RESEARCH ARTICLE

REVISED Connecting complex and simplified models of tipping elements: a nonlinear two-forcing emulator for the Atlantic meridional overturning circulation

[version 2; peer review: 2 approved]

Amaury Laridon[1,2], Victor Couplet[2], Justin Gérard[2], Wim Thiery[1], Michel Crucifix[2]

[1]Water and Climate, Vrije Universiteit Brussel, Brussels, Brussels, 1050, Belgium
[2]Earth and Life Institute, Universite catholique de Louvain, Louvain-la-Neuve, Walloon Region, 1348, Belgium

v2 **First published:** 31 Mar 2025, **5**:87
https://doi.org/10.12688/openreseurope.19479.1
Latest published: 31 Oct 2025, **5**:87
https://doi.org/10.12688/openreseurope.19479.2

Abstract

Background

Despite its far-reaching implications, accurately characterizing the non-linear dynamics of the Atlantic Meridional Overturning Circulation (AMOC) remains a significant challenge. Complex models, including Earth System Models (ESMs) and Earth System Models of Intermediate Complexity (EMICs), offer valuable insights; however, they are computationally expensive and subject to substantial uncertainties in identifying AMOC tipping points. In contrast, simple conceptual models based on simple dynamical systems have been developed to represent tipping elements such as the AMOC. These models can be calibrated against complex models to explore various scenarios and forcing spaces, functioning as emulators. Traditionally, such emulators have focused on a single forcing variable, typically global mean temperature, despite the well-established influence of freshwater fluxes on AMOC dynamics. Moreover, existing two-forcing AMOC emulators lack robust calibration methods against complex models.

Methods

In this study, we develop and validate a two-forcing AMOC emulator that incorporates global mean temperature and freshwater flux, grounded in non-linear dynamics. The emulator is calibrated against

Open Peer Review

Approval Status ✔ ✔

	1	2
version 2 (revision) 31 Oct 2025	✔ view	
version 1 31 Mar 2025	? view	✔ view

1. **Valerian Jacques-Dumas**, Utrecht University, Utrecht, The Netherlands

2. **Dipti Hingmire**, University of Victoria, Victoria, Canada

Any reports and responses or comments on the article can be found at the end of the article.

the AMOC response within the EMIC cGENIE. After validation, the emulator is integrated into SURFER, a reduced-complexity climate model, enabling rapid simulation of AMOC trajectories under diverse emission scenarios.

Results

By accounting for Greenland Ice Sheet melt, the emulator captures three additional collapse trajectories for emission scenarios ranging from SSP3-7.0 to SSP5-8.5. Furthermore, the emulator allows an assessment of the critical forcing manifold of the AMOC in the complex model, enabling the identification of combined forcing thresholds for the AMOC and serving as a tool for comparing the sensitivities of complex models.

Conclusions

With its low computational cost and calibration accuracy, our emulator offers an efficient tool for exploring AMOC dynamics in future climatic scenarios. Finally, the methods used to develop this emulator are generalizable, providing a framework for studying other tipping elements in research.

Plain Language Summary

As global temperatures rise due to greenhouse gas emissions, certain key components of the Earth's climate system are approaching critical thresholds known as tipping points, beyond which large, potentially irreversible changes may occur. One such tipping element is the Atlantic Meridional Overturning Circulation (AMOC), a crucial ocean current responsible for redistributing heat between the hemispheres. If the AMOC were to collapse, it could result in substantial regional changes in temperature, precipitation, and other critical aspects of the climate. However, the exact location of the AMOC tipping point and the time it would take to collapse remain uncertain. Complex models remain uncertain, and running them is computationally expensive, making it difficult to explore a wide range of potential future scenarios. In this study, we developed an emulator—a simplified conceptual model calibrated on a more complex model—to reproduce the behaviour of the AMOC during a potential collapse. This emulator is built on a new methodology that improves the alignment between simplified dynamics and complex models. The resulting tool enables the production of numerous AMOC simulations under various emission scenarios with low computational cost, while remaining consistent with our understanding of the physical processes that govern the AMOC. We present simulations, with this emulator, showing AMOC collapse under future high emission pathways. This emulator and methods enable researchers to better explore the potential responses of the AMOC to future emissions and strengthens our ability to predict and prepare for abrupt changes in the climate system.

Keywords
Climate change, Tipping Points, AMOC, Emulator, Non-linear dynamics, SURFER, cGENIE.

This article is included in the Earth and Environmental Sciences gateway.

This article is included in the Horizon Europe gateway.

This article is included in the Horizon 2020 gateway.

Corresponding author: Amaury Laridon (Amaury.Laridon@vub.be)

Author roles: Laridon A: Conceptualization, Data Curation, Formal Analysis, Funding Acquisition, Investigation, Methodology, Project Administration, Software, Supervision, Validation, Visualization, Writing – Original Draft Preparation, Writing – Review & Editing; **Couplet V**: Conceptualization, Formal Analysis, Methodology, Project Administration, Resources, Software, Supervision, Writing – Review & Editing; **Gérard J**: Resources, Writing – Review & Editing; **Thiery W**: Project Administration, Supervision, Writing – Review & Editing; **Crucifix M**: Conceptualization, Project Administration, Supervision, Writing – Review & Editing

Competing interests: No competing interests were disclosed.

Grant information: This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 820970 project- Tipping Points in the Earth System (TiPES). This project has also received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 101081369 project- Socioeconomic Pathways, Adaptation and Resilience to Changing CLimate in Europe (SPARCCLE).The majority of the technical work in this paper originates from the MSc Thesis of AL, completed in 2024 at UCLouvain (52) under the guidance of MC. The writing and finalization of the research for this article were conducted at the beginning of AL's PhD thesis under the guidance of WT at VUB. The PhD is funded by the FWO under File Number: 1151225N.

How to cite this article: Laridon A, Couplet V, Gérard J *et al.* **Connecting complex and simplified models of tipping elements: a nonlinear two-forcing emulator for the Atlantic meridional overturning circulation [version 2; peer review: 2 approved]** Open Research Europe 2025, **5**:87 https://doi.org/10.12688/openreseurope.19479.2

First published: 31 Mar 2025, **5**:87 https://doi.org/10.12688/openreseurope.19479.1

REVISED **Amendments from Version 1**

In this second version, we have carefully taken into account the reviewers' feedback. The main concern was to improve the readability of the article by rephrasing certain sentences in order to make the text clearer and more accessible. To further enhance both readability and reproducibility, we revised the presentation of several key results and working assumptions—such as the values assigned to emulator parameters, the design of the calibration experiments, and the structure of the supplementary materials.

In terms of scientific content, we carried out a new calibration experiment for the timescale parameter of the emulator. In addition, we performed four new validation experiments to ensure a clear separation between the experiments used for calibration and those used for validation. The general conclusions reported in the previous version remain unchanged, but they are now supported with greater rigor and strengthened scientific reliability.

Finally, regarding specific methodological concerns raised by the reviewers, we provide detailed responses in the "Reply to Reviewers" section included at the end of the article.

Any further responses from the reviewers can be found at the end of the article

1. Introduction

1.1 The AMOC as a tipping element: addressing high uncertainties

The Atlantic Meridional Overturning Circulation (AMOC) is a key component of the climate system. It plays a central role in the transport of heat and salt throughout the global ocean and significantly influences both regional and global climate[1–3]. The AMOC has been identified as a tipping element, a large-scale component of the climate system that can reach a tipping point[4]. A tipping point refers to a critical threshold in a forcing parameter, known as the bifurcation parameter, beyond which a small perturbation of this parameter can cause the tipping element to transition from one equilibrium state to another, resulting in a qualitative change[4]. For the AMOC, the secondary stable equilibrium corresponds to a collapsed state, in which the overturning circulation ceases[5]. A cessation or even a slowdown of the AMOC would have significant consequences for temperatures in the North Atlantic[3–6], as well as impacts on the carbon cycle[7], monsoons[2], and, potentially, other tipping elements[5,8,9].

Accurately simulating the AMOC requires an understanding of its physical dynamics. The AMOC operates through the sinking of large amounts of salty water originating from the South Atlantic. As the water travels northward, it cools and becomes sufficiently dense to sink into the depths, forming the North Atlantic Deep Water (NADW). This water mass then returns to the South Atlantic through the deep layers of the Atlantic Ocean[10,11]. The AMOC is a component of the so-called thermohaline circulation, as it is driven by density differences determined by the temperature and salinity of the water. With global warming, the increase in Atlantic water temperature reduces its density, thereby increasing its buoyancy[12]. This thermal forcing is the dominant mechanism driving AMOC weakening[13–15]. The second forcing mechanism involves a disturbance in the salinity of the water within the Atlantic. As the Greenland Ice Sheet (GIS) melts due to global warming, large quantities of freshwater are added to the deep-water sinking regions in the North Atlantic. This freshwater flux reduces the water's density, increasing its buoyancy and diminishing its ability to sink into the depths, thereby weakening the intensity of the AMOC[15–17]. Future variations in water salinity in the North Atlantic may also result from changes in the precipitation-evaporation (P-E) balance driven by temperature anomalies[18]. The tipping point of the AMOC corresponds to a critical threshold where changes in stratification of North Atlantic waters become self-amplifying, as the AMOC can no longer transport salty waters to the North Atlantic[19]. If the threshold is exceeded for a prolonged period, the AMOC will ultimately reach an irreversible collapsed state because returning to the critical value of the bifurcation parameter will not allow the system to return to its initial equilibrium[5,15,20]. A characteristic hysteresis phenomenon is thus present[4,21].

There is evidence that the AMOC has slowed during the 20th century[22], with reconstructions indicating a 15% decline over the past 70 years[23], bringing it closer to its tipping point. However, observations of AMOC slowdown are subject to considerable uncertainty[24], and although the AMOC may have collapsed in the past[8,25,26], accurately forecasting its future evolution (i.e. when and at what rate it may collapse) remains a significant challenge[5,27,28]. Estimates of the tipping point for the AMOC must therefore rely on models, but the results vary significantly[28]. Some studies place the tipping point at a global mean temperature anomaly of 8°C, while others suggest it may already be as low as 1.4°C[29]. Consequently, some studies estimate that a complete collapse of the AMOC could occur by the end of this century[30], or not until 2300[15]. In addition, there is also sensitivity to the rate of the applied forcing[31,32]. Lastly, the tipping dynamics of the AMOC cannot be fully understood without considering those of the GIS, which has a melting threshold beyond which its decline becomes irreversible[29]. In addition to the destabilizing effect of GIS melting on the AMOC due to freshwater input, a collapse of the AMOC would, conversely, have a stabilizing effect on the GIS. Indeed, the resulting regional cooling following an AMOC shutdown would slow down the melting of the GIS. Thus, a coupling between the AMOC and the GIS exists, potentially giving rise to tipping cascades[33]. In this situation the collapse of one system triggers the failure of another or, conversely, helps stabilize it[34]. Again, significant uncertainties remain in the projections of these dynamics, particularly due to the uncertainty regarding the future of the AMOC[5].

1.2 The need for simplified models capturing first-order dynamics

This significant uncertainty regarding the future evolution of AMOC collapse stems from the diversity of models and approaches employed. To assess the stability of the AMOC, stability diagrams are constructed using hysteresis experiments, in which the forcing is slowly varied along a back-and-forth scenario. Such simulation are possible with state-of-the-art Earth System Models (ESMs)[6], but they remain extremely challenging due to their prohibitive computational demands. As a result, ESMs are generally unable to simulate a large number of full AMOC hysteresis curves[35]. Consequently, Earth System Models of Intermediate Complexity (EMICs) offer a better compromise between explicitly resolving processes and computational efficiency for generating AMOC hysteresis curves[27,36]. Like the more complex models, EMICs exhibit significant variability in the location of the tipping point and, in their projections of the future evolution of the AMOC[27]. However, with current computer technology, even EMICs appear too expensive to efficiently exploring the range of potential forcing scenarios and their associated AMOC responses. The possibility must also be considered that most models, EMICs and ESMs included, exhibit excessive stability due to biases in ocean salinity distribution[37,38]. This explains a sustained interest for conceptual models to explore uncertainties and fundamental aspects of tipping points and tipping cascades[34,39–41].

This approach primarily relies on modelling the dynamics of tipping elements through a double-fold bifurcation structure[39–42], consistent with theory, and also supported by real world measures of stability indicators suggesting that AMOC resides in a bistable regime[43–45]. The ocean circulation system is then modelled possessing two stable equilibria separated by an unstable equilibrium. When the bifurcation parameter reaches the critical value of the tipping point, the system can transition from its initial stable equilibrium to the second. These models mathematically impose that the AMOC behaves as a nonlinear dynamical system with this specific double-fold structure, thereby defining its intrinsic tipping element dynamics. The primary advantage of this approach, beyond its ability to capture the hypothesized dynamical behaviour in general terms, lies in its computational efficiency. By calibrating the simplified dynamics on the output of more complex models, we create what is known in the field as an “emulator,” while the complex model is referred to as the “simulator”[46]. When these tipping element emulators are coupled with a reduced-complexity climate model, they offer a tool that is both process-based and computationally efficient, adequate to study evolution over several millennia driven by realistic concentration or emissions scenarios[39].

1.3 Challenges in designing a two-forcing emulator

An emulator based on the concept of a double-fold bifurcation was introduced by Martinez Montero *et al*.[47] within the reduced-complexity climate model SURFER v2.0. to simulate the dynamics of ice sheets. This emulator considers only a single forcing variable, specifically the temperature anomaly computed from emission scenarios. Couplet *et al*.[39,48], further developed SURFER (v3.0) by allowing for the coupling of tipping elements and hence the inclusion of an additional forcing variable. The challenge, however, lies in the calibration technique for the model parameters, which must use existing literature and additional simulations with complex models. Couplet *et al*.[39] sampled the tipping element parameters from uniform distributions spanning the range of values reported in the literature. In contrast to Couplet *et al*.[39], our approach focuses on calibrating the parameters of a two-parameter physical forcing emulator to align with any given hysteresis curve from a simulator. To our knowledge, this methodology is novel. It enables a more realistic calibration of the emulator while facilitating the systematic exploration of the forcing space across different emission scenarios across multimillennial timescales. Importantly, it preserves the key characteristics of the emulated simulator, allowing for computationally efficient yet physically consistent simulations.

Therefore, this study seeks to address the following objective of developing an AMOC emulator with two forcing parameters — temperature and freshwater flux — that can be calibrated against hysteresis from simulators and integrated into a climate model. To assess the emulator’s performance, we consider three key criteria: (i) its ability to correctly predict whether a collapse occurs in the target simulator, (ii) its ability to reproduce the timing of such a collapse, and (iii) its skill in predicting an AMOC collapse under the combined influence of temperature and freshwater flux, as represented in the simulator. The paper is structured as follows. In Section 2, we introduce the AMOC Tipping Calibration Module (ATCM), a simplified nonlinear dynamical model based on the double-fold bifurcation structure with two forcing parameters: temperature anomaly and freshwater flux. We also describe the calibration module used to fit the double-fold structure to any hysteresis curve derived from a simulator. This novel module is based on the assumption of independent forcing calibration experiments, which allows for a generalization of the method by Martinez Monteiro *et al*.[47]. In Section 3, we apply this calibration process to cGENIE, an EMIC, using three calibration experiments. We then validate the ATCM as an emulator by comparing its results with those from four additional cGENIE experiments. Subsequently, we integrate the ATCM into the SURFER climate model, creating a new configuration that provides a calibrated emulator of the AMOC within a fast climate model. This tool facilitates further investigation of the potential collapse dynamics of the AMOC under realistic emission scenarios. It also allows an assessment of the linear critical manifold of the simulator. Section 4 presents our key findings and discusses the limitations of our approach. Additionally, the section outlines how our methods can be applied to emulate other tipping elements, as well as to couple the impact of an AMOC collapse on GIS dynamics, thereby enabling a tipping cascade emulator. We present our conclusions in Section 5.

2. Methods

We introduce the AMOC Tipping Calibration Module (ATCM), an emulator designed to represent the tipping dynamics of the AMOC under two forcing parameters. The key feature of this emulator is its ability to calibrate simplified AMOC dynamics to match any hysteresis curve derived from a more complex, process-based model. We detail the equations that define the model and describe the methodology used for its calibration. The AMOC model is based on the normal form of a double-fold bifurcation, which has been demonstrated in previous studies to be effective in modelling the tipping element behaviour of the AMOC[39,41]. The calibration technique is built upon the ice sheet emulator implemented in SURFER v2.0[47] and incorporates the assumption of independent forcing in the calibration experiments. Subsequently, in the results section, the ATCM is calibrated on the cGENIE model, enabling it to act as an emulator for the AMOC.

