A COMPETITIVE SECTOR DEPLETION ALLOWANCE IN THE CONTEXT OF AN INTERTEMPORAL DOMINANT-FIRM MODEL

by

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Abstract

A frequently stated objective of non-cartel countries is to reduce their dependence on foreign supplies of a strategic exhaustible resource.

One measure seriously discussed by government officials (and representatives of extractive industries) involves the provision (or liberalization) of depletion allowances aimed at home-country extractive firms dealing in the cartelized product. When a depletion allowance is considered in the context of a dynamic dominant-firm model, it is found that short-run supply constraints may be lifted (and thus dependence assured) only at the expense of longer run dependence on foreign reserves. Further results warn that this policy instrument may involve significant costs and tradeoffs likely to reduce its ability to achieve the desired objective.

I. Introduction

It is conceivable that the reaction of a government excluded from the cartelization of a strategic exhaustible resource (and suffering from a subsequent restriction in its supply) takes the form of a depletion allowance directed at home-country extractive firms in the industry.[(1)] According to federal law, a depletion allowance is a tax reduction on income proportional to the amount of reserves extracted.[(2)] Presumably the aim of such a manoeuver would be to speed up the rate of extraction at home (i.e. tilting the extraction profile of home-country firms towards the present) thereby relieving short-run excess demand and simultaneously reducing dependence on foreign owned and controlled reserves. (Clearly, allusion can be made to the 1973-74 OPEC oil embargo and to the subsequent policy decisions made by governments of non-OPEC oil producing countries).[(3)]

Governments concerned with their dependence on foreign reserves have supported the development of a number of alternative strategies. Among these are stressed (a) conservation of the resource, (b) increased exploration activity and (c) the development of substitutes to the cartel product and/or research into replacement technologies. It is important, therefore, also to consider the simultaneous interaction of a depletion allowance with these strategies.

II. The problem

Our objective is to analyze the reactions of the cartel, and of the non-cartel extractive firms (collectively termed "the competitive fringe"), to the imposition of a depletion allowance in the competitive sector. This is done in the context of a dynamic dominant-firm model first developed by Salant (1976) and more recently extended by Gilbert (1978).[4] Production profiles, price paths, and terminal dates of extraction are derived for each sector and compared with optimal solutions in the absence of a depletion allowance.

A priori we expect the provision of a uniform depletion allowance to the competitive fringe, simultaneous with the event of cartel formation, to tilt non-cartel extraction towards the present.[5] This may be thought of in the following way :

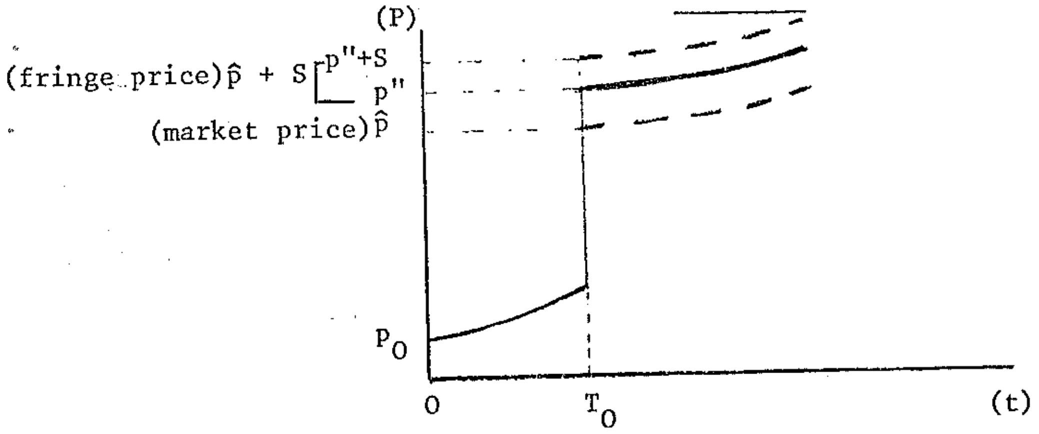
In a competitive market equilibrium all extractive firms initially receive the world price, P_0, for their output which, under the proper set of assumption,[6] rises with the rate of interest i.e. in the period 0 to T_0. (See figure 1). If at time T_0 supply is suddenly constrained by the event of cartel formation, the then world price, P, will jump to say P".

