

Chapter 1

Mesoscopic Transport

*"We all agree that your theory is crazy
but it is not crazy enough to be true."*

Niels Bohr (to a young physicist)

1.1 Introduction

1.1.1 Length scales

The word ‘Mesoscopic’ is derived from the greek words ‘mesos’, meaning ‘middle’, or ‘intermediate’, and ‘skopein’, meaning ‘to look’. Literally, mesoscopic physics looks at systems intermediate between the macroscopic objects that we are familiar with, and the microscopic world of individual atoms and molecules. Obviously, this definition is vague. In some sense, it reflects the fuzziness of the discipline boundaries: its ramifications overlap with many other well-established areas in physics. Actually, the ‘mesoscopic system’ concept must be defined in direct relation with the specific physical properties (optical, mechanical, magnetic, ...) that are examined. As an example, if we focus on the electrical conductivity of a system, the mesoscopic regime is reached when the macroscopic description of its conductivity is no longer valid. New phenomena, which are masked in macroscopic samples, become important, and require new theories for the prediction of their features. Statistics can not be avoided, as these new mesoscopic phenomena are visible in samples with a much too large number of particles to allow for

a treatment using first-principles methods.¹

More precisely, macroscopic laws of conductivity break down when the system size becomes comparable to two characteristic length scales: the mean free path l_μ and the phase coherence length L_ϕ .² Mathematical derivations and formal definitions of l_μ and L_ϕ can be found in many mesoscopic physics textbooks (such as Datta [45]). l_μ and L_ϕ are associated to the corresponding timescales τ_μ and τ_ϕ . In this introductory section, we will simply lean on intuitive concepts to explain the physics behind these length- and time-scales:

- l_μ - τ_μ : irregularities in crystals lead to scattering of the electron waves. Scattering events can be defined as ‘elastic’, when the electron keeps its original energy (*e.g.*, a collision with a displaced atom in the crystal lattice), or ‘inelastic’ when energy is exchanged during the scattering event. The average times between elastic and inelastic collisions are τ_e (or τ_0) and τ_i , respectively. Both electron-phonon and electron-electron interactions contribute to limit τ_i . The corresponding length scales are $l_e = v_F \tau_e$ and $l_i = v_F \tau_i$, where v_F is the Fermi velocity. The momentum relaxation time τ_μ is the characteristic time governing the randomization of the momentum. It is directly related to the electron mobility μ through the relation $\mu = \tau_\mu e / m^*$ (where m^* is the electron effective mass, and e is the electron charge). Fig. 1.1 shows the temperature (T) dependence of μ in our samples. μ is limited at high temperature (above $\sim 50 - 70$ K in our case), by large-angle electron-phonon (inelastic) scattering. At low temperature, it saturates as is limited by elastic collisions (electron-electron interactions do not influence τ_μ and l_μ). The electron mean free path is defined as $l_\mu = v_F \tau_\mu$. If all the dimensions of a sample are smaller than l_μ , transport is in the ballistic regime: electrons ‘bounce’ against the walls of the sample so that the shape of the sample has a strong influence on electron trajectories. The usual analogy for this regime is the billiard table, where billiard balls move along straight lines between specular reflections at the edges of the table.
- τ_ϕ - L_ϕ : L_ϕ and τ_ϕ are directly related to the wave nature of electrons. Indeed, τ_ϕ characterizes the phase randomization. This means that,

¹This is not only due to limited computing capabilities, but also to a lack of informations about initial conditions.

²Additional length scales will appear throughout this thesis, but l_μ and L_ϕ are the main ones.

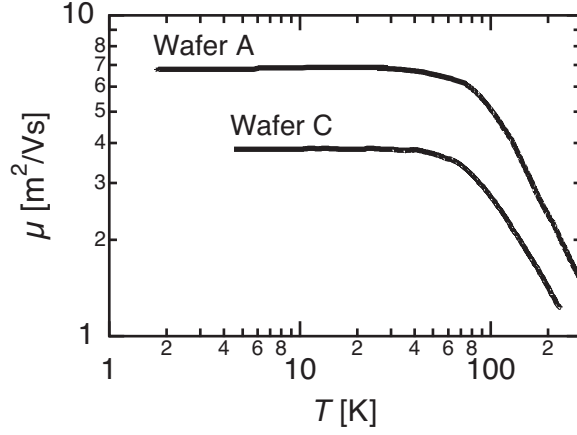


Figure 1.1: Temperature dependence of the electron mobility in wafers A and C.

for a set of electrons with an initial average phase ϕ , the evolution of ϕ with time is an exponential decay, and τ_ϕ is the characteristic time of this decay. In disordered systems, L_ϕ is associated to the time τ_ϕ using the relation $L_\phi = \sqrt{D\tau_\phi}$, where D is the diffusion coefficient.³ Elastic scattering does not affect phase coherence, in opposition to scattering by phonons, or electron-electron interactions (which is the dominant interaction at very low temperature).⁴ In real systems, L_ϕ can be very different from l_μ . L_ϕ and τ_ϕ are much more difficult to evaluate than l_μ and τ_μ in many cases. If the dimensions of a sample are smaller than L_ϕ , signatures of electron interferences appear in its conductance and in the other transport properties. Experimentally, these signatures are often considered as the hallmark of the mesoscopic transport regime.

L_ϕ and l_μ can have very different values depending on the materials and the temperature. Moreover, one, two or the three sample dimensions can be made (almost) arbitrarily small thanks to advances in processing techniques, so that many possibilities can be explored. It is worth focusing on two particular mesoscopic systems: (1) disordered metals where $\lambda_F < l_\mu < a_x, a_y, a_z < L_\phi$ (where λ_F is the Fermi wavelength, and $a_{x,y,z}$ are the

³ $D = v_F^2 \tau / d$, where v_F is the Fermi velocity, τ is a scattering time and d is the dimensionality.

⁴Note, however, that *all* inelastic collisions do not cause phase randomization, which means that τ_ϕ is not necessarily equal to τ_i .

typical dimensions of the system along the three principal directions) and (2) ballistic devices, where $a_z \lesssim \lambda_F < a_x, a_y < l_\mu < L_\phi$.

The transport properties of these two types of mesoscopic systems show deviations from classical predictions for macroscopic systems. At first, deviations can be caused by electron interference effects, which are most of the time averaged out in macroscopic samples. Historically, the main ‘mesoscopic interference effects’ (Aharonov-Bohm oscillations, Weak (Anti) Localization, Universal Conductance Fluctuations) introduced in section 1.1.5, were experimentally demonstrated in disordered systems. Several years later, equivalent effects were evidenced in ballistic systems. However, the microscopic description of interference effects is different in disordered and ballistic systems, as shown in section 1.2.

Additional corrections to the conductivity can emerge in ballistic systems subject to a magnetic field, due to ‘focusing’ trajectories between the measurement probes. Such focusing effects can be observed in relatively large samples: they were discovered in massive bismuth samples more than 30 years ago [147, 148, 90]. The phenomenology of electron focusing effects is introduced in section 1.1.3.

1.1.2 Ballistic systems: building blocks

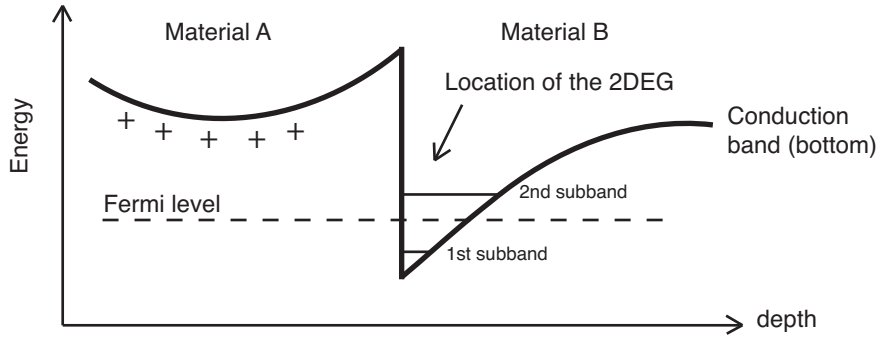


Figure 1.2: Conduction band profile in a typical semiconductor heterostructure. The bold line is the bottom of the conduction band as a function of the distance from the surface of the heterostructure. The dashed line indicates the Fermi level. (see also Ref. [47])

Two-dimensional electron gases (2DEGs) are among the most popular mesoscopic systems and their basic properties are very well documented

in many reference books (we refer, *e.g.*, to Ref. [47]). They form at heterointerfaces in layered or ‘sandwich’ semiconductor structures. Fig. 1.2 represents the energy diagram of the bottom of the conduction band in a typical heterostructure, as a function of the distance from the surface. There is a triangular-shaped well in this diagram at the interface between the two materials, originating from the large difference between their band gaps and the alignment of the Fermi levels in the two materials. Electrons from nearby impurities are transferred to the well where they form a thin conducting layer (the 2DEG), leaving the charged impurities behind them (symbolized by the ‘+’ on Fig. 1.2). Extremely high electron mobilities can be reached in 2DEGs, thanks to the spatial separation between the 2DEG and the impurities which can act as scattering centers, and to the very low level of defect in MBE-grown heterostructures. The confinement of the electron system to a very thin plane implies that electrons are distributed on a discrete number of energy levels, or subbands. In the case of Fig. 1.2, only the lowest subband is populated.

