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Financial contracting along the business cycle.

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Abstract

The paper investigates the effects of macroeconomic conditions on firms' capital structure. We introduce a repeated lender-borrower interaction that allows for debt and equity financing to co-exist as optimal securities in every period. The presence of asymmetric information in the market for loans is responsible for endogenous fluctuations to take place.It is possible to state sufficient conditions for the overall economy debt-equity ratio to exhibit a counter-cyclical behavior.

This result is widely supported by several recent empirical finance works.

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1 Introduction

This paper suggests the existence of a relationship between business cycle fluctuations and modifications in the structure of loan relationships.

We depart from the traditional corporate finance theories in arguing that the characteristics of firms' capital structure (i.e. their debt-equity ratio) are determined by macroeconomic conditions.

There has been a great resurgence of interests in the last decades for the thesis that a deeper understanding of the nature of credit market imperfections may be significantly helpful in characterizing economic cycles. Several different approaches have been developed to support the traditional Gurley and Shaw (1955) conviction that the way in which agents finance their activities, interact with financial institutions and choose their contractual arrangements is to be conceived as the main source of economic fluctuations.

Such intuitions have recently been summarized in different constructions. Generally speaking, we may say that the majority of these studies followed the most standard approach in business cycle theories. This corresponds to represent an intrinsically stable economic system whose dynamics might become erratic and cyclical because of the impact of exogenous stochastic shocks. The shocks, either monetary or real, may crucially alter firms *balance sheets*, limiting in this way their access to financial markets.

This is the reference framework of very influential works like Greenwald and Stigliz (1993), Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1998) and Kiyotaki and Moore (1997).

A somehow less popular research analyzed competitive economies where the presence of asymmetric information in the market for loans is responsible for the existence of endogenous business fluctuations. In such a perspective, the working of financial markets is a source of instability that prevents economic dynamics from gravitating around its stationary position[1].

In the most recent years, a deeper representation of loan relationships, that is, an analysis of the allocations implemented by financial contracts, has been conceived as a natural generalization and reinforcement of this approach. On of the most important works in this direction has been provided by Suarez and Sussman (1997). Given a lender-borrower relationship subject to moral hazard, liquidity effects turn out to be the sources of endogenous fluctuations in an economy where financing takes place through both external and internal revenues. Since firms' effort to subscribe good

[1] In a series of contributions in the late 80s M. Woodford argued that the explicit consideration of the imperfect working of financial markets could alleviate the lack of empirical relevance of the cases of competitive economies exhibiting endogenous fluctuations.

projects is a decreasing function of the amount of external financing, cycles may arise because of the dependence of internal liquidity on prices. During booms external financing will be more and more required, with the main consequence of amplifying moral hazard effects. This will prepare recessions.

The effort of integrating incentive problems in the study of economic fluctuations is anyway very recent[2]. The present work makes a step forward in such a direction trying to properly characterize the role of financial institutions along the business cycle. This is done by embedding a simple two-period contracting problem into an economy á la Suarez and Sussman. The main target has been to make the *form* of the financial contract dependent on macroeconomic conditions, i.e. on equilibrium prices. That is, asymmetric information in the market for loans is still responsible for endogenous fluctuations to take place at equilibrium, but we further argue that business fluctuations affect the nature of firms' financing over time.

We consider a competitive economy populated by an infinite sequence of overlapping generations of borrowers-firms who live for three periods and produce only in the last two periods of their life. Borrowers hold a stochastic production technology but they don't have the resources to carry out their investment projects. The amount of external financing issued at equilibrium is determined by a financial contract signed by every generation of borrowers with a representative consumer-lender. If an optimal dynamic contract exists, then at equilibrium young firms will issue repayments that resemble equity, while old firms will have an incentive to finance through debt arrangements. These alternative forms of financing will coexist in every period: the presence of a dynamic contracting issue is responsible for such a coexistence, while the emergence of standard debt as an optimal financial arrangement comes from the introduction of a Costly State Verification (CSV) problem.

The main feature of our economy turns out to be the interaction between the dynamic contracting problem and the existence of finitely-lived firms. Such an interaction is the key to understand the role of liquidity and the potential alternance between debt and equity financing in our economy.

As a by-product, the debt-equity ratio in the overall economy will vary according to the dynamics of aggregate variables.

This argument suggests a potential departure from the standard pecking-order theory of financing (Myers, 1984) that explains the composition of firms' external indebtness by looking at the agency costs faced by entrepreneurs-borrowers under asymmetric information.

The idea that macroeconomic conditions are an important component

[2] See also Reichlin and Siconolfi (2000).

of the debt-equity choice is receiving an increasing support, both in applied and theoretical studies (see Levy, 2001, for a survey)[3].

The paper is organized as follows: Section 1 discusses the basic Suarez and Sussman framework, Section 2 presents the model and section 3 briefly comments on the main implications of our approach.

2 The Suarez and Sussman economy

2.1 Agents and technology

Suarez and Sussman (SS) consider a competitive economy evolving in discrete time $[t = 0, 1, 2, ..., \infty]$ and populated by two types of agents: entrepreneurs-borrowers and consumers-lenders.

Two goods are exchanged. The first one is the numeraire good, while the second is referred to as coffee: only the former can be invested. Commodities are exchanged in a competitive market where the relative spot price of coffee p_t is determined. Coffee is the only produced good.

Consumers are identical and infinitely lived; on the other hand, there exists an infinite sequence of three-period living overlapping generations of borrowers. Both agents are assumed to be risk-neutral and they both discount future at a unitary rate.

Entrepreneurs have linear preferences with respect to the numeraire and they cannot consume coffee. Each generation of entrepreneurs is constituted by a continuum of agents with unit mass; they have no physical endowment but they hold a stochastic technology for the production of coffee. Production can be activated paying a cost consisting of one unit of numeraire and the production process takes place along two periods: in the first one $Y > 0$ is produced, while second period output is identified by the binary random variable $\tilde{Y}$ that can take the two values $\{0, Y\}$ with probabilities $1 - p$ and p, respectively. Production is subject to a standard moral hazard problem: borrowers can take a costly hidden action (effort) that affects the output probability distribution[4]. More precisely, borrowers' effort π is set equal to p, the probability of getting Y. The disutility of effort is given by the

[3]A growing mass of empirical studies is supporting the intuition that the debt-equity ratio usually exhibits a counter-cyclical behavior. Choe, Masulis and Nanda (1993) state that debt is issued in an anti-cyclical way, while a recent very huge resume by Korajczyk and Levy (2002) documents that macroeconomic conditions do affect the relevant debt-equity ratio only for *non-rationed* firms. Such a counter-ciclicity is also suggested by Tirole (1999).

[4]That is, it has the effect of stochastically improving project's returns.

continuous function $\psi(\pi):[0,1]\rightarrow R_{+}:\psi$ is increasing, strictly convex and satisfies standard boundary assumptions:

$$\psi(0)=0,\psi'\geq 0,\psi''>0,\psi'''\geq 0,\psi'(0)=0,\psi'(1)=\infty \tag{1}$$

There exists a representative consumer-lender with quasi-linear preferences in the numeraire good and unlimited amount of numeraire in every state of the world: such a consumer derives his period-t demand for coffee $D(p_t)$, where $D'(p_t)\leq 0$ for every t and an inelastic supply of the numeraire. The lender is allowed to exchange with every generation of borrowers one unit of the asset he holds (the numeraire) with an activity that will give him a repayment in every production period. The nature of this exchange is properly described by a simple financial contract.

