier is given by[18]

$$\lambda_t^b = -\Phi_b \hat{\pi}_t \tag{1.27}$$

with $\Phi_b \equiv \Delta^{-1} D(yc\Psi + (\gamma_h c + \gamma_c y)\frac{\iota y}{\Psi}) > 0$, $\Delta \equiv \gamma_h c[\gamma_g y \zeta_c + (\gamma_c G + \gamma_g c)\zeta_y] + \gamma_c y[\gamma_g c \zeta_y + (\gamma_h G + \gamma_g y)\zeta_c]$, $\zeta_y \equiv \eta_y + \frac{\gamma_h}{\Psi}(\frac{\iota y}{\theta} + \Omega\beta^{-1})$, $\zeta_c \equiv \eta_c + \frac{\gamma_c}{\Psi}(\frac{\iota y}{\theta} + \Omega\beta^{-1})$ and $D \equiv \gamma_h\gamma_c G + \gamma_h\gamma_g c + \gamma_c\gamma_g y$. According to Assumption 2, if $\hat{b}_0 > 0$, we have that $\lambda_1^b < 0$ and hence, it must be that $\hat{\pi}_1 > 0$. ■

This proposition is related to the *inflationary bias* identified by Leeper et al.

[18] See Appendix 1.7.5 for derivation.

(2020) in the presence of distortionary taxation and time-consistent policy. When the government is confronted to a higher debt burden, inflation increases because of the cost-push effect of labor taxes and the incentive to erode the real value of debt with frontloaded inflation. The benefits of this strategy depend on the CBRF and the impact of future real interest rates on consumption both inside and outside the liquidity trap as described in the following proposition

Proposition 2 *In the liquidity trap, the marginal effect of an increase in government debt on consumption depends on the reaction function of the CB. Qualitatively, we have that*

- *if* $\Phi_c(\Psi) < \gamma_c^{-1}$, *a marginal increase in government debt has a positive effect on current consumption. Formally,* $\frac{\partial \hat{c}_0}{\partial \hat{b}_0} > 0$.
- *if* $\Phi_c(\Psi) > \gamma_c^{-1}$, *a marginal increase in government debt has a negative effect on current consumption. Formally,* $\frac{\partial \hat{c}_0}{\partial \hat{b}_0} < 0$.

where $\Phi_c(\Psi) = \gamma_h G c^{-1} \Phi_b(\Psi) + \gamma_g y \Psi - \gamma_c \gamma_h G c^{-1} \frac{\iota y}{\Psi}$

Proof. For $t > 0$, the target rule for consumption is given by

$$\hat{c}_t = -\Phi_c \hat{\pi}_t \tag{1.28}$$

where Φ_c is defined as in the proposition (see Appendix 1.7.5 for derivation). Substituting this target rule for the expectation term in the time zero Euler equation (1.8) gives

$$\hat{c}_0 = (\gamma_c^{-1} - \Phi_c)\Pi_b \hat{b}_0 - \gamma_c^{-1}(r^* - d_L)$$

Taking the partial derivative of this expression with respect to debt gives $\frac{\partial \hat{c}_0}{\partial \hat{b}_0} = (\gamma_c^{-1} - \Phi_c)\Pi_b$. The fact that $\Pi_b > 0$ according to Proposition 1 completes the proof. ■

As it turns out, $\Phi_c(\Psi) < \gamma_c^{-1}$ is verified only with a very restrictive (and unlikely) set of parameters. Figure 1.1 shows for which value of the inflation weight in the loss function of the CB this condition is verified under baseline calibration (dashed red line).

[Figure 1.1 approximately here]

The vertical line corresponds to the socially optimal weight derived from the second-order approximation of the utility around the efficient steady state ($\delta_\pi = \iota y$). Hence, under fiscal leadership, it is generally the case that, increasing the debt level in the liquidity trap reduces consumption. Moreover, appointing a more conservative CB increases the negative sensitivity of consumption to debt. To understand this last observation, consider the solid blue line that represents the reaction of the budget constraint to a change in inflation, Φ_b. This parameter is increasing in the degree of central bank's conservatism implying that an increase in inflation tightens the budget constraint more because monetary policy stabilises inflation more aggressively at positive interest rates. Hence, the anticipation of higher real rates at the exit of the lower bound weighs on the consumption of forward-looking households.

1.4.2 The role of endogenous government spending

In what follows, consider the common case where $\Phi_c(\Psi) > \gamma_c^{-1}$. So far, we have seen that increasing debt in the liquidity trap boosts inflation expectations but depresses current consumption. The reason is that the CB is expected to respond to excess inflation by setting a negative output gap at the exit of the liquidity trap. What is then the net impact on output and inflation in the liquidity trap?

The answer to this question crucially depends on the marginal response of government spending to a variation of government debt. Totally differentiating the resource constraint gives

$$\underbrace{\frac{\partial \hat{y}_0}{\partial \hat{b}_0}}_{?} = y^{-1}\left(c \underbrace{\frac{\partial \hat{c}_0}{\partial \hat{b}_0}}_{<0} + G \underbrace{\frac{\partial \hat{G}_0}{\partial \hat{b}_0}}_{?} \right)$$

The following proposition characterises the sign of the last term

Proposition 3 *In the liquidity trap, the marginal effect of an increase in government debt on government spending depends on its relative effect on consumption and on the tightness of the ZLB constraint. Issuing more debt stimulates (depresses) government spending if the positive effect from the ZLB constraint tightening outweighs (falls short of) the negative effect from the drop in consumption.*

Proof. The target rule for choosing government spending in the liquidity trap reads (see Appendix 1.7.5 for derivation)

$$\hat{G}_0 = \frac{\gamma_h \eta_c - \gamma_c \eta_y}{D} \psi \pi_0 - \frac{\gamma_h}{D} \lambda_0^{zlb} \tag{1.29}$$

Solving the consumption target rule (1.38) for inflation gives

$$\psi_0 \pi_0 = \frac{(\gamma_h G + \gamma_g y)\gamma_c \lambda_0^{zlb} - D c \hat{c}_0}{\Sigma} \tag{1.30}$$

where $\Sigma \equiv \gamma_g c \eta_y + (\gamma_h G + \gamma_g y)\eta_c > 0$.

Combining (1.29) and (1.30) and, totally differentiating the resulting equation with respect to $\hat{b}_t$ gives

$$\frac{\partial \hat{G}_0}{\partial \hat{b}_0} = \underbrace{\frac{c(\gamma_c \eta_y - \gamma_h \eta_c)}{\Sigma} \frac{\partial \hat{c}_0}{\partial \hat{b}_0}}_{\text{budget constraint effect } (<0)} \underbrace{- \frac{(\gamma_c \eta_y - \gamma_h \eta_c)(\gamma_h G + \gamma_g y)\gamma_c + \Sigma \gamma_h}{D\Sigma} \frac{\partial \lambda_0^{zlb}}{\partial \hat{b}_0}}_{\text{ZLB constraint effect } (>0)} \tag{1.31}$$

The negative sign of the second term stems from Corollary 2 in Appendix 1.7.5. Hence, the marginal effect of debt on government spending is positive if the second term is smaller than the first term (the effect of the ZLB constraint tightening outweighs the effect of the drop in consumption) and negative otherwise. ■

The drop in consumption stemming from a marginal increase in debt has two opposite effects on government spending. On the one hand, the room for a fiscal expansion is reduced due to the shrinkage of the tax base which tightens the budget constraint. On the other hand, additional fiscal support is warranted to mitigate the inflation shortfall. If the second effect dominates, the government reacts to the low state of demand by increasing its own consumption. Whether this fiscal stimulus is strong enough to generate a positive effect on the output gap determines the desirability of issuing more debt in the liquidity trap. In the next section, I investigate this trade-off quantitatively.

1.5 Numerical results

The model is calibrated on the US economy before the Great Recession. A time period represents one quarter of a year. The preference parameters, ν_g and ν_h, are chosen to be consistent with households working one quarter of their time endowment and with a share of government spending equating one fifth of output. The annual interest rate in the intended steady state is set to 2.5% and pins down the time discount factor, β. The parameters of the Phillips Curve are standard to the literature. The intertemporal elasticity of private and public consumption and, the inverse of the Frisch elasticity are equal to 1. The elasticity of substitution between intermediate goods, θ, corresponds to a price markup over the marginal cost of 10%. Given the value of θ, the parameter of the price adjustment cost, ι, matches its Calvo (1983) equivalent (up to a first-order approximation around the deterministic steady-state) when the average duration for setting prices equals one year. The parameter values are summarised in Table 1.1.

[Table 1.1 approximately here]

The market value of debt in the efficient steady state corresponds to 40% of output. This section studies the properties of the optimal time-consistent equilibrium with one-period debt and with longer maturity. An exogenous demand shock of four unconditional standard deviations constitutes the baseline scenario to motivate the liquidity trap episode.[19]

Since the main objective of this paper is to study the optimal use of debt to stabilise the economy in a liquidity trap, it is crucial to properly account for the role of uncertainty in shaping private sector expectations about output and inflation. To this aim, methodological approaches that compute optimal policies along a deterministic path are inadequate because they assume that the duration and depth of the recession are known once the shock has realised. Hence, I instead solve the model using a collocation method on a finite domain for the states. This method not only fully internalises the effect of uncertainty on the decision rules of the agents but also allows to deal with issues related to non-linearity and time-consistency. More details about the algorithm are provided in Appendix 1.7.6.

1.5.1 Deflationary bias at positive nominal rates

Under time-consistent policy, a *deflationary bias* occurs at positive interest rates (see e.g. Nakov, 2008) because the risk of entering a liquidity trap in the future causes forward-looking agents reduce their consumption and prices. When the government cannot commit to higher inflation, this adverse anticipation effect pushes inflation persistently below target and creates an endogenous trad-off between inflation and output gap stabilisation.

In the context of the present model, the optimal coordinated response is for fiscal policy to issue more debt and for monetary policy to accommodate the upward pressure on inflation expectations by keeping nominal rates low. As a result, by issuing only a moderate amount of debt above the steady state, the policymaker lowers expected real rates sufficiently to relax the deflation bias.[20]

In comparison, overcoming the deflation bias is more tedious when the two authorities play a noncooperative game. In this case, the additional debt issued

[19]Since the economy starts in the unstable deterministic steady state, the shock is assumed to occur after a burn-in of 200 periods to phase out the influence of initial conditions.

[20]Those lower expected real rates are akin the ones described by Nakata and Schmidt (2019) with the appointment of a conservative central bank.

on the fiscal side causes monetary policy to contract and pushes up expected real rates. Hence, achieving a desired inflation rate at the risky steady state requires to accumulate more debt than under coordination and the associated output gap falls short of its deterministic counterpart. Table 1.2 shows the difference for selected variables in the risky steady state between coordination and noncooperative game.

[Table 1.2 approximately here]

What is then the risk of a liquidity trap once the economy has settled to the risky steady state? Despite starting from a higher nominal rate at the risky steady state, the stabilisation properties of the noncooperative equilibrium increase the likelihood of visiting the lower bound. In particular, the persistent drop in inflation (expectations) stemming from the debt consolidation in the liquidity trap implies that a smaller decline in demand is needed for the lower bound to become binding, relative to coordination. Simulating a long sample of $100,000$ periods from the risky steady state gives an unconditionnal probability of liquidity trap equal to 13.2% under noncooperative game and to 8.6% under coordination.[21]

1.5.2 Optimal response with short-term liabilities

This subsection emphasises the role of debt in the optimal time-consistent response to the demand shock when debt maturity is one period. To this aim, I set the parameter $\rho = 0$. Figure 1.2 compares the outcome of coordinated policy and noncooperative game by showing the trajectory of nine key variables: the market value of government debt, the nominal interest rate, the output gap, the labor tax rate, inflation, consumption, government spending, the real interest rate and the price of the government bond.

[Figure 1.2 approximately here]

[21] The first $5,000$ observations are discarded to phase out the influence of the initial conditions.

Looking at the first panel, the noncooperative FA (dashed red line) reduces debt at the outset of the liquidity trap by increasing labor taxes and returns it only very slowly to its steady state value. As a result, inflation remains under target long after the nominal rate has lifted-off from the lower bound. This subdued inflation, in turn, causes the independent CB to adopt an expansionary monetary stance by keeping its nominal rate at a lower level than the one which would have prevailed absent of the consolidation. This monetary policy response improves economic outlook as can be seen by the sustained overshooting of the output gap in the top right panel. Since households anticipate real rates to stay below their natural level at the exit of the lower bound, the expectation channel mitigates the drop of consumption in the liquidity trap. This consumption effect, together with the initial cost-push effect of labor taxes, alleviates the inflation shortfall stemming from the negative demand shock. Moreover, the recessive effect of labor taxes is attenuated because the lower bound prevents a monetary policy tightening to dampen the resulting inflation.

This debt consolidation contrasts markedly with the short-lived increase in debt observed under coordination (solid blue line). When the CB coordinates with the FA, it internalises the effect of its policy rate decisions on the government budget constraint. Since the roll-over-cost of debt is more sensitive to the policy rate when the maturity is short, the CB optimally accommodates the inflationary bias stemming from higher debt. Hence, issuing more debt increases inflation (expectations) and relaxes the lower bound constraint. In addition, the issuance finances both a tax cut and a fiscal expansion stimulating the economy sufficiently to create an initially positive output gap in the liquidity trap. The fiscal stimulus is nevertheless slightly overturned at the exit of the ZLB.

Finally, notice that the short-term impact on consumption is very similar under both institutional set-ups but the output loss is smaller under coordination. The accommodative monetary policy creates the ground for a much larger fiscal stimulus which allows the economy to leave the liquidity trap after a shorter time period.

1.5.3 Enhanced coordination with longer debt maturity

To investigate the role of debt maturity in the equilibrium outcome, I increase the value of the parameter ρ from 0 to 0.9434 which corresponds to an average debt maturity of 16 quarters. Figure 1.3 displays the same variables as for the short-term case.

[Figure 1.3 approximately here]

The performance of both institutional set-ups in stabilising the economy is now strikingly similar. In particular, the transition path features an initial small increase in debt, followed by a persistent drop which is slightly more pronounced, and recovers more slowly, when coordination is lacking. This also explains the small difference in inflation. Overall, the dynamics resemble to the noncooperative policy with short-term debt.

What explains this alignment of coordinated and noncooperative policy when the maturity of debt is lengthened? By lengthening the debt maturity, the roll-over-cost of government debt is made less sensitive to the short-term rate. Hence, the coordinated CB focuses less on the fiscal cost of raising the policy rate and more on the benefits in terms of inflation and output gap stabilisation. In other words, it acts more independently from the FA. In this sense, issuing debt of longer maturity provides the authorities with a hedge against coordination issues in dealing with liquidity traps.

1.5.4 Discussion of the results

The analysis in this paper helps to shed light on several dimensions of the optimal time-consistent fiscal and monetary policy when debt matters for the expectations of the agents.

First, the optimality of a debt-financed fiscal stimulus in the liquidity trap lies on the assumption of monetary and fiscal policy coordinating. Previous work such as Eggertsson, 2006; Burgert and Schmidt, 2014 have introduced non-Ricardian policy to show that government debt provides a commitment device to make a low-for-longer interest rate policy time-consistent. As pointed out by Eggertsson (2013), coordination is a central assumption to obtain this result. In his model that assumes collection costs to lump-sum taxes, attributing an independent goal to monetary policy shuts down the expectation channel of deficit-spending and leaves inflation determination to monetary policy alone.[22]

This is an important consideration in practice because perfect coordination is inconsistent with the independent mandate of the Fed to safeguard price stability. This paper shows that if private agents are convinced that monetary policy will never become subservient to fiscal policy, a large accumulation of government debt may not be neutral but even perilous. In the absence of default, a large stock of debt necessarily implies higher interest charges — and thus taxation burden — when the nominal rate lifts-off the lower bound. Should firms pass on the increase in their marginal costs to consumers, the resulting inflation would call for a contractionary monetary policy. Of course, if households are not myopic, they will already raise their savings in the liquidity trap in anticipation of the higher real rates and labor taxes, a scenario that public policy desperately tries to avoid. In Appendix 1.7.8, I show that those dynamics are largely driven by the inflationary bias of public debt when the time-consistent government levies distortionary taxes on labor income. When taxes are kept constant at their steady state level and fiscal policy is thus deprived of its tax instrument, the optimal noncooperative policy consists in accumulating debt in the liquidity trap no matter the maturity of the

[22] To obtain this result, the author assumes that real spending is exogenous and held constant at all time such that Ricardian equivalence holds. Given the ad-hoc objective attributed to the central bank, this implies that debt does not impact inflation and output. This is of course a key difference with my model of endogenous spending. Another difference is the endogenous trade-off between inflation and output that emerges because of distortionary taxation and that is absent of a model with collection costs of lump-sum taxes.

bond. This can be explained by the lack of inflationary effect from public debt that allows the government to increase tremendously its consumption without fearing a monetary tightening as a consequence when the lower bound stops binding. With constant taxes, this large fiscal stimulus mitigates the size of the recession caused by the liquidity trap although at the expense of a larger inflation shortfall due to the lack of cost-push effect from distortionary taxation.

Second, the appointment of an independent central bank improves welfare significantly when debt is short-term. This can be seen in Table 1.3 that computes average welfare over 50 samples of $5,000$ periods each for different institutional set-ups.[23]

[Table 1.3 approximately here]

The occurrence of occasional liquidity trap episodes causes a welfare loss of 0.0377 when fiscal and monetary policy coordinate. This loss falls to 0.0095 when considering a noncooperative game. This result may seem surprising given that the debt consolidation following a large demand shock causes a deeper recession in the near-term. However, central bank independence has a major advantage; it disciplines fiscal policy. The threat of a monetary policy tightening in response to the inflationary bias of debt smoothes inflation in and out the liquidity trap. As a result, standard deviation of inflation (in the second column of the table) is much lower. In contrast, the coordinated fiscal policy accumulates a large amount of debt in the liquidity trap to benefit from the subsequent accomodative stance of monetary policy. Fiscal policy achieves this higher debt level by increasing government spending and cutting labor taxes. While this policy curbs the recession, it does so at the cost of a welfare reducing increase in government spending and inflation volatility. In Appendix 1.7.7, I emphasise that the benefits of nonco-

[23] Welfare is expressed in permanent consumption loss. Let the unconditional expected lifetime utility loss be $\mathcal{S} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\hat{\pi}_t^2]$. A permanent reduction in private consumption by the share $\mathcal{W}$ lowers the utility of the household by an amount equivalent to $\frac{U_c C \mathcal{W}}{1-\beta}$. Hence, the welfare loss is $\mathcal{W} = \frac{1}{2}(1-\beta)C^{-1}\mathcal{S}$.

operative policy in terms of fiscal discipline also exist when abstracting from the lower bound constraint. In my model, the time-consistency assumption produces an expansionary bias of fiscal policy; the government fails to fully internalise the deflationary effects of its combined fiscal stimulus and tax-cut. The welfare effects of this bias are however milder in the absence of the ZLB constraint because of the accrued effectiveness of monetary policy in stabilising target variables.

Third, coordination issues worsen with shorter debt maturity. When the roll-over-cost of debt is more sensitive to the policy rate, the negative effects of monetary tightening arise in the nearer-term and the expectation channel described here is amplified. On the contrary, when debt is long-term, fiscal policy is more insulated from monetary policy decisions and the precise institutional set-up becomes less decisive for stabilisation outcomes. Consequently, welfare losses are virtually the same (see last two lines of Table 1.3).

In Appendix 1.7.9, I show that long-term debt (and thus noncooperative policy) produces IRFs to a large demand shock that are also closer to the first-best outcome observed under commitment. The long-lasting consolidation of debt is similar to the near random walk outcome observed in optimal policy problems that involve consumption smoothing under commitment (e.g. Schmitt-Grohé and Uribe, 2004). In a liquidity trap, forward guidance allows to curb the recession by decreasing expected real interest rates and boosting (expected) inflation (e.g. Eggertsson and Woodford, 2003). Long-term debt pushes fiscal policy to engineer a time-consistent path of debt that somewhat replicates forward guidance while monetary policy keeps inflation in check. In this respect, the recent shortening of the privately held government debt stemming from quantitative easing policies may constitute a cause of concerns because it provides an incentive for fiscal policy to accumulate more debt at the expense of inflation stability.

1.6 Conclusion

I have demonstrated, both analytically and numerically, that debt-financed stimuli in a liquidity trap are undesirable when fiscal and monetary policy are conducted noncooperatively in a time-consistent manner. Large stocks of government debt aggravate the inflationary bias and the contractionary response of monetary policy at the exit of the lower bound. Anticipation of higher real rates and lower output provides forward-looking households with an incentive to reduce consumption and thus jeopardises the recovery. In this context, long-lasting consolidation of debt in the liquidity trap turns out optimal.

This policy insight raises a number of questions regarding the current macroeconomic situation in the US and elsewhere. First, the high stock of government debt may weigh negatively on the duration of lower bound episodes if households expect the central bank to remain adamant in defending its inflation target when the economy recovers. This insight is consistent with the soaring personal savings observed during the COVID-19 crisis.[24] Second, the maturity shortening of privately held debt observed those last years because of quantitative easing tends to amplify this expectation channel and thus the coordination issues in stabilising the economy. Of course, some of those results may be sensitive to the central bank being able to influence private sector expectations through government debt purchases. Future research should therefore study the impact of quantitative easing on the strategies played by the two authorities.

[24]See FRED data (https://fred.stlouisfed.org/graph/?g=AwzQ). Evidently, sanitary restrictions and precautionary savings are complementary explanations for this phenomenon.

1.7 Appendix

1.7.1 Tables and figures

Table 1.1: **Baseline calibration.**

Symbol	Description	Value
β	Subjective discount factor	0.99
θ	Elasticity of substitution among goods	11
ϵ	Price adjustment cost	117.805
γ_c	Intertemporal elasticity of c	1
γ_g	Intertemporal elasticity of G	1
γ_h	Intertemporal elasticity of h	1
ν_h	Utility weight on labor	20
ν_g	Utility weight on labor	0.25
ρ_r	AR coefficient on demand shock	0.77
σ	S.D. of demand shock (%)	0.4

Table 1.2: **Effect of institutional set-up on the risky steady state**

Variable	Deterministic	Risky	
		Coord.	Noncoop.
Mrkt val debt	40	52.8	103.8
Nominal rate	2.48	1.89	2.60
Real rate	2.48	1.81	2.40
Inflation	0	0.08	0.20
Output	0	0.007	-0.187
Tax rate	17.35	17.32	17.35
Gov. spending	20	19.9	19.8

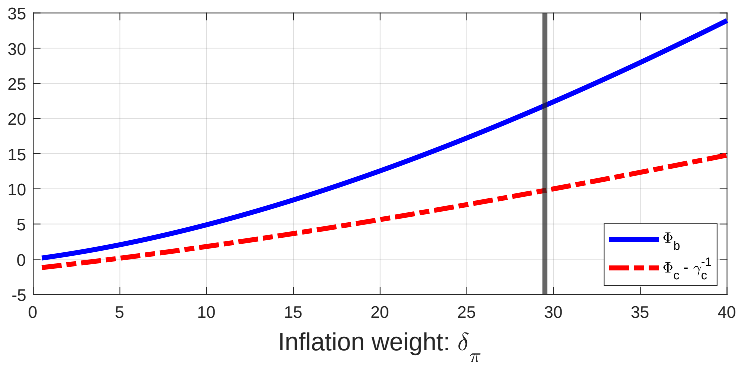
Note: Market value of debt is in percentages of annual output. Nominal, real rates and inflation are in annual percentages. Output is in relative difference from deterministic steady state. Tax rate is in absolute percentages. Government spending is in percentages of output.

Table 1.3: **Welfare losses**

Framework		Std dev.			
	$\mathcal{W}$	$\hat{\pi}_t$	$\hat{c}_t$	$\hat{y}_t$	$\hat{G}_t$
Short-term debt					
Coordination	0.0377	0.6823	0.5578	0.4172	3.1073
Noncooperative game	0.0095	0.0686	0.5242	0.3629	0.6094
Long-term debt					
Coordination	0.0028	0.0465	0.5400	0.3643	0.3992
Noncooperative game	0.0027	0.0421	0.5421	0.3678	0.3896

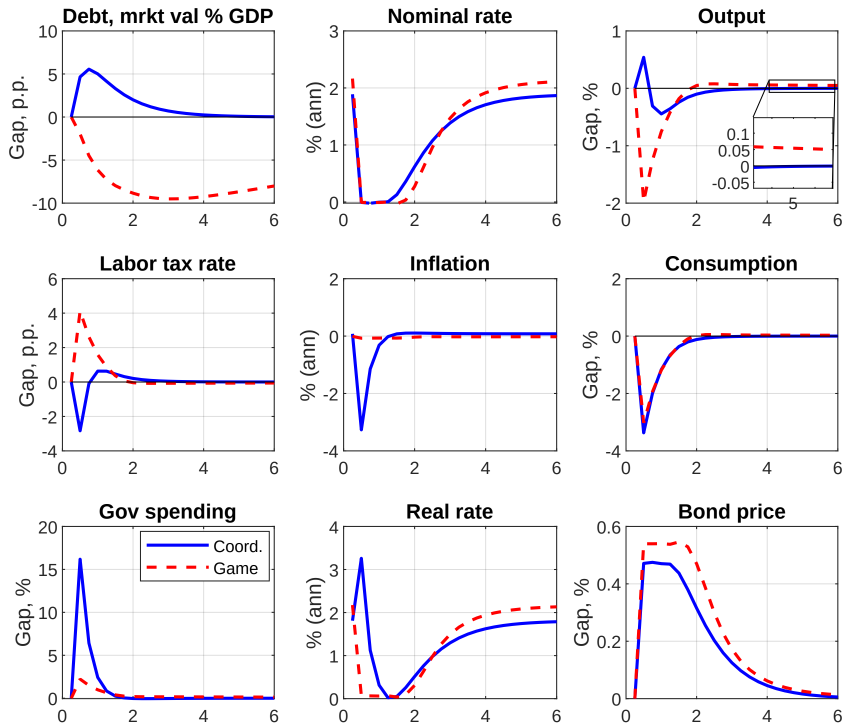
Note: Welfare loss ($\mathcal{W}$) is in percentage of permanent consumption equivalent

Figure 1.1: **Marginal effects in the analytical example as a function of the inflation weight in the CB's objective function.**



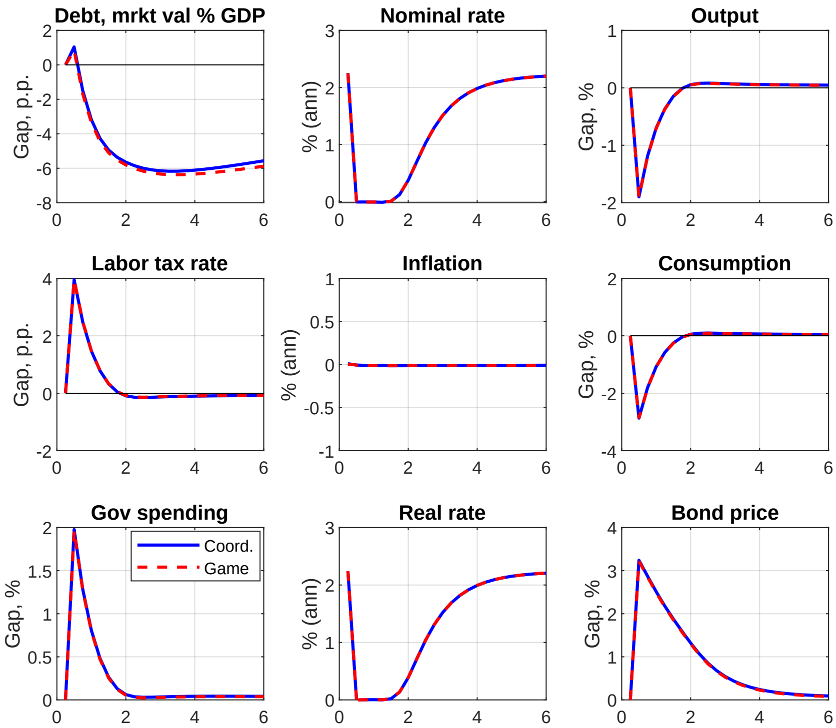
Notes: The solid blue line represents the reaction of the budget constraint to a change in inflation and the dashed red line the reaction of consumption to a change in government debt in the liquidity trap. The vertical line is the socially optimal weight derived from the second-order approximation of the utility around the efficient steady state ($\delta_\pi = \iota y$).

Figure 1.2: **IRFs of key variables to a negative demand shock driving the economy in a liquidity trap: short-term debt.**



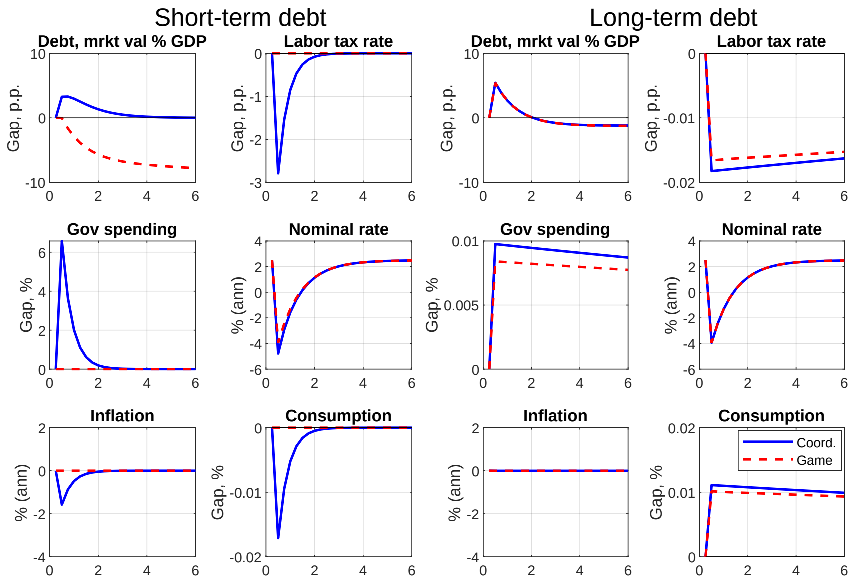
Notes: Short-term debt is debt lasting one quarter ($\rho = 0$). The solid blue line corresponds to the optimal time-consistent policy under perfect coordination between the two authorities and the dashed red line to the noncooperative game with Stackelberg leadership of the FA. The simulation starts after the economy has reached its risky steady state.

Figure 1.3: **IRFs of key variables to a negative demand shock driving the economy in a liquidity trap: long-term debt.**



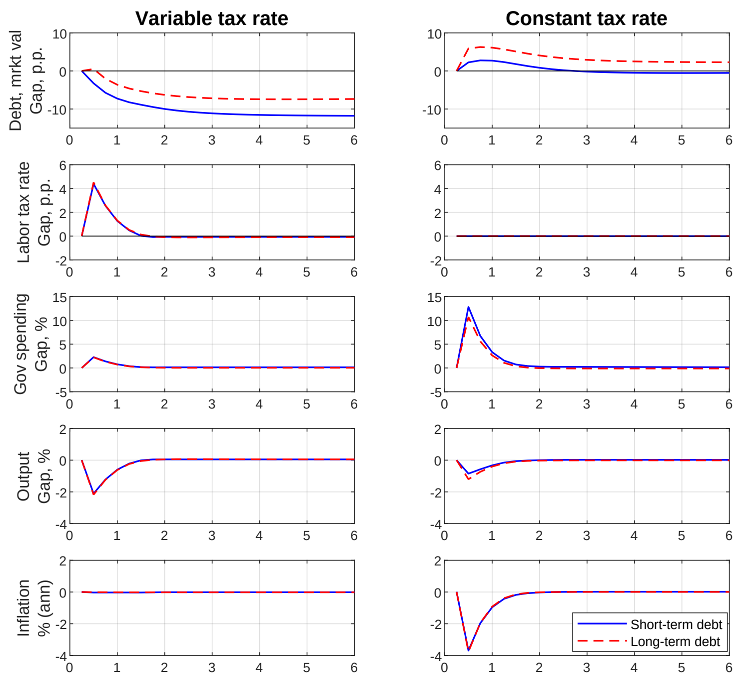
Notes: Long-term debt is debt lasting sixteen quarters ($\rho = 0.9434$). The solid blue line corresponds to the optimal time-consistent policy under perfect coordination between the two authorities and the dashed red line to the noncooperative game with Stackelberg leadership of the FA. The simulation starts after the economy has reached its risky steady state.

Figure 1.4: **IRFs to a large demand shock without the ZLB constraint**



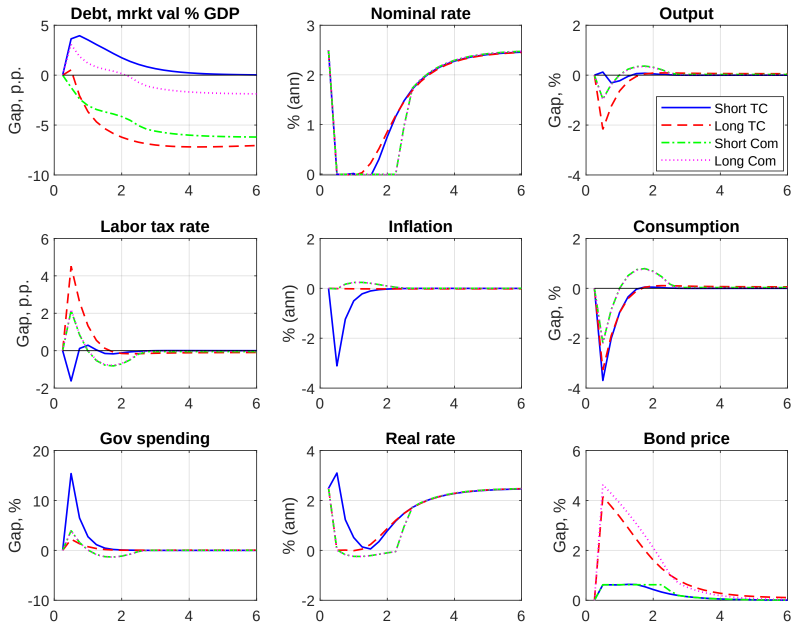
Notes: The solid blue line and the dashed red line are the IRFs for the coordinated policy and the noncooperative game, respectively. The left-hand side panels (two columns) correspond to the case of short-term debt ($\rho = 0$) and the right-hand side panels (two columns) to the long-term debt ($\rho = 0.9434$).

Figure 1.5: **The effects of shutting down the labor tax instrument**



Notes: The solid blue line and the dashed red line are the IRFs for short-term and long-term debt, respectively. The left-hand side panels correspond to the case of all policy instruments available and the right-hand side panels to the constant tax rate.

Figure 1.6: **The optimal joint fiscal and monetary policy under commitment**



Notes: The solid blue line and the dashed red line are the IRFs under time-consistency for short-term and long-term debt, respectively and the dashed-dotted green line and dotted magenta line are the IRFs under commitment for short-term and long-term debt, respectively.

1.7.2 The non-linear economy

Equilibrium conditions The representative household maximises its expected lifetime utility (1.1) subject to its budget constraint (1.3). Attaching multiplier λ_t to the constraint, the FONCs of this problem are standard

$$\begin{aligned}
\nu_h h_t^{\gamma_h} c_t^{\gamma_c} &= (1-\tau_t) w_t \\
R_t^{-1} &= \beta d_t \mathbb{E}_t \left[\frac{c_t^{\gamma_c}}{\pi_{t+1} c_{t+1}^{\gamma_c}} \right] \\
q_t &= \beta d_t \mathbb{E}_t \left[\frac{c_t^{\gamma_c}(1+\rho q_{t+1})}{\pi_{t+1} c_{t+1}^{\gamma_c}} \right]
\end{aligned}$$

The problem (1.5) of the intermediate firm gives rise to the following FONC

$$w_t = \frac{\theta - 1}{\theta(1-s)} - \frac{\iota}{\theta(1-s)} \left(\pi_t(\pi_t - 1) - \beta d_t \mathbb{E}_t \left[\frac{c_t^{\gamma_c}}{c_{t+1}^{\gamma_c}} \frac{y_{t+1}}{y_t} \pi_{t+1}(\pi_{t+1} - 1) \right] \right)$$

Moreover, the budget constraint of the government in real terms reads

$$q_t b_t - (1+\rho q_t)\frac{b_{t-1}}{\pi_t} - G_t + \tau_t w_t y_t - s(w_t y_t - wy) = 0$$

Together with the resource constraint, $y_t = c_t + G_t$, those equations can be log-linearised to obtain the system of equations in the text.

1.7.3 Derivation of the loss function

The loss function is obtained with a second order approximation of the utility of the households around the efficient steady-state. The utility function comprises three components:

1. The first component of utility is $F = \frac{(y_t(1-\frac{\iota}{2}(\pi_t-1)^2)-G_t)^{1-\gamma_c}}{1-\gamma_c}\xi_t$

$$
\begin{array}{ll}
F_y = c^{-\gamma_c}(1-\frac{\iota}{2}(\pi-1)^2)\xi & F_g = -c^{-\gamma_c}\xi \\
F_\pi = -\iota(\pi-1)yc^{-\gamma_c}\xi & F_{yy} = -\gamma_c c^{-\gamma_c-1}[1-\frac{\iota}{2}(\pi-1)^2]^2\xi \\
F_{gg} = -\gamma_c c^{-\gamma_c-1}\xi & F_{\pi\pi} = -(\gamma_c y[\iota(\pi-1)]^2 c^{-\gamma_c-1} + \iota c^{-\gamma_c})y\xi \\
F_{yg} = \gamma_c c^{-\gamma_c-1}(1-\frac{\iota}{2}(\pi-1)^2)\xi & F_{gy} = \gamma_c c^{-\gamma_c-1}(1-\frac{\iota}{2}(\pi-1)^2)\xi \\
F_{g\pi} = -\gamma_c c^{-\gamma_c-1}y(\pi-1)\xi & F_{\pi y} = [y(1-\frac{\iota}{2}(\pi-1)^2)\gamma_c c^{-1}-1]\iota(\pi-1)c^{-\gamma_c}\xi \\
F_{\pi g} = -\iota(\pi-1)y\gamma_c c^{-\gamma_c-1}\xi & F_{y\pi} = [\gamma_c c^{-1}y(1-\frac{\iota}{2}(\pi-1)^2)-1]\iota(\pi-1)c^{-\gamma_c}\xi \\
F_\xi = \frac{c^{1-\gamma_c}}{1-\gamma_c} & F_{y\xi} = c^{-\gamma_c}(1-\frac{\iota}{2}(\pi-1)^2) \\
F_{\xi\xi} = 0 & F_{g\xi} = -c^{-\gamma_c} \\
F_{\xi y} = (1-\frac{\iota}{2}(\pi-1)^2)c^{-\gamma_c} & F_{\xi g} = -c^{-\gamma_c} \\
F_{\xi\pi} = -c^{-\gamma_c}y\iota(\pi-1) & F_{\pi\xi} = -\iota(\pi-1)yc^{-\gamma_c}
\end{array}
$$

given that $\xi = 1$, we have

$$
F \simeq (y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}y^2\hat{y}_t^2 + \gamma_c c^{-1}G\hat{G}_t y\hat{y}_t + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\gamma_c c^{-1}G^2\hat{G}_t^2
$$
$$
-\frac{1}{2}\iota y\hat{\pi}_t^2)c^{-\gamma_c} + t.i.p = y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}(c\hat{c}_t)^2 + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\iota y\hat{\pi}_t^2)c^{-\gamma_c} + t.i.p
$$

2. The second component of utility is $G = \frac{G_t^{1-\gamma_g}}{1-\gamma_g}\nu_g\xi_t$

$$
\begin{array}{ll}
G_g = G^{-\gamma_g}\nu_g\xi & G_{gg} = -\gamma_g G^{-\gamma_g-1}\nu_g\xi \\
G_\xi = \frac{G^{1-\gamma_g}}{1-\gamma_g}\nu_g & G_{\xi\xi} = 0 \\
G_{g\xi} = G^{-\gamma_g}\nu_g & G_{\xi g} = G^{-\gamma_g}\nu_g
\end{array}
$$

given that $\xi = 1$ and $\nu_g = G^{\gamma_g}c^{-\gamma_c}$, we have

$$
G \simeq (G\hat{G}_t - \frac{1}{2}\gamma_g G\hat{G}_t^2 + G\hat{G}_t\hat{\xi}_t)c^{-\gamma_c} + t.i.p
$$

3. The third component of utility is $H = \frac{y_t^{1+\gamma_h}}{1+\gamma_h}\nu_h\xi_t$

$$\begin{aligned}
&H_y = y^{\gamma_h}\nu_h\xi && H_{yy} = \gamma_h y^{\gamma_h - 1}\nu_h\xi \\
&H_\xi = \tfrac{y^{1+\gamma_h}}{1+\gamma_h}\nu_h && H_{\xi\xi} = 0 \\
&H_{y\xi} = y^{\gamma_h}\nu_h && G_{\xi y} = y^{\gamma_h}\nu_h
\end{aligned}$$

given that $\xi = 1$ and $\nu_h = y^{-\gamma_h}c^{-\gamma_c}$, we have

$$H \simeq (y\hat{y}_t + \frac{1}{2}\gamma_h y^2\hat{y}_t^2 + y\hat{y}_t\hat{\xi}_t)c^{-\gamma_c} + t.i.p$$

Collecting all the components and scaling with c^{γ_c}, the second-order approximation is given by

$$\begin{aligned}
U \simeq F + G - H = y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}(c\hat{c}_t)^2 + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\iota y\hat{\pi}_t^2 \\
+ G\hat{G}_t - \frac{1}{2}\gamma_g G\hat{G}_t^2 + G\hat{G}_t\hat{\xi}_t - y\hat{y}_t - \frac{1}{2}\gamma_h y^2\hat{y}_t^2 - y\hat{y}_t\hat{\xi}_t + t.i.p
\end{aligned}$$

Hence, the social objective corresponds to

$$U \simeq -\frac{1}{2}\Big[\big(\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\hat{\pi}_t^2\Big] + t.i.p$$

1.7.4 FONCs of the coordinated problem

The FONCs for the policy problem are detailed below

$$\begin{aligned}
\partial\mathcal{L}/\partial\hat{c}_t \equiv \quad & 0 = \gamma_c c\hat{c}_t + \Lambda_t^r c + \Lambda_t^p\kappa_c - \Lambda_t^q\gamma_c - \Lambda_t^b\frac{y}{\theta}\gamma_c \\
\partial\mathcal{L}/\partial\hat{G}_t \equiv \quad & 0 = \gamma_g G\hat{G}_t + \Lambda_t^r G - \Lambda_t^b G \\
\partial\mathcal{L}/\partial\hat{y}_t \equiv \quad & 0 = \gamma_h y\hat{y}_t - \Lambda_t^r y + \Lambda_t^p\kappa_y - \Lambda_t^b\frac{y}{\theta}(1+\gamma_h) \\
\partial\mathcal{L}/\partial\hat{\pi}_t \equiv \quad & 0 = \iota y\hat{\pi}_t - \Lambda_t^p + \Lambda_t^b\Omega\beta^{-1} \\
\partial\mathcal{L}/\partial\hat{\tau}_t \equiv \quad & 0 = \Lambda_t^p\kappa_\tau + \Lambda_t^b w\tau y\frac{\theta-1}{\theta} \\
\partial\mathcal{L}/\partial\hat{q}_t \equiv \quad & 0 = \Lambda_t^q + \Lambda_t^b\Omega(1-\rho) - \Lambda_t^{zlb} \\
\partial\mathcal{L}/\partial\hat{b}_t \equiv \quad & 0 = \Lambda_t^p\beta\Theta_{1,t} + \Omega(\Lambda_t^b - \mathbb{E}_t\Lambda_{t+1}^b) + \Lambda_t^q\Theta_{2,t} + \Lambda_t^{zlb}(\Theta_{3,t} - \Theta_{2,t})
\end{aligned}$$

After some algebra, the set of first order necessary conditions can be reduced to the following five equations

$$
\begin{aligned}
&y\hat{y}_t = -\frac{\gamma_g y \eta_c + (\gamma_c G + \gamma_g c)\eta_y}{D}\psi\hat{\pi}_t + \frac{\gamma_g \gamma_c y}{D}\Lambda_t^{zlb} \\
&c\hat{c}_t = -\frac{\gamma_g c \eta_y + (\gamma_h G + \gamma_g y)\eta_c}{D}\psi\hat{\pi}_t + \frac{(\gamma_h G + \gamma_g y)\gamma_c}{D}\Lambda_t^{zlb} \\
&(\Omega - \beta\frac{\iota y}{\theta}\Theta_{1,t} - \Omega(1-\rho)\Theta_{2,t})\psi\hat{\pi}_t = \Omega\psi\mathbb{E}_t\hat{\pi}_{t+1} + \Theta_{3,t}\Lambda_t^{zlb} \\
&\Lambda_t^{zlb}(\hat{i}_t - r^*) = 0 \\
&\hat{i}_t > -r^*
\end{aligned}
$$

where $\eta_y \equiv y + \frac{\iota y}{\theta}\kappa_y + \frac{y}{\theta}(1+\gamma_h)$, $\eta_c \equiv \frac{\iota y}{\theta}\kappa_c + \frac{y}{\theta}\gamma_c - c - \gamma_c\Omega(1-\rho)$ and $D \equiv \gamma_h\gamma_c G + \gamma_h\gamma_g c + \gamma_c\gamma_g y$. The parameter $\psi \equiv \frac{y\iota}{y\frac{\iota}{\theta} + \beta^{-1}\Omega} > 0$ determines the inflation response to a change in the shadow value of the budget constraint. The response to a tightening is always positive and the magnitude depends on the relative cost of inflation with respect to its benefits. Finally, $\Theta_{1,t}, \Theta_{2,t}$ and $\Theta_{3,t}$ are defined as in the text.