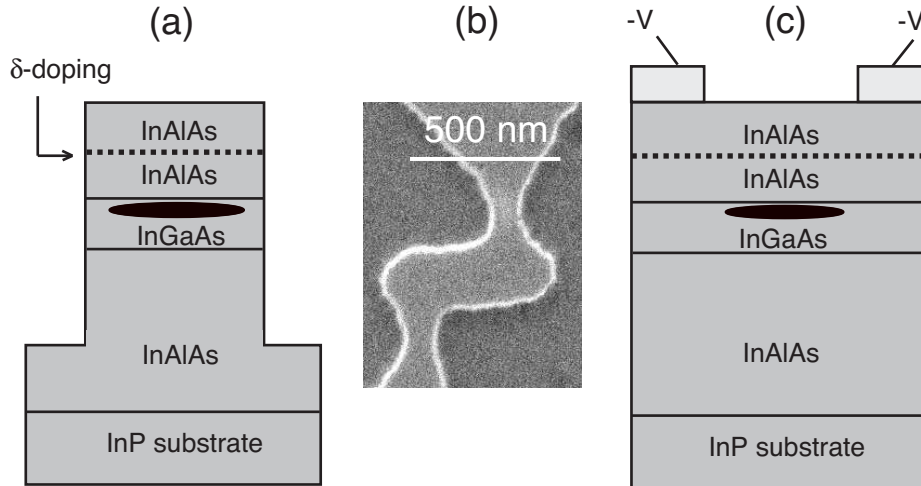


Figure 1.3: Two techniques to obtain a lateral confinement of a 2DEG (in black). (a) The etching technique (lateral view). (b) Top view (SEM micrograph) of an etched quantum dot (dark areas are etched). (c) the top gate technique, side view (gates are in light gray, on top of the structure).

Beside the exploration of the rich properties of 2DEGs, semiconductor heterostructures also made possible the investigation of the properties of one-

and zero-dimensional electron systems. Two main techniques were used to further confine the electron gas. The first one is illustrated in Fig. 1.3(a), and simply consists in removing a part of the material, so that the 2DEG is confined in the remaining part. At the sides of the confined structure, electrons are repelled due to surface charges (the depletion length is the size of this zone). The second technique [Fig. 1.3(c)] makes use of metal (Schottky) contacts, deposited on top of the heterostructure. A negative voltage is applied on the top gates in order to deplete the 2DEG underneath. The dimensions of the remaining 2DEG island can be tuned by varying the applied voltage. This is illustrated in Fig. 1.4, showing the conductance G of a narrow constriction (quantum point contact, or QPC) formed in a 2DEG as a function of the gate voltage V [152].⁵ The conductance shows plateaus at integer multiples of $2e^2/h$, which reflects the gradual depopulation of quantum channels as the width of the constriction decreases.

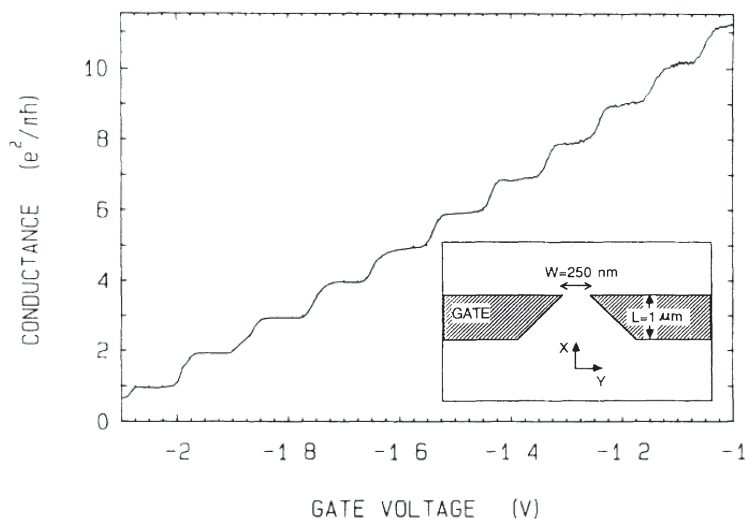


Figure 1.4: Conductance of a point contact as a function of the gate voltage. The inset shows a top view of the gates on top of the AlGaAs/GaAs heterostructure. [From van Wees *et al.* [152]]

⁵In this case, the critical length scale is the electron Fermi wavelength λ_F . When the thermal energy kT , where k is Boltzmann constant, and the level broadening $\Gamma = \hbar/\tau$ (where τ is a scattering time) are ‘sufficiently’ small (*i.e.*, smaller than the separation between energy levels in the constriction), the energy level quantization strongly affects the sample properties.

2DEGs and QPCs are the main building blocks of the ballistic quantum dots (QDs) studied in chapter 3,⁶ and of the cross junctions investigated in chapter 5. Fig. 1.3(b) shows a top-view of a quantum dot defined by the etching technique: it consists of an electron island separated from the electron reservoirs by two QPCs. The size of the dot is inferior to l_μ , so that it is in the ballistic regime. The dot is open, *i.e.* the QPCs carry at least one quantum mode.⁷

In the island, electrons are distributed on a limited number of energy levels,⁸ with an average spacing between the levels given by :

$$\Delta = \frac{2\pi\hbar^2}{m^*A}, \quad (1.1)$$

where A is the QD area and m^* is the electron effective mass. Importantly, we deduce from Eq. (1.1) that the average level spacing is larger in small QDs fabricated from materials with a small m^* .

Another important parameter characterizing a QD is the average time spent by electrons inside the cavity, the ‘dwell time’ τ_d . If trajectories inside the QD bounce many times before it escapes (no direct trajectories between the entrance and exit QPC), and are chaotic, general arguments based on ergodic theory can be used to evaluate τ_d , leading to the following estimation:⁹

$$\tau_d = \frac{2\pi\hbar}{\Delta N}, \quad (1.2)$$

where N is the *total* number of quantum channels in the QPCs (*i.e.* the sum of the number of modes in the QPCs). When the above-mentioned hypothesis are not fulfilled, Eq. (1.2) is only a rough approximation. Alternative expressions were derived by several authors for particular ‘non-ideal’ geometries (see *e.g.* Jensen [95]).

1.1.3 Focusing effects

In the ballistic regime, a magnetic field can be used to ‘focus’ electrons (or holes) between an injecting contact and a collecting contact. A scheme of a typical focusing experiment is shown in Fig. 1.5(a), and consists of a sample

⁶The Bi QD investigated in chapter 4, are not formed from semiconductor heterostructures. However, we will see that it can be considered as zero-dimensional.

⁷In ‘closed’ dots, the QPCs are tunneling barriers for electrons.

⁸Confinement in the QPCs is one-dimensional, and zero-dimensional in the quantum dot. For more informations, see Ref. [47].

⁹The estimation is based on the ratio of the area of the scattering domain to the electron flux through the QPCs [95].

with two contacts, labeled E and C, separated by a distance L . Current is injected in the sample at the contact E and the potential difference between contact C and another reference point on the sample (far from C) is measured as a function of the magnetic field B . The magnetic field is applied in the plane of the sample surface, perpendicular to the line joining E and C.

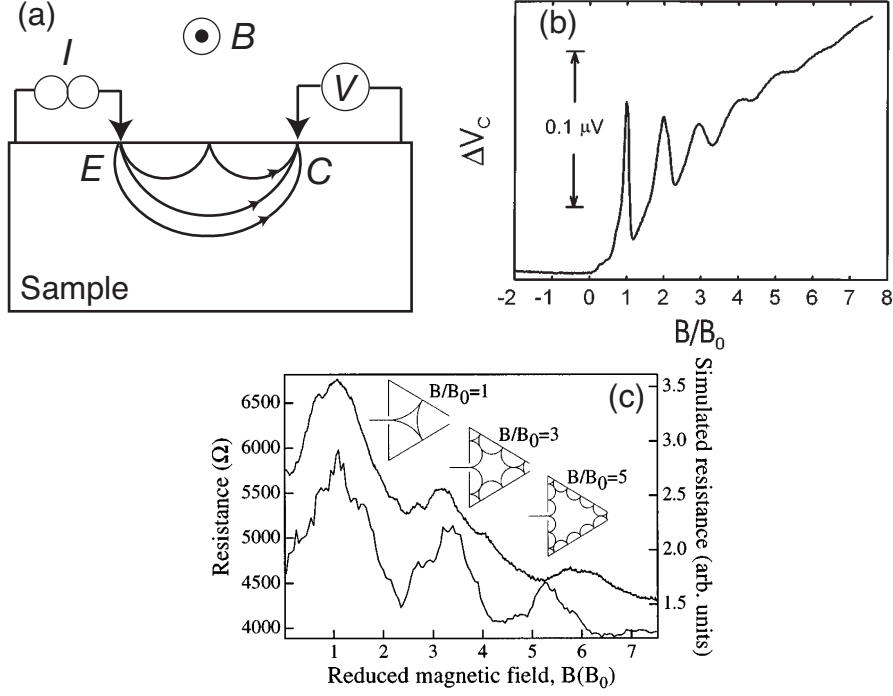


Figure 1.5: (a) Typical geometry for a focusing experiment. (b) Collector voltage as a function of the magnetic field (in units of B_0), in a Bi sample with the trigonal axis perpendicular to the surface ($T = 1.1$ K). (data from Ref. [90]). (c) Comparison of the measured magnetoresistance (top trace) of a triangular quantum dot and the result of a classical simulation of the resistance in the same geometry (from Ref. [106]).