2.2 The contract problem

We examine henceforth the nature of the contract between the representative consumer-lender and a borrower born in $t-1$.

It can be useful to start by characterizing the *first best* allocation. The first best level of effort π^* is identified by the maximization of the project's net present value:

$$\pi^*=\arg\max_{\pi\in[0,1]}\{-1+p_tY+[\pi p_{t+1}Y-\psi(\pi)]\} \tag{2}$$

where -1 is a sunk cost, p_tY the revenues from the first production period and $\pi p_{t+1}Y-\psi(\pi)$ the expected net revenues for the second production period corresponding to an effort π. Given (1), then (2) admits the unique solution $\pi^*(p_{t+1})$ that can be characterized using the first order condition

$$p_{t+1}Y=\psi'(\pi^*) \tag{3}$$

Boundary conditions in (1) guarantee that $\pi^*(p_{t+1})<1$; moreover, using the implicit function theorem, we get $\dfrac{\partial\pi^*(p_{t+1})}{\partial p_{t+1}}>0$.

We remark that a sufficient condition for the first best allocation to be implemented is $p_tY\geq 1$ that defines a situation where borrowers are able to repay the loan with their first period deterministic outcome. Then, we move to the analysis of the structure of incentives under asymmetric information. A contract defines the amount of repayment to the lender in the second production period: we denote this repayment R_{t+1}. Non observability of π

prevents from writing contracts conditional on effort performances, so that the repayment should be conditioned on the realizations of $\tilde{Y}$.

The risk-neutral entrepreneur born in $t-1$ acts as a mechanism designer: he offers a contract R_{t+1} that solves the following

$$\underset{\pi, R_{t+1}}{max} \{p_t Y + [\pi(p_{t+1}Y - R_{t+1}) - \psi(\pi)]\} \tag{4}$$

s.t.

$$R_{t+1} \in [0, p_{t+1}Y] \tag{LL}$$

$$\pi \in \arg\max_{\pi} (p_t Y - R_t) + [\pi(p_{t+1}Y - R_{t+1}) - \psi(\pi)] \tag{IC}$$

$$p_t Y + \pi R_{t+1} \geq 1 \tag{IR}$$

Constraints (IC) and (IR) have the usual meaning: the incentive compatibility (IC) states that the equilibrium effort is an optimal choice for the borrower and the individual rationality (IR) guarantees that the lender participates in the mechanism: notice that the lender has access to a perfectly competitive financial market whose interest rate is normalized to zero. The limited liability (LL) constraint is bounding the set of feasible repayments[5].

In order to characterize the solution we observe that the lender's (IR) constraint is binding[6]; we then replace (IC) with the corresponding first order condition

$$p_{t+1}Y - R_{t+1} = \psi'(\pi) \tag{5}$$

that implicitly defines π as a function of p_{t+1} and R_{t+1}.

Now, substituting (IR) into the objective function (4), borrower's utility U can be rewritten as:

$$U = p_t Y - 1 + \pi p_{t+1} Y - \psi(\pi) \tag{6}$$

[5] It is important to notice that the role of the (LL) constraint in the present framework is not to avoid borrower's negative consumption, rather to provide meaningful solutions to the second best allocations. If such a constraint was not introduced, then bilateral risk neutrality enables to directly implement the first best level of effort (Shavell, 1979).

[6] If it was not, then it would be possible for the borrower to reduce the repayment getting a higher profit under the same set of constraints.

If the feasibility set is non-empty (so that an optimal contract exists), then $\pi^{**} = \pi^{**}(p_t, p_{t+1})$ will be the optimal level of effort. It is straightforward to check that $\pi_1^{**} \geq 0$ and $\pi_2^{**} \geq 0$. These relationships deserve some comments; the first one can be conceived as a *liquidity effect*, effort increases because the increased liquidity make borrowers less dependent on external financing. The other one defines a *profitability effect*: higher prices define higher investment values, and as a consequence the marginal productivity of effort is also higher.

2.3 Equilibrium properties

Referring to period $t+1$, and given that we are dealing with a measure one population of borrowers, so that the law of large numbers applies, the market clearing condition for the coffee market can be written in the following form:

$$[1 + \pi^{**}(p_t, p_{t+1})]\, Y = D(p_{t+1}) \tag{7}$$

Aggregate supply is provided both by entrepreneurs born in $t-1$ and in t. At time $t+1$ the young entrepreneurs (i.e. the ones born in t) offer the amount Y of coffee, while the old ones supply is given by $\pi^{**}(p_t, p_{t+1})Y$. Equation (7) implicitly defines a non-linear first order difference equation in p : under the assumptions in order there exists a continuously differentiable function $g : R_{++} \to R_{++}$ such that:

$$p_{t+1} = g(p_t) \tag{8}$$

Notice that if $p_t Y \geq 1$, then the first best allocation $\pi^*(p_{t+1})$ can be implemented and (7) will depend on p_{t+1} only: (8) will be an algebraic equation in p and we let $\bar{p}$ to be its (unique) solution.

On the other hand, if $p_t Y < 1$ holds, then the second best allocation will be implemented and the implicit function theorem enables to write:

$$g'(p_t) = -\frac{\pi_1^{**}}{\pi_2^{**} - D'(p_{t+1})} \tag{9}$$

This prediction fully explains the role of liquidity in affecting the incentive problem: a higher p_t implies that firms become more liquid, and therefore less dependent on external finance. Thus, they will raise π and the supply of coffee in period $t+1$: this creates the potential for lowering p_{t+1} increasing next period's need of borrowing and so on.

The dynamic behavior of the system, for an assigned initial price, is represented in Figure 1.

Given that g is monotonically decreasing on a definite region, the necessary condition for existence of two-period endogenous fluctuations is satisfied[7]. Also notice that in order for endogenous fluctuations to take place we should have $g'(p^*) < -1$.

This condition holding, the system will be oscillating between *recession* periods, when the price is low and the moral hazard problem is more severe, and *booms* periods when borrowers have additional incentives to exercise effort and reduce the probability of a distress.

In conclusion, we remark that borrowers' liquidity turns out to be a counter-cyclical variable, and several scholars have strongly questioned the empirical relevance of such a prediction[8].

3 Debt, equity and the business cycle

The structure we discussed so far constitutes one of the first attempts to provide a general explanation of how asymmetric information in credit markets may affect the dynamics of economic activity. In addition, the presence of an incentive conflict is unambiguously responsible for the existence of an endogenous reversion mechanism. Liquidity affects entrepreneurs-borrowers economic decisions in sharp contrast with the Modigliani-Miller argument that assigns a central role to net present values.