2.1 AMOC dynamical model

The overall scheme of the ATCM is depicted in Figure 1. The non linear ordinary differential equation that describes the AMOC intensity is given by,

$$\frac{d\Psi}{dt}(\Psi, T, F_{GIS}) = \left(\overbrace{-\Psi^3 + a_1\Psi^2 + b_1\Psi + c_1}^{internal\ dynamics} + \underbrace{d_1 T + e_{12} F_{GIS}}_{forcings} \right) \frac{1}{\tau}, \quad (1)$$

This equation is derived from the normal form of a double-fold bifurcation, enabling the representation of the first-order tipping dynamics of the AMOC. Consequently, the tipping element model acts as a generic dynamical system designed to function as an emulator. The use of a generalized normal form of a double-fold bifurcation is a parsimonious and effective approach to capture bi-stability and hysteresis generally associated with tipping elements in the climate system[39,41,47]. The state variable Ψ is dimensionless and spans the interval [0,1], defining the state of the AMOC relative to its pre-industrial level Ψ (t = 0) = 1. For the AMOC, Ψ is the ratio of its current intensity in Sv with its pre-industrial value. The first group of terms in Equation (1) represents the internal dynamics of the tipping element. Since it is a cubic polynomial of the state variable it allows the tipping element to have 1,2 or 3 stable states depending on its forcings[39]. The coefficients a_1, b_1, c_1, d_1, e_{12} do not correspond directly to specific physical properties, but they control the positions of the bifurcation points and can be calibrated based on hysteresis experiments diagnosed in the simulators. Finally, the parameter τ represents an inverse time scale characterizing the typical duration required for the system to deviate from its equilibrium states. In other words, it quantifies the characteristic timescale of AMOC adjustment following a perturbation. It can be calibrated based on a transient experiment from the simulator and constitutes the final parameter of our emulator.

The first forcing term, T, which appears in Equation (1), represents the global mean surface air temperature anomaly relative to pre-industrial levels and encapsulates the physical effects of warming on AMOC water stratification. The second forcing term, F_{GIS}, represents the freshwater flux resulting from GIS melting, which in turn weakens the AMOC by enhancing water column stratification. In the following subsection, we introduce the method to calibrate all parameters of our emulator, a_1, b_1, c_1, d_1, e_{12} and τ, using three calibration experiments derived from the simulator.

2.2 Calibration module

For the calibration, we aim to find the best fit of our double-fold structure to the complex hysteresis curve, ensuring that our simplified dynamics come as close as possible to the bifurcation points identified in the process-based hysteresis. In the paper describing the SURFER v2.0 model, Martinez-Montero *et al.*[47] also modelled tipping elements using a canonical double-fold normal form. They developed a method to calibrate the emulator using the coordinates of the bifurcation points identified

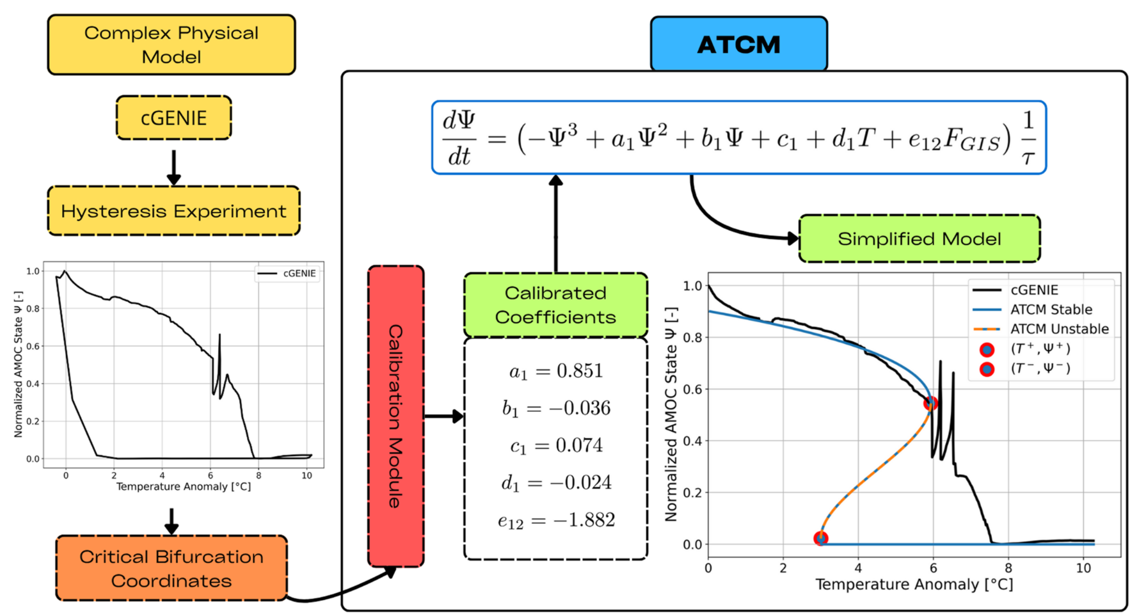

Figure 1. Schematic of the AMOC Tipping Calibration Module (ATCM). The module requires input in the form of bifurcation coordinates derived from complex experiments. These coordinates are used to adjust the calibration coefficients within the simplified tipping element model.

in the simulator experiments. Martinez-Monteiro *et al.*[47] compute explicitly the calibration parameters as a function of the coordinates of the tipping point in the forcing space, as the latter are easier to jutify. This method also allows to test the underlying hypothesis that the leading-order dynamics of tipping elements such as the AMOC can be captured by a double-fold. Our objective is to adhere to this calibration strategy. However, the mathematical framework presented by Martinez-Monteiro *et al.*[47] for modelling each individual ice sheet relies on an ordinary differential equation with only one forcing parameter, namely the temperature anomaly. In our case, for the AMOC, we have two forcing variables: the temperature anomaly and the freshwater flux.

To generalize the method of Martinez-Monteiro *et al.*[46] we need an operational assumption. Specifically, we assume that the simulator used to calibrate the hysteresis of the AMOC allow for independent application of forcing to the AMOC. In other words, with our emulator, we should be able to access process-based models that can force the AMOC solely through globally averaged atmospheric temperature anomalies while keeping the freshwater flux forcing constant, and vice versa. This is technically feasible with most models. If the aim is also to emulate the transition from the collapsed state to the nominal state, an additional constraint arises in the selection of the simulator. Specifically, the simulator used as a reference must be adequate for conducting hysteresis experiments, which involve simulations where the tipping element remains in quasi-equilibrium. For instance, in the cGENIE two first experiments used for calibration of collapse dynamics, simulations span over 20,000 years (see Supplementary Materials). This requirement implies that models categorized as EMICs are generally more suitable candidates to provide reference simulations for emulation[35]. Throughout the text, we will refer to EMICs as the simulator we aim to emulate, although the emulator can be calibrated using any process-based model of the AMOC.

The procedure for calibrating our simplified bifurcation diagram, which is generalizable to N forcings, is as follows. With N forcing variables, we conduct N independent sensitivity experiments, each designed to separately calibrate the bifurcation diagram corresponding to a specific forcing variable. In each case, the remaining $N-1$ forcing variables are held constant, thereby reducing the calibration to a Martinez-Monteiro *et al.*[47]-type model involving only a single forcing dimension. For the AMOC, *EXPA* is defined as the calibration experiment of the AMOC intensity with respect to a temperature forcing in any simulator that satisfies the aforementioned conditions. From this experiment we can retrieve the coordinates of the bifurcation points denoted by,

$$(\Psi^+, T^+), (\Psi^-, T^-). \tag{2}$$

In this context, $\Psi^{\pm}$ represents the normalized values of the AMOC intensity, where the system transitions from its nominal stable equilibrium state ($\Psi^{\pm}$) to its collapsed equilibrium state, and from the collapsed equilibrium state (Ψ^-) back to the nominal equilibrium state. The $T^{\pm}$ values denote the critical temperature anomaly forcing at which the two aforementioned bifurcations occur within the system. It should be noted that the method used to measure the AMOC strength may vary from one simulator to another. In this study using cGENIE, the definition chosen to quantify the maximum AMOC strength will be specified in the Results section. The Equation (1) in the *EXPA* experiment can be written as,

$$\frac{d\Psi}{dt} = \left(-\Psi^3 + a_1\Psi^2 + b_1\Psi + c_1 + d_1 T + e_{12} F_{GIS}^A\right)\frac{1}{\tau}, \tag{3}$$

where F_{GIS}^A represents the arbitrary constant value of the freshwater flux forcing applied during the first calibration experiment. Consequently, $c_1 + e_{12} F_{GIS}^A$ is a constant term in this experiment, effectively reducing the conceptual model to a single-forcing experiment. With this simplification enables we can calibrate the parameters within the technique proposed by Martinez Montero *et al.*[47]:

$$a_1 = \frac{3(\Psi^- + \Psi^+)}{2} \tag{4}$$

$$b_1 = -3\Psi^- \Psi^+ \tag{5}$$

$$c_1 = \frac{T^+ \Psi^{-2}(\Psi^- - 3\Psi^+) - T^- \Psi^{+2}(\Psi^+ - 3\Psi^-)}{2(T^- - T^+)} - e_{12} F_{GIS}^A \tag{6}$$

$$d_1 = -\frac{(\Psi^+ - \Psi^-)^3}{2(T^+ - T^-)} \tag{7}$$

We define *EXPB* as the second calibration experiment examining the AMOC intensity response to freshwater forcing, using the same simulator. The freshwater flux parameterization in the simulator is implemented as a hosing tipping experiment. From this experiment, we can extract the coordinates of the bifurcation points,

$$(\Psi^+, F_{GIS}^+), (\Psi^-, F_{GIS}^-). \tag{8}$$

$F_{GIS}^{\pm}$ represents the critical values of the freshwater flux forcing at which the bifurcation of the AMOC occurs. By defining T^B as the constant value of the temperature anomaly imposed during this second calibration experiment, Equation (1) can be expressed as:

$$\frac{d\Psi}{dt} = (-\Psi^3 + a_1\Psi^2 + b_1\Psi + c_1 + d_1 T^B + e_{12} F_{GIS})\frac{1}{\tau}. \tag{9}$$

Here, $c_1 + d_1 T^B$ represents the constant term, and the method proposed by Martinez Monteiro *et al.*[47] enables the determination of two new values for the following parameters,

$$c_1 = \frac{F_{GIS}^+ \Psi^{-2}(\Psi^- - 3\Psi^+) - F_{GIS}^- \Psi^{+2}(\Psi^+ - 3\Psi^-)}{2\left(F_{GIS}^- - F_{GIS}^+\right)} - d_1 T^B, \tag{10}$$

$$e_{12} = -\frac{(\Psi^+ - \Psi^-)^3}{2\left(F_{GIS}^+ - F_{GIS}^-\right)}, \tag{11}$$

Thus, from the five unknowns (a_1, b_1, c_1, d_1, e_{12}) and two forcing variables, independent calibration experiments allows us to apply the methodological framework of Martinez Monteiro *et al.*[47] to reduce to the case of a single forcing variable. This results in six equations with an over-determination of the independent parameter c_1. It is possible to generalize the approach to include more than two forcing variables. For instance, as demonstrated in the Supplementary Materials (S2), an additional freshwater flux forcing term was incorporated into Equation (1), which can be used for instance to calibrate the impact of variations in glacier melt under future scenarios. Mathematically, the generalization to N forcing variables results in obtaining $4N - 2(N - 1)$ equations with $4 + (N - 1)$ knowns and $(N - 1)$ over-determined equations. As shown in the next section, these over-determinations occur because the coefficient c_1 is the constant shared across all calibrations of different sensitivity experiments. What value, then, should be assigned to c_1? Sensitivity experiments have shown that defining $c_1 = c_1^A$, where c_1^A is determined from Equation (6), yields the most accurate results for emulating the evolution of the AMOC based on the simulator. This is because temperature forcing is the primary driver of AMOC collapse[15], as demonstrated in CMIP5 experiments[13]. Consequently, we prioritize achieving the most accurate calibration for temperature forcing while tolerating a greater error in calibrating freshwater forcing. Therefore, we adopt the value of c_1 as determined by the temperature sensitivity calibration experiment.

The final parameter in the ATCM module that needs to be calibrated is the internal dynamic time scale of the AMOC, denoted as τ. Based on Armstrong McKay *et al.*[49], we assume that the internal dynamic time scale of the AMOC is the same whether it is collapsing or recovering, i.e., $\tau \equiv \tau_\Psi^+ = \tau_\Psi^-$. To calibrate τ, we aim to minimize the discrepancy between the timing of the AMOC collapse in the simulator and in the emulator. This is achieved through classical numerical optimization to determine the optimal τ calibration in the emulator, allowing a maximum tolerance of 1% difference between the projected timings in the two models.

What type of forcing experiment should be conducted with the simulator for this purpose? By definition, τ requires transient experiments measuring AMOC intensity in our simulator. As we show next, this simplified AMOC dynamics framework cannot simultaneously reproduce with equal accuracy the characteristic timescales of AMOC response to both rapid and slow forcing regimes. Hence, the user should chose a forcing function that is consistent with the emulator's intended application. Here, given our emulator's primary purpose of investigating plausible near-term AMOC collapse scenarios, we calibrate τ in the following section using a cGENIE simulation where atmospheric CO_2 concentrations increase by 1% annually until reaching eight times pre-industrial levels. We will subsequently demonstrate the robustness of these calibration results while discussing their inherent limitations.

3. Results

In this section, we calibrate the ATCM using cGENIE, following the methodology developed in Section 2. Next, we evaluate the ATCM by simulating the AMOC under two SSP scenarios and comparing the results with cGENIE simulations. Finally, we integrate the ATCM into SURFER and simulate the evolution of the AMOC under a range of realistic emission scenarios.

3.1 Calibration with cGENIE

To validate the emulator, we applied the ATCM to cGENIE[50], an EMIC[36] that has already demonstrated its ability to simulate AMOC hysteresis[14,51]. cGENIE includes an ocean circulation model (3D), a dynamic-thermodynamic sea ice model (2D) and an atmospheric energy moisture balance model (2D). The ocean model accounts for the horizontal and vertical transport of heat, salinity and biogeochemical tracers. The circulation is simulated through advection, convection, and mixing[51,52]. In cGENIE, the maximum AMOC strength is defined as the maximum value of the overturning stream function below approximately 1000 meters depth in the Atlantic. This depth threshold ensures that shallow convection associated with surface gyres is not included, even in cases where the AMOC has collapsed. cGENIE has a very simple atmospheric model, which results in a minimal effect of net changes in the Precipitation-Evaporation (P-E) balance over the North Atlantic due to global warming. However, if the ATCM is applied to emulate a simulator that account for variations in the P-E balance, these would be captured in the calibration through the temperature anomaly. cGENIE does not include an explicit parameterization of GIS melt. Although such a component is not strictly required in the simulator targeted for emulation, its absence in cGENIE simplifies the calibration process: during the temperature-forcing calibration, we can be confident that no freshwater flux from the GIS affects the AMOC. This, in turn, simplifies the analysis of the AMOC's sensitivity to a single forcing through temperature in cGENIE. Following the experimental framework outlined in the Methods section, we conducted two independent sensitivity experiments to calibrate the $(a_1, b_1, c_1, d_1, e_{12})$ parameters using cGENIE.

The first experiment, labelled *EXPA*, consist of a 20,000-year simulation with a prescribed CO_2 forcing designed to generate a global atmospheric mean temperature anomaly. We thus apply our operational hypothesis by forcing the AMOC in cGENIE solely through temperature in this initial setup. To do so, we do not apply any artificial, time-dependent freshwater input at this stage. Accordingly, we define $F_{GIS}^A = 0\ Sv$ as the constant and arbitrary freshwater forcing value in *EXPA*. The CO_2, hence temperature forcing is parameterized as a linear increase from 280 ppm to 2,800 ppm. Through the internal dynamics of cGENIE, this forcing is translated into a global mean surface air temperature anomaly, starting from $T = 0°C$ for the initial thousands of years and reaching $T = 0°C$ after 20,000 years (see Supplementary Figure S1). This setup enables us to cover the plausible range of the AMOC tipping point location in terms of temperature forcing[5,49] while ensuring near-equilibrium conditions for the AMOC.

