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Action accessible and weakly action representable varieties of algebras

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Abstract
The main goal of this article is to investigate the relationship between action accessibility and weak action representability in the context of varieties of non-associative algebras over a field. Specifically, using an argument of J. R. A. Gray in the setting of groups, we prove that the varieties of k-nilpotent Lie algebras ($k \geqslant 3$) and the varieties of n-solvable Lie algebras ($n \geqslant 2$) do not form weakly action representable categories. These are the first known examples of action accessible varieties of non-associative algebras that fail to be weakly action representable, establishing that a subvariety of a (weakly) action representable variety of non-associative algebras needs not be weakly action representable. Eventually, we refine J. R. A. Gray's result by proving that the varieties of k-nilpotent groups ($k \geqslant 3$) and that of 2-solvable groups are not weakly action representable.

Keywords Action representable category · Action accessible category · Amalgamation property · Split extension · Non-associative algebra · Lie algebra

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Introduction

Internal actions were defined in [1] by F. Borceux, G. Janelidze and G. M. Kelly with the aim of extending the correspondence between actions and split extensions from the context of groups to arbitrary semi-abelian categories [11]. In some of those categories, internal actions are exceptionally well behaved, in the sense that the actions on each object X are *representable*: this means that there exists an object $[X]$ such that the functor $\mathrm{Act}(-, X) \cong \mathrm{SplExt}(-, X)$ which sends an object B to the set of actions of B on X (isomorphisms classes of split extensions of B by X) is naturally isomorphic to the functor $\mathrm{Hom}(-, [X])$. The notion of representability of actions in a semi-abelian category is further studied in [2], where it is explained for instance that the category of commutative associative algebras over a field is not action representable. Later it was shown in [8] that the only action representable varieties of non-associative algebras over an infinite field of characteristic different from 2 are the category **Lie** of Lie algebras and the category **AbAlg** of abelian algebras. The relative strength of the notion naturally led to the definition of closely related weaker notions.

In [3] D. Bourn and G. Janelidze introduced the notion of *action accessible* category in order to include relevant examples that do not fit into the frame of action representable categories (such as rings, associative algebras and Leibniz algebras [16]). A. Montoli proved in [19] that any *category of interest* in the sense of Orzech [21] is action accessible, while in [5] the authors showed that a weaker notion of representing object (which they name as the *universal strict general actor*) is available for any Orzech category of interest.

Recently, G. Janelidze introduced in [10] the concept of *weakly action representable* category. Instead of asking that for any object X in a semi-abelian category $\mathscr{C}$ there is an object $[X]$ and a natural isomorphism $\mathrm{Act}(-, X) \cong \mathrm{Hom}_{\mathscr{C}}(-, [X])$, we require the existence of a *weakly representing object* $T = T(X)$ and a natural monomorphism of functors

$$\tau\colon\ \mathrm{Act}(-, X) \rightarrowtail \mathrm{Hom}_{\mathscr{C}}(-, T).$$

When each X admits such an object T, $\mathscr{C}$ is said to be *weakly action representable*. This is the case for instance of the category of associative algebras [10], the category of commutative associative algebras [4], or the category of Leibniz algebras [7].

In [10] the author proved that any weakly action representable category is action accessibile. However, J. R. A. Gray observed in [9] that the converse of this implication is not true: he showed that the notion of weak action representability is connected to the existence of a so-called *amalgam* [12], which already appeared in [2] in relations with action representability, and he proved that the varieties of n-solvable groups ($n \geqslant 3$) are not weakly action representable.

The notion of weakly representable action was then studied in [4] in the context of varieties of non-associative algebras over a field, where the authors provided new examples of weakly action representable categories, such as the variety of commutative associative algebras and those of 2-nilpotent (commutative, anti-commutative and non-commutative) algebras.

Nevertheless, the authors were not able to find an example of an action accessible variety which is not weakly action representable and they ended up with the following open questions:

(1) Does the converse of the implication

$$\textit{weakly action representable category} \Rightarrow \textit{action accessible category}$$

hold in the context of varieties of non-associative algebras over a field?

(2) How does the condition of weakly representable actions behave under taking subvarieties? Specifically, if a variety of non-associative algebras $\mathscr{V}$ is weakly action representable, does it follow that every subvariety of $\mathscr{V}$ is also weakly action representable?

