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Abstract

With forward ownership, an upstream supplier internalizes the effect of its supply contracts on the downstream firms, which is so far understood to decrease prices. We show that instead downstream prices generally increase if firms use two-part tariffs. The price-increasing effect of forward ownership occurs with both observable and secret two-part tariffs, albeit for different economic reasons. The results arise under both quantity and price competition as well as for different belief refinements. Partial forward ownership can be more profitable and more harmful for consumers than a full vertical merger between an upstream and a downstream firm.

JEL classification: L22, L40, L8
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1 Introduction

Minority shareholdings between firms in a supply relationship (partial vertical ownership) are prevalent in various industries. Examples include cable operators and broadcasters, banks and payment providers, financial exchanges and clearing houses, as well as automobile producers and their suppliers.[1] Empirical studies have shown that ownership arrangements serve as important instruments to manage and facilitate business relationships (see for instance Flath, 1996; Allen and Phillips, 2000; Fee et al., 2006). Whereas the main effects of ownership links between competitors (horizontal ownership) on pricing and output decisions appear to be well understood, the effects of partial vertical ownership are arguably less clear.

The main concern regarding partial vertical ownership is that it can lead to foreclosure. In the case of forward ownership, for instance, if a supplier obtains only a share of its downstream customer's profit but fully controls the target firm, it can be profitable to require the downstream firm not to purchase from another upstream rival.[2] On the positive side, partial forward ownership can lead to efficiencies, such as reducing the double marginalization problem (Flath, 1989). The academic literature on partial forward ownership therefore is broadly in line with the policy view on vertical mergers as reflected in the European Commission's guidelines.[3]

We contribute to the developing literature and demonstrate that partial forward ownership can be detrimental even in the absence of foreclosure concerns. Our focus is on how such forward ownership shares affect the supply contracts in a vertically-related industry. We therefore set up a model that allows for competition in both market segments. The upstream firms produce a homogeneous input good and have different marginal production costs. We allow the efficient supplier to hold passive forward ownership shares of two downstream firms that compete either in prices or quantities. Such passive ownership shares yield a profit participation but do not confer control over the target firm's decisions. Importantly, this setting allows us to abstract from any foreclosure incentives.

In our analysis, we identify novel price-increasing effects of forward ownership that are arguably neglected in recent antitrust treatment of partial vertical ownership acquisitions. First, we confirm the insight of Flath (1989) that forward ownership can reduce prices (i.e., reduced double marginalization) and show this for both observable and secret linear contracts. In stark contrast, however, we demonstrate that the same ownership structure can have price-increasing effects if firms use two-part tariffs. Contrary to prior arguments, forward ownership can therefore lead to strong price and profit increases in the industry

[1]See European Commission (2013) for an overview of partial ownership acquisitions in the EU and some of its member states. Moreover, Brito et al. (2016), Greenlee and Raskovich (2006), Hunold (2020), and Hunold and Shekhar (2018) provide additional details for specific cases.

[2]Levy et al. (2018) refer to this as customer foreclosure. Similarly, input foreclosure, where a supplier refuses to sell input to an independent downstream firm, can be particularly profitable when a downstream firm holds a share of a supplier (backward ownership).

[3]European Commission guidelines on the assessment of non-horizontal mergers (2008/C 265/07) paras. 10 - 18. The guidelines also mention the possibility of coordinated effects from which we abstract in this article.

to the detriment of consumers. These price increases materialize both with observable and unobservable two-part contract offers, albeit for different economic reasons.

If the supplier's contract offers are observable and competing downstream firms have a supply alternative, the supplier can extract more profits from them by selling at marginal prices below the level that maximizes industry profits in order to extract high fixed fees (Caprice, 2006; Farrell and Shapiro, 2008). As forward ownership reduces the supplier's incentive to extract rents from the downstream firms, we find that it induces the supplier to attach a higher weight on increasing the overall industry profit. This leads to marginal input prices that are closer to the industry profit-maximizing level and thereby to higher output prices downstream. We also find that partial forward ownership can be more harmful than a full vertical merger between an upstream and a downstream firm.

With unobservable two-part tariffs, we show that non-controlling partial forward ownership raises prices even in the presence of a monopolist supplier. In this case the downstream firms realize zero profits independent of the ownership share. It might therefore appear puzzling that partial forward ownership affects the equilibrium outcome although the supplier participates in zero downstream profits. Indeed, there is no price effect with an upstream monopolist and observable two-part tariffs. Moreover, the direction of ownership matters: there is no price effect with non-controlling partial backward ownership in this setting. The reason why higher marginal input prices are sustainable in equilibrium is that the supplier internalizes that its opportunistic (off-equilibrium) behavior would harm the downstream firms. This result holds also in the case of upstream competition as well as for the belief refinements of passive and wary beliefs. In this section, we also discuss how the mechanisms of partial and full vertical integration (as analyzed from Hart and Tirole (1990)) differ from each other.

If vertically-related firms establish a partial ownership arrangement, it might be that this makes contract offers observable, such that the supplier can overcome the commitment problem described above. We therefore investigate the case in which partial ownership not only affects the contract terms but also the information structure of the contracting game. For the case of downstream price competition, the *observability effect* affects prices in the same direction as the *internalization effect*: with linear contracts prices tend to decrease and with two-part tariffs they tend to increase. In the case of downstream quantity competition, the two effects may run in different directions, such that the overall price effect depends on the relative strength of both effects.

In summary, we contribute to the developing theoretical literature studying the competitive effects of partial vertical ownership. Beyond established results on foreclosure, we identify novel anticompetitive price effects of partial forward ownership. A central insight of the related literature so far is that forward ownership reduces double marginalization prices compared to vertical separation (Flath, 1989; Fiocco, 2016). In contrast, backward ownership does not have this feature (Greenlee and Raskovich, 2006), but can instead increase prices (Hunold and Stahl, 2016). By demonstrating anticompetitive price effects of forward ownership, we show that both ownership structures can be detrimental and that forward ownership also raises competition policy concerns beyond foreclosure. An important corollary of our results is that a less strict treatment of non-controlling acquisitions compared to controlling ones can be misguided.

The article is structured as follows. We set up the model in Section 2 and analyze public contracting in Section 3 and secret contracting in Section 4. In Section 5 we study the effects of partial ownership acquisitions that make previously secret contracts of rivals observable.

We provide additional analyses in Section 6. In Section 6.1, we extend the analysis to asymmetric shareholdings. This analysis confirms that the overall effect of asymmetric ownership on the price level and consumer surplus tends to go in the same direction as that of symmetric ownership. The section also relates to the strand of the literature which related partial vertical ownership and foreclosure (Baumol and Ordover, 1994; Spiegel, 2013; Levy et al., 2018). Asymmetric ownership induces the supplier to offer more favorable contract terms to a partially-integrated downstream firm than to an independent competitor. In Section 6.2, we discuss when partial vertical ownership links are profitable for the industry as a whole and for the firms involved in a partial ownership acquisition. Finally, we show in Section 6.3 that firms cannot achieve the same effects of forward ownership by means of a profit-sharing supply contract. We conclude in Section 7 with a more detailed discussion of how our results can contribute to a better treatment of partial ownership acquisitions in merger control, which is an ongoing competition policy debate in jurisdictions including the European Union. Besides the novel insight on how partial forward ownership can be anticompetitive in the absence of foreclosure issues, we explain why it is crucial to take into account the pricing schemes of the vertically-related firms to correctly assess the price effects.

2 Industry and ownership structure

Industry structure. To study the competitive effects of forward ownership, we set up a model that allows for both upstream and downstream competition. There are two symmetric downstream firms which need inputs in order to produce substitutable products.

The production of one unit of downstream output requires one unit of a homogeneous input. There are two ways of producing the input goods. The efficient upstream firm U produces the input at marginal costs normalized to zero. In addition, there is a less efficient competitive fringe, denoted by V, which can also produce the input, but at higher marginal costs of $c > 0$.

The cost difference between the efficient supplier and the fringe, c, is a measure of the intensity of upstream competition. If c is large, the competitive fringe is not a relevant competitor and the efficient supplier is de facto a monopolist. For sufficiently small cost differences, the fringe is a relevant supply alternative and its presence constrains the price setting of the efficient supplier. This improves the downstream firms' position *vis-à-vis* the efficient supplier U.

Contracting. The efficient upstream firm offers each downstream firm $i \in \{A, B\}$ a contract that has a linear (marginal) price w_i and (possibly) a non-linear upfront fee f_i that the downstream firm pays upon contract acceptance. For $f_i = 0$, the tariff is linear. The competitive fringe offers the inputs at marginal costs c.[4] We analyze observable contract offers of the efficient supplier in Section 3 and unobservable ones in Section 4.

Partial ownership. The efficient supplier U holds a partial ownership share $\sigma_i \in [0, 1]$ of downstream firm $i \in \{A, B\}$ (forward ownership). We consider passive ownership that does not confer control over the target firm's strategy, such that the partial ownership is essentially a claim on the corresponding share of the downstream firm's profit. This assumption is employed in several articles of the established literature, such as Flath (1989), Greenlee and Raskovich (2006), and Hunold and Stahl (2016). Focusing on the polar case of non-controlling ownership allows us to illustrate novel pricing effects that arise if the supplier internalizes a share of the downstream firms' profits. Hunold and Schlütter (2021) relax this assumption and also allow an acquiring firm to exercise influence over the target's strategy.[5] Mainly for illustrative purposes, we allow values of the profit share of up to 100%, although, in practice, at least shares above 50% might typically be associated with control.

Sections 3 and 4 focus on symmetric ownership with$\sigma_A = \sigma_B = \sigma$ to abstract from foreclosure incentives. We generalize to asymmetric ownership and relate our results to foreclosure in Section 6.1.

Quantification of price effects. We derive our main results for a general model that allows for price and quantity competition in the downstream market under standard assumptions on demand and profits. Additionally, we illustrate some of our results with closed-form solutions based on the quadratic utility (consumer surplus) of a representative consumer (Singh and Vives, 1984; Häckner, 2000):

$$CS\left(q_A, q_B, I\right) = q_A + q_B - \frac{1}{2}\left(q_A^2 + q_B^2 + 2\gamma q_A q_B\right) + I. \tag{1}$$

The consumer obtains utility from consuming the products from downstream firms A and B and a numeraire good I. The parameter $\gamma \in (0, 1]$ captures the degree of product substitutability. The budget-constrained consumer's maximization problem yields the

[4]This modeling choice avoids technical challenges that arise from considering interlocking relationships with several strategic suppliers, such as the inexistence or multiplicity of equilibria (Miklós-Thal et al., 2010).

[5]In particular, building on O'Brien and Salop (2000), this article points out that a forward profit right can have the same effects as a backward control right (and vice versa) in terms of strategic incentives (such as pricing).

inverse linear demand function

$$p_i(q_i, q_{-i}) = 1 - q_i - \gamma q_{-i}. \tag{2}$$

For $\gamma \to 0$, the product markets are separated and for $\gamma = 1$ the products are perfect substitutes. As an example, we compute the price effects of a 15% forward ownership acquisition, which appears to be a typical and relevant acquisition size. For instance, in a report about partial ownership acquisitions in different jurisdictions, the European Commission (2013) provides a quantitative analysis of 91 transactions in the Zephyr database in which the average post-transaction shareholding is 14.3%. Moreover, in some jurisdictions (such as Germany, Austria, Canada, and USA) acquisitions of this size can be subject to merger review proceedings.

3 Observable contracts

In this section, we derive our results for the case of observable contract offers. We first show that symmetric forward ownership of the efficient supplier U in both downstream firms ($\sigma_A = \sigma_B = \sigma$) can lead to *lower* upstream and downstream prices when upstream tariffs are linear and upstream competition is weak (c is sufficiently large). This is in line with the insights of Flath (1989).[6] Crawford et al. (2018) provide empirical support of this theoretical result by showing that partial vertical ownership indeed reduces double marginalization in the US television market where—as they argue—input prices are predominantly linear.

We then show that this ownership structure can instead lead to *higher* marginal input prices when the supplier uses observable two-part tariffs. Forward ownership can therefore raise prices under non-linear tariffs, while it might reduce prices under linear tariffs. This is arguably an important new insight that should find consideration in the assessment of partial forward ownership acquisitions.

For a given ownership structure, we analyze the following non-cooperative game where each firm maximizes its objective function, taking into account that the supplier internalizes a share of the downstream firms' profits:

1. Supplier U offers each downstream firm an input contract with tariff $t_i = (w_i, f_i)$, $i \in \{A, B\}$. The fringe V offers the input at its marginal cost of c.

2. Downstream firms A and B observe the contract offers made by U and simultaneously accept or reject the offer.

3. Each downstream firm sources inputs (from U if it accepted $U's$ contract, otherwise from the fringe V), produces, and sells its products.

[6] We study asymmetric ownership structures in Section 6.1.

We solve for the subgame-perfect Nash equilibrium in which both downstream firms purchase from the efficient supplier by backward induction.

The profit of downstream firm i net of fixed fees is $\pi_i = (p_i - w_i) q_i$. We conduct the analysis in this section using reduced-form downstream flow profit that depend on both firms' input prices $\pi_i(a, b)$, where a denotes the marginal costs of downstream firm i and b denotes the marginal costs of its competitor $-i$. This reduced-form formulation is consistent with both quantity and price competition downstream.

We assume that there exists a unique and stable equilibrium in the downstream market, and that the profits have the following standard properties.[7]

Assumption 1. *The flow profit $\pi_i(a, b)$ of downstream firm i*

(i) decreases in the own input costs: $\partial \pi_i(a, b) / \partial a < 0$,

(ii) increases in the competitor's input costs: $\partial \pi_i(a, b) / \partial b > 0$ (in the range where both firms make positive sales), and

(iii) decreases when all input costs increase: $\partial \pi_i(a, b) / \partial a + \partial \pi_i(a, b) / \partial b < 0$.

(iv) Moreover, $\partial^2 \pi_i(a, b) / (\partial b)^2$ is not too negative.

We verified that the linear demand function introduced above fulfills the assumptions that we impose on the reduced form objective functions and the industry profit in this section. The assumption implies that an increase of the input costs of both firms leads to a lower total output and higher downstream prices (both if the downstream firms' strategic variables are prices or quantities). In the case of linear demand, part (iv) of the assumption is fulfilled as the term $\partial^2 \pi_i(a, b) / (\partial b)^2$ is positive with both Cournot and Bertrand competition.

