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PURELY GEOMETRICAL CONSIDERATIONS DURING THE DESIGN OF BRIDGES IN THE EARLY 20TH CENTURY – THE CASE OF R. MAILLART

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development of structural forms (morphogenesis), geometry, design methods, structural morphology, concrete, graphic statics, Maillart (Robert)

Abstract

By reviewing some of the reinforced-concrete structures built in the early 20th century, this paper questions the geometrical aspects surrounding their design. It is well established that in the case of Robert Maillart's designs, he relied on the logic of thrust lines, as was the case for masonry bridges that were provided by funicular polygons and graphic statics. Maillart also relied on trial sketches to define the mechanical features of successive sections of the bridge and on geometrical considerations to define the line of his bridges. In the case of his stiffened arch bridges, geometrical considerations were confined to following with almost regular thickness the trajectory of the thrust line. However, for some of them and for the whole family of three-hinged arch bridges, formal and geometrical considerations applied, for which some rules are presented here, and their evolution can be seen over time. The challenge in the geometrical organization of concrete around the thrust line is to equilibrate the stresses and manage the group of possible thrust lines depending on various loading cases. A well-designed concrete geometry avoids tensile stresses, which guarantees relatively long-lasting structures.

With Maillart's approach, an almost completely geometrical approach to design is encountered, since forces are also managed by geometry within the scope of graphic statics. Regular geometries and isostatic and symmetrical structures guarantee the possibility of undertaking a complete analysis using graphical approaches. However, when hyperstaticity or lateral forces are taken into account, the analysis is not as straightforward. This paper examines the extent to which the geometrical approach provides answers to these issues, with various hypotheses such as elastic analysis or a plastic lower-bound approach. The paper concludes by questioning this approach to the design of concrete structures in the early 20th century and its relative interest for structural design today.

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INTRODUCTION

The approach taken to designing structures is down to the individual designers and depends, among other things, on the mathematical and conceptual tools available to them. In the early 20th century, designers were dealing with a new material in the form of structural concrete and new issues about which forms to give concrete structures. Beyond reproducing forms inherited from masonry, timber or steel structures, some designers provided new forms devised for their specific conceptual contexts.

At that time, a series of designers devised geometries in concrete from the perspective of systems. A system made of concrete was a set of geometrical arrangements of structural members, dimensionings and a steel reinforcement characterized by its own arrangement and sizing. In relation to the issue of reproducing an existing arrangement from other materials, for example, Hennebique's system transposed masonry piles and supported iron girders to concrete; François Coignet made use of concrete in formworks in the way that rammed earth was set into formworks to build supporting walls. Constructive systems provide a way of building repetitive structures or are part of a more complex structural arrangement like a bridge.

At the start of the era of concrete, bridges inherited their final geometry and arrangement from bridges made from other material. Two families seemed to emerge: iron and steel families involving mostly a structural behavior related to bending forces with rather light structures, and the masonry family involving heavy structures where compressions forces are dominant.

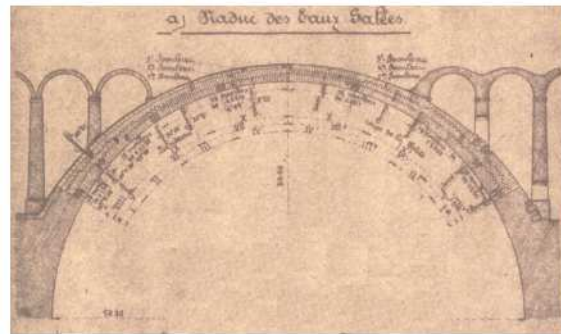


Figure 1: Massive masonry Viaduc des Eaux-Salées (1914)

The dominant science of structures at that time was the theory of elasticity that had been developed for iron structures, managing bending in complex frame structures. As the dominant theory then, it conditioned the way a structure was approached, and therefore the way in which a structural system made of concrete found its form. It led to the development of systems to secure the resistance of concrete beams, as shown by Hennebique's system involving the development of steel reinforcing stirrups.

Nevertheless, something occurred that cannot be included in this theoretical framework. The designer Robert Maillart (1872-1940) produced a series of remarkable bridges that are not easy to interpret as a collection of beams, columns and arches. The underlying thinking of these structures will be seen and the limits of the structural thinking behind it will be shown: this is when other rationales become apparent, some of them linked to purely geometrical considerations.

