



Université Catholique de Louvain
Faculté de Psychologie et des Sciences de l'Éducation
Institut de Recherche en Sciences Psychologiques

Numerical estimation in typical and atypical development: what is the core deficit?



Thèse présentée en vue de l'obtention
du grade de Docteur en Sciences Psychologiques par

Sandrine Mejias

Promoteurs : Pr. Marie-Pascale Noël
Pr. Jacques Grégoire

Jury : Ass. Pr. Bert De Smedt
Pr. Jacqueline Leybaert
Pr. Mauro Pesenti
Pr. Xavier Seron

Louvain-la-Neuve, 14 février 2010



Université Catholique de Louvain
Faculté de Psychologie et des Sciences de l'Éducation
Institut de Recherche en Sciences Psychologiques

Numerical estimation in typical and atypical development: what is the core deficit?

Thèse présentée en vue de l'obtention
du grade de Docteur en Sciences Psychologiques par
Sandrine Mejias

Promoteurs : Pr. Marie-Pascale Noël
Pr. Jacques Grégoire
Jury : Ass. Pr. Bert De Smedt
Pr. Jacqueline Leybaert
Pr. Mauro Pesenti
Pr. Xavier Seron

Louvain-la-Neuve, 14 février 2010

Cover illustration: Nadia Buddle

*For the special exhibition "Attention, please: Numbers!", 1 February – 18
May 2008, The Heinz Nixdorf Museums Forum, Paderborn, Germany.*

A ma famille.

« En mathématiques, on ne comprend pas les choses, on s'y habitue ».

- John von Neumann -

Table of contents

Introduction and overview

Chapter 1: Multiple core systems of magnitude representation	7
1. Models of the approximate representation of numbers	9
2. Models of the exact representation of numbers	15
3. A brief summary	19
Chapter 2: The development of the approximate number system	21
1. Estimates	23
2. Comparisons	28
3. Approximate calculation	39
4. A brief summary	47
5. An alternative view: the existence of a general magnitude system	49
Chapter 3: Developmental dyscalculia	53
1. Definition, prevalence and familial-genetic predisposition	55
2. Neurobiological studies	56
3. General cognitive functions	57
4. MLD, which deficit?	58
4.1. The ANS deficit hypothesis	58
4.2. The access deficit hypothesis	59
4.3. The exact number representation deficit	60
5. Estimates	61
6. Contrasting symbolic and non-symbolic comparison tasks	63
7. Arithmetical procedure	67
8. A larger magnitude system deficit?	69
9. Summary and introduction to the experimental section	69
Chapter 4: Numerical and non-numerical estimation in children with and without mathematical learning disabilities	77
Abstract	79
Study 1	82

1. Method	82
2. Results	91
3. Discussion	98
Study 2	101
1. Method	101
2. Results	105
3. Discussion	105
4. General discussion	106
Chapter 5: Numerical estimation in adults with and without mathematical learning disabilities	111
Abstract	113
1. Method	115
2. Results	116
3. Discussion	124
Chapter 6: Approximate addition in children and adults with or without mathematical learning disabilities: the impact of formats	127
Abstract	129
Study 1	135
1. Method	135
2. Results	142
Study 2	149
1. Method	149
2. Results	152
3. Discussion	159
Chapter 7: General Conclusions	167
References	195
Appendices	213
Acknowledgements	221

List of abbreviations

AES	Absolute error score
AN	Arabic number
ANOVA	Analysis of variance
ANCOVA	Analysis of covariance
ANS	Approximate number system
ATOM	A theory of magnitude
Cov	Coefficient of variation
fMRI	Functional magnetic resonance imaging
HIPS	Horizontal intraparietal segment
HM	Homogeneous-size dots
HT	Heterogeneous-size dots
IQ	Intellectual quotient
Lin	Linear
Log	Logarithmic
MLD	Mathematical learning disabilities
RTs	Response Times
SNARC	Spatial-Numerical Association of Response Codes
w	Weber fraction

Introduction and overview

Numerical information dominates our daily lives. When we wake up with the morning bell, take the bus, buy paint to decorate the house, follow a cake recipe, we are using numbers and arithmetical principles. Life without numerals of some kinds seems impossible. How can math be so natural in our societies? Why is it so hard to understand for some of us?

Humans share with non-human animals a natural ability to process numerical magnitudes (e.g., Izard & Dehaene, 2008; Platt & Johnson, 1971). This is important for the sake of survival (how many predators approaching me?) or to make the best decision (which plate has the largest amount of cookies?). The various characteristics of this ontogenetically and phylogenetically primitive system have been widely studied (Dehaene, 1992). Some parameters that underlie certain kinds of abilities, such as to estimate, have been clearly delineated (e.g., Cordes, Gelman, Gallistel, & Whalen, 2001). However, it seems that the system underlying these abilities or the capacity to access to that system is sometimes damaged. Mathematical learning disabilities (MLD) affects between 5 to 7% of school-age children (von Aster & Shalev, 2007). Those serious difficulties in learning mathematics seem to persist into late adolescence (Shalev, Manor, & Gross-Tsur, 2005). One can easily imagine the negative effect on the lives of those individuals. If sometimes those young adults find ways to compensate to some extent for their mathematical difficulties (by using a calculator for example), others avoid math situations. Obviously, their limiting academic and professional capabilities affects their career opportunities (Rivera-Batiz, 1992). However, we do not know if poor arithmetical skills demonstrated in children rely on a developmental delay (i.e., a minor or major delay in the process of development which may improve with intervention and eventually disappear) or a long lasting deficit (i.e., the developmental milestones are not acquired at the expected times and the lag is not filled in the adulthood).

Multiple hypotheses have been postulated to account for MLD: some authors propounded that MLD are secondary to deficits of general and non-numerical cognitive factors (for a review, see Geary, 2005) while others assume that MLD could results from a deficit of specific numerical cognitive processes (for a review, see Rubinsten & Henik, 2009). The present thesis focuses on the hypothesis of specific deficits of number magnitude representation in MLD children. However, the nature of that representation differs depending on the authors (Butterworth, 1999, 2005; Landerl, Bevan, & Butterworth, 2004; Wilson & Dehaene, 2007). Hence, as the ability to represent and to manipulate magnitudes and the expected deficits in dyscalculia diverge as a function of the model, the different views of the magnitude representation are presented in **Chapter 1**.

For the sake of understanding the specific deficits of the number representation in MLD children, characteristics of that representation in typically achieving children are of importance. Accordingly, in **Chapter 2** are exposed those characteristics, regarding particular experimental designs, that is estimation, comparison and approximation tasks. Moreover, the question of which best mirrored the magnitude representation and more precisely the approximate number system (ANS) is addressed. By reviewing those testing methods of the ANS, with results found in animal cognition and through human development, the various characteristics of this ontogenetically and phylogenetically primitive system are revealed. We also examined how the ANS develops throughout the lifespan, how this development is influenced by environment/education and to what extend this ANS serves as a building block for arithmetical knowledge. This is of interest as our goal is to study the onset and the longevity of developmental dyscalculia. Understanding the basis of a typical development will help to indentify the potential deficits observed in mathematical learning disability and to provide guidelines for remediation. **Chapitre 3**, after providing definition and prevalence of MLD,

presents the different models of specific deficits of number magnitude representation attempting to account for the disturbances observed in dyscalculia. Behavioral data favoring an impairment of the number magnitude representation in dyscalculia are reviewed and discussed in regard of those different models. However, it is currently difficult to interpret the empirical data and to relate them to a particular hypothesis.

In **Chapter 4**, a study is presented focusing on those hypotheses assuming that MLD are due to a basic numerical deficit affecting the ability to represent and to manipulate number magnitude (Butterworth, 1999, 2005; Wilson & Dehaene, 2007) and/or to access that number magnitude representation from numerical symbols (Rousselle & Noël, 2007). The study provides an original contribution to this issue by testing MLD children on numerical estimation tasks with different and contrasting materials.

Following this research, we wondered if the deficits observed in those children were still present in adults and to what extent. Even if some researchers have demonstrated that mathematical difficulties remain present in adulthood (e.g., Rubinsten & Henik, 2005), we do not know if poor arithmetical skills demonstrated in children rely on a developmental delay or a long lasting deficit. **Chapter 5** is devoted to characterize the longevity of numerical impairments into adulthood. These adults who had experienced MLD as children were compared to control adults in numerical estimation tasks.

As MLD individuals show a poor grasp of the meaning and of the properties of arithmetic operations, it would be of interest for them to be able to get a feed-back on their aberrant answers. **Chapter 6** evaluates if they are able to give a plausible approximation of the answer. Moreover, it has been speculated that numbers are part of a larger magnitude system (Walsh, 2003), it seems of interest to examine whether difficulties of MLD children and adults go beyond

the numerical domain and extend to other magnitude representations. Accordingly, Chapter 6 assesses if MLD participants have difficulties in carrying out approximate calculation with different type of numerical quantities. Finally, this study examined if the approximate abilities are related to arithmetical skills in adulthood, as this would have important implications for the diagnosis and rehabilitation of individual with MLD.

In conclusion, **Chapter 7** traces the difficulties presented in MLD participants, as observed through our research, discussed on the potential contribution of our findings to the literature. Finally, we will consider the prospects opened by this work for further investigations on developmental dyscalculia.

Chapter 1:

Multiple core systems of magnitude representation

Humans are able to process numerical quantities in several ways depending on the time available to carry out a specific task, the expected accuracy, and the amount of items presented. When larger numbers of items need to be quantified, two different processes may be used: *the counting*, which is slow and exact, and *the estimation*, which is quick but only approximate. Those quantification processes are said to rely on different systems of representation: a system for representing *exact small numbers of individual objects or events* and a system for representing *large and approximate numerical magnitude* (Feigenson, Dehaene, & Spelke, 2004; Xu, 2003).

The following paragraphs will be devoted to the presentation of those two systems for representing number magnitude. The exact number system will be discussed briefly after focusing on the approximate number representation which constitutes the basements of the present work.

1. Models of the approximate representation of numbers

Common performance signatures across development and across species involved an approximate number system (ANS). Three categories of tasks can be identified, revealing those specific signatures¹.

First, when individuals are asked to guess “how many candies are left in a jar”, their guesses or *estimates* rise linearly with the increase of the target numerosity and the variance of their estimates is proportional to that target. This corresponds to the scalar variability (e.g., Whalen, Gallistel, & Gelman, 1999).

¹ Those three categories of tasks are the subject of the next chapter in order to understand their powers of assessment of the ANS.

Second, when participants are asked to choose the more plentiful of two plates of chocolate pieces, their *comparison* judgment varies with the ratio of the two numerosities. They will present better accuracy for larger differences (i.e., it is easier to distinguish between 10 versus 20 pieces than 7 versus 8 pieces). This suggests that participants form representations of large, approximate numerosities and that their representations accord with Weber's Law² (see for example, Dehaene, 2003): the amount by which two stimuli (two weights, two numerosities, two lengths, etc.) must differ in order to meet a constant criterion of discriminability is proportional to their magnitude.

Third, imagine the situation: a seller makes the coins and you add those coins to the content of your wallet. You have an approximate representation of the quantity of coins in your wallet. Now, if your wallet is torn, the quantity of coins won't meet your expectation. This suggest that adults human are able to *operate* on approximate numerical magnitudes (see for example, Barth et al., 2006). However, the good discriminability is going to be ratio dependant, i.e., individuals' accuracy increased as the ratio between two set of objects increased (it is more easy to distinguish that coins are missing in the scenario "8 coins left from 16 coins" than "8 from 12", corresponding respectively to a 2:1 and 3:2 ratio).

Those ANS' signatures presented briefly above show that the representation implied by this system is approximate and akin to a "number line" (Restle, 1970). This mental number line metaphor was echoed by two different and now classical theoretical proposals: the linear model, which takes place in the accumulator model (Gallistel & Gelman, 1992, 2000) and the logarithmic model (e.g., Dehaene, 1992, 1997), which falls within the model of the triple-code. However, their behavioral predictions are highly similar. This makes their confrontation difficult.

² Weber's law attempts to describe the relationship between the physical magnitudes of stimuli and the perceived intensity of the stimuli (Dehaene, 2003).

1.1. The linear model

Gallistel and Gelman (1992, 2000) proposed a summation coding system in which numbers are represented as similar magnitudes on an analog linear representation with scalar variability (see also Gallistel & Gelman, 2005). This model is based on Meck and Church's (1983) accumulator model accounting for the numerical and temporal competences observed in animals (see Figure 1).

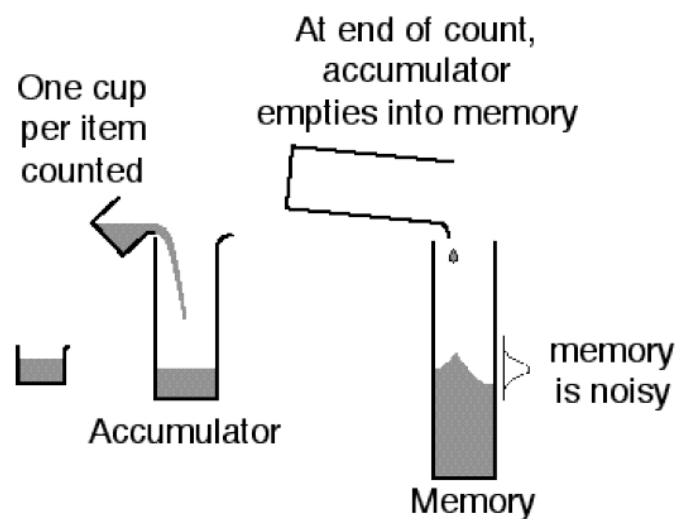


Figure 1: The accumulator model (adapted from Gallistel & Gelman, 2005).

In this model, each focus on an event (for time perception) or on an object (for numerosity) increases the mental accumulator by one level or “one cup”. After successive focuses on each item of a set, the magnitude of the whole can be evaluated in terms of how “full” the accumulator is. Under numerosity detection, the final state of the accumulator corresponds to the cardinal value of the counted target, which is read into noisy memory. Gallistel and Gelman assumed that humans share with animals this nonverbal counting process in which close numbers are represented with overlapping distributions accounting for the distance effect. Moreover, as the model assumes scalar variability in the mapping from sensory inputs to the

linear representation, the variability increases with number magnitude leading to more noisy representations of larger numbers compared to smaller ones (leading to the size effect).

1.2. The logarithmic model

Dehaene (e.g., 1992, 1997) proposed a place coding system where numbers are represented successively with fixed variability on a logarithmic scale (i.e., which is more compressed for larger magnitude). Accordingly, larger numbers are represented by distributions that overlap increasingly with nearby magnitude. Finally, this mental number line is spatially oriented (i.e., numbers are represented from left-to-right).

The logarithmic compression implies that, for a given distance, comparison difficulty increases with increasing size (i.e., larger numbers are represented less accurately than smaller ones, this is the “size effect”), and because of the overlapping, close numbers are more difficult to compare than numbers further apart (called the “distance effect”). The spatial orientation supposed that subjects respond faster to large numbers with the choice on the right than with the choice on the left, whereas the reverse hold true for small numbers (this corresponds to the Spatial-Numerical Association of Response Codes or “SNARC effect”, Dehaene, Bossini, & Giraux, 1993).

This mental number line takes place within the model of the triple-code (Dehaene, 1992, 2001; Dehaene & Cohen, 1995) which assumes that in adults, numerical information is mentally manipulated in three formats. One of them is an analogical representation, in which numbers are represented as distributions of activation on the oriented analogical mental number line obeying Weber’s law (e.g., Dehaene, Dupoux, & Mehler, 1990). The two other formats support the processing of symbolic representations (verbal numerals and Arabic numerals). Transcoding

procedures enable numerical information to be translated directly from one representation to the other. Moreover, each numerical procedure is attached to a specific input and output representation (e.g., in an Arabic numbers comparison task, Arabic numbers are coded as quantities on the mental number line).

In both logarithmic and linear models, larger numerosities are represented by distributions that overlap increasingly with nearby magnitudes. Thus, at a behavioral level, both models lead to the same predictions. The fact that some data are better plotted onto a logarithmic scale than a linear scale will be discussed in the following paragraphs.

1.3. Logarithmic or linear mental number line?

The mental number line metaphor was echoed by the logarithmic (e.g., Dehaene, 1992, 1997) and the linear models (Gallistel & Gelman, 1992, 2000), and both proposals predict that large numbers are represented by distributions that overlap increasingly with nearby magnitudes.

Nowadays however, behavioral and neural measures in animals seem to provide support for the logarithmic model. Nieder and collaborators (Nieder, Freedman, & Miller, 2002; Nieder & Miller, 2003, 2004) studied neurons of the lateral prefrontal cortex, the posterior parietal cortex and the anterior inferior temporal cortex of rhesus monkeys. During a visual task, two monkeys were trained to release a lever if two visual displays presented successively contained the same number of items. Many neurons in their lateral prefrontal cortex and in their intraparietal sulcus were tuned and responded most to a specific number of items³. This corresponds to a tuning

³ Further analyses comparing the activity in the prefrontal cortex and in the intraparietal sulcus suggested that numerosity was first extracted in the intraparietal sulcus then transmitted to the prefrontal cortex (Nieder & Miller, 2004).

function and adjacent quantities evoked relatively similar activity and there was a progressive drop-off as numerical distance increased. The numerical distance effect was revealed as the discrimination of two numerosities with equal distance worsens monkeys' scores as numerical size increased. The neuron data mirrored the size effect as the neuronal tuning obeys Weber's law: the widths of the tuning curves increase linearly with target numerosities, i.e., the neuronal numerical representation becomes less precise with the increase of target magnitude. Monkeys' behavioral performances showed linear increase in their discrimination thresholds as the numerosity increased: they made more errors for adjacent numerosities and improved their scores with increasing numerical distance. Finally, and this is critical in differentiating between the logarithmic compression and the scalar variability, the behavioral and neural distributions (the widths of the tuning curves) were asymmetrically positively skewed when plotted on a linear scale (i.e., when a logarithmically compressed scale is transformed into a linear scale, the distributions are asymmetric with a shallower slope toward higher numbers relative to the mean; see Figure 2 a and b). However, those curves were following a normal distribution when plotted on a logarithmic scale.

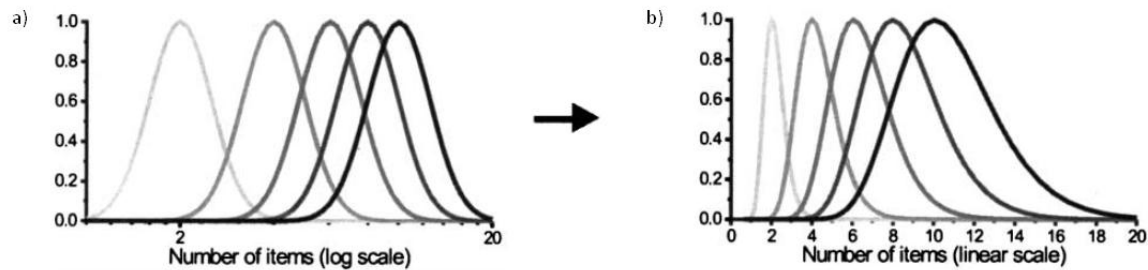


Figure 2: From a logarithmic scale into a linear scale. (a) Quantities are represented on a logarithmically compressed scale with constant variability across different numbers. The accuracy of the representations stays invariable with the increase of the target magnitude; (b) when applying a transformation into a linear scale, the distributions are asymmetric with a shallower slope toward higher numbers relative to the mean (adapted from Nieder & Miller, 2003).

Besides these models to represent the concept of the number line hypothesis by using approximate numerical magnitude representation, two other models postulate an exact numerical representation: one of them represent numbers according to place coding (Verguts & Fias, 2004; Verguts, Fias, & Stevens, 2005), the second one according to summation coding (Zorzi & Butterworth, 1999; Zorzi, Stoianov, & Umiltà, 2005). They will be presented in following paragraphs briefly as these are not central in our research.

2. Models of the exact representation of numbers

In the following paragraphs, the exact representation of numerical information is addressed briefly throughout different computational models. Computational modeling allows one to test, evaluate, and compare the predictive value of cognitive theories as well as to make experimental predictions. Also, going through these models will allow us to understand their approaches to the representation of symbolic and non-symbolic quantities. It is also of interest

for the understanding of the potential deficits observed in dyscalculia as the different hypotheses to account for MLD are characterized by the nature of that representation.

Two models postulate a “small-exact numbers” representation: the place coding model (Verguts & Fias, 2004; Verguts et al., 2005) and the summation coding model (Zorzi & Butterworth, 1999; Zorzi et al., 2005). Place coding (Figure 3, panel a) refers to the idea that a number activates a specific position on the number line, while it is a segment of the number line which is activated in summation coding (Figure 3, panel b, see Roggeman, Verguts, & Fias, 2007).

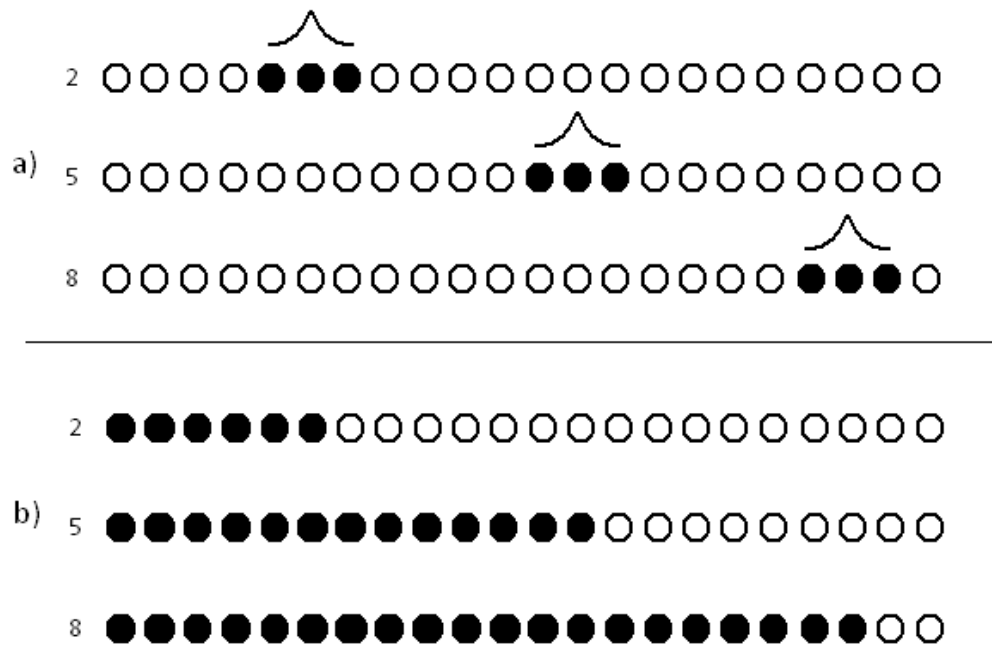


Figure 3: Schematic representation of the number line metaphor and numbers activation a) according place coding, b) according summation coding (adapted from Roggeman et al., 2007).

2.1. Place coding model

Verguts & Fias (2004) proposed a computational model⁴ that addresses the development of symbolic and non-symbolic representations of numerical magnitude (Figure 4). Symbolic and non-symbolic inputs are transformed into internal place-coded representations.

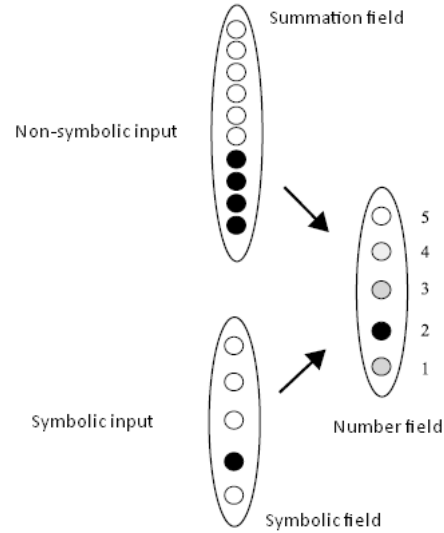


Figure 4: Structure of the computational model which addresses the development of symbolic and non-symbolic representations of numerical magnitude under unsupervised learning (adapted from Verguts and Fias, 2004).

Non-symbolic numerosities enter an input layer and activate nodes, each corresponding to a particular spatial location. Then, a linear combination of the numerical values from the sensory input is transformed by an intermediate step, i.e., the “summation coding” where the noise is proportional to the number of inputs that are being summed. Finally, a linear combination from the hidden field is sent to the output layer.

⁴ This model uses a similar architecture to the numerosity detector network proposed by Dehaene and Changeux (1993).

After pairing symbolic to non-symbolic numerosity, the numerosity detector units became tuned to symbols as well. The model proposes that symbolic representations develop by being mapped onto pre-existing non-symbolic representations and that the learning of symbols leads to changes in the system: the tuning curves were much sharper for symbolic input.

In summary, this numerical magnitude system for small-exact numbers is assumed to behave differently according to the type of input:

a) The representation of non-symbolic quantities is characterized by place-coding with an intermediate step of summation coding, where the noise is proportional to the number of inputs that are being summed so an increasing variability is observed regarding the target size (the system behaves in approximate way with this kind of stimuli).

b) The representation of small symbolic numerical input is characterized by place-coding and constant variability, so the system behaves in an exact way with this kind of stimuli (Fias & Verguts, 2004; Verguts et al., 2005).

Finally, according to Verguts and collaborators (2005), a compression with increasing variability in the range of large numbers is explained by the exposure to numbers. The magnitude representations for small numbers are precise and exact because of their greater frequencies, whereas the magnitude representations for large numbers become approximate because those numbers have low frequency. Moreover, with development and exposure to numbers, the exact magnitude representations should increase further for a wider range of numbers.

2.2. Summation coding model

According to the numerosity coding model developed by Zorzi and collaborator (1999, 2005), numbers are coded by “summation coding”. In this model, the representation of cardinals is a linear and discrete representation. A given symbolic number activates a segment of the number line. Then according to this model, “2” is a subset of the representation of “3”. According to the authors, the numerosity code model assumes a linear representation of numerosity, in a way similar to an accumulator or a thermometer representation: Each set of numerosities is represented by a corresponding number of nodes which contains the smaller subsets. According to the authors, an exact representation of numerosity seems to better capture our intuitive understanding of integer numbers but should coexist with an approximate representation.

3. A brief summary

Through this first chapter, the theoretical proposals to conceptualize the behavioral effects observed when processing numerical magnitude were exposed. The mental number line metaphor was taken over echoed by two different and now classical theoretical proposals: the linear model which takes place in the accumulator model (Gallistel & Gelman, 1992, 2000) and the logarithmic model (e.g., Dehaene, 1992, 1997). These two theoretical proposals conceptualize in a different way how large numerosities are subject to a noisy representation. If their behavioral predictions are highly similar as both reflect the same effects (the distance, size and the SNARC effects), recent behavioral and neural data in animals seem to support the logarithmic model (for a review, see Nieder & Dehaene, 2009). Besides these models, computational models (Verguts & Fias, 2004; Verguts et al., 2005; Zorzi & Butterworth, 1999) addressed the development of symbolic and non-symbolic exact numerical representations.

In the third chapter of the present document, we will explore the current theories for dyscalculia and try to disentangle the multiple hypotheses that have been postulated. Indeed, it should be of interest to consider at first the different theoretical proposals as we will focus on the hypotheses that explored specific deficit according this particular framework of the magnitude representation. In the next chapter, we will address the question of which tasks mirror the best that representation.

Chapter 2:

The development of the approximate number system

In this chapter, the question of which tasks mirrored the ANS is addressed. That is to examine behavioral performance according to a specific task and to examine which task seems to index at best the analog representation in memory⁵ of that specific numerical representation.

Three categories of experimental tasks are largely used in the literature: estimation, comparison and approximate calculation. By reviewing those testing methods of the ANS, with results found in animal cognition and through human development, the different characteristics of this ontogenetically and phylogenetically primitive system are revealed. We also examined how the ANS develops throughout the lifespan, how this development is influenced by environment/education and to what extent this ANS serves as a building block for arithmetical knowledge.

This is of interest as our goal is to study the onset and to characterize the longevity of developmental dyscalculia. To understand the basis of a typical development would help to identify the potential deficits observed in MLD and to provide guidelines for remediation.

1. Estimates

The first insights into the ANS came from studies in animals. Platt and Johnson (1971) varied the number of bar presses that rats must make in order to obtain a reward (food pellets). They found that the number of presses was normally distributed around target magnitude (rats' mean responses increase as function of target magnitude) and that the standard deviation of the distribution increased in direct proportion to the target. The constant coefficient of variation

⁵ That is a representation in the sense of first and second order isomorphism (the structure of the representation contains information about the structure of the object that is represented and the relations that hold between external objects are supposed to exist in a similar fashion in the corresponding form of mental representations, e.g., Lass, Lürer, Ulrich, & Werner, 1993).

(COV = standard deviation of mean response/mean response) across target sizes reflects the direct relationship between the magnitude of the target number and the variation of the response (see Figure 5). This indicated that the rats had an approximate representation of how many presses were required. It gives direct evidence for scalar variability in the representation of numerosity.

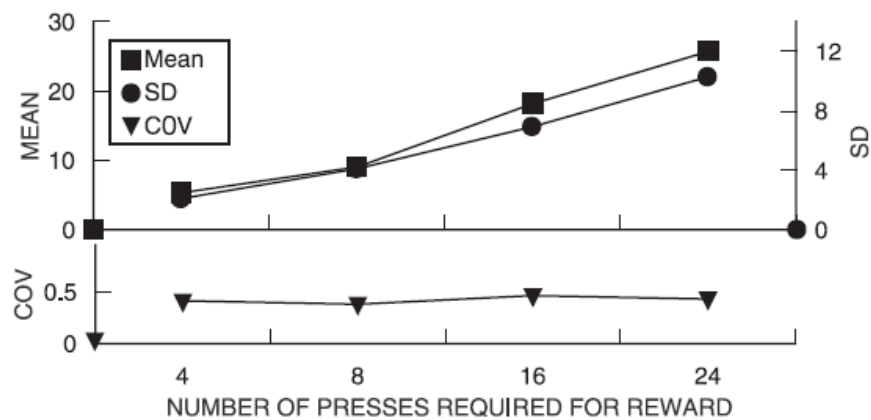


Figure 5: The mean estimate and the variability in responses increased as the magnitude of the target number increased. The means and standard deviations are directly related, resulting in a constant coefficient of variation (COV, adapted from Platt & Johnson, 1971).

This same representation is also used by human children and adults when they are presented with a large number of objects for a short period of time. When the task does not allow them to process numbers in an exact way, they estimate. This implies that children and adults do represent numbers approximately. Evidence for this ability comes from several experiments showing congruencies with animal data.

First, adults can perform numerical estimations whatever the input and output modality/format, that is, whether estimating the number of dots in a set by an Arabic number

(AN) (Izard & Dehaene, 2008) or a verbal number word (Whalen et al., 1999); or by producing key presses in response to written (Cordes et al., 2001; Whalen et al., 1999) or verbal numerical symbols (Castronovo & Seron, 2007). In these conditions, adults show a tendency to underestimate the target number (Izard & Dehaene, 2008) and are generally inaccurate as estimates and response variability both increase with target magnitude, indicating that the underlying representation is less precise for larger numerosities. More specifically, this representation is characterized by a scalar variability which gives rise to a constant COV across target magnitudes (Cordes et al., 2001; Whalen et al., 1999).

Second, studies of typical development have consistently reported increasing precision of the ANS with age, across different types of estimation tasks. This has been shown in 5 to 7 year olds who had to estimate the numerosity of a set of black squares (5 to 11 items) on a number line consisting of a series of Arabic numbers ordered from 0 to 20 (Huntley-Fenner, 2001); in 5 to 9 year olds who had to estimate the numerosity of dot patterns (from 8 to 20 items) either verbally or by drawing lengths (Chillier, 2002) and in 8 to 10 year olds who had to generate dots in a box on a monitor in response to a number presented on a sheet, ranging from 0 to 1000 (Booth & Siegler, 2006). The precision of this analog and approximate representation (calculated as the difference between the participant's answer and the target magnitude) seems to be related to growth, arithmetical learning and the mastery of symbols. For example, in Huntley-Fenner's study (2001), the mean accuracy significantly increased through the age range of 5 to 7 and COV scores were negatively correlated with age in days, showing that estimates were less variable with development. Finally, the author reported that 5 to 7 year olds' COV ranged from .11 to .37, which is higher than adult COV scores (ranging from .12 to .19 in Whalen et al., 1999).

Third, several elements suggest that this ANS increasing precision occurred also with schooling. This was shown through studies using number line placements (i.e., placing an Arabic number ranging from 1 to 100 or 1000 on a horizontal number line) or the generation of 0 to 1000 dots in a box by a computer program to assess the ANS in young children (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003). Results showed that estimation ability depended on children's counting range: unskilled counters could not produce relevant estimates outside their counting range (Lipton & Spelke, 2005; Siegler & Booth, 2004; Siegler & Opfer, 2003). Moreover, according to these authors, a shift on the number magnitude representation occurs during development, going from a logarithmic-based to a linear-based representation: when placing an Arabic number ranging from 1 to 100 or 1000 on a horizontal number line (with 0 at the left end and 100/1000 at the right end), the linearity of estimates correlates with arithmetical achievement at age 7 on a 1 to 100 number line and at age 8 to 10 on a 1 to 1000 number line (Booth & Siegler, 2006; Siegler & Booth, 2004).

Is it the result of children's natural development or the exposure to a culture oriented toward mathematical knowledge? A study of an Amazonian Mundurukú indigene group⁶ with reduced mathematical lexicon shed light on that log-to-lin shift of representation (Dehaene, Izard, Spelke, & Pica, 2008). This study is also the first one that compared symbolic material to non-symbolic visual and auditory numerosities through the use of number line placement tasks as presented here above. Western and Amazonian Mundurukú participants' results were compared: both children and Amazonian indigenes results fit with the idea that numerical

⁶ The Mundurukú Indians reside in the state of Para located in the Amazon Basin of Brazil. When Brazil was discovered and to avoid the conquerors who were interested in using Indians as slave, they moved deeper into the Amazon interior. Thus, the group was able to maintain their languages, most of their customs, and a considerable degree of autonomy, and they have survived as distinct social and cultural entities until the present day (for more information, see <http://www.mnsu.edu/emuseum/cultural/southamerica/munduruku.htm>).

representation obeys Weber's law. Indeed, larger quantities are represented with proportionally greater imprecision, compatible with a logarithmic internal representation. Moreover, a shift from log-to-lin representation occurs during development (as observed in number line task in children studies presented above), and this change seems to be related to schooling and culture. Indeed, at all ages, the Indigenes' mapping of symbolic and non-symbolic number onto space was logarithmic whereas Western adults used a linear mapping with symbolic numbers and small non-symbolic ones. However, Portuguese-schooled members of the Mundurukú placed symbolic Portuguese numerals to space linearly but their responses to Mundurukú numerals and dot patterns remained unchanged. Finally, these results support the hypothesis that the change from log-to-lin numerical representation is not only the result of brain maturation as older Indigenes did not present any linear representation.

To summarize, estimation tasks allow human individuals and non-human species to process and represent approximately quantities, no matter the modality/format of the input and output. This type of task gives due about the variability (COV) and the precision of the ANS. The variability of the representation decreases with age while precision increases. However, if this representation seems to develop naturally, it is not impervious to the influence of mathematical education and the learning of symbols.

In the following paragraphs, we will show that production estimation task has a significant advantage over the comparison of simultaneous sets with respect to the estimation processes, as it minimizes the influence of the perceptual variables on the estimates. However all these tasks may not be a perfect operationalization of the ANS and may constrain participants' performance in the estimation task in undesired ways. For example, the physical number line has attributes that may encourage participants to use specific strategies relative to the

endpoints. Numbers close to the right end of the physical number line tend to be better estimated than numbers in the middle of the line, as they do not benefit from external marks (Siegler & Opfer, 2003). Such a bias should not be present in tasks designed to assess the approximate number representation. Moreover, as reported in studies in adults and typically developing children estimating large numerosities, the approximate number representation can be explored without applying external constraints to the participants' estimates (Castronovo & Seron, 2007; Chillier, 2002; Cordes et al., 2001; Izard & Dehaene, 2008; Whalen et al., 1999). It is important to bear in mind that when participants are provided with a reference point (e.g., "30" dots), this is sufficient to calibrate their estimates on the whole range of stimuli and not just locally around the reference point, as demonstrated by Izard and Dehaene's work (2008). Moreover, a disadvantage of estimation task is that free-production is not possible in infancy.

2. Comparisons

Adults can compare and choose the larger or the smaller of two numerosities. However, this is possible under some conditions only. If an individual is asked to judge which of two dot sets has more elements, his/her judgments are above chance but accuracy varies with the ratio of the two numerosities: the ease of distinguishing between two dot sets depends on the numerical ratio between the two stimuli. This corresponds to the Weber ratio limit and it is calculated using the Weber fraction. More concretely, a Weber fraction (w) of 0.2 implies that two sets need to differ by about 40% ($w * 200$) to be discriminated by a participant. For example, comparing a set of 10 dots versus a set of 12 dots corresponds to a ratio 5:6, where 5 = n_1 and 6 = n_2 . The Weber fraction correspond to $w = (n_2 - n_1)/n_1$. The Weber ratio limit is an important signature of the ANS and studies of typical development have consistently reported w decreases with age.

Through the preceding paragraphs, the idea that individuals are able to process numerical elements in the same and in different sensory modality (visual and auditory samples) and presentation mode (sequential and simultaneous) has been investigated in estimation task. According to some authors, this is an important clue as this lack of modality and presentation mode effects suggests that the ANS is “abstract”. For example, adults can compare two numerosities as accurately as the elements in the two sets are presented in the same and in different modalities (Barth, Kanwisher, & Spelke, 2003). However, according to Pesenti and Andres (2009) these assumptions are fallacious as a non-abstract system could be accessed from different symbolic (e.g., Arabic number) and non-symbolic inputs (e.g., sets of dots) and giving rise to similar effects. Proper ways to assess the format of numerical representations have been suggested by these authors. It would be, for example, to demonstrate overlapping activations in areas attuned to the type of representation and to adopt a lesional approach to demonstrate the usefulness of these areas to perform correctly the tasks relying on the particular representation. Therefore, we will revisit this lack of modality and presentation mode effects briefly through comparison tasks, only in order to evaluate the possible limitations of estimation and comparison tasks.

We will also see that these capabilities are already present in newborns as they were found to be able to discriminate between two numerosities as well, providing clues for this abstract system being present a few hours after birth (Izard, Sann, Spelke, & Streri, 2009). However, if there is much evidence of numerical sensitivity in infants (e.g., Wynn, 1995), accessing the ANS through a task of pure estimate is quite difficult. This is why the existence of such a representation has been evaluated through discrimination procedures on a succession of habituation trials.

Studies have demonstrated that infants can discriminate the numerosity of small sets of objects (e.g., Starkey & Cooper, 1980), or even larger ones when the ratio between them is large enough (e.g., Xu & Spelke, 2000). Concerning numerosities above the subitizing range (> 4), some authors have suggested that quantification is initially based on *non-numerical cues* (i.e., continuous perceptual variables) as preschoolers often base their comparisons on continuous perceptual cues (such as area, volume, contour length, brightness, density, etc.) rather than on number information (Feigenson, Carey, & Spelke, 2002; Mix, Huttenlocher, & Levine, 2002; Rousselle & Noël, 2008b; Rousselle, Palmers, & Noël, 2004). Nevertheless, studies have demonstrated that, with a sufficient ratio between sets, 6 month old infants can discriminate the numerosity of large amounts of well perceptively controlled stimuli (8 from 16 sounds/dots but not from 12 sounds/dots) presented aurally (Lipton & Spelke, 2003) or visually (Xu & Spelke, 2000). However, infants can detect changes in numerosities with smaller ratio at 9 to 10 month olds (8 from 12 sounds/dots but not from 10 sounds/dots and 4 from 6 jumps) (Lipton & Spelke, 2003; Wood & Spelke, 2005; Xu & Arriaga, 2007). The ratio limit is a signature that the ability relies on an approximate numerical representation that obeys Weber's law⁷ but this ratio limit decreases with age.

Finally, a study provides evidence for abstract numerical representations in the first hours of human life: testing infants aging from 7 to 100 hours, Izard and collaborators (2009) showed that newborns responded to numerical quantities across different modalities (visual versus auditory) and formats (sequential versus simultaneous). In this study (presented in Figure 6), infants were familiarized with sequences of syllables (equated for the total duration) repeated a

⁷ It has also been shown that during infancy, discrimination fails for the smallest numerosities : 6 month old infants did not show evidence of discriminating for ratio 2:1 (2 versus 4 sounds/dots/jumps, Lipton & Spelke, 2004; Wood & Spelke, 2005; Xu, 2003). This violation of Weber's law on small numerosities is part of a set of elements that supports the existence of a system dedicated for small numerosities which will be presented later in this manuscript.

fixed number of times (4 or 12, 6 or 18, 4 or 8) in a continuous auditory stream. After 2 minutes of familiarization, the infants were presented with test images (the size of each item and density of the set were controlled) while the auditory stimulus continued to play in background. Then, their looking times were tested with 4 images, alternating between the congruent number (same number of objects as the auditory sequences) and an incongruent number (with a different number of objects). Newborn infants in both familiarization conditions looked longer at the visual image that matched the number of objects, but newborn children discriminate numbers robustly only when they differed by a ratio of 3:1 (12 versus 4 or 18 versus 6) but not for a 2:1 ratio (8 versus 4).

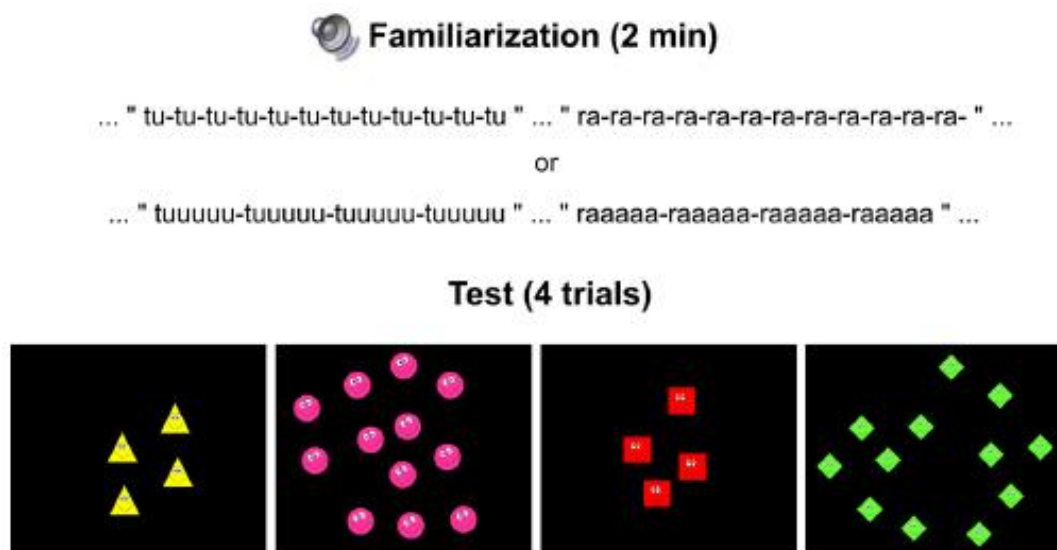


Figure 6: Newborn infants were familiarized with auditory sequences containing a fixed number of syllables, and were then tested with images of the same or a different number of items (here 4 or 12, adapted from Izard et al., 2009).