If the upstream firm U supplies both downstream firms, the total profit of downstream firm i is

$$\pi_i(w_i, w_{-i}) - f_i. \tag{3}$$

In this case the upstream firm U's objective function consists of its own flow profit and the downstream firms' profits, weighted by the ownership shares σ_i:

$$\Omega^U(w_A, w_B, f_A, f_B) = \underbrace{\pi^U(w_A, w_B) + f_A + f_B}_{\text{upstream profit}} + \underbrace{\sum_{i \in \{A,B\}} \sigma_i (\pi_i(w_i, w_{-i}) - f_i)}_{\text{internalized downstream profits}}, \tag{4}$$

with $\pi^U = \sum_{i \in \{A,B\}} w_i q_i$. In order to ensure that the supplier's maximization problem has a unique interior solution, we impose

Assumption 2. *$\pi^U(w_A, w_B) + \sum_{i \in \{A,B\}} \sigma_i \pi_i(w_i, w_{-i})$ is strictly concave in w_A and w_B for all $\sigma_i \in [0, 1]$.*

[7] See, for instance, Farrell and Shapiro (2008) and Levy et al. (2018) for similar assumptions on reduced form profits.

This assumption ensures concavity of the supplier's objective function for the case of linear tariffs ($f_i = 0$). As we will see in Section 3.2, Assumptions 1 and 2 together imply that also with two-part tariffs the supplier's problem is strictly concave in the marginal input prices. Finally, this assumption implies that the industry profit is strictly concave in the marginal input prices such that there exists a unique pair of input prices that maximize the industry profit:

$$\left(w_A^I, w_B^I\right) = \arg\max_{w_i} \sum_{i \in \{A,B\}} p_i q_i. \tag{5}$$

These input prices do not depend on the degree of forward ownership. The sum of all firms' profits (industry profit) increases if the input price approaches w_i^I (either from above or below).

3.1 Linear tariffs

We now fix $f_i = 0$, which implies that supply contracts are linear. Under vertical separation, linear tariffs result in double marginalization (Cournot, 1838). For an upstream monopoly (as with $c = \infty$), this means that the linear input prices (and the resulting downstream prices) are above the level that maximizes the industry profit. Partial forward internalization can alleviate this problem by reducing the upstream prices. The problem of supplier U is to

$$\max_{w_A, w_B} \Omega^U (w_A, w_B) = \pi^U (w_A, w_B) + \sigma (\pi_A (w_A, w_B) + \pi_B (w_B, w_A)), \tag{6}$$

subject to the constraint $w_i \leq c$, which ensures that both downstream firms source from U. The corresponding first-order condition is

$$\frac{\partial \Omega^U}{\partial w_i} = \frac{\partial \pi^U (w_i, w_{-i})}{\partial w_i} + \sigma \left(\frac{\partial \pi_i (w_i, w_{-i})}{\partial w_i} + \frac{\partial \pi_{-i} (w_{-i}, w_i)}{\partial w_i} \right) = 0, \; i \neq -i \in \{A, B\} \tag{7}$$

Denote with $w_A^l(\sigma) = w_B^l(\sigma) = w^l(\sigma)$ the symmetric linear input price that solves the above system of first-order conditions. As the supplier internalizes a share of each downstream firm's profit, there is an incentive to decrease the symmetric linear input price: $\partial w^l(\sigma)/\partial\sigma < 0$. This internalization effect yields input and downstream prices below the level of vertical separation and is therefore procompetitive.

If upstream competition is fierce, the (unconstrained) optimal input price $w^l(\sigma)$ may be above the marginal cost of the competitive fringe. In this case, the internalization effect derived above does not materialize and upstream firm U sets the input price to the highest possible level of c. Hence, the equilibrium input price is $w = \min\left\{w^l(\sigma), c\right\}$. If the competitive fringe is an effective constraint under vertical separation, a partial forward ownership share only affects the equilibrium if it is sufficiently large, such that the unconstrained price $w^l(\sigma)$ falls below c. We summarize in

Proposition 1. *Let the efficient supplier U charge observable linear input prices ($f_i = 0$) and internalize a share $\sigma > 0$ of each downstream firm's profit. This leads to lower input* *prices and downstream prices than vertical separation if upstream competition is weak or non-existent (c sufficiently large). If upstream price competition is strong enough, forward ownership does not affect prices.*

Proof. See Appendix A. □

This proposition replicates the result of Flath (1989) for the case in which the supplier sets a linear input price instead of a production quantity. We obtain qualitatively the same result if the efficient supplier is unrestricted in its price setting. With competition from an efficient enough supply alternative, there is no reduction in the linear input price if the partial forward ownership share is small.

Quantification of price effects with linear demand. With linear demand as defined in Equation (2) and quantity competition in homogeneous products ($\gamma = 1$), the optimal unconstrained input price is $w^l(\sigma) = (3 - 2\sigma)/(6 - 2\sigma)$, which equals 1/2 for $\sigma = 0$ and decreases in σ. This implies that any marginal increase in σ reduces prices if $c > 1/2$. For $c > 1/2$, we find that a 15% forward ownership share decreases final consumer prices by 2.6%. For $c < 1/2$, forward ownership is competitively neutral in an interval of σ starting at $\sigma = 0$ but it can affect prices once σ is large enough (such that $w^l(\sigma) < c$). For instance, at a share $\sigma = 0.25$, we obtain $w^l(0.25) = 0.45$, which implies that forward ownership above 25% only decreases prices if $c > 0.45$.[8]

3.2 Two-part tariffs

We now show that forward ownership can lead to higher marginal input prices and thus industry profits, compared to the case of vertical separation if the supplier uses two-part tariffs. Again, let supplier U internalize a symmetric share $\sigma_A = \sigma_B = \sigma$ of each downstream firm's profit. The maximization problem of supplier U is

$$\begin{aligned} \max_{w_i, f_i,\ i \in \{A,B\}} \Omega^U &= \pi^U(w_A, w_B) + f_A + f_B + \sigma \sum_{i \in \{A,B\}} (\pi_i(w_i, w_{-i}) - f_i) \\ s.t. &\quad \pi_i(w_i, w_{-i}) - f_i \geq \pi_i(c, w_{-i}),\ i, -i \in \{A, B\}, \end{aligned} \tag{8}$$

where the latter participation constraints mean that each downstream firm i must weakly prefer sourcing from U to sourcing from the competitive fringe at linear costs of c, which yields the outside option of $\pi_i(c, w_{-i})$. The competitive fringe is a *relevant supply alternative* if a downstream firm can obtain positive profits when sourcing from the competitive fringe. Otherwise, for a sufficiently large c, a downstream firm's outside option has a value of zero and does not depend on the rival's input costs.

In equilibrium, supplier U sets the fixed fees such that each downstream firm is just indifferent between the contract offer and its outside option such that each firm sources

[8] The detailed computations of all parametric examples are available upon request.

from U. Hence, the reduced maximization problem is

$$\max_{w_A, w_B} \Omega^U = \pi^I (w_A, w_B) - (1-\sigma) (\pi_A (c, w_B) + \pi_B (c, w_A)), \quad (9)$$

where $\pi^I = \pi^U + \sum_{i \in \{A,B\}} \pi_i$ denotes the industry profit. The implied system of first-order conditions is

$$\frac{\partial \Omega^U}{\partial w_i} = \frac{\partial \pi^I (w_i, w_{-i})}{\partial w_i} - (1-\sigma) \frac{\partial \pi_{-i} (c, w_i)}{\partial w_i} = 0, \; i, -i \in \{A, B\}. \quad (10)$$

Denote the symmetric optimal marginal input price of the two-part (tp) tariff by $w_A^{tp}(\sigma) = w_B^{tp}(\sigma) = w^{tp}(\sigma)$. For the extreme case of $\sigma = 1$, Equation (10) is the optimality condition as in the case of vertical integration and the optimal marginal prices maximize the industry profit. The same holds true if $\partial \pi_{-i}(c, w_i) / \partial w_i = 0$, which occurs, for instance, if the competitive fringe is not a supply alternative (i.e., c sufficiently large). In this case, the upstream firm can extract all downstream profits through the fixed fees and simply maximizes the industry profit by setting the marginal input price equal to w^I.

The situation is different, however, if the downstream firms obtain positive profits should they source from the competitive fringe and if the profit internalization due to forward ownership is incomplete ($\sigma < 1$). With such a relevant supply alternative, the outside option profit of a downstream firm increases if its competitor faces higher marginal input costs: $\partial \pi_{-i}(c, w_i) / \partial w_i > 0$. The supplier thus faces a trade-off between a high industry profit π^I and less valuable outside options $\pi_i (c, w_{-i})$ for the downstream firms. As a result, the supplier charges input prices below the industry profit-maximizing level. Reducing the marginal input price allows the supplier to extract higher profits via the fixed fee.[9]

The supplier's marginal profit from lowering a downstream firm's outside option shrinks in the internalization share σ (Equation (10)). Intuitively, partial internalization of the downstream profits makes decreasing these outside option profits less attractive and the supplier puts more emphasis on maximizing the industry profit. We summarize in

Proposition 2. *Let supplier U charge observable two-part tariffs. With upstream competition (c sufficiently small), forward internalization $\sigma > 0$ leads to higher marginal input prices and downstream prices, compared to vertical separation. Without upstream competition, supplier U sets the input price w^I such that downstream prices maximize the industry profit for all $\sigma \in [0, 1]$.*

Proof. See Appendix A. □

The result that, with two-part tariffs, forward internalization leads to higher marginal input prices is in stark contrast to the result of price reductions with linear tariffs (Proposition 1). If the supplier can charge two-part tariffs, double marginalization is not a

[9] For the case of vertical separation, this strategic effect of reducing the downstream firms' outside options by means of low marginal input prices also finds consideration in, for instance, Marx and Shaffer (1999) and Caprice (2006).

concern for the firms. In contrast, with up- and downstream competition, input prices and profits are too low from an industry perspective in the case of vertical separation. This makes an *increase* of the input prices profitable and we show in Proposition 2 that forward ownership makes it feasible and unilaterally profitable for a supplier to do so.

Quantification of price effects with linear demand. Under homogeneous quantity competition with linear demand as defined in Equation (2), the competitive fringe is a relevant supply alternative for the downstream firms if $c < 0.625$. For smaller marginal costs of the competitive fringe, the optimal input price is $w^{tp}(\sigma) = (4c(1-\sigma) + 2\sigma - 1)/2(3-\sigma)$, which increases in $\sigma \in [0, 1]$. If the competitive fringe has larger marginal costs in this example, a downstream firm's outside option is to stay inactive instead of sourcing from the fringe if it refuses U's offer, and partial forward ownership does not affect the marginal input prices. For $c = 1/10$, a forward ownership share of 15% increases final consumer prices by 9.2% in this specification relative to the case of no ownership links. This increase in input prices is considerably larger than the 2.6% decrease of prices in the linear tariffs case for the same ownership share of 15%.

Comparison of non-controlling partial forward ownership and a full vertical merger. Sandonis and Fauli-Oller (2006) study a full vertical merger between an upstream firm and one downstream firm in a comparable setting. They explain that the vertical merger has a positive and a negative effect on total surplus (which corresponds to the sum of consumer surplus and the profits of U, A, and B). The positive effect for total surplus is that the merged entity loses the commitment ability to charge its downstream unit a wholesale price above its own marginal cost. This effect is strong if the competitive fringe has high marginal costs c, because in this case the efficient supplier charges high marginal wholesale prices under vertical separation. The negative effect of a vertical merger on total surplus is that the merged entity completely internalizes the industry profits when making a two-part tariff offer to the independent downstream firm. This yields the incentive to charge a higher wholesale price than under separation to the independent downstream firm because there is no incentive to decrease the outside option of the integrated downstream unit. This effect is more important for small fringe costs c. As a result, Sandonis and Fauli-Oller find that for large enough fringe costs the merger increases total surplus.[10]

Consequently, non-controlling forward ownership, which we find to reduce total surplus and consumer surplus compared to vertical separation (Proposition 2), can be more harmful than a vertical merger. For example, in the linear-demand specification with downstream quantity competition and parameter values of $\gamma = 3/10$ and $c = 4/10$, the efficient supplier holding partial forward ownership of $\sigma > 2/10$ in each downstream firm reduces consumer surplus more than full vertical integration between the efficient supplier and

[10]See their Proposition 4 for quantity competition and their extension in Section 5 for price competition.

one of the two downstream firms and reduces total surplus if $\sigma \geq 0.25$.

The reason for this result is that with non-controlling forward ownership, the supplier does not lose the ability to charge wholesale prices above its marginal costs from both downstream firms and therefore can better control the downstream market outcome. This hurts consumers through higher prices. In general, it turns out that partial forward ownership tends to be more harmful than a vertical merger if

- the partial ownership share is larger,
- the fringe's costs c are larger – as long as the fringe is a relevant alternative – and
- the degree of downstream substitutability γ is smaller.

Partial ownership in a single downstream firm can also be more detrimental than a full merger between the same two firms if the ownership share is large enough.[11] We summarize this result in

Lemma 1. *Non-controlling partial forward ownership can reduce consumer surplus and total surplus more than a full vertical merger between the efficient supplier and one downstream firm.*

Proof. See Appendix A. □

The comparison of partial ownership and a full merger emphasizes that the effects of partial ownership are not merely the convex combination of the polar cases of vertical separation and vertical integration. Strikingly, non-controlling partial acquisitions can be more harmful than full and controlling ones. This shows that a less strict treatment of non-controlling acquisitions in antitrust law might lead to detrimental and unintended effects. If, for instance, a competition authority prohibits a full vertical merger but allows a supplier to acquire non-controlling shares of the downstream firms, consumers can be worse off compared to the full vertical integration case. Furthermore, these results show that partial forward ownership can be detrimental even if it is fully non-controlling. In this case, typical arguments regarding market foreclosure where the partial owner uses control over the target to achieve foreclosure at limited profit losses do not apply.

4 Secret contracts

In this section, we analyze contracting in the vertical chain if the supplier's contract offers are unobservable. In particular, we adjust the information structure to the case in which a downstream firm cannot observe the contract terms and acceptance decision of the rival downstream firm before making its own sourcing and sales decisions.[12] We find that

[11] In the specification above for instance, this is the case if the ownership share in one downstream firm i exceeds $\sigma_i > 40\%$.

[12] This is the case of interim unobservability in the terms of Rey and Vergé (2004).

the direction of the main competitive effects derived for observable contracts—both with linear and two-part tariffs—prevails. With two-part tariffs, they apply even more broadly also to the case of upstream monopoly.

It is well known that the construction of the equilibrium with unobservable contracts depends on whether downstream firms compete in prices or quantities (Rey and Vergé, 2004). In this section, we focus on quantity competition and provide the analysis of two-part tariffs and downstream price competition in Appendix B where we obtain qualitatively similar results.

We solve for a symmetric perfect Bayesian-Nash equilibrium as contract unobservability leads to an incomplete information game. Each downstream firm needs to form a belief about the other downstream firm's contract offer. We assume that the downstream firms hold passive beliefs about their rival's contract offer (see, e.g., Hart and Tirole, 1990; McAfee and Schwartz, 1994; Rey and Vergé, 2004). This implies that a downstream firm thinks that its competitor is served at the equilibrium contract and that it does not update its belief about the other contract if it receives an out-of-equilibrium offer.[13] In the perfect Bayesian-Nash equilibrium the on-path beliefs are correct. Denote a downstream firm's belief about the competitor's rival in capital letters as $T_{-i} = (W_{-i}, F_{-i})$. Based on this belief, there is a one-to-one mapping regarding the downstream firm's expectation about the quantity Q_{-i} that its rival sells.