1.7.5 Analytical example

Derivation of the target rule for the budget constraint multiplier in the proof of proposition 1 From equations (1.20), (1.23) and (1.24), we know respectively that outside the ZLB $\lambda_t^r = \lambda_t^b - \gamma_g G^{-1}(y\hat{y}_t - c\hat{c}_t)$, $\lambda_t^p = -\frac{\iota y}{\theta}\lambda_t^b$ and $\lambda_t^q = -\Omega\lambda_t^b$.

Using those expressions in equation (1.19) and rearranging gives

$$
c\hat{c}_t = \left(\gamma_g c G^{-1} y\hat{y}_t + \zeta_c\lambda_t^b + \gamma_c\frac{\iota y}{\Psi}\hat{\pi}_t\right)\left(\gamma_c + \gamma_g c G^{-1}\right)^{-1} \tag{1.32}
$$

and doing the same with equation (1.21), we have

$$
y\hat{y}_t = \left(\gamma_g y G^{-1} c\hat{c}_t + \zeta_y\lambda_t^b + \gamma_h\frac{\iota y}{\Psi}\hat{\pi}_t\right)\left(\gamma_h + \gamma_g y G^{-1}\right)^{-1} \tag{1.33}
$$

Combining those two expressions gives a target rule for $\hat{y}_t(\hat{\pi}_t, \lambda_t^b)$ and $\hat{c}_t(\hat{\pi}_t, \lambda_t^b)$ with $t > 0$

$$\hat{y}_t = \left(\frac{\iota y}{\Psi} \hat{\pi}_t + [\gamma_g y \zeta_c + (\gamma_c G + \gamma_g c)\zeta_y] D^{-1} \lambda_t^b \right) \tag{1.34}$$

$$\hat{c}_t = \left(\frac{\iota y}{\Psi} \hat{\pi}_t + [\gamma_g c \zeta_y + (\gamma_h G + \gamma_g y)\zeta_c] D^{-1} \lambda_t^b \right) \tag{1.35}$$

Next, use equations (1.34) and (1.35) to substitute respectively for $\hat{y}_t$ and $\hat{c}_t$ in the CBRF (1.18) (with $\mu_t^{zlb} = 0$). Rearranging the resulting expression gives the target rule in proposition 1.

Derivation of the target rule for consumption in the proof of proposition 2 The derivation is straightforward. Rearrange the CBRF (1.18) to obtain: $\hat{y}_t = -\gamma_h^{-1}(\Psi \hat{\pi}_t + \gamma_c \hat{c}_t)$. Next, use this expression, together with equation (1.27), to respectively get rid of $\hat{y}_t$ and λ_t^b in equation (1.32). Then, rearranging gives target rule (1.28).

Derivation of the target rule for government spending at the ZLB in the proof of proposition 3 At the ZLB, we have that $\mu_0^{zlb} < 0$. Then, given the complementary slackness condition, it must be that $\lambda_0^s = 0$ and from equation (1.25), it implies that $\lambda_0^m = 0$. From equations (1.20), (1.23) and (1.24), we know respectively that inside the ZLB $\lambda_0^r = \lambda_0^b - \gamma_g G^{-1}(y\hat{y}_0 - c\hat{c}_0)$, $\lambda_0^p = -\frac{\iota y}{\theta} \lambda_0^b$ and $\lambda_0^q = -\Omega \lambda_0^b + \lambda_0^{zlb}$. Substituting these in equation (1.22) gives

$$\lambda_0^b = -\psi \hat{\pi}_0 \tag{1.36}$$

Using those expressions in equation (1.19) and rearranging gives

$$c\hat{c}_0 = (\gamma_g c G^{-1} y \hat{y}_0 - \eta_c \psi \hat{\pi}_0 + \gamma_c \lambda_0^{zlb})(\gamma_c + \gamma_g c G^{-1})^{-1}$$

and doing the same with equation (1.21), we have

$$y\hat{y}_0 = (\gamma_g y G^{-1} c\hat{c}_0 - \eta_y \psi \hat{\pi}_0)(\gamma_h + \gamma_g y G^{-1})^{-1}$$

Combining the two previous equations, we obtain two targeting rules

$$y\hat{y}_0 = -\frac{\gamma_g y \eta_c + (\gamma_c G + \gamma_g c)\eta_y}{D}\psi\hat{\pi}_0 + \frac{\gamma_g \gamma_c y}{D}\lambda_0^{zlb} \tag{1.37}$$

$$c\hat{c}_0 = -\frac{\gamma_g c \eta_y + (\gamma_h G + \gamma_g y)\eta_c}{D}\psi\hat{\pi}_0 + \frac{(\gamma_h G + \gamma_g y)\gamma_c}{D}\lambda_0^{zlb} \tag{1.38}$$

Taking the difference between the first and second one gives target rule (1.29).

Two corollaries used in the proof of proposition 3

Corollary 1 *For equilibria exhibiting monotone dynamics outside the liquidity trap, we have that* $\Omega - \beta\frac{\iota y}{\theta}\Theta_1 - \Omega\Theta_3 > 0$.

Proof. *Consider two consecutive periods outside the liquidity trap. Then, using target rule* (1.27) *in equation* (1.26)*, we get* $\mathbb{E}_t\hat{\pi}_{t+1} = \frac{\Gamma}{\Omega}\hat{\pi}_t$ *where* $\Gamma \equiv \Omega - \beta\frac{\iota y}{\theta}\Theta_1 - \Omega\Theta_3$*. This is an AR(1) process with auto-regressive parameter* $\frac{\Gamma}{\Omega}$*. Hence, for any monotonic dynamics, it must be that* $0 < \frac{\Gamma}{\Omega} < 1$ *and so,* $\Gamma > 0$. ■

Corollary 2 *In the liquidity trap, a marginal increase in debt tightens the ZLB constraint. Formally,* $\frac{\partial\lambda_0^{zlb}}{\partial\hat{b}_0} < 0$.

Proof. *Using target rules* (1.27) *and* (1.36) *in equation* (1.26) *and rearranging*

$$\psi\hat{\pi}_0 = \frac{\Omega\Phi_b}{\Gamma}\mathbb{E}_0\hat{\pi}_1 + \frac{\Theta_3}{\Gamma}\lambda_0^{zlb} \tag{1.39}$$

where $\Gamma \equiv \Omega - \beta\frac{\iota y}{\theta}\Theta_1 - \Omega\Theta_3 > 0$ *(see corollary 1) and* $\Theta_3 \equiv (\gamma_c\mathcal{C}_b + \Pi_b) = \gamma_c\Pi_b(\gamma_c^{-1} - \Phi_c) < 0$ *(see equation* (1.28)*).*

Combining (1.39) *and* (1.38) *and, totally differentiating the resulting equation* *with respect to* $\hat{b}_t$ *gives*

$$\frac{\partial \lambda_0^{zlb}}{\partial \hat{b}_0} = \underbrace{\frac{D\Gamma c}{\Xi}\frac{\partial \hat{c}_0}{\partial \hat{b}_0}}_{consumption\ effect\ (<0)} - \underbrace{\frac{(\gamma_g c\eta_y + (\gamma_h G + \gamma_g y)\eta_c)\Omega\Phi_b}{\Xi}\Pi_b}_{inflation\ expectation\ effect\ (>0)}$$

where $\Xi \equiv \Gamma(\gamma_h G + \gamma_g y)\gamma_c - (\gamma_g c\eta_y + (\gamma_h G + \gamma_g y)\eta_c)\Theta_3 > 0.$ ■

1.7.6 Solution method

The non-linearities introduced by the occasionally binding ZLB constraint preclude traditional local methods. Instead, I approximate the linear policy functions using a projection method (collocation) on a finite number of points in the domain. This method allows to account fully for the effects of uncertainty at the ZLB.[25] Since the policy functions are linear with a kink, I rely on cubic splines for the basis function. In solving the model, the noncooperative policy game involves some additional steps which are emphasised distinctly. The algorithm proceeds by checking convergence on two nested loops as follows:

1. Construct the grid for the state variables. Use a Gaussian quadrature scheme to discretise the normally distributed innovations to private demand.

2. Use the linear policy functions of the commitment solution from Dynare (outside the ZLB) to obtain an initial guess for the basis coefficients.

3. *Outer loop*: With the current guess of the basis coefficients, approximate the partial derivative of the expectation terms with respect to government debt.

4. *Inner loop*: To solve the system of equilibrium equations, I proceed as follows.

   (i) Use the guess of the basis coefficients to recover government debt from the budget constraint at the collocation nodes.

[25] See Nakata (2017) for an assessment of the impact of uncertainty on macroeconomic volatility at the ZLB.

(ii) Build the new grid and approximate the expectation terms associated with next period's decisions.

(iii) Solve the linear system outside the ZLB with matrix inversion.

(iv) **(only for policy game)** Verify KKT condition on the ZLB constraint of the CB. If $\lambda_t^s < 0$, KKT is violated and the CBRF is not a binding constraint for the Fiscal Leader. Hence, solve the alternative system with the corresponding equation muted.

(v) Verify whether ZLB constraint binds. In the affirmative, solve the alternative system at the ZLB and check KKT conditions.

(vi) Update the guess for the basis coefficients based on the decision rules for the current period. If the difference between the old and the new guess is smaller than $1.49e^{-8}$, the inner loop has converged. Otherwise, go back to step (i).

5. Update the guess for the the partial derivative of the expectation terms with respect to government debt based on basis coefficients obtained from (4). If the difference between the old and the new guess is smaller than $1.49e^{-8}$, the outer loop has converged. Otherwise, go back to step (3).

I implement this procedure in Matlab by relying on the CompEcon toolbox of Miranda and Fackler (2002) for the Gaussian quadrature and function evaluation at the collocation nodes.

1.7.7 Stabilisation properties abstracting from the ZLB

As explained in the main text, the so-called *divine coincidence* does not hold even in the absence of the ZLB constraint because of the presence of distortionary taxes. Hence, welfare gains from the lack of coordination may also arise in this environment. An additional question concerns the policy biases identified by Adam and Billi (2008) under time-consistency that may induce overspending and overly accomodative monetary policy at positive rates. In my model, the jointly optimal policy is equivalent to a sequential policy when the two authorities have the same objective and **the same set of constraints**.

[Figure 1.4 approximately here]

Figure 1.4 shows that the jointly optimal policy responds to the demand shock by increasing government spending and cutting taxes when debt is short-term. This policy attempts to mitigate the inefficiently low level of consumption and output but fails to fully internalise the negative effect on inflation. In this regard, the noncooperative policy improves inflation stabilisation outcomes because the fiscal policy response is muted due to the threat of monetary tightening; a result that recalls the benefits of monetary conservatism underlined by Adam and Billi (2008). When debt is long-term, this threat is less effective because the price of the long-bond is less directly related to the short-term rate. Hence, fiscal policy does react to the demand shock although to a lesser extent than with short-term debt. This policy achieves a low volatility of target variables independently of the institutional framework.

1.7.8 Optimal noncooperative policy with constant taxes

The *inflationary bias* of government debt stems from the cost-push effect of labor taxes: the government wants to stabilise debt at a more rapid pace to avoid subsequent increases in inflation. A natural extension is thus to understand how optimal fiscal and monetary policy change when taxes are kept constant at their steady state value and the incentive to inflate debt away is shut down. The assumption of constant taxes is also relevant in a context of high levels of public indebtedness because it can proxy a situation in which taxes are unresponsive to the level of government debt for example because the top of the Laffer curve as been reached.[26] I avoid repeating the FONCs of the noncooperative fiscal policy here and only point out that they are equivalent to the ones in the text except for the derivative of the Lagrangian with respect to labor taxes that is removed.

Figure 1.5 compares the IRFs to a large negative demand shock with variable taxes (left-hand side panels) and with constant taxes (right-hand side panels) under perfect foresight. The optimal policy departs strikingly from the benchmark case with control over all instruments. When taxes are constant, the government accumulates debt in the liquidity trap no matter the maturity of the bonds issued. Given that the additional debt is not going to be repaid with labor taxes, this policy does not increase inflation expectations in itself. Hence, the government can increase its public consumption tremendously without fearing a monetary tightening at the exit of the liquidity trap that would jeopardise its effort to stimulate consumption. This expansionary fiscal policy allows to mitigate the drop in output when compared to the benchmark case. However, the lack of cost-push effect from labor taxes implies a larger inflation shortfall during the downturn.

[Figure 1.5 approximately here]

Finally, notice that the government accumulates more debt following the drop in demand when the maturity is longer. The relaxing of the budget constraint from the interest rate drop is less pronounced in this case because of the lower sensitivity of the long-bond price to the short-term rate. The government then smoothes the extra burden across time by keeping the level of debt higher than the steady state for a prolonged period of time.

1.7.9 Time-consistency versus commitment

Analysing the case of commitment is useful to understand the key role played by government debt in influencing policy outcomes when time-consistency is imposed

[26]For a paper tackling the constraint exerted by the Laffer curve on optimal time-consistent fiscal policy with distortionary taxation, see Debortoli et al. (2021).

and the first-best is out of reach. Under commitment, the government chooses the complete time path of variables in the initial period subject to the equilibrium conditions. In particular, the budget constraint only has to be satisfied intertemporally while the period-by-period level of debt is obtained residually given the optimal plan for other variables. Attaching the multipliers ϕ_t^r, ϕ_t^p, ϕ_t^q, ϕ_t^i and ϕ_t^b respectivelly to the equilibrium conditions and multiplier ϕ_t^{zlb} to the ZLB constraint, the FONCs read:

$$\begin{aligned}
\partial\mathcal{L}/\partial\hat{c}_t \equiv \quad & 0 = \gamma_c c\hat{c}_t + \phi_t^r c + \phi_t^p \kappa_c - \phi_t^q \gamma_c + \phi_{t-1}^q \gamma_c \beta^{-1} - \phi_t^b \frac{y}{\theta}\gamma_c \\
\partial\mathcal{L}/\partial\hat{G}_t \equiv \quad & 0 = \gamma_g G\hat{G}_t + \phi_t^r G - \phi_t^b G \\
\partial\mathcal{L}/\partial\hat{y}_t \equiv \quad & 0 = \gamma_h y\hat{y}_t - \phi_t^r y + \phi_t^p \kappa_y - \phi_t^b \frac{y}{\theta}(1+\gamma_h) \\
\partial\mathcal{L}/\partial\hat{\pi}_t \equiv \quad & 0 = \iota y\hat{\pi}_t - \phi_t^p + \phi_{t-1}^p + \phi_{t-1}^q \beta^{-1} + \phi_t^b \Omega\beta^{-1}\Psi \\
\partial\mathcal{L}/\partial\hat{\tau}_t \equiv \quad & 0 = \phi_t^p \kappa_\tau + \phi_t^b w\tau y\frac{\theta-1}{\theta} \\
\partial\mathcal{L}/\partial\hat{q}_t \equiv \quad & 0 = \phi_t^q - \phi_{t-1}^q \rho + \phi_t^b \Omega(1-\rho) - \phi_t^{zlb} + \phi_{t-1}^{zlb}\rho \\
\partial\mathcal{L}/\partial\hat{b}_t \equiv \quad & 0 = \phi_t^b - \mathbb{E}_t\phi_{t+1}^b
\end{aligned}$$

and the lower bound constraint $\phi_t^{zlb} < 0$ as well as complementary slackness condition $(\rho\beta\mathbb{E}_t q_{t+1} - \hat{q}_t)\phi_t^{zlb} = 0$.

As can be seen from the last FONC on government debt, the multiplier attached to the government budget constraint is a near random walk. This characteristic reflects the consumption smoothing motive of the planner in the presence of distortionary taxes on labor income (see e.g. Schmitt-Grohé and Uribe, 2004). Figure 1.6 compares the IRFs to a large negative demand shock (five standard deviations) under commitment and under time-consistency.[27] Clearly, the drop in

[27] Under commitment the number of state variables increases to five. This renders the projection method on a grid unhandy. Hence, I solve the model using the piece-wise solution method described in Guerrieri and Iacoviello (2015). For the sake of the comparison and since this method assumes perfect foresight, I turn off uncertainty in my projection method by setting the variance of the demand shock equal to zero.

consumption is curbed under commitment because of the positive effect of forward guidance that lowers expectations about real interest rates (e.g. Eggertsson and Woodford, 2003). The recession is deeper in the near-term when compared with the optimal time-consistent policy with short-term debt. This result is explained by the lower government spending stimulus in the liquidity trap and increased tax rate. This policy optimally stabilises distortions stemming from taxes on labor income and the inflation rate because of the cost-push effect of taxes that reduces the real rate at the lower bound.

Interestingly, the path of the choice variables is the same for short-term and long-term debt under commitment. This equivalence holds because of the time zero implementation of the optimal allocation under commitment. Since the price of the government bond depends on its maturity, the path of debt differs between both set-ups. However, over the infinite horizon, this difference is confined to debt and does not affect other variables. In particular, the initial increase in debt observed with longer maturity reflects the lower sensitivity of the long bond price to the drop in the short-term rate but is compensated by the lower consolidation needed over the longer-term.

[Figure 1.6 approximately here]

The presence of debt thus reduces the welfare loss from time-consistency issues because of the history dependence it introduces in the optimal allocation. This beneficial effect is maximised when the two authorities lack coordination. This is evident both from the welfare analysis in Section 1.5.4 and also from the path of the variables in Figure 1.6 that mimics more closely the firs-best allocation (in particular with the consolidation of debt). This is so because fiscal policy must engineer its own forward guidance while monetary policy keeps inflation in check.

Chapter 2

Debt Management in a World of Fiscal Dominance

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Abstract We study the impact of debt maturity management in an economy where monetary policy is 'passive' and subservient to fiscal policy. We setup a tractable model, to characterize analytically the dynamics of inflation, as well as other macroeconomic variables, showing their dependence on the monetary policy rule and on the maturity of debt. Debt maturity becomes a key variable when the monetary authority reacts to inflation and the appropriate maturity of debt can restore the efficacy of monetary policy in controlling inflation. This requires debt management to focus on issuing long bonds. Moreover, we propose a novel framework of Ramsey optimal coordinated debt and monetary policies, to derive analytically the interest rate rule followed by the monetary authority as a function of debt maturity. The optimal policy model leads to the same prescription, long term debt financing enables to stabilize inflation. Lastly, the relevance of debt maturity in reducing inflation variability is also confirmed in a medium scale DSGE model estimated with US data.

> *"The Federal Reserve cannot make rational decisions of monetary policy without knowing what kind of debt the Treasury intends to issue. The Treasury cannot rationally determine the maturity structure of the interest-bearing debt without knowing how much debt the Federal Reserve intends to monetise."*
>
> – James Tobin (1963), 'An essay on the principles of debt management'

2.1 Introduction

The large rise in government debt levels observed in many countries since the 2008-9 recession and which will continue in the coming years as a consequence of COVID 19, raises numerous concerns regarding the conduct of monetary policy and in particular its ability to control inflation. Since it is questionable that governments will be able to generate the large required surpluses to finance this debt, resorting to inflation for this purpose, may become the only option for many advanced economies for resorbing that debt. If a such a scenario materializes, then monetary policy will become subservient to fiscal policy, giving up (at least partially) its control over inflation.

Numerous papers have studied the interactions between monetary and fiscal policies using the workhorse model of the fiscal theory of the price level (e.g. Leeper, 1991; Cochrane, 2001; Bianchi and Melosi, 2017, 2019; Bianchi and Ilut, 2017; Sims, 2011; Cochrane, 2018; Bhattarai et al., 2014; Leeper and Leith, 2016). One of the key findings in this literature is that when monetary policy becomes subservient to fiscal policy, shocks filtered through the government's budget can impact inflation, and conventional ways to react to these shocks (e.g. lowering interest rates in response to a demand contraction) will not be effective.

These important findings are supported by very elaborate models which account for numerous margins of policy and features of the way monetary and fiscal policies are conducted in practice. Yet, we believe that there is a policy margin whose effects the literature has not adequately explored and which is potentially crucial: *debt maturity management*. With few exceptions (that we discuss in detail below) the literature has abstracted from modelling explicitly the maturity of debt, assuming (mainly for simplicity) that governments issue debt in one asset, either a short or a long term bond. This is not an innocuous assumption: when debt can be issued in more than one maturity, then with the appropriate issuance of short and long bonds, debt management can insulate the government's budget from shocks, and thus reduce inflation volatility and even partially restore the efficacy of monetary policy in dealing with shocks.

In this paper we setup a tractable model to analyze how debt management can complement monetary policy in cases where the latter is subordinate to a fiscal authority that does not adjust the budget surplus to finance debt. Our arguments are rooted into a growing number of papers studying debt management in macroeconomic models (e.g. Angeletos, 2002; Buera and Nicolini, 2004; Lustig et al., 2008; Faraglia et al., 2016, 2019; Debortoli et al., 2017; Bhandari et al., 2017; Bouakez et al., 2018) and in which debt portfolios are chosen to enable governments to smooth distortionary taxes over time. Our paper, instead, investigates what types of debt can better enable monetary policy to stabilize inflation.

In Section 2.2 we setup a simplistic model consisting of the standard New Keynesian block of equations augmented with a fiscal block - the consolidated budget constraint and a policy rule which determines taxes as a function of the lagged level of debt (e.g Leeper, 1991). Our model is broadly similar to Leeper, 1991; Bianchi and Melosi, 2019, what is different is that debt can be issued in two types of bonds, long and short. Monetary policy becomes subservient to fiscal policy when the nominal interest rate adjusts weakly to inflation and when the response of taxes to debt is also weak, the standard configuration of a passive monetary/ active fiscal regime (Leeper, 1991).

Using this setup we first focus on how, depending on the maturity structure of debt, shocks to government spending and to demand are propagated through the economy; we consider two alternative specifications for the monetary policy rule. In the first case, we assume that the nominal interest rate only reacts to inflation, through the usual systematic response, and in the second case we assume that the nominal rate also reacts the movement of the real rate, through the standard 'stochastic intercept term'. For each case, we derive analytically the path of inflation, and show that it hinges crucially on whether the government finances debt long or short term, and also hinges on the specification of the monetary policy rule. We then turn to characterize optimal debt policy, to find the maturity structure that minimizes intertemporally the variability of inflation deriving from shocks to spending and demand. We establish that to deal with fiscal shocks the policymaker issues as much long debt as possible, financing the position with short term assets. To ward off demand shocks, the optimal portfolio may feature positive short term debt, when monetary policy only reacts to inflation, but long term debt is optimal when the nominal rate tracks the real rate. Debt maturity becomes irrelevant when interest rates do not respond at all to inflation.

To understand these properties note first that under passive monetary policy, inflation becomes a backward looking process. Reacting systematically to inflation through raising the nominal rate, will generally not accomplish to reduce inflation, and rather will lead inflation to increase persistently. Issuing long term debt is optimal to finance a spending shock, as this enables to reduce the real value of debt both through current and future inflation, reducing the overall impact of the shock on inflation. Hence the optimal strategy is to issue long and finance the position with short term assets.

On the other hand, inflation dynamics following a demand contraction shock can become complex, inflation may drop initially to negative territory and then (depending on debt maturity) switch sign to turn positive. This happens because a demand shock is filtered through both the government budget constraint and the Euler equation. Following a negative shock, intertemporal solvency requires that the market value of debt increases, to be equal to the value of surpluses that compensate for debt, and so inflation needs to fall. Eventually, however, the (opposite) Euler equation effect dominates and inflation rises.

Whether it is desirable to issue short or long term debt to deal with the demand shock, hinges crucially on the response of monetary policy. When the nominal interest rate only reacts to inflation, then short term debt accomplishes to shield the budget constraint from future inflation, avoiding an even larger initial drop in the price level that would otherwise be required to balance the budget intertemporally. In contrast, when monetary policy tracks the movement in the real rate, the impact of the shock on the Euler equation is muted. Then issuing long debt is optimal and in fact we find a portfolio that can fully insulate government's budget from the demand shocks, so that in equilibrium inflation is fully stabilized. This key finding suggests that the efficacy of monetary policy to control inflation can be restored by debt management, in spite of the fact that we are in a 'passive money' world.

These results emerge from a model with standard ad hoc monetary policy rules, the benchmark fiscal theory framework and this provides a useful link to the literature as well as tractable analytics to investigate the interactions between monetary policy and debt maturity. Since these interactions turn out to be non-trivial we next study how monetary policy would react to any given debt structure, when it can optimally set the nominal interest rate. In Section 2.3 we turn to a model where a 'Ramsey planner' chooses interest rates and the debt portfolio to minimize distortions stemming from inflation and derive analytically an optimal interest rate rule that takes the maturity of debt into account. The resulting policy rule is broadly similar to the ad hoc rules assumed in Section 2.2, featuring a systematic response to inflation and a stochastic intercept term, however, now key parameters are tied down by the maturity of debt. More specifically, the coefficient that governs the response of interest rates to inflation reflects the maturity of the long term bonds issued, and the 'stochastic intercept' that determines how the nominal rate reacts autonomously to spending and demand shocks also depends on maturity. The optimal debt portfolio then eliminates the dependence of monetary policy on the composition of debt through eliminating the impact of shocks on the consolidated budget. We find that to accomplish this, debt management needs to again focus on issuing long term debt.

Section 2.4 demonstrates the relevance of considering the maturity structure of debt as a key variable for macroeconomic stabilization under passive monetary policy, using a more realistic setup, a medium scale dynamic stochastic general equilibrium (DSGE) model. Our quantitative model extends the baseline with preferences exhibiting habit formation, and adds more shocks to the economy, in particular shocks to total factor productivity (TFP), to markups, to government transfers, besides the spending and demand shocks. The model has a rich structure to match US data and it is broadly similar to the models of Bianchi and Ilut (2017). We estimate the quantitative model with standard Bayesian techniques using the post 1980 sample of the US historical data, when as is well known, monetary policy was not subservient to fiscal policy. We then change the policy parameters to produce a 'passive monetary/active fiscal' regime and study the propagation of shocks under different maturity structures. Our analysis reveals that not only does the maturity of debt matter for the propagation of shocks in the macroeconomy, but also that the effect of debt management policy in reducing the volatility of macroeconomic variables can be considerable. This holds in particular for fiscal shocks. According to our experiments, issuing long term debt is optimal to deal with fiscal shocks.

This paper brings several insights to the literature and is related to several strands. First, our considering debt management as a policy margin that can complement monetary policy in pursuing its inflation stabilization goals, is perhaps at odds with the current institutional setup in the US and other OECD countries. According to current practice, the mandate of debt management is to finance debt at low cost given tolerable levels of rollover risk (see e.g. Blommestein and Turner, 2012) and therefore, debt managers do not pursue macroeconomic stabilization objectives as we will assume in this paper. Nevertheless, the idea that monetary policy and debt management should coordinate is not new, and dates back (at least) to the time James Tobin wrote on this subject. According to Tobin (1963) both authorities have powers to influence the entire spectrum of debt, and coordinating actions and aligning objectives seems natural.

Admittedly, changing the mandate of debt management is not a trivial shift in policy and to make the claim that it is optimal involves, at least, modelling explicitly how it would affect private sector expectations over inflation and the costs of debt issuance. [1] We will not consider any of the (possibly numerous) trade-offs in this paper. Moreover, it is worth noting, from the point of view of the model, a full alignment of objectives is perhaps not even necessary. Inflation is determined by the net debt in the hands of the private sector, and since the consolidated budget constraint is sufficient for an equilibrium, quantitative easing can implement the optimal policy outcomes that we identify. However, this property relies on the simplicity of our setting. In a world where both monetary and debt management (DM) policies operate at the long end of the maturity structure and with conflicting goals, it is not clear that one authority will not undermine the actions of the other. (See e.g. Greenwood et al., 2015). If this is so, then changing the mandate of debt management to include macroeconomic stabilization goals may become important. These are important elements that we leave to future work.

Ours is not the first paper to investigate the role of maturity within the context of the fiscal theory of the price level. Cochrane (2001) was the first to introduce

[1] For instance, when monetary policy is not subordinate of fiscal policy, then coordination between monetary and debt policies could be seen as implicit debt monetization or manipulating interest rates to reduce the cost of financing debt. Our focus however is on a passive monetary policy equilibrium, where both of these elements are already present.

long debt to this model, and study the implications for inflation and optimal policy. Cochrane (2018), revisits this analysis to show that the issuance of long term debt gives rise to a 'stepping on a rake' property of monetary shocks: prices drop following a rise in the nominal interest rate but the sign of inflation is reversed subsequently. In addition, a few recent papers have studied optimal debt portfolios using models in which monetary and fiscal policies are coordinated. Lustig et al. (2008) explore how a Ramsey planner will set inflation, tax and debt issuance policies under full commitment whereas Leeper and Leith (2016) and Leeper et al. (2021) focus on equilibria without commitment. Bhattarai et al. (2015) consider the role of maturity in shaping the inflation output trade-off facing the planner during liquidity trap episodes.

These papers are related to ours, but there are some key differences. First and foremost, Ramsey models cannot be easily compared to the fiscal theory of the price level model (Leeper and Leith, 2016). Typically, under Ramsey policy the planner has a dual objective to smooth taxes and inflation across time, the resulting combination of optimal policies maybe far from the 'passive monetary/active fiscal' regime which we focus on here. When we turn to study Ramsey policy in Section 2.3, we constrain the tax rate to be constant, thus fully focusing on an environment where only inflation can adjust to ensure debt solvency. Moreover, the optimal monetary policy rule that we obtain analytically is indeed a standard passive policy rule.

Second, most of these papers consider optimal policies in nonlinear models using global approximation methods,[2] and this implies that they cannot look at the interactions between debt and monetary policy, when the latter is specified with the simple empirically relevant rules that the DSGE literature has employed and identified from the data. In contrast, we utilize a linear model and this enables us to connect with the standard fiscal theory framework of monetary/fiscal interactions

[2] An exception is Cochrane (2001) who draws insights from both linear and nonlinear models.

(e.g. Leeper, 1991), but also to derive sharp analytical results, and in Section 2.4 to apply our findings in a medium scale DSGE model.

Finally, our optimal policy model in Section 2.3 is broadly related to the optimal fiscal/monetary policy literature under *incomplete markets* (e.g. Aiyagari et al., 2002; Schmitt-Grohé and Uribe, 2004; Lustig et al., 2008; Faraglia et al., 2013, 2016 and many others). As in these papers, the lags of the Lagrange multiplier on the government budget constraint are state variables influencing policy through capturing the planner's commitments to a path of the policy variables that enable to adjust appropriately the real value of debt. Our analytical results showing that from complicated Ramsey policy optimality conditions we can derive simple and transparent monetary policy rules, should be of interest.

2.2 Theoretical Framework

We lay out our baseline model which consists of the standard New Keynesian block of equations, the consolidated budget constraint and a fiscal policy rule that sets (distortionary) taxes as a function of debt outstanding. The model draw from previous studies (e.g. Leeper, 1991; Bianchi and Melosi, 2019), the main difference is that we consider debt issued in both long and short term bonds. For brevity, we define here the competitive equilibrium equations in log-linear form. In the Appendix 2.6.5 we describe the background non-linear model and derive the equations from the optimality conditions of the households' and firms' optimization problems. Our model in this section can be seen as a simplified version of the model we setup in Section 2.4.

We let $\hat{x}$ denote the log deviation of variable x from its steady state value, $\overline{x}$. The system of the competitive equilibrium equations is the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t + \beta E_t \hat{\pi}_{t+1}, \tag{2.1}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\gamma_h > 0$, $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}}{(1-\overline{\tau})} > 0$,

$$\hat{i}_t = \hat{\xi}_t + E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1}\right) \tag{2.2}$$

$$\overline{p}_S\overline{b}_S\hat{b}_{t,S} + \overline{p}_S\overline{b}_S\hat{p}_{t,S} + \overline{p}_\delta\overline{b}_\delta\hat{b}_{t,\delta} + \overline{p}_\delta\overline{b}_\delta\hat{p}_{t,\delta} + \frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h+1)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right) - \overline{G}\hat{G}_t$$
$$= \overline{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \overline{p}_\delta\overline{b}_\delta(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\overline{b}_\delta\overline{p}_\delta\hat{p}_{t,\delta} \tag{2.3}$$

$$\overline{p}_\delta\hat{p}_{t,\delta} = \sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j} - \hat{\xi}_t\right)\right] \tag{2.4}$$

$$\overline{p}_S\hat{p}_{t,S} = \beta E_t(\hat{\xi}_{t+1} - \hat{\xi}_t - \hat{\pi}_{t+1}) \tag{2.5}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \epsilon_{i,t} \tag{2.6}$$

$$\hat{\tau}_t = \phi^R_{\tau,b}\hat{D}_{t-1} \tag{2.7}$$

$$\left(\overline{s}_{b_\delta}(1-\overline{s}_{b_\delta})\right)\left(\hat{b}_{t,\delta} - \hat{b}_{t,S}\right) = \upsilon\hat{D}_{t-1} \tag{2.8}$$

(2.1) is the Phillips curve at the heart of our model. $\hat{\pi}_t$ denotes inflation, $\hat{Y}_t$ denotes the output gap and $\hat{\tau}_t$ is a distortionary tax levied on the labor income of households. Parameters $\eta < 0$ and $\theta > 0$ that determine the constants κ_1 and κ_2 govern the markup over marginal costs of production set by firms and the degree of price stickiness respectively.[3] β is the usual discount factor in the background nonlinear model.

(2.2) is the log-linear Euler equation which prices a short term nominal asset. The price of this asset is denoted $\hat{p}_{S,t}$ and it is equal to minus the nominal rate $\hat{i}_t$. $\hat{\xi}$ is a standard demand shock which in the background non-linear model reflects a disturbance to preferences changing the relative valuation of current vs. future utility by the household. In standard fashion, a drop in $\hat{\xi}$ makes the household relatively patient, willing to substitute current for future consumption and thus leading to a drop in the demand for output.

(2.3) is the consolidated budget constraint. We assume that debt is issued

[3] We assume price adjustment costs as in Rotemberg (1982). θ governs the magnitude of these costs. When θ equals zero, prices are fully flexible.

in two maturities, a short bond with real issuance denoted $\hat{b}_S$ and long bond, a perpetuity with decaying coupons, denoted $\hat{b}_\delta$ where δ is the coupon decay factor. The term $\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\Bigg((\gamma_h+1)\hat{Y}_t+\frac{\hat{\tau}_t}{1-\overline{\tau}}\Bigg)$ denotes the revenues of the government from taxation. Parameter γ_h is (in the non-linear background model) the inverse of the Frisch elasticity of labor supply. $\hat{G}_t$ denotes government spending. (2.3) equates the value of new debt issuance (top line) and the value of the government's surplus to the real market value of debt outstanding in period t. Notice that the prices of long and short bonds are objects $\hat{p}_{t,S}$ and $\hat{p}_{t,\delta}$ defined in (2.4) and (2.5) respectively. Substituting into (2.3) these competitive equilibrium prices and the steady state analogues $\overline{p}_S=\beta$ and $\overline{p}_\delta=\frac{\beta}{1-\beta\delta}$. we get:

$$\beta\overline{b}_S\hat{b}_{t,S}+\beta\overline{b}_S E_t(\hat{\xi}_{t+1}-\hat{\xi}_t-\hat{\pi}_{t+1})+\frac{\beta\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta} \tag{2.9}$$

$$+\overline{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}-\hat{\xi}_t\right)\right]+\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h+1)\hat{Y}_t+\frac{\hat{\tau}_t}{1-\overline{\tau}}\right)-\overline{G}\hat{G}_t$$

$$=\overline{b}_S(\hat{b}_{t-1,S}-\hat{\pi}_t)+\frac{\overline{b}_\delta}{1-\beta\delta}(\hat{b}_{t-1,\delta}-\hat{\pi}_t)+\delta\overline{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}-\hat{\xi}_t\right)\right] \tag{2.10}$$

where (2.10) more succinctly summarizes the consolidated budget as an equilibrium object, since we dispensed with prices.

Monetary policy is modelled as interest rate rule (2.6) featuring a systematic response of the nominal interest rate to inflation, governed by parameter ϕ_π, and a disturbance term $\epsilon_{i,t}$ which represents the monetary policy shock. Note that in what follows we will consider two polar scenarios: In one case we will set $\epsilon_{i,t}=0$ thus letting monetary policy react to inflation only through the systematic component $\phi_\pi\hat{\pi}_t$, and in the second case we will set $\epsilon_{i,t}=\hat{\xi}_t-E_t\hat{\xi}_{t+1}$ thus letting the interest rule track the movement of the real rate, the latter being determined fully by the demand shock $\hat{\xi}$. Expressing the policy rule as in (2.6) allows us to summarize these two cases.

Moreover, fiscal policy is rule (2.7) setting the tax rate as a function of the lagged face value of government debt, defined here $\hat{D}_{t-1}=\overline{b}_S\hat{b}_{S,t-1}+\frac{\overline{b}_\delta}{1-\delta}\hat{b}_{\delta,t-1}$.[4]

[4]The decaying coupon bond is priced in the model exactly as a portfolio of zero coupon bonds, the principal on maturity j is δ^{j-1}. Thus the total face value is the quantity $\overline{b}_\delta\hat{b}_{\delta,t-1}$

Parameter $\phi_{\tau,b}$ measures the size of the adjustment of taxes to debt.