To this point, research in infancy has shown that the precision of discrimination increases with development as attested by the decrease of the estimated Weber fraction with age.

Studies presented in Table 1 panel a, provide evidence that numerosity discrimination is characterized by the Weber signature even in infancy. Infants' performances are determined by the ratio and the critical discrimination ratio globally narrows with age.

Table 1: Estimated Weber fraction (w) in different human studies.

Age group	W	Ratio	Stimulus used	References
a. Infancy				
Few hours	2	3:1	Cross-modal	Izard et al., 2009
6 months	1	2:1	Auditory	Lipton & Spelke, 2003,2004
			Visual-spatial	Xu & Spelke, 2000
			Visual-spatial	Xu, 2003
			Sequences of action	Wood & Spelke, 2005
9 months	.50	3:2	Auditory	Lipton & Spelke, 2003,2004
			Sequences of action	Wood & Spelke, 2005
10 months	.50	3:2	Visual-spatial	Xu & Arriaga, 2007
b. Childhood				
3 years	.53	3:2	Visual-spatial	Halberda & Feigenson, 2008
4 years	.38	4:3	Visual-spatial	Halberda & Feigenson, 2008
	.34	4:3	Visual-spatial	Piazza et al., 2010
5 years	.23	5:4	Visual-spatial	Halberda & Feigenson, 2008
6 years	.18	7:6	Visual-spatial	Halberda & Feigenson, 2008
		3:2	Mapping tasks	Mundy & Gilmore, 2009
10 years	.25	5:4	Visual-spatial	Piazza et al., 2010
c. Adulthood				
Western adults (U.S.)	.11	10:9	Visual-spatial	Halberda & Feigenson, 2008
(French)	.12	9:8	Visual-spatial	Pica et al., 2004
(Dutchman)	.14	8:7	Visual-spatial	van Oeffelen & Vos, 1982
(Italian)	.15	8:7	Visual-spatial	Piazza et al., 2010
(French)	.17	7:6	Visual-spatial	Piazza et al., 2004
Indigene adults	.17	7:6	Visual-spatial	Pica et al., 2004

The cause of this w reduction with age is not known. It could result of brain maturation or a refinement linked to the educational environment, such as the learning of arithmetic.

Examining older children's results might give some hints concerning the cause of the refinement. Those studies have reported increasing precision of the ANS with age as well but have provided some nuances.

An increase in precision was reported by Piazza and collaborators (2010) using a classical comparison task of two dot sets in 5 and 10 year old children. Mundy and Gilmore (2009) showed also this increase in another comparison paradigm: the children had to map a symbolic target (i.e., Arabic symbols presented with prerecorded number words) with one of the two alternative non-symbolic choices (dot sets of 20 to 50 dots) or had to do the reverse mapping (a non-symbolic target numerosity with one of the two alternative symbolic choices). They observed an increase of performance between 6 and 8 years of age with a better performance for the mapping from dots to Arabic numbers than for the reverse.

In the first study, children's non-symbolic number acuity did not correlate with (symbolic) arithmetical scores (Piazza et al., 2010). In the second one, scores on the mapping tasks did not correlate with arithmetical scores either when the overall mapping score was considered, when symbolic to non-symbolic or non-symbolic to symbolic problems were considered separately (Mundy & Gilmore, 2009). Results found in those two studies do not plead in favor of a refinement in numerical precision with the exposure to an educational environment (i.e., the learning of arithmetic with schooling). However, contradictory results are found in the literature. It has been reported that the ANS precision seems to be related to performance in exact calculation and number processing: a correlation has been found between the accuracy of performance on a non-symbolic numerical comparison task, presented to students at 14 years of age, and school mathematics performance from kindergarten to sixth grade (Halberda, Mazocco, & Feigenson, 2008).

Contrariwise, the school mathematics performance of 6 to 8 year old children was found to be unrelated to the magnitude of the numerical distance effect exhibited by the children on a comparison task involving non-symbolic numerical displays (Holloway & Ansari, 2009) although

such a relation was obtained on a similar comparison task with symbolic numbers in 6 year olds (i.e., children with a smaller distance effect showed higher mathematics performance, De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009). If those studies supported the increase in precision with age (see Table 1, panel b) the reason of it remains unclear.

Finally, to investigate if the teaching of counting and arithmetic do have an influence on the ANS refinement, a study compared French adults to Amazonian Mundurukú indigenes with little or no instruction in arithmetic and with a rudimentary numerical lexicon (Pica et al., 2004). Pica and collaborators showed through a magnitude comparison of collections task that Mundurukú are able to indicate the larger set of dots even if the number of dots was far beyond their counting range. In Mundurukú, the w was equal to 0.17 which is not far from what has been found in Western adults but also quite close to 6-year-old children (see Table 1 panel b and c). Moreover, the idea that ANS is language-independent as similar results have been observed in animals, infants and adults is reinforced by this study as Mundurukú also show the Weber signature of the large approximate number system.

To summarize, comparison tasks and discrimination paradigms give an insight of the ANS even in newborn infants. A limitation of these comparison tasks comes from the fact that controlling for all of the properties that co-vary with numerosity is quite difficult compared to estimation tasks. Nevertheless, studies that have controlled non-numerical properties at best suggest that human adults, children and infants respond similarly to a wide range of numerical stimuli (objects, sounds, events, stationary and moving items) presented simultaneously or sequentially.

This type of task gives due about the Weber fraction and the Weber ratio limit. This ratio limit is one important signature of the ANS and studies of typical development have consistently

reported w decreases with age. However, the reason of it is not yet clear: does it come from brain maturation or refinement of the ANS supported by the educational environment of the child such as the learning of symbols and arithmetic? Some of the actual observations seem to favor the maturational hypothesis as the critical ratio needed for numerosity discrimination in babies was found to decrease within the first year of life. When looking at Figure 7, the Weber fraction gives the impression to decrease with development. Looking closer, it appears that it decreases in steps: Among the first group of studies collecting data from the infants and the second group gathering data from the preschool children, a significant drop of the w is observed (but no drop is observed between 6 months and 3 years). The same is observed when comparing those groups to the adult's one. The most plausible explanation to account for this phenomenon is the tasks which have been used. Indeed, the concern in babies' studies is to evaluate if they are able to detect changes between large numerosities. In children studies, several ratios are used in order to allow the fit of their performances into a curve and to quantify the amount of imprecision in their internal representation. By contrast, Mundy and Gilmore's results (2009) show threshold accuracy in 6 year olds equivalent to 9 to 10 month infants. However, these children were presented with only two different ratios, excluding the possibility to draw a curve of their performance.

Contrary to the maturational hypothesis, some other studies have demonstrated a correlation between comparison processes and mathematical abilities. However, this relationship has been controversial and according to some authors, it is only revealed with the use of symbolic material (e.g, Holloway & Ansari, 2009). As a matter of fact, these results are congruent with those observed in the Portuguese-schooled members of the Mundurukú subgroup who had the opportunities to learn Portuguese numerals. They placed symbolic Portuguese numerals to space linearly but their responses to Mundurukú numerals and dot

patterns remained logarithmic (Dehaene et al., 2008). This is in accordance with Verguts and collaborators' model (Verguts & Fias, 2004; Verguts et al., 2005): symbolic representations develop by being mapped onto pre-existing non-symbolic representations and the learning of symbols leads to changes in the system as tuning curves become much sharper, but only regarding symbolic inputs. This is congruent with the Mundurukú data as no transfer between materials is observed. Moreover, the authors proposed that the magnitude representations for small numbers are precise and exact because of their greater frequencies in everyday life. Finally, with development and exposure to numbers, the exact magnitude representations should increase further for a wider range of numbers. This proposal is also congruent with the increasing variability in the range of large numbers that become approximate because they have lower frequencies.

Then, to conclude with a natural development of the ANS or with an ANS refinement with the exposure to symbols seems to be hasty to this point. One can imagine that the relationship between that system and formal mathematics may only be observed when tested in older children who have had years of experience with symbolic mathematics (see for example, Halberda et al., 2008). Only studies to be conducted on the long term, i.e. with a longitudinal perspective, may answer this question.

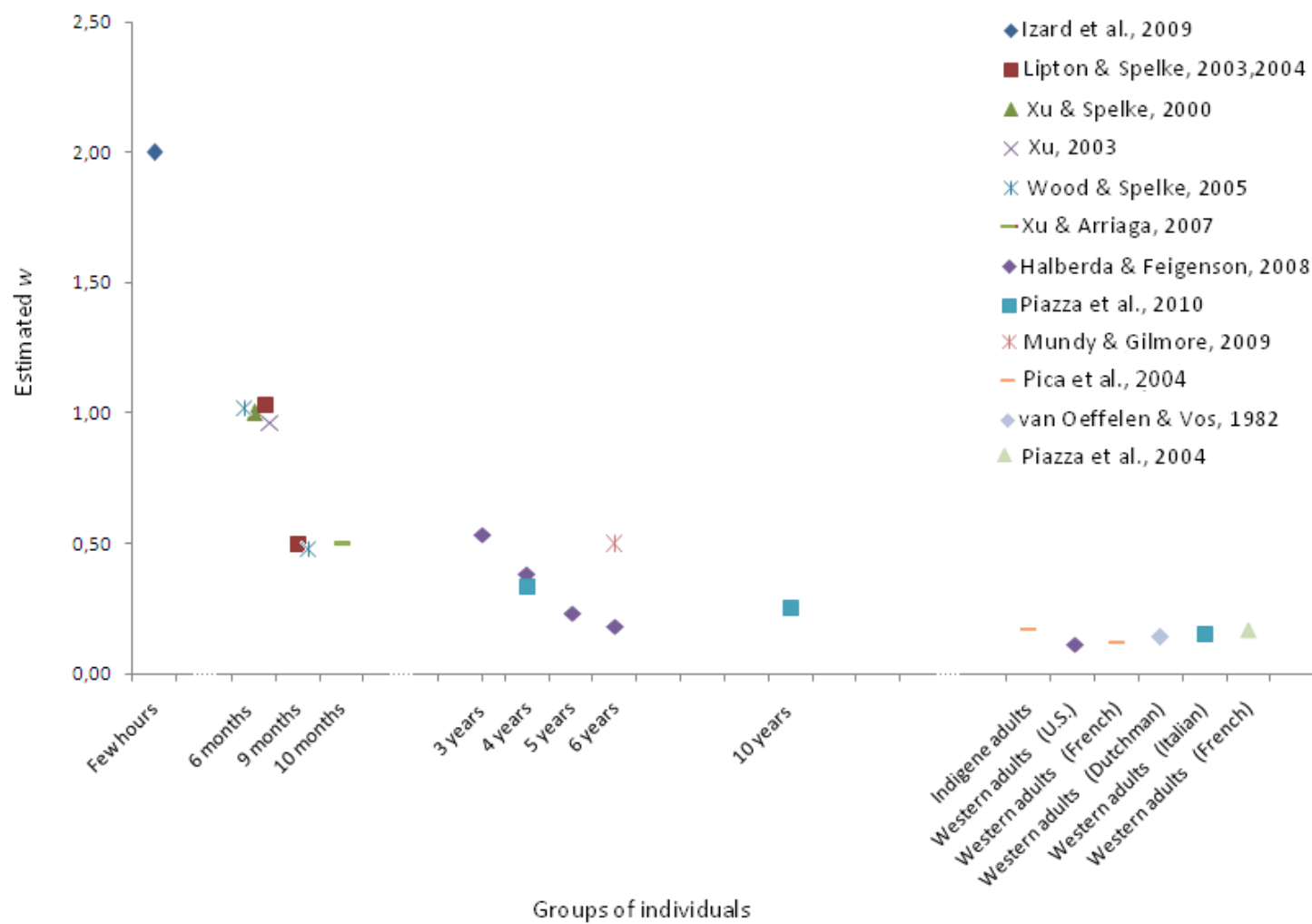


Figure 7: Values of w estimated in different groups of age through different papers using comparison tasks.

3. Approximate calculation

There is evidence that individuals are able to quickly add or subtract two large symbolic or non-symbolic numerosities and to compare the resulting sum or difference to a third set. This has been observed in non-human primates, human infants, human children and adults with or without formal education. A majority of these studies are conducted to examine the existence of a magnitude-based estimation system for representing numerosities that also supports procedures for numerical computation outside formal education. A second goal is to find out whether children use that ANS as building block of their symbolic arithmetic learning.

Barth and collaborators (2006) showed that human adults can mentally add two sets of dots/sounds, subtract one set from the other, and then compare the sum or difference to a third set of large non-symbolic numerosities. Across the different tasks, participants performed above chance. The authors also noted that participants were as accurate in addition than in comparison tasks and whether the two addends appeared in the same or in different modalities/formats. Moreover, participants' performance showed the ANS ratio signature and improved their score rate as the ratio between the true and the proposed outcomes increased.

First evidence for non-symbolic arithmetic abilities outside formal education comes from animal studies. For example, Flombaum, Junge, and Hauser (2005) found that rhesus macaques can spontaneously add two quantities of lemons: by using an expectancy violation looking time procedure (e.g., $3 + 1 = 4$ or 8), rhesus monkeys spontaneously compute addition operations controlled for continuous extent. The limit on this ability is set by a 1:2 ratio difference between the two numbers, since the monkeys failed on numbers differing by a 2:3 ratio (e.g., $3 + 1 = 4$ or 6). This accuracy modulated by the ratio is a signature of the ANS, as already showed in the comparison experiment.

Similar results of non-symbolic numerical operation abilities were found during infancy: McCrink and Wynn (2004) found that 9 month old preverbal infants can successfully discriminate correct and incorrect outcomes of large-number problem sets that exceed object-tracking limits (such as $10 - 5$, or $5 + 5$). When continuous variables (such as area and contour length) are well controlled, 9 month old infants successfully add and subtract numbers of moving objects (going behind an occluder or moving from behind an occluder, see Figure 8). In this study, infants were given correct and incorrect outcomes to a mathematical operation ($5 + 5$ or $10 - 5$). Infants who saw the addition operation looked longer to an outcome of 5 than to an outcome of 10, and infants who saw the subtraction operation looked longer to an outcome of 10 than to an outcome of 5.

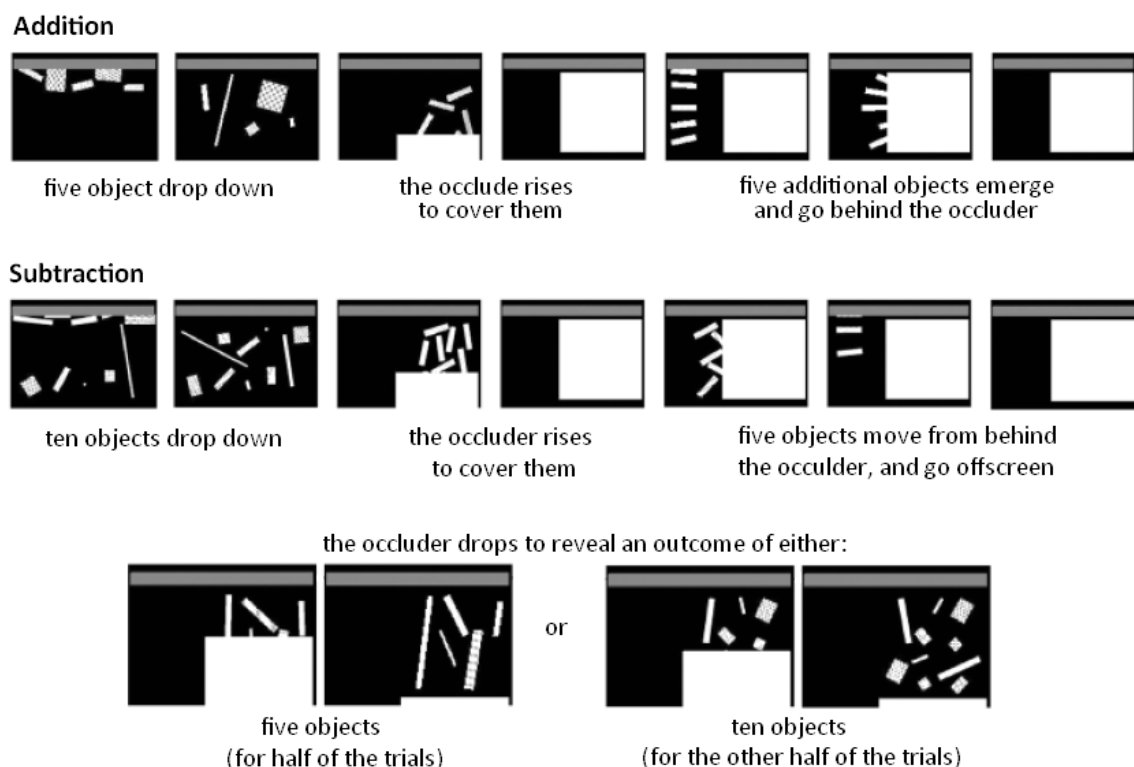


Figure 8: Schematic of the addition and subtraction test movies
(from McCrink & Wynn, 2004).

In studies of preschool children (Barth, La Mont, Lipton, & Spelke, 2005, see also Barth, Beckmann, & Spelke, 2008) there is evidence that young children can add two large sets of elements without counting (within a single modality or across two modalities). Five to 6 year old children were presented with movies and added, for example, two successive dot sets and compared the sum to a third set (see Figure 9 panel a): a first set of blue dots appears and then goes behind an occluder, a second set of blue dots appears and disappears behind the same occluder, then an set of red dots appears. Participants were asked to choose the most numerous of the two dot sets, the blue or the red one. In another experiment, a blue dot set was presented followed by a sequence of tones, so that participants had to integrate quantity information presented aurally and visually (see Figure 9 panel b). Participants were asked if there were more blue or red dots.

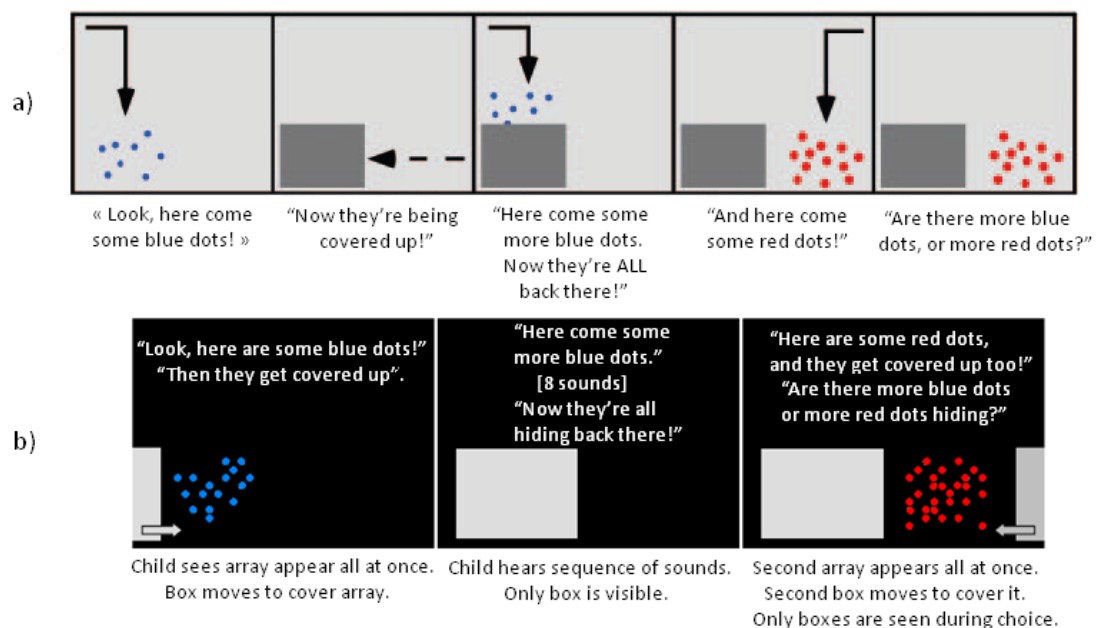


Figure 9: Schematic depictions of procedures and narrative for the two non-symbolic tasks; a) Addition and comparison of visual sets (adapted from Barth et al., 2005); b) Comparison of visual sets and auditory sequences (adapted from Barth et al., 2008).

Children performed all tasks successfully, without resorting to guessing strategies or responding to continuous variables. Again and as observed in animal or in youngest humans, their accuracy

varied with the ratio of the two quantities (the correct answer and the proposed one): a signature of large, approximate number representation.

However, before going further in the literature review, a limitation to the use of such a paradigm should be raised. Indeed, it cannot be excluded that the distance effect revealed here can be explained by a comparison of the proposed outcome with one of the operands without performing the approximate sum or difference (e.g., $8 + 3 = 5$ or $12?$). Then, an alternative experimental design was proposed by McCrink, Dehaene and Dehaene-Lambertz (2007). After they showed short videos of mathematical operations (half addition and half subtraction problems), adults participants were presented with a range of seven outcomes (varying from one half of the true outcome to twice the true outcome) and they had to choose whether each outcome seemed correct or not (see Figure 10). This procedure enables the authors to plot the percentage of times participants perceived each outcome as correct. For each operation, a peak at the approximate location of the true outcome appeared and the variability increases in proportion to the problem size. This finding indicated that human adults can add and subtract numerical magnitudes via an ANS that accord with Weber's Law.

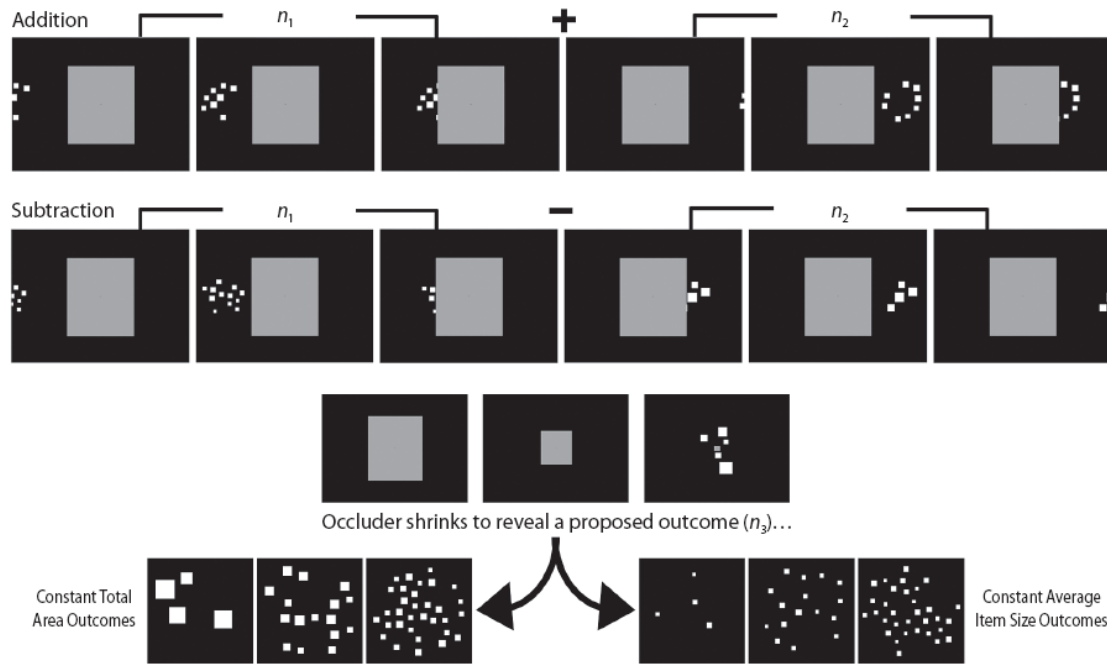


Figure 10: A schematic of addition and subtraction movies and the two test outcome types. A set of n_1 objects moves onto the screen and go behind the occluder. In the addition movies, a set of n_2 objects appears onscreen and joins the first set of objects behind the occluder. In the subtraction movies, a subset of n_2 objects continues to move from behind the occluder and then leaves off-screen. In both cases, the occluder then shrinks and disappears to reveal a proposed outcome set (n_3), which can be either a constant-area outcome or a constant average item-size outcome (adapted from McCrink, Dehaene, & Dehaene-Lambertz, 2007).

After these experimental studies (Barth et al., 2006; Barth et al., 2005), Gilmore, McCarthy and Spelke (2010) evaluated 5 to 6 year old children during their first year of school. They related their performance on large-number non-symbolic approximate addition (as presented in Figure 9) to their mastery of their school's mathematics curriculum at the end of that first year of formal instruction (counting objects, recognizing Arabic numbers, length, symbolic and, non-symbolic comparisons, ... all numerical tasks using numbers smaller than 10). As the relation between non-symbolic numerical abilities and school mathematics might not be specific, they also investigated through hierarchical regression analyses intelligence and literacy achievement's influence. Finally, they compared the performance of children at a range of socio-economic levels, from lower-and middle-socio-economic backgrounds to more advantaged peers. Children's performance on Barth and collaborators' test of non-symbolic, large-number approximate additions predicted their mathematics achievement at the

end of the school year, independently of achievement in reading or intelligence. Non-symbolic arithmetic performance was also related to children's mastery of number words and symbols, no matter the socio-economic backgrounds. So during the first year of schooling, the ANS' efficiency and the good learning of symbols seem dependent.

Similar investigations were conducted in Mundurukú (Pica et al., 2004). They were presented with animations illustrating an addition of two sets of dots, had to approximate the result and had to compare it to a third set. All Mundurukú participants performed above chance, and their precision was identical to that of the French controls with performance affected by the ratio.

Another study provides suggestive evidence for arithmetic operation on large, abstract and symbolic numerical quantities. It is the earliest evidence for approximate arithmetic on symbolic numbers that was reported in children with no formal arithmetic education, who had learned no algorithms for manipulating numerical symbols but who had mastered verbal counting: Gilmore, McCarthy and Spelke (2007) showed that 5 to 6 year old children could perform approximate addition and subtraction of two symbolically presented numbers (presented both as number words and Arabic symbols, see Figure 11 panel a for approximate addition and panel b for approximate subtraction). In this experiment, children were asked to compare the result of the operation to a third symbolic number and to tell who has more (e.g. "Sarah has 21 candies and get 30 more. John has 34 candies. Who has more?"). Results showed that the 5 to 6 year old children who took part to the experiment have no exact calculation knowledge. Indeed, children failed to provide the exact solution to the present problems, to solve those problems when the comparison quantity was altered so as to require an exact representation of number, or to solve the problems through rounding strategies allowing for single-digit addition or comparison. Children's approximate arithmetic performance evidently does not depend on knowledge of exact number.

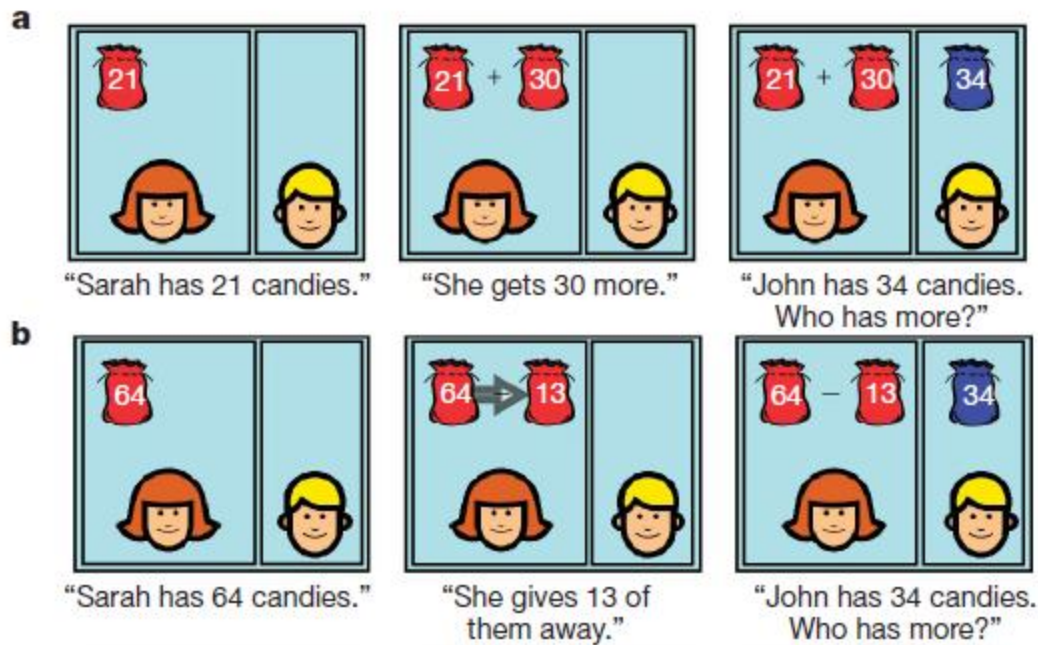


Figure 11: Example of (a) addition and (b) subtraction symbolic, approximate arithmetic problems (adapted from Gilmore et al., 2007).

According to the authors, as children perform significantly above chance these experimental tasks suggest that they build on their ANS to solve these symbolic addition and subtraction problems, just as non-symbolic addition abilities observed in children who took part to Barth and collaborators' studies (Barth et al., 2005).

However, the use of approximation with symbolic numbers in older children probably depends on different processes: they are more likely to use an exact-calculation/comparison process. Some studies however reported convincing evidence of the use of approximate calculation in a verification task in first and second grade children beginning their primary school (Hamann & Ashcraft, 1985). Nevertheless, they probably calculate the correct result and then compare it with the presented answers instead of approximating the result via the ANS exclusively. For the sake of avoiding exact calculation, Rousselle & Noël (2008b) used a verification task with 7 to 8 year old participants. They were asked to make true/false decisions on simple and complex addition problems while the distance between the proposed and the correct answer was manipulated. They showed that children

were exhibiting typical signs attesting the use of approximate calculation: children were found (a) to reject extremely incorrect answers faster than (b) to accept exact answers or (c) to reject dose ones, i.e., $38 + 43 =$ (a) 120, (b) 81, (c) 83. This indicates that they did not run a complete exact calculation procedure before responding. Nevertheless, the use of partial calculation strategies could still account for their faster reaction times for implausible sums. In that case, their abilities would not rely on the computation of approximate number magnitude as it is expected in *non-symbolic* approximate calculation.

The nature of this type of approximate calculation on symbolic numbers seemed particular and different from approximate calculation on non-symbolic notations. Up to date, we do not possess any direct proof that approximate calculation on symbols relies on the analog and approximate number representation. Conversely, self-reported strategies used to approximate calculation on Arabic numbers (usually labeled “computational estimation strategies”) are well studied. Individuals are found to use some type of procedure to generate an approximate answer to an arithmetical problem when calculation of an exact answer is too difficult or unnecessary. However, this ability is difficult for both children and adults and it appears only with formal schooling. Dowker (1997, 2003) reported that children from elementary school demonstrated the ability to generate reasonable approximate answer to arithmetic problems (e.g., $23 + 35$). Nevertheless, this ability is linked to their formal arithmetic skills and is restricted to additions that are just beyond their ability to mentally calculate the exact answer. The common strategy for those younger children is to round the units or the decades’ segments down to the nearest decade ($30 + 50 = 80$ to approximate $32 + 53$) or hundred values ($200 + 600 = 800$ to approximate $213 + 632$). Older children and adults use more sophisticated strategies. They are found to round one or each operands (“46” become “50”), to truncate them (“46” become “40”), and to compensate by adding or subtracting to compensate for rounding (Lemaire, Lecacheur, & Faroli, 2000). These partial calculation strategies consist of performing an exact calculation algorithm on simplified symbolic representations to find an

approximate answer. They are therefore expected to be faster but less precise than exact calculation. To conclude, even if further studies need to be conducted on symbolic approximate calculation, the most plausible but highly speculative hypothesis is that both kinds of approximate calculation processes coexist (i.e., rely on computational estimation strategies and on approximate number magnitudes).

In summary, the ANS serves to represent the approximate cardinal values of large sets of objects or events. Parallel signatures are found in studies of primates, human infants, preschool children and adults to provide evidences for the existence of a magnitude-based estimation system for representing symbolic and non-symbolic numerical magnitude that also supports procedures for numerical computation even outside formal education. According to some authors (e.g., Barth et al., 2005), this is congruent with the fact that the ANS serves as building block of symbolic arithmetic learning. However, to this point, this assumption is challenged by the absence of a correlation between non-symbolic numerical magnitude comparison and mathematical performance at some point of the child's development (e.g., Holloway & Ansari, 2009).

4. A brief summary

In this chapter, the question of which tasks indexed the best the ANS was addressed. Moreover, through behavioral performance, we saw that individuals can estimate, compare and add or subtract two large symbolic or non-symbolic numerosities, whatever the material or the format of the stimuli used. This has been observed in non-human animals, human infants, human children and adults with or without formal education. A majority of those studies conducted in animals, preschoolers or individuals without formal education were carried out to examine the existence of a magnitude-based estimation system for representing numerosities that also supports procedures for numerical computation outside formal education.

However, some questions remain open. For example, the cause of the ANS refinement with age remains unknown. It could reflect pure maturational processes (congruent with the infants' data) and/or the contribution from arithmetical education and the use of symbols in our culture (congruent with some of the Mundurukú's data). However, divergent findings observed in comparison studies make it impossible to decide between the two hypotheses. Indeed, sometimes the relationship between non-symbolic numerical abilities and the learning of symbolic arithmetic is established (e.g., Halberda et al., 2008), sometimes it is not (e.g., Holloway & Ansari, 2009). The contradictory results may be explained by the age range which varies from one study to another. Testing sometimes children who are on the threshold of arithmetic instruction or who have had years of experience with symbolic mathematics does not help disentangling this question. Then, further studies and especially longitudinal ones, are needed to clarify the relationship between non-symbolic and symbolic numerical abilities.

Another question asked was to what extent the ANS serves as a building block for arithmetical knowledge. As reported in the previously mentioned studies, nonhuman animals, infants, children and adults can approximate numerical quantities. Barth and collaborators (Barth et al., 2008; Barth et al., 2006) claimed that this abstract number representation, which follows Weber's law, might serve as the underpinning foundation for formal mathematics as preschoolers with no training in arithmetic are able to process non-symbolic (dots or sounds) (e.g., Barth et al., 2005, Gilmore et al., 2010) and approximate symbolic (Arabic numbers) problems (Gilmore et al., 2007). However, this assumption is also challenged by the absence of a correlation between non-symbolic numerical magnitude comparison and mathematical performance at some point of the child's development (e.g., Holloway & Ansari, 2009; Mundy & Gilmore, 2009).

Finally, another question is addressed concerning the tasks used to evaluate the ANS. To this point, estimation task, with its free-production aspect, seems to be the most advantageous one. Indeed, by switching from a material/format to another, the influence of the perceptual variables on

participant's estimates is minimized. Moreover, there is no constraint of proposing a limited choice of responses (i.e., a limited choice of different ratios) and this allows for a finest capture of the ANS.

Before considering the literature about developmental dyscalculia and how this deficit might interferes with the magnitude representation, the omnipresence of the Weber characteristic through the presented data might suggest that other magnitude than number, such as time and space, are represented via the same approximate system. Accordingly, a theory of magnitude, named "ATOM" and proposed by Walsh (2003), considered the existence of a general magnitude system.

5. An alternative view: the existence of a general magnitude system

Through the preceding paragraphs, the idea that individuals are able to process numerical elements in the same and in different sensory modality (from visual to auditory samples) and presentation mode (spatial or temporal) has been investigated in estimation, comparison and addition tasks. This lack of modality and format effects suggests that it is possible to access the ANS through different modalities and/or formats. As reviewed bellow, temporal and visuo-spatial discrimination in infancy appear to follow the same developmental trajectory.

Studies of analog magnitude in animals have shown that rats are able to classify noise according to time and numerical dimensions and that can discriminate between 2 and 8 seconds/tones (Meck & Church, 1983). As the ANS for discrete (countable – such as sounds) quantities presents the same characteristics compared to the magnitude system that represents continuous (uncountable – such as duration) quantities, this might indicated that these quantities are represented via one unique magnitude system.

Similar performances are found in infancy. Indeed, Brannon, Lutz and Cordes (2006) habituated 5 to 6month olds to stimuli with a single small or large puppet face and tested the infants with a single small and large face. The researchers varied the ratio of the surface area. Infants were able to

discriminate a twofold change (ratio 2:1) in the size but not a 2:3 ratio change. It is also important to note that the looking time increased with the ratio of the novel to familiar area (the larger is the difference, the longer is the looking time).

Accordingly, vanMarle and Wynn (2006) have showed that temporal discrimination in 6 month old infants are ratio dependent as well: habituated to a dancing and sounding cat puppet, then tested with the same dancing and sounding puppet for the habituated duration or a novel duration, infants looked longer at the static puppet that had danced/sounded for the novel duration when the values differed by a 1:2 ratio (at two different sets of absolute values). Brannon, Suanda, and Libertus (2007) replicated those results using a mooing cow puppet. They also demonstrated that between 6 and 10 months of age, temporal discrimination increases in precision: 10 month infants succeed at discriminating a 2:3 ratio. These results showed that discrimination of surface area and temporal changes follows the same developmental progression compared to numerical discrimination observed previously.

Table 2: Quantities representation through infancy.

Age group	W	Ratio	Stimulus used	References
6 months	1	2:1	Area 	Brannon et al., 2006
	1	2:1	Duration 	vanMarle & Wynn, 2006
	1	2:1	Duration 	Brannon et al., 2007
	1	3:1	Liquid 	Gao et al., 2000
9 months	1	3:1	Liquid 	Gao et al., 2000
	.50	2:3	Cross- dimensional 	Lourenco & Longo, 2010
10 months	.50	2:3	Duration 	Brannon et al., 2007

Other interesting findings need to be mentioned as well. Using a habituation, Gao, Levine and Huttenlocher (2000) examined if procedure 6 month olds were capable to distinguish different amounts of red liquid presented in a clear plastic container (half of the infants were presented with a 1/4 full container and the other half with a 3/4 full container). Infants looked significantly longer at a novel quantity than at the familiar quantity. In a second experiment, Gao and collaborators used a violation-of-expectation paradigm. Are 9 month old infants expecting a change after seeing liquid from a 1/2 full container being added to container already 1/4 full of liquid? Shorter looking time for possible event was revealed. These findings indicate that infants are sensitive to changes with amount of liquid as well.

Finally, cross-dimensional experiments showed that 9 month olds transfer associative learning across magnitude. Lourenco and Longo (2010) showed that when infants were habituated to an arbitrary mapping between one magnitude dimension and color-pattern cues as “small white objects with dots” and “black large rectangle with stripes”, infants expected the same color-pattern mapping to hold for numerosity (i.e., greater numerosity: black with stripes; smaller numerosity: white with dots) and duration (i.e., longer-lasting objects: black with stripes; shorter-lasting objects: white with dots). Cross-dimensional transfer occurred bidirectionally for all combinations of size, numerosity, and duration. Even in pre-verbal infants, it seems that representations of magnitude information might be abstracted from specific dimensions.

In Table 2, we note that for various quantity dimensions, infants’ success with ratio changes is mostly congruent. If one goes back and look at Table 1 examining the numerosity discrimination pattern, we can see that infants’ discrimination thresholds are mostly consistent for various dimensions of quantity.

Collectively, those findings indicated that in human and non-human animals, the same mechanism may underlie the ability to represent the different magnitudes. This has led to speculation about the origins of general magnitude representation and that magnitudes are represented quite generally in a common representational format (Feigenson, 2007; Walsh, 2003)

Then, it seems that it would be worth conducting studies assessing the capacities in children with mathematical disability facing other magnitude representations such as space and time. Indeed, their difficulties might go beyond the numerical domain.

Chapter 3:

Developmental dyscalculia

Through the previous chapters, we examined the approximate number magnitude representation and to a lesser extent, the exact number one. To date, it is not clear yet what are the innate part and the learned part of both systems and to what extent those systems are independent.

However, this debate is of interest in the field of dyscalculia. Through the following paragraphs, after providing definition and prevalence, models of specific deficit of the number magnitude representation in dyscalculia are presented. Behavioral data favoring an impairment of the number magnitude representation in MLD are reviewed and discussed in regard to those different models.

1. Definition, prevalence and familial-genetic predisposition

“Developmental dyscalculia” (Shalev, 2004; Shalev & Gross-Tsur, 2001), “mathematical learning disabilities” (Geary, 1993, 2004) or “Mathematics Disorder” (American Psychiatric Association, 1994) are terms that referred to a specific learning disability affecting the acquisition of arithmetic skills and numerical competences, despite normal intelligence and in the absence of neurological injuries (Temple, 1992). This disorder is presumed to be due to specific impairment in brain function (Shalev & Gross-Tsur, 2001).

Epidemiological studies indicate that MLD affects 5-7% of the school-age population (Shalev, 2007; von Aster & Shalev, 2007) with a variation from 1% to 11%, depending on the diagnostic criteria (see for a review, Butterworth, 2005). The cut-off criterion used to determine the population with MLD across studies ranged from the 5th percentile (e.g., Shalev et al., 2005) to the 15th (e.g., Rousselle & Noël, 2007), the 25th (e.g., Koontz & Berch, 1996), the 35th (e.g., Jordan, Hanich, & Kaplan, 2003b) and the 45th percentile (e.g., Geary, Bow-Thomas, & Yao, 1992). However, higher cutoffs (e.g., from 25th percentile) are inconsistent with the estimate prevalence of 5-7%. So it is

important to keep in mind that MLD or dyscalculia may be terms that refer to different deficits because of the authors' definition criteria⁸.

Furthermore, observed deficits often persist into MLD individuals' late adolescence (Shalev et al., 2005), affecting their career opportunities because of their limiting academic and professional capabilities (Rivera-Batiz, 1992). Despite the importance of adequate arithmetic skills in everyday life, little is known about the longevity of numerical impairments.

Familial-genetic predisposition seems to play a role in the apparition of mathematical disabilities. A study conducted by Shalev and collaborators (2001) assessed siblings and parents of children with MLD. It revealed that MLD has a familial predisposition: at least one other first-degree family member was identified with this disorder, i.e., similar percentages for parents (53%), brothers (53%), and sisters (52%). The familial prevalence of dyscalculia is almost tenfold greater than for the school-age population. That evidence for the role of genetics is congruent with the data provided by a study on twins (which confound monozygotic and same-sex dizygotic twin). They found that if a co-twin is diagnosed as dyscalculic, the other co-twin has 50% of probability to present developmental dyscalculia (Alarcon, DeFries, Light, & Pennington, 1997).