On the other hand, if a uniform depletion allowance (i.e. a negative severance tax), V, (where $V < 0$), is guaranteed to competitive producers at the moment of cartel formation, the price perceived by the fringe would no longer be P", but rather P"-V, or alternatively, P"+S (where S=-V).[7]

Initially then, price-taking competitive firms may be induced to shift production towards the present, thus driving down the price increase at T_0 until, at the margin, the world price increase settles at $\hat{P}$, where $\hat{P} < P''$.

Sequentially, if (a) a cartel forms raising the market price of an exhaustible resource to P", and (b) governments of non-cartel countries provide a depletion allowance to firms in the industry equivalent to S on each unit of output, then (c) the price faced by the competitive fringe, $P''+S$, will induce fringe extractive firms to supply more, resulting in (d) a fall in the market price to $\hat{P}$, such that $\hat{p} < p''$ (subject to the condition that $p''+S > \hat{p}+S \geq p'', \forall S > 0$).

Figure 1



Note : $\hat{p} < p'' \Longrightarrow \hat{p} + S < p'' + S$, So $p'' \leq \hat{p} + S < p'' + S, \forall S > 0$.

Here we note some rather crucial results of Salant's dominant-firm model. If both sectors begin with positive stocks, they must both operate simultaneously in the market for a period of time. When the price reaches a "termination" price, P^*, the competitive fringe completes its sales and abandons the market to the cartel. After this point the cartel operates alone. However, as long as competitors hold stocks, the price of the resource rises at the rate of interest (afterward it can rise at a smaller or equal rate).(8) (In fact, the price of an exhaustible resource can never rise by more than the rate of interest in equilibrium)(9).

A series of outcomes can be predicted to arise from this set of observations.

First, if the fringe faces a marginally higher price $\hat{P}+S$ (when $S > 0$) which at the instant of cartel formation is greater than the "old" price (i.e. in the absence of a depletion allowance, p'') then we expect the effective price path observed by the fringe, $(\hat{P}+S)e^{rt}$, to be everywhere greater than the price path $(P'')e^{rt}$ over the duration of the first period.(10) Therefore, profit-maximizing competitive firms will be induced to extract more of the resource at each point in time when facing the marginally higher price path.

Next, as a result of the increased extraction, and thus more rapid depletion of the resource, the fringe will exhaust its reserves at an earlier date (i.e. fringe extraction will be tilted towards the present). The date at which the cartel obtains total control of the market is thus moved forward.

Conversely, both the decline in the length of the first period, and the lower "world" price faced by the cartelized sector, $\hat{P}$, (where $\hat{P} < P$") induces conservation of the resource by the cartel. Thus, a priori, we expect the total amount of the resource extracted by the cartel in the first period to be reduced.

III. The model

Four equations characterize the Salant dominant-firm model. The first two,

(1) $$P^{*}e^{-ru} = P\left[Q^{M}(u, P^{*}) + Q^{c}(u, P^{*})\right]$$

(2) $$MR(P^{*})e^{-ru} = P^{*}e^{-ru} - Q^{M}(u, P^{*}) \cdot a(P^{*}e^{-ru})$$

where Q^{M} = cartel sales
Q^{c} = competitive sector sales
P^{*} = fringe termination price
MR = marginal revenue
a(P) = absolute value of the slope of the consumer demand curve

can be solved simultaneously to obtain the sales of each sector u moments before termination at P^{*}. In the case of a linear demand curve of the form

$$P = F - a(Q^{M} + Q^{c}),$$

(1) and (2) imply

(1') $$Q^{M}(u, P^{*}) = 1/a(F-P^{*})e^{-ru}$$

(2') $$Q^{c}(u, P^{*}) = F/a(1-e^{-ru})$$

Only these sales paths will permit price and marginal revenue to grow at the rate of interest until they reach respectively P^{*} and $MR(P^{*})$. Since competitive sales are assumed independent of the termination price,

(3) $$\int_{0}^{t^{*}} Q^{c}(u, P^{*})\, du = R^{c}$$

where R^{c} = competitive sector reserves

uniquely determines the duration of the first phase, i.e. T_0 to t^{*}, (where we set $T_0 = 0$).

