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Numerical Cognition

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INTRODUCTION

The numerical domain is unique in at least three ways. First, numbers represent a particular aspect of reality (i.e., numerosity). Second, numbers are the object of specific processing such as calculation, parity judgment, magnitude comparison, and so on. And third, numbers can be represented in different formats: Arabic numbers (i.e., digit strings, e.g., *25*), written numbers (i.e., letter strings; e.g., twenty-five) or spoken numbers (i.e., sequences of sounds; e.g., TWENTY-FIVE), Roman numerals (XXV), and so on[1].

These characteristics have raised specific questions. Four of them will be considered in this chapter. First, are there functionally independent systems to process the different formats for numbers? Second, how does the human subject translate a number presented in one format into another format? Third, what are the basic functional components for calculation? And fourth, in which internal code(s) are arithmetic facts stored in memory?

THE BASIC COMPONENTS OF THE NUMBER PROCESSING SYSTEM

Number Comprehension and Production

Any task with numbers requires understanding and/or producing numbers. In examining these basic processes in patients, interesting dissociations have emerged. In particular, Benson and Denckla (1969) reported the case of a patient who could understand the quantity expressed by a number but could no longer produce numbers. So, he could point to the Arabic number corresponding to a spoken number or choose the answer to simple arithmetic problems among numbers written on a sheet of paper but failed in tasks such as counting, writing, reading numbers, or repeating them (i.e., all tasks that required producing numbers).

This difficulty in producing numbers contrasted with the problems in comprehending numbers reported by Gardner, Strub, and Albert (1975). More specifically, their patient's deficit appeared in tasks requiring the comprehension of spoken numbers (e.g., writing numbers to dictation, matching spoken and written numbers, or solving simple arithmetic problems pre-

[1]Spoken numbers will be put into quotes, written Arabic digits will be italicized, and written verbal numbers will be capitalized.

sented orally) whereas tasks using written Arabic numbers were correctly executed (e.g., comparing the magnitude of two Arabic numbers).

These two case studies were more anecdotal reports than detailed single-case studies but they nevertheless point to interesting dissociations between the ability to comprehend versus produce numbers, and between the comprehension of written Arabic versus spoken numbers.

Syntactic Mechanisms

Later case studies have tried to deepen the analysis of patient performance by examining not only the number of errors produced but also their nature. For instance, Noël and Seron (1992) described the case of a demented patient (NR) who produced many more errors in tasks requiring comprehension of Arabic numbers than in those involving comprehension of spoken numbers (e.g., in positioning numbers on an analogical scale,[2] she made 82% errors for 3-digit Arabic numbers but 35% for spoken numbers; in giving the number directly following a target; e.g., if presented *230*, she had to say "two hundred and thirty-one." She made 95% errors if the target was an Arabic number but only 15% if the target was presented orally). An analysis of her difficulty showed that identification of the digits composing the number was nearly perfect but that the global size of the Arabic number was not well captured. For instance, *236* was read as "two thousand three hundred and six," *489* was judged bigger than *3000* and when required to say the number coming after *268*, she said "two thousand six hundred and nine." We concluded that NR's deficit lay in the syntactic mechanisms that allowed an understanding of the combinatorial rules of Arabic digits. Indeed, identifying the quantity to which an Arabic number refers, does not simply require an understanding of the value of each individual digit (e.g., *732* and *327* do not refer to the same quantity). Our Arabic number system is a strict positional base-10 system. The quantity expressed by each digit depends upon its position within the number (2 means two in 72, twenty in 720, two hundred in 7231, and so on). More precisely, the quantity expressed by an Arabic number corresponds to the sum of the quantity expressed by each individual digit multiplied by a power of ten corresponding to the digit's position within the number (starting from the right). For instance, 273 = [3 × 10exp0] + [7 × 10exp1] + [3 ×10exp2]. In such a positional system, it is essential to have clear indications of the power of ten associated with each basic quantity (e.g., to be able to distinguish [3 x 10exp0] + [7 x 10exp1] from [3 x 10exp0] + [7 x 10exp4]). The digit zero has this function: it holds the positions which are not occupied by basic quantities (e.g., zeros allow us to distinguish 73 from 70,003). In this context, NR's difficulties can be seen as related to some part of this process since she generated the same representation of quantity for 236 and 2306.

NR's impairment to the syntactic mechanisms for comprehending Arabic numbers[3] can be nicely contrasted with DM's syntactic difficulties in producing Arabic numbers. DM (Cipolotti, Butterworth, & Warrington, 1994) could comprehend spoken numbers, but was incorrect in writing the corresponding Arabic forms. For instance, he transcoded FOUR THOUSAND THREE HUNDRED AND TWO as *4000 302*, thus selecting the right digits but not the correct number of zeros so that the magnitude of the number was not respected (for similar descriptions, see Singer & Low, 1933, or Noël & Seron, 1995). By contrast, DM's comprehension of Arabic numbers seemed preserved: He could read aloud or compare the magnitude of Arabic numbers. The cases NR and DM thus illustrate the fact that syntactic processes for Arabic numbers can be selectively lesioned. Furthermore, within the syntactic processing of Arabic numbers, DM (but not in NR) showed a dissociation between production and comprehension mechanisms.

[2]A scale with its ends marked 0 on the left and 3000 on the right was used. Three marks were drawn on the scale and the patient had to select the one corresponding to the position of the number presented (e.g., 1492 should be located near the middle of the scale).

[3]NR's difficulties with Arabic numbers were not restricted to the comprehension process. When asked to produce Arabic numbers (e.g., to dictation or from written verbal numbers), she was also error prone.

Difficulties with the syntax of the verbal system have also been described. Sokol and McCloskey's (1988) patient had difficulty when required to produce verbal numbers in both written and spoken modalities. For instance, he read 407013 as "four hundred thousand seven, thirteen" or wrote *106230* as ONE HUNDRED THOUSAND SIX TWO HUNDRED THIRTY one hundred thousand six two hundred thirty (although he was able to comprehend the Arabic numbers given as input). Most of the errors corresponded to the production of ill-formed verbal numbers. The other important observation made by McCloskey et al. (1988) was the high similarity (both in terms of nature and rate) between the written and spoken verbal responses. From that observation, the authors argued that the syntactic mechanisms for producing verbal numbers are shared by spoken and written modalities.

Lexical Mechanisms

The processing of the individual elements that make up the numbers, namely, the Arabic digits or the words, has also attracted the attention of many authors. One of the most striking observations concerning the processing of individual Arabic digits (their identification or their production) is their very strong resistance to brain damage. Indeed, all the cases reported so far show relatively spared processing of single digits in patients who are severely impaired with the reading of other written material such as words, verbal numbers, or letters (see Cohen & Dehaene, 1995, or Holender & Peereman, 1987). According to Dehaene (1992), this strong resistance to brain damage is due to the fact that Arabic digits are processed by both of the two cerebral hemispheres so that only patients with bilateral lesions should show problems with individual Arabic digits.

Difficulties in the processing of individual verbal numbers are more frequent. McCloskey, Sokol, and Goodman (1986) described the case of HY, a patient who made errors in reading aloud Arabic numbers. For instance *902* was read as "nine hundred and *six*", *5* as *seven* or *17* as "*thirteen.*" These errors stemmed from a difficulty in producing spoken numbers.[4] The vast majority consisted of the subtitution of one word for another. But these word substitutions were far from random! Indeed, according to Deloche and Seron (1982a, 1982b, 1987; Seron & Deloche, 1983, 1984), the words making up the verbal numbers are organized in ordered lexical classes with the *Units* (from one to nine), the *Teens* (from eleven to nineteen), the *Tens* (from ten to ninety), and the *Multipliers* (such as hundred, thousand,), see Table 20.1. Taking this theoretical proposal, McCloskey et al. (1986) discovered that most of the time, HY substituted a units word by another units word, a tens word by another tens word and a teens word by another teens word. Following Deloche and Seron's terminology, HY was thus producing *position errors,* namely, correct selection of the lexical class (unit, teens, tens) but not of the position within this class (e.g., thirteen and fifteen are both teens words but one holds the third position and the other holds the fifth).

Another type of word substition was observed in the reading of JG (McCloskey et al., 1986). For instance, this patient read *960* as "nineteen hundred sixteen" or *620* as "six hundred and *two.*" Here, the substitutions respected the position within the class but not the lexical class itself (i.e., *nine,* the expected word in 960, and *nineteen,* the produced word, both hold the ninth position in their respective lexical class but *nineteen* is a teens word whereas a unit word is expected).

The observation of these two specific type of errors (i.e., position and class errors) indicated that accessing the representation of a number word requires two pieces of information: the lexical class and the position within the class.

[4]Indeed, they were not observed in tasks that required comprehension of Arabic numbers without production of spoken numbers and they were much less frequent if a written, rather than a spoken, verbal number had to be produced.

Table 20.1
Lexical Structure of the Number Words.