The magnetic field causes a curvature of the charged particles trajectories, so that electrons (and holes) move along a circle, with a radius (called ‘cyclotron radius’) given, *in the case of a 2DEG*, by:

$$r_c = \frac{\hbar \sqrt{2\pi n_s}}{eB}, \quad (1.3)$$

where n_s is the surface electron density in the 2DEG.¹⁰ When the condition $2r_c = L$ is reached (corresponding to a magnetic field B_0), electrons injected in E are transmitted directly to the collector, provided that they are not scattered (*i.e.* transport is ballistic over the length of the trajectory). At $B = B_0$, a spike is observed in the collector voltage due to this effect. For $B = 2B_0$, electron follow trajectories with one reflection along the device edge. If this reflection is specular, electrons are brought to the collector, causing another spike in the collector voltage. Similarly, trajectories with n bounces are associated with spikes at $B = nB_0$, as shown on Fig. 1.5(a). Note that in Bi samples, spikes, or ‘commensurability oscillations’ associated with electrons *and* holes can be observed using this technique, but they appear for opposite signs of the magnetic field, since the charges of these particles are opposed. The above analysis, strictly valid in the case of electrons injected perpendicularly to the line E-C, still applies to the general case of an electron beam with an angular divergence.¹¹

Commensurability effects with a similar origin were evidenced in many different ballistic systems, including open quantum dots. Fig. 1.5(c) shows the result of Linke *et al.* [106], associating maxima in the magnetoresistance of triangular quantum dots to classical focusing electron orbits inside the QDs, very similar to the orbits on Fig. 1.5(a). Trajectories implying a reflection of the electron to its origin point contact dominate the magnetoresistance in this case.

The temperature dependence of the focusing peaks in InGaAs-based 2DEGs was studied in Ref. [73]. Focusing peaks were found to remain at high temperature (above 100 K) in these systems. Experimentally, the amplitude A_p of the peaks has been reported to vary exponentially with the ratio of the distance L between the probes (or, equivalently, with the trajectory length L_e) and a typical ‘focusing mean free path’ L_f [151, 75, 73, 72]:

$$A_p = A_0 \exp\left(\frac{-L_e}{L_f}\right) \quad (1.4)$$

where A_0 is a constant. This form was obtained by varying L [75]. As far as we know, there is no published theoretical justification for this form.

¹⁰The assumption of a circular electron motion in a magnetic field is not totally correct in real materials: actually, they move along orbits of constant energy on the Fermi surface; these trajectories are determined by the intersection of the Fermi surface with a plane perpendicular to the direction of the magnetic field. As an example, in the case of bismuth, this corresponds to ellipses (the Fermi surface for electrons and holes in Bi is shown on Fig. 2.8).

¹¹Support for this, and additional background informations on transverse focusing can be found in the review by Tsoi *et al.* [148].

However, intuitively, this exponential decay seems logical, as the probability for an electron to keep its initial momentum between the entrance and exit of the system (a cavity for example) decreases exponentially with the relevant scattering time in the Boltzmann formalism.

Strikingly, the amplitude A_p was also found to vary at low temperature, in a range where l_μ is temperature-independent. Based on this observation and Eq. (1.4), it is natural to consider that L_f is different from l_μ , and that L_f depends on the temperature even if l_μ is temperature-independent. The difference between the T -dependence of L_f and l_μ at low- T was attributed to a different sensitivity to small-angle scattering [75, 73, 72]. The mean free path l_μ is less affected by this type of scattering because the contribution of each scattering event in the reduction of l_μ is weighted by $(1 - \cos(\theta))$, where θ is the scattering angle. Focusing phenomena, on the contrary, are perturbed by small-angle scattering, and these scattering events are relatively more important at low temperature, since phonons have smaller momenta. In summary, both l_μ and L_f depend on elastic scattering and electron-phonon scattering. However, as L_f is more sensitive to small angle scattering, one has $L_f < l_\mu$ [75].

1.1.4 Chaos

One of the reasons for the interest in transport through ballistic cavities stems from the fact that they offer the possibility to study experimentally the predictions of the ‘quantum chaos’ theory. In classical dynamics, chaotic behavior is defined as an exponential sensitivity to initial conditions, which therefore makes any prediction on the system’s evolution almost impossible. Playing a classical pinball game in a macroscopic billiard, it is quite easy for a skilled player to predict the trajectory of the ball, taking into account the initial conditions of the shot, and also to repeat several time the same shot. In a stadium-shaped billiard, however, this is much more difficult to achieve, because very small changes in the initial conditions dramatically change the ball’s trajectory. In classical systems, these simple arguments show that the shape of the billiard fully determines whether the ball will move in a chaotic or a regular way.

What happens if the billiard is scaled down and the billiard balls are replaced by electrons, as in ballistic quantum dots fabricated from 2DEGs ? Strangely, even in quantum dots whose shape is classically chaotic, both numerical calculations [4] and recent experiments with a scanning probe [44] provided evidences that ‘scars’ of some particular periodic classical orbits dominate transport. These scars manifest themselves as highly structured

‘scarred’ electron wavefunctions. This implies that statistical predictions based on an ergodic hypothesis¹² may fail in the case of quantum systems whose classical analog is chaotic. It also means that the most common situation in quantum dots is a mixed (chaotic and regular) phase space.

Classically chaotic and regular cavities have different distributions of escape times τ , trajectory lengths L and effective areas Θ .¹³ In the case of chaotic dynamics, the distributions are exponential, each governed by a single parameter [92, 91, 95]:

$$P(\tau) \propto \exp(-\gamma\tau); P(L) \propto \exp(-\gamma_{cl}L); P(\Theta) \propto \exp(-\alpha_{cl}|\Theta|), \quad (1.5)$$

where $\gamma_{cl} = \gamma/v$, γ is the escape rate from the cavity (inverse of the typical escape time), v is the velocity of the particles and α_{cl} is the inverse of a typical effective area enclosed by a trajectory. On the contrary, in regular cavities, the distributions of τ , L and Θ are power laws. γ is determined by the difference between the mean Lyapunov exponent λ (characterizing the exponential rate at which trajectories near the stable periodic orbits diverge from them), and the rate at which chaotic escaping trajectories are folded back into the scattering region (the Kolmogorov-Sinai entropy $h_{KS} = \lambda d$ [95], where d is the fractal dimension of the ‘repeller’, *i.e.* the set of unstable periodic orbits)[91, 95]: $\gamma = \lambda(1 - d)$.

1.1.5 Quantum interferometry in ballistic systems (phenomenological approach)

Aharonov-Bohm oscillations

Aharonov-Bohm (AB) oscillations can be viewed as the archetype of the quantum interference effect. Although AB oscillations are not investigated in this thesis, a simplified explanation of this effect is a very illustrative introduction to the other manifestations of the wave nature of electrons in the transport properties of mesoscopic samples. Consider the interferometer shown in Fig. 1.6(a), constituted by a ring with two openings, fabricated in a semiconductor heterostructure hosting a 2DEG. Electrons enter in the ring from the contact at the left, divide into partial waves each going around an arm of the ring, recombine and interfere on the right, before leaving through

¹²This hypothesis means that electrons ‘visit every possible places in the cavity’ (this is obviously a very informal definition).

¹³As scattering trajectories are open, the enclosed area is replaced by an effective area, defined from a path integral on the vector potential. For a trajectory s , path C_s with a perpendicular magnetic field B with a vector potential A , one has $\Theta_s = \frac{2\pi}{B} \int_{C_s} A dr$.

the right contact. A magnetic field B is applied to the ring. The vector potential $\mathbf{A}$ associated with B induces a change in the phase of electrons. The phases accumulated by electrons going through the upper and lower arm (ϕ_1 and ϕ_2 , respectively) due (only) to the presence of the magnetic field are given by the integrals over the electron paths :

$$\phi_1 = \frac{e}{\hbar} \int_{\text{upper paths}} \mathbf{A} d\mathbf{l}, \text{ and } \phi_2 = \frac{e}{\hbar} \int_{\text{lower paths}} \mathbf{A} d\mathbf{l}. \quad (1.6)$$

The difference in phase is given by $\Delta\phi = \phi_1 - \phi_2 = 2\pi e\Phi/h$ (where Φ is the magnetic flux). Interference of electron partial waves at the right junction, governed by $\Delta\phi$, will be constructive when Φ is a multiple of h/e and destructive halfway between. As a consequence, the transmission of electrons from the left to the right lead (and therefore the conductance or resistance of the sample) will periodically oscillate as a function of the magnetic field, as shown on Fig. 1.6(b).

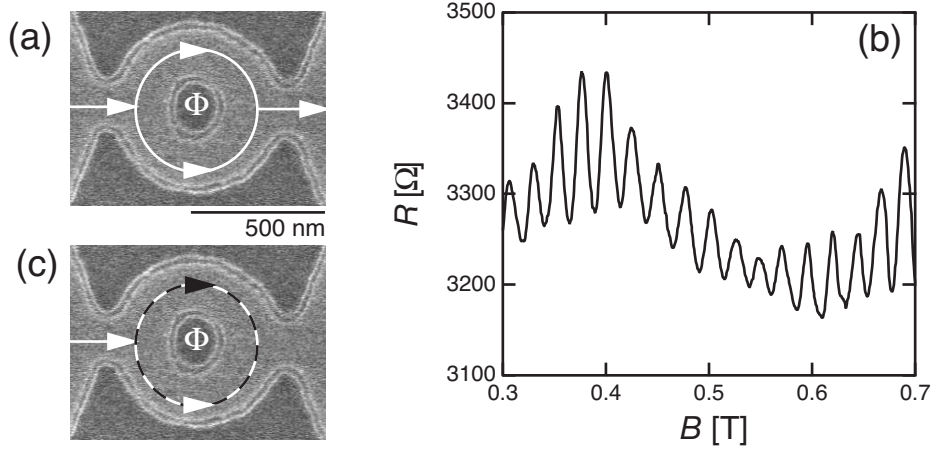


Figure 1.6: (a) SEM micrograph of a quantum ring patterned from an In-GaAs/InAlAs heterojunction (wafer C). Dark regions have been etched. In white: schematic path of electron partial waves transmitted through the device, giving rise to h/e oscillations. (b) Resistance R vs magnetic field B at 1.7 K in the quantum ring shown in (a). (c) In black and white: schematic paths of two electron partial waves, giving rise to $h/2e$ oscillations.