2. Neurobiological studies

Beside genetic and epidemiological data, neurobiological studies tend to indicate that MLD is a brain-based disorder (Kosc, 1974; Shalev & Gross-Tsur, 2001). Indeed, functional neuro-imaging data have suggested differences in cerebral activity (Kadosh et al., 2007; Kucian et al., 2006; Molko et al., 2003; Mussolin et al., 2010a; Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007; Soltesz, Szucs, Dekany, Markus, & Csepe, 2007) and abnormal anatomical development (Isaacs, Edmonds, Lucas, & Gadian, 2001; Molko et al., 2003; Rotzer et al., 2008) of the left and/or the right intraparietal sulci in

⁸ In this manuscript, we will either use "mathematical learning disabilities" (MLD) or "dyscalculia" to refer to the same specific deficit in numerical abilities.

children with MLD. Those regions, and more precisely the horizontal segment of the intraparietal sulcus (Dehaene, Molko, Cohen, & Wilson, 2004; Feigenson et al., 2004) is responsible for approximate representations of number magnitude (*or analog magnitude*): healthy participants show an activation of the horizontal intraparietal segment (HIPS) when comparing numbers or performing arithmetic operations on symbols (for a review, see Dehaene, 2009; Dehaene, Piazza, Pinel, & Cohen, 2003).

These numerical activities and even more basic processes are disrupted in individuals with MLD. Various hypotheses have been postulated to account for dyscalculia: some authors propound that MLD are secondary to deficits of general and non-numerical cognitive factors (as working memory for example, for review, see Geary, 2005) while others assume that it results from a specific deficit of numerical cognitive processes (for review, see Rubinsten & Henik, 2009). In the following sections, only numerical explanations which give a causal relationship to MLD occurrence will be described in details.

3. General cognitive functions

Different authors have been studying which general cognitive processes weaknesses concurrence with mathematical learning difficulties. For example, working memory or long term memory deficits (see Geary, 2005 for a review) could generate a fact retrieval deficit (e.g., Barrouillet, Fayol, & Lathuliere, 1997; Hanich, Jordan, Kaplan, & Dick, 2001; Hitch & McAuley, 1991; Jordan, Hanich, & Kaplan, 2003a; Jordan & Montani, 1997) or the use of immature procedures, such as fingers counting and counting-all algorithm, in order to reduce the load in working memory (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Noël, Seron, & Trovarelli, 2004). Also, fast information decays or procedural errors caused by a non efficient working memory may disrupt the association between a problem and its correct answer and then the storage of an arithmetical fact into the long term memory (Geary, 2005). However, even if adequate general cognitive processes are important to be able to

perform mathematical activities adequately, impairments at that level do not give a causal relationship to MLD occurrence.

4. MLD, which deficit?

In the preceding chapter, we have been through the different systems of magnitude representation and largely depicted the ANS. However, the nature of the numerical magnitude representation differs depending on the authors (for example, Dehaene & Changeux, 1993 versus Zorzi et al., 2005).

In the following paragraphs, we will see that it has been suggested that children with MLD could have very specific deficit to the magnitude representation or to access an intact representation. However, the magnitude representation, which would be deficient in MLD, differs depending on the authors, the expected deficits regarding MLD diverge as well. These different hypotheses for dyscalculia are presented hereafter.

4.1. The ANS deficit hypothesis

According to Dehaene and collaborator (Dehaene, 1997; Wilson & Dehaene, 2007), the core system of numerical representation, which is innate and relies on approximate representations of analog magnitude and which is located in the HIPS (Dehaene et al., 2004; Feigenson et al., 2004) is impaired in children who experience difficulties with the meaning of numbers, i.e., with symbolic and non-symbolic comparison, approximation and arithmetical tasks.

The idea that this HIPS region is a neural substrate of the ANS is underpinned by neural evidences. Concerning acalculia (i.e., an acquired deficit in calculation in adults), the role of the bilateral inferior parietal lobule has been identified as dedicated to quantitative numerical knowledge. Lesion evidence comes from several studies (e.g., Lemer, Dehaene, Spelke, & Cohen, 2003, Delazer, Karner, Zamarian, Donnemiller, & Benke, 2006). One case example is MAR, a left-

handed patient with a right inferior parietal lesion. He did not perform well at mental manipulation of numerical quantities, bisecting numbers lines and subtraction problems (Dehaene & Cohen, 1997). Imaging evidence comes from recent studies in normal adults performing tasks involving quantity representation such as comparing numbers, estimating, subtracting, and approximating. In those types of tasks related to the ANS, the HIPS appears activated (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Pesenti, Thioux, Seron, & De Volder, 2000; Piazza et al., 2004; Pinel, Piazza, Le Bihan, & Dehaene, 2004; Stanescu-Cosson et al., 2000; Venkatraman, Ansari, & Chee, 2005).

Neuropsychological and neuroimaging data suggest that the HIPS may play a role in developmental dyscalculia. According to Wilson and Dehaene (2007), the ANS, which becomes more precise over the development and is central in adult numerical cognition, is a good candidate for a core deficit in MLD. Related symptoms would correspond to difficulties in understanding the meaning of numbers (in tasks involving *non-symbolic materials* such as comparing sets of dots; in tasks involving *symbolic material* such as numerical comparison, addition and subtraction) and lower automatic activation of quantity from symbols (Arabic number and number words). This analog deficit would cause a developmental delay in the entire mathematical field (except the highly verbal processes of counting and fact retrieval).

4.2. The access deficit hypothesis

Although children are born with an innate approximate number representation, which is non-symbolic by nature and similar to that number system present in animals, later on they develop an exact symbolic representation, with the learning of number words and Arabic digits.

To date, even if the idea of the existence of a numerical magnitude system is widely accepted, debate about the nature of that system as well as how exact symbolic abilities develop is still on. In this context, two other theories on the causes of developmental dyscalculia have been proposed. One of them postulates an impaired access to the ANS from symbols (Rousselle & Noël, 2007).

Rousselle and Noël (2007) found that 7 to 8 year old children with MLD (associated or not with reading problems) performed worse than typically achieving children in comparing Arabic numbers but not in comparing sets of sticks. According to these authors, those results reveal an access deficit to numerosity concept from a symbolic notation: MLD children have an intact core representation of numbers as they are able to process non-symbolic numerosity but MLD children present an access deficit to number magnitude representation from symbols uniquely, as they are in trouble when comparing Arabic numbers.

4.3. The exact number representation deficit

The other theory postulates an impaired exact number system. This theory is derived from a “number module” (Butterworth, 1999, 2005) which uses a “numerosity code” (Zorzi & Butterworth, 1999; Zorzi et al., 2005).

Zorzi and Butterworth (1999; Zorzi et al., 2005) assume a number module which uses an internal “numerosity code” that codes number symbols linearly into number magnitudes (see Chapter 1 for more details on this model). This representation of the cardinal meaning is discrete and exact.

Concerning dyscalculia, Butterworth and his collaborators (Butterworth, 1999, 2005; Landerl et al., 2004) proposed that MLD children have a “defective number module” leading to difficulties in representing sets and their abstract properties, including their numerosity, and in processing exact symbolic numerosities (Iuculano, Tang, Hall, & Butterworth, 2008). One might therefore expect that MLD children would exhibit difficulties when processing number magnitude exactly but would show normal performance when approximating numerosity.

This hypothesis is supported by an anecdotic case study reported by Butterworth (1999); Charles, a 30 years old young man who presented basic deficiency of numerical representation, was of normal intelligence but complained about persisting and handicapping mathematical problems. His

important difficulties kept him using immature strategies as counting on his fingers to realise simple additions, which were not automated and he couldn't perform complex calculations. Basic quantity representation seems also inefficient in the case of Charles as his RTs in an Arabic numbers magnitude comparison task were four times longer than typically achieving individuals.

In accordance with this hypothesis, empirical studies on groups showed deficits in children with developmental dyscalculia in a variety of tasks assessing very basic numerical capacity, even as simple as number naming, reciting number sequences from different bounds or apprehending small numerosity (e.g., Hanich et al., 2001; Jordan et al., 2003a; Koontz & Berch, 1996; Landerl et al., 2004; van der Sluis, de Jong, & van der Leij, 2004). In addition, luculano and collaborators (2008) have shown that low-numeracy children performed at normal range in approximate tasks (comparison, addition and subtraction of moving dots) but exhibited poorer results than control children in the symbolic number comparison or symbolic arithmetic tasks.

Recent studies have focused on the hypothesis that MLD involves a basic deficit in the core capacity to represent and manipulate symbolic and non-symbolic number magnitudes.

Bearing in mind the three different hypotheses presented here above, we now examine to what extent data from the MLD literature supports them through tasks indexing the ANS and/or the access to that system.

5. Estimates

As reviewed in chapter 1, several studies of estimation processes have used number line placements to assess magnitude representations in typically developing children (Siegler & Booth, 2004; Siegler & Opfer, 2003). To our knowledge, only three studies analysed MLD children's estimation abilities (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Landerl, Fussenegger, Moll, & Willburger, 2009). Geary and collaborators (2007)

presented 6 year old children with a number line that included the endpoints 0 and 100 and had to place 30 Arabic numbers presented in a random order. The numbers below 30 were over-sampled to allow for fitting of a logarithmic model of the children's estimates. All trials were classified as linear or logarithmic based on whether the child's estimate was closer to the predicted value for the linear model (e.g., 23 for 23) or the logarithmic one (e.g., 45 for 23). Analyses of trial-by-trial variation indicated more frequent reliance on the formal linear representation by control children (only 30% of their number line trials relied in logarithmic representation) and heavy reliance on the logarithmic representation for low achieving (54%) and MLD children (63%). Moreover, children with MLD were less accurate than the low achieving children even when both groups used the logarithmic representation. Similar results were obtained one year later in a follow-up study (Geary et al., 2008).

It has been reported that reliance to linear representation in such a task correlates with mathematical achievement, at least from the 1st to the 4th grade, and should rely on a number-magnitude representation (Booth & Siegler, 2006). Testing 8 to 10 year olds, Landerl and collaborators (2009) showed that the effect of arithmetic ability was more pronounced in the 1-1000 condition than in the 1-100 condition. Dyscalculic (with and without associated reading problem) and typically achieving children's median estimates were better fit by a linear function than a logarithmic one for the 1-100 condition. For the 1-1000 condition, only the control group showed a better fit of the linear function. Finally, Geary and collaborators (2007) noted that central executive system (evaluated through the listening recall, counting recall, and backward digit span) was a partial mediator of the contrast between low achieving and MLD children. According to the authors, attentional control or other subcomponents of this system are involved in the construction of linear representations and contribute to the slow learning of children with MLD.

Moreover, physical number line task may not be a perfect operationalization of the core system and may constrain participants' performance in the estimation task in undesired ways. For example, the physical number line has attributes that may encourage participants to use specific strategies

relative to the endpoints. Numbers close to the right end of the physical number line do indeed tend to be better estimated than number in the middle of the line, that do not benefit from external marks (Siegler & Opfer, 2003). Due to the close intertwining of spatial and numerical representations in the number line task, it is possible that this type of task is sustained by cerebral network involved in visuo-spatial abilities and would be more likely to be defective in a type of dyscalculia linked to the visuo-spatial impairment. This idea is congruent with functional imaging studies. According to Pinel and colleagues (1999), the right fusiform gyrus is involved in the identification of Arabic numerals. The left fusiform gyrus was found to be involved in visual-spatial abilities (when dealing with shape configuration for example) and both fusiform gyri in spatial processes (Hahn, Ross, & Stein, 2006). Accordingly, data seem to show that visual-spatial processing deficits are associated with MLD which leads to vertical and horizontal spatial confusion when dealing with written addition or subtraction (Badian, 1983).

Anyway, that number line type of task is expected to be impaired in MLD children, regarding the exact number representation deficit, where number symbols are coded linearly into number magnitudes, as well as an access deficit to the core system or the core system itself.

6. Contrasting symbolic and non-symbolic comparison tasks

The few experiments that have investigated a possible deficit of the analog and approximate representation (i.e., the ANS) or the access to this system in MLD children used symbolic or non-symbolic comparison tasks. They focused on the processing speed or on the accuracy to compare numerosities. However, the observed results are currently contradictory (see Table 3 for an overview).

Rousselle and Noël (2007) found that 7 to 8 year old children with MLD (with or without associated reading problem) performed worse than typically achieving children in comparing Arabic numbers but not in comparing sets of sticks: MLD children were slower and less accurate compared

to their control peers while they were as fast and as accurate as controls in comparing sticks collections. Similar results were found in Landerl and collaborators' study testing the same age-range children in similar symbolic and non-symbolic tasks (Landerl et al., 2004; Landerl & Kölle, 2009).

Testing the same age-range children, Iuculano, Tang, Hall, and Butterworth (2008) contrasted in symbolic and non-symbolic comparison tasks 23 typically achieving children, 11 low-numeracy children (LN, showing poor performance in exact arithmetic tasks) and 2 MLD children. Those two children were identified via Butterworth's "Dyscalculia Screener" (2003) on the basis of poor performance on the symbolic number task (first case) and the dot enumeration task (the child is asked to count the dots, from 1 to 9, then to compare it to an AN; second case). LN children were as accurate as typically achieving children in the small and large sets comparison tasks using non-symbolic material, although they did less well than control children in the symbolic number comparisons task. Concerning the dyscalculic children, the first one presented the same pattern of result compared to LN children, the second one was also impaired in the large non-symbolic comparison task.

Findings from Rousselle and Noël study as well as the profile result exhibited by LN children and the first case of dyscalculic child from Iuculano and collaborators' study support an access deficit hypothesis. Indeed, the arithmetical impairment presented by these children appears to be underpinned by a disconnection between numerosity concepts and their symbolic notations.

Finally, according to the authors of the second study (Iuculano et al., 2008), LN children had no particular difficulty in representing exact numerosities while the second MLD child failed the dot enumeration task and an exact addition task from the "Dyscalculia Screener". They concluded that as this child showed impairment on the small sets comparison tasks using non-symbolic material, he suffered from a more deep-rooted deficit in the capacity to represent exact symbolic material.

However, the hypothesis of an ANS deficit cannot be ruled out as the child was also impaired in the large non-symbolic comparison task.

Other studies with older MLD children showed this impairment in non-symbolic numerical tasks tapping the analog and approximate number representation. MLD children of 11-12 years of age made more errors than control children in a non-symbolic numerical comparison of small sets (1-9 items) when the numerosities were close to one another (distance of 1-to-3; Price et al., 2007). Similarly, Mussolin, Mejias, and Noël (2010b) assessed the performance of 10 to 11 year old MLD and typically developing children on symbolic and non-symbolic numerical comparison tasks with small numerosities (1-to-9). The MLD children had longer latencies and higher error rates for numbers which were very close (distance of 1) than the control children, irrespective of the number format.

Finally, Piazza and collaborators (2010) showed that the ANS acuity of 8 to 12 year olds with MLD, as measured in the simultaneous comparison of two large numerosities (12 to 20 dots with a standard at 16 ; and 24 to 40 dots with a standard at 32) was only at the level of typically developing 4 to 5 year old kindergartners.

To sum up and as reported in Table 3, it is not clear whether or not children with MLD have a deficit in their approximate number representation. Some studies did not report any difference between MLD and control children on tasks tapping this representation, but only on tasks requiring exact number processing using symbolic numbers (Iuculano et al., 2008; Rousselle & Noël, 2007). Accordingly, it has been proposed that the deficit compromises the access to the analog magnitude from symbols (Rousselle & Noël, 2007) but not the ANS itself.

Table 3: RT, accuracy and performance in MLD compared to controls children in symbolic and non-symbolic comparison tasks.

Age group	Stimulus used	Reported deficits	References
7 to 8 years	1–9 digits	RT : MLD > Controls Accuracy : MLD < Controls	Rousselle & Noël, 2007
	6–28 sets of sticks	RT : MLD = Controls Accuracy : MLD = Controls	
	1–9 digits	Performance: LN < Controls MLD 1 < Controls MLD 2 ? Controls	Iuculano et al., 2008
	1–9 sets of squares	LN = Controls MLD 1 = Controls MLD 2 < Controls	
	10–58 sets of dots	LN = Controls MLD 1 = Controls MLD 2 < Controls	
8 to 9 years	1–9 digits	RT : MLD > Controls	Landerl et al., 2004
2 nd , 3 rd , 4 th grade	1–9 digits	RT : MLD > Controls	Landerl & Kölle, 2009
7 to 10 years (supposed)	21–98 digits	RT : MLD > Controls Accuracy : MLD < Controls	
	20–72 sets of squares	RT : MLD = Controls	
10 to 11 years	1–9 digits	RT : MLD > Controls Accuracy : MLD < Controls	Mussolin et al., 2010b
	1–9 sets of sticks	RT : MLD > Controls Accuracy : MLD < Controls	
11 to 12 years	1–9 sets of squares	Accuracy : MLD < Controls	Price et al., 2007
8 to 12 years	12–20 ; 24–40 sets of dots	Acuity (w): MLD > Controls	Piazza et al., 2010

However, other studies showed impairment in non-symbolic numerical comparison tasks in children with MLD (Mussolin et al., 2010b; Piazza et al., 2010; Price et al., 2007). At present, in each of these studies, an alternative account of this non-symbolic impairment cannot be completely ruled. Indeed, Mussolin and collaborators (2010b) only used small numbers (1–9) and observed significant difference between group of children only when the sets were close to each other (when sets of 4 or 6 items were compared to set of 5 items). A smaller subitizing range in MLD children could possibly explain this pattern (see Koontz & Berch, 1996). The comparison task used by Piazza and

collaborators (2010) and by luculano and collaborators (2008) is highly dependent on visuo-spatial ability. However, this ability was not measured in those studies and could possibly have affected the results as visuo-spatial ability might be impaired in MLD children (Rourke & Conway, 1997). Another important caveat is that when using non-symbolic material, especially in comparison task (i.e., when the two sets to be compared are presented simultaneously), it is always possible that the child's decision is influenced by, or even based on, perceptual variables that co-vary with numerosities. Indeed, it has been shown that it is much easier for children to compare two sets in which the surface area occupied by the items is confounded with the numerosity than to compare two sets in which this dimension is controlled (Rousselle & Noël, 2007; Rousselle et al., 2004). Piazza and collaborators (2010) tried to control for these variables by maintaining the dot size constant in half the trials, and the size of the occupied area in the other half. However, the results of these two conditions were analyzed together, so their influence on children's performance cannot be examined. This could, however be an important factor: it has been shown that density and surface area influence non-symbolic comparisons in MLD children but not in typically developing children (Mussolin et al., 2010b but not in Rousselle & Noël, 2007).

7. Arithmetical procedure

Converging lines of studies in primates, human infants, preschool children, healthy and brain-damaged adults discussed in Chapter 1 provide evidence for the ability to perform approximate calculation on symbolic and non-symbolic quantities, tapping the ANS. For many authors, this ontogenetically and phylogenetically primitive system might serve as a building block of children symbolic arithmetic learning (see for example, Barth et al., 2005).

However, compared to their typically achieving peers, children with MLD show a poor grasp of the meaning and of the properties of arithmetic operations: it could result from a foremost disability to deal with exact numbers and/or the ability to operate on them. Mainly, they show difficulties in

the execution of arithmetical procedures, often related to their misunderstanding of some counting principles (Geary et al., 1992), and which could lead to aberrant answers (e.g., $19 + 22 = 211$; Russell & Ginsburg, 1984). They also show a persistence of immature problem solving strategies even for the execution of simple calculation (e.g., $2 + 5 = 8$; e.g., Geary, Hamson, & Hoard, 2000).

Approximate calculation abilities in MLD children have been mainly investigated with symbolic stimuli. Several studies showed that MLD children were less precise in verification tasks when selecting the result closest to the correct answer from two incorrect ones (e.g., $4 + 9 = 12$ or 19), (Hanich et al., 2001; Jordan & Hanich, 2003; Jordan et al., 2003b). In those studies, even if an impairment of the approximate calculation approach is suggested, only a small number of items were presented (i.e., 10 addition and 10 subtraction problems), and reaction times were not recorded. Moreover, nothing prevents children from calculating the correct result and comparing it with the presented answers. As the use of an exact calculation strategy instead of an approximate one could not be ruled out, the approximate calculation abilities in MLD children cannot be evaluated. Recently, one study established that MLD children probably used approximate calculation, or at least computational estimation strategies, but not as frequently and efficiently as typically achieving children (Rousselle & Noël, 2008b). Finally and congruent with those behavioral data, a brain activation study showed that in 11 year old MLD and age-matched peers showed similar frontal-parietal network activation during approximate calculation ($3 + 8 = 10$ or 5), exact calculation ($2 + 5 = 9$ or 7), and non-symbolic magnitude comparison (sets of fruits, Kucian et al., 2006). However, during approximate calculation, their brain activation was weaker in the left and right intraparietal sulcus, as well as in the inferior and middle frontal gyri. This was interpreted by the authors as a disrupted ANS. However, this hypothesis did not account for the lack of group differences in intraparietal sulcus activation during non-symbolic comparison. Then, a deficit to access the ANS from numerical symbols rather than a deficit in the ANS itself seems more appropriate, even if a deficient ANS

cannot be totally excluded as Kucian and collaborators did not properly control perceptual variables that co-varied with the numerosity like density or total area.

Finally, non-symbolic approximate calculation abilities have been studied in two study, using Barth's paradigm (2005, 2006). Luculano and collaborators have showed that 7 to 8 year low-numeracy children or MLD children did not present any difficulties realizing approximate calculation (i.e., addition and subtraction tasks of dots moving in/from an occluder and then judging whether the result of this addition/subtraction was greater or less than a comparison set of dots). Using a similar task, De Smedt and Gilmore (2011) did not found differences in 6 to 7 year old children with severe and mild forms of mathematical difficulties accuracy compared to typically achieving children.

8. A larger magnitude system deficit?

Brain-imaging studies have reported similar involvement of the HIPS in both numbers and luminance processing (Kadosh et al., 2005; Pinel et al., 2004), suggesting that a particular cerebral network is involved in processing magnitudes (see Kadosh, Lammertyn, & Izard, 2008 for a discussion). This is congruent with Walsh's theory (2003). As he speculated that numbers are part of a larger magnitude system, it seems of interest to examine whether MLD children have difficulties in approximating the sum of continuous non-numerical quantities as well. Accordingly, Mussolin and collaborators (2010a), recently observed that, compared to controls, MLD children showed a weaker involvement of regions dedicated to number and color magnitude processing in comparison tasks: MLD children seem to show impairment treating color as a category (i.e., with a symbolic label) but not as continuum (non-symbolic).

9. Summary and introduction to the experimental section

This third chapter aimed at deepening our understanding of the basic numerical deficit that underlies MLD. Beside general factors deficits (e.g., Geary, 2005), children with dyscalculia present

specific difficulties in the acquisition of arithmetic skills and to deal with numerical representation. However, listed difficulties are varying in relation to participant's age, but probably according to the cut-off criteria as well (see Murphy, Mazzocco, Hanich, & Early, 2007 for a discussion). More important, the definition of what is behind the concept of "numerical representation" varies from one author to another and then, multiple hypotheses have been put forward to account for MLD. However, if the existence of multiple core systems for numerical representation seemed to be the predominant idea (e.g., Feigenson et al., 2004), a common system for the representation of time, space and other magnitudes is also proposed (Bueti & Walsh, 2009; Walsh, 2003).

Along the perspective of a specific deficit of numerical cognitive processes (see for review, Rubinsten & Henik, 2009), two hypotheses postulate that MLD children have a very specific deficit of the number magnitude representation but the nature of that representation differs depending the authors. Firstly, according to Butterworth and his collaborators (Butterworth, 1999, 2005; Landerl et al., 2004), humans are born with a "number module" that represents numerosities exactly (e.g., exact "sixness") and MLD would be due to a deficit at that level. Consequently, MLD children would be impaired when processing exact number magnitude but would be able to represent magnitudes approximately, that is to add or subtract two large symbolic or non-symbolic magnitudes and to compare the resulting sum or difference to a third set (Iuculano et al., 2008).

Conversely, many authors (e.g., Dehaene, 2003; Gallistel & Gelman, 1992) assume that number magnitude is not exact but approximate, obeying Weber's law. Nevertheless, exact arithmetic would develop on the basis of this innate ANS as the precision of this approximate representation is related to performance in exact calculation and number processing (Halberda et al., 2008, but see Holloway & Ansari, 2009) and a deficit at that level would lead to MLD (Berch, 2005; Spelke & Kinzler, 2007; Wilson & Dehaene, 2007). Hence, all numerical tasks, including those requiring children to approximate symbolic or non-symbolic magnitudes or to process exact calculations would lead to impaired performance in children with MLD.

In contrast with these two views assuming a deficit at the level of the number magnitude representation, Rousselle and Noël (2007) have proposed that the deficit of children with MLD lies in the access from symbols (such as Arabic digits) to an intact ANS. MLD children would thus have normal performance in numerical tasks using non-symbolic items such as dot collections but would be impaired relative to controls when the numerical task uses symbolic numbers as input or output.

Current findings are contradictory and it is still debated whether or not children with MLD have a deficit in representing and manipulating exact or approximate number magnitudes or disturbed connections between the ANS and the numerical symbols.

The access deficit hypothesis received support from studies showing normal performance of MLD children in numerical tasks using non-symbolic stimuli but impaired performance in tasks using symbolic stimuli. In particular, De Smedt and Gilmore (2011) found that 6 to 7 year olds with MLD performed worse than controls in comparing AN but not in comparing non-symbolic numerosities (sets of dots). Same results were found by Rousselle and Noël (2007) in 7 to 8 year old MLD children. Testing children in the same age-range, Luculano and colleagues (2008) also showed that low-numeracy children performed within the normal range in approximation tasks using non-symbolic material but exhibited poorer results than control children in tasks using AN. Those results support the access deficit hypothesis: the ANS would be intact but an impaired access to the ANS from symbols would lead to dyscalculia.

However, studies testing older MLD children showed impairment in non-symbolic numerical tasks associated with lower acuity of the ANS, supporting the ANS deficit hypothesis. MLD children of 11 to 12 years of age made more errors than control children in a non-symbolic numerical comparison of small sets (1-9 items) when the numerosities were close to one another (distance of 1 to 3; Price et al., 2007). Similarly, Mussolin and collaborators (2010b) assessed the performance of 10 to 11 year old MLD and typically developing children on symbolic and non-symbolic numerical

comparison tasks with small numerosities (1 to 9; stimuli presented simultaneously). MLD children had longer latencies and higher error rates than control children for pairs of very close magnitude (distance of 1), irrespective of the number format (i.e., number words, Arabic digits or collections). Finally, Piazza and collaborators (2010) showed that 8 to 12 year olds with MLD were less precise than control children when they were required to select the larger of two simultaneously presented dot sets (12 to 20 with a standard at 16; and 24 to 40 with a standard at 32) and actually only performed at the level of typically developing 5 year olds.

Yet, in these studies, an alternative account of this non-symbolic impairment can be proposed. As noted previously, Mussolin et al. (2010b) only used small numbers (1-9) and observed significant difference between MLD and typically developing children only when the sets were at a distance of one from one another, i.e., when sets of 4 or 6 items had to be compared to 5. A smaller subitizing range in MLD children could possibly explain this pattern (see Koontz & Berch, 1996) without resorting to the hypothesis of a defective number magnitude representation. The comparison task used by Piazza and collaborators (2010) is highly dependent on visuo-spatial abilities as the participants had to compare two sets presented simultaneously. As noted by the authors themselves, visuo-spatial abilities are known to be weak in MLD children (Rourke & Conway, 1997), but they were not measured in the study and could possibly explained the results. Furthermore, when using non-symbolic material, especially when the two sets of dots to be compared are spatially spread and presented simultaneously, it is always possible that the child's decision is influenced by, or even based on, perceptual variables that co-vary with numerosities, (e.g., cumulative area of the items, their contour length, or their density but see Dormal and Pesenti, 2009 for the use of linear array of dot arrangements which prevent from such a bias). Indeed, it has been shown that it is much easier for children to compare two sets in which the surface area occupied by the items is confounded with the numerosity than to compare two sets in which this dimension is controlled (Rousselle & Noël, 2007; Rousselle et al., 2004). Piazza and collaborators (2010) tried to control for

these variables by maintaining the dot size constant in half the trials, and the size of the occupied area in the other half. However, the results of these two conditions were analyzed together, so their influence on children's performance was not examined. This could be an important factor as density and surface area might influence non-symbolic comparisons more in MLD children than in typically developing children (see Mussolin et al., 2010b but Rousselle & Noël, 2007).

Finally, only a few studies have tested adults with MLD. Using a number Stroop task, these studies have shown that, contrarily to control adults, MLD participants do not seem to automatically activate the number magnitude of AN (Ashkenazi, Rubinsten, & Henik, 2009; Rubinsten & Henik, 2005). The authors concluded that adults with MLD have impaired access to magnitude representation from AN. However, as the processing of non-symbolic information has not been investigated, it leaves open the question of an ANS deficit in MLD adults rather than an access deficit to that representation from symbols.

Measuring the ANS by numerical estimation tasks

As mentioned through Chapters 1 and 2, the ANS obeys Weber's law. One of the most direct ways to tap this representation is to use numerical estimation tasks.

Several studies of estimation have used the placement of an AN on a number line to assess magnitude representations in typically developing (i.e., placing an Arabic number ranging from 1 to 100 or 1000 on a horizontal number line; Berteletti et al., 2010; Siegler & Booth, 2004; Siegler & Opfer, 2003) and MLD (Geary et al., 2007; Geary et al., 2008) children or the generation of 0 to 1000 dots in a box by a computer program (Booth & Siegler, 2006). However, these tasks may not be a perfect operationalization of the ANS and may constrain participants' performance in the estimation task in undesired ways. For example, the physical number line has attributes that may encourage participants to use specific strategies relative to the endpoints. Numbers close to the right end of the physical number line do indeed tend to be better estimated than number in the middle of the line,

that do not benefit from external marks (Siegler & Opfer, 2003). Such a bias is present in the box task as well (it is said and show to the child that the full box corresponds to “1000”) and should not be present in tasks designed to assess the ANS.

Given the limitations of comparison tasks and number line tasks, we opted for an estimation production task of large numerosities (i.e., beyond the subitizing range), as used in the literature on adults (Castronovo & Seron, 2007; Cordes et al., 2001; Izard & Dehaene, 2008; Whalen et al., 1999) and in some studies with typically developing children (e.g., Chillier, 2002). This type of task allows the ANS to be explored without applying external constraints to the participants’ estimates and with minimizing the influence of the perceptual variables on the estimates.

When participants have to estimate the number of dots in a set presented too fast to be counted exactly, estimates and response variability both increase with target magnitude, indicating that the underlying representation is less precise for larger numerosities (e.g., Huntley-Fenner, 2001; Platt & Johnson, 1971). More specifically, this representation is characterized by a scalar variability which gives rise to a constant coefficient of variation ($COV = \text{standard deviation of mean response} / \text{mean response}$) across target magnitudes (Cordes et al., 2001; Whalen et al., 1999). Therefore, the precision of the participant answer (measured as the difference between a given target and their answer to that specific target) should diminish for larger numerosities. This pattern appears whatever the input and output modality/format, that is, whether estimating the number of dots in a set by an Arabic number (AN) (Izard & Dehaene, 2008) or a verbal number word (Whalen et al., 1999), or producing key presses in response to written (Cordes et al., 2001; Whalen et al., 1999) or verbal numerical symbols (Castronovo & Seron, 2007).

In summary, current results have consistently shown deficits in MLD children in tasks using symbolic numbers. Yet, to disentangle the hypothesis of a defective access to the ANS from numerical symbols, from the hypothesis of a deficit at the level of the magnitude representation

itself, new data are needed especially regarding MLD children's (in)ability to handle the numerical magnitude when non-symbolic stimuli are used. Accordingly, the main purpose of Chapter 4 is to examine the integrity of the ANS in children with MLD.

The aim of Chapter 5 is to examine and characterize the longevity of numerical impairments throughout adulthood in MLD.

As MLD individuals show a poor grasp of the meaning and properties of arithmetic operations, it would be of interest for them to be able to get a feed-back on their aberrant answers. The last experimental study presented in chapter 6 investigates, among other, if MLD participants have difficulties in carrying out approximate calculation regarding different magnitude representations. Finally, this study examined if the approximate abilities are related to arithmetical skills through childhood and adulthood, as this would have important implications for the diagnosis and rehabilitation of people with MLD.

Chapter 4:

Numerical and non-numerical estimation in children with and without mathematical learning disabilities⁹

⁹ This section is a modified version of an article by Mejias, Mussolin, Rousselle, Grégoire and Noël with identical title which has been submitted to *Child Neuropsychology*. In order to avoid redundancy with the preceding sections, the Introduction and Discussion sections of this article have been truncated, but the Method and Results sections are fully reproduced.

Abstract

There are currently multiple explanations for mathematical learning disabilities (MLD).

The present study focused on those assuming that MLD are due to a basic numerical deficit affecting the ability to represent and to manipulate number magnitude (Butterworth, 1999, 2005; Wilson & Dehaene, 2007) and/or to access that number magnitude representation from numerical symbols (Rousselle & Noël, 2007). The present study provides an original contribution to this issue by testing MLD children (carefully selected on the basis of preserved abilities in other domains) on numerical estimation tasks with contrasting symbolic (Arabic numerals) and non-symbolic (collection of dots) numbers used as input or output. MLD children performed consistently less accurately than control children on all the estimation tasks. However, MLD children were even weaker when the task involved the mapping between symbolic and non-symbolic numbers than when the task required a mapping between two non-symbolic numerical formats. Moreover, in the estimation of non-symbolic numerosities, MLD children relied more than control children on perceptual cues such as the cumulative area of the dots. Finally, the task requiring a mapping from a non-symbolic format to a symbolic format was the best predictor of mathematical performance. In order to explain these present results, as well as those reported in the literature, we propose that the impoverished number magnitude representation of MLD children may arise from an initial mapping deficit between number symbols and that magnitude representation.

The major aim of this study is to decide between the different hypotheses: do MLD children have a deficit in the ANS (Wilson & Dehaene, 2007), a deficit in accessing an intact ANS from symbolic numbers (Rousselle & Noël, 2007) or do they present an exact magnitude representation deficit hypothesis (Butterworth, 1999, 2005; Luculano et al., 2008)?

Secondary, this study will also aim at determining whether MLD children are more influenced than control children by the perceptual variables that may co-vary with numerosities as we manipulated the level of perceptual control of the stimuli. Finally, we will also explore whether children are more accurate in mapping a non-symbolic representation to a symbolic representation than vice versa, as observed by Mundy and Gilmore (2009). To compare these hypotheses, we varied both the presentation format of the stimuli to be estimated and the production format of the responses, contrasting non-symbolic and symbolic input and output formats. Accordingly, four estimation tasks were proposed to MLD and control adult participants.

In a first task, collections of homogeneous-size dots were presented to the participants who had to estimate their number by producing an AN. The reverse task was also used as Mundy and Gilmore (2009) showed that for children, mapping from a symbol to a non-symbolic representation is harder than the reverse. Yet, since the dots were of equal size, the estimate could be influenced by the total surface area covered by the dots (Rousselle et al., 2004). Accordingly, a heterogeneous-size dots condition was also used in which collections were made of dots of different sizes that led to a constant cumulative surface area across numerosities. In order to evaluate the impact of the use of a symbolic output on the performance of the participants, they had to produce the AN corresponding to the cardinal of the collection. Finally, a completely non-symbolic task was used in which participants were presented with collections of heterogeneous-size dots and had to produce a collection of the same size but with homogeneous-size dots.

According to the ANS deficit hypothesis, MLD children should show lower performance in all these numerical estimation tasks since they all involve the ANS. Conversely, MLD children should perform as accurately as control according the exact magnitude representation deficit hypothesis. The hypothesis of an access deficit from symbol to a preserve ANS predicts a deficit in the AN to homogeneous-size dots task. Finally, this last proposition can be extended to a failure in MLD to link an intact ANS to an impaired symbolic representation; in that case, MLD children should show lower accuracy in the three tasks involving symbols as input or output.

According to a developmental delay, we can expect that MLD children would take advantage of the task where the cumulative area co-varied perfectly with numerosity (i.e., the homogeneous-size dots task compared to the heterogeneous-size dots task) as younger children do not process numerical information if perceptive variables are sufficient to perform the task (Feigenson et al., 2002; Mix et al., 2002; Rousselle et al., 2004).

Finally, according to Mundy & Gilmore (2009) recent results, children should present better mapping abilities from non-symbolic numerosities to AN than from AN to non-symbolic numerosities.

Study 1

1. Method

1.1. Participants

A total of 46 Caucasian children aged 9 to 10 years took part in the experiment: 23 of them were identified as having MLD (9 males and 14 females) and 23 were classified as controls (14 males and 9 females). The two groups did not differ in terms of gender ($\chi^2(1) = 2.17, p > .23$) or age (see Table 4).

Participants selection procedure and classification scheme

A pool of 390 fourth grade children attending general education classes at nine different public middle schools in Belgium was assessed. Parental consent was obtained for each of the children. This pool of middle class Caucasian children constituted the normative sample for calculation fluency. They were collectively given a sheet of 81 simple single-digit arithmetical problems and were asked to solve as many as possible in 90 seconds. This fluency task was carried out three times, once with additions, once with subtractions and once with multiplications. The children were also administered an untimed mathematical battery that assessed school performance in mathematics (Simonart, 1998). The testing lasted approximately 30 minutes. As the three scores of the calculation fluency tasks and the score of the mathematical battery were highly correlated ($r^2 > .64; p \leq .001$, for the four correlations), the sum of the four Z-scores was computed to provide a “mathematical performance Z-score”. Finally, teachers completed a three-point scale to indicate whether individual students showed severe, some, or no math difficulty.

To be classified as MLD, children had to be judged by their teacher as having some or severe difficulties and to score below percentile 15 (based on the distribution over the 390 children) on both the calculation fluency test and the mathematical battery. This 15th percentile cut-off is more conservative than the one used in most of the studies of mathematical disabilities (Geary et al., 2004;

Geary, Hoard, & Hamson, 1999; Hanich et al., 2001; Jordan et al., 2003b). Thirty-four children met this criterion. For each of these MLD children, a control child was selected from the same class. He or she had to be assessed by the teacher as having no learning difficulties in mathematics, and had to score in the normal range (between the 25th and the 85th percentiles) on the calculation fluency test. Finally, as the factors underlying MLD may differ according to the presence or absence of an associated reading disability (i.e., weak phonological processing rather than weak number sense, see Robinson, Menchetti, & Torgesen, 2002), we excluded children with poor reading skills. All the participants were administered the LUM reading subtest of the LMC-R battery (Khomsy, 1998). This subtest is a one-minute word-reading task involving words of increasing complexity. Seven children who scored less than one standard deviation below the mean (-1σ) on this reading test were removed from the pool. This method of selection gave rise to the same dispersion in the MLD and control groups of children [-1σ ; $+2\sigma$].

The Similarities and Picture Concepts subtests of the Wechsler Intelligence Scale for Children IV (Wechsler, 2005) were also administered to the children, and provided an estimate of their IQ score (Sattler, 1982). Ten participants who scored below 80, and five participants who scored over 145 were removed from the pool. The final group comprised 46 children who took part in our experiments, 23 children identified as having MLD (5.9% of the original pool of children), and 23 control children.

In addition to these selection tests, as our experiment supposed good visuo-spatial abilities, children went through a memory span for pattern assessment. This task, a paper and pencil version of that developed by Wilson, Scott and Power (1987), involved the presentation of matrices in which some cells were randomly completed; the participants had to recall which cells had been filled in. The complexity increased every time the participant was successful in two out of three attempts. Participants made their responses in a booklet of blank matrices, corresponding in size to the target patterns to be recalled. The initial level of complexity involved filling in two cells. As we did not have

normative data for this test and because our tasks were particularly demanding on visuo-spatial memory, the MLD and control groups were compared on this ability (Table 4).

Table 4: Descriptive information and mean scores for the MLD and control groups on the selection tasks for study 1

	Control group (N=23)		MLD group (N=23)		Statistical analyses	
	<i>Mean (SD)</i>	<i>Range</i>	<i>Mean (SD)</i>	<i>Range</i>	<i>t-tests: $t_{(44)}$</i>	<i>Mean difference (95% confidence interval of the difference)</i>
<i>Descriptive information</i>						
Age (in months)	118.13 (4.55)	112–131	118.43 (6.62)	111–143	-.182	1.68 (-3.68–3.07)
Memory span	6.30 (1.06)	4–8	5.73 (.92)	4–7	1.93	.57 (-.02–1.15)
<i>IQ</i>						
Similarities	11.26 (2.32)	7–16	10.36 (2.19)	6–15	1.33	.91 (-.41–2.24)
Concept identifications	10.21 (2.43)	7–15	9.86 (2.90)	5–14	.44	.35 (-1.22–1.92)
Subtest mean scores	10.73 (1.62)	8.5–14	10.11 (1.48)	8–13	1.38	.63 (-.29–1.55)
<i>Mathematics</i>						
Fluency addition	33.91 (6.01)	26–50	16.00 (5.33)	5–29	10.69**	17.91 (14.53–21.29)
Fluency subtraction	31.13 (5.39)	19–41	10.74 (3.25)	5–18	15.54**	20.39 (17.75–23.03)
Fluency multiplication	21.17 (7.37)	10–34	9.61 (3.61)	4–17	6.76**	11.56 (8.12–15.02)
Mathematical scale	14.08 (2.75)	11–20	4.09 (1.53)	1–6	15.25**	10.00 (8.68–11.32)
Math performance Z-score	3.36 (1.43)	1.15–6.05	-3.36 (.72)	-4.82–-2.46	10.17**	6.72 (6.05–7.40)

Note. (*) = MLD significantly differs from healthy participants' at $p < .05$; (**) at $p < .01$.

As shown in Table 4, the two groups did not differ significantly in terms of IQ or memory span for pattern. As expected, the differences between the groups were significant for all the mathematical subtests.

1.2. Estimation tasks

Four estimation tasks were developed. In the “homogeneous-sized dots to AN” (HM to AN) task (Figure 12 A), participants were presented with a set of dots of the same size and were instructed to approximate (without counting) the number of dots in the sets by producing the corresponding AN. Those homogenous-sized dots (visual angle: 0.88°) appeared within a rectangle which was always the same size (visual angle: $27.5^\circ \times 23.1^\circ$). Six different configurations were presented for each numerosity. Input dots covered the entire rectangle and were at least one radius away from each other. The AN stimuli use Arial font (visual angle: 2.2°).

In the “heterogeneous-sized dots to AN” (HT to AN) task (Figure 12 B), participants performed the same task, but on sets of different-sized dots. In each set, the size of the dots was manipulated so that the total blackened area was the same for each numerosity. The heterogeneous-sized dots appeared inside a square (square’s visual angle: 18.48° ; smallest dot’s visual angle: 0.44° and biggest dot’s visual angle: 4.62° ; dot’s visual angle increment: 0.11°). Six different configurations were presented for each numerosity. Input dots covered the entire rectangle and were at least one radius away from each other. To avoid larger collections also being those with smaller dots, each set contained a mixture of the different sizes of dots with smallest and largest dots presented in all sets.

In the “heterogeneous to homogeneous-sized dots” (HT to HM) task (Figure 12 C), participants were presented with sets of heterogeneous-sized dots and asked to produce a collection of homogeneous-sized dots containing approximately the same number of dots. Output dots covered the entire rectangle and were at least one radius away from each other.