Finally, equation (2.8) specifies a rule for the share of the face value of long term debt over the total face value issued in t. $\overline{s}_{b_\delta}$ denotes the steady state share of long bonds in the government's portfolio and υ governs the response of the share to the face value $\hat{D}$.

The above equations define a model that is similar to the baseline fiscal theory framework commonly employed in the literature. As is well known, in this model parameters $\phi_{\tau,b}$ and ϕ_π are the crucial objects that determine whether monetary policy is subordinate to fiscal policy. Unlike the baseline model of the literature, our model assumes that taxes are distortionary and also that the government issues debt in two different types of assets, short and long term bonds. Since these elements appear to be novel, we now precisely characterize the regions for parameters ϕ_π and $\phi_{\tau,b}$ which give us an equilibrium where fiscal policy dominates monetary policy. The following proposition gives the result:

Proposition 1: Policy configuration

The unique equilibrium when monetary/fiscal policy is passive/active is obtained when the following is satisfied: $\phi_\pi < 1$ *and* $\phi_{\tau,b} < \frac{(\beta^{-1}-1)}{\overline{R}(1-\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})})}$.

Proof: See Appendix 2.6.2.

Notice that as in Leeper (1991) the 'passive money' equilibrium emerges when coefficient ϕ_π is less than unity. On the other hand, the cutoff for fiscal policy $\phi_{\tau,b}$ to become 'active' is a function of steady state taxes, $\frac{1}{\gamma_h}$ and $\overline{R}$, the Frisch elasticity and the steady state revenue respectively. Since in our model taxes are distortionary, these parameters influence the behavior of hours in response to changes in taxes. Note further that $1 - \frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})} > 0$ needs to hold otherwise the

times the principal payments (1 $+\delta + \delta^2 + ...$).

economy is on the wrong side of the Laffer curve. Thus, the threshold $\frac{(\beta^{-1}-1)}{\overline{R}(1-\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})})}$ is positive.

Our analysis below sets $\phi_{\tau,b} = 0$. We make this assumption to focus on a scenario where fiscal policy does not fiance debt through taxation and also for tractability, to be able to derive analytical results. However, it is worth noting that under plausible values for parameters $\overline{\tau}$ and γ_h the cutoff defined in Proposition 1 will actually be close to zero, so that even assuming $\phi_{\tau,b} > 0$ would not change our results.

2.2.1 Debt Maturity in a passive money world

Changes in the maturity of debt will in our model exert an influence of the equilibrium, as Ricardian equivalence does not hold. Since the model is linear the crucial quantities are $\overline{b}_S, \overline{b}_\delta$, the steady state values of the two types of debt. These are the objects that influence equilibrium inflation since, as is evident from (2.10), they interact with inflation in the government budget constraint. In contrast, the quantities $\hat{b}_\delta, \hat{b}_S$ (the log deviations from steady state) have no bearing on the equilibrium.[5]

Due to this property, that (zero order) steady state portfolios are only relevant, we can fully characterize macroeconomic outcomes through studying the responses of the economy to one off shocks in spending, demand and $\epsilon_{i,t}$. We now derive analytical formulae describing the dynamics of inflation, when these shocks can occur in period t, assuming that all innovations are i.i.d and no further shocks can occur thereafter. Without loss of generality we set initial debt $\hat{b}_{t-1,\delta} = \hat{b}_{t-1,S} = 0$. The following proposition provides a general formula for the path of inflation:

[5]For the same reason, the specification of the debt issuance rule (2.8) is also not important. In other words, our framework is not suitable to talk about the optimal rebalancing of debt portfolios over the business cycle. This would require a non-linear model. It is worth noting however, that a very persistent finding in the theoretical debt management literature (see for example Angeletos, 2002; Buera and Nicolini, 2004; Lustig et al., 2008; Faraglia et al., 2019) is that optimal portfolios are constant (or roughly constant), a property that also appears to be relevant in practice. Whether or not this would hold in our framework can be explored in future work.

Proposition 2. *Assume that a shock occurs in period t. The path of inflation is given by:*

$$\hat{\pi}_t = \eta_1 \epsilon_{i,t} + \eta_2 \hat{G}_t + \eta_3 \hat{\xi}_t$$

$$\hat{\pi}_{t+\bar{t}} = \phi_\pi^{\bar{t}-1} \Big[(1+\phi_\pi \eta_1)\epsilon_{i,t} + \eta_2 \phi_\pi \hat{G}_t + (\eta_3 \phi_\pi - 1)\hat{\xi}_t \Big], \quad \bar{t} = 1, 2, ...$$

where η_1, η_2, η_3 are:

$$\begin{aligned}
\eta_1 &\equiv - \left[\frac{\overline{b}_\delta \beta \delta}{(1-\beta\delta)(1-\phi_\pi \beta\delta)} \right] \Big/ \left[\overline{R} \frac{(\gamma_h + 1)}{\kappa_1} + \overline{b}_S + \frac{\overline{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right] \\
\eta_2 &\equiv \overline{G} \Big/ \left[\overline{R} \frac{(\gamma_h + 1)}{\kappa_1} + \overline{b}_S + \frac{\overline{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right] \\
\eta_3 &\equiv \left[\beta \overline{b}_S + \frac{\overline{b}_\delta \beta (1-(1-\delta)\delta\beta\phi_\pi)}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right] \Big/ \left[\overline{R} \frac{(\gamma_h + 1)}{\kappa_1} + \overline{b}_S + \frac{\overline{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right]
\end{aligned}$$

and where $\overline{R} = \frac{\tau(1+\eta)\overline{Y}}{\eta}$ denotes the government's revenue in steady state.

Proof. See Appendix 2.6.2

These expressions give us the responses as functions of the model's deep parameters, $\gamma_h, \kappa_1, \beta, \phi_\pi$... and of the portfolio $\overline{b}_S$ or $\overline{b}_\delta$. They thus reveal the impact of maturity on inflation, and also that of varying the debt level, (i.e. increasing autonomously $\overline{b}_S$ or $\overline{b}_\delta$). In our analysis we will mostly focus on the impact of maturity holding debt constant. Given this, we can further simplify the expressions for the ηs by making use of the steady state relation $\frac{\overline{S}}{1-\beta} = \overline{b}_S + \frac{\overline{b}_\delta}{1-\beta\delta}$ (equating the present value of the surplus to the value of debt in steady state) as:

$$\begin{aligned}
\eta_1 &\equiv - \left[\frac{\overline{b}_\delta \beta \delta}{(1-\beta\delta)(1-\phi_\pi \beta\delta)} \right] \Big/ \left[\overline{R} \frac{(\gamma_h + 1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\overline{b}_\delta \phi_\pi \beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right] \\
\eta_2 &\equiv \overline{G} \Big/ \left[\overline{R} \frac{(\gamma_h + 1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\overline{b}_\delta \phi_\pi \beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right] \\
\eta_3 &= \beta \left[\frac{\overline{S}}{(1-\beta)} + \frac{\beta\delta^2 \phi_\pi \overline{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right] \Big/ \left[\overline{R} \frac{(\gamma_h + 1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\overline{b}_\delta \phi_\pi \beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right]
\end{aligned} \tag{2.11}$$

Notice that now the impact of varying the maturity of debt on inflation, holding the debt level constant, can be easily seen from the partial derivatives of η_1, η_2, η_3 with respect to $\overline{b}_\delta$. These expressions, then reveal a few well known properties of the passive money model, with regard to the response of inflation to shocks.

Consider the case of a monetary policy shock, $\epsilon_{i,t} > 0$, assuming that this is not yet tied down to the demand shock. If the government issues only short term debt, $\overline{b}_\delta = 0$, a rise in the nominal rate will have no effect on inflation in t (since $\eta_1 = 0$) but will increase inflation in $t+1, t+2, ...$. In contrast, in the case where $\overline{b}_\delta > 0$, a rise in the policy rate, will trigger first a drop in inflation to subsequently increase inflation, since $1 + \eta_1 \phi_\pi > 0$. The switching sign of inflation in response to an interest rate shock is what Sims (2011) and Cochrane (2018) refer to as 'stepping on a rake'.[6]

Moreover, consider the case of a shock to the spending level. In the passive money model, such a shock is inflationary, and this is verified here by η_2 being positive. An increase in spending needs to be compensated by higher inflation (to make government debt solvent) and the longer is debt maturity, the smaller is the required impact of the shock in t since a persistent increase in inflation can reduce the real pay out of long term debt (e.g. Cochrane, 2001). As (2.11) reveals, this well known property requires $\phi_\pi > 0$ since when $\phi_\pi = 0$ the response of inflation to the spending shock will be concentrated in period t only.

A positive shock in $\hat{\xi}$, a standard innovation to demand, will increase inflation in period t (assuming that $\overline{b}_\delta$ is not too negative), but it will lower inflation in $t+1, t+2, ...$. The effect of this shock is thus qualitatively similar to that of an expansionary monetary policy shock, but not quantitatively, since the coefficients η_1 and η_3 in general differ. This difference arises here because demand shocks are filtered through the government's budget whereas monetary policy shocks are not. We will later show that due to this difference in the coefficients, monetary

[6]In Appendix 2.6.3, we show that the unconventional transmission of shocks in the passive money model with debt is E-stable for most of the portfolios considered in this paper. Hence, if agents were learning, their adaptive expectations would converge to the Rational Expectations Equilibrium when they have the correct Perceived Law of Motion for the economy.

policy's ability to mitigate demand shocks is impaired. However, we will also show, that a suitable DM strategy can restore the ability of the monetary authority to ward off $\hat{\xi}$ shocks, when portfolios can absorb the impact of these shocks from the consolidated budget.

Finally, note that knowing the path of inflation analytically enables us to obtain the path of output in the model. Combining the solution for $\hat{\pi}$ with the Phillips curve we can show that:

$$\hat{Y}_t = \frac{1}{\kappa_1}\bigg((\eta_1 - \beta(1+\phi_\pi\eta_1))\epsilon_{i,t} + \eta_2(1-\phi_\pi\beta)\hat{G}_t + (\eta_3 - \beta(\eta_3\phi_\pi - 1))\hat{\xi}_t\bigg)$$
$$\hat{Y}_{t+\bar{t}} = \phi_\pi^{\bar{t}-1}\frac{1-\phi_\pi\beta}{\kappa_1}\bigg((1+\phi_\pi\eta_1)\epsilon_{i,t} + \eta_2\phi_\pi\hat{G}_t + (\eta_3\phi_\pi - 1)\hat{\xi}_t\bigg),\quad \bar{t}\geq 1 \tag{2.12}$$

which demonstrates that the same forces that influence the response of inflation to the shocks (summarized in coefficients η) also influence the path output.

2.2.2 The objective of DM

Optimal maturity management in our model consists in choosing $\bar{b}_\delta$ to minimize the volatility of inflation, given the equilibrium paths we derived in the previous paragraph. Formally, we will solve the following problem:

$$\bar{b}_\delta = \arg\min \sum_{\bar{t}\geq 0} \beta^{\bar{t}}\sigma^2_{\hat{\pi},t+\bar{t}} \tag{2.13}$$

where the measure of volatility is the conditional variance of inflation, before the shocks are realized. Given the previous formulae, and continuing to assume that shocks are i.i.d we can derive $\sigma^2_{\hat{\pi},t+\bar{t}}$ as:

$$\sigma^2_{\hat{\pi},t} = \eta_1^2\sigma^2_{\epsilon_i} + \eta_2^2\sigma^2_{\hat{G}} + \eta_3^2\sigma^2_{\hat{\xi}} + 2\eta_1\eta_3\sigma_{\epsilon_i,\xi} \tag{2.14}$$

$$\sigma^2_{\hat{\pi},t+\bar{t}} = \phi_\pi^{2(\bar{t}-1)}\Big[(1+\phi_\pi\eta_1)^2\sigma^2_{\epsilon_i} + (\eta_2\phi_\pi)^2\sigma^2_{\hat{G}} + (\eta_3\phi_\pi - 1)^2\sigma^2_{\hat{\xi}} + 2(\eta_3\phi_\pi - 1)(1+\phi_\pi\eta_1)\sigma_{\epsilon_i,\xi}\Big] \tag{2.15}$$

$$\bar{t} = 1, 2, ...$$

As discussed previously, we will separately consider $(\sigma^2_{\epsilon_i}, \sigma_{\epsilon_i,\xi}) \in \{(0,0), (\sigma^2_{\hat{\xi}}, \sigma^2_{\hat{\xi}})\}$ so that in one case monetary policy reacts to the demand innovation through the systematic component of the interest rate rule only, and in the other case monetary policy also responds to the demand shock, by changing the nominal rate to match the movement in the real interest rate.

2.2.3 Demand shocks

We begin with focusing on demand shocks assuming no other policy response to these shocks, beyond the systematic response to inflation in the policy rule. Consider the case where a demand shock occurs in period t. The relevant coefficient that measures the response of inflation to the shock is η_3 in (2.11).

For simplicity, let us focus on the case where $\delta = 1$ (long bonds are consols). η_3 becomes

$$\eta_3 = \beta\Big[\frac{\overline{S}}{(1-\beta)} + \frac{\beta\phi_\pi\bar{b}_1}{(1-\beta)(1-\phi_\pi\beta)}\Big]/\Big[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_1\phi_\pi\beta}{(1-\beta)(1-\phi_\pi\beta)}\Big] \tag{2.16}$$

To investigate this formula notice first that under the assumption that the government is a debtor, so that $\overline{S} > 0$ and $\overline{R} > 0$, we have that $\eta_3 = 0$ when $\bar{b}_1 = -\frac{\overline{S}}{\beta\phi_\pi}(1-\phi_\pi\beta) < 0$. Moreover, the derivative $\frac{d\eta_3}{d\bar{b}_1}$ is strictly positive and in the limit when $\bar{b}_1$ becomes infinite, $\eta_3 = \beta$. In other words, issuing negative long term debt (equivalently positive short debt) makes inflation zero in period t, and issuing more long bonds increases the response of inflation to the demand shock.

To gain insights into why the long bond position affects inflation in this way, consider the intertemporal constraint of the government in period t. Assuming wlog that debt before the shock, in $t-1$, is at its steady state value we can write:

$$\sum_{j\geq 0}\beta^j\overline{R}(\gamma_h+1)\hat{Y}_{t+j}-\overline{S}\hat{\xi}_t\frac{\beta}{1-\beta}=-\overline{b}_S\hat{\pi}_t-\overline{b}_1\sum_{j=0}^{\infty}\beta^j\left[\sum_{l=0}^{j}\hat{\pi}_{t+l}\right]-\overline{b}_1\hat{\xi}_t\frac{\beta}{1-\beta} \tag{2.17}$$

The two terms on the LHS of this equation capture the impact of the shock on the government's intertemporal surplus. The leading term on the LHS represents the effect on output which changes government revenues whereas the second term is the effect of a shock $\hat{\xi}$ on the present discounted value of the surplus holding output constant. Analogously, the RHS of the equation tells us how the real value of debt will adjust to the shock. The first two terms measure the impact of inflation on the real payout of debt (short and long respectively), whereas the last term captures the shock impact on the real long bond price.

(2.17) has to hold for debt to be solvent and notice that there are two different forces that jointly ensure that this will be so. First, a standard fiscal theory argument, inflation will adjust, given debt maturity, so that (2.17) is satisfied and second, a debt management argument, given inflation we can find $\overline{b}_S, \overline{b}_1$ to satisfy intertemporal solvency.

Consider first how inflation responds to the shock given maturity. Notice that a negative demand shock affects the LHS of (2.17) in two ways. Since the shock is probably going to be contractionary, the term $\sum_{j\geq 0}\beta^j\overline{R}(\gamma_h+1)\hat{Y}_{t+j}$ is going to fall. In contrast, $-\overline{S}\hat{\xi}_t\frac{\beta}{1-\beta}$ will rise, since the shock will reduce the real interest rate at which future surpluses are discounted. Whichever of these two effects dominates will determine whether ultimately the RHS of (2.17) needs to rise or fall to make debt solvent.

Consider first the case where only short term debt is issued. Using $\hat{Y}_{t+j}=$

$\frac{1}{\kappa_1}(\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1})$ from the Phillips curve we can write (2.17) as

$$\overline{R}\frac{(\gamma_h + 1)}{\kappa_1}\hat{\pi}_t - \overline{S}\hat{\xi}_t\frac{\beta}{1-\beta} = -\bar{b}_S\hat{\pi}_t \tag{2.18}$$

and thus inflation must turn negative (equal to $\frac{\beta}{1-\beta}\frac{\overline{S}\hat{\xi}_t}{\bar{b}_S + \overline{R}\frac{(\gamma_h+1)}{\kappa_1}}$) to balance the intertemporal budget. The demand shock thus increases the value of the surplus and negative inflation is required to also increase the real value of debt.

Now suppose that both short and long bonds are issued. Going back to (2.17), it is evident that the final term on the RHS $-\bar{b}_1\hat{\xi}_t\frac{\beta}{1-\beta}$ exceeds 0 and thus compensates for the higher surplus on the LHS, whereas the term in the middle, $\sum_{j=0}^{\infty}\beta^j\left[\sum_{l=0}^{j}\hat{\pi}_{t+l}\right]$ will likely be of opposite sign than $\hat{\pi}_t$, since as we previously showed, inflation will switch sign in $t+1$. If this is indeed so, then issuing long bonds will imply a reduction in the real payout of long term debt after the shock, which will need to be compensated by more negative inflation in t to satisfy (2.17).

We can illustrate this, by considering the case where only long term debt is issued, $\bar{b}_1 = \overline{S}$. Using $\hat{\pi}_{t+\bar{t}} = \phi_\pi\hat{\pi}_{t+\bar{t}-1} - \hat{\xi}_t\mathcal{I}_{\bar{t}=1}$, we can write (2.17) as

$$\overline{R}\frac{(\gamma_h + 1)}{\kappa_1}\hat{\pi}_t = \underbrace{-\frac{\bar{b}_1}{(1-\beta)(1-\beta\phi_\pi)}\left(\hat{\pi}_t - \hat{\xi}_t\beta\right)}_{-\bar{b}_1\sum_{j=0}^{\infty}\beta^j\left(\sum_{l=0}^{j}\hat{\pi}_{t+l}\right)} \tag{2.19}$$

yielding

$$\hat{\pi}_t = \frac{1}{\left(\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)(1-\beta\phi_\pi)}\right)}\frac{\beta\overline{S}}{(1-\beta)(1-\beta\phi_\pi)}\hat{\xi}_t < \frac{\beta}{1-\beta}\frac{\overline{S}\hat{\xi}_t}{\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{1-\beta}} \tag{2.20}$$

where the final inequality is inflation under short debt issuance in (2.18). According to (2.20), focusing on issuing long bonds produces a larger drop in $\hat{\pi}_t$ than when only short debt is issued. More balanced portfolios with both short and long bonds are in between these two cases.

Moreover, notice that the finding that issuing long bonds makes inflation in t more negative (to compensate for future positive inflation) also explains why $\hat{\pi}_t = 0$ when $\overline{b}_1 = -\frac{\overline{S}}{\beta\phi_\pi}(1-\phi_\pi\beta) < 0$. When the government saves in a long term asset, the value of long term wealth will drop due to positive future inflation, so that $\hat{\pi}_t$ does not have to turn negative to increase the value of short term debt.

Consider now, the debt management argument, how given the path of inflation, the maturity structure of debt can be targeted to satisfy (2.17). Suppose that we can fix coefficient $\eta_3 \in (0,\beta)$ to not depend on debt maturity. Then, using $\hat{\pi}_t = \eta_3\hat{\xi}_t$ along with the previous derivations, we can show that intertemporal solvency requires:

$$\overline{R}\frac{(\gamma_h+1)}{\kappa_1}\eta_3\hat{\xi}_t - \overline{S}\hat{\xi}_t\frac{\beta}{1-\beta} = -\frac{(\overline{S}-\overline{b}_1)}{1-\beta}\eta_3\hat{\xi}_t - \frac{\overline{b}_1}{(1-\beta)(1-\beta\phi_\pi)}\hat{\xi}_t\Big(\eta_3-\beta\Big) - \overline{b}_1\hat{\xi}_t\frac{\beta}{1-\beta} \tag{2.21}$$

We can easily find the value of $\overline{b}_1$ that satisfies this equation.

Our exercise compiles both forces as we will look for portfolios that minimize the variability of inflation when inflation simultaneously adjusts to satisfy (2.17). We now turn towards this optimal debt policy when shocks to demand are driving inflation.

The optimal DM policy An optimal debt policy will trade off the costs of negative inflation in t with the costs of having positive inflation rates after t. Maintaining the assumption that long bonds are consols the optimal debt management program becomes:

$$\overline{b}_1^* = \arg\min \sum_{\overline{t}\geq 0}\beta^{\overline{t}}\sigma^2_{\hat{\pi},t+\overline{t}} = \arg\min\ \sigma^2_{\hat{\xi}}\left(\eta_3^2 + \frac{\beta}{1-\phi_\pi^2\beta}(\eta_3\phi_\pi-1)^2\right) \tag{2.22}$$

given η_3 defined in (2.16). The following proposition gives us the optimal coefficient $\eta_3(b_1^*)$ on inflation.

Proposition 3. *The optimal portfolio solves $\eta_3(b_1^*) = \phi_\pi\beta$ when $\phi_\pi > 0$.*

Proof. See Appendix 2.6.2

DM will thus choose the optimal portfolio so that the impact effect of the shock on inflation is proportional to coefficient ϕ_π. This optimal policy produces the following path of inflation:

$$\hat{\pi}_t = \phi_\pi \beta \hat{\xi}_t$$

$$\hat{\pi}_{t+\bar{t}} = \phi_\pi^{\bar{t}-1}(\phi_\pi^2 \beta - 1)\hat{\xi}_t \quad \bar{t} = 1, 2, ...$$

and thus policy trades-off the fall in inflation in t (in response to a negative shock $\hat{\xi}_t$) with the subsequent rise in inflation. Completely eliminating deflation in period t is never optimal.

What portfolio implements this optimal policy? When long bonds are consols we have:

$$\bar{b}_1^* = \frac{(1-\beta)(1-\phi_\pi\beta)}{\beta\phi_\pi(\beta-\phi_\pi)}\left[\phi_\pi \overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}(\phi_\pi-\beta)}{(1-\beta)}\right]$$

which implies that the long bond issuance could be positive or negative depending on the relative magnitude of β and ϕ_π.

To gain insight into what $\bar{b}_1^*$ is, over a plausible range of values of model parameters, we calibrate in Table 2.1. We set $\beta = 0.995$ (assuming a quarterly model horizon), $\theta = 17.5$ and $\eta = -6.88$ which give us a Phillips curve with slope coefficient of around 0.3 and a markup equal to 17% in steady state.[7] We normalize output to be equal to 1 in steady state and set $\overline{G}$ equal to 20 percent of $\overline{Y}$. Finally, we set the debt to GDP ratio equal to 60 percent of annual output. Table 2.1 reports the corresponding value of taxes that satisfy the budget constraint in steady state where inflation is zero.

[7]These values are consistent with the rest of the literature (see for example Schmitt-Grohé and Uribe, 2004; Faraglia et al., 2013).

Table 2.1: **Calibration**

Parameter	Value	Label
β	0.995	Discount factor
θ	17.5	Price Stickiness
η	-6.88	Elasticity of Demand
γ_h	1	Inverse of Frisch Elasticity
$\overline{\tau}$	0.248	Tax Rate
$\overline{Y}$	1	Output
$\overline{G}$	0.2	Spending

Notes: The table reports the values of model parameters. β notes the discount factor chosen to target a steady state annual real interest rate of 2 percent. Parameter η is calibrated to target markups of 17 percent in steady state. θ is calibrated as in Schmitt-Grohé and Uribe (2004). Finally, the steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 20 percent of aggregate output which is normalized to unity in steady state.

Given this parameterization of the model we get $\overline{b}_1^* = -0.042$ when $\phi_\pi = .2$ ($\overline{b}_S = 10.84$). When $\phi_\pi = .5$ we obtain $\overline{b}_1^* = -0.005$ ($\overline{b}_S = 3.56$). Finally, in the case where $\phi_\pi = .95$ we have $\overline{b}_1^* = 0.007$ and $\overline{b}_S = 1.00$. The optimal policy thus requires to finance debt short term.

2.2.4 Monetary Policy Shocks and Demand Shocks

The previous paragraph assumed that monetary policy responds to demand shocks only through the systematic component of the Taylor rule. We now turn to study the impact on debt maturity when monetary policy responds to the demand shock through setting $\epsilon_{i,t} = \hat{\xi}_t$. It is well known, that in the standard NK model, where inflation does not have to satisfy the intertermporal budget, such a policy accomplishes to eliminate the impact of the shock, a property widely known as 'divine coincidence'.[8]

However, under passive money, this well known property does not generally

[8]To be precise, divine coincidence would not generally hold under 'active' monetary policy since we assume distortionary taxes. It would only hold under perfect tax smoothing, when taxes do not respond to demand shocks. In our model this would require debt management to 'complete the market' (Angeletos, 2002; Buera and Nicolini, 2004).

hold. Monetary policy is not able to ward off the shock completely, as the demand disturbance exerts an influence on the consolidated budget constraint. Since inflation needs to adjust for the constraint to hold, monetary policy faces an additional constraint that needs to be satisfied.

Consider the path of inflation predicted by the model. Combining the analytical expressions in Proposition 2 we get:

$$\hat{\pi}_{t+\bar{t}} = \phi_{\pi}^{\bar{t}}(\eta_3 + \eta_1)\hat{\xi}_t, \quad \bar{t} = 0, 1, 2, ...$$

Notice first that now the response of inflation to the shock is monotonic. Inflation will not switch sign in $t+1$. Moreover, inflation will generally be different from zero, unless policy can set $\eta_1 = -\eta_3$ which accomplishes to fully stabilize inflation.

This condition generally fails to hold in the model. Using previous derivations we can obtain:

$$\eta_1 + \eta_3 = \beta\left[\frac{\overline{S}}{(1-\beta)} - \frac{\delta\overline{b}_\delta}{(1-\beta\delta)}\right] / \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\overline{b}_\delta\phi_\pi\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]$$

Therefore, to fully insulate inflation from the demand shock, we need to set $\frac{\overline{S}}{(1-\beta)} = \frac{\delta\overline{b}_\delta}{(1-\beta\delta)}$. In the case where $\delta = 1$ as in the previous paragraph, this means that debt management should focus on issuing exclusively long term debt.

To understand why this is so, consider again (2.17), which we can now express as

$$\overline{R}\frac{(\gamma_h+1)}{\kappa_1}\hat{\pi}_t - \overline{S}\hat{\xi}_t\frac{\beta}{1-\beta} = -\overline{b}_S\hat{\pi}_t - \overline{b}_1\frac{\hat{\pi}_t}{(1-\beta)(1-\beta\phi_\pi)} - \overline{b}_1\hat{\xi}_t\frac{\beta}{1-\beta} \tag{2.23}$$

Recall that the final term on the LHS measures how a change in the discount rate affects the value of surpluses, whereas the final term on the RHS is the change market value of debt due to the rise of the real long bond price that follows a negative $\hat{\xi}_t$ shock. When debt management set $\overline{b}_1 = \overline{S}$ these two terms will cancel out and evidently inflation in t will have to be zero for (2.23) to hold.

We highlight our findings with the following proposition:

Proposition 4. *Consider the case where the monetary policy rule is of the form* $\hat{i}_t = \phi_\pi \hat{\pi}_t + \hat{\xi}_t$*, and the nominal interest rate responds to changes in the real rate. There exists a debt management strategy,* $\frac{\overline{S}}{(1-\beta)} = \frac{\delta \bar{b}_\delta}{(1-\beta\delta)}$ *that accomplishes to fully stabilize inflation. When* $\delta = 1$ *(long bonds are consols) the optimal debt management strategy is to issue long term debt only.*

2.2.5 Fiscal Shocks

We consider now how maturity influences the impact of a fiscal shock. From (2.11), coefficient η_2 is decreasing in $\overline{b}_\delta$ insofar as $\phi_\pi > 0$. Moreover, from Proposition 2, inflation continues to respond to the fiscal shock beyond period t according to $\hat{\pi}_{t+\bar{t}} = \phi_\pi^{\bar{t}} \eta_2 \hat{G}_t$. Thus, it is clear that the more long debt is issued, the lower is the effect of spending on inflation in all periods.

This result is intuitive: In our model, the real interest rate is not a function of the spending levels, and a shock to spending impacts inflation only through its effect on the consolidated budget. Then, a rise in spending reduces the government surplus, and requires an increase in inflation to bring the intertermporal budget into balance. When the monetary authority attempts to fight back inflation (but still its reaction is 'too weak', raising the nominal rate less than inflation) it only accomplishes to maintain persistently higher price level growth. With long debt, higher future inflation translates into a larger reduction in the real payout of debt and a smaller inflation rate is required in all periods to bring the intertermporal budget into balance.

2.2.6 Discussion: The importance of the monetary policy rule

Our analytical findings highlighted that the impact of varying debt maturity on macroeconomic outcomes depends on the specification of the monetary policy rule. As we saw, when the monetary authority only responded to inflation through the systematic component, then full inflation stabilization in response to demand shocks was not possible. In contrast, in the case where the policy rule changed the nominal rate to match the movement of the real interest rate, there was a debt management strategy that fully stabilized inflation. In both cases considered DM could complement monetary policy in pursuing the objective of reducing inflation variability.

Moreover, our analytical formulae revealed ϕ_π as another key parameter that influences the interplay between debt maturity and inflation. When $\phi_\pi = 0$, there is basically no difference between long and short term financing and debt maturity exerts no influence on inflation. When the nominal rate does not respond to inflation, then inflation displays no persistence and the response of inflation to shocks is concentrated in period t. Then the effect of inflation on the real value of long and short bonds is the same.

The important lesson that we can draw from these findings is that it is not only debt policy that can help monetary policy to control inflation, but also it is important that monetary policy adopts the right kind of rule to allow debt policy to exert an influence. According to our findings, this rule should feature both a strong systematic response to inflation setting $\phi_\pi >> 0$, but also allow the nominal rate to respond to fluctuations in the real interest rate.

These are of course only partial results. We have not assumed that monetary policy is optimal, allowing it to respond to shocks and to the maturity structure of debt. A fully optimal monetary policy could turn out to follow a very different interest rate rule and analogously the optimal debt portfolios could also be different than the ones we have found in this section. We will turn to the case of optimal monetary policy in Section 2.3 of the paper.

2.2.7 Extensions

Optimal portfolios with simultaneous shocks Thus far we have studied demand and spending shocks in isolation. However, since both types of shocks can hit the economy in period t, it is purposeful to discuss debt management strategies that can deal with the occurrence of both types of shocks. We saw previously that in order deal with fiscal shocks the optimal DM strategy calls for making the long bond position as large as possible, and financing with short term assets. In contrast for demand shocks optimal long bond positions were finite. There is thus a tension when both shocks can occur simultaneously, and DM needs to find a portfolio that balances the benefit from insulating inflation against demand shocks with the benefit of dealing effectively with fiscal shocks.

In Appendix 2.6.2 we derive the FONC of the optimal policy problem when the two types of shocks coexist. The following proposition summarizes our results.

Proposition 5. *Assume long bonds are consols. Consider the case where monetary policy reacts to the demand shock only through the systematic reaction to inflation and $\phi_\pi > 0$. The optimal portfolio solves:*

$$\eta_3 = \phi_\pi \beta + \eta_2 \frac{\overline{G}}{\beta \overline{R} \frac{(1+\gamma_h)}{\kappa_1}} \frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}} \tag{2.24}$$

The optimal issuance becomes:

$$\overline{b}_1^*(\hat{\xi}, \hat{G}) = \frac{(1-\beta)(1-\phi_\pi\beta)}{\beta\phi_\pi(1-\phi_\pi)} \left[\phi_\pi \overline{R} \frac{(\gamma_h+1)}{\kappa_1} - \frac{\overline{S}(1-\phi_\pi)}{(1-\beta)} + \frac{\overline{G}^2}{\beta^2 \frac{\overline{R}(\gamma_h+1)}{\kappa_1}} \frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}} \right]$$

Alternatively, in the case where monetary policy offsets the demand shock set-

ting $\epsilon_{i,t} = \hat{\xi}_t$, the optimal portfolio is:

$$\overline{b}_1^*(\hat{\xi}, \hat{G}) = \overline{S} + \overline{G}^2 \frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}} \frac{1-\beta}{\beta(1-\beta\phi_\pi)} \frac{\phi_\pi}{\left(\frac{\overline{S}}{1-\beta\phi_\pi} + \overline{R}\frac{(1+\gamma_h)}{\kappa_1}\right)} \tag{2.25}$$

Proof: See Appendix 2.6.2.

Proposition 5 is easily comparable to our previous analytical results. We previously saw that in the presence of only the $\hat{\xi}$ shock, the optimal portfolio is $\eta_3 = \phi_\pi\beta$ when $\epsilon_{i,t} = 0$. According to (2.24) we have $\eta_3 > \phi_\pi\beta$ implying that the issuance of long term debt now increases. Moreover, when monetary policy tracks the real rate, the optimal portfolio no longer sets $\overline{b}_1 = \overline{S}$, now the issuance of the long bond is higher as is illustrated by (2.25).

These results are of course to be expected. Spending shocks become a factor that pulls the long bond issuance towards infinity (since for spending shocks the larger the position in long debt the better). To deal with these types of shocks the policymaker has to tolerate a higher impact of demand disturbances on inflation. The larger is the relative variance of the spending shock the larger is the weight that the optimal formula attributes to it (the term $\frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}}$ in (2.24) and (2.25)).

Finally note that according to (2.25), ϕ_π becomes a crucial parameter in the case where monetary policy directly responds to the demand shock through the stochastic intercept term $\epsilon_{i,t}$. To interpret (2.25), recall that when ϕ_π is close to zero, then increasing the long bond issuance will not help much in stabilizing inflation in response to the spending shock, because inflation displays very little persistence. Then, optimal policy prefers to focus on stabilizing inflation responding to the demand disturbance. High values of ϕ_π imply the opposite; it is now optimal ward off the inflationary effects of spending shocks, by issuing large amounts of long term debt.

Macroeconomic Volatility Does debt management help reduce considerably macroeconomic volatility? So far we have relied on analytical expressions to find qualitative results. However, in order to suggest that DM is an important tool to mitigate the impact of shocks to inflation we need to quantitatively assess the impact of shocks under alternative maturity structures. We use the parameter values reported in Table 1 and further assume $(\sigma_\xi, \sigma_G) = (0.4\%, 4.5\%)$.[9]

[Figure 2.1 approximately here]

Figures 2.1 and 2.2 show the impulse responses to one standard deviation shocks in demand and spending. Figure 2.1 assumes $\epsilon_{t,i} = 0$ whereas in Figure 2.2 we let the monetary authority lower the nominal rate 1 for 1 with the real rate. The left panels consider the case of the demand shock, whereas on the right we plot the responses to a spending shock. We consider alternative maturity structures as follows: The dashed red lines assume that DM finances debt short term. The dotted lines assume only long term financing, whereas solid lines correspond to the optimal portfolios.[10] Finally, the dashed-dotted green lines set $\bar{b}_\delta$ to the optimal policy defined in Proposition 5, when both shocks can simultaneously occur.

As is evident from the figures, even with our simplistic i.i.d structure of shocks we obtain large impacts from varying the maturity of debt. For example, financing short implies that a spending shock raises inflation by 1 percent on impact whereas long term financing reduces this impact effect considerably and the larger is coefficient ϕ_π the less is the volatility displayed by inflation (e.g. bottom panels). Analogously, whereas issuing long debt only, fully stabilizes inflation following a demand shock in Figure 2.2, when short debt is issued, inflation drops to -1% and continues being negative for several periods along the transition.

Finally, when debt management attempts to deal with both types of shocks

[9] The value for σ_ξ is chosen so that a one standard deviation shock does not drive the level interest rate below zero. $\sigma_G = 0.045$ is a standard calibration for the variance of the spending shock.

[10] In the case of the spending shock we make $\bar{b}_\delta$ a very large number (10 times GDP).

adopting the portfolios derived in Proposition 5, inflation turns positive following a demand contraction in Figure 2.2, since now the long bond issuance exceeds 100 percent of the value of the portfolio and so $\eta_1 + \eta_3 > 0$.

[Figure 2.2 approximately here]

The Impact of Debt Levels and the slope of the Phillips curve We close this section by briefly considering how our results would change if we assumed a higher initial debt level and if the slope of the Phillips curve (parameter κ_1) is lower. Both impacts of these parameter changes can be easily read off from our previous formulae. Higher debt implies higher values of $\overline{S}$ and $\overline{R}$. In the case of fiscal shocks, it remains optimal to issue as much long debt as possible, and as the expression for η_2 reveals, for any $\overline{b}_\delta$ the effect of spending shocks on inflation is now less. Intuitively, at higher debt levels, a given increase in inflation reduces more the real payout of debt and so less inflation is needed to satisfy the intertemporal constraint. In the case of demand shocks, and focusing on the case where $\epsilon_{i,t} = \hat{\xi}_t$ and $\delta = 1$ we still obtain $\overline{b}_1 = \overline{S}$. Financing long remains optimal and clearly the quantity of long bonds issued increases in the debt level.

Parameter κ_1 exerts a similar influence, appearing in the denominator of the fractions η_1, η_2, η_3. Changes in the slope of the Phillips curve do not affect the optimal portfolios identified previously.

2.3 Optimal Monetary and DM policies

Our analysis thus far has relied on a model where monetary policy is summarized through an ad hoc interest rate rule. One of our key findings was that DM can restore the efficacy of monetary policy in shielding inflation from the impact of demand shocks. This was the case when debt was long term and the monetary policy rule responded to the demand shock directly, through tracking the real rate. Another important finding was that for debt management to be able to contribute in stabilizing inflation a significant systematic response of interest rates to inflation was needed.

We now abandon the assumption that monetary policy follows ad hoc interest rate rules and turn to a model where both monetary and debt policies are optimal. We do so for several reasons, including to investigate whether the above findings generalize to the case where the monetary authority can set interest rates to be optimal for any debt maturity, but also to see whether optimal policy will tie down parameter ϕ_π and the stochastic intercept term to the maturity of debt issued. Moreover, under optimal interest rates, perhaps a different maturity of debt than the one we identified previously will be desirable.

To investigate this, we solve a Ramsey program assuming coordinated DM and monetary authorities. The 'planner' sets the path of inflation, output and interest rates, and the debt portfolio to minimize the variability of inflation in response to the spending and demand shocks and we further assume full commitment to announced state contingent paths for the policy variables. Under these assumptions, we will show that optimal monetary policy can be summarized through an interest rate rule, broadly similar to the ad hoc rule that we employed in Section 2.2. However, the inflation coefficient ϕ_π and the stochastic intercept will indeed reflect debt maturity. Optimal portfolios will however, be the same as before, as focusing on long term debt will enable to stabilize inflation.

2.3.1 Ramsey Program and Optimality

Even though we will maintain the same structure of shocks as in the previous section (i.i.d. and after t no further shocks occur), to make the analysis more general, we will first solve the optimal policy program without imposing these assumptions. The Ramsey planner in our model chooses sequences $\left\{\hat{\pi}_{t+\bar{t}}, \hat{Y}_{t+\bar{t}}, \hat{i}_{t+\bar{t}}, \hat{b}_{t+\bar{t},S}, \hat{b}_{t+\bar{t},\delta}\right\}_{\bar{t}\geq 0}$ and the steady state portfolio $\bar{b}_\delta$ to minimize the variability of inflation subject to the competitive equilibrium equations (the Euler and Phillips curve equations and the consolidated budget), assuming also that taxes are held constant.[11] Noting that since $\hat{i}_{t+\bar{t}}$ will only appear on the LHS of the Euler equation (as we do not tie down interest rates to follow a specific functional form) we can dispense with this constraint. We then can state the policy problem formally as:

$$\max_{\left\{\hat{\pi}_{t+\bar{t}},\hat{Y}_{t+\bar{t}},\hat{b}_{t+\bar{t},S},\hat{b}_{t+\bar{t},\delta}\right\}_{\bar{t}\geq 0},\ \bar{b}_\delta} -E_t \sum_{\bar{t}\geq 0} \beta^{\bar{t}} \hat{\pi}^2_{t+\bar{t}}$$

subject to

$$\hat{\pi}_{t+\bar{t}} = \kappa_1 \hat{Y}_{t+\bar{t}} + \beta E_{t+\bar{t}} \hat{\pi}_{t+\bar{t}+1}$$

and

$$\begin{aligned}
&\beta\bar{b}_S\hat{b}_{t+\bar{t},S} + \beta\bar{b}_S E_{t+\bar{t}}(\hat{\xi}_{t+\bar{t}+1} - \hat{\xi}_{t+\bar{t}} - \hat{\pi}_{t+\bar{t}+1}) + \frac{\beta\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t+\bar{t},\delta} \\
&+\bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left[E_{t+\bar{t}}\left(-\sum_{l=1}^{j} \hat{\pi}_{t+\bar{t}+l} + \hat{\xi}_{t+\bar{t}+j} - \hat{\xi}_{t+\bar{t}}\right)\right] + \frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}(\gamma_h+1)\hat{Y}_{t+\bar{t}} - \overline{G}\hat{G}_{t+\bar{t}} \\
&= \bar{b}_S(\hat{b}_{t+\bar{t}-1,S} - \hat{\pi}_{t+\bar{t}}) + \frac{\bar{b}_\delta}{1-\beta\delta}(\hat{b}_{t+\bar{t}-1,\delta} - \hat{\pi}_{t+\bar{t}}) \\
&+\delta\bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left[E_{t+\bar{t}}\left(-\sum_{l=1}^{j} \hat{\pi}_{t+\bar{t}+l} + \hat{\xi}_{t+\bar{t}+j} - \hat{\xi}_{t+\bar{t}}\right)\right]
\end{aligned}$$

given also that $\bar{b}_S = \frac{\overline{S}}{1-\beta} - \frac{\delta\bar{b}_\delta}{1-\beta\delta}$.