Now, we believe that a further and meaningful departure from that argument can be provided here, that is a characterization of the debt-equity ratio as an endogenous variable, strongly dependent on business cycle fluctuations. In order to take into account this phenomenon, we will allow for a proper design of financial institutions, trying to generalize the financial contracting analysis. In other words, we want to restate the finding that asymmetric information in credit markets can be a source of endogenous fluctuations, but we introduce a positive argument: existence of (endogenous) cyclical dynamics are affecting the nature of financial arrangements.

SS build their contract problem under three crucial assumptions: there exists moral hazard with hidden action, both lenders and borrowers are risk neutral, and the stochastic realizations of output are identified by the binary variable $\{0, Y\}$; the implications of such assumptions should be more carefully examined.

The first point to stress is that the (LL) constraint forces the repayment

[7] It is well known that a monotonic first order difference equation cannot produce cycles of an order higher than two.

[8] See Reichlin (2001), p.18.

R_{t+1} to have the form $\{0, x\}$ where x is the unknown to be determined by the contract problem itself. This kind of contract cannot in principle be interpreted as a well defined financial instrument: it is neither a debt, nor an equity, given that it turns out to be (weakly) monotone over states, but it is defined over two states only. In other words, referring to such a simplified binary structure prevents the proper design of a financial arrangement, the form of the optimal contract being de facto determined by the working of limited liability alone.

Furthermore, we have seen that (LL) is a sufficient condition to induce monotonicity of the repayment structure in output levels. Unfortunately, under very general assumptions on the distribution function, monotonicity cannot be predicted anymore. This result has been shown by Innes (1990) and the basic intuition is straightforward: whenever lenders cannot observe effort, then the outcome realization will be interpreted as a signal of borrowers' action. The repayment will therefore be high (taking the form of a punishment) when output is low and viceversa. That is, we get a *live or die* contract. Under bilateral risk neutrality the only way to avoid such an uncomfortable result and to predict a debt-like contract is to explicitly assume monotonicity through the introduction of a monotonicity constraint on repayments[9]. The role of this constraint is of course played by limited liability in the $\{0, Y\}$ case.

Hence, our impression is that in order to allow for more robust predictions on the nature of financial relationships, we should modify the basic representation of asymmetric information. We believe that the most recent researches on financial contracting have made significant steps in showing how the so called Costly State Verification (CSV) paradigm can constitute a solid foundation for debt contracts. The basic, one-stage CSV structure refers to situations where *symmetric* information is assumed as the time of contracting: stochastic returns from a project can be observed at no cost by borrowers only, that is, we have *ex-post* private information. Borrowers are assumed to report their private information through a message. Lenders can verify the state only through the performance of a costly auditing activity.

In a very influential paper, Gale and Hellwig (1985) showed the optimality of debt contracts in such a framework; more recently, Krasa and Villamil (2000) provided general conditions for the optimality of debt when limited commitment and stochastic auditing are allowed[10].

[9] The potential rationales for such a constraint are briefly discussed in Innes (1990), p.50.

[10] Krasa and Villamil results may overcome the issue raised by Mookherje and Png (1989), among others: whenever stochastic auditing is preformed, then debt will not

At the same time, Hellwig (2001) has restated the conditions ensuring co-existence of optimal debt contracts and intermediaries performing a delegated monitoring activity under CSV. In other words, he defines a set of sufficient conditions that ensure at the same time optimality of debt contracting and optimality for debt to be intermediated by banks.

The present work does not discuss in detail the properties of the CSV apparatus[11], it rather tries to develop a simple dynamic contracting structure allowing for debt and equity financing to co-exist at equilibrium. It is important to point out that the basic CSV framework is already allowing for a form of equity financing: insiders hold all the equity, with the consequence that only *internal* equity is considered[12].

A departure from the benchmark CSV environment becomes necessary if we want to account for the role of outside equity investors: "This sort of capital structure does not look much like that employed in the United States or most other developed nations. In particular, major corporations typically have a large class of outside equity holders who are not necessarily any better informed than debt holders"[13].

The following sections will try to develop these intuitions.

3.1 The model

We keep the focus on the interaction between a representative consumer-lender and a population of three-period living overlapping generations of entrepreneurs-borrowers. In order to avoid considering risk sharing issues, bilateral risk neutrality is assumed. We also parallel SS in considering a discrete time, two-goods economy, where only one good is produced; we let p_t for $t = [0, 1, 2, ...]$ to be the price of this good.

Borrowers have exclusive access to a stochastic technology that is productive for two periods.

Production is assumed to have the following characteristics:

Assumption 1: Production takes place with a *stochastic linear* technology in both periods. One unit of numeraire at $t-1$ is transformed into $y = \{y_1, y_2, ..., y_N\}$ of the final good with probabilities $\{\pi_1, \pi_2, ..., \pi_N\}$. Furthermore, x units of the final good will be produced at $t+1$; x is continuous

emerge as the optimal security.

[11] A more general evaluation of the most recent developments in CSV schemes in the light of the traditional critiques moved to those constructions (and summarized in the Hart's 1995 book) is proposed in Attar and Campioni (2003).

[12] That is, funds owned by borrowers and directly invested in the project are exchanged for a residual claim that has an equity form.

[13] Boyd and Smith (1999), p. 271.

with the continuous distribution function $F(x) : [0, \bar{x}] \rightarrow [0, 1]$. In addition, x and y are independently distributed.

Assumption 2: Borrowers born, say, in $t-1$ are allowed to choose their amount of investment in the project $i_{t-1} \in [0, \bar{\imath}] \subset R_+$. Every generation of borrowers is also endowed with a positive amount of the numeraire good that we denote $w < 1$.

Assumption 3: A sunk cost of one unit of numeraire should be paid for production to be activated. Since $w < 1$, entrepreneurs should rely on external finance to fund their investments. For a borrower born in $t-1$ (for $t = 0, 1, 2, ...$) external finance is obtained through a loan contract that specifies the investment made in $t-1$ and the amount of numeraire repaid to the lender in t and in $t+1$, respectively.

Assumption 4: The investment project has a positive net present value. Letting $\hat{y} \equiv \sum_n y_n \pi_n$, $\hat{x} \equiv \int_0^{\bar{x}} x dF(x)$ and $\hat{x} > \hat{y}$, we should have: $NPV \equiv p_t \hat{y} + p_{t+1} \hat{x} - 1 > 0$. Furthermore, in order for the incentive problem to be meaningful, we assume $p_t \hat{y} < 1$ for every p_t.

The information structure is such that y is fully observable by the two parties, while second period realizations are observable by borrowers only. The lender can become informed of the true state through the performance of a costly auditing activity. Thus, a standard Costly State Verification problem (CSV) arises in the second production period. As a consequence, both the lender-borrower relationship and the nature of the implementation problem will be different from the ones tackled in the previous section.