FAMILIES OF BELONGING AND BEHAVIORS IN MAILLART'S BRIDGES

As shown in Zastavni (2013), Maillart developed several typologies for concrete bridges that can be categorized into six families: three-hinged arch bridges, massive classical arch bridges, arches with a strongly off-center thrust line, stiffened arch bridges, continuous girder bridges and, later on, cantilever bridges. He did not invent these typologies, but developed them for construction with structural concrete in line with its specific characteristics.

Maillart's three-hinged bridges and massive classical arch bridges are the translation of masonry bridges into concrete ones.

Maillart's stiffened arch-bridges are the translation of inverted suspension bridges into a concrete arrangement.

Maillart's continuous girder bridges come from arrangements where the span is viewed as a beam, but managed on a geometric basis that is associated with another structural types.

Maillart's arch with a strongly off-center thrust line is hybrid, a kind of classical compressed arch whose behavior escapes the original logic, going beyond compression to become subject to major bending forces.

Maillart's cantilever bridges are in line with modern boxed sections subject to bending forces and designed like cantilever beams.

The latter two typologies do not correlate with the combined influence of geometries and structural behavior since a geometrical adjustment cannot be shown, with underlying structural behavior remaining practically independent of the geometry. However the first four will be detailed and analyzed below in specific chapters.

MAILLART'S MASSIVE ARCH BRIDGES AND THREE-HINGED ARCH BRIDGES

Maillart's concrete massive arch bridges inherited their features from masonry bridges which had emerged out of the theory of masonry vaulting that originated in the 18th century with Philippe de la Hire among others. This theory uses the principle of thrust lines that are presumed to remain in the material of the arch. In France, the "Epure de Mery" (1840) applied this theory to bridges. The geometry of the arc of a massive arch made of concrete is mostly an arc of a circle. The reason for this is simply the ease of describing the geometry that needs to be used on the building site.

Looking at Maillart's massive arch bridges, a circular geometry was used (Maillart's Aareburg Bridge, 1911-12). However with three-hinged arch bridges, the geometries used will vary and demonstrate the nature of the geometrically-oriented thinking.

Maillart started with very classical bridges before 1900: rounded circular arch-bridges made of concrete but strictly implementing the arrangement of a massive masonry bridge. Maillart's Stauffacher Bridge in Zurich (1899) marked the start of a gradual adaptation. This bridge is hollow above the concrete thickness needed to support the anticipated loadings.

Maillart's Zuoz Bridge (1901) was the first time a concrete box section appeared (Billington 1979), where in fact two systems coexist: a supporting U-shaped arch intended to guide thrust forces through its materiality and a hollow box complex integrating the U-shaped arch, lateral walls and the deck's slab. The former system refers to thrust lines and almost parabolic geometries close to funicular geometries of equilibrium. The latter – a hollow concrete box, also known

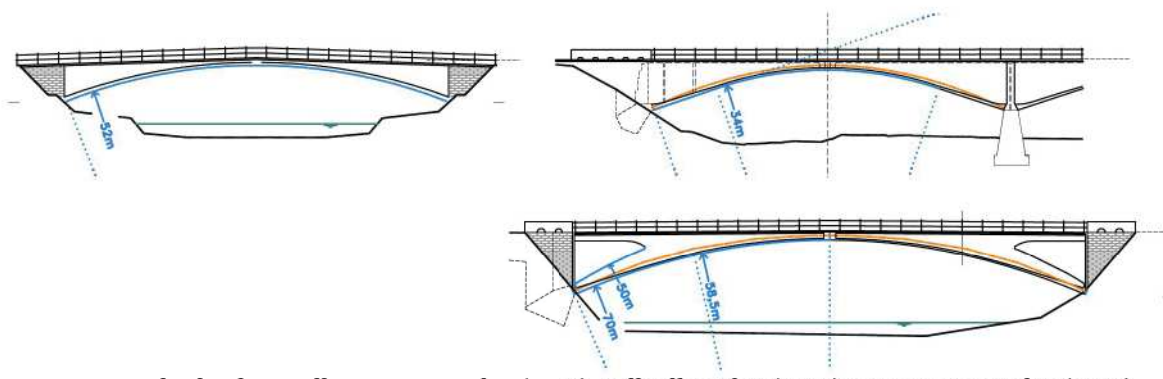


Figure 2: geometrical rules for Maillart's Zuoz Bridge (1901), Billwill Bridge (1904) & Tavanasa Bridge (1905)