Note that since $\bar{b}_\delta$ is chosen, the above is not a linear-quadratic program. To solve for the optimal policies, we proceed in two steps: We first hold constant $\bar{b}_\delta$ and solve a linear quadratic program to determine the optimal sequence $\left\{\hat{\pi}_{t+\bar{t}}, \hat{Y}_{t+\bar{t}}, \hat{b}_{t+\bar{t},S}, \hat{b}_{t+\bar{t},\delta}\right\}_{\bar{t}\geq 0}$ through solving a system of first order conditions. Second, we vary $\bar{b}_\delta$ to determine the portfolio through the upper envelope defined by the optimality conditions in the first step.

[11]Notice that the usual resource constraint that is included in Ramsey programs is already accounted for, since we could replace consumption with output and spending.

Optimality In Appendix 2.6.4 we setup a Lagrangian to derive the optimal paths of output inflation and bond quantities (in log deviation from steady state). Attach a multiplier $\psi_{\pi,t+\bar{t}}$ to the Phillips curve and $\psi_{g,t+\bar{t}}$ to the consolidated budget; the first order conditions are the following:

$$-\hat{\pi}_{t+\bar{t}} + \Delta\psi_{\pi,t+\bar{t}} + \bar{b}_S \Delta\psi_{gov,t+\bar{t}} + \frac{\bar{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t+\bar{t}-l} = 0 \tag{2.26}$$

$$-\psi_{\pi,t+\bar{t}}\kappa_1 + \frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y}(1+\gamma_h)\psi_{gov,t+\bar{t}} = 0 \tag{2.27}$$

$$\frac{\beta\bar{b}_\delta}{1-\beta\delta}\left(\psi_{gov,t+\bar{t}} - E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1}\right) = 0 \tag{2.28}$$

$$\beta\bar{b}_S\left(\psi_{gov,t+\bar{t}} - E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1}\right) = 0 \tag{2.29}$$

(2.26) is the FONC with respect to $\hat{\pi}_{t+\bar{t}}$; (2.27), (2.28) , (2.29) are first order conditions with respect to $\hat{Y}_{t+\bar{t}}$, $\hat{b}_{t+\bar{t},\delta}$ and $\hat{b}_{t+\bar{t},S}$ respectively.

Several comments are in order. First, note that since the model is linear (2.28) and (2.29) will not pin down an optimal rule for the share of long over short term bonds. As before, quantities $\hat{b}_{t+\bar{t},\delta}$ and $\hat{b}_{t+\bar{t},S}$ will not be important; thus, arbitrarily setting $\hat{b}_{t+\bar{t},\delta}$ to zero and financing with $\hat{b}_{t+\bar{t},S}$ is consistent with the first order conditions since both (2.28) and (2.29) define that $\psi_{gov,t+\bar{t}}$ evolves like a random walk.

Second, the random walk property of the multiplier $\psi_{gov,t+\bar{t}}$ is a standard feature in the optimal policy literature. Since financing debt impinges distortions to the economy, ours is a model of optimal policy under incomplete markets as in Aiyagari et al., 2002; Schmitt-Grohé and Uribe, 2004; Lustig et al., 2008; Faraglia et al., 2013, 2016. Whereas in these papers debt can be financed through taxes, in our model taxes are held constant and the planner uses distortionary inflation to satisfy budget solvency. In both contexts, shocks to the economy translate to changes in the excess burden of distortions, and the multiplier, which measures the magnitude of these distortions, behaves like a random walk, since the planner wants to spread evenly the costs across periods.

Third, as (2.26) reveals, when government debt is long term, all the lags of the multiplier enter into the state vector and influence inflation. Combining (2.26) and (2.27) to substitute out ψ_π we can obtain the following expression which pins down $\hat{\pi}_{t+\bar{t}}$ as a function of these state variables

$$\hat{\pi}_{t+\bar{t}} = \overline{R}\frac{(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t+\bar{t}} + \bar{b}_S\Delta\psi_{gov,t+\bar{t}} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t+\bar{t}-l} \tag{2.30}$$

To interpret (2.30), note that a shock to either demand or spending will change the value of the multiplier according to its impact on the consolidated budget. For instance, an increase in spending will tighten the constraint and thus increase the value of $\psi_{gov,t}$. According to (2.30) this will increase inflation on impact, but also continue exerting an influence on inflation in the future, i.e. through the term $\delta^{\bar{t}}\Delta\psi_{gov,t}$ in the final part on the RHS. When debt is long term, the planner desires to spread inflation across periods, through promising higher inflation rates in the future. This feature of optimal policy in our model is common with Lustig et al., 2008; Faraglia et al., 2013, 2016.

2.3.2 The Optimal Interest rate rule

We now show that optimal policy in the Ramsey model can be implemented with an interest rate rule that is similar to the ad hoc rules we employed in Section 2.2. We summarize this policy rule in the following proposition:

Proposition 6. *The optimal interest rate policy that implements the Ramsey outcome is:*

$$\hat{i}_{t+\bar{t}} = \delta\hat{\pi}_{t+\bar{t}} - \delta\omega_Y\Delta\psi_{gov,t+\bar{t}} + \hat{\xi}_{t+\bar{t}} \tag{2.31}$$

where $\omega_Y \equiv \frac{\overline{S}}{1-\beta} - \frac{\bar{b}_\delta\beta\delta}{1-\beta\delta} + \overline{R}\frac{(1+\gamma_h)}{\kappa_1}$.

Proof: See Appendix 2.6.2.

Notice first that optimal interest rates are determined by two components. The first is a standard systematic response to inflation, $\delta\hat{\pi}_{t+\bar{t}}$, the second is the stochastic intercept term $-\delta\omega_Y\Delta\psi_{gov,t+\bar{t}}+\hat{\xi}_{t+\bar{t}}$. Let us focus first on the systematic response. Using, for comparison, the notation we employed in Section 2.2, we now have $\phi_\pi = \delta < 1$. Thus, the Ramsey planner fights inflation by raising the nominal rate, but the magnitude of the reaction of the interest rate is now tied down by the decaying coupon factor.

To interpret this feature, recall, that parameter ϕ_π governs the persistence of inflation. When long bonds are issued, both current and future inflation can contribute towards adjusting the real value of debt. But if the duration of long bonds is not considerable (or δ is a small number) then letting inflation deviate persistently from target (setting $\phi_\pi > \delta$) will not help in terms of making debt more sustainable. Thus, under the optimal plan, inflation distortions persist for as long as long debt coupon payments last.

Notice however, that (perhaps counterintuitively) the term $\delta\hat{\pi}_{t+\bar{t}}$ does not depend on whether the long term debt is actually issued, i.e. when $\bar{b}_\delta > 0$. We will now explain that the stochastic intercept term will compensate for this, and inflation will be frontloaded if all debt is short.

Consider the term $\delta\omega_Y\Delta\psi_{gov,t+\bar{t}}$. Notice that the maturity of debt influences it. Coefficient ω_Y is decreasing in the quantity of long bonds issued. Moreover, as we argued previously, shocks filtered through budget constraint, will induce fluctuations in $\Delta\psi_{gov,t+\bar{t}}$. For example, a positive spending shock occurring in period $t+\bar{t}$, will result in $\Delta\psi_{gov,t+\bar{t}} > 0$, since the consolidated budget tightens. Then, if debt is short term, $\omega_Y > 0$ and the planner will keep the interest rate lower in period $t+\bar{t}$ (than $\delta\hat{\pi}_{t+\bar{t}}$). In contrast, if a sufficiently high quantity of long bonds is issued then the opposite will hold, the nominal rate will be set higher when the shock hits, since now $\omega_Y < 0$.

What is the planner trying to do? Notice that lowering the nominal rate will have as effect to frontload inflation. When debt is financed short, this enables a larger reduction of real debt in response to the shock. Conversely, increasing $\hat{i}_{t+\bar{t}}$ will accomplish to spread inflation across periods, which is optimal when debt is long term.

Lastly, according to (2.31), the nominal rate tracks movements in the real rate. $\hat{\xi}_{t+\bar{t}}$ exerts a direct influence on the policy rule, but also an indirect influence through the intertemporal budget and $\Delta\psi_{gov,t+\bar{t}}$.

2.3.3 One off Shocks: The dynamic path of inflation

To investigate further these properties let us now go back to the case where shocks in t are i.i.d and there are no further shocks to the economy after t. Under these assumptions it is possible to derive analytical expressions for the multiplier ψ_{gov}. From (2.28) and (2.29) (removing conditional expectations after t) we have $\Delta\psi_{gov,t+\bar{t}} = 0$ for $\bar{t} \geq 1$ and $\Delta\psi_{gov,t} \neq 0$.[12] the optimal interest rate path becomes:

$$\begin{aligned} \hat{i}_t &= \delta\hat{\pi}_t - \delta\omega_Y\Delta\psi_{gov,t} + \hat{\xi}_t \\ \hat{i}_{t+\bar{t}} &= \delta\hat{\pi}_{t+\bar{t}} \end{aligned} \tag{2.32}$$

and we can further show that:

$$\Delta\psi_{gov,t} = \left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)^2 + \frac{\overline{b}_\delta^2\beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1}\left[\overline{G}\hat{G}_t + \left(\frac{\overline{\beta S}}{1-\beta} - \frac{\beta\delta\overline{b}_\delta}{1-\beta\delta}\right)\hat{\xi}_t\right] \tag{2.33}$$

(see Appendix 2.6.2).

(2.33) expresses $\Delta\psi_{gov,t}$ as a function of spending and demand shocks; the loadings on the shocks are functions of the maturity of debt. As is evident, higher

[12] We can also assume initial conditions $\psi_{gov,t-1} = \psi_{gov,t-2} = ...\psi_{gov,-1}$ such that the planner does not want to inflate away debt in the absence of shocks.

long debt issuance reduces the response of the multiplier to the spending shock. Moreover, the effect of the demand shock is zero when the government sets $\frac{\beta\delta\bar{b}_\delta}{1-\beta\delta} = \frac{\overline{S}}{1-\beta}$.

Using (2.30) we can now show that the path of inflation is given by

$$\hat{\pi}_t = \left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\Delta\psi_{gov,t}$$

$$\hat{\pi}_{t+\bar{t}} = \frac{\bar{b}_\delta}{1-\beta\delta}\delta^{\bar{t}}\Delta\psi_{gov,t}, \quad \bar{t} \geq 1 \tag{2.34}$$

Combining (2.33) and (2.34) we can derive the impact effects (the counterparts of coefficients η in Section 2.2) for spending and demand shocks as:

$$\eta_2^* = \left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)^2 + \frac{\bar{b}_\delta^2\beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1}\overline{G}\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right) \tag{2.35}$$

$$(\eta_1^* + \eta_3^*) = \tag{2.36}$$

$$\left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)^2 + \frac{\bar{b}_\delta^2\beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1}\left(\frac{\overline{\beta S}}{1-\beta} - \frac{\beta\delta\bar{b}_\delta}{1-\beta\delta}\right)\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)$$

where the stars denote that monetary policy is now optimal. Consider first the response to the spending shock, η_2^* and let us compare this to the analogous response in the model of Section 2.2. As is clear from (2.11) and (2.35), η_2^* is not the same as η_2 even when we set $\phi_\pi = \delta$. The terms that appear in the denominator in (2.11) appear also in (2.35), however, it is now the squares that show up in the denominator, rather than the levels. The numerators of η_2^* and η_2 are also different.

Analogously, in the case of the demand shock, letting $\phi_\pi = \delta$ we will have

$$\eta_1 + \eta_3 = \beta\left[\frac{\overline{S}}{(1-\beta)} - \frac{\beta\delta\bar{b}_\delta}{(1-\beta\delta)}\right] \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_\delta\beta\delta^2}{(1-\beta\delta)(1-\beta\delta^2)}\right]$$

where again the coefficient differs from the response under optimal policy in (2.35).

Where do these differences come from? Simple inspection of the two policy rules suggests that the s term $-\delta\omega_Y\Delta\psi_{gov,t}$ is responsible. As explained, this term determines whether the optimal policy desires to frontload/backload inflation as a function of the quantity of long bonds issued.

In Figure 2.3 we show the impulse responses of inflation to the demand and spending shocks under optimal policy. The dotted lines plot the responses when all of debt is short term and the dashed lines the case where all debt is long. Under short term debt both demand and spending shocks have only a temporary effect on inflation. This is the term $-\delta\omega_Y\Delta\psi_{gov,t}$ at work. In contrast, when debt is long term ($\bar{b}_S = 0$ in the figure), inflation deviates persistently from zero after the spending shock.

[Figure 2.3 approximately here]

2.3.4 Optimal Maturity

The optimal maturity $\bar{b}_\delta$ is easy to find. Consider first the case of spending shocks only. We can show that the objective function of DM becomes:

$$-E_t\sum_{\tilde{t}\geq 0}\beta^{\tilde{t}}\hat{\pi}^2_{t+\tilde{t}} = -\left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1}+\frac{\overline{S}}{1-\beta}\right)^2+\frac{\bar{b}^2_\delta\beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1}\overline{G}^2\sigma^2_{\hat{G}}$$

and as in the model of Section 2.2, the optimal policy is to issue as much long term debt as possible. Moreover, from (2.33) it is obvious that the policy that fully stabilizes inflation sets $\frac{\beta\delta\bar{b}_\delta}{1-\beta\delta} = \frac{\overline{S}}{1-\beta}$, the optimal structure of debt we identified in Section 2.2.

Notice, finally, that under optimal debt we will have $\Delta\psi_{gov,t} = 0$ (in the limit in the case of spending shocks). The optimal debt policy thus eliminates the dependence of the monetary policy rule on the composition of debt, through eliminating the impact of shocks on the consolidated budget. For this to happen debt management needs to again focus on issuing long term debt.

2.4 A Medium Scale DSGE model

Our core analysis in this paper has relied on a simplistic model to characterize transparently the interplay between debt maturity and inflation. Yet to claim that the interactions between debt and monetary polices are indeed relevant, it is important to extend our findings to a more empirically relevant setup, a medium scale DSGE model. We now work with a model with a wider set of shocks, including shocks to productivity, markups and government transfers, to study how debt policy can influence inflation in the presence of these additional disturbances. Moreover, the representative household in the model has preferences featuring habit formation, this implies more realistic adjustment of asset prices to shocks.

We first briefly describe our medium scale model, which is broadly similar to Bianchi and Ilut (2017). Then, we describe the estimation of the model and the results we get in subsection 2.4.2. Lastly, we will investigate the effects of maturity on equilibrium outcomes in subsection 2.4.3.

2.4.1 The Model

Household Preferences and Optimality The economy is populated by a single household with preferences of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\log(C_t - \Omega C^a_{t-1}) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

where C_t denotes the consumption of the household, ΩC^a_{t-1} is an external habit stock, where $0 < \Omega < 1$ and C^a_{t-1} denotes the average level of consumption in $t-1$. The household derives disutility from exerting labor effort h_t. Parameters χ and γ_h govern the household's preferences over leisure. ξ_t is a preference shifter which impacts the relative discounting of current and future utility flows.

The household maximizes utility subject to the flow budget constraint:

$$P_t C_t + P_{t,\delta} B_{t,\delta} + P_{t,S} B_{t,S} = (1-\tau_t) W_t h_t + P_t Tr_t + B_{t-1,S} + (1+\delta P_{t,\delta}) B_{t,\delta} + P_t Div_t$$

$B_{t,\delta}$ is a long government bond. $P_{t,\delta}$ is the price of the asset. $B_{t,S}$ denotes the quantity of short term (one-period) debt and $P_{t,S}$ its price at issuance.

W_t denotes the nominal wage and P_t is the price level. Div_t is real dividends paid by monopolistically competitive firms and Tr_t denotes lump-sum transfers given to the household by the fiscal authority. The household maximizes utility subject to the flow budget constraint.

The household maximizes utility subject to the budget constraint. The first order conditions for long and short bonds and the labor supply condition are given by:

$$\frac{1}{C_t - \Omega C_{t-1}^a} = \beta R_t E_t \frac{1}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C_t^a} \tag{2.37}$$

$$\frac{1}{C_t - \Omega C_{t-1}^a} P_{L,t} = \beta E_t \frac{1 + \delta P_{L,t+1}}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C_t^a} \tag{2.38}$$

$$\chi h_t^{\gamma_h} (C_t - \Omega C_{t-1}^a) = (1 - \tau_t) \frac{W_t}{P_t} \tag{2.39}$$

Firms, Production and the Phillips curve We assume that output is produced by a continuum of monopolistically competitive firms which operate technologies with labor as the sole input. Aggregate output is produced by a representative, perfectly competitive, final-good producer that aggregates the intermediate products of firms according to $Y_t = \left(\int_0^1 Y_t(j)^{\frac{1+\eta_t}{\eta_t}} dj \right)^{\frac{\eta_t}{1+\eta_t}}$. η_t is a (time varying) parameter that governs the elasticity of substitution. The production function of the generic good firm j is $Y_t(j) = A_t h_t(j)^{1-\alpha}$; A_t denotes the level of TFP in the economy. We assume that the growth rate of A_t (expressed in logarithms) follows an AR(1) process:

$$\ln\left(\frac{A_t}{A_{t-1}}\right) \equiv a_t = (1 - \rho_a)\gamma + \rho_a a_{t-1} + \epsilon_{a,t}$$

Parameter γ denotes the steady-state growth rate of A_t.

Firms face price adjustment costs as in Rotemberg (1982). The cost function of firm j is: $AC_t(j) = \frac{\theta}{2}(\frac{P_t(j)}{P_{t-1}(j)} - \pi)^2 Y_t$. $\theta \geq 0$ again governs the degree of price stickiness. π is the steady state level of gross inflation.

The profit maximization problem of the generic firm j is defined by :

$$\begin{aligned}
\max_{P_t(j)} \quad & E_t \sum_{s=0}^{\infty} Q_{t,t+s}\Big(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{MC_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - AC_{t+s}(j)\Big) \\
\text{s.t.} \quad & Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}}\Big)^{\eta_t} Y_{t+s} \\
& AC_{t+s}(j) = \frac{\theta}{2}\Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \overline{\pi}\Big)^2 Y_{t+s}
\end{aligned}$$

where $Q_{t,t+s} \equiv \beta^s E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+s} - \Omega C^a_{t+s-1}}$ is the household's discount factor and MC_{t+s} denotes marginal costs of production. $AC_{t+s}(j)$ is the quadratic price adjustment cost incurred by the firm.

In Appendix 2.6.5 we show that solving this problem and imposing a symmetric equilibrium, gives rise to the following (non-linear) New-Keynesian Phillips curve:

$$\theta(\pi_t - \overline{\pi})\pi_t = (1 + \eta_t)(1 - \frac{MC_t}{P_t}) + \beta\theta E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+1} - \Omega C^a_t} \frac{Y_{t+1}}{Y_t}(\pi_{t+1} - \overline{\pi})\pi_{t+1}.$$

MC_t denotes marginal costs of production.

Fiscal and Monetary Policy The government levies distortionary taxes and issues debt to finance spending G_t and transfers Tr_t. The flow budget constraint of the government is:

$$P_{t,S}B_{t,S} + P_{t,\delta}B_{t,\delta} = B_{t-1,S} + (1 + \delta P_{t,\delta})B_{t-1,\delta} + P_t(G_t + Tr_t) - \tau_t W_t h_t + \Lambda_t \tag{2.40}$$

Notice that following Bianchi and Ilut (2017) we augment the flow budget with an exogenous shock variable Λ_t capturing features of government finances that we have left outside the model.[13] $\tau_t W_t h_t$ denotes the fiscal revenues of the government.

In Appendix 2.6.5, we rewrite all model equations, expressing variables as ratios

[13]Note that this shock is also necessary in order to estimate the model since debt, spending and transfers will be treated as observables.

over nominal GDP and take a log linear approximation around the non-stochastic steady state. To define the policy rules of the fiscal and the monetary authorities, and the stochastic processes for transfers and the shocks we will now use the log-linear format.

Labour income taxes are set according to:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\Big[\phi_{\tau,b}\hat{D}_{t-1} + \phi_{\tau,y}(\hat{Y}_t - \hat{Y}_t^n) + \phi_{\tau,g}(g^{-1}\hat{g}_t + \hat{tr}_t)\Big] + \epsilon_{\tau,t} \tag{2.41}$$

where again $\hat{D}_{t-1}$ denotes the face value of debt issued in $t-1$, $\hat{Y}_t$ is output and $\hat{g}_t$ is the log deviation of scaled spending, $\frac{1}{1-\frac{G}{Y}_t}$, from its steady state value. Y_t^n is the natural level of output, that obtains under flexible prices.

Notice that in (2.78) labour income taxes are not only allowed to adjust to debt, but also to the cycle, through the term $\phi_{\tau,y}(\hat{Y}_t - \hat{Y}_t^n)$, and we further allow for spending and transfers to influence $\hat{\tau}_t$.

Transfers in the model evolve according to:

$$\hat{tr}_t = \rho_{tr}\hat{tr}_{t-1} + (1-\rho_{tr})\phi_{tr,y}(\hat{Y}_t - \hat{Y}_t^n) + \epsilon_{tr,t} \tag{2.42}$$

where ρ_{tr} governs the persistence of transfers and $\phi_{tr,y}$ measures the response to the output gap. Government spending (as a fraction of GDP) follows a simple AR(1) process:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t} \tag{2.43}$$

The shocks $\epsilon_{\tau,t}$, , $\epsilon_{tr,t}$ and $\epsilon_{g,t}$ are all assumed to be i.i.d.

Monetary policy is assumed to set the nominal rate according to:

$$\hat{i}_t = \rho_r \hat{i}_{t-1} + (1-\rho_r)\Big(\phi_\pi \hat{\pi}_t + \phi_y(\hat{Y}_t - \hat{Y}_t^n)\Big) + \epsilon_{i,t} \tag{2.44}$$

Finally, the remaining shocks are all assumed to follow first order autoregressive processes (see Appendix 2.6.5).

2.4.2 Estimation

We estimate parameter values and stochastic processes so that the model matches the data observations from the US economy. As it is well known, since the beginning of the 1980s, US monetary policy was not subservient to fiscal policy. The passive regime is more likely to have prevailed in the 1970s and before (see e.g. Bianchi and Ilut, 2017). Since our model is not designed to deal with regime fluctuations, we thus have to choose either to estimate the model under the assumption that monetary policy is 'active' using post 1980 data, and then set policy parameters to produce a passive regime, or estimate the model with observations prior to 1980 which would allow us to identify these policy parameters directly from the data.

We choose to do the former. The main reason is that we do not see why, if the passive monetary regime is to resurface in the future, the interest rate rule coefficients have to be equal to the values that fit the pre 1980 data. In our experiments below we will treat these coefficients as free parameters, considering various specifications of the passive monetary policy rule. Moreover, using the more recent observations allows us to obtain more accurate estimates of structural parameters that are likely to have shifted over the decades.[14].

Our sample is 1980:Q1 - 2008Q4. We truncate the sample to the 4th quarter of 2008 since the short term nominal rate in the US reached zero in the first quarter or 2009. Dealing with the non-linearities implied by the non-negativity constraint on the nominal interest rate, in estimation is beyond the scope of this exercise.

We include the following variables in the estimation of the model: Real GDP growth, GDP deflator inflation, the federal funds rate, federal revenues as a fraction

[14]Most notably, parameters related to the Phillips curve have shifted as recent literature advocates (see Del Negro et al., 2015)

of GDP, total government expenditures (including transfers) to GDP, the market value of debt to GDP ratio and finally government spending (for consumption and investment) to GDP. The details on the sources of these variables and the measurement equations that are employed to link data variables with their model counterparts are spelled out in Appendix 2.6.5.

To proceed with estimation we first select prior distributions for the parameters we wish to estimate and pick values for parameters that we want to fix in estimation. Table 2.2 summarizes the calibrated values of the parameters that we fix and the right side of Table 2.3 reports our choice of prior distributions for the parameters we estimate with Bayesian techniques. The priors are in line with previous papers in the literature (see e.g. Bianchi and Ilut, 2017).

[Table 2.2 approximately here]

We fix the values of the labor share, α, the elasticity of labor supply, $\frac{1}{\gamma_h}$ and the demand elasticity parameter, η. We assume $\alpha = 0.66$ and $\gamma_h = 1$. Moreover, η is chosen so that that markups are 15 percent in steady state. Finally, we normalize the steady-state value of output to unity. Parameter δ is also set exogenously and we assume $\delta = 0.95$.[15]

[Table 2.3 approximately here]

The left side of Table 2.3 reports the posterior estimates of the model parameter distributions. Our estimates are reasonably close to the analogous objects reported in Bianchi and Ilut (2017) for the post 1980s sample. For example, according to both our estimates and theirs, the monetary policy rule displays a strong reaction of the nominal interest rate to inflation, and the estimated mean of the response to output is around .6. The parameter that governs interest rate smoothing exceeds

[15]Note that, to simplify, we allow for only long term debt in the estimation. As discussed, in the linear model the behavior of bond portfolios over the cycle does not matter, and moreover, under active monetary policy the steady state portfolios will not impact (directly) inflation.

0.9. Moreover, the estimated response of taxes to the lagged value of debt is relatively low ($\phi_{\tau,b} = 0.064$ at the mean of the posterior distribution). Though with this value fiscal policy is active, debt is close to being an explosive process.

Other key parameters, e.g. the slope of the New Keynesian Phillips curve (which is relatively flat - the mean estimate being $\kappa = 0.009$) and the habit parameter (Ω, which is centered around roughly 0.5) are also in line with the results reported in Bianchi and Ilut (2017).

Finally, before turning to the main focus of this exercise, which is to evaluate how debt maturity affects the properties of inflation under passive monetary, we study the impulse responses to shocks in the active monetary policy regime that we have estimated.

[Figure 2.4 approximately here]

Figure 2.4 plots the responses of inflation and output to demand, TFP, markup shocks and shocks to fiscal variables[16] and notice that these responses are now measured in percentage points. Therefore, 1 is a 1 percent increase of a variable relative to the balanced growth path, 0.1 is a 0.1 percent increase, etc. As is evident from the figure shocks related to technology and to fiscal variables exert essentially no influence on inflation (though they do impact the output gap) and it is rather shocks to markups and demand shocks driving inflation volatility. This is to be expected. Since monetary policy is not concerned with debt sustainability in this model fiscal shocks have effectively no bearing on inflation. On the other hand, the fact that markup shocks are key is a common finding in the literature.[17]

2.4.3 Debt maturity in the medium scale DSGE model

Our theoretical analysis explored how the maturity of debt influences the properties of inflation. In some cases, we were able to show that choosing the right

[16]We leave outside the figure the tax shock and the shock to the interest rate rule, since we will also later not consider them in the passive monetary policy model.

[17]See, for example, Fratto and Uhlig (2020).

maturity of debt completely insulated inflation from the impact of random shocks, and restored the ability of monetary policy to control inflation. Though it is possible to repeat this type of analysis in the medium scale model of this section, since we now have many shocks, we choose to focus on a set of simpler experiments, studying how three alternative debt management strategies ('only short', 'only long', 'borrow long and save in short') impact the dynamic adjustment of inflation to the shocks. The aim throughout this section is to verify, in the medium scale model, that debt maturity is important for inflation variability.

To bring the economy to the passive regime we assume first that taxes are constant through time and second, we assume that monetary policy follows rules of the form (2.44) but now parameters are such that the nominal interest rate responds weakly to inflation. In Figures 2.5 and 2.6 we set $\phi_\pi = 0.9$ leaving the remaining parameters of the monetary policy rule be equal to the reported means of the posterior distributions in Table 2.3. Each of the figures shows impulse responses to three types of shocks. The top panels trace the responses of inflation whereas the bottom panels concern the adjustment of output. The dashed red lines show impulse responses when all debt is short term. The solid (blue) lines assume that all debt is long term and the dotted (black) lines set long term debt to be 10 times larger than the total value of debt financing the position with short term savings.

[Figure 2.5 approximately here]

Consider first Figure 2.5 which covers shocks to TFP, markups and shocks to the government budget (Λ). Notice that even though the impact of TFP shocks is not large, it is clearly evident that shorter debt maturity increases the magnitude of the response of inflation to the shock. In contrast, debt maturity does not seem to matter for the propagation of markup shocks, which essentially lead to the same impulse responses in Figure 2.5 in Figure 2.4 under the active policy regime.

[Figure 2.6 approximately here]

In contrast to the active regime, where markup shocks are the most important driver of inflation, under passive policy inflation displays a strong reaction to fiscal shocks. This is clearly shown in the right panel of Figure 2.5 but also in Figure 2.6 where we plot the reaction of inflation to $\hat{G}$ shocks in the left panel. Moreover, for both types of shocks the maturity structure of debt impacts the response of inflation, and the longer is maturity, the less is the response. Under a very long maturity structure (when the long bond issuance is 10 times the value of debt) we obtain for both shocks a response of inflation that is less than half of the response under short term debt. This prediction is obviously in line with our theoretical analysis where we found that issuing large amounts of long term debt is optimal to deal with fiscal shocks.

Finally, consider the middle panel of Figure 2.6, where we plot the response to a negative demand shock. When debt is only short, the demand shock causes a drop in inflation and a recession initially, however, a few periods down the line, inflation turns positive. This also happens with long term debt. Because of the switch in sign, debt maturity cannot mitigate the response of inflation to the shock, it could only be chosen to balance the cost of initial deflation with the cost of future inflation, as we saw in Section 2.2 of the paper.

To reiterate debt management is effective in stabilizing inflation in the face of fiscal shocks, which drive most of the excess inflation volatility under passive monetary policy. Thus, according to our findings it is preferable to issue large amounts of long bonds.

Appendix 2.6.1 extends further this analysis. We experiment with interest rate rules that feature no interest rate smoothing, and also consider rules that track the real interest rate. In these cases as well, large long bond issuances reduce the impact of fiscal shocks and it is preferable to tilt the maturity structure towards long debt.

2.5 Conclusion

We have provided a tractable framework to think about the interactions between debt maturity and inflation when monetary policy is subservient to fiscal policy. Our analytical results showed that these interactions are non-trivial and that debt management can complement monetary policy in pursuing its objective to control inflation. We drew analogous insights from a model where monetary and debt policies are jointly optimal. A particularly interesting analytical finding we obtained from this model, is that optimal monetary policy can be summarized in a simple interest rate rule that clarifies how interest rate policies depend on the maturity of debt issued.

Lastly, using a medium scale DGSE model we have tested whether indeed debt maturity matters when monetary policy is passive and found large effects when we looked at fiscal shocks and their impact on inflation.

Future work could apply these insights to models with regime fluctuations when monetary policy can transition between the active and passive regimes. In this case, debt maturity could exert a significant influence even in the active policy scenario. Moreover, such an exercise could be useful to pin down the optimal maturity when there is a temporary switch in the regime to e.g. monetize part of debt.

Of course a meaningful next step would also be to make the model non-linear, such that it can rely on more realistic yield curves, when inflation risk premia matter. Finally, in a non-linear model with regime fluctuations one could consider whether changing the mandate of debt management to align objectives with monetary policy as we assumed in this paper, is indeed desirable.

2.6 Appendix

2.6.1 Tables and figures

Table 2.2: **Calibrated parameters**

	Parameter	Value
y	steady state output (normalization)	1
$1-\alpha$	labor share	0.66
δ	decaying rate of coupon bonds	0.95
η	demand Elasticity	-7.66
γ_h	inverse of Frisch elasticity	1

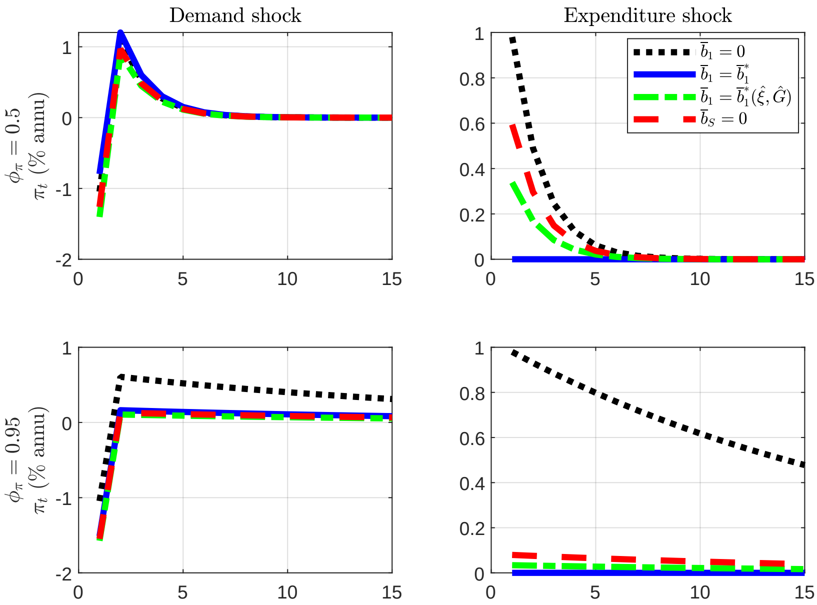
Notes: The table reports model parameters whose values we fix in estimation. See text for details.

Table 2.3: **Prior and posterior distributions**

	Parameter	Posterior		Prior		
		mean	90 % interval	distrib	par A	par B
Quarterly trends and ss values						
100γ	trend growth	0.361	[0.29 ; 0.433]	G	0.4	0.05
$100 \log \pi$	inflation	0.607	[0.516 ; 0.692]	G	0.5	0.05
$100(\beta^{-1} - 1)$	discount rate	0.139	[0.051 ; 0.228]	G	0.25	0.1
g	g-to-gdp	1.07	[1.06 ; 1.08]	N	1.06	0.04
$b_L/4$	annual mv debt-to-gdp	0.223	[0.17 ; 0.282]	N	0.25	0.05
tax	taxes-to-gdp	0.043	[0.041 ; 0.046]	N	0.045	0.0025
Households and firms						
Ω	habits	0.501	[0.407 ; 0.593]	B	0.8	0.1
κ	slope nkpc	0.009	[0.001 ; 0.017]	G	0.3	0.15
Monetary policy						
ϕ_π	taylor, inflation	2.264	[1.69 ; 2.802]	N	2.5	0.3
ϕ_y	taylor; output	0.649	[0.467 ; 0.844]	G	0.4	0.1
ρ_r	i.r. smoothing	0.964	[0.954 ; 0.975]	B	0.5	0.2
Fiscal rules						
$\phi_{\tau,b}$	tax response to b	0.064	[0.037 ; 0.09]	G	0.07	0.02
$\phi_{\tau,y}$	tax response to y	0.307	[-0.021 ; 0.641]	N	0.4	0.2
$\phi_{\tau,g}$	tax response to g	0.491	[0.17 ; 0.811]	N	0.5	0.2
$\phi_{tr,y}$	tr response to y	-0.641	[-0.771 ; -0.5]	N	-0.4	0.2
ρ_{tr}	tr smoothing	0.212	[0.131 ; 0.293]	B	0.2	0.05
ρ_τ	tax smoothing	0.969	[0.95 ; 0.99]	B	0.5	0.2
Shocks, persistence						
ρ_η	markup	0.563	[0.483 ; 0.637]	B	0.5	0.2
ρ_ξ	preference	0.954	[0.935 ; 0.973]	B	0.5	0.2
ρ_a	tfp	0.299	[0.162 ; 0.439]	B	0.5	0.2
ρ_g	gov. spending	0.976	[0.959 ; 0.993]	B	0.5	0.2
ρ_λ	gov b.c	0.155	[0.038 ; 0.265]	B	0.5	0.2
Shocks, standard deviations						
σ_τ	taxes	3.603	[2.607 ; 4.632]	IG	1	2
σ_g	gov. spending	3.742	[3.342 ; 4.144]	IG	1	1
σ_η	markup	0.227	[0.202 ; 0.251]	IG	1	1
σ_ξ	preference	0.178	[0.156 ; 0.199]	IG	1	1
σ_a	tfp	0.894	[0.691 ; 1.096]	IG	1	1
σ_m	mon. policy	0.088	[0.078 ; 0.096]	IG	0.5	0.5
σ_λ	gov b.c	0.371	[0.313 ; 0.43]	IG	0.5	0.5
σ_{tr}	transfers	0.285	[0.247 ; 0.324]	IG	0.5	0.5

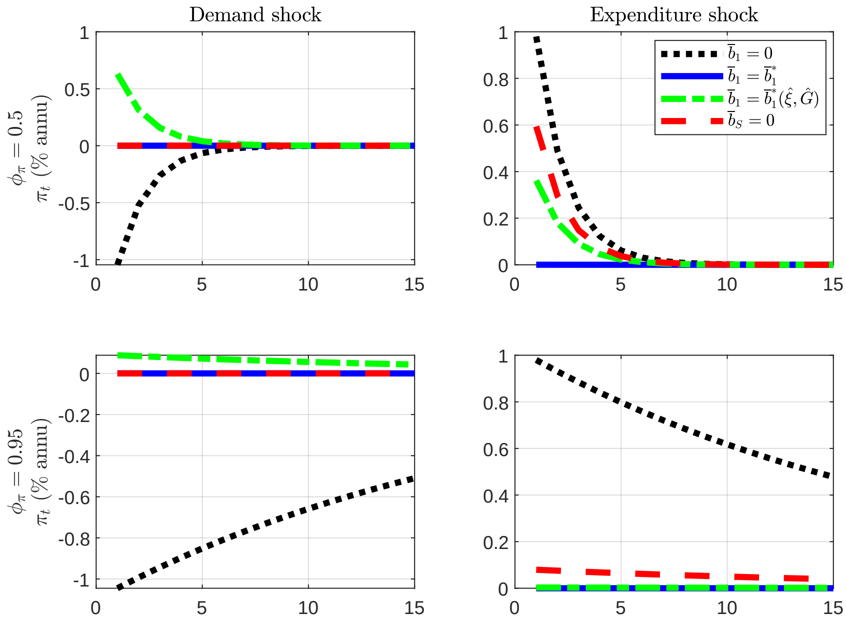
Notes: The table reports the prior and posterior distributions of the estimated parameters. The first column reports the mean of the posterior of each parameter, obtained from Monte-Carlo simulations of the posterior distribution using the MH algorithm. The second column reports the 90% HPD intervals obtained from the same draws. The third column indicates the assumed prior distribution (B: beta, G: gamma, IG: inverse gamma, N: normal). The fourth and fifth columns report the first and second moments of the priors.

Figure 2.1: **Responses to shocks when interest rates respond to inflation only.**



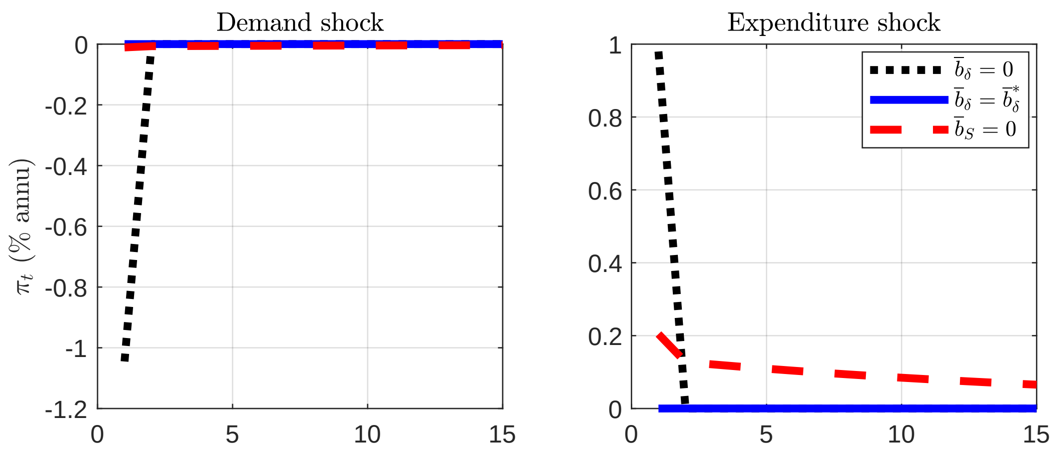
Notes: The solid lines show the response of inflation under the optimal portfolio. The dotted lines represent the responses when all debt is long term, whereas the dashed lines the case where debt is short term. The dashed-dotted lines assume the optimal portfolio in Proposition 5, when spending and demand shocks can simultaneously occur. We assume a monetary policy rule of the form $\hat{i}_t = \phi_\pi \hat{\pi}_t$.