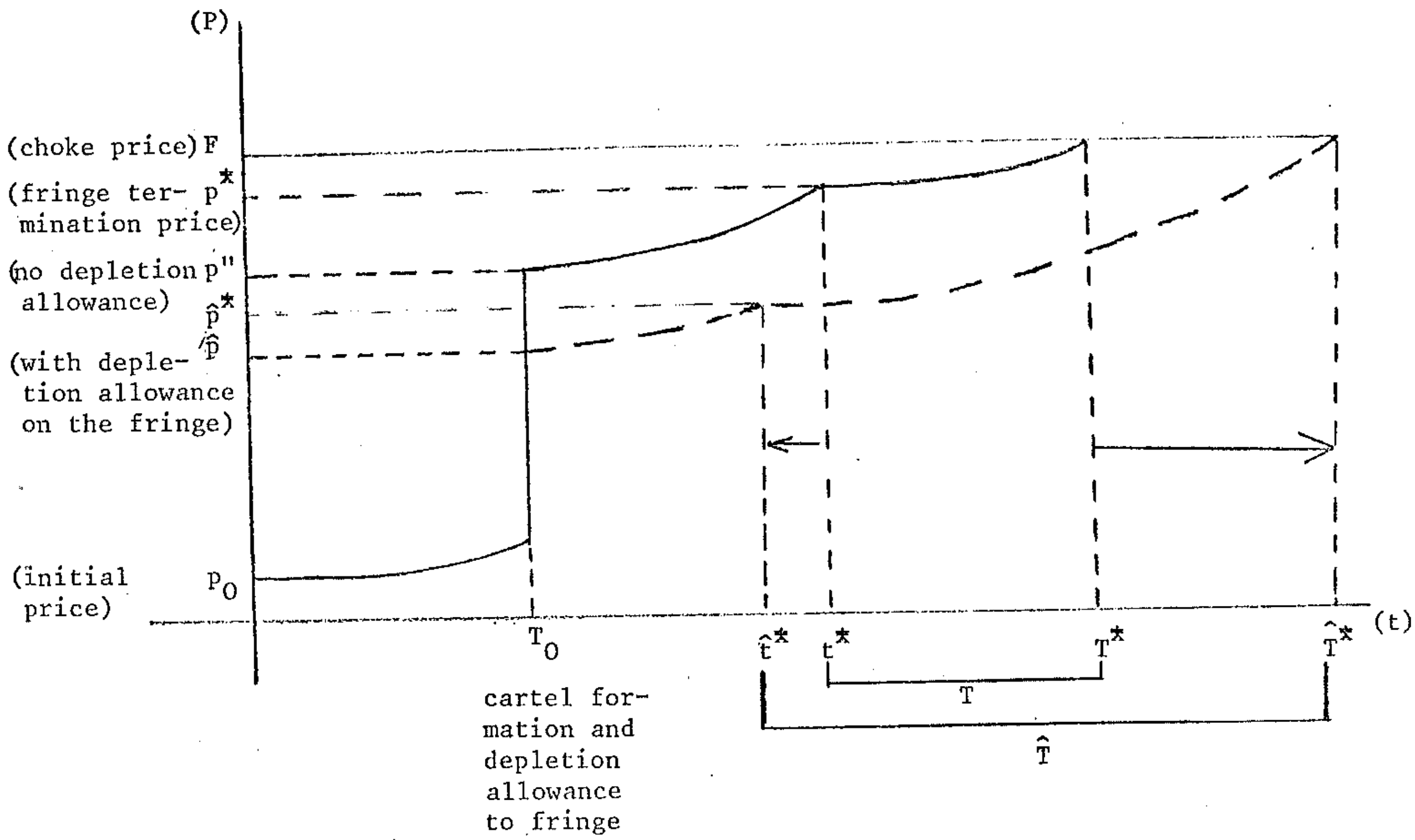
$$(4) \qquad \int_0^{t^*} Q^M(u,\ P^*)\ du + \Delta(P^*) = \bar{R}^M$$

where R^M = cartel reserves
$\Delta(P^*)$ = cartel sales in phase 2

may then be used to determine the termination price, P^*, of that phase.

Whenever an equilibrium exists, the price path will have the appearance as shown in Figure 2. The price begins at some level P", grows for t^* periods at the rate of interest, and then grows more gradually for T additional periods until the choke price, F, is reached.

Figure 2



Note : the relative magnitude of the changes due to a depletion allowance depend on the parameters involved.

