Conclusions.

This dissertation addresses several aspects of modern asset liability management from an individual and institutional point of view. Through the first part of the thesis (chapters 2 and 3), devoted to the annuity puzzle issue, I've studied the stochastic control theory and the numerical methods solving the HJB equations. Despite their apparent simplicity, those methods have to be applied cautiously and a fine tuning of discretization parameters is required to obtain a compromise between time of computations and precision of the solution. Future research can make the annuity puzzle model more realistic by allowing stochastic interest rates and inflation.

The second part of the dissertation focuses on the asset liability management of pension funds exposed to mortality risk. In chapters 4, 5 and 6, I've tried to show that the martingale approach has still sense in an actuarial context. Due to the presence of mortality, the optimal wealth process is not perfectly hedgeable. However the best replicating strategy may be found by the dynamic programming principle. Chapter 4 presents the method applied in the two next chapters to defined contributions and defined benefits pension plans. A future theme of research could be to emphasize the relation between risk minimizing strategies and the investement strategies obtained by our approach. Another possible extension could be to consider more than one cohort of affiliates.

In chapter 7, I've also developed a mortality model based upon Lévy processes and inspired from the financial literature over credit risk. The redaction of this chapter was the starting point of a journey in the theory of Lévy processes and its applications. My contribution is to show that Lévy processes, mainly used to model the fat tail behaviour of assets, are adapted to model other random events such as mortality. An interesting application could be to insert those Lévy mortality processes in the issues of chapters 4 to 6, instead of a deterministic force of mortality.

The third part of the thesis is still devoted to the institutional asset liability management but in the setting of complete markets. In chapter 8, I've determined the investment strategy maximizing the expected power utility of terminal wealth under a value at risk constraint.The completeness of the market entails that the optimal wealth process is fully replicable by an adapted investment strategy. The novelty of the work is to consider this issue with stochastic interest rates. An interesting extension could be to bound the conditional value at risk rather than the value at risk.

Finally, in chapter 9, I've proposed a model to valuate a fair profit sharing rate for participating policies. This chapter differs from the previous one on two points. Firstly, given that the market is complete, the investor's utility is not needed to achieve the pricing. Secondly,

only static investment strategies are considered. The FFT algorithm detailled in this part is a powerful tool to find formulas for options price in term of the characteristic function of assets logprices. A future research could be to use the same methodology to price option on a mortality index, modelled by Lévy processes similar to those of chapter 7.

Pour celui qui ne sait pas où il va, il n'est nul vent favorable.

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