KEY PLAYERS IN NETWORK-ORIENTED INTERVENTIONS

Ata Atay, Ana Mauleon, Simon Schopohl, Vincent Vannetelbosch

REPRINT | 3278

UCLouvain

CORE | LIDAM
Louvain Institute of Data Analysis and Modeling in economics and statistics

CORE

Voie du Roman Pays 34, L1.03.01

B-1348 Louvain-la-Neuve

Tel (32 10) 47 43 04

Email:lidam-library@uclouvain.be

https://uclouvain.be/en/research-institutes/lidam/core/core-reprints.html

Journal of Economic Interaction and Coordination (2023) 18:879–907
https://doi.org/10.1007/s11403-023-00394-9

REGULAR ARTICLE

Key players in network-oriented interventions

Ata Atay[1] · Ana Mauleon[2] · Simon Schopohl[2] · Vincent Vannetelbosch[3]

Received: 20 October 2022 / Accepted: 3 July 2023 / Published online: 8 August 2023
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2023

Abstract
Individuals who are embedded in a social network decide non-cooperatively how much effort to exert in supporting victims of misbehavior. Each individual's optimal effort depends on the contextual effect, the social multiplier effect and the social conformity effect. We characterize the Nash equilibrium, and we derive an inter-centrality measure for finding the key player who once isolated increases the most the aggregate effort. An individual is more likely to be the key player if she is influencing many other individuals, she is exerting a low effort because of her characteristics, and her neighbors are strongly influenced by her. The key player policy increases substantially the aggregate effort, and the targeted player should never be selected randomly. The key player is likely to remain the key player in presence of social workers except if she is becoming much less influential due to her closeness to social workers. Finally, we consider alternative policies (e.g., training bystanders for supporting victims) and compare them to the policy of isolating the key player.

Keywords Social networks · Social support · Sexual and gender-based violence · Bullying · Peer effects · Key player · Conformity

✉ Vincent Vannetelbosch
vincent.vannetelbosch@uclouvain.be

Ata Atay
aatay@ub.edu

Ana Mauleon
ana.mauleon@uclouvain.be

[1] Department of Mathematical Economics, Finance and Actuarial Sciences, and Barcelona Economic Analysis Team (BEAT), University of Barcelona, Barcelona, Spain

[2] CORE/LIDAM and CEREC, UCLouvain, Louvain-la-Neuve, Belgium

[3] CORE/LIDAM, UCLouvain, Louvain-la-Neuve, Belgium

JEL Classification A14 · C72 · D85 · Z13

1 Introduction

Social support and the positive influence of social network members are critical for victims of sexual and gender-based violence (SGBV).[1] SGBV in close relationships is a widespread phenomenon found in societies all over the world. Almost one out of three women who have been in a relationship is estimated to have been abused by a partner during her lifetime (WHO).[2] Victims often do not report the violence or harassment they suffer. Shame, a desire to protect the perpetrator, stigma, guilt and fear are the main reasons that women subjected to SGBV give when explaining why they barely report SGBV. The abuse often concerns various aspects of women's everyday lives, affecting their social and economic situations, but also their physical and mental health (Boethius and Åkerström 2020). Hence, it is important to implement successful network-oriented interventions to mitigate the effects of SGBV.

Another form of misbehavior is bullying at school.[3] Bullying has a negative effect on the physical and mental health of bullied students, on their school work, on their relationships with friends and family, and on how they feel about themselves. Hawkins et al. (2001) find that more than half of bullying situations stop when peers or friends intervene on behalf of the student being bullied. In addition, the decision of one bystander to exert more effort in reporting and helping bullied students positively influence the behavior of her peers. This is the so-called social multiplier effect.[4] One type of interventions implemented to reduce the negative effects of bullying is peer norms interventions that make clear to young people that most of their peers oppose mistreatment.[5] Once bystanders are influenced by peer norms, it becomes even more costly for them to exert less effort than their peers in reporting and helping bullied students. This is the so-called social conformity effect.

[1] SGBV is defined as any act that is perpetrated against a person's will and is based on gender norms and unequal power relationships. See UNHCR S. Gender-Based Violence prevention and response. Available in https://emergency.unhcr.org/entry/60283/sexual-and-gender-based-violence-sgbv-prevention-and-response, vol. 26; 2017.

[2] WHO report 'Global and regional estimates of violence against women: prevalence and health effects of intimate partner violence and non-partner sexual violence.' Department of Reproductive Health and Research (https://www.who.int/publications/i/item/9789241564625).

[3] More than one out of every five students report being bullied in the USA (National Center for Educational Statistics 2016). Rates of bullying for 12–18-year-old students are around 35% for traditional bullying involvement and 15% for cyberbullying involvement (see Modecki et al. 2014). Of these students who report being bullied at school, 33% indicate that they are bullied at least once or twice a month during the school year. A slightly higher portion of female than of male students report being bullied at school. But, a higher percentage of male than of female students report being physically bullied. The reasons for being bullied reported most often by students are related to students' characteristics like race/ethnicity, gender, disability, religion, sexual orientation.

[4] Thornberg et al. (2012) find that beliefs of bystanders in their social self-efficacy are positively associated with defending and negatively associated with passive behavior from other bystanders.

[5] See Davis and Nixon (2013). Other interventions often implemented are building staff-students connections, disciplinary responses to negative peer behaviors, encouraging bystanders to confront and discourage the unkind behavior, and social skills training.

In this paper, we propose the key player strategy as network-oriented intervention for mitigating the effects of any form of misbehavior or mistreatment such as SGBV or bullying.[6] The key player strategy aims at finding and isolating optimally a negatively influencing individual in order to increase the social support to victims of misbehavior as well as effective prevention and intervention efforts by peers within the social network.[7]

We adopt an approach similar to Ballester et al. (2006) and Ballester and Zenou (2014) for identifying key players. We develop a network game[8] where individuals are connected through a network and they decide non-cooperatively how much effort to exert in supporting victims of misbehavior. The individual effort to support victims of misbehavior may be affected by (i) her individual characteristics and the characteristics of her neighbors (i.e., the contextual effect), (ii) the effort levels of her neighbors (i.e., the social multiplier or network spillovers effect), and (iii) the norms of conduct set by neighbors[9] (i.e., the social conformity effect), and (iv) unobservable correlated effects. Beside peer-led interventions, we allow for professionally led interventions by service providers, like social workers. Social workers are assumed to exert more effort than what they would optimally do if they were standard individuals.

We derive the unique Nash equilibrium of the network game. The equilibrium effort of each individual is proportional to her Katz-Bonacich weighted centrality. We look for the key player. The key player is defined as the individual who once isolated generates the greatest augmentation in the total effort for supporting victims of misbehavior. To do so, we propose a new measure of contextual inter-centrality that determines the key player to be isolated. This measure captures three effects: (i) the change in effort due to the change in the context when some individual is isolated, (ii) the change in effort due to the network structure change after the isolation, and (iii) the effort exerted by the individual who is isolated.[10]

We illustrate the policy of finding and isolating the key player by means of a specific network structure that is rich enough to disentangle the effects of network

[6] Network-oriented interventions are widely used in public health (e.g., smoking cessation, cancer screening, diet management, HIV prevention). See Shelton et al. (2019). Hogan et al. (2002) provide evidence for the overall usefulness of social support interventions for improving long-term health outcomes. With respect to women in violent relationships, Goodman and Smyth (2011) argue for a shift in domestic violence services toward a more network-oriented approach, one that favors partnerships between professionals and victims' social support networks.

[7] Mouttapa et al. (2004) found the friendship network and the pattern of friendships among individuals within a group are important aspects of adolescent school bullying.

[8] Jackson and Zenou (2015) provide a comprehensive introduction to network games.

[9] That is, individuals are penalized if they deviate from the effort level of their neighbors in the social network. See, e.g., Patacchini and Zenou (2012), Boucher (2016), Landini et al. (2016), Boguslaw (2017), Lee et al. (2021) about peer effects and conformism in social networks.

[10] Only the first two effects are present in the contextual inter-centrality measure of Ballester and Zenou (2014) where the key player is removed rather than being isolated and the planner's objective is to reduce the total effort level. In our context, the objective of the planner is to increase the total effort in supporting victims of misbehavior and an isolated individual may still exert some positive effort depending on her own characteristics. Hence, the effort of the isolated individual matters for determining who is the key player.

spillovers, social conformity and players' characteristics on the resulting equilibrium outcomes and the key player strategy. We find that the key player is not necessarily the individual who is the most central within the network nor the individual who is doing less effort than all other individuals. The most central individual with a negative attribute (i.e., someone who is reluctant to support victims, like a perpetrator) is not necessarily the key player to be isolated. In fact, an individual is more likely to become the key player if (i) she is influencing many other individuals (i.e., she has many neighbors), (ii) she is exerting a low effort because of her characteristics, and (iii) her neighbors are strongly influenced by her (i.e., her neighbors have few links). Implementing the key player strategy always increases the total effort exerted by all individuals except when individuals are homogeneous or their characteristics are correlated with their centrality. Comparing the total effort obtained by isolating the key player to the total effort that would be exerted if the target was selected randomly, we observe that the key player policy increases substantially the total effort. Hence, the planner should never target randomly the individual to be isolated, except if it is too costly to get the information about the network structure.

Indeed, it is assumed that the planner has perfect knowledge about the network structure as well as the characteristics of all individuals. Acquiring such knowledge might be costly, and this is why we compare the policy of isolating the key player with either doing nothing or isolating randomly some individual. We advocate to implement the policy of isolating the key player only if it substantially increases the aggregate effort compared to doing nothing and isolating randomly some individual. Thus, given some imperfect knowledge of the network structure, the planner should first estimate whether the benefits of isolating the key player versus selecting some player randomly overcome the costs for obtaining more precise information about the network structure.

In the presence of social workers, an individual is more likely to become the key player if she is influencing negatively many other players and she is not too influenced by social workers. Hence, an individual who was the key player without social workers is likely to remain the key player except if she is becoming less central and less influential due to her closeness to social workers.

We also consider alternative policies, and we compare them to the policy of isolating the key player. A first alternative policy consists of finding the key player who once turned into a social worker generates the highest possible increase in aggregate effort level. To do so, we obtain the benevolent change inter-centrality measure. A second alternative policy consists of finding the key player who once trained for supporting victims of misbehavior generates the highest possible increase in aggregate effort level. Training some targeted individual modifies her characteristics so that she is now eager to exert more effort in supporting victims. The planner has always incentives to implement both policies instead of doing nothing. However, both policies perform only slightly better than selecting randomly some individual. Thus, if the data collection about the network relationships and the characteristics of the individuals is too costly, the planner might prefer to target randomly some individual who could be either turned into a social worker or trained for supporting victims instead of implementing the policy of isolating the key player.