	Lexical Class		
Position	Units	Teens	Tens
1st	One	Eleven	Ten
2nd	Two	Twelve	Twenty
3rd	Three	Thirteen	Thirty
4th	Four	Fourteen	Forty
5th	Five	Fifteen	Fifty
6th	Six	Sixteen	Sixty
7th	Seven	Seventeen	Seventy
8th	Eight	Eighteen	Eighty
9th	Nine	Nineteen	Ninety

A Functional Architecture for Number Processing

The analysis of brain-damaged patients has shown multiple dissociations and associations. On this basis, McCloskey, Caramazza, and Basili (1985) proposed a general framework for number processing (see Figure 20.1) in which they distinguished between comprehension and production modules for Arabic and verbal formats. The comprehension modules are used to generate a semantic representation of the number, namely, a representation of the quantity to which the number refers. The production modules convert the semantic representation of the number into the appropriate output (Arabic or verbal) format. For McCloskey et al. (1985) the semantic representation is a base-10 representation: It specifies in abstract form the basic quantities and the power of ten associated with each. For instance, *5060* and "five thousand sixty" both activate the same base-10 semantic representation: {5}10EXP3, {6}10EXP1. Within both the comprehension and production components, lexical and syntactic processing mechanisms are identified. Lexical processing involves the comprehension or the production of individual elements in the number (i.e., the words, e.g., SIX, the digits, e.g., *6*). Separate lexicons are supposed to contain the representations of the digits (Arabic lexicon), the written verbal numbers (graphemic lexicon), and the spoken numbers (phonologic lexicon). The processing of the relations between these lexical elements is made through syntactic mechanisms. In these, a distinction is introduced between the number formats: the Arabic syntactic mechanisms process Arabic numbers and the verbal syntactic mechanisms process both the written and the spoken verbal numbers.

This architecture provides a general framework that allows the interpretation of most case studies of number processing disorders. However, the content of some of these components still deserves investigation. In particular, the lexicon might contain more units than just the lexical primitives. For instance, Cohen, Dehaene, and Verstichel (1994) have argued that very familiar numbers such as famous dates, brands of cars, and so on might also be represented in the lexicon of the comprehension system. Others, such as Noël, Fias, and Brysbaert (1997) have suggested that numbers corresponding to products of simple multiplication (e.g., 36) might also be represented as wholes in the lexicon of the production system. The syntactic mechanisms also need to be described in greater detial.

Another important issue is how these different components are related to one another. For instance, how do we write the words corresponding to an Arabic number or read it out loud?

TRANSCODING PROCESSES

The Issue

Transcoding processes refer to the translation of a number from one format to another one, such as reading aloud Arabic numbers, writing a written number word to dictation, and so on.

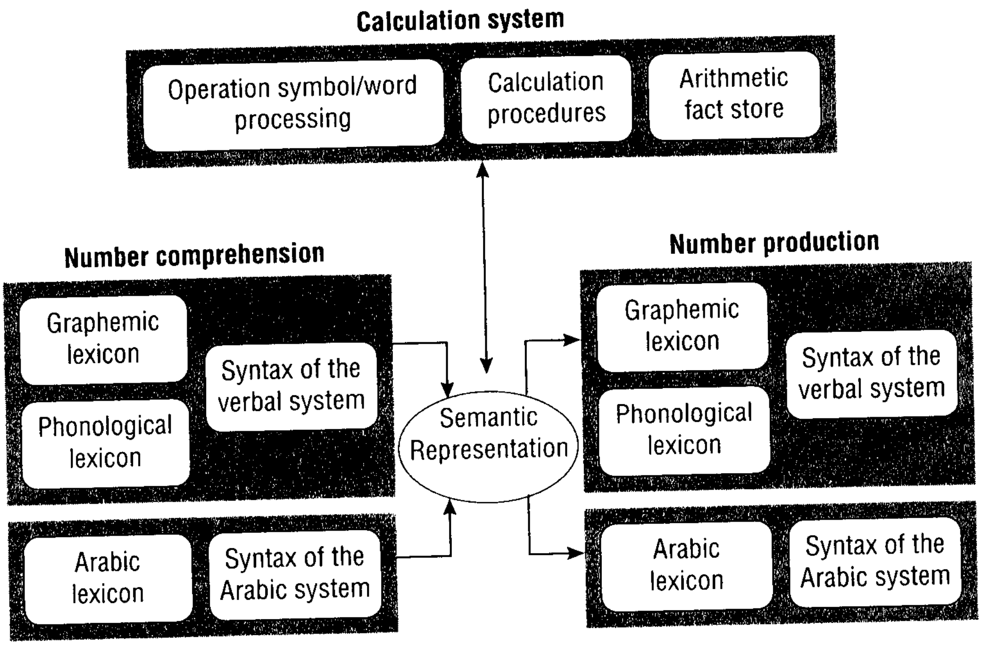

Figure 20.1: McCloskey, Caramazza, & Basili's (1985) architecture for calculation and number processing. Adapted from "Cognitive Mechanisms in Number Processing and Calculation: Evidence from Dyscalculia," by M. McCloskey, A. Caramazza, and A. Basili, 1985, *Brain and Cognition*, 4, 173, 174, & 190.

In McCloskey et al.'s (1985) model, transcoding is assumed to be realized by the successive actions of the comprehension and production modules: The former converts the input number into the corresponding base-10 semantic representation and the later translates this base-10 semantic representation into the expected production format. For instance, reading aloud *52* requires first generating the base-10 semantic representation {5}10EXP1, {2}10EXP0 through the Arabic number comprehension system and then, translating this base-10 semantic representation into the corresponding sequence of spoken words ("fifty-two") through the verbal number production system. In this framework, the base-10 semantic representation of numbers constitutes the obligatory bottle-neck between the satellite input and output systems for transcoding operations.

Some authors however, such as Deloche and Seron (1982a, 1982b; Seron & Deloche, 1983, 1984), Cipolotti and Butterworth (1995) or Dehaene (1992) have argued that other transcoding mechanisms should also be posited that would not require the generation of a semantic representation of the corresponding quantity and are therefore referred to as *asemantic transcoding algorithms*. An important debate has thus emerged between those postulating only semantic transcoding paths (the *semantic models*) and those assuming the existence of additional asemantic transcoding mechanisms (the *asemantic models*).

Neuropsychological Data[5]

At first glance, two types of dissociations should be looked for in order to demonstrate the existence of parallel semantic and asemantic transcoding algorithms: (a) patients showing

[5]This section has been largerly inspired by Seron and Noël's (1995) review of that question.

impaired semantic processing of numbers, though being able to transcode them and (b) patients with preserved semantic transcoding route but impaired asemantic transcoding. Yet, in this latter case, if the semantic pathway were intact, it could still be used to transcode any number and so, no transcoding errors should be expected. The situation is thus more complicated than at first thought: it seems indeed that only one side of the dissociation could be encountered, i.e., the first one. Yet, as we will see, the real situation of the data collected so far is quite unexpected since none of the cases reported correspond to this first side of the dissociation.

Three types of arguments for the existence of asemantic transcoding mechanisms have been proposed: (a) traces of the characteristics of the input in the output form, (b) impaired transcoding with spared number comprehension and production, and (c) transcoding performance varying according to the task demands.[6]

TRACES OF THE INPUT'S CHARACTERISTICS ON THE OUTPUT FORM

According to McCloskey et al. (1985), number production is based on the base-10 semantic representation corresponding to a number. Since this representation is independent of the presentation format of the input number, then errors in number production should be free of the characteristics of the format of the input number.

Yet, these assumptions were put into question by Cohen and Dehaene (1991) who argued for the existence of transcoding mechanisms acting directly on a visual-spatial code, and by Noël and Seron (1995) who proposed that the transcoding process could directly operate on the verbal input.

Cohen and Dehaene's (1991) patient, YM (whose left temporal tumor extended into the left temporal lobe and the lower part of the splenium of the corpus callosum) produced a substantial rate of errors (22%) in reading aloud Arabic numbers. These were mainly substitutions (87%) which tended to be visually similar to the corresponding correct digits. Furthermore with the standard horizontal presentation of Arabic numbers, YM was more likely to err on the leftmost digit than on the others, but when Arabic numbers were presented vertically, this spatial bias disappeared. The visual and spatial characteristics of these errors suggested a deficit located at a very peripheral stage of the visual encoding of the Arabic numbers. Yet, in a magnitude comparison task with pairs of Arabic numbers, YM was perfect, which indicated a preservation of the Arabic number comprehension mechanisms. This paradoxical profile led Cohen and Dehaene (1991) to argue that the reading of Arabic numbers was using asemantic mechanisms based on a visuo-spatial code which preserved the visual and the spatial characteristics of the stimulus and which were impaired in YM, while the comparison was mediated by an unimpaired semantic system.

