URSI-B EMTS 2025, Bologna, 23–27 June

Efficient Method of Moments Analysis of Periodic Metasurfaces Combining Spectral and Spatial Domain Approaches

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Abstract

This paper presents an efficient Method of Moments (MoM) formulation for the analysis of periodic metasurface (MTS) antennas. Whereas the periodic MoM matrix can be computed in spatial or spectral domains, its slowly convergent behavior in both domains directly impacts its computation time. In this work, it is proposed to accelerate the MoM matrix computation time through an appropriate combination of spectral and spatial domain approaches. This will be particularly well suited for the analysis of embedded element patterns of infinite periodic structures as the latter can be obtained using the periodic MoM for each direction of observation. Validation results are shown for two different MTS implementations, with details of the computation times.

1 Introduction

Infinite arrays of printed structures have been the subject of intense research for several decades [1]. Those were traditionally studied for applications in the field of phased arrays, but over the past decade, metasurfaces (MTSs) have represented a growing field of application for this type of structures [2, 3, 4]. For periodic structures, the possibility of introducing periodic boundary conditions, and hence to limit the numerical analysis to the unit cell, represents an important advantage. This can for instance be carried out with the Method of Moments (MoM) [5], using periodic Green's functions [6, 7, 8, 9]. Such an approach allows the full inclusion of the effects of mutual coupling, except for deviations that may appear near the edges of the finite periodic structure (array truncation). However, modulated MTSs are usually comprised of a large number of inclusions that are very small compared to the wavelength, such that the number of basis functions, and hence the number of unknowns per unit cell, may become prohibitive, especially if the unit cell is made of a variety of different patches covering several squared wavelengths. This paper presents an efficient way of formulating and implementing this problem in the framework of the infinite-array approximation. It makes use of a combination of spectral and spatial domain approaches for the interaction between basis and testing functions. It is based on a technique exploited in [10] for the efficient calculation of the free-space periodic Green's function. In a nutshell, the testing functions are artificially shifted to speed up the spectral-domain interactions while a space-domain correction is introduced.

The remainder of this paper is organized as follows. Section 2 reminds the periodic MoM formulation in spectral domain. Then, an acceleration technique for the MoM matrix computation is proposed in Section 3. Validation results as well as details of the computation times are shown in Section 4. They are given for the embedded element currents, i.e. currents on the whole periodic structure when only one unit cell is excited. Conclusions are drawn in Section 5.

2 Periodic Method of Moments Formulation

This paper focuses on the analysis of periodic MTS antennas with the MoM. This type of structure consists of a grounded dielectric slab on which sub-wavelength metal patches are periodically repeated with periods a and b along x and y, respectively. The unit cell may contain several different patches. The structure is excited with a 2-D phased array of unitary vertical elementary dipoles (VEDs) located in the middle of the substrate, with the same periodicity as the MTS, and with phase shifts $-\psi_x$ and $-\psi_y$ along x and y, respectively. This type of problem is typically solved using the periodic MoM [6, 7, 8, 9]. In this work, one wants to obtain the embedded element current of the infinite structure. This quantity can be obtained by solving the periodic MoM for each direction of observation with the appropriate phase shift [11], which further emphasizes the need for a very efficient periodic MoM. The MoM linear system of equations is obtained as a Galerkin solution of the electric field integral equation, and can be expressed as $\underline{\underline{Z}}\ \underline{i} = \underline{v}$, where $\underline{\underline{Z}}$ is the MoM impedance matrix, $\underline{v}$ is the excitation vector, and $\underline{i}$ is the vector containing the weights associated with the current distribution decomposition into basis functions [5]. To obtain the MoM impedance matrix, the convolution integrals in the space domain can be expressed as products in the spectral domain [7, 8, 12]. This approach relies on the knowledge of the Fourier transforms of basis functions, and of the dyadic Green's function in spectral domain [13]. For coplanar elements, it can be written as

$$Z(m,n) = \frac{1}{ab} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \underline{\tilde{F}}_t^m(-k_x^p, -k_y^q)^T \\ \underline{\underline{\tilde{G}}}(k_x^p, k_y^q)\ \underline{\tilde{F}}_b^n(k_x^p, k_y^q), \tag{1}$$