Finally, in the “AN to homogeneous-sized dots” (AN to HM) task (Figure 12 D), participants were presented with an AN and asked to produce the collection of homogeneous-sized dots that best corresponded to that AN.

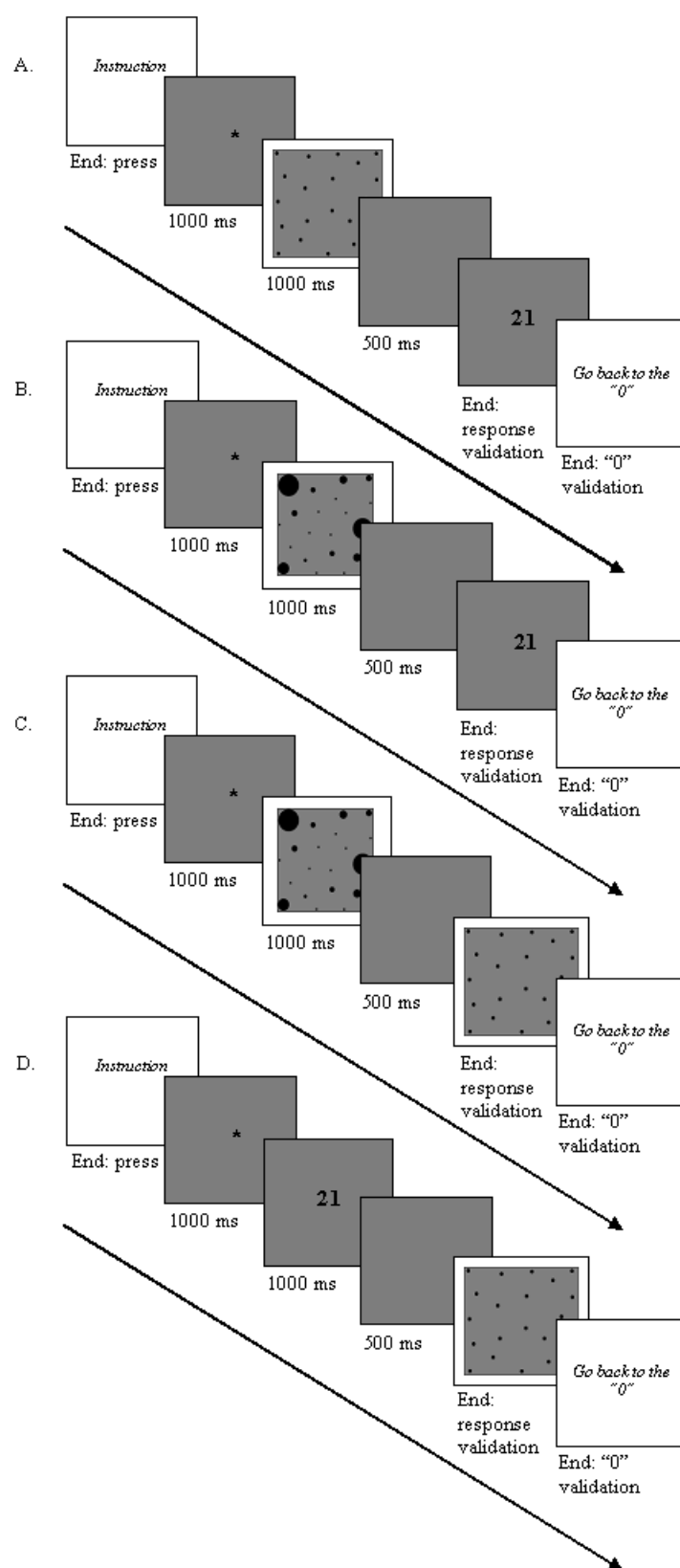


Figure 12: Time course of the experiment for each task: (A) HM to AN; (B) HT to AN; (C) HT to HM and (D) AN to HM.

1.3. Stimuli

Seven numerical quantities (8, 12, 16, 21, 26, 34, and 64) were presented six times to participants, providing a total of 42 stimuli in each of the 4 tasks. The stimuli were displayed in black on a grey screen.

1.4. Experimental procedure

The participants were asked to estimate, as fast as they could and without counting, the numbers/numerocities presented on a computer screen in each of the four tasks and produce the corresponding estimates. Presented following a Latin-square order between participants, four blocks (one for each task) of 42 randomly presented trials were administered in a single session. For each trial, the procedure was the same: a fixation cross appeared for 1000 ms, which was then replaced by the number/numerosity to be estimated for 1000 ms; after that, an empty grey screen was displayed for 500 ms, followed by a “0”/empty rectangle from which the participant had to produce a symbolic/non-symbolic response using a potentiometer. The potentiometer was a 10 x 5 cm² joy pad with a track wheel and a button (see Figure 13). By turning the track wheel clockwise, participants flashed images on the screen (AN or homogeneous-sized black dots, depending on the task) ranging from 0 to 254, with an increment of 1. Participants validated their answer by pressing the button of the potentiometer. Then, the instruction to go back to the “0” position was given. A microcontroller is responsible for measuring the position of the potentiometer, to save the state of the pushbutton and interact with the PC application. Response times were recorded from the presentation of the “0”/empty rectangle to the moment participants validated their answers (see <http://www.ipsp.ud.ac.be/recherche/projets/Potentiometre/> for a complete description of the potentiometer). All trials took place on a PC-compatible portable (screen size: 30.5 × 23 cm) running E-Prime software (Schneider, Eschmann, & Zuccolotto, 2002).

Before beginning the experiment, a training session of 16 trials was presented, once for each task. The training session used different numerosities (15, 25, 50, 75) from the experiment. Immediately after each trial in the training session, the expected response was provided to the participant on the PC screen in order to calibrate their estimation (Izard & Dehaene, 2008). Data from these trials are not reported here.

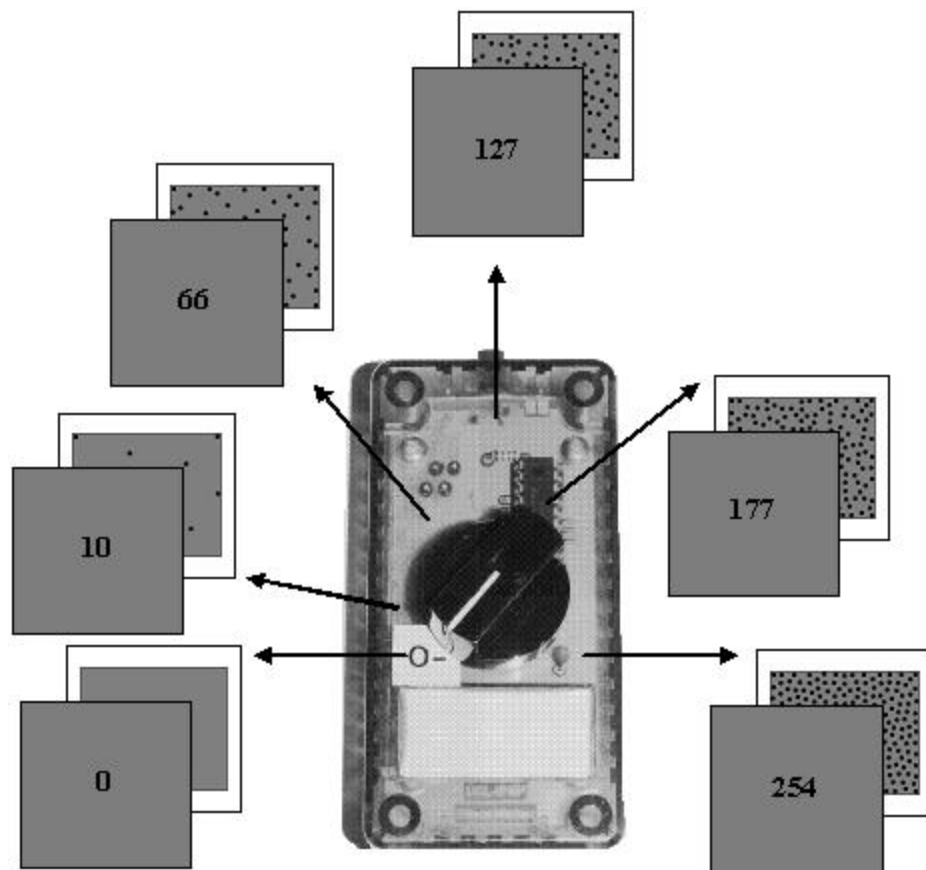


Figure 13: The potentiometer used as the response interface, surrounded by examples of the Arabic numbers and homogeneous-sized black dots. By turning the track wheel clockwise, participants flashed images on the screen ranging from 0 to 254, with an increment of one.

2. Results

2.1. The variability of number estimation

For each group and each task, regression analyses were performed on the children's means and standard deviations. Both measures increased in direct proportion to the target magnitude with slopes close to 1 (see Table 5 panel a and b) and that COV (i.e., the ratio of the standard deviation to the mean) was relatively constant across target size (as the slopes from the regression analysis did not significantly differ from 0 as showed by *t*-test analyses). However, MLD children had larger COVs ($M = .38$, $SD = .11$) compared to control children ($M = .29$, $SD = .08$; $t(44) = -3.29$, $p = .002$). This indicates a greater variability of the ANS in MLD compared to control children (see Appendix A for detailed analysis).

Table 5: Results of the linear regression between the predictor variable (target results) and (a) the mean of the estimates, (b) the standard deviations of the estimates, and (c) the coefficients of variation (COV) of the estimates in the four different tasks for MLD and control groups. The mean COV scores are also shown (d).

HM to AN		HT to AN		HT to HM		AN to HM	
Control group	MLD group	Control group	MLD group	Control group	MLD group	Control group	MLD group
<i>(a) Mean of estimates</i>							
$r^2 = .990$	$r^2 = .990$	$r^2 = .981$	$r^2 = .973$	$r^2 = .990$	$r^2 = .975$	$r^2 = .967$	$r^2 = .943$
$\theta_0 = .999$	$\theta_0 = .995$	$\theta_0 = .990$	$\theta_0 = .986$	$\theta_0 = .995$	$\theta_0 = .987$	$\theta_0 = .983$	$\theta_0 = .971$
$t(5) = 50.57^{**}$	$t(5) = 22.55^{**}$	$t(5) = 15.94^{**}$	$t(5) = 13.31^{**}$	$t(5) = 22.72^{**}$	$t(5) = 13.87^{**}$	$t(5) = 12.15^{**}$	$t(5) = 9.11^{**}$
<i>(b) Standard deviation</i>							
$r^2 = .991$	$r^2 = .945$	$r^2 = .873$	$r^2 = .941$	$r^2 = .988$	$r^2 = .911$	$r^2 = .974$	$r^2 = .932$
$\theta_0 = .995$	$\theta_0 = .972$	$\theta_0 = .934$	$\theta_0 = .970$	$\theta_0 = .994$	$\theta_0 = .955$	$\theta_0 = .987$	$\theta_0 = .965$
$t(5) = 23.30^{**}$	$t(5) = 9.26^{**}$	$t(5) = 5.85^{**}$	$t(5) = 8.95^{**}$	$t(5) = 20.09^{**}$	$t(5) = 7.16^{**}$	$t(5) = 13.58^{**}$	$t(5) = 8.27^{**}$
<i>(c) COV</i>							
$r^2 = .024$	$r^2 = .079$	$r^2 = .048$	$r^2 = .130$	$r^2 = .418$	$r^2 = .372$	$r^2 = .008$	$r^2 = .020$
$\theta_0 = -.156$	$\theta_0 = -.282$	$\theta_0 = -.218$	$\theta_0 = -.361$	$\theta_0 = -.646$	$\theta_0 = -.610$	$\theta_0 = .092$	$\theta_0 = -.143$
$t(5) = -.35$	$t(5) = -.66$	$t(5) = -.50$	$t(5) = -.87$	$t(5) = -1.66$	$t(5) = -1.72$	$t(5) = .21$	$t(5) = -.32$
<i>(d) COV score means (SD)</i>							
0.291 (.08)	0.389 (.12)	0.317 (.10)	0.422 (.15)	0.299 (.07)	0.397 (.13)	0.251 (.11)	0.328 (.12)

Note. (*) = significant difference between MLD and control group at $p < .05$; (**) at $p < .01$.

2.2. The precision of number estimation

The precision of participants' numerical estimation was calculated as an absolute error score (AES) computed as follows:

$$\text{AES} = |x_i - T|$$

where x_i corresponds to the participant's estimate on trial i , T to the target value. The absolute value of the sum provides a measure of overall accuracy in performance, irrespective of the direction of the differences between the participant's answers and the target, it is thus sensitive to the extent to which the subject was "off target" (Schmidt & Lee, 2005). Mean AES per group, target and task are depicted in Figure 14.

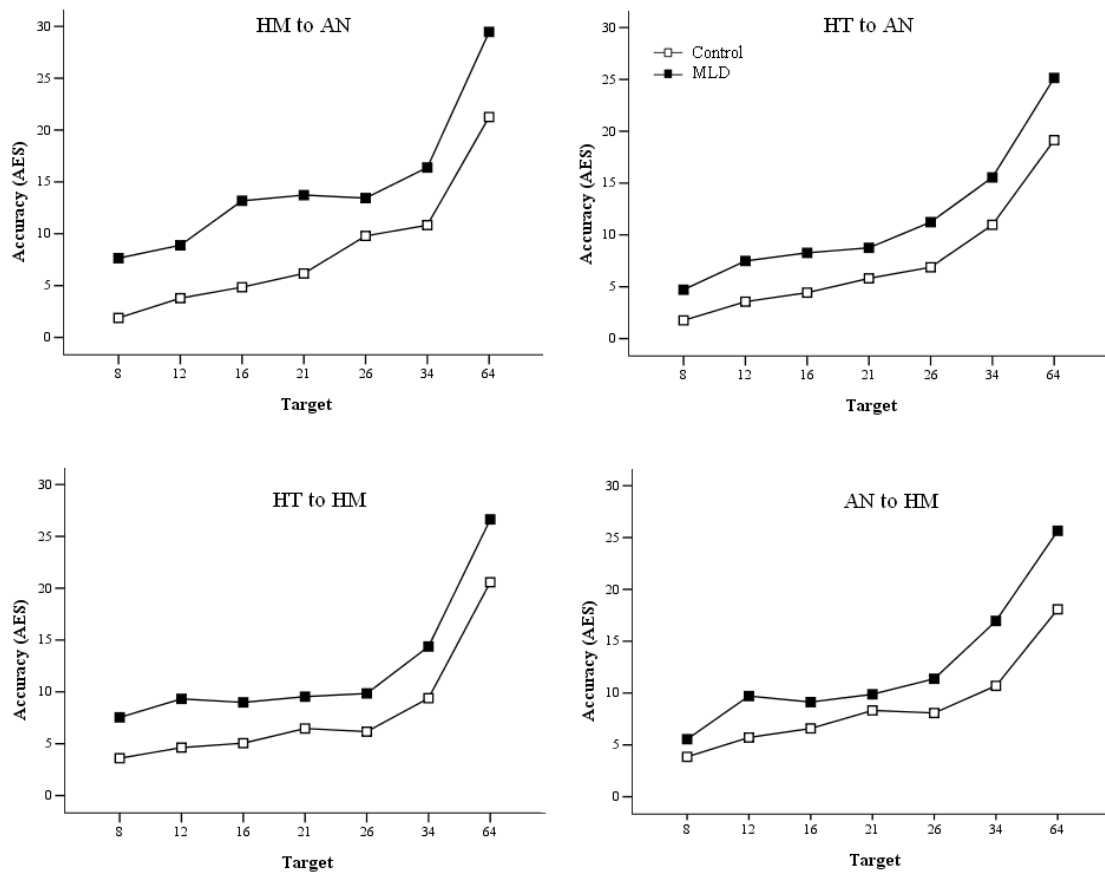


Figure 14: The mean accuracy (AES) for MLD and control children for each target magnitude on the four different tasks.

To address the question regarding the access from symbols to the ANS as well as the influence of the mapping direction, and see whether we could replicate Mundy and Gilmore's (2009) finding that children are more accurate in mapping from a non-symbolic representation to a symbolic representation, we contrasted the performance in the HM to AN and the AN to HM tasks. An ANOVA was run with these two tasks and the seven targets as within-subject factors and the group as the between-subjects factor. This analysis displayed the expected target effect ($F(6, 264) = 87.60, \eta^2 = .67, p < .001$) and a group effect ($F(1, 44) = 16.80, \eta^2 = .28, p < .001$), indicating that precision decreased with the target magnitude, and that MLD children were less accurate than control children (Figure 15). They were no task effect ($F(1, 44) = 1.86, \eta^2 = .04, p = .179$) nor interaction between the task and the group ($F(1, 44) = .02, \eta^2 < .01, p = .888$) suggesting that both group exhibited the same accuracy profile regarding the task.

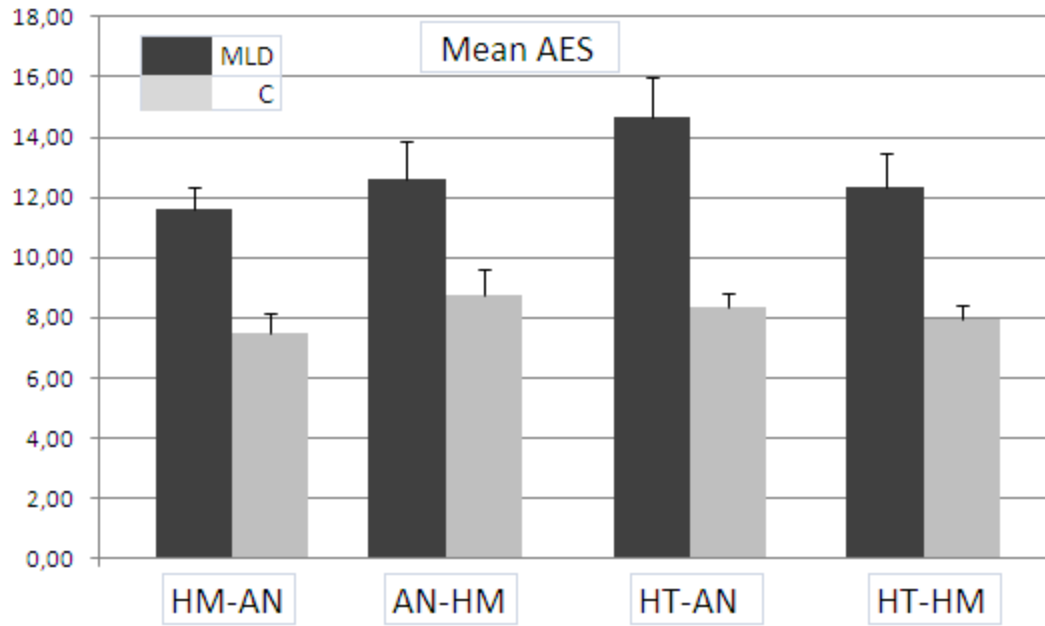


Figure 15: The accuracy of MLD and control children on the four estimation tasks (Error bars indicate the standard errors of the mean).

However, although the interaction between the target and the task was not significant ($F(6, 264) = .63, \eta^2 = .14, p = .705$), Figure 16 suggests that children were more precise in the non-symbolic to symbolic mapping task, at least on the small numerosities (which does make sense over the age of the subject). The same ANOVA was therefore run again, on the four smaller targets (8, 12, 16, 21) only. This analysis revealed, together with the target effect ($F(3, 132) = 21.83, \eta^2 = .28, p < .332$) and the group effect ($F(1, 44) = 10.24, \eta^2 = .19, p < .001$), a task effect ($F(1, 44) = 5.05, \eta^2 = .08, p = .050$) indicating that, for this number range, children were more accurate in mapping a non-symbolic stimulus to a symbolic representation than in the reverse task (no significant effect was revealed on larger targets).

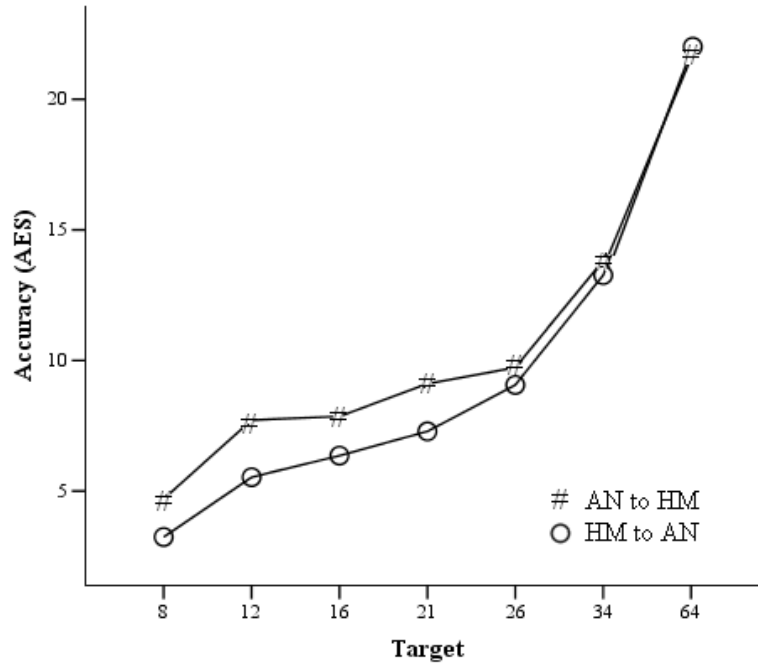


Figure 16: Children's accuracy (AES) on the estimation task involving symbolic to non-symbolic mapping and vice versa.

To address our second question and measure the impact of perceptual cues on estimation, we contrasted performance on the HT to AN and the HM to AN tasks. An ANOVA was run with these two tasks and the seven targets as within-subject factors, and the group (control and MLD) as the between-subjects factor. This analysis showed a target effect ($F(6, 264) = 85.76$, $\eta^2 = .66$, $p < .001$), indicating that the precision decreased with the magnitude of the targets. There was also a task effect ($F(1, 44) = 9.98$, $\eta^2 = .19$, $p = .003$), indicating that both groups were more accurate when moving from an homogeneous dot pattern to an Arabic number than from a heterogeneous pattern (Figure 15). In addition there was a significant group effect ($F(1, 44) = 26.67$, $\eta^2 = .38$, $p < .001$), indicating that MLD children ($M = 13.14$, $SD = 4.45$) were less accurate than control children ($M = 7.94$, $SD = 1.87$). The task by group interaction was marginally significant ($F(1, 44) = 3.19$, $\eta^2 = .07$, $p = .087$). Separate analyses by group revealed only a task effect in the MLD group of children ($F(1, 22) = 9.13$, $\eta^2 = .29$, $p = .006$). This suggests that only MLD children were making significant use of the perceptive cues which correlated with numerosity.

Finally, to address the question of the impact of a symbolic output on the performance of MLD children, we compared the HT to AN and the HT to HM tasks. The effect of target was significant ($F(6, 264) = 85.88, \eta^2 = .66, p < .001$), as was the group effect, $F(1, 44) = 21.06, \eta^2 = .32, p < .001$, confirming that MLD children ($M = 13.50, SD = 5.28$) were less accurate than control children ($M = 8.18, SD = 1.73$). There was a significant task effect ($F(1, 44) = 6.61, \eta^2 = .13, p = .014$), revealing that both groups were more precise when a non-symbolic output was required (Figure 15). Finally, the task by group interaction was marginally significant ($F(1, 44) = 3.42, \eta^2 = .07, p = .071$). Separate analyses by group were performed. Thus only the MLD children showed significant task effect ($F(1, 22) = 5.78, \eta^2 = .21, p = .025$): they were significantly more accurate on the completely non-symbolic HT to HM task ($M = 12.32, SD = 5.34$) than on the task including an Arabic number ($M = 14.68, SD = 6.19$).

2.3. Number estimation as a predictor of mathematical achievement

Pearson correlation analyses between COV, AES and mathematical performance Z-score showed that individual COV scores and AES were all significantly related to participants' mathematical performance (see Table 6 panels a and b). A multiple regression analysis was then performed to select the best predictor of mathematical scores. Eight predictors (one COV score and one AES score per task) were considered as independent variables. The AES in the HM to AN task was selected as the best predictor of mathematical performance ($F(8, 45) = 4.73, p < .001$) and accounted for 33% of the variance. No other predictor made an additional, statistically significant, contribution (all $ps > .11$).

Table 6: The correlations between mathematical performance and children's COV scores (panel a) and AES (panel b) on the four estimation tasks

	HM to AN	HT to HM	HT to AN	AN to HM
(a) COV				
Mathematical performance	-.394**	-.414**	-.350*	-.296*
(b) AES				
Mathematical performance	-.589**	-.466**	-.576**	-.353*

Note. (**) Pearson correlations coefficient significant at the 0.01 or (*) at 0.05 level (2-tailed).

3. Discussion

This study aimed at testing whether MLD children present or not a deficit in their approximate number magnitude representation, whether their difficulties lie in accessing that preserve ANS from symbolic codes or more generally correspond to a failure to link an intact ANS to an impaired symbolic representation.

We also wanted to test whether MLD children are more influenced than control children by the perceptual variables that may co-vary with numerosities. Finally, we wanted to examine whether children are more accurate in mapping a non-symbolic representation to a symbolic representation than the reverse, as observed by Mundy and Gilmore (2009).

The estimate production tasks developed and used here allowed us to measure the precision and the variability of the estimates produced in both symbolic and non-symbolic formats, while also varying the input format. The results indicated that both MLD and control children were able to estimate the size of a collection of homogeneous or heterogeneous-sized dots by producing the corresponding AN or a comparable collection, and to produce a collection that corresponded to a given AN. In each of these tasks, their estimates increased linearly with the target number. The variability of their estimates also increased with the target magnitude, which led to stable COVs as reported in the literature (for example, Whalen et al., 1999).

Our results indicated that on all four tasks, the estimates produced by MLD children were less accurate and more variable than those produced by control children. More detailed analyses showed that MLD but not control children were sensitive to the presence or absence of symbolic code in the task. MLD children were more precise in a completely non-symbolic task (HT to HM) than in a task involving symbols (HT to AN). They were also more sensitive to the perceptual variables that co-vary with numerosities: their estimates were less accurate in the task using heterogeneous-size dots (controlling for surface area occupied by the stimuli) than in the one using homogeneous-size dots (where cumulative dots area is directly proportional to numerosity).

The fact that MLD children had generally poorer results than control children is in agreement with the hypothesis of an impairment in the approximate representation of numbers magnitude in MLD children (Wilson & Dehaene, 2007). However, their even weaker performance when the task involved symbolic numbers than when it was completely non-symbolic might indicate the presence of a supplementary deficit in the symbolic representation. It is also interesting to note that, although performance in all the estimation tasks was significantly correlated with the participants' mathematical performance, the best predictor was the measure of accuracy on the Homogeneous-size dots to Arabic numerals task—a task that involves the link between the approximate magnitude representation and the symbolic one. Finally, the greater sensitivity of MLD children to the interference of perceptive cues in the numerical estimation suggests a developmental immaturity of the number processing as such an influence has been widely observed in younger children (Feigenson et al., 2002; Mix et al., 2002; Rousselle & Noël, 2008b; Rousselle et al., 2004).

This study also gave us the opportunity to replicate Mundy and Gilmore's (2009) finding that the mapping from a non-symbolic representation to a symbolic one was more accurate than the reverse mapping. Our data showed that the children's performance were sensitive to the mapping direction only when small targets were used. For numerosities below the mid-20s, we replicated the findings of Mundy and Gilmore (2009): children in both groups were more accurate in the estimation task

that involved mapping from a non-symbolic stimulus (HM) to a symbolic (AN) representation, than the reverse. For larger numerosities, no difference was observed, possibly because of the very large variance of the estimations.

In summary, our main results support the hypothesis that the approximate number magnitude representation is impaired in 10-year-old MLD children (Wilson & Dehaene, 2007). There also appears to be a supplementary deficit in the link between that representation and symbolic numbers (Rousselle & Noël, 2007) and a greater reliance on perceptual cues when processing numerosities. Yet, at this point, an alternative explanation should be considered. As MLD children performed worse than control children on all the estimation tasks, it is possible that they present a general problem with the estimation process itself. A second experiment was thus conducted to examine this possibility by looking for group differences in a non-numerical estimation task. More specifically, we decided to compare the groups on a color estimation task.

Study 2

1. Method

1.1. Participants

A subgroup of children who participated in the first experiment also took part, 11 months later, in this second experiment. Twelve MLD children and 12 control children went through the mathematical tasks again to make sure they still had the same profile. The differences between the groups (see Table 7) were significant on all the mathematical subtests, showing that our selection criteria were still valid at the time of the second experiment. The other pre-test scores (i.e., the IQ and memory scores) were taken from the first session. The two groups differed significantly in their IQ scores (see Table 7), and IQ was therefore included as a covariate in subsequent analyses. The two groups were similar in terms of age in months, but differed slightly in terms of gender (9 males and 3 females in the control group, compared to 4 males and 8 females in the MLD group, $\chi^2(1) = 4.19, p > .04$).

Table 7: Descriptive information and mean scores for the MLD and control groups on the color tasks (study 2).

	Control group (N=12)		MLD group (N = 12)		Statistical analyses		
	Mean (SD)	Range	Mean (SD)	Range	t-tests: $t_{(22)}$	Mean difference	95% confidence interval of the difference
Descriptive information							
Age (in months)	129.25 (4.83)	123–141	129.33 (2.54)	125–134	-.053	-1.28	-4.23–1.68
Memory span ^{^^}	6.50 (1.17)	4–8	5.92 (0.99)	4–7	1.316	.42	-.64–1.49
IQ							
Similarities	11.67 (2.19)	8–16	10.25 (2.05)	7–13	1.637	1.53	-.54–3.60
Concept identifications	11.17 (2.72)	8–15	9.50 (2.11)	6–13	1.675	1.83	-.46–4.12
Subtest mean scores	11.42 (1.62)	8.5–14	9.88 (0.98)	8.5–12	2.819*	1.68	.48–2.88
Mathematics							
Fluency addition	51.58 (13.32)	32–77	23.25 (6.54)	13–33	6.66**	16.08	10.83–21.33
Fluency subtraction	39.50 (7.59)	27–55	20.17 (5.57)	13–27	7.12**	21.03	17.21–24.85
Fluency multiplication	32.83 (7.32)	21–43	12.92 (6.39)	4–26	7.10**	10.11	5.25–14.97
Mathematical scale	18.42 (3.68)	13–23	8.42 (3.66)	3–16	6.68**	9.94	7.75–12.14
Math performance Z-score	2.97 (1.29)	1.15–5.80	-3.39 (.77)	-4.48–2.47	14.46**	6.36	5.44–7.27

Note. (*) = MLD significantly differs from control participants' at $p < .05$; (**) at $p < .01$; (^) tests taken at the time of this experimental session; (^^) tests taken one year before this experimental session

1.2. Experimental color task

The children had to estimate the hues of colored patterns on the computer screen.

1.3. Stimuli

Input and output colors were presented to the participants within a rectangle of a constant size, with a visual angle of $17.6^\circ \times 14^\circ$ on a grey PC screen. The colors were created using the "Munsell colour model," a perceptual color system, which specifies colors based on three variables: hue, value and chroma (HVC) (Figure 17). The model was created so that colors could be arranged in equal intervals of visual perception. This aids interpretation of the results.

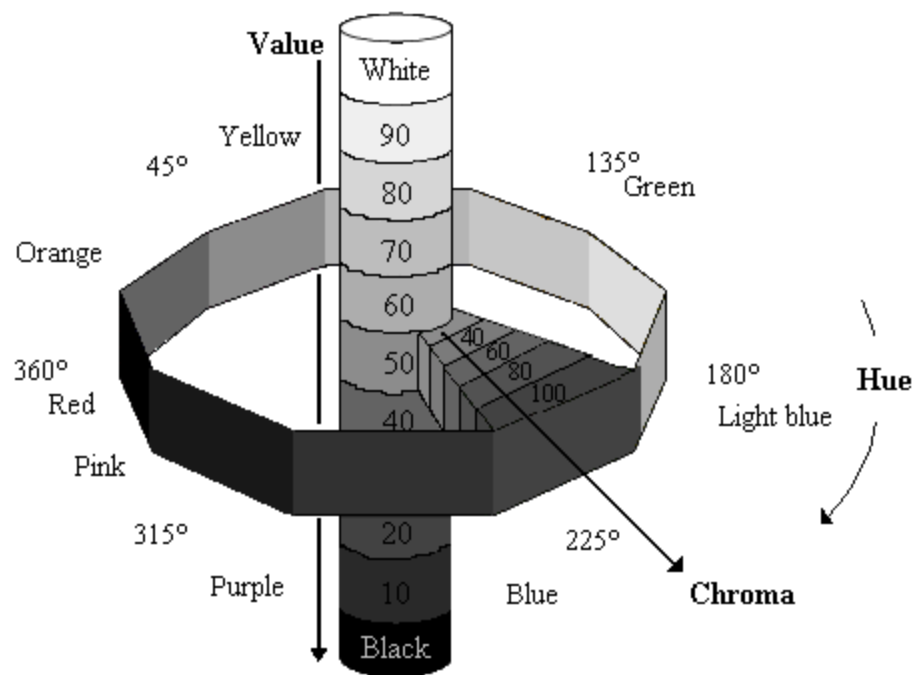


Figure 17: The Munsell color model, represented by a cylindrical coordinate system (adapted from <http://www.britannica.com>).

The hue is the quality by which one color is distinguished from another (i.e., red from yellow, green, blue, etc.). This dimension corresponds to a position on the color wheel, expressed in degrees

(between 0° and 360°). The value is a measure of where a particular color lies along the lightness/darkness axis (0% corresponds to absolute black and 100% to absolute white). The chroma is the measurement of how pure a hue is in terms of grey (0% corresponds to no color, and 100% to total saturation).

In our experiment the value and the chroma were fixed at 100%. White was used at the 0 position on the potentiometer. To match hues and potentiometer positions, a 1.41 ratio was calculated (as the hues have a range of 0 to 360, and the potentiometer only allowed scores of 0 to 254). Table 8 describes some specific hues in terms of the degrees (from 0° to 360°) assigned to the potentiometer's fixed positions (from 0 to 255) and the usual name for the color.

Table 8: Some colors with the specific angles assigned to the potentiometer's fixed positions in study 2.

Potentiometer position	Hue (in degrees)	Usual name
0	0	White
8**	348	Red
20*	331	Pink
50**	290	Purple
90*	234	Blue
115**	198	Light blue
163*	133	Green
216**	60	Yellow
250*	12	Orange

Notes. (*) Training trials; (**) Test trials.

1.4. Experimental procedure

The time course of the experiment, as well as the response interface, was the same as in Experiment 1. From the continuum of 254 hues, four input colors (corresponding to the 8 (red), 50 (purple), 115 (light blue), and 216 (yellow) positions, were selected, and were presented six times in random order. This gave a total of 24 test trials. Prior to the test trials, the participants practiced with

four different colors (pink, blue, green, orange), each presented once. Participants received the correct response as feedback. Data from these trials are not included in the analyses.

The potentiometer track wheel allowed the participants to flash colors on the screen, so as to show their answer in terms of hues, on a continuum from 0 to 254. When they were satisfied with their answer, the children had to validate it by pressing the potentiometer button. Then, the instruction to go back to the “0” position was given.

2. Results

A qualitative assessment of children’s answers confirmed that the answers for a specific hue did in fact correspond to what, for example, matched the label “red”. The accuracy of the participants’ estimates was assessed by calculating the AES. A repeated measure ANCOVA with color (red, purple, light blue, yellow) as the within-subject factor and group (MLD or control) as a between-subjects factor was calculated, with IQ as covariate. No significant effect was found: no group effect ($F(1, 21) = .51, p = .48$), and no interaction between color and group ($F(3, 63) = 1.05, p = .38$).

It should also be mentioned that the absence of a significant group effect is not due to the small size of the sample. Indeed, power of the group effect was sufficient (0.80) and the negligible difference actually observed indicates a better precision for MLD children than for controls. Furthermore, when we considered the performance of these 24 children on the numerical estimation tasks, the significant group effect was replicated.

3. Discussion

Experiment 2 provides evidence that both MLD and control children were able to estimate the hue of a colored pattern. In this experiment, the accuracy of the two groups of children was similar, indicating that the MLD children’s difficulties in estimating numerosities were not due to a general deficit with estimation tasks.

4. General discussion

The principal aim of this paper was to evaluate which deficit underlying MLD. The first issue was to determine whether children with MLD present a basic numerical deficit in their approximate representation of number magnitude as postulated by Wilson and Dehaene (2007) or whether their deficit affects the connections between that representation and numerical symbols, as proposed by Rousselle and Noël (2007) or both representations. This question was addressed as current results in the literature are incoherent and sometimes subject to discussion. For that purpose, we used numerical estimation tasks as they allow a very direct measure of the precision and variability of the underlying number magnitude representation. Tasks of this type have been used widely with adults, sometimes with children but have never been proposed to MLD participants. Our results indicated that both MLD and control children were able to perform the task as their estimates increased linearly with the target and they showed the same profile of a stable COV, as reported in adult studies, supporting the idea of a magnitude representation obeying to Weber's law. Yet, on all four numerical estimation tasks, MLD children's estimates were consistently more variable and less accurate than those produced by control children. This pattern of performance does not result from a general and non-specific deficit of estimation as MLD children were able to estimate the hue of color patches (Study 2) as well as their typically developing peers. These results support Wilson and Dehaene's (2007) hypothesis of a deficit in the number magnitude representation in children with MLD.

However, finer analyses contrasting symbolic and non-symbolic estimation of dot sets (HT to HM versus HT to AN), indicated that MLD children exhibited greater difficulty in the task involving symbolic numbers than in the purely non-symbolic task. This indicates that children with MLD have an additional deficit in the connections between the approximate magnitude representation of numbers and the symbolic format, as proposed by Rousselle and Noël (2007) or with the symbolic numerical representation.

A second issue concerned whether MLD children are more influenced than control children by the perceptual variables that may co-vary with numerosities. Our results indicate that only MLD children took advantage of the perceptive cues which correlated with numerosity and made better estimates in the HM task (where the cumulative area co-varied perfectly with numerosity) than in the HT task (where the cumulative area was constant across numerosities). Previous studies have shown that, in comparison tasks, young children do not process numerical information if perceptive variables are sufficient to perform the task (Feigenson et al., 2002; Mix et al., 2002; Rousselle et al., 2004). However, as children get older they gradually focus more and more on the numerical information (Cuneo, 1982; Mix et al., 2002; Rousselle & Noël, 2008a). Accordingly, our results might indicate that MLD corresponds to a developmental delay.

The last question addressed concerned the influence of the direction of mapping. Mundy and Gilmore (2009) had reported, in a population of typically developing children (6 to 8 year olds), that the mapping from a symbolic format to a non-symbolic one was less precise than the reverse mapping from a non-symbolic code to a symbolic one. Using a different paradigm, and testing older children, we replicated these findings and observed that both MLD and typically-developing children were better at estimating dot sets by producing an AN than the reverse. This might be due to life experiences as we are often required to make estimates of the ‘how many cookies are left in the jar?’ type, while the reverse mapping is less frequent, and therefore less practiced.

By demonstrating MLD children’s impairment in a purely non-symbolic numerical estimation task, the present study adds to the small amount of existing evidence for a deficit affecting the approximate number magnitude representation (Mussolin, et al., 2010b; Piazza, et al., 2010; Price, et al., 2007).

However, at this point, two questions are left unanswered. First, in our study, the impairment of the number magnitude representation in MLD children seems to go along with an additive problem

in connecting numerical symbols to that representation or with a poorer symbolic representation. Are these two independent problems or is one simply the consequence of the other? Second, if the deficit causing MLD is a defective number magnitude representation, why did some studies fail to find any difference between MLD and control children in tasks tapping the approximate magnitude representation with non-symbolic numbers (Iuculano et al., 2008; Rousselle & Noël, 2007)?

One possible way to address these questions is to adopt a developmental perspective. As we know, 6 month old babies can discriminate between sets of different numerosities as long as they differ from one another by a ratio of 1:2 (Lipton & Spelke, 2003; Xu & Spelke, 2000). This suggests that there may be an innate, or at least a very precocious, approximate number magnitude representation. A few months later, at the age of 9 months, their discrimination abilities has increased up to a 2:3 ratio (Lipton & Spelke, 2004; Wood & Spelke, 2005) which suggests that the number magnitude representation gets more precise with the child's brain maturation or experience. Two years later, the children learn to count, and still later they learn the Arabic symbols. These symbols allow the children to develop a much more precise representation of number magnitude: several studies have shown that the accuracy of the ANS increases during that developmental period as well (Chillier, 2002; Halberda & Feigenson, 2008). It has been suggested that learning symbols and processing exact numbers by counting and calculation might indeed contribute to increase the number magnitude acuity (Dehaene, 2009; Piazza et al., 2010).

This refinement hypothesis nicely accounts for the current data and for the incoherent profiles reported in previous research. However, it should be tested directly by, for instance, replicating this study on younger groups of MLD children. This hypothesis indeed predicts that impairment in MLD children should first be observed in tasks requiring a mapping between symbolic codes and number magnitude, and only later on should it extend to non-symbolic tasks. The fact that, in our study, the largest discrepancy between MLD and control children was observed in the HT to AN task shows that mapping between approximate representation and symbols is still a major difficulty for these

children, especially when there are no perceptual cues co-varying with numerosities to help them. It is also interesting to note that the best predictor of math performance was also a condition requiring this mapping between the approximate magnitude representation and Arabic symbols.

In summary, our results support the hypothesis that the number magnitude representation is less precise in MLD children than in their normally developing peers. Nevertheless, we argue that this difference arises from an initial deficit in learning to link the approximate representation of number magnitude with exact symbolic systems, which, in turn, produces a weaker refinement of number magnitude representation over time, leaving MLD children with an impoverished number acuity compared to typically achieving children.

Chapter 5:

Numerical estimation in adults with and without

mathematical learning disabilities¹⁰

¹⁰ This section is a modified version of an article by Mejias, Grégoire and Noël with identical title which has been submitted to *Learning and individual differences*. In order to avoid redundancy with the preceding sections, the Introduction, Method and Discussion sections of this article have been truncated, but the Results section is fully reproduced and completed with the comparison of the results observed in Chapter 4.

Abstract

The number-magnitude representation system or the access to/from that system from/to symbolic numbers is problematic in children experiencing mathematical disabilities (MLD). In the present study, adults who had experienced MLD as children were compared to control adults in numerical estimation tasks. The performances of the two groups were evaluated in the production of an estimated answer, when using numerical information presented in symbolic and non-symbolic formats as input and output. Consistently, in all the numerical tasks, the MLD participants performed less accurately than the control participants, irrespective of the format of the estimate. Adults who had experienced MLD as children still show specific difficulties in numerical estimation in adulthood, which indicates that they have a less precise number-magnitude representation.