Electron partial waves, instead of being transmitted through the ring, can also travel along the whole ring before interfering at the junction near

the incoming lead (as illustrated in Fig. 1.6(c)). Such trajectories are time-reversed trajectories. Compared to the half-ring trajectories discussed above, these trajectories enclose twice as much magnetic flux, which implies that the maxima of the transmission will occur for $\Phi = nh/2e$ (n integer). The associated $h/2e$ resistance oscillations are known as Al'tshuler-Aronov-Spivak (AAS) oscillations [9, 10]. h/e and $h/2e$ effects differ by their behaviour at $B = 0$. Indeed, if there is a small difference between the length of the two arms of the ring, it may happen that $\Delta\phi$ is not a multiple of 2π at $B = 0$. In this case, the h/e effect does not lead to a maximum transmission (minimum resistance) at $B = 0$. However, the AAS effect is not affected by the geometrical asymmetry, and hence always leads to a maximum backscattering (maximum resistance) at $B = 0$.

Ballistic transport is not a necessary condition for the observation of these effects. Historically, they were first evidenced in disordered systems: cylindrical metal films for $h/2e$ oscillations [136], and individual Au rings for h/e oscillations [155]. The only condition required to observe the AB or AAS oscillations is that L_ϕ is longer than the minimum path length L through the ring. More precisely, the amplitude of the oscillations is proportional to $\exp(-L/L_\phi)$.

Universal Conductance Fluctuations

The shape of the device shown in Fig. 1.7(a) is very similar to that of the quantum ring discussed above, except for the central part: there is no scatterer at the center of the device. As a consequence, there is a much larger set of possible ‘classical paths’ for electron partial waves between the two openings of the device. A different phase is accumulated for each path. In a semiclassical approach (see section 1.2.3) it is customary to isolate pairs of paths, as illustrated in Fig. 1.7(a). Each pair will generate interferences leading to oscillations in the magnetoconductance with a particular period (specific to each pair). In the magnetoconductance (G vs B) of the sample, the sum of all the particular oscillations with different period will sum up, yielding a complex pattern of ‘aperiodic’, but reproducible, conductance fluctuations shown in Fig. 1.7(b). In quantum dots, fluctuations not only show up as a response to changes in magnetic field, but can also be generated by changes of the shape of the cavity, or of the Fermi energy [21, 97].

In quantum dots, these fluctuations are called Universal Conductance Fluctuations (UCFs), because their amplitude is universal, *i.e.* independent of the system size and shape, provided that the underlying classical dynamics is chaotic (see section 1.1.4). It is worth recalling that the ‘generic’ situation

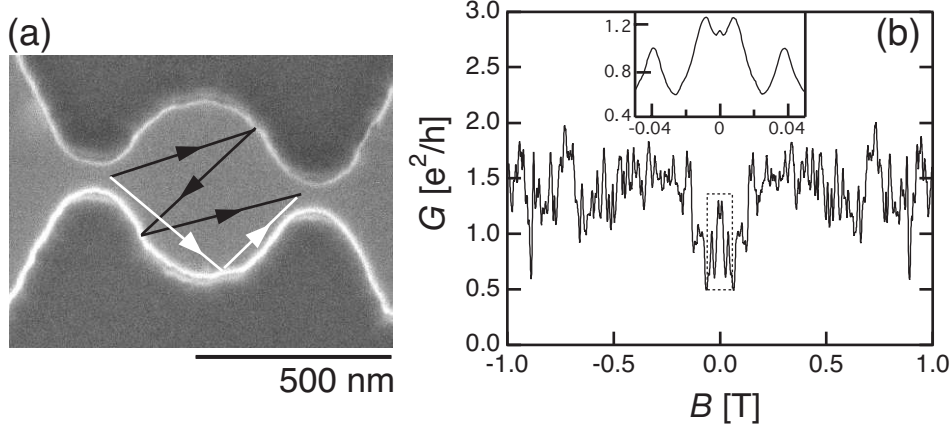


Figure 1.7: (a) SEM micrograph of a quantum billiard (sample B_1) patterned from an InGaAs/InAlAs heterojunction. Dark regions have been etched. In black and white: schematic paths of electron partial waves transmitted through the device, giving rise to Universal Conductance Fluctuations. (b) Conductance G vs B at 0.3 K in sample B_1 , showing UCFs. The inset shows G vs B at very low B .

in quantum dots is a mixed regular-chaotic phase space, with a power-law escape from the cavity. In this case theory predicts that the fluctuations of conductance are statistically self-similar, so that they are characterized by a fractal dimension [98]. This was experimentally confirmed soon after the theory was published [146, 134, 145, 133, 119].

The term UCFs was first used for conductance fluctuations observed in large disordered systems. In these systems, the contribution of a very large number of randomly scattered phase coherent loops ensemble averages to produce conductance fluctuations that are always of the order of e^2/h in amplitude in the limit $T \rightarrow 0$, irrespective of the degree of disorder of the sample [103]. The specific microscopic details of the sample determines the specific pattern of fluctuations but not their mean amplitude. While the mechanism giving rise to the fluctuations is similar in quantum dots, there is a difference: a smaller number of phase coherent loops contribute to the effect, and the fluctuations do not have a ‘universal’ mean amplitude of $\sim e^2/h$ (see section 1.2.4). Therefore, the term ‘UCFs’ is somewhat abusive in the case of quantum dots, but, to maintain consistency with common

usage, we will use it for QDs in the following.¹⁴

Both for disordered and ballistic devices, statistical methods are needed to describe UCFs properties. Indeed, the details of an individual measurement for a given set of experimental parameters are too complicated to be predicted. The calculation details are somewhat different in disordered and ballistic systems, due to the different nature of scattering in both cases (see section 1.2).

Weak Localization

As in quantum rings, some pairs of trajectories within billiards lead to a coherent backscattering of electrons. An example of such a pair of trajectories is shown in Fig. 1.8(a). The black and white paths accumulate the same phase shift and therefore interfere constructively at their origin. The difference with the quantum ring situation lies in the larger number of such pairs in a billiard. The effect of the magnetic field on each of these pairs is to break the time reversal symmetry : the paired paths accumulate a different phase, so that backscattering is no longer coherent. As a consequence, one observes a minimum of conductance (maximum of resistance) at $B = 0$, one of the signatures of the Weak Localization (WL) effect.¹⁵

Such a WL minimum can be observed in Fig. 1.7. However, in this sample, the effect is mixed with UCFs, and both WL and UCFs have a similar amplitude. In billiards whose shape can be controlled by electrostatic gates, it is possible to separate the WL feature from UCFs in G vs B . The idea is to measure G vs B for several values of voltage V_g applied to a shape-distorting gate. The pattern of UCFs will be different for each value of V_g , but the WL feature will remain the same in each curve. Therefore, averaging over a sufficient number of such curves will leave only the WL correction to G vs B , as illustrated in Fig. 1.8(b-c).

Due to the finite coherence length in realistic samples at nonzero temperature, coherence is broken for the longest pairs of paths, which reduces the WL effect, as well as the UCFs amplitude (we will see below that some other UCFs properties are affected by the limited coherence). This sensitivity to L_ϕ (or τ_ϕ) makes WL and UCFs two very useful tools to evaluate L_ϕ and τ_ϕ and their evolution with temperature, both in ballistic and disordered

¹⁴There is a similar problem for the Weak Localization effect discussed hereafter.

¹⁵The weak localization concept is not restricted to the ‘small world’ of mesoscopic systems. As an example, weak localization was recently evidenced for seismic waves in a natural environment [101].

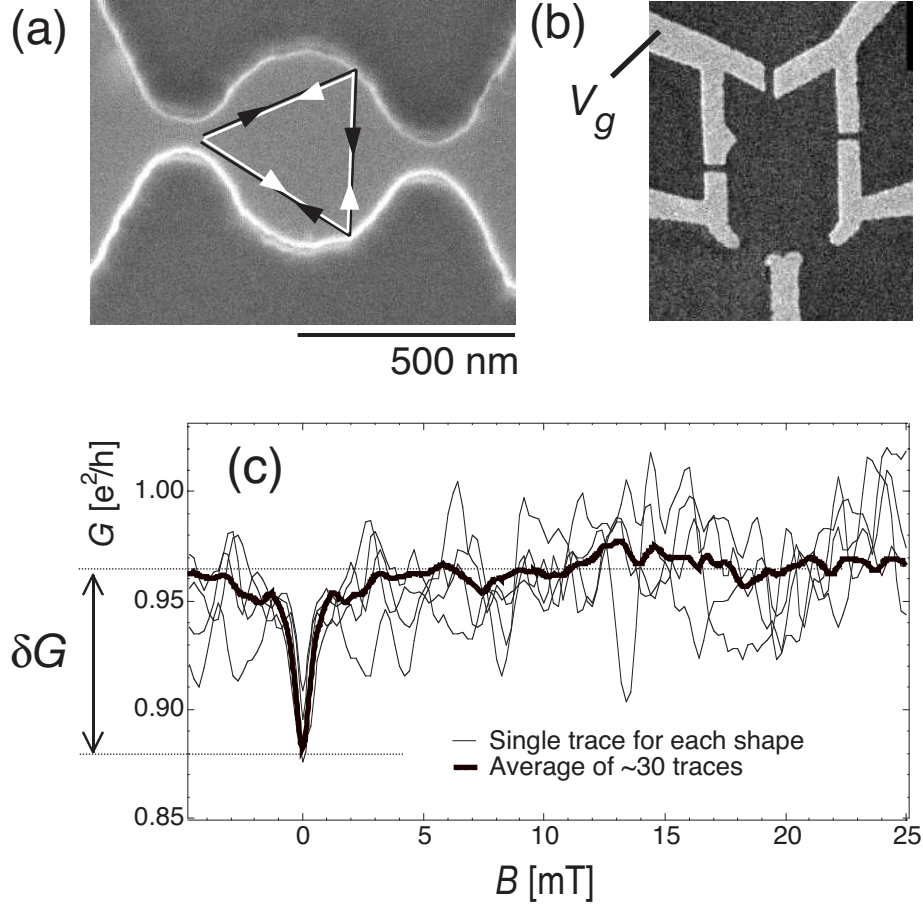


Figure 1.8: (a) SEM micrograph of sample B₁, with schematic paths of two electron partial waves (black and white), leading to coherent backscattering. (b) SEM micrograph of a quantum billiard defined by metal gates (light gray) on top of an AlGaAs/GaAs heterostructure. (c) Thin curves : G vs B in the cavity shown in (b), for different values of V_g . The bold curve is a shape-averaged G vs B [(b) and (c) from Ref. [83]].