Figure 2.2: **Responses to shocks when interest rates track the real rate.**



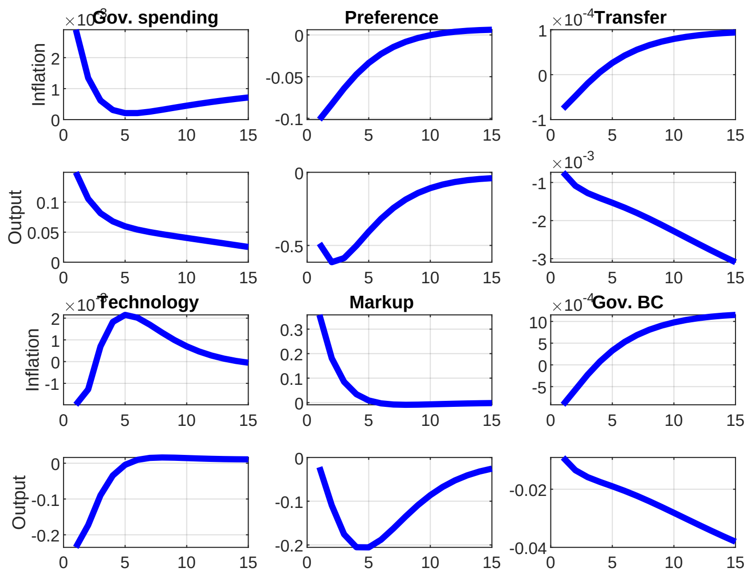
Notes: See Figure 2.1 for a description of the portfolio that corresponds to each graph. We assume a monetary policy rule of the form $\hat{i}_t = \phi_\pi \hat{\pi}_t + \hat{\xi}_t$.

Figure 2.3: **Responses to shocks under optimal policy.**



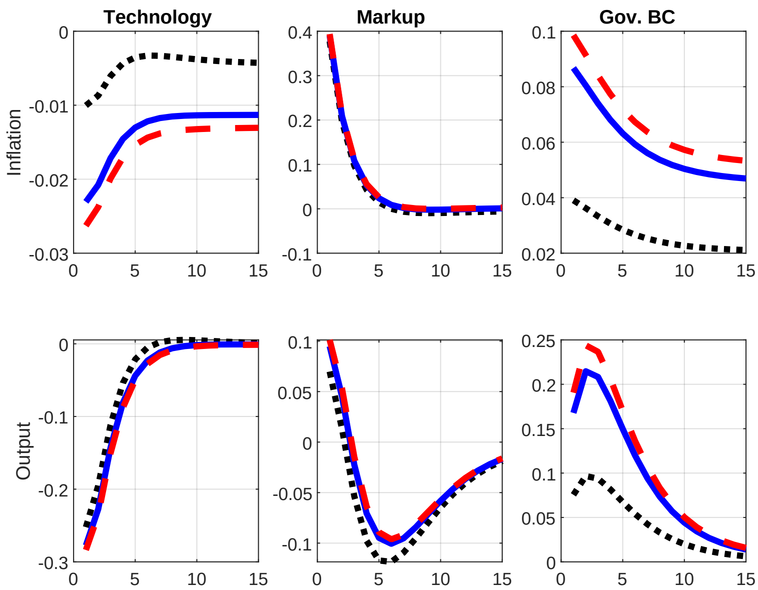
Notes: The left panel shows the response of inflation to a negative demand shock when monetary policy is optimal. The dashed line assumes long term financing, whereas the dotted line assumes only short term debt. The solid line corresponds to the optimal portfolio. The right panel plots the responses to a spending shock.

Figure 2.4: **Responses to shocks in the estimated model under active monetary policy.**



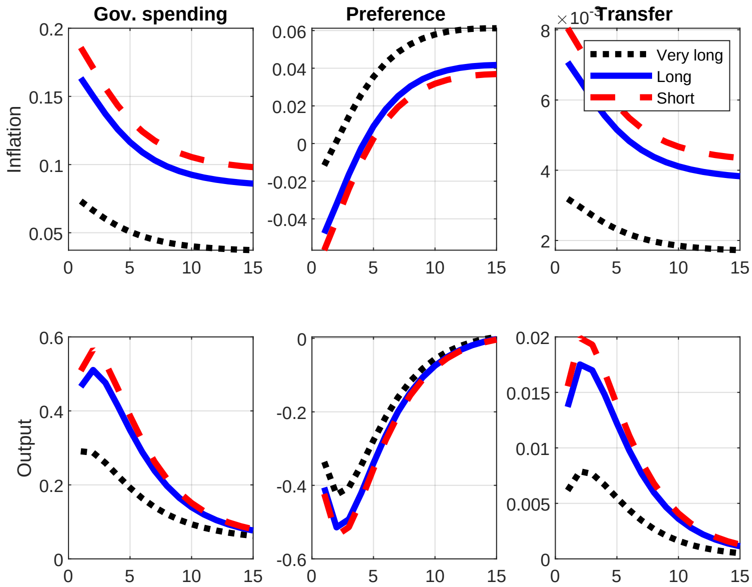
Notes: The graphs show impulse responses to one standard deviation shocks from the posterior distributions of the medium scale model. For each of the shocks in the model we plot inflation and output responses. Debt is only long term, as we assume in estimation.

Figure 2.5: **Responses to shocks in the medium scale model I.**



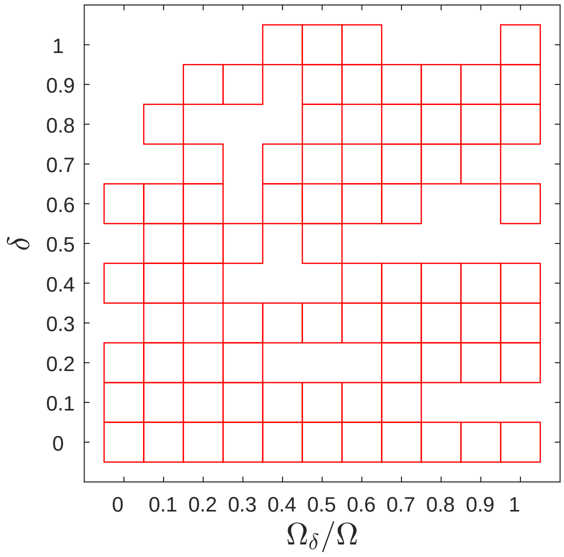
Notes: The figure plots the impulse responses of inflation and output to TFP, markup shocks and shocks to the government budget Λ. The dashed lines correspond to the case where all debt is short term, the solid lines to the case where all debt is long and the dashed-dotted lines assume that long term debt is 10 times the value of total debt.

Figure 2.6: **Responses to shocks in the medium scale model II.**



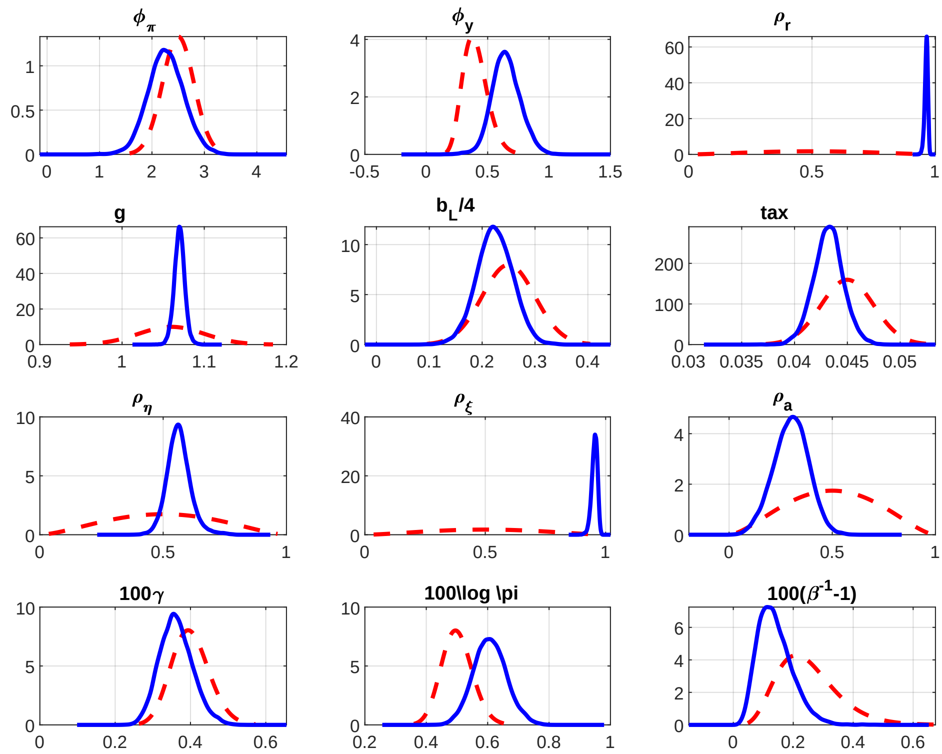
Notes: The figure plots the impulse responses of inflation and output to spending, demand (preference) and transfer shocks. The dashed lines correspond to the case where all debt is short term, the solid lines to the case where all debt is long and the dashed-dotted lines assume that long term debt is 10 times the value of total debt.

Figure 2.7: **E-stability in the passive money model**



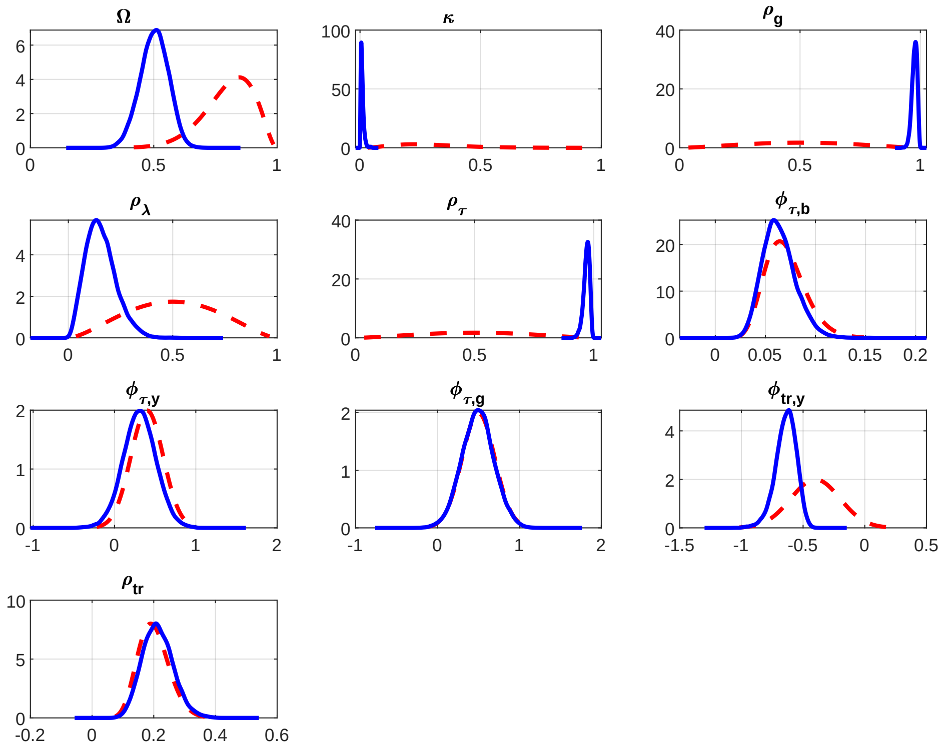
Notes: The figure plots the E-stable solutions (red square) depending on the maturity of the long bond (vertical axis) and the share of the long bond in the steady state portfolio (horizontal axis).

Figure 2.8: **Posterior distributions – Parameters (1)**



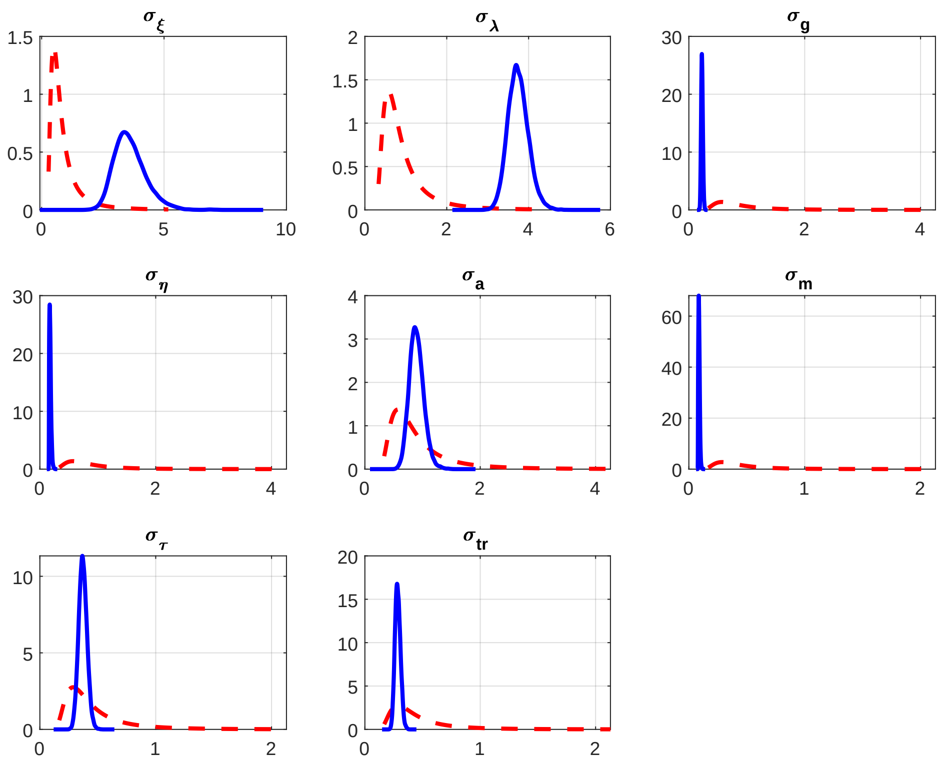
Notes: Solid blue lines represent posterior distributions of model parameters, obtained from the simulation of two chains of the Metropolis Hastings algorithm with 100,000 parameter draws. Dotted red lines give the prior distributions.

Figure 2.9: **Posterior distributions – Parameters (2)**



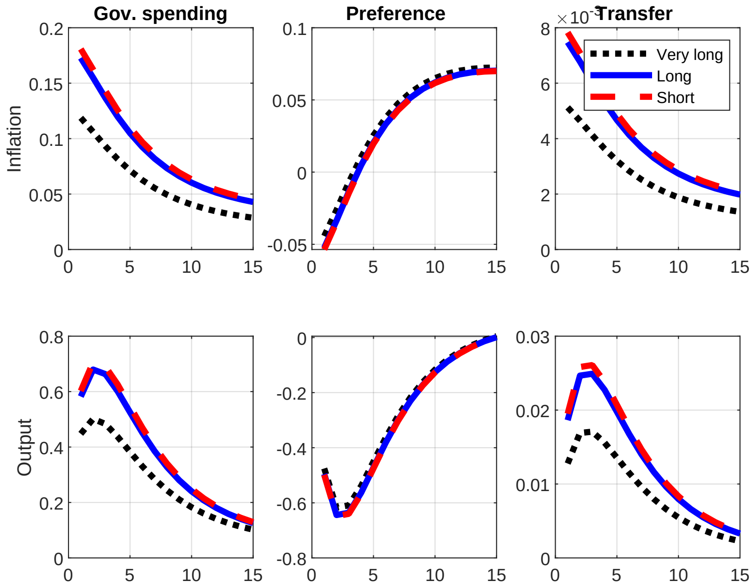
Notes: Solid blue lines show the posterior distributions of model parameters, obtained from the simulation of two chains of the Metropolis Hastings algorithm with 100,000 parameter draws. Dotted red lines show the prior distributions.

Figure 2.10: **Posterior distributions – Standard deviations of shocks**



Notes: Solid blue lines show the posterior distribution of the standard deviations of the i.i.d shock processes of the model, obtained from the simulation of two chains of the Metropolis Hastings algorithm with 100,000 parameter draws. Dotted red lines show the prior distributions.

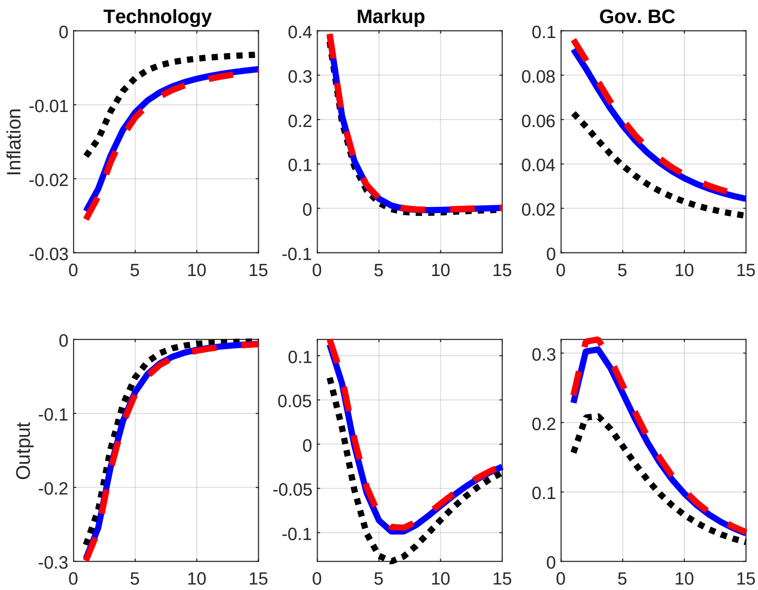
Figure 2.11: **Responses to shocks in the medium scale model under alternative policy rules: rule I (1)**



Notes: The figure displays the response of inflation and output to government spending, preference and transfer shocks in the DSGE model described in the paper and in this Appendix, using the passive Taylor rule estimated in Bianchi and Ilut (2017):

$$\hat{i}_t = 0.7480\hat{i}_{t-1} + (1 - 0.7480)\Big(0.5343\hat{\pi}_t + 0.1412(\hat{y}_t - \hat{y}_t^n)\Big) + \epsilon_{m,t} \ .$$

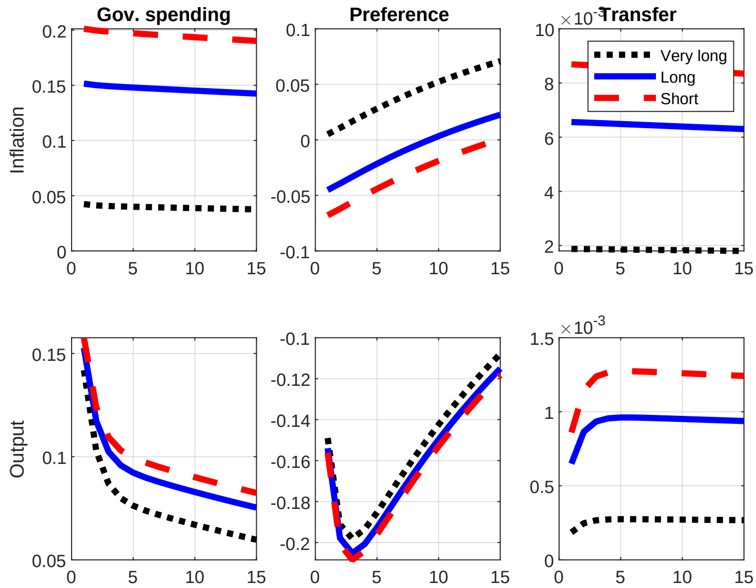
Figure 2.12: **Responses to shocks in the medium scale model under alternative policy rules: rule I (2)**



Notes: The figure displays the response of inflation and output to technology, markups, and government budget constraint shocks in the DSGE model described in the paper and in this Appendix, using the passive Taylor rule estimated in Bianchi and Ilut (2017):

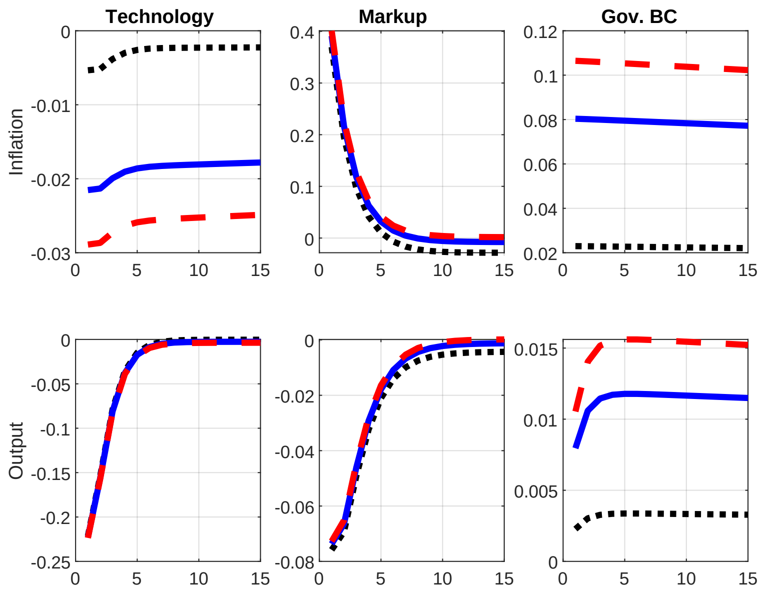
$$\hat{i}_t = 0.7480\hat{i}_{t-1} + (1 - 0.7480)\Big(0.5343\hat{\pi}_t + 0.1412(\hat{y}_t - \hat{y}_t^n)\Big) + \epsilon_{m,t} \ .$$

Figure 2.13: **Responses to shocks in the medium scale model under alternative policy rules: rule II (1)**



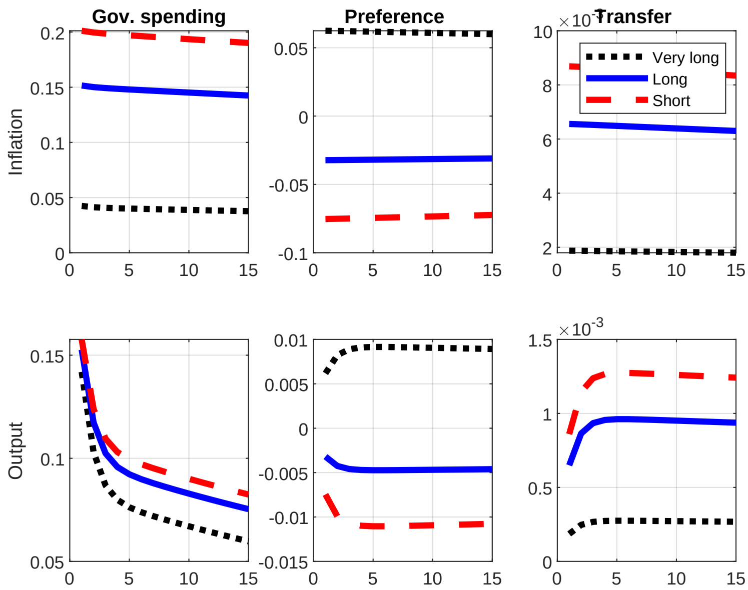
Notes: The figure displays the response of inflation and output to government spending, preference and transfer shocks in the DSGE model described in the paper and in this Appendix, using a version of our passive Taylor rule without persistence in the nominal interest rate, i.e. setting $\rho_r = 0$.

Figure 2.14: **Responses to shocks in the medium scale model under alternative policy rules: rule II (2)**



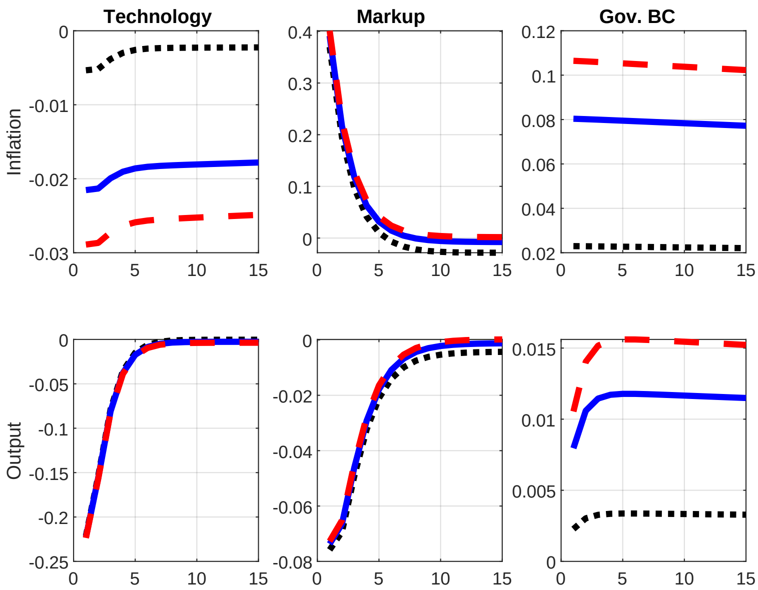
Notes: The figure displays the response of inflation and output to technology, markups, and government budget constraint shocks in the DSGE model described in the paper and in this Appendix, using a version of our passive Taylor rule without persistence in the nominal interest rate, i.e. setting $\rho_r = 0$.

Figure 2.15: **Responses to shocks in the medium scale model under alternative policy rules: rule III (1).**



Notes: The figure displays the response of inflation and output to government spending, preference and transfer shocks in the DSGE model described in the paper and in this Appendix, using a Taylor rule (i) implying no persistence in interest rates, i.e. setting $\rho_r = 0$, and (ii) tracking the fluctuations in the real interest rate implied by the preference shock: $\hat{i}_t = \hat{\xi}^b_t - E_t\hat{\xi}^b_{t+1} + \phi_\pi\hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}^n_t) + \epsilon_{m,t}$.

Figure 2.16: **Responses to shocks in the medium scale model under alternative policy rules: rule III (2)**



Notes: The figure displays the response of inflation and output to technology, markups, and government budget constraint shocks in the DSGE model described in the paper and in this Appendix, using a Taylor rule (i) implying no persistence in interest rates, i.e. setting $\rho_r = 0$, and (ii) tracking the fluctuations in the real interest rate implied by the preference shock: $\hat{i}_t = \hat{\xi}^b_t - E_t\hat{\xi}^b_{t+1} + \phi_\pi \hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}^n_t) + \epsilon_{m,t}$.

2.6.2 Derivations from the analytical model

Proof of proposition 1 Let $\hat{V}_t = \Omega_S \hat{b}_{t,S} + \Omega_\delta \hat{b}_{t,\delta}$ be the log deviation of the total market value of debt from its steady state value where $\Omega_S \equiv \beta b_S$ and $\Omega_\delta \equiv \frac{\beta b_\delta}{1-\beta\delta}$. According to rule (2.7), the tax rate responds to the market value of debt.[18] Then, the system of equations period-to-period can be written as:

$$\begin{aligned}
\hat{V}_t - \Omega_S \hat{i}_t + \Omega_\delta(1-\delta)\hat{p}_{t,\delta} \\
-\beta^{-1}\hat{V}_{t-1} - \beta^{-1}(\Omega_S+\Omega_\delta)\hat{\pi}_t + \frac{\tau(1+\eta)y}{\eta}\Big([1-\gamma_h]\hat{y}_t + \frac{\hat{\tau}}{1-\tau}\Big) - G\hat{G}_t = 0 \\
\hat{p}_{t,\delta} + \hat{i}_t - \beta\delta E_t \hat{p}_{t+1,\delta} = 0 \\
\hat{i}_t - E_t(\hat{\pi}_{t+1} - \hat{\xi}_{t+1}) - \hat{\xi}_t = 0 \\
\hat{\pi}_t - \kappa_1 \hat{y}_t - \kappa_2 \hat{\tau}_t - \beta E_t \hat{\pi}_{t+1} = 0
\end{aligned}$$

where $\hat{p}_{t,\delta}$ denotes the price of the long-term bond. Applying substitutions with rules (2.7) and (2.6), the system can be reduced to the following three equations:

$$\begin{aligned}
\hat{V}_t - \Omega_S \epsilon_{i,t} + \Omega_\delta(1-\delta)\hat{p}_{t,\delta} - \left(\beta^{-1} - \frac{\tau y(1+\eta)(\kappa_1 - (1-\tau)(1+\gamma_h)\kappa_2)}{\eta(1-\tau)\kappa_1}\phi^R_{\tau,b}\right)\hat{V}_{t-1} - G\hat{G}_t \\
+\left(\beta^{-1}(\Omega_S+\Omega_\delta) + \frac{\tau(1+\eta)y(1+\gamma_h)}{\eta\kappa_1} - \Omega_S\phi_\pi\right)\hat{\pi}_t - \frac{\tau(1+\eta)y(1+\gamma_h)}{\eta\kappa_1}\beta E_t\hat{\pi}_{t+1} = 0 \\
\hat{p}_{t,\delta} + \phi_\pi\hat{\pi}_t + \epsilon_{i,t} - \beta\delta E_t\hat{p}_{t+1,\delta} = 0 \\
\phi_\pi\hat{\pi}_t + \epsilon_{i,t} - E_t\hat{\pi}_{t+1} + E_t\hat{\xi}_{t+1} - \hat{\xi}_t = 0
\end{aligned}$$

Let's denote $\Omega \equiv \Omega_S + \Omega_\delta$ and $\Phi \equiv \frac{\tau(1+\eta)y(1+\gamma_h)}{\eta\kappa_1}\beta$. For any variable x_t, define the rational expectations errors by $\eta^x_t \equiv \hat{x}_t - E_{t-1}\hat{x}_t$ and replace x_t with $E_{t-1}x_t + \eta^x_t$.

[18] Notice that the precise definition of $\hat{V}_t$ does not matter for the dynamics of other macroeconomic variables provided appropriate scaling.

Assuming i.i.d. shocks, we can then write the system in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta\delta & 0 \\ -\Phi & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t\hat{\pi}_{t+1} \\ E_t\hat{p}_{t+1,\delta} \\ \hat{V}_t \end{bmatrix}$$

$$= \begin{bmatrix} \phi_\pi & 0 & 0 \\ \phi_\pi & 1 & 0 \\ \left(\Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) & \left(\beta^{-1} - \frac{\kappa_1-(1-\tau)(1+\gamma_h)\kappa_2}{(1-\tau)(1+\gamma_h)\beta}\Phi\phi^R_{\tau,b}\right) \end{bmatrix} \begin{bmatrix} E_{t-1}\hat{\pi}_t \\ E_{t-1}\hat{p}_{t,\delta} \\ \hat{V}_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ \Omega_S & G & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t} \\ \hat{G}_t \\ \hat{\xi}_t \end{bmatrix} + \begin{bmatrix} \phi_\pi & 0 \\ \phi_\pi & 1 \\ \left(\Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) \end{bmatrix} \begin{bmatrix} \eta^\pi_t \\ \eta^{p_\delta}_t \end{bmatrix}$$

Or, more compactly:

$$AZ_t = BZ_{t-1} + CX_t + DY_t$$

where X_t is the vector of exogenous variables. Now, premultiply each side by:

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (\beta\delta)^{-1} & 0 \\ \Phi & 0 & 1 \end{bmatrix}$$

The resulting system reads

$$\begin{bmatrix} E_t\hat{\pi}_{t+1} \\ E_t\hat{p}_{t+1,\delta} \\ \hat{V}_t \end{bmatrix}$$

$$= \begin{bmatrix} \phi_\pi & 0 & 0 \\ \phi_\pi(\beta\delta)^{-1} & (\beta\delta)^{-1} & 0 \\ \left(\Phi\phi_\pi + \Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) & \left(\beta^{-1} - \frac{\kappa_1-(1-\tau)(1+\gamma_h)\kappa_2}{(1-\tau)(1+\gamma_h)\beta}\Phi\phi^R_{\tau,b}\right) \end{bmatrix} \begin{bmatrix} E_{t-1}\hat{\pi}_t \\ E_{t-1}\hat{p}_{t,\delta} \\ \hat{V}_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & -1 \\ (\beta\delta)^{-1} & 0 & 0 \\ \Phi + \Omega_S & G & -\Phi \end{bmatrix} \begin{bmatrix} \epsilon_{i,t} \\ \hat{G}_t \\ \hat{\xi}_t \end{bmatrix} + \begin{bmatrix} \phi_\pi & 0 \\ \phi_\pi(\beta\delta)^{-1} & (\beta\delta)^{-1} \\ \left(\Phi\phi_\pi + \Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) \end{bmatrix} \begin{bmatrix} \eta^\pi_t \\ \eta^{p_\delta}_t \end{bmatrix}$$

The system has three eigenvalues:

$$
\begin{aligned}
e_1 &= \phi_\pi \\
e_2 &= (\beta\delta)^{-1} \\
e_3 &= \beta^{-1} - \frac{\tau y(1+\eta)(\kappa_1 - (1-\tau)(1+\gamma_h)\kappa_2)}{\eta(1-\tau)\kappa_1}\phi^R_{\tau,b}
\end{aligned}
$$

Moreover, there are two non-predetermined variables, $\mathbb{E}_t\hat{p}_{t+1,\delta}$ and $\mathbb{E}_t\hat{\pi}_{t+1}$. For determinacy, we hence need two of the eigenvalues to lie outside the unit circle. For any non-explosive maturity of debt, we have that $\delta \leq 1$ and thus, $e_2 > 1$. Consequently, we need either $\phi_\pi > 1$ or $\phi_{\tau,b} < \frac{\eta(1-\tau)\kappa_1(\beta^{-1}-1)}{\tau(1+\eta)y(\kappa_1-(1-\tau)(1+\gamma_h)\kappa_2)}$. ■

Proof of proposition 2 Assume that $\phi^R_{\tau,b} = 0$ and $\phi_\pi < 1$. Combining the Euler equation with the monetary policy rule (and assuming no shocks after period t) we get:

$$
\hat{\pi}_{t+\bar{t}} = \phi_\pi \hat{\pi}_{t+\bar{t}-1} + (\epsilon_{i,t} - \hat{\xi}_t)\mathcal{I}_{\bar{t}=1} \tag{2.45}
$$

From the Phillips curve we have

$$
\hat{Y}_{t+\bar{t}} = \frac{1}{\kappa_1}\left(\hat{\pi}_{t+\bar{t}} - \beta\hat{\pi}_{t+1+\bar{t}}\right) = \frac{1}{\kappa_1}(1-\beta\phi_\pi)\hat{\pi}_{t+\bar{t}} - \frac{\beta}{\kappa_1}(\epsilon_{i,t} - \hat{\xi}_t)\mathcal{I}_{\bar{t}=1}
$$

Given these expressions to characterize inflation in t we use the date t intertemporal budget constraint of the government. We can write the constraint as:

$$
\begin{aligned}
&\sum_{j\geq 0}\beta^j E_t\left(\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}((\gamma_h+1)\hat{Y}_{t+j} + \hat{\xi}_{t+j}) - \overline{G}(\hat{G}_{t+j} + \hat{\xi}_{t+j})\right) \\
&= \overline{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} + \overline{b}_\delta E_t\sum_{j=0}^{\infty}\beta^j\delta^j\left[-\sum_{l=0}^{j}\hat{\pi}_{t+l}\right] + \hat{\xi}_t(\overline{b}_\delta + \overline{b}_S)
\end{aligned}
$$

The RHS can be written as:

$$\bar{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} - \bar{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j \left[\frac{(1-\phi_\pi^{j+1})}{1-\phi_\pi}\hat{\pi}_t + \frac{(1-\phi_\pi^j)}{1-\phi_\pi}(\epsilon_{i,t} - \hat{\xi}_t) \right] + \hat{\xi}_t(\bar{b}_\delta + \bar{b}_S)$$

The LHS is

$$\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}(\gamma_h + 1)\frac{1}{\kappa_1}\hat{\pi}_t - \overline{G}\hat{G}_t + \left(\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta} - \overline{G} \right)\hat{\xi}_t$$

Simplifying further the RHS can be written as:

$$\bar{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} - \frac{\bar{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\hat{\pi}_t$$
$$- \frac{\bar{b}_\delta\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}(\epsilon_{i,t} - \hat{\xi}_t) + \hat{\xi}_t(\bar{b}_\delta + \bar{b}_S)$$

Equating LHS and RHS letting $\overline{R} = \overline{\tau}(1+\eta)\overline{Y}\eta$ and noting that the steady state surplus can be written as $\left(\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta} - \overline{G} \right) = (1-\beta)\bar{b}_S + \frac{1-\beta}{1-\beta\delta)}\bar{b}_\delta$ and rearranging we get:

$$\left[\overline{R}\frac{(\gamma_h + 1)}{\kappa_1} + \bar{b}_S + \frac{\bar{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right]\hat{\pi}_t$$
$$= -\left[\frac{\bar{b}_\delta\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right]\epsilon_{i,t} + \overline{G}\hat{G}_t + \left[\beta\bar{b}_S + \frac{\bar{b}_\delta\beta(1-(1-\delta)\delta\beta\phi_\pi)}{(1-\beta\delta)(1-\phi_\pi\beta\delta)} \right]\hat{\xi}_t$$

It is easy to obtain coefficients η_1, η_2 and η_3 from the above expression. From (2.45) it is possible to characterize the entire path of inflation. ■

Proof of Proposition 3. The FONC of the minimisation program (2.13) with only demand shocks is:

$$\left(\eta_3 + \frac{\phi_\pi\beta}{1-\phi_\pi^2\beta}(\eta_3\phi_\pi - 1) \right)\frac{d\eta_3}{d\bar{b}_1}\sigma_{\hat{\xi}}^2 = 0$$

When $\phi_\pi = 0$, it holds that $\frac{d\eta_3}{d\bar{b}_1} = 0$. The maturity of debt is irrelevant.

In the case where $\phi_\pi > 0$ we have that $\frac{d\eta_3}{d\bar{b}_1} \neq 0$ and thus, the FONC imposes that $\eta_3 + \frac{\phi_\pi \beta}{1-\phi_\pi^2 \beta}(\eta_3 \phi_\pi - 1) = 0$ which corresponds to $\eta_3 = \beta\phi_\pi$ ■

Proof of Proposition 5. Let's consider first the case where the interest rate respond to the demand shock only through its endogenous response to inflation. The FONC for the combined shock is:

$$\eta_2 \eta_2' \sigma_G^2 \left(1 + \frac{\beta\phi_\pi^2}{1-\beta\phi_\pi^2}\right) + \eta_3' \sigma_\xi^2 \left(\eta_3 + \frac{\beta\phi_\pi(\eta_3\phi_\pi - 1)}{1-\beta\phi_\pi^2}\right) = 0$$

where $\eta_2' \equiv \frac{d\eta_2}{d\bar{b}_1}$ and $\eta_3' \equiv \frac{d\eta_3}{d\bar{b}_1}$. Rearranging this expression, we obtain:

$$\eta_3 = \beta\phi_\pi - \eta_2 \frac{\eta_2'}{\eta_3'} \frac{\sigma_G^2}{\sigma_\xi^2} \tag{2.46}$$

Notice that η_2 and η_3 can be written as:

$$\eta_2 = \frac{\overline{G}}{\Lambda}$$
$$\eta_3 = \frac{\beta\left(\Lambda - \frac{\overline{R}(\gamma_h+1)}{\kappa_1}\right)}{\Lambda}$$

where $\Lambda \equiv \overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_\delta \phi_\pi \beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}$. And thus,

$$\eta_2' = \frac{-\Lambda_{b_\delta}' \overline{G}}{\Lambda^2}$$
$$\eta_3' = \frac{\frac{\beta\overline{R}(\gamma_h+1)}{\kappa_1}\Lambda_{b_\delta}'}{\Lambda^2}$$

Substituting these derivatives in (2.46) allows to easily recover the formula in the text.

Next, let us consider the case of monetary policy responding one-to-one to the demand shock i.e. $\epsilon_{i,t} = \hat{\xi}_t$. Since this implies that $\sigma_{\epsilon_i,\hat{\xi}} = \sigma^2_{\hat{\xi}}$, the FONC becomes:

$$\eta_1\eta_1'\sigma_G^2 + (\eta_2\eta_2' + \eta_3\eta_3' + \eta_1'\eta_3 + \eta_3'\eta_1)\sigma_\xi^2 + \frac{\beta\phi_\pi\eta_2\eta_2'}{1-\beta\phi_\pi}\sigma_G^2$$
$$+\left(\frac{(1+\phi_\pi\eta_1)\eta_1'}{1-\beta\phi_\pi} + \frac{(\eta_3\phi_\pi - 1)\eta_3'}{1-\beta\phi_\pi}\right)\beta\phi_\pi\sigma_\xi^2 = \frac{\phi_\pi\eta_1 - \eta_3\phi_\pi - \eta_1'\eta_3\phi_\pi^2 - \eta_3'\eta_1\phi_\pi^2}{1-\beta\phi_\pi}\beta\sigma_\xi^2$$

Simplifying this expression, we arrive to:

$$\eta_1 + \eta_3 = -\frac{\eta_2\eta_2'}{\eta_1' + \eta_3'}\frac{\sigma_G^2}{\sigma_\xi^2}$$

where $\eta_1' \equiv \frac{d\eta_1}{d\bar{b}_1}$.