To the best of our knowledge, no study characterizes the longevity of numerical impairment in adult with MLD. We do not know if the poor abilities of MLD children in basic numerical tasks reflect a developmental delay which recovers later on, or corresponds to a long-lasting deficit that is still present in adulthood. Extending the previous studies conducted on children, we want to explore whether the weak acuity of the ANS present in older children is still present in adults or whether they only show difficulties in tasks using number symbols. By comparing MLD and control participants' performance on numerical estimation tasks contrasting the use of symbolic and non-symbolic numbers, we will confront the presence of an ANS deficit or the existence of a failure to link an intact ANS with symbolic representation in adulthood.

Therefore, the four estimation tasks used with the children in Chapter 4 were proposed to MLD and control adult participants.

According to the ANS deficit hypothesis, MLD participants should show lower performance in all these numerical estimation tasks since they all involve the ANS. Conversely, an impaired connection between the ANS and the symbolic representation predicts a deficit in the symbolic tasks involving a mapping between the ANS and numerical symbols.

1. Method

1.1. *Participants*

A total of 44 Caucasian adults took part in the experiment: 22 of them were identified as having MLD and 22 were classified as controls (C). The two groups of adults did not differ in terms of gender (6 males and 16 in each groups) or age (see Table 9).

Participants selection procedure and classification scheme

Participants were recruited via an advertisement saying that we were looking for (a) individuals who had had significant difficulties in learning mathematics during childhood and/or who had been diagnosed as “dyscalculic” ; (b) individuals who had never had specific learning difficulties. Then both groups went through a questionnaire and were included in a group if they filled particular criteria (see Appendix B for details of the recruitment and the matching procedures).

1.2. *Experimental tasks*

Arithmetical and memory span for pattern assessments

After we ran the experiment (to avoid generating math anxiety) and in order to confirm that arithmetical difficulties persists into adulthood, participants went through a battery of arithmetical tests which is part of the standardized battery developed by Shalev and collaborators (2001) and adapted by Rubinsten and Henik (2005, see Appendix C).

Moreover, as our experiment supposed good visuo-spatial abilities, participants went through a memory span for pattern assessment. This task, a paper and pencil version of that developed by Wilson, Scott and Power (1987), involved the presentation of matrices in which some cells were randomly completed; the participants had to recall which cells had been filled in. The complexity increased every time the participant was successful in two out of three attempts. Participants made

their responses in a booklet of blank matrices, corresponding in size to the target patterns to be recalled. The initial level of complexity involved filling in two cells.

Estimation tasks, stimuli and experimental procedure

The four estimation tasks, the stimuli and the experimental procedure used were those presented in Chapter 4.

2. Results

2.1. Arithmetical and memory span for pattern assessments

As reported in Table 9, differences between the MLD and C groups were significant for eleven of the fourteen arithmetic sub-scores, both for the total response time and the percentage of errors. On all but two of the 14 subtests, the mean score of the MLD and the C groups fell into the score range of respectively the MLD and C group of Rubinsten and Henik's (2005)¹¹. Finally, our MLD population's overall error rate fell under the 4th percentile of the control group.

Although scores on the memory span for patterns were in the normal range for both groups, Cs have higher scores than MLD participants (Table 9). Therefore, scores on the memory span for pattern were used as covariate in further analyses.

¹¹ Except for the complex additions in the MLD (our group was scoring -2.43 standard deviation below) and for the subtraction of fractions in the C (our group was +1.78 standard deviation above).

Table 9: Descriptive information and arithmetic scores with range values for the MLD and C groups.

	Control group (N=22)		MLD group (N=22)		Statistical analyses
<i>Descriptive information</i>	<i>Mean (SD)</i>	<i>Range</i>	<i>Mean (SD)</i>	<i>Range</i>	<i>t-tests: $t_{(42)}$</i>
Age (in months)	24.41 (10.27)	17–54	24.41 (9.61)	19–54	.00
Memory span	7.14 (0.77)	6–9	9.14 (1.88)	7–13	-4.60**
Arithmetical battery (no of items)	Mean number of errors (SD)	Range	Mean number of errors (SD)	Range	
<i>Number facts</i>					
Addition (5)	0.41 (0.59)	0–2	0.05 (0.21)	0–1	2.72*
Subtraction (5)	0.50 (0.74)	0–2	0.00 (0.00)	0–0	3.17**
Multiplication (5)	0.27 (0.46)	0–1	0.09 (0.29)	0–1	1.57
Division (5)	0.18 (0.39)	0–1	0.14 (0.35)	0–1	.403
<i>Complex arithmetic</i>					
Addition (8)	1.32 (1.46)	0–5	0.45 (0.60)	0–2	2.57*
Subtraction (8)	1.36 (1.26)	0–5	0.41 (0.59)	0–2	3.23**
Multiplication (8)	2.95 (2.50)	0–7	0.77 (0.92)	0–3	3.84**
Division (8)	2.59 (2.15)	0–7	0.55 (0.74)	0–2	4.22**
<i>Decimals</i>					
Addition (4)	0.68 (0.95)	0–3	0.14 (0.35)	0–1	2.54*
Subtraction (4)	1.14 (1.17)	0–3	0.23 (0.53)	0–2	3.33**
<i>Fractions</i>					
Addition (5)	0.50 (0.67)	0–2	0.05 (0.21)	0–1	3.02**
Subtraction (5)	0.77 (1.31)	0–4	0.27 (0.55)	0–2	1.65
Multiplication (5)	2.27 (1.91)	0–5	0.27 (0.88)	0–4	4.46**
Division (5)	2.82 (2.11)	0–5	0.41 (1.05)	0–4	4.80**
Overall error rate (%)	22.22 (11.62)	2.5–40	4.77 (3.98)	0–20	6.67**
Total response time (sec)	1326.41 (390.54)	468–2010	752.59 (266.04)	388–1316	5.70**

Note. (*) = MLD differs significantly from Cat $p < .05$; ** at $p < .01$

2.2. Pattern of estimates

Regression analyses performed by group and by task on the participants' means and on the standard deviations showed that both increased in direct proportion to the target magnitude with slopes close to 1 (Table 10 panels a and b) and that COV (i.e., the ratio of the standard deviation to the mean) was relatively constant across target size (as the slopes from the regression analysis did not significantly differ from 0¹² as showed by the t-test analyses, see Table 10 panel c).

¹² Only a small percentage of participants did not present a constant COV score: In the HT to HM task, 9% of C participants (i.e., 2 participants) and in the AN to HM dots task, 9% of MLD and 23% of C participants (i.e., 2 and 5 participants respectively).

Table 10: Linear regression between the target numerosities and (a) the mean estimate, (b) the standard deviation of the estimates, (c) the coefficients of variation for the MLD and C groups on the four tasks

HM to AN		HT to AN		HT to HM		AN to HM	
C group	MLD group	C group	MLD group	C group	MLD group	C group	MLD group
<i>(a) Mean of estimates</i>							
$r^2 = .992$	$r^2 = .988$	$r^2 = .989$	$r^2 = .968$	$r^2 = .994$	$r^2 = .986$	$r^2 = .999$	$r^2 = .997$
$\theta_0 = .996$	$\theta_0 = .993$	$\theta_0 = .994$	$\theta_0 = .984$	$\theta_0 = .997$	$\theta_0 = .993$	$\theta_0 = 1.00$	$\theta_0 = .999$
$t(5) = 24.53^{**}$	$t(5) = 20.30^{**}$	$t(5) = 21.02^{**}$	$t(5) = 12.32^{**}$	$t(5) = 29.05^{**}$	$t(5) = 18.88^{**}$	$t(5) = 96.86^{**}$	$t(5) = 41.49^{**}$
<i>(b) Standard deviation of estimates</i>							
$r^2 = .940$	$r^2 = .953$	$r^2 = .850$	$r^2 = .813$	$r^2 = .990$	$r^2 = .968$	$r^2 = .992$	$r^2 = .986$
$\theta_0 = .970$	$\theta_0 = .976$	$\theta_0 = .922$	$\theta_0 = .902$	$\theta_0 = .995$	$\theta_0 = .984$	$\theta_0 = .996$	$\theta_0 = .993$
$t(5) = 8.86^{**}$	$t(5) = 10.09^{**}$	$t(5) = 5.33^{**}$	$t(5) = 4.67^{**}$	$t(5) = 22.31^{**}$	$t(5) = 12.31^{**}$	$t(5) = 24.48^{**}$	$t(5) = 18.96^{**}$
<i>(c) Coefficients of variation</i>							
$r^2 = .105$	$r^2 = .011$	$r^2 = .065$	$r^2 = .370$	$r^2 = .719$	$r^2 = .147$	$r^2 = .628$	$r^2 = .661$
$\theta_0 = .324$	$\theta_0 = -.106$	$\theta_0 = .255$	$\theta_0 = -.609$	$\theta_0 = .848$	$\theta_0 = -.383$	$\theta_0 = .793$	$\theta_0 = .813$
$t(5) = .77$	$t(5) = -.24$	$t(5) = .59$	$t(5) = -1.72$	$t(5) = 3.57^*$	$t(5) = -.93$	$t(5) = 2.91^*$	$t(5) = 3.13^*$

Note. (*) = significant difference between MLD and control group at $p < .05$; (**) at $p < .01$.

2.3. Comparison of the two groups

The COV scores were entered into a repeated measures analysis of covariance (ANCOVA) with 4 Tasks (HM to AN, HT to AN, HT to HM, AN to HM) and 7 Targets (8, 12, 16, 21, 26, 34, 36) as the within-subject factors, 2 Groups (MLD and C adults) as a between-subjects factor and the memory span for pattern as a covariate. This analysis revealed a main effect of group, $F(1, 41) = 7.67$, $\eta^2 = .16$, $p = .008$, showing that adults with MLD had higher COV scores (*mean COV* = .203) than the C adults (*mean COV* = .161) (Figure 18 panel a). There were no other significant results. This indicates a greater variability of the ANS in MLD compared to C adults.

The same analysis was run on the absolute error score. Absolute error score (AES) is a measure of response precision. For each number target, the AES is computed as follows:

$$AES = \sum |x_i - T|$$

where x_i corresponds to the participant's estimate on trial i , T to the target value. The absolute value of the sum provides a measure of overall accuracy in performance, irrespective of the direction of the differences between the participant's answers and the target, it is thus sensitive to the extent to which the subject was "off target" (Schmidt & Lee, 2005).

The ANCOVA on the AES mean sum was calculated for each target (because the Mauchly test of sphericity showed that sphericity could not be assumed in this case, an adjustment according to the Greenhouse–Geisser procedure was used to correct the degrees of freedom in the following F -test) and showed increased imprecision with the magnitude of the target, $F(1.32, 53.93) = 3.78$, $\eta^2 = .08$, $p \leq .040$ (see Table 11) and a significant group effect, $F(1, 41) = 7.94$, $\eta^2 = .16$, $p = .007$, indicating that MLD adults ($M = 7.36$, $SD = 4.25$) were less accurate than C adults ($M = 5.53$, $SD = 2.94$). There were no other significant results (Figure 18 panel b).

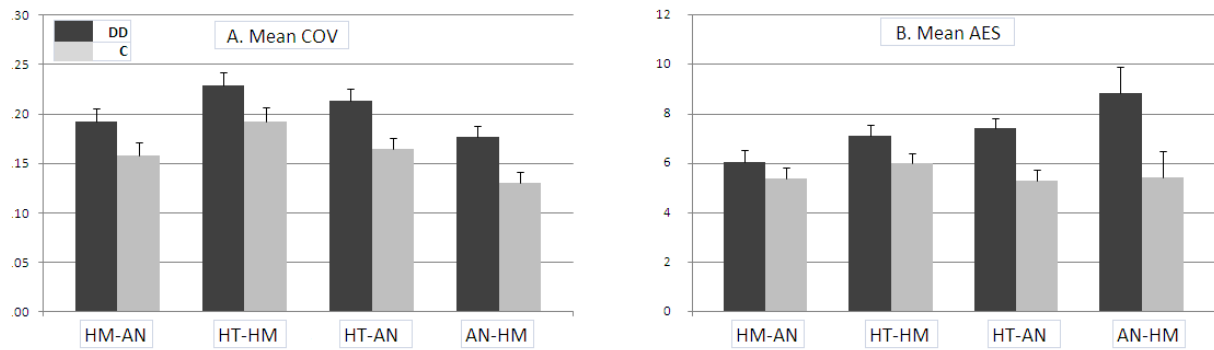


Figure 18 : Slopes of A. the coefficients of variation (COV; ratios of standard deviation and mean estimate) and B. the responses precision (AES; absolute error score). Error bars indicate the standard errors of the mean.

Finally, response times were entered in the same global ANCOVA and showed no significant results. MLD and C adults thus did not differ in terms of RTs ($M = 4081$, $SD = 1172$; $M = 4618$, $SD = 1848$, for MLD and C adults respectively; $F(1, 41) = .65$, $\eta^2 = .02$, $p = .423$). Congruent with Piazza and collaborators' TR results (2010), this indicates that the poorer accuracy of MLD compared to C adults does not stem from the use of different response strategies but reflects differences in the magnitude representation.

Table 11: Statistics for the mean AES for each target magnitude in the four different tasks and per group.

	<i>Targets</i>							
	8	12	16	21	26	34	64	<i>Total</i>
<i>HM to AN</i>								
MLD's Mean (SD)	1.29 (1.46)	2.96 (1.83)	3.67 (1.89)	5.18 (2.62)	6.15 (2.82)	7.93 (3.92)	17.04 (9.51)	6.32 (2.08)
C's Mean (SD)	.62 (.60)	1.72 (1.32)	2.81 (1.22)	4.15 (2.63)	4.75 (2.47)	7.70 (4.16)	14.12 (6.98)	5.12 (1.81)
<i>HT to AN</i>								
MLD's Mean (SD)	2.61 (3.72)	3.03 (2.42)	3.78 (1.62)	5.46 (3.51)	6.09 (2.12)	8.40 (2.58)	19.92 (6.95)	7.04 (1.81)
C's Mean (SD)	1.63 (1.20)	2.34 (1.37)	3.19 (1.85)	4.08 (1.97)	5.69 (3.19)	7.57 (2.79)	18.02 (7.08)	6.08 (1.70)
<i>HT to HM</i>								
MLD's Mean (SD)	1.56 (1.03)	2.47 (1.30)	4.15 (2.30)	6.09 (2.55)	6.17 (2.58)	9.47 (3.31)	21.59 (8.27)	7.36 (1.94)
C's Mean (SD)	.72 (.71)	1.36 (.59)	2.52 (1.10)	4.49 (2.15)	5.06 (2.18)	6.86 (2.75)	16.66 (8.54)	5.38 (1.50)
<i>AN to HM</i>								
MLD's Mean (SD)	2.06 (1.62)	4.58 (4.26)	4.91 (3.42)	6.43 (5.41)	7.21 (6.01)	10.59 (7.31)	26.19 (22.59)	8.85 (5.58)
C's Mean (SD)	1.20 (1.14)	1.86 (1.50)	2.95 (2.49)	3.81 (2.62)	5.06 (2.91)	7.49 (5.46)	15.47 (9.23)	5.40 (2.92)

2.4. Correlation between math performance and estimation abilities

Pearson product-moment correlation analyses between mathematical performances (i.e., the overall error rate on the battery of arithmetical tests) and individual COV scores and AESs showed all significant relationships (see Table 12 panels a and b).

Table 12: Correlations between the percentage of errors on the overall battery of arithmetical tests and (a) adults personal COV scores and (b) their AESs in the four estimation tasks

	HM to AN	HT to HM	HT to AN	AN to HM
(a) COV				
Correlation	.542**	.427**	.756**	.540**
(b) AES				
Correlation	.337*	.319*	.449**	.433**

Note. * Correlation coefficient significant at the .05 level (2-tailed); ** significant at .01 level (2-tailed).

2.5. Children's and adults' performances

The same estimation tasks had been used in the previous chapter with C and MLD 10 year old children. By comparing the results of the study with children to those of the present work on adults, we can explore the changes in these measures through development in the C and MLD groups. Accordingly, ANOVAs were carried out separately for COV and AES¹³, with 2 cohorts (children and adults) and 2 groups (C and MLD participants) as between factors. For both COV and AES there was a significant group effect (COV: $F(3, 87) = 21.93$, $\eta^2 = .20$, $p \leq .001$; AES: $F(3, 87) = 38.60$, $\eta^2 = .31$, $p \leq .001$), indicating that the variability and imprecision of the underlying representation were larger in MLD than in C participants. There was also a cohort effect (COV: $F(3, 87) = 98.40$, $\eta^2 = .53$, $p \leq .001$; AES: $F(3, 87) = 61.07$, $\eta^2 = .41$, $p \leq .001$), showing that variability and imprecision were greater in children than in adults. The group X cohort interaction was also significant for AES ($F(3, 87) = 6.60$, η^2

¹³ For each participant, a mean COV and AES were computed across the four estimation tasks, as the correlations between the tasks were high ($r > .420$, all p s $\leq .001$).

= .07, $p = .012$). Indeed, the difference between MLD and C participants was stronger in children ($M = 10.52$, $SD = 3.77$) than in adults ($M = 6.45$, $SD = 1.97$) ($t(89) = 6.31$, $p \leq .001$) (see Figure 19).

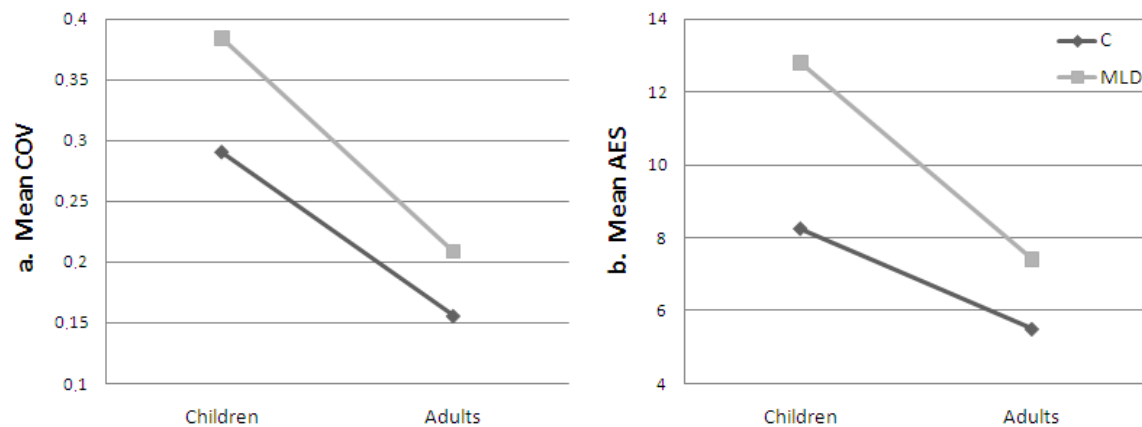


Figure 19: The Cohort (Children and Adults) x Group (C and MLD) interaction for the COV (panel a) and the AES (panel b).

3. Discussion

Previous data have reported long-lasting difficulties in MLD children in a 8 year follow-up study (Shalev et al., 2005). However, very few is known about the longevity of MLD in adulthood. Two hypotheses are currently proposed to account for MLD: one assuming a deficit in the ANS (Wilson & Dehaene, 2007), the other assuming a defective connection between this preserved system and numerical symbols (Rousselle & Noël, 2007). Current data seem to favor the later in MLD children who start arithmetical learning (Iuculano et al., 2008; Rousselle & Noël, 2007) while data in older MLD children showed that the ANS is deficient (Mussolin et al., 2010b; Piazza et al., 2010; Price et al., 2007). In adults, Ashkenazi and collaborators (2009) and Rubinsten and Henik (2005) have proposed, on the basis of a number Stroop task, that they are impaired in accessing the ANS from AN but no study has ever tried to see whether they are or not impaired in tasks tapping the ANS. Accordingly,

this study aimed at measuring both the integrity of the ANS and its connections with symbolic numbers in adult participants who experienced MLD as a child.

The performance of adult participants with a history of MLD in childhood was compared with the one of a C group matched in terms of age and professional activity. Four different numerical-estimation tasks, varying the mapping side between symbolic and non-symbolic formats or between two non-symbolic formats was used. According to the defective-access hypothesis, MLD participants should be impaired solely in estimation tasks involving symbolic numbers. On the contrary, impairment in all the estimation tasks is expected on the basis of ANS impairment.

First, our results showed that, although they had been recruited on the basis of self-reported history of childhood difficulties with mathematics, MLD participants scored significantly worse than the C participants on the arithmetical test on both response time and accuracy. These results confirm our recruitment procedure and show the persistence of MLD from childhood to adulthood (as also reported by Rubinsten & Henik, 2005).

Second, both C and MLD participants showed the expected indications that they accessed the ANS to handle the tasks (see Feigenson et al., 2004). Their mean estimates and the variation of their answers increased proportionally to the target, yielding a constant coefficient of variation across target size. Moreover, there was no significant difference between estimates in the HT to AN task and the HM to AN task which argues in favor of a real numerical estimation in both tasks rather than a perceptual estimation based on the cumulative surface in the later condition.

Finally, although they took the same time to perform the tasks, MLD participants showed more variability (measured by the COV) and lower precision (measured by the AES) in their numerical estimations than the C participants, and this was true for all four numerical tasks, including those that did not involve mapping between symbolic numbers and the ANS. Moreover, results indicate that this reduced acuity is correlated with arithmetical skills.

These results replicated those from the study comparing 9 to 10 year old MLD and C children presented in Chapter 4 (Mejias, Mussolin, Rousselle, Gregoire, & Noël, submitted). Using the same estimation tasks, we observed that MLD children had larger COV than C children and larger AES in all four numerical tasks. The results of this study indicate a reduced acuity of approximate number representation in MLD. The results of the current study as well as those collected on children thus indicate that this reduced acuity is evident throughout development (i.e., from age 10 to adulthood) and performance correlates with arithmetical skills. By comparing the results of the two studies, we can see that both adult groups were more accurate than the related children group, but MLD participants seem to benefit more from daily exposure to numbers than C participants, as their acuity increased proportionately more. The most likely explanation seems to be that C adults reach a level of acuity close to the limit of the human system. However, these results should be confirmed by a longitudinal study. This acuity improvement is well in line with results from training studies. It has been shown that children with MLD increase their acuity when exposed to training on numerical comparison where symbolic and non-symbolic materials are presented together in order to reinforce the link between them and the representation of numbers (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006).

The results of the current study favor the ANS deficit hypothesis and show that the magnitude representation of numbers would be both more variable and less precise in MLD adults than in C participants. MLD participants' difficulties seemed to reflect a long-lasting deficit that continued through adulthood rather than a developmental delay that has been filled. These results might also have some implications for the rehabilitation of people with MLD. Indeed, at present, rehabilitation of MLD focuses mainly on procedural and arithmetic skills. Training estimation abilities in order to increase the acuity of the ANS might be beneficial for MLD individuals and might even allow them to avoid very distant errors related to poor knowledge of procedures ($15 + 48 = 513$).

Chapter 6:

Approximate addition in children and adults with or without mathematical learning disabilities: the impact of formats¹⁴

¹⁴ This section is an article in preparation. In order to avoid redundancy with the preceding sections, the Introduction as well as the Discussion sections have been truncated.

Abstract

The present study focused on those multiple explanations for dyscalculia assuming that this disability could be due to a) a basic numerical deficit affecting the representation and the manipulation of number magnitude (Butterworth, 1999, 2005; Wilson & Dehaene, 2007), b) an access deficit to that number magnitude from numerical symbols (Rousselle & Noël, 2007) or eventually due to c) a deficit affecting a larger magnitude system (Walsh, 2003). Study 1 tests the different hypotheses by comparing MLD and C children's abilities to provide an approximate answer to additions contrasting symbolic (AN) and non-symbolic (pattern of dots) numerical magnitudes as well as continuous (quantity of liquid) magnitudes. MLD children performed consistently less accurately than C children but were even weaker when the task involved symbolic notations. Study 2 compares C adults and adults who had experienced MLD in childhood. Results showed a global magnitude deficit in the MLD group of children while deficit are limited to an access deficit to the number magnitude from symbols in the MLD group of adults. Finally, the comparison between adults and children performance provides the characteristic and the longevity of their difficulties and has implications for the diagnosis and rehabilitation of people with MLD.

MLD is characterized by long lasting difficulties in the realization of simple calculation (e.g., $2+5=8$; Geary et al., 2000) and in the execution of arithmetical procedures (e.g., $15+48 = 513$; Russell & Ginsburg, 1984). As MLD children show a poor grasp of the meaning and properties of arithmetic operations (Geary et al., 1992), it would be in their interest to be able to get a feed-back on their aberrant answers. But are they able to give a plausible approximation of the answer? This is the first question addressed in the present study, knowing that this ability requires (1) the understanding of the meaning and properties of the arithmetic operation; (2) the access to the ANS.

The second purpose of the present study is to disentangle between the different hypotheses accounting for MLD. Is dyscalculia resulting from a deficit affecting the approximate or the exact magnitude representation (Butterworth, 1999, 2005; Wilson & Dehaene, 2007), an access deficit to an intact ANS from numerical symbols (Rousselle & Noël, 2007) or a deficit affecting a larger magnitude system (Walsh, 2003)? By using pure non-symbolic numerosities (i.e., with no symbolic/non-symbolic notation mix between the input and the output as proposed in Chapters 4 and 5), we can be sure that individuals are going to access the ANS without eventually referring to an interfering symbolic representation.

Accordingly, four approximate addition tasks were proposed to MLD and control participants. In two non-symbolic approximate addition tasks, participants had to produce a pattern of dots which roughly corresponds to the sum of three set of dots presented sequentially. In the first version of the task, sets were made of homogeneous-size dots. In the second version of the task, sets were made of heterogeneous-size dots that led to a constant cumulative surface area across numerosities (i.e., avoiding the participants' answers to be based on the total surface area covered by the dots; Rousselle et al., 2004). In the third task, continuous magnitudes were used: participants were asked to produce a quantity of liquid which roughly corresponds to the sum of three quantities of liquid presented sequentially. Finally, the fourth task was a symbolic approximate addition in which

participants had to produce an AN corresponding to the sum of three-addend presented sequentially.

If MLD participants' difficulties result from an ANS deficit (Wilson & Dehaene, 2007), they should present lower performance than C in both symbolic and non-symbolic numerical approximate calculation tasks. If MLD participants' difficulties result from an access deficit to the ANS from symbols (Rousselle & Noël, 2007), MLD should perform more poorly than controls in the symbolic approximate calculation only. If MLD participants' difficulties result from a larger magnitude system deficit (Walsh, 2003), they should present lower performance in each task (including the one on continuous quantities). Finally, no particular difficulties regarding these experimental tasks should be observed if MLD is due to an exact number representation deficit (Butterworth, 1999, 2005; Luculano et al., 2008).

The atypical development of approximation abilities

Literate adults and individuals outside formal education are able to adopt approximate calculation strategies to solve a symbolic or non-symbolic arithmetic problem (e.g., Barth et al., 2008; Barth et al., 2005; Dehaene, 1992; Flombaum et al., 2005; Lemaire et al., 2000; McCrink & Wynn, 2004; Pica et al., 2004). Examining participants' symbolic approximate calculation abilities give some thoughts about their control processes and their ability to avoid aberrant responses. Examining participants' performances on non-symbolic approximate calculation is known to mirror the ANS (see Chapter 2).

Symbolic approximate addition performances in 6 to 7 year old MLD and low-achieving children were examined by De Smedt and Gilmore (2011) using Gilmore and collaborators' paradigm (2007; see Chapter 2, paragraph 3 "*Approximate calculation*" of this manuscript). On a trial, children were presented with an AN (presented simultaneously in its verbal format) which appeared below a cartoon-character. Then, a second AN appeared below the same character. Finally, a third AN

appeared below another character and the children were asked, “Who has more?”. Compared to the control group of children, MLD and low-achieving children performed much poorly in this type of verification task.

Without using any story context, several studies showed similar results: 7 to 9 year old MLD children were less precise in verification tasks when selecting the result closest to the correct answer from two incorrect ones (e.g., $4 + 9 = 12$ or 19 ; Hanich et al., 2001; Jordan & Hanich, 2003; Jordan et al., 2003b). Yet, in those studies, only a small number of items were presented (i.e., 10 addition and 10 subtraction problems), and response times were not recorded. Moreover, nothing prevents children from calculating the correct result and then to comparing it with the proposed answers. As the use of an exact calculation strategy could not be ruled out, we can not know to what extent the approximate strategies have been instigated to perform the task. Recently, one study established that 8 year old MLD children probably used approximate calculation, or at least computational estimation strategies, but not as frequently and efficiently as typically achieving children (Rousselle & Noël, 2008b).

To our knowledge, non-symbolic approximate arithmetic abilities in MLD children have been investigated in two studies using also verification tasks. De Smedt and Gilmore (2011) showed that 6 to 7 year old children with severe and mild forms of mathematical difficulties performed at the range of control children in the verification of an approximate addition. This task is the non-symbolic version of the task used by Gilmore and collaborators (2007) in which AN are replaced by moving dots (marbles). Large numbers (from 5 to 58) were selected such that the sum of the moving dots were greater than the comparison number on half of the trials. The sum differed from the comparison number by 3 different ratios (4:7, 4:6, or 4:5). However, children who responded below chance (regardless the ratio) were removed from the experimental pool. As the study was evaluating which types of numerical representations difficulties are present in MLD children, this choice can bias the results. Indeed, a quarter of the MLD population was removed from the experimental pool; all of

them failed the non-symbolic task (versus a sixth of the control population). Even if the authors' choice is understandable, one cannot exclude the fact that MLD children poor performance on non-symbolic materials was related to a severe and deep rooted deficit of the ANS.

A similar study has been realized with slightly older children by luculano and collaborators (2008). Again, they showed that 7 to 8 year old low-numeracy children performed at normal range in the verification of the approximate addition and subtraction of moving dots. In their study, only two MLD children were examined and showed the same results.

If those studies suggested that MLD children are impaired on tasks that involved the access of the ANS from symbols, the existence of an ANS deficit cannot be completely ruled out.

The current studies

MLD participants' abilities to give a plausible approximation for a given calculation was examined in free estimate production tasks using complex three-addend additions presented in symbolic and non-symbolic notations. Complex three-addend additions were chosen to promote the use of approximate arithmetic instead of exact computation. Moreover, using a free production task to assess approximate arithmetic is not only closer to scholar context of learning, but also gives the opportunity to examine participants' response accuracy with no external constraint (as opposed to the verification task where multiple answers are proposed, according a ratio). The ability to give a plausible approximation of the answer is assumed to place great emphasis on the manipulation of magnitude representation, providing direct assessment of the ANS (Dehaene & Cohen, 1991, 1997; Stanescu-Cosson et al., 2000). The participants' performance should show a specific signature associated with the access to the ANS: the imprecision of their answers should increase with the size of the correct answer (e.g., RTs were also examined to ensure that participants did not compute the correct answer for symbolic stimuli).

In the first study, MLD children's performance will be compared to control children's performance. In the second study, performance of adult participants who have experienced dyscalculia in their childhood will be compared to performance of control adults with no history of learning disabilities. The comparison of these different profiles will allow us to assess the longevity of the basic impairment of MLD participants. Finally, studies in this chapter examined also if the approximate abilities are related to arithmetical skills through childhood and adulthood as this would have important implications for the diagnosis and rehabilitation of people with MLD.

Study 1

1. Method

1.1. Participants

A total of 38 Caucasian children aged 10 to 11 years took part in the experiment: 19 of them were identified as having MLD (9 males and 10 females) and 19 were classified as controls (C, 10 males and 9 females). The two groups did not differ in terms of gender ($\chi^2(1) = .11, p > .75$) or age (see Table 13).

Participants selection procedure and classification scheme

A pool of 494 Caucasian children attending general education classes from thirteen different French-speaking Belgian public elementary schools was assessed. Parental consent was obtained for each of the children. This pool of middle class Caucasian children constituted the normative sample for the following tasks: children were collectively administered a sheet of 81 simple single-digit arithmetical problems and were asked to solve as many as possible in 90 seconds. This fluency task was carried out three times, once with additions, once with subtractions and once with multiplications. The children were also administered an untimed mathematical test that assessed school performance in mathematics (Simonart, 1998). The testing lasted approximately 30 minutes. As the three scores of the calculation fluency tasks and the score of the mathematical battery were highly correlated ($r^2 > .79; p \leq .001$, for the four correlations), the mean of the four Z-scores was computed to provide a mean “mathematical performance Z-score”. Finally, teachers completed a three-point scale to indicate whether individual students showed severe, some, or no math difficulty.

Based on the distribution over the 494 children, children were classified as MLD if they scored below percentile 15 on both the calculation fluency test and the mathematical battery and were judged by their teacher as having some or severe difficulties. This 15th percentile cut-off is more

conservative than that typically used in most of the studies on mathematical disabilities (Geary et al., 2004; Geary et al., 1999; Hanich et al., 2001; Jordan et al., 2003b). For each of these MLD children, a C child was selected from the same classroom. To be included in the C group, children had to score in the normal range on the multiplication fluency test (with a score between the 25th and the 85th percentiles) and to be assessed by the teacher as having no learning difficulties in mathematics. Finally, as the factors underlying MLD might differ according to the presence or absence of an associated reading disability (i.e., weak phonological processing versus weak number sense, see Robinson et al., 2002), we excluded children with poor reading achievement level. All the participants were administered the LUM reading subtest of LMC-R battery (Khomsj, 1998). This subtest is a one-minute word-reading task involving words of increasing spelling complexity. Only participants who were scoring above the limits of lower standards (-1σ) were selected for the following assessments. This method of selection gave rise to the same dispersion in the MLD and C groups of children [-1σ ; $+2\sigma$].

The Similarities and Picture Concepts subtests of the Wechsler Intelligence Scale for Children -IV (Wechsler, 2005) were also administered to the children, and provided an estimate of their IQ score (Sattler, 1982). Participants who scored below a mean standard note of 8, and participants who scored above a global standard note of 14 were removed from the pool. The final group included a total of 38 participants who took part in our experiments, 19 children being identified as MLD and 19 as C children.

In addition to these selection tests, children's memory span for pattern was assessed using a paper and pencil version of the task developed by Wilson, Scott, and Power (1987). This task consisted of a series of matrices in which some cells were randomly filled in. The complexity increased every time the child was successful with two out of three attempts. The initial level of complexity was two filled cells. Children draw their answers in a booklet of blank matrices, using a matrix of the same size as each of the target patterns to be recalled. As we did not have normative

data for this test and because our tasks were particularly demanding on visuo-spatial memory, the MLD and C groups were compared on this ability (Table 13).

As shown in Table 13, the two groups did not differ significantly in terms of IQ or memory span for pattern. As expected, the differences between the groups were significant for all the mathematical subtests.

Table 13: Descriptive information and mean scores for the MLD and C groups of children on the selection tasks for study 1.

	C group (N=19)		MLD group (N=19)		Statistical analyses	
	<i>Mean (SD)</i>	<i>Range</i>	<i>Mean (SD)</i>	<i>Range</i>	<i>t-tests: $t_{(36)}$</i>	Mean difference
						<i>[95% confidence interval of the difference]</i>
Descriptive information						
Age (in months)	128.00 (7.08)	112-141	126.79 (7.86)	114-141	.50	1.21 [-3.71–6.13]
Memory span	6.47 (1.02)	4-8	6.21 (0.98)	5-8	.81	.26 [-.39–.92]
IQ						
Similarities	11.32 (2.00)	8-14	10.32 (1.86)	7-13	-.76	1.0 [-.27–2.27]
Concept identifications	10.36 (1.92)	8-13	10.89 (2.35)	7-15	1.60	-.53 [-1.9–.89]
Subtest mean scores	10.84 (1.54)	8-13	10.61 (1.44)	8.5-14	.49	.24 [-.75–1.22]
Mathematics						
Fluency addition	49.05 (11.85)	32-77	21.16 (5.42)	13-30	9.33**	27.89 [21.82–33.96]
Fluency subtraction	40.42 (7.49)	27-55	18.11 (4.69)	10-27	11.01**	22.32 [18.21–26.42]
Fluency multiplication	32.53 (8.70)	21-47	11.21 (4.08)	4-18	9.67**	21.32 [16.84–25.79]
Mathematical scale	17.36 (3.89)	11-23	8.21 (3.01)	3-16	8.07**	9.16 [6.85–11.46]
Math performance Z-score	.83 (.54)	.03–1.72	-.83 (.19)	-1.16–.59	12.67**	1.66 [1.40–1.93]

Note. (*) = MLD significantly differs from healthy participants' at $p < .05$; (**) at $p < .01$.

1.2. Experimental tasks

Four different approximate addition tasks were developed: the addends were either represented as amounts of homogeneous or heterogeneous-size dots, volumes of water or Arabic numbers.

The first task used discrete and non-symbolic materials: children were presented with three sets of homogeneous-size dots and had to produce a collection of homogeneous-size dots that correspond to the sum of those three sets of dots (Figure 20 panel a). Considering that dots were of equal size, children's estimation could be influenced by the surface area and correlated continuous variables which co-vary with numerosity (Rousselle et al., 2004). Accordingly, the second task used discrete and non-symbolic materials as well but sizes of dots presented as inputs were heterogeneous: children were presented with three sets of heterogeneous-size dots so that the total blackened area was equated over the numerosities across trials (Figure 20 panel b). Moreover, dots with the smallest and the largest size were presented in all sets to avoid smaller sets being also those with bigger elements. Yet, they had to produce a collection of homogeneous-size dots that correspond to the sum. The third task is non-numeric and used non-symbolic continuous stimuli: children were successively presented with three glasses containing different volume of water and had to produce a total equivalent amount of liquid in an empty glass (Figure 20 panel c). Finally, the fourth task was symbolic: children were presented with 3 successive Arabic numbers and had to produce the approximate Arabic number resulting from their sums (Figure 20 panel d).

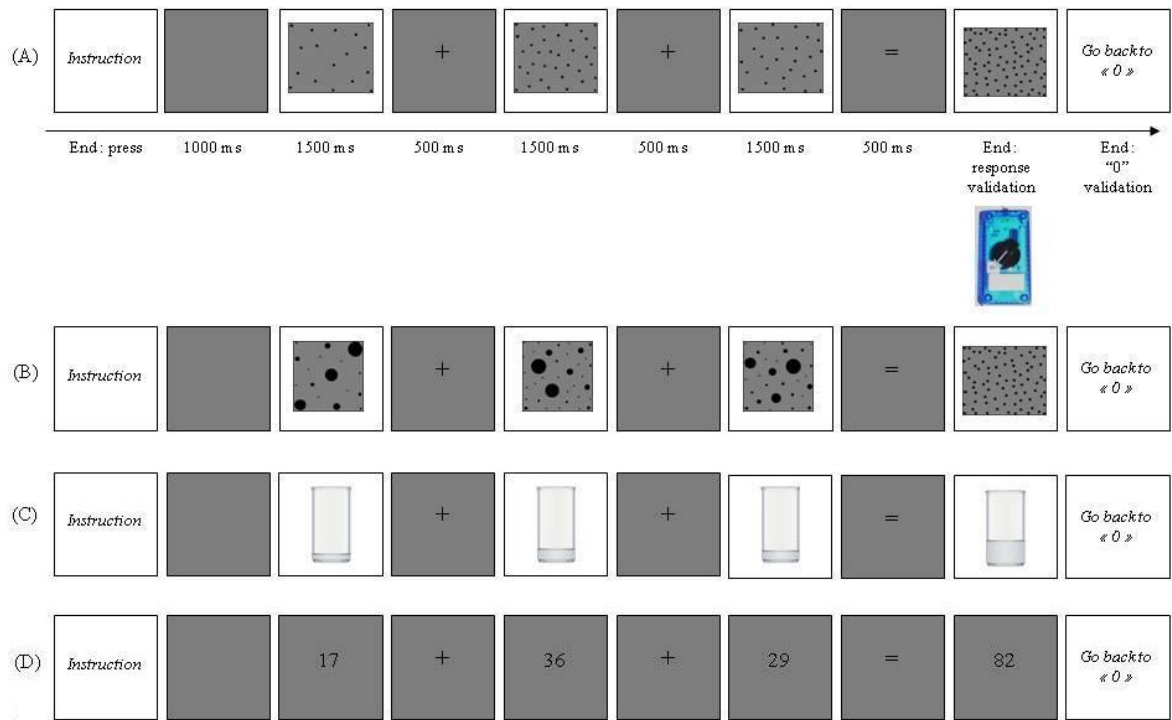


Figure 20: Time course of each trial in each approximate addition tasks, respectively, (A) sets of equal size dots to be approximated by equal size dots, (B) sets of heterogeneous-size dots to be approximated by equal size dots, (C) glasses of water to be approximated by a quantity of liquid, (D) Arabic numbers to be approximated by an Arabic number.

1.3. Stimuli

Correct responses corresponded to four different target quantities (34, 52, 82, and 132). Six different additions per target response were proposed to participants. The same six different additions were used in the four different tasks and were, for example, $7 + 8 + 19 (= 34)$; $16 + 17 + 19 (= 52)$; $29 + 17 + 36 (= 82)$; $27 + 56 + 49 (= 132)$. To avoid exact calculation, all additions involve carrying over. Stimuli were presented to participants in black on a grey screen, except for the volumes of water. Arabic numbers were displayed in Arial font with a visual angle of 2.2° . Input or output homogeneous-size dots, with a visual angle of 0.88° , were presented inside a fixed size rectangle, which covered a $27.5^\circ \times 23.1^\circ$ visual angle. Sets of heterogeneous-size dots presented as input appeared inside a square, which covered a visual angle of 18.48° . Dots were spread of in a way to cover the entire area. The smallest dot had a visual angle of 0.44° and the biggest one a visual

angle of 4.62° . The distance between neighboring dots were at least one radius away from each other. Input or output glasses of water covered a $10.6^\circ \times 4.8^\circ$ visual angle.

1.4. Experimental procedure

In all tasks, participants were instructed to approximate, as fast as they could and without counting, the total sum of the 3 quantities presented sequentially and to produce the corresponding quantity using the potentiometer.

Within a single experimental session, the four tasks were administered following a Latin-square order between participants. For each trial, the exact timing of stimuli appearance is detailed in Figure 20. After the instruction, stimuli to be added (i.e., collections of dots, amount of liquids or Arabic numbers) were sequentially presented for 1500 ms. Between stimuli, the addition sign was presented for 500 ms. The last term to be added was followed by the equal sign for 500ms. Then, a "0"/empty rectangle from which the participant had to produce a response using the potentiometer appeared. Thus, depending on the task, participants flashed homogeneous-size black dots, amount of liquid in a glass or AN, ranging from 0 to 254 (for liquids, "254" correspond to a full glass; otherwise, portions of 254th are shown). When they were satisfied with their answer, they had to validate it by pressing the button of the potentiometer. Then, the instruction to go back to the '0' position was given.

Before beginning this experimental session, the participants attempted a computerized exact calculation task with the experimental setting presented above: seven two-term additions, for example $8 + 6 (= 15)$ and seven three-term additions, for example $6 + 9 + 18 (= 87)$ on Arabic numbers. Concerning those three-term additions, the correct answer on four out of the seven additions were dosed to experimental additions' correct answer. In this session no approximate answer was asked, the participants were told to process exact calculation by using the potentiometer as the response interface.