One of the first application of the key player strategy was developed for delinquent networks. Ballester et al. (2010) propose a delinquent network game where players decide about how much effort to exert in criminal activities. They derive both the key player (i.e., optimal single player removal for reducing criminal activities) and the key group (i.e., optimal group removal). Zenou (2016) gives an overview of the recent literature on key players in social and economic networks. There is an empirical literature that support key player policies. Using data from adolescents in the USA, Lee et al. (2021) show that contextual effects matter since the key player in crime may be different when one uses either Ballester et al. (2006) inter-centrality measure or Ballester and Zenou (2014) contextual inter-centrality measure. Moreover, compared to a policy that removes the most active delinquent from the network, they show that the key player strategy leads to a much higher delinquency reduction. Similarly, using a data set of co-offenders in Sweden, Lindquist and Zenou (2019) find that the key player strategy outperforms alternative policies like targeting the most active delinquent or targeting the most central delinquents in the criminal network.

Compared to other applications of the key player strategy, the present paper has the following innovations: (i) the key player is the individual who, once isolated from the rest of individuals, increases the most the aggregate effort exerted in the whole population, (ii) the key player is isolated rather than being removed from the network, (iii) individuals can be victims, bystanders, perpetrators or social workers, and (iv) alternative key player strategies such as turning individuals into social workers or training individuals for helping victims are considered.

Based on our model, Ogbe et al. (2021) analyze the potential impact of alternative network-oriented interventions for survivors of SGBV among asylum seekers in Belgium. They observe the importance and role of peer support as well as the importance of understanding the nature of the social network of asylum seekers before implementing a peer support or peer-led intervention.[11] Our objective is not to determine who are perpetrators, victims or bystanders nor to determine why individuals become perpetrators, victims or bystanders. We simply focus on the social support and the influence of peers for mitigating the effects of misbehavior such as SGBV or bullying. Social network ties may try to stop the abuse, may help the victim by offering means of escape, or they may help report the violence. A network-oriented intervention, like the key player strategy, does not aim at eradicating any form of misbehavior but rather aims at relieving or helping victims.

The paper is organized as follows. In Sect. 2 we present the network game and we determine the Nash equilibrium effort levels of this game. In Sect. 3 we derive the contextual inter-centrality measure for finding the key player to be isolated and we study the relative performance of this key player policy. In Sect. 4 we consider alternative key player policies where the key player is either turned into a social worker or trained for helping victims. Finally, we draw conclusions.

[11] In Belgium, Ogbe et al. (2021) find that network-oriented interventions involving asylum seekers and members of their network have the potential for improving physical and mental health outcomes of asylum seekers who are SGBV survivors.

2 Network-oriented interventions

2.1 The network game

Let $N = \{1, \dots, n\}$ be the finite set of individuals, S be the finite set of benevolent players (or social workers), and $N^+ = N \cup S$ denote the set of all players with $\#N = n$ and $\#S = s$. Players are arranged in a network g where a link between player i and player j is denoted by $g_{ij} = 1$. If i and j are not linked, then $g_{ij} = 0$. By convention, $g_{ii} = 0$. Let $N_i = \{j \in N^+ \mid g_{ij} = 1\}$ be the set of neighbors of player i in g. We keep track of social connections in network through the row-normalized adjacency matrix $\mathbf{G}^* = \left(g^*_{ij}\right)$. It is a directed and weighted network with $g^*_{ij} = g_{ij} / \sum_{j=1}^{n+s} g_{ij}$, and so for each player $i \in N^+$, $\sum_{j \in N^+} g^*_{ij} = 1$. If there is a link between i and j, then $g^*_{ij} > 0$. Otherwise, $g^*_{ij} = 0$.

Players in the network decide how much effort to exert in reporting misbehavior (harassment, bullying, etc.) and/or in helping victims of misbehavior. We denote by x_i the effort level of player i, with $0 \le x_i \le \bar{x}$, and by $\mathbf{x} = (x_1, \dots, x_{n+s})$ the population profile. Each individual $i \in N$ exerts an effort level that maximizes her utility:

$$u_i\left(\mathbf{x}, \mathbf{G}^*\right) = \phi_i \cdot x_i + \lambda_1 \cdot \left(\sum_{j \in N^+} g^*_{ij} \cdot x_j\right) \cdot x_i - \frac{\lambda_2}{2} \cdot \sum_{j \in N^+} g^*_{ij} \cdot (x_i - x_j)^2 - \frac{\lambda_3}{2} \cdot x_i^2 \tag{1}$$

where

$$\phi_i = \mathbf{y}'_i \cdot \boldsymbol{\beta}_1 + \left(\sum_{j \in N^+} g^*_{ij} \cdot \mathbf{y}'_j\right) \cdot \boldsymbol{\beta}_2 + \xi + \epsilon_i$$

is the contextual effect of individual i.[12] The first term of the utility function, $\phi_i \cdot x_i$, describes the direct benefit from exerting effort in reporting or helping. Player i's contextual effect, ϕ_i, depends not only on her own attribute (e.g., age, gender, education, ...) but also on the weighted average attribute of her neighbors. The vector $\mathbf{y}_i$ contains all observable characteristics of player i and is transformed into a positive real number by means of its coefficient vector $\boldsymbol{\beta}_1$, while $\sum_{j \in N^+} g^*_{ij} \cdot \mathbf{y}'_j$ captures the weighted average characteristics of her neighbors $j \in N_i$ and is transformed into a positive real number by means of its coefficient vector $\boldsymbol{\beta}_2$. Finally, ξ is the unobservable (to the researcher) exogenous network characteristics and ϵ_i is the unobservable (to the researcher) characteristics of individual i.

The second term of the utility function, $\lambda_1 \cdot \left(\sum_{j \in N^+} g^*_{ij} \cdot x_j\right) \cdot x_i$, captures the positive spillover effect from the effort exerted by the neighbors, weighted by λ_1. The parameter $\lambda_1 \ge 0$ is the social multiplier coefficient and it captures the strength of social multiplier effect. The decision of one individual to exert more effort can

[12] Throughout the paper we use the term "player" for a generic agent, "individual" for a player who maximizes her utility given in (1) and "benevolent player" for a player who exerts an exogenously given level of effort.

directly influence the behavior of her neighbors or peers. That is, from each neighbor j that exerts a positive effort, player i obtains a spillover according to the weight of the link between i and j that induces her to exert a higher effort.

The third term of the utility function, $(\lambda_2/2) \cdot \sum_{j \in N^+} g^*_{ij} \cdot (x_i - x_j)^2$, captures the social conformity effect. The parameter $\lambda_2 \geq 0$ is the social conformity coefficient, and it captures the strength of social conformity. Individuals are influenced by the social norm, and so there is a cost for deviating from the social norm which is increasing with the distance from the effort levels done by peers. That is, each individual would like that her effort matches with the effort of her peers. Hence, the further the effort of individual i and her neighbor j are away from each other, the larger is the conflict. Again, the weight of the link g_{ij} is used as an indicator of how much individual i cares about having a conflict with individual j. Notice that the efforts of network neighbors are strategic complements:

$$\frac{\partial^2 u_i(\mathbf{x}, \mathbf{G}^*)}{\partial x_i \partial x_j} = (\lambda_1 + \lambda_2) \cdot g^*_{ij} \geq 0,$$

with $\lambda_1, \lambda_2 \geq 0$.

The final term of the utility function, $(\lambda_3/2) \cdot x_i^2$, is the direct cost to exert the effort x_i in reporting misbehavior and/or helping victims of misbehavior. The higher the effort x_i, the higher the direct cost. From now on, we assume that $\bar{x} \geq \phi_i/(\lambda_3 - \lambda_1)$ holds for each individual $i \in N$. This condition ensures that the equilibrium effort level of individual i is within the interval $[0, \bar{x}]$.

If an individual $i \in N$ has no neighbor at all, she chooses an effort x_i, with $0 \leq x_i \leq \bar{x}$ to simply maximize

$$u_i(x_i) = \left(\mathbf{y}'_i \cdot \boldsymbol{\beta}_1 + \xi + \epsilon_i\right) x_i - \frac{\lambda_3}{2} \cdot x_i^2.$$

Benevolent players (or social workers) are the ones who aim that misbehavior is reported and victims are helped. Thus, each benevolent player or social worker $i \in S$ always exerts an exogenous large effort x_S with $0 < x_S \leq \bar{x}$.

2.2 Nash equilibrium

We first derive the optimal effort level of each individual $i \in N$.

Lemma 1 *The best response function of an individual $i \in N$ is given by*

$$x_i = \begin{cases} \max\left\{0, \left(\mathbf{y}'_i \cdot \beta_1 + \xi + \in_i\right)/\lambda_3\right\} & \text{if } N_i = \emptyset \\ \alpha_i + \gamma \cdot \sum\limits_{j \in N} g^*_{ij} x_j & \text{if } N_i \cap N \neq \emptyset \\ \alpha_i & \text{if } \emptyset \neq N_i \subseteq S \end{cases} \tag{2}$$

with

$$\gamma = \frac{\lambda_1 + \lambda_2}{\lambda_2 + \lambda_3} \text{ and}$$
$$\alpha_i = \frac{\phi_i}{\lambda_2 + \lambda_3} + \gamma \cdot \sum_{j \in S} g^*_{ij} x_S.$$

Proof We take $\partial u_i(x_i)/\partial x_i = 0$ and solve it for x_i. This is straightforward for i such that $N_i = \emptyset$. For i such that $N_i \cap N \neq \emptyset$, we get $\partial u_i(x_i)/\partial x_i = 0$

$$\begin{aligned}
\Leftrightarrow\ & \phi_i + \lambda_1 \cdot \sum_{j \in N^+} g^*_{ij} x_j - \frac{1}{2} \cdot \lambda_2 \cdot \sum_{j \in N^+} g^*_{ij} \cdot (2x_i - 2x_j) - \frac{1}{2} \cdot \lambda_3 \cdot 2x_i &= 0 \\
\Leftrightarrow\ & \phi_i + (\lambda_1 + \lambda_2) \cdot \sum_{j \in N^+} g^*_{ij} x_j - (\lambda_2 + \lambda_3) \cdot x_i &= 0 \\
\Leftrightarrow\ & \frac{\phi_i}{\lambda_2 + \lambda_3} + \frac{\lambda_1 + \lambda_2}{\lambda_2 + \lambda_3} \cdot \sum_{j \in N^+} g^*_{ij} x_j &= x_i \\
\Leftrightarrow\ & \frac{\phi_i}{\lambda_2 + \lambda_3} + \frac{\lambda_1 + \lambda_2}{\lambda_2 + \lambda_3} \cdot \sum_{j \in S} g^*_{ij} \underbrace{x_j}_{=x_S} + \frac{\lambda_1 + \lambda_2}{\lambda_2 + \lambda_3} \cdot \sum_{j \in N} g^*_{ij} x_j &= x_i \\
\Leftrightarrow\ & \underbrace{\frac{\phi_i}{\lambda_2 + \lambda_3} + \frac{\lambda_1 + \lambda_2}{\lambda_2 + \lambda_3} \cdot \sum_{j \in S} g^*_{ij} x_S}_{\equiv \alpha_i} + \underbrace{\frac{\lambda_1 + \lambda_2}{\lambda_2 + \lambda_3}}_{\equiv \gamma} \cdot \sum_{j \in N} g^*_{ij} x_j &= x_i.
\end{aligned}$$

For i such that $\emptyset \neq N_i \subseteq S$, the second term of the last expression vanishes. □

Notice that the best response function of all connected individuals is independent of the effort of disconnected individuals. To find the Nash equilibria, we assume without loss of generality that there are no disconnected individuals.[13] Then, we get the $n \times 1$ vector $\mathbf{x} = \boldsymbol{\alpha} + \gamma \mathbf{G}^*_n \cdot \mathbf{x}$, where $\boldsymbol{\alpha} = (\alpha_1, \dots \alpha_n)$ is a $n \times 1$ vector and $\mathbf{G}^*_n$ is the matrix $\mathbf{G}^*$ limited to the first n lines and n columns. Let $\mathbb{1}_n$ be the n-dimensional vector of ones.