systems.¹⁶

Spin and Spin-Orbit coupling effects

Beside its phase and charge, an electron also carries a spin. Recently, there has been a rising interest in the control of the spin degree of freedom in mesoscopic system, initially motivated by the possible realization of ‘spin transistors’ and spin filters [46].¹⁷ An important ingredient in the control of the spin is the spin-orbit (SO) coupling. The SO interaction is a relativistic effect, arising when an electron is moving in an effective electric field. This electric field has different contributions. One of these contributions, the ‘Dresselhaus term’ in the Hamiltonian of the system [49], originates from the asymmetry of the crystal structures (*e.g.* in the zinc-blend structure of III-V compounds, such as GaAs). In heterostructures, the lack of inversion symmetry is enhanced at interfaces, which is the source of an electric field perpendicular to the heterostructure interface contributing to the SO effect, as evidenced by Rashba [33, 131]. Moreover, heavy atoms usually have a strong contribution to the SO effect.¹⁸ As a result of the presence of these SO terms in the hamiltonian, spin-up and spin-down energy levels are split (lifted spin degeneracy).

Both UCFs and the weak localization effect can be modified in the case of strong SO coupling. Folk and coworkers recently uncovered the first evidence for a modification of the UCFs amplitude related to spin effects in open ballistic quantum dots fabricated from AlGaAs/GaAs heterostructures [55]. They showed that the amplitude of UCFs is reduced by a factor of 1.9-5.5 in the presence of a strong in-plane magnetic field (measurements performed in our group on similar samples revealed a reduction factor in the range ~ 3 -10 [68]). This observation was explained in terms of Zeeman splitting and enhanced SO interaction at high B [70], each contribution reducing the UCFs amplitude by a factor of 2 with respect to UCFs amplitude at low B .

Conventionally, SO coupling manifests itself in G vs B curves around $B = 0$, in the form of a small positive correction to G suppressed by a magnetic field: the weak antilocalization (WAL) effect (the counterpart of weak

¹⁶Note that, in disordered systems, the observation of the WL effect does not require that the size of the system is inferior to L_ϕ . However, in general, the effect is enhanced in low-dimensional systems.

¹⁷In such devices, the current is modulated by a spin precession due to the spin-orbit coupling.

¹⁸The strength of the SO coupling is approximately proportional to Z^4 , where Z is the atomic number. A detailed investigation of the dependence of SO coupling on Z can be found, *e.g.*, in Ref. [60].

localization). Naturally, one can wonder why weak localization is modified into weak antilocalization in the presence of spin-orbit coupling. This was intuitively explained by Bergmann [20]¹⁹ in the context of disordered thin metal films. An electron has its spin rotated during each scattering event in the presence of SO coupling. Starting from a spin state s , two complementary scattering series will result in two different final rotated spin $s' = T_r s$ and $s'' = T_r^{-1} s$ (T_r being a ‘rotation operator’). It can be shown that if s' and s'' are related by a spin rotation of 2π , the interference of the complementary electron wavefunctions is destructive. Theory shows that such destructive interferences predominates when SO coupling is strong, leading to the opposite of the backscattering effect observed in the absence of spin rotation. Similar to the WL effect, this ‘forward scattering’ effect is destroyed in the presence of a magnetic field, so that a maximum is observed at $B = 0$ in the case of strong SO coupling. The theory of weak antilocalization in disordered systems was established by Hikami, Larkin and Nagaoka [77]. Fitting G vs B around $B = 0$ with this theory allows to extract a new length scale, L_{so} , corresponding to the spin-orbit scattering length. Note that this length and the associated time τ_{so} ($L_{so} = \sqrt{D\tau_{so}}$) are temperature independent.

While the ‘ballistic weak localization’ was observed in quantum dots in the early 90s, ‘ballistic weak antilocalization’ was not observed until very recently. The effect was first experimentally demonstrated in GaAs/AlGaAs quantum dots by Zumbuhl and coworkers [158], closely followed by our observation of weak antilocalization in a quasi-ballistic Bi quantum dot [69] (cfr. chapter 4). Most importantly, in GaAs/AlGaAs quantum dots, WAL can be controlled by an external gate voltage and is suppressed in dots with a lateral size inferior to L_{SO} , as recently observed [158].

1.2 Calculation tools

1.2.1 Disordered and ballistic systems

In disordered metals, electrons are scattered by defects of atomic dimensions, and propagate in a semiclassical way between scattering events. The quantum interference effects in such systems were treated within the framework of perturbation theory for the disorder, with the conductivity obtained as a current-current correlation function (Kubo formula) [20, 102]. One of the most important successes in this field is the very general Hikami for-

¹⁹This article is also the reference for an intuitive explanation of weak localization in disordered systems.

mula [77] [Eq. (4.6) in section 4.6], expressing both the temperature and magnetic field dependence of the low- B conductance of low-dimensional disordered metals, taking into account all the scattering mechanisms, and its remarkable experimental confirmation in many different systems.

The theoretical treatment of quantum transport in ballistic systems requires specific calculation tools, slightly different from those used in disordered systems. Indeed, classical trajectories solely depend on the geometry, and the notion of impurity average does not have any sense: it must be replaced by an average over the shape or the energy. Furthermore, scattering events do not have a random outcome as in the diffusive case. Therefore, existing tools had to be adapted, or new methods had to be created to understand the statistical features of the quantum interference phenomena in ballistic samples.

The purpose of this section is to summarize the approach of two of these tools, the semiclassical (SC) approach and the random matrix theory (RMT), without omitting the assumptions of the underlying models. Most of the data in chapters 3 and 4 were analyzed using the results of these theories. Some other models exist, such as the supersymmetric approach [52], whose predictions are in agreement with the results of the SC theory and the RMT, for the properties of interest in this work. Here, we will keep the formal (mathematical) aspects at the lowest possible level, since this thesis is primarily experimental. We will also emphasize the main results obtained using the different theoretical approaches, and examine some of their consequences. A pedagogical presentation of the semiclassical method and its applications is given by Jalabert in Ref. [91] and the RMT was reviewed by Beenakker in [19]. Alhassid [6] presents the main statistical methods and their applications with an emphasis on closed quantum dots.

1.2.2 Scattering formalism

In the Landauer-Büttiker approach [30, 45, 91], a typical open quantum dot sample in the phase coherent and ballistic regime is represented as shown in Fig. 1.9. The whole sample is composed of three main elements: electron reservoirs, leads, and the cavity itself. Physically, the two electron reservoirs are the measuring devices, and are characterized by chemical potential μ_1 and μ_2 (which are not affected by incoming or outgoing electrons), with $\mu_1 - \mu_2 = eV$. The leads, connecting the cavity to the reservoirs, can be approximated as having a finite width W in the y direction and being infinite in the x direction, so that a finite number of channels propagate into them. The eigenstates in the leads are the product of the wavefunctions

$\phi_a = \sqrt{2/W} \sin(\pi a y/W)$ (a integer) and plane waves of the type $e^{\pm i k_a x}$ with wavevectors k_a satisfying $\epsilon = [(a\pi/W)^2 + k_a^2] \hbar^2 / (2m^*)$. There are M transverse momenta satisfying the latter relation with $k_a^2 > 0$, and therefore a total of $2M$ propagating channels (incoming and outgoing lead states).

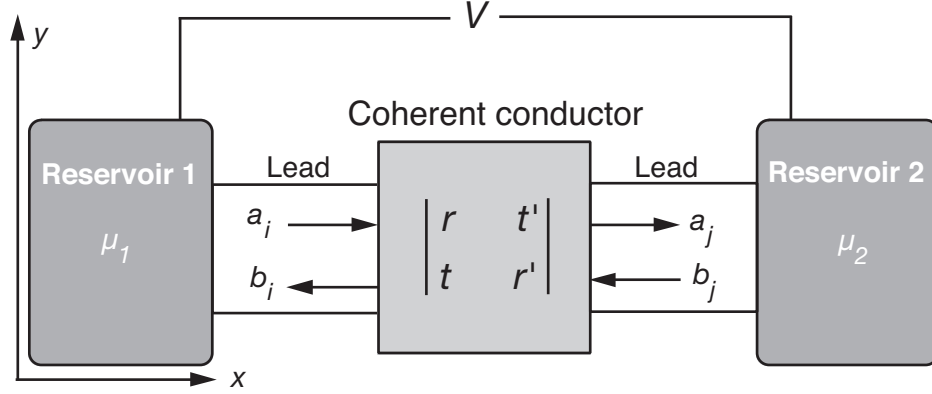


Figure 1.9: Scheme of a coherent conductor related by leads to electron reservoirs. The coherent part of the device is characterized by the S-matrix, relating the incoming mode amplitudes a_i to the outgoing amplitudes b_i .