as the “Système Maillart” – is related nowadays to sections intended to sustain bending forces and therefore refers to another structural behavior. The ensuing question is therefore what geometry should be used to be consistent with expected and devised structural behaviors? With the Zuoz Bridge, the answer remains simple: this bridge was in keeping with the spirit of massive bridges even if it was hollowed out, and therefore a simple arc of a circle was used for the bottom line of the arch.

With his next three-hinged arch-bridge – the Billwil Bridge (1904) – this thinking was taken further. According to the logic of working with compression arcs, the arch tries to get closer to a parabola (with a line and an arc of a circle) and the transversal box section simplified. Lateral walls remain complete on both sides.

In 1905, Maillart designed the Tavanasa Bridge. A new typology appeared because this time the lateral walls were hollowed near to the supporting hinges. The core of the arch, the deployment of its line, was therefore intended to remain close to a thrust line that is almost parabolic.

The notion of thrust line refers to the idea of a load path and is closely related to graphic statics. The thrust line is drawn according to the loadings encountered and via the funicular polygon in the space diagram in the context of graphic statics representing the structure. Each thrust line is specific to a given loading and varies accordingly. The challenge for a design acting mainly on compression is for the thrust line to remain as close as possible to the thrust line with the core of the structure's geometry, and therefore minimize bending moments. The problem is therefore this: a structure is supposed to resist various loading cases, while the ideal thrust line and geometry matches only one loading case to become the reference loading case. Which one should be selected? An analysis has to be performed to determine which loading is dominant. In the case of heavy masonry bridges, the dead load is far greater than the live load, which is frequently ignored in the design of the geometry of the arch bridge. Maillart's view of the way in which the structure is expected to behave depended on the reference thrust line and the corresponding loading case. For the series of Maillart's early three-hinged arch bridges, the live loads were 250kg/m^2 and a punctual mobile loading was less than or equal to 6 metric tons, which is relatively insignificant. In Maillart's Salginatobel Bridge (1929), loading modestly increased to 350kg/m^2 and 8 metric tons. These bridges were therefore designed in the spirit of heavy masonry bridges. Another interesting answer to the issue of the reference loading case will be provided later with Maillart's stiffened arch-bridges that will be analyzed in a specific chapter below.

Constructive devices such as hinges impact structural behavior and also influence the structural morphology. The example of hinges within an arch is explicit: the arch can become thinnest around the hinges that are being crossed by the thrust, while the thrust can move throughout the whole thickness of the arch in unarticulated arches. This means that thickness is very important everywhere in this scenario. So for three-hinged bridges, the arch becomes thinnest around the hinges and thickens further away from them. The issue is one of regulating the geometry to fit the expected structural behavior, while simultaneously considering the need for a regulating geometry to frame the construction phase. These are the reasons why it can be appropriate to go beyond purely structural reasons to give the structure its final geometrical arrangement. In the case of the Tavanasa Bridge, the core geometry for the middle line of the arch is the one of the thrust line linked to the trajectory of forces bound to the structural behavior in compression. The bottom of the arc is a lenticular dilatation of the mean parabolic thrust line and is geometrized as a succession of two arcs of a circle with diminishing radii. In Tavanasa, these are 58,5 and 70 meters. Loadings also indicate the geometries to be given to the foundation blocks. If the arch is rather flat with a low sag and heavy loadings, the thrust at the support is considerable due to the

very important horizontal component of the supporting forces. Since the solution for masonry bridges is to associate this quite oblique thrust with a heavy loading of the foundations to obtain a verticalized resultant that can fit the capacity of the supporting ground, this impacts the scale of the supporting blocks directly. From there, the geometry of the foundations will be drawn with characteristic orientations that are horizontal, and perpendicular thrust lines and oblique lines making a symmetrical angle around the resulting thrust line.