In order to focus on the alternative representation of financial arrangements that we are suggesting here, we adopt the same representation of consumers as SS

3.2 Consumers

We refer to an infinite-living, representative consumer lender. His behavior can be stated as follows:

$$\max_{\{c_t, b_t\}_{t=1,2,...}} E_t \left[\sum_{s=0}^{\infty} (b_{t+s} + u(c_{t+s})) \right] \tag{10}$$

s.t.

$$b_t + p_t c_t + i_t = e + R_{t-1} i_{t-1} \qquad \text{for} \quad t = 1, 2, ... \tag{11}$$

$$b_t \geq 0, \;\; c_t \geq 0 \;\; \text{and} \;\; a_0 \;\; \text{given} \tag{12}$$

We let b_t and c_t to be the demands for the numeraire and the consumption good, respectively; i_t is the amount of loan offered, R_{t-1} is repayment that the consumer received at time $t-1$ acting as a lender and e is the consumer's endowment.

We will henceforth assume that $u : R_+ \rightarrow R$ is strictly increasing and strictly concave. We will follow SS in focusing on interior solutions to (10)-(12), so that lenders behavior is described by the sequence of demands for consumption good $D(p_t)$ for $t = 1, 2, ...$ with $D'(p_t) \leq 0$ for every t.

Nonetheless, we maintain the SS assumption of borrowers having linear preferences in the consumption good, with a discount factor equal to one.

This, together with the risk-neutral behavior, makes the time of borrowers' consumption irrelevant and we focus on borrowers who consume in the last period of their life.

Thus, lenders can exchange with borrowers born, say, in $t-1$ one unit of the safe asset they hold (namely, the numeraire) with an activity that will give them a repayment in any of the two production periods. The nature of this exchange is properly described by a financial contract.

To sum up: we introduced the same assumptions of SS on agents' preferences and on the structure of markets. We are concerned with a representation of asymmetric information that makes our implementation problem significantly different from the SS one.

3.3 Financial contracting

A financial contract subscribed by a borrower born in $t-1$ defines the allocation of resources that will be delivered as repayments to the lender in periods t and $t+1$, namely R_t and R_{t+1}, together with the amount invested in the stochastic technology i_{t-1}. Moreover, a contract should specify a set of states where auditing is performed. Observability of y allows to write contracts conditioned on first period realizations, while lenders strategies should be specified conditionally to the message $m \in M \subset R_+$ sent by borrowers who observe x.

Finally, a contract defines the amount of wealth $v \leq w$ that the borrower decides to invest in the project.

The general mechanism involving a representative consumer-lender and an entrepreneur-borrower born in $t-1$ is defined by:

$$\Gamma = [i_{t-1}, R_t(y), M, R_{t+1}(m, y), A_t(y), v] \tag{13}$$

where M is the borrower's message space, $A_t(y) \subseteq M$ is the set of audited reports and $B_t(y)$ is its complement. Notice that we are restricting to the class of deterministic auditing schemes: for any given message $m \in M$ auditing is either performed with probability one or not performed at all. We believe that the most recent work in the CSV literature has shown how this hypothesis can constitute a reasonable simplification.

Events are taking place according to the following sequence:

- Time 0: The contract Γ is offered
- Time 1: y realizes; the repayment R_t is issued
- Time 2: x realizes; entrepreneurs send a message m and the contracts is executed

The simple nature of the dynamic interaction enables us to restrict on the subset of *direct* mechanisms, that is to the set of mechanisms where the message space and the state space coincide. Then, we are allowed to set $M = [0, \bar{x}]$ without loss of generality[14]. Before stating the contract problem, two further assumptions are introduced:

Assumption 5: Both agents have access to a deterministic (storage) technology, giving one unit of the consumption good for any unit of the numeraire invested. Letting $z_n \equiv p_{t+1}x + [p_t y_n - R_t(y_n)]$, borrower's liquidity at time $t+1$ will be given by $c_n = z_n i_{t-1}$.

Assumption 6: Auditing costs are a well behaved, increasing and strictly convex function of borrower's investments. We define the auditing cost function as $\gamma(i) : [0, \bar{\imath}] \to R_+$, with $\gamma' > 0$ and $\gamma'' > 0$.

These two assumptions are very close to the ones introduced in the analysis of dynamic CSV problems by Chang (1990). We also remark that the lender is living in a perfectly competitive capital market whose interest rate has been normalized to zero. The contract problem is defined by the maximization of the expected utility of the borrower (who designs the mechanism) over a well defined feasibility set. Given Assumptions 1-6, every optimal contract is a solution of the following:

$$\underset{\Gamma}{Max}\ i_{t-1} \sum_n \pi_n \left[p_t y_n - R_t(y_n) + \int_{A_t} (p_{t+1}x - R_{t+1}(x, y_n))\, dF(x) \right] +$$

$$+ i_{t-1} \sum_n \pi_n \left[\int_{B_t} (p_{t+1}x - R_{t+1}(x, y_n))\, dF(x) \right] + (w - v)$$

[14] Given that a contract can be written conditionally to first period realizations, then there are no incentives for renegotiation. In order to ensure that the *Revelation Principle* holds it is sufficient to assume that the parties are committed to contractual prescriptions.

s.t.

$$R_t(y_n) \in [0, p_t y_n] \qquad \text{for n=1,2,...,N} \tag{14}$$

$$R_{t+1}(x, y_n) \in [0, p_{t+1}x + p_t y_n - R_t(y_n)] \qquad \text{for n=1,2,...,N} \tag{15}$$

$$R_{t+1}(x, y_n) = R_{t+1}(y_n) \qquad \text{for n=1,2,...,N, } \forall x \in B_t \tag{16}$$

$$R_{t+1}(y_n) \geq R_{t+1}(x, y_n) \qquad \text{for n=1,2,...,N, } \forall x \in A_t \tag{17}$$

$$\sum_n \pi_n \left[R_t(y_n) + \int_{A_t} \left(R_{t+1}(x, y_n) - \frac{\gamma(i_{t-1})}{i_{t-1}} \right) dF(x) \right] +$$

$$\sum_n \pi_n \int_{B_t} R_{t+1}(x, y_n) dF(x) \geq 1 - v \tag{18}$$

The second period *Limited Liability* constraint (15) embeds the additional liquidity coming from retained earnings. The same caveat holds for the second period *Incentive Compatibility* constraints (16) and (17); notice that the former imposes that when verification doesn't happen, then date two repayments will be independent of the reported cash flow. The latter states that in verified states it is unfeasible for the borrower to give back the fixed repayment $R_{t+1}(y_n)$[15]. Constraints (16) and (17) taken together constitute a necessary and sufficient condition to induce borrower's *truthful revelation* at equilibrium.

Auditing costs are assumed to be paid by the lender; the lender has access to a competitive capital market with interest rate normalized to zero: his reservation utility will therefore given by $(1 - v)i_{t-1}$. The *Individual Rationality* constraint guaranteeing lender's participation to the mechanism is given by (18) that has been written down in per-unit terms[16]. The magnitude $\frac{\gamma(i_{t-1})}{i_{t-1}}$ defines the amount of auditing costs per unit of consumption good invested in the stochastic technology.

In order to characterize the solution to Problem 2 we will proceed through *backward induction*, applying the results of the one-period CSV to the last

[15] A formal derivation of incentive compatibility constraints like (16) and (17) is proposed in Townsend (1979).