Proposition 1 *Assume that the spectral radius ρ of $\mathbf{G}^*_n$ satisfies $\rho(\mathbf{G}^*_n) \cdot \gamma < 1$. The unique Nash equilibrium in pure strategies is given by*

$$\mathbf{x}^* = \left(\mathbb{1}_n - \gamma \mathbf{G}^*_n\right)^{-1} \cdot \boldsymbol{\alpha}.$$

Proof The unique Nash equilibrium is obtained by solving the best response for $\mathbf{x}$. From Lemma 1, we have

[13] If there are k disconnected individuals, we simply remove the entries corresponding to the disconnected individuals from the vector $\boldsymbol{\alpha}$ and the matrix $\mathbf{G}^*_n$ and we obtain $\boldsymbol{\alpha}$ as a $n - k \times 1$ vector and $\mathbf{G}^*_n$ as a $n - k \times n - k$ matrix.

$$\begin{aligned}\mathbf{x}^* &= \boldsymbol{\alpha} + \gamma \mathbf{G}_n^* \mathbf{x}^* \\ \mathbf{x}^* - \gamma \mathbf{G}_n^* \mathbf{x}^* &= \boldsymbol{\alpha} \\ \left(\mathbb{1}_n - \gamma \mathbf{G}_n^*\right) \cdot \mathbf{x}^* &= \boldsymbol{\alpha} \\ \mathbf{x}^* &= \left(\mathbb{1}_n - \gamma \mathbf{G}_n^*\right)^{-1} \cdot \boldsymbol{\alpha}\end{aligned}$$

We require that $\rho(\mathbf{G}_n^*) \cdot \gamma < 1$ to ensure that $\left(\mathbb{1}_n - \gamma \mathbf{G}_n^*\right)^{-1}$ is well-defined and non-negative (Debreu and Herstein 1953). □

Let $(\mathbf{G}_n^*)^k$ be the k-th power of $\mathbf{G}_n^*$, with coefficients $g_{ij}^{[k]}$, where k is some integer. The matrix $(\mathbf{G}_n^*)^k$ keeps track of the weighted indirect connections in the network: $g_{ij}^{[k]} \geq 0$ measures the weight of walks of length $k \geq 1$ from i to j that go only through individuals who are not benevolent players. Given a scalar $\gamma \geq 0$ and $\mathbf{G}_n^*$, we define the following matrix:

$$\mathbf{M} = \left(\mathbb{1}_n - \gamma \mathbf{G}_n^*\right)^{-1} = \sum_{k=0}^{\infty} \gamma^k (\mathbf{G}_n^*)^k.$$

Given a $n \times 1$ vector $\boldsymbol{\alpha}$, we define the Katz–Bonacich $\boldsymbol{\alpha}$-weighted centrality (due to Bonacich 1987) of parameter γ as

$$\mathbf{b}(g, \boldsymbol{\alpha}, \gamma) = \sum_{k=0}^{\infty} \gamma^k (\mathbf{G}_n^*)^k \boldsymbol{\alpha} = \left(\mathbb{1}_n - \gamma \mathbf{G}_n^*\right)^{-1} \cdot \boldsymbol{\alpha}.$$

Corollary 1 *Assume that the spectral radius ρ of $\mathbf{G}_n^*$ satisfies $\rho(\mathbf{G}_n^*) \cdot \gamma < 1$. Then, the unique Nash equilibrium in pure strategies is given by*

$$\mathbf{x}^* = \mathbf{b}(g, \boldsymbol{\alpha}, \gamma).$$

Thus, at the Nash equilibrium, each individual $i \in N$ exerts an effort equal to her weighted Katz–Bonacich centrality.[14]

Notice that in the case all players are homogeneous (i.e., all individuals have the same characteristics and no benevolent player), then each individual will exert the same equilibrium effort given by

$$x_i^* = \frac{\phi_i}{\lambda_3 - \lambda_1} \text{ for all } i \in N,$$

with $\phi_j = \phi_k$ for all $j, k \in N$. As expected, the social conformity coefficient λ_2 does not affect the equilibrium outcomes when all neighbors are identical in their attributes or characteristics. In addition, regardless of the network structure and the

[14] It is not excluded that different individuals would exert the same effort at equilibrium. However, if the network structure exhibits homophily (i.e., individuals tend to associate with those similar to them) then it becomes much more likely that individuals exert effort levels close to similar individuals. See Currarini et al. (2009) for a study of homophily in friendship networks.

number of individuals connected, the equilibrium efforts of all individuals are identical and depend only on λ_1, λ_3 and ϕ. Hence, aggregate utilities and total efforts are the same for different network architectures connecting all homogeneous players. This result is driven by the fact that we keep track of social connections through the row-normalized weighted adjacency matrix.

3 Network-oriented policies

3.1 Finding and isolating the key player

We denote the entries of $\mathbf{M}$ by m_{ij} and the entries of the $n \times 1$ vector $\mathbf{b}$ by b_k. It holds that

$$b_k(g, \boldsymbol{\alpha}) = \sum_{j \in N} m_{jk} \alpha_j.$$

The planner's objective is to find the key player, that is, the individual who once isolated generates the highest possible increase in aggregate effort level. Individual i is said to be isolated if $N_i = \emptyset$. To find the key player, we have to compare the original network with the network where an individual is removed. Let $g^{[-i]} = g \setminus \{jk \mid jk \in g \text{ and } j = i\}$ be the network obtained from g by deleting all the links of individual i in g. Let $\mathbf{G}^{*[-i]}$ denote the adjacency matrix in which individuals i has been removed from the network. This adjacency matrix $\mathbf{G}^{*[-i]}$ is obtained from $\mathbf{G}^*$ by removing the ith row and the ith column and by adjusting the weights such that the weights of all outgoing links sum up to 1: $g_{jk}^{*[-i]} = g_{jk} / \sum_{k \in N^+ \setminus \{i\}} g_{jk}$. When individual i is removed from the network, the matrix $\mathbf{M}$ becomes

$$\mathbf{M}^{[-i]} = \left(\mathbb{1}_{n-1} - \gamma \mathbf{G}_n^{*[-i]} \right)^{-1}$$

with $m_{jk}^{[-i]}$ being the entries of matrix $\mathbf{M}^{[-i]}$. Let $c_k(g, \boldsymbol{\alpha}) = \sum_{j \in N} m_{jk} \alpha_k$, where the index of $\boldsymbol{\alpha}$ in the summation is the only difference between b_k and c_k, and let

$$\alpha_j^{[-i]} = \frac{\phi_j^{[-i]}}{\lambda_2 + \lambda_3} + \gamma \cdot \left(\sum_{k \in S} g_{jk}^{*[-i]} x_b \right)$$

$$\phi_j^{[-i]} = \mathbf{y}_j' \cdot \boldsymbol{\beta}_1 + \left(\sum_{k \in N^+, k \neq i} g_{jk}^{*[-i]} \cdot \mathbf{y}_k' \right) \cdot \boldsymbol{\beta}_2 + \xi + \epsilon_j.$$

Let $B(g, \boldsymbol{\alpha})$ denote the total effort of all individuals. It holds that

$$B(g, \boldsymbol{\alpha}) = \sum_{j \in N} b_j(g, \boldsymbol{\alpha}) = \sum_{j \in N} c_j(g, \boldsymbol{\alpha}).$$

The total effect of isolating individual i is given by

$$B(g^{[-i]}, \boldsymbol{\alpha}^{[-i]}) - B(g, \boldsymbol{\alpha}) + \max\left\{0, \frac{\mathbf{y}_i' \cdot \boldsymbol{\beta}_1 + \xi + \epsilon_i}{\lambda_3}\right\} = -\delta_i^1$$

To find who is the key player to be isolated, we simply need to solve $\min_{i \in N} \delta_i^1$.