Maillart's next three-hinged arch bridge was the Salginatobel Bridge (1929). From Maillart's working drawings, the constraints for the design appear similar: a narrow bridge with a width of 3,5 meters, a 350kg/m² live load and a punctual mobile load equal to 8 metric tons. The reference loading case was the dead load with a continuous arched line at the center. Live loads represent only 8% of the total loading. This is why Maillart used a continuous circular line for the middle of the bridge at its center. However there were two specific constraints that led Maillart to re-work the geometries and exceed the simple rule of an arch design according to two radii: the bridge is long and it is high.

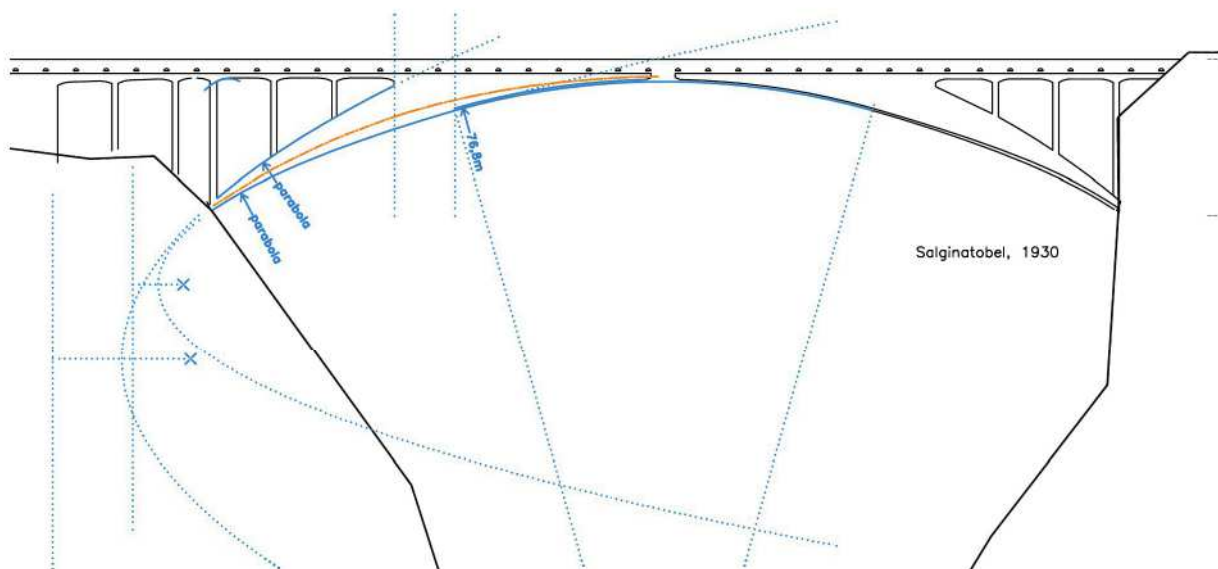


Figure 3: geometrical rules for Maillart's Salginatobel Bridge (1929)

Its length meant that the deck could not be supported only by its contact at the center with the supporting arch and at both sides by abutments. Maillart needed additional support designed as columns, as implemented in classic massive concrete bridges such as Maillart's Aareburg Bridge (1911-12) and Hennebique's Châtellerault Bridge (1899). A rectangular frame made of columns and beams supporting the deck was used, but there is something interesting about their connection. A parabolic transition curve was used and this refers to Maillart's broad study on connections between columns and mushroom slabs (Zastavni, 2012). Given its height, Maillart had to deal with the need to connect the central arc of a circle with the supports, while carving the intrado of the arch to stay close to the thrust line and limit bending. The result is that he used a parabolic line for the quarter spans on both sides, but the directrix of the parabola was... horizontal (Fivet & Zastavni, 2012)! It shows that, in terms of the bordering geometry, the parabola was used as a purely graphic means that is disconnected with any structural logic linked to funiculars and thrust line. This specific use of a parabola as a graphic means can also be observed elsewhere on the ground plan, on sides of the elevation of piers, and for the limiting upper curve of the supporting arch. It emphasizes therefore the difference between the inherent structural ge-

ometries, where funiculars or parabolas define the core middle line of the average geometry, and the geometrical adaptations of the limits of the arrangement by way of Euclidean or quadric figures.