IV. The results

Turning now to the results of our analysis, we observe some interesting changes in the equilibrium solutions due to the inclusion of a depletion allowance in the competitive sector. (See Table 1).[11]

TABLE 1

(1') $Q^M u = 1/a(F-P^*)e^{-ru}$

(1") $\hat{Q}^M u = 1/a(F-P^*)e^{-ru} + v/a$

(2') $Q^c u = F/a(1-e^{-ru})$

(2") $\hat{Q}^c u = F/a(1-e^{-ru}) - 2v/a$

where $v < 0$, $a > 0$

Comparing equation (1') to (1") we find that $\hat{Q}^M u < Q^M u$(where a "^" represents the solution when a competitive fringe depletion allowance, $v < 0$, is considered). In other words, if u moments before t^* cartel extraction is some value X, then with a depletion allowance on the competitive fringe, extraction at that same moment u would be $X + v/a$ Since $v < 0$ and $a > 0$, the model predicts that a cartel will extract less of the resource in the first phase when a depletion allowance is provided to the competitive sector.

On the other hand, comparing equations (2') and (2") we find that $\hat{Q}^c u > Q^c u$. This indicates that u moments before t^* the competitive fringe extracts more of the resource than it does in the absence of a depletion allowance.

Utilizing analytical technique we verify another of our earlier predictions. From the competitive fringe stock equation we find that

$$\frac{dt^*}{dv} = \frac{-2t^*/F}{\left[\frac{2v}{F} - (1 - e^{-rt^*})\right]} > 0$$

indicating that $\hat{t}^* < t^*$ under the assumptions $r > 0$, $F > 0$.

A depletion allowance on the competitive fringe thus encourages more rapid exploitation of the resource in that sector. This results in a more rapid depletion of the stock (i.e. $\hat{t}^* < t^*$) with the competitive fringe dropping out sooner. This allows the cartel to control the market at an earlier date.[12]

Solving for the fringe termination price we discover that it is lower when a depletion allowance is provided to competitive firms in that sector (i.e. $\hat{P}^* < P^*$).[13] This result enables us to establish that the price path $(\hat{P})e^{rt}$ must be everywhere below the price path $(P'')e^{rt}$ (where $\hat{P}$ is the initial price at T_0 with a depletion allowance on the fringe and P'' is the initial price in the absence of such a depletion allowance).[14]

Finally, we determine that the ultimate date at which the resource is exhausted, is extended by the provision of a depletion allowance to the competitive fringe. This suggests further that the market price of the resource will be everywhere lower, not only in the first phase, but in the second phase as well. (See Figure 2).

V. Policy conclusions

The preceding analysis reveals a strong correlation between the predicted outcomes and actual solutions derived from the model. Our results suggest that the competitive sector will exhaust its reserves at an earlier date if a depletion allowance is provided to non-cartel extractive firms. Thus, there is reason to believe that a "tilting" of the extraction profiles of fringe firms may occur, thereby relieving (in the short-run !) the excess demand of non-cartel countries and simultaneously reducing their dependance on foreign reserves.

However, of major concern is a consideration of the cost to the fringe of this temporary relief. Our results lead us to conclude that an important element of this cost may be reflected by the inefficiencies of having the market controlled at an earlier date by the cartel (which is then in the position of a monopolist). So, although a depletion allowance may alleviate short-term dependence on foreign supplies, the ultimate date at which the cartel gains control of the market is moved forward.

The sub-optimally of this event, as felt by non-cartel countries, may be moderated to some extent by the final result (derived from this model). The "artificially" lower price, P, faced by the cartel when a depletion allowance is provided to competitive firms, induces conservation of the resource by the cartel, postponing the date at which the choke price is attained. This is simply a variant of Hotelling's result that a monopolist will be relatively conservationist viz a viz the resource (see Hotelling 1931). Therefore, both the conservationist nature of the cartel, together with its premature control of the market, are sufficient conditions to guarantee that the resource stock will be stretched beyond its "natural lifetime" i.e. in the absence of a depletion allowance.

On the other hand, the cost (to the fringe) of "buying time" is reflected not only in the uncertainty and inefficiency associated with a more complete control of the market by the cartel, but also through the undermining of alternative strategies due to the existence of a lower market price. More specifically, a perverse incentive exists : (a) to consume more of the resource, (b) to limit exploration activity (unless the firm can be assured of a government subsidy), and (c) to reduce investment in the development of replacement technologies and/or substitutes to the cartel product. Thus, the results of this model would suggest that the imposition of a depletion allowance be followed by appropriately placed taxes and subsidies : taxes to restrict consumption of the resource, and subsidies to provide incentives for (i) exploration, (ii) the development of substitutes to the cartel product, and (iii) research into replacement technologies.