The strength of these data has, however, been questioned by McCloskey (1992) and Noël (1994) who, among other things, argue that perfect performance in the comparison of Arabic digits does not necessarily imply that the Arabic number comprehension component is intact.[7]

[6]A fourth type of argument has to do with the observation of better reading of familar Arabic numbers (e.g., 1789, the French revolution) than unfamiliar ones (e.g., 8179; see, Beauvois & Derouesné, 1979; Cohen et al., 1994; Delazer & Girelli, 1997). An effect of the familairty status of the numbers can not be explained within the semantic model of McCloskey et al. (1985). However, it is problemantic for the other models as well. To account for such a profile, Cohen et al. (1994) have proposed that the referent of the number in various domains such as dates, ages, brands of cars, etc. (what they called *encyclopedic knowledge*) should also be coded for at the semantic level. Transcoding of familiar numbers could thus be mediated by this specific semantic representation of encyclopedic knowledge; whereas non-familiar numbers could not. The question here is thus the type of semantic representation that is activated during transcoding and not whether semantic mediation is necessary for transcoding. Therefore, we will not go into this type of data in this chapter.

[7]Let us suppose that *5* is read "four" and *8* as "six." This double error in digit identification could still lead to correct comparison of the magnitude: In both the Arabic numbers presented and those incorrectly identified, the second number is the larger.

Table 20.2
Examples of LR's Arabic Production from Different Verbal Structures that Represent the Same Numerosity.

Thousand-Sum Relationships	Response
One thousand two hundred	1200
One thousand three hundred	1000300
One thousand seven hundred	1000700
One thousand seven hundred and thirty-eight	100070038
One thousand five hundred and twenty-three	100050023
Hundred-Product Relationships	**Response**
Twelve hundred	1200
Thirteen hundred	1300
Seventeen hundred	1700
Seventeen hundred and thirty-eight	170038
Fifteen hundred and twenty-three	150023

The case presented by Noël and Seron (1995), LR (who had a probable dementia of the Alzheimer's type) made many errors in writing Arabic numbers corresponding to written or spoken verbal numbers (respectively, 50% and 33% of errors). These errors consisted of writing too many zeros (e.g., THREE THOUSAND SEVEN HUNDRED AND ONE was transcoded 3000701). Tasks tapping the comprehension of verbal numbers gave rise to only a few errors. But up to 75% of errors were obtained in tasks requiring the production of comparable Arabic numbers (i.e., writing the Arabic number corresponding to the quantity expressed by sets of tokens of different values). Furthermore, the errors produced were of the exact same type as those obtained in transcoding. On this basis, it was concluded that LR's transcoding errors were due to a deficit in the Arabic-number production system. However, a fine-grained analysis revealed that LR's errors were actually a function of the syntactic structure of the verbal numbers to transcode. Indeed, errors appeared mainly on sum relationships with Thousand (e.g, MILLE DEUX means MILLE + DEUX, or one thousand plus two) but not on product relations (e.g., DEUX MILLE means DEUX × MILLE, or two times one thousand). This gave rise to astonishing differences when comparing the transcoding of two verbal numbers that contact the same base-10 semantic representation but that have different syntactic structure. For instance, DOUZE CENTS (twelve hundred) refers to the same quantity as MILLE DEUX CENTS (one thousand two hundred) but only the later involves a sum relation with Thousand. Errors were more frequent in the case of thousand-sum structures (such as ONE THOUSAND TWO HUNDRED) than in hundred-product structures (such as TWELVE HUNDRED). And when errors were produced for the two comparable verbal forms, the faulty production of Arabic numbers differed according to the presented item (see Table 20.2 for a few examples). Such a profile is totally unexpected in a model such as McCloskey's which assumes that the same base-10 semantic representation (e.g., {1}10exp3; {2} 10exp2 for TWELVE HUNDRED or for ONE THOUSAND TWO HUNDRED) is activated by the two types of verbal numbers[8]. Accordingly, a deficit in the Arabic production system should lead to the same type and rate of errors regardless of the syntactic structure of the verbal input which has generated this base-10 semantic representation. Noël and Seron (1995) interpreted this case by suggesting that transcoding was not operating on the basis of base-10 semantic representations but on verbal number representations.

[8]LR could indeed understand the verbal numbers with both structures: He could select tokens to represent the corresponding magnitude, indicate how many times one hundred was contained in the numbers, or compare the magnitude of two numbers with the same verbal structure.

Transcoding Errors without Any Comprehension or Production Deficit

According to McCloskey et al. (1985), transcoding results from the succession of comprehension and production processes so that errors in transcoding should come from either impaired comprehension or production. Yet, several case studies have been reported with impaired transcoding but no underlying functional deficit in comprehension and production mechanisms. This, of course, is very problematic for the architecture proposed by McCloskey at al. (1985).

The case of AT (Blanken, Dorn, & Sinn, 1997) illustrates this situation (but see also SF; Cipolotti, 1995) but criticized by Seron and Noël (1995) or SAM (Cipolotti & Butterworth, 1995). AT is a German patient (CVA in the territory of the left middle cerebral artery) who produced errors in reading aloud Arabic numbers which consisted in the failure to apply correctly the inversion rule of the German number system. Thus, when asked to read *28*, Germans say "eight and twenty" but AT said "two and eighty." This type of error also occurred in verifying the equivalence between spoken and Arabic numbers or in selecting among 4 Arabic numbers, the one that matched a spoken verbal number (e.g., for "six and seventy" he selected *67* instead of *76*).

According to Blanken et al., 1997 these inversion errors were not due to an early visual deficit as AT could make a same/different judgment with pairs of Arabic numbers. Neither were they due to an Arabic comprehension deficit: AT was indeed fast and flawless in comparing the magnitude of pairs of Arabic numbers and could arrange triplets of Arabic numbers according to their ascending magnitude (e.g., *43, 34, 37*). Finally, the inversion errors could not be located at the level of the verbal production system since they did not appear in repetition, reading aloud verbal numbers, or when required to speak out loud the approximate age of 30 human faces.

Blanken et al. thus concluded that these inversion errors could not find a proper explanation in the semantic transcoding model of McCloskey et al. (1985). Rather, they interpreted AT's profile by suggesting that Arabic number reading was made through asemantic transcoding processing based on visual input. The inversion errors would be motivated by the conflict between the left-to-right visual-spatial arrangement of the Arabic digits and the reverse temporal sequence of German spoken numbers. In contrast, the magnitude tasks with Arabic numbers or the reading tasks of written verbal numbers were based on a strict left-to-right parsing where the inversion rule did not interfere.

In summary, AT was not able to read aloud Arabic numbers, but could still generate the semantic representation of Arabic numbers and produce spoken numbers from semantic representations. But, if the comprehension and production systems were unimpaired, why did this patient not use the seemingly intact semantic transcoding route instead of the supposedly defective asemantic one? This question has been tentatively answered by Cipolotti (1995) who argued that the instruction to "read the Arabic numbers" would preferentially activate the "asemantic route" and that, furthermore, there is reciprocal inhibition between the semantic and the asemantic transcoding routes, so that, activation of one inhibits the other. This hypothesis concerning the role of task demand in the selection between semantic or asemantic transcoding routes was more extensively studied by Cohen and Dehaene (1995).

Effects of Task Demands

Cohen and Dehaene (1995) reported the cases of SMA and GOD, two patients who were mildly impaired in reading aloud single digits (8% and 18% errors for SMA and GOD respectively). Yet, their reading performance differed when exactly the same Arabic numbers had to be processed in the context of magnitude comparison (e.g., *read these two digits aloud and tell me which is larger*: 10–15% errors) or for subsequent addition (*read these two digits aloud and give me their sum*: 31–45% errors). For Cohen and Dehaene, this suggested that at the very first stages of digit identification, task demands already influence the use of one processing route or the other.

The authors explained these patterns by assuming the existence of two distinct visual identification processes for digits, one in each cerebral hemisphere. However, as they also assumed that each hemisphere is not equally implicated in every numerical task, the task determines whether it is the left or the right digit identification system that is used. Accordingly, if one of the digit identification systems is impaired, then different error rates in digit identification might be observed depending on task demands.