where $\underline{\underline{\tilde{G}}}$ is the spectral dyadic Green's function of the dielectric slab, and where $\underline{\tilde{F}}_b^n$ and $\underline{\tilde{F}}_t^n$ are the Fourier transforms of the n-th basis and testing function, respectively. The superscripts T and $\sim$ refer to the transpose operator and to the spectral domain through a Fourier transform, respectively. The sum is carried over the Floquet wavenumbers $k_x^p = \psi_x/a + p2\pi/a$ and $k_y^q = \psi_y/b + q2\pi/b$. In a similar way, the excitation vector can be computed as

$$v(m) = \frac{-1}{ab} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \underline{\tilde{F}}_t^m(-k_x^p, -k_y^q)^T \underline{\tilde{E}}_i(k_x^p, k_y^q), \tag{2}$$

where $\underline{\tilde{E}}_i$ is the spectral incident electric field associated with each VED excitation. This summation is rapidly converging thanks to the vertical shift of about a tenth of a wavelength between the feeding point (at the middle of the dielectric layer) and the MTS layer. However, the spectral series of (1) typically has slow convergence because sources and fields are at the same height. To tackle this issue, an acceleration technique combining spectral and spatial domain approaches is presented in Section 3.

3 Acceleration Technique

In order to accelerate the MoM matrix calculation, an average-medium solution may be extracted from (1) [14, 15]. This corresponds to the interaction in a homogeneous medium with a permittivity equal to the average of that of the substrate and that of the air. One has then a simpler situation with a term corresponding to the average-medium interaction and a rapidly converging term that accounts for the presence of the substrate. The latter reads as

$$Z_1(m,n) = \frac{1}{ab} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \underline{\tilde{F}}_t^m(-k_x^p, -k_y^q)^T \left(\underline{\underline{\tilde{G}}}(k_x^p, k_y^q) - \underline{\underline{\tilde{G}}}_{avg}(k_x^p, k_y^q)\right) \underline{\tilde{F}}_b^n(k_x^p, k_y^q), \tag{3}$$

where $\underline{\underline{\tilde{G}}}_{avg}$ is the dyadic Green's function of the homogeneous average medium in spectral domain. However, the corresponding average-medium solution which needs to be added afterwards may still be converging too slowly. This is where we can take benefit from the fact that the metallization is now limited to a plane in a homogeneous medium. Indeed, in that case, the spectral integration converges much faster if the testing functions are slightly shifted upward, by a distance Δz of the order of 5% of the wavelength [10]. Hence, in a first instance, the homogeneous-medium solution is computed in spectral domain with that shift:

$$Z_2(m,n) = \frac{1}{ab} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \underline{\tilde{F}}_t^m(-k_x^p, -k_y^q)^T \underline{\underline{\tilde{G}}}_{avg}(k_x^p, k_y^q)\, e^{-jk_{z,avg}|\Delta z|}\, \underline{\tilde{F}}_b^n(k_x^p, k_y^q), \tag{4}$$

where $k_{z,avg} = \sqrt{k_{avg}^2 - (k_x^p)^2 - (k_y^q)^2}$ with k_{avg} the wavenumber in the average homogeneous medium, and $\mathrm{Im}(k_{z,avg}) < 0$. The faster convergence can be traced back to the presence of the $e^{-jk_{z,avg}|\Delta z|}$ factor in the Green's function, where $k_{z,avg}$ becomes imaginary and hence the exponential turns into a rapidly converging real exponential. The above approach calls for a correction, which can be computed efficiently in space domain. That correction corresponds to the difference between (i) the solution with the testing functions in the same plane and (ii) the solution with the shifted testing functions, as explained above. This solution compensates the error generated by the spectral-domain solution with shift Δz referred to above. It is given by

$$Z_3(m,n) = \sum_{u=-\infty}^{+\infty} \sum_{v=-\infty}^{+\infty} e^{-j(\psi_x u + \psi_y v)} (I_0^{u,v}(m,n) - I_{\Delta z}^{u,v}(m,n)), \tag{5}$$