The main aim of this article is to show that the answer to these questions is "no". We use Proposition 2.2. of [9] to prove that the varieties $\mathbf{Nil}_k(\mathbf{Lie})$ of k-nilpotent Lie algebras and $\mathbf{Sol}_n(\mathbf{Lie})$ of n-solvable Lie algebras, which are subvarieties of **Lie**, are not weakly action representable for any $k \geqslant 3$ and $n \geqslant 2$. As an additional result, we use similar methods to refine J. R. A. Gray's result [9] by proving that the varieties of k-nilpotent groups ($k \geqslant 3$) and that of 2-solvable groups are not weakly action representable.

1 Preliminaries

Let $\mathscr{C}$ be a semi-abelian category [11] and let B, X be objects of $\mathscr{C}$. We recall that a *split extension* of B by X is a diagram

$$0 \longrightarrow X \xrightarrow{k} A \underset{\beta}{\overset{\alpha}{\rightleftarrows}} B \longrightarrow 0$$

in $\mathscr{C}$ such that $\alpha \circ \beta = \mathrm{id}_B$ and (X, k) is a kernel of α.

For any object X of $\mathscr{C}$, one may define the functor

$$\mathrm{SplExt}(-, X)\colon \mathscr{C}^{op} \to \mathbf{Set}$$

which maps any object B of $\mathscr{C}$ to the set $\mathrm{SplExt}(B, X)$ of isomorphism classes of split extensions of B by X, and any arrow $f\colon B' \to B$ to the *change of base* morphism $f^*\colon \mathrm{SplExt}(B, X) \to \mathrm{SplExt}(B', X)$ given by pulling back along f.

Given a semi-abelian category, one may define the notion of internal action [1]. Internal actions on an object X give rise to a functor

$$\mathrm{Act}(-, X)\colon \mathscr{C}^{op} \to \mathbf{Set}$$

and one may prove there exists a natural isomorphism $\mathrm{Act}(-, X) \cong \mathrm{SplExt}(-, X)$ (see [2]). Here we don't describe explicitly internal actions, since split extensions are more handy to work with, especially in the context of non-associative algebras. This justifies the terminology in the definition that follows.

Definition 1.1 [2] A semi-abelian category $\mathscr{C}$ is said to be *action representable* if for any object X in $\mathscr{C}$, the functor $\mathrm{SplExt}(-, X)$ is representable. This means that there exists an object $[X]$ of $\mathscr{C}$ and a natural isomorphism of functors

$$\mathrm{SplExt}(-, X) \cong \mathrm{Hom}_{\mathscr{C}}(-, [X]).$$

The prototype examples of action representable categories are the category **Grp** of groups and the category **Lie** of Lie algebras over a commutative unitary ring. In the first case, it is well known that every action of B by X is represented by a group homomorphism $B \to \mathrm{Aut}(X)$, where $\mathrm{Aut}(X)$ is the group of automorphisms of X. In the second case, any split extension of B by X is represented by a Lie algebra homomorphism $B \to \mathrm{Der}(X)$, where $\mathrm{Der}(X)$ is the Lie algebra of derivations of X.

However the notion of action representable category has proven to be quite restrictive: for instance, in [8] the authors proved that the only examples of action representable varieties of non-associative algebras (over an infinite field $\mathbb{F}$ of characteristic different from 2) are the category **AbAlg** of abelian algebras and the category **Lie** of Lie algebras.

Definition 1.2 [10] A semi-abelian category $\mathscr{C}$ is said to be *weakly action representable* if for every objext X in $\mathscr{C}$, the functor $\mathrm{SplExt}(-, X)$ admits a weak representation. This means that there exist an object $T = T(X)$ of $\mathscr{C}$ and a natural monomorphism of functors

$$\tau\colon \mathrm{SplExt}(-, X) \rightarrowtail \mathrm{Hom}_{\mathscr{C}}(-, T).$$

A morphism $(\varphi\colon B \to T) \in \mathrm{Im}(\tau_B)$ is called *acting morphism*.