In conclusion, the structural idea behind this series of bridges ending with the Salginatobel Bridge, despite their feature of implementing a concrete box section, is that they are the heirs of massive masonry bridges where the dead load is dominant, where live loads are almost disregarded and where thrust lines are used to define the average geometry. When it comes to the final geometrical arrangement, arcs of a circle and eventually parabolas were chosen to match the average geometry when Maillart's practice matured.

MAILLART'S LATER THREE-HINGED ARCH BRIDGES

Maillart's later three-hinged arch-bridges were built between 1932 and 1940. An evolution in the use of bridges motivated the changes required in Maillart's three-hinged prototypes. These were the increase in live load and in the widths of bridges. Similarly, Maillart distanced himself from his paradigms about what a concrete arch bridge was supposed to look like.

From his 1932 bridges onwards, the widths of his bridges were between 8 and 10 meters. This increased the total loading and required the occurrence of arched ribs on the underside of the bridges to be increased. Arches became just two or three simple rectangular boxes on simple U-shapes instead of one.

A highly significant element is the relative importance of live loads. Compared to the Salginatobel Bridge (8% of the total loading), it became 38% of total loading in the Vessy Bridge. This meant that it was not yet a light bridge where both loadings were of the same order of magnitude, but it was nevertheless relatively significant. Maillart therefore had to expect more variations in possible thrust lines. This had a very obvious geometrical impact: the height of the arch increased everywhere except around the hinges, so that the geometrical envelope had to expand by a lenticular side extension of the reference loading case, except at the hinges. As a consequence, the connection between both curves of each half-bridge were broken and the form slightly ogival. To draw the geometrical envelop of the arches, Maillart rationalized the hinges and curves. The geometrical rule remains the same: two arcs of a circle with increasing radii when getting close to the support or straight lines. For each design, Maillart had to find the right sequence of the two centers and the corresponding radii, with measurements in round numbers. The design of the Vessy bridge is eloquent: it was first sketched as the succession of an arc of a circle whose radius is 150 meters followed by a straight line towards the support. Then when

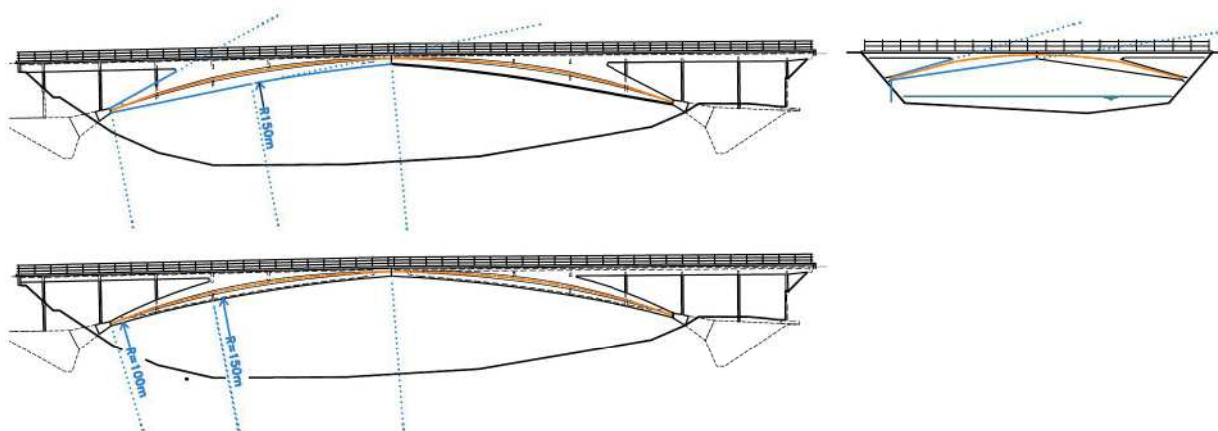


Figure 4: geometrical rules for Maillart's Vessy Bridge as sketched and as built (1936) & Garstadt Bridge (1940)

Maillart adjusted the capacity of resistance to a lower loading, it was sketched as the succession of an arc of a circle whose radius is 150 meters followed by an arc whose radius is 100 meters. Since these geometries are implemented around a reference middle line linked to a thrust line, and considering the increasing magnitude of the live load, the question is: what is its corresponding loading case? The analyses show that it remains a dead load only and that the variations due to the distributed live loads and mobile punctual loadings have almost no influence.