The second independent calibration experiment, labelled *EXPB*, involves a 20,000-year hosing simulation with freshwater flux forcing linearly increasing from 0 Sv to 0.2 Sv (see Supplementary Figure S2). For the constant value of the global mean temperature anomaly in this experiment, we set $T^B = 0°C$. The freshwater forcing values are chosen to align with the plausible range of AMOC tipping point locations associated with freshwater perturbations[17,26] and to be sufficient to trigger an AMOC collapse in cGENIE. The total salinity of the oceans is maintained constant in the simulation by applying a compensatory freshwater flux over the Pacific. The freshwater hosing is applied between 20°*N* and 50°*N* across the full width of the Atlantic (see Figure S3) to reproduce the hosing region by Rahmstorf *et al.*[27]. The durations of the *EXPA* and *EXPB* simulations are chosen to ensure that the AMOC is forced sufficiently slowly, allowing it to remain in near a stable state throughout the calibration experiments.

The two trajectories of the AMOC intensity obtained with cGENIE in the *EXPA* and *EXPB* experiments are shown in Figure 2. This three-dimensional representation, combining the two-dimensional forcing space with the additional AMOC intensity dimension, makes it possible to visualise the full range of possible forcing conditions for the AMOC, which the emulator will subsequently be able to explore. In the Supplementary Materials, Figures S4 and S5 show the bifurcation diagrams of the AMOC in the two-dimensional plane of the *EXPA* and *EXPB* experiments.

Since the primary objective of the ATCM is to accurately calibrate the collapse branch, we focus exclusively on simulating collapse scenarios with cGENIE. This allows to extend the duration of the simulations with the complex model and thus, obtain an AMOC as close as possible to its equilibrium in the simulator. In other words, a complete hysteresis cycle was not performed to refine our calibration of the collapse branch, but this does not undermine the validity of the results. As demonstrated in Laridon[53], the ATCM can also successfully emulate the AMOC trajectory of the simulator through the second bifurcation point. However, to achieve an optimal fit for the collapse trajectory, slight deviations from the bifurcation point coordinates provided by the simulator may enhance the emulator's accuracy in capturing the collapse dynamics. Accordingly, the values of $\Psi^- = 0.022$, $T^- = 3°C$ and $F^-_{GIS} = 0.037$ *Sv* were selected, as they minimise the calibration error along the trajectory of the nominal stable equilibrium. However, if the research objective is to accurately calibrate the recovery transition, it is possible to provide

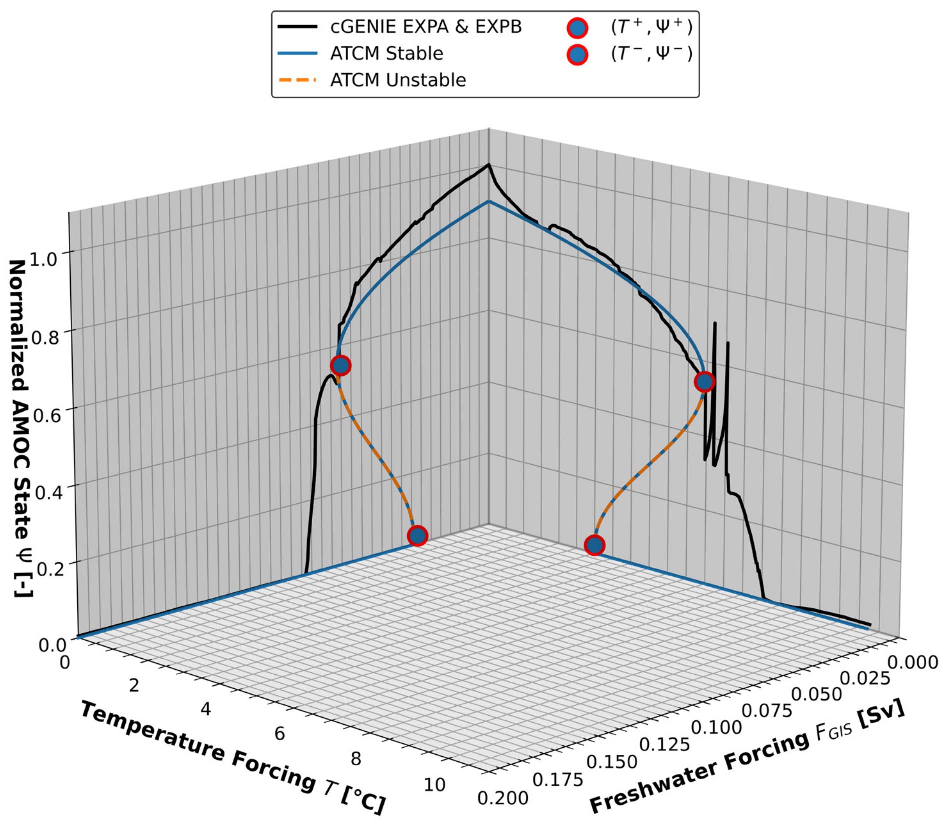

Figure 2. Bifurcation diagrams of cGENIE and ATCM in the (*T*, F_{GIS}) forcing space. The two simulations produced by cGENIE during the calibration experiments *EXPA* and *EXPB* are shown in their respective planes as solid black lines. The identification of the bifurcation points based on these collapsed branches is marked in red, while the simplified hysteresis produced by the ATCM is shown in blue. The branch of the unstable equilibrium, which separates the two stable equilibria, is represented by the orange dashed line.

the emulator with the values of T^- and F_{GIS}^- corresponding to those identified in a full hysteresis experiment. A trade-off must therefore be made in the choice of the bifurcation point coordinates, in order to distribute the emulator's calibration error where it is most relevant with respect to the scientific objective.

All coordinates of the bifurcation points used in this study are given in Table 1. There is no standardised method for determining the precise location of bifurcation points. Consequently, the selection should be based on expert judgment and a comprehensive understanding of the simulator behaviour. In the ATCM model, Equation (1) imposes that $\Psi_A^+ = \Psi_B^+ \equiv \Psi^+$ and $\Psi_A^- = \Psi_B^- \equiv \Psi^-$ to keep the number of unknows to a manageable level. Moreover, we aim to find the best fit for temperature forcing, so we define $\Psi^\pm \equiv \Psi_A^\pm$. Using these values, we compute the parameters (a_1, b_1, c_1, d_1, e_{12}), according to Equation (4–7, 11), and the results are presented in Table 2. Based on the calibration of these parameters, the simplified hysteresis loop emulated by the ATCM is shown in Figure 2. Since we calibrated the ATCM using the c_1 value from *EXPA*, the double-fold structure passes precisely through the upper bifurcation point (T^+, Ψ^+) identified on the cGENIE curve from the temperature forcing calibration experiment. This is not the case for *EXPB*, as will be explained in the Discussion section. Indeed, there is a $0.002Sv$ difference between the F_{GIS}^+ tipping point identified from cGENIE and the value computed by the ATCM, resulting in a relative difference between the two of 2.53%. Overall, the collapse dynamics demonstrate a reasonably good calibration against the cGENIE simulations.

Finally we must set τ, the characteristic timescale of AMOC dynamics. Following the experimental protocol outlined in the Methods section, we performed a third calibration experiment with cGENIE, labelled *EXPC*, spanning 1,250 years. The simulation was initialized at 280 ppm atmospheric CO_2 (pre-industrial level) and increased concentrations by 1% per year until reaching 2240 ppm (8× pre-industrial levels), maintaining this concentration thereafter (see Supplementary Figure 6). This calibration experiment applied only thermal forcing, without freshwater hosing. In this cGENIE simulation AMOC collapses, reaches a minimum intensity ($\Psi = 0.03$) by 2190 CE, then partially recovers to less than 10% of its initial strength (Figure 3). This behaviour reflects residual, persistent deep convection below $100m$ in our simulator, which gradually diminishes over longer time scales, as confirmed by additional analyses. We therefore define AMOC collapse in cGENIE as the event occurring by 2190 CE. We find that a value of $\tau = 18.94\ yr^{-1}$ best reproduces the timing of this event in the emulator (Figure 3b).

While reproducing the timing of collapse, significant differences in the trajectories emerge between the emulator and the simulator outputs. This discrepancy stems primarily from: (i) the emulator's simplified single-equation framework, which lacks spatial resolution compared to cGENIE's 3D dynamical ocean physics, and (ii) the emulator's inability to capture rate-dependent AMOC modifications (R-tipping phenomena[31]) that are present in cGENIE. We further address these aspects in the validation experiments and discussion. Notably, the choice of *EXPC* imposes operational constraints—if the emulator is intended for studying slower forcing regimes, alternative transient simulations would provide a more suitable calibration baseline (see Figure S7). The ATCM's minimal parameterization does not allow for universal calibration across all forcing timescales. However, as demonstrated in

Table 1. Bifurcation coordinates. The values for Ψ^+, T^+, F_{GIS}^+ are retrieved from the cGENIE calibration sensitivity experiments *EXPA* and *EXPB* shown in Figure 2. The Ψ^-, F_{GIS}^- and T^- values are manually fixed during the calibration procedure and adjusted to improve the fit of the collapse branch. The F_{GIS}^A and T^B values correspond to the arbitrary constant forcing applied during the sensitivity experiment.

	Ψ^+ [*Adim*]	Ψ^- [*Adim*]	T^+ [°*C*]	T^- [°*C*]	F_{GIS}^+ [*Sv*]	F_{GIS}^- [*Sv*]	F_{GIS}^A [*Sv*]	T^B [°*C*]
EXPA	0.545	0.022	5.93	3	/	/	0	/
EXPB	0.545	0.022	/	/	0.075	0.037	/	0

Table 2. Internal dynamics parameter. The parameters are computed with the ATCM using Equation (7–10,14) using coordinates of the bifurcation points given by Table 1.

a_1	b_1	c_1	d_1	e_{12}
0.851	-0.036	0.074	-0.024	-1.882

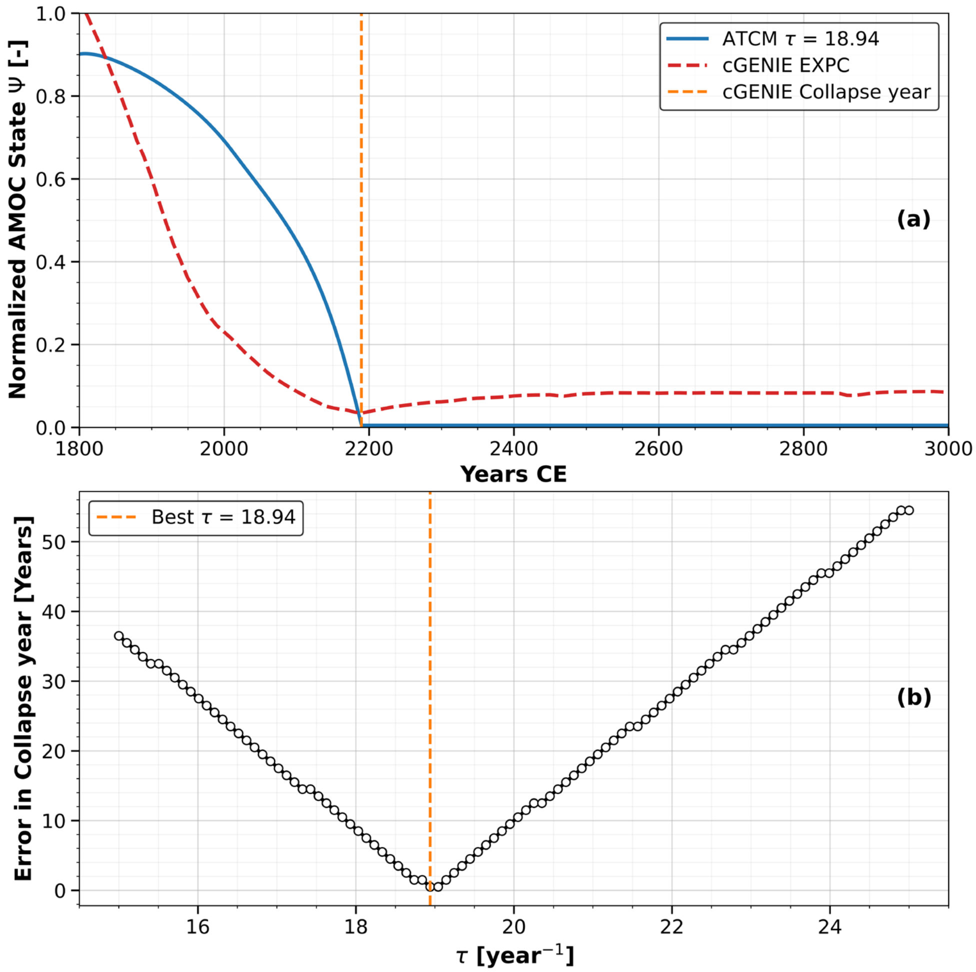

Figure 3. (**a**) Normalized AMOC intensity in the *EXPC* calibration experiment within the ATCM (blue) and cGENIE (red). (**b**) ATCM Optimization of the τ parameter to replicate the collapse year in cGENIE.

the following section, our 1% CO_2 forcing in EXPC enables robust emulator performance when applied to SSP scenarios, which constitute the primary focus of this study.

3.2 Validation of the emulator

To validate the emulator, we assess the ability of the ATCM to reproduce the AMOC trajectory simulated by cGENIE under four forcing combinations that differ from the ones used in the calibration experiments. Since the primary purpose of the emulator is to generate simulations for relatively near-term and realistic future scenarios, we selected two SSP simulations, namely SSP2-4.5 and SSP5-8.5. For each of these two SSP simulations, which prescribe distinct CO_2 concentration pathways and thus different temperature forcings, we additionally consider a configuration in which a constant hosing of 0.06 *Sv* is applied in the same region as in previous cGENIE experiments. The resulting temperature anomalies in cGENIE are presented in Supplementary Figure S8. The four normalized AMOC trajectories from these validation experiments are shown in Figure 4 as solid lines. In cGENIE, the AMOC does not collapse under SSP2-4.5 without hosing, but does collapse under SSP5-8.5 without hosing. The most striking result is that the AMOC does collapse under SSP2-4.5 when freshwater forcing is applied. Under SSP5-8.5, the collapse is naturally faster when hosing is included than when it is not.

We now examine the trajectories produced by the ATCM, shown in Figure 4 as solid lines. First, we note that whenever the AMOC collapses in cGENIE, it also collapses in the emulator, and similarly, when the AMOC remains active in the simulator, it remains active in the emulator. This constitutes the minimal criterion for the emulator's skill—namely, the ability to correctly predict whether a collapse occurs in the simulator. Second, regarding the timing of the collapse, the emulator consistently predicts it to occur earlier than in cGENIE. This discrepancy is most pronounced for the SSP2-4.5 scenario

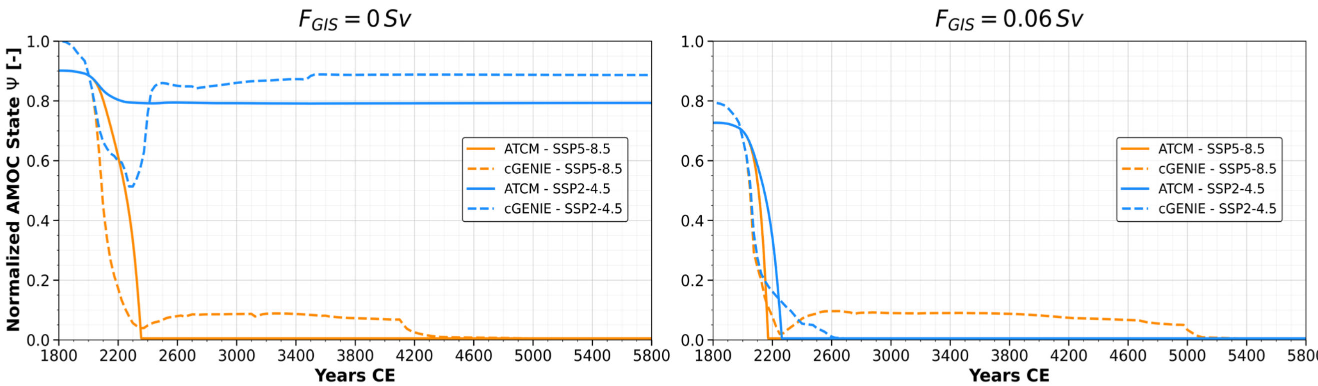

Figure 4. Trajectories of the AMOC in the validation experiments with the emulator and the simulator under SSP scenarios without additional hosing (left) and with additional hosing (right). Solid lines correspond to the ATCM, while dashed lines correspond to the simulator.

with hosing, which represents the lowest forcing rate among the validation experiments. We recall that the parameter τ was calibrated using the *EXPC*, based on a transient experiment with a 1% increase in atmospheric CO_2 concentration per year. This forcing pathway has a rate of forcing closer to that of SSP5-8.5. This explains why the emulator performs particularly well for the SSP5-8.5 scenario without hosing, reproducing the timing of the complete collapse within only six years of cGENIE (2350 CE in cGENIE versus 2356 CE in the ATCM).