We assume that the inverse demand $p_i\left(q_i, q_{-i}\right)$ for each downstream firm $i \in \{A, B\}$ is symmetric and satisfies $\frac{\partial p_i}{\partial q_i} < \frac{\partial p_i}{\partial q_{-i}} < 0$. We further impose

Assumption 3. $2\frac{\partial p_i(q_i,q_{-i})}{\partial q_i} - \frac{\partial p_{-i}(q_i,q_{-i})}{\partial q_i} + \frac{\partial^2 p_i(q_i,q_{-i})}{\partial^2 q_i} q_i < 0,$

which ensures that the downstream firms' profit functions are strictly concave and that downstream profits decrease for a uniform increase in the marginal input price. It is therefore the analog to Assumption 1 in the section on observable contract offers. In addition, we assume that the downstream firm's second-order conditions are fulfilled in the relevant range in order to guarantee a unique and stable symmetric equilibrium in the downstream market.[14]

Anticipating that the other downstream firm produces the equilibrium quantity Q_{-i}, the expected profit of downstream firm i is

$$\pi_i = \left(p_i\left(q_i, Q_{-i}\right) - w_i\right) q_i - f_i, \tag{11}$$

where it expects to sell at a price $p_i\left(q_i, Q_{-i}\right)$ that depends on its own quantity choice q_i and its beliefs about the quantity Q_{-i} that its competitor will produce. We follow Rey and Vergé (2004) and denote the optimal strategic decision of a downstream firm as a function of its input price:

$$q_i\left(w_i\right) \quad = \quad \arg\max_{q_i} \left(p_i\left(q_i, Q_{-i}\right) - w_i\right) q_i - f_i. \tag{12}$$

[13]For the case of two-part tariffs, we analyze the belief refinement of wary beliefs in Appendix C.

[14]This assumption holds if $\partial^2 \pi_i / \partial^2 q_i + \left|\partial^2 \pi_i / \partial q_i \partial q_{-i}\right| < 0$.

Importantly, due to contract unobservability, the optimal strategic choice of downstream firm i cannot adjust if the rival's input price w_{-i} changes. It therefore only depends on the downstream firm's marginal input price w_i and its belief about the rival's quantity choice Q_{-i}. For the sake of a concise exposition, we do not display the downstream firm i's belief about its rival marginal input price W_{-i} or the corresponding quantity choice Q_{-i} as an additional argument in $q_i(w_i)$. Note that the strategic decision $q_i(w_i)$ is independent of the fixed fee f_i and therefore the same with linear and two-part tariffs under secret contracting.

4.1 Linear tariffs

We first analyze the case of unobservable linear tariffs with $f_i = 0$. This section builds on Gaudin (2019) who shows that under vertical separation double marginalization also arises under secret contracting. There is thus scope for forward ownership to reduce double marginalization, as in the case of observable linear tariffs.

Given the downstream firms' beliefs about each others' contract offers and corresponding quantity choices, the supplier's objective function is

$$\Omega^U = \sum_{i,-i\in\{A,B\}} w_i q_i(w_i) + \sigma\left[\left(p_i\left(q_i(w_i), q_{-i}(w_{-i})\right) - w_i\right) q_i(w_i)\right]. \tag{13}$$

Similar to Assumption 2 for the case of observable contracts, we impose

Assumption 4. *The supplier's objective function in Equation* (13) *is strictly concave in* w_A *and* w_B *for all* $\sigma_i \in [0,1]$.

Note that whereas the downstream firms expect to sell at the price $p_i(q_i(w_i), Q_{-i})$, the supplier knows both contract offers as well as the beliefs, and anticipates a downstream price of $p_i(q_i(w_i), q_{-i}(w_{-i}))$.[15] The difference to the case with observable contract offers is that a change in w_i only affects the quantity choice of downstream firm i. When changing w_i, the other downstream firm $-i$ is only affected through the effect of i's quantity choice on the aggregate price. Taking this strategic behavior into account, the supplier's first-order condition, $\partial\Omega^U/\partial w_i = 0\ i \in \{A,B\}$, is

$$\begin{aligned} & q_i(w_i) + w_i \frac{\partial q_i(w_i)}{\partial w_i} \\ + \quad & \sigma\left[\underbrace{-q_i(w_i) + q_{-i}(w_{-i})\frac{\partial p_{-i}(q_{-i}(w_{-i}), q_i(w_i))}{\partial q_i}\frac{\partial q_i(w_i)}{\partial w_i}}_{<0}\right] = 0. \end{aligned} \tag{14}$$

Denote the symmetric input price that solves the system of first-order conditions with $w_A^{lu}(\sigma) = w_B^{lu}(\sigma) = w^{lu}(\sigma)$, where $w^{lu}(\sigma) = W_i,\ i \in \{A,B\}$. The superscript lu stands

[15] Recall that $q_i(w_i)$ implicitly also depends on i's belief about its rival marginal input price W_{-i} and the corresponding quantity choice Q_{-i}, but only w_i is observable for i and therefore directly influences $q_i(w_i)$.

for linear and unobservable contracts. If the marginal costs of the competitive fringe are sufficiently large, this is the equilibrium input price. We show in the proof to the next proposition that the term in brackets is negative. Hence, by the same reasoning as in the proof of Proposition 1, we obtain that a larger degree of partial forward internalization leads to lower input prices.

If the competitive fringe has marginal costs $c < w^{lu}(\sigma)$, the supplier charges an input price as high as the fringe's marginal costs, that is, $w^{lu}(\sigma) = c$. In this case, partial forward ownership does not affect the linear input price. We summarize in

Proposition 3. *Let the efficient supplier U charge unobservable linear input prices ($f_i = 0$) and internalize a share $\sigma > 0$ of each downstream firm's profit. With quantity competition and passive beliefs, this leads to lower input prices and downstream prices than vertical separation if upstream competition is weak or non-existent (c sufficiently large). Otherwise, forward ownership does not affect prices.*

Proof. See Appendix A. □

Quantification of price effects with linear demand. With the same linear demand specification as in Section 3.1 (quantity competition in homogeneous products with $\gamma = 1$), a forward ownership share of 15% decreases final consumer prices by 2.1% if the competitive fringe is sufficiently inefficient. For unobservable linear contracts, the competitive fringe is sufficiently inefficient if $c > 2/5$. The reduction of 2.1% is smaller than in the case of observable linear tariffs where the reduction amounts to 2.6%. Note that this is consistent with Gaudin (2019) who shows for the case of vertical separation ($\sigma = 0$) that double marginalization is less severe under secret contracts if the downstream firms' actions are strategic substitutes (i.e., under quantity competition in our setting).

4.2 Two-part tariffs

With secret two-part tariffs and under vertical separation, the supplier suffers from the well-known commitment problem (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994).[16] It can secretly offer each downstream firm a contract with a low linear price as this maximizes the bilateral profit, which it can extract through the fixed fee. Under vertical separation, this can lead to marginal prices equal to the supplier's marginal costs, and in turn low downstream prices and industry profits. In this section we show that partial forward ownership acts as a commitment device for the supplier to charge higher marginal input prices. Relatedly to this result, Flath (1996) provides empirical evidence from Japanese business groups (keiretsu) that minority shareholdings can deter opportunism.[17]

[16] Fiocco (2016) studies secret contracting and partial backward ownership, but excludes that an upstream firm supplies two competing downstream firms. Hence, there is no commitment problem à la Hart and Tirole (1990).

[17] Note that the mechanism to reduce opportunism identified in Flath (1996) differs from the one that is the focus of this section. In particular, Flath (1996) shows theoretically that partial ownership holdings

In equilibrium, the supplier offers a contract such that the downstream firms are indifferent between accepting the contract and their outside option. We denote the outside option profit that a downstream firm expects to realize if it sources from the competitive fringe at an input price of c as $\pi_i(c, W_{-i})$; Equation (36) contains the formal definition. If the marginal costs of the competitive fringe are too large, supplier U is a monopolist and the downstream firm's outside option profit is equal to zero. From a downstream firm's profit in Equation (11), it follows that the fixed fee is equal to

$$f_i = (p_i(q_i(w_i), Q_{-i}) - w_i) q_i(w_i) - \pi_i(c, W_{-i}). \tag{15}$$

The determination of the fixed fee is conceptually the same as with observable two-part tariffs: it extracts the downstream firm's profit on the equilibrium path up to the outside option $\pi_i(c, W_{-i})$. The only difference is that the fee now depends on a downstream firm's expectation about its rival marginal input price W_{-i} and the corresponding quantity choice Q_{-i}.

The supplier, who internalizes a share σ of each downstream firm's profit, has the following objective function for given beliefs of the downstream firms about each others' output decisions:

$$\begin{aligned} \Omega^U(w_A, w_B, f_A, f_B) \quad = \quad & \sum_{i\in\{A,B\}} w_i q_i(w_i) + f_i \\ & + \sigma\left((p_i(q_i(w_i), q_{-i}(w_{-i})) - w_i) q_i(w_i) - f_i\right). \end{aligned} \tag{16}$$

Whereas f_i depends only on the contract offered to downstream firm i (and its belief), the profit that the supplier internalizes depends on the contract offers to both downstream firms. Intuitively, if the supplier changes the contract with one downstream firm, this has no effect on the fixed fee that it can extract from the second downstream firm simply because the latter firm cannot observe this change. In contrast, changing the contract for one firm has an effect on the eventually realized profits and the supplier internalizes this effect with a weight of σ.

Our starting point is that the well-known equilibrium with marginal cost pricing does not exist with partial forward ownership.

Lemma 2. *Let supplier U charge unobservable two-part tariffs. The downstream firms compete in quantities and hold passive beliefs. With forward ownership, there always exists a unique symmetric perfect Bayesian-Nash equilibrium with passive beliefs. In equilibrium, the marginal input prices are <u>not</u> equal to the supplier's marginal costs.*

Proof. See Appendix A. □

In order to establish a perfect Bayesian-Nash equilibrium under partial forward ownership with secret two-part tariffs, it is generally not sufficient that the first-order conditions hold.

strengthen a firm's ability to penalize opportunistic behavior by selling the shares. In contrast, our results do not rely on the threat of selling the shares.

It is also necessary to verify that the supplier has no incentive to change both contracts simultaneously (a multilateral deviation). For vertical separation, Rey and Vergé (2004) show that an equilibrium exists if there is downstream quantity competition or price competition that is not too intense. The lemma above extends the existence result for quantity competition to the case of partial forward ownership.

In the following proposition, we establish that partial forward ownership leads to an increase of the prices also with secret two-part tariffs.

Proposition 4. *Under the conditions in Lemma 2, the equilibrium marginal input prices (and thus the downstream prices) increase in the degree of forward internalization σ.*

Proof. See Appendix A. □

Proposition 4 shows that the profit internalization of partial forward ownership reduces the supplier's commitment problem that secret contracting causes under vertical separation. The reason is that the supplier internalizes a share of the loss of one downstream firm if it secretly offers a lower input price to the downstream rival. Hence, such an ownership structure is an effective commitment to higher input (and thus downstream) prices. Importantly, the price-increasing effects of partial forward ownership occur regardless of whether there is a relevant supply alternative or not. This implies that partial forward ownership may also have price effects in the case of an upstream monopolist that makes secret contract offers, while it is neutral in this case under observable tariffs (Proposition 2).

Moreover, Lemma 2 establishes that an equilibrium in passive beliefs exists if downstream firms compete in quantities. The contracts, however, do not affect the supplier's maximization problem in a separable way, as is the case under vertical separation. This implies that the main justification for employing passive beliefs does not hold if the supplier internalizes a share of the downstream profits. In Appendix C, we therefore additionally analyze the belief refinement of *wary beliefs* whereby—in the case of an out-of-equilibrium offer—a downstream firm anticipates that the supplier might also have an incentive to change the contract offer to the downstream rival. As with passive beliefs, we find that forward internalization yields higher input prices than vertical separation and therefore reduces the commitment problem.

Quantification of price effects with linear demand. Under homogeneous quantity competition with linear demand as defined in Equation (2) ($\gamma = 1$), a forward ownership share of 15% increases final consumer prices by 9.5%. This is comparable to the price increase of 9.2% with observable two-part tariffs and the competitive fringe with $c = 0.1$ (Section 3.2).

Comparison of non-controlling partial forward ownership and a full vertical merger. Hart and Tirole (1990) show that a full vertical merger can solve the supplier's commitment problem. This theory, however, relies on the premise that vertical integration shifts the residual control rights and removes all conflicts regarding prices and trading policies between the merging up- and downstream firms. Moreover, in Hart and Tirole (1990) the direction of integration does not matter for the competitive effects to occur. However, passive partial forward ownership arrangements differ in important ways, such that one cannot assume the same effects as in the case of a full vertical mergers to occur. First, passive forward ownership only confers partial profit rights but no control rights over the target firm, such that there are still conflicts of interests between the firms when trading. Second, it is important that with an monopoly supplier making take-it-or-leave-it offers, the downstream firms make zero profit in equilibrium because the supplier can extract all profits through the fixed fees. This occurs irrespective of whether the supplier holds a forward ownership share or not. As the downstream profit that the monopoly supplier partially internalizes through its forward ownership stake is equal to zero in either case, it is therefore not straightforward that partial ownership has any effect. Indeed, there is no price effect of partial forward ownership when an unconstrained monopoly supplier charges observable two-part tariffs (see Proposition 2). Third, in contrast to full vertical integration, the direction of ownership matters in the case of partial vertical ownership. Importantly, passive backward ownership does not lead to anticompetitive price effects in the same setting (Hunold and Schlütter, 2019).

5 Discussion of the competitive effects with observable and secret tariffs

Different channels of price increases due to forward profit internalization. Whereas the profit internalization due to forward ownership can be procompetitive with linear tariffs (Sections 3.1 and 4.1), our analysis shows that it weakly increases the supplier's input prices with both observable and unobservable two-part tariffs (Sections 3.2 and 4.2). See Table 2 for an overview. In the case of linear tariffs (first two columns), the reason for the price increases is similar under both information structures. When an upstream firm internalizes part of the downstream margin, it has an incentive to cut its input prices. This effect may not materialize, however, if the competitive fringe is sufficiently efficient because the optimal price is then defined by a corner solution. We say that there is a relevant supply alternative if under vertical separation the efficient supplier sets the linear input price w equal to the fringe cost c because a higher price would yield zero sales. In this case, small degrees of forward ownership have no effect on the downstream prices. If the forward share is so large that the optimal unconstrained linear input price falls below c, such an acquisition reduces prices.

Upstream market structure	Supply contracts			
	Linear tariffs		Two-part tariffs	
	Observable	Secret	Observable	Secret
No relevant supply alternative[18]	*Down*	*Down*	*None*	*Up*
Relevant supply alternative	*None or down*	*None or down*	*Up*	*Up*

Table 2: Effects on downstream prices of partial forward ownership relative to vertical separation for different tariffs and upstream market structures.