[16] That is, both sides of (18) have been divided by i_{t-1}.

stage of the game[17]. At first, we show that the repayment issued in the second period takes the form of a standard debt contract (SDC)[18]. In the traditional financial contracts literature a standard debt contract is identified by the following attributes:

- Repayment is fixed over non bankruptcy states.
- When bankruptcy takes place, then the lender recovers the whole amount of the project and the borrower becomes a residual claimant.

Proposition 1 *Any feasible contract* Γ *is dominated by a contract implying:* $R_{t+1}(x, y_n) = p_{t+1}x + p_t y_n - R_t(y_n)$ $\forall x \in A_t$, $\forall y_n$ *and* $R_{t+1}(x, y_n) = R_{t+1}(y_n)$ $\forall x \in B_t$, $\forall y_n$.

Proof. The second part of the proposition is implied by (17). In order to complete the proof we proceed in three steps.

First we show that we can focus without loss of generality on contracts implying borrower's maximum equity participation. In other words, we can always assume that $w = v$: otherwise the optimal contract Γ could be replaced by a contract $\Gamma' \equiv \left[i_{t-1}, R_t, M, R'_{t+1}, A_t, v'\right]$ such that $R'_{t+1} = R_{t+1} - (v - w)$ and $v' = w$ which still satisfies all the relevant constraints. Then, we can argue that the optimal contract predicts the auditing region to coincide with the bankruptcy region. That is, if the array $[R_{t+1}(y_n), R_{t+1}(x, y_n), A_t(y_n)]$ belongs to an optimal contract, then we should have:

$$A_t(y_n) = \{x : p_{t+1}x + p_t y_n - R_t(y_n) \leq R_{t+1}(y_n)\}$$

Now, if $p_{t+1}x + p_t y_n - R_t(y_n) \leq R_{t+1}(y_n)$, then $R_{t+1}(x, y_n) \leq R_{t+1}(y_n)$ by (15).

The other direction is proved by contradiction: if $R_{t+1}(x, y_n) \leq R_{t+1}(y_n)$ implied $p_{t+1}x + p_t y_n - R_t(y_n) > R_{t+1}(y_n)$ on an interval $C_t \subset A_t$, then two cases would be possible:

(i) $R_{t+1}(x, y_n) = R_{t+1}(y_n)$; then $p_{t+1}x + p_t y_n - R_t(y_n) > R_{t+1}(x, y_n)$, violating (15).

(ii) $R_{t+1}(x, y_n) > R_{t+1}(y_n)$. In this case we can define a new feasible triple $\left[R'_{t+1}(y_n), R'_{t+1}(x, y_n), A'_t(y_n)\right]$ such that $A'_t = A_t / \{C_t\}$ and $R'_{t+1}(x, y_n) < R_{t+1}(x, y_n)$, so that borrower's payoff is strictly greater[19].

[17] Existence of significative solutions for a more general version of our Problem 2 has been already shown by Chang (1990).

[18] Optimality of debt will be shown here in a way that slightly differs from the Gale and Hellwig one.

[19] A precise formulation for R' has been for instance suggested in Gale (2001).

The last step is to show that *maximum recovery* is taking place in bankruptcy states: for every $x \in A_t$ optimal contracting implies that $R_{t+1}(y_n) = p_{t+1}x + p_t y_n - R_t(y_n)$. That is, the lender gets the entire proceeds from the investment. Now, in order to complete the proof, we remark that the lender's Individual Rationality constraint of Problem 2 is binding. As a consequence the borrower's objective function becomes:

$$i_{t-1}\left[(p_t\hat{y} + p_{t+1}\hat{x} - 1) - \sum_n \pi_n \int_{A_t} \frac{\gamma(i_{t-1})}{i_{t-1}} dF(x)\right]$$

Then, every feasible contract $\{R_{t+1}(y_n), A_t\}$ is dominated by the (feasible) contract $\Gamma' = \{R'_{t+1}(y_n), A'_t\}$ with $R'_{t+1}(y_n) = p_{t+1}x + p_t y_n - R_t(y_n)$ $\forall x \in A'_t : \Gamma'$ is minimizing the extension of the bankruptcy region[20] and, at the same time, it is increasing the borrower's objective function. ■

Proposition 3 implies that:

$$A_t(y_n) = [0, \phi_n], \qquad \phi_n = \frac{R_{t+1}(y_n) + R_t(y_n) - p_t y_n}{p_{t+1}} \tag{19}$$

The extension of the auditing (bankruptcy) region is negatively correlated with consumption good prices: a rise either in p_t or in p_{t+1} makes the borrower more liquid, reducing the probability of a default.

Relying on second period equilibrium and substituting (18) into the borrower's objective function, Problem 2 can be rewritten as:

$$\underset{i_{t-1},R_t(y_n),R_{t+1}(y_n)}{Max} \; i_{t-1}\left[(p_t\hat{y} + p_{t+1}\hat{x} - 1) - \sum_n \pi_n \int_{A_t} \frac{\gamma(i_{t-1})}{i_{t-1}} dF(x)\right] \tag{20}$$

s.t.

$$R_t(y_n) \in [0, p_t y_n] \qquad \text{for n=1,2,...,N} \tag{21}$$

$$\sum_n \pi_n \left[R_t(y_n) + \int_0^{\phi_n} \left(p_{t+1}x + p_t y_n - R_t(y_n) - \frac{\gamma(i_{t-1})}{i_{t-1}}\right) dF(x)\right] + \tag{22}$$
$$+ \sum_n \pi_n \left[\int_{\phi_n}^{\bar{x}} R_{t+1}(y_n) dF(x)\right] = 1$$

[20] Given that borrower's consumption is protected by *Limited Liability*, the bankruptcy region cannot be reduced farther.

The problem is reformulated as the minimization of last period bankruptcy (auditing) costs under the *Individual Rationality* and the first period *Limited Liability* constraints. It is notable that our problem is *not* monotone in i_{t-1}: the convexity of γ implies that $\int_0^{\phi_n} \frac{\gamma(i_{t-1})}{i_{t-1}} dF(x)$ is increasing in i_{t-1}. A further restriction is introduced:

Remark We focus on the class of density functions $f(x)$ implying that (20) is a strictly concave function[21].