This said, even under SSP2-4.5, the ~350-years difference in the timing of total collapse between the two models is acceptable, especially considering that by 2240 CE both models predict an AMOC intensity below 15%, which already indicates an abrupt and nearly complete collapse. Third, the emulator successfully reproduces the AMOC collapse under SSP2-4.5 when hosing is applied. This achievement results from the novel representation of the AMOC in the ATCM, which incorporates an additional freshwater flux forcing variable calibrated against the target simulator. We see this as a significant result. Finally, the differences between the emulator and cGENIE trajectories arise from the spatial and physical dynamics represented in cGENIE but absent from the emulator. In particular, cGENIE captures the fact that the changes in AMOC intensity not only depends on the absolute magnitude of the forcings but also on their rate of change over time. For instance, under the SSP2-4.5 scenario without hosing, between 2000 CE and 2400 CE when the rate of warming is the highest, the emulator projects an AMOC strength nearly 30% higher than in cGENIE. Similarly, in the other simulations, the emulator systematically overestimates the AMOC strength during the collapse. In practice, calibrating τ using the *EXPC* —which involves a relatively high forcing rate compared to the validation experiments—partly compensates for this limitation.

3.3 ATCM Integration within the SURFER climate model

We now incorporate the ATCM emulator into the reduced-complexity climate model SURFER to explore AMOC's response under a range of realistic future emission scenarios (see Figure 5). SURFER features a process-based carbon cycle capable of reliably simulating atmospheric CO_2 concentrations and global mean temperature changes. This reduced-complexity model also simulates sea-level rise and various ocean acidification metrics in response to anthropogenic greenhouse gas emissions, while enabling simulations over timescales ranging from decades to millions of years[47,48]. Version 3.0 of SURFER comprises 17 differential equations that describe carbon exchanges among six reservoirs: the atmosphere, terrestrial systems, upper, intermediate, and deep ocean layers, and deep-sea sediments[48]. Additionally, it models temperature anomalies across ocean layers, ice sheet volumes for Greenland and Antarctica, and sea-level changes due to glacier dynamics.

SURFER has proven effective for integrating tipping element dynamics into climate simulations, as demonstrated by Couplet *et al.*[39]. At the time of this project, version 3.0 of SURFER was not yet available. Therefore, we chose to integrate the ATCM into a preliminary version of v3.0 (see Data Availability), which we refer to here as pre3.0. There are no major differences between pre3.0 and v3.0. Some changes in the parameterization of SURFER's carbon cycle remain, but they are not central to the objectives of this study. To integrate the ATCM into SURFER pre3.0, we chose to deactivate all components associated with tipping elements other than the AMOC and GIS in order to specifically isolate the contributions of the new AMOC representation.

Within SURFER we obtain dynamic projections of the mean atmospheric temperature anomaly T and the freshwater flux associated with GIS melting, all computed within a limited time. Indeed, the parameterisation of F_{GIS} follows that of Couplet *et al.*[39] and is defined as:

$$F_{GIS} = -\alpha_{GIS} \frac{dV}{dt}. \tag{12}$$

The freshwater contribution of the GIS is proportional to the temporal variation of its dimensionless volume V. As with the

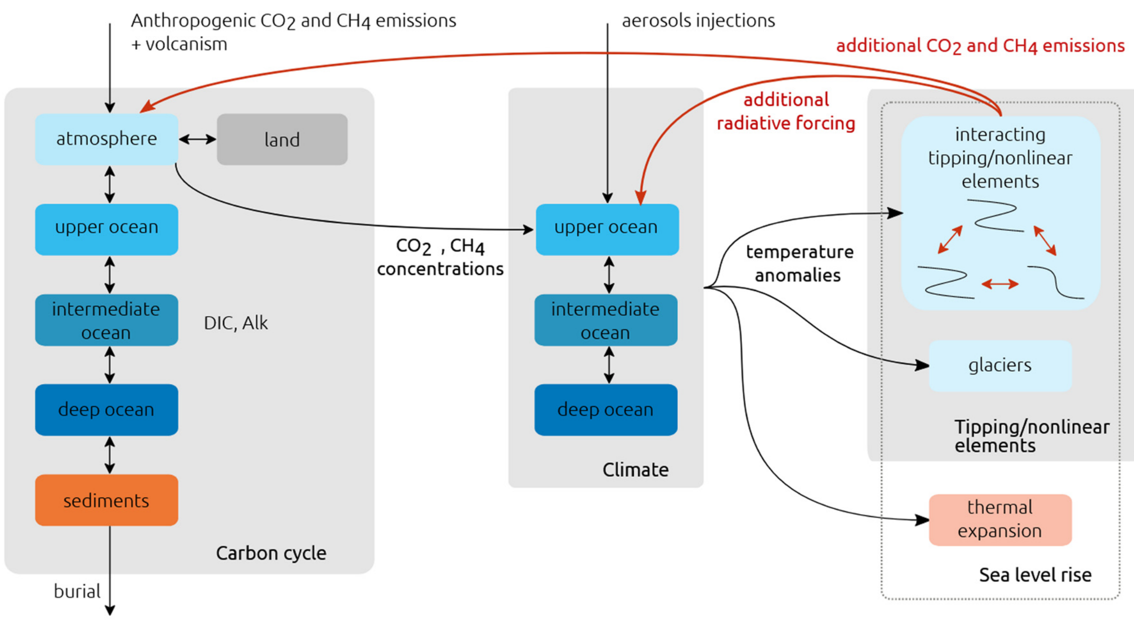

Figure 5. Conceptual diagram of SURFER, including interacting tipping elements and their feedback on climate. Figure reproduced with permission from Couplet *et al.*[39]. The ATCM is integrated into the 'Tipping Elements' box of SURFER.

normalization of Ψ, the value of V is normalized by the volume of the GIS during the pre-industrial period. The parameter α_{GIS} relates the temporal variation of the dimensionless fraction of the GIS to a freshwater flux. The details of its computation and value are provided in Couplet *et al*.[39]. In this pre-3.0 version of SURFER, we also use the same equation and calibration coefficients as in SURFER v2.0[47] to describe the temporal evolution of the GIS volume,

$$\frac{dV}{dt} = \left(\overbrace{-V^3 + a_2 V^2 + b_2 V + c_2}^{internal\ dynamics} + \underbrace{d_2 T}_{forcing} \right) \mu_V(V). \tag{13}$$

The term $\mu_V(V)$ encodes the dynamic time scale of the GIS, analogous to that of the AMOC, except that this value is parameterized differently depending on whether the GIS is regaining ice volume ($\tau_V^+ = 5500\ yr^{-1}$) or melting it ($\tau_V^- = 470\ yr^{-1}$). These values are those used in SURFER v2.0[46] and are themselves derived from a calibration against a complex GIS model. Note that it is possible to add in Equation (13) a second forcing term proportional to $(1 - \Psi)$ to account for the stabilising effect of a weakened AMOC on GIS melting. In Section S1 of the Supplementary Materials, we show how this two-forcing-variable equation can be calibrated, similarly to the ATCM. This tool can then be used to investigate tipping cascades[32,34] between these two systems with what we could call a tipping cascade emulator.

Figure 6 illustrates the fifteen emission scenarios selected as inputs to simulate the evolution of the AMOC within the emulator once integrated into SURFER. These emission trajectories follow the historical record from 1750 to 2010 CE, and are then extended using a logistic equation so that the cumulative emissions from 1750 to 2500 CE range from 1000 to 5000 PgC[54]. In this way, they cover the spectrum from SSP1-2.6 to SSP5-8.5.

3.4 Critical manifold and sampling the forcing space

Based on the calibration of the ATCM to cGENIE, we can represent in the forcing variable space (T, F_{GIS}), the two calibration experiments, *EXPA* and *EXPB*, as well as the validation experiment that consider hosing (see Figure 7). This representation demonstrates the use of the emulator as a tool for mapping the forcing space of a given simulator. The critical manifold W_c is defined by the ATCM as follows,

$$W_c(T) = \frac{d_1}{e_{12}}(T^+ - T). \tag{14}$$

This manifold delineates the forcing space into two regions: one where the linear combinations of the variables T and F_{GIS} do not lead to an AMOC tipping in the emulator, and another where such combinations would, at equilibrium, drive the AMOC into the collapsed stable state. Despite the approximations inherent in the calibration process, the ATCM provides a valuable pre-diagnostic tool for analysing forcing combinations that, if sustained for a sufficient duration[31], are expected to lead to a complete tipping of the AMOC. For example, we can use this critical manifold to identify the tipping point according to the emulator for the SSP experiments with a 0.06 Sv hosing applied in the specific region within cGENIE. In this way, the ATCM provides an estimate indicating that the

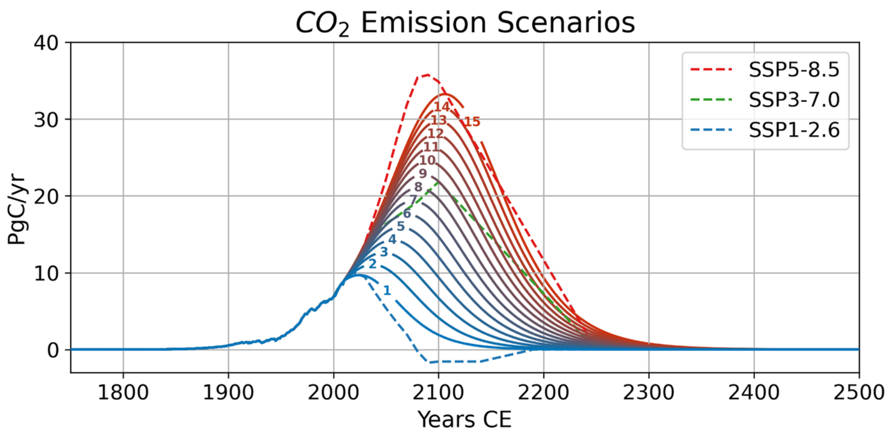

Figure 6. Fifteen emission scenarios used as input to SURFER. These scenarios account for cumulative release of CO_2 in the atmosphere between 1000 to 5000 PgC since 1750 CE, and cover a similar range as the SSP scenarios. The emission trajectories range from blue for scenarios with the lowest emissions to red for those with the highest greenhouse gas emissions, with their corresponding labels placed directly along the curves.

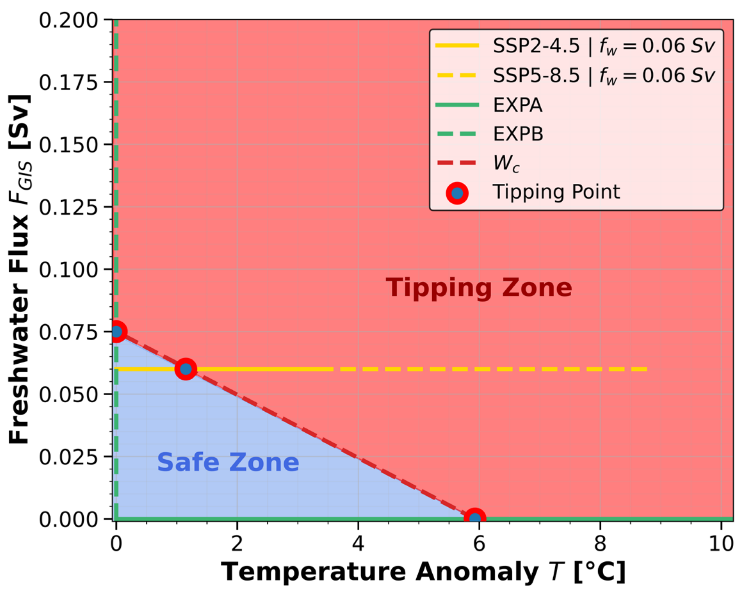

Figure 7. Forcing space (*T, F_{GIS}*) showing calibration experiments *EXPA* (solid green line) and *EXPB* (dashed green line), along with two SSP validation experiments under hosing conditions (solide yellow line for SSP5-8.5 and dashed yellow line for SSP2-4.5). The tipping points identified in *EXPA* and *EXPB* are indicated by the blue and red markers. The red dashed line, W_c, represents the critical bifurcation manifold in the ATCM. The "Safe Zone" in the forcing space corresponds to the combinations of the two forcing variables that do not reach the bifurcation point, whereas the "Tipping Zone" corresponds to conditions exceeding this bifurcation threshold in the ATCM.

tipping point associated with the temperature anomaly is near 1.2°C. This estimate is thus obtained through the emulator without the need to perform a new computationally expensive hysteresis experiment in cGENIE, which would combine both temperature and freshwater flux forcing.

As an application, we investigated the response of the ATCM integrated into SURFER under 15 emission scenarios, ranging from SSP1-2.6 to SSP5-8.5 (see Figure 6). Figure 8 shows the trajectories of these 10,000-year simulations in the forcing space (T, F_{GIS}). The critical manifold W_c is of course identical

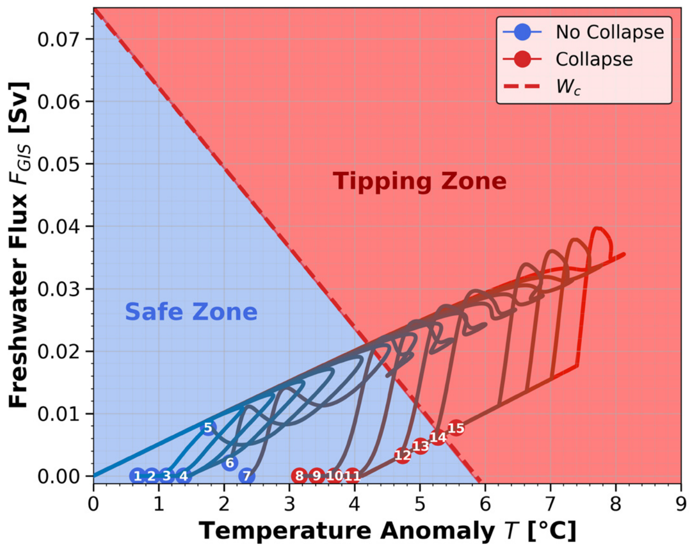

Figure 8. Evolution of Temperature (T) and Greenland meltwater flux (F_{GIS}) in SURFER under 15 emission scenarios (see Figure 6). The dots indicate the state of the AMOC at the end of the 10,000-year simulation: a blue dot represents $\Psi > \Psi^{+}$, corresponding to an active AMOC state, while a red dot represents $\Psi < 0.1$, indicating a collapsed state. The labels corresponding to their associated emission scenarios (see Figure 6) are written in white inside these dots. The red dashed line, W_c, denotes the critical bifurcation manifold in the ATCM, dividing the forcing space into the same two regions as defined in Figure 7.

to that in Figure 7, derived from the ATCM through the calibration experiments *EXPA* and *EXPB*. Nine of the 15 trajectories (labelled 7–15 in Figure 6) exceed the bifurcation threshold of the AMOC when considering the linear combination of the two forcing variables; in other words these trajectories cross W_c Notably, trajectory 7 surpasses this threshold only briefly and weakly, resulting in the overshoot-without-tipping behavior described by Ritchie *et al.*[30]. Hysteresis effects are evident: trajectories 8–14 lead to AMOC collapse and do not recover circulation despite returning to the safe space. Lastly, these results underscore the importance of accounting for both temperature and freshwater flux, as the inclusion of freshwater forcing lowers the temperature-associated tipping point due to their combined influence, as reflected in the slope of W_c.