Sources: Propositions 1 and 3 contain the results for linear tariffs, Propositions 2 and 4 the results for two-part tariffs.

With two-part tariffs, the reasons for the price increases depend on the information structure. Under observable two-part tariffs (Column 3 in Table 2) and a relevant supply alternative for the downstream firms, the supplier strategically depreciates the outside option profit of the downstream firms in order to extract a larger share of the total profits via the fixed fee. This strategic reduction of the downstream firms' outside options comes at the cost that the total profit is below the joint profit maximum. With partial forward ownership, the supplier attaches a lower weight on strategically reducing the downstream profits (as it internalizes a share of these profits). In turn, it attaches a higher weight to achieving a high total profit that is closer to the joint profit maximum, which increases marginal input and final prices.

If the supplier is an unconstrained monopolist (no relevant supply alternative for the downstream firms), it already implements the monopoly prices with observable two-part tariffs absent any vertical ownership links. The supplier can extract all profits through the fixed fee and sets marginal input prices in order to maximize the industry profit. Obtaining a fraction σ, say 15%, of the downstream profits of zero through the ownership participation therefore does not affect this optimization, such that the equilibrium prices are unaffected.

With unobservable two-part tariffs (Column 4 in Table 2), the starting point under vertical separation is different as the supplier has an opportunism problem which generally leads to prices below the monopoly level, even when the supplier is an unconstrained monopolist. As Hart and Tirole (1990) have shown, this problem can lead to marginal wholesale prices equal to costs. Importantly, as with observable contracts, the monopolistic supplier extracts all profits from the downstream firms through the fixed fees, irrespective of whether the supplier internalizes a share of these profits or not. It may therefore seem odd at first glance that forward ownership actually affects the equilibrium input prices as the monopolist supplier does not receive any profit from its forward shareholding on the equilibrium path. Nevertheless, we show that forward ownership allows the supplier

[18] The critical threshold for the fringe's marginal costs c such that it is a relevant supply alternative generally varies between the cases.

to commit to higher input prices as it now internalizes a part of the losses that an off-equilibrium deviation (i.e., its opportunism in terms of a lower marginal input price for the competitor) would cause. This commitment effect leads to higher downstream prices and lower consumer surplus.

We find the same price effects if the efficient supplier faces fringe competition. The main difference is that the downstream firms have a positive outside option and therefore receive a lower fixed fee compared to the case with a monopolist supplier, but the downstream firms' outside option does not affect the marginal input prices.

Crucially, while partial forward ownership reduces the supplier's commitment problem identified by Hart and Tirole (1990) and leads to higher prices, partial backward acquisitions do not have this feature.[19] This is an important difference to the case of full vertical integration in which the direction of acquisition does not matter for the competitive effects.

Partial ownership can help to make contract offers observable. In general, vertical ownership links may also change the information structure in the vertical chain.[20] In particular, it is possible that contracts are unobservable under vertical separation but partial ownership links allow the firms to exchange information in a credible way and build trust, such that the opportunism problem due to contract unobservability ceases to exist.[21] While we do not explicitly model how partial ownership makes contracts observable, our analysis is nevertheless instructive for this case. In particular, it allows us to compare the equilibrium under contract unobservability and vertical separation with the equilibrium under contract observability and partial forward ownership shares. Distinguishing between downstream price and quantity competition turns out to be important.

In the following tables, we summarize for each mode of downstream competition (prices and quantities) the price effects that occur from partial forward ownership if this acquisition additionally makes otherwise secret contracts observable. The internalization effect refers to the results that we have derived in Sections 3 and 4 for observable linear and two-part tariffs. We define as observability effect the price change that occurs under vertical separation when the information structure changes from secret contracts (with passive beliefs) to observable contracts. We collect results for the observability effect from the related literature, notably Caprice (2006), Hunold and Stahl (2016), and Gaudin (2019).

With downstream price competition, the effect of making contracts observable unambiguously reinforces our results on the effects of partial forward ownership: both the price-reducing effects of internalizing shares of the downstream profits with linear tariffs

[19] In particular, Hunold and Schlütter (2019) show that passive backward ownership does not confer additional commitment power to the supplier and the equilibrium input costs remain at the same level as under vertical separation.

[20] For instance, Fee et al. (2006) provide evidence that partial ownership aids in bonding trading parties together.

[21] See also Do and Miklós-Thal (2021) who study a dynamic recontracting game and relate the extent of the opportunism problem to fundamentals of the negotiation process, such as the parties' reaction speeds and their patience.

as well as the price-increasing internalization effects with two-part tariffs are reinforced (see Table 3).

For linear supply contracts (first two rows of tables 3 and 4) and the case of vertical separation, Gaudin (2019) shows that double marginalization is more (less) severe under secret contracts than public ones if the downstream firms' actions are strategic complements (substitutes).[22] In our setting, this means for the case of downstream price competition that the observability effect tends to reduce prices. Hence, partial forward ownership that additionally makes linear contracts observable is at worst neutral (if the competitive fringe is very efficient such that neither the observability nor the internalization effect materializes) and decreases prices otherwise.

In rows 3 and 4 of Table 3, we summarize the effects for the case of two-part tariffs. If the contract offers are secret (and under the belief refinement of passive beliefs), marginal input prices are equal to the supplier's marginal costs under vertical separation (Hart and Tirole, 1990; Rey and Vergé, 2004). This implies that, unless the supplier offers marginal input prices below its marginal costs if contracts are observable, the change in the information structure leads to higher prices. If the efficient supplier is a monopolist in the upstream market (no relevant supply alternative), it clearly does not charge marginal input prices below its marginal costs.[23] The same holds true in the case of upstream competition by a less efficient competitive fringe and downstream price competition, which implies that both the observability effect and the internalization effect increase prices in this case.[24]

[22]In our analysis strategic substitutes relates to the case of Cournot competition and strategic complements to the case of Bertrand competition in the downstream market.

[23]In the extreme of downstream monopolists, the supplier optimally sells the input at marginal costs and extracts the monopoly profit via the fixed fee. For downstream competition, the supplier charges higher marginal input prices in order to induce the downstream firms to set the monopoly price. This holds both for downstream price and quantity competition.

[24]For fringe costs close to zero ($c \to 0$), Hunold and Stahl (2016) show in Proposition 5 that U sets $w > 0$. Implicit differentiation of Equation (A5) therein (and using the downstream firms' first-order conditions) yields $\partial w / \partial c > 0$. Hence, for all c, U sets $w > 0$.

		Effects on downstream prices due to		
	Upstream market structure	profit internalization	contract observability	both (total effect)
Linear tariffs	Relevant supply alternative	*None or down*	*None*	*None or down*
	No relevant supply alternative	*Down*	*Down*	*Down*
Two-part tariffs	Relevant supply alternative	*Up*	*Up*	*Up*
	No relevant supply alternative	*None*	*Up*	*Up*

Table 3: Effects on the downstream prices of observability and profit internalization (under price competition)

See Table 2 for details on the internalization effects. The observability effect describes how prices change if the information structure changes from secret to observable contracts (under vertical separation).

With downstream quantity competition, the comparison is more subtle both with linear and two-part tariffs (see Table 4). The first two rows cover the case of linear supply contracts. With a relevant supply alternative, the results are the same as with price competition. If the efficient supplier is not restricted by the competitive fringe, however, Gaudin (2019) shows that the observability effect leads to higher prices if the downstream firms compete in quantities. The observability effect and the internalization effect therefore generally go in opposite directions in this case and the overall effect is unclear.[25]

		Effects on downstream prices due to		
	Upstream market structure	profit internalization	contract observability	both (total effect)
Linear tariffs	Relevant supply alternative	*None or down*	*None*	*None or down*
	No relevant supply alternative	*Down*	*Up*	*Unclear*
Two-part tariffs	Relevant supply alternative	*Up*	*Up or down (depending on c)*	*Up or down (depending on c)*
	No relevant supply alternative	*None*	*Up*	*Up*

Table 4: Effects on the downstream prices of observability and profit internalization (under quantity competition)

See Table 2 for details on the internalization effects. The observability effect describes how prices change if the information structure changes from secret to observable contracts (under vertical separation).

The last two rows of Table 4 describe the case of two-part tariffs. With an upstream monopoly, the internalization effect again is neutral while the observability effect leads

[25]For the linear demand function in Equation (2) with $\gamma = 1$, we find that the detrimental effect of making contracts observable is compensated by the procompetitive effect of forward ownership if $\sigma > 1/2$.

to higher prices (by the same reasoning as with downstream price competition). If the competitive fringe is relatively efficient (c small enough), both effects go in opposite directions. In particular, Caprice (2006) shows that with downstream quantity competition the (observable) marginal input prices can in fact be below the supplier's marginal costs if the competitive fringe is sufficiently efficient. In this case, the observability effect and the internalization effect therefore go in opposite directions.[26] We summarize these findings in

Summary 1. The competitive effect of making contracts observable tend to go in the same direction as the internalization results derived in Sections 3 and 4: with linear contracts, prices tend to decrease, whereas they tend to increase. As described above, these results are clear-cut in the case of downstream price competition. Subject to some qualifiers, the same holds for the case of downstream quantity competition.

6 Further analyses

6.1 Asymmetric ownership

The focus on symmetric forward ownership in the main analysis helps to keep the model tractable. The results that we present for asymmetric ownership structures are twofold.

- First, for a given market structure, asymmetric ownership tends to affect the price level (and thus consumer surplus) in the same direction as symmetric ownership.
- Second, and consistent with the existing literature, our analysis confirms that asymmetric partial ownership may have foreclosure effects: a downstream firm tends to pay a higher input price than its competitor when a supplier (partially) internalizes the competitor's profit.

We start by discussing the case of linear tariffs and turn to two-part tariffs afterwards. As in the analysis of symmetric ownership structures it is necessary to distinguish between the cases of upstream competition and upstream monopoly. For upstream competition, we know from Proposition 1 that the procompetitive effect of forward ownership does not materialize, as the downstream firms' input costs remain at the same level as under vertical separation. For the same reason, asymmetric profit internalization thus does not change marginal input prices either, provided that upstream competition is sufficiently strong.

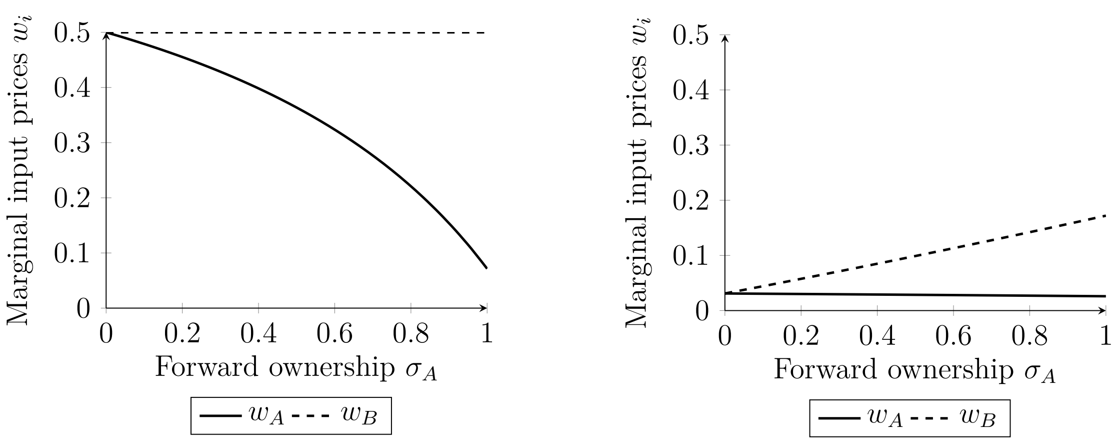
For the case of an upstream monopoly and linear tariffs, symmetric forward internalization (as derived in Proposition 1) induces U to charge the downstream firms lower input prices than under vertical separation. For the sake of tractability, we analyze the case of

[26]For the linear demand function in Equation (2) with $\gamma = 1$ and $c = 0.05$, we find that the procompetitive effect of making contracts observable is compensated by the detrimental effect of forward ownership if $\sigma > 0.44$.

asymmetric ownership on the downstream firms' input prices with Cournot competition and linear demand (Eq. (2), with $\gamma = 1/2$). The results are robust to varying degrees of product substitutability (γ) and are qualitatively similar for Bertrand competition.

In particular, we assume that the supplier holds a partial forward ownership $\sigma_A \in [0, 1]$ of downstream firm A but no shares in B. Panel (a) of Figure 1 illustrates that asymmetric ownership induces U to set lower input prices for downstream firm A compared to vertical separation. In contrast, as U has an incentive to divert profits from firm B to firm A, the linear input price w_B remains at the same high level as under vertical separation.

Figure 1: Marginal input prices with forward ownership of firm A



(a) Linear tariffs (b) Two-part tariffs

The left panel shows the optimal observable linear input prices for an upstream monopoly (i.e., c sufficiently large). The right panel shows the optimal marginal input prices under observable two-part tariffs when the competitive fringe is a relevant supply alternative (with $c = 0.3$). Both panels show the input prices for $\sigma_A \in [0, 1]$ and $\sigma_B = 0$. Competition is in quantities with demand defined in Eq. (2) and $\gamma = 1/2$.

With observable two-part tariffs, an unconstrained upstream monopolist obtains the maximal industry profit already under vertical separation. Partial ownership thus appears to be less relevant in this case and does not affect prices. With upstream competition and observable two-part tariffs, our main result is that forward ownership induces the supplier to increase the input prices toward the level that maximizes the industry profit (Proposition 2). Using again the linear demand framework introduced above, we find that with asymmetric ownership ($\sigma_A \in [0, 1]$ and $\sigma_B = 0$) the supplier wants to increase the channel profit with downstream firm A whereas it still wants to keep B's outside option profit low as in the case of vertical separation. Panel (b) of Figure 1 shows that this is possible by increasing the input costs of downstream firm B as this allows A to achieve a higher profit and increases total profits. In contrast, there is only a minor change of A's input costs.

Summary 2. In the parametric example based on the linear demand in Equation (2), asymmetric partial ownership leads to an *unequal treatment* of the downstream firms. On average, it affects the input prices in the same direction as symmetric forward ownership.

Literature on asymmetric integration and foreclosure. The analysis of asymmetric ownership relates to the literature on the vertical integration and foreclosure of competitors. In this regard, our model is most closely related to Spiegel (2013) who also considers that the supplier may offer better contract terms to a partially-integrated downstream firm. Spiegel (2013) analyzes the effects of these discriminatory input prices on the downstream firms' investment incentives and, in turn, the propensity to be vertically foreclosed. Vertical foreclosure occurs if one downstream firm successfully improves the product whereas the other one fails to do so. In contrast to Spiegel (2013), who fixes the input price for the integrated downstream firm at the level under vertical separation, we allow the upstream firm to adjust the contract that it offers to the partially-integrated downstream firm and compare the effects under linear and two-part tariffs. Moreover, our focus is not on investment incentives in the downstream market.