All the experiments took place on a PC-compatible portable (screen size: 30.5 cm × 23 cm) running E-Prime software (Schneider et al., 2002).

2. Results

2.1. Analysis of the computerized exact calculation task

The percentage of correct answer computed for the two-and-three-term additions is more important in the C group of children ($M = 52.63$, $SD = 7.61$) compared to the MLD group of children ($M = 46.24$, $SD = 10.75$) ($t(36) = 2.116$, $p = .04$). Reaction times for correct responses (calculated in milliseconds) are faster in the C group of children ($M = 12715$, $SD = 4737$) compared to the MLD group of children ($M = 17756$, $SD = 7983$) ($t(36) = -2.37$, $p = .02$).

2.2. The precision of approximate additions

To evaluate children's accuracy in the four approximate addition tasks, an absolute error score (AES) was calculated ($| \text{participant's estimate answers} - \text{target magnitude} |$). The median of this score was calculated on each target quantities. As we planned to run a repeated measures Analysis of variance (ANOVA) which assumed normal data distribution and variance homogeneity, we first verified those assumptions with Shapiro-Wilk's normality test and Levene's variance homogeneity test. Both indicated that AES did not completely fit these parametric assumptions. Therefore, logarithmic transformation was used in AES analyses ($\log(\text{AES})$) and met the normality conditions.

An ANOVA was performed on $\log(\text{AES})$ with target (34, 52, 82, 132) and task (homogeneous-size dots, heterogeneous-size dots, glasses of water and AN) as within-subject factors and group (C or MLD) as the between-subjects factor. The target effect was highly significant, $F(3, 108) = 77.10$, $\eta^2 = .68$, $p \leq .001$ and indicated that imprecision increased with the target magnitude ($\log(\text{AES}) = 1.00$, 1.10, 1.26 and 1.47 for targets 34, 52, 82 and 132, respectively). A significant effect of group was also found, $F(1, 36) = 28.71$, $\eta^2 = .44$, $p \leq .001$, revealing that MLD children ($M = 1.32$, $SD = 0.15$) were less

accurate (i.e., had higher log(AESs)) than their C peers ($M = 1.09$, $SD = 0.10$). The task effect was significant as well, $F(3, 108) = 56.15$, $\eta^2 = .61$, $p \leq .001$. Pairwise comparisons showed that all the differences, but the one between the homogeneous-size dots task and the glasses of water task, were significant. Indeed, participants were more accurate in the AN task ($\log(\text{AES}) = .93$) compared to the glasses of water task ($\log(\text{AES}) = 1.24$) ($p \leq .001$) which did not differ from the homogeneous-size dots task ($\log(\text{AES}) = 1.26$) ($p = 1$). Finally, they were more accurate in the homogeneous-size dots task than in the heterogeneous-size dots task ($\log(\text{AES}) = 1.42$) ($p \leq .001$).

Several interactions were significant and will be discussed below, guided by the questions raised in the introduction to this study: the tasks were found to interact with the groups, $F(3, 108) = 8.63$, $\eta^2 = .19$, $p \leq .001$ and with the target magnitudes, $F(9, 324) = 7.79$, $\eta^2 = .18$, $p \leq .001$.

To investigate further the Task $\times$ Group interaction, t -tests were performed in order to compare groups in each task. Groups were different on the homogeneous-size dots task ($t(36) = -2.75$, $p = .009$), glasses of water task ($t(36) = -3.22$, $p = .003$) and the AN task ($t(36) = -6.09$, $p \leq .001$), but not on the heterogeneous-size dots task ($t(36) = -1.07$, $p = .10$). A look at the Figure 21 showed that the observed difference between groups seems even more important in the AN task: the ANOVA comparing non-symbolic tasks (2 non-symbolic tasks $\times$ 2 groups) did not reveal an interaction of the task with the group (i.e., homogeneous-size dots and glasses of water, $F(1,36) = .24$, $\eta^2 \leq .01$, $p = .62$). However, the task interacted with the group significantly when including the AN task ($F(2,72) = 10.47$, $\eta^2 = .23$, $p \leq .001$). A linear regression analysis showed that the difference between non-symbolic and symbolic tasks is partly linked to the participant's exact arithmetic skill level (the percentage of correct answers in the exact tasks predicted the observed difference between the homogeneous-size dots tasks and the symbolic AN task, $r^2 = .339$; $F(1, 37) = 18.47$, $p \leq .001$). Finally, it is worth noting that in the C group, each task differs from the other ($p \leq .028$), except the homogeneous-size dots from the heterogeneous-size dots task ($p = 1.00$); in the MLD group, only the

heterogeneous-size dots task differs from the AN task ($p \leq .001$) and from the glasses of water task ($p = .035$).

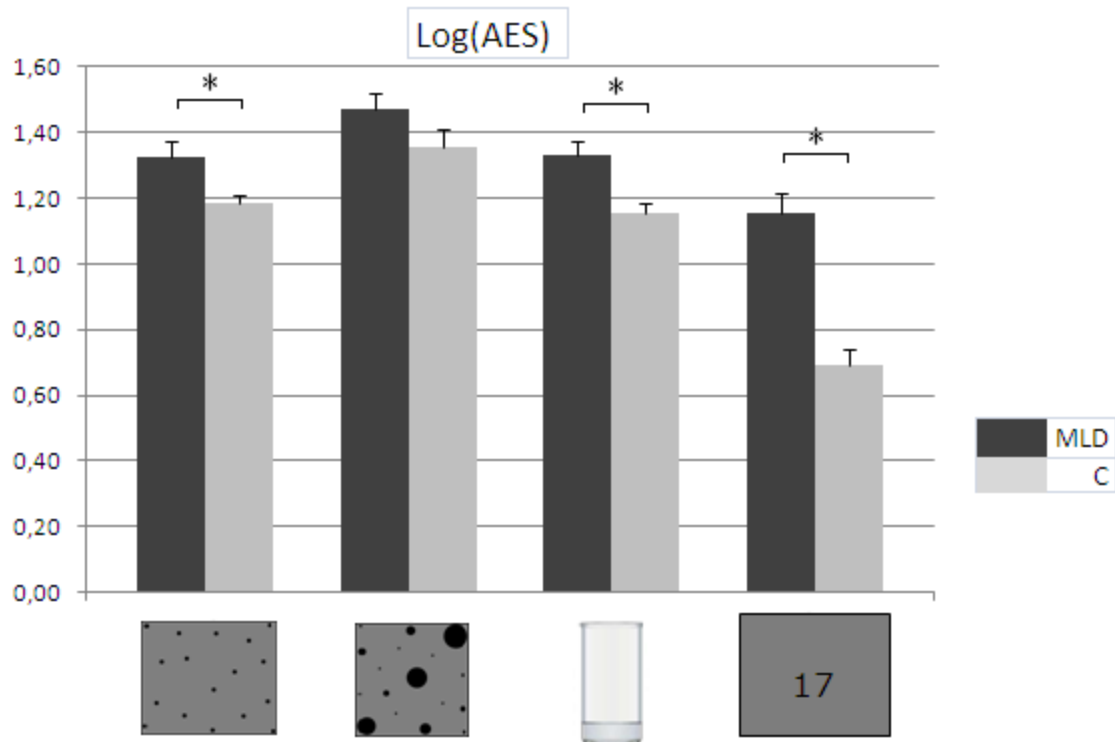


Figure 21: Log(AES) per group in each task (error bars indicate the standard errors of the mean). Significant difference between groups is represented by the stars.

To investigate further the *Target magnitude* $\times$ *Task* interaction, four separate ANOVAs testing the target effect were run on the log(AES) per task. The target magnitude effect was highly significant in all the numerical tasks (in the homogeneous-size dots task, $F(3, 111) = 45.75$, $\eta^2 = .55$, $p \leq .001$; heterogeneous-size dots $F(3, 111) = 22.81$, $\eta^2 = .38$, $p \leq .001$; and AN, $F(3, 111) = 44.01$, $\eta^2 = .54$, $p \leq .001$), but was only marginal in the glasses of water task ($F(3, 111) = 2.47$, $\eta^2 = .06$, $p = .065$). This indicates that this continuous magnitude is not exactly processed as the discrete numerical ones (Figure 22).

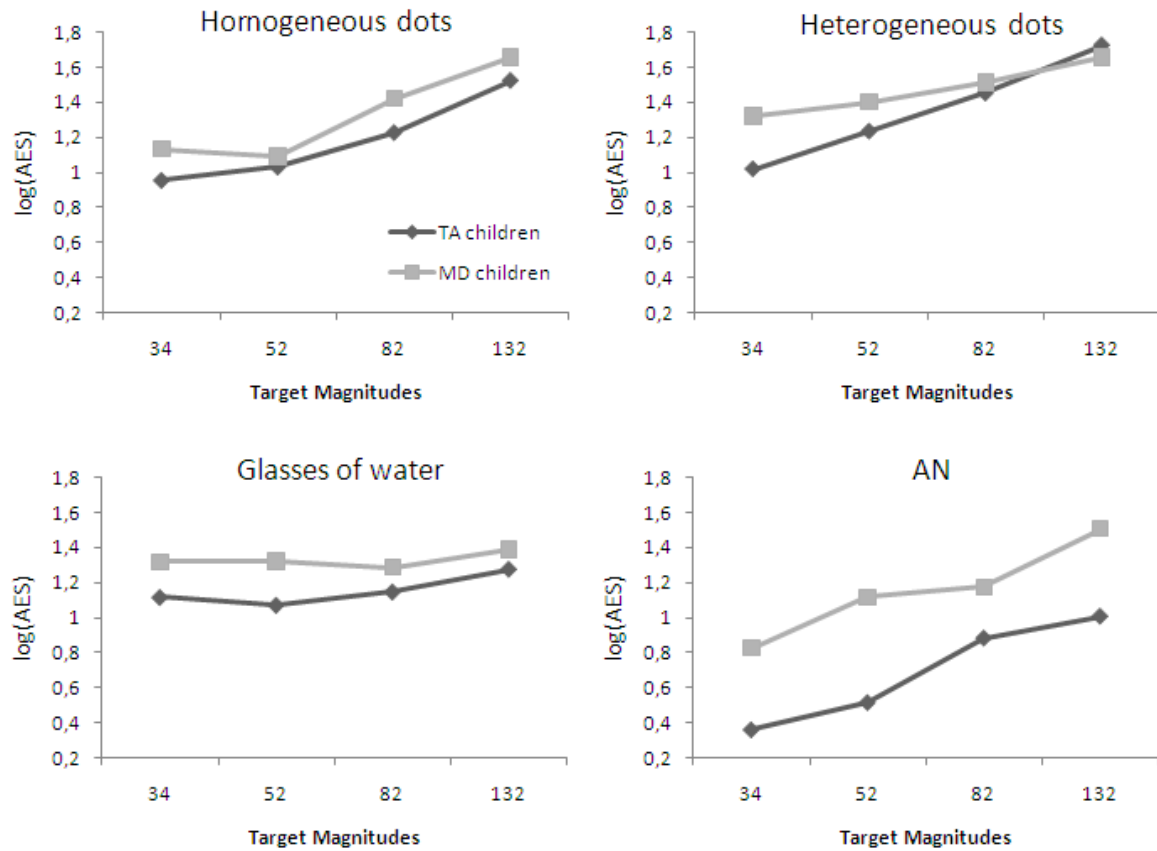


Figure 22: Log(AES) for MLD and C children in each task according to the target magnitude.

In order to control that participants did not use an exact calculation strategy to perform the tasks, we run an ANOVA on mean response times comparing the four approximation tasks and the exact calculation task. The ANOVA thus included the five tasks as within-subject factors and the two groups as between-subject factors. It revealed a group effect ($F(1, 56) = 4.36, \eta^2 = .11, p = .044$), a task effect ($F(4, 144) = 68.69, \eta^2 = .66, p \leq .001$) and a significant interaction between them ($F(4, 144) = 3.76, \eta^2 = .10, p = .006$). Control and MLD children did not differ in their response time in the 4 approximate tasks (all $ps > .17$) but in the exact calculation MLD children were slower than the C children ($t(36) = -2.37, p = .023$) (see Figure 23). Moreover, children were significantly slower to perform the exact calculation task compared to the symbolic approximate task ($p \leq .001$), suggesting the use of different strategies (exact versus computational estimation strategies). Finally, in the approximation

tasks, latencies did not differ between the three non-symbolic tasks ($p > .08$) but these led to shorter response time than the symbolic approximate task ($ps \leq .001$), revealing the possibility of different strategies use.

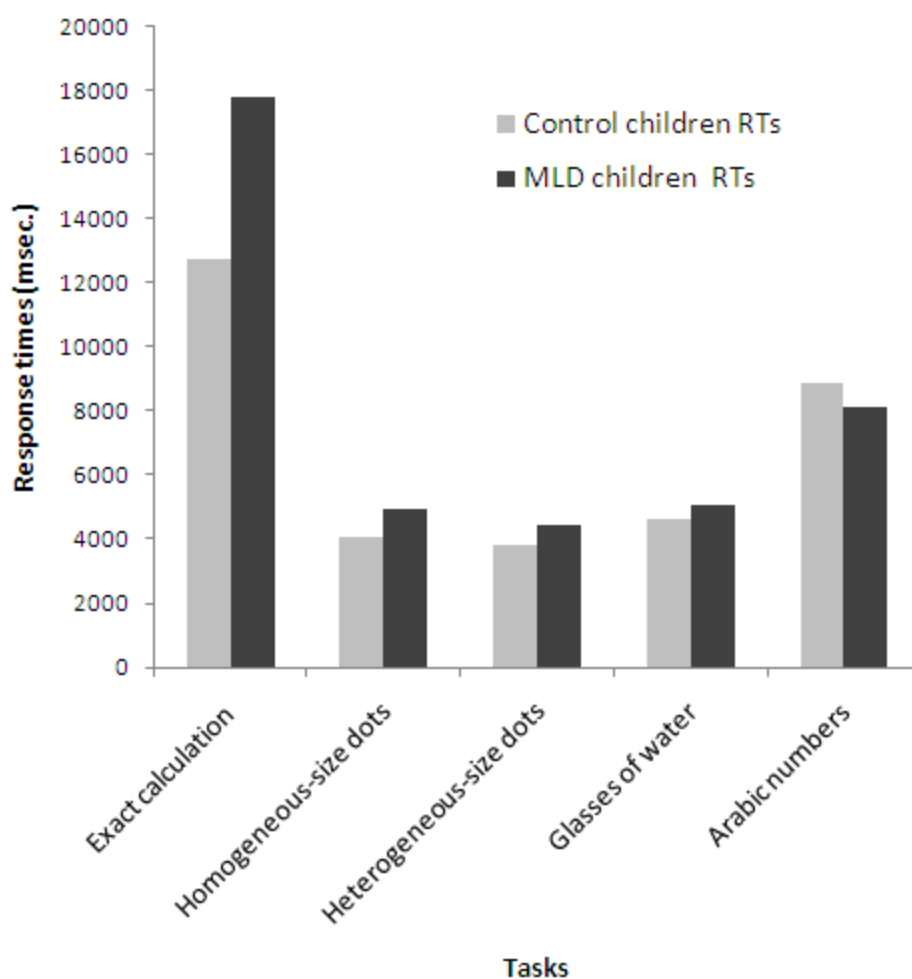


Figure 23 : Responses times (msec.) for MLD and C children in the exact calculation task and in each of the four approximate tasks.

Finally, those results are congruent with children's *reported strategies for the approximation tasks*. Although this has not been a systematic interview, self-reported strategies used by the children have been recorded for 10 children of both groups. In the AN task, children reported

systematically computational estimation strategies such as rounding or truncating the operands, and to adjust their response by turning more or less the track wheel. In the dots tasks, the one including heterogeneous size-dots is spontaneously reported as being more difficult than the homogeneous in 33% of the children equally distributed in each group. No child reported strategies based on the use of verbal numeral to resolve the task. Children who reported to verbalize (50%) used words like "a few dots plus more dots, plus a few dots again". The fifty other percents of the children reported to use perceptive strategies such as adding the three amounts of dots seen on the screen or bringing together all the dots. Those children used a lot of gestures to accompany their explanations. Strategies reported were equally distributed in both groups. Finally, for the glasses of water task, the strategy use seems to be entirely perceptive as children reported to add the three quantities presented into the empty glass, and often used their fingers to show the quantity.

Summary

Study 1 shows the typical AES increase with the target magnitude in both groups of children in all the three numerical estimation tasks but not in the glasses of water task that used continuous quantity.

MLD children were less accurate compared to C children no matter the format of the task, except for the heterogeneous-dots task, which seems to be really difficult for both group. If children are found to be less effective in the non-numerical continuous type of task, the glasses of water material may be not process as a numerical magnitude. Indeed, imprecision do not grow with the target as expected and must be further investigated. The actual profile of results observed in MLD children fit with the prediction of an ANS deficit as proposed by Wilson and Dehaene (2007) which may be extended to a larger magnitude system deficit (Walsh, 2003). However, it is worth noting that MLD children are even more in difficulties when it comes to the symbolic type of task.

Finally, the RT analyses showed that children did not compute the exact sum in the approximation tasks. However, they were significantly slower in the AN approximation task relative to the three other approximation tasks which suggest that they might have used different strategies in that condition, such as using exact arithmetic after rounding or truncating the operands (as reported in their comments).

Study 2

1. **Method**

1.1. Participants

A total of 34 Caucasian adults took part in the experiment: 17 of them were identified as having MLD and 17 were classified as controls (C). The two groups of adults did not differ in terms of gender (4 males and 13 females in the C group, compared to 3 males and 14 females in the MLD group, $\chi^2(1) = .18, p > .67$) or age (see Table 14).

Participants selection procedure and classification scheme

Adults' selection procedure and classification scheme was exactly as described in Chapter 5. As shown in Table 14, differences between groups were significant for ten among the fourteen arithmetic sub-scores, both for the total response time and the percentage of errors. On all but one of the 14 subtests, the mean score of the MLD and the C groups fell into the score range of respectively the MLD and C group of Rubinsten and Henik's (2005)¹⁵. Although scores on the memory span for pattern were in the normal range for both groups, C participants had higher scores than MLD participants. Therefore, scores on the memory span were used as covariate in further analyses.

1.2. Experimental tasks

The exact task and four approximation tasks as well as the instructions used were exactly the same as those used in study 1 of the present chapter.

¹⁵ Except for the complex additions in the MLD (our group was scoring -2.5 standard deviation below the mean of Rubinsten and Henik's MLD participants).

1.3. Stimuli

Only the number of stimuli used was different: correct responses corresponded to seven different target quantities (34, 42, 52, 64, 82, 104, 132). Six different additions per target response were proposed to participants.

Table 14 : Descriptive information and mean scores for the MLD and C groups of adults on the selection tasks for study 2.

	C group (N=17)		MLD group (N=17)		Statistical analyses	
	Mean (SD)	Range	Mean (SD)	Range	<i>t</i> -tests: $t_{(32)}$	Mean difference (95% confidence interval of the difference)
Descriptive information						
Age (in months)	25.59 (10.72)	19–54	25.82 (11.34)	18–54	-.06	-.24 (-7.94–7.47)
Memory span	8.76 (1.79)	7–12	7.17 (0.64)	6–9	3.45**	.46 (.65–2.53)
Battery of arithmetical tests (no of items)	Mean number of errors (SD)	Range	Mean number of errors (SD)	Range		
<i>Number facts</i>						
Addition (5)	0.06 (0.24)	0–1	0.29 (0.47)	0–1	-1.84	-.24 (-.50–.03)
Subtraction (5)	0.00 (0.00)	0–0	0.35 (0.61)	0–2	-2.40*	-.35 (-.65–.05)
Multiplication (5)	0.12 (0.33)	0–1	0.23 (0.44)	0–1	-.88	-.12 (-.39–.15)
Division (5)	0.12 (0.33)	0–1	0.23 (0.44)	0–1	-.88	-.12 (-.39–.15)
<i>Complex arithmetic</i>						
Addition (8)	0.53 (0.62)	0–2	1.35 (1.46)	0–5	-2.14*	-.82 (-1.61–.04)
Subtraction (8)	0.41 (0.62)	0–2	1.47 (1.28)	0–5	-3.07**	-1.06 (-1.76–.37)
Multiplication (8)	0.47 (0.51)	0–1	2.47 (2.32)	0–7	-3.47**	-2.00 (-3.17–.83)
Division (8)	0.71 (0.77)	0–2	2.18 (1.81)	0–5	-3.08**	-1.47 (-2.44–.50)
<i>Decimals</i>						
Addition (4)	0.18 (0.39)	0–1	0.71 (0.99)	0–3	-2.06*	-.53 (-1.05–.01)
Subtraction (4)	0.29 (0.59)	0–2	1.06 (1.14)	0–3	-2.45*	-.76 (-1.40–.13)
<i>Fractions</i>						
Addition (5)	0.06 (0.24)	0–1	0.47 (0.72)	0–2	-2.24*	-.41 (-.79–.04)
Subtraction (5)	0.47 (0.62)	0–2	0.71 (1.45)	0–4	-.62	-.24 (-1.01–.54)
Multiplication (5)	0.29 (0.98)	0–4	2.12 (2.00)	0–5	-3.37**	-1.82 (-2.92–.72)
Division (5)	0.53 (1.18)	0–4	2.94 (2.11)	0–5	-4.12**	-2.41 (-3.60–1.22)
Overall error rate (%)	5.59 (5.64)	1.25–26.25	20.66 (11.22)	2.5–40	-4.95**	-15.07 (-21.28–8.87)
Total response time (sec)	755.88 (284.22)	388–1316	1235.65 (370.65)	468–1758	-4.24**	-479.76 (-710.51–249.01)

Note. (*) = MLD significantly differs from healthy participants' at $p < .05$; (**) at $p < .01$.

2. Results

2.1. Analysis of the computerized exact calculation task

The percentage of correct answers computed for the two-and-three-term additions is higher in the C group of adults ($M = 81.5$, $SD = 13.61$) than in the MLD group of adults ($M = 71.85$, $SD = 14.61$; $t(32) = 1.99$, $p = .055$). Reaction times (in milliseconds) are lower in the C group ($M = 8752$, $SD = 2486$) than in the MLD group ($M = 11074$, $SD = 5187$), although this difference is not significant ($t(32) = -1.66$, $p = .10$).

2.2. The precision of approximate additions

The log (AES) was calculated as the measure of overall accuracy just as it has been done in Study 1. An ANCOVA was performed on log(AES) with targets (8, 12, 16, 21, 26, 34, 64) and tasks (homogeneous-size dots, heterogeneous-size dots, glasses of water, AN) as within-subject factors, 2 groups (C and MLD adults) as a between-subjects factor and visuo-spatial memory span as a covariate. The analysis revealed a significant target effect, $F(6, 186) = 2.41$, $\eta^2 = .07$, $p < .029$, showing that imprecision increased with the magnitude of the target magnitude (log(AES) = .787, .863, .915, .963, 1.035, 1.131, and 1.210 for target 34, 42, 52, 64, 82, 104 and 132 respectively). A significant effect of group was found, $F(1, 31) = 4.85$, $\eta^2 = .16$, $p = .035$, indicating that MLD adults ($M = 1.038$, $SD = .088$) were less accurate (i.e., had higher AESs) than C adults ($M = .934$, $SD = .135$). There was also a significant task effect, $F(3, 93) = 4.16$, $\eta^2 = .12$, $p = .008$. Pairwise comparison showed that participants were more accurate in the AN task (log(AES) = .41) compared to glasses of water task (log(AES) = 1.12) ($p < .001$) that did not differ from the homogeneous-size dots task (log(AES) = 1.16) ($p = 1$). Finally, the homogeneous-size

dots task differ only marginally from the heterogeneous-size dots task ($\log(\text{AES}) = 1.25$) ($p = .065$).

Two interactions were also significant and will be discussed below: the tasks by groups interaction, $F(3, 93) = 3.21$, $\eta^2 = .09$, $p = .027$; and the target magnitudes by tasks by groups interaction, $F(18, 558) = 1.80$, $\eta^2 = .06$, $p = .023$.

To investigate further the Task $\times$ Group interaction, t -tests were performed in order to compare groups in each task. Groups significantly differed only on the AN task ($t(32) = 2.12$, $p = .042$; but not on the homogeneous-size dots task, $t(32) = .46$, $p = .65$, glasses of water task, $t(32) = 1.89$, $p = .070$, and the heterogeneous-size dots task, $t(32) = .43$, $p = .67$).

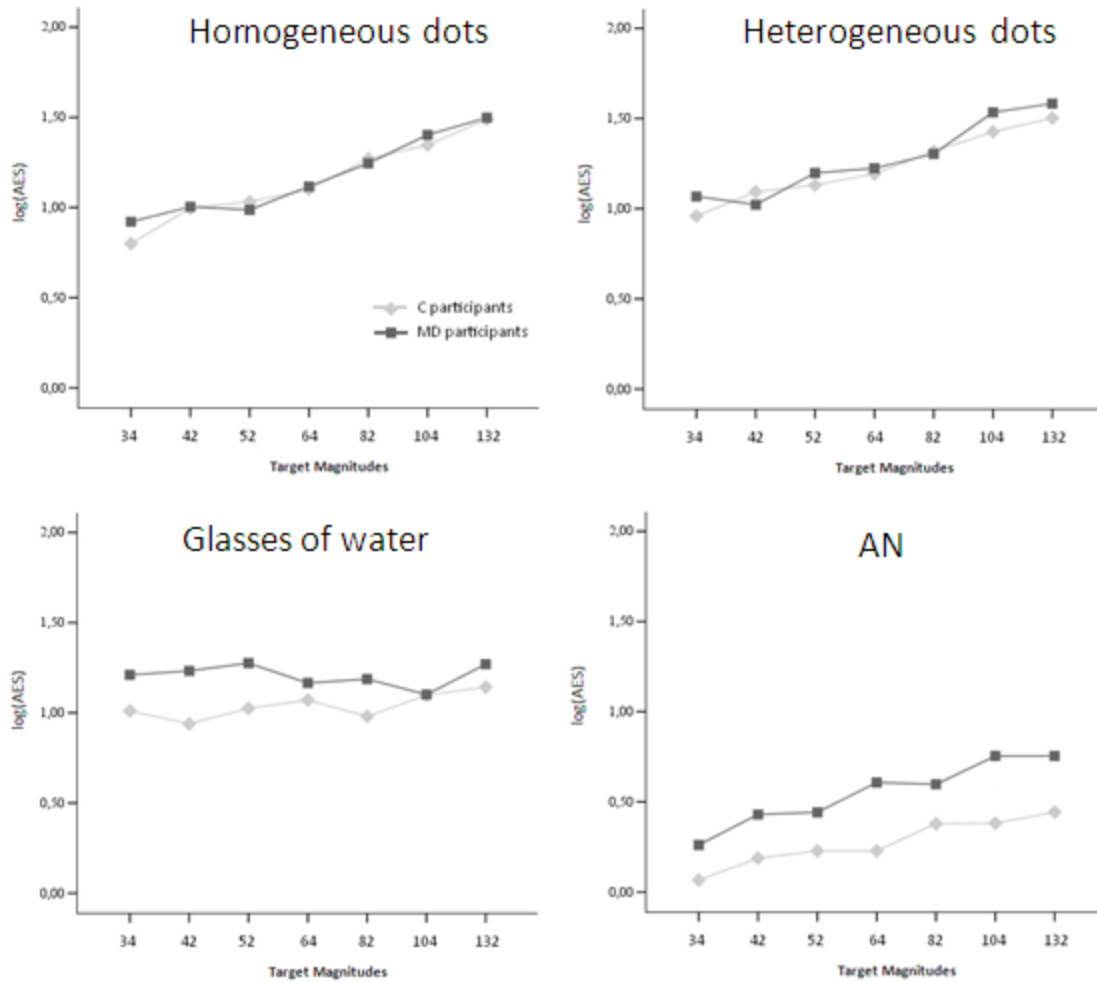


Figure 24: Log(AES) for MLD and control participants in each task according to the target magnitude.

To investigate further the triple interaction *Target magnitudes x Tasks x Groups*, ANCOVAs (7 targets and memory span as a covariate) for separated group on each task were performed. However, both groups showed congruent results: MLD and C adults participants performance in the four different tasks revealed a target magnitude effect in the three numerical tasks: the imprecision of estimates increased globally with the target magnitude in the homogeneous, heterogeneous-size dots and AN tasks (for the C group: $F(6, 90) = 42.18$, $\eta^2 = .57$, $p < .001$, $F(6, 90) = 24.21$, $\eta^2 = .43$, $p < .001$ and $F(6, 90) = 11.54$, $\eta^2 = .27$, $p < .001$, respectively and for the

MLD group: $F(6, 90) = 42.18$, $\eta^2 = .57$, $p < .001$, $F(6, 90) = 24.21$, $\eta^2 = .43$, $p < .001$ and $F(6, 90) = 11.54$, $\eta^2 = .27$, $p \leq .001$, respectively), but not in the glasses of water task (for the C group: $F(6, 90) = 2.02$, $\eta^2 = .12$, $p = .11$; for the MLD group: $F(6, 90) = 1.18$, $\eta^2 = .07$, $p = .32$, see Figure 24). When performing ANCOVA by task (2 groups x 7 targets), the interaction was significant in the glasses of water task ($F(6, 186) = 2.81$, $\eta^2 = .08$, $p = .012$). However, no specific effect is identified (such as an increase in the log(AES) based on the target).

In order to control that participants did not use an exact calculation strategy to perform the tasks, we run an ANCOVA on mean response times comparing the four approximation tasks and the exact calculation task. The ANCOVA included the five tasks as within-subject factors, the two groups as between-subject factors and the visuo-spatial memory span as covariate. It only revealed a task effect ($F(4, 124) = 4.98$, $\eta^2 = .14$, $p \leq .001$). Pairwise comparison showed that participants were significantly slower to perform the exact calculation task ($M = 9913$, $SD = 4175$) compared to the AN approximate task ($M = 5982$, $SD = 2688$, $p \leq .001$), suggesting the use of different strategies (exact versus computational estimation strategies). Finally, the AN approximate task led to longer response times compared to the three non-symbolic tasks ($p \leq .001$) revealing again the set-up of different strategies, while latencies to perform the two non-symbolic numerical tasks were not different (homogeneous-size dots tasks, $M = 3616$, $SD = 1682$, versus heterogeneous-dots tasks ($M = 3566$, $SD = 1548$, $p > .1$). Finally, the glasses of water task ($M = 4255$, $SD = 1901$) took more time to the participants to answer comparing to the homogeneous-size dots task ($p = .043$), but not compared to the heterogeneous one ($p = .13$).

Summary

Study 2 shows the typically AES increase with the target magnitude in both groups of adults in the three numerical tasks but not in the glasses of water task.

MLD adults were less accurate compared to C adults in the AN task. Besides, adults were found to be marginally less effective in the non-numerical continuous type of task; that is the glasses of water one. However, this task seems to be processed differently as imprecision do not grow with the target as expected. The profile of results observed in MLD adults fits with the prediction of an access deficit as proposed by Rousselle and Noël.

Regarding the RT analyses, similar results as those observed in children are found, i.e., exact calculation strategies are not used regarding the process of approximate arithmetic on non-symbolic materials. Different strategies seem to be set up regarding the type of material.

Comparison of adult and children's data

Children's and adults' performances. As the same estimation tasks were used with children and adults, results give us the opportunity to explore the changes in these measures through development in the C and MLD groups. Accordingly, ANCOVAs were carried out for (log)AES, with 4 targets (34, 52, 82, 132) and 4 tasks (homogeneous-size dots, heterogeneous-size dots, glasses of water, AN) as the within-subject factors, and 2 cohorts (children and adults) and 2 groups (C and MLD participants) as between-subjects factor and visuo-spatial span as a covariate.

A significant target effect was revealed, $F(3, 201) = 4.46$, $\eta^2 = .06$, $p = .005$, showing that imprecision increased with the magnitude of the target (log(AES) = .898, 1.006, 1.149 and 1.340 for target 34, 52, 82, and 132 respectively). A significant effect of group was found, $F(1, 67) = 28.44$, $\eta^2 = .30$, $p \leq .001$, indicating that MLD participants ($M = 1.181$, $SD = .196$) were less accurate (i.e., had higher AESs) than C participants ($M = 1.015$, $SD = .128$). A cohort effect was found as well, $F(1, 67) = 33.56$, $\eta^2 = .34$, $p \leq .001$, indicating that children ($M = 1.201$, $SD = .170$)

were less accurate than adults ($M = .995$, $SD = .128$). There was also a significant task effect, $F(3, 201) = 3.81$, $\eta^2 = .05$, $p = .011$. Pairwise comparison showed that participants were more accurate in the AN task ($\log(\text{AES}) = .66$) compared to glasses of water task ($\log(\text{AES}) = 1.19$) ($p \leq .001$) which did not differ from the homogeneous-size dots task ($\log(\text{AES}) = 1.21$) ($p = 1$). Finally, the homogeneous-size dots task differ from the heterogeneous-size dots task ($\log(\text{AES}) = 1.34$) ($p \leq .001$).

Moreover, the tasks by groups interaction was significant ($F(3, 201) = 7.69$, $\eta^2 = .10$, $p \leq .001$). This is congruent with previous analyses: MLD participants are less precise than C participants in the homogeneous-size dots task ($t(70) = -2.03$, $p = .046$), the glasses of water task ($t(70) = -4.281$, $p \leq .001$) and the AN task ($t(70) = -4.22$, $p \leq .001$) but not in the heterogeneous-size dots task ($t(70) = -1.881$, $p = .064$).

The tasks by cohort interaction was significant as well ($F(3, 201) = 15.68$, $\eta^2 = .19$, $p \leq .001$). The imprecision was greater in children than in adults in the homogeneous-size dots task ($t(70) = 2.52$, $p = .014$), the heterogeneous-size dots task ($t(70) = 3.50$, $p = .001$), the glasses of water task ($t(70) = 2.17$, $p = .003$) and the AN task ($t(70) = 7.49$, $p \leq .001$). The difference between participants was stronger in the AN task (mean $\log(\text{AES})$ difference: .53) highlighting a important gain in adults in this task, compared to the homogeneous-size dots task (mean $\log(\text{AES})$ difference: .10), the heterogeneous-size dots task (mean $\log(\text{AES})$ difference: .16) and the glasses of water task (mean $\log(\text{AES})$ difference: .10).

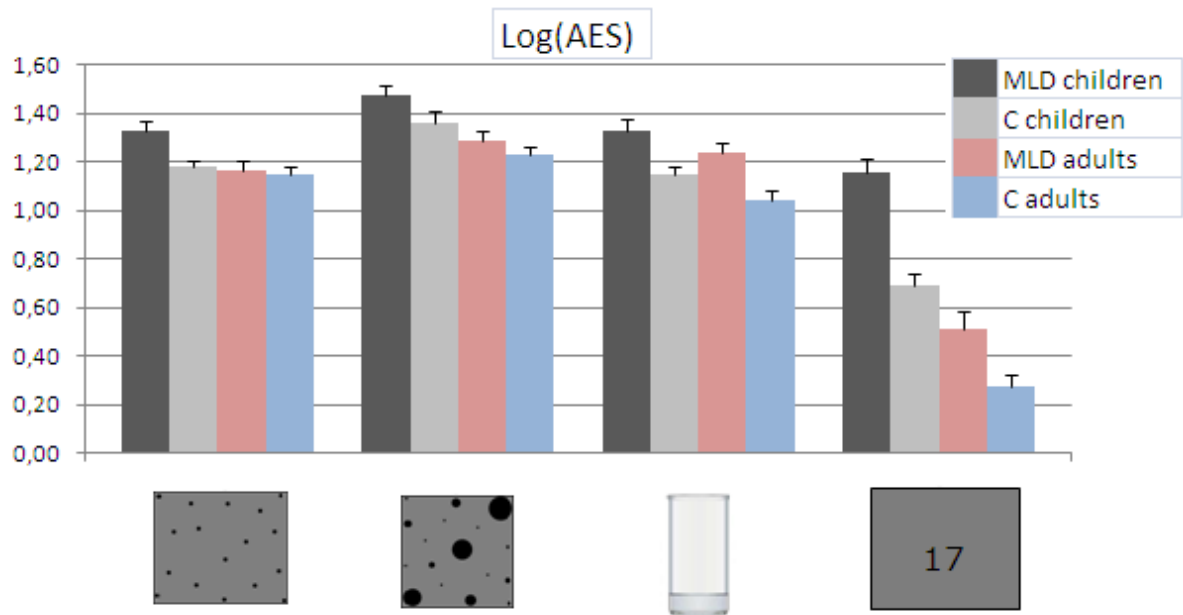


Figure 25: Log(AES) per group and cohort in each task.

2.3. Correlation between math performance and approximate abilities

Pearson product-moment correlation analyses were calculated between mathematical performances (i.e., the performance on the mathematical scale for the group of children and the overall error rate on the battery of arithmetical tests for the adults) and individual log(AES)s. Significant correlations were obtained between math performances in children and their precision in the Homogeneous-size dots, glasses of water and AN estimation tasks. No significant relationships were found in adults (see Table 15).

Table 15: Correlations between mathematical performance in children and adults and their log(AES)s in the four approximation tasks

				
Correlation for children	.327*	.296	.437**	.680**
Correlation for adults	.047	.05	.167	.051 ¹⁶

Note. * Significant at the .05 level (2-tailed); ** significant at .01 level (2-tailed).

3. Discussion

It is well known that MLD individuals exhibit poor arithmetical abilities and that it is in their own interest to get a quick feed-back on their aberrant answers. The competency to give a plausible approximate answer to an arithmetical operation requires the understanding of the meaning and properties of that operation. It also requires the access to the ANS. Examining MLD and typically developing individuals' performance on different type of approximate operations gives some thoughts about the integrity of their magnitude representations. This is of interest regarding the multiple explanations for dyscalculia: they assumed a basic deficit affecting the representation of a number magnitude (Butterworth, 1999, 2005; Wilson & Dehaene, 2007) which could potentially extend to a larger magnitude system (Walsh, 2003) or affecting the access to a number magnitude from numerical symbols (Rousselle & Noël, 2007). However, to date, it is impossible to disentangle between the different hypotheses accounting for MLD.

¹⁶ A correlation between performance in mathematics and the AN approximation task in adults could not be demonstrated either. However, this correlation emerged as significant for the MLD subgroup of participants ($r = .639$; $p = .006$). The correlation could not be demonstrated by the control subgroup ($r = .353$; $p = .165$) because the ceiling effect arising from the arithmetical test which was insufficiently challenging.

The two studies presented in this chapter aimed at examining children and adult's abilities to approximate the result of different types of complex additions involving numerical and non-numerical magnitude. Accordingly, it gives us the opportunity to evaluate the integrity of magnitude representations as well as the access to them.

Accessing the approximate number system

First of all, this paper shows that children and adults are able to approximate the sum of three collections of dots, quantity of water or AN presented sequentially. Moreover, the three numerical types of tasks modelize their approximate number representation: the shape of participants' answers gave rise to an increasing AES (i.e., the imprecision of their answers increases with the size of the correct answer), except in the continuous non-numerical type of material. This confirms that numerical tasks and the different populations who were approached in the present studies give us the opportunities to access and examine that ANS separately for MLD and C children and adults, when varying numerical material notations. However, these results give the non-numerical task special status.

Contrasting the different hypotheses

By studying individuals' accuracy when operating on approximate numerical and non-numerical magnitudes gives us the opportunity to consider directly the impact of the magnitude format on MLD and C participants and to confront the different hypotheses. First, are MLD participants suffering from an access deficit to the ANS when dealing with symbolic material uniquely as proposed by Rousselle and Noël (2007)? Second, is their ability to approximate preserved compare to their ability to process exact calculation task (Butterworth, 1999, 2005; luculano et al., 2008)? Third, are they suffering from an analog deficit? If they do, they are

expected to be impaired in accessing the ANS either with symbolic and non-symbolic numerical materials (Wilson & Dehaene, 2007). Finally, are they having difficulties in carrying out tasks of approximating non-numerical quantities as well? This is expected if the ANS supports a system that exceeds the field of numerical magnitudes (Walsh, 2003).

Current findings are contradictory and it is still debated whether or not individuals with MLD have a deficit in representing and manipulating exact or approximate number magnitudes or disturbed connections between the core representation and the number symbols.

Our findings, regarding the performances of children with MLD, are in line with the analog deficit hypothesis (Wilson & Dehaene, 2007). This deficit can be extended to a larger magnitude representation as they also were impaired when dealing with continuous quantities. Indeed, they were accessing an approximate representation giving rise to less accuracy in their answers compared to C children in the four different tasks involving numerical and non-numerical magnitudes.

This is in accordance with previous results from non-symbolic numerical comparison tasks testing the same age range subjects. Piazza and collaborators (2010), showed that 10 year old MLD children scored at the level of 5 year old typically achieving children. Similarly, same age-range MLD children showed higher error-rates for numbers which were close together, resulting in a steeper slope of the numerical distance (Mussolin et al., 2010b; Price et al., 2007). Taken together, results from comparison tasks and our present results favor the hypothesis of a weaker ANS for MLD relative to C peers, whatever the format of magnitude presentation.

However, our findings concerning MLD children are contradictory compared to recent ones, which favour the access deficit hypothesis. By using the verification of approximate additions on

symbolic and non-symbolic numerical materials, De Smedt and Gilmore (2011) showed that 6 year old MLD children were impaired with the symbolic material. By using the novel type of tasks to investigate the MLD participants' ANS, that is a free production tasks using 3-addend complex additions presented in symbolic and non-symbolic notation, we believe to be closer to the scholar context of learning. This is of interest to make sure to stick with the reality of a mathematical disability as diagnosed by the available and actual tests. Accordingly, experimental tasks were found to highly correlate with children's mathematical performances. More importantly, it gives the opportunity to index participants' ANS with no external constraint contrarily to the usually proposed verification tasks (where the sum differed from the comparison numbers by several pre-defined ratios). On one hand, it gives us the opportunity to confound different perceptual variables and to ensure that participants do not use perceptive cues instead of using the numerical ones (i.e., by mixing the use of heterogeneous-size dots in the input and using homogeneous ones in the output prevent from such a bias). On the other hand, the use of a free production paradigm allowed us to grasp more finely the magnitude representation (e.g., Castronovo & Seron, 2007; Cordes et al., 2001). Previous research might not found MLD versus C group differences because the type of task they used.