with

$$I_{\Delta z}^{u,v}(m,n) = \frac{-j\omega\mu}{4\pi} \iint_{S'} \iint_{S} \frac{e^{-jk_{avg}|\underline{r}_{u,v} - \underline{r}' - \Delta z \hat{\underline{z}}|}}{|\underline{r}_{u,v} - \underline{r}' - \Delta z \hat{\underline{z}}|} \left(\underline{F}_b^n(\underline{r}_{u,v}) \cdot \underline{F}_t^m(\underline{r}') - \frac{1}{k_{avg}^2} \nabla \cdot \underline{F}_b^n(\underline{r}_{u,v}) \nabla \cdot \underline{F}_t^m(\underline{r}')\right) dS\, dS', \tag{6}$$

where $\underline{r}_{u,v} = (x + ua, y + vb, 0)$ and $\underline{r}' = (x', y', 0)$ refer to the location of the basis and testing function, respectively, and where the integrals are carried out along $dS = dxdy$ and $dS' = dx'dy'$. The permeability of the medium is μ, and $\omega = 2\pi f$ for frequency f. The space-domain approach in (5) supposes that the interactions can be obtained with all the cells of the infinite array. However, since what is needed corresponds to the difference between fields tested on two slightly different heights, the convergence can be very fast. Besides, if the interactions between cells are kept in memory, results for any phase shift in (5) can be obtained by superimposing those interactions with the proper phase factor. This technique is very convenient for generating embedded element currents, i.e. currents on the whole periodic MTS when only one unit cell is excited. To sum up, the very slowly converging summation in (1) can be efficiently accelerated combining spectral and spatial domain approaches as

$$Z(m,n) = \underbrace{Z_1(m,n)}_{spectral} + \underbrace{Z_2(m,n)}_{spectral} + \underbrace{Z_3(m,n)}_{spatial}. \tag{7}$$

4 Simulation Results

The approach detailed in Section 3 is validated using a code based on scalar potentials for the established of the MoM matrix for periodic printed structures. The latter consists of a spatial approach with tabulation of the Green's function and extraction of the average homogeneous medium. This code has already been validated against commercial software in [16, 17]. The hybrid approach presented in Section 3 has been implemented in MATLAB R2023a language and runs on a conventional desktop computer equipped with an Intel Core i7 CPU and 32 GB of RAM. The operating frequency is fixed to 24 GHz. The dielectric slab has a thickness of 1.524 mm and a relative permittivity of $\varepsilon_r = 3.66$.

The upward shift of testing functions in (4) and (5) is chosen to be $\Delta z = \lambda_0/20$, with λ_0 being the free-space wavelength. Firstly, a unit cell of size $\lambda_0/8 \times \lambda_0/8$ composed of a single coffee bean patch with a diameter of about $\lambda_0/10$ is considered. The latter is meshed with 348 Rao-Wilton-Glisson (RWG) basis functions [18] and shown in Figure 1. The corresponding embedded element current obtained with the proposed methodology (labeled "combined") is compared with the one obtained with the code relying on a spatial approach introduced above (labeled "spatial") in Figure 1, where (k_x, k_y) are the spectral coordinates and $k_0 = 2\pi/\lambda_0$ is the free-space wavenumber. There is a very good agreement between the curves, which contributes to validating this method.