Other articles study whether a firm refuses to participate in the market for the intermediate good as a supplier or customer. With full vertical integration, Salinger (1988) and Ordover et al. (1990) show that this form of foreclosure can be profitable as it can raise the rivals' costs.[27] As regards partial vertical ownership, Baumol and Ordover (1994) establish that a downstream firm that fully controls a bottleneck supplier, but only gets part of its profit, can have higher incentives to foreclose a downstream rival than under full vertical integration. The partial owner has to bear only a fraction of the upstream costs of foreclosure (foregone input sales), but internalizes the full benefit of relaxed downstream competition. Levy et al. (2018) show that the profitability of such a foreclosure strategy depends on the initial ownership structure.

6.2 Profitability of partial ownership and consumer surplus

We now study how the price effects of partial forward ownership affect the firms' profits, consumer surplus and, ultimately, what ownership structure is likely to arise.[28] If the firms can arrive at efficient agreements with each other over the ownership structure ("Coasian bargaining"), they should implement an ownership arrangement that maximizes their joint (industry) profits. In our setting, starting from the reference case of vertical separation, symmetric partial forward ownership increases industry profits if it moves the downstream price toward the monopoly level. This is generally the case for linear input prices in the presence of an upstream monopolist as well as for two-part tariffs with upstream competition (and in the case of unobservable two-part tariffs even independently of the degree of upstream competition). The industry should thus have an incentive in these cases to choose an ownership structure that yields monopoly prices, or if this is not

[27]The analysis of Ordover et al. (1990) has been criticized on the grounds that the integrated supplier needs to commit itself to refusing to supply any of the non-integrated downstream firms (Hart and Tirole, 1990; Reiffen, 1992). Among others, Allain et al. (2016) propose a model that does not rely on this form of commitment. They also study the case of partial ownership in an extension and find that forward integration increases the incentive to degrade the conditions offered to the downstream rival.

[28]Of course, there may be other motives for partial vertical ownership, such as research and development cooperation, and the pricing implications may be additional effects arising from the acquisition.

attainable, it should choose the highest possible internalization.

For instance, effective merger control may render ownership structures that maximize the industry profit (lead to monopolization) unattainable. It might still be feasible and profitable for pairs of firms to bilaterally establish an ownership link, however. As in the previous section, we therefore assess the effect of asymmetric ownership structures on the firms' profits as well as on the consumer surplus based for the case of downstream quantity competition with homogeneous goods and linear demand (Equation (2) with $\gamma = 1$).[29]

In Table 5, we report the results of a 15% partial forward ownership share between supplier U and downstream firm A whereas there is no ownership link between U and B. It compares the firms' profits $\pi_A^U = \pi^U + \pi_A$ and π_B, and the consumer surplus CS under this ownership structure to the benchmark case of vertical separation.

Table 5: Firm profits and consumer surplus under asymmetric ownership

	π_A^U	π_B	Consumer surplus CS
Linear tariffs (upstream monopoly)	3.1%	-10.8%	5.6%
Two-part tariffs (upstream comp., $c = 0.1$)	8.3%	-10.8%	-3.5%

The table shows the relative changes in firm profits and consumer surplus for a partial forward ownership share of $\sigma_A = 15\%$ and $\sigma_B = 0\%$ compared to vertical separation. The first line shows the results under observable linear input prices and for an upstream monopoly (i.e., c sufficiently large). The second line shows the results for observable two-part tariffs when the competitive fringe is a relevant supply alternative (with $c = 0.1$). Competition is in quantities with demand defined in Eq.(2) and $\gamma = 1$.

The first line of Table 5 shows the results for observable linear tariffs if the efficient supplier can act as an upstream monopolist. Forward ownership between U and A has a positive effect on the joint profit of these firms (+3.1%) and a strong negative effect of -10.8% on B's profit. The consumer surplus (CS) increases by 5.6% due to the lower price level. This result is in line with Panel (a) in Figure 1, which documents that under linear tariffs the input prices for A decreases in the forward ownership share.

The second line in the table shows the results for observable two-part tariffs. We consider the case that the competitive fringe has marginal costs of $c = 0.1$ and is therefore a relevant supply alternative for the downstream firms. The forward ownership share of 15% increases the joint profit of U and A by 8.3%. In contrast, the profit of downstream firm B decreases by -10.8%. As the overall price level increases if the supply contract is nonlinear, forward ownership decreases consumer surplus in this case (-3.5%).

Summary 3. In the parametric example based on the linear demand in Equation (2), partial forward ownership between upstream firm U and downstream firm A increases the

[29]The results are qualitatively the same for varying degrees of product substitutability γ and also for the case of price competition downstream.

joint profits of these firms to the detriment to the non-integrated downstream firm B, both with linear and two-part tariffs. Consumer surplus increases with linear tariffs but decreases with two-part tariffs.

6.3 Partial ownership versus profit-sharing supply contracts

One might wonder whether the price-increasing effects of forward ownership and observable two-part tariffs (Proposition 2) can emerge without an ownership arrangement, but with a supply contract which involves a marginal input price w_i, a fixed fee f_i, and additionally entitles the supplier to a share σ_i of the downstream profits (similar to "revenue sharing"). We show that this is not the case.[30]

Suppose that the efficient supplier U offers each downstream firm a supply contract (w_i, f_i, σ_i). If the firms are vertically separated, the supplier's maximization problem is

$$\begin{aligned} \max \ \Omega^U &= \pi^U (w_i, w_{-i}) + \sum_{i \in \{A,B\}} \sigma_i (\pi_i (w_i, w_{-i}) - f_i) + f_i \\ &s.t. \quad (1-\sigma_i)(\pi_i (w_i, w_{-i}) - f_i) \geq \pi_i (c, w_{-i}), \text{ for } i, -i \in \{A, B\} \end{aligned} \tag{17}$$

If the participation constrains are binding and solving them for $\sigma_i \pi_i (w_i, w_{-i}) + (1-\sigma_i) f_i$, yields the reduced maximization problem

$$\max_{w_A, w_B} \Omega^U = \pi^I (w_A, w_B) - \pi_A (c, w_B) - \pi_B (c, w_A), \tag{18}$$

which is the same reduced problem as for the case of vertical separation and observable two-part tariffs (the problem in Equation (9) for $\sigma = 0$). Therefore, the supplier has no incentive to charge higher marginal input prices under a profit-sharing contract (w_i, f_i, σ_i) than under a two-part tariff (w_i, f_i) if the firms are vertically separated. Adding a profit-sharing element to an observable two-part tariff does not change the contracting outcome.[31] We summarize this result in

Proposition 5. *The price-increasing result derived for observable two-part tariffs (w_i, f_i) under forward ownership does not emerge under vertical separation if firms add a profit-sharing clause σ_i to the supply contract.*

Proof. Direct implications of the derivation above. □

The reason for the different outcomes of partial vertical ownership and supply contracts with a profit sharing component is that ownership also yields a profit participation for supplier U when the downstream firm rejects its supply contract and sources from the

[30] For unobservable contracts, Hart and Tirole (1990) show for general contracts $t_{ij}(q_i)$ between upstream firm j and downstream firm j that opportunism cannot be solved on a contractual basis.

[31] Moreover, this result shows that σ_i and f_i are equivalent instruments to make the downstream firms indifferent to their outside option.

fringe. With an ownership participation, supplier U thus has less incentives to decrease the outside option profit. Instead, with a profit sharing contract under vertical separation, supplier U only cares about the downstream firm's profits in the case that its supply contract is accepted.

7 Conclusion

We offer novel insights for the assessment of partial ownership in vertically-related industries. For settings where foreclosure is less of a concern, the existing literature so far has revealed predominantly price-decreasing effects of non-controlling partial forward ownership (Flath, 1989; Fiocco, 2016). This suggests a rather generous treatment of such ownership structures by competition authorities and regulators. Importantly, we show that partial forward ownership can lead to large price increases if the supply contracts are nonlinear. These competitive effects occur both if contracts are observable as well as if they are unobservable, albeit for different economic reasons.

With observable two-part tariffs, a supplier strategically sets marginal prices below the level that maximizes industry profits if downstream firms have a relevant supply alternative. This allows the upstream firm to obtain a larger share of the industry profit, at the cost of reducing the total industry profit. This extraction incentive is lower if the supplier internalizes a share of the downstream profits, which implies higher marginal input prices and thus downstream prices as well as higher industry profits than with vertical separation.

Instead, with secret contracting, even an upstream monopolist cannot commit to charging downstream competitors high input prices under vertical separation (Hart and Tirole, 1990). This opens the door for profit-increasing structural arrangements even in the case of an upstream monopoly. Our analysis reveals that partial forward ownership effectively enables the upstream firm to commit to higher input prices, which in turn leads to higher consumer prices. We also show that the profits of the involved firms tend to increase when they establish these price increasing partial-ownership structures.

We also contribute to the analysis of partial forward ownership in the presence of linear supply contracts. First, if downstream firms have a relevant supply alternative the procompetitive effect derived by Flath (1989) may not materialize and partial forward ownership is competitively neutral. Second, we study the case of unobservable linear tariffs. Building on Gaudin (2019), we demonstrate that partial forward ownership can also reduce double marginalization under the alternative information structure of secret contracts.

Finally, we also consider the possibility that a partial vertical ownership arrangement makes otherwise secret contracts observable. This may, for instance, occur because partial ownership links facilitate the exchange of information between vertically-related firms. We illustrate that the price effects of partial forward ownership summarized above are reinforced in most settings when the acquisition makes contracts observable: both the price decreases with linear tariffs as well as the prices increases with two-part tariffs. Only in certain cases of downstream quantity competition, the effect of rendering contracts observable may run in the opposite direction to the internalization effect. In these instances, a more detailed assessment may be required to arrive at prediction of the total effect of the ownership acquisition.

For vertically-related firms, partial ownership links can be a means to facilitate their business relationships by overcoming opportunism problems and fostering research and development cooperation (Flath, 1996; Allen and Phillips, 2000; Fee et al., 2006; Hunold and Shekhar, 2018). Compared to full mergers, minority shareholdings tend to be much easier to establish with less resistance from shareholders, stakeholders, and competition authorities. In particular if no or limited control rights are involved, a merger control review is often times not required. Moreover, both entities continue operating under their own management which can have important effects on strategic decisions such as relationship-specific investments. Our article complements these insights by shedding light on the pricing implications of such ownership links. Importantly, we point out that partial forward ownership can be more detrimental than a full vertical merger between an upstream firm and a downstream firm in an industry. A less strict treatment of partial ownership acquisitions might therefore not be warranted.

Our results are also of relevance for the current competition policy debate on how to treat partial ownership acquisitions in merger control. Under certain conditions, the review of non-controlling partial ownership acquisitions is currently feasible in some jurisdictions. Examples include Austria, Brazil, Germany, Japan, the UK, and the United States.[32] While such acquisitions are currently not covered by the European Union Merger Regulation, the European Commission has in recent years explored the necessity and possibility to control such acquisitions.[33] Our results suggest that—besides established procompetitive effects on research and development cooperation and concerns about foreclosure—the price effects of partial vertical ownership should be taken into account. As our analysis highlights, competition authorities should consider the nature of the supply contracts as well as the competitive forces at the upstream level when assessing a partial ownership acquisition. This insight is particularly important for the case of forward ownership as the economic literature so far predicts price-decreasing effects. In stark contrast, our analysis reveals that such ownership arrangements can lead to higher prices, depending on the degree of upstream competition as well as on the pricing arrangement between upstream and downstream firms.

[32]See European Commission (2013) for an overview of the regulation in these jurisdictions.

[33]Council Regulation (EC) No 139/2004 of 20 January, 2004, on the control of concentrations between undertakings (OJ L 24, 29.1.2004, p. 1).

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Appendix A: Proofs

Proof of Proposition 1. If c is sufficiently large $\left(w^l(\sigma) < c\right)$, the symmetric input price $w^l(\sigma)$ that solves the system of first-order conditions defined in Equation (7) is the equilibrium input price. Assumption 2 ensures that the second-order conditions are fulfilled. Implicit differentiation of the $w^l(\sigma)$ with respect to the degree of forward internalization σ yields

$$\frac{\partial w^l(\sigma)}{\partial \sigma} = -\frac{\partial \pi_i\left(w_i^l, w_{-i}^l\right)/\partial w_i + \partial \pi_{-i}\left(w_{-i}^l, w_i^l\right)/\partial w_i}{\partial^2 \Omega^U/\partial^2 w_i + \partial^2 \Omega^U/\partial w_i \partial w_{-i}} < 0; \tag{19}$$

Under Assumption 1, the numerator is negative; the denominator is negative due to the concavity of the supplier's objective function. In a symmetric interior equilibrium, the equilibrium input price $w^l(\sigma)$ thus decreases in the degree of forward internalization σ if c is sufficiently large. This implies that also downstream prices decrease for an increase in the forward ownership share σ.

A downstream firm's participation constraint is violated if $w^l(\sigma) \geq c$. Given the concave optimization problem, the supplier sets the input price as high as possible in this case; that means $w = c$. This is the same input price as under vertical separation because, if the participation constraint is binding at σ, it is also binding at any $\sigma' \in [0, \sigma]$. Thus, the downstream prices are at the same level as under vertical separation. This establishes the result. □

Proof of Proposition 2. Recall that we denote by $w^{tp}(\sigma)$ the symmetric per-unit input price with observable two-part tariffs that solves the system of first-order conditions in Equation (10). Assumptions 1 and 2 imply that the second-order conditions are fulfilled. Implicit differentiation of $w^{tp}(\sigma)$ with respect to σ yields

$$\frac{\partial w^{tp}(\sigma)}{\partial \sigma} = -\frac{\partial \pi_{-i}\left(c, w_i^{tp}\right)/\partial w_i}{\partial^2 \Omega^U/\partial^2 w_i + \partial^2 \Omega^U/\partial w_i \partial w_{-i}} \geq 0. \tag{20}$$

The denominator is negative due to the concavity of the supplier's objective function. The numerator is strictly positive if the competitive fringe is a relevant supply alternative (Assumption 1). This implies that the inequality in Equation (20) is strict. In this case input prices (and thus downstream prices) increase in the degree of forward internalization σ.