The relevant Lagrangian will therefore be:

$$L = i_{t-1}\left[(p_t\hat{y} + p_{t+1}\hat{x} - 1) - \sum_n \pi_n \int_0^{\phi_n} \frac{\gamma(i_{t-1})}{i_{t-1}} dF(x)\right] +$$

$$+\lambda \sum_n \pi_n \left[R_t(y_n) + \int_0^{\phi_n} \left(p_{t+1}x + p_t y_n - R_t(y_n) - \frac{\gamma(i_{t-1})}{i_{t-1}}\right) dF(x) + \right] +$$

$$+\lambda \sum_n \pi_n \int_{\phi_n}^{\bar{x}} R_{t+1}(y_n) dF(x) - 1 + \mu\left[p_t y_n - R_t(y_n)\right]$$

For every t, necessary conditions for optimality are the following:

$$\frac{\partial L}{\partial i_{t-1}} = 0 \Rightarrow \lambda = i_{t-1}^2 \frac{NPV - \sum_n \pi_n \int_0^{\phi_n} z_n \gamma'(i_{t-1}) dF(x)}{\left[i_{t-1}\gamma'(i_{t-1}) - \gamma(i_{t-1})\right] \sum_n \pi_n \int_0^{\phi_n} dF(x)} \tag{23}$$

$$\frac{\partial L}{\partial R_t} = 0 \Rightarrow (\lambda + i_{t-1}) \int_0^{\phi_n} \gamma'(i_{t-1}) dF(x) = \frac{\mu}{\pi_n} \tag{24}$$

$$\frac{\partial L}{\partial R_{t+1}} = 0 \Rightarrow (\lambda + i_{t-1}) \left[(-\frac{1}{p_{t+1}}) \frac{\gamma(i_{t-1})}{i_{t-1}} f(\phi_n)\right] + \lambda\left[1 - F(\phi_n)\right] = 0 \tag{25}$$

$$\mu \geq 0, \qquad \mu\left[p_t y_n - R_t(y_n)\right] = 0 \qquad \textit{for every } n \tag{26}$$

[21] The restriction that every $f(x)$ must satisfy is defined in Appendix 1. A necessary condition for (20) to be a strictly concave function turns out to be $f'(x) > 0$.

$$\sum_n \pi_n \left[R_t(y_n) + \int_0^{\phi_n} \left(p_{t+1}x + p_t y_n - R_t(y_n) - \frac{\gamma(i_{t-1})}{i_{t-1}} \right) dF(x) \right] +$$

$$+ \sum_n \pi_n \int_{\phi_n}^{\bar{x}} R_{t+1}(y_n) dF(x) = 1 \tag{27}$$

Optimal contracts are hence identified by a system of five non-linear equations in the five unknowns: λ, μ, i_{t-1}, $R_t(y_n)$, $R_{t+1}(y_n)$.

It should be recalled that $\lambda > 0$ holds, given that the lender's *Individual Rationality* constraint is binding. As a by-product, we have:

$$NPV - \sum_n \pi_n \int_0^{\phi_n} z_n \gamma'(i_{t-1}) dF(x) \geq 0 \tag{28}$$

Given Assumption 6 and recalling that $i_{t-1} > 0$ for every t, then from (23) we get $\mu > 0$. We are hence concerned with a boundary solution for $R_t(y_n)$. In the present setting the choice over the dynamic structure of repayments is therefore rather trivial: the borrower always has the incentive to give back the entire first period outcome (with the lender becoming the owner of the firm). This prediction is a direct implication of having assumed *symmetric information* in the first period: as a consequence, there is no room for an incentive cost of repaying in the first period. We will further discuss this issue in the next section. We remark here that from (26) we get:

$$R_t(y_n) = p_t y_n \qquad \textit{for every } n \tag{29}$$

That is, the repayment issued at period t by borrowers born in $t-1$ will take an *equity* form. Hence, the decision taken by a borrower born in $t-1$ whether to repay in the first or in the second period defines the amount of (*external*) equity and debt he issues in t and $t+1$, respectively. Given that we are dealing with three-period lived overlapping generations of borrowers, it will be possible to determine the relative amount of debt and equity issued in every relevant period. Also notice that in every period *internal equity*, that coincides with the entrepreneurial wealth invested in the project, is co-existing with *external equity* identified by the amount of repayment taking a *strictly monotone* form. The relative composition of securities issued at equilibrium is of course dependent on prices and costs behavior over time.

A first step in understanding the properties of equilibrium economic dynamics is given by an analysis of the responsiveness of investments to consumption's good prices. Standard calculus enables us to state the following

Proposition 2 *Optimal contracts predict i_{t-1} to positively depend both on p_t and on p_{t+1}, for every t.*

Proof. See Appendix 2. ■

3.4 Equilibrium and dynamics

In defining a market clearing condition for the consumption good in $t+1$ we should take into account that in each period two generations of producers are actually active, namely the ones born in t and the ones born in $t-1$. They are offering an amount $\hat{y}$ and $\hat{x}$ of the consumption good, respectively. The consumption good market clearing condition for $t+1$ will therefore be:

$$i_{t-1}(p_t, p_{t+1})\hat{x} + i_t(p_{t+1}, p_{t+2})\hat{y} = D(p_{t+1}) \tag{30}$$

Given that i and D are continuously differentiable functions, there exists an implicit relationship among equilibrium prices. This relationship is given by the continuously differentiable function $q(p_t, p_{t+1}) : \mathcal{R}_{++} \times \mathcal{R}_{++} \rightarrow \mathcal{R}_{++}$ such that:

$$p_{t+2} = q(p_t, p_{t+1}) \qquad \textit{for every } t \tag{31}$$

In other words, the sequence of market clearing prices corresponds to the solution of a second order and non linear difference equation with two given initial conditions on p.

The first step in the analysis of the dynamical system (31) is to check whether a stationary solution exists. That is, we should verify the existence of a fixed point $\bar{p}$ of the q function such that $\bar{p} = q(\bar{p})$. Looking at (30), if such a $\bar{p}$ does exist, then:

$$i_{t-1}(\bar{p}, \bar{p})\hat{x} + i_t(\bar{p}, \bar{p})\hat{y} = D(\bar{p}) \tag{32}$$

Now, given that the right hand side of (32) is increasing in $\bar{p}$ and the left hand side is decreasing in $\bar{p}$, then, once the boundary value $D(0)$ has been properly defined, there must exist a $\bar{p} \in R_{++}$ such that (32) is verified.

The properties of the solution of (31) can be better understood by considering the correspondent planar system of first order difference equations. In the present context, (31) can be represented by the family of dynamical systems $v_{t+1} = Q(v_t, k)$, where $v_t \equiv (z_t, p_t)$ and $z_t \equiv p_{t+1}$. Also, $k \in K \subset R^2_+$ stays for a vector of parameters indexing the system; more precisely, we have $k \equiv (\hat{x}, \hat{y})$. Under our assumptions it is possible to state that a two-period cycle exists

Proposition 3 *The system* $v_{t+1} = Q(v_t, k)$ *with* $Q : R^2_{++} \times K \to R^2_{++}$ *with assigned initial conditions admits the existence of period doubling (flip) bifurcations in a sufficiently smooth neighborhood of the non-hyperbolic equilibrium* $(\bar{p}, \bar{k})$ *where* $\bar{p}$ *is the stationary solution of the system parametrized by* $\bar{k}$

Proof. See Appendix 2. ■

Notice that we can confirm SS finding that the presence of cyclical dynamics is still induced by asymmetric information in loan relationships. This can be easily shown if we think of the first best as the allocation implemented when auditing costs are set equal to zero. In this case, given Assumption 5, the investment will be set at $\bar{\imath}$, independently of the price structure. Then, (30) will be an equation in p_{t+1} only and the market clearing equilibrium will be associated to a stationary outcome, as in SS.