In conclusion, there appears to be a set of "trade-offs" which non-cartel countries must face when considering whether or not to provide a depletion allowance to home-country extractive firms. Within the context of this model, it has been shown that :

1) The fringe depletion allowance will indeed reduce short-run dependence on foreign supplies, but
2) the cartel will take over the market at an earlier date. Curiously, this fact also insures that
3) the cartel will tend to conserve the resource, thereby stretching its lifetime over a wider horizon. However,
4) the existence of a lower market price over the lifetime of the resource, suggests that
5) the development of alternative strategies may be inhibited by a perverse incentive to make use of the resource. Therefore,
6) government intervention may subsequently be required to correct the distortions introduced by the original strategy to provide a depletion allowance to competitive firms.

FOOTNOTES

(1) "Sometimes these allowances are very high indeed being intended to encourage production". Scott (1955).

(2) A Depletion allowance plays the same role as a negative severance tax (let $v < 0$ be the depletion allowance rate and $S = -v > 0$ be the severance tax rate).

(3) Note that the outcome of future cartel meetings may result in strategies taken by governments similar to the one outlined in this paper.

(4) See Appendix for derivation of the results.

(5) See for example Scott (1955) and Clark (1976).

(6) Hotelling (1931). For ease of exposition we do not include costs. This, however, should not significantly affect the results.

(7) To deal with a discontinuity is a recognized problem. The following result is derived analytically, since the limitations of the model prevent us from deriving this result explicitly.

(8) See Salant (1976).

(9) Otherwise, competitive speculators would enter and attempt to make unlimited profits by buying in the first period and selling in the next. Similarly, if the price begins to rise by less than the rate of interest, the competitive sector will find selling its inventory before this region is reached to be optimal.

(10) Note : $\hat{p} < p''$, but as $S \to 0$, $\hat{p} \to p''$ $(\hat{p} + S \geq p'',\ S > 0)$

(11) In this exercise we consider a linear demand curve of the form $p = F - a(Q^M + Q^c)$.

(12) The same point may be illustrated graphically by comparing the graphs of e^{-rt} and

$$1 + \frac{ar\ R^c}{F} - rt^*(1 - \frac{2v}{F}).$$

(13) See appendix Table 2.

(14) This is true since any given initial price must rise at the rate of interest during the first phase, $\hat{p} < p''$, and the fringe termination price is lower with a depletion allowance than without one $(\hat{p}^* < p^*)$.

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APPENDIX

Utilizing Salant's solution technique we derive analytical results for a particular specification of the model: (1). In the absence of a depletion allowance (i.e. with $v = 0$), and (2) with a depletion allowance ($v < 0$) provided to the competitive fringe.

Consider a linear specification of the world demand for oil of the form

$$p = F - aQ$$

where p = market price, F is the "choke" price, and $Q = Q^M + Q^c$, (where Q^M and Q^c refer to sales of the cartel and competitive fringe respectively). Marginal costs of extraction are assumed to be zero (for ease of exposition).

The cartel takes competitive sector sales as given and on this basis constructs an excess demand curve, which is the demand curve for its product. Using this demand curve, the cartel chooses a sales path which maximizes its discounted profits. The competitive fringe, meanwhile, takes cartel behaviour and the world price as given and attempts to maximize its own discounted profits.

Let p^* and MR^* be price and marginal revenue at time t^* at which the competitors exhaust their stock. Then u moments before t^* we know that, with $v = 0$,

$$(1) \qquad p^* e^{-ru} = F - a(Q_u^M + Q_u^c)$$

and

$$(2) \qquad MR^* e^{-ru} = F - aQ_u^c - 2a\, Q_u^M .$$

On the other hand, with $v < 0$ provided to the competitive fringe, equation (1) becomes, (1') $p^{*}e^{-ru} = (F-v) - a(Q_u^M + Q_u^c)$.

The marginal revenue associated by the cartel with the sale of Q_u^M units is simply the (perceived) price it would receive for selling anothers unit i.e. eq.(1), less the loss it would incur due to the slight reduction in price on the inframarginal units (Q_u^M) it sells.