3.5 New collapse trajectories captured by the emulator

To highlight the importance of developing an AMOC emulator that accounts for both temperature and freshwater flux forcing within a reduced-complexity climate model, we compared two cases using the ATCM in SURFER. In the first case, both forcings are included; in the second, the freshwater flux forcing is deliberately disabled. In both cases, we applied the emission scenarios shown in Figure 6. When the freshwater flux forcing from GIS melting is deactivated (see Figure 9a), only the five highest emission scenarios (labelled 11 to 15) lead to an AMOC collapse during these 10,000-year simulations. However, when the freshwater forcing from the GIS is activated in SURFER, three additional scenarios (labelled 8 to 10) result in an AMOC collapse. These three scenarios correspond to total cumulative emissions ranging from 3000 *PgC* to 3571 *PgC* over the period 1750–2500 CE. In other words, they lie slightly below and above the cumulative total emissions associated with SSP3-7.0, as shown in Figure 6. Thus, in addition to reproducing the key result identified in the validation experiments—namely, that the emulator can simulate collapses that would not occur under temperature-only forcing—the emulator coupled with SURFER now allows the simulation of these collapse trajectories at low computational cost, without the need to rerun the full complex model.

Lastly, in these comparisons of the trajectories with the temperature bifurcation diagram computed by the emulator, two additional noteworthy observations can be made. When the freshwater flux forcing is excluded (see Figure 9a), the AMOC trajectories lie above the bifurcation diagram. In other words, the system is unable to track its stable equilibrium position. Indeed, rate of forcing changes shifts the AMOC significantly off the equilibrium position that would be observed in a hysteresis experiment. In Figure 9b, by contrast, the trajectories consistently lie below the temperature bifurcation diagram. This occurs because the trajectories are driven by two forcings, whereas here we are representing only the temperature bifurcation diagram (for the purpose of comparing the trajectories between Figures 9a and 9b) and not the bifurcation diagram corresponding to the combined effect of both forcings.

4. Discussion

We developed a two-forcing-variable AMOC emulator that can be calibrated using only three calibration experiments. The differences between the ATCM and cGENIE during the

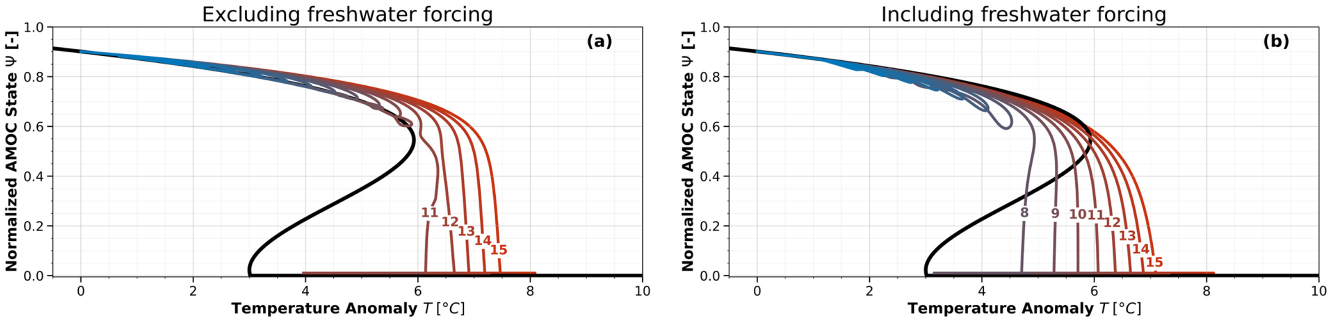

Figure 9. Temperature bifurcation diagram from the ATCM showing the 15 AMOC trajectories in the case where only temperature forcing is applied (**a**) and when freshwater forcing is also included (**b**). The 15 trajectories are computed based on the emissions scenarios described in Figure 6. AMOC trajectories are colored from blue for the lowest-emission scenarios to red for the highest-emission scenarios, with labels corresponding to those in Figure 6. The black curve represents the temperature bifurcation diagram computed by the ATCM based on *EXPA*.

calibration phase can be attributed to two factors: the inherent limitations of the conceptual approach and constraints within the ATCM calibration module. The first factor stems from the intrinsic simplifications involved in fitting a double-fold structure to a complex hysteresis. These limitations were expected. From a technical standpoint, fitting a third-order polynomial to a complex curve is expected to introduce a non-negligible discrepancy between the approximation and the behaviour of the full complex model. The challenge, therefore, is to identify the most effective approach to minimize this inherent error. In the following, we describe the origin of these approximations and the strategies implemented to mitigate their impact.

The first source of error in the calibration module arises from the values of the independent term c_1, which must take the same value across all sensitivity experiments. As outlined in the Methods section, the coefficient is over-determined across the two sensitivity experiments. Consequently, a specific value of c_1 is selected based on one calibration experiment, but this value may not apply to the other. This approach implies that our simplified model is calibrated to the bifurcation points of one calibration experiment (temperature forcing) but not the other (freshwater flux forcing). However, the decision to minimize error for temperature forcing is justified because this forcing is the dominant factor influencing the AMOC[13,14,55].

A second limitation of the ATCM is that we did not impose the value of the AMOC intensity at preindustrial levels, as doing so would introduce an additional constraint and thereby limit experimental flexibility.

Furthermore, in the calibration experiments *EXPA* and *EXPB*, we used the following values for the forcings held constant: $F^A_{GIS} = 0\ Sv$ and $T^B = 0\ °C$. In principle, these parameters could be constant and not zero. However, there is a scientific rationale behind our choice. First, the prevailing approach in the literature when conducting AMOC hysteresis experiments is to vary one forcing while keeping the other (if included) fixed at zero. This helps reduce the spin-up period in the complex model simulations. Our methodological choice is therefore supported by a broader availability of simulation data. Finally, as shown in Figure 7 and Figure 8, the emulator allows us to estimate the critical manifold of the emulated simulator by assuming that this manifold is a straight line. Given a similar uncertainty in the estimated position of each tipping point, the probability of accurately approximating the true line increases when the two bifurcation points used to define it are farther apart. However, if users wish to characterize the critical manifold in more detail, it would be valuable to conduct additional hysteresis experiments using non-zero constant values for F^A_{GIS} or T^B. This would make it possible to test the robustness of the assumption that the critical manifold is linear in the simulator.

In addition to the differences observed in fitting the bifurcation diagram, limitations of the emulator became apparent during the calibration of the τ parameter. Although we developed a method to numerically calibrate this parameter based on the identification of the collapse timing in the simulator, Figure 3(a) reveals substantial differences between the two transient trajectories. This discrepancy arises because, in a complex model such as cGENIE, the AMOC may be sensitive not only to the absolute magnitude of the forcing but also to its rate of change over time. This rate dependence causes cGENIE to simulate a much more abrupt AMOC collapse than the ATCM, despite τ being calibrated so that the timing of the collapse matches in both models.

After calibrating the simplified AMOC dynamics, the emulator was tested in four additional experiments, including two that combined both forcing variables. The results indicate that the emulator performs well according to three criteria: (i) correctly predicting whether a collapse occurs in the simulator, (ii) reproducing the timing of such a collapse, and

(iii) capturing the AMOC response under the combined influence of temperature and freshwater flux. For criteria (i) and (iii), the emulator reproduces the results of the simulator exactly. Regarding criterion (ii), the emulator tends to slightly anticipate the timing of a complete AMOC collapse compared to the simulator. This occurs in scenarios with weaker forcing than in our chosen *EXPC* experiment, which applies a 1% per year increase in CO_2 concentration. Nevertheless, the AMOC intensity in the simulator is below 15% at that time, which can still be considered qualitatively as a collapsed state. These results demonstrate the emulator's capability to provide an accurate representation of near-future AMOC collapse trajectories under realistic emission scenarios. For studies involving much slower forcing rates, it would be advisable to use a transient AMOC experiment with a slower forcing than the 1% CO_2 increase per year *EXPC*. Otherwise, the emulator may overestimate even more the timing of total collapse (see Supplementary Figure S7).

Therefore, we argue that the resulting emulator is relevant. The calibration achieved provides a tool that closely approximates the behaviour of the simulator it emulates, making it suitable for practical use as an emulator. However, simulators, whether EMICs or even ESMs, still carry significant uncertainty regarding the exact location of the AMOC tipping points and, consequently, the timing of their collapse[5,49,55]. In this context, the ATCM should not be regarded as a tool for delivering accurate projections. While it is highly likely that the emulator's values deviate from the process-based results of the simulator, we demonstrate that the emulator can be calibrated sufficiently well to provide plausible and relevant outcomes. This is a key requirement for an AMOC tipping element emulator, as it enables more accurate studies of the AMOC state under plausible future positions within its forcing space. As shown in the Results section (see Figure 7 and Figure 8), the ATCM facilitates realistic sampling of the forcing space once integrated into a reduced-complexity climate model like SURFER. Where complex models face significant computational constraints, the presented emulator enables performing a large number of simulations at very low cost, while maintaining consistency with more complex models. These computationally-efficient simulations also allow for the study of overshoot without tipping scenarios[31] by sampling the bivariate forcing space with higher resolution (Figure 8).

A promising direction for future research is to calibrate the ATCM using an ensemble of EMICs and potentially ESMs, allowing exploration of both the forcing space and the critical thresholds of the models, while also accounting for model uncertainty. Furthermore, the calibration approach presented here can be generalized to incorporate additional forcing variables (see Supplementary Materials S2). Another promising avenue is the application of the ATCM methodology to the GIS (see Supplementary Materials), yielding a non-linear dynamics tool calibrated on simulators to investigate potential cascading collapses between the AMOC and GIS[32,34]. Beyond the GIS, this framework for constructing tipping element emulators can be extended to other tipping elements, starting with the six additional elements whose initial parameterizations are included in an extended version of SURFER v3.0[39]. Finally, this emulator could be integrated into other reduced-complexity climate models, such as FAIR[56], with minimal technical adjustments, as it requires only the global mean temperature anomaly and a parameterization of the GIS freshwater flux as input from the climate model.

5. Conclusion

We have presented the AMOC Tipping Calibration Module (ATCM), an emulator of AMOC dynamics based on a double-fold bifurcation structure. For the first time, this emulator incorporates two forcing variables that can be calibrated using only three experiments from the target complex model, also called the "simulator". The emulator reproduces the AMOC behaviour observed in these experiments, although simplified dynamics introduce differences during the transient phase. For example, simulators such as cGENIE capture rate-induced effects, which can only be partially compensated by calibrating the time scale parameter. Despite these limitations, the ATCM reliably: (i) predicts whether a collapse occurs, (ii) reproduces its timing, and (iii) captures the AMOC response under combined temperature and freshwater forcing. Across four validation experiments, the emulator correctly reproduces the occurrence of a collapse. The emulator systematically predicts the collapse slightly earlier than the simulator under forcing scenarios with slower rates than in the calibration experiment. Most notably, the emulator consistently reproduces AMOC collapse trajectories under simultaneous temperature and freshwater forcing.

In addition to reproducing AMOC behaviour according to these three criteria, the ATCM offers substantial computational advantages over complex models. When integrated into the reduced-complexity model SURFER, it enables efficient exploration of the forcing space across a range of emission scenarios. We demonstrated that three emission trajectories between SSP3-7.0 and SSP5-8.5 trigger an AMOC collapse in SURFER when the two-forcing emulator is included. Furthermore, the emulator provides a linear approximation of AMOC's critical manifold in the complex model, specifying the combinations of temperature and freshwater forcing that result in a collapse at equilibrium. This allows systematic investigation of boundary cases and comparative sensitivity analysis.

Finally, the calibration methodology can be extended to other tipping elements. A next step would be to calibrate a Greenland Ice Sheet (GIS) emulator in a similar manner, enabling the study of potential cascading collapses between the AMOC and GIS at low computational cost. The emulator could also be implemented in other reduced-complexity models with minimal adjustment. Overall, we have established a robust framework for calibrating two-forcing tipping element dynamics, bridging conceptual and complex models. This tool can therefore enhance understanding of tipping element collapse trajectories, which remain poorly understood despite their profound societal implications.

Ethics and consent

In accordance with the F1000 AI Policy, the authors confirm that they have used *ChatGPT (GPT-4, OpenAI, 2025)* during the preparation of this manuscript. The tool was employed exclusively for language editing purposes, specifically to improve the clarity, grammar, and academic tone of English translations. No content generation, conceptual development, or substantive modifications were delegated to the tool. All scientific content, interpretations, and conclusions remain entirely the work of the authors.

No additional ethical approval or consent was required.

Data availability

Laridon, A *et al*. Connecting complex and simplified models of tipping elements: a nonlinear two-forcing emulator for the Atlantic meridional overturning circulation. Zenodo. DOI: https://zenodo.org/records/17379110; 2025. This repository contains the data as well as the scripts, the SURFER model and the ATCM used in this study. The *ATCM.ipynb* Jupyter Notebook contains the Python code where the ATCM is defined, calibrated, and validated using cGENIE simulations. The *SURFER_pre3.0_ATCM.ipynb* Jupyter Notebook contains the integration of the ATCM into the SURFER pre3.0 climate model. These two notebooks were used to generate all the figures presented in this paper. The emission scenario data required to run the simulations implemented in SURFER can be found in the *data/SURFER/* directory within the Zenodo repository. More details on the definition of the emission scenarios used can be found in Appendix A.2 of V. Couplet's PhD thesis[55]. The repository is available under Creative Commons Zero v1.0 Universal.

Laridon A *et al*. Supplementary Materials. Connecting complex and simplified models of tipping elements: a nonlinear two-forcing emulator for the Atlantic meridional overturning circulation. Zenodo. DOI: https://zenodo.org/records/17379239; 2025. This repository contains the supplementary materials referred in this paper (i) GIS calibration (ii) parameterization of the AMOC with three forcing variables (iii) supplementary Figures. The repository is available under Creative Commons Zero v1.0 Universal.

The code for the version of the 'muffin' release of the cGENIE Earth system model used in this paper, is tagged as v0.9.67, and is assigned a DOI: https://doi.org/10.5281/zenodo.17372071[57]. Configuration files for the specific experiments presented in the paper can be found in the directory: genie-userconfigs/PUBS/published/Laridon_et_al_2025. Details of the experiments, plus the command line needed to run each one, are given in the readme.txt file in that directory. All other configuration files and boundary conditions are provided as part of the code release. A manual detailing code installation, basic model configuration, tutorials covering various aspects of model configuration, experimental design, and output, plus the processing of results, is assigned a DOI: https://doi.org/10.5281/zenodo.13377225[58].

Author contributions

AL, VC, and MC conceptualized the study and its objectives, which are based on AL's MSc thesis. AL and VC are responsible for the design of the ATCM methodology. AL conducted the formal and technical analysis of the project. MC and WT managed and coordinated the research activity planning and execution. MC, as the supervisor of AL's MSc thesis and co-supervisor of AL's PhD, and WT, as the supervisor of AL's PhD, provided oversight and guidance. JG assisted in designing the calibration experiments with cGENIE and conducted them. AL analysed, with the help of JG these simulations. AL integrated the cGENIE simulations into the ATCM. AL is responsible for the study's visualization, creation, and presentation of the published work. AL wrote the original draft, produced all the figures and led the manuscript writing process. AL, MC, JG, VC and WT reviewed and revised it.