Proposition 2 *Assume that each player has at least two links. Then, the contribution of isolated individual i to the total effort in the game is given by the contextual inter-centrality*

$$\delta_i^1(g, \boldsymbol{\alpha}) = \underbrace{\sum_{\substack{k \in N \\ k \in N_i}} c_k(g^{[-i]}, \boldsymbol{\alpha}) - c_k(g^{[-i]}, \boldsymbol{\alpha}^{[-i]})}_{\text{Contextual change}} + \underbrace{\frac{b_i(g, \boldsymbol{\alpha})}{m_{ii}} \sum_{j \in N} m_{ij}}_{\text{Intercentrality of individual } i}$$
$$\underbrace{- \max\left\{0, \frac{\mathbf{y}_i' \cdot \boldsymbol{\beta}_1 + \xi + \epsilon_i}{\lambda_3}\right\}}_{\text{Effort of i when isolated}}.$$

Proof Under the assumption that each player has at least two links, the contribution of individual i to the total effort in the game is

$$B(g, \boldsymbol{\alpha}) - B(g^{[-i]}, \boldsymbol{\alpha}^{[-i]}) + \max\left\{0, \frac{\mathbf{y}_i' \cdot \boldsymbol{\beta}_1 + \xi + \epsilon_i}{\lambda_3}\right\}.$$

We have that

$$
\begin{aligned}
B(g,\boldsymbol{\alpha}) - B\big(g^{[-i]},\boldsymbol{\alpha}^{[-i]}\big) &= \sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}\alpha_k - \sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{[-i]}\alpha_k^{[-i]} \\
&= \sum_{j=1}^{n}\left(\sum_{\substack{k=1\\k\neq i}}^{n} m_{jk}\alpha_k - m_{jk}^{[-i]}\alpha_k^{[-i]}\right) + \sum_{j=1}^{n} m_{ji}\alpha_i \\
&= \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq i\\k\in N_i}}^{n} m_{jk}\alpha_k - m_{jk}^{[-i]}\alpha_k^{[-i]} + \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq i\\k\notin N_i}}^{n} m_{jk}\alpha_k - m_{jk}^{[-i]}\alpha_k^{[-i]} + \sum_{j=1}^{n} m_{ji}\alpha_i \\
&= \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq i\\k\in N_i}}^{n} m_{jk}\alpha_k - m_{jk}^{[-i]}\alpha_k + m_{jk}^{[-i]}\alpha_k - m_{jk}^{[-i]}\alpha_k^{[-i]} \\
&\quad + \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq i\\k\notin N_i}}^{n} (m_{jk} - m_{jk}^{[-i]})\alpha_k + \sum_{j=1}^{n} m_{ji}\alpha_i \\
&= \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq i\\k\in N_i}}^{n} m_{jk}^{[-i]}(\alpha_k - \alpha_k^{[-i]}) + \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq i}}^{n}(m_{jk} - m_{jk}^{[-i]})\alpha_k + \sum_{j=1}^{n} m_{ji}\alpha_i \\
&= \sum_{\substack{k=1\\k\neq i\\k\in N_i}}^{n}\sum_{j=1}^{n} m_{jk}^{[-i]}(\alpha_k - \alpha_k^{[-i]}) + \frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j=1}^{n} m_{ij} \\
&= \sum_{\substack{k=1\\k\neq i\\k\in N_i}}^{n} c_k(g^{[-i]},\boldsymbol{\alpha}) - c_k(g^{[-i]},\boldsymbol{\alpha}^{[-i]}) + \frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j=1}^{n} m_{ij},
\end{aligned}
$$

where the fourth equality follows from the fact that $\alpha_k = \alpha_k^{[-i]}$ for all $k \notin N_i$. The sixth equality follows from

$$
\begin{aligned}
\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq i}}^{n}(m_{jk}-m_{jk}^{[-i]})\alpha_k+\sum_{j=1}^{n}m_{ji}\alpha_i &= \sum_{j=1}^{n}b_j(g,\boldsymbol{\alpha})-\sum_{\substack{j=1\\j\neq i}}^{n}b_j^{[-i]}(g^{[-i]},\boldsymbol{\alpha})\\
&= b_i(g,\boldsymbol{\alpha})+\sum_{\substack{j=1\\j\neq i}}^{n}b_j(g,\boldsymbol{\alpha})-b_j^{[-i]}(g^{[-i]},\boldsymbol{\alpha})\\
&= b_i(g,\boldsymbol{\alpha})+\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{n}m_{jk}\alpha_k-m_{jk}^{[-i]}\alpha_k\\
&= b_i(g,\boldsymbol{\alpha})+\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{n}\frac{m_{ij}m_{ik}}{m_{ii}}\alpha_k\\
&= b_i(g,\boldsymbol{\alpha})+b_i(g,\boldsymbol{\alpha})\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\frac{m_{ij}}{m_{ii}}\\
&= b_i(g,\boldsymbol{\alpha})\left(1+\sum_{\substack{j=1\\j\neq i}}^{n}\frac{m_{ij}}{m_{ii}}\right)\\
&= b_i(g,\boldsymbol{\alpha})\left(\frac{m_{ii}}{m_{ii}}+\sum_{\substack{j=1\\j\neq i}}^{n}\frac{m_{ij}}{m_{ii}}\right)\\
&= \frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j=1}^{n}m_{ij},
\end{aligned}
$$

where the fourth equality follows from Lemma 1 in Ballester et al. (2006), and the fifth equality is obtained from

$$
\begin{aligned}
\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{n}\frac{m_{ij}m_{ik}}{m_{ii}}\alpha_k &= \sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=i}\frac{m_{ij}m_{ik}}{m_{ii}}\alpha_k+\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{\substack{k=1\\k\neq i}}^{n}\frac{m_{ij}m_{ik}}{m_{ii}}\alpha_k\\
&= \sum_{\substack{j=1\\j\neq i}}^{n}\frac{m_{ij}m_{ii}}{m_{ii}}\alpha_i+\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{\substack{k=1\\k\neq i}}^{n}\frac{m_{ij}m_{ik}}{m_{ii}}\alpha_k\\
&= \sum_{\substack{j=1\\j\neq i}}^{n}\frac{m_{ij}}{m_{ii}}\left(m_{ii}\alpha_i+\sum_{\substack{k=1\\k\neq i}}^{n}m_{ik}\alpha_k\right)\\
&= b_i(g,\boldsymbol{\alpha})\cdot\sum_{\substack{j=1\\j\neq i}}^{n}\frac{m_{ij}}{m_{ii}}.
\end{aligned}
$$

□

The decision of exerting an effort for of each individual is affected not only by her own characteristics but also by the characteristics of her friends. The contextual inter-centrality of individual i, $\delta_i^1(g, \boldsymbol{\alpha})$, emphasizes the three effects at work when individual i is isolated from the rest of the players. The first effect is the contextual variable change effect, which is due to the change in the context when individual i is isolated from the network while the network is kept unchanged. The second effect is the network structure change effect, which captures the change in effort due to the network structure change after the removal of individual i. The third effect is simply the effort exerted by individual i when isolated. Only the first two effects are present in the contextual inter-centrality measure of Ballester and Zenou (2014) where the key player is removed rather than being isolated. However, an isolated individual may still exert some positive effort and her effort depends only on her own characteristics. Since individuals may have different characteristics, efforts exerted by isolated individuals matter when identifying the key player to be isolated.

Notice that the key player policy is such that the planner only modifies the network by isolating an individual. Then, all other players adapt their effort after the isolation but they are not allowed to change their links among them. Such an assumption is often justified by the fact that network relationships take more time to adjust than effort levels.[15]

3.2 An illustration

We illustrate the policy of finding and isolating the key player by means of a network g with 10 players given in Fig. 1. This network structure is specific but rich enough to disentangle the effects of network spillovers, social conformity and players' characteristics on the resulting equilibrium outcomes and the key player policy. This network connects all players, and if some player is removed from the network, all other players remain connected. Let $\lambda_1 = \lambda_2 = 1$ and $\lambda_3 = 2$, and so $\gamma = 2/3$. Players 4 and 7 are the most central players in the network, while player 9 is the least central player.[16] To understand the role played by each player's attribute and her position within the network on the key player policy, we consider the following ten cases. In all ten cases the attributes of the players sum up to 100. In addition, attributes are chosen such that all individuals exert a strictly positive effort at equilibrium when the network connect all players. When an individual becomes isolated, she optimally exerts a strictly positive effort if her attribute is positive while she does no effort if her attribute is negative.[17]

A All players have the same attribute: $y_i = 10$ for all $i \in N$.
B Players 3 and 8 have polarized attributes: $y_3 = y_8 = 2$, and $y_i = 12$ for all $i \in N \backslash \{3, 8\}$.

[15] Mauleon and Vannetelbosch (2016) provide a comprehensive overview of solution concepts for solving network formation games.

[16] Katz centrality measures are 0.283 for player 1(6), 0.346 for player 2(5), 0.317 for player 3(8), 0.397 for player 4(7), 0.187 for player 9 and 0.223 for player 10.

[17] Players with negative attributes may be interpreted as individuals who are inclined to misbehave.

C Players 4 and 7 have polarized attributes: $y_4 = y_7 = 2$, and $y_i = 12$ for all $i \in N \backslash \{4, 7\}$.
D Players have decreasing attributes: $y_1 = 19$, and $y_{i+1} = y_i - 2$ for all $i \in N \setminus \{1\}$.
E Players have increasing attributes: $y_1 = 1$, and $y_{i+1} = y_i + 2$ for all $i \in N \setminus \{1\}$.
F Attributes are degree-based: $y_i = \#N_i(100/\sum_{j \in N_j})$. That is, $y_1 = y_6 = 8,33$, $y_2 = y_3 = y_5 = y_8 = 11,11$, $y_4 = y_7 = 13,89$, and $y_9 = y_{10} = 5,56$.
G Players 3 and 8 have different negative attributes: $y_3 = -1$, $y_8 = -3$, and $y_i = 13$ for all $i \in N \backslash \{3, 8\}$.
H Players 3 and 8 have the same negative attribute: $y_3 = y_8 = -2$ and $y_i = 13$ for all $i \in N \backslash \{3, 8\}$.
I Players 4 and 7 have different negative attributes: $y_4 = -1$, $y_7 = -3$, and $y_i = 13$ for all $i \in N \backslash \{4, 7\}$.
J Players 4 and 7 have the same negative attribute: $y_4 = y_7 = -2$ and $y_i = 13$ for all $i \in N \backslash \{4, 7\}$.

For the policy of finding and isolating the key player (P1), Table 1 computes, for each of the ten cases, the value of effort x_i exerted by each individual,[18] the total effort $\sum_{i=1}^{10} x_i = X_{\mathrm{P1}}$, the key player(s), and the total effort when an individual is randomly isolated (X_{RP1}). For the benchmark, i.e., no policy is implemented (NP), Table 2 computes, for each of the ten cases, the value of effort x_i exerted by each individual and the total effort $\sum_{i=1}^{10} x_i = X_{\mathrm{NP}}$. The two bottom lines of Table 1 give us the relative performance of isolating the key player both with respect to doing nothing ($X_{\mathrm{P1}}/X_{\mathrm{NP}}$) and with respect to isolating randomly an individual ($X_{\mathrm{P1}}/X_{\mathrm{RP1}}$).

First, we notice that the key player is not necessarily the player who is the most central within the network nor the player who is doing less effort than all other players. For instance, in the situation where players 3 and 8 have negative attributes (case H), player 8 (or 3) is the key player but players 4 and 7 are more central while player 9 is exerting less effort than player 8. Player 8 (or 3) turns to be the key player because (i) players 4 and 7 are doing a much higher effort than player 8 and (ii) player 9 who is the least central player is the only one to do less effort than player 8. Thus, finding the key player often deviates from simply selecting either the most central player or the player who is exerting the least effort.

Second, we observe that, once the key player is isolated, the efforts exerted by all other players increase while the effort run by the isolated player drastically decreases. Therefore, the total effort exerted by all players (including the key player) may increase or decrease when isolating the key player. When all players have the same attribute, they all exert the same effort and it is useless to isolate some player. If the attribute of a given player decreases (increases), then she will do less (more) effort except if the attributes of her neighbors substantially increase (decrease). Suppose that we start from homogeneous attributes (case A) and we substantially decrease the attributes of players 3 and 8 while we slightly increase the attributes of

[18] Given some players are symmetric in the network, there could be more than one key player. Since the planner can only isolate a single individual, we give in bold the effort done by the selected key player who is being isolated.