A new model for numerical cognition was proposed by Dehaene (1992), accompanied later by an anatomical implementation of this architecture (Dehaene & Cohen, 1995). This model is called the *triple-code model* (see Figure 20.2) as it assumes the existence of three types of internal representations for numbers: Two of them are format-dependent: a *visual Arabic number form* and a *verbal word frame*, and one is format-independent: an *analog magnitude representation*. The two format-dependent representations are directly interfaced by format-specific comprehension and production mechanisms. The visual Arabic number form represents numbers as strings of digits on an internal visual-spatial sketchpad. Arabic number forms can be processed or generated in each of the cerebral hemispheres in the occipital-temporal regions of the ventral visual pathway. The verbal word frames represent numbers as syntactically organised sequences of words and their processing is only present in the left hemisphere within the classic perisylvian language areas. Finally, the analog magnitude representation is supposed to be the real semantic representation of quantity. Based on data from normal subjects (e.g., Dehaene, Dupoux, & Melher, 1990), Dehaene pictured it as an oriented number line with quantities being represented by local distributions of activations. Small numbers are on the left side and large numbers on the right side. In addition, this representation obeys Weber's Law such that it becomes increasingly imprecise as numbers get larger. Both hemispheres possess an analog magnitude representation of numbers in the vicinity of the parieto-occipito-temporal junction. These representations are typically used for tasks requiring the processing of the magnitude such as selecting the bigger of two numbers or giving approximations (e.g., how much is 13% of 1567?). Yet, contrary to the model proposed by McCloskey et al. (1985), the two format-dependent representations, i.e., the visual Arabic number form and the verbal word frame, are not only used for activating the corresponding analog magnitude representation from a sequence of digits or of words. In particular, the visual Arabic number form is supposed to code for parity information (i.e., whether numbers are odd or even) and is used in multi-digit operations. Conversely, the verbal word frames are used for counting and for storing addition and multiplication tables.[9]

Finally, the model specifies how internal representations are connected with one another: there are paths linking the different representations within each of the hemispheres, and paths connecting (through the corpus callosum) the left and right analog magnitude representations and the left and right Arabic visual systems.

[9]Dehaene and collaborators have tested the anatomical implementation of their model with neuroimaging studies but the picture obtained is not clear. Multiplication of two numbers should activate the left language areas. However, activation in the left hemisphere but located at in the inferior parietal cortex was obtained using ERPs (Keifer & Dehaene, 1997). Furthermore, activation of the bilateral inferior parietal lobules (with a small left assymetry) was measured for multiplication (relative to a magnitude comparison task) using PET (Dehaene et al., 1996). Conversely, bilateral activation in the parieto-occipito-temporal junctions is expected in case of magnitude comparison of numbers. Yet, Dehaene et al. (1996) could not find specific cortical activation for comparison (relative to multiplication) in the PET study. Using ERPs, Dehaene (1995) measured significant activation on sites close to the parieto-occipito junction, but with a right lateralization. However, a bilateral activation in the inferior parietal lobules (with a left-sided predominance) was observed by Pinel et al. (1999) with functional magnetic resonance. A bilateral activation was also measured by Thioux, Seron, and Pesenti (1999; with PET) in the intra-parietal lobes for two semantic tasks with numbers (judging if a number was odd or even, or larger and smaller than 5). Let us note that parity judgements are supposed to rely on the visual Arabic number forms (occipito-temporal regions) and not on the analog magnitude representations (parieto-occipito-temporal junctions).

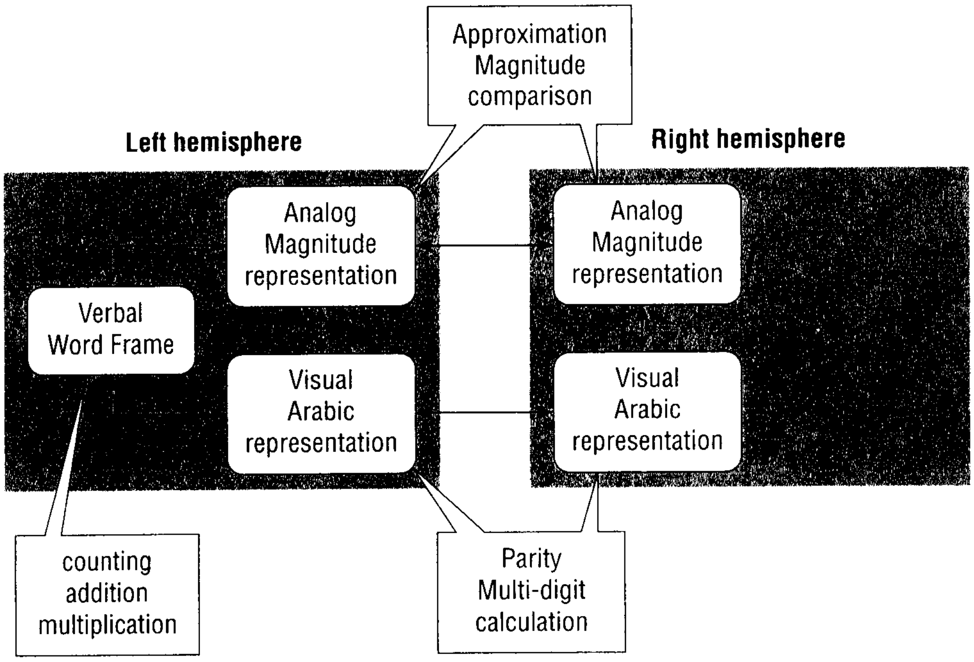

Figure 20.2: Dehaene and Cohen's triple-code model.

Accordingly, transcoding between Arabic and verbal numbers can use several paths. First, there is an asemantic transcoding route in the left hemisphere that directly connects the visual Arabic system (which generates the visual Arabic number representation) to the verbal system (which generates verbal word frames). Second, two paths link the visual Arabic system to the verbal system through the analog magnitude representation. The first of these semantic transcoding routes relates the left visual Arabic system to the verbal system through the left analog magnitude representation. The second semantic transcoding path goes from the right visual Arabic system, connects to the right analog magnitude representation which, in turn, contacts its homologue in the left hemisphere, which itself, is related to the verbal system. However, given that these analog magnitude representations follow Weber's law, these semantic routes can only be used for rounding up and are thus not suitable for precise transcoding of numbers (e.g., the analog magnitude representation of *212* would be rounded down to activate the quantity 200).

The triple-code model differs from McCloskey et al.'s (1985) in three main ways. First, the semantic representation is supposed to be an analog representation of magnitude. Second, the precision of that representation is not perfect so that semantic transcoding decreases in precision with larger numbers (following Weber's law). Third, the semantic representation is not the obligatory bottle-neck of all transcoding nor of all numerical processing (parity judgments or retrieval of arithmetic facts, for instance, do not require the activation of the analog magnitude code).

If we come back to the cases of SAM and GOD, we see that the differences in their Arabic reading according to task demands is nicely accounted for by the triple-code model. Indeed, if one considers that their lesions affect the left visual Arabic system (or a disconnection of this system), then we would predict that tasks mediated by the left hemisphere would lead to more errors than tasks mediated by the right hemisphere. As simple addition is assumed to rely on the left verbal system, reading Arabic digits for subsequent addition would preferentially activate the *left* visual Arabic system, (i.e., the one that is damaged). Conversely, tasks such as magnitude comparison which can be equally handled by both hemispheres would not lead to a preferential use of the left, impaired, visual Arabic system so that reading of Arabic numbers in the context of a magnitude comparison would be better than for a subsequent addition.

The triple-code model has also been used recently by Miozzo and Caramazza (1998) to account for the performance of GV (lesion in the left occipital and posterior temporal areas, extending to the corpus callosum). This patient produced many errors in reading Arabic digits (42% incorrect) although he could access magnitude from Arabic digits (i.e., compare the magnitude of 1- or 2-digit Arabic numbers) and produce verbal numbers in other contexts (i.e., solving simple calculations presented orally, answering numerical questions such as "how many pennies are there in one dollar?"). According to Miozzo and Caramazza, the patient's profile could be interpreted as resulting from an impaired left visual Arabic system. Accordingly, Arabic number magnitude comparisons could be carried out using the intact right visual Arabic system. But reading would be error prone because the transcoding path going from the right visual Arabic system would be insufficient to drive the verbal system in the left hemisphere without error: because the information was incomplete or it was degraded through inter-hemispheric transfer (remember that the patient's lesion extended to the corpus callosum).

However, two aspects of the patient's data do not fit with Dehaene and Cohen's model (1995). First, if Arabic number reading uses the right visual Arabic system, then, it necessarily goes through the analog magnitude representation system (both on the right and then on the left). As these representations give only an approximation of the quantity, errors would be expected for large numbers but they should still tend to respect the global magnitude of the number. However, Miozzo and Caramazza did not find any numerical proximity between the targets and errors he produced.[10] Second, if the right visual Arabic system was preserved, then the patient should have been able to determine whether Arabic numbers are odd or even as Arabic number forms are supposed to code for parity information. Yet, such a task was impossible for the patient.

Data with Normal Subjects

The rapid development of the debate between proponents of semantic and asemantic transcoding models in neuropsychology has finally given rise to a few attempts to address that issue in research with unimpaired subjects. But, as we will see, the data obtained are quite controversial.