Electrons incoming from the leads are scattered inside the cavity. Incoming and outgoing mode amplitudes are related by a $2M \times 2M$ scattering matrix (or S-matrix) composed by $M \times M$ reflection and transmission matrices r and t (r' and t'):²⁰

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}. \quad (1.7)$$

Transmission and reflection coefficients ($\mathcal{T}_{ba}$ and $\mathcal{R}_{ba}$) are obtained from individual elements of the reflection and transmission matrices (t_{ba}, r_{ba}, t'_{ba} and r'_{ba}): $\mathcal{T}_{ba} = |t_{ba}|^2$ and $\mathcal{R}_{ba} = |r_{ba}|^2$. The total transmission $\mathcal{T} = \sum_{a,b} \mathcal{T}_{ba}$ and reflection $\mathcal{R} = \sum_{a,b} \mathcal{R}_{ba}$ coefficients obey $\mathcal{T} + \mathcal{R} = N$ and $\mathcal{T} = \mathcal{T}'$, $\mathcal{R} = \mathcal{R}'$ due to unitarity of the S-matrix. S-matrices can be combined in the case of complex or large samples (see Ref. [45]), which makes this formalism very convenient and flexible. The critical point, however, is the evaluation of the

²⁰ r and r' correspond to reflection from the left lead back to itself and from the right lead back to itself, respectively; t and t' represent respectively transmission from the left to the right lead and conversely.

individual matrix elements. This can be achieved using an integration over Green functions (Eq. (4) in ref. [92]).²¹

Assuming that $\mu_1 > \mu_2$, electrons with an energy in the interval $[\mu_1, \mu_2]$ are injected into right-going states from the left reservoir, but no electrons in this energy range are injected from the right reservoir. Therefore, the net current I is proportional to the number of states in the interval $[\mu_1, \mu_2]$:

$$I = 2e \sum_{a=1}^N v_a \frac{dn_a}{d\epsilon} eV \sum_{b=1}^N \mathcal{T}_{ba} = \frac{2e^2}{h} V \sum_{a,b=1}^N \mathcal{T}_{ba}, \quad (1.8)$$

where N is the number of modes at the energy μ_1 and the one-dimensional density of states $\frac{dn_a}{d\epsilon} = 1/hv_a$. The conductance is therefore $G = I/V = (2e^2/h)\mathcal{T}$. The scattering formalism is a very convenient and useful way to express physical quantities in terms of transmission or reflection coefficients.

1.2.3 Semiclassical theory

The semiclassical approach is a powerful tool to relate the classical and quantum properties of ballistic systems. In this framework, the Green function, critical for the evaluation of the coefficients of the scattering matrix, is approximated as a sum where each term corresponds to a trajectory s [91]. Each term of the sum is also weighted by a stability prefactor (depending on the incoming and outgoing angles), and contributes to a phase given by the classical ‘action integral’ $S_s = \int_{C_s} \mathbf{p} \cdot d\mathbf{q}$ (where C_s is a path). Assuming that the dynamics inside the cavity is classically chaotic, the elements of the transmission and reflection matrices are also expressed as a sum over isolated trajectories (Eq. (5) in Ref. [92]), and the transmission coefficients are given by sums over *pairs* of trajectories.

Using these new expressions for reflection and transmission coefficients, it is then possible to predict statistical properties of the magnetoconductance features discussed in section 1.1.5. Of particular interest for us is the magnetic field correlation function $C(\Delta B) = \langle \delta G(B + \Delta B) \delta G(B) \rangle$, which can be expressed as a function of the transmission coefficients. Suitable assumptions about the sum over paths (*e.g.*, the sum is replaced by an integration over injection angles, which have an exponential distribution) allow

²¹The Green function is a very useful tool whenever a response R is related to an excitation X by a differential operator D_{op} : $D_{op}R = X$. The Green function is defined as $G = D_{op}^{-1}$ so that $R = D_{op}^{-1}X = GX$. It has an exact expression in the leads, which can be used as a starting point for a recursive evaluation in the cavity. More details on the evaluation of this function and its use in mesoscopic physics can be found in [45]. The relation between t_{ab} and the Green functions is given by Eq. (2.8) in Ref. [91].

to obtain the following expression [92]:

$$C(\Delta B) = \frac{C(0)}{[1 + (\Delta B / \alpha_{cl} \phi_0)^2]^2}, \quad (1.9)$$

where α_{cl}^{-1} is the rms area enclosed by trajectories inside the cavity, and ϕ_0 is the flux quantum. The corresponding power spectrum S_G (Fourier transform of the correlation function) as a function of the frequency f , in T^{-1} , is :

$$S_G(f) = S_G(0) [1 + 2\pi \alpha_{cl} \phi_0 f] e^{-2\pi \alpha_{cl} \phi_0 f}. \quad (1.10)$$

Another important result of the semiclassical approach concerns the weak localization correction to the average conductance, whose lineshape is predicted to be lorentzian [16]:²²

$$\langle G(B) \rangle = \langle G(B) \rangle_{B \neq 0} - \frac{\delta G}{1 + (2B / \alpha_{cl} \phi_0)^2}, \quad (1.11)$$

where δG is the amplitude of the weak localization dip. The characteristic field required to break the weak localization effect is therefore $\alpha_{cl} \phi_0$. We emphasize that the results (1.9), (1.10) and (1.11) are valid provided that the classical dynamics inside the cavity is chaotic in the sense explained in section 1.1.4. In the case of a regular dot shape, or non-ergodic trajectories, these results can be altered, as shown experimentally by Chang et al. [34]: while a lorentzian shape was found for the WL feature in chaotic quantum dots, as predicted by the semiclassical theory, a triangular shape was observed in non-chaotic cavities. The predictions for the power spectrum [Eq. (1.10)] was successfully checked experimentally by Marcus and collaborators [112]. An enhancement of the high- f spectral content was observed in integrable cavities.

1.2.4 Random Matrix Theory

Random Matrix Theory (RMT) was originally introduced in the 50's to describe the fluctuation properties of the neutron resonances in the compound nuclei. RMT is justified when there is a lack of knowledge about the exact Hamiltonian of a system, as in mesoscopic systems (due to the too large number of parameters involved). Within the RMT, two approaches can be followed [6]: (i) the Hamiltonian approach, where the Hamiltonian of the system is assumed to be 'random', but belongs to one of the three 'Gaussian'

²²Based on similar arguments as above, using the expression of reflection coefficients

ensembles of matrices obeying the particular time-space symmetries of the problem; (ii) the S-matrix approach, where the S-matrix [Eq. (1.7)] itself is assumed to be random, but belongs to a certain ensemble of matrices (Dyson's circular ensembles). In the latter case, when time-reversal symmetry is broken (*i.e.* when $B \neq 0$), the appropriate ensemble is the CUE (circular unitary ensemble, $\beta = 2$); when $B = 0$ the ensemble is the COE (circular orthogonal ensemble, $\beta = 1$) and in the case of a strong spin-orbit scattering and $B = 0$, the ensemble is the symplectic ensemble ($\beta = 4$). Support for using these random matrices to describe scattering in quantum chaotic systems was given by Blümel and Smilansky in Ref. [24].

RMT was first applied in the context of ballistic open quantum dots by Baranger and Mello [17], and by Jalabert, Pichard and Beenakker [94]. Assuming that M is the number of modes in each lead,²³ they found the expression for the average conductance and the variance of UCFs for $\beta = 1$ ($B = 0$):

$$\langle G \rangle = \frac{2M}{2M+1} \frac{e^2}{h}, \quad (1.12)$$

$$\text{var}(G) = \frac{4M(M+1)^2}{(2M+1)^2(2M+3)} \frac{e^2}{h}, \quad (1.13)$$

and for $\beta = 2$:

$$\langle G \rangle = M \frac{e^2}{h}, \quad (1.14)$$

$$\text{var}(G) = \frac{M^2}{4M^2-1} \frac{e^2}{h}. \quad (1.15)$$

Eq. (1.14) implies that the number of modes can be inferred from $\langle G \rangle$ when time-reversal symmetry is broken. The weak localization correction is the difference between $\langle G \rangle$ for $\beta = 2$ and $\beta = 1$. Moreover, the RMT also allows to compute the full distribution of conductance $P(G)$ (see [94, 17]).

Phase (de)coherence is not included in the above expressions. Within RMT, decoherence can be taken into account using the voltage probe model of Büttiker [30, 31]: a third 'fictitious' voltage probe is attached to the quantum dot with a voltage adjusted to keep a null current in the lead. Electrons escape from the dot through this voltage probe at a rate $\Gamma_\phi = 1/\tau_\phi$ and are reinjected into the quantum dot without phase memory. The effective

²³Note that in chapters 3 and 4, we often speak about the total number of modes N in both QPCs, *i.e.* $N = 2M$.

number of modes in the lead is $N_\phi = 2\pi\hbar/\tau_\phi\Delta$. This model was applied to conductance fluctuations in open quantum dots by Baranger and Mello [18] and by Brouwer and Beenakker [25]. The weak localization correction can be approximated by the following formula in the case of large M :

$$\langle\delta G\rangle \simeq \frac{M}{2M + N_\phi} \frac{e^2}{h} \quad (1.16)$$

and the variance ($\beta = 2$):

$$\text{var}(G) \simeq \frac{M^2}{(\sqrt{4M^2 - 1} + N_\phi)^2}. \quad (1.17)$$

Eq. (1.16) and (1.17) therefore describe the reduction in amplitude of the WL effect and of UCFs, due to finite coherence time. The drawback of this model is to concentrate the loss of phase coherence at the QPC between the quantum dot and the fictitious voltage probe. This was corrected to account for dephasing occurring within the whole dot by Brouwer and Beenakker [26, 27] (distributed ϕ -lead model).

Furthermore, at $T \neq 0$, electrons are spread out in energy (thermal smearing). The conductance is therefore obtained by averaging over energy, with a weighting given by the Fermi envelope ($-f'(E)$, the derivative of the Fermi function with respect to the energy E) [83]:

$$G = - \int f'(E) G(E) dE. \quad (1.18)$$

The weak localization effect is not affected by thermal averaging. Therefore, Eq. (1.16) can be used for a direct conversion of the measured $\langle\delta G\rangle$ into τ_ϕ . This procedure was used to extract τ_ϕ in GaAs/AlGaAs quantum dots by Huibers *et al.* [86].