Focus first on the LHS. Notice that η_1 can be written as:

$$\eta_1 = -\frac{\left[\Lambda - \left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\right]\frac{1}{\phi_\pi}}{\Lambda}$$

and hence, we have:

$$LHS = \eta_1 + \eta_3 = \frac{\beta}{(1-\beta)\Lambda}(\overline{S} - \overline{b}_\delta)$$

Let us now turn to the RHS. The derivative of η_1 is:

$$\eta_1' = -\frac{\left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\frac{\Lambda'_{b_\delta}}{\phi_\pi}}{\Lambda^2}$$

Hence,

$$\eta_1' + \eta_3' = \frac{\left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1}(\beta\phi_\pi - 1) - \frac{\overline{S}}{1-\beta}\right)\frac{\Lambda'_{b_\delta}}{\phi_\pi}}{\Lambda^2}$$

The RHS is thus:

$$RHS = -\frac{\overline{G}^2 \phi_\pi}{\left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1}(1-\beta\phi_\pi) + \frac{\overline{S}}{1-\beta}\right)\Lambda}$$

Equating the LHS and the RHS and rearranging, one can easily retrieve the formula in the text. ■

Proof of Proposition 6. Rewriting equation (2.30) with the lag operator gives:

$$(1-\delta L)\hat{\pi}_{t+\bar{t}} = \frac{(1-L)(1-\delta L)(1+\gamma_h)}{\kappa_1}\bar{R}\psi_{gov,t+\bar{t}} + (1-L)(1-\delta L)\bar{b}_S\psi_{gov,t+\bar{t}} + \frac{1-L}{1-\beta\delta}\bar{b}_\delta\psi_{gov,t+\bar{t}}$$

Forward this equation one period and use the backshift operator $BE_{t+\bar{t}}x_{t+\bar{t}} = E_{t+\bar{t}}x_{t+\bar{t}-1}$ to obtain:

$$\begin{aligned}(1-\delta B)E_{t+\bar{t}}\hat{\pi}_{t+\bar{t}+1} = {} & \frac{(1-B)(1-\delta B)(1+\gamma_h)}{\kappa_1}\bar{R}E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1} \\ & + (1-B)(1-\delta B)\bar{b}_S E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1} + \frac{1-B}{1-\beta\delta}\bar{b}_\delta E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1}\end{aligned}$$

Since $\psi_{gov,t}$ is a random walk, we have that $(1-B)E_t\psi_{gov,t+1} = 0$. Hence, the previous equation boils down to:

$$E_{t+\bar{t}}\hat{\pi}_{t+\bar{t}+1} = \delta\hat{\pi}_{t+\bar{t}} - \delta\left(\frac{(1+\gamma_h)}{\kappa_1}\bar{R} + \bar{b}_S\right)\Delta\psi_{gov,t+\bar{t}} \tag{2.47}$$

Plugging this expression in the Fisher equation $\hat{i}_{t+\bar{t}} = E_{t+\bar{t}}\hat{\pi}_{t+1+\bar{t}} + \hat{\xi}_{t+\bar{t}}$ gives the optimal interest rule in the text. ■

Derivation of equation (2.33) in the text Since we assume that no further shock hits the economy after period t then $\Delta\psi_{gov,t+j} = 0$, $j \geq 1$. Moreover setting wlog $\psi_{gov,t-1} = \psi_{gov,t-2} = ...\psi_{gov,-1} = 0$ we can write (2.30) as

$$\hat{\pi}_t = \left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta} \right) \Delta\psi_{gov,t}$$

Moreover, combining this equation with (2.47) and extrapolating for any generic period gives the following path of future inflation

$$\hat{\pi}_{t+\bar{t}} = \frac{\overline{b}_\delta}{1-\beta\delta}\delta^{\bar{t}}\Delta\psi_{gov,t} \tag{2.48}$$

Next, consider the intertemporal government budget constraint:

$$\sum_{j=0}^{\infty} \beta^j \left(\bar{R}[(1+\gamma_h)\hat{y}_{t+j} + \hat{\xi}_{t+j}] - \bar{G}[\hat{G}_{t+j} + \hat{\xi}_{t+j}] \right)$$
$$= \bar{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} - \bar{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j \sum_{l=0}^{j} \hat{\pi}_{t+l} + (\bar{b}_S + \bar{b}_\delta)\hat{\xi}_t$$

Plugging (2.48) in the Phillips Curve, we have:

$$\hat{y}_t = \frac{\hat{\pi}_t}{\kappa_1} - \frac{\beta\overline{b}_\delta}{\kappa_1(1-\beta\delta)}\delta\Delta\psi_{gov,t}$$

and, for any $j > 0$,

$$\hat{y}_{t+j} = \frac{\overline{b}_\delta}{\kappa_1(1-\beta\delta)}\delta^j\Delta\psi_{gov,t} - \frac{\beta\overline{b}_\delta}{\kappa_1(1-\beta\delta)}\delta^{j+1}\Delta\psi_{gov,t}$$

Hence:

$$\sum_{j=0}^{\infty} \beta^j \hat{y}_{t+j} = \frac{\hat{\pi}_t}{\kappa_1}$$

The LHS of the intertemporal budget constraint can be written as:

$$\frac{\overline{R}(1+\gamma_h)}{\kappa_1}\hat{\pi}_t - \overline{G}\hat{G}_t + \overline{S}\hat{\xi}_t$$

Let's focus now on the RHS. The following decomposition applies:

$$-\bar{b}_\delta\sum_{j=0}^{\infty}\beta^j\delta^j\sum_{l=0}^{j}\hat{\pi}_{t+l} = -\bar{b}_\delta\sum_{j=0}^{\infty}\beta^j\delta^j\hat{\pi}_t - \frac{\bar{b}_\delta^2}{1-\beta\delta}\Delta\psi_{gov,t}\sum_{j=1}^{\infty}\beta^j\delta^j\sum_{l=1}^{j}\delta^l$$

$$= -\frac{\bar{b}_\delta}{1-\beta\delta}\hat{\pi}_t - \frac{\bar{b}_\delta^2}{1-\beta\delta}\Delta\psi_{gov,t}\sum_{j=0}^{\infty}\beta^j\delta^{j+1}\frac{1-\delta^j}{1-\delta} = \left(\bar{b}_S - \frac{\overline{S}}{1-\beta}\right)\hat{\pi}_t - \frac{\bar{b}_\delta^2\beta\delta^2}{(1-\beta\delta)^2(1-\beta\delta^2)}\Delta\psi_{gov,t}$$

where the last equality stems from the steady state identity $\frac{\bar{b}_\delta}{1-\beta\delta} = \bar{b}_S - \frac{\overline{S}}{1-\beta}$. Equating the LHS and the RHS (assuming as before that initial debt is zero), we find:

$$\left(\frac{\overline{R}(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\hat{\pi}_t = \overline{G}\hat{G}_t + (\bar{b}_S + \bar{b}_\delta - \overline{S})\hat{\xi}_t - \frac{\bar{b}_\delta^2\beta\delta^2}{(1-\beta\delta)^2(1-\beta\delta^2)}\Delta\psi_{gov,t}$$

Using (2.30) to substitute for $\hat{\pi}_t$ and rearranging gives the expression in the text.

2.6.3 E-stability of the passive monetary path

Under the passive money regime, the equilibrium path selected departs from the standard outcome with monetary policy stabilising inflation in response to exogenous shocks. Instead, inflation contributes to stabilise the budget constraint of the government. Here, we show under which conditions, this Rational Expectations Equilibrium (REE) is stable under adaptive learning when agents have the correct Perceived Law of Motion (PLM) for the economy but do not know the exact value of the parameters. To examine E-stability conditions, first write the system recursively as:

$$y_t = BE_t y_{t+1} + Cy_{t-1} + Dw_t$$

where $y_t = [\hat{\pi}_t \quad \hat{y}_t \quad \hat{V}_t]'$, $w_t = [\epsilon_{i,t} \quad \hat{G}_t \quad \hat{\xi}_t]'$ and $B = \tilde{A}^{-1}\tilde{B}$, $C = \tilde{A}^{-1}\tilde{C}$, $D = \tilde{A}^{-1}\tilde{D}$ with the following matrices

$$\tilde{A} = \begin{bmatrix} \phi_\pi & 0 & 0 \\ \phi_\pi & 1 & 0 \\ \Xi & -\Omega_\delta(1-\delta) & 1 \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta\delta & 0 \\ -\Phi & 0 & 0 \end{bmatrix}$$
$$\tilde{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\beta^{-1} \end{bmatrix} \quad \tilde{D} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 0 \\ -\Omega_S & -G & 0 \end{bmatrix}$$

where $\Xi \equiv \Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}$ and as before $\Omega \equiv \Omega_S + \Omega_\delta$ and $\Phi \equiv \frac{\tau(1+\eta)\overline{Y}(1+\gamma_h)}{\eta\kappa_1}\beta$ and $\hat{V}_t = \Omega_S\hat{b}_{t,S} + \Omega_\delta\hat{b}_{t,\delta}$ with $\Omega_S \equiv \beta b_S$ and $\Omega_\delta \equiv \frac{\beta b_\delta}{1-\beta\delta}$.

The Minimum State Variable (MSV) form of the system is given by

$$y_t = a + by_{t-1} + cw_t$$

with corresponding expectations

$$E_t y_{t+1} = a + by_t$$

Hence, the MSV solutions satisfy

$$\begin{aligned} (I - Bb - B)a &= 0 \\ Bb^2 + C &= b \end{aligned}$$

The mapping from the PLM to the ALM is thus

$$T(a,b) = \Big((I - Bb)^{-1}Ba, (I - Bb)^{-1}C\Big)$$

The E-stability conditions are obtained from **?**, Proposition 10.3 where

$$DT_a(\bar{a},\bar{b}) = (I - B\bar{b})^{-1}B$$
$$DT_b(\bar{b}) = [(I - B\bar{b})^{-1}C]' \otimes [(I - B\bar{b})^{-1}B]$$

The proposition states that the eigenvalues of those matrices must have real parts less than one for the MSV solution to be E-stable.

[Figure 2.7 approximately here]

Figure 2.7 shows for which value of the parameter δ and share of the long bond in the portfolio Ω_δ/Ω the solution of the system is E-stable. While the REE is E-stable under most parametrization, some comments are due. First, the REE is not E-stable if only short-term debt is issued when consols are available. In contrast, the portfolio is always E-stable if the maturity of the long bond is equivalent to the short bond. Finally, for the baseline calibration of $\delta = .95$, most of the porfolios considered in in the text are E-stable.

2.6.4 Lagrangian of the optimal policy problem

Let $\psi_{\pi,t+\bar{t}}$ and $\psi_{g,t+\bar{t}}$ be the Lagrange multipliers attached to the Phillips curve and the consolidated budget, respectively. We formulate the Lagrangian as

$$\begin{aligned}
\mathcal{L} = E_t \sum_{\bar{t}=0}^{\infty} \beta^{\bar{t}} \Bigg\{ & \hat{\pi}_{t+\bar{t}} + \psi_{\pi,t+\bar{t}}\left[\hat{\pi}_{t+\bar{t}} - \kappa_1 \hat{Y}_{t+\bar{t}} - \beta E_{t+\bar{t}}\hat{\pi}_{t+\bar{t}+1}\right] \\
& + \psi_{g,t+\bar{t}} \Bigg[\beta\bar{b}_S\hat{b}_{t+\bar{t},S} + \beta\bar{b}_S E_{t+\bar{t}}(\hat{\xi}_{t+\bar{t}+1} - \hat{\xi}_{t+\bar{t}} - \hat{\pi}_{t+\bar{t}+1}) + \frac{\beta\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t+\bar{t},\delta} \\
& + \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left(E_{t+\bar{t}} \left(-\sum_{l=1}^{j} \hat{\pi}_{t+\bar{t}+l} + \hat{\xi}_{t+\bar{t}+j} - \hat{\xi}_{t+\bar{t}} \right) \right) + \frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}(\gamma_h + 1)\hat{Y}_{t+\bar{t}} - \overline{G}\hat{G}_{t+\bar{t}} \\
& - \bar{b}_S(\hat{b}_{t+\bar{t}-1,S} - \hat{\pi}_{t+\bar{t}}) - \frac{\bar{b}_\delta}{1-\beta\delta}(\hat{b}_{t+\bar{t}-1,\delta} - \hat{\pi}_{t+\bar{t}}) \\
& - \delta\bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left(E_{t+\bar{t}} \left(-\sum_{l=1}^{j} \hat{\pi}_{t+\bar{t}+l} + \hat{\xi}_{t+\bar{t}+j} - \hat{\xi}_{t+\bar{t}} \right) \right) \Bigg] \Bigg\}
\end{aligned}$$

2.6.5 The quantitative model

In Section 2.4 of the paper, we estimate a New-Keynesian DSGE model broadly similar to Bianchi and Melosi (2017) and Bianchi and Ilut (2017). We depart from these papers in assuming the presence of distortionary labor taxes. In this section, we first provide a concise description of the estimated model and the obtained estimation output. Then, we lay out the model extension which allows for a non-trivial portoflio composition between short and long-term government bonds.

The estimated model

Household preferences are of the following form:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\log(C_t - \Omega C^a_{t-1}) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

where C_t is again consumption and C^a_t is an external habit stock ($0 < \Omega < 1$ determines the degree of habit formation) as discussed in text. Households maximize utility subject to

$$P_t C_t + P_{t,S} B_{t,S} + P_{t,L} B_{t,\delta} \leqslant (1-\tau_t) W_t h_t + P_t D_t + B_{t-1,S} + (1+\delta P_{t,L}) B_{t-1,\delta}$$

The first order conditions give us Euler equations for short and long-term government bonds, and the labor supply condition:

$$\frac{1}{C_t - \Omega C^a_{t-1}} = \beta R_t E_t \frac{1}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C^a_t} \tag{2.49}$$

$$\frac{1}{C_t - \Omega C^a_{t-1}} P_{L,t} = \beta E_t \frac{1+\delta P_{L,t+1}}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C^a_t} \tag{2.50}$$

$$\chi h_t^{\gamma_h} (C_t - \Omega C^a_{t-1}) = (1-\tau_t) \frac{W_t}{P_t} \tag{2.51}$$

Production takes place in monopolistically competitive firms which operate technologies with labour as the sole input. The final good is a CES aggregate of the intermediate goods $Y_t(j)$:

$$Y_t = \Big(\int_0^1 Y_t(j)^{\frac{1+\eta_t}{\eta_t}} dj\Big)^{\frac{\eta_t}{1+\eta_t}} \tag{2.52}$$

where η_t is a time-varying process governing the elasticity of substitution between differentiated goods. Firms set prices to maximize profits subject to the demand curve

$$Y_t(j) = \Big(\frac{P_t(j)}{P_t}\Big)^{\eta_t} Y_t \tag{2.53}$$

and given price adjustment costs. The dynamic profit maximization program is:

$$\begin{aligned}
\max_{P_t(j)} \quad & E_t \sum_{s=0}^{\infty} Q_{t,t+s}\Big(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{MC_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - AC_{t+s}(j)\Big) \\
\text{s.t.} \quad & Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}}\Big)^{\eta_t} Y_{t+s} && (2.54) \\
& AC_{t+s}(j) = \frac{\theta}{2}\Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \overline{\pi}\Big)^2 Y_{t+s} && (2.55)
\end{aligned}$$

where $Q_{t,t+s} \equiv \beta^s E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+s} - \Omega C^a_{t+s-1}}$ is the discount factor of households and MC_{t+s} denotes marginal costs of production. $AC_{t+s}(j)$ in equation (2.55) is the quadratic price adjustment cost incurred by the firm.

Focusing on a symmetric equilibrium the first order condition from the firms's dynamic programs gives us the following economy wide non-linear Phillips Curve:

$$\theta(\pi_t - \overline{\pi})\pi_t = 1 + \eta_t(1 - \frac{MC_t}{P_t}) + \beta\theta E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+1} - \Omega C^a_t} \frac{Y_{t+1}}{Y_t}(\pi_{t+1} - \overline{\pi})\pi_{t+1} \tag{2.56}$$

The production of $Y_t(j)$ is determined according to

$$Y_t(j) = A_t h_t(j)^{1-\alpha} \tag{2.57}$$

where A_t is TFP, whose growth rate (in logs) follows an AR(1) process:

$$\ln\left(\frac{A_t}{A_{t-1}}\right) \equiv a_t = (1-\rho_a)\gamma + \rho_a a_{t-1} + \epsilon_{a,t}$$

Parameter γ denotes the steady-state growth rate if A_t.

Given the above, (real) marginal costs can be expressed as:

$$\frac{MC_t(j)}{P_t} = \frac{W_t}{P_t}\frac{1}{(1-\alpha)A_t h_t(j)^{-\alpha}}$$

Fiscal policy The government levies distortionary taxes and issues debt to finance spending, and transfers to the private sector. The flow budget constraint is:

$$P_{t,L}B_{t,\delta} = (1+\delta P_{t,L})B_{t-1,\delta} + P_t(G_t + TR_t) - \tau_t W_t h_t \tag{2.58}$$

where G_t and TR_t denote respectively government purchases of goods and services and lump-sum transfers rebated to households.

We follow Bianchi and Ilut (2017) and express the government budget constraint as a fraction of output Y_t. Define $b_{\delta,t} \equiv \frac{P_{L,t}B_{\delta,t}}{P_t Y_t}$ as the real market value of debt-to-GDP ratio and $R_t^L \equiv \frac{1+\delta P_{L,t+1}}{P_{L,t}}$ is the holding period return on the long bond. Also let $g_t \equiv \frac{1}{1-\frac{G_t}{Y_t}}$ and $tr_t \equiv \frac{TR_t}{Y_t}$. Using $\frac{W_t h_t}{P_t Y_t} = \frac{MC_t}{P_t} \equiv mc_t$, we can write the (real) flow government budget constraint as:

$$b_{\delta,t} \quad = \quad \frac{R_{t-1}^L b_{\delta,t-1}}{\pi_t}\frac{Y_{t-1}}{Y_t} + 1 - g_t^{-1} + tr_t - \tau_t mc_t \tag{2.59}$$

Equilibrium In the symmetric equilibrium all firms set the same price and produce equal output, hence $Y_t(j) = Y_t$ and $h_t(j) = h_t \ \forall j \in [0,1]$. We can therefore write the aggregate production function as

$$Y_t = A_t h_t^{1-\alpha} \tag{2.60}$$

and the (economy-wide) marginal costs of production are

$$\frac{MC_t}{P_t} = \frac{W_t}{P_t} \frac{1}{(1-\alpha)A_t h_t^{-\alpha}} \tag{2.61}$$

The resource constraint of the economy is

$$Y_t = C_t + G_t \tag{2.62}$$

and rescaling in terms of $g_t = \frac{1}{1-\frac{G_t}{Y_t}}$ we have

$$Y_t = C_t \, g_t \tag{2.63}$$

Detrending Since TFP grows over time we rescale model variables to make them stationary. We define $c_t \equiv \frac{C_t}{A_t}$ and $y_t \equiv \frac{Y_t}{A_t}$. The model equations can be written as follows:

$$\frac{1}{c_t - \Omega c_{t-1}^a e^{-a_t}} = \beta R_t E_t \frac{1}{\pi_{t+1}} \frac{e^{-a_{t+1}}}{c_{t+1} - \Omega c_t^a e^{-a_{t+1}}} \tag{2.64}$$

$$\frac{1}{c_t - \Omega c_{t-1}^a e^{-a_t}} = \beta E_t \frac{1+\delta P_{L,t+1}}{\pi_{t+1}} \frac{e^{-a_{t+1}}}{c_{t+1} - \Omega c_t^a e^{-a_{t+1}}} \tag{2.65}$$

$$\theta(\pi_t - \pi)\pi_t = 1 + \eta_t(1 - mc_t) + \beta\theta E_t \frac{(c_t - \Omega c_{t-1}^a e^{-a_t})}{(c_{t+1} - \Omega c_t^a e^{-a_{t+1}})} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \pi)\pi_{t+1} \tag{2.66}$$

$$mc_t = \frac{\chi y_t^{\frac{\gamma_h + \alpha}{1-\alpha}}}{1-\tau_t} (c_t - \Omega c_{t-1}^a e^{-a_t}) \tag{2.67}$$

$$y_t = c_t g_t \tag{2.68}$$

$$b_{\delta,t} = \frac{R_{t-1}^L b_{\delta,t-1}}{\pi_t} \frac{y_{t-1}}{y_t} e^{-a_t} + 1 - g_t^{-1} + tr_t - \tau_t mc_t \tag{2.69}$$

$$R_t^L = \frac{1+\delta P_{L,t+1}}{P_{L,t}} \tag{2.70}$$

where $R_t \equiv P_{t,s}^{-1}$. Notice that we made use of the production function (2.60) and the optimal labour supply condition (2.51) to dispense with W_t and h_t.

Equations (2.64) to (2.70) (together with the ZLB) provide a full description of the competitive equilibrium of the model economy. Dropping time subscripts we obtain steady state relationships. Define $\overline{x}$, the steady-state value of variable x_t. We can write:

$$\overline{R} = \frac{\overline{\pi} e^{\gamma}}{\beta} \qquad \overline{mc} = \frac{1+\eta}{\eta}$$

$$\overline{P}_L = \frac{\beta e^{-\gamma}}{\overline{\pi} - \beta \delta e^{-\gamma}} \qquad \overline{c} = \frac{\overline{y}}{\overline{g}}$$

$$\overline{R}_L = \frac{1 + \delta \overline{P}_L}{\overline{P}_L}$$

The value of $\overline{Y}$ is normalized to 1. The steady-state values of inflation (π), the market value of government debt-to-GDP (b_δ), government spending-to-GDP (g) and taxes (τ) are all estimated from the data. Parameters χ and the value of tr are adjusted accordingly to satisfy the steady-state equations.

Log-linearized equations Define $\hat{x}_t \equiv \log x_t - \log \overline{x}$ as the log deviation of variable x from its steady-state value. We will apply a log linear transformation to all model variables except to the labor tax rate, the market value of debt-to-gdp ratio, and transfers, in which case we use a linear approximation (i.e. $\hat{x}_t = x_t - \overline{x}$, for $x = \tau, tr, b_L$). Linearizing the system of equations (2.64)-(2.70) gives:

$$(1+\Omega e^{-\gamma})\hat{c}_t = E_t(\hat{c}_{t+1} + \hat{\xi}_{t+1}) - (\hat{i}_t - E_t\pi_{t+1}) + \Omega e^{-\gamma}(\hat{c}_{t-1} + E_t\hat{a}_{t+1} - \hat{a}_t) \quad (2.71)$$
$$+(1-\Omega e^{-\gamma})(\hat{\xi}_t - E_t\hat{\xi}_{t+1})$$
$$\hat{i}_t = E_t\hat{r}_{L,t+1} \quad (2.72)$$
$$\hat{\pi}_t = \kappa \hat{mc}_t + \beta E_t\hat{\pi}_{t+1} + \hat{\eta}_t \quad (2.73)$$
$$\hat{b}_{L,t} = \frac{\overline{b_L}}{\beta}(\hat{r^L}_t - \hat{\pi}_t - \hat{a}_t + \hat{y}_{t-1} - \hat{y}_t) + \frac{\hat{b}_{L,t-1}}{\beta} \quad (2.74)$$
$$-(1-\nu)(\hat{\tau}_t + \tau\hat{mc}_t) + g^{-1}\hat{g}_t + \hat{tr}_t + \hat{\Lambda}_t$$
$$\hat{y}_t = \hat{c}_t + \hat{g}_t \quad (2.75)$$
$$\hat{mc}_t = \frac{1}{1-\Omega e^{-\gamma}}(\hat{c}_t - \Omega e^{-\gamma}(\hat{c}_{t-1} - \hat{a}_t)) + \frac{\hat{\tau}_t}{1-\tau} + \frac{\alpha+\phi}{1-\alpha}\hat{y}_t \quad (2.76)$$
$$\hat{r^L}_t = \delta R^{-1}\hat{p}_{L,t} - \hat{p}_{L,t-1} \quad (2.77)$$

$\hat{\Lambda}_t$ is an exogenous shock to the budget constraint that captures features of the government expenditures and policies which are not explicitly modelled.

Fiscal variables follow simple feedback rules. We adopt the following specifications which are very similar to Bianchi and Melosi (2017) and Bianchi and Ilut (2017):

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + (1-\rho_\tau)\Big[\phi_{\tau,b}\hat{b}_{\delta,t-1} + \phi_{\tau,y}(\hat{y}_t - \hat{y}^n_t) + \phi_{\tau,g}(g^{-1}\hat{g}_t + \hat{tr}_t)\Big] + \epsilon_{\tau,t} \quad (2.78)$$

for the labour income tax rate, assuming that taxes have a first order autoregressive component and respond to the lagged value of debt and to the deviation of output from its natural level.

Transfers evolve as

$$\hat{tr}_t = \rho_{tr}\hat{tr}_{t-1} + (1-\rho_{tr})\phi_{tr,y}(\hat{y}_t - \hat{y}^n_t) + \epsilon_{tr,t}. \quad (2.79)$$

Lastly government spending (as a fraction of GDP) follows a simple AR(1) process:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t} \tag{2.80}$$

The shocks $\epsilon_{\tau,t}$, $\epsilon_{tr^*,t}$, $\epsilon_{tr,t}$ and $\epsilon_{g,t}$ are all assumed to be i.i.d.

The central bank sets the nominal interest rate accroding to the following Taylor rule:

$$\hat{i}_t = \rho_r \hat{i}_{t-1} + (1-\rho_r)\Big(\phi_\pi \hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}_t^n)\Big) + \epsilon_{m,t} \tag{2.81}$$

The natural level of output $\hat{y}_t^n$, is defined as the level of output that would prevail under flexible prices. It satisfies the following:

$$\Big(\frac{1}{1-\Omega e^{-\gamma}} + \frac{\alpha+\gamma_h}{1-\alpha}\Big)\hat{y}_t^n = \frac{1}{1-\Omega e^{-\gamma}}\hat{g}_t + \frac{\Omega e^{-\gamma}}{1-\Omega e^{-\gamma}}(\hat{y}_{t-1}^n - \hat{g}_{t-1} - \hat{a}_{t-1}) - \frac{1}{1-\tau}\hat{\tau}_t \tag{2.82}$$

The remaining shock processes (markups, preferences, technology growth, and shocks to the government budget) are all assumed to follow AR(1) processes:

$$\hat{\eta}_t = \rho_\eta \hat{\eta}_{t-1} + \epsilon_{\eta,t}$$
$$\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \epsilon_{\xi,t}$$
$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a,t}$$
$$\hat{\Lambda}_t = \rho_\lambda \hat{\Lambda}_{t-1} + \epsilon_{\lambda,t}$$

where $\epsilon_{\eta,t}$, $\epsilon_{\xi,t}$, $\epsilon_{a,t}$, $\epsilon_{\lambda,t}$ are i.i.d shocks.

Estimation

Data The dataset we use to estimate our structural model contains series on GDP, inflation, interest rates, government spending, total government expenditures, tax revenues, and the market value of government debt. All the data are for the U.S and are observed at a quarterly frequency for the period 1980Q1-2008Q4.

For the shock filtering exercise we extended the dataset to 2016Q4 to include the Great Recession period.

Unless stated otherwise, all data are taken from the National Income and Products Account (NIPA) collected by the Bureau of Economic Analysis. Real values are obtained using the GDP deflator. The construction of fiscal data series follows Leeper, 2010; Faust and Leeper, 2015; Bianchi and Ilut, 2017.

Output (GDP_t)**:** Nominal GDP (Table 1.1.5, Line 1) is deflated using the GDP deflator, and divided by the Population Index (constructed using the Civilian Noninstitutional Population series (CNP16OV) from the Bureau of Labor Statistics). The data used for estimation is the quarterly growth rate of the resulting series.

Inflation ($INFL_t$)**:** The quarterly inflation rate is defined as the growth rate of the GDP deflator (Table 1.1.4 Line 1).

Nominal interest rate (FFR_t)**:** The quarterly nominal interest rate used for estimation is the Fed Funds rate, retrieved from the FRED database.

Government spending-to-GDP ratio (GOV_t)**:** Government spending is defined as the sum of consumption expenditure (Table 3.2 Line 25), gross government investment (Table 3.2 Line 45), net purchases of non-produced assets (Table 3.2 Line 47), minus consumption of fixed capital (Table 3.2 Line 48).

Total government expenditures-to-GDP (EXP_t)**:** Total government expenditures are taken as the sum of government spending (defined above) and transfers. Transfers are defined as the sum of current transfer payments (Table 3.2 Line 26), subsidies (Table 3.2 Line 36), capital transfer payments (Table 3.2 Line 46), minus current transfer receipts (Table 3.2 Line 19) and capital transfer receipts (Table 3.2 Line 42).

Tax revenues-to-GDP (TAX_t)**:** Tax revenues are defined as current receipts (Table 3.2 Line 42) minus current transfer receipts (Table 3.2 Line 19).

Market-value of debt-to-GDP ($MVDEBT_t$)**:** The data for the market value of U.S government debt is taken from the Dallas Fed. We use the series on the market value of marketable treasury debt. To construct quarterly series we use the stock of debt in the first month of each quarter.

Measurement equations The equations linking model variables to the data series used for estimation are:

$$
\begin{aligned}
GDP_t &= 100\gamma + \hat{y}_t - \hat{y}_{t-1} + \hat{a}_t \\
INFL_t &= 100\log(\Pi) + \hat{\pi}_t \\
FFR_t &= 100\log(R) + \hat{i}_t \\
TAX_t &= (1-\nu)(100\tau + \hat{\tau}_t + \tau\hat{mc}_t) \\
MVDEBT_t &= 0.25(100b_\delta + \hat{b}_{\delta,t}) \\
GOV_t &= 100\log(g) + \hat{g}_t \\
EXP_t &= 100\Big[\frac{\beta-1}{\beta}b_\delta + (1-\nu)\tau\Big] + g^{-1}\hat{g}_t + \hat{tr}_t
\end{aligned}
$$

.

Estimation output Figures 2.8 and 2.9 plot the prior and posterior distributions of estimated model parameters. The dotted red lines and solid blue lines represent respectively prior and posterior densities.

[Figures 2.8, 2.9 and 2.10 approximately here]

Figure 2.10 contains the analogous plot with prior and posterior densities for the standard deviations of exogenous shock processes.

Accounting for debt porfolios

Defining $b_{S,t} = \frac{q_t B_{S,t}}{Y_t}$, and relaxing the assumption that short-term bonds are in zero net supply, equation (2.59) can be rewritten as:

$$b_{\delta,t} + b_{S,t} = \frac{R^L_{t-1} b_{\delta,t-1}}{\pi_t} \frac{Y_{t-1}}{Y_t} + \frac{R_{t-1} b_{S,t-1}}{\pi_t} \frac{Y_{t-1}}{Y_t} + 1 - g_t^{-1} + tr_t - \tau_t mc_t \quad (2.83)$$

The de-trended government budget constraint (2.69) is now:

$$b_{\delta,t} + b_{S,t} = \frac{R^L_{t-1} b_{\delta,t-1}}{\pi_t} \frac{y_{t-1}}{y_t} e^{-a_t} + \frac{R_{t-1} b_{S,t-1}}{\pi_t} \frac{y_{t-1}}{y_t} e^{-a_t} + 1 - g_t^{-1} + tr_t - \tau_t mc_t \quad (2.84)$$

The market-value of debt-to-gdp ratio is expressed as $mv_t \equiv b_{\delta,t} + b_{S,t}$. In the steady-state, we assume that mv is equal to its estimated value (see above). Then, for a given value of b_δ, we have $b_S = mv - b_L$. The log-linearized government budget constraint (2.6.5) becomes:

$$\hat{b}_{\delta,t} + \hat{b}_{S,t} = \frac{\overline{b_\delta}}{\beta}(\hat{r^L}_t - \hat{\pi}_t - \hat{a}_t + \hat{y}_{t-1} - \hat{y}_t) + \frac{\hat{b}_{\delta,t-1}}{\beta} + \frac{\overline{b_S}}{\beta}(\hat{i}_t - \hat{\pi}_t - \hat{a}_t + \hat{y}_{t-1} - \hat{y}_t)$$
$$+ \frac{\hat{b}_{S,t-1}}{\beta} - (1-\nu)(\hat{\tau}_t + \tau \hat{mc}_t) + g^{-1}\hat{g}_t + \hat{tr}_t + \hat{\Lambda}_t \quad (2.85)$$

To solve the model, we need to defined a rule which governs the dynamics of the portfolio. We assume the same rule as in the small-scale model presented in text:

$$\hat{b}_{\delta,t} - \hat{b}_{S,t} = v \hat{mv}_t \quad (2.86)$$

In this model version, we re-express the tax rule (2.78) to respond to the lagged value of the total market value of debt:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\Big[\phi_{\tau,b}\hat{mv}_{t-1} + \phi_{\tau,y}(\hat{y}_t - \hat{y}^n_t) + \phi_{\tau,g}(g^{-1}\hat{g}_t + \hat{tr}^*_t)\Big] + \epsilon_{\tau,t} \quad (2.87)$$

Chapter 3

Government Debt and Expectations-Driven Liquidity Traps

Abstract In a liquidity trap caused by a negative feedback loop on expectations, an expansionary fiscal policy causes output to contract when taxes are confined to labor income. This self-defeating effect is more pronounced when the policy consists in a deficit-financed spending stimulus as opposed to a tax-cut. The negative response of inflation to public spending increases the real payout of government debt and reduces consumption of forward-looking households that anticipate higher labor taxes and contractionary monetary policy during the recovery. In light of this result, the welfare-maximising strategy in an expectations-driven liquidity trap is to shrink the size of the state; the government should cut taxes while mitigating the accumulation of debt by concurrently curbing spending.

3.1 Introduction

The low-rate environment observed since the Great Recession has reduced the cost of debt-financed fiscal stimuli. This opportunity has been exploited extensively by the US government with the ratio of privately held government debt over GDP ballooning to unprecedented levels. Conventional wisdom is that spending policies have large multiplicative effects in a liquidity trap because they alleviate the shortfall in inflation and reduce real rates. This insight concerns liquidity traps that are caused by large contractionary shocks on private demand. However, the long duration of binding lower bound episodes experienced in advanced economies suggests a potential shift in expectations. In this context, central banks have increasingly acknowledged the risk of self-fulfilling deflationary spirals stemming from the asymmetric response of interest rates to inflation.[1] While monetary policy can easily curb inflation by tightening its stance, the presence of an effective lower bound on the policy rate renders the task of anchoring inflation expectations a more delicate one.

Hence, it is crucial to understand the implications of accumulating government debt in a liquidity trap caused by negative feedback loops on expectations. This paper seeks to answer this question based on a New Keynesian model in which occasional shifts in households' confidence lead to a stationary environment characterised by deflation and depressed output even in the absence of a deterioration in economic fundamentals. I label this environment the *unintended* regime as opposed to the *intended* regime where inflation is anchored at the central bank's target and the output gap is zero, and study the macroeconomic implications of a *regime shock* that causes switches from one regime to the other. I show that those dynamics have important consequences for deficit-financed fiscal stimuli when taxes are confined to labor income. More specifically, I consider a *perfectly-timed* spending increase

[1] In its recent strategic review, the Fed has emphasised the importance of staying ahead of the inflation curve to avoid sliding inflation expectations

and tax-cut. There are three main insights stemming from this analysis.

First, the presence of government debt causes a negative shift in confidence to weigh more heavily on economic activity. The lower consumption in the unintended regime shrinks the tax-base and creates a positive endogenous response of the tax rate that reduces inflation in the unintended regime. As a result, real interest rates increase and consumption drops in excess from its counterpart in an economy without debt.

Second, standard fiscal stimuli are self-defeating in an expectations-driven liquidity trap; an expansionary fiscal policy leads to a net drop in output. Moreover, the effects are quantitatively different whether the policy consists in a spending stimulus or a tax-cut. To understand this result, I study a simplified economy without debt and show that a spending stimulus (a tax-cut) is deflationary (inflationary) because the long expected duration of the slump implies that excess demand (supply) is increasing (decreasing) in inflation. The deflationary effect of the spending stimulus causes the government to accumulate more debt in the liquidity trap to compensate for the increase in the real payout. Hence, labor taxes increase in the intended regime and monetary policy tightens in response to the cost-push effect. Since households are forward-looking, the higher expected real rates depress consumption during the downturn. In contrast, debt soars less with a tax-cut because the rise in inflation reduces the real payout of debt and widens the tax-base. Nevertheless, the crowding-out effect of the expected monetary tightening on consumption dominates the crowding-in effect of the lower real rates in the unintended regime and results in a negative net effect on output, albeit lower than the one of the spending stimulus.

Third, an analysis on optimised rules reveals that the welfare-maximising policy consists in a reduction of the size of the state: the government should provide a significant tax-cut combined with a spending-cut. On the one hand, the tax-cut allows to mitigate the inflation shortfall in the unintended regime. On the other hand, the spending-cut reinforces this inflationary effect while slowing down the accumulation of debt. Exiting the liquidity trap with a lower debt burden is effective in stimulating consumption of forward-looking households because it reduces the cost-push effect of labor taxes and the expected real rates. Those benefits are also attested by the positive output effect of a faster debt consolidation in the tax rule.

The interactions between fiscal and monetary policy in a low rate environment have received considerable attention in the New Keynesian literature. Influential studies such as Woodford (2011); Christiano et al. (2011); Erceg and Lindé (2014) have shown that the fiscal multiplier of government spending policies can be large in a liquidity trap driven by fundamental shocks because they create inflation and reduce the real rates.[2] Moreover, in the presence of distortionary taxes on labor income, financing the stimulus with bonds issuance turns out mostly innocuous because the enlargement of the tax-base (partially) offsets the additional debt burden. Unsurprisingly, Burgert and Schmidt (2014) also find that deficit-financed stimuli constitute the optimal policy for a discretionary planner when dealing with large contractionary demand shocks. While those results build a solid case for an expansionary fiscal policy in a fundamental liquidity trap, it remains unclear how they would extend to a liquidity trap driven by shifts in consumers' confidence.

Several papers have studied the characteristics of expectations-driven liquidity traps in a New Keynesian framework. The closest to this paper is Mertens and Ravn (2014) who study the effect of fiscal policy in a stationary environment characterised by deflation and depressed output. However, differently from this paper, their model is non-linear and they assume that lump-sum taxes balance the budget constraint of the government every period. Hence, Ricardian equivalence holds in their model and financing fiscal stimuli with debt or taxes does not affect other macroeconomic variables. More generally, the literature about expectations-driven

[2] In particular, see Appendix B of Christiano et al. (2011) for the case of distortionary taxation most closely related to this paper.

liquidity traps can be divided in two strands. One strand studies the macroeconomic outcomes in this type of environment. For example, Borağan Aruoba et al. (2018) estimate a model subject to both fundamental and expectations-driven liquidity traps and find that the former corresponds more to the US experience while the later can describe the Japan economy since the late 1990s. Schmitt-Grohé and Uribe (2017) show that adding job market frictions can help this kind of models in explaining jobless recoveries. Jarociński and Maćkowiak (2018) combine those insights with the literature on self-fulfilling debt crises to demonstrate the efficacy of mutualised debt in avoiding bad economic outcomes. Nakata and Schmidt (2021) show that traditional ingredients of optimal fiscal and monetary policy are largely powerless in stabilising the economy when the liquidity trap is non-fundamental. The second strand of this literature studies how to suppress the existence of this type of liquidity traps altogether. Building on the insights of Benhabib et al. (2002), those papers have mainly focused on the design of ad-hoc rules in achieving this outcome as in Schmidt, 2016; Tamanyu, 2019; Sugo and Ueda, 2008.

The remainder of the paper is organised as follows. Section 3.2 describes the model and the existence of expectations-driven liquidity traps. Section 3.3 provides a simplified example of an economy without debt to clarify the plain vanilla effects of fiscal policy. Section 3.4 contains the main analysis on the role of debt in the two types of liquidity traps. Section 3.5 considers optimised rules to identify the best fiscal strategy and extends the results of the previous section to recurrent regime shocks. Finally, section 3.6 concludes.

3.2 The model

The model is a cashless New Keynesian economy with monopolistic competition and nominal rigidities. The private sector is composed of an infinitely lived representative household, a representative aggregate good producer and a continuum of intermediate good producers. The public sector is represented by two institutions, a central bank and a government respectively in charge of monetary and fiscal policy decisions.

3.2.1 Households and firms

The representative household derives utility from consuming the private good c_t and the public good G_t while it dislikes hours worked h_t. I assume a separable utility function leading to the following expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big[\frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \Big] \tag{3.1}$$

where $\mathbb{E}_t$ is the rational expectations operator conditional on information in period t, β is the time discount factor. Parameters γ_c and γ_g are respectively the intertemporal eslasticy for private and government consumption and, γ_h is the inverse of the Frisch elasticity. I also attach utility weights ν_g and ν_h to characterise preference for government consumption and hours, relatively to private consumption.

The variable ξ_t is an exogenous process characterising the time preference. Under this specification, time preference between states of two consecutive periods evolves according to $\xi_t/(\beta\xi_{t+1})$. This process characterises the fluctuations at the origin of *fundamental* liquidity traps. Hence, I introduce the variable $d_t \equiv \xi_{t+1}/\xi_t$ that I refer to as a demand shock.

The household sells hours to intermediate firms for a wage W_t net of labor taxes τ_t and may save via a one period nominal and non state-contingent government bond, B_t^h. The household budget constraint (in real terms) is:

$$c_t + \frac{b_t^h}{R_t} = (1-\tau_t) w_t h_t + \frac{b_{t-1}^h}{\pi_t} + \int_0^1 \frac{\Pi_{i,t}}{P_t} di + \frac{T_t}{P_t} \tag{3.2}$$

where R_t is the gross nominal interest rate and π_t is the gross inflation rate. The household chooses $\{c_t, h_t, b_t^h\}_{t=0}^{\infty}$ to maximise expected lifetime utility (3.1) subject to (3.2) and the no-Ponzi scheme conditions on the nominal asset

$$\lim_{j\to\infty} \mathbb{E}_t \left(\left(\prod_{k=0}^{t+j} \frac{1}{R_k} \right) B_{t+j}^h \right) \geqslant 0$$

Intermediate firms operate under monopolistic competition and seek to maximise profits subject to quadratic price adjustment costs à la Rotemberg (1982). From the profit maximisation problem of the final producer, the demand function of the generic firm producing i is given by

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} y_t \tag{3.3}$$

where θ is the marginal rate of substitution between varieties. The program of the firm i is

$$\max_{P_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t \left[\left(\frac{P_{i,t}}{P_t} \right)^{-\theta} y_t \left(\frac{P_{i,t}}{P_t} - (1-s)w_t \right) - \frac{\psi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 \right] \tag{3.4}$$

where λ_t is the multiplier of budget constraint from the household problem and ψ is the price adjustment cost factor.