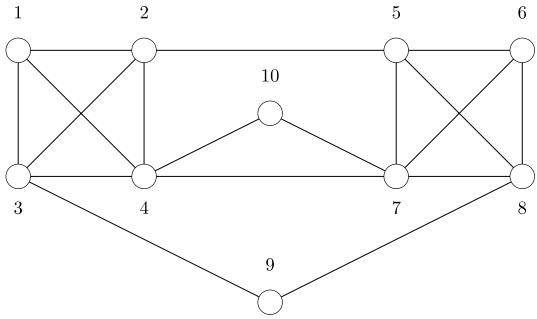

Fig. 1 A network g with 10 players

all other players (case B). Then, players 3 and 8 will exert much less effort while all other players will do more effort with the exception of player 9. In fact, player 9 is decreasing her effort, though her greater attribute, because she is only surrounded by players 3 and 8 whose attributes considerably decrease.

We next analyze the relative performance of the policy of finding and isolating the key player (P1) with respect to the no policy (NP). The objective of the planner is to increase the total effort, i.e., the sum of all effort levels exerted by all players (including the key player). If $X_{\mathrm{P1}}/X_{\mathrm{NP}}$ is strictly greater than 1 then isolating the key player is beneficial for the society. As already mentioned, when attributes are homogeneous, it is always better not to isolate any player. In addition, when attributes are positive and correlated with the centrality of the players, it is better to avoid isolating some player. For instance, when each player's attribute is simply her centrality degree (case F), the key player turns to be player 9 who is the least central player. Since player 9 is only surrounded by two very central players, she is exerting a relatively high effort. But, once she becomes isolated, her effort level drops quite substantially, and this huge decrease is not compensated by the greater effort levels done by all other players. In all other cases, implementing the policy of isolating the key player would increase the total effort exerted by all players. When players 4 and 7 have low attributes while all other players have a much higher attribute (case C), players 4 and 7 exert a low effort and are the most central players. Hence, isolating either player 4 or player 7 will increase the total effort done by all players. Indeed, by isolating such a player who is central and is exerting a low effort, the planner is able to push up the total effort because the isolated player was very influential and was influencing negatively all other players, especially her numerous neighbors. Once she is isolated, her former neighbors are now more influenced by players who are exerting higher effort levels. In general, if the most central players do low effort levels compared to other players, then they are probably the key players, and isolating them is likely to raise the total effort. When players 3 and 8 have low attributes while all other have a much higher attribute (case B), players 3 and 8 exert a low effort but they are not the most central players and so they are not influencing enough negatively the other players to lead to a substantial increase of the effort levels done by all other players once they are isolated. Hence, it is better not to isolate the key player in case B. When players

have decreasing attributes (case D), the key player is player 8 rather than the most central player 7. The reason is that player 7 is a neighbor of player 4 who has a much higher attribute and is linked to players with an even higher attribute, while player 8 is not linked to player 4. In fact, player 8 exerts the lowest effort level among all players and she is sufficiently central to be optimally isolated, leading to an increase of the total effort. When two players have negative attributes (cases G, H, I, J), the key player is the one with the lowest attribute since this player is sufficiently central, and isolating this player substantially increases the total effort (up to 7%).

To assess the relevance of the key player policy (P1), we also compare the increase in total effort following the isolation of the key player with respect to what would

Table 1 Efforts, total efforts, key players and relative performance for the policy of isolating the key player (P1)

	A	B	C	D	E	F	G	H	I	J
x_1	20	23.34	23.46	28.79	14.10	21.70	20.72	21.22	20.41	25.19
x_2	20	22.95	23.01	27.50	**1**	22.20	21.64	21.99	21.41	24.52
x_3	20	**1**	23.37	26.74	16.77	22.20	17.36	17.42	21.34	25.05
x_4	**5**	23.06	**2**	24.33	18.53	22.68	22.23	23.11	17.09	**0**
x_5	20	20.89	20.72	22.11	23.35	22.20	24.53	24.79	24.61	21.08
x_6	20	20.23	20.01	19.78	23.71	21.70	25.08	25.34	25.24	20.01
x_7	20	21.31	17.64	19.24	23.61	22.68	24.69	25.23	**0**	16.46
x_8	20	17.83	20.67	**1.50**	24.75	22.20	**0**	**0**	25.11	21.01
x_9	20	16.55	22.68	23.83	22.84	**2.78**	15.57	15.28	24.15	24.02
x_{10}	20	22.79	16.43	18.19	23.71	21.60	24.31	25.11	15.40	14.64
$\sum_{i=1}^{10} x_i$	185	189.95	188.99	212.00	192.39	201.96	196.13	193.42	194.78	191.98
Key pl	N	3,8	4,7	8	2	9	8	3,8	7	4,7
X_{RP1}	185	179.52	172	192.45	173.05	197.77	177.88	177.88	166.60	166.60
$X_{\mathrm{P1}}/X_{\mathrm{NP}}$	0.925	0.976	1.017	1.005	1.018	0.942	1.022	1.007	1.089	1.074
$X_{\mathrm{P1}}/X_{\mathrm{RP1}}$	1.000	1.058	1.099	1.102	1.112	1.021	1.103	1.087	1.169	1.152

Table 2 Efforts and total efforts under the no policy (NP)

	A	B	C	D	E	F	G	H	I	J
x_1	20	19.87	19.21	28.18	11.82	21.32	20.09	19.80	19.09	18.82
x_2	20	20.21	19.61	26.69	13.31	21.86	20.52	20.32	19.60	19.42
x_3	20	17.52	19.84	25.46	14.54	21.14	16.72	16.27	19.97	19.76
x_4	20	20.67	16.00	23.64	16.36	22.38	21.18	21.01	14.31	14.00
x_5	20	20.21	19.61	19.86	20.14	21.86	20.12	20.32	19.23	19.42
x_6	20	19.87	19.21	17.51	22.49	21.32	19.51	19.80	18.55	18.82
x_7	20	20.67	16.00	17.57	22.43	22.38	20.84	21.01	13.69	14.00
x_8	20	17.52	19.84	16.37	23.63	21.14	15.82	16.27	19.55	19.76
x_9	20	16.34	21.23	18.28	21.72	19.65	14.52	14.52	21.84	21.84
x_{10}	20	21.78	15.33	17.40	22.60	21.40	22.67	22.67	13.00	13.00
$\sum_{i=1}^{10} x_i$	200	194.66	185.89	210.96	189.04	214.44	191.99	191.99	178.83	178.83

happen if the target is selected randomly (RP1). We observe that, in all cases with the exception of homogeneous attributes, the key player policy increases the total effort by at least 2% and by at most 17% compared to the total effort done when the player to be isolated is chosen randomly. Indeed, we have that $X_{\mathrm{P1}}/X_{\mathrm{RP1}}$ is greater than one. In addition, isolating randomly a player (RP1) always decreases the total effort with respect to doing nothing (NP). Thus, one should never target and isolate randomly a player.

3.3 More about the performance of isolating the key player

From Table 1, one could be inclined to conclude that the policy of isolating the key player always increases the total effort when some players have negative attributes. In Table 3 we report the relative performance of the key player policy ($X_{\mathrm{P1}}/X_{\mathrm{NP}}$) for situations where a single player has a negative attribute. Notice that the key player is always the player with a negative attribute. We observe that the relative performance of P1 with respect to NP increases with the centrality of the player who has a negative attribute. When player 4 (or 7) who is the most central player has a negative attribute, the total effort increases by 9.3% from isolating player 4. However, when player 9 who is the least central player has a negative attribute, isolating player 9 would reduce the total effort by 2.5%. The reason is that player 9 is not influencing enough players while she is herself influenced by players who have positive attributes and are themselves mostly influenced by players with positive attributes. Thus, isolating the key player, even if she has a negative attribute, is not always beneficial for the society.

In Table 4 we analyze more deeply the relative performance of the key player policy (P1) when two players have negative attributes. We observe that, when player 4 who is the most central player has a negative attribute, she is the key player. Then, isolating player 4 always increases the total effort with the highest increase obtained when player 10 is the other player to have a negative attribute. Indeed, once player 4 is isolated, player 10 is only influenced directly by player 7 who is now the most central player and has a positive attribute. When players 2 and 9 have negative attributes, player 2 is the key player and isolating player 2 increases the total effort by 1.8%. However, if player 1 has a negative attribute instead of player 2, player 1 is not central enough to lead to an increase of the total effort when being isolated. So, again the planner does not always have incentives to isolate the key player, even if she has a negative attribute.

The most interesting situation arises when players 3 and 5 have negative attributes while all other players have positive attributes. In terms of centrality, player 5 is more central than player 3. However, it turns out that player 3 is the key player. Player 3 is the key player because she is linked to player 9 who is only linked to

Table 3 Relative performance of the isolating key player policy (P1) with respect to the centrality of the player with a negative attribute

	$y_1 = -3.5$ $y_i = 11.5$ $(i \neq 1)$	$y_2 = -3.5$ $y_i = 11.5$ $(i \neq 2)$	$y_3 = -3.5$ $y_i = 11.5$ $(i \neq 3)$	$y_4 = -3.5$ $y_i = 11.5$ $(i \neq 4)$	$y_9 = -3.5$ $y_i = 11.5$ $(i \neq 9)$	$y_{10} = -3.5$ $y_i = 11.5$ $(i \neq 10)$
X_{P1}	207	207	207	207	207	207
X_{NP}	205.06	197.86	195.99	189.41	211.16	212.18
Key player	1	2	3	4	9	10
$X_{\mathrm{P1}}/X_{\mathrm{NP}}$	1.009	1.046	1.056	1.093	0.980	0.976

Table 4 Relative performance of the isolating key player policy (P1) when two players have negative attributes

	$y_2 = y_4 = -2$ $y_i = 13$ $(i \neq 2, 4)$	$y_4 = y_5 = -2$ $y_i = 13$ $(i \neq 4, 5)$	$y_4 = y_9 = -2$ $y_i = 13$ $(i \neq 4, 9)$	$y_4 = y_{10} = -2$ $y_i = 13$ $(i \neq 4, 10)$
X_{P1}	202.21	196.35	212.59	220.75
X_{NP}	193.85	187.27	200.58	201.59
Key player	4	4	4	4
$X_{\mathrm{P1}}/X_{\mathrm{NP}}$	1.043	1.048	1.060	1.095

	$y_2 = y_9 = -2$ $y_i = 13$ $(i \neq 2, 9)$	$y_1 = y_9 = -2$ $y_i = 13$ $(i \neq 1, 9)$	$y_3 = y_5 = -2$ $y_i = 13$ $(i \neq 3, 5)$
X_{P1}	212.80	213.69	198.12
X_{NP}	209.02	216.23	193.85
Key player	2	1	3
$X_{\mathrm{P1}}/X_{\mathrm{NP}}$	1.018	0.988	1.022

players 3 and 8, and so player 9 is strongly influenced by player 3. Isolating player 3 induces a substantial increase of the effort exerted by player 9. Player 5 is not the key player because all her neighbors are also influenced by many players with positive attributes. Thus, the most central player with a negative attribute is not necessarily the key player to be isolated.