References

1. Stocker TF: **Chapter 1 - The ocean as a component of the climate system.** In: Siedler G, Griffies SM, Gould J, Church JA, editors. *International Geophysics.* Academic Press, (Ocean Circulation and Climate). 2013; **103**: 3–30.
Publisher Full Text
2. Jackson LC, Kahana R, Graham T, *et al.*: **Global and European climate impacts of a slowdown of the AMOC in a high resolution GCM.** *Clim Dyn.* 2015; **45**(11): 3299–316.
Publisher Full Text
3. Liu W, Fedorov AV, Xie SP, *et al.*: **Climate impacts of a weakened Atlantic Meridional Overturning Circulation in a warming climate.** *Sci Adv.* 2020; **6**(26): eaaz4876.
Publisher Full Text
4. Lenton T, Held H, Kriegler E, *et al.*: **Tipping elements in the Earth's climate system.** *Proc Natl Acad Sci U S A.* 2008; **105**(6): 1786–93.
PubMed Abstract | **Publisher Full Text** | **Free Full Text**
5. Lenton T, Laybourn L, Armstrong McKay D, *et al.*: **Global tipping point report 2023.** UK: University of Exeter, 2023.
Reference Source
6. Van Westen RM, Kliphuis M, Dijkstra HA: **Physics-based early warning signal shows that AMOC is on tipping course.** *Sci Adv.* 2024; **10**(6): eadk1189.
PubMed Abstract | **Publisher Full Text** | **Free Full Text**
7. Zickfeld K, Eby M, Weaver AJ: **Carbon-cycle feedbacks of changes in the Atlantic Meridional Overturning Circulation under future atmospheric CO_2.** *Glob Biogeochem Cycles.* 2008; **22**(3).
Publisher Full Text
8. Brovkin V, Brook E, Williams JW, *et al.*: **Past abrupt changes, tipping points and cascading impacts in the Earth system.** *Nat Geosci.* 2021; **14**(8): 550–8.
Publisher Full Text
9. Rocha JC, Peterson G, Bodin Ö, *et al.*: **Cascading regime shifts within and across scales.** *Science.* 2018; **362**(6421): 1379–83.
PubMed Abstract | **Publisher Full Text**
10. Buckley MW, Marshall J: **Observations, inferences, and mechanisms of the Atlantic Meridional Overturning Circulation: a review.** *Rev Geophys.* 2016; **54**(1): 5–63.
Publisher Full Text
11. Goosse H: **Climate system dynamics and modelling.** 1st ed. Cambridge University Press, 2015; [cited 2024 Apr 20].
Publisher Full Text
12. Stocker TF, Schmittner A: **Influence of CO_2 emission rates on the stability of the thermohaline circulation.** *Nature.* 1997; **388**(6645): 862–5.
Publisher Full Text
13. Levang SJ, Schmitt RW: **What causes the AMOC to weaken in CMIP5?** *J Clim.*

2020; **33**(4): 1535–45.
Publisher Full Text

14. Gérard J, Crucifix M: **Diagnosing the causes of AMOC slowdown in a coupled model: a cautionary tale.** *Earth Syst Dyn.* 2024; **15**(2): 293–306.
Publisher Full Text
15. Bakker P, Schmittner A, Lenaerts JTM, *et al.*: **Fate of the Atlantic Meridional Overturning Circulation: strong decline under continued warming and Greenland melting.** *Geophys Res Lett.* 2016; **43**(23): 12,252–12,260.
Publisher Full Text
16. Böning CW, Behrens E, Biastoch A, *et al.*: **Emerging impact of Greenland meltwater on deepwater formation in the North Atlantic Ocean.** *Nat Geosci.* 2016; **9**(7): 523–7.
Publisher Full Text
17. Jackson LC, Alastrué de Asenjo E, Bellomo K, *et al.*: **Understanding AMOC stability: the North Atlantic hosing model intercomparison project.** *Geosci Model Dev.* 2023; **16**(7): 1975–95.
Publisher Full Text
18. Romanou A, Rind D, Jonas J, *et al.*: **Stochastic bifurcation of the North Atlantic circulation under a midrange future climate scenario with the NASA-GISS ModelE.** *J Clim.* 2023; **36**(18): 6141–61.
Publisher Full Text
19. Stommel H: **Thermohaline convection with two stable regimes of flow.** *Tellus.* 1961; **13**(2): 224–30.
20. Wang S, Foster A, Lenz EA, *et al.*: **Mechanisms and impacts of earth system tipping elements.** *Rev Geophys.* 2023; **61**(1): e2021RG000757.
Publisher Full Text
21. Jackson LC, Wood RA: **Hysteresis and resilience of the AMOC in an eddy-permitting GCM.** *Geophys Res Lett.* 2018; **45**(16): 8547–8556.
Publisher Full Text
22. Rahmstorf S, Box JE, Feulner G, *et al.*: **Exceptional twentieth-century slowdown in Atlantic Ocean overturning circulation.** *Nat Clim Change.* 2015; **5**(5): 475–480.
Publisher Full Text
23. Caesar L, Rahmstorf S, Robinson A, *et al.*: **Observed fingerprint of a weakening Atlantic Ocean overturning circulation.** *Nature.* 2018; **556**(7700): 191–196.
PubMed Abstract | Publisher Full Text
24. Jackson LC, Biastoch A, Buckley MW, *et al.*: **The evolution of the North Atlantic Meridional Overturning Circulation since 1980.** *Nat Rev Earth Environ.* 2022; **3**(4): 241–54.
Publisher Full Text
25. Clement AC, Peterson LC: **Mechanisms of abrupt climate change of the last glacial period.** *Rev Geophys.* 2008; **46**(4).
Publisher Full Text
26. Rahmstorf S: **Ocean circulation and climate during the past 120,000 years.** *Nature.* 2002; **419**(6903): 207–14.
PubMed Abstract | Publisher Full Text
27. Rahmstorf S, Crucifix M, Ganopolski A, *et al.*: **Thermohaline circulation hysteresis: a model intercomparison.** *Geophys Res Lett.* 2005; **32**(23).
Publisher Full Text
28. Weijer W, Cheng W, Drijfhout SS, *et al.*: **Stability of the Atlantic Meridional Overturning Circulation: a review and synthesis.** *J Geophys Res Oceans.* 2019; **124**(8): 5336–5375.
Publisher Full Text
29. Armstrong McKay DI, Staal A, Abrams JF, *et al.*: **Exceeding 1.5°C global warming could trigger multiple climate tipping points.** *Science.* 2022; **377**(6611): eabn7950.
PubMed Abstract | Publisher Full Text
30. Ditlevsen P, Ditlevsen S: **Warning of a forthcoming collapse of the Atlantic Meridional Overturning Circulation.** *Nat Commun.* 2023; **14**(1): 4254.
PubMed Abstract | Publisher Full Text | Free Full Text
31. Ritchie P, Karabacak Ö, Sieber J: **Inverse-square law between time and amplitude for crossing tipping thresholds.** *Proc Math Phys Eng Sci.* 2019; **475**(2222): 20180504.
PubMed Abstract | Publisher Full Text | Free Full Text
32. Klose AK, Donges JF, Feudel U, *et al.*: **Rate-induced tipping cascades arising from interactions between the Greenland Ice Sheet and the Atlantic Meridional Overturning Circulation.** *Earth Syst Dyn.* 2024; **15**(3): 635–52.
Publisher Full Text
33. Klose AK, Wunderling N, Winkelmann R, *et al.*: **What do we mean, 'tipping cascade'?** *Environ Res Lett.* 2021; **16**(12): 125011.
Publisher Full Text
34. Sinet S, von der Heydt AS, Dijkstra HA: **AMOC stabilization under the interaction with tipping polar ice sheets.** *Geophys Res Lett.* 2023; **50**(2): e2022GL100305.
Publisher Full Text
35. Wood RA, Rodríguez JM, Smith RS, *et al.*: **Observable, low-order dynamical controls on thresholds of the Atlantic Meridional Overturning Circulation.** *Clim Dyn.* 2019; **53**(11): 6815–6834.
Publisher Full Text
36. Claussen M, Mysak L, Weaver A, *et al.*: **Earth system models of intermediate complexity: closing the gap in the spectrum of climate system models.** *Clim Dyn.* 2002; **18**(7): 579–586.
Publisher Full Text
37. Liu W, Xie SP, Liu Z, *et al.*: **Overlooked possibility of a collapsed Atlantic Meridional Overturning Circulation in warming climate.** *Sci Adv.* 2017; **3**(1): e1601666.
PubMed Abstract | Publisher Full Text | Free Full Text
38. Mecking JV, Drijfhout SS, Jackson LC, *et al.*: **The effect of model bias on Atlantic freshwater transport and implications for AMOC bi-stability.** *Tellus Dyn Meteorol Oceanogr.* 2017; **69**(1): 1299910.
Publisher Full Text
39. Couplet V, Wunderling N, Crucifix M: **Tipping interactions and cascades on multimillennial time scales in a model of reduced complexity**. 2024.
40. Dekker MM, von der Heydt AS, Dijkstra HA: **Cascading transitions in the climate system.** *Earth Syst Dyn.* 2018; **9**(4): 1243–60.
Publisher Full Text
41. Wunderling N, Donges JF, Kurths J, *et al.*: **Interacting tipping elements increase risk of climate domino effects under global warming.** *Earth Syst Dyn.* 2021; **12**(2): 601–19.
Publisher Full Text
42. Klose AK, Karle V, Winkelmann R, *et al.*: **Emergence of cascading dynamics in interacting tipping elements of ecology and climate.** *R Soc Open Sci.* 2020; **7**(6): 200599.
PubMed Abstract | Publisher Full Text | Free Full Text
43. Liu W, Liu Z: **A diagnostic indicator of the stability of the Atlantic Meridional Overturning Circulation in CCSM3.** *J Clim.* 2013; **26**(6): 1926–38.
Publisher Full Text
44. Rahmstorf S: **On the freshwater forcing and transport of the Atlantic thermohaline circulation.** *Clim Dyn.* 1996; **12**: 799–811.
Publisher Full Text
45. de Vries P, Weber SL: **The Atlantic freshwater budget as a diagnostic for the existence of a stable shut down of the Meridional Overturning Circulation.** *Geophys Res Lett.* 2005; **32**(9).
Publisher Full Text
46. Oakley JE, Youngman BD: **Calibration of complex computer simulators using likelihood emulation**. *arXiv*, 2014; [cited 2025 Oct 15].
Publisher Full Text
47. Martínez Montero M, Crucifix M, Couplet V, *et al.*: **SURFER v2.0: a flexible and simple model linking anthropogenic CO_2 emissions and solar radiation modification to ocean acidification and sea level rise.** *Geosci Model Dev.* 2022; **15**(21): 8059–84.
Publisher Full Text
48. Couplet V, Martínez Montero M, Crucifix M: **SURFER v3.0: a fast model with ice sheet tipping points and carbon cycle feedbacks for short and long-term climate scenarios.** *EGUsphere.* 2024; 1–77.
Publisher Full Text
49. Armstrong McKay DI, Staal A, Abrams JF, *et al.*: **Exceeding 1.5°C global warming could trigger multiple climate tipping points.** *Science.* 2022; **377**(6611): eabn7950.
PubMed Abstract | Publisher Full Text
50. Ridgwell A: **cGENIE.muffin EMIC Documentation.** 2022.
Reference Source
51. Marsh R, Sóbester A, Hart EE, *et al.*: **An optimally tuned ensemble of the "eb_go_gs" configuration of GENIE: parameter sensitivity and bifurcations in the Atlantic overturning circulation.** *Geosci Model Dev.* 2013; **6**(5): 1729–44.
Publisher Full Text
52. Edwards NR, Marsh R: **Uncertainties due to transport-parameter sensitivity in an efficient 3-D ocean-climate model.** *Clim Dyn.* 2005; **24**(4): 415–33.
Publisher Full Text
53. Amaury L: **Development of a simplified dynamics emulator and investigation of cascading collapses of the AMOC and Greenland Ice Sheet in a climate model.** UCLouvain, 2024.
Reference Source
54. Couplet V: **Tipping cascades, future Earth system trajectories, and the prospect of a hothouse : a study with a reduced-complexity model.** 2025.
Reference Source
55. Liu W, Xie SP, Liu Z, *et al.*: **Overlooked possibility of a collapsed Atlantic Meridional Overturning Circulation in warming climate.** *Sci Adv.* 2017; **3**(1): e1601666.
PubMed Abstract | Publisher Full Text | Free Full Text
56. Smith CJ, Forster PM, Allen M, *et al.*: **FAIR v1.3: a simple emissions-based impulse response and carbon cycle model.** *Geosci Model Dev.* 2018; **11**(6): 2273–97.
Publisher Full Text
57. Ridgwell A, Reinhard C, van de Velde S, *et al.*: **derpycode/cgenie.muffin: v0.9.67**. Zenodo. 2025; [cited 2025 Oct 17].
https://zenodo.org/doi/10.5281/zenodo.17372071
58. Ridgwell A, Hülse D, Peterson C, *et al.*: **derpycode/muffindoc: v0.24-574-NSF**. Zenodo. 2024; [cited 2025 Oct 17].
https://zenodo.org/doi/10.5281/zenodo.13377225

Open Peer Review

Current Peer Review Status:

Version 2

Reviewer Report 12 November 2025

https://doi.org/10.21956/openreseurope.22466.r63213

Valerian Jacques-Dumas
Utrecht University, Utrecht, The Netherlands

Overview
I would like to thank the authors for the revisions of the manuscript that greatly improved its clarity and better supported the conclusions.
However, upon reading the new version, I still have a few comments, which mainly have to do with small mistakes.

Comments
- p.4: « the AMOC has slowed » → « has slowed down » ?
- p.4: « Estimates of the tipping point » → you do not estimate the tipping point, you rather estimate its position e.g.
- p.5: « Such simulation » → « simulations »
- p.5: « As a result, ESMs are generally unable to simulate a large number of full AMOC hysteresis curves » → at first, this formulation is unclear because I wonder why you would like to simulate several hysteresis experiments. A few pages later, I understand that you simulate one hysteresis curve by forcing variable. It is really a detail, but this sentence would benefit from reformulation or clarification.
- p.5: « The ocean circulation system is then modelled possessing two stable equilibria separated by an unstable equilibrium. » → this sentence is not completely correct I think and could be reformulated for readability.
- p.5: « critical value of the tipping point » → this does not make sense since the tipping point is already a critical threshold.
- p.5: « seeks to address the following objective of developing » → this is very heavy and could be reformulated to read better.
- eq.1: there is a problem with the punctuation. The comma before the equation should be removed and the comma after should be a full stop.
- p.7: « jutify » → « justify »
- p.7: « allow for independent… » → « allows »
- p.7: the second paragraph, where you explain the generalisation of the Martinez-Monteiro method, is misleading. Indeed, I understand from it that you choose to emulate cGENIE because

this model allows you to also simulate the transition from the collapsed state to the nominal state. This is the impression it gives when the sentence describing this transition is followed by « For instance, in the cGENIE » (by the way, this sentence is also not correct, there is an issue with punctuation and conjugation of the main verb). So it really came as a surprise that you only simulate the transition to a collapsed AMOC.
- In relation with the former point, you mention in that same paragraph the calibration of « hysteresis experiment ». This would be the case if you also simulated the recovery of the AMOC. Here, you only calibrate a collapse experiment.
- p.7: « With this simplification enables » → this is not correct and too heavy, it could be reformulated for readability.
- p.7: « calibrate the parameters within the technique » → I'm not sure that « within » is the correct word here.
- p.7: « independent calibration experiments allows » → « allow »
- p.8: « consist of a 20000-year long... » → « consists »
- p.8: « and reaching $T=0^\circ C$ after 20000 years » → it should be $T=10^\circ C$, no?
- p.9: « show the bifurcation diagrams » → you only show in Fig.2 the bifurcation diagrams of the ATCM, not of cGENIE.
- p.9: « Accordingly, the values of $\Psi^-$$...» → once again, this is not very clear. In the Methods section, you explain the procedure using a full hysteresis experiment, so I expect you to do the same here. But instead, you fix the values corresponding to the recovery branch. I would suggest to make it clear in the Methods section already that you only focus on the collapse branch.
- p.10: « In this cGENIE simulation AMOC collapses, » → there is a punctuation problem here, there should be a comma after « simulation ».
- p.12: « the ~350-years difference » → « 350-year »
- p.12: « below 15% » → « 15% » in itself does not mean anything, you should say « 15% of the nominal AMOC strength » for instance.
- p.12 : « which already indicates an abrupt » → the way this sentence is formulated, the notion of abruptness here is a bit meaningless because it seems like the abruptness stems from the decrease of the AMOC strength below 15% of its original value.
- p.15: « these trajectories cross W_c Notably » → a full stop is missing at the end of the sentence.
- p.15: « trajectories 8-14 lead to AMOC collapse » → « lead to an AMOC collapse » and what about trajectory 15?
- p.17: « (see Supplementary Materials)» → do you mean Supplementary Materials S1?

- Supplementary Material S1: « forcing exclusively by » appears twice, but it is really heavy and I'm not sure it is correct.
- Supplementary figure 2: You end the caption with « produce an hysteresis experiment » but you do not produce such experiment, only its collapse branch.

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Extreme and rare events, AMOC, nonlinear dynamics, machine learning

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Version 1

Reviewer Report 18 June 2025

https://doi.org/10.21956/openreseurope.21075.r54425

Dipti Hingmire
University of Victoria, Victoria, British Columbia, Canada

Complex systems like AMOC require computationally expensive complex Earth System models to be run to understand its tipping points. Here the authors have developed a nonlinear two forcing emulator for AMOC and it calibrated with reduced complexity climate models. New insights regarding AMOC collapse from SSP3-7.0 to SSP5-8.5 are provided with calibrated emulator.