On the other hand, we depart from the SS prediction that relates low prices with expansions and growing prices with recessions. In other words, we can argue that prices have a *pro-cyclical* behavior. Appendix 1 shows that an optimal contract should imply $\dfrac{\partial i_{t-1}}{\partial p_{t+1}} > \dfrac{\partial i_{t-1}}{\partial p_t}$ for every t : investments planned in $t-1$ are more sensitive to p_{t+1} than to p_t. It directly implies that investments planned when prices are high are higher than the ones planned under a low price regime. In other words, if we assume that a two-period cycle takes place with $p_t < p_{t+1}$, then $i_{t-1}(p_t, p_{t+1}) > i_t(p_{t+1}, p_{t+2})$ for every t.

As a consequence, the following corollary of Proposition 6 holds:

Corollary 4 *Assume that a two-period cycle is taking place. Then, the* $t+1$ *aggregate supply* $i_{t-1}(p_t, p_{t+1})\hat{x} + i_t(p_{t+1}, p_{t+2})\hat{y}$ *will be higher than the* $t+2$ *supply* $i_t(p_{t+1}, p_{t+2})\hat{x} + i_{t+1}(p_{t+2}, p_{t+3})\hat{y}$ *whenever* $p_{t+1} > p_t$ *for every* t.

The potential existence of two-period endogenous fluctuations is a relevant result in order to characterize the repayment dynamics. As a matter of fact, such dynamics may enlight the role of financial institutions in economic fluctuations.

Then, we examine the dynamics of the debt-equity ratio as it is conceived in this economy. Keeping the focus on period $t+1$, we let $R^d_{t+1}(t-1)$ to be the amount of repayment issued in the form of debt by the generation of borrowers born in $t-1$ and $R^e_{t+1}(t-1)$ to be the amount repaid in the form of equity by borrowers born in t. Such a debt-equity ratio defines the proportion of firms that issue equity with respect to those that are issuing debt in the same period.

Actually, both these variables evolve pro-cyclically; the amount of equity issued in $t+1$ is a linear function of p_{t+1}: $R^e_{t+1}(t-1) = p_{t+1}\hat{y}$. A higher p_{t+1} induces a higher $R^d_{t+1}(t-1)$ through the associated reduction of the bankruptcy region, even though bankruptcy costs have become higher because of the corresponding increase in the amount of loan offered[22].

We let this ratio to be[23]:

$$\sigma_{t+1} = \frac{R^d_{t+1}(t-1)}{R^e_{t+1}(t)} \frac{i_{t-1}}{i_t} \tag{33}$$

Notice that $R^d_{t+1}(t-1) = 1 - R^e_t(t-1) = 1 - p_t\hat{y}$. Then, (33) can be rewritten as

$$\sigma_{t+1} = \frac{1 - p_t\hat{y}}{p_{t+1}\hat{y}} \frac{i_{t-1}}{i_t}$$

As a matter of fact, the debt-equity ratio does fluctuate along the business cycle. If the first period expected outcome $\hat{y}$ is high enough, then a rise in the amount of external equity issued during expansions is more than offsetting the corresponding increase in debt financing. Let assume that in a two-period cycle $p_{t+1} = p_{t-1} > p_t$, so that an expansion takes place in $t+1$. Then, in order for σ to exhibit an anti-cyclical behavior we should have that:

$$\frac{1 - p_t\hat{y}}{p_{t+1}\hat{y}} \frac{i_{t+1}}{i_t} < \frac{1 - p_{t+1}\hat{y}}{p_t\hat{y}} \frac{i_t}{i_{t+1}} \tag{34}$$

and simple algebra shows that a sufficient condition for (34) to be satisfied is

$$\hat{y} > \frac{1}{2p_t i_t} \qquad \text{for every } t$$

4 Conclusive remarks

In the present work we tried to provide new support to the idea that structural imperfections in credit markets may originate persistent fluctuations in the level of economic activity. The analysis suggests a positive foundation

[22] The existence of a positive, tough non linear, correlation between p_{t+1} and the amount of debt $R^d_{t+1}(t-1)$ can be checked using again standard differentiation.

[23] Notice that, for simplicity reasons, (33) accounts for external equity only. We remark that in every period there exist internal equities issued by *three* generations of borrowers.

for endogenous cycle schemes: existence of fluctuations affects the structure of financial arrangements over time.

The main feature of our economy turns out to be the interaction between the dynamic contracting problem and the existence of finitely-lived firms. Such an interaction is the key to understand the role of liquidity and the potential alternance between debt and equity financing in our economy.

Nonetheless, an important issue should still be tackled: the present framework does not allow yet for any sort of equity issuing cost[24]. This induces two unpleasant consequences: equities are issued up to their physical maximum and, furthermore, we cannot explain very well the use of equity financing in projects that generate unobservable cash flows.

The next step of the present research is therefore supposed to be an allowance for a positive cost of repaying in the first period, so to get a more significative dynamic choice over repayments.

Finally, we briefly comment on some contributions related to ours. The closest research has been carried on in several works by Boyd and Smith[25]. They defined a framework that can allow for the co-existence of debt and equity financing in a one-stage borrowing relationship. In a further step, they have embedded such a contractual structure into an optimal growth framework in order to explain how equities are more and more issued during economic developments. Anyway, the growth process is entirely driven by investments in the commonly available technology given that the amount invested in non-verifiable projects is always set at its physical maximum. This differs from the business cycle explanation suggested here.

[24] It is known that in the basic Myers and Majluf (1984) paper the informative cost of equity plays a determinant role in defining the *pecking order* in financing choices.

[25] Boyd and Smith (1998), (1999).

Appendix 1

We recall that from (29) we have: $A_t = \dfrac{R_{t+1}}{p_{t+1}}$. The objective function (20) therefore becomes:

$$g(i_{t-1}, R_{t+1}) = i_{t-1}(p_t\hat{y} + p_{t+1}\hat{x} - 1) - \gamma(i_{t-1})\sum_n \pi_n \int_0^{\frac{R_{t+1}}{p_{t+1}}} dF(x) \quad (35)$$

Given that

$$\frac{\partial^2 g}{\partial i_{t-1}^2} = -\gamma''(i_{t-1})\sum_n \pi_n F(\frac{R_{t+1}}{p_{t+1}}) < 0$$

then (34) is strictly concave for all the $f(x)$ satisfying the following inequality:

$$\gamma''(i_{t-1})\gamma(i_{t-1})\sum_n \pi_n F(\frac{R_{t+1}}{p_{t+1}})f'(\frac{R_{t+1}}{p_{t+1}}) > \left[\gamma'(i_{t-1})\right]^2 \sum_n \pi_n \left[f(\frac{R_{t+1}}{p_{t+1}})\right]^2 \quad (36)$$

and a necessary condition for (35) to hold is to have $f' > 0$.

Appendix 2

We have to show that:

$$\frac{\partial i_{t-1}}{\partial p_{t+1}} > 0, \qquad \frac{\partial i_{t-1}}{\partial p_t} > 0 \qquad \textit{for every } t \tag{A2}$$

We consider the system (23)-(27) recalling that $\mu > 0$ implies $p_t y_n = R_t(y_n)$. Then, we have that $\phi_n = \dfrac{R_{t+1}(y_n)}{p_{t+1}}$.