Variations in this geometrical framework can be observed in Maillart's later bridges. Maillart's Garstatt Bridge (1940) has arches that are only composed of straight lines, radicalizing Maillart's geometrical management of three-hinged arch bridges as introduced with the Vessy Bridge.

MAILLART'S STIFFENED ARCH BRIDGES

This bridge corresponds to the structural model of a suspended bridge, but with an inversion between compression and traction forces considering the funicular member. This was suggested by Wilhelm Ritter when Maillart was still a student (Billington, 1997). This option cast an interesting light on the issue of the reference loading case and its variations under live loads.

As shown elsewhere (Billington, 1973), the stiffened arch bridge is the complementary association of a funicular arch supporting permanent loads by compression only, and a rigid deck acting as a girder against live loads (Fig. 11). So the geometrical issue of the middle line is simplified since there is a specific device supposed to sustain bending forces caused by variations induced by live loads. As for Maillart's other bridges, the reference loading case remained dead loads only.

The only geometrical issue was about the form to be given to the funicular supporting arch. The Valterschielbach Bridge (1924) is a simple and unique arc of a circle. Being relatively flat, its thickness between 23 and 29 centimeters enables the corresponding funicular thrust line to remain inside the thickness of the supporting arch. A change occurred in Schwandbach (1933). The supporting arch became simply funicular and therefore polygonal. Its thickness decreased to 20 centimeters. The ideal geometry would be polygonal, but the segments should also be funicular since there are dead loads.

It is remarkable that on no occasion did Maillart try to obtain the ideal geometry coming directly from the ideal structural form that was materialized most of the time by a funicular polygon. With regard to the construction process, there was always a simplification of the obtained geometry to rationalize it.

CONCLUSIONS AND PERSPECTIVES

This paper answers the question of how the geometries are dealt with in reinforced concrete where there is a concern for matching structural behavior as closely as possible with a material as adaptive as concrete. It takes rules from Robert Maillart's work involving several typologies of

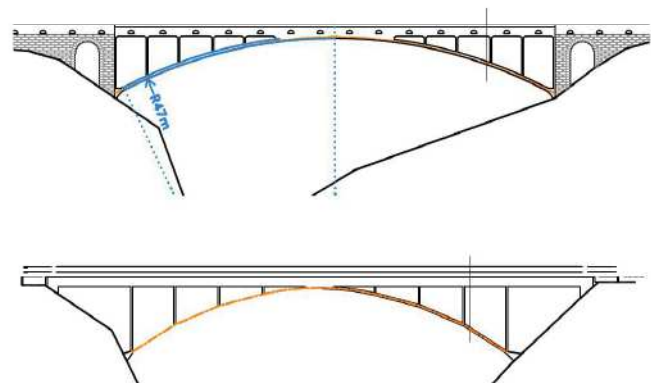


Figure 5: geometrical rules for Maillart's Valterschielbach Bridge (1924) & Schwandbach Bridge (1933)

bridges between 1899 and 1940. The analysis shows that there were two major antagonistic pretexts for doing so: to meet requirements concerning structural efficiency and reliability and to meet the need to build with geometrical rules that are as simple as possible. Maillart's methods show where structural rationales act on the average geometrical framework, mostly by way of a funicular load path obtained with graphic statics, and when a geometrical adaptation with simple lines and figures has been used to define the final detailed geometry. Adaptations are necessary since the core geometry has to be related to a reference loading case that is to be modified by live loads.

A little further on, the final geometries modify or condition the structural behavior and question the core structural geometry. But since reinforced concrete has shown adaptive behaviors in the framework of plastic lower-bound approaches, the challenge becomes one of best matching the expected structural behavior so as not to cause damage during the reorganization of forces. This is where the geometrical thinking demonstrated by Maillart makes such sense. It resolves the issue of hyperstaticity during the design phase, where many behaviors can be expected depending on the relative stiffness of constituting members and boundary conditions as revealed by elastic analyses. Any problem of this kind can be solved by designing structural behaviors using the principle of the lower bound theorem and then place the material at best to match the corresponding expected forces. This is how Maillart proceeded (Ochsendorf, 2005) and this is how the analysis of thrust is managed, as shown with Méry. In this context the geometrical procedure used by Maillart makes completely sense and acquires a very contemporary significance.

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