More and more emulators of this kind should be developed and calibrated for rapid and less expensive way of exploring tipping points. Therefore, this study is important and relevant.

Is the work clearly and accurately presented and does it cite the current literature?
Yes

Is the study design appropriate and does the work have academic merit?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Yes

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: I am expert of large scale climate dynamics including AMOC dynamics, however I lack mathematical expertise to evaluate emulator design and concepts. I request editor please provide extra weightage to other reviewers comment before making decision.

I confirm that I have read this submission and believe that I have an appropriate level of

expertise to confirm that it is of an acceptable scientific standard.

Reviewer Report 16 June 2025

https://doi.org/10.21956/openreseurope.21075.r54758

?

Valerian Jacques-Dumas
Utrecht University, Utrecht, The Netherlands

Short summary
This article presents a simple tipping elements and tipping cascades emulator, which can be tuned to the dynamics of more complex models. The authors model the tipping behaviour of the AMOC and GIS as coupled double-fold bifurcations and tune their parameters to independent hysteresis experiments in the cGENIE model. The methodology for tuning these parameters already existed but was limited to a single forcing variable. The authors extended it to an arbitrary number of forcing variables and show that they are able to reasonably reproduce the dynamics of cGENIE. This simple emulator is then coupled to SURFER, where it is used to explore the response of the AMOC to a large range of SSP scenarios. This approach allows the derivation of the critical manifold in the forcing space; the inclusion of multiple forcings also shows new overshoot and tipping trajectories that could not be observed before.

Review summary: Approved with reservations
Tackling the issue of computational cost is very important to study tipping cascades and, more generally, tipping elements. Using simple emulators tuned to complex dynamics is an appealing way to solve this problem so I find the overall method, the results and perspectives of this study to be interesting and worth publishing. However, I have some comments and general remarks that should be addressed to enhance readability and reproducibility.

Major comments

1) My major concern is the readability of the article. Many sentences are confusing and I had to read them multiple times before understanding. Moreover, some grammatical mistakes also hinder readability. Some examples are given below but I would suggest to read again through the article carefully.

- (Section 1.3) “However, the AMOC is influenced by two forcing variables: temperature anomaly and freshwater flux. Given the necessity of more realistically constraining potential future AMOC trajectories, this study seeks to address the following objective: develop an AMOC emulator with two forcing parameters — temperature and freshwater flux — that can be calibrated against hysteresis from complex models and integrated into a climate model.”

This is redundant and overly complicated. “However” at the beginning is also wrongly used, leading to some confusion.

- (Section 2.2) “Mathematically, the generalization to N forcing variables results in obtaining 4N – 2(N – 1) equations with 4 + (N – 1) knowns, leading to N – 1 over-determined equations. As will be shown in the next section with the emulation of cGENIE, these over-determinations reflect the fact that the coefficient c1 is the shared constant across all calibrations of different sensitivity experiments. It represents the component of the model that must adjust for each sensitivity experiment while remaining consistent across all of them."

This could be made simpler and clearer.

- (Section 2.2) "For instance, the simulation, with a forcing duration of 2000 years to reach equilibrium, can start with a 0 Sv forcing. It then increases the forcing by 0.05 Sv to slow down the AMOC, decreases the forcing by 0.05 Sv until the value reaches -0.05 Sv, and repeats this procedure several times. The goal is to obtain a time series of the AMOC intensity with several instances where the AMOC is being reduced."

This is VERY confusing. It took me several attempts to understand it and I thought it would be a linear ramping-up so Figure 3b came as a surprise. It could be rephrased more simply and more efficiently.

- (Section 3.1) "Nevertheless, they do not compromise the calibration of the parameter τ, as it is designed to replicate the temporal lag between the AMOC intensity at a given moment and its corresponding equilibrium intensity during the four forcing level transitions in EXPC."

The end of this sentence is unclear.

- (Section 3.1)"The goal is to calibrate the parameter τ to reproduce the time lag between the intensity of the AMOC and its corresponding equilibrium intensity under the applied forcing."

This sentence is unclear, partly because of the expression "intensity of the AMOC"

- (Section 4) "While it is highly likely that the model values deviate from exact accuracy, we demonstrate that they are sufficiently well calibrated to remain plausible."

This sentence is very unclear, especially in relation with the previous sentence. What do you mean here? Could you rephrase it?

2) Across the article, authors mention that hysteresis are performed on a single forcing parameter while keeping the other fixed at certain value T^B or F_GIS^A. However, what precise values are given other than these variables are never explicit in the Results section. This must be important, though for the calibration of the ATCM. Can you make it clearer? Figure 2 suggests that these extra parameters are fixed to 0 but it should be in the text. After re-reading, I understood that the fixed values are the ones presented in Table 1, is it correct? If so, it should be made clearer in the text.

3) Moreover, Figure 2 is not very clear. You could consider making it 2 panels, one for each variable.
4) Similarly, I would strongly encourage you to replace Figure S4 with two panels because the bifurcation points in the intertwined planes are not readable at all. In particular, I cannot check from the Figure that the values of T- and F_GIS you mention are the right ones.

5) The other important point is the generalisation of your results. You claim that their emulator is very flexible in reproducing tipping behaviour, which is very appealing, but validate it using the same parameters as for calibration. As far as I understand, the experiment val_exp_1 simply combines the parameter ramp-ups of EXPA and EXPB. It would be very interesting to see how the emulator performs when validated (against cGENIE) for different hosing experiments and global

warming scenarios. This would also support the interesting dynamics observed when the emulator is applied to a whole range of SSP scenarios.

6) In the Supplementary Material S1, you go through the whole calibration procedure for the GIS but do not present any result. I find it a pity that you show the whole method to potentially simulate AMOC-GIS tipping interaction, but stop there. Could you consider adding these results to the paper (if there is enough room)? Otherwise, is it worth keeping this Section of the Supplementary Material? If the AMOC-GIS interaction results are out of the scope of this paper, I think that the calibration of the GIS model is also out of its scope.
7) In relation with this point, at the very end of Section 2.1, you write “In the results section, we calibrate τ + Ψ and τ − Ψ based on a tuning procedure to reproduce the internal timescale dynamics of the AMOC in cGENIE. For the GIS, we set V+ τ = 5500 years and V− τ = 700 years, based on reasonable values given in the reviews by Armstrong McKay et al.[29] and Couplet et al.[40].”

- Why mention the time scales of the GIS if you do not use them at all afterwards? And if you use them later, you should rather give at the same place where you explain the calibration of AMOC time scales (end if Section 2.2).

7) (Section 3.1) “Since the primary objective of the ATCM is to accurately calibrate the collapse branch, we focus exclusively on simulating collapse scenarios with cGENIE.”

- You describe the whole procedure to calibrate all parameters of your emulator (including the return time scale tau_minus) and even provide in Supplementary Material (Section S4) the full hysteresis experiments which would allow you to perform the whole calibration. Why not use it? Wouldn’t it make your results more robust by bringing additional constraint?

8) (Section 4) “Moreover, the emulator allows assessing, under the assumption that the two forcings add linearly, the critical manifold Wc of the emulated complex model.”

- Could you discuss this assumption? Is it a strong assumption? Has it already been discussed elsewhere?

9) (Conclusion) Most of your Conclusion is redundant with your Discussion. Can you consider removing the conclusion and slightly expanding the discussion to avoid repeating the same things twice?

10) (Supplementary Material, Section 1). I don’t understand this section. Is it even possible in cGENIE to increase the temperature while keeping the AMOC strength constant? It seems very strange since you also claim that the P-E anomaly stemming from global warming is the main driver of AMOC weakening. Moreover, you never mention what is the fixed value of the AMOC strength. Similarly, when you vary the AMOC strength (how? Is it a model parameter?), you fix the global mean temperature but don’t explicitly give the value of T_V^D. You can leave most details in the MSc thesis you cite but everything needed for reproduction of your results should be in the article.
11) In relation with this comment, the sentence “For the GIS case, it requires a complex model capable of simulating its volume projections with independent forcing possibilities for the AMOC and temperature.” in Section 2.2 is not very clear.

12)

- (Section 3.1) “However, to achieve an optimal fit for the collapse trajectory, slight deviations from the bifurcation point coordinates provided by the complex model may improve the emulator’s accuracy in representing the collapse trajectory.”
- (Caption of Table 1) “The T– value was adjusted to improve the calibration of the upper branch.”
- (Supplementary Material, Section 4) “Additionally, to improve the calibration of the emulator on the collapse branch, it was shown in Laridon (52) that arbitrarily changing the coordinates of the second bifurcation point improves calibration. For this reason, in this study, we reuse the critical values of the second bifurcation point from these two experiments to focus on longer calibration runs that more effectively capture the AMOC collapse at the first bifurcation point, given that forcing can act more slowly in the calibration experiments described in the main text.”

What does it mean that you “adjust” or “arbitrarily change” the value of T- to improve calibration? Moreover, I do not really understand the sentences quoted here from the Supplementary Material: they are very heavy and not clear at all.

Minor comments

1) (Section 1.2) “However, due to their prohibitive computational demands, complex models, such as Earth System Models (ESMs) are unable to simulate numerous complete AMOC hysteresis curves[36].”

- You already cite van Westen et. al (2024) but not here unfortunately. It would be particularly relevant because they precisely compute a hysteresis curve from an ESM.

2) (Section 2.1) “In Equation (1), this term encapsulates the physical effects of warming on AMOC water stratification, as well as freshwater anomalies resulting from simulated P-E in the complex model.”

- This is confusing because it seems that T represents at the same time temperature and salinity. Moreover, after that, you no longer mention its P-E component. You could thus simplify this sentence and leave the whole P-E flux to the Supplementary Materials.

3) (Section 2.2) “Based on Armstrong McKay et al.29 we assume that the internal dynamic time scale of the AMOC from its nominal stable state to its collapse stable state is the same, i.e., . Ψ ττ − +Ψ ≡ = τ”

- Has this actually been checked in sensitivity experiments?

4) (Section 3.1, paragraph 3) Don’t you mean EXPB instead of EXPA?

5) (Section3.1, paragraph 3) “The freshwater hosing is applied between 20°N and 50°N across the full width of the Atlantic (Figure S3- See underlying data) to reproduce the hosing region by Rahmstorf et al.[27].”

- Could you comment on the influence of this choice? Different hosing areas have since then been compared, for instance in [1].

6) (Section 3.2) “However, after 3,000 years, a sudden drop in AMOC intensity, followed by a partial recovery, is not captured by the ATCM.”

- Do you know why such drop happens? Had it already been observed?

7) (Section 3.4) “As an application, we studied the response of the ATCM once integrated into SURFER under 15 different emission scenarios spanning SSP1-2.6 to SSP5-8.5.”
- Do you have any references on the design of these scenarios?

8) At the end of Section 3.3 and in Section 3.4, you reference Figure 5 twice, although it should be Figure 6.

9) (Supplementary Material, Section 1)“We denote ��������, in the case of the GIS as the first calibration experiment of the GIS volume in relation to temperature.”
- This sentence should be rephrased for clarity.

10) (Supplementary Material, section 2) “The duration of �������� was choose to”
- The verb should be “chosen”.

11) (Supplementary Material, Section 4) “For ��������, it was also a 20,000-year run,”
- This is not consistent with the beginning with the rest of the paragraph. From what I understand from the end of the paragraph, the full EXPB run lasts 20,000 years. But the EXPA run lasts 40,000 years. So in the sentence quoted here, you cannot use “also” and “20,000 years” together, otherwise it’s very confusing. In this same sentence, “-0.2 Sv” is cut, with “-“ on one line and “0.2 Sv” on the next, although they should stay together for clarity.

References

1. Jackson L, Alastrué de Asenjo E, Bellomo K, Danabasoglu G, et al.: Understanding AMOC stability: the North Atlantic Hosing Model Intercomparison Project. *Geoscientific Model Development* . 2023; **16** (7): 1975-1995 Publisher Full Text

Is the work clearly and accurately presented and does it cite the current literature?
Partly

Is the study design appropriate and does the work have academic merit?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Partly

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Extreme and rare events, AMOC, nonlinear dynamics, machine learning

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Author Response 17 Oct 2025

Amaury Laridon

Major comments

1) Readability of the article *"My major concern is the readability of the article. Many sentences are confusing and I had to read them multiple times before understanding. Moreover, some grammatical mistakes also hinder readability. Some examples are given below but I would suggest to read again through the article carefully."* Thank you for raising this concern. We have carefully reviewed the entire manuscript and reformulated numerous sentences to enhance clarity and readability. Special attention was devoted to all highlighted sections, which have been systematically revised.

2) Clear mention of the value of the constant forcing parameter *"Across the article, authors mention that hysteresis are performed on a single forcing parameter while keeping the other fixed at certain value T^B or F_GIS^A. However, what precise values are given other than these variables are never explicit in the Results section. This must be important, though for the calibration of the ATCM. Can you make it clearer? Figure 2 suggests that these extra parameters are fixed to 0 but it should be in the text. After re-reading, I understood that the fixed values are the ones presented in Table 1, is it correct? If so, it should be made clearer in the text."* Indeed, it was an oversight that these fixed values were not specified in the *Results* section nor discussed further in the *Discussion*. We have now addressed this by explicitly stating the values chosen for *FGISA* and *TB* in the *Results* section and by including them in Table 1. In the *Discussion* section, we have added a paragraph presenting the scientific rationale behind our choice of setting *FGIS A=0 Sv* and *TB=0 °C*, as well as the implications of using alternative fixed but non-zero values for these two parameters.

3) Figure 2 not very clear *"Figure 2 is not very clear. You could consider making it 2 panels, one for each variable."* We appreciate the reviewer's suggestion. However, we would prefer to keep Figure 2 in its current form, as we believe that presenting the full forcing space in a single panel effectively conveys the range of possible combinations between temperature and freshwater flux. This unified representation is intended to emphasize that only two calibration experiments are sufficient to approximate the complex system dynamics within the entire forcing space. The specific values of the coordinates corresponding to the bifurcation points identified in the figure are provided in Table 1 for clarity. Following the reviewer's comment, we have added a sentence in the manuscript, when introducing this figure, to further highlight its purpose of providing a visual representation of the now two-dimensional forcing space. This addition further supports our argument that the emulator will subsequently enable sampling across this wide parameter space. Finally, we have included in the Supplementary Materials the two 2D plots of the bifurcation diagrams for temperature and freshwater forcing alone, in order to enhance readability.

4) Figure S4 not very clear and values used for *T*-and *FGIS-* *Similarly, I would strongly encourage you to replace Figure S4 with two panels because the bifurcation points in the intertwined planes are not readable at all. In particular, I cannot check from the Figure that the values of T- and F_GIS you mention are the right ones.* Thank you for raising this concern. We have ultimately decided to remove Section 4 from the Supplementary Materials. The main reason is that this section contained an error and did not contribute to the clarity of the presentation; on the contrary, it was confusing. Specifically, the values chosen for *T*and *FGIS*- are not directly those from the simulation results presented in A. Laridon's MSc thesis. As previously stated—though perhaps not clearly enough—these values were artificially modified in the emulator, based on the rationale that the focus of this study is to demonstrate that a good calibration on the collapse branch, which is of greater near-term interest, can be achieved. It is also now reminded in Section 3.1 that there is a trade-off in calibrating the emulator to minimize the calibration error where it is most relevant given the scientific objective. Section 4 was originally included to show that it is also possible to calibrate the emulator on a full hysteresis curve, but its presentation and the way it was referenced in Section 3.1 were confusing. We have therefore rewritten the explanation of the choices and justifications for the values of *T*-and *FGIS*- , while inviting readers interested in the full hysteresis calibration results to consult the MSc thesis.

5) Generalization of the results *The other important point is the generalisation of your results. You claim that their emulator is very flexible in reproducing tipping behaviour, which is very appealing, but validate it using the same parameters as for calibration. As far as I understand, the experiment val_exp_1 simply combines the parameter ramp-ups of EXPA and EXPB. It would be very interesting to see how the emulator performs when validated (against cGENIE) for different hosing experiments and global warming scenarios. This would also support the interesting dynamics observed when the emulator is applied to a whole range of SSP scenarios.* We thank the reviewer for this very relevant and constructive suggestion. In response, we now validate the emulator against four new scenarios in the revised manuscript: SSP2-4.5 and SSP5-8.5, each with and without an added hosing flux of 0.06 Sv. We provide a more detailed description of the experimental protocol in the updated manuscript. In addition, we performed a new calibration experiment for the timescale-related parameter, which allows a strict separation between the experiments used for calibration and those used for validation. We believe that these additions further reinforce the conclusions already presented in the previous version of the study.