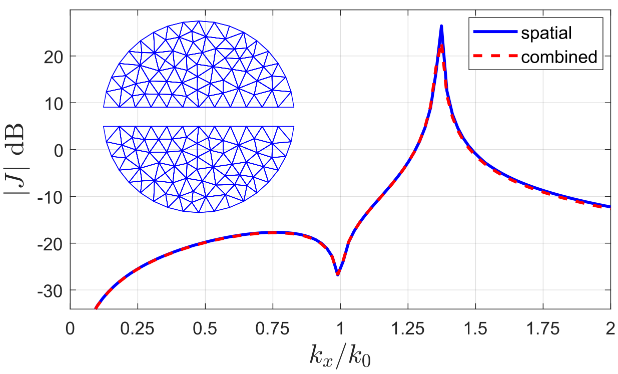
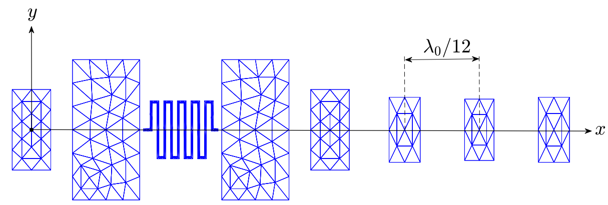
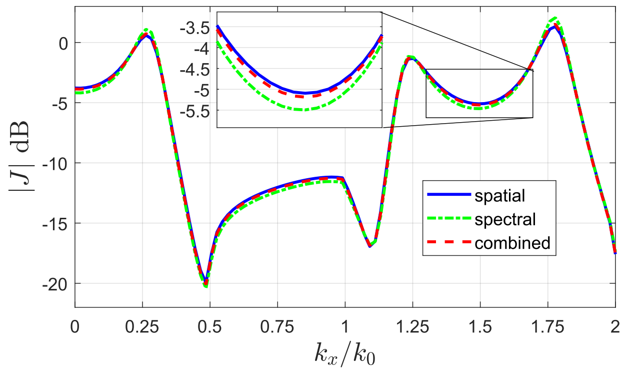


Figure 1. Comparison of the simulated embedded element currents obtained for a coffee bean patch, for $k_y = 0$.

Secondly, the MTS layer introduced in [19] is analysed. It consists of an array of 7 rectangular patches and a meander line repeated with periods $a = 2\lambda_0/3$ and $b = \lambda_0/7$. It is meshed with 524 RWG basis functions. Results are shown in Figure 2 where the embedded element current obtained with a fully spectral approach (i.e. using (1) and $k_{x,y}^{p;q} \in [-400, 400]\, k_0$) has also been simulated (labeled "spectral"). The computation times required to produce the results in Figure 2 are summarized in Table 1. For the hybrid approach proposed here, the spatial integrations (6) can be precomputed once for all pairs of cells (referred as "precomputation time" in Table 1) and added with the appropriate phase shift for each direction of observation, as explained in Section 3. Convergence to order of 0.1 dB has been observed for u and v limited to $[-2, 2]$ in (5), and $k_{x,y}^{p;q} \in [-15, 15]\, k_0$ in (3) and (4). As can be seen, the proposed methodology combining spatial and spectral domain approaches is much more efficient than the two other ones. It can be observed that, in that case, the total computation time is dominated by the precomputation of the space-domain integrals. However, the periodic MoM for each direction being very fast, the combined approach allows the efficient simulation of embedded element patterns even if many directions are required. Finally, it is worth mentioning that the computation time for the combined approach depends on the parameter Δz, and that the trade-off between the time required for the spectral summations and the spatial integrations deserves further investigation.



(a)



(b)

Figure 2. Simulation results for the array of 7 rectangular patches and 1 meander line. (a) Mesh. (b) Comparison of the simulated embedded element currents, for $k_y = 0$.

Table 1. Comparison of the computation times required to produce the results in Figure 2.

	Spatial	Spectral	Combined
Precomputation time			101 s
			$+$
Time for 1 direction	51 s	20.4 s	90 ms
	$\times$	$\times$	$\times$
Number of directions	100	100	100
	$=$	$=$	$=$
Total time	1h25	34 min	110 s

5 Conclusion

An efficient way of computing the periodic MoM matrix for the analysis of MTS antennas has been proposed. It is obtained through a combination of spectral and spatial domain approaches. It is particularly convenient for the analysis of embedded element currents in infinite periodic structures, as it requires the computation of the MoM matrix for every direction of interest. Simulation results for two different types of MTS layers have demonstrated the accuracy and computational time saving of the proposed method. Further works may consider another way of combining spatial and spectral domain approaches based on the extrapolation of the MoM matrix with shifted testing functions toward a result without any shift [20, 21].

6 Acknowledgements

This activity is funded under a Research Fellowship of the Fonds de la Recherche Scientifique - FNRS. It is also part of the Space4ReLaunch project, which is supported by the SPW Economie Emploi Recherche of the Walloon Region, under the grant agreement nr 2210181.

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