Hence, we conclude that a player is more likely to become the key player to be isolated if (i) she is quite influential (i.e., she has many links), (ii) she is exerting a low effort (i.e., she has a low attribute), and (iii) her neighbors are strongly influenced by her (i.e., her neighbors have few links).

3.4 Finding the key player in presence of benevolent players

Suppose now that player 10 is a benevolent player. In Table 5 we report the relative performance of the key player policy ($X_{\mathrm{P1S}}/X_{\mathrm{NPS}}$) for situations where a single player has a negative attribute and player 10 is a benevolent player.[19] Comparing Table 5 with Table 3, we observe that the presence of a benevolent player only slightly reduces the relative performance of the isolating key player policy with respect to isolating nobody. The key player remains still the player with a negative attribute and the relative performance still increases with the centrality of the player who has a negative attribute, except for player 4. The reason is that, once player 10 who is a neighbor of player 4 is a benevolent player, player 4 is directly influenced by the benevolent player and so she is becoming less central and influential than without benevolent players.

In Table 6 we analyze the relative performance of the key player policy (P1S) when two players have negative attributes and player 10 is a benevolent player. Comparing Table 6 with Table 4, we observe that, when players 2 and 4 have negative attributes, player 2 becomes now the key player to be isolated instead of player 4. Player 4 is directly influenced by the benevolent player 10 and player 2, while player 2 is only indirectly influenced by the benevolent player 10. In addition, player 2 directly influences the same number of players as player 4. Isolating player 2 means that player 4 will be even more influenced by the benevolent player 10 and no more influenced at all by player 2. Isolating player 4 would not increase the influence of the benevolent player 10 on player 2. However, when players 1 and 4 have negative attributes, player 4 remains the key player to be isolated since player 1 is still less central and influential than player 4 even though player 4 is closer to the benevolent player.

Thus, in the presence of benevolent players, a player is more likely to become the key player to be isolated if she is influencing negatively many other players and she is not too influenced by benevolent players.

4 Discussion

4.1 Turning an individual into a benevolent player

The planner's objective is now to find the key player, that is, the individual who once turned into a benevolent player generates the highest possible increase in aggregate effort level.[20] Benevolent players choose the effort x_S while standard individuals still maximize their utility. Let

[19] The effort level of the benevolent player 10 is obtained by multiplying by 1.25 her optimal effort she would exert if she was a standard player. For instance, when $y_2 = -3.5$ and $y_j = 11.5$ for all $j \neq 2$, $x^*_{10} = 21.24$ and her effort as a benevolent player is simply $x_s = 1.25 \times 21.24 = 26.56$.

[20] Davis and Davis (2007) provide a wide range of options that bystanders can use before or during or after bullying situations: e.g., specific techniques for teaching empathy and social problem solving skills, limiting the rewards of bullying behavior, and building a partnership between students and staff to create a positive and inclusive peer culture.

Table 5 Relative performance of the isolating key player policy (P1S) with respect to the centrality of the player with a negative attribute when player 10 is a benevolent player

	$y_1 = -3.5$ $y_i = 11.5$ $(i \neq 1)$	$y_2 = -3.5$ $y_i = 11.5$ $(i \neq 2)$	$y_3 = -3.5$ $y_i = 11.5$ $(i \neq 3)$	$y_4 = -3.5$ $y_i = 11.5$ $(i \neq 4)$	$y_9 = -3.5$ $y_i = 11.5$ $(i \neq 9)$
X_{P1S}	214.21	213.68	213.85	203.57	216.07
X_{NPS}	214.65	207.31	205.48	196.75	221.15
Key player	1	2	3	4	9
$X_{\text{P1S}}/X_{\text{NPS}}$	0.998	1.031	1.041	1.035	0.977

Table 6 Relative performance of the isolating key player policy (P1S) when two players have negative attributes and player 10 is a benevolent player

	$y_2 = y_4 = -2$ $y_i = 13$ $(i \neq 2, 4)$	$y_4 = y_5 = -2$ $y_i = 13$ $(i \neq 4, 5)$	$y_4 = y_7 = -2$ $y_i = 13$ $(i \neq 4, 7)$	$y_4 = y_9 = -2$ $y_i = 13$ $(i \neq 4, 9)$
X_{P1S}	198.84	195.57	194.30	209.79
X_{NPS}	195.16	195.16	184.61	209.00
Key player	2	4	4,7	4
$X_{\text{P1S}}/X_{\text{NPS}}$	1.019	1.002	1.052	1.004
	$y_2 = y_9 = -2$ $y_i = 13$ $(i \neq 2, 9)$	$y_1 = y_9 = -2$ $y_i = 13$ $(i \neq 1, 9)$	$y_3 = y_5 = -2$ $y_i = 13$ $(i \neq 3, 5)$	$y_3 = y_8 = -2$ $y_i = 13$ $(i \neq 3, 8)$
X_{P1S}	220.94	222.26	206.05	201.17
X_{NPS}	219.55	226.91	203.90	202.07
Key player	2	1	3	3,8
$X_{\text{P1S}}/X_{\text{NPS}}$	1.006	0.980	1.011	0.996

$$\alpha_j^{[+i]} = \frac{\phi_j}{\lambda_2 + \lambda_3} + \gamma \cdot \left(\sum_{k \in S} g_{jk}^* x_S + g_{ji}^* x_S \right) = \alpha_j + \gamma \cdot g_{ji}^* \cdot x_S$$

where individual i is the one who is turned into a benevolent player. Since all players keep their links, there is no change in the original network and in the contextual effects of the players.

Proposition 3 *Assume that each player has at least two links. Then, turning an individual into a benevolent player increases the total effort in the game by the benevolent change inter-centrality*

$$\delta_i^2(g,\boldsymbol{\alpha}) = \underbrace{\sum_{\substack{k\in N\\ k\in N_i}} c_k(g^{[-i]},\boldsymbol{\alpha}^{[+i]}) - c_k(g^{[-i]},\boldsymbol{\alpha})}_{\text{Benevolent player change}} - \underbrace{\frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j\in N} m_{ij}}_{\text{Intercentrality of player } i}$$

$$= \sum_{\substack{k\in N\\ k\in N_i}}\sum_{j\in N} m_{jk}^{[-i]}\cdot\gamma\cdot g_{kl}^{*}\cdot x_S - \frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j\in N} m_{ij}.$$

Proof Under the assumption that each player has at least two links, the effect on the total effort of turning an individual into a benevolent player is given by

$$B(g^{[-i]},\boldsymbol{\alpha}^{[+i]}) - B(g,\boldsymbol{\alpha}) = \sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{[-i]}\alpha_k^{[+i]} - \sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}\alpha_k$$

$$= \sum_{j=1}^{n}\left(\sum_{\substack{k=1\\ k\neq i}}^{n} m_{jk}^{[-i]}\alpha_k^{[+i]} - m_{jk}\alpha_k\right) - \sum_{j=1}^{n} m_{ji}\alpha_i$$

$$= \sum_{j=1}^{n}\sum_{\substack{k=1\\ k\neq i\\ k\in N_i}}^{n} m_{jk}^{[-i]}\alpha_k^{[+i]} - m_{jk}\alpha_k + \sum_{j=1}^{n}\sum_{\substack{k=1\\ k\neq i\\ k\notin N_i}}^{n} m_{jk}^{[-i]}\alpha_k^{[+i]} - m_{jk}\alpha_k - \sum_{j=1}^{n} m_{ji}\alpha_i$$

$$= \sum_{j=1}^{n}\sum_{\substack{k=1\\ k\neq i\\ k\in N_i}}^{n} m_{jk}^{[-i]}\alpha_k^{[+i]} - m_{jk}^{[-i]}\alpha_k + m_{jk}^{[-i]}\alpha_k - m_{jk}\alpha_k$$

$$+ \sum_{j=1}^{n}\sum_{\substack{k=1\\ k\neq i\\ k\notin N_i}}^{n} (m_{jk}^{[-i]} - m_{jk})\alpha_k - \sum_{j=1}^{n} m_{ji}\alpha_i$$

$$= \sum_{j=1}^{n}\sum_{\substack{k=1\\ k\neq i\\ k\in N_i}}^{n} m_{jk}^{[-i]}(\alpha_k^{[+i]} - \alpha_k) - \left(\sum_{j=1}^{n}\sum_{\substack{k=1\\ k\neq i}}^{n}(m_{jk} - m_{jk}^{[-i]})\alpha_k + \sum_{j=1}^{n} m_{ji}\alpha_i\right)$$

$$= \sum_{\substack{k=1\\ k\neq i\\ k\in N_i}}^{n}\sum_{j=1}^{n} m_{jk}^{[-i]}(\alpha_k^{[+i]} - \alpha_k) - \frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j=1}^{n} m_{ij}$$

$$= \sum_{\substack{k=1\\ k\neq i\\ k\in N_i}}^{n} c_k(g^{*[-i]},\alpha^{[+i]}) - c_k(g^{*[-i]},\alpha) - \frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j=1}^{n} m_{ij}$$

$$= \sum_{\substack{k=1\\ k\neq i\\ k\in N_i}}^{n}\sum_{j=1}^{n} m_{jk}^{[-i]}\cdot\gamma\cdot g_{kl}^{*}\cdot x_S - \frac{b_i(g,\boldsymbol{\alpha})}{m_{ii}}\sum_{j=1}^{n} m_{ij},$$

where the fourth equality follows from $\alpha_i^{[+i]} = \alpha_k$ and the sixth equality is obtained as in the Proof of Proposition 2. □