First, using a physical matching task (e.g., *3* = *3* but *3* ≠ THREE, 3 ≠ FOUR or 3 ≠ 4), Dehaene and Akhavein (1995) noticed a trend for incorrectly responding *same* to physically different pairs representing the same quantity (e.g., 2- TWO). Girelli (1998) replicated this observation: subjects were slower and less accurate when they had to reject the physical identity of an Arabic and a verbal number representing the same quantity (*2* – *TWO*) than when they represented different quantities (*2* - *THREE*). These observations were taken as an argument in favor of connections between the Arabic and the verbal systems. However, are these connections semantically mediated? If so, then some effects typically found in tasks activating the magnitude representation should arise. In particular, a distance effect should be observed: rejecting a pair of two numerically close numbers (e.g., *2* - *THREE)* should be slower than rejecting a pair of numerically distant numbers (e.g., *2* - *SIX*). Yet, Girelli could not find any evidence of such a distance effect and concluded that semantic representations of numbers were not activated during that task, that is, that the connection between verbal and Arabic numbers is not semantically mediated.

[10]The same prediction could also be made for SMA and GOD in the «read and compare task». However, the authors did not analyze the distance between errors and targets. Furthermore, the task only used single-digit Arabic numbers, namely, numbers that allow a precise representation in the analog magnitude system.

In contrast, Fias (1998) proposed two main arguments for the existence of a single, semantically mediated, path for Arabic number reading. First, in a phoneme monitoring task of Arabic digits (i.e., detecting an /e/ sound–which requires Arabic-to-verbal transcoding), Fias, Brysbaert, Geypens, and d'Ydewalle (1996) showed the presence of a SNARC effect (which stands for Spatial-Numerical Association of Response Codes), usually considered to be an indicator of the activation of analog magnitude representations.[11] Second, in reading aloud Arabic digits, the presence of a written verbal number exerted an influence such that there was a faster response if it corresponded to the Arabic digit than if it did not (e.g., *4 – FOUR* vs. *4 - FIVE*). By contrast, reading verbal numbers was not influenced by the accompanying digit. According to Fias et al., these results argue in favour of obligatory semantic mediation in the Arabic to verbal transcoding.

However, this conclusion does not seem to be the only possible one. Indeed, the second experiment showed that Arabic number reading was influenced by a simultaneously presented number word. This means that at some point, the target and the distractor activated representations at the same level of processing. This common level could well be the semantic representation. But the verbal number production system might also be a possible candidate: digits activate it as required by the task whereas number words automatically activate the corresponding phonemes. We think that a stronger case for Fias' conclusion would be to show that not only do words influence the reading of digits but also that this influence shows semantic properties such as a distance effect (i.e., in reading *3*, greater interference from SEVEN vs. FOUR).

An indication of the activation of the analog magnitude representation (the SNARC effect) was observed in the phoneme monitoring task. Yet, a SNARC effect was also obtained by Fias (1998) in low-level visual tasks with Arabic digits such as judging whether there is a closed shape in the visually presented digit or whether a triangle superimposed on the digit is pointing upwards or downwards. Would we conclude from these experiments that those visual processes are semantically mediated? Probably not. Rather, we would conclude that semantic activation is quite automatic from the presentation of an Arabic digit. Accordingly, the semantic activation observed in the phoneme monitoring task indicates that the semantic representation was activated but it does not imply the nonexistence of a parallel asemantic processing route.

Finally, if, as observed by Fias, Arabic digits automatically activate their corresponding semantic representations then, showing the existence of a second (or several), nonsemantic route(s), seems quite difficult in studies with normal subjects and thus points to the great importance of testing patients to clarify the debate.

Conclusion

Should asemantic transcoding algorithms be posited to supplement semantic ones? According to McCloskey (1992) there is, currently, no evidence justifying this addition. Yet, Cipolotti and Butterworth (1995) argue that asemantic transcoding should be added. Dehaene and Cohen (1995) agree with this conclusion but they differ from Cipolotti and Butterworth in that they also assume the existence of multiple semantic transcoding routes (depending on whether it is the left or right hemisphere visual Arabic system that is used and also whether the semantic system involved is the left or right analog magnitude representation). The models also differ in their conception of the semantic representations of numbers. Both McCloskey (1992) and Dehaene and Cohen (1995) assume that these should represent quantity but the former supposes that quantity is represented by a base-10 semantic representation that would be precise even for large numbers, whereas Cohen and Dehaene hypothesize that an analog magnitude representation codes for approximate quantity.

[11]The SNARC effect corresponds to the fact that small numbers are responded to faster with the left side and large numbers faster with the right side (i.e., usually corresponding to left and right hand, respectively), which is consistent with a left-right orientation of the representation of number magnitude.

Three types of evidence have been presented in support of the existence of asemantic transcoding routes: (a) cases in which the production of numbers is influenced by visual or verbal characteristics of the input, (b) cases presenting impaired transcoding but seemingly spared comprehension and production mechanisms, and (c) transcoding performance that is influenced by task demands.

All of these cases can be accounted for by assuming damaged asemantic transcoding mechanisms together with spared semantic pathways. However, many of the cases presented could find alternative explanations. More importantly, no one has ever reported the opposite side of the dissociation (i.e., spared asemantic transcoding in the case of impaired semantic mechanisms, as emphasized by Seron & Noël, 1995). So, no one has ever described the case of a patient who was unable to access the magnitude of numbers though being able to read or write them down. Two cases were reported with difficulties in comprehending numbers, but both also showed difficulties in transcoding! In particular, CG (Cipolotti, Butterworth, & Denes, 1991) showed a dramatic loss of all numerical knowledge for numbers beyond 4 regardless of presentation format, but she was also unable to transcode any of these numbers! Similarly, NR (Noël & Seron, 1993) showed difficulties in comprehending some Arabic numbers and showed striking similarities between the errors produced in transcoding tasks and those appearing in other tasks tapping the semantic system[12]. The description of patients constituting the second side of the double dissociation between semantic and asemantic transcoding paths, will thus be crucial.

THE BASIC COMPONENTS OF THE CALCULATION SYSTEM

Among the operations that are specific to the numerical domain, those that have been the most extensively studied are the single-digit arithmetical operations such as addition, multiplication or subtraction. Once more here, cognitive neuropsychology has proven to be very useful in specifying the different functional components.

Among the first studies of patients with calculation deficits were those of Ferro and Botelho (1980). Their patients, AL (brain trauma) and MA (CVA), produced errors in simple calculation such that the responses provided corresponded to an incorrect operation applied on the two operands presented (e.g., $3 \times 5 = 8$ or $9 \times 3 = 6$). An analysis of the patients' responses indicated that their errors came from impaired identification of the operation symbols (+, −, ×, . . .). However, the words of operations (i.e., plus, minus, times) were correctly understood, by MA at least, as he made no errors when calculation problems were presented orally. The patient's difficulty did not extend to other types of written symbols such as digits, letters, flags and playing cards. This argues for the existence of mechanisms specifically dedicated to the identification of arithmetical operation symbols.

Two years later, Warrington (1982) reported the case of DRC, who also produced errors in calculation (11% errors in addition, 8% for subtraction, and 9% for multiplication) yet was perfectly able to recognize and define the symbols of arithmetic operations. This patient's difficulty was in retrieving the answers of previously familiar single-digit problems. Consequently, he had to reconstruct solutions by using back-up counting procedures. So that, in time-unlimited tasks, he was perfect but erred when time pressure was applied. From this pattern, Warrington concluded that solutions to single-digit problems are usually retrieved from long-term memory, called *arithmetical facts*, but that DRC suffered damage affecting access to this store.

This case study contributed to the debate that was, at that time, between those who assumed that single-digit addition problems were solved by counting procedures (Groen & Parkman,

[12] For instance, the patient read correctly *1258* but said "two thousand three hundred and six" for *236*. Accordingly, *236* was judged bigger than *1258*!

1972) and those who hypothesized a simple retrieval of the solution from memory (Ashcraft & Battaglia, 1978). Subsequent work has largerly supported the latter view, although LeFevre and her collaborators have showed that even educated adults keep using procedural strategies (e.g., counting, decomposition, transformation, etc.) to solve some single-digit problems (LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996).

If answers are merely retrieved from long-term memory, then the issue of how these solutions to single-digit problems are organized in long-term memory can be raised. To handle this question, McCloskey, Aliminosa, and Sokol (1991) analyzed the errors in single-digit multiplication produced by 13 patients. From that pattern, they distinguished two types of problems: the M × N and the 0's problems. The M × N problems (with M and N being any number from 2 to 9) gave rise to a very heterogeneous pattern of errors. Within the same patient, some problems led to many errors while others were perfect. For instance, MD was 95% incorrect on 9 × 3 but only 10% incorrect on 9 × 4 and perfect on 9 × 2. Such an observation creates difficulties for models assuming that arithmetic facts are stored in a table-like structure (e.g., Ashcraft & Battaglia, 1978). Within these models, the presentation of a problem activates the corresponding rows and columns in the table (i.e., the entry nodes for the first and second operand, respectively) and this activation spreads until an intersection at the location where the correct answer is stored. Accordingly, if the node of 9 × 3 is lost (e.g., links are destroyed), the multiplication problems that require activation beyond that point in the table (e.g., 9 × 4) should not be accessible anymore, which, as shown by McCloskey et al.'s patients, was not the case. Instead, these results suggest that each fact (e.g., 9 × 3 or 9 × 4) has its own separate representation in memory that can be selectively damaged (see Ashcraft, 1995, for a review of these models).