Hence, it seems quite possible that the amount of ratio used and the choice of these ratios by De Smedt and Gilmore (2011) have allowed identifying only an important difference between the groups performance; that is concerning the symbolic task. Indeed, two out of the three ratios were quite large and easy to perform: 4:7 is successful by infant under 10 months and 4:6 by children under 3 years. The last one, 4:5, is usually successful with children from ages 5 to 10. Concerning the symbolic task, one can imagine (as data by ratio are not provided in the study) that the different trials of the two first ratios are succeed by the large majority of the children; for the last ratio (i.e., 4:5), those low-achieving children fail almost all trials, the children with

mathematical difficulties succeed some of them and one third of the trials are successfully manage by the C children (according to De Smedt and Gilmore data, this gives rise about 78% of correct answers for control children, 68% of correct answers for MLD children and 66% for the low-achieving children, according the symbolic task, all ratios combined). Therefore, significant difference between groups pops-out (without Ratio x Group interaction). Concerning the non-symbolic task, the majority of the children which remain in the analyses (as the authors remove from the statistical analyses those who had really poor performances on that task, this is one fourth of the children with severe mathematical difficulties) may have failed most trials from the 4:5 (congruent with the Ratio x Task interaction) and some of the 4:6 ratios, the children with severe mathematical difficulties experiending more difficulties compared to the C children (this gives rise about 68% of correct answers for C children, 66% for the low-achieving and 63% for MLD children) but this time, even if a difference exist between MLD and C children, it does not appears as significant. Using a free-estimate production task enabled us to examine the ANS without having to define a specific numerical range to look at.

Moreover, using the median accuracy for a given target (participants produced six answers per target) gives the possibility to leave the outlying data aside (i.e., without ruling out a participant which might present a deep rooted defidit of the analog magnitude representation).

As already mentioned, the present results do not replicate Rousselle and Noël' specific symbolic access deficit hypothesis in MLD children (2007), as MLD children were presenting less accuracy in the non-symbolic tasks as well. However, an important difference between magnitude processing in C compared to MLD children needs to be quoted. That is the symbolic task being best performed than the non-symbolic tasks, mainly by the C children already mentioned above and present by C children in De Smedt and Gilmore studies as well (2011). It is

important to note that a task consisting of Arabic numbers appears to be facilitator when children have a normal learning.

This seems to have important implication to draft appropriate intervention, especially as if MLD children were less accurate in non-symbolic tasks compared to C children, this is not true in adulthood anymore. According the typically achieving population's abilities in the homogeneous-dots task, a maximal accuracy is reached around age 10. This result fits with previous data as it has been found that in non-symbolic comparison task using visuo-spatial stimuli, the estimated Weber fraction (w) is quite similar at age 6 ($w = .18$, Halberda & Feigenson, 2008) compared to French western adults ($w = .17$, Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004) or Indigene adults ($w = .17$, Pica et al., 2004). When using non-symbolic material, especially when two sets to be compared are presented simultaneously and in this configuration of dots pattern, it is always possible that participants' decision is influenced or even based on perceptual variables that co-vary with numerosities (but see Dormal and Pesenti, 2009 for the use of linear array of dot arrangements which prevent from such a bias). Indeed, it has been shown that it is much easier for children to compare two sets in which the surface area occupied by the items is confounded with the numerosity than to compare two sets in which this dimension is controlled (Rousselle & Noël, 2007; Rousselle, Palmers & Noël, 2004). This also can stand as an explanation why previous research might not found MLD versus C group differences because of the materials they used. Their non-symbolic comparison tasks carried out at a particular age (between 1st and 4th grade) were perhaps not discriminating because they might already had reach a maximal accuracy.

Examining the adulthood: which evolution?

Study 2 aimed at characterizing the longevity of difficulties presented by MLD during childhood. Reported data favor the access deficit hypothesis (Iuculano et al., 2008; Rousselle & Noël, 2007). Indeed, the profile of results observed in MLD adults fit with the prediction of this hypothesis as MLD adults were less accurate compared to C adults in the AN approximate and exact tasks. Besides, adults were found to be marginally less effective in the non-numerical continuous type of tasks; the glasses of water one. The impoverished magnitude representation system in MLD children may evolve and the magnitude representation deficit may no longer be present in adults with MLD. This is congruent with the idea of a developmental delay which lies at the origin of this learning deficit.

Moreover, the hypothesis of a long-lasting deficit of the symbolic representation in the MLD adulthood is also supported by several data. Poor arithmetical skills are reported at different stages of development (Shalev et al., 2005), the MLD participants from Study 2 scored significantly worse than the C participants on the arithmetical test and showed persistent deficit regarding the symbolic material. Those results are congruent with the idea that even if symbolic representations develop by being mapped onto the pre-existing ANS, the symbolic representation is separated from that ANS and can be affected separately. The ANS primitive system evolves through development and regarding the limit of the human system. If a deficit occurs, one can expect that the individual will catch its late. On the other side, the symbolic learned system develops only if it is stimulated (see for example the Mundurucu data, Pica et al., 2004). Even if its development depends on the ANS good one, the link between the ANS and the symbolic representation present in children tend to disappear when the arithmetical rules are learned and automatized. Congruently, log(AES) from the non-symbolic approximation tasks are

not any longer correlated with arithmetical abilities in adults. Finally, individuals presenting difficulties regarding the symbolic system are not able to catch their late as easily.

In summary, results from Study 2 reproduce the already mentioned symbolic representation deficit in MLD adults (Ashkenazi et al., 2009; Rubinsten & Henik, 2005). However, we went further by using a wider set of tasks. This long lasting deficit contrasts with the developmental retardation that has been filled regarding the ANS deficit observed in childhood.

Only longitudinal studies could possibly determine if the impoverished magnitude representation observed in children with MLD comes from a first mapping deficit between number symbols and magnitude representation or if two different populations (i.e., with a magnitude representation versus a symbolic representation deficit) have been enrolled in these studies.

As the exact arithmetic would develop on the basis of the innate ANS and as the systems seem to have limited influence on each other (see Verguts & Fias, 2004; Verguts et al., 2005), distinguishing potential profiles resulting in dyscalculia would be of interest for designing an appropriate intervention. Finally and to extend the difficulties highlighted regarding the continuous material, it should be of interest to investigate further through a developmental perspective the MLD aptitude to deal with other continuous material such as time duration (see for example, Droit-Volet, 2000).

Chapter 7:

General Conclusions

The main purpose of the present thesis was to investigate why dyscalculic children present difficulties resolving tasks that tap into the understanding of magnitude representation. We proceeded by examining which of the current models of dyscalculia fits the best with the results reported in the literature and with those presented in this thesis. The definition of the concept of “magnitude representation” varies from one author to another. These distinctions will be emphasized throughout the Discussion. Because of the importance of mathematical abilities for a successful living, we also examined whether the deficits shown by MLD children persist over time. More precisely, we investigated whether adults who had experienced MLD as children still show specific difficulties in magnitude representation. This should allow us to characterize the longevity of their potentially remaining difficulties.

In the ensuing sections, we describe the interest of the novel and original method used through our experiments. Then, an overview of the results reported in Chapters 4, 5 and 6, as well as their theoretical implications are presented. This thesis ends with some suggestions for practice and future research.

1. Methodological considerations

Through Chapter 2, we examined the different tasks used to index the ANS. The review of the data collected with the two tasks commonly used, i.e., numerical comparison and numerical estimation, led us to formulate several criticisms and to propose self-production estimation tasks that show significant advantages.

This type of tasks consisted in the generation of an estimate answer in response to a single target magnitude (Chapters 4 and 5); or the generation of an approximate answer in response to

multiple target magnitudes presented sequentially (Chapter 6). To ensure the comparison of the results obtained with different materials or format of presentations (using symbolic or non-symbolic notations, discrete or continuous materials), the range of the numerosities and the response interface were kept constant across tasks.

Estimation tasks allow individuals to process and represent approximately quantities, irrespective to the input and output notation. This type of tasks models the ANS and gives clues about the variability (i.e., coefficient of variation) and the precision of the ANS (i.e., accuracy of the numerical estimate): classically, participants' answers indicate that the absolute error score increases with the target magnitude. This effect was replicated with our self-production estimation tasks in all our experiments, confirming that the numerical tasks we tested gave us the opportunity to study the ANS in MLD children and adults, across different notations.

The interface of the self-production estimation tasks, named the potentiometer, was provided with a track wheel (for flashing images ranging from 0 to 254, with an increment of 1 on the screen of a computer by turning the wheel clockwise, and a button to validate the answer). It allowed participants of any age to give their answer easily, under the control of Eprime, and its attractive design ensured a high level of motivation. Finally, the response interface included on-line visual feedback, allowing participants to adjust their answers while keeping them approximate (this was ensured by the sensitivity of the track wheel as it was really hard to stop on a magnitude n rather than $n+1$ or $n-1$). The present device therefore presents the advantage to extract the closest estimate of the ANS in each participant.

Moreover, our paradigm implies that the numerosity to be estimated and the numerosity representing the response on the screen are presented sequentially, whereas classical comparison tasks use a simultaneous presentation of the two numerosities to compare. This

allowed us to minimize the influence of the perceptual variables (which was even reinforced by mixing the use of heterogeneous-size dots in the input and using homogeneous ones in the output).

Whereas it has been reported that adults are able to process the numerosity rather than perceptual dimensions of dot arrays (Dormal & Pesenti, 2009) younger children were found to rely on perceptual cues (Rousselle & Noël, 2008). Because it has been proposed that MLD may correspond to a developmental delay (e.g., Wilson & Dehaene, 2007), it was necessary to control for perceptual cues in the stimulus display not only for children but also for adults with MLD.

However, while controlling this issue, previous estimation tasks failed to operationalize the ANS appropriately, because they constrain participants' performance in undesired ways. For example, pointing to a position of a physical number line may encourage participants to use specific strategies relative to the endpoints. Numbers close to the right end of the physical number line tend to be better estimated than numbers in the middle of the line, as they do not benefit from external marks (Siegler & Opfer, 2003). Such a bias should not be present in tasks designed to assess the ANS. Accordingly, we wanted a device with no endpoint or at least a material that gave participants the illusion of being able to produce infinite responses from zero. The endpoint was actually fixed at "254" and was chosen far from the biggest possible answers ("64" in the two first experiments and "132" in the last ones). None of the participants produced an answer in the neighborhood of 254. Another potential bias to avoid comes from another type of self-production estimation task that requires pressing a key in response to a target magnitude. In this task, participants are asked to say aloud a word ("the" for example; see Cordes et al., 2001) simultaneously to each press to prevent the use of subvocal articulation of

counting words. However, as we examined an atypical population that can show associate deficits as dyslexia or impaired executive functions (for a review, see Geary, 2004), we wanted to reduce as possible the cognitive load of the task and the use of linguistic processes which could lead to non-specific effects in magnitude processing.

Finally, the use of a self-production paradigm allowed us to grasp more finely the magnitude representation (e.g., Castronovo & Seron, 2007; Cordes, et al., 2001). Comparison tasks usually include a limited amount of ratios between numerosities because of time constraints. Consequently, because of this restricted choice, only important differences are expected to emerge from the comparison between groups. Using self-estimate production tasks enabled us to examine the ANS without having to define a specific numerical range to look at. The use of the median accuracy for a given target ($N=6$) gave us the possibility to control for the presence of outliers without excluding a participant.

2. Overview of the results

Some authors proposed that MLD is due to a specific deficit of some types of number representations. In line with this perspective, two hypotheses postulate that MLD children have a very specific deficit of the number magnitude representation but the nature of that core system differs depending on the authors (Butterworth, 1999, 2005; Landerl, Bevan, & Butterworth, 2004; Wilson & Dehaene, 2007). First, according to Butterworth and his collaborators (Butterworth, 1999, 2005; Landerl, et al., 2004), we are born with a “number module” that represents numerosities exactly (e.g., exact “sixness”). According to these authors, MLD could be seen as an *exact number representation deficit*. Second, another view shared by many authors (e.g., Dehaene, 2003; Gallistel & Gelman, 1992) assumed that the core magnitude system, shared with animals and already present in infants, is not exact but approximate.

Nevertheless, exact arithmetic would develop on the basis of this innate ANS as the precision of this approximate representation is related to performance in exact calculation and number processing (Halberda, Mazocco, & Feigenson, 2008, but see Holloway & Ansari, 2009). A deficit at that level, named it *the ANS deficit hypothesis*, would lead to MLD (Wilson & Dehaene, 2007). In contrast with these two views assuming a core information deficit, Rousselle and Noël (2007) have proposed the alternative *access deficit hypothesis*, which stands for an access deficit to the intact ANS from symbols (such as Arabic digits or number words). Finally, an alternative view of the magnitude representation proposed that numerical estimates are computed by a common system for the representation of time, space and other magnitudes (Bueti & Walsh, 2009; Walsh, 2003).

According to the ANS deficit hypothesis (Figure 26 panel a), MLD participants should show lower performance in all numerical estimation tasks since they all involve the ANS. The access deficit hypothesis predicts a deficit in task accessing the ANS from symbols (Figure 26 panel b) and a larger deficit hypothesis predicts impairments dealing with different kinds of magnitudes (Figure 26 panel c). Following the exact deficit hypothesis, no difference between typical and atypical developing children should occur as no exact processes are required to perform our experimental tasks.

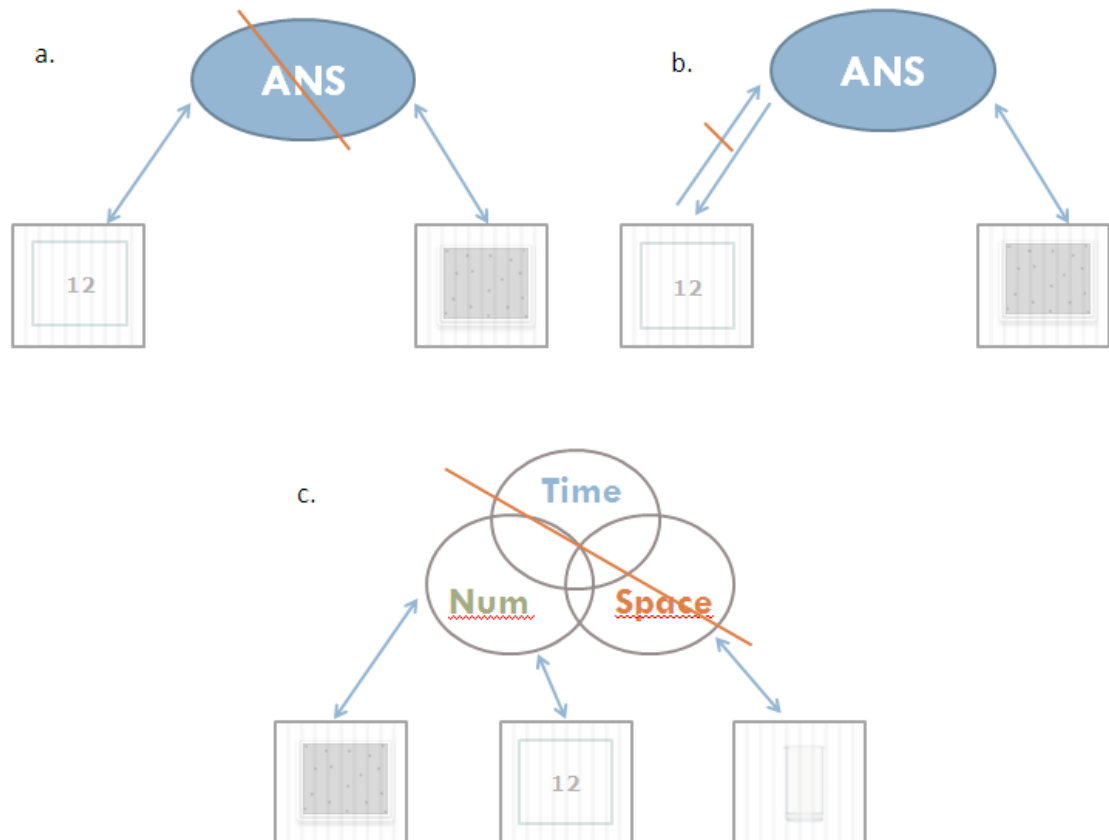


Figure 26: Expected deficits regarding the different hypotheses; a. the ANS deficit hypothesis, b. the access deficit hypothesis and c. a larger magnitude system deficit.

The main goal of **Chapter 4** was to evaluate which from the ANS, the access (Figure 26 panel a and b, respectively) or the exact representation deficit underlie MLD. For that purpose, we used estimation tasks that involve symbolic mapping onto the ANS and vice-versa, as well as estimation tasks that require mapping non-symbolic numerosities together. These tasks allow a very direct measure of the precision and variability of the ANS.

In all numerical estimation tasks, MLD children's estimates were consistently more variable and less accurate than those produced by control children. This pattern of performance does not result from a general and non-specific deficit of estimation processes as MLD children were

able to estimate the hue of color patches (see Study 2 presented under Chapter 4) as well their typically developing peers.

The global impairment observed in the different estimation tasks already allow us to exclude that MLD could be related to an exclusive impairment of an exact number representation as proposed by Butterworth (1999, 2005; see also luculano et al., 2008).

The performance of MLD children is compatible with an ANS deficit (Wilson & Dehaene, 2007). Indeed, the global impairment observed in the different tasks and especially the non-symbolic estimation tasks suggests that MLD is rooted in the core representation of number magnitude (rather than in the access from symbol process) because this task is supposed to address the ANS without the involvement of symbolic systems, providing a “pure” measurement of the ANS. Although these results converge with a small amount of existing evidence (Mussolin, et al., 2010; Piazza, et al., 2010; Price, et al., 2007) to suggest a deficit affecting the ANS, we cannot exclude that the strategy used by our participants deviated from the ANS. Indeed, it is possible that the subjects completed the entire non-symbolic task by symbolizing the presented numerosities. This strategy could be emphasized, in our task, by the training phase¹⁷.

Moreover, the direct comparison between symbolic and non-symbolic estimation of dot sets indicated that MLD children exhibited even greater difficulty in task involving symbolic numbers than in the purely non-symbolic task. This result is difficult to explain under the assumption that MLD is exclusively due to a deficit at the core level of the analogue magnitude processing. Indeed, such a pattern of performance is better explained by an access deficit hypothesis.

¹⁷ We thought avoiding this symbolization mainly in participants who went through the non-symbolic task in the first place (the tasks were counterbalanced according to a Latin-square between individuals). However, all participants went through an initial calibration phase in which they practiced the different tasks. Therefore, the choice -even unconscious- of the process used to achieve a given task could potentially be contaminated at the training phase.

Although the comparison and estimation tasks used in previous studies face several difficulties (see Methodological considerations), results consistently showed that children with MLD are impaired on symbolic tasks whereas no difference is observed with control children when considering non-symbolic tasks only (De Smedt & Gilmore, 2011; Geary et al., 2008; Luculano et al., 2008; Landerl & Kölle, 2009; Rousselle & Noël, 2007). Altogether, these results support an “access deficit to numerical magnitude from symbolic numbers” at least in the early stages of development. Because this result was found for children of 9 years or less, it is currently not possible to generalize it to older children with MLD. Moreover, the results we collected in the estimation of a number of dots in a symbolic format allow us to extend previous results. In other words, difficulties of MLD children are not restricted to the access to ANS from symbols, as initially proposed by Rousselle and Noël (2007), but they also affect the access to symbolic representations from the ANS. This points to an extended deficit in processing symbolic numerical representations: we then postulate that MLD is related to the existence of a mapping deficit between the ANS and the symbolic representation, coming first from a lexical access deficit proper to the arithmetical domain; that is the symbolic representation. This proposal will be discussed further later.

Another question addressed in Chapter 4 concerned the influence of the direction of mapping. Indeed, although both directions of mapping seem to be concerned in MLD, the deficit is not symmetric. Mundy and Gilmore (2009) had reported, in a population of typically developing children (6 to 8 year olds), that the mapping from a symbolic format to a non-symbolic one was less precise than the reverse mapping from a non-symbolic code to a symbolic one. Using a different paradigm, and testing older children, we replicated these findings and observed that both MLD and typically-developing children had more difficulties to access the ANS from symbols than in the reverse direction. It is worth noting that the less impaired

direction requires a matching from an evolutionarily old quantity representation system to a culturally dependent number symbol system, thereby following the trajectory of the phylo- and ontogenesis. We would like to underlie that the link between those two systems is artificial and depends on cultural and educational constraints. In our western culture, this link is particularly reinforced by the “how many” exercises used in kindergarten to teach numbers (see Figure 27). In contrast, the reverse mapping is less frequent, and therefore less practiced. It will be really easy to test this assumption of an environmental influence by examining individuals how are daily exposed to the reverse mapping (for example the market gardeners for who the type of task “give me [five lemons]” is over trained) or to test if a reverse effect can be obtained following intensive practice of the symbolic to non-symbolic mapping.

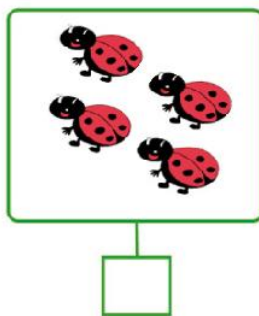


Figure 27: From a kindergarten classroom, a classical example of “How many” type of exercise.

A last result showed that MLD children are more influenced than control children by the perceptual variables that may co-vary with numerosities. Our results indicate that only MLD children took advantage of the perceptive cues which correlated with numerosity and made better estimates in the Homogeneous-sized dots task (where the cumulative area co-varied perfectly with numerosity) than in the Heterogeneous-sized dots task (where the cumulative area was constant across numerosities). MLD children behave as younger ones: they preferentially process perceptive variables rather than numerical ones to perform the task.

Chapter 5 presented performances of adult participants with a history of MLD in childhood. They were compared with the ones of a C group matched in terms of age and professional activity. The four different numerical-estimation tasks were those used in Chapter 4. Our study aimed at characterizing the longevity of the deficit observed in childhood in the previous study and extending the results found by Ashkenazi and collaborators (2009) and Rubinsten and Henik (2005). On the basis of a number Stroop task, they reported an impairment in accessing the ANS from AN. First, our results showed that, although they had been recruited on the basis of self-reported history of childhood difficulties with mathematics, MLD participants scored significantly worse than the C participants in the arithmetical test for both response time and accuracy. These results validate the recruitment procedure based on introspective reports of mathematical difficulties and suggest the persistence of MLD from childhood to adulthood. Second, we showed that the mapping deficit identified between two representations is a deficit that persists over time as the observed reduced acuity is evident throughout development (i.e., from age 10 to adulthood). Finally, mapping abilities correlate with arithmetical skills. Then we proposed that those individuals presenting a mapping deficit coming first from a lexical access deficit proper to the arithmetical domain suffer from a long lasting deficit.

While comparing the results of the two studies (presented in Chapter 4 and in Chapter 5), we can see that both adult groups were more accurate than the related children group, but MLD participants seem to benefit from daily exposure to numbers more than C participants. Indeed, their acuity showed a sharp increase from childhood to adulthood when compared to controls. The most likely explanation is that C adults reach a level of acuity close to the limit of the human system earlier in the development. However, these results should be confirmed by a longitudinal study.

We argue that the deficits observed in studies reported in Chapter 4 and 5 emphasize the importance of the mapping between non-symbolic and symbolic representations in MLD. For this reason, we assume that such mapping plays a key role for the acquisition of good arithmetical skills.

As it is well known that MLD individuals exhibit poor arithmetical skills, **Chapter 6** aimed at examining children and adult's abilities to get a quick feed-back on their aberrant answers by evaluating their competencies to give a plausible *approximation* of the answer. Moreover, this research confronts the different hypotheses accounting for MLD by studying individuals' accuracy when operating on approximate numerical and non-numerical magnitude tasks.

Accordingly, four approximate addition tasks were proposed to MLD and control participants. Two non-symbolic approximate addition tasks were proposed to the participants in order to index their ANS and to confront the ANS deficit hypothesis (Wilson & Dehaene, 2007): participants had to produce a pattern of dots which roughly corresponds to the sum of three set of dots presented sequentially. In the first version of the task, sets were made of homogeneous-size dots. In the second version of the task, sets were made of heterogeneous-size dots that led to a constant cumulative surface area across numerosities (i.e., avoiding the participants' answers to be based on the total surface area covered by the dots; Rousselle et al., 2004). In the third task, continuous magnitudes were used in order to evaluate if MLD participants' difficulties result from a larger magnitude system deficit (Walsh, 2003, see Figure 26 panel c): participants were asked to produce a quantity of liquid which roughly corresponds to the sum of three quantities of liquid presented sequentially. Finally, the fourth task was a symbolic approximate addition in which participants had to produce an AN corresponding to the sum of three-addend presented sequentially. This task taps into the ANS from the symbolic system. If MLD

participants' difficulties result from an access deficit to the ANS from symbols (Rousselle & Noël, 2007) or more precisely from a mapping deficit as proposed above, MLD should perform more poorly than controls in the symbolic approximate calculation only.

In all numerical approximation tasks, MLD children's approximate were consistently less accurate than those produced by control children. MLD adults were less accurate compared to control adults regarding the symbolic task only.

The deficit observed in the MLD participants in the symbolic approximate addition task support the mapping deficit hypothesis. However, the data gathered in Chapter 6 point to another specific difficulty in MLD children. Because they encountered difficulties with non-symbolic numerical and non-numerical magnitudes in approximate calculation tasks, it is likely that they faced a deficit of acuity in the ANS. This fits with the analogue deficit hypothesis, or even a larger magnitude representation deficit. However, because this deficit is not unique to non-symbolic notations and because it was not observed in MLD adults, we argue that it rather reflects a delay in the maturation of the perceptive sphere underlying the appropriate use of the ANS, which added to the mapping deficit. This delay in the maturation of the perceptive sphere is in accordance with non-symbolic numerical comparisons tasks testing the same age range subjects. Piazza and collaborators (2010) showed that 10 year old MLD children scored at the level of five year old typically achieving children when they are asked to compare pattern of dots. Similarly, same age-range MLD children showed higher error-rates for numbers which were close together, resulting in a steeper slope of the numerical distance (Mussolin et al., 2010b; Price et al., 2007).

Finally, the fact that the mapping deficit is a long lasting one is also supported by data in the Study 2 presented in Chapter 6: MLD adults were less accurate compared to controls only in the

symbolic approximate task. Besides, adults were found to be marginally less effective in the non-numerical continuous type of task; the Glasses of water one. The impoverished ANS present in MLD children evolved and the delay was filled once reached adulthood. This reinforces the idea of a developmental delay affecting the ANS.

A last point we would like to discuss concerning the Chapter 6, is the possibility for the ANS and the symbolic representation to be highly dependent, but only during the first years of arithmetical learning. Indeed, if arithmetic performances correlate with non-symbolic approximate abilities in children (i.e., their ANS), no such significant relationship was found in adults. Then, we postulated that this correlation revealed the presence of a strong link between the different representations (ANS and symbolic representation) in children. However, this link disappeared in adults as arithmetical abilities do not enhance the ANS anymore and rely on a symbolic representation which is fully a-semantic as proposed in the triple-code model (Dehaene, 1992, Figure 28). This idea of a link present in children but which disappeared in adults is reinforced by the fact that mapping tasks (from Chapter 4) and approximate tasks (from Chapter 6) correlate in children (correlation analysis realized on children who took part to both experiment, $N = 19$; $r = .594$; $p = .007$) and not in adults ($N = 33$; $r = .053$; $p = .769$).

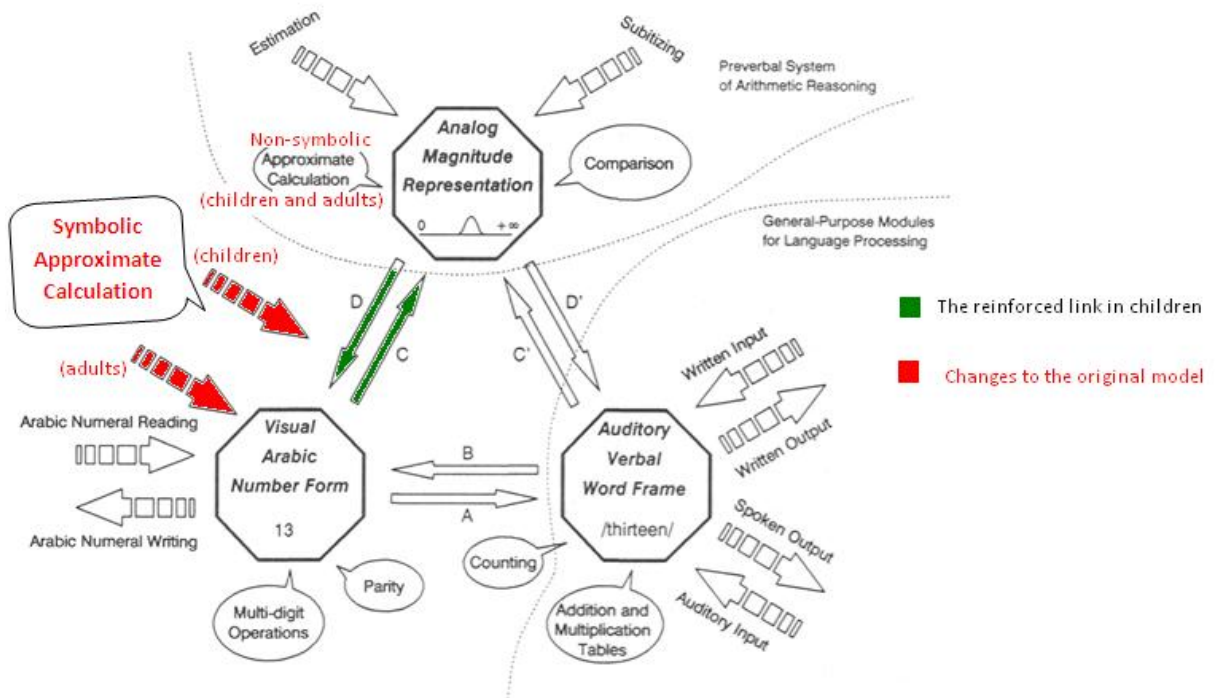


Figure 28: The Triple-code model with some modifications according to the development of numerical competences in children (reproduced from Dehaene, 1992).

In summary, we proposed that MLD is both related to a long lasting mapping deficit coming first from a lexical access deficit proper to the arithmetical domain, that is the symbolic representation, and to a developmental delay affecting the ANS and coming first from a slow maturation of the perceptive sphere.

As MLD children are classically recruited based on their poor arithmetical performances, we believe that the majority of those recruited children have a symbolic representation deficit and some of them an ANS deficit (giving rise to some symbolic deficit as well). Children with a weak symbolic representation should be slower at AN reading, while children suffering from a slow maturation of the perceptive sphere would exhibit difficulties in subitizing (indeed, according to Dehaene, 1992, this task directly rely on the analog representation, see Figure 28). Once

reached adulthood, only those who presented in childhood a lexical access deficit remained in trouble with the math. As we can not exclude that the slow maturation added to the problems of mapping, we believe that it should be possible to observe both deficits independently of one another. However, it would be interesting to assess through longitudinal study whether or not the slow maturation could be the source of the mapping deficit.

Finally, our results challenge the hypothesis that symbolic numerical representation and non-symbolic numerical representation are strongly related throughout life (Dehaene, 1997, 2009) and suggest the existence of artificial link created and reinforced by the needs of our society, present during the school year and which can be reactivated under certain situations in adulthood (as forced mapping).

3. Theoretical implications

In the view of that proposition, we propose the ANS as an encapsulated module which is not submitted to the influence of the culture and to the language but depends on the perceptive abilities of an individual versus the symbolic system, which, on the contrary is influenced by the language and the culture and may develop by being mapped on the ANS. However, the strength of this mapping is also culturally dependent.

Studies of typical development have consistently reported increasing precision of the ANS during development (through discrimination tasks, e.g., Izard et al., 2009; Lipton & Spelke, 2003, 2004; Xu & Arriaga, 2007; comparison tasks, Halberda & Feigenson, 2008; Mundy & Gilmore, 2009; Piazza et al., 2010; and estimation tasks, Huntley-Fenner, 2001, Chillyer, 2002 see also the larger variability and lower acuity in children compared to adults in Chapter 4 of the present document). This increase of precision, indexed by either the decrease of the Weber fraction or

the decrease of the coefficient of variation during development, gives us a thought about how the ANS develops throughout the lifespan. Some of the refinement of this system may be due to simple maturation of the neural circuitry subserving the ANS. However, some authors have suggested that experience and familiarity with symbols can also affect its sharpening: it has been reported that individual abilities to process tasks which index this ANS is correlated to their math performances. Hence, some authors have suggested that this is a proof to the AES being the “start-up tools for symbolic number representation” (for a review, see Piazza, 2010). However, this correlation between arithmetical abilities and the AES acuity is not obvious as it is shown only in particular cases as we will present below. We proposed that this refinement is more related to the mapping between the symbolic and the analogical representation than ANS intrinsic changes.

3.1. The ANS as a building block for arithmetical knowledge?

Gelman and Gallistel (2000) proposed an hypothesis which takes as its starting point the accumulator model as proposed by Meck and Church (1983). Humans would have learned a procedure to decide how to map from a mental magnitude to a numeral and from a numeral to a corresponding magnitude (“*the bi-directional mapping hypothesis*”). Children can progressively make sense of the number words they have heard constantly, as well as the counting routine they have learned. However, it takes them at least a year to realize that this enumeration procedure also determines the number of object present in that set (the cardinal word principle, Wynn, 1990, 1992a). Moreover, children appear to learn the cardinal meanings of smaller number words sequentially before learning the cardinal meanings of larger number words within their counting range (they give “1” item from a pile and they know that the other number words pick out larger sets of objects; then, they give “2” before they can give 3, and so on).

Finally, children notice that the progression in the verbal counting routine corresponds in one hand, to the addition of one item to the set and in the other hand, to the increase of the cardinal value of the set (see for a review, Mix, Sandhofer, & Baroody, 2005). At that time, children seem to be able to apprehend the natural number concepts.

A complementary hypothesis has been recently proposed by Piazza (2010) to explain this slow process of the acquisition of the lexical number word. The lexical acquisition of numbers emerges only if children maturation of their perceptive system permit to discriminate between numerically adjacent sets ($N-1$ and $N+1$): “Children should be able to understand accurately the numeral ‘three’ only when they can reliably distinguish a set of three from a set of two and a set of four (i.e. when they become sensitive to a 3:4 ratio)” (p. 547). Accordingly, ratio data from discrimination experiments reported in Table 1 of this thesis fit with this idea as children become “three-knower” at about the age of 3 (see Wynn, 1992). To confirm this idea it would be of interest to examine children’s ANS acuity during the first years to see if it correlates with their lexical acquisition of numbers.

Beside the perceptive system (and its limits), the language (as a tool of communication made of words to describe the world) seems then to have its importance in the construction of the natural number concepts.

3.2. The role of language in the acquisition of an exact and symbolic representation

The acquisition of particular numerical languages is also determining for the development of the exact and symbolic knowledge. Humans have at their disposal the language that is a complex system of representation and communication. Human individuals and non-humans trained ones (see the famous case of Kanzi, Savage-Rumbaugh & Lewin, 1994) are able to

semantize arbitral material which designates a referent: the relationship between signifiers (the sounds) and signifieds (the content) is arbitrary (there is no physical resemblance between the analog auditory sequence and the content it represents). This sign arbitrariness is a specific characteristic of the language system (de Saussure, 1968). However, some signs are absolutely arbitrary (“twenty”) while others are only relatively arbitrary (“nine-teen”) as some signs have been constructed following some regularities of the system. Contrariwise to animals and by learning the verbal routine, children develop the capacity to communicate, thanks to that system’s regularities (e.g., Vaudair, 1998). Indeed, each element or sign has its significance by contrast and opposition to all other signs: in our system “five” has a particular significance because of its position on the number line and by contrast to “four” or “six”; however, “five” does not have the same significance in each culture. In our Western culture and according to the context, “five” can be precise and determine exactly “five” precise and particular elements (see Fuson, 1988 for an extended discussion of number uses). Contrariwise, in the Mundurukú culture, “five” does not have the same significance as “six” in their language system is only approximate. Arithmetic capabilities in children who learned these languages experienced difficulties in counting and cannot perform more complicated symbolic operations of mathematics (Pica et al., 2004). Then, the acquisition of particular numerical languages determines for the development of the exact and symbolic knowledge.

Regarding our needs according to particular cultural and environmental conditions (as promoting the use of fingers, ecological material as rulers and cubes to learn the function of the Western counting system versus using Oksapmin body part number system), processing differences due to the language of the individual would appear. For example, the standard Oksapmin number system differs from the Western base-ten system (see for example, Saxe, 1999). Oksapmin individuals count beginning with the thumb on one hand, enumerates 27

places around the body upper side, ending on the little finger of the opposite hand. To count further, Oksapmin individuals continue back up to the wrist of the second hand and progress back upward on the body. In traditional practices, Oksapmin used the body system to count (e.g., pigs) and have only analogs of computational processes. For example, Oksapmin traded goods directly in “one-for-one” or “one-for many”. With such analogs computational processes, the answer cannot be determined in the absence of the objects.

According to the weaker form of the Worfian linguistic determinism hypothesis, proposed by Hunt and Agnoli (1991), “the language differentially favors some thought processes over others, to the point that a thought that is easily expressed in one language might virtually never be developed by speakers of another language” (p.378). Then, this corresponds to an indication of the extent to which a particular part of the cognitive system (here the ANS or the symbolic system) can be modulated by the language (the symbolic system) or is an impermeable and encapsulated module (the ANS). Can we found proof of the ANS being refined differently according to a particular use of mathematical language?

3.3. Interactions between language and ANS

It has been suggested that numerical experiences and the acquisition of number symbols improve the representation of numbers (Dehaene, 2009). This thought is supported by data from practice (training on numerical comparison that emphasizes the links between symbolic and non-symbolic representations of number by using repeated association techniques whereby Arabic, verbal and quantity codes are presented together; Wilson et al., 2006). However, things are not that obvious as contradictory data are found through the literature regarding the possible improvement of the ANS with the learning of symbols. Through the following paragraph, we will see that the interactions seem to go in both direction and that this

refinement is more related to the mapping between the symbolic and the analogical representation than ANS intrinsic changes.

According to the idea that acquisition of number symbols improves the analog representation of numbers, the ANS acuity has been related to performance in exact calculation as correlations have been found between the accuracy of performance on a non-symbolic numerical comparison task, presented to students at 14 years of age, and school mathematics performance from kindergarten to sixth grade (Halberda et al., 2008). However, as both measures (i.e., the ANS acuity and mathematics performance) are supposed to evolve during the child development, this is not surprising and in any case correlations are indicative of the cause of such a relation. Then, it remains unclear whether sharper ANS acuity is related to better symbolic performance.

Alternative explanations for this potential refinement do exist. Regarding Verguts and collaborators' computational model (Verguts & Fias, 2004; Verguts et al., 2005), the symbolic representations develop by being mapped onto pre-existing non-symbolic representations and, through development, the learning of counting words leads to changes in the *access* to the ANS from symbols. The predictive issue of this model fits some Mundurukú data. Members of the Mundurukú sub-group who had the opportunities to learn Portuguese numerals placed symbolic Portuguese numerals to space linearly but their responses to Mundurukú numerals and dot patterns remained logarithmic (Dehaene et al., 2008). This challenges again the hypothesis that symbolic numerical representation and the ANS are strongly related (Dehaene, 2009) and suggest the existence of two independent systems as no transfers inter (symbolic/non-symbolic) or intra (symbolic) notations have been observed.

To the question of the role of language in the acquisition of an exact and symbolic representation, this example seems to support the idea that a particular part of the cognitive system can be modulated by language processes, here the symbolic system and the mapping from that system to the ANS, which has been accessed differently according to a particular use of mathematical language (exact for Portuguese, approximate for larger number than 5 for Mundurukú). Then, experiences with number symbols contribute to a better mapping between the systems which may be reflected by the linearization.

It is also interesting to note that, in Chapter 4, even if all mapping tasks were significantly correlated with the participants' mathematical performance (i.e., estimating the cardinality of a dots pattern by an Arabic number or producing dots in response to an Arabic number), the best predictor was the measure of accuracy on the Homogeneous-sized dots to Arabic numerals task—a task that involves the link between the ANS and the symbolic representation. This is also in accordance with the literature: those studies which have pointed a link between the ANS acuity and math achievement are almost exclusively those using a mapping between the ANS and a symbolic representation (that is number line placement at age 7 and 8, , Booth & Siegler, 2006; Siegler & Booth, 2004).

Finally, the idea that the learning of symbols consists in developing specific lexical access proper to the arithmetical domain and then creates a mapping with an ancient analog representation is also supported by neuroanatomical data.

Language and arithmetic are jointly lateralized to the left hemisphere in the majority of right-handed individuals and the left parietal cortex seems to play a role during the arithmetic development: The left hemisphere is essential to process exact calculation (Piazza, Mechelli, Price, & Butterworth, 2006, see also the case report of N.A.U., Dehaene & Cohen, 1991) but is

also involved when processing approximate comparisons (which can be processed either by the left or right posterior parietal cortex since simultaneous bilateral TMS was needed to alter the comparison of digits far from 5; Andres, Seron, & Olivier, 2005). Ansari and Dhital (2006) used functional neuroimaging to compare the neural correlates of non-symbolic magnitude judgments (pattern of squares) between children and adults. The authors showed a greater effect of the distance between numbers during a comparison task in the left intraparietal sulcus in adults compared to 10 year olds, suggesting an increasing involvement of this region with increasing age. These findings suggest that the left intraparietal sulcus undergoes significant age-related changes during the processing of non-symbolic magnitude.

Moreover, it has been shown in different ages children that neural correlates underlying mental arithmetic change between the ages of 8 and 19 years while performing a same arithmetical task (Rivera, Reiss, Eckert, & Menon, 2005). The authors found an increasing activation with age in the left parietal and left lateral occipitotemporal cortex, whereas activation in the dorsolateral and ventrolateral prefrontal cortex and in the anterior cingulate cortex decreased. The authors argue for the development of arithmetic representations in the left intraparietal regions and hypothesize that decreasing engagement of frontal areas may reflect increasing automaticity and decreasing demands on attention and memory resources. However, Diester and Nieder (2007) shed light on those results by simulating a symbolic to non-symbolic mapping in rhesus monkeys : they trained them to associate AN with the numerosity of multiple-dot displays. They found that individual neurons in the prefrontal cortex responded in a tuned fashion to the same numerical values of dot sets and associated AN. They named these neurons “association neurons”. According to the authors, these neurons are responsible for the link between numerosity and AN.

4. Practical implications and Perspectives for further research

Our findings suggest that some dyscalculic individuals with MLD suffered from an ANS developmental delay, coming first from a slow maturation of the perceptive sphere. We also propose that some other individuals present a mapping deficit coming first from a lexical access deficit proper to the arithmetical domain. Those individuals suffer from a long lasting deficit.

It then should be of interest to evaluate children within a longitudinal perspective. We hypothesize that children at risk of dyscalculia related to a slow maturation process of the perceptive sphere would develop difficulties regarding early spatial attention capacity that is the capacity to track individual objects through space and time (see for example Feigenson, 2005) or to deal with small number arrays (or to subitize, see for example Hyde & Spelke, 2009 according an object files perspective). We have showed that difficulties related to ANS also spread over discrete material as liquid. Then, including discrete material as well should be of interest in both evaluating children at risk but also to have them manipulate different kind of materials.