Parameter s is a time-invariant employment subsidy, financed with lump-sum transfers, that offsets steady state distortions stemming from monopolistic competition and distortionary taxation.[3] This subsidy is used in Section 3.5 on optimised rules to guarantee the efficiency of the economy at the steady state and derive the social loss function.

[3] This implies that distortions from labor taxation do occur outside the steady state. See Leith and Wren-Lewis (2013) for a similar use of a steady state subsidy.

3.2.2 Log-linear approximation

The non-linear equilibrium equations are log-linearised around the steady-state.[4] In the benchmark model, the subsidy is set to zero ($s = 0$) and the steady state is distorted. The system of linear equations characterising the private sector decisions reads

$$\beta\tilde{b}_t - \tilde{b}_{t-1} + \Omega(\beta^{-1}\hat{\pi}_t - \hat{i}_t) - G\hat{G}_t + \tau wy\Big(\gamma_c\hat{c}_t + (1+\gamma_h)\hat{y}_t + \frac{\tilde{\tau}_t}{(1-\tau)\tau}\Big) = 0 \quad \text{(BC)} \quad (3.5)$$

$$\hat{c}_t = -\gamma_c^{-1}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{d}_t) + \mathbb{E}_t\hat{c}_{t+1} = 0 \quad \text{(EE)} \quad (3.6)$$

$$\hat{\pi}_t = \kappa_y\hat{y}_t + \kappa_c\hat{c}_t + \kappa_\tau\hat{\tau}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \quad \text{(PC)} \quad (3.7)$$

$$y\hat{y}_t = c\hat{c}_t + G\hat{G}_t \quad \text{(RC)} \quad (3.8)$$

where $\Omega \equiv \beta b$ is the market value of debt and $\kappa_y \equiv \frac{\gamma_h(\theta-1)}{\psi}$, $\kappa_c \equiv \frac{\gamma_c(\theta-1)}{\psi}$ and $\kappa_\tau \equiv \frac{(\theta-1)}{(1-\tau)\psi}$. All variables are in log-deviations from their steady state value, except for the government debt which is expressed as a share of nominal trend annual GDP, $\tilde{b}_t \equiv \frac{b_t-b}{4y}$ and the tax rate which is in deviation from its steady state value, $\tilde{\tau}_t \equiv \tau_t - \tau$.

The first equation is the Budget Constraint (BC) of the government. It states that the net nominal debt position must be financed with an appropriate surplus (or deficit). The government surplus depends positively on inflation and taxes (which include both the tax base and the tax rate) and negatively on interest rates and spending. The law of motion for debt is a constraint in my model because government liabilities are ultimately financed with distortionary taxes on labor income. The second equation is the Euler Equation (EE) that links the real rate of return on assets to the intertemporal trade-off of the households. The third equation is the Phillips Curve (PC) at the heart of the New Keynesian model. Notice that the labor tax rate enters in the Phillips curve. In my model, the marginal cost of the firms is increasing in the tax rate. Hence, labor taxes are similar to a cost-push shock that distorts the trade-off between inflation and output

[4] A description of the non-linear economy and the efficient steady-state can be found in Appendix 3.7.2 and Appendix 3.7.2, respectively.

gap stabilisation. As a result, the so-called *divine coincidence* breaks down in my model even in the absence of a binding lower bound. Finally, the last equation is the Resource Constraint (RC) that holds as an accounting identity.

3.2.3 Expectations-driven liquidity traps

In this paper, I consider the standard case of a monetary policy that follows a Taylor rule of the form $\hat{i}_t = max[-log(\beta^{-1}), \phi_\pi \hat{\pi}_t]$. The max operator implies that monetary policy is constraint by an effective lower bound on the nominal interest rate. As first demonstrated by Benhabib et al. (2001), this type of rule implies the existence of a stationary environment characterised by depressed inflation and consumption and a nominal rate that is constrained by the lower bound. This situation is often seen (Arifovic et al., 2018; Tamanyu, 2019) as resulting from self-fulfilling pessimism. For a reason that is left unspecified, rational agents begin to anticipate a negative economic outlook and reduce their demand for consumption goods. The central bank wants to alleviate the drop in inflation by cutting the nominal rate but the lower bound prevents the response to be sufficiently accomodative. This constrained policy response causes the initial pessimistic expectations to realise and the economy ends up in a deflationary spiral. This kind of *expectations-driven liquidity trap* also exists in my non-Ricardian economy with government debt. Before studying its characteristics, I provide a simplified example in which lump-sum taxes balance the budget constraint every period to gain intuition about the effect of fiscal policy on inflation.

3.3 A simplified economy without debt

In the presence of lump-sum taxes, I can drop the budget constraint from my system of equations. For the rest of the paper, I will focus on two regimes; the *intended* regime and the *unintended* regime. In the intended regime, inflation is on target and the lower bound does not bind. Conversely, in the unintended regime, inflation and consumption are subdued and the nominal rate is constrained by the lower bound. In the presence of the non-linear Taylor rule, I can write the system as a collection of four equations; two for the intended regime and two for the unintended regime

$$\hat{c}_t^i = -\gamma_c^{-1}(\phi_\pi \hat{\pi}_t^i - \mathbb{E}_t^i \hat{\pi}_{t+1}) + \mathbb{E}_t^i \hat{c}_{t+1} \tag{3.9}$$

$$\hat{\pi}_t^i = \kappa_G \hat{G}_t^i + \tilde{\kappa}_c \hat{c}_t^i + \kappa_\tau \tilde{\tau}_t^i + \beta \mathbb{E}_t^i \hat{\pi}_{t+1} \tag{3.10}$$

$$\hat{c}_t^u = -\gamma_c^{-1}(-i^* - \mathbb{E}_t^u \hat{\pi}_{t+1}) + \mathbb{E}_t^u \hat{c}_{t+1} \tag{3.11}$$

$$\hat{\pi}_t^u = \kappa_G \hat{G}_t^u + \tilde{\kappa}_c \hat{c}_t^u + \kappa_\tau \tilde{\tau}_t^u + \beta \mathbb{E}_t^u \hat{\pi}_{t+1} \tag{3.12}$$

where $\kappa_G \equiv \kappa_y G y^{-1}$ and $\overline{\kappa}_c \equiv (\kappa_c + \kappa_y c y^{-1})$.

The superscript i and u denote the intended and unintended regime, respectively. Those superscripts also apply to the rational expectations operator to indicate that the expectations are conditional to the current regime. In the following, I assume that regime switches are triggered by nonfundamental shocks that I simply call *regime shocks*.[5] The origin of a regime shock is not specified explicitly but results in a confidence shift from the private sector. Let's denote the probability of switching from the unintended regime to the intended regime by p_u and the probability of switching from the intended regime to the unintended regime by p_i. I assume the following transition matrix of probabilities

$$\begin{bmatrix} p_i & (1-p_i) \\ (1-p_u) & p_u \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.05 & 0.95 \end{bmatrix}$$

Hence, the intended regime is absorbing and the unintended regime is very

[5] This type of shock is also often referred as *sunspot* shocks in the literature.

persistent. The conditional expectations of a given variable x then read

$$\mathbb{E}_t^i \hat{x}_{t+1} = p_i \hat{x}_t^i + (1 - p_i)\hat{x}_t^u$$
$$\mathbb{E}_t^u \hat{x}_{t+1} = p_u \hat{x}_t^u + (1 - p_u)\hat{x}_t^i$$

This representation allows to recover the value for each endogenous variable and regime easily by solving a system of four equations in four unknowns.

3.3.1 Existence of an expectations-driven liquidity trap

Before analysing the conditions for existence of the unintended regime, I need to calibrate the model. A time period represents one quarter of a year. The preference parameters, ν_g and ν_h, are chosen to be consistent with households working one quarter of their time endowment and with a share of government spending equating one fifth of output. The annual interest rate in the intended steady state is set to 2.5% and pins down the time discount factor, β. The parameters of the Phillips curve are standard to the literature. The intertemporal elasticity of private and public consumption and, the inverse of the Frisch elasticity are equal to 1. The elasticity of substitution between intermediate goods, θ, corresponds to a price markup over the marginal cost of 10%. Given the value of θ, the parameter of the price adjustment cost, ψ, matches its Calvo (1983) equivalent (up to a first-order approximation around the deterministic steady-state) when the average duration for setting prices equals one year. The parameter values are summarised in Table 3.1.

[Table 3.1 approximately here]

Consider first the absence of fiscal stimulus, $\hat{G}_t = 0$ and $\hat{\tau}_t = 0$. The equilibrium values are $\hat{c}_t^i = \hat{\pi}_t^i = 0$ and, $\hat{c}_t^u = -0.24\%$ and $\hat{\pi}_t^u = -0.67\%$. According to the Taylor rule, the corresponding annualized interest rates are 2.5% and -1.52%, respectively. Therefore, the lower bound is binding in the unintended regime.

Figure 3.1 shows how the existence of those two regimes depends on the probability of staying in the unintended regime.

[Figure 3.1 approximately here]

The red line represents the Phillips curve (PC) and the black line represents the Euler equation (EE) which is kinked because of the lower bound constraint. As can be seen from the left panel, a high probability of staying in the unintended regime implies that the PC crosses the EE two times with the intersections corresponding to the equilibrium values in each regime. Notice that in the unintended regime, the EE is upward sloping which means that increasing inflation also increases consumption because the real rate drops when the lower bound is binding. This is not the case in the intended regime where the EE is downward sloping because of the monetary tightening. Finally, notice that a low probability of staying in the unintended regime ($p_u = 0.5$ in the right panel) is incompatible with the existence of an expectations-driven liquidity trap.

3.3.2 An example affirming Mertens and Ravn (2014)

Consider now the case of a fiscal stimulus that can be either government spending or a tax-cut. I assume a *perfectly-timed* stimulus lasting for as long as the economy remains in the unintended regime.[6] I first show the effect of this stimulus analytically. Solving the PC for consumption gives

$$\hat{c}_t^u = \overline{\kappa}_c^{-1}\Big(\hat{\pi}_t^u - \kappa_G \hat{G}_t^u - \kappa_\tau \hat{\tau}_t^u - \beta \mathbb{E}_t^u \hat{\pi}_{t+1}\Big)$$

Combining this expression with the EE gives

$$-\hat{i}^* = \overline{\kappa}_c^{-1}\Big(-\hat{\pi}_t^u + \kappa_G(\hat{G}_t^u - \mathbb{E}_t^u \hat{G}_{t+1}) + \kappa_\tau(\hat{\tau}_t^u - \mathbb{E}_t^u \hat{\tau}_{t+1}) + (\overline{\kappa}_c + 1 + \beta)\mathbb{E}_t^u \hat{\pi}_{t+1} - \beta \mathbb{E}_t^u \hat{\pi}_{t+2}\Big)$$

[6] This is equivalent to considering a fiscal policy shock in the unintendend regime that follows an AR(1) process with autoregressive parameter p_u. See Woodford (2011) for the use of a similar exogenous process.

With probability p_u, the economy remains in the unintended regime, in which case next period inflation is equal to current inflation. Moreover, with probability $(1-p_u)$, a regime shock hits the economy and inflation is equal to its value in the intended regime i.e. $\hat{\pi}_t^i = 0$. Notice that the same logic applies to government spending and labor taxes because of the perfectly-timed assumption. Then, I can write

$$-\hat{i}^* = \overline{\kappa}_c^{-1}\Big(-\hat{\pi}_t^u + \kappa_G(\hat{G}_t^u - \mathbb{E}_t^u\hat{G}_{t+1}) + \kappa_\tau(\hat{\tau}_t^u - \mathbb{E}_t^u\hat{\tau}_{t+1}) + (\overline{\kappa}_c + 1 + \beta)p_u\hat{\pi}_t^u - \beta p_u^2\hat{\pi}_t^u\Big)$$

Which, solving for inflation, gives

$$\hat{\pi}_t = \frac{1}{\Lambda}\Big[-\hat{i}^* - \frac{\kappa_G}{\tilde{\kappa}_c}(1-p_u)\hat{G}_t - \frac{\kappa_\tau}{\tilde{\kappa}_c}(1-p_u)\hat{\tau}_t\Big]$$

where $\Lambda \equiv p_u - \frac{(1-p_u)}{\tilde{\kappa}_c} + \frac{\beta p_u}{\tilde{\kappa}_c}(1-p_u)$. Parameter Λ has two roots: $r_1 = .68 < 1$ and $r_2 = 1.48 > 1$. Focusing on $0 < p_u < 1$, the following sign restrictions apply

$$\Lambda \begin{cases} < 0, & \text{if } p_u \leq 0.68 \\ > 0, & \text{otherwise} \end{cases}$$

The effect of a fiscal stimulus on inflation depends on the persistence of the unintended regime and of the perfectly-timed stimulus. If persistence is low, $p_u \leq 0.68$, inflation is increasing in the spending stimulus and decreasing in the tax-cut. This result echoes standard New Keynesian models with fundamental demand shocks. As argued by Wieland (2018), fiscal policy shocks may generate conventional inflation effects in an expectations-driven liquidity trap if they are sufficiently transitory.[7] In contrast, the calibration consistent with the existence of this type of liquidity trap in my model yields $\Lambda > 0$. Hence, the impact on inflation

[7]The author points out that the multiplier depends only on the expected duration of the spending policy and not on the duration of the liquidity trap. However, this does not hold in my non-Ricardian model because of the wealth effect of government debt.

is negative for an increase in government spending and positive for a decrease in the tax rate. This is the main argument behind the superiority of tax-cuts over government spending in the non-linear model of Mertens and Ravn (2014). In an expectations-driven liquidity trap, excess demand is increasing in inflation because of its expected long-lasting negative effect on real rates. Hence, firms respond to the excess demand stemming from a spending stimulus by lowering their prices and the real rate increases, crowding out consumption. In contrast, a tax-cut increases after-tax salary of the workers and creates excess supply in the labor market. For the same reason, excess supply is decreasing in inflation and firms now choose to increase their prices. The resulting drop in the real rate crowds in consumption. To illustrate this result numerically, Figure 3.2 consider respectively an increase in government spending by 1% of GDP and a decrease in the tax rate of 1 percentage point in the unintended regime.

[Figure 3.2 approximately here]

The real interest rate is lower (higher) with a tax-cut (spending stimulus) and hence, consumption increases (decreases) with respect to the no intervention scenario. All in all, output is higher with the tax-cut even though the latter decreases by only one percentage point.[8]

3.4 Fiscal policy when government debt matters

In this section, I explore how the financing source of fiscal stimuli impacts their effectiveness in an expectations-driven liquidity trap. For this purpose, I assume more realistically that only distorting income taxes are available to the government and hence, its solvency condition becomes binding. More specifically, tax

[8] Output gap turning positive in the liquidity trap with a fiscal stimulus may seem an extreme result. However, as Arifovic et al. (2018) have shown, unintended regimes with low inflation and binding lower bound also exist in an economy buffeted by natural rate shocks. In this case, expectations-driven liquidity traps entail a larger output loss and fiscal stimuli do not create a positive output gap.

variations have two components; an endogenous component that reacts to inherited government debt and an exogenous component that is independent of debt. The labor tax rule reads

$$\tilde{\tau}_t = \phi_\tau \tilde{b}_t + \epsilon_{\tau,t}$$

where $\epsilon_{\tau,t}$ governs the tax stimulus and is contingent to the state of the economy. As a baseline calibration, I choose $\phi_\tau = 0.1$ implying that a 10 percentage points increase in the debt-to-output ratio lifts the labor tax rate by 1 percentage point. This calibration ensures medium-run fiscal solvency, and makes fiscal policy passive in the sense of Leeper (1991). Since the pace of debt consolidation is potentially influential for the marcoeconomic dynamics, I investigate the sensitivity of the results when varying this parameter in Appendix 3.7.6. Moreover, for the numerical exercises below, I need to choose a steady state value of government debt. Baseline calibration assumes that the market value of debt amounts to 50% of annual GDP, a value that is roughly consistent with the average debt ratio observed in the US since the Great Recession (and before the COVID-19 crisis) according to the data of the Federal Reserve Bank of Saint-Louis (FRED). The next section reviews the impact of fiscal stimuli in a fundamental liquidity trap to facilitate the intuition about the role of debt. Section 3.4.2 addresses the case of the expactations-driven liquidity trap.

3.4.1 A review of the fundamental liquidity trap

I define a *fundamental liquidity trap* as one that is triggered by a negative demand shock in equation (3.6). For the ease of comparison, I assume that the demand shock follows the exogenous process

$$\hat{d}_t = \begin{cases} 0.016, & \text{if } t \leq T \\ 0, & \text{otherwise} \end{cases} \tag{3.13}$$

Moreover, private agents are uncertain about when period T will actually occur. This uncertainty is captured by a two-state Markov process.[9] Note that the existence of fundamental liquidity traps require their expected duration to be much lower than expectations-driven liquidity traps (see e.g. the discussion in Nakata and Schmidt, 2019; Kollmann, 2021b). As a benchmark calibration, I thus choose

$$\begin{bmatrix} p_h & (1-p_h) \\ (1-p_l) & p_l \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \end{bmatrix}$$

which implies an expected duration of the low state below two years.[10] The solution of the model is obtained with the same method of undetermined coefficients used for the expectations-driven liquidity trap (see Appendix 3.7.3). Consider the same two perfectly-timed fiscal stimuli studied in Section 3.3; a spending stimulus and a tax-cut. Figure 3.3 compares their respective effect in a fundamental liquidity trap when government debt matters for stabilisation policies.

[Figure 3.3 approximately here]

Consider first that the government increases spending to stimulate the economy (dashed-dotted blue line). The top left panel shows that this strategy is effective to boost output. This result is related to the large fiscal multiplier at the zero lower bound identified by Woodford, 2011; Christiano et al., 2011; Erceg and Lindé, 2014. In parallel to the demand effect of government spending on output, larger deficits trigger an endogenous rise in the labor tax rate. As a result, the cost-push effect of taxes reduces real rates at the lower bound and crowds in consumption in a virtuous loop.[11] In particular, the bottom left panel shows that debt is reduced in response to higher spending which indicates that the stimulus is self-financing

[9] see e.g. Woodford (2011) for a similar characterisation of a fundamental liquidity trap.

[10] Notice that I here refer to the high state and the low state of demand to make a distinction between the intended and unintended regime when liquidity traps are expectations-driven.

[11] As shown by de Beauffort (2020), this effect underlies the optimal increase in labor taxes observed in a liquidity trap when the central bank is independent.

because the positive effect on consumption widens the tax base.

Turning to the tax-cut (the dashed red line), this strategy turns out self-defeating. Cutting taxes aggravates the inflation shortfall because of the negative cost-push effect. Consumption drops due to higher real rates and the tax base shrinks. Hence, the economy is burdened by higher debt levels and lower output gap. Moreover, the cost-push effect of labor taxes when the policy is overturned at the exit of the liquidity trap results in a monetary tightening that is anticipated by forward-looking households and reduces their consumption further at the outset of the liquidity trap.

3.4.2 Fiscal policy in an expectations-driven liquidity trap

I now turn to the expectations-driven liquidity trap. This paper is the first to consider non-Ricardian fiscal policy in this environment. When taxes are confined to labor income, government debt becomes a state variable that matters for the decisions of the households and firms. Hence, I write the policy function for any variable x_t as follows

$$x_t^i = \mathcal{X}_c^i + \mathcal{X}_b^i \tilde{b}_{t-1}$$
$$x_t^u = \mathcal{X}_c^u + \mathcal{X}_b^u \tilde{b}_{t-1}$$

where $\mathcal{X}_c^{u,i}$ is a regime dependent constant and $\mathcal{X}_b^{u,i}$ is a regime dependent policy response to inherited government debt. To recover the value of those coefficients, I rely on a method of undetermined coefficients. Appendix 3.7.3 provides more details about this method. Before considering the role of fiscal policy in stabilising the economy subject to regime shocks, I provide intuition about the role of debt under a neutral policy stance. Figure 3.4 displays the policy functions for different levels of steady state government debt. The vertical dashed red line corresponds to the baseline calibration (50% of annual output).

[Figure 3.4 approximately here]

The figure reveals several important features of the policy functions when debt matters for private agents' decisions. First, the level of debt is positive in the unintended regime even when fiscal policy is neutral. The deflationary spiral increases the real value of government debt. The positive endogenous response of labor taxes weighs negatively on consumption and reduces the tax base. Hence, government debt stabilises at a higher level. Furthermore, this level depends positively on the steady state debt. A same drop in inflation has a proportionally larger effect on the real value of debt when the steady state debt is increased. As a result, inflation and consumption have a lower intercept and are less sensitive variations in the inherited level of debt. Nevertheless, the larger debt accumulation when the steady state debt increases aggravates the recession because of the anticipated contractionary monetary policy in the intended regime.

Impulse response functions How is the effectiveness of fiscal policy changed when government debt introduces history dependence in the private allocation? Figure 3.5 shows the impulse response functions to the same fiscal stimuli as before but in an expectations-driven liquidity trap with endogenous debt accumulation.

Clearly, the inflationary effects of the stimuli are the same as in the economy without debt. This implies that the real interest rate is higher (lower) in the unintended regime with an increase in spending (a tax-cut). However, the effects are sensibly different from Section 3.3. First of all, both stimuli are now *self-defeating*; by attempting to stimulate the economy, fiscal policy instead aggravates the recession. Second of all, the negative effect on consumption and output is more pronounced with a spending stimulus than with a tax-cut. Appendix 3.7.4 shows that those effects may be overturned by reducing the expected duration of the fiscal policy. As explained by Wieland (2018), this expected duration, rather than the type of liquidity trap, determines whether or not the policy expands output. In a fundamentally-driven liquidity trap, the expected duration of a perfectly-timed stimulus is low enough for the policy to be expansionary because of the short expected duration of the lower bound episode itself. In contrast, the high expected duration of the expectations-driven liquidity trap renders the same perfectly-timed stimuli contractionary.

[Figure 3.5 approximately here]

Moreover, in my non-Ricardian economy, financing fiscal stimuli with government debt is not innocuous. The accumulation of debt in the unintended regime directly translates into a higher tax rate that pushes-up inflation in the intended regime and increases the interest rate in the Taylor rule. The bottom right panel shows that this monetary tightening implies that real rates are consistently higher in normal times with depressive effects on consumption and output (first two panels).

Hence, there is a trade-off between providing a stimulus in the unintended regime and reducing the debt burden inherited by the government at the exit of the liquidity trap. Central to this trade-off are the rational expectations of the private sector. Private agents anticipate that real rates will be higher in the intended regime when the government accumulates more debt to finance its stimulus. With forward-looking agents, this expectation channel decreases consumption and inflation in the unintended regime.

In this regard, the spending stimulus has the largest impact on debt accumulation in the unintended regime because of two combined effects on deficits; a direct effect from the increase in spending and an indirect effect from drop in inflation which increases the real payout of debt. Hence, with a spending stimulus, real rates stand in excess from their steady state level in the unintended regime *and* for a prolonged period after the regime shock, strongly crowding out consumption (-2.4%) in the unintended regime. This private consumption effect prevails over the demand effect of the stimulus and produces a negative net effect on output.

In comparison, the tax-cut has a lower impact on debt accumulation. The inflationary effect of the tax-cut partly offsets the drop in revenues by decreasing the real payout of debt and by widening the tax base. This implies that real rates have opposite effects on consumption inside and outside the liquidity trap. Nevertheless, the middle top panel shows that the crowding-out effect of the higher expected real rates and subdued economic activity dominates in the unintended regime. As a result, the tax-cut is recessionary when financed with debt issuance.

Finally, consider how those results are affected by the pace of debt consolidation. Appendix 3.7.6 provides an analysis of the marginal fiscal multiplier for various values of the parameter ϕ_τ. Increasing the consolidation pace has a positive effect on the multiplier which becomes less negative. By increasing taxes more during the downturn, the government decreases the debt burden at positive interest rates. Hence, the cost-push effect of labor taxes is reduced and the lower expected real rates stimulate consumption of forward-looking agents in the unintended regime.

More on the expectation channel In an expectations-driven liquidity trap, government debt thus provides an additional channel that feeds into expectations of the private agents and weakens the effectiveness of expansionary fiscal policies. The strength of this expectation channel depends on the discounting of the households for future consumption and economic activity and also, on the link between the level of debt and its servicing costs at positive interest rates. To illustrate this argument, Appendix 3.7.5 considers a modified version of the model in which government bonds enter in the utility function of the representative household. Under this specification, bond yields are a function of two components; the underlying consumption claims (as previously) and the total supply of government bonds (the *convenience yield*). The modified Euler equation reads

$$\hat{c}_t = -\gamma_c^{-1}(\hat{i}_t - \beta R \mathbb{E}_t \hat{\pi}_{t+1}) + \beta R \mathbb{E}_t \hat{c}_{t+1} + \alpha \tilde{b}_t$$

where $\alpha \equiv \chi \frac{\gamma_b}{\gamma_c} c^{\gamma_c} b^{-\gamma_b - 1}$. Parameter χ governs the degree of preference over safe assets (POSA) and parameter γ_b is the wealth curvature.[12] The wealth curvature controls the elasticity of the bond yield with respect to the bond supply. With linear POSA ($\gamma_b = 0$), the supply of government bonds does not have a direct impact on interest rates. Nevertheless, POSA introduces additional discounting in the Euler equation (this is apparent given that $\beta R < 1$).[13] As a result, macroeconomic outcomes at the outset of the intended regime matter less for the saving and consumption decisions of the households prior to the regime shock.

[Figure 3.6 approximately here]

The left panel of Figure 3.6 shows that the inflationary effect of a deficit-financed tax-cut depends quantitatively and qualitatively on γ_b. With γ_b low enough, the tax-cut remains inflationary with respect to the no intervention policy. However, the positive inflation gap decreases when γ_b increases and even becomes negative for $\gamma_b > 0.06$. In this case, the debt accumulation caused by the tax-cut implies high real rates at the outset of the intended regime because the convenience yield is compressed. Since rational agents are forward-looking, expectations of subsequent depressed consumption and economic activity reduce inflation in the unintended regime. Note that the additional discounting with POSA mitigates this expectation channel but does not overturn its effect for γ_b high enough. Hence, a tax-cut may be deflationary in presence of a convenience yield on safe assets. In this case, the macroeconomic effects of a spending stimulus and a tax-cut are *qualitatively* the same.[14] The right panel shows that the recessionary effects of the spending stimulus are strongly increasing in the wealth curvature because of the negative response of convenience yields to debt accumulation. This result contrasts

[12] For the calibration of the POSA parameters, see Appendix 3.7.5.

[13] This property of the convenience yield is used by Kollmann (2021a) to generate a positive inflation rate in the expectations-driven liquidity traps

[14] This result contrasts markedly with the opposite inflationary effects found by Mertens and Ravn (2014).

with Rannenberg (2021) who finds that the wealth curvature does not matter much for the effectiveness of a perfectly-timed spending stimulus in a fundamental liquidity trap because the high fiscal multiplier implies that debt accumulation is negligible. Evidently, this does not hold in an expectations-driven liquidity trap since, as I have demonstrated, a spending stimulus increases the debt burden significantly. Finally, notice that the results with POSA obtained in this section would also extend to a model with government default in which the perceived probability of default is correlated to the level of debt as in Bonam and Lukkezen (2019).

3.5 Optimised rules

The previous section explores the implications of accumulating debt for the effect of fiscal stimuli when fiscal and monetary policy are conducted according to standard ad-hoc rules. No clear policy recommendation emerges from this exercise except that purely deficit-financed fiscal stimuli appear inadequate because of their recessionary effects in the intended regime and the intertemporal transmission through the expectation channel. In this section, I thus investigate policy recommendations stemming from optimised rules. For this purpose, I assume that the employment subsidy takes a value consistent with an efficient steady-state. This assumption allows me to derive a meaningful welfare criterion to rank policy outcomes. As demonstrated in Appendix 3.7.8, a second-order approximation of the household's utility around the efficient steady state gives an unconditional expected lifetime welfare loss equal to $\frac{1}{2}U_c\mathcal{L}$ with

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\pi_t^2\right] \tag{3.14}$$

Moreover, a permanent reduction in private consumption by the share $\mathcal{S}$ lowers the utility of the household by an amount equivalent to $\frac{U_c C\mathcal{S}}{1-\beta}$. Hence, I express the welfare loss in consumption equivalent as $\frac{U_c C\mathcal{S}}{1-\beta} = \frac{1}{2} U_c \mathcal{L}$, or

$$\mathcal{S} = \frac{1}{2}(1-\beta)C^{-1}\mathcal{L}$$

To assess the effectiveness of fiscal policy in stabilising target variables, I simulate the economy for 600 quarters and 500 samples.[15] The regime shock continues to follow a Markov Chain, however now I assume that none of the regimes is absorbing. Instead, I choose $p_u = p_i = 0.98$, a value that implies an average duration of 12.5 years for each regime. Figure 3.7 plots the welfare loss depending on the fiscal policy implemented in the unintended regime.

[Figure 3.7 approximately here]

Each curve corresponds to a tax policy while the horizontal axis represents the associated spending policy. From the position of the curves, cutting taxes in the unintended regime appears welfare improving provided that the stimulus is not too large. The optimal tax-cut turns out to be -3 p.p. since any tax-cut above this value moves the curve to the north. Notice that the no intervention policy entails the largest consumption loss. Moreover, for any tax-cut larger or equal to zero, decreasing government spending by an adequate amount can further improve welfare. This amount depends positively on the size of the tax-cut. To shed light on the mechanisms underlying those results, Table 3.2 shows the average outcome for key variables in the unintended (U) regime versus the intended (I) regime for selected fiscal policies.

First, notice that the optimised policy alleviates the inflation shortfall in the unintended regime (-2.33%) when compared to the case of no intervention (-2.55%). This result is intuitive for both the tax-cut and the spending-cut are inflationary in an expectations-driven liquidity trap. Hence, this policy supports output and consumption by lowering real rates. However, the tax-cut may not be too large

[15]The first 100 quarters of each sample are discarded as as a burn-in period.

because the increase in debt implies higher real rates when the lower bound stops binding. The decrease in government spending provides some fiscal space in this regard. Nevertheless, government debt is on average 25% (13.5%) above its steady state value in the unintended (intended) regime. Consequently, consumption and output are subdued in the intended regime because monetary policy tightens in reaction to the cost-push effect of labor taxes.

Hence, the optimised policy strikes a balance between the benefits occurring in the unintended regime and the costs occurring in the intended regime. This trade-off largely depends on the conditional expectations of the private agents in each regime. In this regard, the second line of the table shows that inflation in the intended regime is negative when fiscal policy is muted. The pronounced deflation in the unintended regime weighs negatively on inflation expectations even outside the liquidity trap since forward-looking households attach a positive probability (though low) for the deflationary spiral to occur again in the future.

[Table 3.2 approximately here]

This analysis clarifies the role of fiscal policy in an expectations-driven liquidity trap when government debt matters. In this type of liquidity trap, monetary policy is helpless in stabilising the economy. For example, Nakata and Schmidt (2021) show in a Ricardian economy with lump-sum taxes that increasing the inflation target of a time-consistent planner or increasing the weight it attaches to inflation stabilisation in its objective have opposite effects on inflation inside and outside an expectations-driven liquidity trap and the gains in stabilising the economy are therefore unclear. In my model, fiscal policy attempts to anchor inflation (expectations) in the two regimes. In achieving this objective, the government is however constrained by the debt accumulation and its implications for monetary policy once the lower bound stops binding. To illustrate this last point, consider the excessive tax-cut (-4%) in the third line of the table. While inflation is better stabilised in this case, it is at the cost of accumulating a large amount of debt. As a result, consumption and output drop too much in the intended regime and the welfare loss increases by 1.2 percents.

Notice that a spending-cut alone may seem appealing because it increases inflation in the expectations-driven liquidity trap while consolidating debt and lowering the endogenous response of taxes. However, as witnessed by the last line of the table, it turns out to have the worst effect on welfare. The reason is twofold. First, despite lowering the real rates and stimulating consumption, the spending-cut reduces output in the unintended regime because of the negative effect on public demand. Second, in the absence of other policies, the consolidation of debt (-7.5% in the unintended regime) implies that inflation is subdued in the intended regime.

Finally, I close the discussion with an important remark on the optimised strategy. The result of improving welfare in a liquidity trap by cutting taxes and government spending admittedly seems far fetched given the large expansionary fiscal policy that we have observed in reality. First of all, this result holds only in an expectations-driven liquidity trap with a long-lasting intervention. Hence, if the liquidity trap is fundamental, shrinking the size of the state would be counterproductive. To illustrate this point, Appendix 3.7.7 evaluates the optimised rules when the policymaker is uncertain about the type of liquidity trap occurring. To this end, I assume that once in a high state of demand, there is a positive probability to fall in a liquidity trap. However now, this liquidity trap may be either fundamental or expectations-driven. The optimised strategy in this context is to adopt the status-quo (except for a mild spending-cut). Since fiscal policy has opposite effects depending on the type of liquidity trap, the government refrain from intervening. Second of all, the duration of the intervention matters for the results. If the policy is not going to outlast the liquidity trap, then it will never be recessionary in a fundamental liquidity trap. As demonstrated in this paper, this affirmation is not true for an expectations-driven liquidity trap: a perfectly-timed stimulus puts a significant strain on public finances and depresses output during the downturn.

3.6 Conclusion

With the COVID-19 crisis unfolding, deficit-financed fiscal stimuli have grown in popularity in the US and other advanced economies. One of the main insights of the literature on fiscal and monetary policy in a fundamental liquidity trap is that the large multiplier of spending policies tremendously reduces its impact on debt accumulation. However, the soaring debt-to-GDP ratios may be a sign that we are overestimating those self-financing effects. At the same time, central banks are emphasising the risk of long-lasting liquidity traps caused by self-fulfilling deflationary spirals. The reality might be somewhere in-between. Nevertheless, this paper shows that financing government spending with debt issuance can be counterproductive in an expectations-driven liquidity trap because it increases real rates both at the lower bound and in its aftermath. In this context, reducing the size of the state supports economic activity by reaping the benefits of a tax-cut while keeping the accumulation of debt at a manageable level. This policy alleviates the need for a monetary tightening at the exit of the liquidity trap and stimulates consumption of forward-looking households during the downturn by reducing expected real rates.

3.7 Appendix

3.7.1 Tables and figures

Table 3.1: **Baseline calibration.**

Symbol	Description	Value
β	Subjective discount factor	0.99
θ	Elasticity of substitution among goods	11
ϵ	Price adjustment cost	117.805
γ_c	Intertemporal elasticity of c	1
γ_h	Intertemporal elasticity of h	1
ν_h	Utility weight on labor	20
ϕ_π	Inflation parameter in Taylor rule	1.5

Table 3.2: **Welfare loss and macroeconomic outcomes of fiscal policy**

Fiscal policy				$\hat{\pi}$		$\hat{c}$		$\hat{y}$		$\hat{b}$	
	$\hat{G}_u$	$\hat{\tau}_u$	$\mathcal{L}$	U	I	U	I	U	I	U	I
Optimised	-1	-3	0.15	-2.33	0.37	0.35	-0.84	0.08	-0.67	25.08	13.53
No stimulus	0	0	10.57	-2.55	-0.13	-0.11	0.08	-0.09	0.06	0.07	-0.02
Excessive tax-cut	0	-4	1.19	-2.26	0.53	0.22	-1.22	0.17	-0.97	35.43	19.08
Spending-cut only	-5	0	21.74	-2.53	-0.09	0.95	0.35	-0.24	0.28	-7.49	-3.90

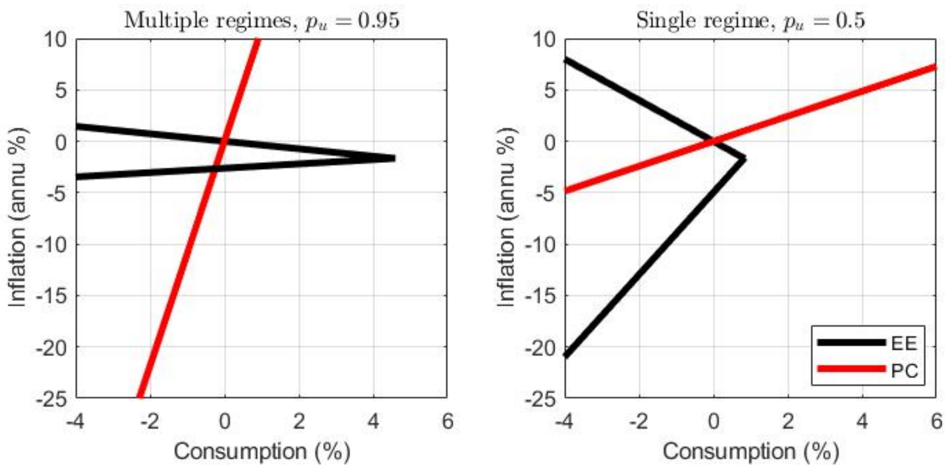
Note: Gov. spending, consumption and output are expressed in percentage deviation from the steady state. Inflation is in annual percentages. Debt is in percentage of annual output and the tax rate in absolute percentages. Welfare loss is in percentage of permanent consumption equivalent for the optimised policy and in percent excess deviation from the optimised policy for the other policies. U and I stand for the unintended and intended regime, respectively.

Table 3.3: **Macroeconomic outcomes with regime uncertainty**

				$\hat{\pi}$			$\hat{c}$			$\hat{b}$		
	$\hat{G}_u$	$\hat{\tau}_u$	$\mathcal{L}$	U	L	H	U	L	H	U	L	H
Optimised	-1	0	0.16	-2.39	-3.99	-0.25	0.25	-2.92	0.10	-2.00	-0.71	-0.70

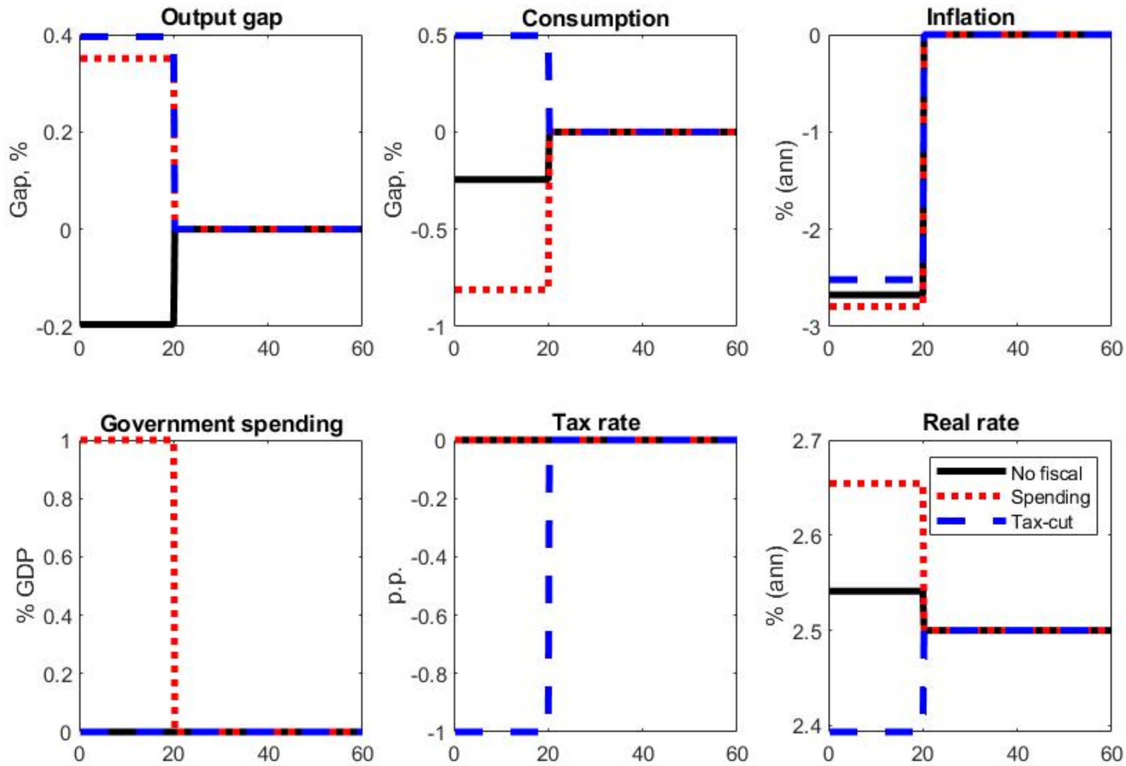
Note: Gov. spending, consumption is expressed in percentage deviation from the steady state. Inflation is in annual percentages. Debt is in percentage of annual output and the tax rate in absolute percentages. Welfare loss is in percentage of permanent consumption equivalent. U, L and H stand for the unintended regime, low regime and high regime, respectively.