By contrast, the *0*'s problems (M × 0 or 0 × M) showed a uniform profile with either global loss or preservation and could dissociate from the M × N problems. For instance, HM was 100% incorrect on *0* × N problems but only 3% incorrect on M × N problems whereas CM made no error on *0* x N problems but 17% errors on M × N problems. Finally, recovery on these 0's problems can be sudden and directly generalized to all 0's problems. For instance, JG was trained on *0 × 8* and *9 × 0* problems: Prior to training, he was uniformly incorrect on all the *0*'s problems but after the training, he was 99.3 % correct on all of them (and not just on *0 × 8* and *9 × 0*). Let us note finally, that rules are also found in other operations (e.g., *0 + N*, *0 / N*) or with numbers different from zero (N / *1*, N - N, N / N, and so on).

Another point that has been raised concerns the relationship between arithmetical facts from different operations. Dagenbach and McCloskey (1992) hypothesized that the arithmetical fact store was segregated by operations. Their proposition was supported by observations of dissociations among arithmetical operations. For instance, their patient RG (CVA in the left posterior parietal region) was more severely impaired in multiplication and addition than in subtraction (respectively, 22%, 25% and 61% correct). Other dissociations among operations were also reported by Pesenti, Seron, and Van der Linden (1994), McCloskey et al. (1991), Hittmair-Delazer, Semenza, and Denes (1994), or Cipolotti and De Lacy Castello (1995).

Yet data from normal subjects indicate that these stores are not totally independent from one another. For instance, confusions among operations are obtained both in production (e.g., producing *20* in response to *4 + 5*) and in verification (rejecting more slowly *4 + 5 = 20* than *4 + 5 = 18*). These data have led Ashcraft (1982) to propose that addition and multiplication are stored in inter-related networks.

The question of the relationship between multiplication and division is also interesting. Indeed, do we solve a division problem (e.g., *68/8*) by retrieving the corresponding division fact or do we rather rely on multiplication (e.g., *8 × ? = 64*)? This question was studied by Campbell (1997; see also Campbell, 1999) with normal adults. He showed that RTs and error characteristics are highly correlated for corresponding division and multiplication problems. Priming between operations was also obtained but asymetrically: multiplication errors (e.g., *7 × 9 = 56*) were primed by previous division trials (*56/7 = 8*), but not the reverse. Campbell suggested that division and multiplication problems involve separate memory representations but that multiplication is often used as a check *after* direct retrieval of the quotient. This checking procedure would explain the assymetry of the priming effect.

Using a different methodology, LeFevre and Morris (1999) showed that multiplication performance benefited from prior solution of division problems more than division benefited from prior solution of multiplication problems. This priming was greater on large multiplication problems than on small ones. Furthermore, participants reported using multiplication to solve division problems (e.g., *56/7 = ?* recast as *7 x ? = 56*) on a substantial proportion of trials, and more often on large than on small division problems. The authors assumed that multiplication and division are stored in independent representations but that solution of division problems is sometimes mediated by direct access to multiplication knowledge.

These data clearly show some relationship between division and multiplication facts. However, it is still not perfectly clear whether multiplication is used to *check* or to *mediate* division. These relations between division and multiplication facts are well in line with the case study of Hittmair-Delazer, et al. (1994). Their patient was impaired for both multiplication and division in such a way that the division problems that were incorrect corresponded to the multiplication problems that could not be solved by the patient. Furthermore, the effects of rehabilitation of multiplication transferred to division which led the authors to conclude that the patient's division performance depended on the availability of the corresponding multiplication facts.

However, the case reported by Cipolotti and de Lacy Costello (1995) contradicts this. This patient (infarct in the left parietal and probably posterior frontal regions) was much better in multiplication (about 70% correct) than in division (20% correct) which suggests that he was not relying on multiplication for carrying out division (e.g., solving *48/6 = ?* by searching 6 × ?= 48). This hypothesis was further supported by the fact that "N times ?" problems (e.g., 6 × ? = 48) were as poor as division problems (48/6 = ?; respectively, 16% and 24% correct, compared with 88% correct for the corresponding multiplication problems).

One way to account for this pattern is to consider the existence of individual differences in this domain. According to Campbell (1999), individuals with a very well-developed memory for division facts presumably rely very little on multiplication. Another possible explanation would be that some patients have completely lost the links between operations. This seemed to be the case for JG (Delazer & Benke, 1997): She was correct on *59/64* multiplication problems but was completely unable to answer division problems. This dissociation was explained by the fact that she had completely forgotten the meaning of the division operation and did not realize the relation between multiplication and division, so that she never recast division problems into multiplication.

In summary, data so far argue for distinguishing, within the calculation system, a subcomponent for processing words or symbols corresponding to the operations as well as interconnected arithmetical fact stores. These components are schematically represented in McCloskey et al.'s model,[13] (see Figure 20.1). Yet, Hittmair-Delazer and colleagues (Delazer & Benke, 1997; Hittmair-Delazer, Sailer, & Benke, 1995; Hittmair-Delazer et al., 1994) pointed to the need to consider another important aspect of calculation abilities that was not part of McCloskey et al.'s (1985) model (i.e., what she calls *conceptual knowledge about arithmetic)*. Her idea came from the study of BE (but see also DA in Hittmair-Delazer et al., 1995), a patient who presented with severe acalculia after a cerebral embolism (left basal ganglia). In multiplication, he was able to give an immediate and correct response to only 28 out of 64 single-digit problems. For the remaining 36 items, he developed complex strategies to generate answers, on the basis of the few multiplication facts that he had kept in memory. For instance, to solve *4 × 9* he

[13]The calculation component also involves a module for "calculation procedures" which contains the algorithms for multi-digit operations. However, this chapter will only consider single-digit calculation. This calculation procedures module will thus not be presented here.

computed $(9 \times 2) + (9 \times 2)$ or $(9 \times 10 / 2) - 9$. The complexity of these (correct) strategies strikingly showed the preservation, in this patient, of a deep understanding of the principles of the mathematical operations involved (e.g., commutativity, associativity, distributivity).

The opposite pattern was also reported (Delazer & Benke, 1997): JG was unable to define arithmetical operations (e.g., multiplication is: "to multiply"), complete a task of arithmetical reasoning in which the answer to a second problem should be inferred from the answer to a first one (e.g., *if* $12 \times 4 = 48$ *then* $4 \times 12 = ?$) or use back-up strategies for solving a calculation for which she had forgotten the answer. However, she could nevertheless realize some calculations (multiplication: 59/64 correct; addition: 28/64 correct; subtraction: 12/28 correct; but no division correct) just by resorting to stored answers. This double dissociation thus argues in favor of the functional independence of arithmetical fact stores on the one hand and conceptual knowledge about arithmetic on the other.

STORAGE FORMAT OF ARITHMETICAL FACTS

Theoretical Perspectives

One of the important issues that has been raised about arithmetical facts is the nature of the stored representations. Two broad categories of hypotheses have been proposed. First, the *multiple-format position* assumes that arithmetical facts are stored in a variety of internal codes as determined by the formats in which they have been encountered (see Campbell & Clark, 1988; Clark & Campbell, 1991). For instance, 3×7 gives rise to strong activation of the visual Arabic representation of $<3 \times 7 = 21>$ as well as to weaker activation of other representations such as the corresponding phonological representation "three times seven is twenty-one." By contrast, the spoken multiplication "three times seven" gives rise to a stronger activation of the corresponding phonological representation and weaker activation of non-verbal representations.

Instead, the *single-format position* assumes that, although problems are encountered in many different formats, arithmetical facts are stored in a unique type of representation. In particular, McCloskey et al.'s (1985, but see also McCloskey & Macaruso, 1995) assume that arithmetic facts are stored in the form of base-10 semantic representations. So that 3×7 or "three times seven" would both generate the same base-10 semantic representation of the problem (i.e., {3}10EXP0 [×] {7}10EXP0 which would then trigger the base-10 semantic representation of the corresponding product {2}10EXP1, {1}10EXP0).

Another single-format position has been proposed by Dehaene (1992) and Dehaene and Cohen (1997): addition and multiplication facts would be stored as *verbal representations*. According to that hypothesis, retrieving the solution to a multiplication problem presented in a nonverbal format (e.g., written in Arabic digits) would first require a translation of the presented form into the corresponding verbal representation to trigger the verbal representation of the answer.

Single or Multiple Formats for Arithmetical Facts?

The single-format and multiple-format hypotheses make different predictions regarding the effects of manipulating presentation format. According to the single-format view, effects of format manipulation could only be located at problem encoding (i.e., when generating the base-10 or verbal representation corresponding to the problem). By contrast, the multiple-format view assumes that manipulating the presentation format of a problem should give rise to differences in the retrieval stage.