Examples of weakly action representable category include the variety **Assoc** of associative algebras [10], where $T = \mathrm{Bim}(X)$ is the associative algebra of *bimultipliers* of X [17]; the variety **Leib** of Leibniz algebras, where $T = \mathrm{Bider}(X)$ is the Leibniz algebra of *biderivations* of X [16, 18]; the varieties of 2-nilpotent (commutative, anti-commutative and non-commutative) algebras (see [4, 13–15]). Note that in these mentioned examples, the weakly representing object is quite easy to consider. However, this is not always the case. The variety **CAssoc** of commutative associative algebras is also weakly action representable, but the algebra that could be expected to be a weakly representing object is not commutative. Nevertheless, in [4] a big colimit merged with the amalgamation property is used to naturally construct a weakly representing object.

Another important observation made by G. Janelidze in [10] is that every weakly action representable category is action accessible. We thus have that

$$\textit{action representability} \Rightarrow \textit{weak action representability} \Rightarrow \textit{action accessibility}.$$

The author ended up with an open question, whether reasonably mild conditions may be found on a semi-abelian category under which the second implication may be reversed.

This question was very interestingly addressed by J. R. A. Gray, who proved in [9] that the varieties of n-solvable groups are not weakly action representable for any $n \geqslant 3$. He showed that the notion of weak action representability is connected with the so-called *amalgamation property* (AP) [12].

We recall that a span of monomorphisms $m\colon S \rightarrowtail B$ and $m'\colon S \rightarrowtail B'$ in a category $\mathscr{C}$ admits an amalgam in $\mathscr{C}$ if there exist two monomorphisms $u\colon B \rightarrowtail D$ and $u'\colon B' \rightarrowtail D$ in $\mathscr{C}$ such that $um = u'm'$. The relations between the representabilty of actions and the (AP) were firstly investigated in [2], where it was proved that for any Orzech category of interest, action representability is equivalent to the (AP) for *protosplit* monomorphisms, i.e., monomorphisms which form the kernel part of a split extension.

Using the (AP), J. R. A. Gray gave sufficient conditions under which a Birkhoff subcategory of an action representable category is not weakly action representable.

Proposition 1.3 [9] *Let $\mathscr{C}$ be an action representable category and let $\mathscr{X}$ be a Birkhoff subcategory of $\mathscr{C}$. Suppose there exist two monomorphisms $m\colon S \rightarrowtail B$, $m'\colon S \rightarrowtail B'$ in $\mathbb{X}$, two monomorphisms $u\colon B \rightarrowtail D$, $u'\colon B' \rightarrowtail D$ in $\mathscr{C}$, an object X of $\mathscr{X}$ and a monomorphism $v\colon D \rightarrowtail [X]$ such that*

(a) *m and m' cannot be amalgamated in $\mathscr{X}$;*
(b) *$um = u'm'$;*
(c) *the split extensions with kernel X corresponding to vu and vu' are in $\mathscr{X}$.*

Then, the category $\mathscr{X}$ is not weakly action representable. □

Thanks to a B. H. Neumann's example of an abelian group S, a 2-nilpotent group B and two monomorphisms $m, m' \colon S \rightarrowtail B$ which cannot be amalgamated in any solvable group D (see [20]), it is possible to apply Proposition 1.3 in order to prove that the varieties of n-solvable groups are not weakly action representable for any $n \geqslant 3$.

2 *k*-nilpotent and 2-solvable groups

In this section we adapt to k-nilpotent ($k \geqslant 3$) and 2-solvable groups the proof stating that the categories of n-solvable groups ($n \geqslant 3$) are not weakly action representable.

Let us first recall the concrete example the example of B. H. Neumann that is used in J. R. A. Gray's proof. Consider the following groups in the form of generators and relations:

$$\begin{aligned} S &= \langle a, b \mid [a, b] = a^5 = b^5 = 1 \rangle, \\ B &= \langle x, a, b \mid [x, a] = b^{-1}, [x, b] = x^5 = a^5 = b^5 = 1 \rangle, \\ B' &= \langle y, a, b \mid [y, b] = a^{-1}, [y, a] = y^5 = a^5 = b^5 = 1 \rangle. \end{aligned}$$

It was proved in [20] that the obvious inclusions of S inside B and B' cannot be amalgamated in any solvable group, and it was observed in [9] that there exist a group X and two monomorphisms $\psi \colon B \rightarrowtail \mathrm{Aut}(X)$ and $\psi' \colon B' \rightarrowtail \mathrm{Aut}(X)$ agreeing on S, such that the corresponding split extensions are 3-solvable.