Subtracting eq. (2) from eq. (1) we get,

(3) $Q_u^M = 1/a\ (p^{*} - MR^{*})\ e^{-ru}$.

Similarly, substracting eq. (2) from eq. (1') gives us,

(3') $Q_u^M = 1/a\ (p^{*} - MR^{*})\ e^{-ru} + v/a$

Since at time t^{*} the fringe exhausts its reserves, i.e. when $u = 0$,

$Q_u^c = 0$ and $v = 0$.

Therefore, (at t^{*}) from the demand equation we have

$p^{*} = F - a(Q^{M*})$

and

$MR^{*} = F - 2a\ (Q^{M*})$

such that

$p^{*} - MR^{*} = aQ^{M*} = F - p^{*}$.

Therefore, equations (3) and (3') become

(4) $\boxed{Q_u^M = 1/a\ (F - p^{*})\ e^{-ru}}$

and

$$(4') \quad \boxed{\hat{Q}_u^M = 1/a\ (F - p^*)\ e^{-ru} + v/a.}$$

Substituting equation (4) into (1) and (4') into (1') we get

$$(5) \quad \boxed{Q_u^c = F/a(1 - e^{-ru})}$$

and

$$(5') \quad \boxed{Q_u^c = F/a(1 - e^{-ru}) - 2v/a}$$

(Equations (4), (4'), (5) and (5') are listed in Table 1 of the paper as (1'), (1"), (2') and (2") respectively).

Since fringe stocks are exhausted at t^* we can write the competitive fringe stock equation as $\int_0^{t^*} Q_u^c\, du = R^c$.

Substituting (5') into the stock equation we find

$$(6) \quad F/ar \left[t^* r + e^{-rt^*} - 1 \right] - \frac{2vt^*}{a} = R^c$$

Totally differentiating eq. (6) we get :

$$\frac{dt^*}{dv} = \frac{-\dfrac{2t^*}{F}}{\left[\dfrac{2v}{F} - (1 - e^{-rt^*}) \right]} > 0$$

Indicating that $\hat{t}^* < t^*$ (where "^" refers to the provision of a fringe depletion allowance) under the assumptions $r > 0$, $F > 0$.

From equation (4') we know that total cartel reserves sold to t^*, i.e. first phase cartel extraction ($\int_0^{t^*} Q_u^M\, du = X_1$), are given by

$$(7) \quad \frac{(F - p^*)\ (1 - e^{-rt^*})}{ar} + \frac{vt^*}{a} = X_1$$

Turning to the second phase of extraction we observe that the cartel now operates alone in the market. Hence, the cartel (now a monopolist) starts with a marginal revenue

$$MR^{*} = F - 2a\ Q^{M^{*}}$$

where $Q^{M^{*}} = Q_u^M = 1/a(F - p^{*})\ e^{-ru}$

for $u = 0$

$$= 1/a(F - p^{*}).$$

This implies that $MR^{*} = 2p^{*} - F$.

From the principle of optimality, the cartel maximizes its profits in phase two (i.e. after the competitive fringe has dropped out) such that

$$MR = \lambda e^{ru}\ ;\ u = t^{*}, \ldots, t^{*} + T$$

Hence $MR^{*} = \lambda e^{rt^{*}}$

or (i) $\boxed{\lambda e^{rt^{*}} = 2p^{*} - F}$

Also, since at terminal time $T^{*} = t^{*} + T$, $Q^{M} = 0$, $MR = F$ such that,

(ii) $\boxed{\lambda e^{r(t^{*} + T)} = F}$

The size of the remaining stock of the resource exploited in the latter phase of the industry's life is given by, $\int_{t^{*}}^{t^{*}+T} Q_u^M\ du = X_2$

$MR = F - 2a\ Q^{M} \Rightarrow Q^{M} = \frac{F - MR}{2a}$ where $MR = \lambda e^{ru}$, such that

$$\int_{t^{*}}^{t^{*}+T} \left[\frac{F - MR}{2a}\right] du = 1/2a \left[FT - 1/r\ (\lambda e^{r(t^{*} + T)} - \lambda e^{rt^{*}})\right]$$