A series of studies with normal subjects has reported the effects of manipulating the presentation format of calculation problems (for a review, see Noël et al., 1997). Some of them, such as LeFevre, Bisanz, and Mrkonjic (1988) or McClain and Shih Huang (1982) failed to find any effect of the manipulation. Others did obtain effects but the question is to determine their functional locus: Is it in the encoding or retrieval stages?

For instance, Gonzalez and Kolers (1982) used a verification task of addition problems presented either in Arabic digits (*4 + 5*), Roman symbols (IV + V) or in a combination of the two formats. They obtained interactions between the presentation format and parameters measuring the difficulty of the addition. Yet, Noël and Seron (1992) showed that these interactions might well be due to factors at play at the encoding level. Indeed, the encoding time of Roman numbers varies: I, II, III, V, and X are processed faster than IV, VI, VII, and VIII. Noël and Seron showed how this variability could explain the pattern of RTs obtained in calculation by Gonzalez and Kolers.

Stronger effects of presentation format were obtained by Campbell (1994; Campbell & Clark 1992). For instance, Campbell and Clark (1992) reported for single-digit multiplication, greater increase in RTs and error rates due to problem difficulty in the written verbal (SEVEN × FIVE) than in the Arabic condition (*7 × 5)*. Yet, once more, the locus of these effects was criticized by McCloskey, Macaruso, and Whetstone (1992). They ran a regression analysis on the RT-differences between the word and the digit formats and found that problem difficulty disappeared as a signficant predictor if the frequency of each word and digit was controlled (for other criticism, see Noël et al., 1997).

Sokol, McCloskey, Cohen, and Aliminosa (1991) used a similar methodology, but on a brain-damaged patient (PS, left-hemispheric CVA) who showed major problems in single-digit multiplication. They varied the presentation format of the problems but found no effect on PS's multiplication performance. The type of errors were the same in all the conditions (see Table 20.3) and PS tended to err on the same problems (the error rate measured for each individual problem was highly correlated between the different presentation formats with r varying between .83 and .90). These results thus supported the single-format assumption but did not address directly the claim that these representations were base-10 semantic versus verbal representations. Yet, Sokol et al. (1991) noted that the errors made by PS reflected numerical rather than phonological similarity between problems, which is more in line with a semantic representation of quantity rather than a verbal one.

In summary, some studies have found interactions between presentation format and factors related to the retrieval stage of arithmetical facts. Yet, in most cases, a reinterpretation of the effects at an encoding or production stage is possible. Thus, as of yet, there is no strong argument in favor of the multiple-format hypothesis and we are thus left with the two single-format proposals: the base-10 semantic hypothesis of McCloskey et al. (1985) and the verbal hypothesis of Dehaene (1992).

Table 20.3
Patient PS's Error Distribution as a Function of the Presentation Format.

Format	Error Type: Operand-related*	Omission	Other
Arabic numbers	37	17	6
Written words	41	15	7
Dots	41	14	8

*Operand-related errors correspond to errors belonging to the table of at least one of the operands of the problem (e.g., 3 × 5 = 10).

Verbal Representations for Addition and Multiplication Facts?

According to Dehaene (1992), addition and multiplication facts are stored as verbal representations such that a problem (regardless of format) is first transcoded into a verbal representation for retrieving the solution. This process is blind to the meaning of the numbers as no semantic processing of the numbers is needed.

Dehaene and Cohen (1997) do not assume stored verbal representations of this sort for complex addition and multiplication (e.g., *13 + 5*) problems nor for subtraction and division because these problems are not normally acquired by rote learning. Arriving at answers for these problems would require "meaningful manipulations" of analog magnitude representations. Semantic processes of this sort would also be used if the verbal representations corresponding to single-digit addition or multiplication problems were lost or inaccessible.

On this basis Dehaene and Cohen (1997) expected that more severe difficulties in addition and multiplication than in subtraction or division should be associated with a more general impairment of rote verbal memory. By contrast, greater impairment for subtraction and division than for addition and multiplication, should be accompanied by difficulties in the activation or manipulation of analog magnitude representations of numbers. Dehaene and Cohen (1997) reported two case studies that illustrate those two situations.

The first case, MAR (left-handed man, infarct in the right, supposedly dominant hemisphere, inferior parietal lobule) had greater difficulties with subtraction (25% correct) and division (46%) than with addition (68%) and multiplication (73%). Accordingly, MAR was also impaired when required to manipulate the magnitude of numbers (i.e., comparing Arabic numbers: 84% correct, selecting between two Arabic numbers the one that was closest to a third one: 80% correct) but was good in tasks tapping rote verbal memory (e.g., counting, reciting the alphabet, the days of the week, the months, the musical notes).

Dehaene and Cohen (1997) concluded that "MAR's deficit might be best described as a complete disconnection between a partially impaired quantitative system and a fully intact verbal system" (p. 241). Yet, how could a "fully intact verbal system" produce 30% errors in addition and multiplication, in other words, operations that are supposedly realized through this verbal system? Furthermore, a disconnection between the partially impaired quantitative system and the verbal system should lead to errors in tasks that rely on the analog magnitude representation of verbal numerals. Yet, the patient was perfect in two tasks of this type: positioning a verbal numeral on an analog scale (a thermometer from 0 to 100) and orally producing cognitive estimates (i.e., Shallice & Evans', 1978, test).

The second case, BOO (left-hemispheric capsulo-lenticular haemorrhage, Dehaene & Cohen, 1997) presents the opposite profile: spared performance in tasks tapping the analog magnitude representation of numbers (e.g., Arabic number comparison or calculation approximation) but impaired rote verbal memories (the patient was no longer able to recite musical notes and letters of the alphabet, or to remember nursery rhymes, prayers, and so on[14]). Accordingly, in arithmetic, BOO should show the inverse profile of MAR, namely, errors mainly in addition and multiplication. Actually, BOO showed significantly more difficulties with multiplication (72% correct) than with addition (94%) or subtraction (94%; the division data were unreliable).

The difference between the expected pattern and the observed one is interpreted as coming from the fact that back-up strategies using semantic processing can take place when verbal representations are lost or inaccessible. This idea fits quite well with the very long RTs measured in addition (mean of 2,792 ms) and multiplication (mean of 2,556 ms) but long RTs were also observed for subtraction (mean RTs of 2,854 ms when it takes usually about 1 s to carry out such calculations), an operation which should not be affected in BOO. Moreover,

[14]Yet, the patient scored 9/9 on automatized sequences and 2/2 on reciting subtests of the Boston Diagnostic Aphasia Examination which is not in agreement with a global loss of verbal routines.

semantic-based processing of multiplication should result in errors that are close in magnitude to the correct solution (e.g., $6 \times 8 = 46, 47, 49, 50$) but the mean distance was large (8.7 ± 6.4) and 13 out of the 15 errors produced belonged to the multiplication table (e.g., $8 \times 8 = 36$, $3 \times 7 = 27$) which suggests that incorrect verbal routines were used.

In summary, the two cases constitute a double dissociation which only grossly fits with the prediction of the model. But they might also be interpreted within the architecture proposed by McCloskey et al., (1985). MAR could be impaired in the comprehension of Arabic numerals[15] (see impaired performance on comparison test with Arabic digits and errors in all the arithmetical operations) as well as in the arithmetical facts store (mainly subtraction). For BOO, Dehaene and Cohen (1997) themselves proposed that: "It may never be fully excluded that her multiplication impairment and her deficits of rote verbal knowledge were due to two independent lesions" (p. 242). Thus, a simple hypothesis of a selective deficit in the multiplication fact store could explain her profile in an architecture such as that of McCloskey et al. (1985). These two cases illustrate the predictions of the triple-code model but they do not constitute strong evidence against the view proposed by McCloskey et al.

To evaluate Dehaene and Cohen's (1997) proposal, we have examined the single cases reported in the literature with deficits in arithmetical-fact retrieval. According to Dehaene and Cohen, greater impairment for addition and multiplication should be accompanied by general problems in rote verbal memories whereas greater difficulty in subtraction or division should be related to difficulties with the manipulation of analog magnitude representation for numbers. Unfortunately, many of the cases considered here were published before Dehaene and Cohen's (1997) proposal, so that the available information is not always as complete and detailed as one would like.

Nearly all the cases reviewed show lower performance in multiplication or addition than in the other operations. One should thus expect associated difficulties with more general rote verbal memories.

This is indeed the type of profile encountered in RG (Dagenbach and McCloskey, 1992: 22% correct in addition, 25% in multiplication, and 61% in subtraction). This patient was unable to recite the days of the week or the months of the year. He could only produce the first letters of the alphabet and the first 9 numbers when asked to count from 1 to 21.