4.2 Training an individual for exerting more effort

The planner's objective is now to find the key player, that is, the individual who once trained generates the highest possible increase in aggregate effort level.[21] Let the training attribute be the t-th entry in $\mathbf{y}$. For player i being trained, let $\alpha_j^{i,\tau}$ be defined as

$$\alpha_j^{i,\tau} = \frac{\phi_j^{i,\tau}}{\lambda_2 + \lambda_3} + \gamma \cdot \left(\sum_{k=n+1}^{n+s} g_{jk}^* x_S \right) = \alpha_j + \frac{\phi_j^{i,\tau} - \phi_j}{\lambda_2 + \lambda_3},$$

with

$$\phi_i^{i,\tau} = \mathbf{y}_i^{\tau\prime} \cdot \boldsymbol{\beta}_1 + \left(\sum_{j=1}^{n+s} g_{ij}^* \cdot \mathbf{y}_j' \right) \cdot \boldsymbol{\beta}_2 + \xi + \epsilon_i = \phi_i + \tau \cdot \boldsymbol{\beta}_1^t, \text{ and}$$

$$\phi_j^{i,\tau} = \mathbf{y}_j' \cdot \boldsymbol{\beta}_1 + \left(g_{ji}^* \mathbf{y}_i^{\tau\prime} + \sum_{\substack{k=1 \\ k \neq i}}^{n+s} g_{jk}^* \cdot \mathbf{y}_k' \right) \cdot \boldsymbol{\beta}_2 + \xi + \epsilon_j = \phi_j + g_{ij}^* \tau \cdot \boldsymbol{\beta}_2^t \text{ for } j \neq i.$$

Proposition 4 *Assume that all players have at least one link. Then, training individual i by τ, i.e., increasing y_i^t to $y_i^t + \tau$, increases the total effort of the game by*

$$\delta_i^3(g, \boldsymbol{\alpha}) = \sum_{\substack{k=1 \\ k \in N_i \cup \{i\}}}^{n} c_k(g, \boldsymbol{\alpha}^{i,\tau}) - c_k(g, \boldsymbol{\alpha}) = \frac{\tau}{\lambda_2 + \lambda_3} \sum_{j \in N} \left(m_{ji} \boldsymbol{\beta}_1^t + \sum_{\substack{k \in N \\ k \in N_i}} m_{jk} g_{ki}^* \boldsymbol{\beta}_2^t \right)$$

$$= \sum_{j \in N} \left(\underbrace{\frac{\tau m_{ji} \boldsymbol{\beta}_1^t}{\lambda_2 + \lambda_3}}_{\text{Direct effect}} + \underbrace{\sum_{\substack{k \in N \\ k \in N_i}} \frac{\tau m_{jk} g_{ki}^* \boldsymbol{\beta}_2^t}{\lambda_2 + \lambda_3}}_{\text{Indirect effect}} \right).$$

[21] Padgett and Notar (2013) report that peer mediation is a strategy where students themselves are taught to help resolve conflicts among their peers. In New Hampshire (USA) where middle school students have been trained as peer mediators, students who are involved in the conflict sign a contract at the end of the mediation about changing their behaviors. In addition, buddy systems encourage reporting. Students are paired with a friend or older student and these buddies would be someone on whom victims can depend for help when bullying occurs.

Proof Under the assumption that each player has at least one link, the effect on the total effort of training individual i by τ is given by

$$
\begin{aligned}
B(g, \boldsymbol{\alpha}^{i,\tau}) - B(g, \boldsymbol{\alpha}) &= \sum_{j=1}^{n} \sum_{k=1}^{n} m_{jk}(\alpha_k^{i,\tau} - \alpha_k) \\
&= \sum_{j=1}^{n} \left(\sum_{\substack{k=1 \\ k \in N_i \cup \{i\}}}^{n} m_{jk}(\alpha_k^{i,\tau} - \alpha_k) + \sum_{\substack{k=1 \\ k \neq i \\ k \notin N_i}}^{n} m_{jk}(\alpha_k^{i,\tau} - \alpha_k) \right) \\
&= \sum_{j=1}^{n} \left(\sum_{\substack{k=1 \\ k \in N_i \cup \{i\}}}^{n} m_{jk}(\alpha_k^{i,\tau} - \alpha_k) + 0 \right) \\
&= \sum_{\substack{k=1 \\ k \in N_i \cup \{i\}}}^{n} c_k(g, \boldsymbol{\alpha}^{i,\tau}) - c_k(g, \boldsymbol{\alpha}) \\
&= \sum_{\substack{k=1 \\ k \in N_i \cup \{i\}}}^{n} \sum_{j=1}^{n} m_{jk}(\alpha_k^{i,\tau} - \alpha_k) = \sum_{j=1}^{n} \sum_{\substack{k=1 \\ k \in N_i \cup \{i\}}}^{n} m_{jk}(\alpha_k^{i,\tau} - \alpha_k) \\
&= \sum_{j=1}^{n} \left(m_{ji} \frac{\phi_i^{i,\tau} - \phi_i}{\lambda_2 + \lambda_3} + \sum_{\substack{k=1 \\ k \neq i \\ k \in N_i}}^{n} m_{jk} \frac{\phi_k^{i,\tau} - \phi_k}{\lambda_2 + \lambda_3} \right) \\
&= \sum_{j=1}^{n} \left(m_{ji} \frac{\tau \boldsymbol{\beta}_1^t}{\lambda_2 + \lambda_3} + \sum_{\substack{k=1 \\ k \neq i \\ k \in N_i}}^{n} m_{jk} \frac{g_{ki}^* \tau \boldsymbol{\beta}_2^t}{\lambda_2 + \lambda_3} \right) \\
&= \frac{\tau}{\lambda_2 + \lambda_3} \sum_{j=1}^{n} \left(m_{ji} \boldsymbol{\beta}_1^t + \sum_{\substack{k \neq i \\ k \in N_i}} m_{jk} g_{ki}^* \boldsymbol{\beta}_2^t \right) \\
&= \sum_{j \in N} \left(\frac{\tau m_{ji} \boldsymbol{\beta}_1^t}{\lambda_2 + \lambda_3} + \sum_{\substack{k \in N \\ k \in N_i}} \frac{\tau m_{jk} g_{ki}^* \boldsymbol{\beta}_2^t}{\lambda_2 + \lambda_3} \right).
\end{aligned}
$$

□

Notice that the total effect of training individual i can be decomposed into two parts: $(\tau m_{ji}\boldsymbol{\beta}_1^t)/(\lambda_2+\lambda_3)$ is the direct training effect of individual i and $\sum_{\substack{k\in N\\ k\in N_i}}(\tau m_{jk}g_{ki}^*\boldsymbol{\beta}_2^t)/(\lambda_2+\lambda_3)$ is the indirect effect due to the contextual effects. All neighbors of individual i get an increased contextual effect from individual i and therefore increase their effort as well.

4.3 Comparing policies

We reconsider the network of Fig. 1, and we suppose that turning an individual into a benevolent player increases her optimal effort by 25%. For the policy of turning an individual into a benevolent player (P2), Table 7 computes, for each of the ten cases, the value of effort x_i exerted by each player,[22] the total effort $\sum_{i=1}^{10} x_i = X_{\mathrm{P2}}$, the benevolent key player(s), and the total effort when a player is randomly selected (X_{RP2}). The optimal target to be turned into a benevolent player depends on both the network and the attributes. For instance, when players 4 and 7 have low attributes compared to all other players (case C), players 4 and 7 are the most central players but they are exerting not enough effort to be turned into a benevolent player, and so it becomes optimal to target either player 3 or player 8. Players 3 and 8 are only slightly less central than players 4 and 7 but they exert a much higher effort. When players have decreasing attributes (case D), player 1 exerts the highest effort but she is not central enough to become the optimal target. Player 4 is the optimal target, and not player 7, even though they are both the most central players, but player 4 is exerting a much higher effort. Thus, P2 does not always target neither the most central player nor the player doing the highest effort. When attribute levels are positively correlated with the centrality of players, it is optimal to target the most central player.

We observe that, the relative performance of turning an individual into a benevolent player (P2) with respect to doing nothing (NP) increases the total effort by at least 7.1% and by at most 9.0%. So, it is always better for the planner to implement P2 rather than doing nothing. What happens if instead of targeting the optimal player for P2, the planner chooses randomly the individual to be turned into a benevolent player. We observe that P2 increases the total effort by at least 0.9% and by at most 2.8% compared to the total effort when the targeted player is chosen randomly (RP2). Thus, P2 only performs slightly better than RP2. Moreover, P2 requires the exact knowledge of the network and the attributes. Hence, if the planner cannot obtain such information or it is too costly to get it, then selecting randomly the individual to become benevolent could be a good alternative. In this case, it may be preferable for the planner to select the less costly individual to be turned into a benevolent player.

[22] Since there could be more than one benevolent key player and the planner can only turn a single individual into a benevolent one, we give in bold the effort done by the selected individual.

Table 7 Efforts and total efforts under the policy of a benevolent key player (P2)

	A	B	C	D	E	F	G	H	I	J
x_1	21.77	21.69	20.88	30.27	12.38	23.30	21.97	21.66	20.77	20.48
x_2	21.48	21.74	20.98	28.43	13.99	23.51	22.08	21.87	20.98	20.78
x_3	21.48	19.05	**24.80**	27.22	15.08	22.81	18.29	17.83	**24.96**	**24.70**
x_4	**25.00**	**25.84**	17.19	**29.55**	17.65	**27.98**	**26.48**	**26.26**	15.51	15.18
x_5	20.60	20.83	20.03	20.57	21.80	22.53	20.75	20.95	19.66	19.84
x_6	20.50	20.38	19.50	18.10	24.47	21.87	20.04	20.32	18.84	19.10
x_7	21.15	21.86	16.39	18.93	**28.04**	23.66	22.06	22.22	14.08	14.39
x_8	20.48	18.02	20.33	16.94	25.30	21.69	16.34	16.78	20.04	20.25
x_9	20.66	17.02	23.04	19.05	22.46	20.39	15.21	15.20	23.67	23.65
x_{10}	22.05	23.90	15.86	19.83	24.90	23.70	24.85	24.83	13.53	13.52
$\sum_{i=1}^{10} x_i$	215.17	210.33	199.00	228.89	206.07	231.43	208.07	207.92	192.02	191.89
Key pl	4,7	4,7	3,8	4	7	4,7	4	4,7	3	3,8
X_{RP2}	212.24	206.60	197.21	223.96	200.54	227.63	203.91	203.78	189.72	189.71
$X_{\mathrm{P2}}/X_{\mathrm{NP}}$	1.076	1.080	1.071	1.085	1.090	1.079	1.084	1.083	1.074	1.073
$X_{\mathrm{P2}}/X_{\mathrm{RP2}}$	1.014	1.018	1.009	1.022	1.028	1.017	1.020	1.020	1.012	1.011

We now look at the policy of training some individual for exerting more effort in supporting victims (P3) in the network of Fig. 1. Suppose that training an individual increases her attribute by 2. Table 8 computes, for each of the ten cases, the value of effort x_i exerted by each player,[23] the total effort $\sum_{i=1}^{10} x_i = X_{\mathrm{P3}}$, the key player(s) to be trained, and the total effort when an individual is randomly trained (X_{RP3}). The optimal target to be trained for exerting more effort in supporting victims is always the most central player. We observe that, the relative performance of training an individual (P3) relative to doing nothing (NP) increases the total effort by at least 2.5% and by at most 3.0%. Again, it is always better for the planner to implement P3 rather than doing nothing. What happens if instead of targeting the optimal player for P3, the planner chooses randomly the individual to be trained? We observe that P3 only slightly increases the total effort by 0.7% compared to the total effort when the targeted player is chosen randomly (RP3). Thus, if collecting the information about the relationships and the attributes of the individuals is costly, the best choice for the planner is to select randomly the individual to be trained.