If the competitive fringe is not a relevant supply alternative, we have $\partial \pi_{-i}(c, w_i)/\partial w_i = 0$, which implies $\partial w^{tp}(\sigma)/\partial \sigma = 0$ and that the downstream prices are the same for all $\sigma \in [0, 1]$. This establishes the result. □

Proof of Proposition 3. If c is sufficiently large $\left(w^{lu}(\sigma) < c\right)$, the system of first-order conditions defined in Equation (14) defines the symmetric equilibrium input price, which we denote as $w^{lu}(\sigma)$. In the symmetric equilibrium, the retailers' beliefs, which are implicitly contained in the first-order conditions, are correct. Implicit differentiation of Equation (14) yields the following comparative static of $w^{lu}(\sigma)$ with respect to the degree of forward internalization σ

$$\frac{\partial w^{lu}(\sigma)}{\partial \sigma} = -\frac{\left[-q_i\left(w^{lu}\right) + q_{-i}\left(w^{lu}\right)\frac{\partial p_{-i}\left(q_{-i}\left(w^{lu}\right),q_i\left(w^{lu}\right)\right)}{\partial q_i}\frac{\partial q_i\left(w^{lu}\right)}{\partial w_i}\right]}{\partial^2\Omega^U/\partial^2 w_i}, \tag{21}$$

where $\partial q_i\left(w^{lu}\right)/\partial w_i$ refers to $\partial q_i(w_i)/\partial w_i$ evaluated at $w_i = w^{lu}$. The denominator is negative under the assumption that the supplier's objective function is strictly concave. Hence, the comparative static $\partial w^{lu}(\sigma)/\partial\sigma$ is negative if the term in the numerator is negative. As $q_i = q_{-i}$ by demand symmetry, the numerator can be written as

$$q_i\left(w^{lu}\right)\left(-1 + \frac{\partial p_{-i}\left(q_{-i}\left(w^{lu}\right), q_i\left(w^{lu}\right)\right)}{\partial q_i}\frac{\partial q_i\left(w^{lu}\right)}{\partial w_i}\right). \tag{22}$$

The term in the brackets of Equation (22) is negative if

$$1 > \frac{\partial p_{-i}\left(q_{-i}\left(w^{lu}\right), q_i\left(w^{lu}\right)\right)}{\partial q_i}\frac{\partial q_i\left(w^{lu}\right)}{\partial w_i}. \tag{23}$$

Applying the implicit function theorem to the downstream firm i's first-order condition, $\partial\pi_i/\partial q_i = 0$, we can derive the comparative static

$$\frac{\partial q_i(w_i)}{\partial w_i} = -\frac{-1}{\left(\partial^2 p_i(q_i,q_{-i})/\partial^2 q_i\right)q_i + 2\left(\partial p_i(q_i,q_{-i})/\partial q_i\right)}, \tag{24}$$

where the denominator is $\partial^2\pi_i/\partial^2 q_i$. Inserting Equation (24) in Equation (23) and using demand symmetry yields

$$2\frac{\partial p_i(q_i,q_{-i})}{\partial q_i} - \frac{\partial p_{-i}(q_{-i},q_i)}{\partial q_i} + \frac{\partial^2 p_i(q_i,q_{-i})}{\partial^2 q_i}q_i < 0, \tag{25}$$

which is negative according to Assumption 3. This establishes the result that $\partial w^{lu}(\sigma)/\partial\sigma < 0$ if c is sufficiently large. In contrast, a downstream firm's participation constraint is violated if $w^{lu}(\sigma) \geq c$. Given the concave optimization problem, the supplier sets the input price as high as possible in this case. Under the same reasoning as in the proof of Proposition 1, downstream prices are thus at the same level as under vertical separation. This establishes the result. □

Proof of Lemma 1. We demonstrate the possibility result of Lemma 1 based on the linear demand function defined in (2). First, we characterize the case with partial forward ownership. Second, we provide the results for the full-vertical-integration case based on Sandonis and Fauli-Oller (2006). For both partial and full integration, we focus on the case that the marginal costs c of the competitive fringe satisfy $c < \frac{2-\gamma}{2}$, which ensures that the competitive fringe is a relevant supply alternative for an independent downstream firm at the equilibrium contract offer.

In the case of partial forward ownership, the downstream firms sell the quantities

$$q_i = \frac{2-\gamma-2w_i+\gamma w_{-i}}{4-\gamma^2},\ i \in \{A,B\}. \tag{26}$$

These equilibrium quantities in stage 3 are independent of the forward ownership share σ as the efficient supplier cannot influence the downstream firms' strategies.

Solving supplier $U's$ optimality condition (10) for the case of linear demand yields the equilibrium wholesale prices

$$w_A = w_B = \frac{\gamma\left(\left(\gamma-2\right)\left(\gamma-2\sigma\right)-4c\left(\sigma-1\right)\right)}{2\left(\gamma^3-\gamma^2\left(2+\sigma\right)+4\right)}. \tag{27}$$

The resulting consumer surplus (see Equation (1)) is

$$\frac{\left(\gamma+1\right)\left(\gamma^3-2\gamma^2+4c\gamma\left(\sigma-1\right)-4\gamma\sigma+8\right)^2}{4\left(\gamma+2\right)^2\left(\gamma^3-\gamma^2(\sigma+2)+4\right)^2}, \tag{28}$$

which decreases in the partial forward ownership share σ for $\gamma > 0$.

For the case of full vertical integration, we consider the merger between the efficient supplier U and downstream firm A. A consequence of the merger is that the downstream unit of the vertically-integrated firm makes its output decision based on the supplier's marginal costs of zero. The objective function of the vertically-integrated firm is

$$\pi_A^{U,VI} = p_A\left(q_A,q_B\right)q_A + w_B q_B + f_B. \tag{29}$$

For the equilibrium in stage 3 this implies

$$\begin{aligned} q_A^{VI} &= \frac{2-\gamma+\gamma w_B}{4-\gamma^2}, \\ q_B^{VI} &= \frac{2-\gamma-2w_B}{4-\gamma^2}. \end{aligned}$$

The optimal contract offer for the independent downstream firm B involves

$$w_B^{VI} = \frac{\left(\gamma-2\right)^2\gamma}{8-6\gamma^2}. \tag{30}$$

The resulting consumer surplus is

$$\frac{8-3\gamma^2-4\gamma}{32-24\gamma^2}. \tag{31}$$

The consumer surplus under full vertical integration (Equation 31) does not depend σ. From Sandonis and Fauli-Oller (2006) we know that the consumer surplus under vertical integration is smaller than under vertical separation; the latter corresponds to the surplus in Equation (28) for $\sigma = 0$. In the second polar case of $\sigma = 1$, this result is reversed and the surplus under full integration exceeds the one in Equation (28). In this case the difference between the consumer surplus in Equation (31) and the one in Equation (28) is

$$\frac{\gamma\left(3\gamma^2+\gamma-4\right)}{8\left(1+\gamma\right)\left(3\gamma^2-4\right)} > 0,\ \forall\gamma\in\left(0,1\right).$$

Together with the observation that the consumer surplus in Equation (28) decreases with σ, this implies that there is an interior threshold $\bar{\sigma}$, such that partial forward ownership yields lower consumer surplus than full vertical integration if $\sigma > \bar{\sigma}$. The exact solution of $\bar{\sigma}$ is involved as Equation (28) depends on σ in a non-linear way. For the specification of $\gamma = 3/10$ and $c = 4/10$, we find the consumer surplus with full vertical integration exceeds the one under partial forward ownership if $\sigma > 0.1934$. For the same specification, total surplus, that is the sum of consumer surplus and the profits of U, A and B, is smaller under partial forward ownership than under full vertical integration if $\sigma > 0.246$. This establishes the result. □

Proof of Lemma 2. First, we derive the supplier's maximization problem and establish equilibrium existence and uniqueness.

Anticipating that its competitor produces the equilibrium quantity Q_{-i}, the profit of downstream firm i when accepting the contract is

$$\pi_i = \left(p_i\left(q_i, Q_{-i}\right) - w_i\right) q_i - f_i. \tag{32}$$

Each downstream firm optimally sets the quantity for a given input price w_i and belief about the other firm's quantity (Q_{-i}) as follows:

$$q_i\left(w_i\right) = \arg\max_{q_i}\left(p_i\left(q_i, Q_{-i}\right) - w_i\right) q_i - f_i, \tag{33}$$

which is implicitly defined by the downstream firm i's first-order condition $\partial\pi_i/\partial q_i = 0$, which implies

$$\left(p_i\left(q_i, Q_{-i}\right) - w_i\right) + \frac{\partial p_i\left(q_i, Q_{-i}\right)}{\partial q_i} q_i = 0. \tag{34}$$

Based on this first-order condition, implicit differentiation of Equation (34) yields that the equilibrium quantity decreases monotonically in w_i. That is,

$$\frac{\partial q_i\left(w_i\right)}{\partial w_i} = -\frac{-1}{\partial^2\pi_i/\partial^2 q_i} < 0, \tag{35}$$

where the denominator is negative due to the concavity of i's objective function (Assumption 3). Hence, there is a one-to-one mapping between the w_i and $q_i\left(w_i\right)$. Below, we can therefore characterize the supplier's problem as effectively selecting quantities in the downstream market.

If a downstream firm does not accept the contract offers and instead sources from the competitive fringe, its profit is

$$\pi_i(c, W_{-i}) = (p_i(q_i, Q_{-i}) - c)\, q_i(c), \tag{36}$$

which indicates that downstream firm i has input costs of c while it anticipates that its competitor accepts the contract and faces an marginal input price of W_{-i}. Recall that W_{-i} denotes the belief about the rival's contract as the offers are secret and there is a one-to-one relation between W_{-i} and the expected quantity Q_{-i}. If the competitive fringe is very inefficient (large c), it is not a relevant supply alternative and the efficient supplier is effectively an upstream monopolist.

Next, we turn to the contract offers of the efficient supplier U. For given beliefs of the downstream firms, the supplier's objective function is

$$\begin{aligned} \Omega^U = \ \textstyle\sum_{i\in\{A,B\}} \ & w_i \cdot q_i(w_i) + f_i \\ & + \sigma\left((p_i(q_i(w_i), q_{-i}(w_{-i})) - w_i)\, q_i(w_i) - f_i\right), \end{aligned} \tag{37}$$

subject to the constraint that each downstream firms is willing to accept its contract offer. The supplier internalizes a share σ of each downstream firm's actual profit (which equals the flow profit $(p_i(q_i(w_i), q_{-i}(w_{-i})) - w_i)\, q_i(w_i)$ minus the fixed fee f_i). It makes the downstream firms, given their beliefs, indifferent to their outside option of $\pi_i(c, W_{-i})$ by setting

$$f_i \ = \ (p_i(q_i(w_i, Q_{-i}), Q_{-i}) - w_i)\, q_i(w_i) - \pi_i(c, W_{-i}). \tag{38}$$

Importantly, note that a downstream firm's outside option $\pi_i(c, W_{-i})$ does not depend on its own marginal input price w_i.

By substituting the fixed fees from (38), we can therefore write the supplier's objective function as follows:

$$\begin{aligned} & \Omega^U \\ = & \sum_{i\in\{A,B\}} \left(\sigma p_i(q_i(w_i), q_{-i}(w_{-i}))\, q_i + (1-\sigma)\left(p_i(q_i(w_i), Q_{-i})\, q_i(w_i) - \pi_i(c, W_{-i})\right)\right). \end{aligned} \tag{39}$$

The first term in Equation (39) is proportional to the industry profit π^I

$$= \sum_{i\in\{A,B\}} p_i(q_i, q_{-i})\, q_i,$$

which is a strictly concave function in w_A and w_B (Assumption 4). The second term contains the supplier's objective function under vertical separation, which is strictly concave in w_A and w_B as in Rey and Vergé (2004) (see Proposition 1 therein). Note that even a positive outside option $\pi_i(c, W_{-i}) > 0$ does not change this result as it does not depend on the contract offer w_i directly but only on the downstream firm's belief about its rival's contract offer W_{-i}. Hence, the objective function in Equation (39) is strictly concave, which means that the sufficient conditions for equilibrium existence and uniqueness are fulfilled.

As derived in Equation (35), there is a one-to-one mapping between the effective input prices and the equilibrium quantities. It is thus convenient to characterize the supplier's maximization problem as selecting the downstream quantities directly.

Next, we show that the equilibrium does not involve marginal input prices equal to the supplier's marginal costs if the supplier internalizes a share $\sigma > 0$ of the downstream firms' profits. The first-order condition with respect to (say) q_A is

$$\begin{aligned}\frac{\partial \Omega^U}{\partial q_A} &= \sigma \left(\underbrace{\frac{\partial p_A(q_A, q_B)}{\partial q_A} q_A + p_A(q_A, q_B)}_{=w_A} + \frac{\partial p_B(q_B, q_A)}{\partial q_B} q_B \right) \\ &+ (1-\sigma) \underbrace{\left(\frac{\partial p_A(q_A, Q_B)}{\partial q_A} q_A + p_A(q_A, Q_B) \right)}_{=w_A} = 0. \end{aligned} \tag{40}$$

Denote the symmetric equilibrium quantity that solves the supplier's first-order conditions with respect to w_A and w_B as $q_A^{tpu}(\sigma) = q_B^{tpu}(\sigma) = q^{tpu}(\sigma)$ and the symmetric input price, which induces this quantity in the downstream market, with $w_A^{tpu}(\sigma) = w_B^{tpu}(\sigma) = w^{tpu}(\sigma)$, where *tpu* indicates that the supplier uses two-part tariffs that are unobservable.

From the downstream firm's first-order condition (Equation (34)), we can infer that the first two terms in the bracket in the first line of Equation (40) equals the marginal input price w_A, as does the term in the brackets in the second line. Note that $p_A(q_A, q_B) = p_A(q_A, Q_B)$ as the beliefs are correct in equilibrium.

Marginal input prices equal to the supplier's marginal costs, that is, $w^{tpu}(\sigma) = 0$ do not solve the first-order condition in Equation (40) if $\sigma > 0$. In particular, we observe that at $w^{tpu}(\sigma) = 0$ the term in the first line of Equation (40) is positive: $(\partial p_B(q_B, q_A)/\partial q_A) \cdot q_B > 0$. Input prices equal to the supplier's marginal costs thus do not fulfill the necessary condition for an equilibrium in this case. This establishes the result. □

Proof of Proposition 4. Lemma 2 establishes the existence and uniqueness of the equilibrium. Moreover, it shows that input costs equal to the supplier's marginal costs are not an equilibrium for $\sigma > 0$. Here, we characterize the equilibrium for $\sigma > 0$.