6) Method developed for AMOC-GIS tipping interaction but not used *In the Supplementary Material S1, you go through the whole calibration procedure for the GIS but do not present any result. I find it a pity that you show the whole method to potentially simulate AMOC-GIS tipping interaction, but stop there. Could you consider adding these results to the paper (if there is enough room)? Otherwise, is it worth keeping this Section of the Supplementary Material? If the AMOC-GIS interaction results are out of the scope of this paper, I think that the calibration of the GIS model is also out of its scope.* We thank the reviewer for this thoughtful comment. In the revised manuscript, we have modified Figure 1 by removing the GIS equation, as it unnecessarily complicated the interpretation of the figure. This change also makes the parameter τ more explicit. In addition, we have moved the former Equation (2), which described the evolution of the GIS, from the Methods section to Subsection 3.3 of the

Results, where we integrate the emulator into SURFER. Finally, we have removed the previous parameterization of the AMOC to GIS interaction, as it was partly arbitrary and based on the MSc thesis of A. Laridon. We now adopt exactly the same equation and calibration as in SURFER v2.0 (Martinez Monteiro et al., (2022)), ensuring greater consistency, transparency, and reproducibility in our results. The primary objective of this study was to demonstrate the feasibility of constructing a two-variable forcing emulator capable of reproducing consistent results when compared with a target complex model. We have included Section S1 in the Supplementary Materials to illustrate how the proposed mathematical framework can also be applied to the construction and calibration of a GIS emulator, thereby enabling the exploration of potential tipping cascades between the AMOC and the GIS. While some example applications of this methodology are presented in the MSc thesis of A. Laridon, we have not yet calibrated the simplified GIS model against a complex GIS model. The purpose of Section S1 is therefore to outline the theoretical and methodological groundwork for such an extension, rather than to present calibrated results. Given the focus of the present paper, a full calibration of the GIS model and a detailed analysis of potential AMOC–GIS interactions are beyond its scope. However, we agree that these represent promising avenues for future research. We have clarified this motivation and limitation in the revised manuscript to better reflect the role of Section S1, in addition to the modifications described above.

7) Choice of the $\tau V+/-$ *In relation with this point, at the very end of Section 2.1, you write “In the results section, we calibrate $\tau + \Psi$ and $\tau - \Psi$ based on a tuning procedure to reproduce the internal timescale dynamics of the AMOC in cGENIE. For the GIS, we set $V+ \tau$ = 5500 years and $V- \tau$ = 700 years, based on reasonable values given in the reviews by Armstrong McKay et al.[29] and Couplet et al.[40].” Why mention the time scales of the GIS if you do not use them at all afterwards? And if you use them later, you should rather give at the same place where you explain the calibration of AMOC time scales (end if Section 2.2).* We thank the reviewer for pointing out this lack of clarity. We have now added the missing information regarding the values of $\tau V+$ et $\tau V-$ in section 3.3. As mentioned in the previous comment, we now use exactly the same equation to describe the dynamics of the GIS, as well as the calibration parameters, as in SURFER v2.0 (cfr. Martinez Monteiro et al.(2022)). In that paper, the GIS equation was calibrated against the results of a complex GIS model. By adopting exactly the same GIS equation here, we are also able to compare the emulator’s results—with and without freshwater flux forcing—more rigorously than in the first version of the manuscript.

8) Assumption that the two forcings add linearly *(Section 4) “Moreover, the emulator allows assessing, under the assumption that the two forcings add linearly, the critical manifold Wc of the emulated complex model.” Could you discuss this assumption? Is it a strong assumption? Has it already been discussed elsewhere?* Assuming that the forcings combine linearly is a common methodological choice, largely because it offers a straightforward and tractable starting point. While there is no definitive evidence that this assumption strictly holds, the fact that few tipping experiments in complex models consider multiple forcings makes it challenging to adopt an alternative approach. The aim of the emulator is therefore to provide a tool that, under the most common and simplified assumption of treating the two dominant AMOC forcings linearly, enables an assessment of AMOC sensitivity across different complex models. As discussed further in the revised manuscript, adopting alternative values for *FGISA* and *TB* would make it possible to test the robustness of the assumption that the two forcings add linearly. However, if future literature supports a new prevailing hypothesis regarding the analytical form of the combination of these two forcings, it would be entirely possible to reformulate the AMOC equation in the emulator and apply the same calibration method to it.

9) Repetitions between the Conclusions and the Discussions *(Conclusion) Most of your Conclusion is redundant with your Discussion. Can you consider removing the conclusion and slightly expanding the discussion to avoid repeating the same things twice?* We acknowledge the reviewer's comment regarding the perceived redundancy between the conclusion and the discussion sections. In response, we have expanded the discussion to include additional points, making it more comprehensive and revised the conclusion.

10) Supplementary Materials, Section 1 – More clear information for reproduction *I don't understand this section. Is it even possible in cGENIE to increase the temperature while keeping the AMOC strength constant? It seems very strange since you also claim that the P-E anomaly stemming from global warming is the main driver of AMOC weakening. Moreover, you never mention what is the fixed value of the AMOC strength. Similarly, when you vary the AMOC strength (how? Is it a model parameter?), you fix the global mean temperature but don't explicitly give the value of T_V^D. You can leave most details in the MSc thesis you cite but everything needed for reproduction of your results should be in the article.* The purpose of Section 1 of the Supplementary Materials is purely to provide a technical demonstration of how the same calibration method can be applied to a simplified model of the GIS in order to construct an emulator. We have added a paragraph at the beginning of this section to clarify its objective and to emphasize that applying this framework requires the availability of a complex model of the GIS. cGENIE, in itself, is not capable of producing the two calibration experiments required to emulate the GIS, assuming its two main forcings are global mean atmospheric temperature anomaly and AMOC intensity. Indeed, cGENIE does not include an explicit representation of the GIS or its melt processes. In this section, we describe in the protocol of *EXPC* that, using a suitable complex GIS model, a first sensitivity experiment should be conducted in which the GIS is forced by the atmospheric temperature anomaly while keeping AMOC intensity fixed. Using the notation of the section, the value is denoted as *ΨV*= *ΨVC*. Similarly, for the second calibration experiment *EXPD*, the atmospheric temperature anomaly is held constant at a generic value *TV*=*TVD*. It is expected that these numerical values are not provided, as these simulations have not been conducted. Once again, the sole purpose of this section is to present the mathematical framework and to highlight the operational constraints that such an application would impose on the complex GIS model to be used. We have also revised the citation of the MSc Thesis to make it clearer to the reader what can be expected from it. In the MSc Thesis, the GIS model was not calibrated based on a complex GIS model either; instead, parameter values were chosen based on reasonable assumptions drawn from the literature (e.g., Armstrong McKay et al. (2022)). However, the work did investigate the coupled emulator framework between the AMOC and GIS, and the potential tipping dynamics of both systems.

11) Section 2.2 – Need of a complex model for the GIS *In relation with this comment, the sentence "For the GIS case, it requires a complex model capable of simulating its volume projections with independent forcing possibilities for the AMOC and temperature." in Section 2.2* *is not very clear.* As explained in our response to comment number 6, we have now relocated and revised the presentation of the GIS dynamics used in the manuscript. It is now strictly based on that of SURFER v2.0, which has already been published and was calibrated against a complex GIS model.

12) Arbitrarily change the value of T- *(Section 3.1) "However, to achieve an optimal fit for the collapse trajectory, slight deviations from the bifurcation point coordinates provided by the complex model may improve the emulator's accuracy in representing the collapse trajectory."* ○ (Caption of Table 1) "The T– value was adjusted to improve the calibration of the upper branch." ○ (Supplementary Material, Section 4) "Additionally, to improve the calibration of the emulator on the collapse branch, it was shown in Laridon (52) that arbitrarily changing the coordinates of the second bifurcation point improves calibration. For this reason, in this study, we reuse the critical values of the second bifurcation point from these two experiments to focus on longer calibration runs that more effectively capture the AMOC collapse at the first bifurcation point, given that forcing can act more slowly in the calibration experiments described in the main text." ○ What does it mean that you "adjust" or "arbitrarily change" the value of T- to improve calibration? Moreover, I do not really understand the sentences quoted here from the Supplementary Material: they are very heavy and not clear at all. As previously stated, we have decided to remove Section 4 of the Supplementary Materials, as it did not contribute to improving the clarity of the manuscript. As mentioned in response to Point (4) above, we have also revised Section 3.1 to clarify which bifurcation point coordinates are derived from the two cGENIE sensitivity experiments that are shown, and which coordinates are manually fixed to improve the calibration of the upper collapse branch. We now provide a clearer justification for this choice in Section 3.1. Accordingly, for the parameters Ψ-, *FGIS-, T-*, we have also updated the description of Table 1 to make it explicit that these values are manually set.

Minor comments

1) Nuance on van Westen et al.(2024) citation *(Section 1.2) "However, due to their prohibitive computational demands, complex models, such as Earth System Models (ESMs) are unable to simulate numerous complete AMOC hysteresis curves[36]." You already cite van Westen et. al (2024) but not here unfortunately. It would be particularly relevant because they precisely compute a hysteresis curve from an ESM.* We have added a reference to the study by van Westen et al. (2024) in this part of Section 1.2, emphasising that although hysteresis curves from ESMs do now exist, they remain scarce due to the high computational cost associated with producing them.

2) P-E mention in the T-forcing *(Section 2.1) "In Equation (1), this term encapsulates the physical effects of warming on AMOC water stratification, as well as freshwater anomalies resulting from simulated P-E in the complex model." This is confusing because it seems that T represents at the same time temperature and salinity. Moreover, after that, you no longer mention its P-E component. You could thus simplify this sentence and leave the whole P-E flux to the Supplementary Materials.* This has been addressed by removing the statement in Section 2.1 suggesting that the variable T accounts not only for the effect of heat on stratification but also for freshwater anomalies related to precipitation-minus-evaporation (P–E). That statement was indeed confusing. However, the explanation that the effect of P–E can be captured if it is also simulated by the complex model has been retained in Section 3.1. In this order, we believe the reading is much clearer.

3) Verification of tau_+ = tau_- in sensitivity experiments *(Section 2.2) "Based on Armstrong McKay et al.29 we assume that the internal dynamic time scale of the AMOC from its nominal stable state to its collapse stable state is the same, i.e., . Ψ ττ − +Ψ ≡ = τ"* ◯ Has this actually been checked in sensitivity experiments? These two parameters by themselves do not represent the total time required for the AMOC to tip from its ON state to its OFF state and vice versa, but rather characterize the internal dynamic time scales associated with the AMOC's strengthening and weakening phases. Based on the analysis presented in Gérard and Crucifix (2024)1, Figures 7a and 7b illustrate hysteresis loops of AMOC intensity in response to atmospheric temperature anomalies and freshwater flux forcing. From these figures, we observe that, to a first approximation, the system spends a comparable amount of time transitioning from the ON to the OFF state as it does from the OFF to the ON state. Specifically, in both cases, the transition appears to take approximately 2000 years, which supports our assumption that τΨ+=τΨ-.
1 Gérard J, Crucifix M. Diagnosing the causes of AMOC slowdown in a coupled model: a cautionary tale. Earth System Dynamics. 2024 Mar 22;15(2):293–306.

4) Typo *(Section 3.1, paragraph 3) Don't you mean EXPB instead of EXPA?* Yes, indeed — thank you for pointing this out. We have corrected it accordingly.

5) Choice of the hosing region *(Section3.1, paragraph 3) "The freshwater hosing is applied between 20°N and 50°N across the full width of the Atlantic (Figure S3- See underlying data) to reproduce the hosing region by Rahmstorf et al.[27]." Could you comment on the influence of this choice? Different hosing areas have since then been compared, for instance in [1].* We selected this region to apply the hosing in cGENIE because it is the one used in previous hosing experiments with the same model (see Rahmstorf et al. (2005)2 and Gérard and Crucifix (2025)). The rationale behind choosing this specific latitude band was to deliberately avoid deep convection regions, in order not to shut down the AMOC too abruptly. Furthermore, Laridon (2024)3 has shown that the differences between this latitude band and a higher one, such as 50°N–70°N, are not significant, given that cGENIE is not a high-resolution model. What matters most when selecting the hosing region in the Atlantic is the consistency of its location across the different complex models that we aim to emulate. This ensures that no indirect bias is introduced in the subsequent calibration and comparison of the AMOC emulator versions.
2 Rahmstorf S, Crucifix M, Ganopolski A, Goosse H, Kamenkovich I, Knutti R, et al. Thermohaline circulation hysteresis: A model intercomparison. Geophysical Research Letters [Internet]. 2005 [cited 2023 Oct 22];32(23). Available from: https://onlinelibrary.wiley.com/doi/abs/10.1029/2005GL023655 3 Laridon Amaury. Development of a Simplified Dynamics Emulator and Investigation of Cascading Collapses of the AMOC and Greenland Ice Sheet in a Climate Model [Internet]. UCLouvain; 2024. Available from: https://dial.uclouvain.be/memoire/ucl/object/thesis:46552

6) Drop in the AMOC intensity after 3000 years *(Section 3.2) "However, after 3,000 years, a sudden drop in AMOC intensity, followed by a partial recovery, is not captured by the ATCM."* ◯ Do you know why such drop happens? Had it already been observed? J. Gérard, one of the co-authors of this paper and the researcher who conducted the cGENIE simulations, has

substantial expertise with this model, which he also employs in his own research. According to his interpretation, the abrupt decrease observed likely corresponds to the AMOC entering the self-sustained oscillatory regime that exists in cGENIE, as previously described in Gérard and Crucifix (2024) 1 (see Figure 7a). However, in our setup, the AMOC is forced not only by atmospheric temperature anomalies but also by freshwater fluxes, which further limits the time the system remains within this limit cycle. As a result, the system exits the cycle rapidly, preventing a full oscillation from becoming apparent. This behaviour is therefore understood to be specific to the cGENIE model.
1 Gérard J, Crucifix M. Diagnosing the causes of AMOC slowdown in a coupled model: a cautionary tale. Earth System Dynamics. 2024 Mar 22;15(2):293–306.

7) References of the design of the emissions scenarios ? *(Section 3.4) “As an application, we studied the response of the ATCM once integrated into SURFER under 15 different emission scenarios spanning SSP1-2.6 to SSP5-8.5.”* ◯ Do you have any references on the design of these scenarios? Indeed, we had not specified the references for these scenarios. The reader can find them in Appendix A.2 of V. Couplet’s PhD thesis. We have now added the corresponding details in the manuscript, as well as in the “Data Availability” section.

8) Typo of references in Figure 5 *At the end of Section 3.3 and in Section 3.4, you reference Figure 5 twice, although it should be Figure 6.* Yes, indeed — thank you for pointing this out. We have corrected it accordingly.

9) Typo in Supplementary Materials *(Supplementary Material, Section 1)“We denote ��������, in the case of the GIS as the first calibration experiment of the GIS volume in relation to temperature.”* ◯ This sentence should be rephrased for clarity. We have revised this sentence to make it clearer.

10) Typo in Supplementary Materials *(Supplementary Material, section 2) “The duration of �������� was choose to”* ◯ The verb should be “chosen”. Yes, indeed — thank you for pointing this out. We have corrected it accordingly.

11) Typo in Supplementary Materials *(Supplementary Material, Section 4) “For ��������, it was also a 20,000-year run,” This is not consistent with the beginning with the rest of the paragraph. From what I understand from the end of the paragraph, the full EXPB run lasts 20,000 years. But the EXPA run lasts 40,000 years. So in the sentence quoted here, you cannot use “also” and “20,000 years” together, otherwise it’s very confusing. In this same sentence, “-0.2 Sv” is cut, with “-“ on one line and “0.2 Sv” on the next, although they should stay together for clarity.* The issue has been resolved by removing Section 4 from the Supplementary Materials, for the reasons previously mentioned. This section was indeed particularly confusing and did not add any substantial or relevant content to the overall presentation.

Competing Interests: No competing interests were disclosed.