On the other hand, thermal smearing has an effect on the variance of the conductance. It must therefore be evaluated by integrating over the conductance correlator, as detailed in section 3.4.1, except for $\Delta \ll kT$. In the latter case, the integration can be well approximated by a simpler expression (see Ref. [83]).

The RMT theory of quantum transport has also been refined to include the effect of non-ideal leads. Generalized formulas for the WL correction and the variance of UCFs in the case of leads with a transmission probability $0 \leq \Gamma_n \leq 1$ for each mode n in the leads were derived in Ref. [26] (see also section 4.5).

The range of application of RMT in low-dimensionnal systems was very recently extended to account for spin-dependent phenomena, including the effect of a spin-orbit coupling and a spin-degeneracy-lifting magnetic field in the plane of the quantum dot. Specifically, Cremers and coworkers treated the case of electrons in a quantum well formed in a zinc-blend type semiconductor, with a Rashba and a Dresselhaus term in the Hamiltonian [42]. In this case, the very general expressions Eq. (21) and (29) in this reference give the zero-field correction to the conductance and the conductance correlator, respectively. As above, a numerical integration of the conductance correlator weighted by a product of Fermi envelopes allows to evaluate the variance of UCFs.

At the end of this section, it is worth recalling some assumptions of the RMT in the framework of ballistic quantum dots : (i) most RMT formulas are valid only for large M ; (ii) the classical dynamics of particles inside the cavity must be chaotic; (iii) the ergodic time²⁴ should be larger than the dwell time τ_d . Nevertheless, even in quantum dots with a small total number of modes, τ_ϕ could be successfully determined from RMT methods [86]. Moreover, as noted above, it is not straightforward to assess the chaotic or integrable character of a quantum dot: the generic situation is a mixed phase space, with partially chaotic and regular behavior.

In addition to the initial constraints (i-iii), several authors pointed out particular situations leading to corrections to the conventional RMT. Alves and Lewenkopf recently demonstrated that in the case of large τ_ϕ ($N_\phi \ll 1$), there is an ambiguity in the method to extract τ_ϕ from the weak localization effect, leading to an underestimation of τ_ϕ [11]. The ambiguity comes from the large sensitivity of the extracted τ_ϕ to the transmission through the QPCs (even for an almost perfect transmission). Moreover, the role of an additional timescale, the Ehrenfest time τ_E has been the subject of intense research in the last few years [5, 3, 89, 149]. $\tau_E = \lambda^{-1} \ln(L/\lambda_F)$ is the time necessary to spread an electron wavepacket of size λ_F to the linear system size L . Calculations show that RMT predictions must be corrected by a factor $\exp(-\tau_E/\tau_\phi)$ for the WL correction and $\exp(-2\tau_E/\tau_\phi)$ for $\text{var}(G)$ [149].

²⁴For a quantum dot with a size L , Fermi velocity v_F , and mean free path l_μ , the ergodic time is the time of diffusion through the dot $\tau_{erg} = L^2/v_F l_\mu$ (for ballistic dots, $\tau_{erg} \sim L/v_F$). In practice, in chaotic quantum dots, the other relevant time scales are larger than τ_{erg} .

1.3 Coherence time in mesoscopic systems

1.3.1 Introduction: mechanisms

A large part of this thesis (chapters 3 and 4) is devoted to the carrier coherence time τ_ϕ in ballistic quantum dots. In order to put our results in perspective, we review in this section the main experimental results concerning the coherence time in some mesoscopic systems. Theories explaining the mechanisms of electron decoherence are also discussed. A more exhaustive picture of experimental and theoretical studies on electron decoherence in mesoscopic structures can be found in the recent review by Lin and Bird [104].

We start by briefly recalling the main mechanisms affecting the electron phase coherence:

- Electron-phonon (e-ph) scattering. This mechanism dominates over all the other decoherence processes at high temperature, but is less important at low temperature. Experiments in 2DEGs revealed a rate $1/\tau_{\phi,e-ph} \propto T^3$ for the loss of phase coherence by e-ph scattering [120]. Theory for disordered mesoscopic systems also predicts a very strong temperature dependence (T^4 in three-dimensional disordered systems [132]).
- Large-energy-transfer electron-electron scattering. Such collisions are strongly inelastic (a single such collision is sufficient to destroy phase memory), so that the scattering time associated to this mechanism is often termed ‘inelastic electron-electron scattering time’ τ_{ee} . Physically, τ_{ee} corresponds to the timescale necessary to relax the excess energy of an excited electron by means of electron-electron interactions [104]. In three dimensions (without disorder), the corresponding scattering rate is $\frac{1}{\tau_{ee}} = \frac{(k_B T)^2}{\hbar E_F}$ [13]. In two-dimensional disordered systems, Fukuyama and Abrahams found [59]:

$$\frac{1}{\tau_{ee}} = \frac{\pi}{2} \frac{k_B T^2}{\hbar E_F} \ln \frac{E_F}{k_B T}. \quad (1.19)$$

- Small-energy-transfer e-e scattering. Altshuler *et al.* [8] pointed out that the phase destruction can also occur without relaxation of an excess energy, but rather through small energy transfer processes, arising from fluctuation in the background electromagnetic field generated by the electron bath. This process is known as Nyquist scattering and

the temperature dependence of the scattering rate is :

$$\frac{1}{\tau_N} = \frac{kT}{2\pi\hbar} \frac{\lambda_F}{l_\mu} \ln \frac{\pi l_\mu}{\lambda_F} \quad (1.20)$$

in the case of a two-dimensional disordered system.

- Some other mechanisms were recently invoked, mainly to explain the low-temperature saturation of τ_ϕ , such as interaction with two-level systems (TLS), or extrinsic effects such as electromagnetic noise sources or hot-electron effects (we refer to [104] and references therein for more information).

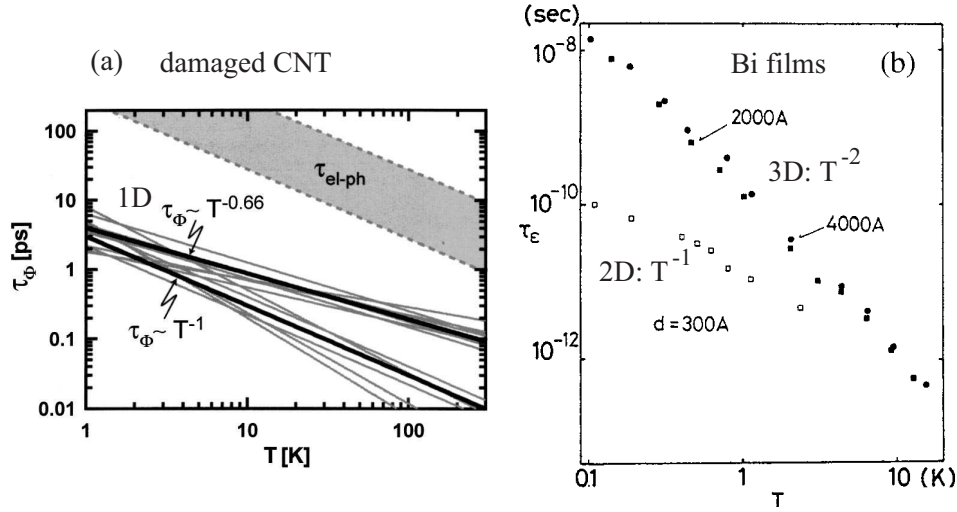


Figure 1.10: (a) τ_ϕ vs T in damaged carbon nanotubes. The gray lines are the experimental data, which cluster around $\tau_\phi \propto T^{-2/3}$ and T^{-1} , depending on the irradiation and the nanotube chirality (From Ref. [12]); (b) τ_ϕ vs T in evaporated Bi films with various thicknesses, indicated on the graph, showing the transition from a 2D to a 3D situation (From [104]).

In general, the total coherence time τ_ϕ can be obtained using Matthiessen's rule (see *e.g.* Ref. [38, 50]):

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{\phi, e-ph}} + \frac{1}{\tau_{ee}} + \frac{1}{\tau_N} + \dots \quad (1.21)$$

Importantly, the temperature dependence of each scattering rate depends on the dimensionality of the sample (*e.g.* $\frac{1}{\tau_N} \propto T^{2/3}$ in 1D samples). As

an example, τ_ϕ vs T in 1D, 2D and 3D systems is shown on Fig. 1.10(a) and (b). The temperature dependence of the measured τ_ϕ can therefore be used as an indicator of the dimensionality of the sample. Note that many authors report discrepancies between the experimental τ_ϕ vs T data and the quantitative predictions of the dephasing models (see *e.g.* ref. [50, 125]). However, in most cases, the experimental temperature dependence (more specifically the exponent p in $\tau_\phi \propto T^{-p}$) is in agreement with the theoretical value.

1.3.2 Dephasing in semiconductor quantum dots

As theory and experiments in 1D, 2D and 3D evidenced a dimensionality-dependence of τ_ϕ vs T , it is natural to wonder about the behaviour of τ_ϕ vs T in zero-dimensional systems, *i.e.* quantum dots, and in particular in the ballistic regime. Up to our knowledge, there has been only few theoretical predictions for this specific case so far. Sivan, Imry and Aronov [138] obtained a rate $\tau_\phi^{-1} \propto T^2$ due to electron-electron scattering in a disordered quantum dot. Takane [143] studied the case of a ballistic chaotic quantum dot with large leads (with a width W satisfying $\lambda_F \ll W$)²⁵ and obtained a rate $\tau_\phi^{-1} = c(\lambda_F/W)(k_B T/\hbar)$, c being of the order of unity, for dephasing by Coulomb electron-electron interactions. In the case of a diffusive cavity, he also predicted a crossover from $\tau_\phi^{-1} \propto T$ to $\tau_\phi^{-1} \propto T^2$ below a temperature $T_{cross} = \Gamma_0/k_B$, where Γ_0 is the lowest eigenvalue of a diffusion equation characterizing the system [144].