The supplier's first-order condition with respect to q_A in Equation (40) consists of the derivative of the industry profit π^I with respect to q_A (i.e., $\partial \pi^I(q_A, q_B)/\partial q_A$, first line in Eq. (40)) and the derivative of the joint profit between U and A with respect to q_A (i.e., $\partial \pi_A^U(q_A, q_B)/\partial q_A$, second line in Eq. (40)). This implies that

$$\frac{\partial \Omega^U(q_A, q_B)}{\partial q_A} = \sigma \frac{\partial \pi^I(q_A, q_B)}{\partial q_A} + (1-\sigma) \frac{\partial \pi_A^U(q_A)}{\partial q_A} = 0. \tag{41}$$

Implicit differentiation of the symmetric downstream quantity $q^{tpu}(\sigma)$ with respect to σ

yields

$$\frac{\partial q^{tpu}(\sigma)}{\partial \sigma} = -\frac{\partial \pi^I(q_A, q_B)/\partial q_A - \partial \pi_A^U(q_A, q_B)/\partial q_A}{\partial^2 \Omega^U(q_A, q_B)/\partial^2 q_A + \partial^2 \Omega^U(q_A, q_B)/\partial q_A \partial q_B} < 0. \tag{42}$$

By Lemma 2, the denominator is negative. For quantities in the range between the level that maximizes the industry profit and the level that maximizes the bilateral profit of the supplier with one downstream firm, we have that $\partial \pi^I(q_A, q_B)/\partial q_A < 0$ and $\partial \pi_A^U(q_A, q_B)/\partial q_A > 0$. Hence, the numerator is negative and the quantity $q(\sigma)$ thus decreases (and downstream prices increase) in σ. This establishes the result. □

Appendix B: Unobservable two-part tariffs and price competition downstream

Preliminaries

In this appendix, we show that partial forward ownership is anticompetitive if the supplier charges unobservable two-part tariffs and the downstream firms compete in prices. The supplier holds a symmetric forward ownership σ of both downstream firms. In order to analyze Bertrand competition, we first introduce additional notation. Building on Rey and Vergé (2004), we assume that the demand for each downstream firm, $q_i(p_i, p_{-i})$ is symmetric with $\frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_{-i}} < 0 < \frac{\partial q_i}{\partial p_{-i}}$. Given that both downstream firms accept the supplier's contract offer, a downstream firm's objective function can be written as

$$\pi_i(p_i, p_{-i}) = (p_i - w_i)\, q_i(p_i, p_{-i}) - f_i. \tag{43}$$

We maintain the assumption that the downstream firms' objective functions are strictly concave (see Assumption 3 in the analysis of Cournot competition).

As emphasized by Rey and Vergé (2004) for the case of vertical separation, the supplier's objective function is not necessarily concave with unobservable contracts and thus a perfect Bayesian-Nash equilibrium may fail to exist. We impose

Assumption 5. *There exists a unique symmetric solution to the first-order conditions of the supplier's maximization problem and the objective function $\Omega^U(w_A, w_B)$ has negative second-order derivatives: $\partial^2\Omega^U/\partial^2 w_i$.*

This assumption is fulfilled under vertical separation (Rey and Vergé, 2004). Assumption 5 ensures that this extends to the case of partial forward ownership. In general, it is satisfied if the second derivatives of demand are not too positive. In particular, it is satisfied with linear demand.[34] For equilibrium existence, we additionally verify that the Hessian is negative definite and hence that non-local deviations are unprofitable.

Recall that we denote a downstream firm's contract offer as $t_i = (w_i, f_i)$ and the passive beliefs about the other downstream firm's contract offer as $T_{-i} = (W_{-i}, F_{-i})$. Similarly,

[34] For instance, Pagnozzi and Piccolo (2012) make a related assumption on the supplier's objective function.

downstream firm i expects that the rival $-i$ sets the equilibrium price P_{-i}. In equilibrium, beliefs are correct.

Analysis and Proofs

We first establish that there is no equilibrium with input prices equal to the supplier's marginal costs for the case of Bertrand competition either.

Lemma 3. *Suppose supplier U charges unobservable two-part tariffs and holds a symmetric forward ownership share σ of both downstream firms. If the downstream firms compete in prices and hold passive beliefs, input prices equal to the supplier's marginal costs are not an equilibrium.*

Proof of Lemma 3. In the case of Bertrand competition, downstream firm i sets the optimal price given passive beliefs about the rival's contract offers W_{-i} and the resulting P_{-i} as follows:

$$p_i(w_i) = \arg\max_{p_i} (p_i - w_i)\, q_i(p_i, P_{-i}), \tag{44}$$

which is implicitly defined by the first-order condition $\partial \pi_i / \partial p_i = 0$, that is,

$$(p_i - w_i)\frac{\partial q_i(p_i, P_{-i})}{\partial p_i} + q_i(p_i, P_{-i}) = 0. \tag{45}$$

The equilibrium price $p_i(w_i)$ increases monotonically in w_i:

$$\frac{\partial p_i(w_i)}{\partial w_i} = \frac{\frac{\partial q_i(p_i, P_{-i})}{\partial p_i}}{(p_i - w_i)\frac{\partial^2 q_i(p_i, P_{-i})}{\partial^2 p_i} + 2\frac{\partial q_i(p_i, P_{-i})}{\partial p_i}} > 0. \tag{46}$$

The inequality holds as the denominator is negative (due to the downstream firm's strictly concave profit function) and as $\partial q_i / \partial p_i < 0$. Denote with $q_i(p_i(w_i), P_i)$ the quantity that downstream firm i expects to sell given its own price $p_i(w_i)$ and the expected competitor's price P_{-i}. The supplier sets the fixed fee such that the downstream firms are indifferent between accepting and rejecting the offer. This implies

$$f_i = (p_i(w_i) - w_i)\, q_i(p_i(w_i), P_{-i}) - \pi_i(c, W_{-i}), \tag{47}$$

where $\pi_i(c, W_{-i})$ denotes a downstream firm's outside option in case the competitive fringe is a relevant supply alternative.

The supplier can correctly infer the downstream prices and quantities, as it knows the input prices that it offers to both downstream firms. Hence, we denote with $q_i(p_i(w_i), p_{-i}(w_{-i}))$ the quantity that downstream firm i ends up selling when the supplier offers input prices of (w_i, w_{-i}). The supplier's objective function can be written as

$$\begin{aligned} \Omega^U(w_A, w_B, f_A, f_B) &= \sum_{i \in \{A,B\}} w_i q_i(p_i(w_i), p_{-i}(w_{-i})) + f_i \\ &+ \sigma\left((p_i(w_i) - w_i)\, q_i(p_i(w_i), p_{-i}(w_{-i})) - f_i\right). \end{aligned} \tag{48}$$

Substituting for the fixed fees (Equation (47)) and suppressing the dependence of p_i on w_i, we can express U's objective function as

$$\begin{aligned}\Omega^U\left(w_A,w_B\right)= \quad & \textstyle\sum_{i\in\{A,B\}} \quad \sigma\cdot\left(p_iq_i\left(p_i,p_{-i}\right)\right)+(1-\sigma)\cdot\left(p_iq_i\left(p_i,P_{-i}\right)\right) \\ & \qquad + \ (1-\sigma)\cdot\left(w_i\left(q_i\left(p_i,p_{-i}\right)-q_i\left(p_i,P_{-i}\right)\right)-\pi_i\left(c,W_{-i}\right)\right).\end{aligned} \tag{49}$$

The unique candidate equilibrium prices solve the system of the supplier's first-order conditions. The first-order condition to the supplier's objective function in Equation (49) with respect to (say) w_A is

$$\begin{aligned}\frac{\partial\Omega^U}{\partial w_A} \quad &= \quad \sigma\left(q_A\left(p_A,p_B\right)+p_A\frac{\partial q_A\left(p_A,p_B\right)}{\partial p_A}\right)\frac{\partial p_A}{\partial w_A} \\ &+ \quad (1-\sigma)\left(q_A\left(p_A,P_B\right)+p_A\frac{\partial q_A\left(p_A,P_B\right)}{\partial p_A}\right)\frac{\partial p_A}{\partial w_A} \\ &+ \quad \left(\left(\sigma p_B+(1-\sigma)\,w_B\right)\frac{\partial q_B\left(p_B,p_A\right)}{\partial p_A}\right)\frac{\partial p_A}{\partial w_A} \\ &+ \quad (1-\sigma)\left(w_A\left(\frac{\partial q_A\left(p_A,p_B\right)}{\partial p_A}-\frac{\partial q_A\left(p_A,P_B\right)}{\partial p_A}\right)\right)\frac{\partial p_A}{\partial w_A} \\ &+ \quad (1-\sigma)\left(q_A\left(p_A,p_B\right)-q_A\left(p_A,P_B\right)\right)=0.\end{aligned} \tag{50}$$

Equation (50) can be simplified by employing that beliefs are correct in equilibrium ($p_i = P_i$). This implies that the terms in the first two lines can be combined to

$$q_A\left(p_A,p_B\right)+p_A\frac{\partial q_A\left(p_A,p_B\right)}{\partial p_A}. \tag{51}$$

Moreover, note that the last two lines of Equation (50) are equal to zero due to the fact that the beliefs are correct in equilibrium. Taking these simplifications into account and dividing the first-order condition by $\partial p_A/\partial w_A$ yields

$$\begin{aligned}\frac{\partial\Omega^U}{\partial w_A}/\left(\partial p_A/\partial w_A\right) \quad &= \quad q_A\left(p_A,p_B\right)+p_A\frac{\partial q_A\left(p_A,p_B\right)}{\partial p_A} \\ &+ \quad \left(\sigma p_B+(1-\sigma)\,w_B\right)\frac{\partial q_B\left(p_B,p_A\right)}{\partial p_A}=0.\end{aligned} \tag{52}$$

From rearranging the downstream firm's first-order condition $\partial\pi_A/\partial p_A = 0$, we obtain

$$q_A+p_A\frac{\partial q_A}{\partial p_A}=w_A\frac{\partial q_A}{\partial p_A}, \tag{53}$$

Substituting the right-hand side from Equation (53) in the second line of Equation (50) yields

$$w_A\frac{\partial q_A\left(p_A,p_B\right)}{\partial p_A}+\left(\sigma p_B+(1-\sigma)\,w_B\right)\frac{\partial q_B\left(p_B,p_A\right)}{\partial p_A} \quad = \quad 0 \tag{54}$$

The first-order condition with respect to w_B can be derived accordingly. Denote the symmetric marginal input price that solves the system of first-order conditions by $w_A^{tpu}(\sigma) = w_B^{tpu}(\sigma) = w^{tpu}(\sigma)$. For $\sigma = 0$ (vertical separation), the unique solution to the system of first-order conditions is $w^{tpu}(0) = 0$. For $\sigma > 0$, however, $w^{tpu}(\sigma) = 0$ does not solve the system of first-order conditions. In particular, in this case the left-hand side of Equation (54) becomes $\sigma p_B \cdot \partial q_B(p_B, p_A)/\partial p_A$, which is positive. Input prices equal to the supplier's marginal costs thus do not solve the first-order condition and the supplier. This establishes the result. □

In the following proposition, we derive the conditions for equilibrium existence and characterize the comparative static of the marginal input price in the forward ownership share σ.

Proposition 6. *Suppose supplier U charges unobservable two-part tariffs and holds a symmetric forward ownership share σ of both downstream firms. If the downstream firms compete in prices and hold passive beliefs, there exists a unique symmetric perfect Bayesian-Nash equilibrium if and only if the cross-elasticity of substitution is sufficiently small. For linear demand, the condition is*

$$\frac{\epsilon}{2} \;>\; \left(1 - \frac{\sigma}{2}\right)\epsilon_s, \tag{55}$$

with ϵ and ϵ_S defined in Equation (57). Equation (61) contains the condition for non-linear demand.
The marginal input prices and downstream prices increase in the forward ownership share σ.

Proof of Proposition 6. We first establish equilibrium existence and then derive the comparative static results for partial forward ownership. As analyzed in Rey and Vergé (2004) for vertical separation (see Proposition 2 therein), an equilibrium exists if the cross-price elasticity of demand is small enough in relation to the own-price elasticity:

$$\epsilon_s < \frac{\epsilon}{2}, \tag{56}$$

where

$$\epsilon \equiv -\frac{\partial q_i(p_i, p_{-i})}{\partial p_i}\frac{p_i}{q_i(p_i, p_{-i})}, \; \epsilon_s \equiv \frac{\partial q_i(p_i, p_{-i})}{\partial p_{-i}}\frac{p_{-i}}{q_i(p_i, p_{-i})}. \tag{57}$$

If the cross elasticity ϵ_s is larger, Rey and Vergé (2004) demonstrate a profitable multilateral deviation from the candidate equilibrium for the supplier, which implies that a perfect Bayesian-Nash equilibrium in passive beliefs does not exist.

We now derive the condition that there is no profitable multilateral deviation from the candidate equilibrium under partial vertical ownership. A sufficient condition that the candidate equilibrium establishes a perfect Bayesian equilibrium is to verify that the Hessian matrix of second-order derivatives $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is negative definite at the candidate equilibrium input prices. In a symmetric equilibrium, it holds that $a = d =$

$\partial^2\Omega^U/\partial^2 w_A$ and $b = c = \partial^2\Omega^U/\partial w_A \partial w_B$ for the elements of the Hessian matrix. We obtain the first element of the Hessian matrix by differentiating Equation (50) with respect to w_A. Evaluating the second-order condition at the symmetric candidate equilibrium yields

$$\begin{aligned}\frac{\partial^2\Omega^U}{\partial^2 w_A} &= \left(2\frac{\partial q_A(p_A,p_B)}{\partial p_A} + p_A\frac{\partial^2 q_A(p_A,p_B)}{\partial^2 p_A}\right)\left(\frac{\partial p_A}{\partial w_A}\right)^2 \\ &+ (\sigma p_A + (1-\sigma) w_A)\frac{\partial^2 q_B(p_B,p_A)}{\partial^2 p_A}\left(\frac{\partial p_A}{\partial w_A}\right)^2. \end{aligned} \tag{58}$$

Note that this derivative simplifies at the symmetric candidate equilibrium at which $w_A = w_B$, $p_A = p_B$ and the beliefs are correct. Moreover, using the downstream firms' first-order conditions further simplifies the terms such that, e.g., no terms with $\partial^2 p_A/\partial^2 w_A$ show up. This second-order derivative is negative under Assumption 5. This assumption is fulfilled, for instance, if $\partial^2 q_i(p_i,p_{-i})/\partial^2 p_i \leq 0$ and $\partial^2 q_i(p_i,p_{-i})/\partial^2 p_{-i} \leq 0$. The second element of the Hessian matrix evaluated at the symmetric candidate equilibrium is

$$\begin{aligned}\frac{\partial^2\Omega^U}{\partial w_A\partial w_B} &= 2\left(1-\sigma+\sigma\frac{\partial p_A}{\partial w_A}\right)\frac{\partial q_A(p_A,p_B)}{\partial p_B}\frac{\partial p_A}{\partial w_A} \\ &+ 2(\sigma p_A + (1-\sigma) w_A)\frac{\partial^2 q_A(p_A,p_B)}{\partial p_A \partial p_B}\left(\frac{\partial p_A}{\partial w_A}\right)^2, \end{aligned} \tag{59}$$

where we again use that the demand and the equilibrium are symmetric ($p_A = p_B$, $\partial q_A(p_A,p_B)/\partial p_B = \partial q_B(p_B,p_A)/\partial p_A$, $\partial p_A/\partial w_A = \partial p_B/\partial w_B$), as well as that the beliefs are correct in equilibrium.