We aim now to show that it is possible to choose X, ψ and ψ' in such a way the split extensions corresponding to ψ and ψ' are 3-nilpotent. Let $X = \mathbb{Z}_5^3$ be the abelian group formed by 3 copies of $\mathbb{Z}_5$. We consider the group monomorphisms $\psi \colon B \rightarrowtail \mathrm{Aut}(X)$ and $\psi' \colon B' \rightarrowtail \mathrm{Aut}(X)$ defined by

$$\psi(x) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \psi(y) = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\psi(a) = \psi'(a) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \psi(b) = \psi'(b) = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

It is immediate to see that the relations are preserved. To finish the proof, we just need to check that the induced split extensions are 3-nilpotent.

Let $B_\psi = B \ltimes_\psi X$ and $B_{\psi'} = B \ltimes_{\psi'} X$ be the semi-direct products induced by ψ and ψ' respectively. We have to check that they are 3-nilpotent groups. Recall that the multiplication in B_ψ is defined by

$$(g, x)(g', x') = (gg', x\psi(g)(x')),$$

for any $g, g' \in B$ and $x, x' \in X$. Since B is 2-nilpotent and X is abelian, one may check that

$$\begin{aligned} [B_\psi, B_\psi] &\cong \langle b \rangle \ltimes_\psi \mathbb{Z}_5^2, \\ [B_\psi, [B_\psi, B_\psi]] &\cong \mathbb{Z}_5 \end{aligned}$$

and $[B_\psi, [B_\psi, [B_\psi, B_\psi]]]$ is the trivial group. Thus, B_ψ is a 3-nilpotent group. The same result can be obtained for the semi-direct product $B_{\psi'}$ and we can state the following.

Theorem 2.1 *The variety of k-nilpotent groups is not weakly action representable for any $k \geqslant 3$.* □

Since 3-nilpotent implies 2-solvable, the same example allows to prove the following.

Corollary 2.2 *The variety of 2-solvable groups is not weakly action representable.* □

3 Subvarieties of Lie algebras

The aim of this section is to prove that, over a field $\mathbb{F}$, the varieties of k-nilpotent Lie algebras ($k \geqslant 3$) and the varieties of n-solvable Lie algebras ($n \geqslant 2$) are not weakly action representable.

Recall that, given any Lie algebra L, its lower central series is defined as

$$L^0 = L, \quad L^k = [L, L^{k-1}],$$

for any $k \in \mathbb{N}$. The Lie algebra L is said to be k-nilpotent if $L^{k-1} \neq 0$ and $L^k = 0$. In addition, its derived series is defined as

$$L^{(0)} = L, \quad L^{(n)} = [L^{(n-1)}, L^{(n-1)}],$$

for any $n \in \mathbb{N}$. The Lie algebra L is said to be n-solvable if $L^{(n-1)} \neq 0$ and $L^{(n)} = 0$.

We denote by $\mathbf{Nil}_k(\mathbf{Lie})$ and $\mathbf{Sol}_n(\mathbf{Lie})$ the subvarieties of **Lie** consisting of all Lie algebras that are s-nilpotent and t-solvable, respectively, for some $s \leqslant k$ and $t \leqslant n$. Note that the classes of all nilpotent and solvable Lie algebras do not, in general, form varieties; however, they do when the degree of solvability or nilpotency is bounded.

Inspired by B. H. Neumann's proof for solvable groups [20], we show that there exist an abelian algebra S, and two monomorphisms $m\colon S \rightarrowtail B$ and $m'\colon S \rightarrowtail B'$, where B and B' are solvable, which cannot be amalgamated in any solvable Lie algebra.

Let $S = \mathbb{F}\{a, b\}$ be the 2-dimensional abelian algebra and consider two copies B, B' of the 3-dimensional Heisenberg algebra. More explicitly, $B = \mathbb{F}\{x, a, b\}$ with the only non-zero bracket $[x, a] = b$, and $B' = \mathbb{F}\{y, a, b\}$ with the only non-zero bracket $[y, b] = a$, where the mononomorphisms are the obvious inclusions.

Now, suppose there exists a solvable Lie algebra D and two monomorphisms $u\colon B \rightarrowtail D$ and $u'\colon B' \rightarrowtail D$, such that they agree on S.