Substituting (i) and (ii) for $\lambda e^{rt^{*}}$ and $\lambda e^{r(t^{*} + T)}$ respectively gives us the amount of cartel reserves extracted in the second phase, or

$$(8) \qquad \frac{F}{ar}\left[\frac{rT}{2} + \frac{p^*}{F} - 1\right] = X_2$$

Total cartel reserves $(X_1 + X_2)$ may then be written as the sum of eq. (7) and eq. (8) :

$$(9) \qquad \boxed{R^M = \frac{(F - P^*)\ (1 - e^{-rt^*})}{ar} + \frac{vt^*}{a} + \frac{F}{ar}\left[\frac{rT}{2} + \frac{p^*}{F} - 1\right]}$$

Since both T (the duration of the second phase) and p^* appear in equation (9) we need a second equation in order to obtain analytical solutions for these variables. We can use the two marginal revenue expressions to give us another equation enabling us to solve the system. Recall that,

$$\text{(i)} \quad \lambda e^{rt^*} = 2p^* - F$$

and

$$\text{(ii)} \quad \lambda e^{r(t^* + T)} = \lambda e^{rt^*}(e^{rT}) = F \text{ or } \lambda e^{rt^*} = F\ e^{-rT}$$

This implies further that

$$\lambda e^{rt^*} = 2p^* - F = F\ e^{-rT}$$

such that

$$(10) \qquad \boxed{p^* = F/2(1 + e^{-rT})}$$

Substituting the expression for p^* into the cartel stock equation gives us

$$(11) \qquad \frac{2ar\ R^M}{F} + e^{-rt^*} - e^{-rt^*}(e^{-rT}) - \frac{2r\ vt^*}{F} - rT = 0.$$

Since the final exhaustion date is $T^* = r^* + T$, we can substitute $T = T^* - t^*$ into eq. (11). Totally differentiating this expression gives us

$$\frac{dT^*}{dv} = \frac{1}{\left[e^{-rT^*} - 1\right]}\left[\frac{2t^*}{F} + \frac{dt^*}{dv}\left[\frac{2v}{F} - (1 - e^{-rt^*})\right]\right]$$

where $\frac{dT^*}{dv} < 0$. This indicates that $\hat{T}^* > T^*$ (i.e. the resource is conserved, or extracted over a longer period of time, with the presence of a fringe depletion allowance).

Rewriting eq. (11) with the knowledge that $T^* = t^* + T$ and totally differentiating the resulting expression gives us :

$$\frac{dp^*}{dT^*} = r\,\frac{F}{2}\,e^{-rT^*}(e^{rt^*})\left[\frac{dt^*}{dT^*} - 1\right]$$

where $\frac{dp^*}{dT^*} < 0$, since $\frac{dt^*}{dT^*} = \frac{dt^*}{dv} \div \frac{dT^*}{dv}$, and from before $\frac{dt^*}{dv} > 0$, $\frac{dT^*}{dv} < 0$.

Furthermore $\frac{dp^*}{dv} = \frac{dp^*}{dT^*} \cdot \frac{dT^*}{dv}$ such that $\frac{dp^*}{dv} > 0$.

This same result can be found by taking the natural logarithm of eq. (11) thereby finding an expression for T in terms of p^* i.e. $T = 1/r\left[\ln F - \ln (2p^* - F)\right]$. This can then be substituted into eq. (9), the cartel stock equation, which upon total differentiation results in the expression :

$$\frac{dp^*}{dv}\left[\Delta'(p^*) - \frac{1}{ar}(1 - e^{-rt^*})\right] = \frac{-t^*}{a} - \frac{dt^*}{dv}\,Q^M(t^*, p^*)$$

where

$$\Delta(p^*) = \frac{F}{2ar}\left[\ln F - \ln(2p^* - F) - 1\right] + \frac{1}{2ar}(2p^* - F)$$

Thus, under the further assumption that $F/2 < p^* < F$ (the upper inequality is implied by the linear demand function, the lower inequality is necessary to guarantee a finite terminal time) we have $\frac{dp^*}{dv} > 0$, again indicating that $\hat{p}^* < p^*$.

Briefly summarizing our results we have :

TABLE 2

(1)		$\hat{Q}^c > Q^c$
(2)		$\hat{Q}^M < Q^M$
(3)	$\frac{dt^*}{dv} > 0 \Longrightarrow$	$\hat{t}^* < t^*$
(4)	$\frac{dp^*}{dv} > 0 \Longrightarrow$	$\hat{p}^* < p^*$
(5)	$\frac{dT^*}{dv} < 0 \Longrightarrow$	$\hat{T}^* > T^*$