From (23) and (25) considered together we get:

$$\left[NPV - \gamma'(i_{t-1})\sum_n \pi_n \int_0^{\phi_n} dF(x)\right]\left[(-\frac{1}{p_{t+1}})\frac{\gamma(i_{t-1})}{i_{t-1}}f(\phi_n) + 1 - F(\phi_n)\right] +$$

$$+(-\frac{\gamma(i_{t-1})}{p_{t+1}})\left[\frac{\gamma'(i_{t-1})i_{t-1} - \gamma(i_{t-1})}{i_{t-1}^2}\right]\sum_n \pi_n F(\phi_n) f(\phi_n) = 0 \tag{37}$$

To simplify notation, we define:

$\theta_1 \equiv NPV - \gamma'(i_{t-1})\sum_n \pi_n \int_0^{\phi_n} dF(x) > 0$ by (22)

$\theta_2 \equiv (-\dfrac{1}{p_{t+1}})\dfrac{\gamma(i_{t-1})}{i_{t-1}}f(\phi_n) + 1 - F(\phi_n) > 0$ by (24)

$\theta_3 \equiv (\dfrac{\gamma(i_{t-1})}{p_{t+1}})\left[\dfrac{\gamma'(i_{t-1})i_{t-1} - \gamma(i_{t-1})}{i_{t-1}^2}\right]\sum_n \pi_n F(\phi_n) f(\phi_n) > 0$

Equation (36) implicitly defines a relationship between i_{t-1}, p_{t+1} and p_t. Rewriting (36) as $h(i_{t-1}, p_{t+1}, p_t) = 0$, implicit differentiation gives us:

$$\frac{\partial i_{t-1}}{\partial p_{t+1}} = -\left[\frac{\partial h/\partial p_{t+1}}{\partial h/\partial i_{t-1}}\right] \qquad \frac{\partial i_{t-1}}{\partial p_t} = -\left[\frac{\partial h/\partial p_t}{\partial h/\partial i_{t-1}}\right]$$

Then, recalling that $f'(x) > 0$ and that γ is a convex function, standard algebra gives us:

$$\frac{\partial \theta_1}{\partial p_{t+1}} > 0, \frac{\partial \theta_2}{\partial p_{t+1}} > 0 and \frac{\partial \theta_3}{\partial p_{t+1}} < 0$$

It follows that: $\dfrac{\partial h}{\partial p_{t+1}} = \theta_1 \dfrac{\partial \theta_2}{\partial p_{t+1}} + \theta_2 \dfrac{\partial \theta_1}{\partial p_{t+1}} - \dfrac{\partial \theta_3}{\partial p_{t+1}} > 0$ and $\dfrac{\partial h}{\partial p_t} = \theta_2 \dfrac{\partial \theta_1}{\partial p_t} = \theta_2 \hat{y} > 0$

Finally:

$$\frac{\partial h}{\partial i_{t-1}} = \theta_1 \frac{\partial \theta_2}{\partial i_{t-1}} + \theta_2 \frac{\partial \theta_1}{\partial i_{t-1}} - \frac{\partial \theta_3}{\partial i_{t-1}} < 0 \tag{38}$$

Hence, if Problem 4 is a concave program, then (A2) is satisfied: a raising price implies, *ceteris paribus*, an increase in the contracted level of investments.

We conclude remarking that the following condition is holding as well:

$$\frac{\partial i_{t-1}}{\partial p_{t+1}} > \frac{\partial i_{t-1}}{\partial p_t} > 0 \qquad \textit{for every } t \tag{39}$$

that is, investments planned in $t-1$ are more sensible to p_{t+1} than to p_t. This can be checked by observing that:

$$\theta_2 \frac{\partial \theta_1}{\partial p_{t+1}} > \theta_2 \hat{y} = \theta_2 \frac{\partial \theta_1}{\partial p_t}$$

Appendix 3

We explain here how period doubling fluctuations may take place in this economy.

We start again from the system

$$i_{t-1}(p_t, p_{t+1})\hat{x} + i_t(p_{t+1}, p_{t+2})\hat{y} = D(p_{t+1})$$

As usual, we transform this second order non linear equation into a planar system. We define $z_t = p_{t+1}$ and we are left with:

$$i_{t-1}(p_t, z_t)\hat{x} + i_t(z_t, z_{t+1})\hat{y} = D(z_t) \tag{40}$$

$$p_{t+1} = z_t \tag{41}$$

Notice that (39) implicitly defines the function $m : z_{t+1} = m(z_t, p_t)$ and, given our previous results, we have that $\dfrac{\partial m}{\partial p_t} < 0$ and $\dfrac{\partial m}{\partial z_t} < 0$. Then, we can approximate the dynamics of the system around its stationary solution looking at the corresponding *Jacobian Matrix*:

$$J = \begin{bmatrix} \dfrac{\partial m}{\partial z_t} & \dfrac{\partial m}{\partial p_t} \\ 1 & 0 \end{bmatrix} \tag{42}$$

Now, the results stated above imply that $trJ < 0$ and $detJ > 0$, that is, J has two negative eigenvalues. Eigenvalues are:

$$\lambda_1 = \frac{m_z + \sqrt{m_z^2 + 4m_p}}{2}, \quad \lambda_2 = \frac{m_z - \sqrt{m_z^2 + 4m_p}}{2} \tag{43}$$

As a matter of fact, we have:

$$\lambda_1, \lambda_2 \leq 0 \qquad \text{and} \qquad \lambda_1 \geq \lambda_2 \tag{44}$$

We also introduce the following notation:

$A \equiv \dfrac{\partial i_{t-1}}{\partial p_t} = \dfrac{\partial i_t}{\partial p_{t+1}}$

$B \equiv \dfrac{\partial i_t}{\partial p_{t+2}} = \dfrac{\partial i_{t-1}}{\partial p_{t+1}}$

It follows that: $m_z = -\dfrac{\hat{x}B + \hat{y}A - D'(p_{t+1})}{\hat{y}B}$ and $m_p = -\dfrac{\hat{x}A}{\hat{y}B}$

In order to analyze the emergence of endogenous fluctuations we start by noticing that:

$$\frac{\partial \lambda_2}{\partial \hat{x}} \leq 0 \text{ whenever } \hat{x} \in K = \left\{ \hat{x} : \hat{x} \geq \frac{\hat{y}A + D'(p_{t+1})}{A - B} \right\}$$

Furthermore, we have that $\lambda_2 = -1$ if $\hat{x} = \hat{y} - \dfrac{D'(p_{t+1})}{A - B}$. Given (43), whenever $\lambda_2 = -1$, then $\lambda_1 \in [-1, 0]$; that is, λ_1 is inside the *unitary circle*. Now, in order for $\hat{y} - \dfrac{D'(p_{t+1})}{A - B}$ to be a bifurcation value for the parameter $\hat{x}$, the eigenvalue λ_2 should cross -1. In other words, we should have:

$$\hat{y} - \frac{D'(p_{t+1})}{A - B} \in K$$

It is straightforward to check that this condition holds if $\hat{y} \leq \dfrac{D'(p_{t+1})A}{(A - B)^2}$, with $\dfrac{D'(p_{t+1})A}{(A - B)^2} > 0$.

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