The lack of a firmly established theory did not slow down the experimental work [113, 41, 23, 86, 84, 130, 119]. In GaAs/AlGaAs open QDs,²⁶ Huibers and coworkers [86, 84] measured the following dependence for the coherence time²⁷: $\tau_\phi \propto T^{-p}$ with $1 < p < 2$, as shown on Fig. 1.11(a). They interpreted this observation to an interplay of both e-e scattering mechanisms described above (Nyquist and large-energy-transfer mechanisms). The temperature dependence of τ_ϕ^{-1} was found to be only *qualitatively* consistent with the theoretical expression established for disordered two-dimensionnal

²⁵Note that this condition is almost never respected in quantum dots, where the conductance of leads are quantified.

²⁶All the experiments on dephasing in open ballistic quantum dots have been performed on GaAs/AlGaAs quantum dots.

²⁷They measured the temperature dependence of the shape-averaged weak localization peak [Eq. (1.16)], and obtained consistent results using methods based on the power spectrum of the fluctuations and the width of the WL peak.

(2D) electron systems [59, 8, 38, 86]:

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_N} = \frac{\pi}{2} \frac{kT^2}{\hbar E_F} \ln \frac{E_F}{kT} + \frac{kT}{2\pi\hbar} \frac{\lambda_F}{l_\mu} \ln \frac{\pi l_\mu}{\lambda_F}. \quad (1.22)$$

However, a *quantitative* agreement with experimental data was only found for an arbitrary value of l_μ , one order of magnitude smaller than l_μ measured in the 2DEG. Most other experimental reports on GaAs/AlGaAs are in overall agreement with the results of Huibers ($\tau_\phi \propto T^{-p}$ with $1 < p < 2$). In a particular cases [130], a weaker dependence $\tau_\phi \propto T^{-2/3}$ was found, but could be related to additional coherence mechanisms occurring in the leads.

1.3.3 Low-temperature saturation of τ_ϕ

Electron decoherence in mesoscopic systems recently generated a large interest and many controversies [104]. At the center of the debate is the saturation of the electron coherence time τ_ϕ at low T observed in a wide variety of systems, as shown on Fig. 1.11(b) [121]. Conventionally, it is expected that the dephasing time should reach an infinite value in the presence of only electron-phonon and electron-electron scattering. A non-diverging τ_ϕ as $T \rightarrow 0$ would have important implications on the validity of the Fermi-liquid picture in the zero-temperature limit [154], and on the explanation of the anomalously large persistent current in mesoscopic rings [100].

In metal films and nanowires, there is a strong controversy on whether the observed saturation of τ_ϕ can be attributed to the presence of dilute magnetic impurities [121, 128, 129, 122]. In the framework of the Hikami theory [77], magnetic impurities induce spin-spin scattering events, which can lead to a reduction of the quantum interference effects. These effects are governed by a temperature-independent spin-spin scattering time τ_s . Several works argue in favor of such a magnetic origin (see [129] and references therein). Measurements performed on Ag and Au wires revealed a dependence of the saturation behavior on the source material purity (*i.e.* a purposely incorporated amount of magnetic impurities induces a saturation, while samples made from the highest purity materials do not show a saturation). In Cu wires, however, saturation was found systematic. Measurements of the Aharonov Bohm effect in Cu rings showed that τ_ϕ increases with B on a field scale proportional to the temperature,²⁸ indicating that dilute impurities are responsible for the observed saturation. In the case of

²⁸At high magnetic field, the authors of ref. [129] assume that spin-flip collisions are frozen out.

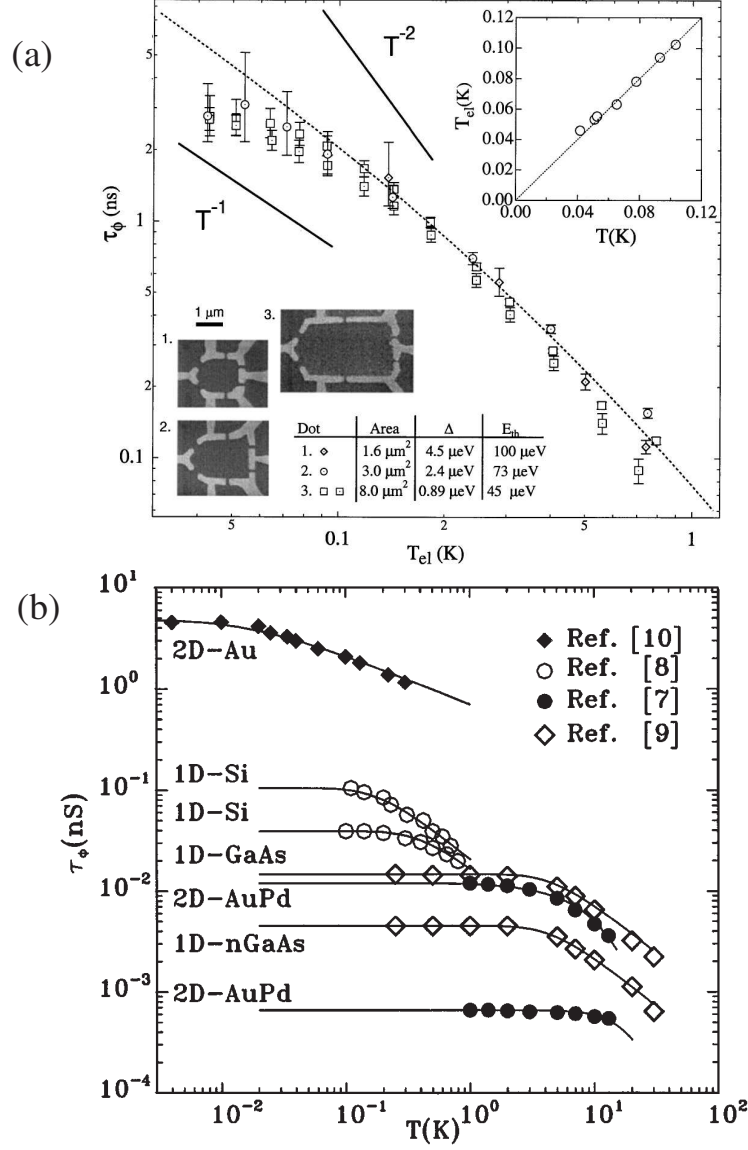


Figure 1.11: (a) τ_ϕ vs T_{el} (temperature of the electronic system). Lines show T_{el}^{-1} and T_{el}^{-2} dependencies. Insets show micrographs of the samples with their parameters, and T_{el} vs T [From Ref. [84]]; (b) low temperature saturation of τ_ϕ vs T in various low-dimensional systems [From Ref. [121]].

copper, impurities are thought to originate from the systematic presence of an oxide layer.

However, a significant number of arguments go against this interpretation. In particular, the recent work of Mohanty and coworkers showed that a saturation of τ_ϕ could be observed in magnetic fields large enough to polarize any magnetic impurity spin [122]. The debate on the possible magnetic origin of the low temperature saturation of τ_ϕ is therefore not closed yet.

Beside the magnetic origin, there are several other suggestions regarding the source of the anomalous dephasing. Natelson *et al.* have studied the saturation of τ_ϕ vs T in AuPd wires differing only by their geometry. The thinnest wires showed a behavior consistent with the standard Nyquist scattering in 1D systems, but a weaker temperature dependence was observed in wider wires, contrary to the expected stronger T^{-1} variation, suggesting that the geometry plays an important role in the saturation behavior. A link between the saturated dephasing time $\tau_{\phi,sat}$ and disorder was also found by Lin and Kao [105] in three dimensional polycrystalline films : $\tau_{\phi,sat} \propto D^{-\alpha}$ with $\alpha \sim 1$ (D is the diffusion constant).

Compared to metal structures, MBE-grown semiconductor heterostructures have a much lower level of defects and are essentially free of any magnetic impurities, which makes them ideal candidates for the investigation of the intrinsic decoherence at very low T . Surprisingly, open quantum dots (QDs) fabricated from high mobility GaAs heterostructures also revealed a saturation of τ_ϕ at low T [41, 23, 84, 130, 119] (the saturation is visible on Fig. 1.11(a), below ~ 100 mK). In these experiments, the onset of saturation was found in the range $80 \text{ mK} < T_{\text{onset}} < 900 \text{ mK}$. As in the case of metal nanostructures, the extrinsic vs intrinsic nature of the saturation has been heavily discussed. On one hand, ending a long debate, careful experiments by Huibers *et al.* showed that the saturation is not caused by unintentional irradiation that would raise the electron temperature [84]. On the other hand, the influence of the QD mean energy-level spacing Δ on T_{onset} was questioned. In some cases it was found that $\Delta \sim kT_{\text{onset}}$ [23, 119], but significant discrepancies with this relation were obtained in other experiments [130, 41, 84] and will be confirmed in this thesis. Surprisingly, in the case of open QDs, the actual *value* of the coherence time in the saturated regime $\tau_{\phi,sat}$, while obviously fundamental to this problem, has attracted much less attention.

1.4 Conclusion

In this chapter, we have introduced the main ingredients needed to understand the physics of mesoscopic systems (in particular, of quantum dots), and the context of our investigations, presented in chapters 3 and 4. We will mainly deal with the various characteristic times and lengths defined in the introductory section of this chapter. Each of them reflects the influence of a particular type of interactions on the electron transport properties. We will evaluate their relative importance in our samples, and try to shed light on unsolved issues introduced in the last part of this chapter. For that purpose, the discussion initiated about the results of the semiclassical and random matrix theory will be somewhat extended and ‘adapted’ to the peculiarities of our quantum dots.