The Hessian is negative definite if $-\partial^2\Omega^U/\partial^2 w_A > \left|\partial^2\Omega^U/\partial w_A\partial w_B\right|$. The second-order derivative $\partial^2\Omega^U/\partial^2 w_A$ is negative by assumption and the sign of $\partial^2\Omega^U/\partial w_A\partial w_B$ can be either positive or negative. By inserting Equations (58) and (59) in the inequality above, we obtain

$$\begin{aligned}&\left(2\frac{\partial q_A(p_A,p_B)}{\partial p_A} + p_A\frac{\partial^2 q_A(p_A,p_B)}{\partial^2 p_A}\right)\left(\frac{\partial p_A}{\partial w_A}\right)^2 + (\sigma p_A + (1-\sigma) w_A)\frac{\partial^2 q_B(p_B,p_A)}{\partial^2 p_A}\left(\frac{\partial p_A}{\partial w_A}\right)^2 \\ > &\left|2\left(1-\sigma+\sigma\frac{\partial p_A}{\partial w_A}\right)\frac{\partial q_A(p_A,p_B)}{\partial p_B}\frac{\partial p_A}{\partial w_A} + 2(\sigma p_A + (1-\sigma) w_A)\frac{\partial^2 q_A(p_A,p_B)}{\partial p_A\partial p_B}\left(\frac{\partial p_A}{\partial w_A}\right)^2\right|. \end{aligned} \tag{60}$$

Simplifying the inequality and using that $\frac{\partial p_A}{\partial w_A} = -\frac{\partial^2\pi_A/\partial p_A\partial w_A}{\partial^2\pi_A/\partial^2 p_A}$ allows us to rewrite this inequality as

$$\begin{aligned}&-\frac{\partial q_A(p_A,p_B)}{\partial p_A} + (\sigma p_A + (1-\sigma) w_A)\frac{\partial^2 q_B(p_B,p_A)}{\partial^2 p_A}\frac{\partial p_A}{\partial w_A} + w_A\frac{\partial^2 q_A(p_A,p_B)}{\partial^2 p_A}\frac{\partial p_A}{\partial w_A} \\ > &\left|2\left(1-\sigma+\sigma\frac{\partial p_A}{\partial w_A}\right)\frac{\partial q_A(p_A,p_B)}{\partial p_B} + 2(\sigma p_A + (1-\sigma) w_A)\frac{\partial^2 q_A(p_A,p_B)}{\partial p_A\partial p_B}\frac{\partial p_A}{\partial w_A}\right|. \end{aligned} \tag{61}$$

If Condition (61) holds at the candidate marginal input price $w^{tpu}(\sigma)$, the necessary and sufficient conditions for a perfect Bayesian equilibrium with passive beliefs and partial forward ownership to exist are fulfilled.[35]

With linear demand, the inequality in (61) simplifies to

$$-\frac{\partial q_A(p_A,p_B)}{\partial p_A} \quad > \quad \left|2\left(1-\frac{\sigma}{2}\right)\frac{\partial q_A(p_A,p_B)}{\partial p_B}\right|, \tag{63}$$

as $\frac{\partial p_A}{\partial w_A} = 1/2$ in this case (see Equation (46)). Using the definitions of the demand elasticities (Equation (57)), Condition 63 can be written as

$$\frac{\epsilon}{2} \quad > \quad \left(1-\frac{\sigma}{2}\right)\epsilon_s. \tag{64}$$

This condition is reported in the proposition. It implies that the condition for equilibrium existence is easier to fulfill with forward ownership than under vertical separation.

Last, we characterize the comparative static of the marginal input price $w^{tpu}(\sigma)$ (which is implicitly defined by Equation (54)) with respect to the forward ownership share σ. Suppose that Condition (61) is fulfilled such that an equilibrium exists. Using the symmetry of the demand and candidate equilibrium (with $w_A^{tpu} = w_B^{tpu} = w^{tpu}$ and $p_A = p_B$), we can write the supplier's first-order condition in Equation (54) as

$$\left(\sigma p_i + (1-\sigma)\, w^{tpu}\right)\frac{\partial q_{-i}(p_i,p_{-i})}{\partial p_i} \quad + \quad w^{tpu}\frac{\partial q_i(p_i,p_{-i})}{\partial p_i} = 0, \tag{65}$$

Implicit differentiation yields

$$\frac{\partial w^{tpu}(\sigma)}{\partial \sigma} = -\frac{\left(p_i - w^{tpu}\right)\frac{\partial q_{-i}(p_i,p_{-i})}{\partial p_i}}{\partial^2\Omega^U/\partial^2 w_i} > 0, \tag{66}$$

which is positive as the denominator is negative if an equilibrium exists, and the numerator is positive as $p > w$ and $\frac{\partial q_{-i}(p_{-i},p_i)}{\partial p_i} > 0$. We conclude that the marginal input price $w^{tpu}(\sigma)$ increases in σ. As there is a one-to-one mapping between the input prices and the downstream prices, this implies that also the symmetric downstream prices $p_A(\sigma)$ and $p_B(\sigma)$ increase in σ. This establishes the result. □

Appendix C: Unobservable two-part tariffs under wary beliefs

In this appendix, we extend the analysis of unobservable two-part tariffs with passive beliefs to the belief refinement of wary beliefs. We focus on the case of Cournot competition as in Section 4. Moreover, we restrict attention to the case of an upstream

[35] Under vertical separation ($\sigma = 0$), the same condition as in Rey and Vergé (2004) emerges (as $w_i = 0$ in this case):

$$-q_A(p_A,p_B) > 2\frac{\partial q_A(p_A,p_B)}{\partial p_B} \quad \Longleftrightarrow \quad \frac{\epsilon}{2} \geq \epsilon_s. \tag{62}$$

monopoly (marginal costs of the competitive fringe c so large that it is not a relevant supply alternative).

With forward ownership, a supplier's optimal contract offer to one firm generally depends on the supply contract it has offered to the other firm, even with downstream quantity competition (see Section 4). This is in contrast to the result that a supplier's contract offer is not affected by the other contract in the case of vertical separation (Hart and Tirole, 1990; Rey and Vergé, 2004). With wary beliefs, we account for the fact that a downstream firm updates its belief about the competitor's contract offer. In particular, given its own contract offer, a downstream firm with wary beliefs anticipates that the supplier will behave optimally as regards the other downstream firm. We focus on wary beliefs that depend only on the wholesale price and that are defined as follows (see McAfee and Schwartz, 1994; Rey and Vergé, 2004):

Definition 1. When downstream firm i receives a contract $t_i = (w_i, f_i)$, it believes that

(i) the manufacturer expects it to accept this contract,

(ii) given that firm i accepts (w_i, f_i), the manufacturer offers downstream firm $j \neq i$ the contract $(W_j(w_i), F_j(w_i))$ that is best for the manufacturer from among all contracts acceptable to firm j, and

(iii) downstream firm j reasons the same way.

In the following proposition, we summarize our results for the belief refinement of wary beliefs. Note that we do not derive equilibrium existence in this case but characterize marginal input prices provided that an equilibrium exists.[36]

Proposition 7. *Suppose supplier U internalizes $\sigma \in (0,1)$ of the downstream firms' profits and charges secret two-part tariffs. Suppose further that the downstream firms compete in quantities and hold wary beliefs (Definition 1). In any equilibrium, the input prices are above the supplier's marginal costs and below the industry-maximizing level. The input prices increase in the degree of forward internalization σ.*

Proof of Proposition 7. Under Definition 1, (say) downstream firm A believes that the supplier charges its competitor B an input price $W_B(w_A)$ that is optimal given w_A:

$$\begin{aligned} W_B(w_A) \quad &= \arg\max\nolimits_{w_B} \quad \pi^U(q_A(w_A), q_B(w_B)) && (67)\\ &\qquad + (1-\sigma)(f_A + f_B) \\ &\qquad + \sigma(\pi_A(q_A(w_A), q_B(w_B)) + \pi_B(q_B(w_B), q_A(w_A))) && (68)\\ s.t. \qquad & f_B \leq \pi_B(q_B(w_B), Q_A(W_A(w_B))), \end{aligned}$$

with

$$\pi^U(q_A(w_A), q_B(w_B)) = \sum_{i\in\{A,B\}} w_i q_i(w_i), \tag{69}$$

[36] For the case of vertical separation, Rey and Vergé (2004) derive equilibrium existence with polynomial beliefs.

and

$$\pi_i\left(q_i\left(w_i\right), q_{-i}\left(w_{-i}\right)\right)=\left(p_i\left(q_i\left(w_i\right), q_{-i}\left(w_{-i}\right)\right)-w_i\right) q_i\left(w_i\right).$$

Solving the participation constraints under equality for the fixed fees and inserting these into the supplier's objective function yields

$$\begin{aligned}
\Omega^U\left(w_A, w_B\right) &= \pi^U\left(q_A\left(w_A\right), q_B\left(w_B\right)\right) \\
&+ (1-\sigma)\left(\pi_A\left(q_A\left(w_A\right), Q_B\left(W_B\left(w_A\right)\right)\right)+\pi_B\left(q_B\left(w_B\right), Q_A\left(W_A\left(w_B\right)\right)\right)\right) \\
&+ \sigma\left(\pi_A\left(q_A\left(w_A\right), q_B\left(w_B\right)\right)+\pi_B\left(q_B\left(w_B\right), q_A\left(w_A\right)\right)\right).
\end{aligned} \tag{70}$$

The resulting first-order condition of Ω^U with respect to w_A is

$$\begin{aligned}
\frac{\partial \Omega^U\left(w_A, w_B\right)}{\partial w_A} &= \left(\frac{\partial \pi^U\left(q_A\left(w_A\right), q_B\left(w_B\right)\right)}{\partial q_A}+\frac{\partial \pi_A\left(q_A\left(w_A\right), Q_B\left(W_B\left(w_A\right)\right)\right)}{\partial q_A}\right) \frac{\partial q_A\left(w_A\right)}{\partial w_A} \\
&+ \left(\sigma\left(1-\frac{\partial W_B}{\partial w_A}\right)+\frac{\partial W_B}{\partial w_A}\right) \frac{\partial \pi_A\left(q_A\left(w_A\right), Q_B\left(W_B\left(w_A\right)\right)\right)}{\partial Q_B} \frac{\partial q_A\left(w_A\right)}{\partial w_A}=0,
\end{aligned} \tag{71}$$

where beliefs are correct in equilibrium. For $\sigma=0$, that is under vertical separation, we know from Rey and Vergé (2004) that a downstream firm's belief about the competitor's input price does not change under quantity competition if its own input price changes: $\partial W_B\left(w_A\right) / \partial w_A=0$. Hence, we obtain, as with passive beliefs, that the supplier optimally sets $w_i=0,\ i \in\{A, B\}$ in order to maximize the bilateral profit $\pi^U+\pi_i$.

For $\sigma>0$, it is not necessarily true that the belief does not depend on the own contract offer. In order to assess the first-order condition for $\sigma>0$, we therefore evaluate how downstream firm A updates its belief $W_B\left(w_A\right)$ if the input price w_A changes. By symmetry, the same argument applies to downstream firm B. The definition of wary beliefs implies $\frac{\partial \Omega^U\left(w_A=W_A\left(w_B\right), w_B\right)}{\partial w_A}=0$ and $\frac{\partial \Omega^U\left(w_A, w_B=W_B\left(w_A\right)\right)}{\partial w_B}=0$ due to the assumption of mutually optimal offers of the supplier. Implicit differentiation of $\frac{\partial \Omega^U\left(w_A, w_B\right)}{\partial w_B}|_{w_B=W_B\left(w_A\right)}=0$ with respect to w_A is hence zero by definition of wary beliefs. That is,

$$\frac{\partial^2 \Omega^U\left(w_A, w_B=W_B\left(w_A\right)\right)}{\partial^2 W_B} \frac{\partial W_B\left(w_A\right)}{\partial w_A}+\frac{\partial^2 \Omega^U\left(w_A, w_B=W_B\left(w_A\right)\right)}{\partial w_B \partial w_A}=0, \tag{72}$$

where the expressions arise from differentiating $\Omega^U\left(w_A, W_B\left(w_A\right)\right)$ with respect to each argument. Rearranging this expression, and using that beliefs are correct in equilibrium, that is $w_i=W_i\left(w_{-i}\right)$, yields

$$\frac{\partial W_B\left(w_A\right)}{\partial w_A}=-\frac{\partial^2 \Omega^U\left(w_A, w_B\right) / \partial w_B \partial w_A}{\partial^2 \Omega^U\left(w_A, w_B\right) / \partial^2 w_B}. \tag{73}$$

Moreover, if the second-order conditions of the supplier's maximization problem are fulfilled (see the concavity requirements explained in the proof of Proposition 6) it holds that

$$\left|\frac{\partial^2 \Omega^U\left(w_A, w_B\right)}{\partial^2 w_B}\right|>\left|\frac{\partial^2 \Omega^U\left(w_A, w_B\right)}{\partial w_A \partial w_B}\right|, \tag{74}$$

which implies

$$\left|\frac{\partial W_B\left(w_A\right)}{\partial w_A}\right| < 1. \tag{75}$$

Based on this result, we can evaluate the comparative static of the marginal input price with respect to the forward ownership share, that is $\partial w_A\left(\sigma\right)/\partial\sigma$. Implicit differentiation of the first-order condition in Equation (71) yields

$$\frac{\partial w_A\left(\sigma\right)}{\partial\sigma} = -\frac{\left(1-\frac{\partial W_B}{\partial w_A}\right)\left(\partial\pi_A\left(q_A\left(w_A\right), Q_B\left(W_B\left(w_A\right)\right)\right)\right)/\partial Q_B}{\partial^2\Omega^U/\partial^2 w_A + \partial^2\Omega^U/\partial w_A\partial w_B} > 0. \tag{76}$$

If an equilibrium exists, the derivative is weakly positive as (i) the denominator is strictly negative (due to the concavity of the supplier's maximization problem) and the numerator is positive as $\partial W_B\left(w_A\right)/\partial w_A \in (-1, 1)$ (see Equation (75)).

Moreover, we can add and subtract $\frac{\pi_B\left(q_B\left(w_B\right), q_A\left(w_A\right)\right)}{\partial q_A}\frac{\partial q_A\left(w_A\right)}{\partial w_A}$ to and from the supplier's first order condition in Equation (71) and re-write it as

$$\begin{aligned} \frac{\partial\Omega^U}{\partial w_A} &= \frac{\partial\pi^I\left(q_A\left(w_A\right), q_B\left(w_B\right)\right)}{\partial q_A}\frac{\partial q_A\left(w_A\right)}{\partial w_A} \\ &+ \left(1-\sigma\right)\underbrace{\frac{\partial\pi_A\left(q_A\left(w_A\right), Q_B\left(W_B\left(w_A\right)\right)\right)}{\partial Q_B}}_{<0}\underbrace{\frac{\partial Q_B\left(W_B\left(w_A\right)\right)}{\partial W_B}}_{<0}\underbrace{\left(\frac{\partial W_B}{\partial w_A}-1\right)}_{<0} = 0. \end{aligned} \tag{77}$$

Recall that the marginal input prices that maximize the industry profit are denoted with w^I. Importantly, for $\sigma < 1$, the second line of Equation (77) is negative. This implies that the optimal input prices are below the industry-maximizing level unless $\sigma = 1$. This establishes the result. □