An entire research field is dedicated to “perceptual learning” which refers to improvements in sensory abilities after training (see for examples, Ahissar & Hochstein, 1997; Seitz & Watanabe, 2005). This corresponds to the process of long-lasting changes in performing perceptual tasks (the process of attaining awareness or understanding of sensory information such as visual, auditory, tactile, etc.). It has been demonstrated that perceptual learning is due to practice and is related to a simple discrimination along a single dimension (to discriminate an orientation in the visual modality) to complex categorizations involving the integration of several dimensions (radiologists develop refined abilities to distinguish subtle patterns of tumors in images that show no pattern to the untrained eye). The ability to improve in such tasks is retained throughout life and it has been shown that some transfers occurred during initial

training inside a dimension (e.g., orientation, Jeter, Doshier, Liu, & Lu, 2010). To evaluate deeper the conditions of such learning and to transfer it to material which can be useful for enhancing the ANS should be of interest.

Concerning the remediation of individuals affected by a mapping deficit, we can turn to those remediation programs that have been proposed in the field of dyslexia. This disorder is characterized by difficulties in language processing, primarily at the level of phonological awareness, i.e., the ability to recognize and manipulate the sound structure of words (Snow, Burns, & Griffin, 1998) coming from a basic temporal processing impairment (Tallal, Miller, & Fitch, 1993).

Remediation programs proposed to dyslexic children focused on auditory processing and oral language skills important for reading with exercises that emphasize different aspects of oral language, including auditory attention, discrimination, memory, as well as phonological processing and listening comprehension. Those programs are not really specific but seem to be effective (see for example, Temple et al., 2003). A similar program was implemented with young 7 to 9 year old MLD children (Wilson et al., 2006). Like the program proposed in dyslexia, it included various numerical tasks which aimed to enhance “the number sense”. Most of all, this program increases the mapping between the ANS, the symbolic system and space. The researchers reported that, at the end of the program, children were faster at enumerating dot for numerosities (in the subitizing range only), they were able to compare faster number magnitudes (with even better performance regarding symbolic notation compared to non-symbolic notation) and they improved their ability in subtraction but not in addition. This is encouraging results. However, this study of remediation has been following by a small set of subjects (9 children) for a small amount of trial sessions (a maximum of ten hours over ten

weeks). It is worth noting that, to be effective with dyslexic children, the therapy had to be applied with a heavy schedule on successive days, and required intense practice schedules (Merzenich et al., 1996).

As the mapping between systems seems crucial for a successful arithmetical learning, giving a chance to such a program seems important. It should be more specific (by targeting the proper task that reinforces the mapping between the system), gradual (by diminishing for example the perceptive cues progressively), using only the basic abilities deficient in a given child (to reduce the time), with a good implementation at home involving the entire family and some rewards to give a chance to such a program to be successful.

References

- Ahissar, M., & Hochstein, S. (1997). Task difficulty and the specificity of perceptual learning. *Nature* 387, 401–406.
- Alarcon, M., DeFries, J. C., Light, J. G., & Pennington, B. F. (1997). A Twin Study of Mathematics Disability. *Journal of Learning Disabilities*, 30(6), 617-623.
- Andres, M., Seron, X., & Olivier, E. (2005). Hemispheric lateralization of number comparison. *Cognitive Brain Research*, 25(1), 283-290.
- Ansari, D., & Dhital, B. (2006). Age-related changes in the activation of the intraparietal sulcus during non-symbolic magnitude processing: an event-related fMRI study. *Journal of Cognitive Neuroscience*, 18, 820-828.
- Ashkenazi, S., Rubinsten, O., & Henik, A. (2009). Attention, automaticity, and developmental dyscalculia. *Neuropsychology*, 23(4), 535-540.
- Association, A. P. (1994). *Diagnostic and statistical manual of mental disorders* (4 ed.). Washington, D.C.
- Badian, N. A. (1983). Dyscalculia and nonverbal disorders of learning. In H. R. Myklebust (Ed.), *Progress in learning disabilities* (Vol. 5, pp. 235-264). New York: Grune & Stratton.
- Barrouillet, P., Fayol, M., & Lathuliere, E. (1997). Selecting between competitors in multiplication tasks: An explanation of the errors produced by adolescents with learning difficulties. *International Journal of Behavioral Development*, 21(2), 253-275.
- Barth, H., Beckmann, L., & Spelke, E. S. (2008). Nonsymbolic, approximate arithmetic in children: Abstract addition prior to instruction. *Developmental Psychology*, 44(5), 1466-1477.
- Barth, H., Kanwisher, N., & Spelke, E. S. (2003). The construction of large number representations in adults. *Cognition*, 86(3), 201-221.
- Barth, H., La Mont, K., Lipton, J. S., Dehaene, S., Kanwisher, N., & Spelke, E. S. (2006). Non-symbolic arithmetic in adults and young children. *Cognition*, 98(3), 199-222.
- Barth, H., La Mont, K., Lipton, J. S., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences of the United States of America*, 102(39), 14116-14121.
- Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities*, 38(4), 333-339.
- Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental Psychology*, 46(2), 545-551.

- Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, 42(1), 189-201.
- Brannon, E. M., Lutz, D., & Cordes, S. (2006). The development of area discrimination and its implications for number representation in infancy. *Developmental Science*, 9(6), F59-F64.
- Brannon, E. M., Suanda, S., & Libertus, K. (2007). Temporal discrimination increases in precision over development and parallels the development of numerosity discrimination. *Developmental Science*, 10(6), 770-777.
- Bueti, D., & Walsh, V. (2009). The parietal cortex and the representation of time, space, number and other magnitudes. *Philosophical Transactions of the Royal Society B-Biological Sciences*, 364(1525), 1831-1840.
- Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.
- Butterworth, B. (2003). Dyscalculia Screener. London: nfer Nelson Publishing Company Ltd.
- Butterworth, B. (2005). Developmental dyscalculia. In J. I. D. Campbell (Ed.), *Handbook of Mathematical Cognition*. (pp. 455-467). New York: Psychology Press.
- Castronovo, J., & Seron, X. (2007). Numerical estimation in blind subjects: Evidence of the impact of blindness and its following experience. *Journal of Experimental Psychology-Human Perception and Performance*, 33, 1089-1106.
- Chillier, L. (2002). La ligne numérique et les codages du nombre chez l'enfant. In J. Bideaud & H. Lehalle (Eds.), *Le développement des activités numériques chez l'enfant* (pp. 129-150). Paris: Hermes, Lavoisier.
- Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin & Review*, 8(4), 698-707.
- Cuneo, D. O. (1982). Children's judgments of numerical quantity: A new view of early quantification. *Cognitive Psychology*, 14, 13-44.
- de Saussure, F. (1968). *Cours de linguistique générale*. Paris: Payot
- De Smedt, B., & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of Experimental Child Psychology*, 108(2), 278-292

- De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, 103(4), 469-479.
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44, 1-42.
- Dehaene, S. (1997). *The number sense*. New York: Oxford University Press.
- Dehaene, S. (2001). Precis of the number sense. *Mind & Language*, 16(1), 16-36.
- Dehaene, S. (2003). The neural basis of the Weber-Fechner law: a logarithmic mental number. *Trends in Cognitive Sciences*, 7(4), 145-147.
- Dehaene, S. (2009). Origins of mathematical intuitions. The case of arithmetic *Year in Cognitive Neuroscience* (Vol. 1156, pp. 232-259). Oxford: Blackwell Publishing.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology-General*, 122(3), 371-396.
- Dehaene, S., & Changeux, J. P. (1993). Development of elementary numerical abilities - a neuronal model. *Journal of Cognitive Neuroscience*, 5(4), 390-407.
- Dehaene, S., & Cohen, L. (1991). 2 mental calculation systems - a case-study of severe acalculia with preserved approximation. *Neuropsychologia*, 29(11), 1045-1074.
- Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, 1, 83-120.
- Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*, 33, 219-250.
- Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital - analogical and symbolic effects in 2-digit number comparison. *Journal of Experimental Psychology-Human Perception and Performance*, 16(3), 626-641.
- Dehaene, S., Izard, V., Spelke, E. S., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in western and amazonian indigene cultures. *Science*, 320(5880), 1217-1220.
- Dehaene, S., Molko, N., Cohen, L., & Wilson, A. J. (2004). Arithmetic and the brain. *Current Opinion in Neurobiology*, 14(2), 218-224.
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20(3-6), 487-506.
- Dehaene, S., Spelke, E. S., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*, 284(5416), 970-974.

- Delazer, M., Karner, E., Zamarian, L., Donnemiller, E., & Benke, T. (2006). Number processing in posterior cortical atrophy: A neuropsychological case study. *Neuropsychologia*, 44, 36-51.
- Diester, I., & Nieder, A. (2007). Semantic associations between signs and numerical categories in the prefrontal cortex. . *PLoS Biology*, 5(11).
- Dormal, V., & Pesenti, M. (2009). Common and specific contributions of the intraparietal sulci to numerosity and length processing. *Human Brain Mapping*, 30, 2466–2476.
- Dowker, A. (1997). Young children's addition estimates. *Mathematical Cognition*, 3(2), 141-154.
- Dowker, A. (2003). Young children's estimates for addition : the zone of partial knowledge and understanding. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills. Constructing adaptive expertise*. (pp. pp243-265). London: LEA.
- Droit-Volet, S. (2000). L'estimation du temps : perspective développementale. *L'année psychologique.*, 100(3), 443-464.
- Feigenson, L. (2005). A double-dissociation in infants' representations of object arrays. *Cognition*, 95(3), B37-B48.
- Feigenson, L. (2007). The equality of quantity. *Trends in Cognitive Sciences*, 11(5), 185-187.
- Feigenson, L., Carey, S., & Spelke, E. S. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology*, 44(1), 33-66.
- Feigenson, L., Dehaene, S., & Spelke, E. S. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8(7), 307-314.
- Flombaum, J. I., Junge, J. A., & Hauser, M. D. (2005). Rhesus monkeys (*Macaca mulatta*) spontaneously compute addition operations over large numbers. *Cognition*, 97(3), 315-325.
- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York, USA: Springer.
- Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, 44(1-2), 43-74.
- Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: from reals to integers. *Trends in Cognitive Sciences*, 4(2), 59-65.
- Gallistel, C. R., & Gelman, R. (2005). Mathematical cognition. In K. Holyoak & R. Morrison (Eds.), *The Cambridge handbook of thinking and reasoning*. (pp. 559-588). New York: Cambridge University Press.

- Gao, F., Levine, S. C., & Huttenlocher, J. (2000). What do infants know about continuous quantity? *Journal of Experimental Child Psychology*, 77(1), 20-29.
- Geary, D. C. (1993). Mathematical disabilities - Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, 114(2), 345-362.
- Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37(1), 4-15.
- Geary, D. C. (2005). Les troubles d'apprentissage en arithmétique : rôle de la mémoire de travail et des connaissances conceptuelles. In M. P. Noël (Ed.), *La dyscalculie: trouble du développement numérique de l'enfant*. Marseille: Solal.
- Geary, D. C., Bow-Thomas, C. C., & Yao, Y. (1992). Counting knowledge and skill in cognitive addition: A comparison of normal and mathematically disabled children. *Journal of Experimental Child Psychology*, 54(3), 372-391.
- Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology*, 77(3), 236-263.
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology*, 88(2), 121-151.
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, 78(4), 1343-1359.
- Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology*, 74(3), 213-239.
- Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology*, 33(3), 277-299.
- Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. *Nature*, 447(7144), 589-U586.

- Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2010). Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. *Cognition*, 115(3), 394-406.
- Hahn, B., Ross, T. J., & Stein, E. A. (2006). Neuroanatomical dissociation between bottom-up and top-down processes of visuospatial selective attention. *Neuroimage*(32), 842-853.
- Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "Number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, 44(5), 1457-1465.
- Halberda, J., Mazocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, 455(7213), 665-U662.
- Hamann, M. S., & Ashcraft, M. H. (1985). Simple and complex mental additions across development. *Journal of Experimental Child Psychology*, (40), 49-72.
- Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, 93(3), 615-626.
- Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning-difficulties. *British Journal of Psychology*, 82, 375-386.
- Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology*, 103(1), 17-29.
- Hunt, E., & Agnoli, F. (1991). The Worfian hypothesis: A cognitive psychology perspective. *Psychological Review*, 98 (3), 377-389.
- Huntley-Fenner, G. (2001). Children's understanding of number is similar to adults' and rats': numerical estimation by 5-7-year-olds. *Cognition*, 78(3), B27-B40.
- Hyde, D. C., & Spelke, E. S. (2009). All numbers are not equal: An electrophysiological investigation of small and large number representations. *Journal of Cognitive Neuroscience*, 21(6), 1039-1053.
- Isaacs, E. B., Edmonds, C. J., Lucas, A., & Gadian, D. G. (2001). Calculation difficulties in children of very low birthweight - A neural correlate. *Brain*, 124, 1701-1707.
- Iuculano, T., Tang, J., Hall, C. W. B., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental Science*, 11, 669-680.

- Izard, V., & Dehaene, S. (2008). Calibrating the mental number line. *Cognition*, 106(3), 1221-1247.
- Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences of the United States of America*, 106(25), 10382-10385.
- Jeter, P. E., Doshier, B. A., Liu, S. H., & Lu, Z. L. (2010). Specificity of perceptual learning increases with increased training. *Vision Research*, 50, 1928–1940.
- Jordan, N. C., & Hanich, L. B. (2003). Characteristics of children with moderate mathematics deficiencies: A longitudinal perspective. . *Learning Disabilities Research and Practice*, 18(4), 213-221.
- Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003a). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology*, 85(2), 103-119.
- Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003b). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development*, 74(3), 834-850.
- Jordan, N. C., & Montani, T. O. (1997). Cognitive arithmetic and problem solving: A comparison of children with specific and general mathematics difficulties. *Journal of Learning Disabilities*, 30(6), 624-634.
- Kadosh, R. C., Henik, A., Rubinsten, O., Mohr, H., Dori, H., van de Ven, V., et al. (2005). Are numbers special? The comparison systems of the human brain investigated by fMRI. *Neuropsychologia*, 43(9), 1238-1248.
- Kadosh, R. C., Kadosh, K. C., Schuhmann, T., Kaas, A., Goebel, R., Henik, A., et al. (2007). Virtual dyscalculia induced by parietal-lobe TMS impairs automatic magnitude processing. *Current Biology*, 17(8), 689-693.
- Kadosh, R. C., Lammertyn, J., & Izard, V. (2008). Are numbers special? An overview of chronometric, neuroimaging, developmental and comparative studies of magnitude representation. *Progress in Neurobiology*, 84(2), 132-147.
- Khomsy, A. (1998). *Epreuve d'évaluation de la compétence en lecture. Lecture de mots et compréhension-révisée. (L.M.C-R)*. Paris: Editions du Centre de Psychologie Appliquée.
- Koontz, K. L., & Berch, D. B. (1996). Identifying simple numerical stimuli: Processing inefficiencies exhibited by arithmetic learning disabled children. *Mathematical Cognition*, 2(1), 1-23.
- Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities*, 7(3), 164-177.

- Kucian, K., Loenneker, T., Dietrich, T., Dosch, M., Martin, E., & von Aster, M. (2006). Impaired neural networks for approximate calculation in dyscalculic children: a functional MRI study. *Behavioral and Brain Function*, 2(31), 1-27.
- Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: a study of 8-9-year-old students. *Cognition*, 93(2), 99-125.
- Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: Two learning disorders with different cognitive profiles. *Journal of Experimental Child Psychology*, 103(3), 309-324.
- Landerl, K., & Kölle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. *Journal of Experimental Child Psychology*, 103(4), 546-565.
- Lass, U., Lüer, G., Ulrich, M., & Werner, S. (1993). Access to analog representations in memory for visually perceived forms: The facilitating effect of declarative knowledge. In G. Strube & K. F. Wender (Eds.), *The cognitive psychology of knowledge. The German Wissenspsychologie project* (pp. 75-96). Amsterdam: North Holland.
- Lemaire, P., Lecacheur, M., & Faroli, F. (2000). Childre's strategy use in computational estimation. *Canadian Journal of Experimental Psychology*, 54(2), 141-148.
- Lemer, C., Dehaene, S., Spelke, E. S., & Cohen, L. (2003). Approximate quantities and exact number words: dissociable systems. *Neuropsychologia*, 41(14), 1942-1958.
- Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense: Large-number discrimination in human infants. *Psychological Science*, 14(5), 396-401.
- Lipton, J. S., & Spelke, E. S. (2004). Discrimination of large and small numerosities by human infants. *Infancy*, 5(3), 271-290.
- Lipton, J. S., & Spelke, E. S. (2005). Preschool children's mapping of number words to nonsymbolic numerosities. *Child Development*, 76(5), 978-988.
- Lourenco, S. F., & Longo, M. R. (2010). General magnitude representation in human infants. *Psychological Science*, 21(6), 873-881.
- McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & Psychophysics*, 69(8), 1324-1333.
- McCrink , K., Dehaene , S., & Dehaene-Lambertz , G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & Psychophysics*, 69(8), 1324-1333.

- McCrink, K., & Wynn, K. (2004). Large-number addition and subtraction by 9-month-old infants. *Psychological Science, 15*(11), 776-781.
- Meck, W. H., & Church, R. M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology-Animal Behavior Processes, 9*(3), 320-334.
- Mejias, S., Mussolin, C., Rousselle, L., Gregoire, J., & Noël, M. P. (submitted). Numerical and non-numerical estimation in children with and without mathematical learning disabilities.
- Merzenich, M. M., Jenkins, W. M., Johnston, P., Schreiner, C., Miller, S. L., & Tallal, P. (1996). Temporal Processing Deficits of Language-Learning Impaired Children Ameliorated by Training. *Science, 271*, 77-81
- Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). Multiple cues for quantification in infancy: Is number one of them? *Psychological Bulletin, 128*(2), 278-294.
- Mix, K. S., Sandhofer, C. M., & Baroody, A. J. (2005). Number words and number concepts: The interplay of verbal and nonverbal quantification in early childhood *Advances in Child Development and Behavior, Vol 33* (Vol. 33, pp. 305-346).
- Molko, N., Cachia, A., Riviere, D., Mangin, J. F., Bruandet, M., Le Bihan, D., et al. (2003). Functional and structural alterations of the intraparietal sulcus in a developmental dyscalculia of genetic origin. *Neuron, 40*(4), 847-858.
- Mundy, E., & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology, 103*(4), 490-502.
- Munsell colour model. Retrieved August 2008. www.britannica.com
- Murphy, M. M., Mazzocco, M. M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities, 40*(5), 458-478.
- Mussolin, C., De Volder, A., Grandin, C., Schlögel, X., Nassogne, M.-C., & Noël, M.-P. (2010a). Neural correlates of symbolic number comparison in developmental dyscalculia. *Journal of Cognitive Neuroscience, 22*(5), 860-874.
- Mussolin, C., Mejias, S., & Noël, M.-P. (2010b). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition, 115*(1), 10-25.
- Nieder, A., & Dehaene, S. (2009). Representation of number in the brain. *Annual Review of Neuroscience, 32*(1), 185-208.

- Nieder, A., Freedman, D. J., & Miller, E. K. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science*, 297(5587), 1708-1711.
- Nieder, A., & Miller, E. K. (2003). Coding of cognitive magnitude: compressed scaling of numerical information in the primate prefrontal cortex. *Neuron*, 37(1), 149-157.
- Nieder, A., & Miller, E. K. (2004). A parieto-frontal network for visual numerical information in the monkey. *Proceedings of the National Academy of Sciences of the United States of America*, 101(19), 7457-7462.
- Noël, M. P., Seron, X., & Trovarelli, F. (2004). Working memory as a predictor of addition skills and addition strategies in children. *Cahiers De Psychologie Cognitive-Current Psychology of Cognition*, 22(1), 3-25.
- Pesenti, M., & Andres, M. (2009). Common mistakes about numerical representations. *Behavioral and Brain Sciences*, 32(3/4), 346-348.
- Pesenti, M., Thioux, M., Seron, X., & De Volder, A. (2000). Neuroanatomical substrates of arabic number processing, numerical comparison, and simple addition: A PET study. *Journal of Cognitive Neuroscience*, 12(3), 461-479.
- Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences*, 14(12), 542-551.
- Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., et al. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33-41.
- Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron*, 44(3), 547-555.
- Piazza, M., Mechelli, A., Price, C. J., & Butterworth, B. (2006). Exact and approximate judgements of visual and auditory numerosity: An fMRI study. *Brain Res*, 23(1), 177-188.
- Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, 306(5695), 499-503.
- Pinel, P., Piazza, M., Le Bihan, D., & Dehaene, S. (2004). Distributed and overlapping cerebral representations of number, size, and luminance during comparative judgments. *Neuron*, 41(6), 983-993.
- Platt, J. R., & Johnson, D. M. (1971). Localization of position within a homogeneous behavior chain: Effects of error contingencies. *Learning and Motivation*, 2(4), 386-414.

- Price, G. R., Holloway, I., Rasanen, P., Vesterinen, M., & Ansari, D. (2007). Impaired parietal magnitude processing in developmental dyscalculia. *Current Biology*, 17(24), R1042-R1043.
- Restle, F. (1970). Speed of adding and comparing numbers. *Journal of Experimental Psychology*, 83(2, Part 1), 274-278.
- Rivera-Batiz, F. L. (1992). Quantitative literacy and the likelihood of employment among young adults in the United States. *The Journal of Human Resources*, 27(2), 313-328.
- Rivera, S. M., Reiss, A. L., Eckert, M. A., & Menon, V. (2005). Developmental changes in mental arithmetic: Evidence for increased functional specialization in the left inferior parietal cortex. *Cerebral Cortex* November, 15, 1779–1790.
- Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research & Practice*, 17(2), 81-89.
- Roggeman, C., Verguts, T., & Fias, W. (2007). Priming reveals differential coding of symbolic and non-symbolic quantities. *Cognition*, 105, 380-394.
- Rotzer, S., Kucian, K., Martin, E., von Aster, M., Klaver, P., & Loenneker, T. (2008). Optimized voxel-based morphometry in children with developmental dyscalculia. *Neuroimage*, 39(1), 417-422.
- Rourke, B. P., & Conway, J. A. (1997). Disabilities of arithmetic and mathematical reasoning: Perspectives from neurology and neuropsychology. *Journal of Learning Disabilities*, 30(1), 34-46.
- Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, 102(3), 361-395.
- Rousselle, L., & Noël, M. P. (2008a). The development of automatic numerosity processing in preschoolers: Evidence for numerosity-perceptual interference. *Developmental Psychology*, 44(2), 544-560.
- Rousselle, L., & Noël, M. P. (2008b). Mental arithmetic in children with mathematics learning disabilities: The adaptive use of approximate calculation in an addition verification task. *Journal of Learning Disabilities*, 41, 498 - 513.
- Rousselle, L., Palmers, E., & Noël, M. P. (2004). Magnitude comparison in preschoolers: What counts? Influence of perceptual variables. *Journal of Experimental Child Psychology*, 87(1), 57-84.

- Rubinsten, O., & Henik, A. (2005). Automatic activation of internal magnitudes: A study of developmental dyscalculia. *Neuropsychology*, 19(5), 641-648.
- Rubinsten, O., & Henik, A. (2009). Developmental dyscalculia: heterogeneity might not mean different mechanisms. *Trends in Cognitive Sciences*, 13(2), 92-99.
- Russell, R. L., & Ginsburg, H. P. (1984). Cognitive analysis of children's mathematics difficulties. *Cognition and Instruction*, 1(2), 217-244.
- Sattler, J. M. (1982). *Assessment of children's intelligence and special abilities*. Boston, MA: Allyn and Bacon.
- Savage-Rumbaugh, S., & Lewin, R. (1994). *Kanzi. The ape at the brink of the human mind*. New York: John Wiley & Sons, Inc.
- Saxe, G. B. (1999). Cognition, development, and cultural practices. In E. Turiel (Ed.), *Culture and Development. New Direction in Child Psychology*. San Francisco: Jossey-Bass.
- Schmidt, R. A., & Lee, T. D. (2005). *Motor control and learning: A behavioral emphasis* (4th ed.). Champaign, IL: Human Kinetics Publishers.
- Schneider, W., Eschmann, A., & Zuccolotto, A. (2002) (Version 1.2). Pittsburgh, USA: Psychology Software Tools Inc.
- Seitz, A., & Watanabe, T. (2005). A unified model of task-irrelevant and task-relevant perceptual learning. *Trends in Cognitive Sciences*, 9(7), 329-334.
- Shalev, R. S. (2004). Developmental dyscalculia. *Journal of Child Neurology*, 19(10), 765-771.
- Shalev, R. S. (2007). Prevalence of developmental dyscalculia. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 49-60). Baltimore, MD: Paul H. Brookes Publishing Co.
- Shalev, R. S., & Gross-Tsur, V. (2001). Developmental dyscalculia. *Pediatric Neurology*, 24(5), 337-342.
- Shalev, R. S., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: a prospective six-year follow-up. *Developmental Medicine & Child Neurology*, 47, 121-125.
- Shalev, R. S., Manor, O., Kerem, B., Ayali, M., Badichi, N., Friedlander, Y., et al. (2001). Developmental dyscalculia is a familial learning disability. *Journal of learning disabilities*, 34(1), 59-65.
- Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, 75(2), 428-444.

- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, 14(3), 237-243.
- Simonart, G. (1998). *Echelle d'apprentissage scolaires primaires*. Braine-le-Château, Belgique: ATM.
- Snow, C. E., Burns, M. S., & Griffin, P. (1998). *Preventing Reading Difficulties in Young Children*. Washington, DC: Natl. Acad. Press.
- Soltesz, F., Szucs, D., Dekany, J., Markus, A., & Csepe, V. (2007). A combined event-related potential and neuropsychological investigation of developmental dyscalculia. *Neuroscience Letters*, 417, 181-186.
- Spelke, E. S., & Kinzler, K. D. (2007). Core knowledge. *Developmental Science*, 10(1), 89-96.
- Stanescu-Cosson, R., Pinel, P., van De Moortele, P. F., Le Bihan, D., Cohen, L., & Dehaene, S. (2000). Understanding dissociations in dyscalculia: a brain imaging study of the impact of number size on the cerebral networks for exact and approximate calculation. *Brain*, 123, 2240-2255.
- Starkey, P., & Cooper, R. G. (1980). Perception of Numbers by Human Infants. *Science*, 210(4473), 1033-1035.
- Tallal, P., Miller, S., & Fitch, R. H. (1993). Neurobiological Basis of Speech: A case for the Preeminence of Temporal Processing *Temporal Information Processing in the Nervous System: Special Reference to Dyslexia and Dysphasia* (Vol. 682, pp. 27-47). New York: T. Tallal; A.M. Galaburda; R.R. Llinas & C. von Euler.
- Temple, C. M. (1992). Developmental dyscalculia. In S. J. S. a. I. Rapin (Ed.), *Handbook of neuropsychology* (Vol. 7, pp. 211-222). New York: Elsevier.
- Temple, E., Deutsch, G. K., Poldrack, R. A., Miller, S. L., Tallal, P., Merzenich, M. M., et al. (2003). Neural deficits in children with dyslexia ameliorated by behavioral remediation: Evidence from functional MRI. *Proceedings of the National Academy of Sciences*, 100(5), 2860-2865.
- van der Sluis, S., de Jong, P. F., & van der Leij, A. (2004). Inhibition and shifting in children with learning deficits in arithmetic and reading. *Journal of Experimental Child Psychology*, 87(3), 239-266.
- van Oeffelen, M. P., & Vos, P. G. (1982). A probabilistic model for the discrimination of visual number. *Perception & Psychophysics*, 32(2), 163-170.

- vanMarle, K., & Wynn, K. (2006). Six-month-old infants use analog magnitudes to represent duration. *Developmental Science*, 9(5), F41-F49.
- Vauclair, J. (1998). *L'homme et le singe. Psychologie comparée*. Paris: Flammarion.
- Venkatraman, V., Ansari, D., & Chee, M. W. L. (2005). Neural correlates of symbolic and non-symbolic arithmetic. *Neuropsychologia*, 43(5), 744-753.
- Verguts, T., & Fias, W. (2004). Representation of number in animals and humans: A neural model. *Journal of Cognitive Neuroscience*, 16(9), 1493-1504.
- Verguts, T., Fias, W., & Stevens, M. (2005). A model of exact small-number representation. *Psychonomic Bulletin & Review*, 12(1), 66-80.
- von Aster, M. G., & Shalev, R. S. (2007). Number development and developmental dyscalculia. *Developmental Medicine and Child Neurology*, 49(11), 868-873.
- Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, 7(11), 483-488.
- Wechsler, D. (2005). *Manuel du WISC-IV*. Paris: ECPA.
- Whalen, J., Gallistel, C. R., & Gelman, R. (1999). Non verbal counting in humans: The psychophysics of number representation. *Psychological Science*, 10(2), 130-137.
- Wilson, A. J., & Dehaene, S. (2007). Number sense and developmental dyscalculia. In D. Coch, G. Dawson & K. Fischer (Eds.), *Human behavior, learning, and the developing brain: Atypical development (2nd ed.)* (pp. 212-237). New York: Guilford Press.
- Wilson, A. J., Revkin, S. K., Cohen, L., Cohen, D., & Dehaene, S. (2006). An open trial assessment of "The Number Race", an adaptive computer game for remediation of dyscalculia. *Behavioral and brain functions*, 2(20).
- Wilson, J. T. L., Scott, J. H., & Power, K. G. (1987). Developmental differences in the span of visual memory for pattern. *British Journal of Developmental Psychology*, 5, 249-255.
- Wood, J. N., & Spelke, E. S. (2005). Infants' enumeration of actions: numerical discrimination and its signature limits. *Developmental Science*, 8(2), 173-181.
- Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, 24, 220-251.
- Wynn, K. (1995). Origins of numerical knowledge. *Mathematical Cognition*, 1(1), 35-60.
- Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition*, 89, B15-B25.

- Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology*, 25, 103-108.
- Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, 74(1), B1-B11.
- Zorzi, M., & Butterworth, B. (1999). *A computational model of number comparison*. Paper presented at the Proceedings of the twenty first annual conference of the cognitive science society.
- Zorzi, M., Stoianov, I., & Umiltà, C. (2005). Computational modelling of numerical cognition. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 67-84). Hove: Psychology Press.

Appendices

APPENDIX A

Experiment 1 (Chapter 4): Detailed analysis of the COV

We analyzed the variability of the underlying representation, using the coefficient of variation (COV; i.e., the ratio of the standard deviation to the mean estimate). The COV was approximately constant in both MLD and control children across target size. Stable COVs is the signature of scalar variability and indicates that mean estimates and standard deviations are proportional to each other and vary in direct proportion to the numerosity (Castronovo & Seron, 2007; Cordes et al., 2001; Huntley-Fenner, 2001; Izard & Dehaene, 2008; Whalen et al., 1999). As shown in Table 5 panel c and Figure A below, the slope of the best linear fit to these mean COV scores did not significantly differ from 0 for any task in either group (all p s > .1). The participants' personal COV scores (the means and SDs of the COV scores are presented in Table 5 panel d) were entered in a repeated measures analysis of variance (ANOVA) with task (HM to AN, HT to AN, HT to HM, and AN to HM) as within-subject factors, and group (control or MLD) as the between-subjects factor. This analysis revealed a main effect of group, $F(1, 44) = 10.83$, $\eta^2 = .19$, $p = .002$, showing that MLD children ($M = .38$, $SD = .11$) had larger COVs than control children ($M = .29$, $SD = .08$). This indicates a greater variability of the ANS in MLD compared to C adults. An effect of task was revealed as well, $F(3, 132) = 8.80$, $\eta^2 = .17$, $p < .001$. T-tests indicated that variability was lower in the AN to HM task (mean COV = .29, $SD = .12$) than in the HT to AN task (mean COV = .36, $SD = .14$) ($t(45) = 4.26$, $p < .001$). However the HT to AN, the HM to AN (mean COV = .34, $SD = .12$) and the HT to HM (mean COV = .35, $SD = .12$) tasks did not differ significantly from each other ($p > .09$). There were no other significant results.

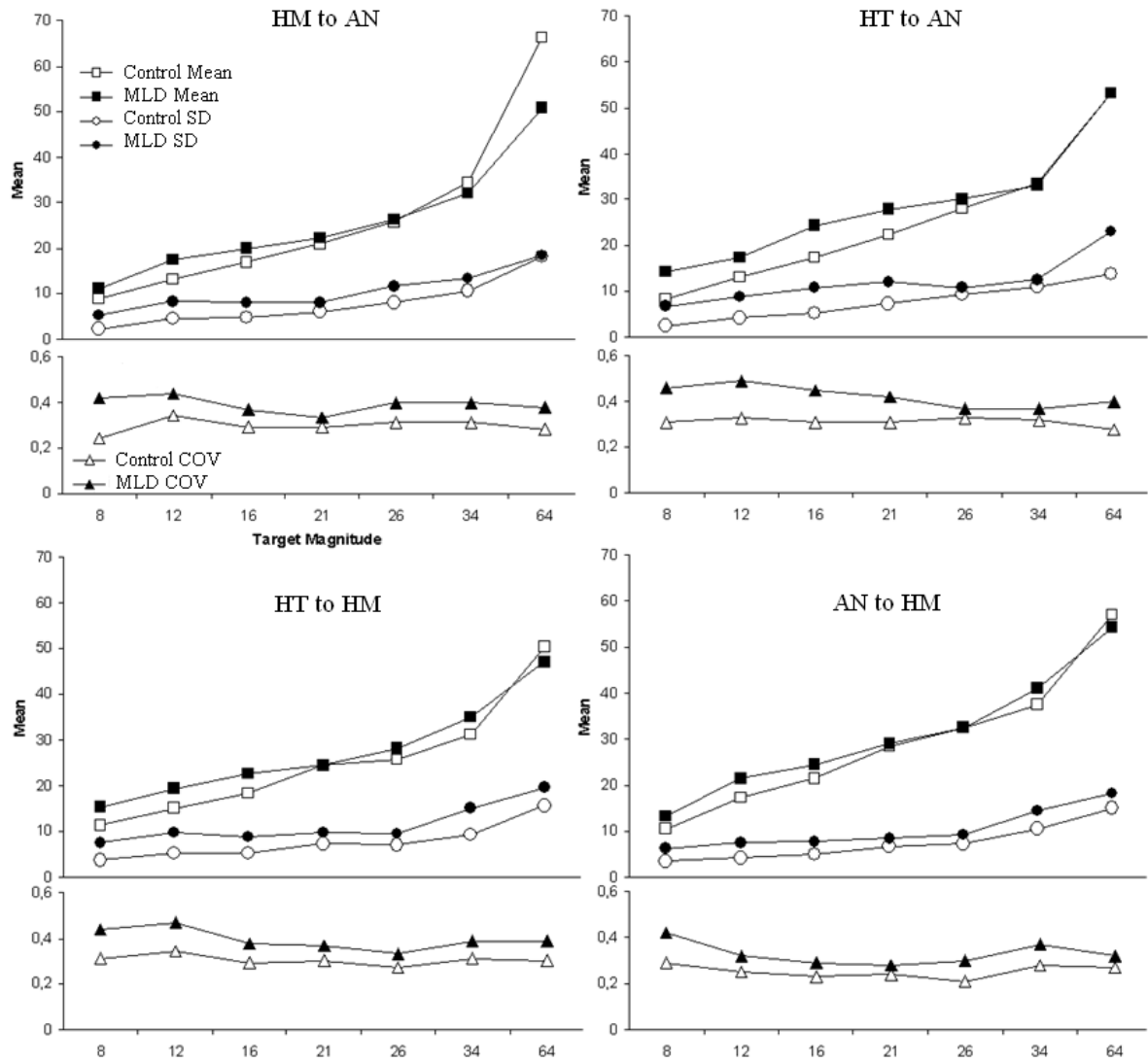


Figure A: In the four estimation tasks, and in both groups, the mean estimate increased linearly with the target magnitude and with the standard deviation. However, the coefficients of variation (COV) – the ratios of the standard deviation to the mean – were approximately constant across the target magnitudes in both groups. The slopes for the two groups differed significantly in the four tasks, indicating that the MLD children showed greater variability than the control children in their numerical estimation.

APPENDIX B

Experiment 2 (Chapter 5): Recruitment and matching procedures

Both MLD and C groups had to fill in a questionnaire where they were asked questions about their development, i.e., “Did you experience learning difficulties? If yes, (a) were you diagnosed in primary school as having learning disabilities (dyscalculia, dysgraphia, attention deficit, etc.); (b) describe with some examples the impact of your learning difficulties in the everyday life, both in the past and the present; (c) what were the consequences of your learning difficulties (remedial classes, special education, repeating a year, etc.)?”

Participants were included in the MLD group if they filled three criteria: (1) they were diagnosed as “dyscalculic” as a child (this concerns only the four youngest participants as this diagnosis is only used for a few years in Belgium) or had experienced problems with mathematics which entailed some remedial action (100% of the selected participants reported so); (2) to report at least two difficulties such as an inability to learn arithmetical facts (100% of the participants), to use calculation in situations such as checking their change in shops (20%), to calculate a discount (20%); to avoid math situations by choosing literary studies (30%), to feel frustrated/anxious in mathematical situations (20%); (3) they should not have experienced other learning disabilities.

The participants in the C group had never experienced any learning disability and had never taken part in any remedial educational program. They were selected to match precisely those in the MLD group. They were paired with respect to age, gender (see Table 9) and current professional or educational activity, i.e., they had to be employed to the same job or to attend

the same field studies (i.e., in each group, they were 4 professional: one secretary, two economists, one speech therapist and 18 students: one in tourism, 12 in psychology, 1 in history, 1 in law, 1 in archeology, 1 in classical literature, 1 in German languages). So for examples, they were two secretaries, one in the MLD group and one in the C group and both of them were 34 year old women, they were two law students, one in each group and both of them were 19 year old men, etc.

APPENDIX C

Experiment 2 (Chapter 5): Arithmetical tests

Participants went through a battery of arithmetical tests which is was part of the standardized battery developed by Shalev and collaborators (2001) and adapted by Rubinsten and Henik (2005). That battery is composed of 20 simple arithmetical exercises (simple additions, subtractions, multiplications, and divisions), 32 complex arithmetical exercises (additions, subtractions, multiplications, and divisions), 8 decimal exercises (complex additions and complex subtractions) and 20 fraction exercises (additions, subtractions, multiplications and divisions).

Acknowledgements

La réalisation d'une thèse, c'est avant tout le fruit de collaborations, à bien des niveaux...

Je tiens tout d'abord à exprimer ma profonde gratitude à l'égard de mes promoteurs, Marie-Pascale Noël et Jacques Grégoire pour leur immense disponibilité malgré leur planning chargé. Marie-Pascale, lorsque tu m'as convoquée dans ton bureau entre deux consultations aux CPS, j'ai évidemment paniqué. Lorsque tu m'as dit que j'avais une nuit pour prendre ma décision de démarrer une thèse à tes côtés et d'entrer dans ce vaste monde du calcul, j'ai été bouleversée. Affolée, j'ai cru que tu n'avais pas remarqué que j'étais en fin de grossesse. Mais tu m'as immédiatement rassurée et tu m'as fait confiance, tu m'as ouvert les portes de la recherche. Merci de m'avoir proposé de travailler avec toi dans ce monde pendant ces quatre années. A l'instar des premières heures, elles ont été un enchaînement de montagnes russes émotionnelles tant sur le plan professionnel que personnel: des joies des premiers résultats au désarroi face aux refus répétés des éditeurs. Mais à chaque étape, tu t'es montrée très présente pour m'apaiser, m'encourager, partager mes joies et mes peines.

Mes remerciements vont également vers les membres de mon comité de thèse, Xavier Seron et Mauro Pesenti, pour leur suivi et la pertinence de leurs remarques et commentaires lors des différentes réunions. Ce fut un grand honneur de bénéficier de votre expertise dans l'accomplissement de cette thèse.

Je tiens aussi à remercier les Professeurs Jacqueline Leybaert et Bert De Smedt pour avoir accepté de faire partie de mon jury.

Travailler au troisième, c'est aussi des amitiés et du bon temps passé entre deux portes, à la cafet', lors des différentes activités scientifiques et culturelles. Merci pour tous ces moments à Anne, Marie, Céline et Christine N.; à Auriane, Sue, Christine, Virginie, Carla et Hélène ; à Salva, Mike (plus une spéciale dédicace pour ton aide et ton soutien), Mauro, Christophe, Sam, Gilles, Pierre et Fred.

Merci à l'équipe des filles du développement, à Laurence pour sa perspicacité et son ticket d'entrée pour Saint-Luc, à Alice et nos longues discussions qui me manquent déjà. A Gaëlle, merci pour tous nos échanges, ton écoute, ta gentillesse, ton grand cœur et d'avoir partagé ce moment unique qu'est la fin de thèse !

Mandy ! Merci pour ton soutien, nos heures de bavardages futiles et essentiellement non futiles, merci pour tous les moments que l'on partage tout simplement avec son amie. Je m'y prends mal, on dirait un adieu... Bon, je te téléphone, attends...

Merci à Pierre Mahau pour son expertise et l'énergie dépensée à construire un matériel nickel, à Nathalie Lefèvre pour ses conseils statistiques avisés, à Pierre Feyereisen, Marie-Anne Schelstraete, et Michel Hupet pour leur soutien. Merci à Yolande, pour sa bonne humeur et sa disponibilité.

Merci à tous nos participants et surtout aux enfants qui ont enduré des heures de testing, qui se sont accrochés et qui ont réalisé les tâches avec le plus grand sérieux. Merci aux parents, aux enseignants et aux directeurs des écoles de nous avoir accordé leur temps, leur confiance et d'avoir permis aux enfants de prendre part aux expériences. Merci aux sujets adultes pour leur patience.

Et puis à tout ceux qui m'entoure : merci à Claire et Pauline pour votre compréhension, votre gentillesse et d'avoir gardé les enfants !

Merci à mes grands parents pour tout ce qu'ils ont fait pour moi pendant toutes ces années et tout l'amour qu'ils m'ont aussi donné.

Et surtout, surtout, ...

Merci à mes parents, pour votre éternelle confiance, votre foi en mes capacités et votre soutien inconditionnel, l'éventuelle librairie, toutes les opportunités, les heures passées en voiture pour me conduire ici où là, les minutes (pardon les années) passées aux téléphones des quatre coins de la planète, aux moments et pour les raisons les plus invraisemblables, pour votre patience, votre incroyable patience dans l'écoute de mes divagations obscures concernant mes testings, mes papiers, mes stats, mes malheurs divers et variés. Evidemment, ça aurait été tellement mieux si nous avions pu tous être réunis aujourd'hui. Merci d'avoir été la meilleure famille où il a fait bon grandir.

Merci à toi Bilou, pour tout et plus encore. Pour ces derniers mois pendant lesquels tu as passés de nombreuses heures à m'écouter radoter sur la richesse/la pénibilité/la joie/le désarroi

de faire de la recherche. A t'occuper des enfants alors que je rédige. A sacrifier les vacances pour que je lise... Surtout merci d'être à mes côtés depuis toutes ces années, pour tout ce que nous avons vécu et partagé. Sans toi, rien ne serait pareil et sans toi, il n'y aurait pas notre merveilleuse famille avec nos deux adorables bambins.

Merci mes petits amours d'être là ! Camille et Basile, vous êtes mes rayons de soleil. Continuez sur votre lancée à pétiller ! Vous êtes ma plus grande fierté.