Consider the Lie subalgebra P of D generated by $u(B)$ and $u'(B')$. Since S is an ideal of B and B', then $U = u(S) = u'(S)$ is an ideal of P. This gives us the adjoint map

$$\mathrm{ad}\colon P \to \mathrm{Der}(U), \quad p \mapsto \mathrm{ad}_p,$$

where $\mathrm{ad}_p = [p, -]$ and $\mathrm{Der}(U) \cong \mathfrak{gl}(2, \mathbb{F})$.

Since P is solvable, $\mathrm{ad}(P) \cong P/\ker(\mathrm{ad})$ is also solvable. Nevertheless, $\mathrm{ad}(P)$ is the Lie algebra generated by

$$\mathrm{ad}_{u(x)} \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \text{ and } \mathrm{ad}_{u'(y)} \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Hence, $\mathrm{ad}(P)$ is isomorphic to the special linear algebra $\mathfrak{sl}(2, \mathbb{F})$, which is a simple Lie algebra, and we get a contradiction. Thus, m and m' cannot be amalgamated in any solvable Lie algebra. Since nilpotency implies solvability, this argument shows that the monomorphisms $m\colon S \rightarrowtail B$ and $m'\colon S \rightarrowtail B'$ cannot be amalgamated in any nilpotent Lie algebra. To conclude the proof, we observe that

(i) **Lie** has the amalgamation property [12]. Thus, there exist a Lie algebra L and two monomorphisms $\psi : B \rightarrowtail L$ and $\psi' : B' \rightarrowtail L$ agreeing on S. For instance, we may consider $L = \mathfrak{gl}(3, \mathbb{F})$ and the two faithful representations defined by

$$\psi(x) = -e_{23}, \quad \psi(a) = e_{12}, \quad \psi(b) = e_{13}$$

and

$$\psi'(y) = -e_{32}, \quad , \psi'(a) = e_{12} \quad , \psi'(b) = e_{13},$$

where e_{ij} is the 3×3 matrix which has 1 in the entry (i, j) and 0 elsewhere. It is immediate to check that the subalgebras $\psi(B)$ and $\psi'(B')$ of $\mathfrak{gl}(3, \mathbb{F})$ are both isomorphic to the 3-dimensional Heisenberg algebra.

(ii) Let X be the 3-dimensional abelian algebra and let $v = \mathrm{id}_L$ be the canonical isomorphism $L \cong \mathrm{Der}(X)$. We aim to prove that the split extension of B by X corresponding to $v \circ \psi = \psi$ and that of B' by X corresponding to $v \circ \psi' = \psi'$ are in $\mathbf{Nil}_k(\mathbf{Lie})$, for any $k \geqslant 3$. To get the result, we need to explicitly check that the semi-direct products $B_\psi = B \ltimes_\psi X$ and $B'_{\psi'} = B \ltimes_{\psi'} X$, induced by ψ and ψ' respectively, are 3-nilpotent Lie algebras. In fact, the semi-direct product B_ψ may be described as the Lie algebra with basis $\{x, a, b, e_1, e_2, e_3\}$ and Lie brackets

$$[x, a] = b, \quad [x, e_3] = -e_2, \quad [a, e_2] = [b, e_3] = e_1.$$

One may easily check that $[B_\psi, B_\psi] = \mathbb{F}\{b, e_1, e_2\}$, $[B_\psi, [B_\psi, B_\psi]] = \mathbb{F}\{e_1\}$ and $[B_\psi, [B_\psi, [B_\psi, B_\psi]]] = 0$, which means that B_ψ is 3-nilpotent. This can be done similarly in the case of $B'_{\psi'}$.

Summarising, the hypothesis of Proposition 1.3 are verified and we can conclude with the following.

Theorem 3.1 *The variety* $\mathbf{Nil}_k(\mathbf{Lie})$ *is not weakly action representable for any* $k \geqslant 3$. □

Note that the same argument does not apply to the case $k = 2$ since the semi-direct products B_ψ and $B_{\psi'}$ are not 2-nilpotent algebras.

Finally, since any 3-nilpotent Lie algebras is also 2-solvable, we get the following.

Corollary 3.2 *The variety* $\mathbf{Sol}_n(\mathbf{Lie})$ *is not weakly action representable for any* $n \geqslant 2$. □

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Declarations

Competing Interest There is no Competing Interest.

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