Figure 3.1: **Regimes in the linear economy**



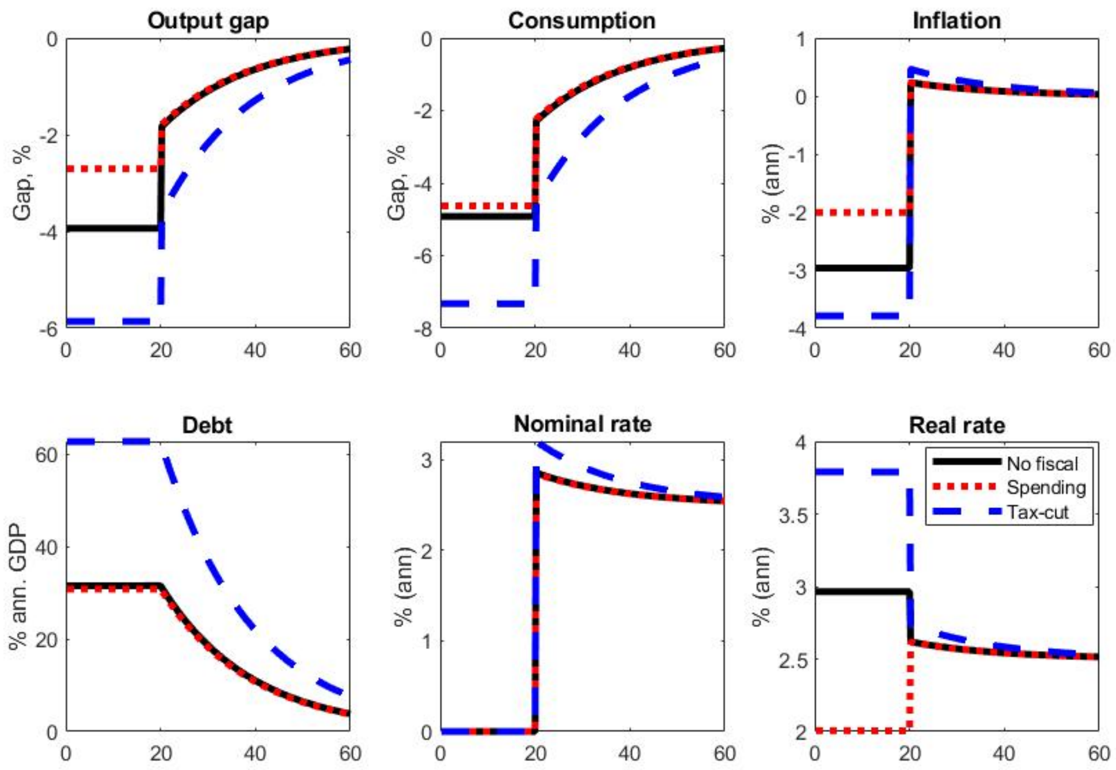
Notes: The black line corresponds to the Euler equation (EE) and the red line to the Phillips curve (PC). The left panel displays two intersections between the curves indicating the existence of two regimes. When p_u is reduced to 0.5, the unintended regime stops to exist.

Figure 3.2: **Effects of a fiscal stimulus in an expectations-driven liquidity trap with Ricardian households**



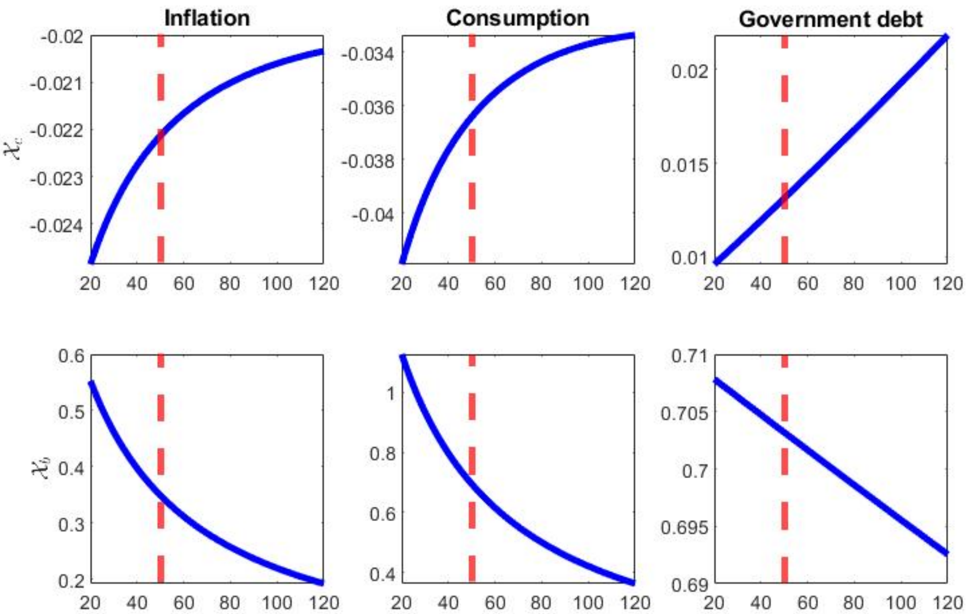
Notes: The solid black line corresponds to a neutral stance of fiscal policy, the dotted red line to a spending increase of one percent of GDP and the dashed blue line to a tax-cut of one percentage point. The economy starts in the unintended regime after debt has converged to its stationary level. The regime shock occurs after 20 years.

Figure 3.3: **Effects of a fiscal stimulus in an fundamental liquidity trap**



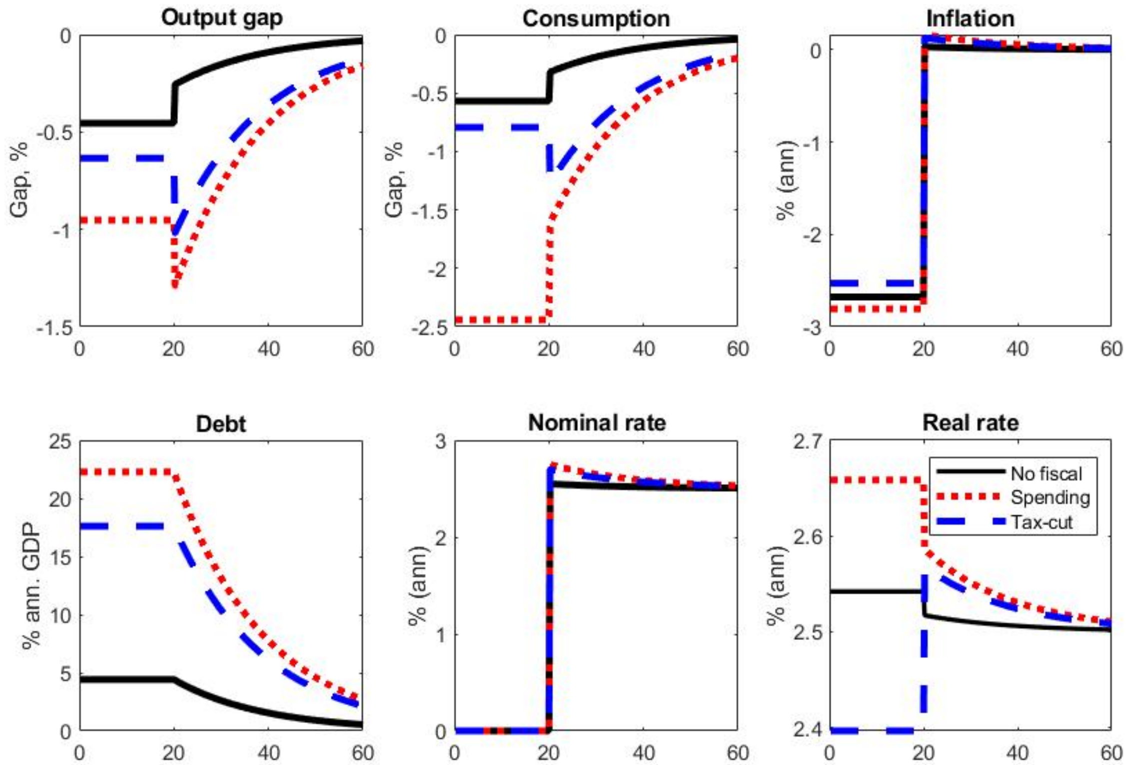
Notes: The solid black line corresponds to a neutral stance of fiscal policy, the dotted red line to a spending increase of one percent of GDP and the dashed blue line to a tax-cut of one percentage point. The economy starts in low state of the world after debt has converged to its stationary level. The demand shock occurs after 20 years.

Figure 3.4: **Policy functions with respect to the steady state debt level**



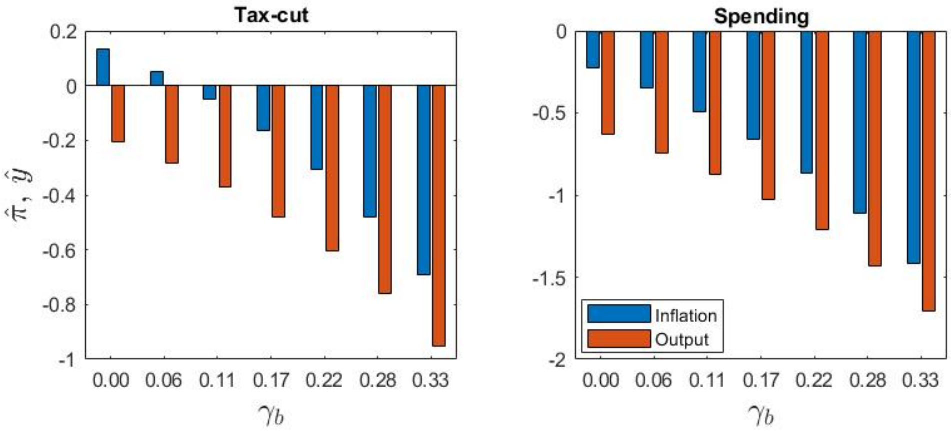
Notes: The solid blue line corresponds to the constant coefficient (top panels) and to the debt coefficient (bottom panels) in the policy function. The horizontal axis is the steady state level of debt and the dashed red line indicates the baseline calibration (50% of annual GDP).

Figure 3.5: **Effects of a fiscal stimulus in an expectations-driven liquidity trap**



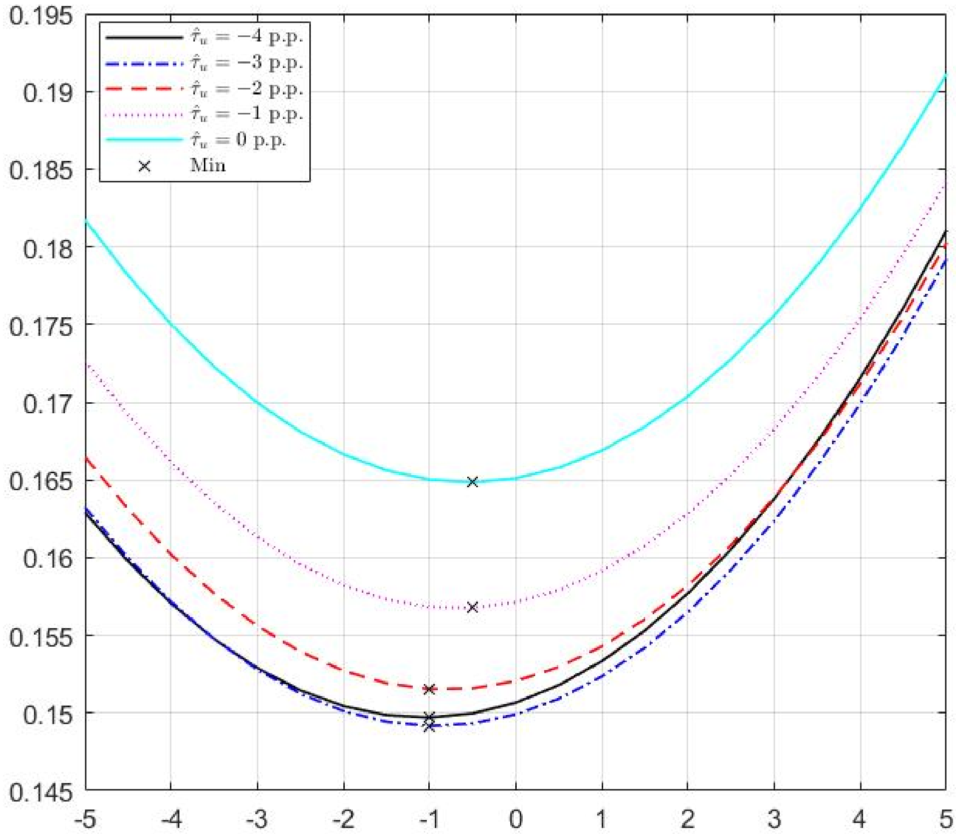
Notes: The solid black line corresponds to a neutral stance of fiscal policy, the dotted red line to a spending increase of one percent of GDP and the dashed blue line to a tax-cut of one percentage point. The economy starts in the unintended regime after debt has converged to its stationary level. The regime shock occurs after 20 years.

Figure 3.6: **Fiscal policy effects as a function of the wealth curvature**



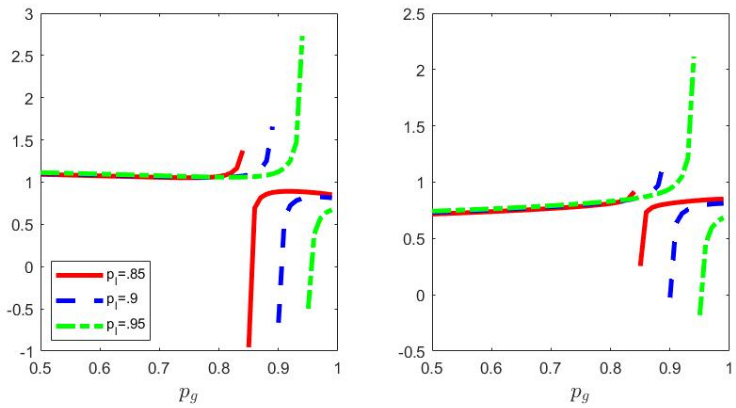
Notes: The bars represent the difference in the unintended regime between the no intervention policy and the relevant fiscal policy for the variable considered. Inflation is expressed in annualised percentage points and output is expressed in percentage points.

Figure 3.7: **Welfare loss for different fiscal policies**



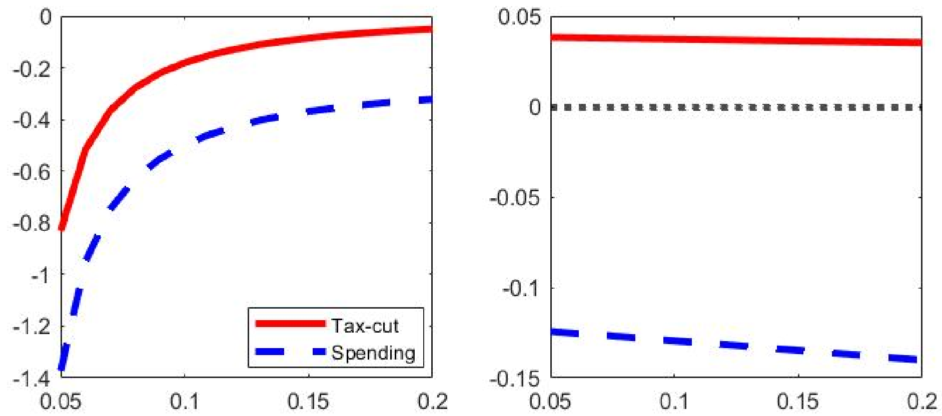
Notes: Each curve corresponds to a different tax-cut ranging from -0.4 p.p. to 0 p.p. The vertical axis captures the associated welfare loss in consumption equivalent depending on a specific spending policy (gap, percents) in the horizontal axis.

Figure 3.8: **Fiscal multipliers as a function of policy duration**



Notes: The left panel draws the output multiplier of government spending and the right panel the one of tax-cut. The solid red line, the dashed blue line and the dashed-dotted green line correspond to the output multiplier when the probability of staying in the unintended regime is respectively 0.85, 0.9 and 0.95. The formula for computing the multipliers are detailed in the text. The horizontal axis is the expected duration of the fiscal experiment in the unintended regime. For high enough duration of fiscal policy, the multiplier turns negative.

Figure 3.9: **Effects of the consolidation pace in an expectations-driven liquidity trap**



Notes: The solid red line and dashed blue line correspond to the output multiplier (left panel) and inflation multiplier (right panel) of the tax-cut and spending stimulus, respectively. The formula for computing the multipliers are detailed in the text. The horizontal axis is the endogenous response of labor taxes to lagged debt, ϕ_τ.

3.7.2 The non-linear economy

Equilibrium conditions The representative household maximises its expected lifetime utility (3.1) subject to its budget constraint (3.2). Attaching multiplier λ_t to the constraint, the FONCs of this problem are standard

$$\nu_h h_t^{\gamma_h} c_t^{\gamma_c} = (1-\tau_t) w_t$$
$$R_t^{-1} = \beta d_t \mathbb{E}_t \left[\frac{c_t^{\gamma_c}}{\pi_{t+1} c_{t+1}^{\gamma_c}} \right]$$

The problem (3.4) of the intermediate firm gives rise to the following FONC

$$w_t = \frac{\theta - 1}{\theta(1-s)} - \frac{\psi}{\theta(1-s)} \left(\pi_t(\pi_t - 1) - \beta d_t \mathbb{E}_t \left[\frac{c_t^{\gamma_c}}{c_{t+1}^{\gamma_c}} \frac{y_{t+1}}{y_t} \pi_{t+1}(\pi_{t+1} - 1) \right] \right)$$

Moreover, the budget constraint of the government in real terms reads

$$\frac{b_t}{R_t} - \frac{b_{t-1}}{\pi_t} - G_t + \tau_t w_t y_t - s(w_t y_t - wy) = 0$$

Together with the resource constraint, $y_t = c_t + G_t$, those equations can be log-linearised to obtain the system of equations in the text.

The first-best allocation The first-best allocation solves the problem

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} + \omega_t(y_t - c_t - G_t) \right]$$

that gives the following FONCs

$$c_t^{-\gamma_c} = \omega_t, \quad \nu_h y_t^{\gamma_h} = \omega_t, \quad \nu_g G_t^{-\gamma_g} = \omega_t$$

Hence, at the first best, it holds that $(1-\tau_t) w_t = 1$.

The steady state (non-efficient and efficient) The non-efficient steady state corresponds to the case where $s = 0$. It is characterised by the following set of equations

$$
\begin{aligned}
&y = c + G \\
&R = \beta^{-1} \\
&w = \frac{\theta - 1}{\theta} \\
&\tau = (wy)^{-1}[G + b(1-\beta)] \\
&\nu_h = y^{-\gamma_h} c^{-\gamma_c} w(1-\tau) \\
&\nu_g = G^{\gamma_g} c^{-\gamma_c}
\end{aligned}
$$

In the *efficient* steady state, the subsidy is set to attain the first-best, $(1-\tau)w = 1$ and is given by

$$s = 1 - \frac{\theta - 1}{\theta}(1 + G + (1-\beta)by^{-1})^{-1}$$

Moreover, the equations for the steady state tax rate and wage become

$$\tau = \frac{\theta s - 1}{\theta - 1} \qquad\qquad w = \frac{\theta - 1}{(1-s)\theta}$$

The other steady state equations remain unchanged.

3.7.3 Solving the expectations-driven liquidity trap

I denote the coefficients related to the constant terms by $\mathcal{C}^r_c$,$\mathcal{B}^r_c$,$\mathcal{Y}^r_c$,$\mathbf{\Pi}^r_c$,$\mathcal{I}^r_c$ and the coefficients related to lagged debt by $\mathcal{C}^r_b$,$\mathcal{B}^r_b$,$\mathcal{Y}^r_b$,$\mathbf{\Pi}^r_b$,$\mathcal{I}^r_b$. Superscript $r \in \{u, i\}$ indicates the regime. The strategy to recover those coefficients is the following. Gather all the constant terms on the left-hand side of each equation and all the terms premultiplying the state variable on the right-hand side of the equation. Then, find the value of the coefficients that implies each side of every equation in the system is equal to zero. This involves two steps. First, solve the system of right-hand side equations. Second, using the coefficients recovered for lagged debt, solve the system of left-hand side equations.

More specifically, the system of right-hand side equations contains a set of equations for the intended regime and for the unintended regime. The system for the intended regime reads

$$
\begin{aligned}
0 &= 1 - \Omega\beta^{-1}\mathbf{\Pi}_b^i + \Omega\mathcal{I}_b^i - w\tau y\Big(\gamma_c\mathcal{C}_b^i + (1+\gamma_h)\mathcal{Y}_b^i + \frac{\phi_\tau}{(1-\tau)\tau}\Big) - \beta\mathcal{B}_b^i \\
0 &= \mathbf{\Pi}_b^i - \kappa_y\mathcal{Y}_b^i - \kappa_c\mathcal{C}_b^i - \kappa_\tau\phi_\tau - \beta\Big(p_i\mathbf{\Pi}_b^i\mathcal{B}_b^i + (1-p_i)\mathbf{\Pi}_b^u\mathcal{B}_b^i\Big) \\
0 &= \gamma_c\mathcal{C}_b^i - \gamma_c\Big(p_i\mathcal{C}_b^i\mathcal{B}_b^i + (1-p_i)\mathcal{C}_b^u\mathcal{B}_b^i\Big) + \mathcal{I}_b^i - \Big(p_i\mathbf{\Pi}_b^i\mathcal{B}_b^i + (1-p_i)\mathbf{\Pi}_b^u\mathcal{B}_b^i\Big) \\
0 &= \mathcal{I}_b^i - \phi_\pi\mathbf{\Pi}_b^i \\
0 &= y\mathcal{Y}_b^i - c\mathcal{C}_b^i
\end{aligned}
$$

The system for the unintended regime reads

$$
\begin{aligned}
0 &= 1 - \Omega\beta^{-1}\mathbf{\Pi}_b^u + \Omega\mathcal{I}_b^u - w\tau y\Big(\gamma_c\mathcal{C}_b^u + (1+\gamma_h)\mathcal{Y}_b^u + \frac{\phi_\tau}{(1-\tau)\tau}\Big) - \beta\mathcal{B}_b^u \\
0 &= \mathbf{\Pi}_b^u - \kappa_y\mathcal{Y}_b^u - \kappa_c\mathcal{C}_b^u - \kappa_\tau\phi_\tau - \beta\Big(p_u\mathbf{\Pi}_b^u\mathcal{B}_b^u + (1-p_u)\mathbf{\Pi}_b^i\mathcal{B}_b^u\Big) \\
0 &= \gamma_c\mathcal{C}_b^u - \gamma_c\Big(p_u\mathcal{C}_b^u\mathcal{B}_b^u + (1-p_u)\mathcal{C}_b^i\mathcal{B}_b^u\Big) + \mathcal{I}_b^u - \Big(p_u\mathbf{\Pi}_b^u\mathcal{B}_b^u + (1-p_u)\mathbf{\Pi}_b^i\mathcal{B}_b^u\Big) \\
0 &= \mathcal{I}_b^u \\
0 &= y\mathcal{Y}_b^u - c\mathcal{C}_b^u
\end{aligned}
$$

Together, this gives a non-linear system of 10 equations in 10 unknowns that I solve using the routine *fsolve* in MATLAB. Once the coefficients of lagged debt have been recovered, the system of constant terms can be solved. As before, it consists in two sets of equations for the intended and unintended regime. For the intended regime, the system reads

$$\begin{aligned}
0 &= -\Omega\beta^{-1}\mathbf{\Pi}_c^i + \Omega\mathcal{I}_c^i - w\tau y(\gamma_c\mathcal{C}_c^i + (1+\gamma_h)\mathcal{Y}_c^i) - \beta\mathcal{B}_c^i \\
0 &= \mathbf{\Pi}_c^i - \kappa_y\mathcal{Y}_c^i - \kappa_c\mathcal{C}_c^i - \beta\Big(p_i(\mathbf{\Pi}_c^i + \mathbf{\Pi}_b^i\mathcal{B}_c^i) + (1-p_i)(\mathbf{\Pi}_c^u + \mathbf{\Pi}_b^u\mathcal{B}_c^i)\Big) \\
0 &= \gamma_c\mathcal{C}_c^i - \gamma_c\Big(p_i(\mathcal{C}_c^i + \mathcal{C}_b^i\mathcal{B}_c^i) + (1-p_i)(\mathcal{C}_c^u + \mathcal{C}_b^u\mathcal{B}_c^i)\Big) \\
&\quad + \mathcal{I}_c^i - \Big(p_i(\mathbf{\Pi}_c^i + \mathbf{\Pi}_b^i\mathcal{B}_c^i) + (1-p_i)(\mathbf{\Pi}_c^u + \mathbf{\Pi}_b^u\mathcal{B}_c^i)\Big) \\
0 &= \mathcal{I}_c^i - \phi_\pi\mathbf{\Pi}_c^i \\
0 &= y\mathcal{Y}_c^i - c\mathcal{C}_c^i
\end{aligned}$$

For the unintended regime, the system reads

$$\begin{aligned}
0 &= -\Omega\beta^{-1}\mathbf{\Pi}_c^u + G\hat{G}^u + \Omega\mathcal{I}_c^i - w\tau y\Big(\gamma_c\mathcal{C}_c^i + (1+\gamma_h)\mathcal{Y}_c^i + \frac{\hat{\tau}^u}{(1-\tau)\tau}\Big) - \beta\mathcal{B}_c^i \\
0 &= \mathbf{\Pi}_c^u - \kappa_y\mathcal{Y}_c^u - \kappa_c\mathcal{C}_c^u - \kappa_\tau\hat{\tau}^u - \beta\Big(p_u(\mathbf{\Pi}_c^u + \mathbf{\Pi}_b^u\mathcal{B}_c^u) + (1-p_u)(\mathbf{\Pi}_c^i + \mathbf{\Pi}_b^i\mathcal{B}_c^u)\Big) \\
0 &= \gamma_c\mathcal{C}_c^u - \gamma_c\Big(p_u(\mathcal{C}_c^u + \mathcal{C}_b^u\mathcal{B}_c^u) + (1-p_u)(\mathcal{C}_c^i + \mathcal{C}_b^i\mathcal{B}_c^u)\Big) \\
&\quad + \mathcal{I}_c^u - \Big(p_u(\mathbf{\Pi}_c^u + \mathbf{\Pi}_b^u\mathcal{B}_c^u) + (1-p_u)(\mathbf{\Pi}_c^i + \mathbf{\Pi}_b^i\mathcal{B}_c^u)\Big) \\
0 &= \mathcal{I}_c^u + log(\beta^{-1}) \\
0 &= y\mathcal{Y}_c^u - c\mathcal{C}_c^u - G\hat{G}^u
\end{aligned}$$

Again, this gives a non-linear system of 10 equations in 10 unknowns that I solve using the routine *fsolve* in MATLAB.

3.7.4 The expected duration of fiscal policy

Wieland (2018) shows that the multiplier of government spending depends on its duration independently of the type of liquidity trap, fundamentally-driven or expectations-driven. More specifically, the size of the multiplier is decreasing in the duration of the policy intervention.

Figure 3.8 shows that this result extends to a spending stimulus when households are non-Ricardian. The figure also displays the multiplier for the tax-cut policy considered previously. Here, I assume the existence of an additional regime, denoted regime n, characterised by a low degree of confidence and a binding lower bound but in which fiscal policy is muted. Conditional on the confidence being low, this regime is assumed to be absorbing. The policy regime considered before is now denoted regime g.

[Figure 3.8 approximately here]

This set-up allows me to consider fiscal policies of expected duration shorter than the expectations driven liquidity trap itself and to analyse the resulting multipliers. The marginal multiplier for a generic period of binding lower bound and operational policy is computed as $\Delta y_t/\Delta G_t$ and $\Delta y_t/(y_t \Delta \tau_t)$ for the spending stimulus and tax-cut, respectively. The figure clearly shows that, for an expected duration of fiscal policy equivalent to the fundamental liquidity trap considered earlier ($p_g = 0.5$), the multiplier of the spending stimulus is positive in the expectations-driven liquidity trap and superior to the tax-cut. Moreover, the multiplier is increasing in the duration of the policy up to a certain point because of the negative effect of cumulative inflation on the real interest rates. However, when the expected duration of the policy is equal or higher than the expected duration of the expectations-driven liquidity trap, the multiplier drops, turning negative initially, and then increasing until reaching a higher bound.

3.7.5 Preference over safe assets

I consider a modified version of the model where government bonds enter in the utility function of the representative household. More specifically, the lifetime utility now becomes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big[\frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} + \chi \frac{b_t^{1-\gamma_b}}{1+\gamma_b} \Big] \tag{3.15}$$

The log-linear system now reads

$$R^{-1}\tilde{b}_t - \tilde{b}_{t-1} + b(\hat{\pi}_t - R^{-1}\hat{i}_t) - G\hat{G}_t + \tau wy\Big(\gamma_c\hat{c}_t + (1+\gamma_h)\hat{y}_t + \frac{\tilde{\tau}_t}{(1-\tau)\tau}\Big)$$

$$\hat{c}_t = -\gamma_c^{-1}(\hat{i}_t - \beta R\mathbb{E}_t\hat{\pi}_{t+1}) + \beta R\mathbb{E}_t\hat{c}_{t+1} + \alpha\tilde{b}_t$$

$$\hat{\pi}_t = \kappa_y\hat{y}_t + \kappa_c\hat{c}_t + \kappa_\tau\hat{\tau}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}$$

$$y\hat{y}_t = c\hat{c}_t + G\hat{G}_t$$

where $\alpha \equiv \gamma_b(\gamma_c b)^{-1}\Theta$ and $\Theta \equiv \chi c^{\gamma_c} b^{-\gamma_b}$. Parameter χ governs the degree of preference over safe assets (POSA).

Under this specification, the individual discount rate exceeds the nominal return on the bond. Formally, if R^* is the risk-free rate in the benchmark model (without POSA), the spread is given by $\frac{R^*}{R} = \frac{\beta+\Theta}{\beta}$ and thus, when $\chi > 0$, the spread is positive. This spread is interpreted as a *convenience yield* on the risk-free bond i.e. a premium that the representative household is willing to pay for the additional safety and liquidity services attributed to this type of asset.[16] To calibrate the additional parameters, I draw on the POSA literature. As in Campbell et al. (2017), I assume that the discounting wedge is $\beta R/\pi = 0.99$. Given my steady state gross rate of $R = 1.0063$ (which corresponds to an annualised interest rate of 2.5%), this gives me an individual discount factor of $\beta = 0.9839$. For the same discounting wedge, Rannenberg (2019) sets the wealth curvature to $\gamma_b = 1/3$. I take this value as an upper bound and choose $\gamma_b \in [0, 1/3]$. The POSA parameter can be recovered as a residual from the steady state Euler equation, $\chi = (R^{-1} - \beta)b^{\gamma_b}$.

3.7.6 Effect of the consolidation pace

Clearly, since taxes on labor income are deflationary in an expectations-driven liquidity trap, the pace at which the government consolidates its debt following an

[16] For a similar interpretation see e.g. Krishnamurthy and Vissing-Jorgensen (2012).

expansionary fiscal shock is likely to play an important role in the effectiveness of the policy. As explained, an increase in government spending impacts the level of debt not only through its direct effect on the budget constraint but also through a reduction in the tax base. The magnitude of the tax response, in turn, depends on the parameter ϕ_τ. In particular, increasing value of this parameter has two opposite effects on output in the liquidity trap. On the one hand, inflation is decreasing in the tax rate and thus higher taxes tend to reduce consumption and output due to their positive effect on the real rate. On the other hand, the stronger consolidation implies a lower debt burden at the exit of the liquidity trap that results in lower real rates and a positive effect on consumption and output through the expectation channel. To investigate this trade-off, Figure 3.9 plots the output and inflation multiplier for a 1% of GDP increase in government spending (the dashed line) and a 1 p.p. tax-cut (the solid line).

First notice that the claim that a higher value of ϕ_τ depresses inflation in the unintended regime is verified. The right panel shows that the higher this parameter, the lower the inflation multiplier of the spending stimulus and the tax-cut.

[Figure 3.9 approximately here]

However, despite increasing the real rate, this deflation does not aggravate the drop in consumption. This can be seen from the left panel. The output multiplier is less negative when ϕ_τ increases reflecting the positive effect of bringing down the debt level faster. A government that consolidates at a slow pace finds herself with a high debt burden at the exit of the liquidity trap. Hence, inflation remains above target for a prolonged period of time after the regime shock and this provides an incentive for forward-looking households to decrease their consumption in anticipation of the contractionary monetary policy and subdued economic recovery. This expectation channel provides the incentive for an additional consolidation effort in the unintended regime.

3.7.7 Optimised rules with regime uncertainty

Section 3.5 on optimised rules assumed that the economy is plagued with rare episodes of expectations-driven liquidity traps. In reality, it might be difficult for the government to identify the true nature of a liquidity trap: fundamental or expectations-driven. Since the perfectly-timed stimuli studied in this paper have contradictory effects depending on the nature of the liquidity trap, it is interesting to evaluate the impact of regime uncertainty on the optimised policy. Intuitively, if fundamental liquidity traps may arise with some probability, then cutting taxes or spending may be less desirable because it sometimes would have recessionary consequences. Hence, I simulate again the economy but this time assuming three regimes; a unintended regime (U) that is expectations-driven, a low regime (L) that is caused by a large demand shock and a high regime (H) with high demand and non-binding lower bound. The transition probability matrix is the following:

$$\begin{bmatrix} p_{UU} & p_{UL} & p_{UH} \\ p_{LU} & p_{LL} & p_{LH} \\ p_{HU} & p_{HL} & p_{HH} \end{bmatrix} = \begin{bmatrix} 0.99 & 0 & 0.01 \\ 0 & 0.5 & 0.5 \\ 0.003 & 0.1 & 0.897 \end{bmatrix}$$

On the one hand, this distribution implies that it is impossible to switch from one binding regime to the other. On the other hand, once in the non-binding regime, it is possible to switch to a binding regime of both kind with a higher probability of fundamental regime.

[Table 3.3 approximately here]

Table 3.3 shows that the optimised outcome still consists in a 1% spending-cut although now no tax-cut is warranted. This policy almost corresponds to a neutral stance of fiscal policy. This result is intuitive. Since expansionary fiscal policy has opposite effects depending on the type of liquidity trap, the best strategy becomes to refrain from intervening. Notice that inflation is lower in the low regime because the spending-cut is deflationary in this case. The opposite is true for the unintended regime.

3.7.8 Derivation of the loss function

The loss function is obtained with a second order approximation of the utility of the households around the efficient steady-state. The utility function comprises three components:

1. The first component of utility is $F = \frac{(y_t(1-\frac{\iota}{2}(\pi_t-1)^2)-G_t)^{1-\gamma_c}}{1-\gamma_c}\xi_t$

$$\begin{array}{ll}
F_y = c^{-\gamma_c}(1-\frac{\iota}{2}(\pi-1)^2)\xi & F_g = -c^{-\gamma_c}\xi \\
F_\pi = -\iota(\pi-1)yc^{-\gamma_c}\xi & F_{yy} = -\gamma_c c^{-\gamma_c-1}[1-\frac{\iota}{2}(\pi-1)^2]^2\xi \\
F_{gg} = -\gamma_c c^{-\gamma_c-1}\xi & F_{\pi\pi} = -(\gamma_c y[\iota(\pi-1)]^2 c^{-\gamma_c-1} + \iota c^{-\gamma_c})y\xi \\
F_{yg} = \gamma_c c^{-\gamma_c-1}(1-\frac{\iota}{2}(\pi-1)^2)\xi & F_{gy} = \gamma_c c^{-\gamma_c-1}(1-\frac{\iota}{2}(\pi-1)^2)\xi \\
F_{g\pi} = -\gamma_c c^{-\gamma_c-1}y(\pi-1)\xi & F_{\pi y} = [y(1-\frac{\iota}{2}(\pi-1)^2)\gamma_c c^{-1}-1]\iota(\pi-1)c^{-\gamma_c}\xi \\
F_{\pi g} = -\iota(\pi-1)y\gamma_c c^{-\gamma_c-1}\xi & F_{y\pi} = [\gamma_c c^{-1}y(1-\frac{\iota}{2}(\pi-1)^2)-1]\iota(\pi-1)c^{-\gamma_c}\xi \\
F_\xi = \frac{c^{1-\gamma_c}}{1-\gamma_c} & F_{y\xi} = c^{-\gamma_c}(1-\frac{\iota}{2}(\pi-1)^2) \\
F_{\xi\xi} = 0 & F_{g\xi} = -c^{-\gamma_c} \\
F_{\xi y} = (1-\frac{\iota}{2}(\pi-1)^2)c^{-\gamma_c} & F_{\xi g} = -c^{-\gamma_c} \\
F_{\xi\pi} = -c^{-\gamma_c}y\iota(\pi-1) & F_{\pi\xi} = -\iota(\pi-1)yc^{-\gamma_c}
\end{array}$$

given that $\xi = 1$, we have

$$F \simeq (y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}y^2\hat{y}_t^2 + \gamma_c c^{-1}G\hat{G}_t y\hat{y}_t + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\gamma_c c^{-1}G^2\hat{G}_t^2$$
$$-\frac{1}{2}\iota y\hat{\pi}_t^2)c^{-\gamma_c} + t.i.p = y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}(c\hat{c}_t)^2 + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\iota y\hat{\pi}_t^2)c^{-\gamma_c} + t.i.p$$

2. The second component of utility is $G = \frac{G_t^{1-\gamma_g}}{1-\gamma_g}\nu_g\xi_t$

$$\begin{array}{ll}
G_g = G^{-\gamma_g}\nu_g\xi & G_{gg} = -\gamma_g G^{-\gamma_g-1}\nu_g\xi \\
G_\xi = \frac{G^{1-\gamma_g}}{1-\gamma_g}\nu_g & G_{\xi\xi} = 0 \\
G_{g\xi} = G^{-\gamma_g}\nu_g & G_{\xi g} = G^{-\gamma_g}\nu_g
\end{array}$$

given that $\xi = 1$ and $\nu_g = G^{\gamma_g} c^{-\gamma_c}$, we have

$$G \simeq (G\hat{G}_t - \frac{1}{2}\gamma_g G\hat{G}_t^2 + G\hat{G}_t\hat{\xi}_t)c^{-\gamma_c} + t.i.p$$

3. The third component of utility is $H = \frac{y_t^{1+\gamma_h}}{1+\gamma_h}\nu_h\xi_t$

$$\begin{aligned}
&H_y = y^{\gamma_h}\nu_h\xi \quad && H_{yy} = \gamma_h y^{\gamma_h - 1}\nu_h\xi \\
&H_\xi = \tfrac{y^{1+\gamma_h}}{1+\gamma_h}\nu_h \quad && H_{\xi\xi} = 0 \\
&H_{y\xi} = y^{\gamma_h}\nu_h \quad && G_{\xi y} = y^{\gamma_h}\nu_h
\end{aligned}$$

given that $\xi = 1$ and $\nu_h = y^{-\gamma_h} c^{-\gamma_c}$, we have

$$H \simeq (y\hat{y}_t + \frac{1}{2}\gamma_h y^2\hat{y}_t^2 + y\hat{y}_t\hat{\xi}_t)c^{-\gamma_c} + t.i.p$$

Collecting all the components and scaling with c^{γ_c}, the second-order approximation is given by

$$\begin{aligned}
U \simeq F + G - H = {}& y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}(c\hat{c}_t)^2 + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\iota y\hat{\pi}_t^2 \\
&+ G\hat{G}_t - \frac{1}{2}\gamma_g G\hat{G}_t^2 + G\hat{G}_t\hat{\xi}_t - y\hat{y}_t - \frac{1}{2}\gamma_h y^2\hat{y}_t^2 - y\hat{y}_t\hat{\xi}_t + t.i.p
\end{aligned}$$

Hence, the social objective corresponds to

$$U \simeq -\frac{1}{2}\Big[\big(\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\hat{\pi}_t^2\Big] + t.i.p$$

Conclusion

This thesis develops theoretical macroeconomic models that contribute to the policy debate by providing new insights on fiscal-monetary interactions. It is composed of three chapters that emphasise the role of government debt (maturity) in shaping private sector's expectations and in stabilising the macroeconomy.

Chapters 1 studies the importance of coordination between fiscal and monetary policy for macroeconomic outcomes during a liquidity trap episode. It emphasises the risks of accumulating debt at low interest rates when private agents anticipate that the central bank will be adamant in defending its inflation target during the recovery. Chapter 2 demonstrates the usefulness of debt maturity portfolios in stabilising the economy when monetary policy is fiscally dominated. The general prescription is that optimal monetary policy should respond to the portfolio of debt and optimal debt management should issue mostly long-term debt, the exact amount depending on the shocks hitting the economy. Chapter 3 offers a new insight on deficit-financed fiscal policy in a low-rate environment by considering liquidity traps that are caused by long-lasting shifts in expectations. Fiscal stimuli should include additional measures to contain debt accumulation in this case.

The conclusions proposed in these three chapters open avenues for future research. Advancing in our understanding of those economic issues will help developing better policies that bring larger benefits at lower risks. This is what I strive for in my academic and professional life. Often though, policy does not wait for theory. But this does not make the two less complementary. Theory may adopt a longer horizon and look at the broader picture. Policy sometimes responds to the urgency of the moment. Nevertheless, modern macroeconomic theory provides a versatile set of tools that guides policymakers and allows them to avoid repetitive mistakes. Personally, this is what I found the most precious with this research journey: structuring my ideas by putting words into equations.

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