The major difference between turning an individual into a benevolent player and training some individual for exerting more effort has to do with the contextual effects. Training some individual increases her attribute and induces her to exert more effort. Through the contextual effects, her neighbors have also incentives to increase their effort levels. In addition, exerting higher effort levels induces neighbors to exert more effort because of the network positive spillover effects and the

[23] Since there could be more than one benevolent key player and the planner can only train a single individual, we give in bold the effort done by the selected individual.

Table 8 Efforts and total efforts under the policy of training the key player (P3)

	A	B	C	D	E	F	G	H	I	J
x_1	20.75	20.61	19.96	28.92	12.03	22.06	20.84	20.55	19.30	19.56
x_2	20.62	20.83	20.23	27.31	13.56	22.48	21.14	20.94	19.85	20.04
x_3	20.63	18.14	20.47	26.09	14.74	21.77	17.34	16.90	20.18	20.39
x_4	**21.11**	**21.79**	**17.11**	**24.75**	16.85	**23.49**	**22.29**	**22.12**	14.80	**15.11**
x_5	20.25	20.47	19.86	20.11	20.77	22.11	20.37	20.57	19.85	19.67
x_6	20.21	20.08	19.42	17.72	23.24	21.53	19.72	20.01	19.30	19.03
x_7	20.49	21.16	16.49	18.06	**23.54**	22.86	21.33	21.50	**14.80**	14.49
x_8	20.20	17.72	20.05	16.57	24.26	21.35	16.03	16.48	20.18	19.97
x_9	20.28	16.62	21.50	18.56	22.00	19.93	14.79	14.79	22.12	22.12
x_{10}	20.87	22.65	16.20	18.27	23.46	22.27	23.54	23.54	13.87	13.87
$\sum_{i=1}^{10} x_i$	205.41	200.07	191.30	216.37	194.45	219.85	197.39	197.40	184.24	184.24
Key pl	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7
X_{RP3}	204.00	198.66	189.89	214.96	193.04	218.44	195.99	195.99	182.83	182.83
X_{P3}/X_{NP}	1.027	1.028	1.029	1.026	1.029	1.025	1.028	1.028	1.030	1.030
X_{P3}/X_{RP3}	1.007	1.007	1.007	1.007	1.007	1.006	1.007	1.007	1.008	1.008

social conformity. However, when some individual is turned into a benevolent player, it has no impact on her attribute and the contextual effects. Only the network spillovers and the social conformity then play a role.

4.4 The cost of finding and isolating the key player

The computation of the contextual inter-centrality measure for each player relies on the knowledge of the network.[24] Thus, implementing the key player strategy obviously has its costs. The relative gains from targeting the key player instead of selecting at random increase with the variability in contextual inter-centrality measures across players. Hence, the key player strategy seems better suited for asymmetric network structures with players having quite different characteristics or attributes. Beside looking for the optimal single player to be isolated from the network to increase aggregate effort in reporting, one could easily extend the analysis to look for the optimal group to be isolated. However, implementing the negative key group strategy is much more demanding since the problem of finding a key group in a network is NP-hard (Ballester et al. 2010). In addition of facing such computational complexity, the costs borne by the society for isolating more than one player are likely to increase with the number of players to be isolated.

[24] Foerster et al. (2021) study network formation games when individuals may have either public links or private ones.

4.5 Conclusion

We have studied a model where individuals are embedded in a network of relationships and they can potentially be victims, bystanders, or perpetrators of misbehavior. Each individual decides non-cooperatively how much effort to exert in supporting or helping victims of misbehavior. Each individual's optimal effort depends on the contextual effect, the social multiplier effect and the social conformity effect. We have characterized the Nash equilibrium and we have derived an inter-centrality measure for finding the key player who once isolated increases the most the aggregated effort within the social network. An individual is more likely to be the key player if she is influencing many other individuals, she is exerting a low effort because of her characteristics, and her neighbors are strongly influenced by her. The key player policy increases substantially the aggregate effort and the targeted player should never be selected randomly. The key player without social workers is likely to remain the key player with social workers except if she is becoming less influential due to her closeness to social workers. Finally, we have considered alternative policies (e.g., training bystanders for helping victims) and we have compared them to the policy of isolating the key player.

Acknowledgements We thank Emilomo Ogbe and Olivier Degomme for in-depth discussions aiming at developing a model to identify network-oriented strategies for mitigating the effects of sexual and gender-based violence (SGBV) among asylum seekers. We thank the editor, an associate editor and an anonymous referee for their valuable comments and suggestions to improve the paper. Ata Atay is a Serra Húnter Assistant Professor (Professor Lector Serra Húnter) under the Pla Serra Húnter. Ana Mauleon and Vincent Vannetelbosch are Research Director and Senior Research Associate of the National Fund for Scientific Research (FNRS), respectively. Financial support from the Belgian French speaking community ARC project 15/20-072 of Saint-Louis University - Brussels, from the Fonds de la Recherche Scientifique - FNRS research grant T.0143.18, from the Spanish Ministerio de Ciencia e Innovación research grant PID2020-113110GB-100/AEI/10.13039/501100011033, from the Generalitat de Catalunya research grant 2021SGR00306, and from the University of Barcelona research grant AS017672 is gratefully acknowledged. This work has been partly supported by COST Action CA16228 European Network for Game Theory.

Declarations

Conflict of interest The authors declare that no conflict of interest.

Consent to participate Not applicable.

Consent for publication Not applicable.

Ethical approval Not applicable.

References

Ballester C, Zenou Y (2014) Key player policies when contextual effects matter. J Math Sociol 38:233–248

Ballester C, Calvó-Armengol A, Zenou Y (2006) Who's who in networks. Wanted: the key player. Econometrica 74:1403–1417

Ballester C, Calvó-Armengol A, Zenou Y (2010) Delinquent networks. J Eur Econ Assoc 8:34–61

Boethius S, Åkerström M (2020) Revealing hidden realities: disclosing domestic abuse to informal others. Nordic J Criminol 21:186–202

Boguslaw J (2017) The key player in disruptive behavior: whom should we target to improve the classroom learning environment. Stockholm University, Stockholm
Bonacich P (1987) Power and centrality: a family of measures. Am J Sociol 92:1170–1182
Boucher V (2016) Conformism and self-selection in social networks. J Public Econ 136:30–44
Bramoullé Y, Djebbari H, Fortin B (2009) Identification of peer effects through social networks. J Econom 150:41–55
Currarini S, Jackson MO, Pin P (2009) An economic model of friendship, homophily, minorities and segregation. Econometrica 77:1003–1045
Davis S, Davis J (2007) From empowering bystanders in bullying prevention. Research Press Publishers, Champaign
Davis S, Nixon CL (2013) Youth voice project: student insights into bullying and peer mistreatment. Research Press Publishers, Champaign
Debreu G, Herstein IN (1953) Nonnegative square matrices. Econometrica 21:597–607
Foerster M, Mauleon A, Vannetelbosch V (2021) Shadow links. J Econ Theory 197:105325
Goodman LA, Smyth KF (2011) A call for a social network-oriented approach to services for survivors of intimate partner violence. Psychol Violence 1(2):79–92
Hawkins DL, Pepler DJ, Craig WM (2001) Naturalistic observations of peer interventions in bullying. Soc Dev 10:512–527
Hogan BE, Linden W, Najarian B (2002) Social support interventions. Do they work? Clin Psychol Rev 22:381–440
Jackson MO, Zenou Y (2015) Games on networks. In: Young P, Zamir S (eds) Handbook of game theory, vol 4. Elsevier, Amsterdam, pp 91–157
Landini F, Montinari N, Pin P, Piovesan M (2016) Friendship network in the classroom: parents bias on peer effects. J Econ Behav Org 129:56–73
Lee L-F, Liu X, Patacchini E, Zenou Y (2021) Who is the key player? A network analysis of juvenile delinquency. J Bus Econ Stat 39:849–857
Lindquist M, Zenou Y (2019) Crime and networks: 10 policy lessons. Oxf Rev Econ Policy 35:746–771
Mauleon A, Vannetelbosch V (2016) Network formation games. In: Bramoullé Y, Rogers BW, Galeotti A (eds) Oxford handbook on the economics of networks. Oxford University Press, Oxford, pp 167–190
Mazzone A (2020) Bystanders to bullying: an introduction to the special issue. Int J Bullying Prev 2:1–5
Modecki KL, Minchin J, Harbaugh AG, Guerra NG, Runions KC (2014) Bullying prevelance across contexts: a meta-analysis measuring cyber and traditional bullying. J Adolesc Health 55:602–611
Mouttapa M, Valente T, Gallaher P, Rohrbach IA, Unger B (2004) Victimization. Adolescence 39:315–355
National Center for Educational Statistics (2016) Students reports of bullying: results from the 2015 school crime supplement to the national crime victimization survey. Web Tables U.S. Department of Education, NCES 2017-015. https://nces.ed.gov/pubs2017/2017015.pdf
Ogbe E, Jbour A, Rahbari L, Unnithan M, Degomme O (2021) The potential role of network-oriented interventions for survivors of sexual and gender-based violence among asylum seekers in Belgium. BMC Public Health 21:25
Padgett S, Notar CE (2013) Bystanders are the key to stopping bullying universal. J Educ Res 1:33–41
Patacchini E, Zenou Y (2012) Juvenile delinquency and conformism. J Law Econ Org 28:1–31
Shelton RC, Lee M, Brotzman LE, Crookes DM, Jandorf L, Erwind D, Gage-Bouchard EA (2019) Use of social network analysis in the development, dissemination, implementation, and sustainability of health behavior interventions for adults: a systematic review. Soc Sci Med 220:81–101
Thornberg T, Tenenbaum L, Varjas K, Meyers J, Jungert T, Vanegas G (2012) Bystander motivation in bullying incidents: to intervene or not to intervene? West J Emerg Med 8:247–252
Zenou Y (2016) Key players. In: Bramoullé Y, Rogers BW, Galeotti A (eds) Oxford handbook on the economics of networks. Oxford University Press, Oxford, pp 244–274

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.