The case of Rossor, Warrington, and Cipolotti (1995) also showed worse performance for multiplication (77% correct) than for addition (96%) and subtraction (100% correct). Furthermore, many of the correct products seemingly derived from back-up strategies (e.g., beside the answer *45* for 9×5, he wrote "$18 + 18 + 9$," similarly, "$16+16$" was written next to 8×4). As expected, this patient also had difficulties with verbal routines: he was profoundly aphasic and was unable to produce or continue automatic sequences.

By contrast, BB (Pesenti et al., 1994), who was more impaired in multiplication and addition than for subtraction (respectively, 43%, 67%, and 92% correct), showed no indication of impaired rote verbal memories: She could produce the days of the week, the letters of the alphabet, count from 1 to 21, and recite, although slowly, the months of the year.

Similarly, Cohen and Dehaene's (1994) patient was severely impaired in multiplication (58% correct) and less in addition (80% correct; no other operations were tested) but "she was fast and accurate in reciting automatic series (days of the week, months of the year, counting aurally, completing familiar proverbs)" (p. 218).

The opposite pattern of deterioration in arithmetic (i.e., better performance in multiplication compared to the other operations) has only been reported in the case of JG (Delazer & Benke, 1997: 92% correct in multiplication, 44% in addition, 32% in subtraction, and division is impossible). We have no specific information about the patient's verbal routines but the patient's verbal skills were reported to be intact, she was fluent and could count up to twenty without

[15] In neither of these cases was there an evaluation of number comprehension and production in the different formats.

error. We also have clinical observations suggesting that the verbal medium was of great importance in carrying out calculation. Indeed, when problems were written in Arabic digits, JG always read them out loud first, produced a spoken answer and finally wrote the corresponding digits. If verbal suppression was used in calculation task (repeating "the"), she was unable to deal with the task, even if she had only to point to the correct answer among several. She said that it was impossible for her to recognize the correct answer if she could not read the problem aloud. However, her need for verbal mediation was not specific to addition or multiplication.

A further prediction is that her poor subtraction should be related to poor manipulation of magnitude representations. In this respect, results are not clear-cut. Indeed, on the one hand, JG had no difficulty in comparing the magnitude of two Arabic numerals but on the other hand, she produced 9/14 as errors when asked to select poker chips of different values that corresponded to a given Arabic numeral and she never used back-up strategies for solving facts for which she could not retrieve an answer.

We could thus consider that this patient was impaired in operations relying on analog magnitude representations (subtraction and division) but good in those relying on verbal routines (multiplication). The poor scores with addition could be accounted for by assuming that this operation was realized through semantic processing rather than verbal routines. Accordingly, the authors wrote that, in their country, addition, subtraction, and division are not taught through verbal recitation.

Another way of testing the verbal representation hypothesis for addition and multiplication is to examine the underlying "translation" hypothesis. Indeed, according to Dehaene and Cohen (1997), a multiplication (or an addition) problem presented in Arabic digits must first be translated into the corresponding verbal representation for accessing the verbal representation of the solution. If, for example, we consider a patient with a deficit specifically affecting the translation of Arabic to verbal, one should expect more errors in addition and multiplication presented in Arabic than in verbal. By contrast, such a manipulation of the presentation format should not affect subtraction or division since they do not require verbal translation.

The case of HAR (McNeil & Warrington, 1994; glioma affecting the left occipital and parietal as well as the right occipital lobes) illustrates this point. In oral presentation, HAR was good in addition (92% correct), multiplication (100% correct), and subtraction (98% correct). Yet, if problems were written in Arabic digits, difficulties appeared in multiplication (48% correct), and addition (63% correct) but not in subtraction (96% correct). This pattern of performance can hardly be interpreted in McCloskey et al.'s (1985) model. In Dehaene's, it can be accounted for by assuming a difficulty in transcoding the Arabic problem into the corresponding verbal representation. The fact that the patient was bad in reading aloud Arabic digits (74% correct for single digits) argues in favor of that hypothesis. But how could we account for HAR's ability to match Arabic digits to spoken verbal numerals (using the same digits as those involved in arithmetical operations)?

However, all cases presenting difficulties in translating an Arabic digit problem into the corresponding verbal form do not show the expected effect of arithmetical operation.

For instance, Whalen (1994) presented the case of a patient who made many errors in reading Arabic numbers. When multiplication problems were presented in the Arabic format, he read correctly only 12% of them. If required to give an oral answer, he was 41% correct but if the answer had to be written in Arabic digits, he was 98% correct. For instance, 9 × 7 was read as "nine times six" but the correct answer *63* was written by the patient. This profile is quite unexpected under the verbal translation hypothesis. Indeed, under this hypothesis, a visually presented problem must to be translated into corresponding verbal representation and it is this verbal representation that activates the corresponding answer. Consequently, the patient's answer (spoken or written) should match the verbal representation of the problem (frequently wrong in this patient) and not the visually presented problem.

A similar pattern was observed in ZA (Delazer & Girelli, 1997). This patient was presented with calculation problems written in Arabic digits and was asked to first give his answer orally and then to write it down in Arabic digits. Here also, errors were more frequent for spoken answers (35% errors) than for written digit ones (7% errors).

These two cases are extremely problematic for Dehaene's (1992) hypothesis of an auditory verbal code for addition and multiplication facts. However, in more recent articles, Dehaene and Cohen (1995, 1997) have abandoned the assumption that facts are stored in an auditory verbal code (i.e., in the form of a phonological representation; e.g., /fIfti-tu/) and have proposed instead that facts are represented in *verbal word frames.* That is, they are stored as syntactically organized sequences of words (e.g., Tens {5}, Ones {2} for FIFTY-TWO) in which Tens {5} and Ones {2} constitute abstract word lemmas (or nodes) that are the basis for *subsequent* access to both phonological and graphemic representations of the word forms. In the context of this more recent proposal, Delazer and Girelli (1997) interpreted ZA's reading errors as a difficulty in accessing the phonological representation of spoken numbers from the verbal word frame. Accordingly, errors appear in all tasks requiring the production of spoken numbers, including oral multiplication. By contrast, translating an Arabic problem into the corresponding verbal representation (i.e., the verbal word frame) would not require the step of lexical access to the phonological word form itself. For that reason, fewer errors would be expected when multiplication problems are responded to in the Arabic modality than in the spoken modality. Unfortunately the authors did not obtain any independent evidence for this locus of impairment.

Conclusions

About half of the cases fit with the hypothesis of verbal representations for addition and multiplication. Yet, in neuropsychology, the question is not to find a majority of patients showing the expected profile but rather, to show that all patterns are consistent with a hypothesis.

Furthermore, the model assumes that addition and multiplication facts are stored as verbal representations. If some cases do indeed show similar performance with these two operations (e.g., MAR, RG), others show greater impairment for multiplication than addition (e.g., BOO, Rossor et al.'s patient, BB; Cohen and Dehaene's (1994) patient, HAR) or sometimes also the reverse (HAR). How should we account for these results? Perhaps, depending on the education system, some people have verbal representations for multiplication but not for addition whereas others have verbal representations for both?

The Dehaene and Cohen (1997) hypothesis assumes that semantic processing takes place for subtraction and division. Yet, the manipulations of analog magnitude representations required for these operations are not defined and are not easy to imagine (e.g., what types of simple manipulations of analog magnitude representations could be used for solving division problems?). These assumptions also lead to certain predictions. For instance, simple retrieval of a stored verbal representation should be faster than semantic processing. A size effect (i.e., RTs and error rate increase as the numbers of the calculation problem get bigger) should be more important for operations involving the analog magnitude representation than for those requiring the simple retrieval of a verbal representation. All these points need to be considered in evaluating the Dehaene and Cohen hypothesis.

Finally, nearly all of the cases presented (but not HAR for instance) could find an interpretation within the architecture of McCloskey et al. (1985) although one must recognize that these interpretations are often less economical and less elegant than those provided in Dehaene and Cohen's model. Furthermore, the McCloskey et al. framework does not say anything about the frequent (but not obligatory) association between deficits in multiplication and addition and difficulty with rote verbal memories: one possibility is that this association is fortuitous, another is that it is not.

GENERAL CONCLUSIONS

In the mathematical cognition domain, the neuropsychological method has been very fruitful in determining the basic components of number processing and calculation. This type of work culminated in McCloskey et al.'s (1985) proposal of a cognitive architecture for calculation and number processing. However, the content of some of the components (e.g., the different number lexicons) are still open to question, whereas others (e.g., the syntactic mechanisms) need to be specified in more detail.

Another important contribution of neuropsychology has been to raise theoretical questions regarding the way in which these basic components are connected to one another, including the role of the semantic system as an obligatory bottleneck. For most of these questions, turning to pathology seems unavoidable as in normal subjects, semantic activation appears to be quite automatic. However, in using the cognitive neuropsychological method, it is important to avoid anecdotal reports and focus insted on detailed investigations of specific hypotheses informed by the literature of published cases.

In the numerical domain, the cognitive neuropsychological approach has thus, not only brought new data into the field, but has also raised questions and generated interesting theoretical frameworks.

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